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Dynamic Toll Pricing using Dynamic Traffic Assignment System with Online Calibration

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Abstract

The paper presents a toll pricing methodology using a dynamic traffic assignment (DTA) system. This methodology relies on the DTA system's capability to understand and predict traffic conditions, thus enhanced online calibration methodologies are applied to the DTA system, featuring a heuristic technique to calibrate supply parameters online. Improved offline calibration techniques are developed to apply toll pricing in a real network consisting of managed lanes and general purpose lanes. The online calibration methodologies are tested using real data from this network, and the results find the DTA system able to estimate and predict traffic flow and speed with satisfactory accuracy under congestion. Toll pricing is formulated as an optimization problem to maximize toll revenue, subject to network conditions and tolling regulations. Travelers are assumed to make route choice based on offline calibrated discrete choice models. Toll optimization is applied in a closed-loop evaluation framework where a microscopic simulator is used to mimic the real network. Online calibration of the DTA system is enabled to ensure good optimization performance. Toll optimization is tested under multiple experimental scenarios, and the methodology is found able to increase toll revenue compared with the condition when online calibration is not available. It should be noted that the toll rates and revenues presented in this paper are obtained in a simulation environment based on the calibration and optimization algorithms, and as the work is ongoing these results are far from being a recommendation to operators of managed lanes.

Congestion management aims to improve transportation system performance and reduce traffic congestion by either altering traffic demand or changing transportation supply. Among congestion management schemes, road pricing (i.e., tolling) is a commonly used strategy, which may aim to generate revenue to recover road construction and maintenance costs and thus incentivize improvement to transportation supply, as well as managing congestion by altering temporal and spatial dimensions of travel behaviors, and travelers' decisions on mode choice or whether to travel (*1*).

Among road pricing strategies, dynamic pricing has been extensively studied in recent years (*2*). Applications of dynamic pricing arise in many cities. The authors advocate two criteria for an effective dynamic pricing scheme: real-time efficiency and proactive decision making. The computation time of any algorithm should be short enough to support real-time decision making, and decisions should be made based on predicted traffic conditions instead of observed ones.

This research implements and tests a dynamic toll pricing framework where decisions on toll are made in real time (every 5 min) based on predicted traffic conditions. The framework is applied in the context of managed lanes and from the viewpoint of the operator with an objective to maximize revenue while offering premium level of service. State estimation and prediction are provided by a DTA system, DynaMIT. Travelers' route choice behaviors are predicted by discrete choice models based on travel time savings and toll rates. Toll optimization is fully integrated with DynaMIT to maximize revenue subject to network conditions given tolling regulations. The impact of toll optimization is evaluated through a closed-loop evaluation framework so the platform optimizing the tolls is different from where the tolls are implemented and evaluated. A microscopic simulator, MITSIM (*3*), is used as the second simulation

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platform for evaluating the tolls, and it serves as the real world in this closed-loop framework.

Effective toll optimization is only possible when the DTA system is capable of understanding and predicting current and future traffic conditions. The authors propose and apply a heuristic online calibration method to calibrate supply parameters and reduce discrepancies in sensor speed between simulation and actual data, and also apply generalized least squares to calibrate origin-destination (OD) demand. Performance of the online calibration methodology is tested by calibrating DynaMIT toward real data available for the case study of managed lanes in Texas. DynaMIT is then deployed in the closed-loop setting to test toll optimization, and the online calibration module calibrates DynaMIT toward simulated sensor measurements provided by MITSIM.

The effectiveness of the toll optimization can only be confirmed when the demand and travel behaviors are represented accurately in the simulation platforms. In this paper offline calibration of the microscopic simulator is also briefly discussed. It is calibrated to the real data before deployment of the closed-loop framework.

The contributions of this study include the design, implementation, and testing of a heuristic online calibration method for supply parameters with complexity of $O(n)$, where n represents the number of segments in the network. Note that the algorithm is parallelizable, and it works simultaneously with the existing generalized least squares (GLS) algorithm for OD calibration. Secondly, the enhanced dynamic toll pricing framework is implemented and tested under multiple scenarios in a closed-loop testing framework. These tests show added benefit to toll optimization because of online calibration.

The subsequent sections of this paper include a literature review on offline and online calibration of traffic simulators, as well as congestion pricing strategies and applications. The optimization framework, calibration methodologies, and closed-loop evaluation framework are then presented. Finally, the results of the online calibration and toll optimization are shown in a case study with data from a real network, followed by conclusions and future research directions.

Literature Review

Earlier literature on toll pricing often relied on a simplified representation of supply, demand, or both. Pricing strategies were mostly reactive instead of proactive, without explicitly predicting traveler behaviors in reaction to pricing strategies. Yin et al. (*4*) proposed dynamic toll pricing approaches in the context of managed lanes with the objective of maximizing throughput. A feedback control approach was applied, such that toll decisions would be reactive to traffic conditions.

More recent literature on toll pricing includes studies applying proactive pricing strategies. Jang et al. (*5*) proposed a closed-form model to predict certain system performance measures with tolling decisions based on the predicted performance. Dong et al. (*6*) studied the benefits of a proactive control strategy where predictions play a role when adjusting tolls based on the deviation from the desired network conditions, and their optimization was integrated into a DTA system DYNASMART. With an attempt to simplify optimization, Chen et al. (*7*) developed a family of surrogate-based models for optimization of dynamic tolls, with optimization of a peak and off-peak toll. They used a DTA system, DynusT, to construct various surrogate models, and applied them to a corridor in Maryland with a composite objective function of travel time, throughput, and revenue.

For a simulation-based proactive toll pricing system, online calibration of the simulator is important to ensure the simulation accurately mimics the real network and toll pricing decisions are based on accurate prediction of traffic conditions. Online calibration of a DTA system usually includes calibration of OD demand parameters, behavior parameters, and supply parameters. GLS is widely used for OD calibration. An iterative calibration framework to calibrate OD, behavioral and supply parameters jointly in a mesoscopic DTA model was studied by Yin and Lou (*4*). The OD demand is calibrated by the GLS method, while behavior and supply parameters are estimated with specific empirical methods.

Hashemi and Abdelghany (*8*) proposed online calibration methods in a traffic management context, using GLS for OD and an empirical method for supply parameters. They then applied these online calibration methods to support traffic management strategy generation (*9*). Hashemi and Abdelghany used a DTA system and a meta-heuristic search algorithm to generate control strategies, and applied their model to a corridor in Dallas, with an objective to reduce total travel time. Their optimization system predicts significant time savings with the optimal strategies generated by it, but the actual impacts of such strategies were not tested in the real network or in a simulation environment that is different from the DTA system itself.

Yang et al. (*3*) developed a microscopic traffic simulator, MITSIM, and proposed a closed-loop testing framework in which traffic management strategies were implemented in the simulator and the performance of such strategies were then evaluated. Lu et al. (*10*) proposed a weighted-SPSA algorithm to calibrate a microscopic traffic simulator to ensure it is a good representation of the real world.

Recent advances in online calibration methods include simultaneous calibration of all parameters with a unified model, of which the extended Kalman filter (EKF) is an

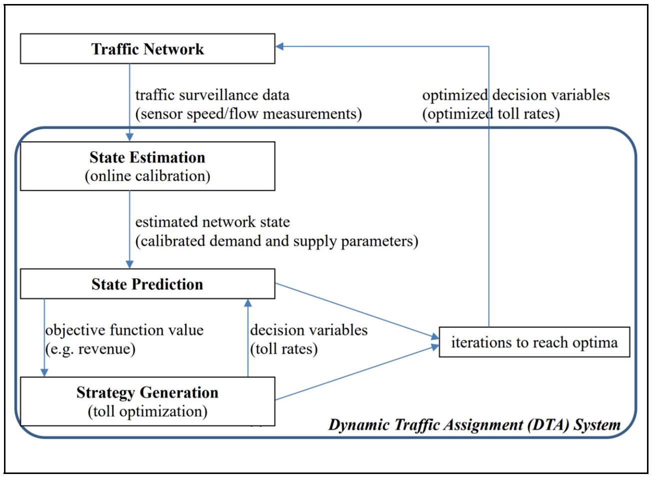

Figure 1. Toll optimization framework.

example. Antoniou et al. (*11*) proposed EKF for online calibration of DynaMIT. By linearizing the relation between all measurements, including speed, and all parameters, including supply parameters, the EKF algorithm can be used to calibrate simultaneously all parameters toward all measurements.

More recently there has been research on large-scale problems. Gupta et al. (*12*) developed a toll optimization method with a generic algorithm based on traffic predictions from DynaMIT, and applied the model to the expressway network in Singapore, where 13 tolls were optimized. Zhang et al. (*13*) developed a metamodel embedding an analytical model of how calibration parameter is related to the objective function. The methodology addresses calibration of OD demand and is demonstrated with a case study of the Berlin metropolitan area network. Prakash et al. (*14*) applied a principal components approach to conduct online calibration using the GLS algorithm. By calibrating the principal components of parameters instead of original parameters, this method greatly scales down the computation effort of large-scale online calibration problems. It slightly worsens estimation accuracy but reaches better prediction accuracy, since principal components capture inherent correlations of the parameters while removing noise. Prakash et al. (*15*) also applied this approach to online calibration using the EKF algorithm and obtained similar results, which implies a potential of reducing dimensionality in large-scale demand–supply simultaneous online calibration problems.

The toll pricing framework in this paper is a real-time proactive system where toll rates are optimized every 5 min based on predicted traffic conditions in the next 15 min. The proof-of-concept has already been demonstrated on a toy network in Wang et al. (*16*), and this paper enhances the methodology by integrating online calibration into the framework to achieve better toll optimization performance in a case study of managed lanes in Texas.

Methodology

Overview of the Dynamic Toll Pricing Framework

A toll optimization framework (*16*) is deployed with DynaMIT (*17*), a mesoscopic DTA system developed in the ITS Laboratory of the Massachusetts Institute of Technology (MIT). DynaMIT reads sensor data, calibrates its parameters to estimate traffic state, and generates control strategies (toll rates) based on predicted traffic conditions (Figure 1). Toll optimization is based on a rolling horizon framework, that is, for each rolling period (e.g., 5 min), it receives new real-time information from the network, runs the estimation and optimization modules, and provides optimized toll rates for the prediction interval (e.g., 15 min) to the network.

Optimization Formulation

The toll for each tolling location i is represented by $\theta_i = (\theta_{i1}, \ldots, \theta_{iT})$, where T is the number of tolling intervals in the optimization horizon. The speed and flow for each tolling location i and tolling interval t are denoted by v_{it} and q_{it}, respectively.

The managed lane operator has to comply with tolling regulations, which need to be taken into account by the optimization model. There is a toll cap per mile and the operator may decide to exceed this toll cap only under certain conditions. Specifically, given average speed ($\bar{v}$) and volume ($\bar{q}$) across all sensor locations and predefined critical values of speed (v^{cr}) and volume (q^{cr}), the following rules are in effect:

- If $\bar{v} \leqslant v^{cr}$, then toll rate is multiplied by a flexible demand factor between a lower bound $\mathrm{DF}_{\mathrm{it}}^{\mathrm{lb}}$ and an upper bound $\mathrm{DF}_{\mathrm{it}}^{\mathrm{ub}}$, and the toll rate will increase compared with the previous toll, that is, $\mathrm{DF}_{\mathrm{it}}^{\mathrm{lb}} \geqslant 1$.
- If $\bar{q} > q^{cr}$, then, depending on the level of $\bar{q}$, there is a set of rules to calculate a fixed demand factor which may result in an increased, decreased, or maintained toll rate.

When either rule is adopted, the managed lanes are operated in mandatory mode. Otherwise they are managed in dynamic mode.

The optimization model therefore includes a binary decision (δ_{it}) of switching or not to the mandatory mode in addition to the decision on the toll vector (θ). The problem is formulated as follows:

$$\max \sum_{i \in I} \sum_{t \in T} q_{it}\theta_{it} + \alpha^{v}_{it} min(v_{it} - v^{cr}, 0) + \alpha^{q}_{it} \min(q^{cr}_{i} - q_{it}, 0) \quad (1)$$

$$\text{s.t.}(v_{it}, q_{it}) = \text{DTA}(\theta)\ \forall i \in I, t \in T \quad (2)$$

$$\delta_{it} \leqslant \eta_{it}\ \forall i \in I, t \in T \quad (3)$$

$$\delta_{it} \geqslant M(\delta_{i(t-1)} - 1) + (1/100)(\theta_{i(t-1)} - \theta^{CAP}_{i})\ \forall i \in I, t \in T \quad (4)$$

$$(DF^{lb}_{it}, DF^{ub}_{it}, \eta_{it}) = f(v_{it}, q_{it})\ \forall i \in I, t \in \{2, \ldots, T\} \quad (5)$$

$$\delta_{it}\theta_{i(t-1)}DF^{lb}_{it} \leqslant \theta_{it} \leqslant (1 - \delta_{it})\theta^{CAP}_{i} + \delta_{it}\theta_{i(t-1)}DF^{ub}_{it}\ \forall i \in I, t \in T \quad (6)$$

$$\theta_{i(t-1)} - \Delta - \delta_{it}M \leqslant \theta_{it} \leqslant \theta_{i(t-1)} + \Delta + \delta_{it}M\ \forall i \in I, t \in T \quad (7)$$

$$\delta_{it} \in (0, 1)\ \forall i \in I, t \in T \quad (8)$$

$$\theta_{it} \geqslant 0\ \forall i \in I, t \in T \quad (9)$$

The objective function (Equation 1) has three terms: one is toll revenue and the other two are penalty terms to account for critical speed and volume pre-specified by the regulations. Namely, the second term is the penalty for going below the critical speed and the third term is the penalty for exceeding the critical volume on the managed lane. The critical speed is the same across the network, however, the critical flow changes based on the number of lanes. In this study, it was decided to formulate these constraints through penalty terms since it is a simulation-based setting. Namely, one cannot constrain the simulator not to give certain speed and flow measurements; instead the solution was evaluated through the resulting measurements based on if and how much it violates the desired conditions. Furthermore, the penalty coefficients α^{v}_{it} and α^{q}_{it} were set empirically.

Constraints (Equation 2) ensure that the predicted speed and volume are provided by the traffic simulator to evaluate the objective function and also for the decisions in future intervals. Constraints (Equation 3) maintain that the system cannot enter mandatory mode (δ cannot be 1) if it is not allowed by measurements (for the next interval) or predictions (for future intervals). Constraints (Equation 4) enable a gradual decrease in the toll when exiting the mandatory mode. If the system is in mandatory mode in *t*-1 and the toll is above the toll cap, then the system needs to stay in mandatory mode in interval *t*. If the conditions are improving, the demand factors from the regulations will go down and the toll will gradually decrease.

Constraints (Equation 5) maintain matching between the predicted traffic conditions and demand factors and the allowance to enter mandatory mode for the future intervals through predetermined functions. Note that η_{it} is input for the next interval based on field measurements and a variable to be optimized for the subsequent intervals based on predicted traffic conditions. Similarly, DF^{lb}_{it} and DF^{ub}_{it} are inputs for the next interval and variables for future intervals. DF^{lb}_{it} and DF^{ub}_{it} will be the same in mandatory mode so that the toll will be equal to the demand factor times the previous toll. On the other hand, when in dynamic mode, DF^{lb}_{it} will be zero and DF^{ub}_{it} will be the toll cap. Constraints (Equation 6) regulate these bounds on the toll such that if the decision is to stay in dynamic mode ($\delta = 0$), then the toll is optimized between 0 and the toll cap, otherwise ($\delta = 1$) toll rates follow the regulations for mandatory mode.

Finally, constraints (Equation 7) control the maximum change in the toll. This constraint is active only in dynamic mode ($\delta = 0$), and not in mandatory mode ($\delta = 1$). Constraints (Equations 8 and 9) define the decision variables as binary and nonnegative continuous, respectively. Currently this problem is solved with simple search heuristics and future work involves other solution algorithms.

Calibration and Prediction in the DTA System

Effective control strategies rely on the DTA system's capability to predict traffic conditions under candidate toll rates. Prediction accuracy depends on state estimation performance. Offline and online calibrations are essential to ensure accurate estimation of the current network state.

A state is a vector consisting of demand and supply parameters. State estimation is the real-time process of incorporating an initial state, historical data, and real-time surveillance data to achieve a more reliable estimation of the current state.

Offline calibration provides a priori values of the parameters which are then calibrated online. This research relies on iterative proportional fitting (IPF) to obtain a historical time-dependent OD demand table based on historical sensor flow measurements. The choice parameters are calibrated empirically so that simulated choice ratios match actual data. For supply parameters, a closed-form model is used, which is described in next section, so it is possible to estimate the model parameters with actual sensor data.

To perform online calibration, the GLS algorithm is used to estimate OD demand from real-time sensor flow measurements. For supply parameters, a heuristic online calibration framework is proposed, to adjust supply parameters in real time, and the simulation results were found to match sensor data with satisfactory accuracy in relation to speed measurements, including when congestion is present.

The state prediction module predicts future states based on the current state, taking into consideration any historical information, strategies (e.g., future toll rates) to be deployed and travelers' responses to guidance information. The prediction model is formulated as an autoregressive process (*11*):

$$x_t^{\text{pred}} - x_t^{\text{hist}} = \sum_{i=1}^{n} f_i \left(x_{t-i}^{\text{est}} - x_{t-i}^{\text{hist}} \right) \tag{10}$$

where
x_t^{pred} is the predicted parameter value for the current interval;
x_t^{hist} is the historical parameter value for the current interval;
n is the autoregressive degree;
f_i is the autoregressive coefficient for degree i;
x_{t-i}^{est} is the estimated parameter value for the i-th interval ahead;
x_{t-i}^{hist} is the historical parameter value for the i-th interval ahead.

For demand, n and f_i were estimated using offline calibrated time-dependent OD parameters. For supply, since time-dependent supply parameters are not obtained offline, the above autoregressive model is simplified as

$$x_t^{\text{pred}} - x^{\text{hist}} = f \left(x_{t-1}^{\text{est}} - x^{\text{hist}} \right) \tag{11}$$

and the coefficient f is empirically determined. The predicted parameters x_t^{pred} were then used as input to simulate traffic for the prediction interval (e.g., 15 min) and to obtain predicted sensor measurements.

To evaluate the calibration and prediction accuracies, RMSN (Root Mean Square error, Normalized) was used to quantify the difference between actual and simulated measurements (*11*). RMSN is defined by the following equation:

$$\text{RMSN} = \sqrt{\frac{1}{M} \sum_{i=1}^{M} \left(y_i^{\text{est}} - y_i^{\text{true}} \right)^2 / y_i^{\text{true}}} \tag{12}$$

where
M is the number of measurements;
y_i^{est} is the estimated value of the i-th measurement;
y_i^{true} is the true value of the i-th measurement.

Algorithm for Online Calibration of Supply Parameters

The optimization module of this study relies heavily on accurate prediction of drivers' choices between managed lanes and general purpose lanes, and travel time (or travel speed) would be an important factor for their decisions. Therefore, it is essential to make sure the state estimation module can accurately reveal the supply parameters and thus simulated travel speed can match actual sensor speed measurements.

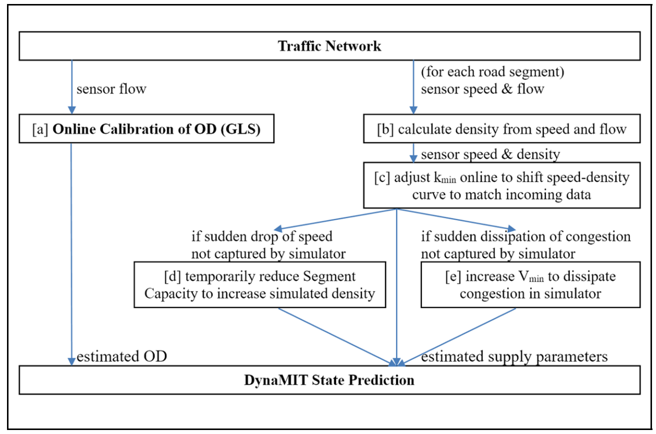

Figure 2. Proposed online calibration process.

In the DynaMIT traffic simulation module, a road segment consists of a queuing part (downstream) and a moving part (upstream) (*17*). A queue will form only if flow on the segment exceeds segment capacity, or if a queue on the downstream segment spills out. Traffic speed on the queuing part is subject to a queuing model. If a queuing part does not exist, or it does not occupy the full segment, then traffic speed on the moving part is described by the following relationships:

$$\begin{aligned} v &= \max(v_{\min}, v_s) & \\ v_s &= v_{\max} & \text{when k} \leq k_{\min} \\ v_s &= v_{\max} \left(1 - \left(\frac{k - k_{\min}}{k_{\text{jam}}} \right)^{\beta} \right)^{\alpha} & \text{when k} > k_{\min} \end{aligned} \tag{13}$$

where k is density, v is speed, v_s is an intermediate variable, and the other six parameters ($v_{\min}, v_{\max}, k_{\min}, k_{\text{jam}}, \alpha, \beta$) as well as *Segment Capacity* are referred to as supply parameters.

For the seven supply parameters of each road segment, this study estimated their a priori values from speed and flow measurement data offline. When deploying real-time toll optimization, a selection of supply parameters was adjusted online in reaction to real-time sensor measurements. Figure 2 illustrates these operations. Steps [b] to [e] constitute the heuristic online calibration method for supply parameters. Note that steps [d], [e], or both, are only used in rare cases to correct simulation errors.

Closed-Loop Evaluation Framework

Before the toll optimization framework is implemented in the real world, the validity and performance of the developed models and algorithms need to be tested in a simulation environment. Therefore, a closed-loop evaluation framework is applied by using a microscopic

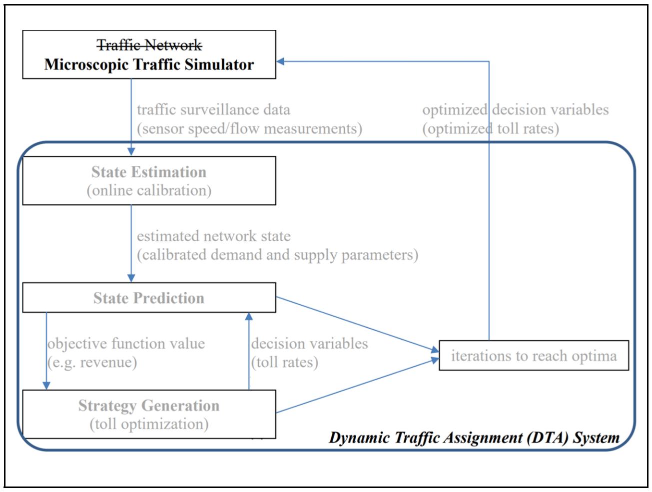

Figure 3. Closed-loop evaluation framework.

simulator as a representation of the actual traffic network (Figure 3).

In this study MITSIM was used as the testbed. MITSIM is a microscopic traffic simulator developed in the ITS Laboratory of MIT (*3*). It incorporates road topography, time-dependent OD demand, driving behavior (car following, lane changing, etc.) models and route choice models, simulates individual vehicle's movements, and generates simulated sensor measurements.

Route choice is modeled as a path-size logit model, which takes into account the similarities between paths that are overlapping. Drivers make route choice decisions based on information on toll rates and travel times. To mimic real-world conditions, drivers are assumed to have access to real-time traffic information, for instance, through mobile navigation applications, so they are aware of current traffic conditions (i.e., travel time) on downstream links. As for toll rates, it is assumed they know real-time toll rates only when they are close to the decision point. Otherwise, the drivers rely on historical toll rates (at that time of day) to make decisions.

The optimized toll rates are implemented in MITSIM. DynaMIT is provided with data from MITSIM sensors rather than a real-world traffic surveillance system. The closed-loop testing framework requires that the microscopic traffic simulator represents the real-world accurately, that is, drivers in MITSIM behave similarly to those in the real world, and demand–supply interactions occur in the same way. This is achieved by calibrating MITSIM toward real data.

Calibration of the microscopic traffic simulator relies on an enhanced weighted-SPSA algorithm (*18*). Demand parameters and selected behavior parameters are calibrated simultaneously to minimize the discrepancies between simulated and actual sensor measurements.

Case Study

The methodology was applied to the NTE TEXpress network, a 13-mile corridor on U.S. Interstate Highway I-820 and Texas State Highway TX-183, which consists of managed lanes (ML) and general purpose lanes (GPL) (Figure 4). The network is equipped with sensors which provide traffic flow and speed measurements, and toll gantries for non-stop tolling.

The private operator of this corridor provided the authors with samples of data collected on nine Fridays in summer 2017, which included sensor flow and speed measurements, toll rates, and automatic vehicle identification (AVI) data.

The tolls are applied on two tolling segments. Segment 1 is highlighted darker in Figure 4, and segment 2 has lighter color. Toll gantries are located at the beginning of each tolling segment, and at entry ramps to ML. A driver pays a toll when entering ML. The toll rate is determined with respect to the entry point but not the exit point. If the driver continues from tolling segment 2 to segment 1 on westbound, he/she pays a second toll.

In this case study the focus is on the westbound (WB) corridor of the network. For ease of analysis, the WB corridor is divided into nine parts based on locations of entry and exit ramps on ML, as shown on Figure 4. Parts 1 to 4 belong to tolling segment 2, and parts 5 to 9 belong to tolling segment 1.

Offline Calibration

The AVI data give an insight into the OD pattern but they include just a fraction of the vehicles. The data are used as seed OD for better offline calibration. The IPF algorithm was used to scale up the AVI-based OD, according to flow at origin and destination nodes. Flow data are available at most origin and destination nodes, either obtained from sensors on corresponding origin and destination links, or calculated from sensor flow on nearby links according to the flow conservation law. The IPF algorithm converges with no more than 0.1% error in fitting origin or destination flow.

The route choice model in DynaMIT is a path-size logit model, where probability of choosing path i is specified as

$$P(i) = \frac{e^{V_i + \ln PS_i}}{\sum_{j \in C} e^{V_j + \ln PS_j}} \tag{14}$$

where C is the set of all possible paths, and PS_i is the path-size variable for path i, specifying the path's degree of overlapping with other paths. V_i is the systematic utility of path i, given by the following equation:

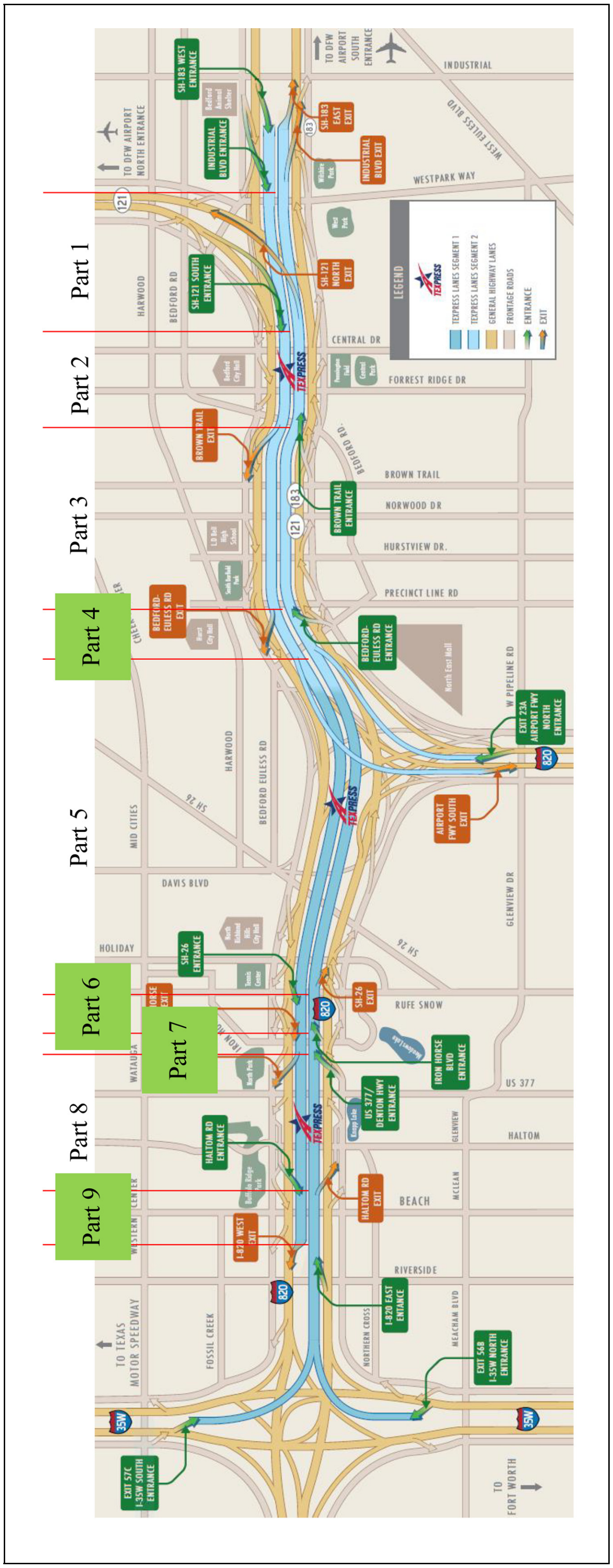

Figure 4. NTE TEXpress network.

$$V_i = -\mu\left(TT_i - \frac{toll_i}{VOT} + c_i\right) \tag{15}$$

where μ is the scaling factor, TT_i and $toll_i$ are travel time and toll cost on path *i*, *VOT* is the specific driver's value of time, and c_i is a constant. It is assumed that different drivers have different *VOT* which is subject to a log-normal distribution. The choice model was estimated empirically to make sure simulated choice ratios match actual data. For successful calibration, the constant term is introduced to capture some network-specific phenomena. The model parameters are also allowed to be different in different periods, which includes morning (05:30 to 09:00), midday (09:00 to 14:00), afternoon (14:00 to 18:00) and evening (18:00 to 21:00). These periods are determined based on historical toll rates on the network.

Supply parameters are estimated with Day 1 data. Firstly, a set of supply parameters was estimated for each type of road segment (ML, GPL, ramp). Using the results as starting values, supply parameters are estimated for each road segment. The statistics of estimated supply parameters are presented in Table 1. Figure 5 shows the data points and estimated supply curve for selected road segments.

After the offline calibration process, a set of parameters is obtained for Day 1, and the simulation results have an error of 19% in RMSN for flow measurements and 15% for speed measurements.

Online Calibration and Prediction

DynaMIT is calibrated offline to Day 1 data and a set of parameters obtained. Using Day 1 parameters as a priori values, DynaMIT is then calibrated online for the other eight days.

For each 5-min time interval, firstly, one simulation is run with predicted parameters from last interval, simulated measurements obtained, and then demand and supply calibration is applied to obtain calibrated demand and supply parameters. Finally DynaMIT simulates traffic with calibrated parameters. The GLS algorithm works well for calibrating OD demand parameters, as long as the error of simulated speed is not large. The heuristic is effective to replicate real-world congestion in the simulator. The results of online calibration are shown in Table 2 as *OC demand&supply*. The *No OC* case is a base case where historical OD and supply parameters are used in the simulation. The *OC demand only* case has OD calibrated by GLS algorithm, but historical supply parameters are used in the simulation.

Table 1. Statistics of Offline Estimated Supply Parameters

	Segment type	v_{min} (mph)	v_{min} (mph)	k_{min} (veh/mile)	k_{jam} (veh/mile)	α	β	Segment capacity (veh/s)
minimum	ML	8	57	0.005	0.08	2.4	1	0.56
	GPL	8	54	0.005	0.08	2.4	1	0.56
	Ramp	8	40	0.002	0.08	2.4	1	0.56
median	ML	8	72	0.009	0.09	3	1	1.11
	GPL	8	65	0.014	0.10	3	1	1.67
	Ramp	8	64	0.007	0.11	2.4	1	1.00
maximum	ML	40	76	0.012	0.16	3	1	1.70
	GPL	31	69	0.019	0.12	3	1	2.78
	Ramp	8	72	0.011	0.16	3	1	2.28

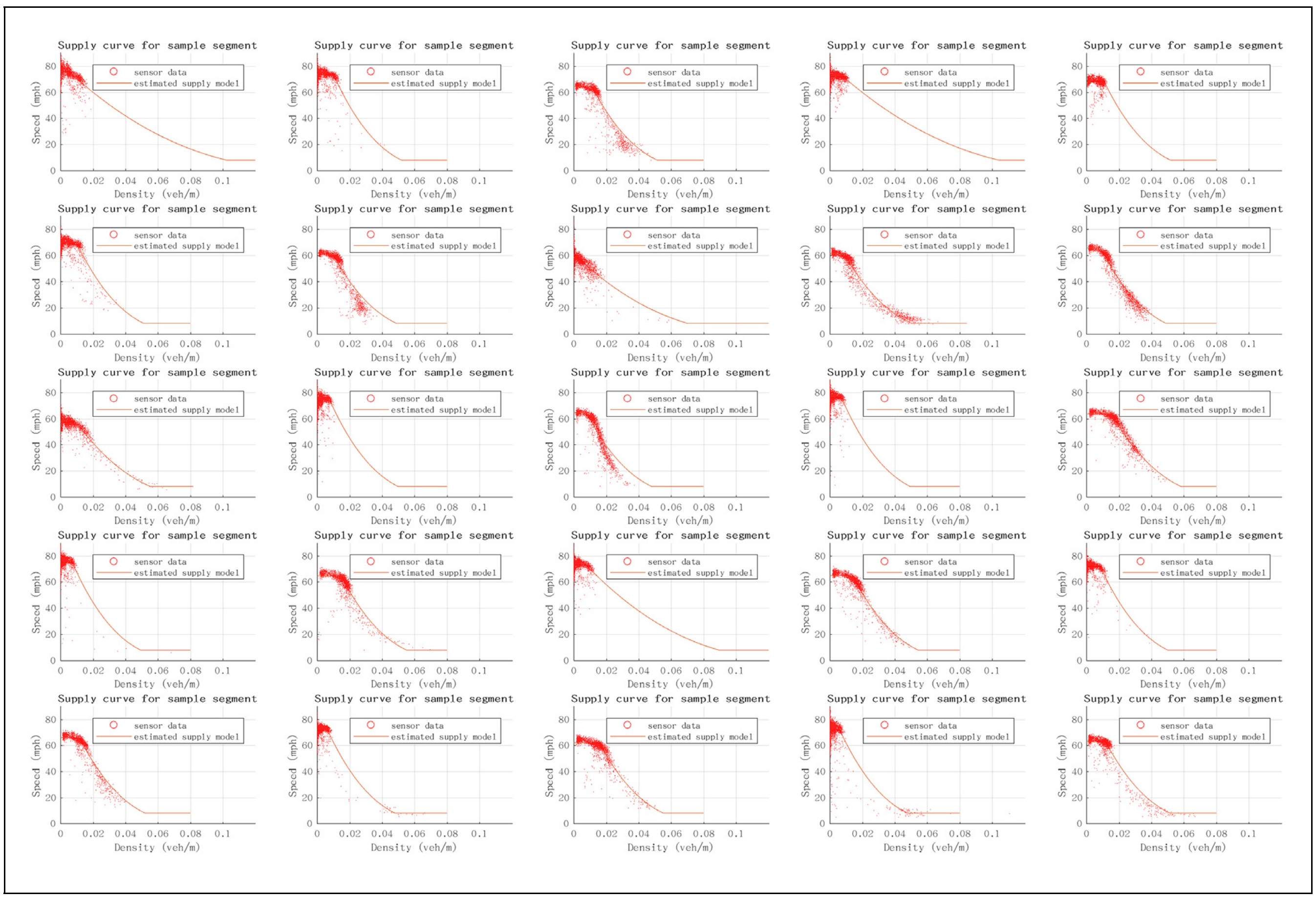

Figure 5. Examples of calibrated supply models.

Taking Day 1 offline calibration results as the baseline, simulations of other days have much larger error for flow if online calibration is not performed, because those days have different demand from Day 1. The error for speed is about the same, because supply parameters were static in these cases and they are similar in different days. Online calibration of demand greatly improves flow accuracy. Addition of supply online calibration then improves speed accuracy, because of its capability to calibrate supply parameters dynamically. In all cases, prediction RMSNs are slightly larger than estimation, which is as expected and acceptable, because the prediction model incorporated additional errors.

More detailed results for Day 6 are presented in Figure 6. It shows the simulated flow and speed after online calibration of demand and supply compared with

Table 2. Calibration and Prediction Accuracies

		Estimation		Prediction (0~15 min later)					
				0–5 min		5–10 min		10–15 min	
RMSN(%)		Flow	Speed	Flow	Speed	Flow	Speed	Flow	Speed
Day 1	Offline calibration results	19	15						
Day 2	No OC	22	16	22	15	22	15	22	15
	OC demand only	12	16	16	15	19	15	19	15
	OC demand&supply	12	13	17	11	19	12	22	12
Day 3	No OC	23	12	23	14	23	14	22	14
	OC demand only	12	12	16	14	18	14	19	14
	OC demand&supply	12	10	16	10	19	11	21	11
Day 4	No OC	23	13	23	15	23	15	23	15
	OC demand only	12	13	16	15	18	15	19	15
	OC demand&supply	13	11	17	11	19	12	22	12
Day 5	No OC	38	22	38	23	38	24	38	23
	OC demand only	16	23	23	24	25	24	26	24
	OC demand&supply	18	19	24	17	26	18	29	18
Day 6	No OC	33	17	33	14	33	14	33	14
	OC demand only	13	17	19	14	22	14	23	14
	OC demand&supply	15	15	21	10	23	11	25	12
Day 7	No OC	23	14	23	14	23	14	23	14
	OC demand only	12	14	16	14	18	14	19	14
	OC demand&supply	12	12	16	10	19	10	22	11
Day 8	No OC	23	12	24	13	24	13	24	13
	OC demand only	14	12	18	13	20	13	21	13
	OC demand&supply	14	10	19	10	21	10	23	10
Day 9	No OC	22	12	22	13	23	13	23	13
	OC demand only	11	12	16	13	18	13	19	13
	OC demand&supply	14	9	19	9	21	10	22	10
Average (Day2–9)	No OC	26	15	26	15	26	15	26	15
	OC demand only	13	15	18	15	20	15	21	15
	OC demand&supply	14	12	19	11	21	12	23	12

Note: RMSN = root mean square error, normalized; OC = online calibration.

true measurements. Each small plot shows average flow or speed on one of the nine parts of the GPL.

It can be seen that the proposed online calibration methods are successful in replicating flow and speed fluctuations in each part of the WB GPL, although in some cases simulated congestion is still not as severe as in actual measurements. ML has less congestion overall and the plots for ML are omitted.

The results below demonstrate that it is possible to understand and predict traffic conditions when congestion occurs. The predictions used for toll optimization in the DTA system are accurate, thus evaluation of the objective function is accurate. Therefore the system is able to make informed decisions on toll rates.

Toll Optimization

The toll optimization framework was further evaluated in a closed loop. First, MITSIM was calibrated toward the sensor measurements of Day 6. RMSN of the calibration result was 19% for flow and 17% for speed. The toll optimization framework was then applied and the optimized toll rates implemented in MITSIM.

These toll rates are compared with a base toll, which is obtained with the same toll optimization methodology, except that online calibration is not enabled. In this situation, DynaMIT is fed with parameters calibrated offline toward Day 1 data. Comparing optimized toll with this base toll highlights the added benefit of online calibration in the prediction-based dynamic tolling.

Higher toll revenue is observed when evaluating optimized toll rates in closed loop, compared with base toll rates. The toll optimization framework was also evaluated under certain experimental scenarios, and these experiments generated improved revenue in the simulation environment.

There are five gantries on the WB lanes of the network. The toll optimization model generates toll rates for the two gantries located at the beginning of each tolling segment. The toll rate at each of the other three gantries is a fraction of the gantry at the beginning of the

Figure 6. Comparison of actual and simulated flow and speed in GPL of WB corridor of the network: (*a*) flow, and (*b*) speed.

corresponding tolling segment. According to the tolling regulations, the toll rates may change dynamically every 5 min, and the amount of change cannot exceed ±$ 0.50. Toll rates on tolling segments 1 and 2 are subject to an upper bound of $5.30 and $5.70, respectively, except when the ML become congested. A further constraint was added that toll rates on the two tolling segments cannot be different by more than $1.00, both for practical considerations and for consistency with historical toll rate data.

For this study, a search algorithm was used which searches three toll values for each tolling segment, that

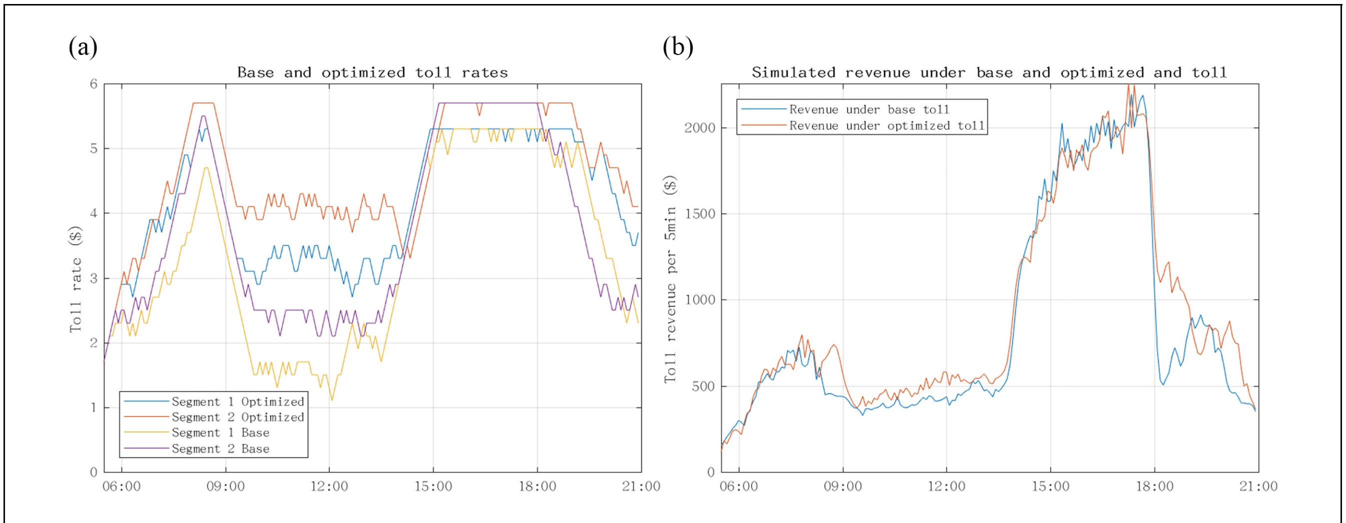

Figure 7. Comparison of base and optimized toll rates and revenues: (*a*) toll rates, and (*b*) revenues.

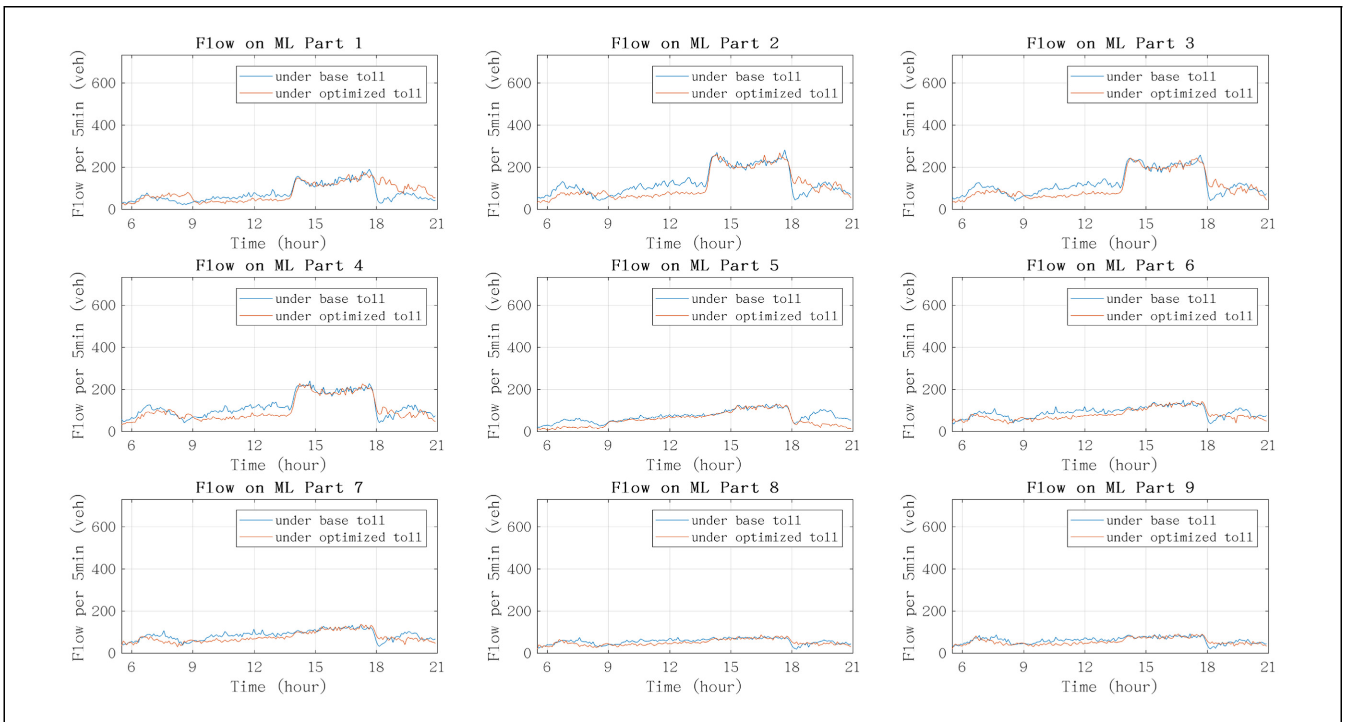

Figure 8. Flow on ML under base and optimized toll rates.

is, reduce by $ 0.20, stay the same, or increase by $ 0.20. The algorithm then evaluates objective function by calculating toll revenue in the next 15 min. Figure 7 shows the optimized toll rates for each tolling segment compared with base toll, and per-5-min revenue under these two tolls. Note that the revenues shown are calculated from simulation results by MITSIM, the testbed for evaluating the toll optimization framework in this study. Figures 8 and 9 show flow on ML and speed on GPL, comparing the simulation results under optimized toll rates and under base toll rates.

The optimization results suggest, in general, higher toll rates compared with the base toll except during evening peak hours, when they both reach the upper bound,

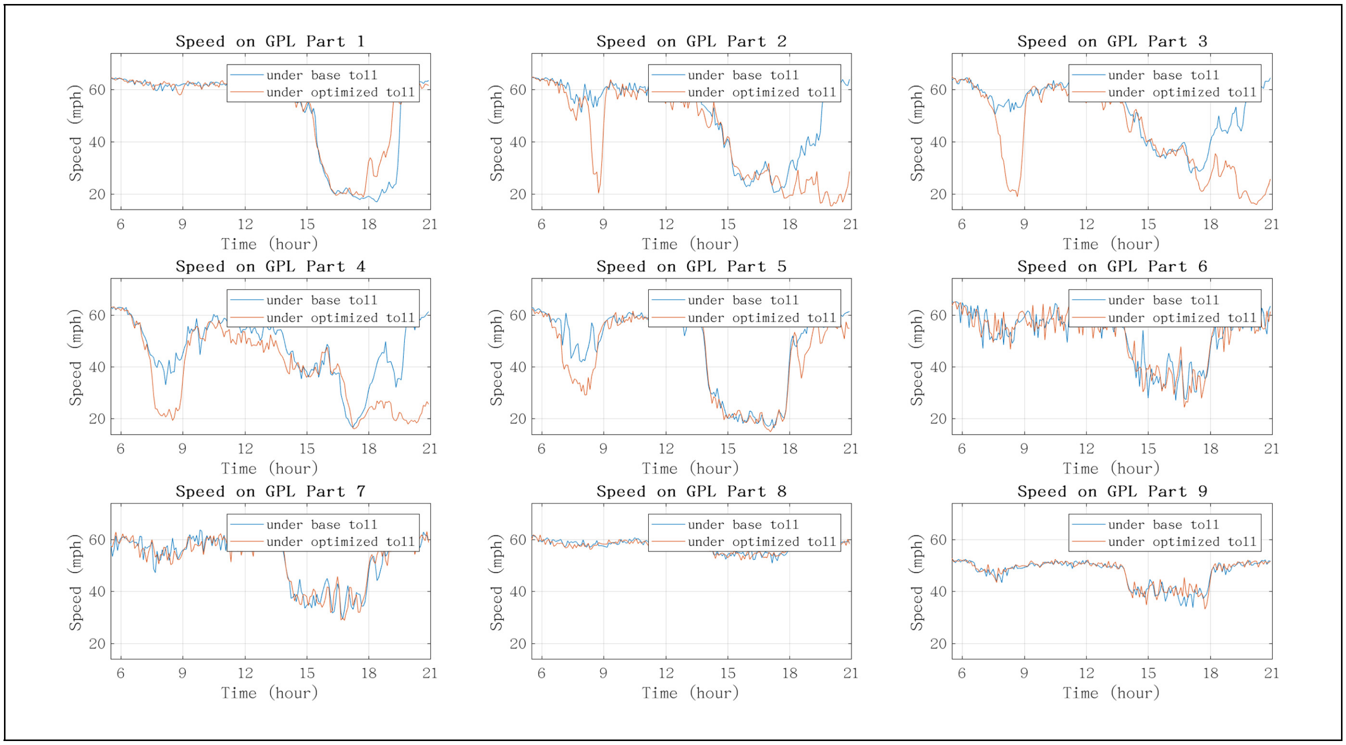

Figure 9. Speed on GPL under base and optimized toll rates.

because online calibration successfully captures most congestion, and the travelers' route choice model in this system shows room for increase in toll when congestion occurs. According to the simulation of the 05:30 to 21:00 period in the closed-loop framework, revenue is 8.1% higher under optimized toll rates. Under optimized toll, flow on ML is generally lower since toll rates are higher, and thus flow on GPL increases and speed on GPL decreases. However, on tolling segment 2 (parts 1~4) GPL becomes so congested after 17:00 that optimized toll rates remain at high levels even after the evening peak period. Meanwhile, there is still higher flow on ML at part 1, which leads to much higher revenue during that period. Note that this framework does not address congestion on GPL. Based on the present evaluation in the closed-loop framework, the above results demonstrate that the dynamic toll pricing framework with online calibration is promising for improved revenue.

Flow on GPL is not shown because it is complementary to flow on ML. Speed on ML is not shown because ML are generally not congested. With optimized toll rates, speed on ML is maintained at a high level. Since different model parameter values are use in four periods of the day, there may be sudden changes of simulated flow between periods.

Limitations include a narrow search range for the toll rates. If the algorithm allows toll rates to change by a higher value in each interval, then the revenue under optimized toll rates might be even higher.

Toll Optimization under Different Scenarios

Toll optimization was further evaluated under some experimental scenarios:

1. Toll rates are not subject to an upper bound.
2. Demand is 20% lower.
3. Drivers' braking behaviors are more conservative so that deceleration rates are 50% lower.

Optimized toll rates under these scenarios are presented in Figure 10. These experiments are conducted for the period 05:30 to 18:00.

Under scenario 1, when no upper bound on toll is in effect, toll rates during morning and evening peak periods would potentially increase to as much as twice the original upper bound, generating a revenue gain of 5.3% during the simulation period of 05:30 to 18:00, which is a slightly larger gain compared with 4.0%, the case where there is an upper bound. This indicates there is still room for raising the toll rates above the upper bound, based on travelers' elasticity to toll rates as implied by the route choice model in this study. Nevertheless, the rate of increase reduces as the toll increases, since the response

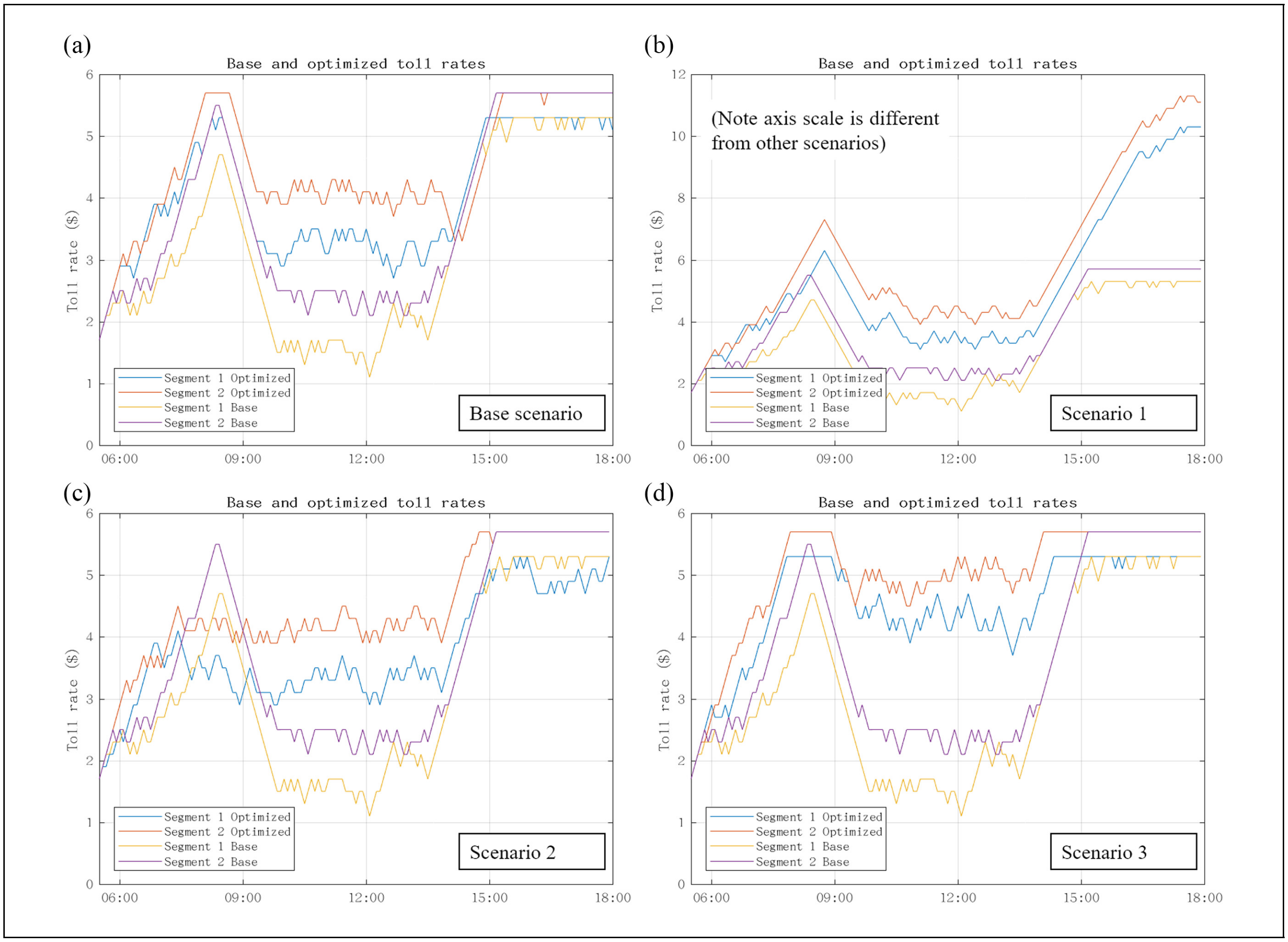

Figure 10. Optimized toll rates under base and experimental scenarios: (*a*) base scenario, (*b*) scenario 1, (*c*) scenario 2, (*d*) scenario 3.

of travelers is eventually effective and the supply–demand interaction is working properly under the proposed framework.

Scenario 2 represents a day with 20% less demand. Optimized toll rates become lower than the base scenario because of less congestion on GPL, but still higher than base toll during the midday period. Since midday is not congested anyway, reducing demand does not affect optimized toll rates. Toll revenue would be lower than base scenario because of fewer trips, but applying toll optimization with online calibration still increases revenue by 1.7% compared with applying the base toll rates that are not adjusted dynamically.

Scenario 3 simulates drivers driving in a more conservative way, potentially because of bad weather. Because of slower deceleration rates, headway between vehicles has to increase, thus the overall capacity of the highway decreases. Because of more congestion, the toll optimization algorithm chooses to maintain much higher toll rates compared with base case, and similar flow on ML is maintained, thus generating a revenue gain of 9.8% compared with base toll rates. Under heavier congestion, drivers choose ML even when toll rates are much higher, because of greater savings in travel time, and the proposed toll optimization framework benefits from online calibration to estimate and predict congestion.

The above tests under the simulation environment demonstrate the important role of online calibration in the prediction-based dynamic toll pricing framework. When online calibration is enabled and it is possible to estimate and predict traffic conditions with satisfactory accuracy, decisions on toll rates made by the DTA-based optimization are better than in the case where no online calibration is available. The added benefit of online calibration is especially large when there is significant congestion on the network, and is less evident when no congestion is present, which confirms that online calibration of supply parameters in an effort to match estimated and actual traffic speed is key to the success of the prediction-base tolling framework.

Conclusion

This paper presents calibration and optimization methodologies for a dynamic toll pricing framework. This framework is integrated with a DTA system to optimize toll rates by evaluating toll revenues under predicted traffic conditions. Thus online calibration is important to ensure the DTA system accurately understands and predicts traffic conditions. A heuristic online calibration algorithm is proposed to adjust supply parameters in the DTA system dynamically in response to real-time surveillance data. This algorithm is tested with real sensor data from a corridor consisting of ML and GPL, and the calibration accuracy is impressive, even when significant congestion is present. With online calibration enabled, the toll optimization is tested in a closed-loop evaluation framework. A microscopic simulator is calibrated offline toward real data, and integrated in the closed-loop evaluation framework as a representation of real network. The DTA-based optimization framework generates optimized toll rates, which are then implemented in the microscopic simulator instead of in the real network. The closed-loop toll optimization test is done under a base scenario and three experimental scenarios. In each scenario, optimized toll rates are consistent with the authors' expectations, and higher toll revenue is obtained when optimized toll rates are implemented, compared with the base toll rates generated in a system without online calibration. It is also observed that the system is maintained in real time, that is, the optimized tolls are always obtained in less than 5 min.

It should be noted that this research is conducted in a simulation environment relying on a discrete choice model to predict travelers' route choices under different traffic conditions and toll rates, and parameters in that model are known to the DTA system optimizing the toll. Recent research by Burris and Brady (*19*) suggests travelers' route choice behaviors may be more complex than a route choice model which only considers travel time and monetary cost. Further research is necessary before the proposed methodology can claim to be valid in the real world. Future research includes a comprehensive and personalized model for travelers' decisions to use ML, as well as online calibration of the choice model parameters.

Future research on toll optimization algorithms may potentially improve the effectiveness of toll optimization and obtain larger revenue gain, or the algorithm may be extended to incorporate other objectives. The current algorithm is a simple search algorithm and should be improved without sacrificing computational efficiency. Robust toll optimization algorithms may be another future direction to account for the situation in which the DTA system may not have perfect knowledge of travelers' choice behaviors and future network conditions.

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Author Contributions

The authors confirm contribution to the paper as follows: study conception and design: YZ, AA, BA, MB-A; data collection: YZ, BA; analysis and interpretation of results: YZ, AA, BA; draft manuscript preparation: YZ, BA, AA. All authors reviewed the results and approved the final version of the manuscript.

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