Delft University of Technology
Department of Aerospace Engineering

Memorandum M-456

# NUMERICAL EXPERIMENTS ON THE ESTIMATION OF RELATIVE STATION POSITIONS USING SINGLE PASSES OF LAGEOS AND STARLETTE LASER RANGE OBSERVATIONS

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SUMMARY

Within the Crustal Dynamics Project the European efforts to detect and monitor crustal motions and deformations are focussed on the Mediterranean region. In the near-future a number of mobile laser systems will be deployed in this tectonically active region to track LAGEOS and STARLETTE. One of the options to process the laser range observations is to apply a semi-geometric data reduction mode, in which the relative station positions are determined from a number of single satellite passes where the stations track the satellite almost simultaneously.

To gain experience in this type of data processing, this technique was applied to an existing regional network of laser stations, of which the coordinates were already known. All laser tracking data acquired since 1979 and available in the databank of the Kootwijk laser ranging station in the Netherlands were scanned for single passes, in which one of the satellites BEACON-C, GEOS-1, GEOS-2, GEOS-3, SEASAT, LAGEOS and STARLETTE had been tracked simultaneously by at least 3 laser stations. The data comprise European tracking data until the end of 1981, and American tracking data until the end of the MERIT short-campaign in October 1980.

It turned out that there were only 2 occasions in which a satellite had been tracked by 3 European stations simultaneously. Also for the American stations relatively few networks could be identified with sufficient quasi-simultaneous tracking. Finally, two networks consisting of 5 stations located in the USA were selected; for both networks 9 or 10 short arcs consisting of LAGEOS or STARLETTE tracking data were formed. One network consists of the stations at Greenbelt (7063), McDonald (7086), Haystack (7091), Owens Valley (7114) and Goldstone (7115). For the 10 selected LAGEOS arcs at least 4 out of these 5 stations have tracked the satellite simultaneously. The other network consists of the stations at Quincy (7051), San Diego (7062), Greenbelt (7063), Bear Lake (7082) and Mount Hopkins (7921). For the 6 LAGEOS and 3 STARLETTE arcs selected, 3 stations have always tracked a satellite simultaneously. Although the strength of the first network was considered better, it was realized that for both networks no real accurate results could be expected, because of the poor geometry and the fact that most of the ground tracks during the observation periods were parallel to each other and more-or-less perpendicular to the majority of the interstation baselines.

For each network the coordinates of 4 out of the 5 stations have been determined, while the coordinates of a base station were held fixed at the University of Texas UT 8112.2 solution. In most of the computations the tailored LAGEOS GEM-L1 gravity model was used. To investigate the sensitivity of the solution to the gravity model applied, coordinate solutions have also been generated for the tailored STARLETTE PGS-1331 gravity model. In addition, to gain some insight in geometry effects, for each network the computations were repeated for another base station.

It was found that for Greenbelt, McDonald, Quincy, San Diego and Bear Lake the overall laser range residual rms of the orbital fits is about 9 cm. For Owens Valley and Goldstone somewhat higher values of about 12 cm were obtained, while for Haystack and Mount Hopkins the laser range residual rms's are about 17 cm and 22 cm, respectively. The selection of different base stations proved to have important effects on the adjusted positions of the other stations. Coordinate differences of several meters occurred. Of course, this is a direct consequence of the poor tracking network geometry and the availability of mainly parallel passes. Even for these extremely short arcs, the application of different gravity models resulted in changes of the adjusted coordinates of up to 70 cm.

Comparing the station coordinate solutions with the University of Texas UT 8112.2 solution shows that for both networks coordinate errors of many meters are possible. As was expected, the most accurate interstation baselines were obtained for the network in which at least 4 stations have tracked LAGEOS simultaneously. Taking into account that most of the adjusted baselines show relatively large formal standard deviations, in fact no significant deviation from the University of Texas baselines solution was found.

The study will be continued by selecting other regional laser station networks with better ground track geometries with respect to the station configuration.

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*Paper presented at the 3rd Crustal Dynamics Working Group meeting, NASA Goddard Space Flight Center, October 28-29, 1982.*

## Summary of computation model.

### Dynamic model

- Gravity field
  GEM - L1; for some comparisonsalso PGS - 1331 (STR1),
  truncated at degree and order 41
  $a_e$ = 6378.137 km; GM = 398600.44 $km^3/s^2$
  c = 299792.458 km/s
- Atmospheric drag for STARLETTE
  Jacchia 1971 atmosph. model
  S = 0.04524 $m^2$; m = 47.25 kg; $C_D$ = 2.2
- Solar and lunar attraction
  JPL DE-96 ephemeris
- Solar radiation pressure
  Stand. 1 AU press. = 4.5 x $10^{-6}$ $N/m^2$
  LAGEOS : S = 0.28263 $m^2$; m = 407.821 kg; $C_R$ = 1.1
  STARLETTE: S = 0.04524 $m^2$; m = 47.25 kg; $C_R$ = 1.5
- Solid earth tides
  Force: $k_2$ = 0.2821; $\phi_2$ = 2.018°
  Tracking station displ.: $h_2$ = 0.6; $l_2$ = 0.075
- Indirect acceleration due to moon - $J_2$ interaction
- Univ. of Texas UT 8112.2 solution for coordinates of 1 station
- BIH polar motion and UT1 data

### Estimated parameters

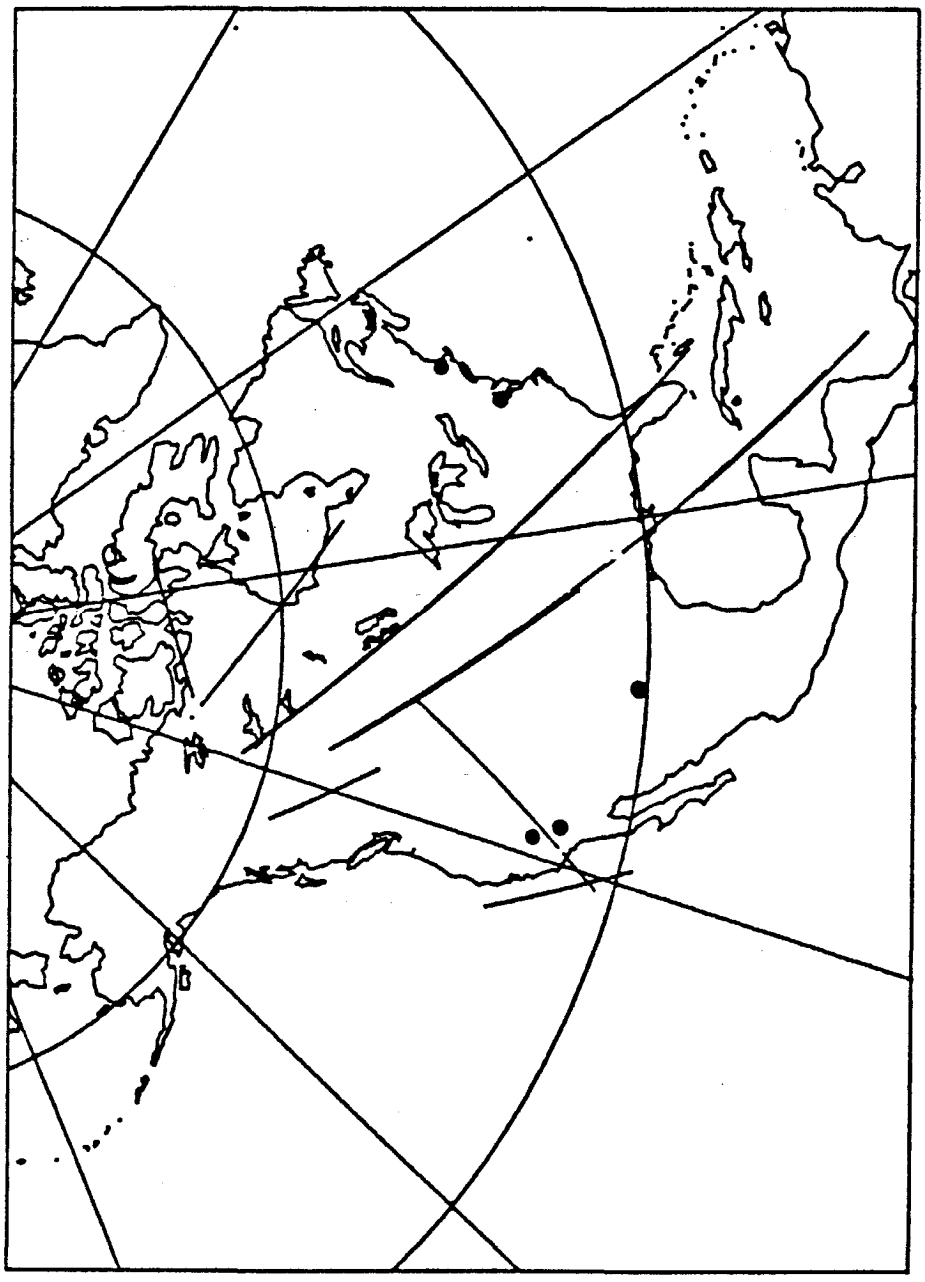
- State vector at epoch
- Coordinates of 4 stations

### Observations

- Assumed measurement errors
  Weight sigma for all stations = 12 cm
  Editing sigma for all stations = 50 cm
- Cut-off elevation = 20°
- Preprocessing: troposph. refract.; transit-time delay; center-of-mass correction

Periods of simultaneous LAGEOS tracking by at least 4 of the laser stations at Greenbelt (7063), McDonald (7086), Haystack (7091), Owens Valley (7114) and Goldstone (7115). For each pass both the total number of observations available and the actual number of observations selected for the station coordinate solutions are indicated. The reduction of the number of data points was necessary to balance the weight of the different stations.

| Arc id. | Date (yymmdd) | Start (hhmmss) | Length (min) | Observations 7063 | 7086 | 7091 | 7114 | 7115 |
|---|---|---|---|---|---|---|---|---|
| L1 | 791022 | 035958 | 4.9 | 48/48 | - | 70/70 | 180/90 | 60/60 |
| L2 | 791023 | 092622 | 22.1 | 509/50 | - | 854/85 | 139/69 | 726/72 |
| L3 | 791031 | 090141 | 26.9 | 104/52 | 153/51 | 1226/122 | 589/58 | 864/86 |
| L4 | 791101 | 010717 | 11.7 | - | 72/72 | 204/68 | 19/19 | 93/93 |
| L5 | 800205 | 035614 | 10.8 | - | 56/56 | 442/44 | 46/46 | 275/55 |
| L6 | 800206 | 061237 | 4.6 | 52/52 | 40/40 | 76/76 | - | 313/62 |
| L7 | 800213 | 070436 | 5.5 | - | 183/61 | 13/13 | 263/52 | 68/68 |
| L8 | 800304 | 041220 | 27.0 | 471/47 | 1255/125 | - | 639/63 | 1065/106 |
| L9 | 800312 | 035057 | 7.7 | 255/51 | 286/57 | 65/65 | 73/73 | 176/88 |
| L10 | 800320 | 034604 | 12.0 | - | 28/28 | 46/46 | 169/84 | 589/58 |
| Overall | | | 133.2 | 1439/300 | 2073/490 | 2996/589 | 2117/554 | 4229/748 |



The positions of the 5 tracking stations and the LAGEOS ground tracks during the observation periods of the 10 short arcs.

Summary of the LAGEOS range residual statistics for the coordinate solution in which the position of Greenbelt is held fixed. Listed are mean/rms values in centimeters (GEM-L1 gravity model).

| Arc id. | Greenbelt | McDonald | Haystack | Owens Val. | Goldstone | Overall |
|---|---|---|---|---|---|---|
| L1 | 0/7 | - | 0/13 | -1/11 | 1/10 | 0/11 |
| L2 | 1/9 | - | 0/19 | 4/18 | -1/12 | 0/15 |
| L3 | -1/9 | 0/11 | 0/16 | 0/13 | 0/12 | 0/14 |
| L4 | - | 0/8 | 0/19 | 9/14 | -2/11 | 0/13 |
| L5 | - | 0/7 | 0/14 | -2/12 | 1/13 | 0/12 |
| L6 | 1/12 | -1/8 | 0/15 | - | 0/14 | 0/13 |
| L7 | - | -1/8 | 1/28 | -2/10 | 2/8 | 0/9 |
| L8 | 0/10 | 0/8 | - | -2/12 | 2/14 | 0/11 |
| L9 | -4/9 | 0/7 | 2/18 | -1/12 | 1/11 | 0/12 |
| L10 | - | 4/10 | 0/20 | 2/12 | -4/12 | 0/14 |

The station position shifts from the combined LAGEOS short arcs relative to the Univ. of Texas UT 8112.2 solution. Three solutions are presented in which successively the coordinates of Greenbelt, Haystack or Goldstone were held fixed at the UT 8112.2 values. Listed are shift/formal standard deviation of the adjusted coordinate in meters (GEM-L1 gravity model).

| Station | Greenbelt fixed | Haystack fixed | Goldstone fixed |
|---|---|---|---|
| Greenbelt | | | |
| $\Delta\phi$ | - | 1.53/0.05 | -11.48/0.39 |
| $\Delta\Lambda$ | - | -2.04/0.06 | -2.03/0.07 |
| $\Delta h$ | - | -0.39/0.19 | -0.96/1.49 |
| McDonald | | | |
| $\Delta\phi$ | 8.92/0.25 | 8.46/0.29 | -4.04/0.14 |
| $\Delta\Lambda$ | -4.32/0.15 | -6.26/0.23 | -2.12/0.08 |
| $\Delta h$ | -1.06/0.98 | -1.50/1.41 | 0.09/0.29 |
| Haystack | | | |
| $\Delta\phi$ | -1.86/0.05 | - | -12.64/0.42 |
| $\Delta\Lambda$ | 2.10/0.05 | - | -0.95/0.06 |
| $\Delta h$ | 0.36/0.15 | - | -1.05/1.84 |
| Owens Valley | | | |
| $\Delta\phi$ | 13.23/0.36 | 11.72/0.40 | 0.52/0.02 |
| $\Delta\Lambda$ | -3.59/0.14 | -5.85/0.24 | 0.75/0.03 |
| $\Delta h$ | -1.47/1.26 | -1.87/1.69 | -0.07/0.06 |
| Goldstone | | | |
| $\Delta\phi$ | 12.74/0.35 | 11.34/0.39 | - |
| $\Delta\Lambda$ | -4.11/0.15 | -6.27/0.25 | - |
| $\Delta h$ | -1.46/1.28 | -1.86/1.72 | - |

Effects of deleting Greenbelt observations on the positions of the laser tracking stations as recovered from the combined LAGEOS short arcs. In both solutions the coordinates of Goldstone were held fixed at the Univ. of Texas UT 8112.2 values; the position shifts are relative to this UT 8112.2 solution. Listed are shift/formal standard deviation of the adjusted coordinate in meters (GEM-L1 gravity model).

| Station | Greenbelt observ. included | All Greenbelt observ. deleted |
|---|---|---|
| Greenbelt | | |
| $\Delta\phi$ | -11.48/0.39 | - |
| $\Delta\Lambda$ | -2.03/0.07 | - |
| $\Delta h$ | -0.96/1.49 | - |
| McDonald | | |
| $\Delta\phi$ | -4.04/0.14 | -0.73/0.15 |
| $\Delta\Lambda$ | -2.12/0.08 | -0.33/0.08 |
| $\Delta h$ | 0.09/0.29 | 0.02/0.34 |
| Haystack | | |
| $\Delta\phi$ | -12.64/0.42 | -2.88/0.46 |
| $\Delta\Lambda$ | -0.95/0.06 | 0.19/0.06 |
| $\Delta h$ | -1.05/1.84 | 1.02/2.03 |
| Owens Valley | | |
| $\Delta\phi$ | 0.52/0.02 | 0.09/0.02 |
| $\Delta\Lambda$ | 0.75/0.03 | 0.18/0.03 |
| $\Delta h$ | -0.07/0.06 | -0.01/0.07 |

Effects of applying a different gravity model on the positions of the laser tracking stations as recovered from the combined LAGEOS short arcs. In both solutions the coordinates of Greenbelt were held fixed at the Univ. of Texas UT 8112.2 values; the position shifts are relative to this UT 8112.2 solution and are listed in meters.

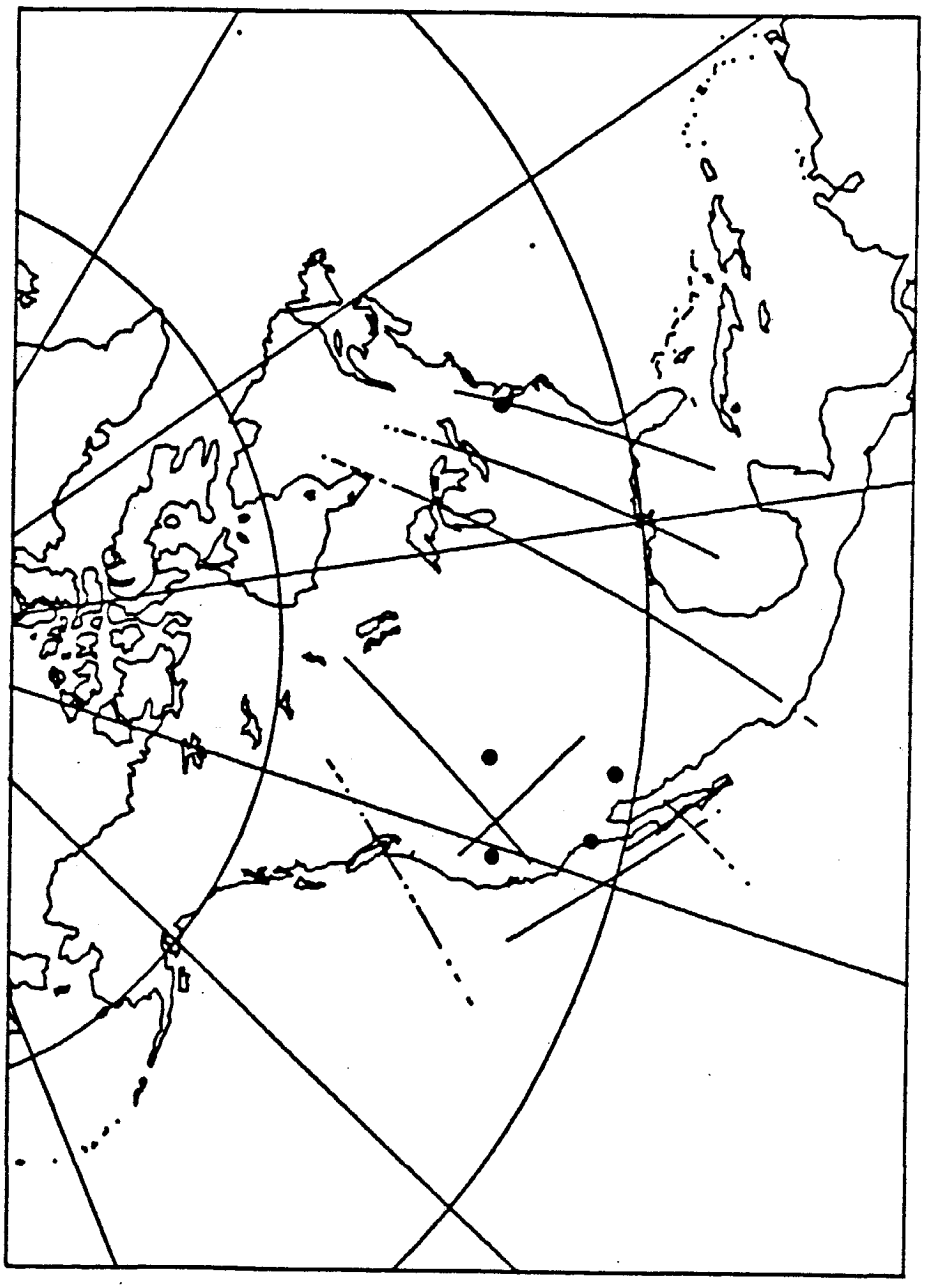
| Station | GEM-L1 gravity model | PGS-STR1 gravity model |
|---|---|---|
| McDonald | | |
| $\Delta\phi$ | 8.92 | 9.39 |
| $\Delta\Lambda$ | -4.32 | -4.58 |
| $\Delta h$ | -1.06 | -1.21 |
| Haystack | | |
| $\Delta\phi$ | -1.86 | -1.96 |
| $\Delta\Lambda$ | 2.10 | 2.18 |
| $\Delta h$ | 0.36 | 0.38 |
| Owens Valley | | |
| $\Delta\phi$ | 13.23 | 13.92 |
| $\Delta\Lambda$ | -3.59 | -3.84 |
| $\Delta h$ | -1.47 | -1.68 |
| Goldstone | | |
| $\Delta\phi$ | 12.74 | 13.40 |
| $\Delta\Lambda$ | -4.11 | -4.38 |
| $\Delta h$ | -1.46 | -1.67 |

Comparison of the interstation baseline shifts relative to the Univ. of Texas UT 8112.2 solution. Solutions are presented in which the coordinates of Greenbelt, Haystack or Goldstone were held fixed at the UT 8112.2 values; in the latter case the effects of deleting all Greenbelt observations are also indicated. Listed are baseline shift/formal standard deviation in centimeters (GEM-L1 gravity model).

| Baseline | Greenbelt fixed | Haystack fixed | Goldstone fixed | |
|---|---|---|---|---|
| | | | Greenbelt observ. included | All Greenbelt observ. deleted |
| Greenbelt - McDonald | -56/34 | -49/32 | -81/65 | 62/404 |
| Greenbelt - Haystack | 44/11 | 42/11 | 43/11 | -165/1015 |
| Greenbelt - Owens Valley | -62/49 | -55/48 | -110/87 | -19/88 |
| Greenbelt - Goldstone | -61/48 | -52/47 | -106/89 | - |
| McDonald - Haystack | 6/35 | 12/33 | -20/70 | 42/76 |
| McDonald - Owens Valley | 4/18 | -1/18 | -7/12 | -1/13 |
| McDonald - Goldstone | 2/16 | -1/16 | -10/11 | 3/12 |
| Haystack - Owens Valley | -2/51 | 7/49 | -52/100 | 65/110 |
| Haystack - Goldstone | 1/50 | 11/47 | -47/100 | 66/110 |
| Owens Valley - Goldstone | 4/4 | 2/4 | 4/5 | -2/5 |

Periods of simultaneous LAGEOS or STARLETTE tracking by 3 of the laser stations at Quincy (7051), San Diego (7062), Greenbelt (7063), Bear Lake (7082) and Mount Hopkins (7921). For each pass both the total number of observations available and the actual number of observations selected for the station coordinate solutions are indicated. The reduction of the number of datapoints was necessary to balance the weight of the different stations.

| Arc id. | Date (yymmdd) | Start (hhmmss) | Length (min) | Observations 7051 | 7062 | 7063 | 7082 | 7921 |
|---|---|---|---|---|---|---|---|---|
| L1 | 790402 | 094230 | 5.5 | 12/12 | 106/21 | - | - | 6/6 |
| L2 | 790419 | 072234 | 15.6 | - | 198/66 | 729/72 | 296/59 | - |
| L3 | 790420 | 093612 | 12.6 | 148/74 | 111/55 | 474/94 | - | - |
| L4 | 790430 | 063115 | 12.4 | - | - | 554/55 | 56/56 | 9/9 |
| L5 | 790430 | 100729 | 12.5 | - | 17/17 | - | 58/58 | 14/14 |
| L6 | 790518 | 063304 | 26.4 | 934/93 | - | 437/87 | 283/56 | - |
| S1 | 790402 | 123010 | 2.1 | 69/69 | 85/85 | - | - | 11/11 |
| S2 | 790405 | 113616 | 5.2 | 159/53 | 74/74 | - | - | 16/16 |
| S3 | 790420 | 053500 | 3.2 | - | 46/46 | - | 10/10 | 21/21 |
| Overall | | | 95.5 | 1322/301 | 637/364 | 2194/308 | 703/239 | 77/77 |



The positions of the 5 tracking stations and the LAGEOS or STARLETTE ground tracks during the observation periods of the 9 short arcs.

Summary of the LAGEOS and STARLETTE laser range residual statistics for the coordinate solution in which the position of Greenbelt is held fixed. Listed are mean/rms values in centimeters (GEM-L1 gravity model).

| Arc id. | Quincy | San Diego | Greenbelt | Bear Lake | Mt. Hopkins | Overall |
|---|---|---|---|---|---|---|
| L1 | 0/8 | 0/10 | - | - | 0/32 | 0/15 |
| L2 | - | 0/12 | 0/6 | 0/6 | - | 0/9 |
| L3 | 0/9 | 0/7 | 0/7 | - | - | 0/8 |
| L4 | - | - | 0/6 | 0/8 | 0/14 | 0/7 |
| L5 | - | 0/12 | - | 0/9 | 0/33 | 0/16 |
| L6 | 0/10 | - | 0/10 | 0/13 | - | 0/11 |
| S1 | 0/10 | 0/6 | - | - | 0/14 | 0/8 |
| S2 | 0/12 | 0/5 | - | - | 0/19 | 0/10 |
| S3 | - | 0/6 | - | 0/7 | 0/15 | 0/9 |

The station position shifts from the combined LAGEOS and STARLETTE short arcs relative to the Univ. of Texas UT 8112.2 solution. Two solutions are presented in which the Greenbelt or San Diego coordinates were held fixed at the UT 8112.2 values. Listed are shift/formal standard deviation of the adjusted coordinate in meters (GEM-L1 gravity model).

| Station | Greenbelt fixed | San Diego fixed |
|---|---|---|
| Quincy | | |
| $\Delta\phi$ | 22.98/1.05 | -0.99/0.11 |
| $\Delta\Lambda$ | -7.84/0.29 | -1.36/0.32 |
| $\Delta h$ | 3.44/3.82 | 0.74/1.17 |
| San Diego | | |
| $\Delta\phi$ | 22.31/0.99 | - |
| $\Delta\Lambda$ | -11.34/0.46 | - |
| $\Delta h$ | 2.74/3.71 | - |
| Greenbelt | | |
| $\Delta\phi$ | - | 3.90/1.10 |
| $\Delta\Lambda$ | - | 2.91/0.15 |
| $\Delta h$ | - | -0.42/4.74 |
| Bear Lake | | |
| $\Delta\phi$ | 18.92/0.84 | 0.68/0.17 |
| $\Delta\Lambda$ | -4.34/0.12 | -1.96/0.41 |
| $\Delta h$ | 1.49/2.63 | -0.47/1.49 |
| Mount Hopkins | | |
| $\Delta\phi$ | 20.27/0.87 | 1.34/0.18 |
| $\Delta\Lambda$ | -10.61/0.42 | 0.38/0.05 |
| $\Delta h$ | 4.76/3.75 | 2.12/1.35 |

Effects of applying a different gravity model on the positions of the laser tracking stations as recovered from the combined LAGEOS and STARLETTE short arcs. In both solutions the coordinates of Greenbelt were held fixed at the Univ. of Texas UT 8112.2 values; the position shifts are relative to this UT 8112.2 solution and are listed in meters.

| | GEM-L1 gravity model | PGS-STR1 gravity model |
|---|---|---|
| Quincy | | |
| $\Delta\phi$ | 22.98 | 23.58 |
| $\Delta\Lambda$ | -7.84 | -7.93 |
| $\Delta h$ | 3.44 | 3.39 |
| San Diego | | |
| $\Delta\phi$ | 22.31 | 22.89 |
| $\Delta\Lambda$ | -11.34 | -11.58 |
| $\Delta h$ | 2.74 | 2.69 |
| Bear Lake | | |
| $\Delta\phi$ | 18.92 | 19.40 |
| $\Delta\Lambda$ | -4.34 | -4.33 |
| $\Delta h$ | 1.49 | 1.46 |
| Mount Hopkins | | |
| $\Delta\phi$ | 20.27 | 20.79 |
| $\Delta\Lambda$ | -10.61 | -10.87 |
| $\Delta h$ | 4.76 | 4.77 |

Comparison of the interstation baseline shifts relative to the Univ. of Texas UT 8112.2 solution. Solutions are presented in which the coordinates of Greenbelt or San Diego were held fixed at the UT 8112.2 values. Listed are baseline shift/ formal standard deviation in centimeters (GEM-L1 gravity model).

| Baseline | Greenbelt fixed | San Diego fixed |
|---|---|---|
| Quincy - San Diego | -39/28 | -26/25 |
| Quincy - Greenbelt | 340/252 | 323/254 |
| Quincy - Bear Lake | 26/87 | -15/66 |
| Quincy - Mount Hopkins | -6/17 | -11/16 |
| San Diego - Greenbelt | 256/187 | 259/203 |
| San Diego - Bear Lake | -61/35 | -31/42 |
| San Diego - Mount Hopkins | 38/38 | 20/31 |
| Greenbelt - Bear Lake | 284/255 | 320/249 |
| Greenbelt - Mount Hopkins | 247/189 | 265/204 |
| Bear Lake - Mount Hopkins | -96/78 | -40/35 |