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Impact of High Levels of Wind Penetration on the Exercise of Market Power in the Multi-Area Systems

Ekaterina Moiseeva

ESCUELA TÉCNICA SUPERIOR DE INGENIERÍA

Impact of High Levels of Wind Penetration on the Exercise of Market Power in the Multi-Area Systems

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Keywords: wind integration, market power, game theory, mathematical programming

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SETS Joint Doctorate

The Erasmus Mundus Joint Doctorate in **Sustainable Energy Technologies and Strategies**, SETS Joint Doctorate, is an international programme run by six institutions in cooperation:

- Comillas Pontifical University, Madrid, Spain
- Delft University of Technology, Delft, the Netherlands
- Florence School of Regulation, Florence, Italy
- Johns Hopkins University, Baltimore, USA
- KTH Royal Institute of Technology, Stockholm, Sweden
- University Paris-Sud 11, Paris, France

The Doctoral Degrees issued upon completion of the programme are issued by Comillas Pontifical University, Delft University of Technology, and KTH Royal Institute of Technology.

The Degree Certificates are giving reference to the joint programme. The doctoral candidates are jointly supervised, and must pass a joint examination procedure set up by the three institutions issuing the degrees.

This Thesis is a part of the examination for the doctoral degree.

The invested degrees are official in Spain, the Netherlands and Sweden respectively.

SETS Joint Doctorate was awarded the Erasmus Mundus **excellence label** by the European Commission in year 2010, and the European Commission's **Education, Audiovisual and Culture Executive Agency**, EACEA, has supported the funding of this programme.

Abstract in English Language

Author: Ekaterina Moiseeva
Affiliation: KTH Royal Institute of Technology
Title: Impact of high levels of wind penetration on the exercise of market power in the multi-area systems
Language: English
Keywords: wind integration, market power, game theory, mathematical programming

New European energy policies have set a goal of a high share of renewable energy in electricity markets. In the presence of high levels of renewable generation, and especially wind, there is more uncertainty in the supply. It is natural, that volatility in energy production induces the volatility in energy prices. This can create incentives for the generators to exercise market power by traditional means: withholding the output by conventional generators, bidding not the true marginal costs, or using locational market power. In addition, a new type of market power has been recently observed: exercise of market power on ramp rate.

This dissertation focuses on modeling the exercise of market power in power systems with high penetration of wind power. The models consider a single, or multiple profit-maximizing generators. Flexibility is identified as one of the major issues in wind-integrated power systems. Therefore, part of the research studies the behavior of strategic hydropower producers as main providers of flexibility in systems, where hydropower is available.

Developed models are formulated as mathematical and equilibrium problems with equilibrium constraints (MPECs and EPECs). The models are recast as mixed-integer linear programs (MILPs) using discretization. Resulting MILPs can be solved directly by commercially-available MILP solvers, or by applying decomposition. Proposed Modified Benders Decomposition Algorithm (MBDA) significantly improves the computational efficiency.

Abstract in Spanish Language

Autor: Ekaterina Moiseeva
Afiliación: KTH Royal Institute of Technology
Título: Impacto de los altos niveles de penetración del viento en el ejercicio del poder de mercado en los sistemas de múltiples zonas
Idioma: Ingles
Palabras claves: integración del viento, poder de mercado, teoría de juegos, programación matemática

Las nuevas políticas energéticas europeas han establecido como objetivo un alto nivel de intercambio de energías renovables, en el Mercado eléctrico. Con la presencia de altos niveles de generación de energías renovables, especialmente la producida por el viento, aumenta la incertidumbre en la transmisión de la energía procedente de esta fuente. Esto es un efecto natural dónde la volatilidad de la producción de esta energía se refleja en la volatilidad de los precios de mercado eléctrico. Este hecho puedo repercutir en incentivos económicos para los productores de energía a la hora de ejercer más poder de mercado por medios tradicionales: retener la producción convencional, atando los costes marginales o ejerciendo más poder en mercado localizados. Además, se ha observado un nuevo tipo de poder de mercado: el poder de mercado que refiere a *ramp rate*.

Esta tesis se centra en el modelado de las prácticas de poder de mercado en los sistemas de potencia donde existe alta generación de energía eólica. Los modelos consideran la maximización de beneficio con uno o múltiples productores de energía. La flexibilidad se identifica como uno de los mayores problemas en los sistemas de potencia con integración eólica. Por este motive, parte de este trabajo de investigación estudia el comportamiento de estrategias de productores de energía hidroeléctrica cómo los principales proveedores de flexibilidad en los sistemas de potencia, allá dónde la hidroeléctrica esté presente.

Los modelos desarrollados son formulados como problemas matemáticos y problemas de equilibrio con restricciones de equilibrio (MPECs y EPECs). Los modelos son reformulados como mixed-integer programas lineales enteros mixtos (MILP). Los MILPs resultantes pueden ser resueltos directamente por algoritmos MILP comerciales, o aplicando descomposición. Esta tesis propone un nuevo *Modified Benders Decomposition Algorithm* (MBDA), el cual mejora significativamente la eficiencia computacional.

Abstract in Swedish Language

Författare: Ekaterina Moiseeva
Anslutning: Kungliga Tekniska Högskolan
Titel: Inverkan av höga nivåer av vindkraft på utövande av marknadsinflytande i flerområdes kraftsystem
Språk: Engelska
Nyckelord: vind integration, marknadsinflytande, spelteori, matematisk programmering

Ny europeisk energipolitik har som mål att öka andelen förnybar elproduktion. Förnybar elproduktion från vindkraft har till skillnad från konventionell kraftproduktion, en större osäkerhet i utbudet. Det är naturligt, att den här volatila osäkerheten i energiproduktionen skapar volatilitet även i energipriserna. Detta kan skapa möjligheter för producenter att utöva marknadsinflytande genom att undanhålla produktionen av konventionella kraftkällor, genom att använda fabricerade marginalkostnader eller med hjälp av lokalt marknadsinflytande. Dessutom har en ny typ av marknadsinflytande nyligen observerats, vilket är att utöva marknadsinflytande med hjälp av ramphastigheter.

Denna avhandlings fokus är på modellering av utövandet av marknadsinflytande i kraftsystem med hög andel av vindkraft. Modellerna beaktar en enda eller flera vinstmaximerande generatorer. Flexibilitet har identifierats som en av de stora problemen vid integrering av hög andel vindkraft i ett kraftsystem. Storskalig vattenkarft är väldigt flexibel och därför fokuserar endel av forskningen på vattenkraftsproducenternas strategiska möjligheter att leverera flexibilitet i kraftsystem med hög andel elproduktion från vindkraft.

De utvecklade modellerna har formulerats som matematiska jämviktsproblem med jämviktsbegränsningar (MPECs och EPECs). Modellerna är omarbetade till så kallade *mixed-integer linear programs* (MILPs) med hjälp av diskreta metoder. Resulterande MILPs kan lösas direkt med hjälp av kommersiellt tillgängliga MILP-lösare, eller genom att applicera så kallad decomposition. Den föreslagna *Modified Benders Decomposition Algorithm* (MBDA) förbättrar effektiviteten i beräkningarna avsevärt.

Abstract in Dutch Language

Auteur: Ekaterina Moiseeva
Instituut: KTH Royal Institute of Technology
Titel: Impact van hoge niveaus van windpenetratie bij de uitoefening van marktmacht in de multizone systemen
Taal: Engels
Trefwoorden: windintegratie, marktmacht, speltheorie, wiskundige programmering

Het nieuwe Europese energiebeleid heeft als doel gesteld om een groot aandeel duurzame energie op de elektriciteitsmarkten te realiseren. In aanwezigheid van hoge niveaus van hernieuwbare elektriciteitsproductie, vooral op basis van wind, is er een grotere voorzieningsonzekerheid. Volatiliteit in de energieproductie veroorzaakt vanzelfsprekend volatiliteit in de energieprijzen. Dit kan prikkels voor producenten creëren om de marktmacht op traditionele wijze uit te oefenen: reductie van de conventionele productie-eenheden, inbieden op andere dan de marginale productiekosten, of plaatselijke marktmacht te gebruiken. Daarnaast is recentelijk een nieuw type marktmacht waargenomen: uitoefening van de machtspositie op de *ramp rate*.

Dit proefschrift richt zich op het modelleren van de uitoefening van de marktmacht in energiesystemen met een hoog aandeel windvermogen. De modellen beschouwen een enkele of meerdere winst-maximerende producenten. Flexibiliteit wordt geïdentificeerd als een van de belangrijkste problemen in zulke energiesystemen. Daarom richt een deel van dit onderzoek zich op het strategisch gedrag van waterkrachtproducenten als belangrijkste leveranciers van flexibiliteit, wanneer tenminste waterkracht beschikbaar is.

De ontwikkelde modellen worden geformuleerd als wiskundige en *equilibrium* problemen met *equilibrium constraints* (MPEC's en EPEC's). De modellen worden herschikt als *mixed-integer* lineair programmeren opgaven (MILP's) met discretisatie. De resulterende MILP's kunnen worden opgelost door commercieel verkrijgbare MILP-*solvers*, of door ontleding toe te passen. Het voorgestelde *Modified Benders Decomposition Algorithm* (MBDA) verbetert de berekeningsefficiëntie significant.

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List of symbols

Nomenclature for Chapter 4:
Exercise of market power in wind-integrated power systems

Indices	
t	Time periods, $t \in (1..T)$,
c	Possible contingencies, $c \in (1..C)$,
i	Generating units, $i \in (1..I)$,
i_s	Strategic generating units, $i_s \in (1..I_s), I_s \subset I$,
s	Bidding strategies, $s \in (1..S)$,
l	Transmission lines, $l \in (1..L)$,
n	System nodes, $n \in (1..N)$.
Variables	
π_i	Profit of generator i,
g_{itc}	Dispatched output by generator i,
$\hat{R}_i^{up}$	Ramp-up bid of generator i,
$\hat{R}_i^{dn}$	Ramp-down bid of generator i.
Parameters	
K_i	Maximum output by unit i,
c_i	Marginal cost of unit i,
p_c	Probability of contingency c,
R_i^{up}	Ramp-up limits for unit i in Δt
R_i^{dn}	Ramp-down limits for unit i in Δt
H_{ln}	Power Transfer Distribution Factor (PTDF),
F_l	Flow limit on line l,
d_n	Demand at node n.

Lagrange multipliers (LM)

μ^{A1}_{itc}	LM of unit i lower capacity limit,
μ^{A2}_{i}	LM of unit i initial state upper capacity limit,
μ^{A3}_{itc}	LM of unit i transient states upper capacity limit,
λ^{B}_{tc}	LM of energy balance,
μ^{C}_{ltc}	LM of a flow constraint on line l,
μ^{D1}_{itc}	LM of unit i ramp-up constraint,
μ^{D2}_{itc}	LM of unit i ramp-down constraint.

Nomenclature for Chapter 5:
Market power in power systems with high share of wind power and hydropower

Indices

i	Generating units
t	Time periods
l	Transmission lines
n	System nodes
k	Bid alternatives
w	Stochastic scenarios
j	Benders iterations

Sets

I	Generating units
I^h	Strategic hydropower units, $I^h \subset I$
I^{up},I^{dn}	Upstream and downstream units, I^{up},$I^{dn} \subset I^h$

Parameters (upper-case letters)

M^0_i	Initial water level of unit $i \in I^h$, m^3
M_i	Maximum water level of unit $i \in I^h$, m^3
V_{itw}	Inflow to the reservoir of unit $i \in I^h$ in t, m^3
Γ_i	Production equivalent of unit $i \in I^h$, m^3/MWh
Q_i	Maximum generation of unit i, MW
R_i	Maximum ramp rate of unit i, MW/h
C_i	Maximum price bid of unit $i \in I^h$, \$/MWh
C^M_i	Marginal cost of unit i, \$/MWh

Λ_i^f	Future price for unit $i \in I^h$, \$/MWh
P_w	Probability of scenario w
H_{ln}	Power Transfer Distribution Factor
F_l	Power flow limit on line l, MW
D_{ntw}	Demand at node n in t, MWh

Variables (lower-case letters)

π	Profit of strategic hydropower producer, \$
q_{itw}	Dispatched generation of unit i in t, MWh
s_{itw}	Spillage of unit $i \in I^h$ in t, m^3
m_{itw}	Water level of unit $i \in I^h$ in t, m^3
x_{itk}^q	Binary variable of quantity bidding decision
x_{ik}^r	Binary variable of ramp-rate bidding decision
x_{itk}^c	Binary variable of price bidding decision
$\hat{q}_{it}$	Quantity bid of unit i in t, MW
$\hat{r}_i$	Ramp-rate bid of unit i, MW/h
$\hat{c}_{it}$	Price bid of unit i in t, \$/MWh

Lagrange multipliers (LM)

$\mu_{itw}^{A_1}$, $\nu_{itw}^{A_1}$	LM of unit i lower capacity limit constraint
$\mu_{itw}^{A_2}$, $\nu_{itw}^{A_2}$	LM of unit i upper capacity limit constraint
λ_{tw}^B, ϕ_{tw}^B	LM of energy balance constraint
μ_{ltw}^C, ν_{ltw}^C	LM of the flow constraint of line l
$\mu_{itw}^{D_1}$, $\nu_{itw}^{D_1}$	LM of unit i ramp-up constraint
$\mu_{itw}^{D_2}$, $\nu_{itw}^{D_2}$	LM of unit i ramp-down constraint
μ_{itw}^E, ν_{itw}^E	LM of spillage nonnegativity constraint
$\mu_{itw}^{F_1}$, $\nu_{itw}^{F_1}$	LM of lower water-level-limit constraint
$\mu_{itw}^{F_2}$, $\nu_{itw}^{F_2}$	LM of upper water-level-limit constraint
λ_{itw}^G, ϕ_{itw}^G	LM of water balance constraint

Operator

$\mathbf{I}(\mathit{condition})$	"If" operator

List of abbreviations

BNE	Best Nash Equilibrium
DC	Dispatch Cost
EPEC	Equilibrium Problem with Equilibrium Constraints
KKT	Karush-Kuhn-Tucker conditions
LM	Lagrange multiplier
LMP	Locational Marginal Price
LP	Linear program
MBDA	Modified Benders Decomposition Algorithm
MILP	Mixed-Integer Linear Program
MPEC	Mathematical Problem with Equilibrium Constraints
OPcOP	Optimization Problem constrained by Optimization Problem
PTDF	Power Transfer Distribution Factor
SC	Social Cost
SRED	Short-Run Economic Dispatch
STD	Standard Deviation
WNE	Worst Nash Equilibrium

Chapter 1

Introduction

This chapter motivates the topic of the dissertation and defines the background for the studies in Sections 1.1 and 1.2. The list of publications in journals with Journal Citation Report (JCR) and peer-reviewed conference papers is provided in Section 1.3. Finally, the outline for the remaining chapters is given in Section 1.4.

1.1 Background

The deregulation of electric power industry has started in 1981 in Chile, followed by England and Wales (1990), Norway (1991), and Argentina (1992) [1]. The aim of liberalization was to bring economic benefits in the long term, delivering timely and well-located investments by private companies. Liberalization was also expected to improve efficiency in the operation of generation plants, networks, and distribution services. Competition was seen as a driving force behind these changes [2]. This required significant changes in the way the electricity industry is organized and operated.

Electricity industry reform has apparently improved the efficiency and productivity of the industry. In Australia there are evidences of greater efficiency and reliability amongst generating plants [3]. The liberalization in Europe has allowed increasing opportunities for electricity market integration and cross-border trade [4].

However, the liberalization of the electricity markets was not always a smooth process. The crisis and market breakdown that hit California in 2001, only a few years after the new market was launched [2], have raised significant skepticism in the society. During the crisis, the electricity prices went to extreme levels, forcing the disconnection of some of the loads. The crisis had serious consequences for consumers, who were disconnected from the grid in rotating blackouts, and for electric utilities, which suffered major financial distress. Unilateral market power has been identified as one of the major causes of these events [5]. Another example is the liberalization of power industry in England and Wales. Even though the reform there has led to significant improvements in many dimensions, the decision to create only three generating companies out of the state-owned CEGB has led to significant market power, persisting for several years [6].

Market power is the ability of electricity generating firms to influence market prices through their unilateral actions [5, 7]. Electricity markets have many features, which make them prone to the exercise of market power: binding transmission constraints, largely inelastic demand, limited number of competing firms, repeated bidding [8]. There are no large scale technologies for storing electrical energy. These factors give rise to physical and economic withholding by dominant generating firms: the companies bid capacity or price, which differ from their true characteristics. The lessons learned from the beginning of the liberalization process have led to the development of market power indicators – measures, showing the presence of market power in electricity markets. Some indicators, e.g. HHI and four-firm concentration ratio, are based on calculating the share of the largest companies in the market [3, 9]. Other classic measures calculate the margin between the price bids and real marginal costs of producers [10].

While these measures had an important role in the beginning of the liberalization process, modern power industry has considerably evolved. In particular, the share of renewable resources has considerably increased in many countries around the world bringing new challenges. The European Union (EU) has set a target to reduce greenhouse gas emissions by 80-95% in 2050 as compared to 1990 levels [11]. The governmental and social support resulted in a significant increase in the installed capacity of renewable power sources, in particular wind power. Wind power was installed more than any other form of power generation in 2015. It accounted for 44.2% of total power capacity installations [12]. The amount of new wind power turbines installations continues to increase every year. One of the primary challenges in the integration of wind power is the problem of intermittency. Even a carefully predicted wind power output may suddenly depart from the forecast level. This calls for an increased amount of flexibility required in the system.

Flexibility expresses the capability of a power system to maintain continuous service, even when exposed to rapid and large swings in supply or demand [13]. There are many levels at which flexibility can be offered in power systems: flexibility of generation resources, flexibility of transmission and distribution systems [14], flexibility of the market to incentivize the power system to account for variability [15], and demand side flexibility [16]. Ramp rates largely define flexibility from the generation side.

There is some evidence of strategic behavior with respect to ramp rates occurring in practice. In the Australian National Electricity Market (NEM), when transmission constraints arise within a certain region, the generators are paid the regional price for their output rather than the correct local marginal price. Generators in this situation are said to be “constrained on” or “constrained off”. When the regional price is high, a generator, which is constrained off, will strategically manipulate its bid in a variety of ways in order to maintain a high output target from the dispatch engine. This typically involves offering the generator’s output at the price floor (-1,000 $/MWh). Alternatively, some generators also routinely reduce their offered ramp rate in order to maintain their dispatch level. In an attempt to prevent this, in 2009 a new rule was introduced which requires generators to offer a minimum ramp rate of 3 MW per minute (or 3% of unit capacity). More recently the Australian Energy Regulator (AER) has proposed a rule, which requires generators to offer a ramp rate which matches their technical capability.

1.2 Research motivation

The theoretical foundation behind the exercise of market power in power systems with high penetration of wind power is still very weak. The incentives for the exercise of market power in such systems should be carefully studied. There is a need for new models, able to capture the increased need for flexibility in wind-integrated systems. In present dissertation this challenge is addressed. New models are proposed, capturing the exercise of market power in wind-integrated systems. Such models can be used for full-scale market modeling and analysis.

1.3 List of publications

The following articles were published (to be published) during the PhD studies:

Papers published in journals with Journal Citation Report (JCR):

[J1] E. Moiseeva, M.R. Hesamzadeh, D.R. Biggar, "Exercise of Market Power on Ramp Rate in Wind-Integrated Power Systems," *IEEE Transactions on Power Systems*, Vol. 30, No. 3, pp. 1614-1623, May 2015 (Invited paper to special section on Wind & Solar Energy: Uncovering and Accommodating Their Impacts on Electricity Markets).

[J2] E. Moiseeva, S. Wogrin, M.R. Hesamzadeh, "Generation Flexibility in Ramp Rates: Strategic Behavior and Lessons for Electricity Market Design," *European Journal of Operational Research*, accepted February 2017.

[J3] E. Moiseeva, M.R. Hesamzadeh, "Strategic Bidding of a Hydropower Producer under Uncertainty: Modified Benders Approach," *IEEE Transactions on Power Systems*, accepted April 2017.

Paper under review in journal with JCR:

[J4] E. Moiseeva, M.R. Hesamzadeh, "Nash Equilibria in Hydro-Dominated Systems under Uncertainty: Modified Benders Approach," *IEEE Transactions on Sustainable Energy*, submitted February 2017.

Working paper:

[J5] E. Moiseeva, M.R. Hesamzadeh, D. Bunn, D.R. Biggar "Modeling the Hedging Decisions of a Generator with Market Power in Systems with High Penetration of Wind Power," *European Journal of Operational Research.*

Peer-reviewed conference papers:

[C1] E. Moiseeva, M.R. Hesamzadeh, "Modeling the Unilateral Multi-part Strategic Withholding in Electricity Markets," Australasian Universities Power Engineering Conference, Wollongong, Australia, 27-30 September 2015.

[C2] E. Moiseeva, M.R. Hesamzadeh, "Strategic Bidding by a Risk-Averse Firm with a Portfolio of Renewable Sources," IEEE PowerTech Conference, Eindhoven, the Netherlands, 29 June-2 July 2015.

[C3] E. Moiseeva, M.R. Hesamzadeh, I. Dimoulkas, "Tacit Collusion with Imperfect Information: Ex-Ante Detection," IEEE Power & Energy Society General Meeting, National Harbor, MD, USA, 27-31 July 2014.

[C4] E. Moiseeva, M.R. Hesamzadeh, "Modeling the Hedging Decisions in Electricity Markets Using Two-stage Games," IEEE ISGT Europe 2013 Conference, Copenhagen, Denmark, 6-9 October 2013.

[C5] E. Moiseeva, M.R. Hesamzadeh, "Impact of Energy Storage Devices on Energy Price in Decentralized Wind-Diesel Utilities," 10th International Conference on the European Energy Market, Stockholm, Sweden, 28-30 May 2013.

1.4 Thesis outline

The remaining chapters of this dissertation are organized as follows:

Chapter 2	provides the mathematical foundations for the dissertation, including the relevant concepts from game theory, optimization, and stochastic programming.
Chapter 3	gives a brief description of power system modeling conventions and assumptions.
Chapter 4	reviews exercise of market power in wind-integrated systems and identifies flexibility as one of the drivers for strategic behavior. The chapter is based on the publication [J1].
Chapter 5	focuses on the exercise of market power in hydro-dominated power systems with high share of wind power. Since hydropower producers are often the main providers for flexibility, their strategic behavior is carefully reviewed. The chapter is based on publications [J3] and [J4].

Chapter 6	reviews market power from the market design perspective. Two market design possibilities are compared and the impact of each of the design on the propensity of strategic generators to exercise market power in wind-integrated systems is discussed. The chapter is based on publication [J2].
Chapter 7	concludes the dissertation and provides the possible directions for future research.

Chapter 2

Mathematical foundation

This chapter reviews the main mathematical principles, forming the theoretical foundation of this dissertation. The necessary chapters of game theory are reviewed in Section 2.1, optimization concepts are discussed in Section 2.2. Section 2.3 describes the uncertainty modeling, utilized in this dissertation. Section 2.4 describes Benders decomposition technique, which is used to solve large optimization problems' instances.

2.1 Game theory

In this dissertation game theory is used to model the interaction of strategic players. The players are assumed to be rational and posses perfect information regarding the set of competitors' strategies. The section focuses on simultaneous-move and sequential games.

2.1.1 Simultaneous-move games

Simultaneous-move games are often used to model the interaction of strategic players in the markets. The situation with multiple profit-maximizing firms and no collusion can be modeled as Cournot [17–19] or Bertrand [20] games. Alternatively, conjectured-price response parameter is sometimes used to express a variety of competition intensities [21].

In Bertrand games each firm chooses a single price for each generator, or each area served, and believes that other firms will change prices in response [20, 22, 23]. The limitation of the Bertrand models is that even with a large number of companies in presence of capacity limit and transmission system the prices in the market can rise above marginal costs and even fluctuate without convergence [22, 24].

Cournot is another form of competition, where firms choose quantities, as best response to the anticipated competitors' strategies. Its simplicity and valuable qualities have made Cournot a popular concept in power market models [17,25–29]. It has been shown that even in the markets with relatively large number of competing firms Cournot models yield prices well above the competitive levels [30].

Conjectured-price response captures various degrees of strategic behavior in the spot market. Conjectural variations reflect the firm's conjecture about other firms' reaction to a change in its production [21,31]. This representation allows us to express the special cases of oligopolistic behavior ranging from perfect competition to a Cournot oligopoly [32]. Since this method can be seen as a "shortcut" for more complicated behaviors in implicit dynamic games, it has been a subject of theoretical controversies [33]. However, the conjectural variations appear versatile, when used in industrial applications. They can capture the competition structure, which is neither perfectly competitive, nor Cournot [34].

Nash equilibrium

Nash equilibrium is a solution concept for a non-cooperative game, in which each player is assumed to know the equilibrium strategies of the other players, and no player has an incentive to deviate from its equilibrium strategy [35]. This solution concept was introduced by John Nash [36, 37] and has been widely used in economics and industrial organization. Nash equilibrium concept is used in game-theoretic models of simultaneous-move games in electricity markets [38–41].

The definition of Nash equilibrium can be expressed mathematically as following:

$$\pi_i(s_i^*, s_{-i}^*) \geq \pi_i(s_i, s_{-i}^*), \quad \forall i. \tag{2.1}$$

This condition guarantees that for each strategic actor i the profit in the candidate strategy combination s_i^* must be greater or equal than the profit under alternative choice of strategy s_i, while the strategies of the competitors s_{-i}^* are held fixed.

Multiple Nash equilibria

Depending on a case study, one problem may have several Nash equilibria [42]. In this dissertation two methods are utilized to deal with this multiplicity of solutions: finding all Nash equilibria or focusing on the extremal Nash equilibrium.

- **Finding all Nash equilibria** can be done by formulating an optimization problem, aiming to find one Nash equilibrium and extending it with an integer cut [43]. An integer cut removes each newly obtained equilibrium from the feasible set. The problem is solved multiple times, until there are no more Nash equilibria. This technique is demonstrated in [J4].

- **Extremal Nash equilibrium** was introduced in [44], where it was defined as Nash equilibrium that maximizes or minimizes a certain objective function, in the context of a selfish routing game. In [45] Worst and Best extremal Nash equilibria (WNE and BNE) are applied to the social cost. If S^* is the set of all Nash equilibria strategies and $SC(s_i^*)$ is the social cost of each Nash equilibrium, $s_i^{*\text{worst}}$ is the worst Nash equilibrium of the game if and only if

$$s_i^{*\text{worst}} \in arg \max_{s^* \in S^*} SC(s_i^*). \tag{2.2}$$

  In a similar way Best Nash equilibrium is a Nash equilibrium at which social costs are minimized. The concept of extremal Nash equilibria allows to differentiate between multiple Nash equilibria, according to the defined criterion (in this case – social cost).

Nash equilibrium under uncertainty

In this dissertation proposed models often include uncertainty. Bayesian and robust Nash equilibria are two ways of finding Nash equilibrium under uncertainty:

- For **Bayesian Nash equilibrium** each player is assumed to have a subjective uncertainty probability distribution function [46]. This assumption is applicable to the most of the uncertainties observed in the power system, such as wind, reservoir inflows, demand uncertainty.

When introducing scenarios w describing the uncertainty, the Nash equilibrium (2.1) becomes:

$$E_w[\pi_{iw}(s_i^*, s_{-i}^*)] \geq E_w[\pi_{iw}(s_i, s_{-i}^*)], \quad \forall i. \tag{2.3}$$

- Finding **robust Nash equilibrium** does not require the prior knowledge of probability distribution function for the incomplete information [47]. This is very useful, when certain scenarios have no historic data or when probabilities of the scenarios are difficult to compute. Robust Nash equilibrium uses the worst-case approach, where (2.1) is reformulated as follows:

$$\min_w[\pi_{iw}(s_i^*, s_{-i}^*)] \geq \min_w[\pi_{iw}(s_i, s_{-i}^*)], \quad \forall i. \tag{2.4}$$

2.1.2 Sequential-move game

Another type of model used in this dissertation is Stackelberg game. In Stackelberg game one player is the leader of the game – it acts first. Other players are observing the action of the leader and reacting using their available actions [48]. This structure is often used to represent a dominant firm, deciding on its strategic bids. The bids are received by the system operator, who dispatches the firm and the competitive fringe [49, 50]. More information on the electricity market organization can be found in Section 3.2. Stackelberg game can be formulated as a bilevel optimization problem. This type of problems will be reviewed in Section 2.2.3.

2.2 Optimization

An optimization problem or mathematical programming problem is a mathematical entity that allows maximizing or minimizing a certain objective (i.e. objective function) subject to restrictions, typically in the form of equality or inequality constraints [51]. An optimization problem has general form:

$$\underset{x}{\text{minimize}} \quad f(x) \tag{2.5a}$$

$$\text{subject to:} \quad h(x) = 0 \tag{2.5b}$$

$$g(x) \leq 0, \tag{2.5c}$$

where $x \in \mathbb{R}^n$ is the optimization variable vector, $f(x) : \mathbb{R}^n \rightarrow \mathbb{R}$ is the objective function to be minimized, $h(x) : \mathbb{R}^n \rightarrow \mathbb{R}^{m_E}$ are the functions for the equality constraints and $g(x) : \mathbb{R}^n \rightarrow \mathbb{R}^{m_I}$ for the inequality constraints.

2.2.1 Linear optimization problems

Linear programming problems (LP) is a particular class of optimization problems. An LP is generally formulated as:

$$\underset{x}{\text{minimize}} \quad c^T x \tag{2.6a}$$
$$\text{subject to:} \quad Ax \geq b \tag{2.6b}$$
$$x \geq 0. \tag{2.6c}$$

The dual problem of a linear problem is formulated as:

$$\underset{\lambda}{\text{maximize}} \quad \lambda^T b \tag{2.7a}$$
$$\text{subject to:} \quad \lambda^T A \leq c^T \tag{2.7b}$$
$$\lambda \geq 0. \tag{2.7c}$$

According to the Strong Duality Theorem [52] if x is an optimal solution of the primal problem (2.6) and λ is an optimal solution of the dual problem (2.7), then

$$c^T x = \lambda^T b. \tag{2.8}$$

Additionally, it can be shown that at an optimal solution

$$\lambda_j = \frac{\Delta(c^T x)}{\Delta b_j} \quad \forall j. \tag{2.9}$$

It means that λ_j is the sensitivity of the objective function of the primal problem with respect to the right-hand-side parameter b_j of that primal problem. This result will be important for the Benders decomposition technique.

The Karush-Kuhn-Tucker (KKT) conditions are conditions that the optimal solutions of a broad range of optimization problems should satisfy. For linear problems the KKT conditions are both sufficient and necessary for the optimality [51]. The KKT conditions of problem (2.6) are:

$$c^T - \lambda^T A = 0 \tag{2.10a}$$
$$Ax \geq b \tag{2.10b}$$
$$x \geq 0 \tag{2.10c}$$
$$\lambda^T (b - Ax) = 0 \tag{2.10d}$$
$$\lambda \geq 0 \tag{2.10e}$$

The KKT conditions include stationary conditions (2.10a), primal feasibility conditions (2.10b)-(2.10c), complementary slackness conditions (2.10d), and dual feasibility conditions (2.10e). For linear problems only complementary slackness conditions are non-linear. KKT conditions can be written as linear or mixed-integer system of equations using one of the three following techniques to avoid nonlinear complementary slackness conditions (CSCs).

Disjunctive Constraints

The disjunctive constraints, or BigM technique, is a commonly used technique, first introduced in [53], to linearize the expressions of the form: $y^T g(x, y) = 0$, where both y and $g(x, y)$ are positive continuous variables. Introducing a binary variable b, the expression can be rewritten as $yb + g(x, y)(1 - b)$, which in turn can be expressed with a set of constraints:

$$
\begin{aligned}
&0 \geq y \geq \bar{K}(1-b) \\
&0 \geq g(x,y) \geq \bar{K}b
\end{aligned}
$$

The value of $\bar{K}$ is a pre-determined parameter and should be chosen in such a way that the value of $y^T g(x, y)$ is bounded above by it. However, the value should not be chosen too high, as it makes the optimization task, where such technique is implemented, ill-conditioned, and, therefore, computationally difficult [54]. It also should not be chosen too low to impose extra bounds on involved variables. The advantage of the method is that it is straight-forward in implementation [55].

SOS1-based Approach

The SOS1-based approach for solving mathematical problems with equilibrium constraints is explicitly discussed in [56]. The method is applied to the problem with equilibrium constraints in the form: $y^T g(x, y) = 0$, where $y \geq 0$, $g(x, y) \geq 0$, and x, y are optimization variables. After the introduction of SOS1 variables v^+ and v^- the equivalent constraint set is:

$$
\begin{aligned}
&y \geq 0 \\
&g(x,y) \geq 0 \\
&v^+ + v^- = (y + g(x,y))/2 \\
&v^+ - v^- = (y - g(x,y))/2
\end{aligned}
$$

Strong duality

Another way of reformulating a lower-level optimization problem is by using the strong duality property similar to (2.8) in the KKT conditions. To do this, the problem should satisfy Slater's sufficient condition for strong duality, namely the primal problem should be convex and strictly feasible. This technique is commonly used for the solution of MPECs [57], as it allows avoiding the complementary slackness conditions and, therefore, nonlinearities in the lower level.

Comparison

The common problem with using the disjunctive constraints is that, while seemingly easy to implement, parameters $\bar{K}$ need to be chosen carefully, as described in detail in [54]. Additionally, using this method of linearization requires adding a number of binary variables, which increases the computational time for the large-scale mixed-integer problems [58].

In contrast, using Schur decomposition and SOS type 1 technique does not require preliminary design. Authors in [56] show that under certain conditions, the nonlinear terms, arising in KKT conditions, can be linearized using the SOS1 technique. This, in turn, requires the introduction of new variables, but the method is shown to outperform the disjunctive constraints technique in terms of computational efficiency.

The drawback of the additional variables is canceled out in the strong duality formulation used to avoid the nonlinear terms in the formulation [59]. The conditions for applying this technique typically hold in the problems arising from modeling the electricity markets. The strong duality holds if the weak Slater's condition holds.

2.2.2 Mixed-integer linear optimization problems

In mixed-integer linear optimization problems (MILP) some of the variables are integer. An example of MILP can be formulated as follows:

$$\underset{x,y}{\text{minimize}} \quad c^T x + d^T y \tag{2.11a}$$

$$\text{subject to:} \quad A_1 x \geq b_1 \tag{2.11b}$$

$$A_2 y \geq b_2 \tag{2.11c}$$

$$x \geq 0, \quad y \in \{0, 1\}. \tag{2.11d}$$

MILP can be solved to the global optimum by commercial solvers. MILP solvers are usually based on modern modifications of grid search algorithm [60], simplex-like method [61], or branch and bound [62]. However, due to the solution procedure, even small mixed-integer problems can be very computationally intensive and require significant amounts of physical memory.

2.2.3 Equilibrium problems

Equilibrium problems are commonly used to model the game-theoretic situations. Two models discussed in this disseratation are Mathematical Problems with Equilibrium Constraints (MPEC) and Equilibrium Problems with Equilibrium Constraints (EPEC).

MPEC problems

The leader-follower structure of the Stackelberg game can be expressed using optimization problem constrained by optimization problem (OPcOP). The mathematical representation can be as following:

$$\underset{x,y}{\text{minimize}} \quad f(x,y,z) \tag{2.12a}$$

$$\text{subject to:} \quad h(x,y,z) = 0 \tag{2.12b}$$

$$g(x,y,z) \leq 0 \tag{2.12c}$$

$$z \in arg \ \underset{z}{\text{minimize}} \quad f_1(x,y,z) \tag{2.12d}$$

$$\text{subject to:} \quad h_1(x,y,z) = 0 \tag{2.12e}$$

$$g_1(x,y,z) \leq 0 \tag{2.12f}$$

If the lower level problem satisfies the constraint qualification, OPcOP can be directly reformulated to MPEC, by taking the KKT conditions of the lower-level optimization problem [51]. The structure of two problems is compared in Figure 2.1. Complementary slackness conditions can be further reformulated by one of the techniques presented in Section 2.2.1.

EPEC problems

The previous section describes MPEC models as representing leader-follower structure, when a single leader anticipates the equilibrium reaction of the followers, who in turn naively believe that the leader's decisions are exogenous

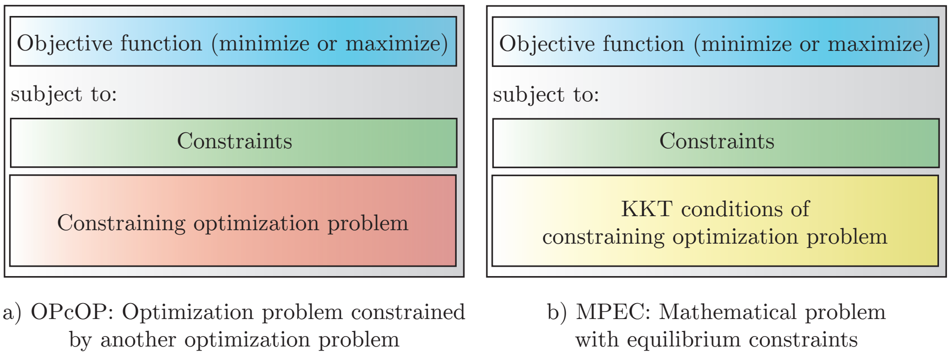

Figure 2.1: Structure of optimization problem constrained by optimization problem (OPcOP) as compared to mathematical problem with equilibrium constraints (MPEC)

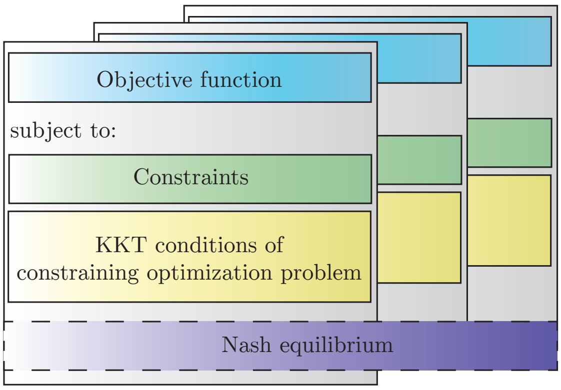

Figure 2.2: Structure of equilibrium problem with equilibrium constraints (EPEC)

and fixed. EPEC models are used to model games when there is more than one leader. The aim is to find an equilibrium between multiple leaders – in the context of electricity markets, Nash equilibrium between strategic players. An illustration to EPEC structure is presented in Figure 2.2.

EPEC can be thought as a collection of MPEC problems. Accordingly, a common way to solve such problem is diagonalization [63–65]. Using this method MPEC problems corresponding to different producers can be solved iteratively, by fixing the decision variables for all but one strategic agents and looking for a stable point, when neither of the players wants to change its strategy unilaterally. This method is used in [J2]. However, the method is only applicable to smaller case studies, due to a computationally intensive convergence procedure. Additionally, a post-check is required to guarantee that obtained stable point is a Nash equilibrium.

Other possible methods can include solving the whole EPEC formulation, and checking the second-order sufficient condition for each player's MPEC, as in [42]. The scalability of the problem can be limited. One way to avoid the scalability difficulties is to combine Bertrand and Cournot models of competition, or to disregard some of the constraints [66].

In this dissertation EPEC instances are solved by discretization of the solution space [45,67–69]. The method is demonstrated in [J1] and [J4]. Using this method Nash equilibrium constraint and constraints for all strategic generators, including the formulation for the alternative strategies are specified as in (2.1). EPEC problem for finding a Nash equilibrium can be formulated as a MILP optimization problem:

$$\underset{\Omega^{\text{MILP}}}{\text{minimize}}\ \Delta\pi = \sum_{i \in I^h} \epsilon_i \tag{2.13a}$$

$$\text{subject to:}\quad \pi_i(s_i^*, s_{-i}^*) + \epsilon_i \geq \pi_i(s_i, s_{-i}^*),\ \ \forall i \in I^{\text{strategic}}, \tag{2.13b}$$

$$\epsilon_i \geq 0,\ \ \forall i \in I^{\text{strategic}}, \tag{2.13c}$$

$$\Omega^{\text{MILP}} \in \mathbf{X}. \tag{2.13d}$$

Here Ω^{MILP} is a set of variables including upper-level and lower-level variables. Set $\mathbf{X}$ describes the feasible set of the whole problem including the upper level constraints and KKT conditions of the lower level. Variable ϵ_i is a deviation of current strategy from the profit-maximizing strategy, expressed in profit difference. At Nash equilibrium $\epsilon_i = 0$, $\forall i$.

2.3 Uncertainty modeling

Uncertainty is a crucial element of the models used in this dissertation. The relevant sources of uncertainty, including wind power generation uncertainty, can be represented using scenarios. Sources of uncertainty, identified in this dissertation, and utilized scenario generation techniques are described in the following sections.

2.3.1 Sources of uncertainty

The following sources of uncertainty are identified as relevant for the models under consideration:

- Uncertainty in wind power production – can be modeled using sampling from a probability distribution function, or using a moment-matching technique. These scenario-generation techniques are described in the following section. This source of uncertainty is considered in all models [J1], [J2], [J3], and [J4].

- Uncertainty in demand – demand is an important source of uncertainty in the short term. While deviations are usually mild, they may affect the strategy of generating company.

- Inflow uncertainty – is identified as the most important source of uncertainty in the models including hydropower producers [70–72]. This source of uncertainty is considered in the models of hydro-dominated systems in [J3] and [J4].

- Uncertainty in competitors' offers – is used to represent possible deviations in the bids of the fringe generators. This source of uncertainty is relevant for MPEC models of a single dominant firm [73–75].

2.3.2 Scenario generation techniques

A single-period stochastic programming model can be formulated [76] as:

$$\underset{x}{\text{minimize}} \quad g_0(x, \bar{\epsilon}) \tag{2.14a}$$

$$\text{subject to:} \quad g_i(x, \bar{\epsilon}) \leq 0 \ \ \forall i \tag{2.14b}$$

$$x \in \mathbf{X} \subset \mathbb{R}^n, \tag{2.14c}$$

where $\bar{\epsilon}$ is a random vector. Except for some trivial cases (2.14) can not be solved with continuous distributions. Hence, continuous distribution of the stochastic parameters have to be approximated by discrete distributions with a limited number of outcomes. Such discretization is often called a scenario tree [77]. In this dissertation two scenario-generation methods are used, described below.

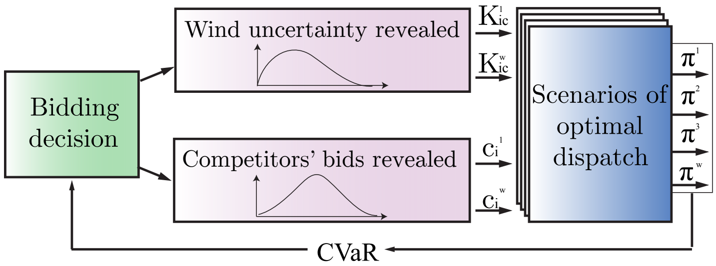

Figure 2.3: Scenario creation for a problem of risk-concerned profit-maximizing producer.

Conditional sampling

Conditional sampling is the most common method for generating scenarios [77]. At each stage of the scenario tree several values are sampled from the stochastic process $\{\bar{\epsilon}\}$, sampling each marginal (the univariate component) separately. The samples are then combined all-against-all, resulting in a vector of independent random variables. An example of this approach, utilized for the publication [C4] is presented in Figure 2.3.

Resulting scenario tree grows exponentially with the dimension of the random vector. Sampling w scenarios for k marginals, the obtained number of scenarios is w^k. To mitigate this problem a scenario reduction technique can be applied [78, 79]. Many of the scenario reduction techniques are readily implemented in optimization software (GAMS), and therefore the number of scenarios can be scaled according to computational requirements [80].

Moment matching

Conditional sampling has two important limitations: it can only be applied if the exact probability distribution functions of random parameters are provided, and it does not take into account the correlation between multiple uncertain parameters. These drawbacks are mitigated by the moment-matching technique [81]. Marginals can be described by the moments (mean, variance, skewness, kurtosis, etc.), obtained from the real data.

In [J3] and [J4] Nord Pool data is used to generate scenarios reflecting the correlations and statistical properties of the real data. Other examples of moment-matching technique usage can be found in [82–85].

2.4 Benders decomposition

EPEC and MPEC problems, introduced in Section 2.2.3 can be reformulated to MILP using strong duality condition as in Section 2.2.1 and discretization. MILP problems can be solved in a centralized manner using the powerful solvers nowadays available [86]. Alternatively, MILP problems can be decomposed to separate integer and continuous variables. For stochastic MPEC problems the resulting continuous problems can be further decomposed by blocks per scenarios. In further section several modifications of Benders decomposition are presented, which exploit the special structure of MILP problems.

2.4.1 Primal Benders decomposition

Primal Benders decomposition is a short name for the "Benders decomposition based on primal problem", adopted in this dissertation. Assume a MILP problem of a form

$$\underset{x,y}{\text{minimize}} \quad \sum_i c_i x_i + \sum_j d_j y_j \tag{2.15a}$$

$$\text{subject to:} \quad \sum_i a_{li} x_i + \sum_j e_{lj} y_j = b_l \ \ \forall l \tag{2.15b}$$

$$x_i \in \{0,1\} \ \ \forall i, \ \ y_j \in \mathbb{R} \ \ \forall j. \tag{2.15c}$$

Primal master problem ($\mathbf{MP}^{\text{primal}}$) is formulated as:

$$\underset{x,\alpha}{\text{minimize}} \quad \sum_i c_i x_i + \alpha \tag{2.16a}$$

$$\text{subject to:} \quad \alpha \geq \sum_j d_j y_j^{(k)} + \sum_i \lambda_i^{(k)} (x_i - x_i^{(k)}), \ \ k = 1, ..., v-1 \tag{2.16b}$$

$$x_i \in \{0,1\} \ \ \forall i \tag{2.16c}$$

$$\alpha \geq \alpha^{\text{down}}. \tag{2.16d}$$

Here $x_i^{(k)}$ is the value of x_i in the previous iteration (expression (2.16b) does not exist for the first iteration), α^{down} is a value for the lower bound of the problem. It can be chosen arbitrarily low: $\alpha^{\text{down}} \to -\infty$.

Corresponding primal linear Benders subproblem ($\mathbf{SP}^{\text{primal}}$) can be formulated as follows:

$$\underset{x,y}{\text{minimize}} \quad \sum_j d_j y_j \tag{2.17a}$$

$$\text{subject to:} \quad \sum_j e_{lj} y_j = b_l - \sum_i a_{li} x_i \ \ \forall l \tag{2.17b}$$

$$x_i = x_i^{(v)} : \ \ \lambda_i \ \ \forall i \tag{2.17c}$$

$$x_i \in \mathbb{R}, \ \ y_j \in \mathbb{R}. \tag{2.17d}$$

Here (2.17c) is used to fix the value of x_i to the parametric value $x_i^{(v)}$ obtained in iteration (v). The solution of this problem is y_j and λ_i.

The procedure for solving the problem using this implementation of Benders decomposition, requires solving $\mathbf{MP}^{\text{primal}}$ and $\mathbf{SP}^{\text{primal}}$ iteratively, until the lower bound ($z^{(v)}_{\text{down}} = \sum_{i=1}^{n} c_i x_i^{(v)} + \alpha^{(v)}$) and upper bound ($z^{(v)}_{\text{up}} = \sum_{i=1}^{n} c_i x_i^{(v)} + \sum_{j=1}^{m} d_j y_j^{(v)}$) would match. This type of decomposition is easy in implementation, as it requires just minimal reformulation of the initial MILP problem. However, very often the models formulated in this dissertation contain disjunctive constraints. This method has a very limited performance on such problems. Obtained Benders cuts are loose and sometimes contain irrelevant information, which causes numerical difficulties limiting the use of the method [87].

2.4.2 Dual Benders decomposition

Dual Benders decomposition is short for “Benders decomposition based on dual problem”. It is the initial Benders procedure, described in [88]. This type of decomposition is especially useful for MILP with disjunctive constraints. With this formulation the disjunctive parameter is only included in the objective function.

Assume an initial MILP problem with disjunctive constraints:

$$\underset{x,y}{\text{minimize}} \quad \sum_j d_j y_j \tag{2.18a}$$

$$\text{subject to:} \quad \sum_j e_{lj} y_j \geq b_l \ \ \forall l \tag{2.18b}$$

$$\sum_j \bar{e}_{ij} y \geq \bar{b}_i - H(1 - x_i) \ \ \forall i \tag{2.18c}$$

$$y_j \in \mathbb{R}, \ \ x_i \in \{0, 1\}. \tag{2.18d}$$

Here H is a disjunctive parameter – a large constant which relaxes or enforces constraint (2.18c) depending on the value of variable x_i. Fixing binary variables to a candidate vector x_i, a general linear subproblem (similar to $\mathbf{SP}^{\text{primal}}$) is:

$$
\begin{aligned}
\underset{y}{\text{minimize}} \quad & \sum_j d_j y_j \\
\text{subject to:} \quad & \sum_j e_{lj} y_j \geq b_l \quad : u_l \;\; \forall l \\
& \sum_j \bar{e}_{ij} y \geq \bar{b}_i - H(1 - x_i) \quad : \bar{u}_i \;\; \forall i \\
& y_j \in \mathbb{R}.
\end{aligned}
$$

Here u_l are Lagrange multipliers of general (non-disjunctive) constraints, $\bar{u}_i$ are the Lagrange multipliers for the disjunctive constraints. The dual of this problem ($\mathbf{SP}^{\text{dual}}$) is:

$$
\underset{u,\bar{u}}{\text{maximize}} \quad \sum_l u_l b_l + \sum_i \bar{u}_i (\bar{b}_i - H(1 - x_i)) \tag{2.19}
$$

$$
\text{subject to:} \quad \sum_l u e_{lj} + \sum_i \bar{u} \bar{e}_{ij} \leq d_j \;\; \forall j \tag{2.20}
$$

$$
u_l, \bar{u}_i \geq 0. \tag{2.21}
$$

Notice that the feasible region of problem $\mathbf{SP}^{\text{dual}}$ is free from the disjunctive parameter H. The extreme points of the feasible region can be denoted as $\{u^p, \bar{u}^p\}_p$, the initial MILP (2.15) can be restated as $\mathbf{MP}^{\text{dual}}$:

$$
\begin{aligned}
\underset{x,\alpha}{\text{minimize}} \quad & \alpha \\
\text{subject to:} \quad & \alpha \geq \sum_l u_l^p b_l + \sum_i \bar{u}_i^p (\bar{b}_i - H(1 - x_i)), \;\; \forall p \\
& x_i \in \{0, 1\} \;\; \forall i \\
& \alpha \geq \alpha^{\text{down}}.
\end{aligned}
$$

Similarly to the previous formulation, master and subproblems are solved iteratively, until the convergence is attained [88].

2.4.3 Modified Benders decomposition (MBDA)

Modified Benders decomposition is based on the reformulation of Benders procedure, proposed in [89] and additionally modified in this dissertation to improve the computational properties. Subproblem can be reformulated as follows ($\mathbf{SP}^{\text{MBDA}}$):

$$\begin{aligned} \underset{u,\bar{u}}{\text{maximize}} \quad & \sum_l u_l b_l + \sum_i \bar{u}_i \bar{b}_i \\ \text{subject to:} \quad & \sum_l u e_{lj} + \sum_i \bar{u} \bar{e}_{ij} \leq d_j \ \ \forall j \\ & \sum_i \bar{u}_i (1 - x_i) = 0 \\ & u_l, \bar{u}_i \geq 0. \end{aligned}$$

Notice that subproblem becomes completely independent from the disjunctive parameter H. The lemma that allows removing the disjunctive parameter from the objective function is stated in [J3]. The corresponding master problem $\mathbf{MP}^{\text{MBDA}}$ is based on set-partitioning reformulation of problem $\mathbf{MP}^{\text{dual}}$:

$$\begin{aligned} \underset{x,\omega_p}{\text{minimize}} \quad & \sum_p K^p \omega_p \\ \text{subject to:} \quad & \sum_{i \in \Omega^p} x_i \leq |\Omega^p| - 1 + \sum_{p' \geq p} \omega_{p'}, \ \ , \ \ \forall p, \\ & \sum_p \omega_p = 1, \\ & \omega_p, x_i \in \{0, 1\}. \end{aligned}$$

Here Ω^p is the index set corresponding to the strictly positive $\bar{u}$, K^p is the calculated value of the objective function at different extreme points. The author in [89], also shows that the properties of Benders decomposition (existence and uniqueness of the solution) hold for MBDA using tree search algorithm. Applications of MBDA are explored in [J3] and [J4]. They are further discussed in Chapter 5.

Chapter 3

Power System Modeling

This chapter reviews the models utilized for representing the elements of power system. Section 3.1 describes the DC power flow assumption, Section 3.2 focuses on the electricity market organization and mathematical formulation of the optimal dispatch. Section 3.3 presents the models for generating technologies, operating in the market. Section 3.4 describes the assumptions regarding the consumers in electricity markets.

3.1 DC power flow

According to [90], the exact expression for the real power flow from node i to node j (measured at node i, in the direction of node j) is:

$$F_{ij} = G_{ij}[V_i^2 - V_i V_j \cos(\delta_i - \delta_j)] + \Omega_{ij} V_i V_j \sin(\delta_i - \delta_j),$$

where $G_{ij} = R_{ij}/(R_{ij}^2 + X_{ij}^2)$, $\Omega_{ij} = X_{ij}/(R_{ij}^2 + X_{ij}^2)$, R_{ij} and X_{ij} are correspondingly resistance and reactance of the line from i to j. V_i and V_j are voltages at nodes, δ_i and δ_j are the phase angles relative to the reference node. For the models used in this paper the following can be assumed:

1. Line resistances are negligible compared to line reactances ($R_{ij} << X_{ij}$ for all lines).

2. The voltage amplitude is equal for all nodes in per unit values: $|V_N| \approx 1$ p.u.

3. Voltage angle differences between neighboring nodes are small: $\sin(\delta_i - \delta_j) \approx (\delta_i - \delta_j)$ and $\cos(\delta_i - \delta_j) \approx 1$.

These assumptions allow us to use a linear formulation for the power flow: $F = HZ$, where H is a Power Transfer Distribution Factors (PTDF) matrix, and Z is a matrix of injections. It is empirically and theoretically shown in the literature that DC power flow equations can be used for all operating points of the grid as long as the grid topology is retained [91, 92]. In the following formulations the DC power flow assumption is used in order to represent the network flows in a convenient linear manner.

3.2 Electricity market organization

Liberalized electricity markets are run by the system operator. In general, market participants, producers and consumers, submit their bids on price and production/consumption to the system operator. The system operator collects the bids and orders them to maximize the productive and allocative efficiency: the producers offering their production at the lowest price are dispatched first, also the consumers, who value their consumption the most, are supplied first. The dispatch, which minimizes the costs, subject to the system constraints is also called optimal dispatch.

3.2.1 Optimal dispatch

Assume $\hat{C}_{it}$ and $\hat{Q}_{it}$ are given values for the price and production bids of generators. Optimal dispatch for scenarios w and time steps t can be formulated as a linear optimization problem:

$$\underset{q_{itw}}{\text{minimize}} \sum_{i,t,w} P_w \hat{C}_{it} q_{itw} \tag{3.1a}$$

$$\text{subject to: } 0 \leq q_{itw} \leq \hat{Q}_{it} \ \ \forall itw, \tag{3.1b}$$

$$\sum_i q_{itw} = \sum_n D_{ntw} \ \ \forall tw, \tag{3.1c}$$

$$\sum_n H_{ln}(\sum_{i:n} q_{itw} - D_{ntw}) \leq F_l \ \ \forall ltw. \tag{3.1d}$$

Here (3.1a) is the objective of the system operator, minimizing the total cost of dispatch, (3.1b) is the capacity constraint, setting limit on the dispatched production, (3.1c) represent the system balance, and (3.1d) is an expression setting the limit on flows. This set of constraint can be further expanded, in order to reflect the presence of special generating technologies, or additional system constraints.

3.2.2 Security-constrained economic dispatch

The increasing uncertainty and variability in power system conditions call for a revised market design, where a market operator forecasts contingencies and carries out an efficient security-constrained dispatch. The authors in [93] proposed a short-run economic dispatch approach to the security-constrained economic dispatch problem. The proposed short-run economic dispatch approach models (1) the probabilities of contingencies, and (2) the trade-off between the preventive and corrective actions, in (3) a convex optimization structure. The system is dispatched in a way that fast-ramping generators can react to the contingencies in the most economic way. The advantages of the proposed model are discussed in details in [94]. Short-run economic dispatch can be formulated as follows:

$$\underset{q_i, q_{itc}}{\text{minimize}} \; (1 - \sum_c P_c) \sum_i \hat{C}_i q_i + \sum_{c,i,t} P_c \hat{C}_i q_{itc} \tag{3.2a}$$

$$\text{subject to: } 0 \leq q_i \leq \hat{Q}_i \;\; \forall i, \tag{3.2b}$$

$$0 \leq q_{itc} \leq \hat{Q}_{ic} \;\; \forall itc, \tag{3.2c}$$

$$\sum_i q_{itc} = \sum_n D_{ntw} \;\; \forall tc, \tag{3.2d}$$

$$\sum_n H_{ln} (\sum_{i:n} q_{itc} - D_{nt}) \leq F_l \;\; \forall lt, \tag{3.2e}$$

$$q_{itc} - q_{i(t-1)c} \leq \hat{R}_i^{up} \;\; \forall itc, \tag{3.2f}$$

$$- q_{itc} + q_{i(t-1)c} \leq \hat{R}_i^{dn} \;\; \forall itc. \tag{3.2g}$$

Here t is the number of time periods needed for the system to recover fully from the contingency. Time step 1 is considered a non-contingency stage, therefore: $q_{i(t=1)c} = q_i$. Generation in the following time steps in case of a contingency c is denoted as q_{itc}. There are new constraints, compared to (3.1): (3.2c) defines the capacity limits of units in case of contingency, (3.2f)-(3.2g) set the ramp limits of units.

This formulation expresses the need of flexibility in ramps in case the contingency, or wind power fluctuation occurs. Strategic behavior in such setup is studied in [J1].

3.3 Generating technologies

The formulations (3.1) and (3.2) above can be used to model conventional generating technologies, assuming not taking into account start-up and shut-down costs. These formulations can be extended to include additional constraints, representing certain generation technologies [75].

3.3.1 Wind power

Wind power is a rapidly growing renewable source usually characterized by considerable investment costs and relatively low maintenance and operation costs. Spilling the energy produced by a wind power unit is usually cost-inefficient, but could be reasonable in order to manage the network congestions or as a part of profit-maximizing strategy [95]. To model this undesirable, but possible power spillage wind turbines are modeled as power producers with close to zero marginal costs. The intermittency of a wind turbine output is modeled by introducing the scenarios of available capacity as in (3.1b).

3.3.2 Hydropower

Hydropower producers are the main providers of flexibility in the systems, where they are present. Hydropower producers can be modeled using the following expressions:

$$0 \leq s_{itw} \;\; \forall(i \in I^h)tw, \tag{3.3a}$$

$$0 \leq m_{itw} \leq M_i \;\; \forall(i \in I^h)tw, \tag{3.3b}$$

$$\begin{aligned} m_{itw} - m_{i(t-1)w} &= M_i^0 \mathbf{I}(t{=}1) + V_{itw} - \Gamma_i q_{itw} - s_{itw} \\ &+ \sum_{i \in I^{up}} (\Gamma_i q_{i(t-T_i)w} + s_{i(t-T_i)w}) \;\; \forall(i \in I^h)tw. \end{aligned} \tag{3.3c}$$

Here in time step t, scenario w: s_{itw} is spillage of hydropower producer, m_{itw} is the water level, V_{itw} is an inflow. M_i is the maximum water level, and M_i^0 is the initial water level taken into account if the constraint is formulated for the first time step ($\mathbf{I}(t = 1)$). In hydropower plants with a large storage capacity, head variation has negligible influence on operating efficiency in the short-term, therefore a constant production equivalent Γ_i can be assumed [96]. Constraint (3.3a) limits spillage, (3.3b) sets the limits on water level, (3.3c) describes the hydrological balance.

A system of connected reservoirs is considered. Therefore the hydrological balance for unit i, includes the water used for production or spilled by an upstream unit $i \in I^{up}$. Additionally, depending on the waterways' structure, there might be time lag in water relocation T_i.

In certain applications it may be also needed to model a problem of hydropower producer. Since hydropower producers have limited volume of the reservoir, it might be profitable to save some water for the future. The future water value is included in the profit formulation of the producer:

$$\pi = \sum_{(i \in I^h),w} P_w \Big(\sum_t (\lambda_{ntw} - C_i^M) q_{itw} + m_{iTw} \Lambda_i^f \frac{1}{\Gamma_i} \Big) \tag{3.4}$$

Here the first term in paranthesis describes the market profit of the hydropower producer: λ_{ntw} is the locational marginal price, and C_i^M is the marginal costs of the generator. The second term describes the value of the water kept in the reservoir by the end of the modeling horizon: Λ_i^f is a parameter, forecast by the hydropower producer and defining the value of water in the future.

3.3.3 Energy storage

Energy storage can act as a load within its capacity $L_i \leq 0$, consuming electricity during the periods of a lower price, or like a generator, smoothing out the demand spikes [97]. For energy storage constraint (3.2c) becomes:

$$\text{for } i \in I^{\text{storage}}: \quad L_{ic} \leq q_{itc} \leq K_{ic}. \tag{3.5}$$

The conversion cannot be regarded as completely free, as there are certain losses. Power flowing in the battery, $P_{itc}^{in} \geq 0$, amounts to $\frac{P_{itc}^{in}}{\eta^{in}}$ energy stored, where η^{in} is an efficiency factor. Analogically, $P_{itc}^{out} \geq 0$ flowing out of a battery becomes $P^{out}\eta^{out}$ after conversion. Therefore, the output of energy storage unit can be written as:

$$\text{for } i \in I^{\text{storage}}: \quad q_{itc} = -\frac{P_{itc}^{in}}{\eta^{in}} + P_{itc}^{out}\eta^{out} \tag{3.6}$$

Other constraints for the battery are the minimum E^{min} and maximum E^{max} states of charge. Considering the initial state of charge E^0 they can be written as:

$$E^0 + \sum_t \left(\frac{P^{in}_{itc}}{\eta^{in}} - P^{out}_{itc} \eta^{out} \right) \leq E^{max} \tag{3.7a}$$

$$E^0 + \sum_t \left(\frac{P^{in}_{itc}}{\eta_{in}} - P^{out}_{itc} \eta^{out} \right) \geq E^{min} \tag{3.7b}$$

To avoid the simultaneous charging and discharging of the battery the objective function is updated with penalizing terms. For example, the objective function (3.2a) can be rewritten as:

$$\begin{aligned} \underset{q_i, q_{itc}}{\text{minimize}} \quad & (1 - \sum_c P_c)(\sum_i \hat{C}_i q_i + P^{in}_{i(t=1)c} c^{in} + P^{out}_{i(t=1)c} c^{out}) + \\ & \sum_{c,i,t} P_c (\hat{C}_i q_{itc} + P^{in}_{i(t>1)c} c^{in} + P^{out}_{i(t>1)c} c^{out}) \end{aligned} \tag{3.8}$$

Here c^{in} and c^{out} are small penalizing costs. These costs are ensuring the correct operation of the storage device.

3.4 Demand

In the formulations (3.1) and (3.2) it is assumed that demand is inelastic, expressed with a parameter D_{ntw}. This assumption is common in power systems, where consumers often do not receive price signals and therefore do not respond by reducing their consumption [98]. However, demand response can be modeled in this framework as a "virtual" generator with high costs C^{DR}_{itw} and capacity equivalent to the capacity of demand-responsive consumers. This generator can be dispatched, when the price reaches high-enough levels, imitating the disconnection of flexible consumption. The symmetry between total surplus maximization and generation cost minimization is proved in [3].

Chapter 4

Exercise of market power in wind-integrated systems

This chapter focuses on modeling strategic behavior on ramps in wind-integrated power system. The findings are based on the publication [J1]. We propose a model for describing the strategic behavior on ramp rates and demonstrate illustrative and numerical results.

4.1 Introduction

With an increasing penetration of wind power, there is likely to be an increasing need for fast-ramping generating units. These generators ensure that no load is lost if supply drops due to the uncertainties in wind power generation. However, it is observed in practice that, in a presence of network constraints, fast-ramping generating units are prone to act strategically and exercise market power by withholding their ramp rates. In the Australian National Electricity Market (NEM), when the prices increase to very high levels, some generators were observed to decrease their declared ramp-rate capabilities, in order to maintain high output for a longer period of time.

Another evidence of strategic behavior was observed in South Australia, an area with very high penetration of wind power. A sudden reduction in wind output there must be matched by a rapid increase in the output of thermal generators. On several occasion the price in the area spiked up to \$12000/MWh. These occasions tended to coincide with times when the offered ramp rate from thermal units was less than their technical capability.

It appears that at times generators may have a commercial incentive to limit the rate at which they ramp up in response to a fall in wind power output.

In this chapter we propose a model, capable to model ramp-rate game in wind integrated power systems. The model covers an important gap in the literature. The authors of [99, 100] consider ramp rates as a part of the bidding information of profit-maximizing generating companies, but they do not assume any strategic behavior relating to ramp rates. The authors of [101] provide an analysis of strategic ramp-rate bidding, but significantly simplify the market clearing problem and do not take into account possible contingencies or network constraints.

The model in this chapter is set up using the concept of the multiple leaders-follower game as discussed in Section 2.2.3. The follower is a market operator who runs the short-run economic dispatch problem, discussed earlier in Section 3.2.2. SRED allows to model a trade-off between preventive and corrective measures for possible contingencies, e.g. wind power outages, in convex structure. The leaders are the profit-maximizing generators, strategic on both ramp rate and generation capacity. The whole setup is modeled as an Equilibrium Problem with Equilibrium Constraints (EPEC). The result of the EPEC model is a set of Nash equilibria of the ramp-rate game. To tackle the multiple Nash equilibria problem, we use the concept of the extremal-Nash equilibria, introduced in Section 2.1.1. We use different techniques for the linearization of the problem and motivate the best one [59]. The final formulation of the proposed game-theoretic model is a single-stage Mixed-Integer Linear Program (MILP). To show the distinctive features of the proposed game model, the whole formulation is applied to the illustrative two-node example system and to the IEEE 24-node system.

4.2 Modeling

This section gives the detailed formulation of the model used in the simulations and the assumptions made. We model a ramp-rate Stackelberg game with multiple leaders and a single follower. The leaders are strategic generators, seeking to maximize their profits by offering strategic bids to the market. The follower is a market operator performing a security-constrained short-run economic dispatch. We assume that all forward contracts [102] have been released before the dispatch takes place.

4.2.1 Lower level

The short-run economic dispatch (SRED) represents the lower level of the ramp-rate game. The setup of this dispatch, the motivation behind it, and the adopted assumptions are discussed in Section 3.2.2. The linear problem representing the optimal short-run economic dispatch in this chapter is as follows:

$$\underset{g_i, g_{itc}}{\text{minimize}} \quad (1 - \sum_c p_c) \sum_i (c_i g_i) + \sum_{c,i,t} p_c (c_i g_{itc}) \tag{4.1a}$$

$$\text{subject to:} \quad g_i, g_{itc} \geq 0 \qquad \leftrightarrow \mu^{A1}_{itc} \tag{4.1b}$$

$$g_i \leq K_i \qquad \leftrightarrow \mu^{A2}_{i} \tag{4.1c}$$

$$g_{itc} \leq K_{ic} \qquad \leftrightarrow \mu^{A3}_{itc} \tag{4.1d}$$

$$\sum_{n, i \in n} g_{itc} = \sum_n d_n \qquad \leftrightarrow \lambda^{B}_{tc} \tag{4.1e}$$

$$\sum_n H_{li} \sum_{n, i \in n} (g_{itc} - d_n) \leq F_l \qquad \leftrightarrow \mu^{C}_{ltc} \tag{4.1f}$$

$$g_{itc} - g_{i(t-1)c} \leq \hat{R}^{up}_i \qquad \leftrightarrow \mu^{D1}_{itc} \tag{4.1g}$$

$$- g_{itc} + g_{i(t-1)c} \leq \hat{R}^{dn}_i \qquad \leftrightarrow \mu^{D2}_{itc} \tag{4.1h}$$

The short-run economic dispatch problem in (4.1a)-(4.1h) is convex and satisfies the weak Slater's condition. Accordingly, the Karush-Kuhn-Tucker (KKT) optimality conditions can be written as linear or mixed-integer system of equations using one of the three techniques to avoid nonlinear complementary slackness conditions described in Section 2.2.1: strong duality, SOS type 1 technique, or BigM technique.

We will denote the total number of primal feasibility equations (4.1b)-(4.1h) as a star ($*$). Three discussed techniques for linearizing the KKT system are compared in Table 4.1 in terms of constraints, variables and constant parameters. Simulating these techniques shows that computational time depends strongly on these values. As shown in Table 4.1, the strong duality technique is the most computationally efficient technique for linearizing our KKT system. Therefore, to ensure the scalability and computational efficiency of the problem formulation, we adopt the strong duality technique in our further simulations.

Table 4.1: Linearization techniques for KKT system. $(*) = (4TIC - 3IC + I + TC + LTC)$ – number of primal feasibility constraints. Capital letters – number of elements in sets, see Nomenclature.

	Strong duality	SOS type 1	BigM technique
Constraints	$2(*) + 1 - TC$	$4(*) - 3TC$	$4(*) - 3TC$
Binary variables	0	0	$(*) - TC$
SOS1 variables	0	$2(*) - 2TC$	0
BigM constants	0	0	$(*) - TC$

4.2.2 Upper level

The upper level of the ramp-rate game is the profit-maximization task solved by the strategic generators. The profit formulation for the generator i can be written as:

$$\pi_i = (p_n - c_i)g_i, \tag{4.2}$$

where p_n is the price at the connection node n of generator i. The nodal price p_n can be expressed as a summation of a system price, λ^B_{tc}, and transmission congestion price, $\sum_l \mu^C_{ltc}H_{li}$, ($p_n = \lambda^B_{tc} - \sum_l \mu^C_{ltc}H_{li}$). The expression (4.2) for the initial state and for each time step t is then:

$$\pi_{itc} = (\lambda^B_{tc} - \sum_l \mu^C_{ltc}H_{li} - c_i)g_{itc}. \tag{4.3}$$

Expressing the λ^B_{tc} from stationary conditions, following the logic in [45], we recast the profit expression as:

$$\pi_{itc} = \mu^{A2}_i g_i + (\mu^{A3}_{itc} + \mu^{D1}_{itc} + \mu^{D2}_{itc})g_{itc}. \tag{4.4}$$

This expression contains nonlinear terms in primal and dual variables. However, using the complementary slackness conditions, we can equivalently write:

$$\begin{aligned} &\mu^{A2}_i g_i = \mu^{A2}_i K_i, \quad \mu^{A3}_{itc} g_{itc} = \mu^{A2}_{itc} K_{ic} \\ &\sum_t \mu^{D1}_{itc} g_{itc} = \sum_t \mu^{D1}_{itc} \hat{R}^{up}_i \\ &\sum_t \mu^{D2}_{itc} g_{itc} = \sum_t \mu^{D2}_{itc} \hat{R}^{dn}_i \end{aligned} \tag{4.5}$$

For the nonstrategic generators the bidding levels of ramp rates are true ramping capabilities, ($\hat{R}_i^{up} = R_i^{up}$ and $\hat{R}_i^{dn} = R_i^{dn}$). For the strategic generators, we assume a strategic choice on ramping level. We can model this strategic choice, by introducing a vector of binary variables x_{ik}^{up} and x_{ik}^{dn}:

$$\hat{R}_i^{up} = (b_0 + \sum_k b_k x_{ik}^{up}) R_i^{up}, \tag{4.6a}$$

$$\hat{R}_i^{dn} = (b_0 + \sum_k b_k x_{ik}^{dn}) R_i^{dn}. \tag{4.6b}$$

Here b_0 and b_k are vectors of constants, such that $b_0 + \sum_k b_k = 1$. This way, $x_{ik}^{up,dn} = \mathbf{1}$ means that the generator bids the full ramp-rate capability to the market. If $x_{ik} = \mathbf{0}$, then $\hat{R}_i^{up} = b_0 R_i^{up}$ and $\hat{R}_i^{dn} = b_0 R_i^{dn}$, which means the generator bids the minimum possible ramp rate level. This way a strategy set $S_i = \{s_1, s_2 ... s\}$ for the strategic unit i is obtained as a set of all k possible combinations for the vector $x_{ik}^{up,dn}$.

We substitute the expressions (4.5)-(4.6) in the profit formulation (4.4) and use the disjunctive constraints [53] to linearize the product of binary variables $x_{ik}^{up,dn}$ and continuous variables $\mu_{itc}^{D1,D2}$, by introducing a new variable $z_{ik}^{D1,D2} = x_{ik}^{up,dn} \mu_{itc}^{D1,D2}$. Assuming for conciseness that $R_i^{up} = R_i^{dn} = R_i$, we obtain the following expression for the profit:

$$\begin{aligned}\pi_i = & \mu_i^{A2} K_i + \sum_{tc} \Big(\mu_{itc}^{A3} K_{ic} + b_0 (\mu_{itc}^{D1} + \mu_{itc}^{D2}) \\ & + \sum_k (z_{itck}^{D1} + z_{itck}^{D2}) b_k R_i \Big).\end{aligned} \tag{4.7}$$

The $z_{ik}^{D1,D2}$ terms are linearized with a following set of constraints:

$$\begin{aligned} & z_{ik}^{D1,D2} \leq \bar{K}_{1,2} x_{ik}^{up,dn}, \\ & z_{ik}^{D1,D2} \leq \mu_{itc}^{D1,D2}, \\ & z_{ik}^{D1,D2} \geq \mu_{itc}^{D1,D2} - \bar{K}_{1,2}(1 - x_{ik}^{up,dn}). \end{aligned} \tag{4.8}$$

Here $\bar{K}_1$ and $\bar{K}_2$ are big-enough disjunctive constants, designed according to the recommendations in [54].

The Nash equilibrium between the strategic generators on ramp rate is reached, when given the ramp-rate strategy of other generators, no player wants to deviate from the chosen strategy. Or, expressed mathematically:

$$\sum_{itc} \pi_{itc}(s_i^*, s_{-i}^*) \geq \sum_{itc} \pi_{itc}(s_i, s_{-i}^*). \tag{4.9}$$

To avoid the problem of multiple Nash equilibria, we use the concept of extremal Nash equilibria, as in 2.1.1. The whole ramp-rate game model consists of the optimality conditions derived in Section 4.2.1, the profit formulation (4.7), disjunctive constraints (4.8), an expression ensuring the Nash equilibrium outcome (4.9), and an extremal-Nash equilibrium expression. As an example, the mixed-integer linear programming problem for finding the best-Nash equilibrium is set out in below:

$$\underset{x_{ik}^{up,dn}}{\text{Minimize}} \quad (1 - \sum_{c} p_c) \sum_{u} (c_i g_i) + \sum_{c,i,t} p_c (c_i g_{itc}) \tag{4.10}$$

$$\text{subject to:} \quad g_i, g_{itc} \geq 0, \quad g_i \leq K_i, \quad g_{itc} \leq K_{ic} \tag{4.11}$$

$$\sum_{n,i \in n} g_{itc} = \sum_{n} d_n \tag{4.12}$$

$$\sum_{n} H_{li} \sum_{n,i \in n} (g_{itc} - d_n) \leq F_l \tag{4.13}$$

$$g_{itc} - g_{i(t-1)c} \leq \hat{R}_i^{up} \tag{4.14}$$

$$- g_{itc} + g_{i(t-1)c} \leq \hat{R}_i^{dn} \tag{4.15}$$

$$\mu_{itc}^{A1}, \mu_i^{A2}, \mu_{itc}^{A3}, \mu_{ltc}^{C}, \mu_{itc}^{D1}, \mu_{itc}^{D2} \geq 0 \tag{4.16}$$

Profit formulation and linearization (4.7)-(4.8)

Nash equilibrium condition (4.9)

Stationary conditions

Strong duality implication

4.3 Illustrative case study

We consider a network with two distinctive areas. The areas are represented by 2 nodes: G1, G2, G3 are placed in node 1, G4, G5, G6 and demand are placed in node 2. We consider a capacity-constrained line, connecting these 2 nodes. Node 1 represents a generation surplus area, while the demand is mostly concentrated in node 2. An example of this setup is the Swedish power system, where generation is mainly concentrated in North and the load centers are situated in South. The northern and southern parts are only connected by several constrained links. The unit data is presented in Table 4.2.

While units G1 to G3, G5 and G6 have low probabilities of failure, unit G4, representing the aggregation of wind power units, has a 1% probability of going off the network, due to the extreme wind conditions. The demand is predicted to be 1500 MW during the short dispatch period under consideration. The dispatch is shown in Figure 4.1.

We consider 2 cases: case (a) represents a base-case, when generators are bidding their true ramping capabilities. In case (b) the generators G5 and G6 have 4 bidding strategies on their ramping capabilities. They can offer 4 levels of ramp rate, from 25% to 100% of their true ramping capability.

In case (b) the bidding decisions of strategic generators G5 and G6 differ. Table 4.2 presents that in case of the best-Nash equilibrium (BNE) generator G6 withholds 25% of its ramping capability. The optimal dispatch of the system is shown in Figure 4.1-(b). We see that the lower ramp rate of G6 forces the market operator to dispatch G5 even in a no-contingency state (time step 1). In the case of the worst-Nash equilibrium (WNE) both generators withhold. The corrective actions are taking more time and dispatch costs increase. As shown in Table 4.3, the costs of market power in cases of BNE and WNE are €6862 and €13365, respectively. This means 21.6% and 42% increase in dispatch costs as compared to the dispatch cost of the competitive case. The average market clearing prices (MCPs), calculated through 7 periods, increase as well.

We see that the concept of extremal-Nash equilibria, introduced in Section

Table 4.2: Unit data for the 2-nodes system. R_i^{up}=R_i^{dn}=R_i In bold – identified cases of withholding.

Unit,	Capacity,	Costs,	Ramp rates, (MW/hr)		
i	K_i (MW)	c_i (€/MW)	R_i	BNE $\hat{R}_i$	WNE $\hat{R}_i$
G1	500	10	1000	1000	1000
G2	500	100	100	100	100
G3	800	50	100	100	100
G4	800	0.1	100	100	100
G5	500	200	100	100	**75**
G6	500	1000	500	**375**	**375**

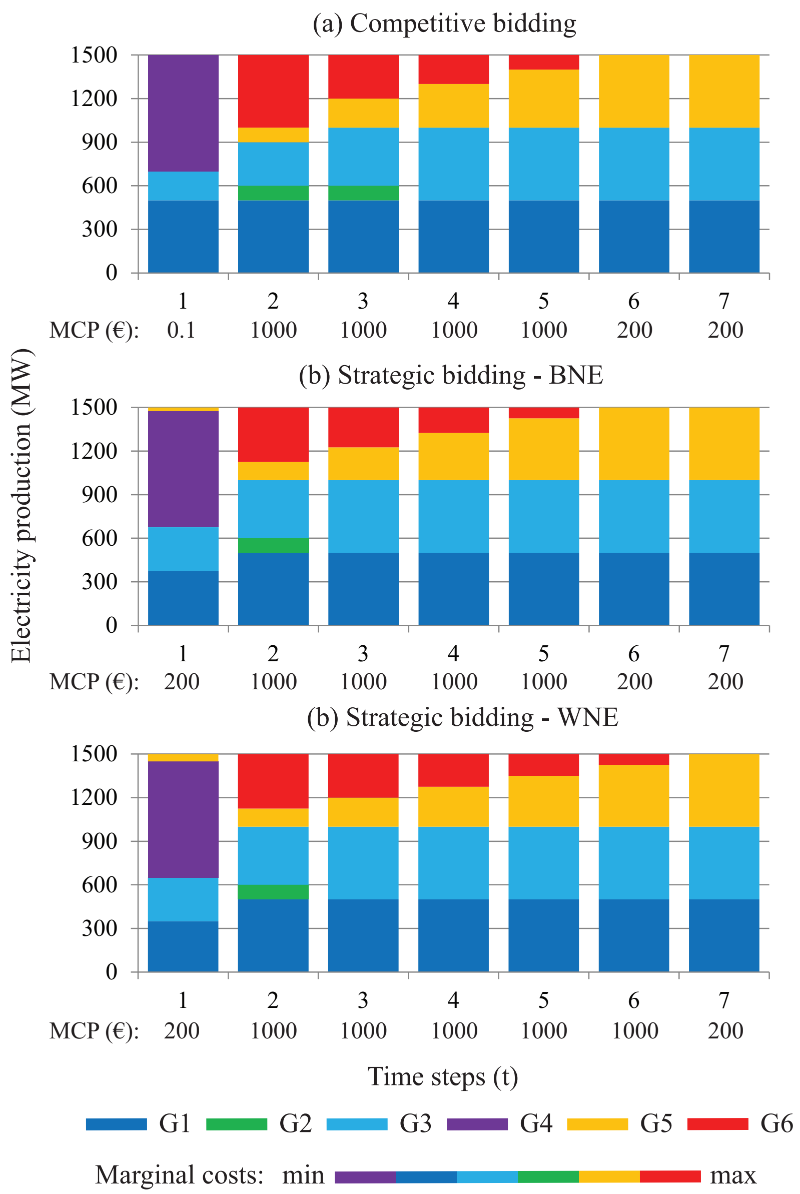

Figure 4.1: Short-run economic dispatch and market clearing prices (MCPs) for the cases of (a) competitive and (b) strategic bidding. BNW: best-Nash equilibrium case, WNE: worst-Nash equilibrium case.

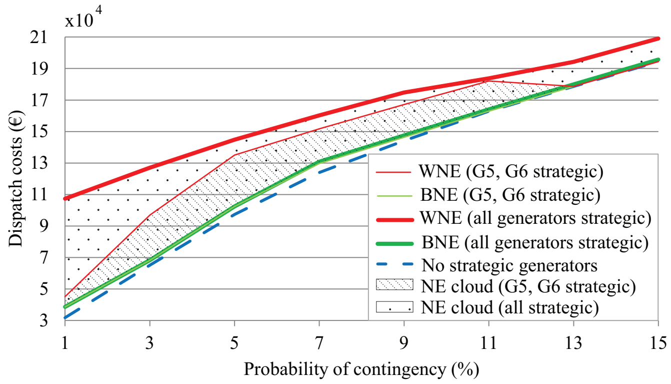

Figure 4.2: The concept of Nash-equilibria band with BNE and WNE as the lower and upper bounds. Clouds of Nash equilibria solutions for the cases, when generators G5 and G6 are strategic, and when all generators are strategic.

Table 4.3: Cost of market power on ramp rate for the illustrative case study (loss of G4 with probability of 1%, G5 and G6 strategic)

	No gaming	Best NE	Worst NE
Dispatch costs (€)	31,779	38641	45144
Cost of market power (€)		6862	13365
Relative increase in costs		21.6%	42%
Average electricity price (€)	629	657	771

2.1.1 sets the bounds for the solution space (Nash-equilibria cloud) in the case of strategic generators. The solutions in the cloud are equiprobable.

Figure 4.2 presents the evolution of dispatch costs over different contingency probabilities. The cloud of solutions, when all generators are strategic, is a super set for the cloud of solutions, when only generators G5 and G6 are strategic. Both bounds are important for the market power assessment; however, depending on the application we could be interested in a lower or upper bound. Another observation from the figure is that the cloud becomes thinner and tends to the non-strategic costs, when the probability of contingency is higher. This can be explained by the trade-off between preventive and corrective actions. As a severe contingency is more probable, system operator takes more preventive actions to minimize the costs and often the most expensive generators are dispatched in the preventive action.

4.4 Numerical results

Here we present only the main findings of the ramp-game modeling on a larger network, for more results we refer the reader to [J1]. EPEC problems are known to be computationally intesive [103], the purpose of this section is to demonstrate that the developed formulation is scalable to larger system, in this case 24-node IEEE system [104]. We assume that G13 is an aggregation of several wind power units. The output of G13 is predicted to be 300 MW, however there is a certain probability that the real output will be different.

In this section we consider a joint game for ramp and quantity. To do so, we introduce a variable $\hat{K}_{ic}$, which describes the production bid of the strategic generator:

$$\hat{K}_{ic} = (b_0 + \sum_k b_k x_{ik}^{cap}) K_i. \tag{4.17}$$

Here x_{ik}^{cap} is a vector of binary variables. Constraints (4.11) are then changed to the following expressions:

$$g_i, g_{itc} \geq 0, \quad g_i \leq \hat{K}_{ic}, \quad g_{ic} \leq \hat{K}_{ic}. \tag{4.18}$$

Following the same logic as in ramp-game, we change the expression for the profit:

$$\begin{aligned} \pi_i = \; & b_0 \mu_i^{A2} + \sum_k b_k z_{ik}^{A2} + \sum_{tc} \Big(\sum_k b_k z_{itck}^{A3} K_{ic} + b_0 (\mu_{itc}^{A3} + \mu_{itc}^{D1} \\ & + \mu_{itc}^{D2}) + \sum_k (z_{itck}^{D1} + z_{itck}^{D2}) b_k R_i \Big). \end{aligned} \tag{4.19}$$

Here $z_{ik}^{A2} = \mu_i^{A2} x_{ik}^{cap}$ and $z_{itck}^{A3} = \mu_{itc}^{A2} x_{ik}^{cap}$ are linearized the same way as described in (4.8). It should be noted that the formulation can be extended further to include other strategic parts of the bid, as for example in [105].

The wind-integrated power systems are characterized by variability of wind power output. Therefore, we investigate different probabilities and sizes for wind power contingencies. Figure 4.3 shows the dispatch costs in the case of two-step demand response with 7 strategic generators (G1, G2, G3, G5, G6, G11 and G12). Both BNE and WNE costs are plotted. In the BNE case, when the actual output of the wind generating unit is more or

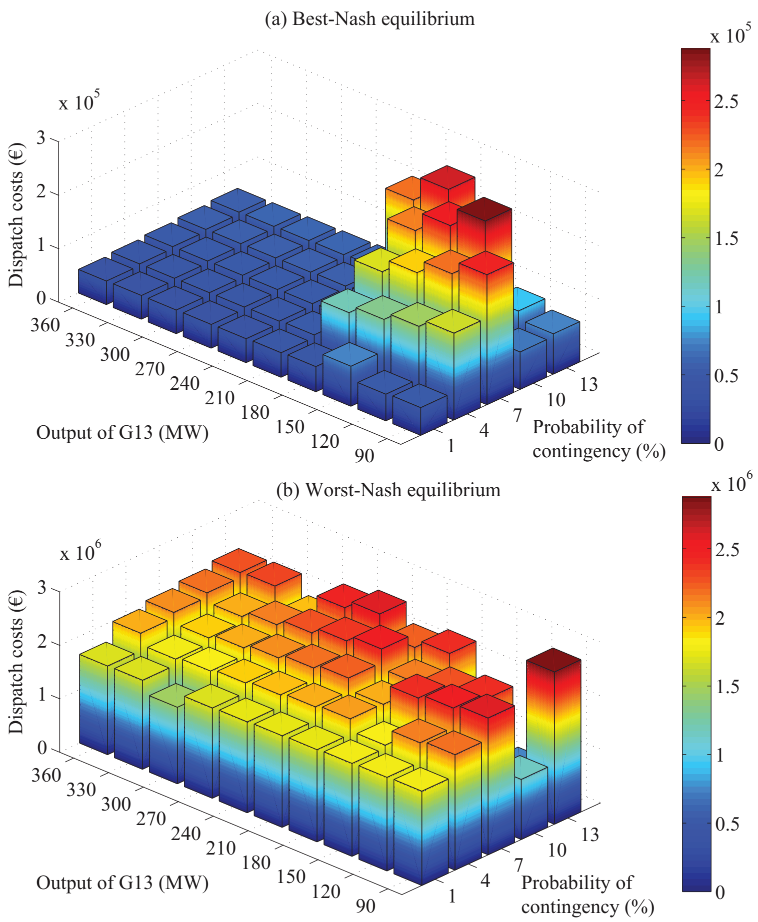

Figure 4.3: Dispatch costs for the cases with different output deviation predicted with different probabilities. (a) Best-Nash equilibrium case, (b) Worst-Nash equilibrium case.

around the predicted value we observe a linear increase in the dispatch costs with the increased probability of congestion. However, when the severity of contingency is high – the actual wind output is less than predicted value by 25% or more – we observe rapid increases in the dispatch costs caused by the gaming behavior of strategic generators. In the WNE case, the dispatch costs are high in all studied cases.

According to the obtained results we can observe that when the severity of contingency is high, which may depend on the system flexibility, the generators are prone to exercise market power by bidding strategically on their capacity and ramp rate levels. This strategic behavior affects considerably the dispatch costs and prices in the system, especially when the demand response is limited. This result supports the necessity of ramp-game modeling in the systems with intermittent supply.

The model is implemented on GAMS platform and solved by CPLEX solver. The GAMS code runs on 2.8 GHz Intel Processor with 2 cores and 8 GB RAM. As an example, the computation time for the BNE case of two-steps demand response and the wind generators output predicted to be 120 MW with probability of 1% is 38 minutes.

4.5 Conclusion

In this chapter we model the strategic behavior in wind-integrated systems, where the intermittency is high, so that the security-constrained short run economic dispatch is a welfare-maximizing way of operating the system. We assume a market operator who collects the bid information, including marginal costs, available capacity and ramp rates. Strategic generators, willing to maximize their profit, can bid lower than their true ramp rates and capacities. The outcome of the market is a Nash equilibrium.

Illustrative case study demonstrates that in presence of network constraints and a major contingency, the generators are prone to exercise market power and bid a lower ramp rate to the market. Further, using a 24-node case study we show that if the severity of contingency is high, strategic generators are prone to maximize their profits by bidding strategically to the market.

This study shows that contingencies, introduced by a strong intermittency, have a significant effect on the proneness of generators to bid strategically, which in turn affects the dispatch costs and, therefore, the system social welfare. Here, the ramping capabilities of the units are particularly important, as they define the speed, with which the system reacts to the contingencies.

Chapter 5

Market power in power systems with high share of wind power and hydropower

In this chapter we study the strategic behavior of hydropower producers as main providers of generation flexibility in wind-integrated systems. The chapter is based on publications [J3] and [J4]. In this chapter we will study the cases of a single dominant hydropower producer and of Nash equilibrium of multiple hydropower producers.

5.1 Introduction

As we have seen in Chapter 4, flexibility plays a crucial role in the systems with high penetration of wind power. Fast-ramping producers compensate the fluctuations of wind power and may have incentives to withhold this flexibility in order to obtain higher profits. Hydropower producers are often regarded as flexibility providers in the systems, where this type of power is available.

The annual hydropower share in electricity generation in Norway is 95-99%. It is also high in Brazil (80%), Iceland (88%), New Zealand (65%), Austria (70%), Canada (62%) and Sweden (42%) [106]. Even though hydropower producers have such an important market share, they have been traditionally regarded as price-takers, optimizing their production schedule based on the price expectation. However, the price-taking assumption for a hydropower producer does not always hold [3]. Reference [107] shows that in hydrothermal systems the strategic behavior can lead to significantly higher prices. In winter 2002-2003, extraordinary high prices were observed in Nordic market. A number of observers voiced the concern that the high prices were a result of strategic behavior by hydropower producers. The hydropower producers may have used an opportunity to spend more stored water in summer in order to create scarcity and, therefore, high prices, in winter [70].

Hydropower producers can use their unique characteristics (such as a capability to store energy, hydrological coupling between units and near-zero marginal cost) to behave strategically in the market [106], [108]. With an increasing penetration of wind and solar generation, hydropower producers can ramp fast to cover the generation fluctuations. Withholding such ramping capability adds another dimension to the strategic behavior of hydropower producers.

Modeling the hydro-dominated systems requires accounting for hydro-specific constraints. The few studies that have looked at market power assessment involving the hydropower producers, have significantly simplified the modeling. Usually only a single profit-maximizing hydropower producer is considered, and the optimal dispatch conditions are simplified as a residual demand curve [109], [110]. In reference [107] the authors consider the market power assessment in a hydrothermal power system, but employ a simplifying residual demand approach, expressing price as a function of price-makers' production, rather than as a result of the optimal dispatch. The authors in [28] include the equilibrium constraints, but their model neglects the uncertainty in reservoir inflows and contains nonlinearities.

In this chapter we model the cases of single dominant hydropower producer and multiple hydropower producers in power systems with high share of wind power. The respective MPEC and EPEC models are recast as MILP and solved using a Modified Benders Decomposition Algorithm (MBDA), discussed earlier in Section 2.4.3. We compare the numerical results with other types of Benders decomposition and monolith solution by CPLEX solver.

5.2 Modeling

In this section we discuss the models of a single profit-maximizing hydropower producer, formulated as an MPEC, and multiple producers, formulated as an EPEC.

5.2.1 Single profit-maximizing hydropower producer

The price-making hydropower producer is modeled in (5.1).

$$\underset{\hat{q}_{it},\hat{r}_i,\hat{c}_{it}}{\text{maximize}}\, \pi = \sum_{(i\in I^h),w} P_w\Big(\sum_t(\lambda_{ntw} - C_i^M)q_{itw} + m_{iTw}\Lambda_i^f\frac{1}{\Gamma_i}\Big) \tag{5.1a}$$

$$\text{subject to: } 0 \le \hat{q}_{it} \le Q_i,\ 0 \le \hat{r}_i \le R_i,\ 0 \le \hat{c}_{it} \le C_i, \forall(i\in I^h)t, \tag{5.1b}$$

where $\lambda_{ntw}, q_{itw}, s_{itw}, m_{itw} \in$

$$\arg\Big\{\underset{q_{itw},s_{itw},m_{itw}}{\text{minimize}} \sum_{i,t,w} P_w\hat{c}_{it}q_{itw} \tag{5.1c}$$

$$\text{subject to: } 0 \le q_{itw} \le \hat{q}_{it} \quad : \mu_{itw}^{A_1}, \mu_{itw}^{A_2},\ \forall i, \tag{5.1d}$$

$$\sum_i q_{itw} = \sum_n D_{ntw} \quad : \lambda_{tw}^B,\ \forall tw, \tag{5.1e}$$

$$\sum_n H_{ln}(\sum_{i:n} q_{itw} - D_{ntw}) \le F_l \ : \mu_{ltw}^C,\ \forall ltw, \tag{5.1f}$$

$$-\hat{r}_i \le q_{i(t-1)w} - q_{itw} \le \hat{r}_i : \mu_{itw}^{D_1}, \mu_{itw}^{D_2},\ \forall itw, \tag{5.1g}$$

$$0 \le s_{itw} \quad : \mu_{itw}^E,\ \forall(i\in I^h)tw, \tag{5.1h}$$

$$0 \le m_{itw} \le M_i \quad : \mu_{itw}^{F_1}, \mu_{itw}^{F_2},\ \forall(i\in I^h)tw, \tag{5.1i}$$

$$m_{itw} - m_{i(t-1)w} = M_i^0\mathbf{I}(t{=}1) + V_{itw} - \Gamma_i q_{itw} - s_{itw} + \sum_{i\in I^{up}}(\Gamma_i q_{itw} + s_{itw}) \ : \lambda_{itw}^G,\ \forall(i\in I^h)tw\Big\}. \tag{5.1j}$$

Here (5.1a) is the profit formulation, where λ_{ntw} is a locational marginal price (LMP) and it can be expressed as $\lambda_{ntw} = \lambda_{tw}^B - \sum_{l,n}\mu_{ltw}^C H_{ln}$. We model hydropower producer using the assumptions made in Section 3.3.2. The term $m_{iTw}\Lambda_i^f\frac{1}{\Gamma_i}$ describes the future value of water left in the reservoir by the end of the modeling horizon (at time step T). Strategic hourly bids $(\hat{q}_{it}, \hat{c}_{it})$, and ramp-rate bid $\hat{r}_i$ of the hydropower producer are modeled by the constraints (5.1b). The LMPs and the dispatch variables are the output of the lower-level economic dispatch problem (5.1c)-(5.1j). Expressions (5.1d) are the generation constraints. Expression (5.1e) represents the energy balance constraint, (5.1f) accounts for the network constraints and (5.1g) is setting the constraints on ramp rate. A system of connected reservoirs is considered for the hydropower producer. We model the constraints on spillage (5.1h), water level (5.1i) and hydrological balance condition (5.1j).

Since the lower-level problem (5.1c)-(5.1j) is linear, we can equivalently rewrite it as Karush-Kuhn-Tucker (KKT) conditions. We avoid the complementary slackness conditions by substituting them with the strong duality condition, as it was shown to be more convenient for linearization in Chapter 4. Accordingly, primal feasibility constraints (5.1d)-(5.1j), dual feasibility constraints, stationarity constraints and strong duality constraint form the optimality conditions of the lower-level problem. Now, we can reformulate the profit by deriving the LMP from the stationary conditions:

$$\pi = \sum_{(i \in I^h),w} P_w \Big(\sum_t (\lambda^B_{tw} - \sum_{l,n} \mu^C_{ltw} H_{ln} - C^M_i) q_{itw} + m_{iTw} \Lambda^f_i \frac{1}{\Gamma_i} \Big) =$$
$$\sum_{(i \in I^h),w} P_w \Big(\sum_t (\hat{c}_{it} - \mu^{A_1}_{itw} + \mu^{A_2}_{itw} + \mu^{D_1}_{itw} - \mu^{D_2}_{itw} - \mu^{D_1}_{i(t+1)w} + \mu^{D_2}_{i(t+1)w}$$
$$- \Gamma_i \lambda^G_{itw} + \Gamma_i \sum_{i \in I^{dn}} \lambda^G_{itw} - C^M_i) q_{itw} + m_{iTw} \Lambda^f_i \frac{1}{\Gamma_i} \Big). \tag{5.2}$$

We simplify expression (5.2) using the complementary slackness (CS) conditions. The term $(*) = (-\Gamma_i \lambda^G_{itw} + \Gamma_i \sum_{i \in I^{dn}} \lambda^G_{itw}) q_{itw}$ appeared from the hydrological balance constraint (5.1j) and it exists only for hydropower units. The final profit expression becomes:

$$\pi = \sum_{(i \in I^h),w} P_w \Big(\sum_t \Big(\mu^{A_2}_{itw} \hat{q}_{it} + (\mu^{D_1}_{itw} + \mu^{D_2}_{itw}) \hat{r}_i + \hat{c}_{it} q_{itw} - C^M_i q_{itw}$$
$$+ \mu^{F_2}_{it} M_i - \lambda^G_{itw} (M^0_i \mathbf{I}(t{=}1) + V_{it}) \Big) + m_{iTw} \Lambda^f_i \frac{1}{\Gamma_i} \Big). \tag{5.3}$$

The equivalent one-level program can be formulated by combining the upper-level constraints and the lower-level optimality conditions. The variables of one-level optimization include primal variables and dual variables of the lower-level problem, and variables of the upper-level problem: $\Omega^{\text{NLP}} = \{q_{itw}$, s_{itw}, m_{itw}, $\mu^{A_1}_{itw}$, $\mu^{A_2}_{itw}$, $\mu^{D_1}_{itw}$, $\mu^{D_2}_{itw}$, μ^{E}_{itw}, $\mu^{F_1}_{itw}$, $\mu^{F_2}_{itw}$, μ^{C}_{ltw}, λ^B_{tw}, λ^G_{itw}, $\hat{q}_{it}$, $\hat{r}_i$, $\hat{c}_{it}\}$. The final profit expression (5.3) and strong duality condition contain bilinear terms $\mu^{A_2}_{itw} \hat{q}_{it}$, $\mu^{D_1}_{itw} \hat{r}_i$, $\mu^{D_2}_{itw} \hat{r}_i$ and $\hat{c}_{it} q_{itw}$. For the non-strategic units we assume: $\hat{q}_{it} = Q_i$, $\hat{r}_i = R_i$, $\hat{c}_{it} = C^M_i$, $\forall i \in (I \backslash I^h)$. For the strategic units these terms need to be linearized.

We approximate the generation capacity Q_i by a pre-defined number of discrete capacities $\hat{Q}_{ik}$. Therefore: $\mu^{A_2}_{itw} \hat{q}_{it} = \mu^{A_2}_{itw} \sum_k x^q_{itk} \hat{Q}_{ik} = \sum_k z^{A_2}_{itwk}$, where $\sum_k x^q_{itk} = 1$. Variable $z^{A_2}_{itwk}$ can be linearized as follows [53]:

$$-\bar{K}^q(1-x^q_{itk}) \leq z^{A_2}_{itwk} - \hat{Q}_{ik}\mu^{A_2}_{itw} \leq \bar{K}^q(1-x^q_{itk}) : (\overline{\alpha}^{A_2}_{itwk}, \underline{\alpha}^{A_2}_{itwk}), \ \forall(i{\in}I^h)twk, \tag{5.4a}$$

$$-\bar{K}^q x^q_{itk} {\leq} z^{A_2}_{itwk} {\leq} \bar{K}^q x^q_{itk} : (\overline{\beta}^{A_2}_{itwk}, \underline{\beta}^{A_2}_{itwk}), \forall(i{\in}I^h)twk. \tag{5.4b}$$

In the constraints above $\bar{K}^q$ is a suitably large constant, not too high to create computational instabilities, and not too low to put extra bounds on the variables [111]. Introducing $\bar{K}^r$, we rewrite (5.1d) and (5.1g) as follows.

$$q_{itw} \leq \hat{Q}_{ik} + \bar{K}^q(1-x^q_{itk}) : (\nu^{A_2}_{itwk}), \forall(i{\in}I^h)twk, \tag{5.5a}$$

$$q_{i(t-1)w} - q_{itw} \leq \hat{R}_{ik} + \bar{K}^r(1-x^r_{ik}) \ : (\nu^{D_1}_{itwk}), \forall(i{\in}I^h)twk, \tag{5.5b}$$

$$q_{itw} - q_{i(t-1)w} \leq \hat{R}_{ik} + \bar{K}^r(1-x^r_{ik}) : (\nu^{D_2}_{itwk}), \forall(i{\in}I^h)twk. \tag{5.5c}$$

We repeat the linearization steps for other bilinear terms, introducing $z^{D_1}_{itwk} = \mu^{D_1}_{itw} \sum_k x^r_{ik} \hat{R}_{ik}$, $z^{D_2}_{itwk} = \mu^{D_2}_{itw} \sum_k x^r_{ik} \hat{R}_{ik}$, $z^{obj}_{itwk} = q_{itw} \sum_k x^c_{itk} \hat{C}_{ik}$. We form a new set of variables Ω^{MILP} by adding x^q_{itk}, x^r_{ik}, x^c_{itk}, $z^{A_2}_{itwk}$, $z^{D_1}_{itwk}$, $z^{D_2}_{itwk}$, z^{obj}_{itwk} to Ω^{NLP}. The proposed stochastic MILP model is set out in (5.6).

$$\underset{\Omega^{\text{MILP}}}{\text{maximize}} \ \pi = \sum_{(i\in I^h),w} P_w \Big(\sum_t (z^{A_2}_{itwk} + z^{D_1}_{itwk} + z^{D_2}_{itwk} + z^{obj}_{itwk} - C^M_i q_{itw} + \mu^{F_2}_{itw} M_i - \lambda^G_{itw}(M^0_i \mathbf{I}(t{=}1) + V_{it})) + m_{iTw} \Lambda^f_i \frac{1}{\Gamma_i} \Big) \tag{5.6a}$$

subject to: (5.1e), (5.1f), (5.1h)-(5.1j), (5.4), (5.5),

$$\text{(5.1d), (5.1g), stationary conditions } \forall i \in I \backslash I^h, \tag{5.6b}$$

$$\mu^{A_1}_{itw}, \mu^{A_2}_{itw}, \mu^{D_1}_{itw}, \mu^{D_2}_{itw}, \mu^{E}_{itw}, \mu^{F_1}_{itw}, \mu^{F_2}_{itw}, \mu^{C}_{ltw} \geq 0, \tag{5.6c}$$

Linearization of $z^{D_1}_{itwk}$, $z^{D_2}_{itwk}$, z^{obj}_{itwk} as in (5.4), $\forall i{\in}I^h$,

Linearization of strong duality implication as in (5.4).

5.2.2 Multiple profit-maximizing hydropower producers

Section 5.2.1 described the formulation of a stochastic MILP model for a single profit-maximizing hydropower producer. In this section we discuss, how we can formulate the game of multiple strategic hydropower producers.

In Section 2.2.3 we discussed that Nash equilibrium can be formulated as an optimization problem:

$$\underset{\Omega^{\text{MILP}}}{\text{minimize}}\ \Delta\pi = \sum_{i\in I^h} \epsilon_i \tag{5.7a}$$

$$\text{subject to:}\quad \pi_i(s_i^*, s_{-i}^*) + \epsilon_i \geq \pi_i(s_i, s_{-i}^*),\ \ \forall i{\in}I^h, \tag{5.7b}$$

$$\epsilon_i \geq 0,\ \ \forall i{\in}I^h \tag{5.7c}$$

At a Nash equilibrium point the deviation ϵ_i is zero for all players. Under uncertainty the profit π_i can be different in different scenarios. There are several ways of how to extend the definition of Nash equilibrium in the case of uncertainty.

Bayesian Nash equilibrium

Each player is assumed to have a subjective uncertainty probability distribution function [46]. This assumption is applicable to the most of the uncertainties observed in the power system, such as wind, reservoir inflows, demand uncertainty. When introducing scenarios w describing the uncertainty, the Nash equilibrium (5.7b) becomes:

$$E_w[\pi_{iw}(s_i^*, s_{-i}^*)] + \epsilon_i \geq E_w[\pi_{iw}(s_i, s_{-i}^*)],\ \ \forall i{\in}I^h. \tag{5.8}$$

Robust Nash equilibrium

Finding robust Nash equilibrium does not require the prior knowledge of probability distribution function for the incomplete information [47]. This is very useful, when certain scenarios have no historic data or when probabilities of the scenarios are difficult to compute. Robust Nash equilibrium uses the worst-case approach, where (5.7b) is reformulated as follows:

$$\min_w[\pi_{iw}(s_i^*, s_{-i}^*)] + \epsilon_i \geq \min_w[\pi_{iw}(s_i, s_{-i}^*)],\ \ \forall i{\in}I^h. \tag{5.9}$$

We introduce π_w^{min} to formulte a mixed-integer reformulation of (5.9):

$$\pi_i^{min}(s_i^*, s_{-i}^*) + \epsilon_i \geq \pi_i^{min}(s_i, s_{-i}^*),\ \ \forall i \in I^h \tag{5.10}$$

Introducing binary variables x^{rob}_{iw} we ensure that π^{min}_i takes the smallest value in scenarios $\min_w[\pi_{iw}]$:

$$\pi^{min}_i \le \pi_{iw}, \forall (i \in I^h) ws \tag{5.11a}$$

$$\pi^{min}_i \ge \pi_{iw} - \bar{K}(1 - x^{rob}_{iw}), \forall (i \in I^h) ws, \tag{5.11b}$$

$$\sum_w x^{rob}_{iw} = 1. \tag{5.11c}$$

The profit for the hydropower producer can be derived in a similar way as in (5.2). Extending the formulation by considering the profit in alternative strategies $\pi_{iw}(s_i, s^*_{-i})$ we can write the whole optimization problem as a stochastic MILP problem (5.12).

$$\underset{\Omega^{\text{MILP}}}{\text{minimize}} \quad \Delta\pi = \sum_{i \in I^h} \epsilon_i \tag{5.12a}$$

subject to: NE definition (5.8), or (5.10)-(5.11), $\epsilon_i \ge 0$,

For s^*_i and (s_i, s^*_{-i}), $\forall i \in I^h$:

$$\{\pi_{iw} = \sum_t (\sum_k (z^{A_2}_{itwk} + z^{D_1}_{itwk} + z^{D_2}_{itwk} + P_w z^{obj}_{itwk}) - C^M_i q_{itw} + \mu^{F_2}_{itw} M_i$$

$$-\lambda^G_{itw}(M^0_i \mathbf{I}(t=1) + V_{it})) + m_{iTw}\Lambda^f_i \frac{1}{\Gamma_i}, \forall (i \in I^h) w, \tag{5.12b}$$

(5.1e), (5.1f), (5.1h)-(5.1j), (5.4), (5.5),

(5.1d), (5.1g), stationary conditions $\forall i \in I \backslash I^h$,

$\mu^{A_1}_{itw}, \mu^{A_2}_{itw}, \mu^{C}_{ltw}, \mu^{D_1}_{itw}, \mu^{D_2}_{itw}, \mu^{E}_{itw}, \mu^{F_1}_{itw}, \mu^{F_2}_{itw} \ge 0$,

Linearization of $z^{D_1}_{itwk}$, $z^{D_2}_{itwk}$, z^{obj}_{itwk} as in (5.4), $\forall i \in I^h$,

Linearization of strong duality as in (5.4)$\}$.

5.3 Solution Approach

In optimization problem (5.6), if we fix binary variables x^q_{itk}, x^r_{ik} and x^c_{itk}, the problem separates in a series of linear programs which can be solved in parallel. For the stochastic MILP problem in (5.12) fixing the binary variables results in a linear program, partly decomposable by blocks in case of the robust Nash equilibrium.

Benders decomposition is commonly used for mixed-integer program, as it allows dealing with complicating variables. In this section we propose

Modified Benders Decomposition Algorithm (MBDA). In Section 5.5 we compare it numerically with alternative Benders approaches (as described in Section 2.4) and with solution by CPLEX solver. In this section we discuss subproblem and master problem based on the model of a single profit-maximizing generator. The corresponding derivations for the case of multiple strategic hydropower producers are provided in [J4].

Modified subproblem corresponding to initial MILP (5.6) can be formulated as follows:

$$\underset{\Omega^{\mathrm{SP}}}{\text{maximize}}\ \pi^{dual} = \sum_w P_w \sum_t \Big(\phi^B_{tw} \sum_n D_{ntw} - \sum_l \nu^C_{ltw}(F_l + \sum_n H_{ln} D_{ntw}) - \sum_{i\in I\setminus I^h} \phi^{stq}_{itw} C_i - \sum_{i\in I^h} \phi^{stq}_{itw} \sum_k \hat{C}_{ik} \check{x}^c_{ik} - \sum_{i\in I\setminus I^h} \phi^{A_2}_{itw} Q_i - \sum_{i\in I^h} \phi^{A_2}_{itw} \sum_k \hat{Q}_{ik} - \sum_{i\in I\setminus I^h} (\phi^{D_1}_{itw} + \phi^{D_2}_{itw}) R_i - \sum_{i\in I^h} (\phi^{D_1}_{itw} + \phi^{D_2}_{itw}) \sum_k \hat{R}_{ik} - \sum_{i\in I^h} \phi^{F_2}_{itw} M_i + \sum_{i\in I^h} \phi^G_{itw}(M^0_i \mathbf{I}(t{=}1) + V_{itw})\Big)\Big), \tag{5.13a}$$

$$\text{subject to: } \sum_{(i\in I^h),t,w,k}\Big(((1-\check{x}^q_{itk})(\nu^{A_2}_{itwk} + \overline{\alpha}^{A_2}_{itwk} + \underline{\alpha}^{A_2}_{itwk}) + \check{x}^q_{itk}(\overline{\beta}^{A_2}_{itwk} + \underline{\beta}^{A_2}_{itwk})) + ((1-\check{x}^r_{ik})(\nu^{D_1}_{itwk} + \overline{\alpha}^{D_1}_{itwk} + \underline{\alpha}^{D_1}_{itwk}) + \check{x}^r_{ik}(\overline{\beta}^{D_1}_{itwk} + \underline{\beta}^{D_1}_{itwk})) + ((1-\check{x}^r_{ik})(\nu^{D_2}_{itwk} + \overline{\alpha}^{D_2}_{itwk} + \underline{\alpha}^{D_2}_{itwk}) + \check{x}^r_{ik}(\overline{\beta}^{D_2}_{itwk} + \underline{\beta}^{D_2}_{itwk})) + ((1-\check{x}^c_{itk})(\overline{\alpha}^{obj}_{itwk} + \underline{\alpha}^{obj}_{itwk}) + \check{x}^c_{itk}(\overline{\beta}^{obj}_{itwk} + \underline{\beta}^{obj}_{itwk}))\Big) = 0, \tag{5.13b}$$

$$\Omega^{\mathrm{SP}} \in X. \tag{5.13c}$$

The feasible region X includes stationary conditions for all variables of (5.6) with fixed binary variables and dual feasibility constraints. We observe that both feasible region and the objective function of the subproblem are free from the disjunctive parameters.

Using the special form of the disjunctive constraints, we can formulate the modified master problem. We use an observation that disjunctive constraints require that solutions to a mathematical program satisfy a subset of given constraints. Therefore, all constraints can be distributed in two sets: relaxed and enforced constraints, depending on the value of the binary variables x^q_{itk}, x^r_{ik}, x^c_{itk}. We propose the following formulation, where disjunctive parameters are removed.

$$\underset{x^q_{itk},x^r_{ik},x^c_{itk},\theta_j}{\text{minimize}} \quad K_0\theta_0 + \sum_j K_j\theta_j \tag{5.14a}$$

$$\text{subject to:} \quad \sum_{i,k\in\Omega^p_j}\Big(\sum_t((1-x^q_{itk})+(1-x^c_{itk}))+(1-x^r_{ik})\Big)$$

$$\leq |\Omega^p_j| - 1 + \sum_{j'\in J'}\theta_{j'}\mathbf{I}(K_j \leq K_{j'}), \forall j, \tag{5.14b}$$

$$\sum_k x^q_{itk}{=}1, \sum_k x^r_{ik}{=}1, \sum_k x^c_{itk}{=}1, \forall(i \in I^h)t, \tag{5.14c}$$

$$\theta_0 + \sum_j \theta_j = 1, \quad \{x^q_{itk}, x^r_{ik}, x^c_{itk}, \theta_j\} \in \{0,1\}. \tag{5.14d}$$

In optimization problem (5.14), parameter $K_j{=}\sum_w P_w\breve{\pi}^{dual}_{wj}$ where $\breve{\pi}^{dual}_{wj}$ is a calculated objective function of (5.13). Also parameter $K_0{=}-\infty$ is the lower bound of (5.14). θ_j is an auxiliary binary variable, created at every iteration. J' is the index set of previous iterations and j is the index of current iteration. If we define $\Omega = \{\overline{\alpha}^{A_2}_{itwk}, \underline{\alpha}^{A_2}_{itwk}, \overline{\beta}^{A_2}_{itwk}, \underline{\beta}^{A_2}_{itwk}, \overline{\alpha}^{D_1}_{itwk}, \underline{\alpha}^{D_1}_{itwk}, \overline{\beta}^{D_1}_{itwk}, \underline{\beta}^{D_1}_{itwk}, \overline{\alpha}^{D_2}_{itwk}, \underline{\alpha}^{D_2}_{itwk}, \overline{\beta}^{D_2}_{itwk}, \underline{\beta}^{D_2}_{itwk}, \overline{\alpha}^{obj}_{itwk}, \underline{\alpha}^{obj}_{itwk}, \overline{\beta}^{obj}_{itwk}, \underline{\beta}^{obj}_{itwk}, \nu^{A_2}_{itwk}, \nu^{D_1}_{itwk}, \nu^{D_2}_{itwk}\}$, then Ω^p_j is the index set of strictly positive variables of Ω in iteration j. $|\Omega^p_j|$ is the cardinality of this set.

5.4 Illustrative case studies

The models formulated in Section 5.2 allow us to model the strategic behavior of hydropower producers in the systems with high levels of wind power uncertainty. Publication [J3] presents several illustrative case studies, highlighting the possibilities for the exercise of market power by hydropower producer. In [J4] we explore possible Nash equilibria, which may occur under uncertainty. Below we present some of our findings.

5.4.1 The impact of forecast future water price on price bidding of hydropower producer

The opportunity cost of the hydropower producer depends on the future expected prices. For hydropower producer the cost today is the benefit

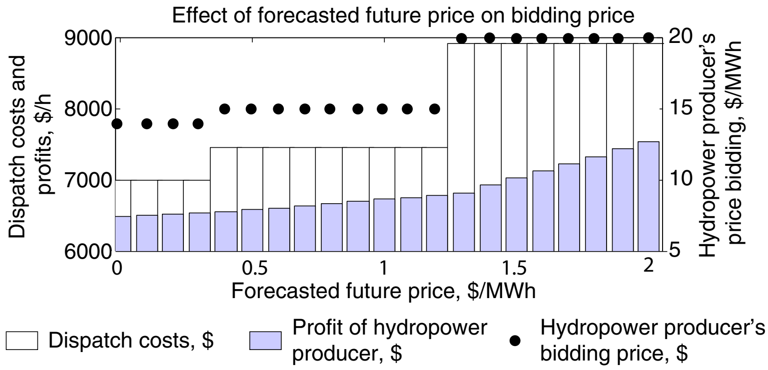

Figure 5.1: Study of future price effect on dispatch cost and profits.

obtained by using the water tomorrow [106]. This benefit is calculated using the best available optimization techniques and historic data, and determines how much water should be saved today in order to be used tomorrow. The price-quantity bids of a hydropower producer are based on the result of these calculations.

The stochastic MILP model (5.6) is used to study the impact of forecast future price on the price bidding of a hydropower producer. We should note that forecast is very much dependent on the availability of data on wind power. In wind-integrated power systems this forecast may be very rough. In Figure 5.1, the forecast future price Λ_i^f in (5.6) is varied between \$0/MWh and \$2/MWh. We observe that the price bid of the strategic hydropower producer depends on the forecast future price.

The price bid of the strategic hydropower producer changes between three values of \$14, \$15 and \$20/MWh. This means that in proximity of these steps the sensitivity of price bid to the forecast future price is very high. The same holds for the dispatch cost. For example, the strategic hydropower producer bids \$15 and \$20/MWh for forecast future prices \$1.2/MWh and \$1.3/MWh, respectively. This results in \$1450 (+19.5%) increase in the dispatch cost for just \$0.1/MWh (+7.7%) increase in the forecast future price. This shows the sensitivity of dispatch cost with respect to the forecast future price. Accordingly, the accuracy of forecast future price can have a severe impact on price bidding of a strategic hydropower producer.

5.4.2 Strategic behavior in price, quantity and ramp-rate bids

The optimization model (5.6) can be used to derive the price, quantity and ramp-rate bids of the hydropower producer (Figure 5.2-(a)). The strategic bids increase the profit of hydropower producer above its competitive level. In Figure 5.2-(b), 5.2-(c), and 5.2-(d) the profit increase with respect to wind generation level and its standard deviation is shown.

We observe that all three types of bid have an important effect on the dispatch cost. Strategic behavior in quantity and price (Figure 5.2-(b) and 5.2-(c)) produce similar results. This is because both of them can be used by the strategic hydropower producer to drive the price to the highest possible value. We also see that strategic behavior in ramp-rate bidding has an important

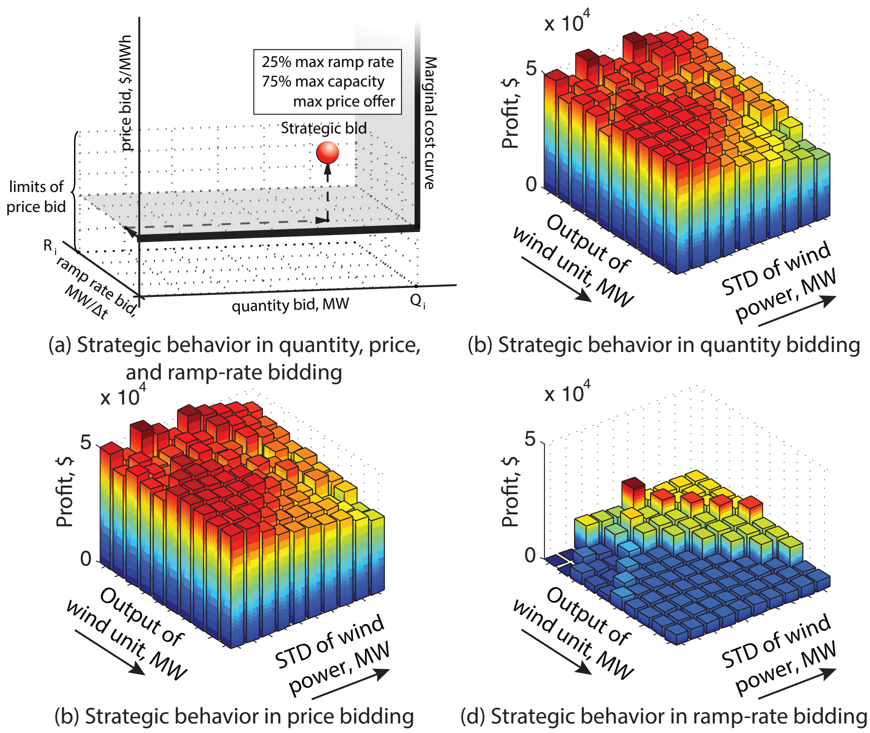

Figure 5.2: Increase in hydropower producer's profit, resulting from strategic bidding on quantity, price and ramp rate. STD: standard deviation.

effect even when the other two types of strategic bids are not employed (Figure 5.2-(d)). This happens in the systems with limited flexibility, where there are not many fast-ramping generators. In our simulations, the strategic hydropower producer is the main provider of system flexibility. Therefore its strategic behavior in ramp-rate bidding leads to the increase in its profit and market prices.

5.4.3 Comparison of Nash equilibria

Since a multitude of solutions is difficult in interpretation, the policy makers and market participants would often prefer to have a single Nash equilibrium reference that can characterize the market situation. This is sometimes implemented with a concept of the best/worst Nash equilibrium [45]. We implement this by extending the objective function (5.12a) by minimization/maximization of the total social costs in the system correspondingly. In this section we analyze the strategic behavior, as different concepts of Nash equilibrium introduced in Section 5.2 are applied.

We compare the results from the best and worst Nash equilibria in Table 5.1: we provide the bids offered by the strategic hydropower producers to the market operator (bold values show withholding), profits, and dispatch cost corresponding to the different Nash equilibria. The following observations can be made:

- **Bids:** Withholding occurs in both types of Nash equilibria. There is more withholding if we consider the worst Nash equilibria, resulting in higher profits for the strategic generators. Worst Nash equilibria (high social costs) – is usually the result from high market prices.

- **Profits of strategic generators:** The profits in Bayesian Nash equilibrium are higher, than in the robust Nash equilibrium. In robust Nash equilibrium strategic bid is optimal for the worst-case scenario, while for Bayesian Nash eqilibrium all scenarios are taken into account.

- **Social costs span:** The difference between the worst and the best Nash equilibria is the largest for the robust Nash equilibria. For robust Nash equilibrium we only consider the worst-case scenario, therefore we do not consider if the chosen strategy holds as a Nash equilibrium for another scenario. This provides more flexibility for the strategic generator when choosing its strategy.

Table 5.1: Comparison of Bayesian and robust best and worst Nash equilibria. DC: dispatch cost.

		Bayesian		Robust	
		best	worst	best	worst
Capacity bid, $\hat{q}$, MW	G3	100	100	100	**75**
	G4	**25**	**50**	**75**	100
Ramp-rate bid, $\hat{r}$, $\frac{MW}{\Delta t}$	G3	10	10	10	**2.5**
	G4	10	**7.5**	10	**5**
Price bid, $\hat{c}$, $\frac{\$}{MWh}$	G3	**20**	**20**	**15**	**20**
	G4	20	**35**	20	**35**
Profit, $\sum_w \pi_{iw}$, $	G3	4637.5	5400	2845	4500
	G4	287.5	287.5	100	250
DC, $		4950	5250	3890	5700

Different types of Nash equilibrium can be used depending on the modeling goals. We can interpret from the results that Bayesian Nash equilibrium is more consistent with realistic behavior, assuming that the probability distributions of the uncertain parameters are known. Robust Nash equilibrium can be applied, when the probability distribution functions of the uncertain parameters are impossible to obtain (e.g., if the event does not have a historical data).

5.5 Numerical results

The performance is evaluated in [J3] and [J4] using the 4-, 24-, 118- and 300-node case studies, obtained from MATPOWER [112]. Simulations for [J3] are performed on a computer with two 2.80 GHz CPU and 8 GB of RAM, while for [J4] we use a computer with 18 cores with hyper-threading Intel Xeon E5-2699 CPU and 128 GB of RAM. In order to improve the performance, the parallelized MBDA is implemented in GAMS using the grid solve and Gather-Update-Scatter-Solve (GUSS) facilities. A detailed explanation of these facilities is provided in [113].

We assume 4 time periods and strategic hydropower producer has 3 actions for each type of the bid (19683 possible combinations for bidding). Additionally, uncertainty in each time period is represented by 20 stochastic scenarios. The scenarios for our simulations have been generated from the real data using the moment-matching technique [81], obtained from Nord

Pool [114] and processed to fit the case studies. The numerical results in [J3] and [J4] prove the computational efficiency of the proposed MBDA approach. We conclude that disjunctive parameter, which is present in primal and dual Benders approaches has an important numerical effect. Primal and dual Benders do not converge after 10 hours of simulation.

We also compare MBDA to the state-of-the-art MILP solver – CPLEX. While CPLEX is better for smaller case studies it considerably underperforms on larger case studies, e.g. 118-node IEEE case study. The reason is in the solution procedure, having large memory requirements. Full evaluation of the numerical results and time quotes are provided in [J3] and [J4].

5.6 Conclusion

One of the reasons, why exercise of market power can be present in wind-integrated power systems, is due to strong fluctuations in supply. Such fluctuations need to be balanced by fast-ramping generators. The role of such flexible generators (especially in the Nordic region) is on hydropower producers, able to ramp up and down quickly to cover the wind power intermittency. It is commonly assumed that hydropower producers are price-takers, deciding their price bids based on price forecasts and future value of water in the reservoirs. However, in this chapter we find out that the crucial role of hydropower producers implies an advantageous position, which can be exploited in order to exercise market power.

In this chapter we derive two stochastic bilevel programs for strategic bidding of hydropower producers. The upper level is one or multiple strategic hydropower producers, bidding their price, quantity and ramp rate to the lower-level system operator. Using a disjunction-based linearization technique the stochastic bilevel program is reformulated as a stochastic MILP with disjunctive constraints. To solve the reformulated stochastic MILP model, the modified Benders decomposition algorithm is proposed. The proposed solution algorithm does not require the optimal tuning of disjunctive parameters and it can be parallelized. The computational efficiency of the parallelized MBDA is demonstrated using the 118-node and 300-node case studies. We have also compared our parallelized MBDA with the state-of-the-art CPLEX solver.

Through an illustrative example system and the developed stochastic MILP model, we identify possible strategies specific to hydropower producers for maximizing the profit. We also study the outcome of Nash equilibrium under uncertainty by applying Bayesian and robust Nash equilibria approaches.

Chapter 6

Lessons for market design in wind-integrated power systems

In previous chapters we have seen that exercise of market power on ramp rate is an important issue in wind-integrated power systems. In this chapter we study how market design can create incentives for generators to behave strategically or competitively. This chapter is based on publication [J2].

6.1 Introduction

With a high wind power share, when generation suddenly departs from the dispatched level it is very important that there are enough of fast-ramping generators, able to sustain the energy balance [115]. We have seen in the previous chapters that a lack of such generators, or their strategic behavior may result in price spikes. The higher the ramping gradient that a power plant can cover, the fewer plants are needed to meet a given net load ramp, thus leading to less minimum generation per ramping [116]. There is a variety of market designs created in the deregulated framework taking into account the limitations of the units: markets in PJM Interconnection, New York Independent System Operator (NYISO) and New England electricity markets involve multi-dimensional auctions, so that participants specify technical constraints during the dispatch.

There is a thorough analysis in [117], showing that preventing gaming on ramp rates is a difficult task and there are no commonly adopted instruments that guarantee to prevent gaming behavior. One of the techniques to avoid the manipulation is to allow ramp-rate constraints to change only once a month, or some other suitable time interval, as adopted, for example, in California [118]. The "NYISO Market Participants User's Guide" states that regulation movement (MW/6 sec) response rate can be updated in three business days [119]. While this has a very little theoretical foundation, the purpose is to prevent generators from responding to the market conditions with false ramp rates. The expected effect would be that, exposed to the uncertainty and high competition, generators would bid the true ramp rate constraints. However, such a mitigating strategy allows a company to specify a deceptive constraint for the whole period in which the ramp constraint can be changed: a regulatory agency often cannot certify the constraint as generators are usually allowed to bid to several markets and may divide their ramping capability between the markets [117]. A different strategy is adopted for example in Nordic electricity market, where generators are free choosing the ramp rate between their flexible orders. To the best of our knowledge there is no research evaluating the efficiency of separating the market stages of production and ramp bids with respect to market power considerations.

In this paper we study the practical implications of separating the stages of a strategic decision as a technique to mitigate the exercise of market power. We model a day-ahead market and allow generators to specify the ramp-rate level as a part of their offers. We use two types of models: a single-stage equilibrium model, corresponding to taking the strategic decision regarding a ramp-rate level and produced quantity simultaneously; and a two-stage model, in which generators choose their ramp rates in the first stage and compete in quantities in the second stage. A single-level setup can be reflected by a complementarity problem formulation (linear – LCP, or mixed – MCP) [120, 121], while a bilevel setup takes a form of an equilibrium problem with equilibrium constraints (EPEC) [103].

6.2 Methodology

Equilibrium models are often employed to represent the nature of market interactions and competition between the strategic generators [122]. In this section we explore the impact of flexibility of power generation on market outcomes. In particular, we compare two different market setups represented by two models: a model that considers the case when ramp-rate and production decisions are taken simultaneously; and a model in which ramp rate is decided first, and then production decisions are taken. We employ a conjectured-price response parameter, which is immediately obtained from the conjectural variations model, capturing various degrees of strategic behavior in the spot market. Conjectural variations [21, 31] reflect the firm's conjecture about other firms' reaction to a change in its production. This representation allows us to express the special cases of oligopolistic behavior ranging from perfect competition to a Cournot oligopoly [32].

6.2.1 Simultaneous ramp and quantity bidding: the one-stage model

In the single-stage model, every generating unit i faces a profit-maximization problem: in time period t it chooses the level of production q_{it}. The ramp-rate level r_i is chosen simultaneously with the production level in the first time period and stays constant throughout the modeling horizon. The objective of the generator is to maximize the revenues in the production stage minus the costs. Parameter c represents the symmetric marginal production cost.

In order to derive the analytic results in this section, let us assume that there are two consecutive time periods: t_1 and t_2. The optimization problem of a company i can be formulated as follows:

$$\forall i: \begin{cases} \underset{q_i, r_i}{\text{maximize}}\ (p_{t_1}(q_{it_1}, q_{-it_1}) - c)q_{it_1} + (p_{t_2}(q_{it_2}, q_{-it_2}) - c)q_{it_2} & \text{(6.1a)} \\ \text{subject to:}\quad \hat{Q} \geq q_{it} \geq 0 \quad : \overline{\mu}_{it}, \underline{\mu}_{it} \quad \forall t & \text{(6.1b)} \\ \qquad\qquad \hat{R} \geq r_i \geq 0 \quad : \overline{\lambda}_i, \underline{\lambda}_i & \text{(6.1c)} \\ \qquad\qquad r_i \geq q_{it_2} - q_{it_1} \quad : \overline{\gamma}_i & \text{(6.1d)} \\ \qquad\qquad q_{it_1} - q_{it_2} \geq r_i \quad : \underline{\gamma}_i. & \text{(6.1e)} \end{cases}$$

In the optimization problem above $\hat{Q}$ is the symmetric installed capacity of the generating unit i and $\hat{R}$ is the maximum technically possible ramp rate. Constraints (6.1d)-(6.1e) show that the change of production between two time periods is limited by the ramp rate. These constraints are specified for the time steps following the first one ($|t| > 1$). The variables on the right are the dual variables associated with constraints (6.1b)-(6.1e).

Prices $p_t(q_i, q_{-i})$ come from the market equilibrium (ME) conditions that link optimization problems of all producers: energy balances for both load periods and the affine definition of the elastic demand d_t, where α is the elasticity parameter.

$$\text{ME}: \;\; d_t = \sum_i q_{it}, \quad d_t = D_t^0 - \alpha p_t(q_{it}, q_{-it}) \;\; \forall t. \tag{6.2}$$

The ME conditions mimic the economic dispatch problem of the system operator. These conditions define the price $p_t(q_{it}, q_{-it})$, dependent on the production q_{it} of the generating companies. The demand elasticity α reflects the price responsiveness of the consumers in the market. This formulation aims at reflecting the market setups used in most of the countries with liberalized electricity markets [51]. Choosing arbitrary the case of increasing demand allows us to remove the constraint (6.1e) from our model as it will be inactive. Since only two time periods are considered, the declared up-rate will be equal to the increased production without loss of generality. The production in the second time period can therefore be expressed as:

$$q_{it_2} = q_{it_1} + r_i. \tag{6.3}$$

Since electricity is a perfectly substitutable good, we can capture a range of behavioral outcomes by conjectural variations in the short-run market formulation, as first introduced in [123]. Conjectural variation is the belief that one firm has about the way its competitors may react if it varies its output or price. This approach attempts to reproduce the dynamic pricing in a "reduced-form" static competition, therefore we are specifying exogenously the reaction of the firms to their rivals. This approach is criticized for the the exogenous nature of the conjectural variations [33], but it allows us to capture a range of behavioral outcomes – from competitive to cooperative and has one parameter which has a simple economic interpretation. We define a conjectured price response parameter θ_i as a company i's belief concerning its influence on price $p_t(q_{it}, q_{-it})$ in a short-term spot market. We express the conjectured price response as:

$$\theta_i = -\frac{dp_t(q_{it}, q_{-it})}{dq_{it}}.$$

This parameter only captures the effect of change in q_{it} and not r_i. By considering only conjectured price response on production we express the type of competition in either of the ramping and the energy markets.

Different levels of the conjectured price response parameter correspond to different market structures. We assume θ_i varying from 0, which corresponds to perfect competition in the market to $(1/\alpha)$, which represents the Cournot oligopoly [124]. By choosing the conjectured price response parameter to vary in this range we study the behavioral outcomes most common in electricity markets. Further, in this section we assume symmetric conjectured price responses $\theta = \theta_1 = \theta_2$.

The Lagrangian function of problem (6.1) for producer i is:

$$\mathcal{L}_i = (p_{t_1}(q_{it_1}, q_{-it_1}) - c)q_{it_1} + (p_{t_2}(q_{it_2}, q_{-it_2}) - c)q_{it_2} + \sum_t \big(\overline{\mu}_{it}(\hat{Q} - q_{it}) + \underline{\mu}_{it} q_{it}\big) + +\overline{\lambda}_i(\hat{R} - r_i) + \underline{\lambda}_i r_i) + \overline{\gamma}_i(r_i - q_{it_2} + q_{it_1}).$$

Although parameter θ is implicit in formulation (6.1) it becomes explicit when we solve the problem by taking the KKT conditions, which are equivalent to the original optimization problem.

We substitute the first derivative of price by θ and use the dual multipliers introduced previously. Note that $\perp$ denotes the complementarity between the constraint and its respective dual variable. The KKT conditions of the problem (6.1) are as follows:

$$\forall i: \begin{cases} \frac{\partial \mathcal{L}_i}{\partial q_{it_1}} = p_{t_1} - \theta q_{it_1} - c + \underline{\mu}_{it_1} - \overline{\mu}_{it_1} + \overline{\gamma}_i = 0 & (6.4a) \\ \frac{\partial \mathcal{L}_i}{\partial q_{it_2}} = p_{t_2} - \theta q_{it_2} - c + \underline{\mu}_{it_2} - \overline{\mu}_{it_2} - \overline{\gamma}_i = 0 & (6.4b) \\ \frac{\partial \mathcal{L}_i}{\partial r_i} = \underline{\lambda}_i - \overline{\lambda}_i + \overline{\gamma}_i = 0 & (6.4c) \\ 0 \leq \underline{\mu}_{it} \perp q_{it} \geq 0 \quad \forall t & (6.4d) \\ 0 \leq \overline{\mu}_{it} \perp \hat{Q} - q_{it} \geq 0 \quad \forall t & (6.4e) \\ 0 \leq \underline{\lambda}_i \perp r_i \geq 0 & (6.4f) \\ 0 \leq \overline{\lambda}_i \perp \hat{R} - r_i \geq 0 & (6.4g) \\ 0 \leq \overline{\gamma}_i \perp r_i - q_{it_2} + q_{it_1} \geq 0 & (6.4h) \end{cases}$$

$$d_t = \sum_i q_i, \quad d_t = D_t^0 - \alpha p_t \quad \forall t. \tag{6.5}$$

For the case, when $\hat{Q} > q_{it} > 0$ and $\hat{R} > r_i > 0$, the dual multipliers $\underline{\mu}_{it}$, $\overline{\mu}_{it}$ and $\underline{\lambda}_i$, $\overline{\lambda}_i$ are equal to zero and we can solve a system of equations:

$$\forall i: \begin{cases} \frac{D_{t_1}^0 - \sum_i q_{it_1}}{\alpha} - \theta q_{it_1} - c + \overline{\gamma}_i = 0, \\ \frac{D_{t_2}^0 - \sum_i (q_{it_1} + r_i)}{\alpha} - \theta(q_{it_1} + r_i) - c - \overline{\gamma}_i = 0, \\ \overline{\gamma}_i = 0. \end{cases}$$

This set of equations can be simplified to the following:

$$\forall i: \begin{cases} D^0_{t_1} - \sum_i q_{it_1}\alpha - \alpha\theta q_{it_1} - \alpha c = 0, \\ D^0_{t_2} - \sum_i (q_{it_1} + r_i) - \alpha\theta(q_{it_1} + r_i) - \alpha c = 0 \end{cases}$$

We can derive a closed-form solution for the optimal production level and the ramp rate, as presented below. Note that SL indicates "single level", since the solution corresponds to the single-stage equilibrium model where ramp and production decisions were taken simultaneously:

$$r_i^{SL} = \frac{D^0_{t_2} - D^0_{t_1}}{\alpha\theta + 2}, \tag{6.6a}$$

$$q_{it_1}^{SL} = \frac{D^0_{t_1} - \alpha c}{\alpha\theta + 2}, \quad q_{it_2}^{SL} = q_{it_1}^{SL} + r_i^{SL}. \tag{6.6b}$$

6.2.2 Separated stages of ramp and quantity bidding: the two-stage model

The market setup, where the generators choose their ramp rate in the first stage and compete in quantities in the second stage, can be modeled using a bilevel equilibrium formulation – EPEC. The bilevel structure aims to represent the market situation when ramp bidding occurs before the actual quantity game. The EPEC problems, known for their complexity, are hard to solve in a closed form. Similar to the previous single-level model, we study the case of two load periods with increasing demand and two symmetric power producers.

In the first stage of the game, the generating units take their decisions regarding the ramp-rate levels that will maximize their profits taking into account the optimal production decisions from the second stage. The upper level of a bilevel model representing the ramp game can be formulated as follows:

$$\forall i: \begin{cases} \underset{r_i}{\text{maximize}} (p_{t_1}(q_{it_1}, q_{-it_1}) - c)q_{it_1} + (p_{t_2}(q_{it_2}, q_{-it_2}) - c)q_{it_2} & (6.7a) \\ \text{subject to:} \quad \hat{R} \geq r_i \geq 0 \quad : \overline{\lambda}_i, \underline{\lambda}_i & (6.7b) \\ \qquad q_{it} \in \Omega^{LL}. & (6.7c) \end{cases}$$

Quantity q_{it} is an outcome of the lower-level Ω^{LL} market equilibrium game:

$$\forall i: \begin{cases} \underset{q_i}{\text{maximize}}(p_{t_1}(q_{it_1}, q_{-it_1}) - c)q_{it_1} + (p_{t_2}(q_{it_2}, q_{-it_2}) - c)q_{it_2} & (6.8a) \\ \text{subject to:} \quad \hat{Q} \geq q_{it} \geq 0 \quad : \overline{\mu}_{it}, \underline{\mu}_{it} \quad \forall t & (6.8b) \\ \qquad r_i \geq q_{it_2} - q_{it_1} \quad : \overline{\gamma}_i. & (6.8c) \end{cases}$$

The market equilibrium conditions link together the optimization problems of the generators:

$$\text{ME}: \; d_t = \sum_i q_i, \quad d_t = D_t^0 - \alpha p_t \quad \forall t. \tag{6.9}$$

Let us first focus on the lower-level problem (6.8). Following the same logic as in the single-level model we can show that the second derivative of the objective function is negative and the constraint qualification holds. Hence, the problem (6.8) can equivalently be written and solved as a set of KKT conditions. The KKT conditions take a form:

$$\forall i: \begin{cases} \frac{\partial \mathcal{L}_i}{\partial q_{it_1}} = p_{t_1}(q_{it_1}, q_{-it_1}) - \theta q_{it_1} - c + \underline{\mu}_{it_1} - \overline{\mu}_{it_1} + \overline{\gamma}_i = 0 & (6.10a) \\ \frac{\partial \mathcal{L}_i}{\partial q_{it_2}} = p_{t_2}(q_{it_2}, q_{-it_2}) - \theta q_{it_2} - c + \underline{\mu}_{it_2} - \overline{\mu}_{it_2} - \overline{\gamma}_i = 0 & (6.10b) \\ 0 \leq \underline{\mu}_{it} \perp q_{it} \geq 0 \quad \forall t & (6.10c) \\ 0 \leq \overline{\mu}_{it} \perp \hat{Q} - q_{it} \geq 0 \quad \forall t & (6.10d) \\ 0 \leq \overline{\gamma}_i \perp r_i - q_{it_2} + q_{it_1} \geq 0 & (6.10e) \end{cases}$$

In order to obtain a closed-form solution we follow a backward induction. We first find the optimal solution of the lower level parameterized by the upper-level variable. Then, we plug this expression in the upper-level and deduce a subgame-perfect Nash equilibrium (SPNE) – an equilibrium, which is optimal for both stages and for the game as a whole, [125]. This concept is often used for describing the sequence of decisions in the electricity market, as for example in the case of forward price caps in [126]. We express the prices in time periods t_1 and t_2 from the market equilibrium conditions (6.9) and use them to complement the equations (6.10).

Simplifying and solving the system (6.10) for the optimal level of quantity parameterized by the r_i variable, we get:

$$\forall i: \begin{cases} \frac{\partial \mathcal{L}_i}{\partial q_{it_1}} = \frac{D_{t_1}^0 - \sum_i q_{it_1}}{\alpha} - \theta q_{it_1} - c + \overline{\gamma}_i = 0 \\ \frac{\partial \mathcal{L}_i}{\partial q_{it_2}} = \frac{D_{t_2}^0 - \sum_i (q_{it_1} + r_i)}{\alpha} - \theta (q_{it_1} + r_i) - c - \overline{\gamma}_i = 0 \end{cases}$$

Summing up the expressions for two time periods, and solving the resulting system of equations for two producers we get the following:

$$q_{it_1}^{BL} = \frac{D_{t_1}^0 + D_{t_2}^0 - 2\alpha c - r_i(2+\alpha\theta)}{2\alpha\theta + 4}, \quad q_{it_2}^{BL} = q_{it_1}^{BL} + r_i \quad \forall i. \tag{6.11}$$

Note that BL indicates "bilevel", since this solution for quantities corresponds to the bilevel equilibrium model, and helps distinguish it from q^{SL}.

We can substitute q_{it_1} in (6.7) by the derived lower-level optimal expression for the production level (6.11). The expression for the bilevel-optimal ramp-rate level becomes:

$$\begin{cases} -D_{t_1}^0 + D_{t_2}^0 - 2r_{i_1} - r_{i_2} = 0 \\ -D_{t_1}^0 + D_{t_2}^0 - r_{i_1} - 2r_{i_2} = 0 \end{cases}$$

$$\rightarrow \quad r_i^{BL} = \frac{D_{t_2}^0 - D_{t_1}^0}{3}. \tag{6.12}$$

We observe that the expression for the optimal level of ramp rate in two-stage model is independent from the conjectured price response parameter θ. In the next section we discuss this result and compare the single-stage and two-stage models.

6.2.3 Discussing the results

The solutions derived in Sections 6.2.1 and 6.2.2 provide us with important insights on strategic decisions regarding the ramp-rate flexibility. In the following propositions we sum up our observations.

Proposition 1.1. *For two time periods, two symmetric generators with affine cost functions and perfectly substitutable products we find that the optimal level of ramp rate for the two-stage model is independent from the conjectured price response parameter θ, representing any market structure from perfect competition to the Cournot oligopoly. In particular, this can be observed for the ramp-rate level as given in* (6.12).

Proof. Section 6.2.2 proves the above proposition by deriving the closed-form solution to the two-stage model. The expression is derived assuming the nontrivial solution $\hat{Q}_i > q_{it} > 0$ and $\hat{R}_i > r_i > 0$. □

Proposition 1.2. *Using the same assumptions as for the Proposition 1, we observe that in the one-stage model the level of ramp rate offered to the market varies with the level of competition, represented by the conjectured price response: (a) the levels of ramp-rate of single- and two-stage setups coincide when the market structure approaches Cournot ($\theta = 1/\alpha$); (b) in the case of perfect competition the ramp-rate level in a single-stage model is higher than in a two-stage model.*

Proof. (a) The optimal two-stage model ramp rate is given by expression (6.12) and is independent of the conjectured price response θ. The expression (6.6a) for an optimal ramp-rate level in a one-stage model depends on θ. However, if in the second expression we substitute $\theta = 1/\alpha$, which is the conjectured price response corresponding to the Cournot oligopoly, expressions (6.6a) and (6.12) coincide. (b) We can show that for any other choice of $\theta < 1/\alpha$ the optimal level of ramp rate in a single-stage model is higher, as the denominator of the expression (6.6a) is smaller than the denominator of the expression (6.12). Expressed mathematically, $\alpha\theta + 2 \leq 3$. □

According to Proposition 1.1, if ramp-rate and quantity levels are decided sequentially and the producers hold conjectures on production, strategic producers are going to withhold the ramp rate regardless of their beliefs on the competition level in the spot market, since doing this improves total profits for the generators. This contradicts the logic of regulatory approaches that separate the two decision stages in order to incentivize producers to allow more ramp-rate flexibility in the second stage. Comparing expressions (6.6a) and (6.12) we see that the maximum value for $\alpha\theta$ in the denominator of (6.6a) is 1, and therefore if the generators are deciding their ramp rates before the spot market, the formulation for the optimal ramp rate takes the form of (6.12). We show theoretically that if the generators are able to behave strategically they do so in the first stage, withholding their ramp rate, therefore limiting the amount of production in the second stage for any degrees of market competition. We investigate this further using a case study in Section 6.4.

Proposition 1.2 shows that the optimal ramp-rate levels in two-stage and single-stage setups coincide in the case of Cournot oligopoly. However, for any other belief regarding the competition level in the market we can show that allowing offering the production and ramp simultaneously yields higher ramp-rate flexibility offered by the generators. The result, shown for the case of two load periods and symmetric generators may hold for a bigger case study as it will be shown in Section 6.4.

In contrast, in the two-stage model ramping creates an additional layer of strategic behavior. Production variables are connected with ramp-rate variable through an inequality constraint. The intuition can be that in the two-stage model strategic firm tries to recover in the upper level the market power lost with more competitive energy markets expressed in the conjectural variations in the lower level. A conclusion of this intuition would be that there is nothing to recover in the Cournot model and hence the results of the single and two stage model coincide for that assumption. Ramping is the only way to recover the lost market power in perfect competition, this is possibly why the two results diverge the most.

6.3 Extension of the models

We can extend the models presented in Section 6.2 to consider a more realistic market situation with an arbitrary number of load periods and asymmetric firms.

6.3.1 Extended formulation of the single-level equilibrium problem

We introduce a parameter c_i^R, a cost of pre-committing the ramp rate. There are two reasons for including this cost. Including such a cost in the objective function reflects the wear-and-tear of the fast-ramping generating units if considered by the generating company. The single-level formulation, introduced in Section 6.2.1 can then be written as:

$$\forall i : \begin{cases} \underset{q_{it}, r_{it}}{\text{maximize}} & \sum_t \Big((p_t(q_{it}, q_{-it}) - c_i) q_{it} - c_i^R r_{it} \Big) & (6.13a) \\ \text{subject to:} & \hat{Q}_i \geq q_{it} \geq 0 : \overline{\mu}_{it}, \underline{\mu}_{it} \quad \forall t & (6.13b) \\ & \hat{R}_i \geq r_{it} \geq 0 : \overline{\lambda}_{it}, \underline{\lambda}_{it} & (6.13c) \\ & r_{it} \geq q_{it} - q_{i(t-1)} : \overline{\gamma}_{it} & (6.13d) \\ & r_{it} \geq q_{i(t-1)} - q_{it} : \underline{\gamma}_{it} & (6.13e) \end{cases}$$

$$d_t = D_t^0 - \alpha p_t(q_{it}, q_{-it}) \quad \forall t \qquad (6.14a)$$

$$d_t = \sum_i q_{it} \quad \forall t. \qquad (6.14b)$$

Here (6.13b) defines the production bid limits, (6.13c) limits the bid on ramp rate. Equations (6.13d) and (6.13e) define ramping constraints for every two sequential time periods. We consider a general load profile, where demand can increase or decrease, and therefore both upward and downward ramping constraints are necessary. The variables, following after the colon are the Lagrange multipliers associated with the respective constraints.

The KKT conditions of problem (6.13a)-(6.13a) are also the optimality conditions. The resulting equilibrium problem is as follows:

$$\forall i: \begin{cases} \frac{\partial \mathcal{L}_i}{\partial q_{it}} = p_t(q_{it}, q_{-it}) - \theta_i q_{it} - c_i + \underline{\mu}_{it} - \overline{\mu}_{it} - \underline{\gamma}_{it} + \overline{\gamma}_{it} \\ \qquad + \underline{\gamma}_{i(t+1)} - \overline{\gamma}_{i(t+1)} = 0 \quad \forall t & (6.15a) \\ \frac{\partial \mathcal{L}_i}{\partial r_{it}} = -c_i^R + \underline{\lambda}_{it} - \overline{\lambda}_{it} + \underline{\gamma}_{it} + \overline{\gamma}_{it} = 0 \quad \forall t & (6.15b) \\ 0 \leq \overline{\mu}_{it} \perp \hat{Q}_i - q_{it} \geq 0 \quad \forall t & (6.15c) \\ 0 \leq \underline{\mu}_{it} \perp q_{it} \geq 0 \quad \forall t & (6.15d) \\ 0 \leq \underline{\gamma}_{it} \perp r_{it} - q_{it} + q_{i(t-1)} \geq 0 \quad \forall t & (6.15e) \\ 0 \leq \overline{\gamma}_{it} \perp r_{it} + q_{it} - q_{i(t-1)} \geq 0 \quad \forall t & (6.15f) \\ 0 \leq \overline{\lambda}_{it} \perp \hat{R}_i - r_{it} \geq 0 \quad \forall t & (6.15g) \\ 0 \leq \underline{\lambda}_{it} \perp r_{it} \geq 0 \quad \forall t & (6.15h) \end{cases}$$

$$D_t^0 - \alpha p_t(q_{it}, q_{-it}) - d_t = 0 \quad \forall t \tag{6.16a}$$

$$\sum_i q_{it} - d_t = 0 \quad \forall t. \tag{6.16b}$$

The introduced setup can be directly programmed as a mixed complementarity problem (MCP) in GAMS and solved until optimality with PATH solver [127].

6.3.2 Extended formulation of the bilevel equilibrium problem

The ramp-bidding problem for a single generating company can be formulated as an MPEC. The corresponding bilevel program for generating company i^* optimizing over a set of variables $\Omega_{i^*} = \{r_{i^*t}, q_{it}, p_t(q_{it}, q_{-it}), d_t, \underline{\mu}_{it}, \overline{\mu}_{it}, \underline{\gamma}_{it}, \overline{\gamma}_{it}\}$

is as follows:

$$\underset{\Omega_{i^*}}{\text{maximize}} \quad \sum_t \Big((p_t(q_{it}, q_{-it}) - c_{i^*}) q_{i^*t} - c^R_{i^*} r_{i^*t} \Big) \tag{6.17a}$$

$$\text{subject to:} \quad \hat{R}_{i^*} \geq r_{i^*t} \geq 0 \quad \forall t \tag{6.17b}$$

$$\hat{Q}_i \geq q_{it} \geq 0 \quad \forall (it) \tag{6.17c}$$

$$r_{it} - q_{it} + q_{i(t-1)} \geq 0 \quad \forall (it) \tag{6.17d}$$

$$r_{it} - q_{i(t-1)} + q_{it} \geq 0 \quad \forall (it) \tag{6.17e}$$

$$\underline{\mu}_{it} \geq 0,\ \overline{\mu}_{it} \geq 0,\ \underline{\gamma}_{it} \geq 0,\ \overline{\gamma}_{it} \geq 0 \quad \forall (it) \tag{6.17f}$$

$$p_t(q_{it}, q_{-it}) - \theta_i q_{it} - c_i + \underline{\mu}_{it} - \overline{\mu}_{it} - \underline{\gamma}_{it} + \overline{\gamma}_{it} + \underline{\gamma}_{i(t+1)} - \overline{\gamma}_{i(t+1)} = 0 \tag{6.17g}$$

$$\underline{\mu}_{it} q_{it} = 0 \quad \forall (it) \tag{6.17h}$$

$$\overline{\mu}_{it} (\hat{Q}_i - q_{it}) = 0 \quad \forall (it) \tag{6.17i}$$

$$\underline{\gamma}_{it} (-q_{it} + q_{i(t-1)} + r_{it}) = 0 \quad \forall (it) \tag{6.17j}$$

$$\overline{\gamma}_{it} (q_{it} - q_{i(t-1)} + r_{it}) = 0 \quad \forall (it) \tag{6.17k}$$

$$D^0_t - \alpha p_t(q_{it}, q_{-it}) - d_t = 0 \quad \forall t \tag{6.17l}$$

$$\sum_i q_{it} - d_t = 0 \quad \forall t. \tag{6.17m}$$

Here constraints (6.17b) correspond to the upper-level constraints, constraints (6.17c)-(6.17e) are the lower-level primal constraints. The corresponding Lagrange multipliers are nonnegative as outlined in the constraints (6.17f). Constraint (6.17g) is the stationarity condition. Conditions (6.17h) to (6.17k) are the complementary slackness conditions. Constraints (6.17l) and (6.17m) correspond to the equalities shared by all generators in the lower level.

With the state-of-the-art solvers (e.g., COUENNE), the problem can be attempted directly as a nonlinear problem, as in this paper. We can obtain a solution to the whole EPEC problem by solving the MPECs iteratively: we sequentially fix the strategic variables of all generators except of one, which is free to choose any level in response to the levels fixed for all other units. This procedure is repeated for all generators in loop until convergence to an equilibrium point, at which no producer wants to change the strategic decision unilaterally [111].

6.3.3 Additional extensions

Additionally, we consider two main extensions:

- We reflect highly concentrated markets we consider a firm ownership of the generating units: a generating company may own several generating units with possibly different production costs. The objective functions in (6.13a) and (6.17a) become:

$$\pi_f = \sum_{i^* \in f,t} \Big((p_t(q_{it}, q_{-it}) - c_{i^*}) q_{i^*t} - c_{i^*}^R r_{i^*t} \Big). \tag{6.18}$$

  Here, unit i^* belongs to the firm f and the profit is maximized for the whole firm.

- We consider an important extension of the model to the decision-making under uncertainty. The uncertainty in wind-integrated power systems can be modeled with a set of scenarios W, where each scenario $w \in W$ has a certain probability $prob_w$. The profit formulation (6.18) introduced in the previous subsection is then extended as follows:

$$\pi_f = \sum_{i^* \in f,t,w} prob_w \Big((p_{tw}(q_{itw}, q_{-itw}) - c_{i^*}) q_{i^*tw} - c_{i^*}^R r_{i^*t} \Big). \tag{6.19}$$

  We also add a nonanticipativity constraint to ensure that ramp rate cannot be changed for every realization of the scenario in the two-stage model.

6.4 Case studies

The developed model can be a tool for analyzing how a market setup affects the flexibility bidding in the system. In this section we check, whether Propositions 1.1 and 1.2, introduced in Section 6.2.3, are still valid for the extended version of the models by performing the simulations. We also model the stochastic case study with firms owning several units. More case studies can be found in [J2].

6.4.1 Comparison of the duopoly models

The duopoly case study lets us analyze the importance of information revealed in the ramp and quantity game. As derived in Section 6.2, the closed-form

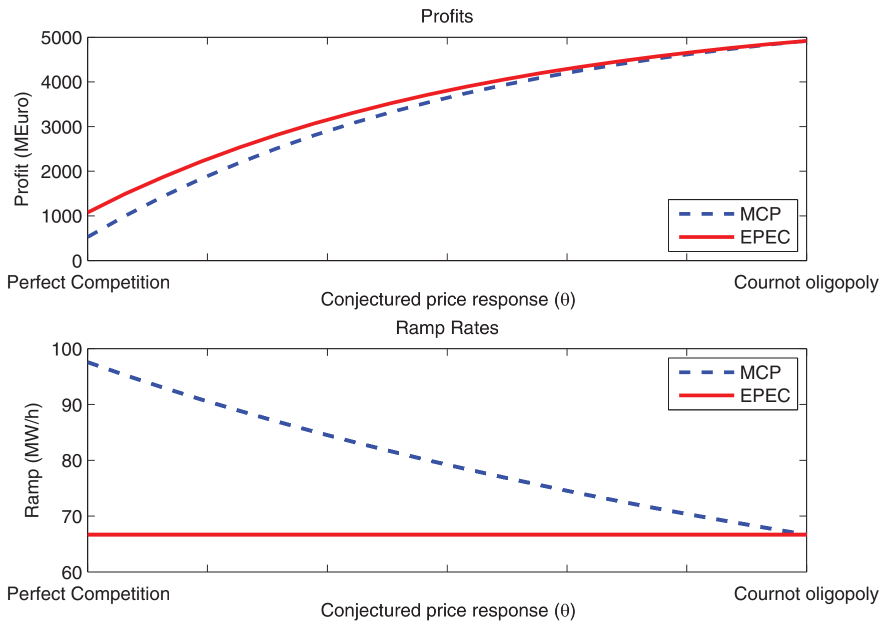

Figure 6.1: Single-stage and two-stage equilibrium solutions for a system with two symmetric generating companies.

expression for the ramp-rate bids in the case of a single-level (SL) and bilevel (BL) optimization problem formulations are:

$$r_i^{SL} = \frac{D_{02} - D_{01}}{\alpha\theta + 2}, \tag{6.20a}$$

$$r_i^{BL} = \frac{D_{02} - D_{01}}{3}. \tag{6.20b}$$

To check this result we formulate the models explicitly as optimization problems and perform the simulations. In this simple case study the capacity and ramp limits are considered nonbinding. The production costs are symmetric and equal to 10 €/MW. We assume two load periods with the demand intercepts $D_t^0 = [200, 400]$ MW. The elasticity of demand $\alpha = 7.2$ MW^{2}h/€.

Figure 6.1 shows the simulation results for the different values of conjectured price parameter. We observe that the ramp-rate level in the single-stage model steadily decreases with a growing conjectured price response parameter until it reaches the value of a two-stage model bid in the point, which

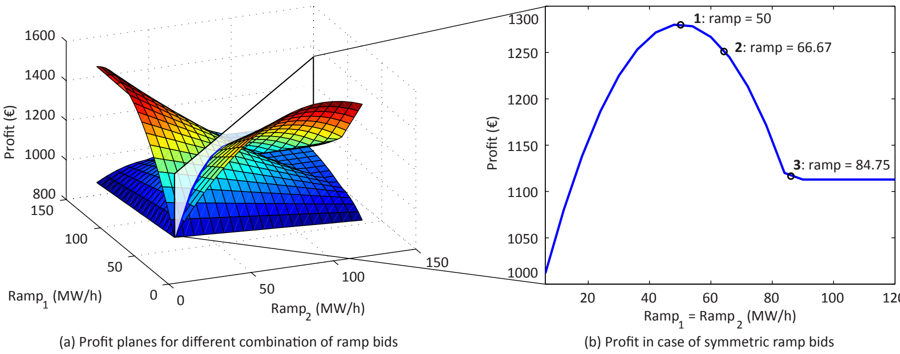

Figure 6.2: Symmetric duopoly case study results.

corresponds to the Cournot competition. The optimal level of ramp rate does not depend on how the unit perceives the competitiveness in the market. The producers prefer to withhold the flexibility and choose a relatively low level of ramp.

Figure 6.2-(a) is obtained by running the one-stage model for a fixed combination of ramp-rate levels of two players for an arbitrary level of the conjectured price response parameter $\theta = 0.05$ €/MW^{2}h. Since the companies are symmetric, we can expect the equilibrium to lie on the middle line, where the bids of the both players are equal. This line is shown in Figure 6.2-(b). We observe that the highest profits for the both units are reached when $r_1 = r_2 = 50$ MW/h (point **1** in Figure 6.2-(b)). This solution would be a stable point if the producers were cooperating, as it maximizes the sum of their profits. However, as we can see in Figure 6.3, in which a profit curve of a single producer is shown for the ramp-rate level of other player fixed, a generator can obtain a higher profit by unilaterally changing its level of the ramp rate (which can be visualized by moving along the x axis of the plane following the arrow). A response to such strategy will be an increase of a ramp-rate level by the second producer. An equilibrium point is finally reached when no unit wants to change its bid unilaterally. In Figure 6.3 we can see that point **2** coincides with the point of the highest profit for a given combination of the ramp rates. This logic illustrates reaching a Nash equilibrium between strategic producers.

We can obtain the same point by fixing the decision of one of the players and observing the response, or reaction, of the other player. By repeating

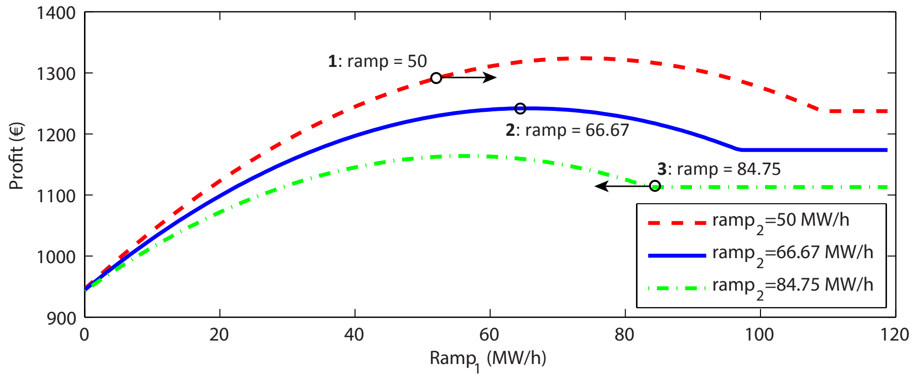

Figure 6.3: Stability of point **2** as an equilibrium point

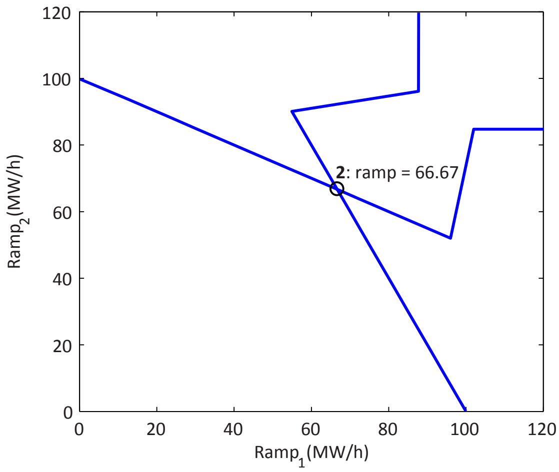

Figure 6.4: Reaction functions of both of the players. The intersection of the reaction function – Nash equilibrium.

this for a range of values and for both players we obtain reaction functions, shown in Figure 6.4. The reaction functions intersect in the only point, which is a Nash equilibrium (point **2** in Figures 6.2-(b), 6.3, and 6.4). In this simple case of smooth profit functions we can prove numerically that this point is the only Nash equilibrium.

Point **3** corresponds to the solution of the single-stage model for the given choice of parameter θ. We can see from Figure 6.2 that this point offers the highest flexibility in ramp. Since the producers are competing in quantities and ramps at the same time, they choose to leave the ramp-rate level flexible, so they can react to the competitors' strategies by withholding or providing more production to the market.

The equivalence of the single-stage and two-stage models for the choice of the conjectured price response representing Cournot competition can be found in some other games. In [128] the authors study the capacity-investment problem and show that for the case of a one load-period game and 2 symmetric producers the investments are generally higher for the single-stage problem, converging to the same value when competition structure is approaching Cournot.

The above study confirms an intuition that market design impacts the generators' behavior and the social welfare. A two-stage setup, which may seem fair intuitively, can lead to higher withholding and therefore social welfare losses.

6.4.2 Stochastic case study

Authors in [117] find that by letting generators restate their ramp rates only once in a longer time period, policy makers can prevent them from responding to market conditions with false ramp rates. In this case study we show that uncertainty, and in particular, wind uncertainty may actually lead to the opposite results in certain circumstances. This means that there are numerical examples, where uncertainty leads to lower levels of flexibility than the deterministic case. In Table 6.1 we present the demand data used for this case study. Note that the deterministic data corresponds to the average of all stochastic scenarios. Figure 6.5 shows the comparison of ramp-rate levels obtained in the deterministic and stochastic cases (for single-stage MCP model we plot the maximum value of ramp rate in scenarios). We observe that while the expected value of the demand intercept in the stochastic scenarios is equal to the one in the deterministic case, the bids follow a very different pattern. Several observations can be made:

- Counter-intuitively, we can observe that stochasticity yields smaller total ramping levels than the deterministic scenario in both two- and single-stage models – this can be explained by the fact that there is a chance of getting higher profit from the expensive unit, if the high-demand scenario occurs. In this case study, the generating companies facing the uncertainty choose to withhold their ramp rates to ensure a certain level of profits in every scenario. However, we should note that it is not the general case, and the results may be different depending on the parameters of the model;

- The initial gap for the low values of the conjectured price response corresponds to the situations close to perfect competition, when the less expensive generator provides the whole capacity to the market, so the second generator is not expected to ramp;

- The kinks in the simulations of the bilevel model can be explained by the solution procedure, as the EPEC is solved via diagonalization. While in a single-level model the result is unique, the two-level model can often have several Nash equilibria as solutions (several points at which no company wants to deviate unilaterally). This is a reason for the jumps in the optimal ramping level. In such case, when there are multiple equilibria, they can be evaluated based on criterion (e.g., social welfare - best/worst Nash equilibrium as discussed in [45]) to choose the equilibrium that yields the best result with respect to this criterion.

- Apart from these particularities that happen close to perfect competition, the ramping flexibility levels follow the same trend as in the duopoly case. The two-stage model levels, for the most part, do not seem to be affected by competitive market behavior. Moreover, the ramping levels obtained in the single-level model under Cournot competition, and the bilevel ramping levels for arbitrary market competition seem to converge to similar values.

These observations allow us to conclude that the outcome similarity of the single- and two-stage models at Cournot holds even for the case, when we consider the portfolio bidding and uncertainty. We also observe that in

Table 6.1: Optimization scenarios

Model	Scenario probability, p.u.				$D^0_{t_1}$, MW				$D^0_{t_2}$, MW			
Deterministic	1				700				1000			
Stochastic	s1:	s2:	s3:	s4:	s1:	s2:	s3:	s4:	s1:	s2:	s3:	s4:
	0.25	0.25	0.25	0.25	700	900	500	700	1000	1000	800	1200

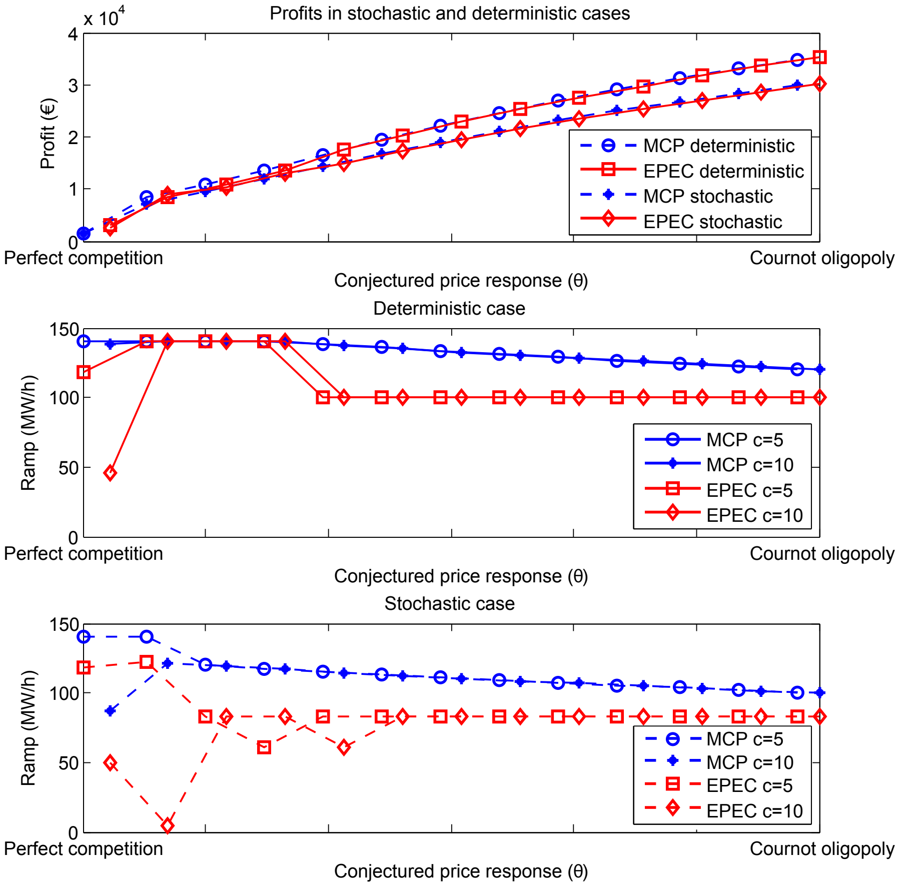

Figure 6.5: Comparison of the stochastic and deterministic cases

this particular case the ramp-rate levels in case of simultaneous bidding are steadily higher than in the case of the separated bidding stages.

It is worth noting that in the stochastic case the results do not show the same stability as in two previous case studies. For example, we can find cases, when the social welfare is higher in two-stage models, because it is more profitable for companies to produce more, so the prices decrease. The intuition for these observations is possibly in the structure of the stochastic model. In the stochastic two-stage model ramp rate is declared before the uncertainty is realized and production takes place. Therefore, the ramp rate constraint does not hold as an equality between ramp rate and the difference in the production, and ramp rate cannot be always used by strategic generators to affect the production levels. However, the existence of the cases that the generators choose to limit their ramp rates even when facing the uncertainty questions considerably the logic of separating the stages of deciding the production and flexibility levels.

6.5 Conclusion

Generation-side flexibility becomes increasingly important as the share of wind generation becomes larger and there are frequent swings in generation supply. In this chapter we present how generating companies decide ramp-rate flexibility and production in a one-stage and a two-stage processes, and we analyze the impact of these different market setups on the results.

We present two models – a single- and a bilevel model – to represent two different types of market setups: bidding the ramp and quantity levels simultaneously; or doing it in two stages, where the generators choose their ramp rates in the first stage, and compete in quantities in the second stage. A market structure, where the bids on ramp rate are submitted before the actual market clearing happens is a common regulatory practice, aiming to minimize the manipulation on the ramping constraints. In this chapter we show that such a regulatory intuition can actually lead to a higher level of withholding. We provide a comparison for a range of different conjectured price responses, capturing company's beliefs regarding its influence on market prices. The proposed model shows that for a market, the structure of which is more competitive than Cournot oligopoly, it may be advantageous to allow simultaneous bidding of ramp rates and quantities. We observe that otherwise, the producers are likely to exercise their market power in the pre-commitment stage as such strategy locks higher profits in the second stage.

Chapter 7

Conclusion

In the final chapter the key conclusions are drawn and the directions for the future research are outlined.

7.1 Concluding remarks

This dissertation studies the exercise of market power in power systems with high penetration of wind power. In this dissertation the problem of increased generation flexibility requirements in wind-integrated systems is tackled. The background for the studies and mathematical modeling, required to represent the problem, are presented in Chapters 1-3.

Starting from Chapter 4 a model of profit-maximizing generators in a system with high penetration of wind power is presented. The model captures the need for flexibility in such systems, due to fluctuations of wind power. This study is motivated by real examples of the strategic behavior, and demonstrates how strategic generators may choose to withhold the ramp rate in order to maximize their profit. The developed model represents the Nash equilibrium between multiple strategic generators and is therefore formulated as an EPEC. EPEC model is then recast as a single-stage MILP, which can be solved by commercial solvers. The case studies presented in the chapter emphasize the importance of modeling the ramp-rate withholding, as it turns out to be crucial in the wind-integrated systems.

Chapter 5 extends the findings of the previous chapter to power systems, where hydropower is present. Hydropower producers possess unique characteristics and are usually used to balance the fluctuations in supply. These unique characteristics can create additional advantages for hydropower producers if they have incentives to exercise market power. Several case studies of single profit-maximizing hydropower producer and several hydropower producers are described in this chapter. Also, the definition of Nash equilibrium under uncertainty is discussed. The models of single and multiple producers can be formulated as MPEC and EPEC respectively. The equilibrium problems can then be reformulated as MILP and solved using modified Benders decomposition approach. This approach allows solving larger case studies by decomposing the formulation in smaller instances, avoiding the disjunctive parameter.

Finally, Chapter 6 opens the topic of regulatory strategies to limit the exercise of market power on ramp rate. In this chapter two market designs are compared: a design where ramp-rate bid is decided at the same time with production-bid (single-stage model), and a design, where generators first declare their ramp rate and then in the second stage decide their production bids. While the intention of the first design is exactly to limit the strategic behavior on ramp rates, a simple case study with two symmetric generators provides a close-form solution, which actually confirms the opposite. The results of the two-generators case study show that generator may choose to "lock in" higher profits by declaring lower ramp rate in the first stage. It may be more advantageous from the system perspective to allow producers declare their ramp rate at the same time with production bidding.

7.2 Future work

One of the major directions for future research is studying the techniques for mitigating market power in wind-integrated power systems. The study, described in Chapter 6, shows that the currently used techniques may not have a strong theoretical foundation and need reconsideration. One of the common mitigation approach is introduction of the forward contracts. This approach needs to be studied further in the context of wind-integrated power systems. Working paper [J5] will discuss the theoretical implications of forward contracts in wind-integrated power systems.

Another possible direction is studying the effect of new promising technologies for demand response. With advances in IT systems, cheaper storage technologies and price-observing consumers, there are more possibilities for demand side to mitigate the strategic behavior of producers and smooth out the price peaks.

Presented models are also very computationally demanding. As case studies are getting larger, the computation time increases. In order to use the developed models for larger case studies, e.g. on European level, proposed decomposition technique can be further improved, or new techniques can be proposed.

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Complete List of Publications

Papers published in journals with Journal Citation Report (JCR):

[J1] E. Moiseeva, M.R. Hesamzadeh, D.R. Biggar, "Exercise of Market Power on Ramp Rate in Wind-Integrated Power Systems," *IEEE Transactions on Power Systems*, Vol. 30, No. 3, pp. 1614-1623, May 2015 (Invited paper to special section on Wind & Solar Energy: Uncovering and Accommodating Their Impacts on Electricity Markets).

[J2] E. Moiseeva, S. Wogrin, M.R. Hesamzadeh, "Generation Flexibility in Ramp Rates: Strategic Behavior and Lessons for Electricity Market Design," *European Journal of Operational Research*, accepted February 2017.

[J3] E. Moiseeva, M.R. Hesamzadeh, "Strategic Bidding of a Hydropower Producer under Uncertainty: Modified Benders Approach," *IEEE Transactions on Power Systems*, accepted April 2017.

Paper under review in journal with JCR:

[J4] E. Moiseeva, M.R. Hesamzadeh, "Nash Equilibria in Hydro-Dominated Systems under Uncertainty: Modified Benders Approach," *IEEE Transactions on Sustainable Energy*, submitted February 2017.

Working paper:

[J5] E. Moiseeva, M.R. Hesamzadeh, D. Bunn, D.R. Biggar "Modeling the Hedging Decisions of a Generator with Market Power in Systems with High Penetration of Wind Power," *European Journal of Operational Research.*

Peer-reviewed conference papers:

[C1] E. Moiseeva, M.R. Hesamzadeh, “Modeling the Unilateral Multi-part Strategic Withholding in Electricity Markets,” Australasian Universities Power Engineering Conference, Wollongong, Australia, 27-30 September 2015.

[C2] E. Moiseeva, M.R. Hesamzadeh, “Strategic Bidding by a Risk-Averse Firm with a Portfolio of Renewable Sources,” IEEE PowerTech Conference, Eindhoven, the Netherlands, 29 June-2 July 2015.

[C3] E. Moiseeva, M.R. Hesamzadeh, I. Dimoulkas, “Tacit Collusion with Imperfect Information: Ex-Ante Detection,” IEEE Power & Energy Society General Meeting, National Harbor, MD, USA, 27-31 July 2014.

[C4] E. Moiseeva, M.R. Hesamzadeh, “Modeling the Hedging Decisions in Electricity Markets Using Two-stage Games,” IEEE ISGT Europe 2013 Conference, Copenhagen, Denmark, 6-9 October 2013.

[C5] E. Moiseeva, M.R. Hesamzadeh, “Impact of Energy Storage Devices on Energy Price in Decentralized Wind-Diesel Utilities,” 10th International Conference on the European Energy Market, Stockholm, Sweden, 28-30 May 2013.

Curriculum Vitæ

Ekaterina Moiseeva was born on June 13th, 1989 in Tomsk, Russian Federation. She started her B.Sc. studies in the Energy Institute of Tomsk Polytechnic University in 2006. She has completed a double-degree M.Sc. program between Tomsk Polytechnic University (TPU) and Czech Technical University (CTU) in Prague in 2012. Ekaterina has graduated with a degree of M.Sc. in Power Engineering and M.Sc. in Management and Economics of Power Engineering, both with honors.

During the B.Sc. and M.Sc. studies Ekaterina had research and study visits to Czech Technical University (Prague, Czech Republic), Technical University of Vienna (Vienna, Austria), Catholic University of Louvain (Louvain-la-Neuve, Belgium). She has been twice selected the best student of TPU, and obtained a dean's prize from CTU.

In 2012, Ekaterina was selected as a PhD candidate of the Erasmus Mundus Joint Doctorate in Sustainable Energy Technologies and Strategies (SETS) and awarded an Erasmus Mundus Fellowship. She joined Electricity Market Research Group (EMReG) in KTH under the supervision of Associate Professor Mohammad Reza Hesamzadeh. The research topic covered the exercise of market power in the multi-area systems with high penetration of wind power. Research method included modeling of market power exercise in power system using equilibrium models. Also, the solution techniques using decomposition techniques and advanced optimization were developed.

In 2014-2015 Ekaterina has visited Institute for Research in Technology in Comillas Pontificial University. She worked with Assistant Professor Sonja Wogrin on close-form solutions to bilevel equilibrium models of market power exercise and on market design aspects.

The research interests of Ekaterina include large-scale optimization, wind power integration, power market modeling, and decomposition techniques.

Publication J1

Exercise of Market Power on Ramp Rate in Wind-Integrated Power Systems

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***Abstract*—With an increasing penetration of wind power, there is likely to be an increasing need for fast-ramping generating units. These generators ensure that no load is lost if supply drops due to the uncertainties in wind power generation. However, it is observed in practice that, in a presence of network constraints, fast-ramping generating units are prone to act strategically and exercise market power by withholding their ramp rates. In this paper we model this gaming behavior on ramp rates. We assume a market operator who collects bids in form of marginal costs, quantities, and ramp rates. He runs a ramp-constrained economic dispatch given the generators' bids, forecasted demand, and contingencies. Following the game-theoretic concepts, we set up a multi-level optimization problem. The lower-level problem is the ramp-constrained economic dispatch and the higher-level represents the profit maximization problems solved by strategic generators. The whole problem is formulated as an Equilibrium Problem with Equilibrium Constraints (EPEC). The outcome of the EPEC problem is a set of Nash equilibria. To tackle the multiple Nash equilibria problem, the concept of the extremal-Nash equilibria is defined and formulated. We model the concept of extremal-Nash equilibria as a single-stage mixed-integer linear programming problem (MILP) and demonstrate the application of this mathematical framework on an illustrative case and on a more realistic case study with tractable results.**

***Index Terms*—Market power; Ramp rate; Wind power.**

NOMENCLATURE

The main notation is presented below for a quick reference. Additional symbols are introduced throughout the text.

Indices

t	Time periods, $t \in (1..T)$,
c	Possible contingencies, $c \in (1..C)$,
i	Generating units, $i \in (1..I)$,
i_s	Strategic generating units, $i_s \in (1..I_s), I_s \subset I$,
s	Bidding strategies, $s \in (1..S)$,
l	Transmission lines, $l \in (1..L)$,
n	System nodes, $n \in (1..N)$.

Variables

π_i	Profit of generator i,
g_{itc}	Dispatched output by generator i,
$\hat{R}_i^{up}$	Ramp-up bid of generator i,
$\hat{R}_i^{dn}$	Ramp-down bid of generator i.

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D.R. Biggar is with the Australian Competition and Consumer Commission, Melbourne, Australia.

Parameters

K_i	Maximum output by unit i,
c_i	Marginal cost of unit i,
p_c	Probability of contingency c,
R_i^{up}	Ramp-up limits for unit i in Δt
R_i^{dn}	Ramp-down limits for unit i in Δt
H_{ln}	Power Transfer Distribution Factor (PTDF),
F_l	Flow limit on line l,
d_n	Demand at node n.

Lagrange multipliers (LM)

μ_{itc}^{A1}	LM of unit i lower capacity limit,
μ_i^{A2}	LM of unit i initial state upper capacity limit,
μ_{itc}^{A3}	LM of unit i transient states upper capacity limit,
λ_{tc}^{B}	LM of energy balance,
μ_{ltc}^{C}	LM of a flow constraint on line l,
μ_{itc}^{D1}	LM of unit i ramp-up constraint,
μ_{itc}^{D2}	LM of unit i ramp-down constraint.

I. INTRODUCTION

THE share of wind power in electricity generation is forecasted to continue increasing rapidly worldwide [1]. One of the primary challenges in the integration of large amounts of wind power is the problem of intermittency. Even a carefully predicted wind power output may, with some positive probability, depart suddenly from the forecast level due to contingencies or weather conditions. This increasing uncertainty and variability in power system conditions calls for a revised market design, where a market operator forecasts contingencies and carries out an efficient security-constrained dispatch both before and after a contingency occurs. Such dispatch, which takes into account transmission constraints and ramping limits, will maximize the overall social welfare in a wind-integrated system. By predicting the dispatch of the system after each possible contingency, such an approach ensures the optimal operation of the system ex ante and ex post, minimizes the reserve capacity required, and facilitates the integration of larger amounts of wind generation.

The advantages of security-constrained economic dispatch have been well-discussed in literature [2]–[6]. The recent advances in its formulations include modeling the wind generation uncertainty [7] and reserve procurement, based on the contingency prediction [8]. More recently, [9] proposed a short-run economic dispatch approach to the security-constrained economic dispatch problem. The proposed short-run economic dispatch approach models (1) the probabilities of contingencies, and (2) the trade-off between the preventive and corrective actions, in (3) a convex optimization structure.The system is dispatched in a way that fast-ramping generators can react to the contingencies in the most economic way. The advantages of the proposed model are discussed in details in [10]. An important feature of this dispatch model is that the ramp-rates can be defined for longer time periods. This feature of the model has two advantages: first, it reflects the reality. In practice (e.g., in California electricity market), each electricity producer is allowed to game on its (i) price offer, (ii) quantity offer and (iii) ramp offer. However, unlike the price and quantity offers, the producers usually indicate their ramp constraints for a longer time horizon, e.g., for a six-month time period, [11]. Second, this setting is expected to help mitigating the exercise of market power on ramp-rate, [12] Following such dispatch, the power system operator can adjust the output of controllable units over very short time scales by probabilistic trade-off between preventive and corrective actions. The generating units' bid information includes not only the marginal costs and the available capacity, but also the ramp rates of the units defined for a longer time-period – as they affect the ability of generators to respond to different contingencies.

Following the short-run economic dispatch framework in [10], we assume an electricity market design where bid information includes available generation capacity, marginal generation cost, and generator's ramp rate. A generator, seeking to maximize its profit, takes a strategic decision regarding its bid. In particular a ramp rate, affecting the rate at which a generator reacts to a contingency, has an effect on electricity spot price. A generator may choose not to bid its full ramp rate, if it rationally expects such decision to maximize its revenue.

There is some evidence of strategic behavior with respect to ramp rates occurring in practice. In the Australian National Electricity Market (NEM), when transmission constraints arise within a certain region, we observe a situation where generators are paid the regional price for their output rather than the correct local marginal price. Generators in this situation are said to be "constrained on" or "constrained off". When the regional price is high, a generator, which is constrained off, will strategically manipulate its bid in a variety of ways in order to maintain a high output target from the dispatch engine. This typically involves offering the generator's output at the price floor (-1,000 $\$/MWh$). Alternatively, some generators also routinely reduce their offered ramp rate in order to maintain their dispatch level. In an attempt to prevent this, in 2009 a new rule was introduced which requires generators to offer a minimum ramp rate of 3 MW per minute (or 3% of unit capacity). More recently the Australian Energy Regulator (AER) has proposed a rule, which requires generators to offer a ramp rate which matches their technical capability.

The strategic manipulation of ramp rates, which has proved troublesome in the NEM, is primarily a consequence of the mis-pricing that arises from the regional pricing structure of the NEM. Although it is a form of strategic behavior it is not a consequence of the exercise of market power. Nevertheless, it is possible to find examples in the NEM, where market power in ramp rates is likely to be present and may have been exercised. For example, the South Australian region of the NEM features significant wind generation. At times of high wind generation there may be only one or two conventional thermal units generating in South Australia. If the interconnector into this area is operating at its limit, a reduction in wind output must be matched by a rapid increase in output from those thermal units. On several occasions these ramp-up limits have been binding, resulting in a spike in the 5-minute wholesale price to above 12,000 $\$/MWh$. These occasions have tended to coincide with times when the offered ramp rate from the thermal units is less than their technical capability. It appears that some generators may at times have a commercial incentive to limit the rate at which they ramp up in response to a fall in wind output.

This paper formulates a ramp-rate game in wind-integrated power systems. The game is set up using the concept of the leader-follower game in applied mathematics. The follower is a market operator who runs the short-run economic dispatch problem. The leaders are the profit-maximizing generators, strategic on both ramp-rate and generation capacity. The whole set-up is modelled as an Equilibrium Problem with Equilibrium Constraints (EPEC). The result of the EPEC model is a set of Nash equilibria of the ramp-rate game. To tackle the multiple Nash equilibria problem, the concept of the extremal-Nash equilibria is introduced [16]. Different linearization techniques are tested and the best one is selected [17]. The final formulation of the proposed game-theoretic model is a single-stage Mixed-Integer Linear Program (MILP). To show the distinctive features of the proposed game model, the whole formulation is applied to the illustrative two-node example system and to the IEEE 24-node system. The simulation results clearly show the strategic behaviors on ramp rates.

The developed model could be used by the market operators and regulators for predicting market power abuse on ramp-rate. We assume that the market operator or regulator has a good knowledge of their systems. They can estimate the key parameters of the model using the historical data to which they have access.

The possibility of strategic behavior regarding the ramp rate is poorly covered in literature. The authors of [13], [14] consider ramp rates as a part of the bidding information of profit-maximizing generating companies, but they do not assume any strategic behavior relating to ramp rates. The authors of [15] provide an analysis of strategic ramp-rate bidding, but significantly simplify the market clearing problem and do not take into account possible contingencies or network constraints. To the authors' best knowledge, there is no reference, which provides a full game-theoretic model for strategic behaviors on ramp-rate capabilities along with the strategic behaviors on quantity and transmission related strategies by generating units. The proposed multi-period EPEC formulation in this paper clearly models and predicts this complex bidding behavior in wind-integrated power systems.

The paper is organized as follows. Section II presents the model and the assumptions used. The application of the model to the illustrative case is demonstrated in Section III and to the realistic case in Section IV. Section V presents the possibilities for the future extension. Finally, the conclusions are drawn in Section VI. Appendix I presents the details on the computation challenges, Appendix II provides the derivation of KKT conditions.

II. Ramp-Rate Game Model

This section gives the detailed formulation of the model used in the simulations and the assumptions made. We have a ramp-rate Stackelberg game with multiple leaders and a single follower. The leaders are strategic generators, seeking to maximize their profits by offering strategic bids to the market. The follower is a market operator performing a security-constrained short-run economic dispatch. We assume that all forward contracts [18] have been released before the dispatch takes place.

A. Short-Run Economic Dispatch

The short-run economic dispatch (SRED) represents the lower level of the ramp-rate game. The set-up of this dispatch, the motivation behind it, and the adopted assumptions are fully described in [9] and [10]. We assume that with a non-zero probability p_c, contingency c will occur (e.g., wind power unit goes off or the output of a wind power unit dramatically changes with respect to a previous hour), and with probability $(1-\sum_c p_c)$ no contingency occurs and the system operates in a normal mode. Therefore, the objective function of the market operator consists of two parts: the cost function before the contingency occurs and the cost function after the contingency occurs:

$$\underset{g_i, g_{itc}}{\text{minimize}} \quad (1-\sum_c p_c)\sum_i (c_i g_i) + \sum_{c,i,t} p_c (c_i g_{itc}) \tag{1}$$

Here t is the number of periods needed for the system to recover fully from the contingency. We consider time step 1 as a non-contingency stage and denote: $g_{i(t=1)c} = g_i$. Generation in the following time steps t in case of a contingency c is denoted as g_{itc}.

The constraints of the dispatch problem and the corresponding Lagrange multipliers are:

a) Production constraints:

$$\begin{aligned} g_i, g_{itc} \geq 0 \quad &\leftrightarrow \quad \mu^{A1}_{itc} \\ g_i \leq K_i \quad &\leftrightarrow \quad \mu^{A2}_{i} \\ g_{itc} \leq K_{ic} \quad &\leftrightarrow \quad \mu^{A3}_{itc} \end{aligned} \tag{2}$$

b) Energy balance constraints:

$$\sum_{n,i \in n} g_{itc} = \sum_n d_n \quad \leftrightarrow \quad \lambda^B_{tc} \tag{3}$$

c) Network flow constraints:

$$\sum_n H_{li} \sum_{n,i \in n} (g_{itc} - d_n) \leq F_l \quad \leftrightarrow \quad \mu^C_{ltc} \tag{4}$$

d) Ramping constraints:

$$\begin{aligned} g_{itc} - g_{i(t-1)c} \leq \hat{R}^{up}_i \quad &\leftrightarrow \quad \mu^{D1}_{itc} \\ -g_{itc} + g_{i(t-1)c} \leq \hat{R}^{dn}_i \quad &\leftrightarrow \quad \mu^{D2}_{itc} \end{aligned} \tag{5}$$

The short-run economic dispatch problem in (1)-(5) is convex and satisfies the weak Slater's condition. Accordingly, the Karush-Kuhn-Tucker (KKT) optimality conditions can be written as linear or mixed-integer system of equations using one of the three following techniques to avoid nonlinear complementary slackness conditions (CSCs):

- CSCs linearized through *disjunctive constraints*.
  The disjunctive constraints [19], or BigM technique, is a commonly used technique to linearize the expressions of the form: $y^T g(x,y) = 0$, where both y and $g(x,y)$ are positive continuous variables by introducing a binary variable b and a disjunctive constant $\bar{K}$. This approach is frequently used in equilibrium models, for example MPEC model in [20], as it allows for straight-forward reformulation of bilevel models.
- CSCs linearized through *SOS type 1 variables*.
  As it is described in [21], the terms of the form, $y^T g(x,y) = 0$, where $y \geq 0$ and $g(x,y) \geq 0$, arising in equilibrium problems formulations, can be efficiently linearized by introducing the SOS type 1 variables v^+ and v^-.
- *Strong duality theorem* implication. In this approach, we replace all CSCs with the condition that primal and dual objective functions give equal results at the optimum (Strong duality condition). The strong duality expression is derived in (28).

Some further details and a comparison of these techniques can be found in Appendix I. We will denote the total number of primal feasibility equations (2)-(5) as a star $(*)$. Three discussed techniques for linearizing the KKT system are compared in Table I in terms of constraints, variables and constant parameters. Simulating these techniques shows that computational time depends strongly on these values. As shown in Table I, the strong duality technique is the most computationally efficient technique for linearizing our KKT system. Therefore, to ensure the scalability and computational efficiency of the problem formulation, we adopt the strong duality technique in our further simulations.

For the conciseness and tractability reasons the full derivation of KKT conditions is shown in Appendix II. The final set of optimality conditions for the short-run economic dispatch problem includes:

- The primal feasibility constraints of the problem: (2)-(5)
- The dual feasibility constraints – the non-negativity of Lagrange multipliers corresponding to the equations (2), (4) and (5)
- The stationary conditions: (27)
- The strong duality term: (28)

TABLE I
LINEARIZATION TECHNIQUES FOR KKT SYSTEM.
$(*) = (4TIC - 3IC + I + TC + LTC)$ – NUMBER OF PRIMAL FEASIBILITY CONSTRAINTS. CAPITAL LETTERS – NUMBER OF ELEMENTS IN SETS, SEE NOMENCLATURE.

	Strong duality	SOS type 1	BigM technique
Constraints	$2(*) + 1 - TC$	$4(*) - 3TC$	$4(*) - 3TC$
Binary variables	0	0	$(*) - TC$
SOS1 variables	0	$2(*) - 2TC$	0
BigM constants	0	0	$(*) - TC$

B. Profit Maximization and Extremal-Nash Equilibria

The upper-level of the ramp-rate game is the profit-maximization task solved by the strategic generators. The profit formulation for the generator i can be written as:

$$\pi_i = (p_n - c_i)g_i, \tag{6}$$

where p_n is the price at the connection node n of generator i. The nodal price p_n can be expressed as a summation of a system price, λ^B_{tc}, and transmission congestion price, $\sum_l \mu^C_{ltc} H_{li}$, ($p_n = \lambda^B_{tc} - \sum_l \mu^C_{ltc} H_{li}$). The expression (6) for the initial state and for each time step t is then:

$$\pi_{itc} = (\lambda^B_{tc} - \sum_l \mu^C_{ltc} H_{li} - c_i)g_{itc}. \tag{7}$$

Expressing the λ^B_{tc} from stationary conditions (27), following the logic in [16], we recast the profit expression as:

$$\pi_{itc} = \mu^{A2}_i g_i + (\mu^{A3}_{itc} + \mu^{D1}_{itc} + \mu^{D2}_{itc})g_{itc}. \tag{8}$$

This expression contains nonlinear terms in primal and dual variables. However, using the complementary slackness conditions (26), we can equivalently write:

$$\begin{aligned} &\mu^{A2}_i g_i = \mu^{A2}_i K_i, \quad \mu^{A3}_{itc} g_{itc} = \mu^{A2}_{itc} K_{ic} \\ &\sum_t \mu^{D1}_{itc} g_{itc} = \sum_t \mu^{D1}_{itc} \hat{R}^{up}_i \\ &\sum_t \mu^{D2}_{itc} g_{itc} = \sum_t \mu^{D2}_{itc} \hat{R}^{dn}_i \end{aligned} \tag{9}$$

For the nonstrategic generators the bidding levels of ramp rates are true ramping capabilities, ($\hat{R}^{up}_i = R^{up}_i$ and $\hat{R}^{dn}_i = R^{dn}_i$). For the strategic generators, we assume a strategic choice on ramping level. We can model this strategic choice, by introducing a vector of binary variables x^{up}_{ik} and x^{dn}_{ik}:

$$\hat{R}^{up}_i = (b_0 + \sum_k b_k x^{up}_{ik}) R^{up}_i, \tag{10}$$

$$\hat{R}^{dn}_i = (b_0 + \sum_k b_k x^{dn}_{ik}) R^{dn}_i. \tag{11}$$

Here b_0 and b_k are vectors of constants, such that $b_0 + \sum_k b_k = 1$. This way, $x^{up,dn}_{ik} = \mathbf{1}$ means that the generator bids the full ramp-rate capability to the market. If $x_{ik} = \mathbf{0}$, then $\hat{R}^{up}_i = b_0 R^{up}_i$ and $\hat{R}^{dn}_i = b_0 R^{dn}_i$, which means the generator bids the minimum possible ramp rate level. This way a strategy set $S_i = \{s_1, s_2 ... s\}$ for the strategic unit i is obtained as a set of all k possible combinations for the vector $x^{up,dn}_{ik}$.

We substitute the expressions (9)-(11) in the profit formulation (8) and use the disjunctive constraints [19] to linearize the product of binary variables $x^{up,dn}_{ik}$ and continuous variables $\mu^{D1,D2}_{itc}$, by introducing a new variable $z^{D1,D2}_{ik} = x^{up,dn}_{ik} \mu^{D1,D2}_{itc}$. Assuming for conciseness that $R^{up}_i = R^{dn}_i = R_i$, we obtain the following expression for the profit:

$$\begin{aligned} \pi_i =& \mu^{A2}_i K_i + \sum_{tc} \Big(\mu^{A3}_{itc} K_{ic} + b_0(\mu^{D1}_{itc} + \mu^{D2}_{itc}) \\ &+ \sum_k (z^{D1}_{itck} + z^{D2}_{itck}) b_k R_i \Big). \end{aligned} \tag{12}$$

The $z^{D1,D2}_{ik}$ terms are linearized with a following set of constraints:

$$\begin{aligned} &z^{D1,D2}_{ik} \le \bar{K}_{1,2} x^{up,dn}_{ik}, \\ &z^{D1,D2}_{ik} \le \mu^{D1,D2}_{itc}, \\ &z^{D1,D2}_{ik} \ge \mu^{D1,D2}_{itc} - \bar{K}_{1,2}(1 - x^{up,dn}_{ik}). \end{aligned} \tag{13}$$

Here $\bar{K}_1$ and $\bar{K}_2$ are big-enough disjunctive constants, designed according to the recommendations in [22].

The Nash equilibrium between the strategic generators on ramp-rate is reached, when given the ramp-rate strategy of other generators, no player wants to deviate from the chosen strategy. Or, expressed mathematically:

$$\sum_{itc} \pi_{itc}(s^*_i, s^*_{-i}) \ge \sum_{itc} \pi_{itc}(s_i, s^*_{-i}). \tag{14}$$

Definition: *The Extremal-Nash equilibria*
The Nash equilibrium s^*_W is called the worst-Nash equilibrium of the ramp-rate game if and only if $s^*_W = arg\ maximize\ DC(s^*)$ where DC refers to the dispatch cost in the short-run economic dispatch formulation. Alternatively, the s^*_B is called the best-Nash equilibrium of the ramp-rate game if and only if $s^*_B = arg\ minimize\ DC(s^*)$. The extremal-Nash equilibrium is then defined as s^*_W or s^*_B.

It should be noted that the number of strategies, available for the actors, directly affects the number and the existence of Nash equilibria. There are three possible situations, (i) no equilibrium: in which case the game-theoretic model is not useful, (ii) one equilibrium: which is an idealistic case, (iii) multiple equilibria: in which case we propose focusing on the above-mentioned extremal-Nash equilibria.

The whole ramp-rate game model consists of the optimality conditions derived in Section II-A, the profit formulation (12), disjunctive constraints (13), an expression ensuring the Nash equilibrium outcome (14), and an extremal-Nash equilibrium expression. As an example, the mixed-integer linear programming problem for finding the best-Nash equilibrium is set out in below:

$$\underset{x^{up,dn}_{ik}}{\text{Minimize}} \quad (1 - \sum_c p_c) \sum_u (c_i g_i) + \sum_{c,i,t} p_c (c_i g_{itc}) \tag{15}$$

$$s.t.: \quad g_i, g_{itc} \ge 0, \quad g_i \le K_i, \quad g_{ic} \le K_{ic} \tag{16}$$

$$\sum_{n,i\in n} g_i = \sum_n d_n \tag{17}$$

$$\sum_n H_{li} \sum_{n,i\in n} (g_i - d_n) \le F_l \tag{18}$$

$$g_i(t) - g_i(t-1) \le \hat{R}^{up}_i \tag{19}$$

$$-g_i(t) + g_i(t-1) \le \hat{R}^{dn}_i \tag{20}$$

$$\mu^{A1}_{itc}, \mu^{A2}_i, \mu^{A3}_{itc}, \mu^C_{ltc}, \mu^{D1}_{itc}, \mu^{D2}_{itc} \ge 0 \tag{21}$$

Profit formulation and linearization (12)-(13)

Nash equilibrium condition (14)

Stationary conditions (27)

Strong duality implication (28)

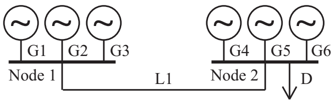

Fig. 1. The two-area system. The generation and demand are aggregated to 2 nodes with a constrained link between them.

For the worst-Nash equilibrium formulation, the minimization problem above will be changed to the maximization problem.

III. Illustrative Example

In this section, we demonstrate the developed model on a simple example system. We also explain the concept of the extremal-Nash equilibria as the upper and lower bounds of the Nash equilibria set.

We consider a network with two distinctive areas. The areas are represented by 2 nodes, aggregating the units located in the associated areas. We consider a capacity-constrained line, connecting these 2 nodes, as shown in Fig. 1. Node 1 represents a generation surplus area, while the demand is mostly concentrated in node 2. An example of this set-up is the Swedish power system, where generation is mainly concentrated in North and the load centers are situated in South. The northern and southern parts are only connected by several constrained links. We use this set-up for a transparent demonstration of generators' bidding behaviors on ramp-rate in presence of network and unit constraints. The unit data is presented in Table II.

While units G1 to G3, G5 and G6 have low probabilities of failure, unit G4, representing the aggregation of wind power units, has a 1% probability of going off the network, due to the extreme wind conditions. The demand is predicted to be 1500 MW during the short dispatch period under consideration. The dispatch is shown in Fig. 2.

The market operator collects bids, an information about the expected level of demand and possible contingencies. Here, we consider 2 cases: case (a) represents a base-case, when generators are bidding their true ramping capabilities. In case (b) the generators G5 and G6 have 4 bidding strategies on their ramping capabilities. They can offer 4 levels of ramp rate, from 25% to 100% of their true ramping capability.

In case (b) the bidding decisions of strategic generators G5 and G6 differ. Table II presents that in case of the best-Nash equilibrium (BNE) generator G6 withholds 25% of his ramping capability. The dispatch then follows the pattern showed in Fig. 2-(b) BNE. We see that the lower ramp rate of G6 forces the market operator to dispatch G5 even in a no-contingency state (time step 1). In the case of the worst-Nash equilibrium (WNE) both generators withhold. The corrective actions are taking more time and dispatch costs increase. As shown in Table III, the costs of market power in cases of BNE and WNE are €6,862 and €13,365, respectively. This means 21.6% and 42% increase in dispatch costs as compared to the dispatch cost of the competitive case. The average market clearing prices (MCPs), calculated through 7 periods, increase as well.

We see that the concept of extremal-Nash equilibria, introduced in Section II sets the bounds for the solution space (Nash-equilibria cloud) in the case of strategic generators. The

TABLE II
Unit data for the 2-nodes system. R_i^{up}=R_i^{dn}=R_i In bold – identified cases of withholding.

Unit, i	Capacity, K_i (MW)	Costs, c_i (€/MW)	Ramp rates, (MW/hr) R_i	BNE $\hat{R}_i$	WNE $\hat{R}_i$
G1	500	10	1000	1000	1000
G2	500	100	100	100	100
G3	800	50	100	100	100
G4	800	0.1	100	100	100
G5	500	200	100	100	**75**
G6	500	1000	500	**375**	**375**

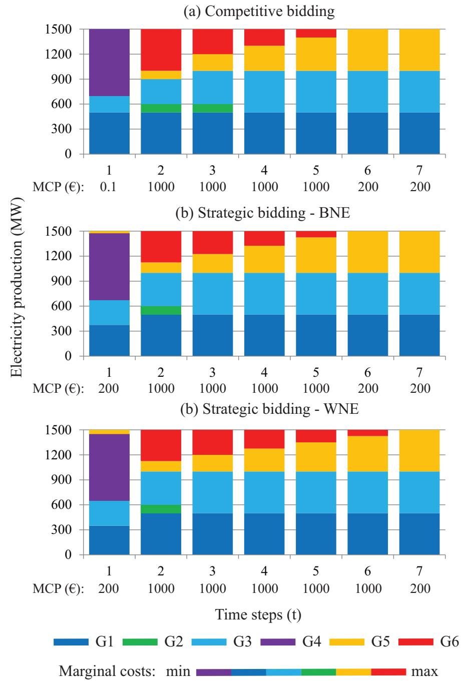

Fig. 2. Short-run economic dispatch and market clearing prices (MCPs) for the cases of (a) competitive and (b) strategic bidding. BNW: best-Nash equilibrium case, WNE: worst-Nash equilibrium case.

TABLE III
Cost of market power on ramp rate for the illustrative case study (loss of G4 with probability of 1%, G5 and G6 strategic)

	No gaming	Best NE	Worst NE
Dispatch costs (€)	31,779	38,641	45,144
Cost of market power (€)		6,862	13,365
Relative increase in costs		21.6%	42%
Average electricity price (€)	629	657	771

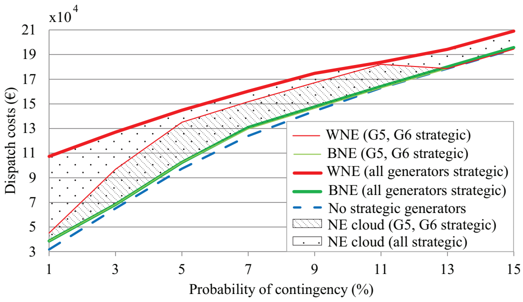

Fig. 3. The concept of Nash-equilibria band with BNE and WNE as the lower and upper bounds. Clouds of Nash equilibria solutions for the cases, when generators G5 and G6 are strategic, and when all generators are strategic.

solutions in the cloud are equiprobable.

Fig. 3 presents the evolution of dispatch costs over different contingency probabilities. The cloud of solutions, when all generators are strategic, is a super set for the cloud of solutions, when only generators G5 and G6 are strategic. Both bounds are important for the market power assessment; however, depending on the application we could be interested in a lower or upper bound. Another observation from the figure is that the cloud becomes thinner and tends to the non-strategic costs, when the probability of contingency is higher. This can be explained by the trade-off between preventive and corrective actions. As a severe contingency is more probable, system operator takes more preventive actions to minimize the costs and often the most expensive generators are dispatched in the preventive action.

IV. Case study

The EPEC problems are widely known to be computationally hard and not easily scalable, [23]. The formulation, developed in Section II, is numerically efficient due to the linearization techniques employed. In Section IV-A, we first extend the ramp-rate game model to include the strategic bidding on generators' capacities. Then the full model is applied to the IEEE 24-node example system. Section IV-B describes the results of simulations on the test system.

A. Ramp and Quantity Gaming Model

We extend the model described in Section II by considering the game both on quantity and ramp rate. To do so, we introduce the variable $\hat{K}_{ic}$, which is the bid on generation capacity, strategically chosen by the generator:

$$\hat{K}_{ic} = (b_0 + \sum_k b_k x_{ik}^{cap}) K_i. \tag{22}$$

Here x_{ik}^{cap} is a vector of binary variables. Constraints (16) are then changed to the following expressions:

$$g_i, g_{itc} \geq 0, \quad g_i \leq \hat{K}_{ic}, \quad g_{ic} \leq \hat{K}_{ic}. \tag{23}$$

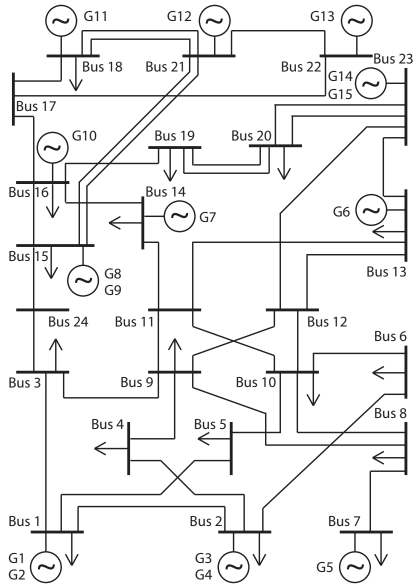

Fig. 4. The IEEE 24-nodes example system.

Following the same logic as in ramp-game, we change the expression for the profit:

$$\begin{aligned} \pi_i = & b_0 \mu_i^{A2} + \sum_k b_k z_{ik}^{A2} + \sum_{tc} \Big(\sum_k b_k z_{itck}^{A3} K_{ic} \\ & + b_0 (\mu_{itc}^{A3} + \mu_{itc}^{D1} + \mu_{itc}^{D2}) \\ & + \sum_k (z_{itck}^{D1} + z_{itck}^{D2}) b_k R_i \Big). \end{aligned} \tag{24}$$

Here $z_{ik}^{A2} = \mu_i^{A2} x_{ik}^{cap}$ and $z_{itck}^{A3} = \mu_{itc}^{A2} x_{ik}^{cap}$ are linearized the same way as described in (13):

$$\begin{aligned} & z_{ik}^{A2,A3} \leq \bar{K}_{3,4} x_{ik}^{cap}, \\ & z_{ik}^{A2,A3} \leq \mu_{itc}^{A2,A3}, \\ & z_{ik}^{A2,A3} \geq \mu_{itc}^{A2,A3} - \bar{K}_{3,4}(1 - x_{ik}^{cap}), \end{aligned}$$

where $\bar{K}_{3,4}$ are disjunctive constraints.

B. Simulation results

In this section, we will model the strategic behaviors both on ramp rate and generation capacity in the IEEE 24-node test system. The model is implemented on GAMS platform and solved by CPLEX solver. The GAMS code runs on 2.8 GHz Intel Processor with 2 cores and 8 GB RAM. All two cores are used for the CPLEX solver. The data for the test system can be found in [24]. In our study we consider Bus 13 as a slack bus.

The test system shown in Fig. 4 has 15 generating units, where G13 is an aggregation of several wind power units. The output of G13 is predicted to be 300 MW, however there is a certain probability that the real output will be different.

There is a probability of 1% that the output of the aggregation of wind turbines is reduced to 120 MW. We assume the strategic behavior of generators G1, G2, G3, G5, G6, G11 and G12 and compare their profits in case of different demand response scenarios. The assumed cases of demand response (DR) are:

- Case 1: we introduce the value of lost load – if some demand is not covered, the price in the corresponding node spikes to a very high value (one-step DR).
- Case 2: there are two steps of demand response – a limited response with medium high prices, and a value of lost load (two-step DR).
- Case 3: there are three steps with low and medium prices and a value of lost load (three-step DR).

Table IV presents the profits, obtained by generators in the cases of best-Nash equilibrium (BNE) and worst-Nash equilibrium (WNE). The dispatch costs in different cases of demand response are also presented in this table. Table V presents the bid values of ramp and generation capacities in the cases of different demand response levels and on different bounds of Nash equilibria cloud. We observe that in all cases of demand response there are occurrences of withholding behavior (market power) either on generation capacity bid, ramp rate bid or both. However, both the dispatch costs and profits of strategic generators decrease as we introduce several steps of demand response. As probability of contingency is relatively small, system operator does not take preventive actions, which can be reflected in dispatch costs.

The wind-integrated power systems are characterized by variability of wind power outputs. Therefore, we investigate different probabilities and sizes for wind power contingencies. Fig. 5 shows the dispatch costs in the case of two-step demand response with 7 strategic generators. Both BNE and WNE costs are plotted. In the BNE case, when the actual output of the wind generating unit is more or around the predicted value we observe a linear increase in the dispatch costs with the increased probability of congestion. However, when the severity of contingency is high – the actual wind output is less than predicted value by 25% or more – we observe rapid increases in the dispatch costs caused by the gaming behavior of strategic generators. In the WNE case, the dispatch costs are high in all studied cases.

According to the obtained results we can observe that when the severity of contingency is high, which may depend on the system flexibility, as introduced in [10], the generators are prone to exercise market power by bidding strategically on their capacity and ramp rate levels. This strategic behavior affects considerably the dispatch costs and prices in the system, especially when the demand response is limited. This result supports the necessity of ramp-game modeling in the systems with intermittent supply.

As discussed in Section II, the strong duality implementation results in a smaller number of constraints and binary

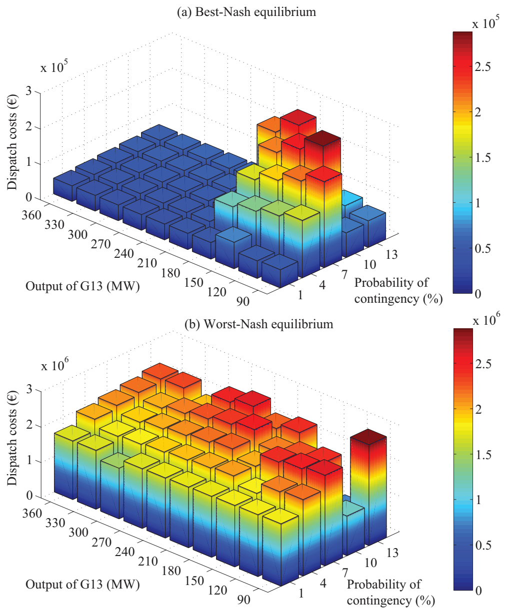

Fig. 5. Dispatch costs for the cases with different output deviation predicted with different probabilities. (a) Best-Nash equilibrium case, (b) Worst-Nash equilibrium case.

TABLE V
RAMP AND CAPACITY WITHHOLDING IN CASE OF BEST- AND WORST-NASH EQUILIBRIUM (BNE /WNE). IN BOLD – IDENTIFIED WITHHOLDING.1SDR: ONE-STEP DEMAND RESPONSE, 2SDR: TWO-STEP DEMAND RESPONSE, 3SDR: THREE-STEP DEMAND RESPONSE.

Unit,	Capacity bid, $\hat{K}_i$, (MW)			Ramp rate bid, $\hat{R}_i$, (MW/hr)		
i_s	1sDR	2sDR	3sDR	1sDR	2sDR	3sDR
G1	**3** /6	6 **/2**	6 **/3**	40 /40	**20** /40	**30 /20**
G2	4 **/1**	4 **/3**	2 **/1**	152 /152	152 /152	152 **/76**
G3	**3** /6	6 **/3**	6 **/2**	**10** /40	**30** /40	**20 /20**
G5	21 /21	21 **/5**	21 **/5**	**150** /300	**150 /150**	300 **/150**
G6	9 **/2**	9 **/2**	9 **/2**	**295 /443**	**295 /148**	**443 /148**
G11	20 /20	20 **/5**	20 **/5**	400 **/200**	400 **/200**	400 **/200**
G12	20 **/15**	**10 /5**	**15 /5**	400 **/100**	400 **/200**	400 **/200**

TABLE IV
PROFITS AND DISPATCH COSTS IN CASE OF BEST- AND WORST-NASH EQUILIBRIUM (BNE /WNE). 1SDR: ONE-STEP DEMAND RESPONSE, 2SDR: TWO-STEP DEMAND RESPONSE, 3SDR: THREE-STEP DEMAND RESPONSE.

Unit, i_s	Compe- -tetive	1sDR BNE /WNE	2sDR BNE /WNE	3sDR BNE /WNE
Profits of strategic generators, 10^3·€				
G1	0.4	0 /407	59 /407	0.23 /203
G2	5.1	407 /1,562	467 /1,563	7.4 /781
G3	0.4	1,563 /407	89 /407	0.3 /203
G5	2.9	1,538 /2,307	457 /1,538	0.8 /1,538
G6	0.7	3,013 /4,543	898 /1,514	6.7 /1,514
G11	17.7	102 /2,059	1,233 /2,059	22.4 /2,059
G12	17.7	411 /4,543	1,233 /2,059	22.6 /2,059
Dispatch costs, 10^3·€				
System	49	99 /1,096	67 /1,750	49 /1,640

variables. This allows us to apply the model for bigger case-studies. The computation time for this system varies with the number of strategic units (dominant market actors), and severity of the contingency. As an example, the computation time for the BNE case of two-steps demand response and the wind generators output predicted to be 120 MW with probability of 1% is 38 minutes.

V. Future Work

The presented model opens an interesting subject for the studies on ramp rates gaming in electricity markets. The model can be extended to consider a set of strategic decisions on ramp rates, bidding quantities and prices. Such extension will reflect the existing markets and make it possible to introduce and test market power mitigation policies, such as an extension of the bidding period for ramp rates. Improving the numerical efficiency of the proposed model and considering a larger strategy set is another extension to this work.

Further, modeling the stochastic volatility of wind power output using the stochastic programming techniques can help in better understanding of gaming on ramp rates. Finally, the model can be extended to consider gaming with incomplete information.

VI. Conclusion

This paper presents a ramp-rate game model. We model the strategic behavior in wind-integrated systems, where the intermittency is high, so that the security-constrained short run economic dispatch is a welfare-maximizing way of operating the system. We assume a market operator who collects the bid information, including marginal costs, available capacity and ramp rates. Strategic generators, willing to maximize their profit, can bid lower than their true ramp rates and capacities. The outcome of the market is a Nash equilibrium. We introduce the concept of Nash-equilibria band to tackle the multiple Nash equilibria problem and formulate the whole model as a mixed-integer linear program.

We demonstrate an application of the developed model on an illustrative case study. We show that in presence of network constraints and a major contingency, the generators are prone to exercise market power and bid a lower ramp rate to the market.

Further, we extend the mathematical formulation by adding the possibility of strategic behavior on capacity and compare the dispatch costs in the cases with different demand response levels using a 24-nodes IEEE example system. We show that if the severity of contingency is high, strategic generators are prone to maximize their profits by bidding strategically to the market. We also quote the computation time.

Obtained results are important for the studies of increasing penetration of wind generation. This study shows that contingencies, introduced by a strong intermittency, have a significant effect on the proneness of generators to bid strategically, which in turn affects the dispatch costs and, therefore, the system social welfare. Here, the ramping capabilities of the units are particularly important, as they define the speed, with which the system reacts to the contingencies.

Appendix I: Computational challenges

Solving EPEC problems is a well-known computational challenge. Finding the optimal solution is often complicated either by a big number of binary variables or by the heuristic decomposition techniques, which may not guarantee finding the global optimum. The search in the solution space is usually performed by branch-and-cut, with a preceding decomposition or reformulation. In this section we demonstrate several reformulation techniques, applicable for the described problem and compare their computational performance for finding the global optimal solution.

A. Disjunctive Constraints

The disjunctive constraints, or BigM technique, is a commonly used technique, first introduced in [19], to linearize the expressions of the form: $y^T g(x,y) = 0$, where both y and $g(x,y)$ are positive continuous variables. We introduce a binary variable b, so that the expression can be rewritten as $yb + g(x,y)(1-b)$, which in turn can be expressed with a set of constraints:

$$0 \geq y \geq \bar{K}(1-b)$$
$$0 \geq g(x,y) \geq \bar{K}b$$

The value of $\bar{K}$ is a pre-determined parameter and should be chosen in such a way that the value of X is bounded above by it. However, the value should not be chosen too high, as it makes the optimization task, where such technique is implemented, ill-conditioned, and, therefore, computationally difficult [22].

B. SOS1-based Approach

The SOS1-based approach for solving mathematical problems with equilibrium constraints is explicitly discussed in [21]. The method is applied to the problem with equilibrium constraints in the form: $y^T g(x,y) = 0$, where $y \geq 0$, $g(x,y) \geq 0$, and x, y are optimization variables. After the introduction of SOS1 variables v^+ and v^- the equivalent constraint set is:

$$y \geq 0$$
$$g(x,y) \geq 0$$
$$v^+ + v^- = (y + g(x,y))/2$$
$$v^+ - v^- = (y - g(x,y))/2$$

C. Strong duality

Another way of reformulating a lower-level optimization problem is by using the strong duality property in the KKT conditions. To do this, the problem should satisfy Slater's sufficient condition for strong duality, namely the primal problem should be convex and strictly feasible. This technique is commonly used for the solution of MPECs [25], as it allows avoiding the complementary slackness conditions and, therefore, nonlinearities in the lower level.

D. Comparison

The common problem with using the disjunctive constraints is that, while seemingly easy to implement, they need to be chosen carefully, as described in detail in [22]. Additionally, using this method of linearization requires adding a number of binary variables, which increases the computational time for the large-scale mixed-integer problems [26].

In contrast, using Schur decomposition and SOS Type1 technique does not require preliminary design. Authors in [21] show that under certain conditions, the nonlinear terms, arising in MPEC formulation can be linearized using the SOS1 technique. This, in turn, requires the introduction of new variables, but the method is shown to outperform the disjunctive constraints technique in terms of computational efficiency

The drawback of the additional variables is canceled out in the strong duality formulation used to avoid the nonlinear terms in the formulation [17]. The conditions for applying this technique typically hold in the EPEC problems arising from modeling the electricity markets. We say that the strong duality holds if the weak Slater's condition holds.

All three modeling approaches were tried for the formulation of the problem. It is observed that due to the decreased number of variables, the strong duality technique significantly outperforms two other techniques, making the scaling of EPEC possible.

Appendix II: KKT conditions of the optimal dispatch

In this section we denote the time step 1 as a no-contingency state. Therefore, $g_{i(t=1)c} = g_i$. To derive the KKT conditions of the problem we first write the Lagrangian corresponding to a linear problem given in (1)-(5):

$$\begin{aligned} L = & -(1 - \sum_c p_c) \sum_i (c_i g_i) + \sum_{c,i,t} p_c (c_i g_{itc}) \\ & + \sum_{itc} \mu_{itc}^{A1} g_{itc} + \sum_i \mu_i^{A2} (K_i - g_i) \\ & + \sum_{itc} \mu_{itc}^{A3} (K_{ic} - g_{itc}) + \sum_{tc} \lambda_{tc}^{B} \sum_{n,i \in n} (g_{itc} - d_n) \\ & + \sum_{ltc} \mu_{ltc}^{C} (F_l - \sum_n H_{li} \sum_{n,i \in n} (g_{itc} - d_n)) \\ & + \sum_{itc} \mu_{itc}^{D1} (\hat{R}_i^{up} - g_{itc} + g_{i(t-1)c}) \\ & + \sum_{itc} \mu_{itc}^{D2} (\hat{R}_i^{dn} + g_{itc} - g_{i(t-1)c}). \end{aligned} \tag{25}$$

The dual feasibility conditions are:

$$\mu_{itc}^{A1}, \mu_i^{A2}, \mu_{itc}^{A3}, \mu_{ltc}^{C}, \mu_{itc}^{D1}, \mu_{itc}^{D2} \geq 0$$

The corresponding complementary slackness conditions are:

$$\begin{aligned} & \mu_{itc}^{A1} g_{itc} = 0, \\ & \mu_i^{A2} (K_i - g_i) = 0, \\ & \mu_{itc}^{A3} (K_{ic} - g_{itc}) = 0, \\ & \mu_{ltc}^{C} (F_l - \sum_n H_{li} \sum_{n,i \in n} (g_{itc} - d_n)) = 0, \\ & \mu_{itc}^{D1} (\hat{R}_i^{up} - g_{itc} + g_{i(t-1)c}) = 0, \\ & \mu_{itc}^{D2} (\hat{R}_i^{dn} + g_{itc} - g_{i(t-1)c}) = 0. \end{aligned} \tag{26}$$

The stationary conditions can then be formulated as:

$$\begin{aligned} & \frac{dL}{dg_{itc}} = -(1 - \sum_c p_c) \sum_i c_i + \sum_{itc} p_c c_i \\ & + \sum_{itc} \mu_{itc}^{A1} + \sum_i \mu_i^{A2} + \sum_{itc} \mu_{itc}^{A3} \\ & + \sum_{tc} \lambda_{tc}^{B} + \sum_{nltc} \mu_{ltc}^{C} H_{li} \\ & - \sum_{itc} \mu_{itc}^{D1} + \sum_{itc} \mu_{itc}^{D2} \\ & + \sum_{i(t+1)c} \mu_{itc}^{D1} - \sum_{i(t+1)c} \mu_{itc}^{D2} = 0. \end{aligned} \tag{27}$$

The implication of a strong duality theorem is that the primal objective equals dual objective at optimum. This can be expressed mathematically as:

$$\begin{aligned} & (1 - \sum_c p_c) \sum_u (c_i g_i) + \sum_{c,i,t} p_c (c_i g_{itc}) = - \sum_i \mu_i^{A2} K_i \\ & - \sum_{itc} \mu_{itc}^{A3} K_{ic} - \sum_{tc} \lambda_{tc}^{B} \sum_n d_n \\ & - \sum_{ltc} \mu_{ltc}^{C} \sum_n H_{li} (F_l + \sum_{n,i \in n} d_n) \\ & - \sum_{itc} \mu_{itc}^{D1} \hat{R}_i^{up} - \sum_{itc} \mu_{itc}^{D2} \hat{R}_i^{dn}. \end{aligned} \tag{28}$$

The strong duality equation (28), added to the KKT conditions, is equivalent to the complementarity conditions. This makes complementarity conditions unnecessary in the problem formulation.

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Publication J2

Generation Flexibility in Ramp Rates: Strategic Behavior and Lessons for Electricity Market Design

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Abstract

A ramp rate usually defines the speed at which an electric power producer can decrease or increase its production in limited time. The availability of fast-ramping generators significantly affects the economic dispatch, especially in the systems with high penetration of intermittent energy sources, e.g. wind power, since the fluctuations in supply are common and sometimes unpredictable. One of the regulatory practices of how to impel generators to provide their true ramp rates is to separate the stages of submitting the bids on ramp rate and production. In this paper we distinguish two types of market structures: one-stage – when electric power producers are deciding their production and ramp rate at the same time, or two-stage – when generators decide their ramp rate first, and choose their production levels at the second stage. We employ one-stage and two-stage equilibrium models respectively to represent these market setups and use a conjectured price response parameter ranging from perfect competition to the Cournot oligopoly to investigate the effect of the market competition structure on the strategic decisions of the generators. We compare these two market setups in a symmetric duopoly case with two time periods and prove that in the two-stage market setup the level of ramp rate is independent of the strategic behavior in the spot market and generally lower than the one offered in the one-stage setup. We also show that the ramp-rate levels in one- and two-stage models coincide at the Cournot oligopoly. We extend the model to asymmetry, several load periods, portfolio bidding, and uncertainty, and show that withholding the ramp rate still occurs in both models. Our findings prove that market regulators cannot rely on only separating the decision stages as an effective measure to mitigate market power and in certain cases it may lead to an adverse effect.

Keywords: OR in energy, ramp rate, strategic decision-making, bilevel programming, equilibrium problems

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1. Introduction

The deregulation of electric power industry has started in 1981 in Chile, followed by England and Wales (1990), Norway (1991), and Argentina (1992), (Pérez-Arriaga, 2014). As a result of the changes, the countries moved away from a centralized planner deciding the investments and committing the units for the dispatch, to a system where private companies compete to supply energy in a market framework. Investments in the market are partly induced by the market processes and partly follow the regulatory incentives. The latter became one of the main reasons why in recent years the share of wind power has been increasing rapidly, and it is widely acknowledged that flexibility of the power system is the key for integrating variable renewable energy sources.

In general sense, power system flexibility describes the extent to which a power system can adapt the patterns of electricity generation and consumption in order to maintain the balance between supply and demand in a cost-efficient manner. Flexibility expresses the capability of a power system to maintain continuous service, even when exposed to rapid and large swings in supply or demand (Müller, 2014). There are many levels at which flexibility can be offered in power systems: flexibility of generation resources, flexibility of transmission and distribution systems, flexibility of the market to incentivize the power system to account for variability, and demand side flexibility (Strbac, 2008).

One of the most important aspects of power system's flexibility is an availability of a sufficient share of rapidly dispatchable generators. With a high wind power share, when generation suddenly departs from the dispatched level it is very important that there are enough of fast-ramping generators, able to sustain the energy balance (Holttinen et al., 2011). A lack of such generators may result in price spikes, when the generation is insufficient. The higher the ramping gradient that a power plant can cover, the fewer plants are needed to meet a given net load ramp, thus leading to less minimum generation per ramping (Brauner et al., 2014). There is a variety of market designs created in the deregulated framework taking into account the limitations of the units: markets in PJM Interconnection, New York Independent System Operator (NYISO) and New England electricity markets involve multi-dimensional auctions, so that participants specify technical constraints during the dispatch. In Contreras et al. (2001) the authors describe the day-ahead electricity auction in the Spanish system. The bidders are allowed to specify ramp-rate constraints, which are incorporated into the market clearing formulation and solved using a heuristic algorithm. Another design, called short-run economic dispatch, taking into account the generation flexibility, is studied in details in Hesamzadeh et al. (2014). The authors propose an optimal dispatch approach, where possible system contingencies are internalized and flexibility of the units is taken into account to reach a balance between preventive and corrective measures for these contingencies.

There are multiple studies and models, trying to evaluate the system's flexibility. For example, IMRES is an electricity generation-planning model for low-carbon power systems (De Sisternes, 2013) and BID3 is a model developed by Pöyry Management Consulting (UK) for future scenarios evaluation. Ramp constraints are included in the constraints set in paper describing the profit-maximizing bidding strategies in Li et al. (1999) and operational costs minimization in Muñoz et al. (2016).

However, there is an important drawback in these models since they do not consider the possibility of strategic behavior involving ramp constraints. It was first shown using an overly simplified model in Kai et al. (2000) that generators can act strategically when declaring their bids on ramp rate to ensure higher profits in the production stage. There is a thorough analysis in Oren and Ross (2005), showing that preventing such gaming is a difficult task and there are no commonly adopted instruments that guarantee to prevent gaming behavior. A game-theoretical bilevel model for analyzing the market power on ramp rate is employed in Moiseeva et al. (2015), however it is assumed that ramp-rate and quantity decisions are taken simultaneously and the market structure is fixed as a Cournot oligopoly.

One of the techniques to avoid the manipulation is to allow ramp-rate constraints to change only once a month, or some other suitable time interval, as adopted, for example, in California (CAISO, 2009). The "NYISO Market Participants User's Guide" states that regulation movement (MW/6 sec) response rate can be updated in three business days (NYISO, 2013). While this has a very little theoretical foundation, the purpose is to prevent generators from responding to the market conditions with false ramp rates. The expected effect would be that, exposed to the uncertainty and high competition, generators would bid the true ramp rate constraints. However, such a mitigating strategy allows a company to specify a deceptive constraint for the whole period in which the ramp constraint can be changed: a regulatory agency often cannot certify the constraint as generators are usually allowed to bid to several markets and may divide their ramping capability between the markets (Oren and Ross, 2005). A different strategy is adopted for example in Nordic electricity market, where generators are free choosing the ramp rate between their flexible orders. To the best of our knowledge there is no research evaluating the efficiency of separating the market stages of production and ramp bids with respect to market power considerations.

In this paper we study the practical implications of separating the stages of a strategic decision as a technique to mitigate the exercise of market power. We model a day-ahead market and allow generators to specify the ramp-rate level as a part of their offers. We use two types of models: a single-stage equilibrium model, corresponding to taking the strategic decision regarding a ramp-rate level and produced quantity simultaneously; and a two-stage model, in which generators choose their ramp rates in the first stage and compete in quantities in the second stage. A single-level setup can be reflected by a complementarity problem formulation (linear – LCP, or mixed – MCP) (Hobbs, 2001, García et al., 2006), while a bilevel setup takes a form of an equilibrium problem with equilibrium constraints (EPEC) (Ralph and Smeers, 2006).

In order to study the effect of the market structure we employ a conjectured-price response parameter, which is immediately obtained from the conjectural variations model, capturing various degrees of strategic behavior in the spot market. Conjectural variations (Fudenberg and Tirole, 1989, Figuières et al., 2004) reflect the firm's conjecture about other firms' reaction to a change in its production. This representation allows us to express the special cases of oligopolistic behavior ranging from perfect competition to a Cournot oligopoly (Daxhelet and Smeers, 2001). The method attempts to capture the dynamic game features in a "reduced-form" static competition model by specifying a certain parameter, which represents the beliefs of the firm. Since this method can be seen as a "shortcut" for more complicated behaviors in implicit dynamic games, it has been a subject of theoretical controversies (Tirole et al., 1988). However, the conjectural variations appear as a versatile tool, when analyzing the behavior of strategic agents in markets with different degrees of competitiveness, as in reality markets are neither perfectly competitive, nor can be characterized as Cournot oligopolies (de Haro et al., 2007). This is also the reason why conjectural variations are consistently used in industrial applications.

Multi-level decision-making is a widely studied research topic. Drawing on the existing research in the area, our paper is similar in spirit to the work in Wogrin et al. (2013b), where the authors compare the open-loop and closed-loop models for the case of capacity expansion. Providing a similar analysis of the differences between the models, our formulations also consider the inter-temporal ramp-rate constraints. We also demonstrate an extension of the models to the stochastic case (Gabriel et al., 2009), when there are several scenarios of the market outcomes. Further, we reflect the concentration of the real-world markets (Gabriel and Leuthold, 2010) using a portfolio formulation, when several units may be owned by a single generating firm.

Single-level problems, being easier in the solution and the interpretation, are widely used in practice. In Murphy and Smeers (2005) the authors interpret a single-level capacity expansion problem as simultaneously building the capacity and selling it in long-term contracts, so there is no spot market. In Allaz (1992) the authors represent a two-stage game of forward contracts and spot market competition with a single-level model, using conjectural variation parameter to reflect the dynamics of the game. However, as noted in Wogrin et al. (2013b), closed-loop models reflect sequentiality of the markets better. The difficulty of solving the closed-loop equilibrium models leads to the necessity of using either iterative algorithms as in Gabriel et al. (2012), or *ex post* solution analysis (Pozo and Contreras, 2011). In this paper we also mention the arising difficulties of solving EPECs, including the fact that possible multiple solutions can appear (Huppmann and Egerer, 2015). We also show that with a particular value of conjectured price response (at Cournot oligopoly) the solutions of the single-level and the bilevel models are similar, and, therefore, the easier single-level model can be solved in order to approximate the complicated EPEC.

To the best of our knowledge this paper is the first providing the theoretical comparison behind performing the ramp-rate and quantity bidding in a single or two distinct stages. In contrast with the existing literature on the comparison of open-loop and closed-loop formulations, we include intertemporal ramp-rate constraints and extend the model to a realistic case of portfolio bidding and bidding under uncertainty. We show that generators prefer to withhold the ramp rate in the first stage. In such a way they maintain the level of output in the production stage, even when a big change is required by the market, and receive high profits.

To improve the generality of the findings, we perform the analysis assuming a range of competitive market behaviors – from perfect competition to a Cournot oligopoly. Our findings show that the results of a bilevel equilibrium model and a single-level model are similar (or coincide in the case of a symmetric duopoly) only for the case with a very limited competition – a Cournot oligopoly.

The contribution of the current paper is therefore threefold:

- A closed-form expression demonstrating the propensity of the strategic generators to withhold ramp rates in a two-stage decision-making process;
- For the proposed one-stage and two-stage models reflecting two different market setups: an analysis of the strategic behavior for market competition intensity ranging from perfect competition to the Cournot oligopoly;
- Development of a two-stage (EPEC) and one-stage (MCP) extended equilibrium models considering the portfolio bidding and strategic decision-making under uncertainty.

The paper is organized as follows. Section 2 describes the formulation of a single- and two-stage equilibrium models, based on a small example of two symmetric generators and two load periods. In Section 3 we extend the model to an arbitrary number of generators and load periods. We also discuss the uniqueness of the solution and consider the portfolio ownership of the units and bidding under uncertainty as the extensions. Section 4 provides the case studies and practical results of the simulations. Section 5 concludes the paper.

2. Methodology

Equilibrium models are often employed to represent the nature of market interactions and competition between the strategic generators Anderson and Cau (2011). In this section we explore the impact of flexibility of power generation on market outcomes. In particular, we compare two different market setups represented by two models: a model that considers the case when ramp-rate and production decisions are taken simultaneously; and a model in which ramp rate is decided first, and then production decisions are taken.

In Section 2.1 we formulate a market setup, in which the players compete in ramp rates and quantities simultaneously. We derive a closed-form expression for a simple case of two load periods and two symmetric generating companies. In Section 2.2 we formulate a two-stage competition model: in the first stage the generators submit the levels of ramp rates, in the second – they compete in quantities. By deriving the theoretical results for the both cases we demonstrate and discuss in Section 2.3 how the difference in the compared market setups can lead to a less liquid market. In both models we use a conjectural variation parameter to study how different levels of competition in the market affect the strategic decisions of the players.

2.1. Simultaneous ramp and quantity bidding: the one-stage model

In the single-stage model, every generating unit i faces a profit-maximization problem: in time period t it chooses the level of production q_{it}. The ramp-rate level r_i is chosen simultaneously with the production level in the first time period and stays constant throughout the modeling horizon. The objective of the generator is to maximize the revenues in the production stage minus the costs. The cost function of the generator can be any convex function, as this will not affect the convexity of the whole problem. For the simplicity of

Indices		*Variables*	
i, i^*, j	Generating units, $i \in (1..I)$	q_{it}	Production level of generator i in time period t
t	Time periods, $t \in (1..T)$	r_{it}	Ramp-rate level of generator i in time period t
w	Scenarios, $w \in (1..W)$	$p_t(q_{it}, q_{-it})$	Price in time period t
f	Firm portfolios, $f \in (1..F)$	d_t	Demand in the time period t
Parameters		*Lagrange multipliers (LM)*	
$\hat{Q}_i$	Capacity of generator i	$\underline{\mu}_{it}$	LM of i's lower capacity limit
$\hat{R}_i$	Maximum ramp of generator i	$\overline{\mu}_{it}$	LM of i's upper capacity limit
c_i	Generation costs of generator i	$\underline{\lambda}_{it}$	LM of i's lower ramp limit
c_i^R	Ramp pre-committing costs	$\overline{\lambda}_{it}$	LM of i's upper ramp limit
α	Demand slope	$\underline{\gamma}_{it}$	LM of i's ramp-down constraint
D_t^0	Demand intercept	$\overline{\gamma}_{it}$	LM of i's ramp-up constraint
θ_i	Conjectured price response parameter		
$prob_w$	Probability of scenario w		

mathematical derivations and proofs we have assumed a linear cost function $C(q_{it})$. Below, parameter c represents the symmetric marginal production cost, which is the simplest example of an affine cost function.

In order to derive the analytic results in this section, let us assume that there are two consecutive time periods: t_1 and t_2. The optimization problem of a company i can be formulated as follows:

$$\forall i: \begin{cases} \underset{q_i, r_i}{\text{maximize}} & (p_{t_1}(q_{it_1}, q_{-it_1}) - c)q_{it_1} + (p_{t_2}(q_{it_2}, q_{-it_2}) - c)q_{it_2} & \text{(1a)} \\ \text{subject to:} & \hat{Q} \geq q_{it} \geq 0 \quad : \overline{\mu}_{it}, \underline{\mu}_{it} \quad \forall t & \text{(1b)} \\ & \hat{R} \geq r_i \geq 0 \quad : \overline{\lambda}_i, \underline{\lambda}_i & \text{(1c)} \\ & r_i \geq q_{it_2} - q_{it_1} \quad : \overline{\gamma}_i & \text{(1d)} \\ & q_{it_1} - q_{it_2} \geq r_i \quad : \underline{\gamma}_i. & \text{(1e)} \end{cases}$$

In the optimization problem above $\hat{Q}$ is the symmetric installed capacity of the generating unit i and $\hat{R}$ is the maximum technically possible ramp rate. Constraints (1d)-(1e) show that the change of production between two time periods is limited by the ramp rate. These constraints are specified for the time steps following the first one ($|t| > 1$). The variables on the right are the dual variables associated with constraints (1b)-(1e).

Prices $p_t(q_i, q_{-i})$ come from the market equilibrium (ME) conditions that link optimization problems of all producers: energy balances for both load periods and the affine definition of the elastic demand d_t, where α is the elasticity parameter.

$$\text{ME}: \quad d_t = \sum_i q_{it}, \quad d_t = D_t^0 - \alpha p_t(q_{it}, q_{-it}) \quad \forall t. \tag{2}$$

The ME conditions mimic the economic dispatch problem of the system operator. These conditions define the price $p_t(q_{it}, q_{-it})$, dependent on the production q_{it} of the generating companies. The demand elasticity α reflects the price responsiveness of the consumers in the market – the higher is its value, the less is demand in the market, when the price is high. This formulation aims at reflecting the market setups used in most of the countries with liberalized electricity markets (Gabriel et al., 2012).

For the two time period example analyzed in this section, there only exist three different cases of demand profiles. Case **1**: increasing demand – corresponds to the case where demand in the time period 2 is higher than in the time period 1 and hence, constraint (1d) is active; case **2**: decreasing demand – when demand in the time period 2 is lower than the one in time period 1 and constraint (1e) is active; or case **3**: stagnating demand, when neither of these constraints is active, which corresponds to demand that stays the same during both time periods. In the third case the problem simplifies to a standard quantity game. Cases **1** and **2** are symmetric, and hence we arbitrarily choose case **1** to illustrate our derivations, however, these derivations would be equivalent for case **2** (with the only difference that the upward ramp becomes a downward ramp). Choosing the case of increasing demand allows us to remove the constraint (1e) from our model as it will be inactive.

Since only two time periods are considered, the declared up-rate will be equal to the increased production without loss of generality. Moreover, the inequality constraint (1d) will hold as an equality at equilibrium, since there are no gains associated with keeping the ramp rate higher than the change of production between two time periods. In real applications, there also might be costs associated with keeping the ramp rate high (e.g., machine fatigue, higher maintenance costs). Then at equilibrium the profit-maximizing producer will keep the ramp rate only as high as required for the change of production rate, in order to avoid unnecessary costs. If the ramp rate will be higher than necessary, the producer can reduce it (which would reduce the corresponding cost in the objective function) without changing its output or the market price. This would be a contradiction to an optimal solution and therefore will not hold as an equilibrium. The production in the second time period can therefore be expressed as:

$$q_{it_2} = q_{it_1} + r_i. \tag{3}$$

The equilibrium problem combining optimization problem (1) for every generator i and the ME conditions (2) can be written as a single convex quadratic optimization problem (Basic version of the Market equilibrium as an Optimization Model – BMOM) as it was shown for the case of the capacity expansion in (Barquin et al., 2004). Using the same assumptions – affine cost functions and single-level strategic decision – we can form a new objective function, which resembles the social welfare maximization. The model is demonstrated in Appendix A. It is easy to verify that the KKT conditions of this optimization problem will coincide exactly with the KKT conditions of single-stage equilibrium problem.

Since electricity is a perfectly substitutable good, we can capture a range of behavioral outcomes by conjectural variations in the short-run market formulation, as first introduced in Bowley et al. (1924). Conjectural variation is the belief that one firm has about the way its competitors may react if it varies its output or price. This approach attempts to reproduce the dynamic pricing in a "reduced-form" static competition, therefore we are specifying exogenously the reaction of the firms to their rivals. This approach is criticized for the the exogenous nature of the conjectural variations (Tirole et al., 1988), but it allows us to capture a range of behavioral outcomes – from competitive to cooperative and has one parameter which has a simple economic interpretation.

We define the conjectural variation parameters as $\Phi_{j,i}$. These represent agent i's belief about how another agent j changes its production in response to a change in i's production. Therefore:

$$\Phi_{j,i} = \frac{dq_j}{dq_i}, \quad i \neq j$$
$$\Phi_{i,i} = 1.$$

Using these equations in a price function, when price is a function of supply in the market $p(q_i, q_{-i})$, such as the function given in (2), we obtain:

$$\frac{dp(q_i, q_{-i})}{dq_i} = -\frac{1}{\alpha}\sum_{i,j}\Phi_{j,i} = -\frac{1}{\alpha}(1 + \sum_{j\neq i}\Phi_{j,i})$$

In our application we consider a global conjectural variation Φ which represents the reaction of all competitors combined: $\Phi = \sum_{j\neq i}\Phi_{j,i}$.

We can now define a conjectured price response parameter θ_i as a company i's belief concerning its influence on price $p_t(q_{it}, q_{-it})$ in a short-term spot market. We express the conjectured price response as:

$$\theta_i = \frac{1}{\alpha}(1 + \Phi) = -\frac{dp_t(q_{it}, q_{-it})}{dq_{it}}.$$

This parameter is more convenient to depict the firms' beliefs regarding the influence of change in their production on price, as opposed to a firm's influence on production by competitors. This parameter only captures the effect of change in q_{it} and not r_i. By considering only conjectured price response on production we express the type of competition in either of the ramping and the energy markets.

Different levels of the conjectured price response parameter correspond to different market structures. We assume θ_i varying from 0, which corresponds to perfect competition in the market to $(1/\alpha)$, which represents the Cournot oligopoly (Daxhelet, 2008). By choosing the conjectured price response parameter to vary in this range we study the behavioral outcomes most common in electricity markets. Further, in this section we assume symmetric conjectured price responses $\theta = \theta_1 = \theta_2$.

The Lagrangian function of problem (1) for producer i is:

$$\begin{aligned}\mathcal{L}_i =& (p_{t_1}(q_{it_1}, q_{-it_1}) - c)q_{it_1} + (p_{t_2}(q_{it_2}, q_{-it_2}) - c)q_{it_2} + \sum_t \left(\overline{\mu}_{it}(\hat{Q} - q_{it}) + \underline{\mu}_{it}q_{it}\right) + \\ &+ \overline{\lambda}_i(\hat{R} - r_i) + \underline{\lambda}_i r_i) + \overline{\gamma}_i(r_i - q_{it_2} + q_{it_1}).\end{aligned}$$

Although parameter θ is implicit in formulation (1) it becomes explicit when we solve the problem by taking the KKT conditions[1]. The θ parameter is also explicit in the quadratic BMOM formulation presented in Appendix A. The constraint qualification holds, as the problem satisfies the Slater's condition (the feasible region has an interior point). Therefore, the KKT conditions are equivalent to the original optimization problem.

We substitute the first derivative of price by θ and use the dual multipliers introduced previously. Note that $\perp$ denotes the complementarity between the constraint and its respective dual variable. The KKT conditions of the problem (1) are as follows:

$$\forall i : \begin{cases} \frac{\partial \mathcal{L}_i}{\partial q_{it_1}} = p_{t_1} - \theta q_{it_1} - c + \underline{\mu}_{it_1} - \overline{\mu}_{it_1} + \overline{\gamma}_i = 0 & \text{(4a)} \\ \frac{\partial \mathcal{L}_i}{\partial q_{it_2}} = p_{t_2} - \theta q_{it_2} - c + \underline{\mu}_{it_2} - \overline{\mu}_{it_2} - \overline{\gamma}_i = 0 & \text{(4b)} \\ \frac{\partial \mathcal{L}_i}{\partial r_i} = \underline{\lambda}_i - \overline{\lambda}_i + \overline{\gamma}_i = 0 & \text{(4c)} \\ 0 \leq \underline{\mu}_{it} \perp q_{it} \geq 0 \quad \forall t & \text{(4d)} \\ 0 \leq \overline{\mu}_{it} \perp \hat{Q} - q_{it} \geq 0 \quad \forall t & \text{(4e)} \\ 0 \leq \underline{\lambda}_i \perp r_i \geq 0 & \text{(4f)} \\ 0 \leq \overline{\lambda}_i \perp \hat{R} - r_i \geq 0 & \text{(4g)} \\ 0 \leq \overline{\gamma}_i \perp r_i - q_{it_2} + q_{it_1} \geq 0 & \text{(4h)} \end{cases}$$

$$d_t = \sum_i q_i, \quad d_t = D_t^0 - \alpha p_t \quad \forall t. \tag{5}$$

For a closed-form solution to the arising equilibrium problem given in (4) and (5), the solution satisfies the following:

1. We assume that at equilibrium the price is higher than the marginal costs of the producers and the production levels of both generating companies are nonzero, i.e., $r_i > 0$ and $q_{it} > 0$. Complementarity conditions imply that the Lagrange multipliers corresponding to the lower limits $\underline{\mu}_{it}$ and $\underline{\lambda}_i$ are equal to zero.
2. For two identical players with the same optimization problems it may occur that the number of solutions is infinite, even if the optimization problems are convex. However, if two generating companies have the same marginal costs it is realistic to assume that in electricity markets they will be dispatched equally, so that generating companies offering the production at equal prices are remunerated equally (Weedy et al., 2012).
3. We use the definition of production in the second period (3). The prices in (4a) and (4b) can then be expressed from the market equilibrium conditions (5): $p_{t_1}(q_{it_1}, q_{-it_1}) = (D_{t_1}^0 - \sum_i q_{it_1})/\alpha$, $p_{t_2}(q_{it_1}, q_{-it_1}) = (D_{t_2}^0 - \sum_i (q_{it_1} + r_i))/\alpha$.

[1]The concavity of each producer's maximization problem holds for any value of θ_i in $[0, 1/\alpha]$: taking the derivative of the objective function (1a) with respect to q_{it} yields $p_t(q_{it}, q_{-it}) - \theta_i q_i - c$. The second derivative is, therefore, $(\theta_i - \theta_i) = 0$. The second derivative with respect to r_i is zero, therefore the Hessian of the profit expression can be seen as a negative semi-definite matrix and concavity holds.

The upper capacity limits as in (4e) and (4g) represent the physical limitations of the generating companies. Disregarding them would simplify the analysis but will contradict the finite nature of the problem. Therefore the inequalities $\hat{Q} \geq q_i > 0$ and $\hat{R} \geq r_i > 0$ cannot be omitted in general case and we can separate 4 cases:

C1: $q_{it_2} = \hat{Q}, r_i = \hat{R}, q_{it_1} = \hat{Q} - \hat{R}$. In this case the system of equations (4) for every generator becomes:

$$\forall i : \begin{cases} p_{t_1}(\hat{Q}, \hat{R}) - \theta(\hat{Q} - \hat{R}) - c + \overline{\gamma}_i = 0, \\ p_{t_2}(\hat{Q}) - \theta\hat{Q} - c - \overline{\mu}_{it_2} - \overline{\gamma}_i = 0, \\ -\overline{\lambda}_i + \overline{\gamma}_i = 0. \end{cases}$$

The system of three equations with three unknown variables can be solved for $\overline{\mu}_{it_2}$, $\overline{\lambda}_i$, and $\overline{\gamma}_i$.

C2: $q_{it_2} = \hat{Q}, r_i < \hat{R}, q_{it_1} = \hat{Q} - r_i$. With these assumptions the system of equations (4) simplifies to three equations for each generator, solvable for $\overline{\mu}_{it_2}$, $\overline{\gamma}_i$ and r_i:

$$\forall i : \begin{cases} \frac{D^0_{t_1} - \sum(\hat{Q} - r_i)}{\alpha} - \theta(\hat{Q} - r_i) - c + \overline{\gamma}_i = 0, \\ \frac{D^0_{t_2} - \sum \hat{Q}}{\alpha} - \theta\hat{Q} - c - \overline{\mu}_{it_2} - \overline{\gamma}_i = 0, \\ \overline{\gamma}_i = 0. \end{cases}$$

The first equation can be solved to obtain the expression for r_i:

$$r_i = \frac{\alpha(\theta\hat{Q} + c)}{\theta\alpha + 2}.$$

C3: $q_{it} < \hat{Q}, r_i = \hat{R}$. The system of equations becomes:

$$\forall i : \begin{cases} \frac{D^0_{t_1} - \sum q_{it_1}}{\alpha} - \theta q_{it_1} - c + \overline{\gamma}_i = 0, \\ \frac{D^0_{t_2} - \sum(q_{it_1} + \hat{R})}{\alpha} - \theta(q_{it_1} + \hat{R}) - c - \overline{\gamma}_i = 0, \\ -\overline{\lambda}_i + \overline{\gamma}_i = 0. \end{cases}$$

We can express the optimal level of q_{it_1}:

$$q_{it_1} = \frac{D^0_{t_1} + D^0_{t_2} - 2\hat{R} - \alpha(\theta\hat{R} + 2c)}{2\theta\alpha + 4}.$$

C4: $\hat{Q} > q_{it} > 0$ and $\hat{R} > r_i > 0$. In this case the dual multipliers $\underline{\mu}_{it}$, $\overline{\mu}_{it}$ and $\underline{\lambda}_i$, $\overline{\lambda}_i$ are equal to zero and we can solve a system of equations:

$$\forall i : \begin{cases} \frac{D^0_{t_1} - \sum_i q_{it_1}}{\alpha} - \theta q_{it_1} - c + \overline{\gamma}_i = 0, \\ \frac{D^0_{t_2} - \sum_i(q_{it_1} + r_i)}{\alpha} - \theta(q_{it_1} + r_i) - c - \overline{\gamma}_i = 0, \\ \overline{\gamma}_i = 0. \end{cases}$$

This set of equations can be simplified to the following:

$$\forall i : \begin{cases} D^0_{t_1} - \sum_i q_{it_1}\alpha - \alpha\theta q_{it_1} - \alpha c = 0, \\ D^0_{t_2} - \sum_i (q_{it_1} + r_i) - \alpha\theta(q_{it_1} + r_i) - \alpha c = 0 \end{cases}$$

We can derive a closed-form solution for the optimal production level and the ramp rate, as presented below. Note that SL indicates "single level", since the solution corresponds to the single-stage equilibrium model where ramp and production decisions were taken simultaneously:

$$r_i^{SL} = \frac{D^0_{t_2} - D^0_{t_1}}{\alpha\theta + 2}, \tag{6a}$$

$$q_{it_1}^{SL} = \frac{D^0_{t_1} - \alpha c}{\alpha\theta + 2}, \quad q_{it_2}^{SL} = q_{it_1}^{SL} + r_i^{SL}. \tag{6b}$$

Even though all 4 cases are possible solutions that can occur in reality, we choose to focus on interior solution, i.e. C4 (when the capacity and ramp limits are not binding), so that we can see the analytical expression for all strategic variables. Also, in this case we can observe the effect of market structure (the effect of firm i's belief concerning its influence on price with change in electricity production level θ). The cases C1-C3 can easily be included in the study, but the value for some of the strategic parameters depends on the choice of data ($\hat{Q}$, $\hat{R}$), and the results do not vary with market structure since they are already at the bound. Therefore, the unbounded case appears as a more interesting one.

2.2. Separated stages of ramp and quantity bidding: the two-stage model

The market setup, where the generators choose their ramp rate in the first stage and compete in quantities in the second stage, can be modeled using a bilevel equilibrium formulation – EPEC. The bilevel structure aims to represent the market situation when ramp bidding occurs before the actual quantity game. The EPEC problems, known for their complexity, are hard to solve in a closed form. Similar to the previous single-level model, we study the case of two load periods with increasing demand and two symmetric power producers.

In the first stage of the game, the generating units take their decisions regarding the ramp-rate levels that will maximize their profits taking into account the optimal production decisions from the second stage. As in Section 2.1, without loss of generality we choose to illustrate the case when the demand is increasing. The upper level of a bilevel model representing the ramp game can be formulated as follows:

$$\forall i : \begin{cases} \underset{r_i}{\text{maximize}} \quad (p_{t_1}(q_{it_1}, q_{-it_1}) - c)q_{it_1} + (p_{t_2}(q_{it_2}, q_{-it_2}) - c)q_{it_2} & (7a) \\ \text{subject to:} \quad \hat{R} \geq r_i \geq 0 \quad : \overline{\lambda}_i, \underline{\lambda}_i & (7b) \\ \qquad q_{it} \in \Omega^{LL}. & (7c) \end{cases}$$

Quantity q_{it} is an outcome of the lower-level Ω^{LL} market equilibrium game:

$$\forall i: \begin{cases} \underset{q_i}{\text{maximize}} & (p_{t_1}(q_{it_1}, q_{-it_1}) - c)q_{it_1} + (p_{t_2}(q_{it_2}, q_{-it_2}) - c)q_{it_2} & \text{(8a)} \\ \text{subject to:} & \hat{Q} \geq q_{it} \geq 0 \quad : \overline{\mu}_{it}, \underline{\mu}_{it} \quad \forall t & \text{(8b)} \\ & r_i \geq q_{it_2} - q_{it_1} \quad : \overline{\gamma}_i. & \text{(8c)} \end{cases}$$

The market equilibrium conditions link together the optimization problems of the generators:

$$\text{ME}: \; d_t = \sum_i q_i, \quad d_t = D_t^0 - \alpha p_t \quad \forall t. \tag{9}$$

Let us first focus on the lower-level problem (8). Following the same logic as in the single-level model we can show that the second derivative of the objective function is negative and the constraint qualification holds. Hence, the problem (8) can equivalently be written and solved as a set of KKT conditions. The KKT conditions take a form:

$$\forall i: \begin{cases} \frac{\partial \mathcal{L}_i}{\partial q_{it_1}} = p_{t_1}(q_{it_1}, q_{-it_1}) - \theta q_{it_1} - c + \underline{\mu}_{it_1} - \overline{\mu}_{it_1} + \overline{\gamma}_i = 0 & \text{(10a)} \\ \frac{\partial \mathcal{L}_i}{\partial q_{it_2}} = p_{t_2}(q_{it_2}, q_{-it_2}) - \theta q_{it_2} - c + \underline{\mu}_{it_2} - \overline{\mu}_{it_2} - \overline{\gamma}_i = 0 & \text{(10b)} \\ 0 \leq \underline{\mu}_{it} \perp q_{it} \geq 0 \quad \forall t & \text{(10c)} \\ 0 \leq \overline{\mu}_{it} \perp \hat{Q} - q_{it} \geq 0 \quad \forall t & \text{(10d)} \\ 0 \leq \overline{\gamma}_i \perp r_i - q_{it_2} + q_{it_1} \geq 0 & \text{(10e)} \end{cases}$$

In order to obtain a closed-form solution we follow a backward induction. We first find the optimal solution of the lower level parameterized by the upper-level variable. Then, we plug this expression in the upper-level and deduce a subgame-perfect Nash equilibrium (SPNE) – an equilibrium, which is optimal for both stages and for the game as a whole, (Fudenberg and Tirole, 1991). This concept is often used for describing the sequence of decisions in the electricity market, as for example in the case of forward price caps in Yao et al. (2007). We express the prices in time periods t_1 and t_2 from the market equilibrium conditions (9) and use them to complement the equations (10).

We would like to stress that the production and ramp level of the generating companies in reality have strictly defined technical limits. However, for the sake of theoretical analysis we consider an interior case[2]. Also, we take into account that in two time periods the generator will not precommit a higher ramp rate than required, therefore constraint (8c) holds as an equality. Simplifying and solving the system (10) for the optimal level of quantity parameterized by the r_i variable, we get:

$$\forall i: \begin{cases} \frac{\partial \mathcal{L}_i}{\partial q_{it_1}} = \frac{D_{t_1}^0 - \sum_i q_{it_1}}{\alpha} - \theta q_{it_1} - c + \overline{\gamma}_i = 0 \\ \frac{\partial \mathcal{L}_i}{\partial q_{it_2}} = \frac{D_{t_2}^0 - \sum_i (q_{it_1} + r_i)}{\alpha} - \theta (q_{it_1} + r_i) - c - \overline{\gamma}_i = 0 \end{cases}$$

[2] As in the case of single-level game we can distinguish four separate cases. However, in three of the cases the expression contains data-dependent parameters $\hat{Q}$ and $\hat{R}$. Therefore, for conciseness reasons we omit the discussion of the cases, as it is performed in the way similar to the single-level model.: $\hat{Q} > q_{it} > 0$ and $\hat{R} > r_i > 0$, when $Q_i > q_{it} > 0$, hence $\overline{\mu}_{it}, \underline{\mu}_{it} = 0$

Summing up the expressions for two time periods, and solving the resulting system of equations for two producers we get the following:

$$\begin{cases} D^0_{t_1} - \sum_i q_{it_1} - \alpha\theta q_{i_1t_1} - 2\alpha c + D^0_{t_2} - \sum_i (q_{it_1} + r_i) + \alpha\theta(q_{i_1t_1} + r_{i_1}) = 0 \\ D^0_{t_1} - \sum_i q_{it_1} - \alpha\theta q_{i_2t_1} - 2\alpha c + D^0_{t_2} - \sum_i (q_{it_1} + r_i) + \alpha\theta(q_{i_2t_1} + r_{i_2}) = 0 \end{cases}$$

$$\rightarrow \quad q^{BL}_{it_1} = \frac{D^0_{t_1} + D^0_{t_2} - 2\alpha c - r_i(2+\alpha\theta)}{2\alpha\theta + 4}, \quad q^{BL}_{it_2} = q^{BL}_{it_1} + r_i \quad \forall i. \tag{11}$$

Note that BL indicates "bilevel", since this solution for quantities corresponds to the bilevel equilibrium model, and helps distinguish it from q^{SL}.

We can substitute q_{it_1} in (7) by the derived lower-level optimal expression for the production level (11). We use $q^{BL}_{it_2} = q^{BL}_{it_1} + r_i$ to define the production in the second time step. This substitution yields an unconstrained convex optimization problem. Therefore, if the function has an optimum point, this point is unique. We can find the optimal solution by setting the derivative of Lagrangian, corresponding to the optimization problem (7) to zero. The derivative of the Lagrange function of a generating company i becomes:

$$\begin{aligned} \frac{\partial \mathcal{L}_i}{\partial r_i} &= \sum_t \Big(\frac{\partial (p_t(q^{BL}_{it}, q^{BL}_{-it}))}{\partial r_i} q^{BL}_{it} + p_t(q^{BL}_{it}, q^{BL}_{-it}) \frac{\partial q^{BL}_{it}}{\partial r_i} - c\frac{\partial q^{BL}_{it}}{\partial r_i} \Big) = \Big(\frac{1}{2\alpha} q^{BL}_{it_1} - \frac{1}{2} p_{t_1}(q^{BL}_{it_1}, q^{BL}_{-it_1}) \\ &+ \frac{1}{2}c \Big) + \Big(-\frac{1}{2\alpha}(q^{BL}_{it_1} + r_i) + \frac{1}{2} p_{t_2}(q^{BL}_{it_1}, q^{BL}_{-it_1}, r_i, r_{-i}) - \frac{1}{2}c \Big) = \frac{1}{2}\Big(p_{t_2}(q^{BL}_{it_1}, q^{BL}_{-it_1}, r_i, r_{-i}) \\ &- p_{t_1}(q^{BL}_{it_1}, q^{BL}_{-it_1}) \Big) - \frac{1}{2\alpha} r_i = 0. \end{aligned} \tag{12}$$

Since the derivative of Lagrangian (12) of a generating company i depends on the production and ramp rate levels of the generating company $(-i)$ we solve expressions for both generating companies as a system of equations. The expression for the bilevel-optimal ramp-rate level becomes:

$$\begin{cases} -D^0_{t_1} + D^0_{t_2} - 2r_{i_1} - r_{i_2} = 0 \\ -D^0_{t_1} + D^0_{t_2} - r_{i_1} - 2r_{i_2} = 0 \end{cases}$$

$$\rightarrow \quad r^{BL}_i = \frac{D^0_{t_2} - D^0_{t_1}}{3}. \tag{13}$$

We observe that the expression for the optimal level of ramp rate in two-stage model is independent from the conjectured price response parameter θ. In the next section we discuss this result and compare the single-stage and two-stage models.

2.3. Discussing the results

The solutions derived in Sections 2.1 and 2.2 provide us with important insights on strategic decisions regarding the ramp-rate flexibility. In the following propositions we sum up our observations.

Proposition 1.1. *For two time periods, two symmetric generators with affine cost functions and perfectly substitutable products we find that the optimal level of ramp rate for the two-stage model is independent from the conjectured price response parameter θ, representing any market structure from perfect competition to the Cournot oligopoly. In particular, this can be observed for the ramp-rate level as given in* (13).

Proof. Section 2.2 proves the above proposition by deriving the closed-form solution to the two-stage model. The expression is derived assuming the nontrivial solution $\hat{Q}_i > q_{it} > 0$ and $\hat{R}_i > r_i > 0$. □

Proposition 1.2. *Using the same assumptions as for the Proposition 1, we observe that in the one-stage model the level of ramp rate offered to the market varies with the level of competition, represented by the conjectured price response: (a) the levels of ramp-rate of single- and two-stage setups coincide when the market structure approaches Cournot ($\theta = 1/\alpha$); (b) in the case of perfect competition the ramp-rate level in a single-stage model is higher than in a two-stage model.*

Proof. (a) The optimal two-stage model ramp rate is given by expression (13) and is independent of the conjectured price response θ. The expression (6a) for an optimal ramp-rate level in a one-stage model depends on θ. However, if in the second expression we substitute $\theta = 1/\alpha$, which is the conjectured price response corresponding to the Cournot oligopoly, expressions (6a) and (13) coincide. (b) We can show that for any other choice of $\theta < 1/\alpha$ the optimal level of ramp rate in a single-stage model is higher, as the denominator of the expression (6a) is smaller than the denominator of the expression (13). Expressed mathematically, $\alpha\theta + 2 \leq 3$. □

According to Proposition 1.1, if ramp-rate and quantity levels are decided sequentially and the producers hold conjectures on production, strategic producers are going to withhold the ramp rate regardless of their beliefs on the competition level in the spot market, since doing this improves total profits for the generators. This contradicts the logic of regulatory approaches that separate the two decision stages in order to incentivize producers to allow more ramp-rate flexibility in the second stage. Comparing expressions (6a) and (13) we see that the maximum value for $\alpha\theta$ in the denominator of (6a) is 1, and therefore if the generators are deciding their ramp rates before the spot market, the formulation for the optimal ramp rate takes the form of (13). We show theoretically that if the generators are able to behave strategically they do so in the first stage, withholding their ramp rate, therefore limiting the amount of production in the second stage for any degrees of market competition. We investigate this further using a case study in Section 4.

Proposition 1.2 shows that the optimal ramp-rate levels in two-stage and single-stage setups coincide in the case of Cournot oligopoly. However, for any other belief regarding the competition level in the market we can show that allowing offering the production and ramp simultaneously yields higher ramp-rate flexibility offered by the generators. The result, shown for the case of two load periods and symmetric generators may hold for a bigger case study as it will be shown in Section 4. The intuition for this result could be that ramping does not introduce any additional strategic behavior in the single-stage model. It is shown in Appendix B that the KKT conditions of single-stage model with ramp and quantity as strategic variables are equivalent to the KKT conditions of quantity-only single-stage model.

In contrast, in the two-stage model ramping creates an additional layer of strategic behavior. Production variables are connected with ramp-rate variable through an inequality constraint. The intuition can be that in the two-stage model strategic firm tries to recover in the upper level the market power lost with more competitive energy markets expressed in the conjectural variations in the lower level. A conclusion of this intuition would be that there is nothing to recover in the Cournot model and hence the results of the single and two stage model coincide for that assumption. Ramping is the only way to recover the lost market power in perfect competition, this is possibly why the two results diverge the most.

3. Model extension

In this section we extend the models presented in Section 2 to consider a more realistic market situation with an arbitrary number of load periods and asymmetric firms, as formulated in Section 3.1 for the single-level model and in Section 3.2 for the bilevel equilibrium model. Since the extended models cannot be solved analytically as in the previous section, we also discuss the resolution methods and the uniqueness of the equilibrium solution in case of the extended bilevel equilibrium model.

Additionally, we consider two main extensions. In Section 3.3 in order to reflect highly concentrated markets we consider a firm ownership of the generating units: a generating company may own several generating units with possibly different production costs. Secondly, in Section 3.4 we consider an important extension of the model to the decision-making under uncertainty. In the areas with high penetration of intermittent renewable sources (e.g. wind power) it is especially important to consider the set of possible scenarios that can considerably affect the decision.

3.1. Extended formulation of the single-level equilibrium problem

To extend the formulation of Section 2.1 to an arbitrary number of load periods, we consider q_{it} as a production level of unit i in the load period t and r_{it} as a ramp-rate level limiting the change of production between two sequential time periods.

We also introduce a parameter c_i^R, a cost of pre-committing the ramp rate. There are two reasons for including this cost. First, including such a cost in the objective function reflects the wear-and-tear of the fast-ramping generating units if considered by the generating company. Thermal shock, metal fatigue, corrosion, erosion, and heat decay are common damage mechanisms that result from frequent ramping. It can also be interpreted as an increase of production costs due to consumption of the useful life of generating equipment. This is the cost of the company for being flexible in the market, as for example described as supplier's ancillary cost in Tsitsiklis and Xu (2015). Authors in Troy et al. (2012) consider the wear-and-tear of generators' components by dynamically modeling cycling costs that they define as frequent start-ups or ramping of units. The authors discuss the difficulty of estimating such costs, as they change dynamically and require long-term historic data, but they estimate ramping costs to 10-20% of the marginal costs. Intertek Aptech has analyzed cycling costs for over 300 generating units and found that the cost of cycling a conventional fossil-fired power plant can be as much as \$2500-500000 per start/stop cycle (Lefton, 2004). Second, setting the ramp rate cost very small also excludes a multitude of equilibria if the company is indifferent about a ramp-rate level. If the optimal solution lies on a facet of the feasible set, including a cost of pre-commiting the ramp rate forces the solution to be a corner solution. The single-level formulation can then be written as:

$$\forall i: \begin{cases} \underset{q_{it}, r_{it}}{\text{maximize}} & \sum_t \Big((p_t(q_{it}, q_{-it}) - c_i) q_{it} - c_i^R r_{it} \Big) & \text{(14a)} \\ \text{subject to:} & \hat{Q}_i \geq q_{it} \geq 0 : \overline{\mu}_{it}, \underline{\mu}_{it} \quad \forall t & \text{(14b)} \\ & \hat{R}_i \geq r_{it} \geq 0 : \overline{\lambda}_{it}, \underline{\lambda}_{it} & \text{(14c)} \\ & r_{it} \geq q_{it} - q_{i(t-1)} : \overline{\gamma}_{it} & \text{(14d)} \\ & r_{it} \geq q_{i(t-1)} - q_{it} : \underline{\gamma}_{it} & \text{(14e)} \end{cases}$$

$$d_t = D_t^0 - \alpha p_t(q_{it}, q_{-it}) \quad \forall t \tag{15a}$$

$$d_t = \sum_i q_{it} \quad \forall t. \tag{15b}$$

Here (14b) defines the production bid limits, (14c) limits the bid on ramp rate. Equations (14d) and (14e) define ramping constraints for every two sequential time periods[3]. In this section we consider a general load profile, where demand can increase or decrease, and therefore both upward and downward ramping constraints are necessary. The variables, following after the colon are the Lagrange multipliers associated with the respective constraints.

We rewrite the optimization problems in (14a)-(15b) by their KKT conditions for all companies. Such equilibrium problem can also be formulated as a quadratic optimization problem, as shown in Barquin et al. (2004). Hence, the KKT conditions are also the optimality conditions. The resulting equilibrium problem is as follows:

$$\forall i: \begin{cases} \frac{\partial \mathcal{L}_i}{\partial q_{it}} = p_t(q_{it}, q_{-it}) - \theta_i q_{it} - c_i + \underline{\mu}_{it} - \overline{\mu}_{it} - \underline{\gamma}_{it} + \overline{\gamma}_{it} + \underline{\gamma}_{i(t+1)} & \\ \qquad - \overline{\gamma}_{i(t+1)} = 0 \quad \forall t & \text{(16a)} \\ \frac{\partial \mathcal{L}_i}{\partial r_{it}} = -c_i^R + \underline{\lambda}_{it} - \overline{\lambda}_{it} + \underline{\gamma}_{it} + \overline{\gamma}_{it} = 0 \quad \forall t & \text{(16b)} \\ 0 \leq \overline{\mu}_{it} \perp \hat{Q}_i - q_{it} \geq 0 \quad \forall t & \text{(16c)} \\ 0 \leq \underline{\mu}_{it} \perp q_{it} \geq 0 \quad \forall t & \text{(16d)} \\ 0 \leq \underline{\gamma}_{it} \perp r_{it} - q_{it} + q_{i(t-1)} \geq 0 \quad \forall t & \text{(16e)} \\ 0 \leq \overline{\gamma}_{it} \perp r_{it} + q_{it} - q_{i(t-1)} \geq 0 \quad \forall t & \text{(16f)} \\ 0 \leq \overline{\lambda}_{it} \perp \hat{R}_i - r_{it} \geq 0 \quad \forall t & \text{(16g)} \\ 0 \leq \underline{\lambda}_{it} \perp r_{it} \geq 0 \quad \forall t & \text{(16h)} \end{cases}$$

[3]We assume this constraint exists for the time period 2 onwards, as we do not take into account the production level preceding the first time step: $q_{i(t=1)-1}$.

$$D_t^0 - \alpha p_t(q_{it}, q_{-it}) - d_t = 0 \quad \forall t \tag{17a}$$

$$\sum_i q_{it} - d_t = 0 \quad \forall t. \tag{17b}$$

3.1.1. Resolution of a single-stage equilibrium model

As it was shown above, the formulated extended single-stage equilibrium model is convex. The introduced setup can be directly programmed as a mixed complementarity problem (MCP) in GAMS and solved until optimality with PATH solver (Dirkse and Ferris, 1995). It has been shown in Billups et al. (1997) that state-of-the-art solvers are able to solve large-scale MCP problems to global optimality in reasonable time, and therefore the problem is highly scalable.

3.2. Extended formulation of the bilevel equilibrium problem

In this section we propose the extension of the two-stage model proposed in Section 2.2 to a general case with multiple load periods and possibly asymmetric power producers. The bilevel model corresponds to the case, when generators first make their decisions regarding their ramp rates. After this information is revealed, generators participate in the second stage of the game, where they compete in quantities.

3.2.1. Formulation of the lower level

The lower level, or the production stage, represents the conjectured-price response market equilibrium. The market equilibrium problem can be written as:

$$\forall i : \begin{cases} \underset{q_{it}}{\text{maximize}} & \sum_t (p_t(q_{it}, q_{-it}) - c_i) q_{it} & (18a) \\ \text{subject to:} & \hat{Q}_i \geq q_{it} \geq 0 : \overline{\mu}_{it}, \underline{\mu}_{it} \quad \forall t & (18b) \\ & r_{it} \geq q_{it} - q_{i(t-1)} : \underline{\gamma}_{it} \quad \forall t & (18c) \\ & r_{it} \geq q_{i(t-1)} - q_{it} : \overline{\gamma}_{it} \quad \forall t & (18d) \end{cases}$$

$$d_t = D_t^0 - \alpha p_t(q_{it}, q_{-it}) \quad \forall t \tag{19}$$

$$d_t = \sum_i q_{it} \quad \forall t. \tag{20}$$

We rewrite the market equilibrium, and substitute the optimization problem (18a) by its KKT conditions for all companies. As before, as the structure of the problem is similar to the single-level problem solved before and to the capacity-expansion problem recast as a quadratic problem in Barquin et al. (2004), we can write the KKT conditions, which are also sufficient and necessary:

$$
\Omega_t^{LL}
\begin{cases}
\frac{\partial \mathcal{L}_i}{\partial q_{it}} = p_t(q_{it}, q_{-it}) - \theta_i q_{it} - c_i + \underline{\mu}_{it} - \overline{\mu}_{it} - \underline{\gamma}_{it} + \overline{\gamma}_{it} + \underline{\gamma}_{i(t+1)} \\
\quad -\overline{\gamma}_{i(t+1)} = 0 \quad \forall i & \text{(21a)} \\
0 \le \overline{\mu}_{it} \perp \hat{Q}_i - q_{it} \ge 0 \quad \forall i & \text{(21b)} \\
0 \le \underline{\mu}_{it} \perp q_{it} \ge 0 \quad \forall i & \text{(21c)} \\
0 \le \underline{\gamma}_{it} \perp r_{it} - q_{it} + q_{i(t-1)} \ge 0 \quad \forall i & \text{(21d)} \\
0 \le \overline{\gamma}_{it} \perp r_{it} - q_{i(t-1)} + q_{it} \ge 0 \quad \forall i & \text{(21e)} \\
D_t^0 - \alpha p_t(q_{it}, q_{-it}) - d_t = 0 & \text{(21f)} \\
\sum_i q_{it} - d_t = 0. & \text{(21g)}
\end{cases}
$$

This optimality conditions reflect the market equilibrium in the quantity games and complement the upper level, formulated below.

3.2.2. Formulation of the upper level

In the upper level, the generating companies maximize their profits, the outcome of the quantity game in the lower level, by setting the ramp-rate levels that will maximize it. The optimization problem of every generating company i^*, a member of set I, is therefore:

$$
\begin{aligned}
\underset{r_{i^*t}}{\text{maximize}} \quad & \sum_t (p_t(q_{it}, q_{-it}) - c_{i^*}) q_{i^*t} - c^R_{i^*} r_{i^*t} && \text{(22a)} \\
\text{subject to:} \quad & \hat{R}_{i^*t} \ge r_{i^*t} \ge 0 \quad \forall t && \text{(22b)} \\
& q_{i^*t} \in \Omega_t^{LL}. && \text{(22c)}
\end{aligned}
$$

The objective function of the lower level optimization problem (18a) and upper level ramp decision (22a)-(22b) are the same, but the problems are different because they consider different optimization variables. However, separating them to different levels represents the awareness of generators that their ramp bid decision will affect the market equilibrium happening in the lower level. In the next subsection we combine the two levels and form a mathematical program with equilibrium constraints (MPEC).

3.2.3. Reformulation as an MPEC

The ramp-bidding problem for a single generating company can be formulated as an MPEC by combining the upper level, derived in Section 3.2.2, with the lower level and market equilibrium conditions from Section 3.2.1. The corresponding bilevel program for generating company i^* optimizing over a set of variables $\Omega_{i^*} = \{r_{i^*t}, q_{it}, p_t(q_{it}, q_{-it}), d_t, \underline{\mu}_{it}, \overline{\mu}_{it}, \underline{\gamma}_{it}, \overline{\gamma}_{it}\}$ is as follows:

$$
\begin{aligned}
\underset{\Omega_{i^*}}{\text{maximize}} \quad & \sum_t \Big((p_t(q_{it}, q_{-it}) - c_{i^*}) q_{i^*t} - c^R_{i^*} r_{i^*t} \Big) && \text{(23a)} \\
\text{subject to:} \quad & \hat{R}_{i^*} \ge r_{i^*t} \ge 0 \quad \forall t && \text{(23b)} \\
& \hat{Q}_i \ge q_{it} \ge 0 \quad \forall (it) && \text{(23c)}
\end{aligned}
$$

$$r_{it} - q_{it} + q_{i(t-1)} \geq 0 \quad \forall(it) \tag{23d}$$

$$r_{it} - q_{i(t-1)} + q_{it} \geq 0 \quad \forall(it) \tag{23e}$$

$$\underline{\mu}_{it} \geq 0,\ \overline{\mu}_{it} \geq 0,\ \underline{\gamma}_{it} \geq 0,\ \overline{\gamma}_{it} \geq 0 \quad \forall(it) \tag{23f}$$

$$p_t(q_{it}, q_{-it}) - \theta_i q_{it} - c_i + \underline{\mu}_{it} - \overline{\mu}_{it} - \underline{\gamma}_{it} + \overline{\gamma}_{it} + \underline{\gamma}_{i(t+1)} - \overline{\gamma}_{i(t+1)} = 0 \tag{23g}$$

$$\underline{\mu}_{it} q_{it} = 0 \quad \forall(it) \tag{23h}$$

$$\overline{\mu}_{it}(\hat{Q}_i - q_{it}) = 0 \quad \forall(it) \tag{23i}$$

$$\underline{\gamma}_{it}(-q_{it} + q_{i(t-1)} + r_{it}) = 0 \quad \forall(it) \tag{23j}$$

$$\overline{\gamma}_{it}(q_{it} - q_{i(t-1)} + r_{it}) = 0 \quad \forall(it) \tag{23k}$$

$$D_t^0 - \alpha p_t(q_{it}, q_{-it}) - d_t = 0 \quad \forall t \tag{23l}$$

$$\sum_i q_{it} - d_t = 0 \quad \forall t. \tag{23m}$$

Here constraints (23b) correspond to the upper-level constraints, constraints (23c)-(23e) are the lower-level primal constraints. The corresponding Lagrange multipliers are nonnegative as outlined in the constraints (23f). Constraint (23g) is the stationarity condition. Conditions (23h) to (23k) are the complementary slackness conditions. Constraints (23l) and (23m) correspond to the equalities shared by all generators in the lower level.

The resulting problem is often called a mathematical program with complementarity constraints (MPCC), and is nonregular and nonconvex. With the state-of-the-art solvers (e.g., COUENNE), the problem can be attempted directly as a nonlinear problem, as in this paper. The solvers for nonlinear programs are usually based on branch-and-bound, branch-and-reduce, interior point method, or constraint qualification techniques. We control the solution of our nonlinear program by checking the optimality flag provided by GAMS. For Nash equilibrium we are interested in a point that is at least locally optimal for all generating firms.

3.2.4. EPEC

The entire extended bilevel equilibrium problem consists of the optimization problems, i.e. MPECs, of every unit i^* taken together, as shown in Figure 1. Therefore, MPEC derived in Section 3.2.3 can be used to obtain the solution for the whole two-stage equilibrium model. By using a diagonalization method (Gabriel et al., 2012) we can obtain an EPEC solution by solving the MPECs iteratively: we sequentially fix the strategic variables of all generators except of one, which is free to choose any level in response to the levels fixed for all other units. This procedure is repeated for all generators in loop until convergence to an equilibrium point, at which no producer wants to change the strategic decision unilaterally. Even though this method is widely used in literature, there is no guarantee for convergence.

EPEC problems are usually highly nonconvex – even if the diagonalization algorithm leads us to a solution, there is no guarantee that this solution is global. In the next subsection we explain the methods of exploring the solution space.

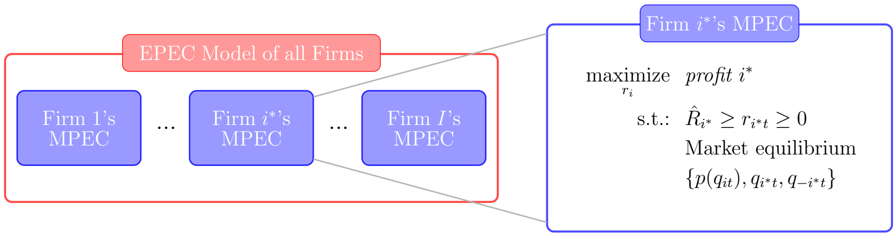


Figure 1: Equilibrium problem with equilibrium constraints (EPEC) is formed by solving the mathematical problems with equilibrium constraints (MPECs) of profit-maximizing producers together.

3.2.5. Multiple Nash equilibria

Since the MPEC is a nonconvex problem, it could have multiple local optima. As a result, EPEC could have several different equilibrium points. The problem of multiple Nash equilibria arising from solving an EPEC problem is addressed by many authors, and we do not focus on this in this research. Some of the methods proposed in the literature, which can be used to explore all Nash equilibria, are shortly discussed below:

- **Barrier function** can be used in the objective function of the problem constrained by the EPEC stationarity conditions (KKT conditions of respective MPECs taken together). The value of the barrier function on a point increases to infinity as the point approaches the boundary of the feasible region (Wright and Nocedal, 1999). The function is used as a penalizing term to prevent reaching an identical solution, so we can explore the feasible region further and enumerate all equilibria. Since the KKT conditions of the nonlinear problem (MPEC) can only provide a stationary point, the exploration process can be computationally very hard.

- **Hypersphear linear constraints** are introduced in the method proposed in Pozo and Contreras (2011). Following this method, the EPEC problem is run in a loop: for an every newly found equilibrium a new constraint is added, "cutting" this point out of the feasibility region. The authors show how in the case of binary decisions the added constraints are linear. In a general case the "hole" added is a quadratic constraint.

- **Discretization of the solution space** is a widely used method (Wogrin et al., 2013a, Hesamzadeh and Biggar, 2012) for finding the multiple equilibria. By recasting a continuous problem as a mixed-integer linear problem (MILP) we can guarantee finding all possible solutions. However, it is not always possible to discretize the problem and save the initial set of solutions. Also the complexity of a problem increases drastically with every additional binary variable, so higher precision discretization procedure is costly computationally.

The best choice of a strategy for finding all Nash equilibria depends on the exact problem under consideration and the size of the case study.

3.2.6. Resolution of a bilevel equilibrium model

The problem addressed in this paper has been solved using the iterative diagonalization method, as introduced earlier. Once diagonalization has converged we carry out an ex-post analysis to verify that we have found an equilibrium. This method is very convenient for larger case studies as it avoids formulating the entire EPEC. However, its outcome might depend on the provided initial point.

Other possible methods can include solving the whole EPEC formulation, and checking the second-order sufficient condition for each player's MPEC, as in Hu and Ralph (2007). Both ways, the scalability of the problem can be limited. One way to avoid the scalability difficulties is to combine Bertrand and Cournot models of competition, or to disregard some of the constraints (Neuhoff et al., 2005).

3.3. Firm ownership

Up until now we have considered a model where every strategic producer had a single production unit. In reality a firm can own several assets whose production levels can vary and whose costs are different as well.

To reformulate the problem we introduce a set F, representing the strategic firms owning portfolios of several units, and update the respective objective functions to consider the profit of a strategic portfolio. The objective functions in (14a) and (23a) become:

$$\pi_f = \sum_{i^* \in f,t} \Big((p_t(q_{it}, q_{-it}) - c_{i^*}) q_{i^*t} - c^R_{i^*} r_{i^*t} \Big). \tag{24}$$

Here, unit i^* belongs to the firm f and the profit is maximized for the whole firm.

3.4. Stochastic modeling

Real-world decision-making always involves a high level of uncertainty of the parameters. Different realizations of the demand, uncertainty in fuel costs, and other parameters can significantly affect the optimal decision. The uncertainty in wind-integrated power systems can be modeled with a set of scenarios W, where each scenario $w \in W$ has a certain probability $prob_w$. The profit formulation (24) introduced in the previous subsection is then extended as follows:

$$\pi_f = \sum_{i^* \in f,t,w} prob_w \Big((p_{tw}(q_{itw}, q_{-itw}) - c_{i^*}) q_{i^*tw} - c^R_{i^*} r_{i^*t} \Big). \tag{25}$$

Facing the uncertainty represented with a set of scenarios W, a generating company decides on the best level of ramp rate. Then, in each scenario a certain demand level is realized and the generating companies produce accordingly. We are hence assuming that in the energy market we know the demand level.

Note that we also add a nonanticipativity constraint to ensure that ramp rate cannot be changed for every realization of the scenario. Therefore, the strategic decision is taken before the uncertainty is realized:

$$r_{i,t,w} = r_{i,t,-w} = r_{it}. \tag{26}$$

Nonanticipativity constraint for two-stage model reflects that the ramp-rate decision has to be made before the uncertainty is realized, which agrees with the logic of the model. In one-stage model the ramp rate decision is made at the same time as production decision. This corresponds to different ramp-rate values in different scenarios. Therefore, we do not include a nonanticipativity constraint in the single-stage model.

4. Case study

The developed model can be a tool for analyzing how a market setup affects the flexibility bidding in the system. In this Section we check, whether Propositions 1.1 and 1.2, introduced in Section 2.3, are still valid for the extended version of the models by performing the simulations. We also provide the case studies for the extended model with assymetric generating companies, firms, owning several units, and for the stochastic case.

4.1. Comparison of the duopoly models

The duopoly case study lets us analyze the importance of information revealed in the ramp and quantity game. As derived in Section 2, the closed-form expression for the ramp-rate bids in the case of a single-level (SL) and bilevel (BL) optimization problem formulations are:

$$r_i^{SL} = \frac{D_{02} - D_{01}}{\alpha\theta + 2}, \tag{27a}$$

$$r_i^{BL} = \frac{D_{02} - D_{01}}{3}. \tag{27b}$$

As outlined in Propositions 1.1 and 1.2 in Section 2.3, we observe that $r^{SL} = r^{BL}$ if $\theta = 1/\alpha$, which, according to (Fudenberg and Tirole, 1989), corresponds to the case of Cournot competition. For the values of the conjectured price response corresponding to higher levels of competition ($\theta < 1/\alpha$) the optimal level of ramp rate is higher in the case of the single-stage model as opposed to the two-stage model.

To check this result we formulate the models explicitly as optimization problems and perform the simulations. In this simple case study the capacity and ramp limits are considered nonbinding. The production costs are symmetric and equal to 10 €/MW. We assume two load periods with the demand intercepts $D_t^0 = [200, 400]$ MW. The elasticity of demand $\alpha = 7.2$ MW^{2}h/€.

Figure 2 shows the simulation results for the different values of conjectured price parameter. We observe that the ramp-rate level in the single-stage model steadily decreases with a growing conjectured price response parameter until it reaches the value of a two-stage model bid in the point, which corresponds to the Cournot competition. The optimal level of ramp rate does not depend on how the unit perceives the competitiveness in the market. The producers prefer to withhold the flexibility and choose a relatively low level of ramp.

Figure 3-(a) is obtained by running the one-stage model for a fixed combination of ramp-levels of two players for an arbitrary level of the conjectured price response parameter $\theta = 0.05$ €/MW^{2}h. Since the companies are symmetric, we can expect the equilibrium to lie

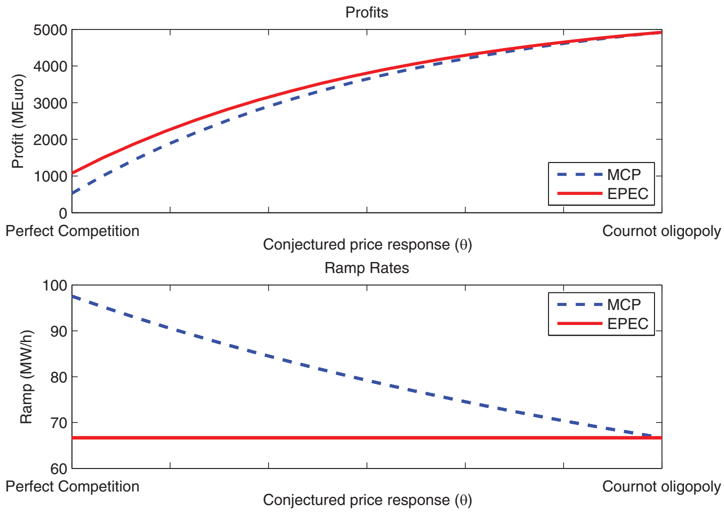


Figure 2: Single-stage and two-stage equilibrium solutions for a system with two symmetric generating companies.

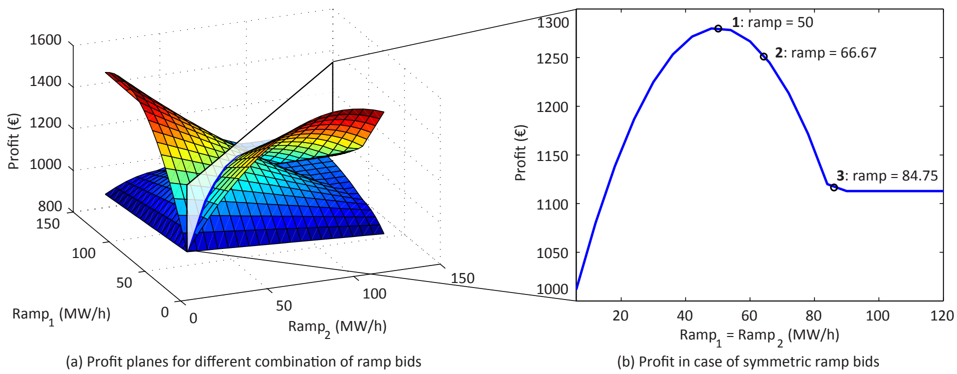


Figure 3: Symmetric duopoly case study results.

on the middle line, where the bids of the both players are equal. This line is shown in Figure 3-(b). We observe that the highest profits for the both units are reached when $r_1 = r_2 = 50$ MW/h (point **1** in Figure 3-(b)). This solution would be a stable point if the producers were cooperating, as it maximizes the sum of their profits. However, as we can see in Figure 4, in which a profit curve of a single producer is shown for the ramp-rate level of other player fixed, a generator can obtain a higher profit by unilaterally changing his level of the ramp rate (which can be visualized by moving along the x axis of the plane following the arrow).

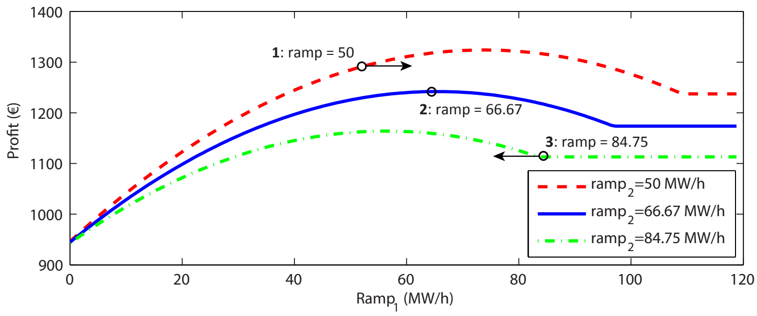


Figure 4: Stability of point **2** as an equilibrium point

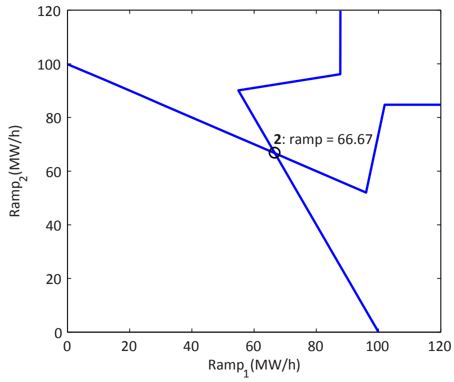


Figure 5: Reaction functions of both of the players. The intersection of the reaction function – Nash equilibrium.

A response to such strategy will be an increase of a ramp-rate level by the second producer. An equilibrium point is finally reached when no unit wants to change its bid unilaterally. In Figure 4 we can see that point **2** coincides with the point of the highest profit for a given combination of the ramp rates. This logic illustrates reaching a Nash equilibrium between strategic producers.

We can obtain the same point by fixing the decision of one of the players and observing the response, or reaction, of the other player. By repeating this for a range of values and for both players we obtain reaction functions, shown in Figure 5. The reaction functions intersect in the only point, which is a Nash equilibrium (point **2** in Figures 3-(b), 4, and 5). In this simple case of smooth profit functions we can prove numerically that this point is the only Nash equilibrium.

Point **3** corresponds to the solution of the single-stage model for the given choice of parameter θ. We can see from Figure 3 that this point offers the highest flexibility in ramp. Since the producers are competing in quantities and ramps at the same time, they choose to leave the ramp-rate level flexible, so they can react to the competitors' strategies by withholding or providing more production to the market.

The equivalence of the single-stage and two-stage models for the choice of the conjectured price response representing Cournot competition can be found in some other games. In (Wogrin et al., 2013b) the authors study the capacity-investment problem and show that for the case of a one load-period game and 2 symmetric producers the investments are generally higher for the single-stage problem, converging to the same value when competition structure is approaching Cournot.

The above study confirms an intuition that market design impacts the generators' behavior and the social welfare. A two-stage setup, which may seem fair intuitively, can lead to higher withholding and therefore social welfare losses.

4.2. Modeling duopoly over a 24-hour horizon

We extend the proposed models to consider the 24-hour modeling period. For a realistic demand profile, we obtain real Swedish hourly demand data for the 17th October 2016 and scale it to fit the duopoly case study (Nord Pool, 2016). We compare two models: in a one-stage model strategic firms choose their ramp and production level every time period, in a two-stage model companies choose their ramp rate for the whole time horizon, and then choose their production level at each time step.

The results of simulations are presented in Figure 6. For the single-stage model we plot the maximum ramp rate offered through the time horizon. We observe that Proposition 1.2 holds for this case study of a longer modeling horizon. The ramp rates offered in a one-stage model are higher, than the ones offered in two-stage model. The ramp rates coincide, when competition level corresponds to Cournot oligopoly. The profits of generating companies are higher in the two-stage model.

4.3. Asymmetric duopoly

In this section we verify that the results of Propositions 1.1 and 1.2 in Section 2.3 hold for the case of asymmetric generating companies. Figure 7 shows the simulation results for two cases: (a) with marginal costs $c_1 = 5$ and $c_2 = 7$ €/MW (quasi-symmetric firms), and (b) with marginal costs $c_1 = 5$ and $c_2 = 15$ €/MW (asymmetric firms). We assume two load periods with the demand intercepts $D_t^0 = [200, 400]$ MW. The elasticity of demand $\alpha = 7.2$ MW^{2}h/€.

In Figure 7 we first observe that in both of the cases $r^{SL} = r^{BL}$ if $\theta = 1/\alpha$ (under Cournot). For the values of the conjectured price response corresponding to higher levels of competition ($\theta < 1/\alpha$), the optimal level of ramp rate is higher in the case of the single-stage model as compared to the two-stage model. These findings support the Proposition 1.2 even for quasi-symmetric and asymmetric firms.

4.4. Firm ownership

We study the effect of having multiple units merged into one portfolio by considering a small case study. The system consists of two firms, owning two units each: a cheap unit with $c_1 = 5$ €/MW and an expensive one with $c_2 = 10$ €/MW. The ramp-rate limits are equal

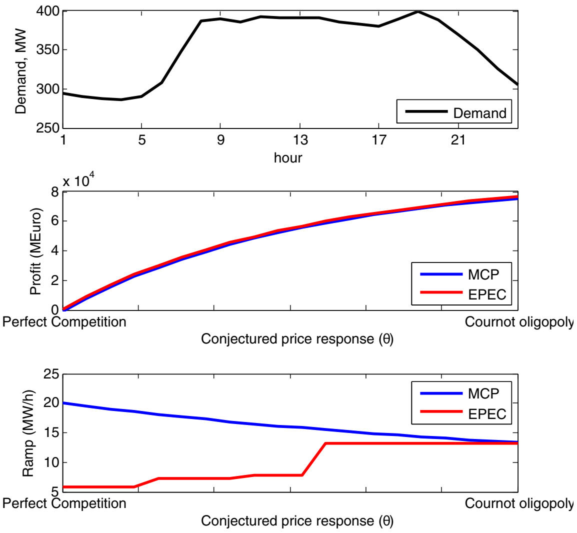
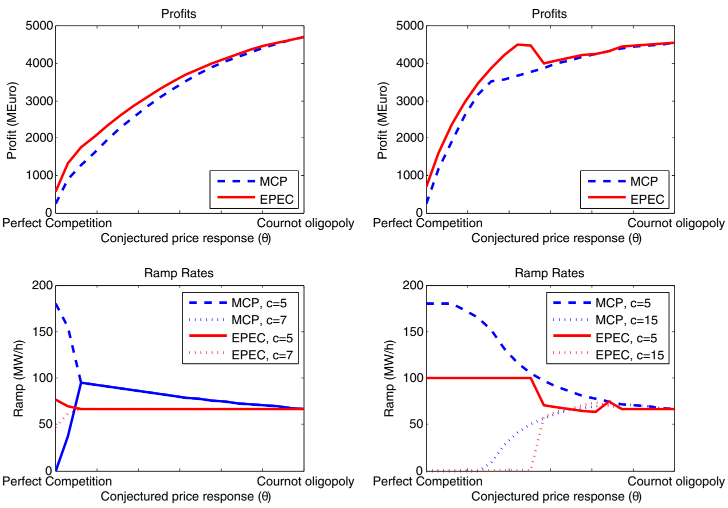


Figure 6: Single-stage and two-stage equilibrium solutions for a system with two asymmetric generating companies and 24 hours modeling horizon.



(a) Generating companies with marginal costs 5 and 7 €/MW (quasi-symmetric case)

(b) Generating companies with marginal costs 5 and 15 €/MW (asymmetric case)

Figure 7: Single-stage and two-stage equilibrium solutions for a duopoly system with quasi-symmetric and asymmetric generating companies.

to $\hat{R}_i = 140$ MW/h and the capacity limits are considered nonbinding in this particular case in order to facilitate the interpretation of the results.

Figure 8 demonstrates the advancement of the ramp-rate levels for this case. The ramp-rate levels follow a pattern similar to the case of duopoly, studied in Section 2. In case of two-stage bidding there are two flat lines corresponding to two different choices of ramp-rate level, depending on the costs of the units. The cheap unit maintains the ramp rate at the level $r = 66.67$ MW/h, while the level of the second unit is zero. The generating company chooses a zero flexibility of the expensive unit, so it is dispatched in both time periods – when demand is lower, and when it increases. In case of simultaneous ramp and quantity bidding generators steadily decrease the ramp rate of a cheaper unit and increase the ramp rate of a more expensive unit with increasing θ. The more market power, the more generating companies restrict their cheaper units in order to increase their profits, when the expensive units are dispatched. The rates converge to a single value when the market structure becomes less competitive. At this point the generators maximize their profits by withholding their capacity and flexibility, so the price increases in both of the models. It is worth noting that the final value of the single-stage bidding is similar to the sum between the bids of two units in the two-stage bidding model.

We observe that even in the extended case with firms owning multiple units, we have similar trends as in the simplified version, shown in Section 2. In particular, given that the firms conjecture only on energy, it can be observed that the levels of ramping flexibility do not depend on strategic spot market behavior in the bilevel model, which supports our findings that a sequential-bidding market structure allows generating companies to exert

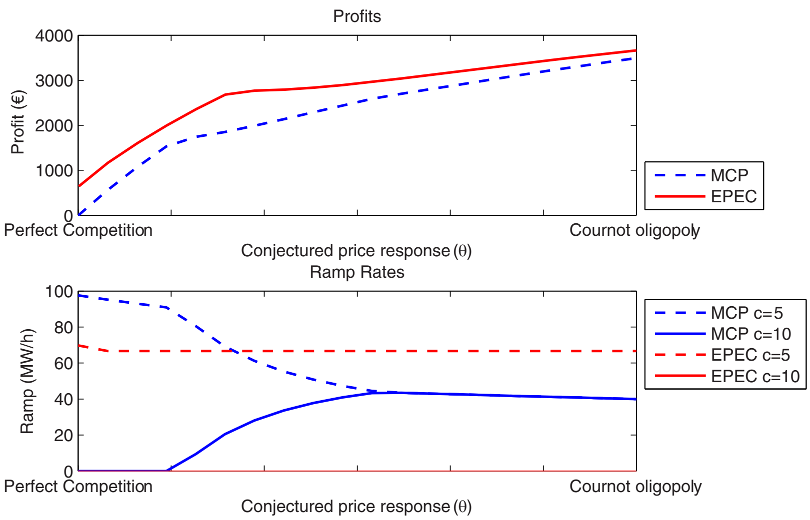


Figure 8: Single-stage and two-stage solutions for a system of two symmetric competing firms with portfolios of 2 units.

Table 1: Social welfare calculations.

Case study		Perfect competition		Cournot	
		1-stage	2-stages	1-stage	2-stages
	Total profits, €+	0	617	4 919	4 919
Duopoly case	Consumer surplus, €	22 137	20 594	9 839	9 839
	= Social welfare, €	22 137	21 211	14 758	14 758
	Total profits, €+	0	617	3 494	3 667
Firm ownership	Consumer surplus, €	22 137	20 594	12 536	11 993
	= Social welfare, €	22 137	21 211	16 030	15 660

market power to a larger extent than the simultaneous-bidding market structure. We observe that the difference between the total ramping level is maximum at perfect competition and decreases towards the values of θ representing Cournot. The single-level model yields profits that are always below the profits obtained in the bilevel model, which indicates that even in this extended case the sequential-bidding market structure allows for the exercise of market power of generating companies.

It should be noted that, while the single-stage model can be solved to the global optimality, the result of a two-stage model is a Nash equilibrium (as opposed to a local/global optimum), a point at which no company wants to deviate unilaterally by modifying its strategy. There could be multiple Nash equilibria and the solution algorithm can converge to any of them. However, from what we have observed in our case study, we always converge to the same equilibrium independent from the starting point, which might suggest that the equilibrium is unique.

Table 1 provides a comparison of social welfare for the case studies above. We observe that separating the stages of deciding the flexibility and production levels results in a lower social welfare. For duopoly the values coincide at Cournot. We can also observe that the social welfare component of companies' profit increases, when the level of competitiveness reaches Cournot, while the consumer surplus decreases considerably.

4.5. Stochastic case study

Authors in Oren and Ross (2005) find that by letting generators restate their ramp rates only once in a longer time period, policy makers can prevent them from responding

Table 2: Optimization scenarios

Model	Scenario probability, p.u.				$D^0_{t_1}$, MW				$D^0_{t_2}$, MW			
Deterministic	1				700				1000			
Stochastic	s1:	s2:	s3:	s4:	s1:	s2:	s3:	s4:	s1:	s2:	s3:	s4:
	0.25	0.25	0.25	0.25	700	900	500	700	1000	1000	800	1200

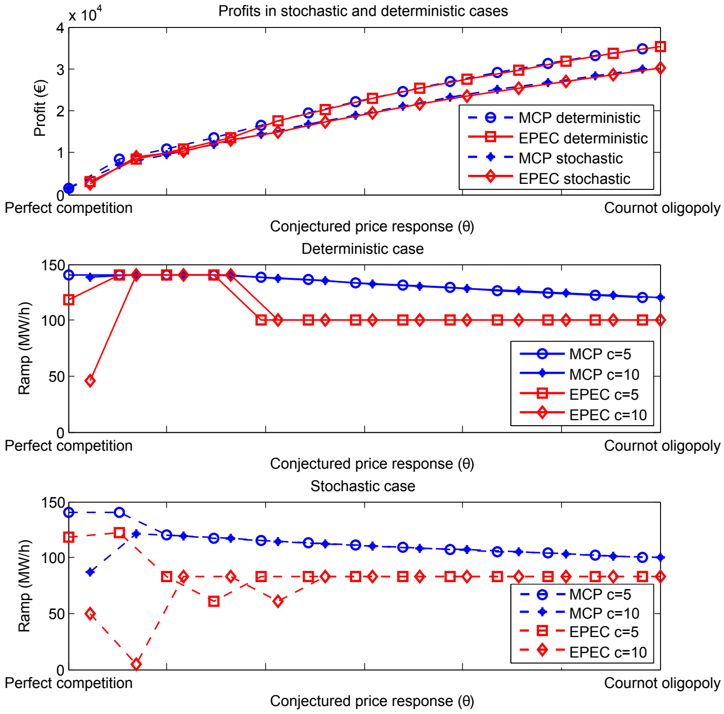


Figure 9: Comparison of the stochastic and deterministic cases

to market conditions with false ramp rates. In this case study we show that uncertainty, and in particular, wind uncertainty may actually lead to the opposite results in certain circumstances. This means that there are numerical examples, where uncertainty leads to lower levels of flexibility than the deterministic case. In Table 2 we present the demand data used for this case study. Note that the deterministic data corresponds to the average of all stochastic scenarios. Figure 9 shows the comparison of ramp-rate levels obtained in the deterministic and stochastic cases (for single-stage MCP model we plot the maximum value of ramp rate in scenarios). We observe that while the expected value of the demand intercept in the stochastic scenarios is equal to the one in the deterministic case, the bids follow a very different pattern. Several observations can be made:

- Counter-intuitively, we can observe that stochasticity yields smaller total ramping levels than the deterministic scenario in both two- and single-stage models – this can be explained by the fact that there is a chance of getting higher profit from the expensive unit, if the high-demand scenario occurs. In this case study, the generating companies facing the uncertainty choose to withhold their ramp rates to ensure a certain level of profits in every scenario. However, we should note that it is not the general case, and the results may be different depending on the parameters of the model;

- The initial gap for the low values of the conjectured price response corresponds to the situations close to perfect competition, when the less expensive generator provides the whole capacity to the market, so the second generator is not expected to ramp;
- The kinks in the simulations of the bilevel model can be explained by the solution procedure, as the EPEC is solved via diagonalization. While in a single-level model the result is unique, the two-level model can often have several Nash equilibria as solutions (several points at which no company wants to deviate unilaterally). This is a reason for the jumps in the optimal ramping level. In such case, when there are multiple equilibria, they can be evaluated based on criterion (e.g., social welfare - best/worst Nash equilibrium as discussed in (Hesamzadeh and Biggar, 2012)) to choose the equilibrium that yields the best result with respect to this criterion.
- Apart from these particularities that happen close to perfect competition, the ramping flexibility levels follow the same trend as in the duopoly case. The two-stage model levels, for the most part, do not seem to be affected by competitive market behavior. Moreover, the ramping levels obtained in the single-level model under Cournot competition, and the bilevel ramping levels for arbitrary market competition seem to converge to similar values.

These observations allow us to conclude that the outcome similarity of the single- and two-stage models at Cournot holds even for the case, when we consider the portfolio bidding and uncertainty. We also observe that in this particular case the ramp-rate levels in case of simultaneous bidding are steadily higher than in the case of the separated bidding stages.

It is worth noting that in the stochastic case the results do not show the same stability as in two previous case studies. For example, we can find cases, when the social welfare is higher in two-stage models, because it is more profitable for companies to produce more, so the prices decrease. The intuition for these observations is possibly in the structure of the stochastic model. In the stochastic two-stage model ramp rate is declared before the uncertainty is realized and production takes place. Therefore, the ramp rate constraint does not hold as an equality between ramp rate and the difference in the production, and ramp rate cannot be always used by strategic generators to affect the production levels. However, the existence of the cases that the generators choose to limit their ramp rates even when facing the uncertainty questions considerably the logic of separating the stages of deciding the production and flexibility levels.

Figure 10 shows the bidding behavior of the producers when exposed to the stochastic scenarios with different demand patterns – increasing, decreasing, or mixed demand pattern scenarios. We observe that the behavior in cases of increasing and decreasing demand is absolutely symmetric. In the case of mixed demand patterns we have two different values for ramp bid – ramping-up bid and ramping-down bid. If we force the bids to be equal the bidding pattern will be the same as in cases a) and c).

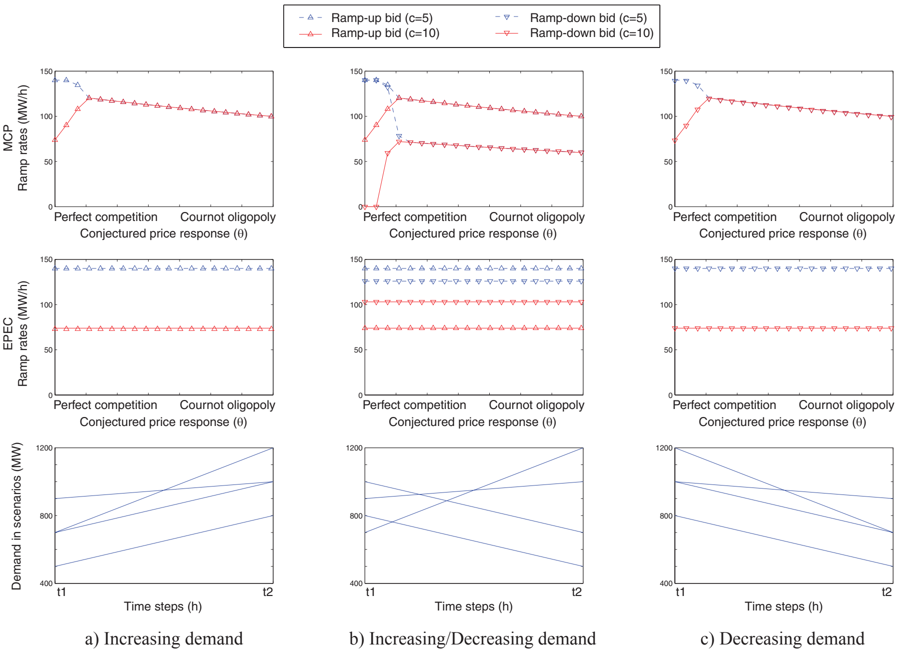


Figure 10: Comparison of stochastic scenarios with different demand patterns.

5. Conclusions

Generation-side flexibility becomes increasingly important as the share of wind generation becomes bigger and there are frequent swings in generation supply. In this paper we present how generating companies decide ramp-rate flexibility and production in a one-stage and a two-stage processes, and we analyze the impact of these different market setups on the results.

We present two models – a single- and a bilevel model – to represent two different types of market setups: bidding the ramp and quantity levels simultaneously; or doing it in two stages, where the generators choose their ramp rates in the first stage, and compete in quantities in the second stage. A market structure, where the bids on ramp rate are submitted before the actual market clearing happens is a common regulatory practice, aiming to minimize the manipulation on the ramping constraints. In this paper we show that such a regulatory intuition can actually lead to a higher level of withholding. We provide a comparison for a range of different conjectured price responses, capturing company's beliefs regarding its influence on market prices.

In the case of duopoly, the observed results show that for the markets that are more competitive than Cournot oligopolies separating the flexibility from production decisions leads to a higher level of withholding. Essentially, when ramp rate and production are decided simultaneously then firms need to set ramping capability as large as required by the market. However, if ramping capability is decided previously (as in the two-stage model), then once the market stage happens, generation companies are "limited" by the ramping levels previously decided – which can of course be less than what would be required by the market. Therefore, a strategy to separate the stages of choosing the flexibility and production results in higher profits for the companies but the social welfare decreases considerably, contrary to the regulatory practices, used by the NYISO (NYISO, 2013), or in California (CAISO, 2009).

We extend the model to an arbitrary number of load periods and include uncertainty. We observe many of the duopoly-case observations to hold in the extended single-level and bilevel equilibrium models. Even in the general case separating the stages of ramp rate and quantity bidding does not necessarily lead to a higher social welfare. The proposed model shows that for a market, the structure of which is more competitive than Cournot oligopoly, it may be advantageous to allow simultaneous bidding of ramp rates and quantities. We observe that otherwise, the producers are likely to exercise their market power in the pre-commitment stage as such strategy locks higher profits in the second stage.

Future work can include introducing a concept of conjectural variations in ramp rate. If system operator recognizes the problem of strategic behavior in ramp rates, when declared before the actual moment of production, we need to study, what are the possible implications, and how we can limit this new type of strategic behavior. Including this concept will move the analysis to studying the Generalized Nash Equilibrium.

Acknowledgements

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Appendix A. Basic market equilibrium as an optimization problem

The one-stage equilibrium model of simultaneous ramp rate and quantity bidding presented in Section 2.1 can also be formulated as an optimization model:

$$\begin{aligned}
\underset{q_i, r_i, d_t}{\text{minimize}} \quad & \sum_{it} \Big(c q_{it} + \frac{1}{2}\theta q_{it}^2 - \frac{1}{\alpha}\Big(D_t^0 d_t - \frac{d_t^2}{2} \Big) \Big) \\
\text{subject to:} \quad & \hat{Q} \geq q_{it} \geq 0 \quad : \overline{\mu}_{it}, \underline{\mu}_{it} \forall t \\
& \hat{R} \geq r_i \geq 0 \quad : \overline{\lambda}_i, \underline{\lambda}_i \\
& r_i \geq q_{it} - q_{i(t-1)} \quad : \overline{\gamma}_{it} \\
& q_{i(t-1)} - q_{it} \geq r_{it} \quad : \underline{\gamma}_{it} \\
& d_t = \sum_i q_{it} \quad : p_t
\end{aligned}$$

It can easily be verified that the KKT conditions of this optimization problem are equivalent to (4).

Appendix B. Supplementary mathematical derivations

We can reformulate problem (1)-(2), as if only the production variable is strategic:

$$\forall i : \begin{cases} \underset{q_i}{\text{maximize}} & (p_{t_1}(q_{it_1}, q_{-it_1}) - c)q_{it_1} + (p_{t_2}(q_{it_2}, q_{-it_2}) - c)q_{it_2} & \text{(B.1a)} \\ \text{subject to:} & \hat{Q} \geq q_{it} \geq 0 \quad : \overline{\mu}_{it}, \underline{\mu}_{it} \quad \forall t & \text{(B.1b)} \\ & r_i \geq q_{it_2} - q_{it_1} \quad : \overline{\gamma}_i & \text{(B.1c)} \\ & q_{it_1} - q_{it_2} \geq r_i \quad : \underline{\gamma}_i. & \text{(B.1d)} \end{cases}$$

$$\text{ME}: \ d_t = \sum_i q_{it}, \quad d_t = D_t^0 - \alpha p_t(q_{it}, q_{-it}) \ \ \forall t. \tag{B.2}$$

The KKT conditions of this problem can be formulated as follows:

$$\forall i : \begin{cases} \frac{\partial \mathcal{L}_i}{\partial q_{it_1}} = p_{t_1} - \theta q_{it_1} - c + \underline{\mu}_{it_1} - \overline{\mu}_{it_1} + \overline{\gamma}_i - \underline{\gamma}_i = 0 & \text{(B.3a)} \\ \frac{\partial \mathcal{L}_i}{\partial q_{it_2}} = p_{t_2} - \theta q_{it_2} - c + \underline{\mu}_{it_2} - \overline{\mu}_{it_2} - \overline{\gamma}_i + \underline{\gamma}_i = 0 & \text{(B.3b)} \\ 0 \leq \underline{\mu}_{it} \perp q_{it} \geq 0 \ \ \forall t & \text{(B.3c)} \\ 0 \leq \overline{\mu}_{it} \perp \hat{Q} - q_{it} \geq 0 \ \ \forall t & \text{(B.3d)} \\ 0 \leq \overline{\gamma}_i \perp r_i - q_{it_2} + q_{it_1} \geq 0 & \text{(B.3e)} \\ 0 \leq \underline{\gamma}_i \perp r_i - q_{it_1} + q_{it_2} \geq 0 & \text{(B.3f)} \end{cases}$$

$$d_t = \sum_i q_i, \quad d_t = D_t^0 - \alpha p_t \ \ \forall t. \tag{B.4}$$

With the same assumptions as in Section 2.1, KKT conditions (B.3)-(B.4) coincide exactly with KKT conditions (4)-(5) and the same expressions (6) can be obtained.

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Strategic Bidding of a Hydropower Producer under Uncertainty: Modified Benders Approach

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***Abstract*—This paper proposes a stochastic bilevel program for strategic bidding of a hydropower producer. The price, quantity and ramp-rate bids are considered. The uncertainty of wind power generation, variation of inflows for the hydropower producer and demand variability are modeled through the moment-matching scenario generation technique. Using discretization the stochastic bilevel program is reformulated as a stochastic mixed-integer linear program (MILP) with disjunctive constraints. We propose a modified Benders decomposition algorithm (MBDA), which fully exploits the disjunctive structure of reformatted MILP model. More importantly, the MBDA does not require optimal tuning of disjunctive parameters and it can be efficiently parallelized. Through an illustrative 5-node example, we identify possible strategies (specific to a hydropower producer) for maximizing profit, which in turn leads to market insights. We also use the IEEE 24-node, 118-node and 300-node case studies to show how our parallelized MBDA outperforms the standard Benders decomposition algorithm. The parallelized MBDA is also compared to the state-of-the-art CPLEX solver.**

***Index Terms*—Stochastic bilevel program, disjunctive constraint, Modified Benders decomposition algorithm**

NOMENCLATURE

The main notation is presented below for a quick reference. Additional symbols are introduced throughout the text.

Indices

i	Generating units
t	Time periods
l	Transmission lines
n	System nodes
k	Bid alternatives
w	Stochastic scenarios
j	Benders iterations

Sets

I	Generating units
I^h	Strategic hydropower units, $I^h \subset I$
I^{up},I^{dn}	Upstream and downstream units, I^{up},$I^{dn} \subset I^h$

Parameters (upper-case letters)

M_i^0	Initial water level of unit $i \in I^h$, m^3
M_i	Maximum water level of unit $i \in I^h$, m^3
V_{itw}	Inflow to the reservoir of unit $i \in I^h$ in t, m^3
Γ_i	Production equivalent of unit $i \in I^h$, m^3/MWh
Q_i	Maximum generation of unit i, MW
R_i	Maximum ramp rate of unit i, MW/h
C_i	Maximum price bid of unit $i \in I^h$, \$/MWh
C_i^M	Marginal cost of unit i, \$/MWh
Λ_i^f	Future price for unit $i \in I^h$, \$/MWh
P_w	Probability of scenario w
H_{ln}	Power Transfer Distribution Factor
F_l	Power flow limit on line l, MW
D_{ntw}	Demand at node n in t, MWh

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Variables (lower-case letters)

π	Profit of strategic hydropower producer, \$
q_{itw}	Dispatched generation of unit i in t, MWh
s_{itw}	Spillage of unit $i \in I^h$ in t, m^3
m_{itw}	Water level of unit $i \in I^h$ in t, m^3
x_{itk}^q	Binary variable of quantity bidding decision
x_{ik}^r	Binary variable of ramp-rate bidding decision
x_{itk}^c	Binary variable of price bidding decision
$\hat{q}_{it}$	Quantity bid of unit i in t, MW
$\hat{r}_i$	Ramp-rate bid of unit i, MW/h
$\hat{c}_{it}$	Price bid of unit i in t, \$/MWh

Lagrange multipliers (LM)

$\mu_{itw}^{A_1}$, $\nu_{itw}^{A_1}$	LM of unit i lower capacity limit constraint
$\mu_{itw}^{A_2}$, $\nu_{itw}^{A_2}$	LM of unit i upper capacity limit constraint
λ_{tw}^B, ϕ_{tw}^B	LM of energy balance constraint
μ_{ltw}^C, ν_{ltw}^C	LM of the flow constraint of line l
$\mu_{itw}^{D_1}$, $\nu_{itw}^{D_1}$	LM of unit i ramp-up constraint
$\mu_{itw}^{D_2}$, $\nu_{itw}^{D_2}$	LM of unit i ramp-down constraint
μ_{itw}^E, ν_{itw}^E	LM of spillage nonnegativity constraint
$\mu_{itw}^{F_1}$, $\nu_{itw}^{F_1}$	LM of lower water-level-limit constraint
$\mu_{itw}^{F_2}$, $\nu_{itw}^{F_2}$	LM of upper water-level-limit constraint
λ_{itw}^G, ϕ_{itw}^G	LM of water balance constraint

Operator

$\mathbf{I}$(*condition*) "If" operator

I. INTRODUCTION

ABOUT 16% of the electricity in the world is generated by hydropower producers. The annual hydropower share in electricity generation in Norway is 95-99%. It is also high in Brazil (80%), Iceland (88%), New Zealand (65%), Austria (70%), Canada (62%) and Sweden (42%) [1]. Even though hydropower producers have such an important market share, they have been traditionally regarded as price-takers, optimizing their production schedule based on the price expectation. However, the price-taking assumption for a hydropower producer does not always hold [2]. Reference [3] shows that in hydrothermal systems the strategic behavior can lead to significantly higher prices. In winter 2002-2003, extraordinary high prices were observed in Nordic market. A number of observers voiced the concern that the high prices were a result of strategic behavior by hydropower producers. The hydropower producers may have used an opportunity to spend more stored water in summer in order to create scarcity and, therefore, high prices, in winter [4].

Hydropower producers can use their unique characteristics (such as a capability to store energy, hydrological coupling between units and near-zero marginal cost) to behave strategically in the market [1], [5]. With an increasing penetration of wind and solar generation, hydropower producers can ramp fast to cover the generation fluctuations. Withholding such ramping capability adds another dimension to the strategic behavior of hydropower producers [6].

The lack of literature on hydropower producer's strategic behavior is partially explained by the difficulty of modeling hydro-dominated systems. The model often includes nonlinearities [7] or exponentially increasing number of scenarios [8]. There are only a few works that consider price-making hydropower producers, but they significantly simplify the modeling, leaving out some important aspects. The authors in [3] perform a market power assessment in hydrothermal systems by taking into account residual demand curve (RDC) and ignoring the uncertainty for the sake of tractability. This approach leaves out the effect of transmission congestion and possibly results in misleading optimal dispatch. A similar RDC approach is employed in [9] for a single hydropower producer, resulting in the same drawbacks. References [10] and [11] ignore hydro-specific constraints such as cascaded configuration and hydrological balance in their analysis. Other works have only looked at very small case studies, as for example duopoly in [12]. The authors in [13] take into account a single price-making hydropower producer. While having a sufficiently complete modeling of hydro-specific constraints, the study does not take into account transmission constraints.

Strategic behavior in power systems is often a consequence of an advantageous position of one, or several strategic power producers [2]. There is a number of models, created in order to reflect limited competition in the market. Using the game-theoretic concepts, the situation with multiple profit-maximizing firms can be modeled as Cournot [14] or Bertrand [15] games. However, in our studies we focus on the advantageous position of a single hydropower generating firm. This firm can be thought as a leader of Stackelberg game [16]. The leader chooses its bids based on what it anticipates the other firms and the system operator would do [17]. The Stackelberg model closely reflects the bilevel nature of strategic interaction in electricity markets and allows to model the strategic position of hydropower producer relative to other market participants.

Accordingly, the current paper proposes a stochastic bilevel program for analyzing the strategic bidding of a hydropower producer. The program has a Stackelberg leader-follower structure. The upper level is a price-making hydropower producer bidding on price, quantity and ramp rate. The lower level is the optimal dispatch with explicit modeling of opportunity cost. We take into account constraints specific to the hydropower producer, such as hydrological coupling of the reservoirs, spillage and water level constraints. We do not take into account the start-up and shut-down costs of the generators.

Uncertainty is a crucial element, which should be considered for the model of hydropower producer. Uncertainty in inflows and demand determines the bidding behavior of strategic hydropower producer. The wind uncertainty determines the amount of ramping needed in the system and the amount of cheap wind power, available for the dispatch. Therefore, we use stochastic programming to reflect the process of decision-making under uncertainty. There are several ways to represent that the exact values of the crucial parameters are not available for the decision-maker at the time of decision. Possible techniques include chance constraints or robust formulation [18]. We use scenario representation to model the uncertainty in the realization of some specific parameters. The stochasticity of demand, inflows and wind generation is modeled through moment-matching scenario generation technique [19]. In contrast with other methods, such as e.g. conditional sampling, moment-matching technique takes into account the crucial statistical characteristics of the probability distribution functions and the correlations between uncertain parameters [20].

Solving stochastic MPECs is a difficult numerical problem. Some of the solution approaches include implicit nonsmooth approaches, piecewise sequential quadratic programming, or perturbation and penalization methods [21]. However, nonlinear methods do not guarantee a global solution, and they are usually computationally intensive. In this paper we use discretization to reformulate the stochastic bilevel program as a stochastic mixed-integer linear program (MILP) with disjunctive constraints [22]. We assume that strategic hydropower producer can choose between discrete levels of capacity, price, or ramp bid. Resulting problem can be solved to the global optimum. There are two main approaches to solving a MILP: (1) Solving directly (monolith solution) or (2) Applying the decomposition technique.

Using the first method, the MILP can be solved directly using state-of-the-art commercial solvers. MILP solvers are usually based on modern modifications of grid search algorithm [23], simplex-like method [24], or branch and bound [25]. CPLEX solver uses a branch and cut algorithm which solves a series of LP subproblems. The realization of CPLEX solver in GAMS supports parallel computing technique, which means that the branch and cut procedure can be performed on several threads. CPLEX solver was shown to be the most efficient solver for finding an optimal solution [26]. However, a single stochastic mixed integer problem generates many LP subproblems, therefore, even small mixed integer problems can be very computationally intensive and require significant amounts of physical memory.

Decomposition techniques were historically created to provide a faster solution to MILPs and overcome the memory requirements. Using Benders decomposition technique, the problem can be separated into a master problem (a mixed-integer program) and a collection of subproblems (linear programs), representing stochastic scenarios. The solution is based on the sensitivities of the subproblems to the candidate solutions from master problem. Benders decomposition guarantees a globally optimal solution for MILP [27]. In this paper, the modified Benders decomposition algorithm (MBDA) is proposed to solve the stochastic MILP model. The MBDA retains the global optimality property of Benders procedure and exploits the disjunctive structure of the stochastic MILP model. It does not require optimal tuning of the disjunctive parameters and it can be efficiently parallelized. We apply state-of-the-art modeling modifications [28], in order to make the decomposition computationally efficient.

Through an illustrative 5-node example, we carefully study the market aspects of strategic bidding of a hydropower producer. The IEEE 24-node, 118-node and 300-node systems are used to show the computational efficiency of the parallelized MBDA. We show through the numerical results that the parallelized MBDA outperforms the standard Benders algorithm for large case studies. We have also compared the parallelized MBDA to the solution of a monolith problem by the state-of-the-art CPLEX solver.

Therefore, the contributions of this paper are: (1) It develops a stochastic MILP model for analyzing the strategic behavior of a hydropower producer with endogenous electricity price formation and explicit modeling of opportunity costs; (2) The parallelized MBDA is proposed to solve the stochastic MILP.

The paper is organized as follows. Section II presents the stochastic bilevel model and its reformulation as a stochastic MILP. Section III provides the modified Benders decomposition algorithm. Section IV illustrates the hydro-specific strategies in electricity markets. The computational properties of the parallelized MBDA are detailed in Section V. Finally, Section VI concludes the paper. Supplementary mathematical expressions are provided in Appendices A, B, and C.

II. The mathematical model (stochastic MILP)

The price-making hydropower producer is modeled in (1).

$$\underset{\hat{q}_{it},\hat{r}_i,\hat{c}_{it}}{\text{maximize}}\,\pi = \sum_{(i\in I^h),w} P_w\Big(\sum_t(\lambda_{ntw}-C_i^M)q_{itw} + m_{iTw}\Lambda_i^f\frac{1}{\Gamma_i}\Big) \quad \text{(1a)}$$

$$\text{subject to: } 0\le\hat{q}_{it}\le Q_i,\ 0\le\hat{r}_i\le R_i,\ 0\le\hat{c}_{it}\le C_i,\quad \forall(i\in I^h)t, \quad \text{(1b)}$$

$$\text{where } \lambda_{ntw}, q_{itw}, s_{itw}, m_{itw}\in \arg\Big\{\underset{q_{itw},s_{itw},m_{itw}}{\text{minimize}}\sum_{i,t,w}P_w\hat{c}_{it}q_{itw} \quad \text{(1c)}$$

$$\text{subject to: } 0\le q_{itw}\le\hat{q}_{it}\quad:\mu^{A_1}_{itw},\mu^{A_2}_{itw},\ \forall i, \quad \text{(1d)}$$

$$\sum_i q_{itw}=\sum_n D_{ntw}\quad:\lambda^B_{tw},\ \forall tw, \quad \text{(1e)}$$

$$\sum_n H_{ln}(\sum_{i:n}q_{itw}-D_{ntw})\le F_l\ :\mu^C_{ltw},\ \forall ltw, \quad \text{(1f)}$$

$$-\hat{r}_i\le q_{i(t-1)w}-q_{itw}\le\hat{r}_i:\mu^{D_1}_{itw},\mu^{D_2}_{itw},\ \forall itw, \quad \text{(1g)}$$

$$0\le s_{itw}\quad:\mu^E_{itw},\ \forall(i\in I^h)tw, \quad \text{(1h)}$$

$$0\le m_{itw}\le M_i\quad:\mu^{F_1}_{itw},\mu^{F_2}_{itw},\ \forall(i\in I^h)tw, \quad \text{(1i)}$$

$$m_{itw}-m_{i(t-1)w}=M_i^0\mathbf{I}(t{=}1)+V_{itw}-\Gamma_i q_{itw}-s_{itw} + \sum_{i\in I^{up}}(\Gamma_i q_{itw}+s_{itw})\ :\lambda^G_{itw},\ \forall(i\in I^h)tw\Big\}. \quad \text{(1j)}$$

Here (1a) is the profit formulation, where λ_{ntw} is a locational marginal price (LMP) and it can be expressed as $\lambda_{ntw} = \lambda^B_{tw}-\sum_{l,n}\mu^C_{ltw}H_{ln}$. The term $m_{iTw}\Lambda_i^f\frac{1}{\Gamma_i}$ describes the future value of water left in the reservoir by the end of the modeling horizon (at time step T). Strategic hourly bids $(\hat{q}_{it},\hat{c}_{it})$, and ramp-rate bid $\hat{r}_i$ of the hydropower producer are modeled by the constraints (1b). The LMPs and the dispatch variables are the output of the lower-level economic dispatch problem (1c)-(1j). Expressions (1d) are the generation constraints. We assume the technical minimal outputs of generating units are equal to zero. Expression (1e) represents the energy balance constraint, (1f) accounts for the network constraints and (1g) is setting the constraints on ramp rate. A system of connected reservoirs is considered for the hydropower producer. We model the constraints on spillage (1h), water level (1i) and hydrological balance condition (1j). In hydropower plants with a large storage capacity, head variation has negligible influence on operating efficiency in the short-term, therefore we assume a constant production equivalent Γ_i [29].

Since the lower-level problem (1c)-(1j) is linear, we can equivalently rewrite it as Karush-Kuhn-Tucker (KKT) conditions. We avoid the complementary slackness conditions by substituting them with the strong duality condition (14), as it was shown to be more convenient for linearization [6]. Accordingly, primal feasibility constraints (1d)-(1j), dual feasibility constraints, stationarity constraints (13a)-(13c) and strong duality constraint (14) form the optimality conditions of the lower-level problem. Now, we can reformulate the profit by deriving the LMP from the stationary conditions (13a):

$$\pi = \sum_{(i\in I^h),w}P_w\Big(\sum_t(\lambda^B_{tw}-\sum_{l,n}\mu^C_{ltw}H_{ln}-C_i^M)q_{itw}+ m_{iTw}\Lambda_i^f\frac{1}{\Gamma_i}\Big)\overset{\text{(13a)}}{=}\sum_{(i\in I^h),w}P_w\Big(\sum_t(\hat{c}_{it}-\mu^{A_1}_{itw}+\mu^{A_2}_{itw}+\mu^{D_1}_{itw} - \mu^{D_2}_{itw}-\mu^{D_1}_{i(t+1)w}+\mu^{D_2}_{i(t+1)w}-\Gamma_i\lambda^G_{itw}+\Gamma_i\sum_{i\in I^{dn}}\lambda^G_{itw} - C_i^M)q_{itw}+m_{iTw}\Lambda_i^f\frac{1}{\Gamma_i}\Big). \quad (2)$$

We simplify expression (2) using the complementary slackness (CS) conditions [6]. The term $(*) = (-\Gamma_i\lambda^G_{itw} + \Gamma_i\sum_{i\in I^{dn}}\lambda^G_{itw})q_{itw}$ appeared from the hydrological balance constraint (1j) and it exists only for hydropower units. We derive the linear reformulation of it in (3). The final profit expression becomes:

$$\pi = \sum_{(i\in I^h),w}P_w\Big(\sum_t\Big(\mu^{A_2}_{itw}\hat{q}_{it}+(\mu^{D_1}_{itw}+\mu^{D_2}_{itw})\hat{r}_i+\hat{c}_{it}q_{itw}-C_i^M q_{itw} +\mu^{F_2}_{it}M_i-\lambda^G_{itw}(M_i^0\mathbf{I}(t{=}1)+V_{it})\Big)+m_{iTw}\Lambda_i^f\frac{1}{\Gamma_i}\Big). \quad (4)$$

The equivalent one-level program can be formulated by combining the upper-level constraints and the lower-level optimality conditions. The variables of one-level optimization include primal variables and dual variables of the lower-level problem, and variables of the upper-level problem: Ω^{NLP}={q_{itw}, s_{itw}, m_{itw}, $\mu^{A_1}_{itw}$, $\mu^{A_2}_{itw}$, $\mu^{D_1}_{itw}$, $\mu^{D_2}_{itw}$, μ^E_{itw}, $\mu^{F_1}_{itw}$, $\mu^{F_2}_{itw}$, μ^C_{ltw}, λ^B_{tw}, λ^G_{itw}, $\hat{q}_{it}$, $\hat{r}_i$, $\hat{c}_{it}$}. The final profit expression (4) and strong duality condition (14) contain bilinear terms $\mu^{A_2}_{itw}\hat{q}_{it}$, $\mu^{D_1}_{itw}\hat{r}_i$, $\mu^{D_2}_{itw}\hat{r}_i$ and $\hat{c}_{it}q_{itw}$. For the non-strategic units we assume: $\hat{q}_{it} = Q_i$, $\hat{r}_i = R_i$, $\hat{c}_{it} = C_i^M$, $\forall i \in (I\backslash I^h)$. For the strategic units these terms need to be linearized.

We approximate the generation capacity Q_i by a predefined number of discrete capacities $\hat{Q}_{ik}$. Therefore: $\mu^{A_2}_{itw}\hat{q}_{it} = \mu^{A_2}_{itw}\sum_k x^q_{itk}\hat{Q}_{ik}$=$\sum_k z^{A_2}_{itwk}$, where $\sum_k x^q_{itk} = 1$. Variable

$z_{itwk}^{A_2}$ can be linearized as follows [30][1]:

$$-\bar{K}^q(1-x_{itk}^q) \le z_{itwk}^{A_2} - \hat{Q}_{ik}\mu_{itw}^{A_2} \le \bar{K}^q(1-x_{itk}^q): (\overline{\alpha}_{itwk}^{A_2}, \underline{\alpha}_{itwk}^{A_2}),\ \forall(i{\in}I^h)twk, \quad (5a)$$

$$-\bar{K}^q x_{itk}^q {\le} z_{itwk}^{A_2} {\le} \bar{K}^q x_{itk}^q : (\overline{\beta}_{itwk}^{A_2}, \underline{\beta}_{itwk}^{A_2}), \forall(i{\in}I^h)twk. \quad (5b)$$

In the constraints above $\bar{K}^q$ is a suitably large constant, not too high to create computational instabilities, and not too low to put extra bounds on the variables [31]. Introducing $\bar{K}^r$, we rewrite (1d) and (1g) as follows.

$$q_{itw} \le \hat{Q}_{ik}{+}\bar{K}^q(1{-}x_{itk}^q) : (\nu_{itwk}^{A_2}), \forall(i{\in}I^h)twk, \quad (6a)$$

$$q_{i(t-1)w} - q_{itw} \le \hat{R}_{ik}{+}\bar{K}^r(1{-}x_{ik}^r) \ : (\nu_{itwk}^{D_1}), \forall(i{\in}I^h)twk, \quad (6b)$$

$$q_{itw} - q_{i(t-1)w} \le \hat{R}_{ik}{+}\bar{K}^r(1{-}x_{ik}^r) : (\nu_{itwk}^{D_2}), \forall(i{\in}I^h)twk. \quad (6c)$$

We repeat the linearization steps for other bilinear terms, introducing $z_{itwk}^{D_1}{=}\ \mu_{itw}^{D_1}\sum_k x_{ik}^r \hat{R}_{ik}$, $z_{itwk}^{D_2}{=}\ \mu_{itw}^{D_2}\sum_k x_{ik}^r \hat{R}_{ik}$, $z_{itwk}^{obj}{=}\ q_{itw}\sum_k x_{itk}^c \hat{C}_{ik}$. We form a new set of variables Ω^{MILP} by adding x_{itk}^q, x_{ik}^r, x_{itk}^c, $z_{itwk}^{A_2}$, $z_{itwk}^{D_1}$, $z_{itwk}^{D_2}$, z_{itwk}^{obj} to Ω^{NLP}. The proposed stochastic MILP model is set out in (7).

$$\underset{\Omega^{\text{MILP}}}{\text{maximize}}\ \pi{=}\sum_{(i\in I^h),w} P_w \Big(\sum_t (z_{itwk}^{A_2}{+}z_{itwk}^{D_1}{+}z_{itwk}^{D_2}{+}z_{itwk}^{obj}{-}C_i^M q_{itw} {+}\mu_{itw}^{F_2}M_i{-}\lambda_{itw}^G(M_i^0\mathbf{I}(t{=}1){+}V_{it})){+}m_{iTw}\Lambda_i^f\frac{1}{\Gamma_i}\Big) \quad (7a)$$

subject to: (1e), (1f), (1h)-(1j), (13), (5), (6),

$$\text{(1d), (1g)}, \forall i \in I \backslash I^h, \quad (7b)$$

$$\mu_{itw}^{A_1}, \mu_{itw}^{A_2}, \mu_{itw}^{D_1}, \mu_{itw}^{D_2}, \mu_{itw}^{E}, \mu_{itw}^{F_1}, \mu_{itw}^{F_2}, \mu_{ltw}^{C} \ge 0, \quad (7c)$$

Linearization of $z_{itwk}^{D_1}$, $z_{itwk}^{D_2}$, z_{itwk}^{obj} as in (5), $\forall i{\in}I^h$,
Linearization of (14) as in (5).

In the next section we show how the stochastic MILP model (7) can be solved efficiently using the parallelized MBDA.

III. The solution algorithm (parallelized MBDA)

In optimization problem (7), if we fix binary variables x_{itk}^q, x_{ik}^r and x_{itk}^c, the problem separates in a series of linear programs which can be solved in parallel. By employing Benders decomposition algorithm, a MILP master problem **MP1** is formulated that provides the lower bound for the global solution.

$$\textbf{MP1:} \underset{\alpha,x_{itk}^q,x_{ik}^r,x_{itk}^c}{\text{minimize}} \quad \alpha \quad (8a)$$

$$\text{subject to: } \alpha \ge \sum_w P_w \check{\pi}_{wj} + \sum_{i,k,w} \check{\lambda}_{ikwj}^r (x_{ik}^r - \check{x}_{ikj}^r) + \sum_{i,t,k,w} \Big(\check{\lambda}_{itkwj}^q (x_{itk}^q - \check{x}_{itkj}^q) + \check{\lambda}_{itkwj}^c (x_{itk}^c - \check{x}_{itkj}^c) \Big), \forall j,$$

$$\sum_k x_{itk}^q = 1, \sum_k x_{ik}^r = 1, \sum_k x_{itk}^c = 1, \forall(i \in I^h)t, \quad (8b)$$

$$\alpha \ge \alpha^{down}, \ \ \{x_{itk}^q, x_{ik}^r, x_{itk}^c\} \in \{0,1\}. \quad (8c)$$

Here α^{down} sets the lower bound of **MP1**. This parameter is set arbitrarily low: $\alpha^{down}{\to} -\infty$. Parameter $\check{\pi}_{wj}$ is the profit in scenario w and iteration j. The $\check{x}_{itkj}^q$, $\check{x}_{ikj}^r$, $\check{x}_{itkj}^c$ are bids in iteration j and $\check{\lambda}_{itkwj}^q$, $\check{\lambda}_{ikwj}^r$, $\check{\lambda}_{itkwj}^c$ are sensitivities of profit function with respect to fixing bid variables x_{itk}^q, x_{ik}^r, x_{itk}^c to $\check{x}_{itkj}^q$, $\check{x}_{ikj}^r$, $\check{x}_{itkj}^c$ respectively. The Benders subproblem is:

$$\textbf{SP1:} \underset{\Omega^{\text{LP}}}{\text{minimize}} \quad -\pi_w \quad (9)$$

$$\text{subject to: (7b)},$$
$$\tilde{x}_{itk}^q = \check{x}_{itk(j-1)}^q{:}\lambda_{itkwj}^q, \tilde{x}_{ik}^r = \check{x}_{ik(j-1)}^r{:}\lambda_{ikwj}^r,$$
$$\tilde{x}_{itk}^c = \check{x}_{itk(j-1)}^c{:}\lambda_{itkwj}^c, \forall(i{\in}I^h)tk, \{\tilde{x}_{itk}^q, \tilde{x}_{ik}^r, \tilde{x}_{itk}^c\}{\in}\mathbb{R}.$$

Here set Ω^{LP} is the same as Ω^{MILP}, except that binary variables x_{itk}^q, x_{ik}^r and x_{itk}^c are replaced by continuous variables $\tilde{x}_{itk}^q$, $\tilde{x}_{ik}^r$, $\tilde{x}_{itk}^c$. The standard Benders decomposition algorithm consists of iteratively solving master problem and subproblem, until the convergence is attained [32]. However, in problem (9) the sensitivities λ_{itkwj}^q, λ_{ikwj}^r and λ_{itkwj}^c are poor, due to disjunctive parameters in the subproblems. The cuts obtained are loose and sometimes contain irrelevant information, which causes numerical difficulties limiting the use of the method [33].

A. The modified subproblem

For fixed $\check{x}_{itk}^q$, $\check{x}_{ik}^r$ and $\check{x}_{itk}^c$, the optimization problem (7) is a linear program and its dual is derived in (10).

$$\textbf{SP2:} \underset{\Omega^{\text{SP}}}{\text{maximize}}\ \pi^{dual} = \sum_w P_w \sum_t \Big(\phi_{tw}^B \sum_n D_{ntw} - \sum_l \nu_{ltw}^C (F_l {+} \sum_n H_{ln} D_{ntw}) {-} \sum_{i\in I\backslash I^h} \phi_{itw}^{stq} C_i {-} \sum_{i\in I^h} \phi_{itw}^{stq} \sum_k \hat{C}_{ik}\check{x}_{ik}^c - \sum_{i\in I\backslash I^h} \phi_{itw}^{A_2} Q_i {-} \sum_{i\in I^h} \phi_{itw}^{A_2} \sum_k \hat{Q}_{ik} {-} \sum_{i\in I\backslash I^h} (\phi_{itw}^{D_1}{+}\phi_{itw}^{D_2}) R_i {-} \sum_{i\in I^h} (\phi_{itw}^{D_1}{+}\phi_{itw}^{D_2}) \sum_k \hat{R}_{ik} {-} \sum_{i\in I^h} \phi_{itw}^{F_2} M_i {+} \sum_{i\in I^h} \phi_{itw}^G (M_i^0 \mathbf{I}(t{=}1) + V_{itw}) \Big)\Big) - \sum_{(i\in I^h),t,w,k} \Big(\bar{K}^q ((1-\check{x}_{itk}^q)(\nu_{itwk}^{A_2} + \overline{\alpha}_{itwk}^{A_2} + \underline{\alpha}_{itwk}^{A_2}) + \check{x}_{itk}^q (\overline{\beta}_{itwk}^{A_2} + \underline{\beta}_{itwk}^{A_2})) + \bar{K}^r ((1-\check{x}_{ik}^r)(\nu_{itwk}^{D_1} + \overline{\alpha}_{itwk}^{D_1}$$

[1] The Lagrange multipliers in parentheses will be used in Section III.

$$(*) \overset{(13b)}{=} \sum_{(i\in I^h),t,w} \mu_{itw}^E \Gamma_i q_{itw} \overset{(1j)}{=} \sum_{(i\in I^h),t,w} \mu_{itw}^E (-m_{itw} + m_{i(t-1)w} + M_i^0\mathbf{I}(t{=}1) + V_{itw} - s_{itw} + \sum_{i\in I^{up}} (-m_{itw} + m_{i(t-1)w} + M_i^0\mathbf{I}(t{=}1) + V_{itw})) \overset{\text{CS}}{=} \sum_{(i\in I^h),t,w} \mu_{itw}^E (-m_{itw} + m_{i(t-1)w} + M_i^0\mathbf{I}(t{=}1) + V_{itw} + \sum_{i\in I^{up}} (-m_{itw} + m_{i(t-1)w} + M_i^0\mathbf{I}(t{=}1) + V_{itw})) \overset{(13b),(13c)}{=} \sum_{(i\in I^h),t,w} (\mu_{itw}^{F_1} - \mu_{itw}^{F_2} + \mu_{i(t+1)w}^{F_1} - \mu_{i(t+1)w}^{F_2} - \sum_{i\in I^{dn}} (\mu_{itw}^{F_1} - \mu_{itw}^{F_2} + \mu_{i(t+1)w}^{F_1} - \mu_{i(t+1)w}^{F_2}))(-m_{itw} + m_{i(t-1)w} + M_i^0\mathbf{I}(t{=}1) + V_{itw} + \sum_{i\in I^{up}} (-m_{itw} + m_{i(t-1)w} + M_i^0\mathbf{I}(t{=}1) + V_{itw})) \overset{\text{CS}}{=} \sum_{(i\in I^h),t,w} (\mu_{itw}^{F_2} M_i + (\mu_{itw}^{F_1} - \mu_{itw}^{F_2} + \mu_{i(t+1)w}^{F_1} - \mu_{i(t+1)w}^{F_2})(M_i^0\mathbf{I}(t{=}1) + V_{it})) \overset{(13c)}{=} \sum_{(i\in I^h),t,w} (\mu_{itw}^{F_2} M_i - \lambda_{itw}^G (M_i^0\mathbf{I}(t{=}1) + V_{it})). \quad (3)$$

$$+\underline{\alpha}^{D_1}_{itwk}) + \check{x}^r_{ik}(\overline{\beta}^{D_1}_{itwk} + \underline{\beta}^{D_1}_{itwk})) + \bar{K}^r((1-\check{x}^r_{ik})(\nu^{D_2}_{itwk} + \overline{\alpha}^{D_2}_{itwk} + \underline{\alpha}^{D_2}_{itwk}) + \check{x}^r_{ik}(\overline{\beta}^{D_2}_{itwk} + \underline{\beta}^{D_2}_{itwk}))+ \bar{K}^c((1-\check{x}^c_{itk})(\overline{\alpha}^{obj}_{itwk}+\underline{\alpha}^{obj}_{itwk}) + \check{x}^c_{itk}(\overline{\beta}^{obj}_{itwk}+\underline{\beta}^{obj}_{itwk}))\Big), \quad (10a)$$

$$\text{subject to: } \Omega^{\text{SP}} \in X. \quad (10b)$$

Here Ω^{SP} is the set of decision variables in (10). The feasible region X is represented by the corresponding stationary, primal feasibility and dual feasibility conditions of optimization problem (7), when x^q_{itk}, x^r_{ik} and x^c_{itk} are fixed to $\check{x}^q_{itk}$, $\check{x}^r_{ik}$ and $\check{x}^c_{itk}$. Full formulation of Ω^{SP} and of the feasible set X are provided in Appendix B. Note that the feasible region of SP2 is independent of the disjunctive parameters $\bar{K}^q$, $\bar{K}^r$, $\bar{K}^c$. The disjunctive parameters appear only in the objective function (10a). To remove these disjunctive parameters, we introduce the following lemma.

Lemma 1. *If disjunctive parameters $\bar{K}^q$, $\bar{K}^r$ and $\bar{K}^c$ in (10) are tuned optimally, the exact disjunction holds. Then*

$$(1-\check{x}^q_{itk})(\nu^{A_2}_{itwk} + \overline{\alpha}^{A_2}_{itwk} + \underline{\alpha}^{A_2}_{itwk}) + \check{x}^q_{itk}(\overline{\beta}^{A_2}_{itwk} + \underline{\beta}^{A_2}_{itwk})=0,$$
$$(1-\check{x}^r_{ik})(\nu^{D_1}_{itwk} + \overline{\alpha}^{D_1}_{itwk} + \underline{\alpha}^{D_1}_{itwk}) + \check{x}^r_{ik}(\overline{\beta}^{D_1}_{itwk} + \underline{\beta}^{D_1}_{itwk})=0,$$
$$(1-\check{x}^r_{ik})(\nu^{D_2}_{itwk} + \overline{\alpha}^{D_2}_{itwk} + \underline{\alpha}^{D_2}_{itwk}) + \check{x}^r_{ik}(\overline{\beta}^{D_2}_{itwk} + \underline{\beta}^{D_2}_{itwk})=0,$$
$$(1-\check{x}^c_{itk})(\overline{\alpha}^{obj}_{itwk} + \underline{\alpha}^{obj}_{itwk}) + \check{x}^c_{itk}(\overline{\beta}^{obj}_{itwk} + \underline{\beta}^{obj}_{itwk})=0.$$

Proof. We define $T1 = (1-\check{x}^q_{itk})(\nu^{A_2}_{itwk}+\overline{\alpha}^{A_2}_{itwk}+\underline{\alpha}^{A_2}_{itwk})$ and $T2 = \check{x}^q_{itk}(\overline{\beta}^{A_2}_{itwk}+\underline{\beta}^{A_2}_{itwk})$. If $\check{x}^q_{itk} = 1$ we have $T1 = 0$. Also, constraint (5b) will be non-binding (since $\bar{K}^q$ is tuned optimally and the constraint is effectively relaxed) and Lagrange multipliers corresponding to a non-binding inequality is equal to zero, $\overline{\beta}^{A_2}_{itwk}, \underline{\beta}^{A_2}_{itwk} = 0$. Therefore, $T2 = 0$ and this means $T1 + T2 = 0$. In the second case, if $\check{x}^q_{itk} = 0$ then $T2 = 0$. Also, the constraints (5a) and (6a) will be non-binding which results in $T1 = 0$. Again we have $T1 + T2 = 0$. The same derivation holds for three other zero-equality terms. □

Using Lemma 1, the disjunctive parameters can be removed from the optimization problem (10). Accordingly, the modified subproblem **SP3** has the same structure as **SP2**, except that the terms related to the disjunctive parameters are removed from the objective function (10a) and added to the constraint set (10b). Additionally, **SP3** has a decomposable structure, therefore we solve it for each scenario w independently. The exact formulation of **SP3** is provided in Appendix C.

B. The modified master problem

The master problem **MP2** corresponding to the dual subproblem **SP2** is formulated in (11).

$$\textbf{MP2:} \quad \underset{\alpha, x^q_{itk}, x^r_{ik}, x^c_{itk}}{\text{minimize}} \quad \alpha \quad (11a)$$

$$\text{subject to: } \alpha \geq \pi^{dual}_j, \quad \forall j, \quad (11b)$$

$$\sum_k x^q_{itk}=1, \sum_k x^r_{ik}=1, \sum_k x^c_{itk}=1, \forall (i \in I^h)t, \quad (11c)$$

$$\alpha \geq \alpha^{down}, \quad \{x^q_{itk}, x^r_{ik}, x^c_{itk}\} \in \{0,1\} \quad (11d)$$

Here π^{dual}_j has the same formulation as π^{dual} in **SP2** with the difference that the decision variables in Ω^{SP} are fixed to the calculated values from **SP3** in the respective iteration j and $\check{x}^q_{itk}$, $\check{x}^r_{ik}$, $\check{x}^c_{itk}$ are replaced by binary variables x^q_{itk}, x^r_{ik}, x^c_{itk}. Note that the calculated decision variables of **SP3** are not affected by the disjunctive parameters. However, the disjunctive parameters are still present in the cuts (11b) and they can cause numerical problems.

Using the special form of the disjunctive constraints, we can reformulate (11). We use an observation that disjunctive constraints require that solutions to a mathematical program satisfy a subset of given constraints. Therefore, all constraints can be distributed in two sets: relaxed and enforced constraints, depending on the value of the binary variables x^q_{itk}, x^r_{ik}, x^c_{itk}. We propose the following formulation, where disjunctive parameters are removed.

$$\textbf{MP3:} \quad \underset{x^q_{itk}, x^r_{ik}, x^c_{itk}, \theta_j}{\text{minimize}} \quad K_0\theta_0 + \sum_j K_j\theta_j \quad (12a)$$

$$\text{subject to:} \sum_{i,k\in\Omega^p_j}\Big(\sum_t((1-x^q_{itk})+(1-x^c_{itk}))+(1-x^r_{ik})\Big) \leq |\Omega^p_j| - 1 + \sum_{j'\in J'} \theta_{j'}\mathbf{I}(K_j \leq K_{j'}), \forall j, \quad (12b)$$

$$\sum_k x^q_{itk}=1, \sum_k x^r_{ik}=1, \sum_k x^c_{itk}=1, \forall(i \in I^h)t, \quad (12c)$$

$$\theta_0 + \sum_j \theta_j = 1, \quad \{x^q_{itk}, x^r_{ik}, x^c_{itk}, \theta_j\} \in \{0,1\}. \quad (12d)$$

In optimization problem (12), parameter $K_j{=}\sum_w P_w \check{\pi}^{dual}_{wj}$ where $\check{\pi}^{dual}_{wj}$ is a calculated objective function of **SP3**. Also parameter $K_0{=}-\infty$ is the lower bound of **MP3**. θ_j is an auxiliary binary variable, created at every iteration. J' is the index set of previous iterations and j is the index of current iteration. If we define $\Omega = \{\overline{\alpha}^{A_2}_{itwk}, \underline{\alpha}^{A_2}_{itwk}, \overline{\beta}^{A_2}_{itwk}, \underline{\beta}^{A_2}_{itwk}, \overline{\alpha}^{D_1}_{itwk}, \underline{\alpha}^{D_1}_{itwk}, \overline{\beta}^{D_1}_{itwk}, \underline{\beta}^{D_1}_{itwk}, \overline{\alpha}^{D_2}_{itwk}, \underline{\alpha}^{D_2}_{itwk}, \overline{\beta}^{D_2}_{itwk}, \underline{\beta}^{D_2}_{itwk}, \overline{\alpha}^{obj}_{itwk}, \underline{\alpha}^{obj}_{itwk}, \overline{\beta}^{obj}_{itwk}, \underline{\beta}^{obj}_{itwk}, \nu^{A_2}_{itwk}, \nu^{D_1}_{itwk}, \nu^{D_2}_{itwk}\}$, then Ω^p_j is the index set of strictly positive variables of Ω in iteration j. $|\Omega^p_j|$ is the cardinality of this set.

Lemma 2. *If constraint (12b) includes all extreme points of (7), then **MP3** is an alternative formulation of (7)*[2].

Proof. See [34]. □

The optimization problem (12) is a set-partitioning problem. The binary variables are determining, which of the constraints from (5), (6) are enforced and, which are relaxed. Using this disjunctive structure allows us to remove the disjunctive parameter. The optimization problem (12) can be efficiently solved by the tree-search algorithm detailed in Algorithm 1. By using this algorithm we fix the value for θ_j before solving **MP3** for finding feasible x^q_{itk}, x^r_{ik}, x^c_{itk}. This reduces the number of binary variables in **MP3**.

We now specify the parallelized MBDA in Algorithm 2 for solving (7). In the parallelized MBDA, subproblems **SP3** for different scenarios are solved in parallel. Then, the master problem **MP3** is solved.

[2]Please note, in our studies the subproblem is always feasible. If it is not the case (e.g., insufficient capacity), the feasibility cut should be added [34].

Algorithm 1 The tree-search algorithm

Step 0: Set θ_0 to 1.
Step 1: Search for feasible x^q_{itk}, x^r_{ik}, x^c_{itk}. If none exist, go to Step 2. Else, go to Step 3.
Step 2: Set θ_j corresponding to the best previously obtained solution to 1 and go to Step 1. If no such θ_j exists, **MP3** is infeasible. Stop.
Step 3: The feasible x^q_{itk}, x^r_{ik}, x^c_{itk} is the optimal solution. The optimal value is K_j.

Algorithm 2 Parallelized MBDA

Step 0: (Initialization) Choose an initial $\check{x}^q_{itk}$, $\check{x}^r_{ik}$, $\check{x}^c_{itk}$, iteration counter $j=0$, $z^{up}=+\inf$, $z^{lo}=-\inf$ and ϵ.
Step 1: (Parallelized LP phase) Solve **SP3** independently for each scenario w with $\check{x}^q_{itkj}$, $\check{x}^r_{ikj}$, $\check{x}^c_{itkj}$. Obtain Ω^p_j. Update $z^{up}{=}\sum_w P_w \pi^{dual}_{wj}$.
Step 2: (MP phase) Update iteration counter $j{\leftarrow}j{+}1$. Using information from Step 1 add a cut to **MP3**. Using the tree-search algorithm solve **MP3** and obtain new solutions $\check{x}^q_{itkj}$, $\check{x}^r_{ikj}$, $\check{x}^c_{itkj}$. Update $z^{lo}{=}\alpha$.
Step 3: (Termination test) If $\bar{z}^{up}{-}z^{lo}{\lesssim}\epsilon$, stop. Otherwise go to Step 1.

Using the parallelized MBDA, the disjunctive parameters are removed from both **MP3** and **SP3**. **MP3** is reformulated as a set-partitioning problem, which has a better relaxation than **MP2** and can be solved very fast [34]. This improves the computational efficiency of the proposed solution algorithm, which will be shown in Section V.

C. Existence and uniqueness of the solution

Since MBDA is a reformulation of the original Benders procedure, the original properties will hold [34]. If master problem **MP3** and subproblems **SP3** can be solved to optimality (if problem (7) is feasible), the original Benders procedure guarantees to reach a global optimum [27]. This means that the solution will be optimal and unique.

IV. Illustrative case study

To analyze the strategic behavior of a hydropower producer, the 5-node case study in [35] is used (Fig.1). We assume that generators $G4$ and $G5$ are hydropower units connected in a cascaded configuration with $G4$ being downstream. $G2$ is assumed to be a wind power unit. We perform two types of simulations using the stochastic MILP (7): (a) Detecting the strategic behaviors; and (b) Optimal bids of hydropower producer. To describe the effect of strategic bidding on the system we define dispatch cost as price bids of producers multiplied by the dispatched output.

A. Detecting the strategic behaviors

1) Relocation of water in time: One specific feature of hydropower producer is that its production can be relocated between the time periods to maximize its profit. This situation was suspected in Norway, when the hydropower producers were claimed to use too much water in summer in order to create scarcity (and high prices) in winter [4]. We perform the analysis of this type of strategic behavior using the MILP model (7) proposed in this paper. We assume that there are two load periods with more demand in period 2. This is illustrated using bathtub diagram in Fig. 2, where two stepwise curves are the residual demand curves, formed using the data of fringe generators [1]. The residual demand curve for the hydropower producer is that portion of market demand that is not supplied by other firms in the market. Thus, it is the market demand function minus the quantity supplied by other firms at each price [2]. We assume, that the hydropower producer is limited by the initial water in the reservoir $M^0_{G4,G5}{\equiv}800$ MWh (no inflows).

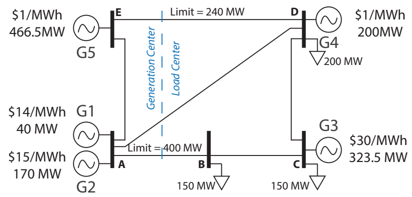

Fig. 1. The 5-node system.

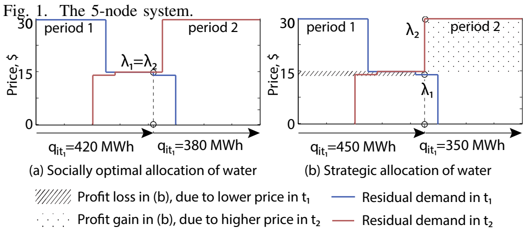

Fig. 2. The effect of relocation of water in time by hydropower producer.

Two situations are presented: (a) socially optimal allocation of water between two time periods and (b) strategic allocation. In the case of strategic behavior the prices become different for the periods (λ_1=\$14/MWh and λ_2=\$30/MWh), in contrast to the uniform price in the socially optimal solution ($\lambda_1{=}\lambda_2$=\$15/MWh). By offering more water in the first time period, strategic hydropower producer has limited amount of water left in the reservoir, which maximizes its profit in the second period. The gain for the hydropower producer from this strategic behavior is \$3750 which is the difference between two hatched areas in Fig. 2-(b).

2) Using waterways instead of transmission network: A hydropower producer with hydrologically coupled units has a unique opportunity to relocate its production between nodes in highly congested transmission networks. Instead of using the electricity network, which can be congested in certain hours, the hydropower producer can use waterways and produce power when needed.

In the illustrative case study, $G4$ and $G5$ are located in different nodes but they are connected by a waterway. The transmission limits between the generation and load centers (capacity limits of lines ED, AD and AB) are set very low, so that the transmission network becomes congested. We simulate 4 cases: with and without hydrological coupling between generators $G4$ and $G5$, and considering strategic and non-strategic hydropower producer. The results of the simulations are presented in Fig. 3.

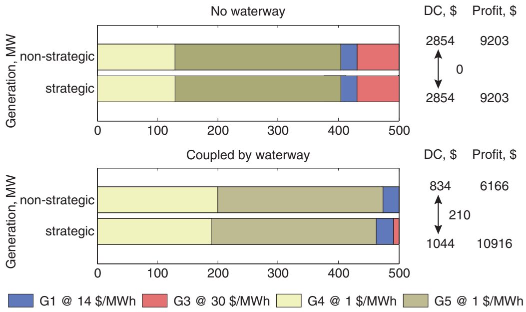

Fig. 3. Strategic hydropower producer with hydrologically coupled and non-coupled units. DC: Dispatch Cost.

First, we analyze the advantages of a waterway for the hydropower producer (both strategic and non-strategic) by comparing the top and bottom charts in Fig. 3. Hydropower producer with hydrologically coupled units relocates around 80 MWh water from upstream constrained-off generator ($G5$) to the one located downstream, in the load pocket ($G4$) – the production of $G4$ increases. When there is no waterway, this relocation of water is not possible. As a result, strategic hydropower producer receives an extra profit of $1713 ($10916 - $9203). In the non-strategic case, the waterway connection allows to decrease the dispatch cost by $2020 ($2854 - $834).

However, we also observe that hydropower producer under waterway case behaves strategically and raises the dispatch cost by $210. It does not have such opportunities for exercising unilateral market power under no-waterway case: the dispatch is the same under the strategic and non-strategic situations.

B. *Optimal bids of the strategic hydropower producer*

1) The impact of forecasted future water price on price bidding: The opportunity cost of the hydropower producer depends on the future expected prices. For hydropower producer the cost today is the benefit obtained by using the water tomorrow [1]. This benefit is calculated using the best available optimization techniques and historic data, and determines how much water should be saved today in order to be used tomorrow. The price-quantity bids of a hydropower producer are based on the result of these calculations.

The stochastic MILP model (7) is used to study the impact of forecasted future price on the price bidding of a hydropower producer. In Fig. 4, the forecasted future price Λ_i^f in (7) is varied between $0/MWh and $2/MWh. We observe that the price bid of the strategic hydropower producer depends on the forecasted future price. As future price increases, the hydropower producer chooses to save more water for future, by setting a higher price bid for its output. The system operator minimizes the dispatch costs: the price bids of all generators multiplied by the dispatched production of these generators. The higher is the price bid of the hydropower producer, the less is the production, dispatched by the system operator. The profit of generator consists of the future value of saved water and profit from the market: it linearly increases with the increase in the future price, and in steps with increasing price bid.

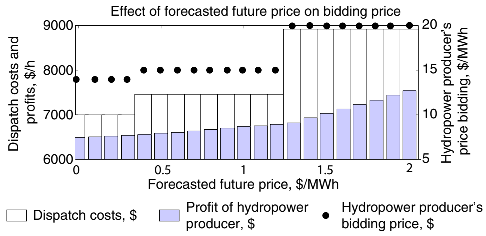

Fig. 4. Study of future price effect on dispatch cost and profits.

TABLE I
STUDYING THE INFLOW UNCERTAINTY EFFECT.

Residual demand curve (RDC)	Expected profit, $	Water offered in $t2$, MWh	Expected dis-patch cost, $
(a) Convex RDC	36 736	**596.50** / 666.5	22 187
(b) Concave RDC	**40 756**	543.75 / 666.5	**24 049**

The price bid of the strategic hydropower producer changes between three values of $14, $15 and $20/MWh. This means that in proximity of these steps the sensitivity of price bid to the forecasted future price is very high. The same holds for the dispatch cost. The dispatch costs are calculated using the offer of the hydropower producer, therefore they increase with an increased price bid. For example, the strategic hydropower producer bids $15 and $20/MWh for forecasted future prices $1.2/MWh and $1.3/MWh, respectively. This results in $1450 (+19.5%) increase in the dispatch cost for just $0.1/MWh (+7.7%) increase in the forecasted future price. This shows the sensitivity of dispatch cost with respect to the forecasted future price. Accordingly, the accuracy of forecasted future price can have a severe impact on price bidding of a strategic hydropower producer.

The case study demonstrates the effect that the uncertainty regarding future water value can create on short term operation. As a possible future extension of the model this uncertainty can be represented as a stochastic value. Further, risk management tools can be applied to deal with uncertainty in future water value.

2) Effect of inflow uncertainty: When analyzing the strategic behavior of hydropower producer, the authors in [4] use a duopoly case study to focus on the effect of inflow uncertainty. The authors show that the shape of the demand curve has the highest influence on the strategic behavior in presence of inflow uncertainty. When the residual inverse demand function is convex, the companies save more water for the later periods. It is opposite, when the demand function is concave. We test this hypothesis using MILP (7), forming the demand curve by modifying the fringe generators. We assume that, as in [4], the strategic hydropower producer is exposed to inflow uncertainty. We introduce two inflow scenarios with low and high realizations for the second time period. Our analysis aims at checking if the general theoretic analysis performed in [4] will hold for our case study.

Our results support the findings in [4], as presented in Table I. Even though in both cases the strategic hydropower producer withholds capacity from the market, the one facing a convex inverse demand function and inflow uncertainty saves

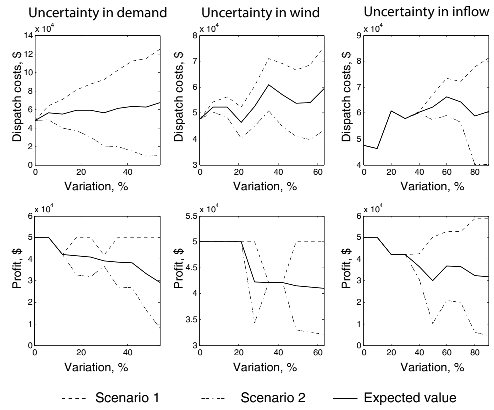

Fig. 5. Effect of uncertainty on the dispatch cost and profit of the strategic hydropower producer.

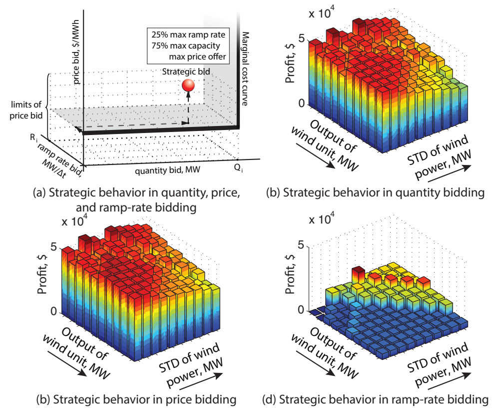

Fig. 6. Increase in hydropower producer's profit, resulting from strategic bidding on quantity, price and ramp rate. STD: standard deviation.

more water for the later period. The profit of hydropower producer and system dispatch cost are higher in the case of concave residual demand curve.

3) General effect of uncertainty: To study the effect of increasing uncertainty, we use MILP (7) and create two scenarios for each of the uncertain parameters (demand, wind and inflow) at different standard deviations. The expected values of these two scenarios are kept the same over different standard deviations. The dispatch cost of the 5-node system and profit of the strategic hydropower producer under uncertainty are plotted in Fig. 5. We observe that the expected profit of the strategic hydropower producer decreases with more variation in uncertain parameters. This is partly because it is much harder for the hydropower producer to behave strategically, when the future is uncertain.

4) Strategic behavior in price, quantity and ramp-rate bids: The optimization model (7) can be used to derive the price, quantity and ramp-rate bids of the hydropower producer (Fig. 6-(a)). The strategic bids increase the profit of hydropower producer above its competitive level. In Fig. 6-(b), 6-(c), and 6-(d) the profit increase with respect to wind generation level and its standard deviation is shown.

We observe that all three types of bid have an important effect on the dispatch cost. Strategic behavior in quantity and price (Fig. 6-(b) and 6-(c)) produce similar results. This is because both of them can be used by the strategic hydropower producer to drive the price to the highest possible value. We also see that strategic behavior in ramp-rate bidding has an important effect even when the other two types of strategic bids are not employed (Fig. 6-(d)). This happens in the systems with limited flexibility, where there are not many fast-ramping generators [6]. In our simulations, the strategic hydropower producer is the main provider of system flexibility. Therefore its strategic behavior in ramp-rate bidding leads to the increase in its profit and market prices.

V. Numerical Results

In this section we focus on the computational performance of the parallelized MBDA for mid and large case studies. The performance is evaluated using the 4-, 24-, 118- and 300-node case studies, obtained from MATPOWER [35]. All simulations are performed on a computer with two 2.80 GHz CPU and 8 GB of RAM. We use 4 computing threads. In order to improve the performance, the parallelized MBDA is implemented in GAMS using the grid solve and Gather-Update-Scatter-Solve (GUSS) facilities. A detailed explanation of these facilities is provided in [28].

We assume 4 time periods and strategic hydropower producer has 3 actions for each type of the bid (19683 possible combinations for bidding). Additionally, uncertainty in each time period is represented by 20 stochastic scenarios. The scenarios for our simulations have been generated from the real data using the moment-matching technique [19], obtained from Nord Pool [36] and processed to fit the case studies.

A. Computational aspects

We compare the following implementations of Benders approach: (1) **MP1** and **SP1**: Standard Benders decomposition algorithm with primal subproblem; (2) **MP2** and **SP2**: Standard Benders decomposition algorithm with dual subproblem; (3) **MP3** and **SP3**: Modified Benders decomposition algorithm (MBDA); (4) **MP3** and **SP3**: Parallelized MBDA with GAMS grid facility and (5) **MP3** and **SP3**: Parallelized MBDA with GUSS facility. In order to ensure the consistency of the results obtained with different methods, we set the optimality gap for Benders procedures to 0.

The computational results of the simulations are compared in Table II. The results clearly demonstrate the superiority of the MBDA and parallelized MBDA over the standard Benders algorithms. The disjunctive parameters, which were set arbitrarily large, affect the results even in the small case study. The formulations with disjunctive parameters in subproblem or master problem (standard Benders approach 1 and 2) do not converge after 10 hours for some of the case studies. This can be explained by the computational difficulties associated with the disjunctive parameter in the larger case studies. The

TABLE II
COMPARISON OF BENDERS PERFORMANCE ON DIFFERENT CASE STUDIES. *: NO RESULTS OBTAINED AFTER 10 HOURS.

Benders algorithm	4-node, $G2 \in I^h$			24-node, $G1 \in I^h$			118-node, $G51 \in I^h$			300-node, $G69 \in I^h$		
	it	t, s	size, MB	it	t, min	size, MB	it	t, min	size, MB	it	t,h	size, MB
1) Standard Benders: primal subproblem	*	*	350	82	36.3	1102	*	*	203	*	*	32000
2) Standard Benders: dual subproblem	17	207	3	82	28.2	8	12	12.7	43	*	*	205
3) Modified Benders approach (MBDA)	17	185	5	82	22.7	8	10	11.2	43	83	3.8	183
4) Parallelized MBDA with grid facility	17	94	4	82	11.2	7	11	8.0	43	82	3.5	184
5) Parallelized MBDA with GUSS	17	10	4	82	3.9	7	10	7.1	43	82	3.4	185
CPLEX		4	350		5.0	1102		9.6	1019		*	>5000

difficulty of tuning the disjunctive parameters for stochastic problems is another complicating factor.

The size of the solution tree for one subproblem in MB is also reported in Table II. The size reaches 32 GB in the case of 300-node example system, making it very hard to solve due to memory requirements. Although the size is reduced under the standard Benders with dual subproblem (approach 2), the algorithm cannot find the global solution after 10 hours. For the 300-node case study, the MBDA finds the global solution in 3.8 hours. The parallelized MBDA with grid facility finds the global solution in 3.5 hours. Under the parallelized MBDA with GUSS, the solution time is reduced to 3.4 hours. As another test, the state-of-the-art CPLEX solver is used to solve the stochastic MILP model (7) for the same case studies for a zero optimality gap. The CPLEX solver did not report any result after 10 hours for the 300-node case study. The parallelized MBDA with GUSS finds the global solutions in 3.4 hours.

B. Market aspects

Market implications of strategic behavior are summarized in Table III. We compare the results when hydropower producer bids its true capacity, costs, and ramp rate, and when it submits a strategic bid. The bottom part of the table shows the increase in dispatch cost from competitive to strategic case. We can see that a single hydropower producer has less effect on the dispatch cost for larger networks. This can be explained by the decreasing share of the strategic generator. If there are many competing generators, strategic hydropower producer has limited opportunities for exercising unilateral market power. We observe that expected profit sometimes remains the same, while dispatch cost increases. Such cases correspond to the situations, when system operator has to dispatch more expensive generators, e.g. due to binding transmission or ramp constraints, but the price-quantity trade-off results in the same profit for the strategic hydropower producer.

TABLE III
MARKET POWER EXERCISE IN MID AND LARGE CASE STUDIES. DC: DISPATCH COST.

	4-node, $G2{\in}I^h$	24-node, $G1{\in}I^h$	118-node, $G51{\in}I^h$	300-node, $G69{\in}I^h$
No market power				
Expected profit, \$	81.9	152.4	3100.8	8274.4
Expected DC, \$	2907.4	24101.3	113518.7	652105.0
After market power				
Expected profit, \$	104.5	152.4	3964.3	8274.4
Expected DC, \$	4905.5	24842.9	116979.5	654078.4
Effect of market power on dispatch cost				
ΔDC, \$	1998.1	741.6	3460.8	1973.4
Increase in DC, %	68,72%	3,08%	3,05%	0,30%

VI. CONCLUSION

This paper proposes a stochastic bilevel program for strategic bidding of a hydropower producer. The upper-level is the strategic hydropower producer, bidding its price, quantity and ramp-rate to the lower-level system operator. Using a disjunction-based linearization technique the stochastic bilevel program is reformulated as a stochastic MILP with disjunctive constraints. To solve the reformulated stochastic MILP model, the modified Benders decomposition algorithm is proposed. The proposed solution algorithm does not require the optimal tuning of disjunctive parameters and it can be parallelized. Through an illustrative example system and the developed stochastic MILP model, we identify possible strategies specific to a hydropower producer for maximizing its profit. The computational efficiency of the parallelized MBDA is demonstrated using the 118-node and 300-node case studies. We have also compared our parallelized MBDA with the state-of-the-art CPLEX solver.

The future work can include extending the formulation to the case with multiple dominant hydropower producers. Such extension will require solving an equilibrium problem with equilibrium constraints.

APPENDIX A
SUPPLEMENTARY MATHEMATICAL EXPRESSIONS

The stationary conditions of the problem (1c)-(1j) are:

$$\frac{dL}{dq_{itw}} = -P_w\hat{c}_{it} + \mu^{A_1}_{itw} - \mu^{A_2}_{itw} + \lambda^B_{tw} - \sum \mu^C_{ltw} H_{ln} - \mu^{D_1}_{itw} + \mu^{D_2}_{itw} + \mu^{D_1}_{i(t+1)w} - \mu^{D_2 l,n}_{i(t+1)w} + \Gamma_i \lambda^G_{itw} - \Gamma_i \sum_{i \in I^{dn}} \lambda^G_{itw} = 0 \ : \phi^{stq}_{itw}, \ \forall itw, \tag{13a}$$

$$\frac{dL}{ds_{itw}} = \mu^E_{itw} + \lambda^G_{itw} - \sum_{i \in I^{dn}} \lambda^G_{itw} = 0 : \phi^{sts}_{itw}, \forall (i{\in}I^h)tw, \tag{13b}$$

$$\frac{dL}{dm_{itw}} = \mu^{F_1}_{itw} - \mu^{F_2}_{itw} + \lambda^G_{itw} - \lambda^G_{i(t+1)w} = 0 : \phi^{stm}_{itw}, \ \forall (i{\in}I^h)tw. \tag{13c}$$

The strong duality theorem results in:

$$\sum_{i,t} P_w \hat{c}_{it} q_{itw} = -\sum_{i,t} \mu^{A_2}_{itw} \hat{q}_{itw} + \sum_t \lambda^B_{tw} \sum_n D_{ntw} - \sum_{l,t} \mu^C_{ltw} (F_l + \sum_n H_{ln} D_{ntw}) - \sum_{i,t} \mu^{D_1}_{itw} \hat{r}_i - \sum_{i,t} \mu^{D_2}_{itw} \hat{r}_i \tag{14}$$

$$-\sum_{(i\in I^h),t}\mu^{F_2}_{itw}M_i + \sum_{(i\in I^h),t}\lambda^G_{tw}(M^0_i\mathbf{I}(t{=}1)+V_{itw}) : \phi^{dual}_{itw}.$$

Appendix B
Dual subproblem formulation: SP2

In order to take a dual of problem (7) we associate Lagrange multipliers with every constraint of **SP1**:

BigM linearization of $z^{A_2}_{itwk}$, $z^{D_1}_{itwk}$, $z^{D_2}_{itwk}$, z^{obj}_{itwk} :
$\underline{\alpha}^{A_2}_{itwk}$, $\overline{\alpha}^{A_2}_{itwk}$, $\underline{\alpha}^{D_1}_{itwk}$, $\overline{\alpha}^{D_2}_{itwk}$, $\underline{\alpha}^{obj}_{itwk}$, $\overline{\alpha}^{obj}_{itwk}$,
$\underline{\beta}^{A_2}_{itwk}$, $\overline{\beta}^{A_2}_{itwk}$, $\underline{\beta}^{D_1}_{itwk}$, $\overline{\beta}^{D_2}_{itwk}$, $\underline{\beta}^{obj}_{itwk}$, $\overline{\beta}^{obj}_{itwk}$

Primal feasibility: (6a)-(6c), (1d),(1e), (1f), (1h)-(1j):
$\nu^{A_1}_{itw}$, $\nu^{A_2}_{itw}$, ν^{C}_{itw}, $\nu^{D_1}_{itw}$, $\nu^{D_2}_{itw}$, ν^{E}_{itw}, $\nu^{F_1}_{itw}$, $\nu^{F_2}_{itw}$, ϕ^{B}_{tw}, ϕ^{G}_{itw}

Dual feasibility: (7c):
$\nu^{A_1^+}_{itw}$, $\nu^{A_2^+}_{itw}$, $\nu^{C^+}_{ltw}$, $\nu^{D_1^+}_{itw}$, $\nu^{D_2^+}_{itw}$, $\nu^{E^+}_{itw}$, $\nu^{F_1^+}_{itw}$, $\nu^{F_2^+}_{itw}$,
$\nu^{z^{A_2^+}}_{itwk}$, $\nu^{z^{D_1^+}}_{itwk}$, $\nu^{z^{D_2^+}}_{itwk}$, $\nu^{z^{obj^+}}_{itwk}$

Stationary conditions (13a)-(13c): $\phi^{stq}_{itw}, \phi^{sts}_{itw}, \phi^{stm}_{itw}$

Strong duality condition (14): ϕ^{dual}_{w}.

The resulting set of decision variables in **SP2** is: Ω^{SP} = $\{\underline{\alpha}^{A_2}_{itwk}$, $\overline{\alpha}^{A_2}_{itwk}$, $\underline{\alpha}^{D_1}_{itwk}$, $\overline{\alpha}^{D_2}_{itwk}$, $\underline{\alpha}^{obj}_{itwk}$, $\overline{\alpha}^{obj}_{itwk}$, $\underline{\beta}^{A_2}_{itwk}$, $\overline{\beta}^{A_2}_{itwk}$, $\underline{\beta}^{D_1}_{itwk}$, $\overline{\beta}^{D_2}_{itwk}$, $\underline{\beta}^{obj}_{itwk}$, $\overline{\beta}^{obj}_{itwk}$, $\nu^{A_1}_{itw}$, ν^{A2}_{itw}, ν^{C}_{itw}, $\nu^{D_1}_{itw}$, ν^{D2}_{itw}, ν^{E}_{itw}, $\nu^{F_1}_{itw}$, $\nu^{F_2}_{itw}$, ϕ^{B}_{tw}, ϕ^{G}_{itw}, ϕ^{stq}_{itw}, ϕ^{sts}_{itw}, ϕ^{stm}_{itw}, ϕ^{dual}_{w}, $\nu^{A_1^+}_{itw}$, $\nu^{A_2^+}_{itw}$, $\nu^{C^+}_{ltw}$, $\nu^{D_1^+}_{itw}$, $\nu^{D_2^+}_{itw}$, $\nu^{E^+}_{itw}$, $\nu^{F_1^+}_{itw}$, $\nu^{F_2^+}_{itw}$, $\nu^{z^{A_2^+}}_{itwk}$, $\nu^{z^{D_1^+}}_{itwk}$, $\nu^{z^{D_2^+}}_{itwk}$, $\nu^{z^{obj^+}}_{itwk}\}$.

The stationary conditions of **SP2** are:

$$\frac{dL^{SP2}}{dq_{itw}} = (-P_wC^M_i + \sum_k(\hat{C}_{ik}(-\overline{\alpha}^{obj}_{itwk}+\underline{\alpha}^{obj}_{itwk}) - \nu^{D1}_{itwk} + \nu^{D2}_{itwk} + \nu^{D1}_{i(t+1)wk} - \nu^{D2}_{i(t+1)wk} - \nu^{A_2}_{itwk}) + \Gamma_i\phi^G_{itw} - \Gamma_i\sum_{i\in I^{dn}}\phi^G_{itw})\mathbf{I}(i\in I^h) + (-\nu^{D1}_{itw}+\nu^{D2}_{itw}+\nu^{D1}_{i(t+1)w} - \nu^{D2}_{i(t+1)w} - \nu^{A_2}_{itw} - \phi^{dual}_wC^M_i)\mathbf{I}(i\in I\backslash I^h) + \nu^{A_1}_{itw} + \phi^B_{tw} - \sum_{l,n}\nu^C_{ltw}H_{ln} = 0; \tag{15a}$$

$$\frac{dL^{SP2}}{d\lambda^B_{tw}} = -\sum_i\phi^{stq}_{itw} + \phi^{dual}_w\sum_n D_{ntw} = 0; \tag{15b}$$

$$\frac{dL^{SP2}}{d\mu^{A_1}_{itw}} = \nu^{A_1^+}_{itw} - \phi^{stq}_{itw} = 0; \tag{15c}$$

$$\frac{dL^{SP2}}{d\mu^{A_2}_{itw}} = \nu^{A_2^+}_{itw} + \sum_k(-\overline{\alpha}^{A_2}_{itwk}\hat{q}_{ik} + \underline{\alpha}^{A_2}_{itwk}\hat{q}_{ik})\mathbf{I}(i\in I^h) - \phi^{dual}_w\hat{Q}_i\mathbf{I}(i\in I\backslash I^h) + \phi^{stq}_{itw} = 0; \tag{15d}$$

$$\frac{dL^{SP2}}{d\mu^{D_1}_{itw}} = \nu^{D_1^+}_{itw} + \sum_k(-\overline{\alpha}^{D_1}_{itwk}\hat{R}_{ik} + \underline{\alpha}^{D_1}_{itwk}\hat{R}_{ik})\mathbf{I}(i\in I^h) - \phi^{dual}_w\hat{R}_i\mathbf{I}(i\in I\backslash I^h) - \phi^{stq}_{itw} + \phi^{stq}_{i(t-1)w} = 0; \tag{15e}$$

$$\frac{dL^{SP2}}{d\mu^{D_2}_{itw}} = \nu^{D_2^+}_{itw} + \sum_k(-\overline{\alpha}^{D_2}_{itwk}\hat{R}_{ik} + \underline{\alpha}^{D_2}_{itwk}\hat{R}_{ik})\mathbf{I}(i\in I^h) - \phi^{dual}_w\hat{R}_i\mathbf{I}(i\in I\backslash I^h) + \phi^{stq}_{itw} - \phi^{stq}_{i(t-1)w} = 0; \tag{15f}$$

$$\frac{dL^{SP2}}{d\mu^{C}_{ltw}} = \nu^{C^+}_{ltw} - \phi^{dual}_w(F_l + \sum_n H_{ln}D_{ntw}) + \phi^{stq}_{itw}\sum_{l,n}H_{l,n} = 0; \tag{15g}$$

$$\frac{dL^{SP2}}{dz^{A_2}_{itwk}}\mathbf{I}(i\in I^h) = \nu^{z^{A_2^+}}_{itwk} + P_w + \overline{\alpha}^{A_2}_{itwk} - \underline{\alpha}^{A_2}_{itwk} + \overline{\beta}^{A_2}_{itwk} - \underline{\beta}^{A_2}_{itwk} - \phi^{dual}_w = 0; \tag{15h}$$

$$\frac{dL^{SP2}}{dz^{D_1}_{itwk}}\mathbf{I}(i\in I^h) = \nu^{z^{D_1^+}}_{itwk} + P_w + \overline{\alpha}^{D_1}_{itwk} - \underline{\alpha}^{D_1}_{itwk} + \overline{\beta}^{D_1}_{itwk} - \underline{\beta}^{D_1}_{itwk} - \phi^{dual}_w = 0; \tag{15i}$$

$$\frac{dL^{SP2}}{dz^{D_2}_{itwk}}\mathbf{I}(i\in I^h) = \nu^{z^{D_2^+}}_{itwk} + P_w + \overline{\alpha}^{D_2}_{itwk} - \underline{\alpha}^{D_2}_{itwk} + \overline{\beta}^{D_2}_{itwk} - \underline{\beta}^{D_2}_{itwk} - \phi^{dual}_w = 0; \tag{15j}$$

$$\frac{dL^{SP2}}{dz^{obj}_{itwk}}\mathbf{I}(i\in I^h) = \nu^{z^{obj^+}}_{itwk} + P_w + \overline{\alpha}^{obj}_{itwk} - \underline{\alpha}^{obj}_{itwk} + \overline{\beta}^{obj}_{itwk} - \underline{\beta}^{obj}_{itwk} - \phi^{dual}_w = 0; \tag{15k}$$

$$\frac{dL^{SP2}}{dm_{itw}}\mathbf{I}(i\in I^h) = P_w\Lambda^f_i\mathbf{I}(t{=}T) + \nu^{F_1}_{itw} - \nu^{F_2}_{itw} + \phi^G_{itw} - \phi^G_{(t+1)w} = 0; \tag{15l}$$

$$\frac{dL^{SP2}}{ds_{itw}}\mathbf{I}(i\in I^h) = \nu^E_{itw} + \phi^G_{itw} - \sum_{i\in I^{dn}}\phi^G_{itw} = 0; \tag{15m}$$

$$\frac{dL^{SP2}}{d\mu^{F_1}_{itw}}\mathbf{I}(i\in I^h) = \nu^{F_1^+}_{itw} - \phi^{stm}_{itw} = 0; \tag{15n}$$

$$\frac{dL^{SP2}}{d\mu^{F_2}_{itw}}\mathbf{I}(i\in I^h) = \nu^{F_2^+}_{itw} + \phi^{stm}_{itw} - \phi^{dual}_w M_i = 0; \tag{15o}$$

$$\frac{dL^{SP2}}{d\mu^{E}_{itw}}\mathbf{I}(i\in I^h) = \nu^{E^+}_{itw} - \phi^{sts}_{itw} = 0; \tag{15p}$$

$$\frac{dL^{SP2}}{d\lambda^{G}_{itw}}\mathbf{I}(i\in I^h) = (-P_w+\phi^{dual}_w)(M^0_i\mathbf{I}(t{=}1)+V_{itw}) - \Gamma_i\phi^{stq}_{itw} + \sum_{i\in I^{up}}(\Gamma_i\phi^{stq}_{itw}+\phi^{sts}_{itw}) - \phi^{stm}_{itw} + \phi^{stm}_{i(t-1)w} - \phi^{sts}_{itw} = 0; \tag{15q}$$

The whole feasible set of **SP2** is represented by: $X = [(15)$, $\{\underline{\alpha}^{A_2}_{itwk}$, $\overline{\alpha}^{A_2}_{itwk}$, $\underline{\alpha}^{D_1}_{itwk}$, $\overline{\alpha}^{D_2}_{itwk}$, $\underline{\alpha}^{obj}_{itwk}$, $\overline{\alpha}^{obj}_{itwk}$, $\underline{\beta}^{A_2}_{itwk}$, $\overline{\beta}^{A_2}_{itwk}$, $\underline{\beta}^{D_1}_{itwk}$, $\overline{\beta}^{D_2}_{itwk}$, $\underline{\beta}^{obj}_{itwk}$, $\overline{\beta}^{obj}_{itwk}$, $\nu^{A_1}_{itw}$, ν^{A2}_{itw}, ν^{C}_{itw}, $\nu^{D_1}_{itw}$, ν^{D2}_{itw}, ν^{E}_{itw}, $\nu^{F_1}_{itw}$, $\nu^{F_2}_{itw}$, $\nu^{A_1^+}_{itw}$, $\nu^{A_2^+}_{itw}$, $\nu^{C^+}_{ltw}$, $\nu^{D_1^+}_{itw}$, $\nu^{D_2^+}_{itw}$, $\nu^{E^+}_{itw}$, $\nu^{F_1^+}_{itw}$, $\nu^{F_2^+}_{itw}$, $\nu^{z^{A_2^+}}_{itwk}$, $\nu^{z^{D_1^+}}_{itwk}$, $\nu^{z^{D_2^+}}_{itwk}$, $\nu^{z^{obj}}_{itwk}\} \geq 0$].

Appendix C
Dual subproblem formulation: SP3

Problem **SP3** has a following structure:

SP3: $\underset{\Omega^{\text{SP}}}{\text{maximize}}$ $\pi^{dual} = \sum_w P_w\sum_t\Big(\phi^B_{tw}\sum_n D_{ntw} - \sum_l\nu^C_{ltw}(F_l + \sum_n H_{ln}D_{ntw}) - \sum_{i\in I\backslash I^h}\phi^{stq}_{itw}C_i - \sum_{i\in I^h}\phi^{stq}_{itw}\sum_k\hat{C}_{ik}\check{x}^c_{ik} - \sum_{i\in I\backslash I^h}\phi^{A_2}_{itw}Q_i - \sum_{i\in I^h}\phi^{A_2}_{itw}\sum_k\hat{Q}_{ik} - \sum_{i\in I\backslash I^h}(\phi^{D_1}_{itw}+\phi^{D_2}_{itw})R_i - \sum_{i\in I^h}(\phi^{D_1}_{itw}+\phi^{D_2}_{itw})\sum_k\hat{R}_{ik} - \sum_{i\in I^h}\phi^{F_2}_{itw}M_i + \sum_{i\in I^h}\phi^G_{itw}(M^0_i\mathbf{I}(t{=}1) + V_{itw})\Big)\Big),$ (16a)

subject to: $\sum_{(i\in I^h),t,w,k}\Big(((1-\check{x}^q_{itk})(\nu^{A_2}_{itwk} + \overline{\alpha}^{A_2}_{itwk} + \underline{\alpha}^{A_2}_{itwk}) + \check{x}^q_{itk}(\overline{\beta}^{A_2}_{itwk} + \underline{\beta}^{A_2}_{itwk})) + ((1-\check{x}^r_{ik})(\nu^{D_1}_{itwk} + \overline{\alpha}^{D_1}_{itwk}$

$$+\underline{\alpha}^{D_1}_{itwk}) + \check{x}^r_{ik}(\overline{\beta}^{D_1}_{itwk} + \underline{\beta}^{D_1}_{itwk})) + ((1-\check{x}^r_{ik})(\nu^{D_2}_{itwk}+ \overline{\alpha}^{D_2}_{itwk} + \underline{\alpha}^{D_2}_{itwk}) + \check{x}^r_{ik}(\overline{\beta}^{D_2}_{itwk} + \underline{\beta}^{D_2}_{itwk})) + ((1-\check{x}^c_{itk})(\overline{\alpha}^{obj}_{itwk} +\underline{\alpha}^{obj}_{itwk}) + \check{x}^c_{itk}(\overline{\beta}^{obj}_{itwk}+\underline{\beta}^{obj}_{itwk}))\Big) = 0, \tag{16b}$$

$$\Omega^{\text{SP}} \in X. \tag{16c}$$

The objective function and the feasible set of **SP3** are free from the disjunctive parameter $\bar{K}$.

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Nash Equilibria in Hydro-Dominated Systems under Uncertainty: Modified Benders Approach

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***Abstract*—In this paper we use experience of our studies of a single profit-maximizing hydropower producer and model strategic interaction of multiple producers in hydro-dominated power systems under uncertainty as an equilibrium problem with equilibrium constraints (EPEC), reformulated as a stochastic mixed-integer linear program (MILP) with disjunctive constraints. We model strategic hydropower producers, who can affect the market price by submitting strategic bids in quantity, price, and ramp rate. The bids are submitted to the system operator, who minimizes the dispatch cost. We take into account the hydro-specific constraints and uncertainty in the system. Solving the problem results in finding Nash equilibria. We discuss two types of Nash equilibria under uncertainty: Bayesian and robust Nash equilibria. Large EPEC instances can be solved using a decomposition method – Modified Benders Decomposition Approach (MBDA). This method eliminates the problem of tuning the disjunctive parameter and reduces the memory requirements, resulting in improved computation time.**

***Index Terms*—EPEC, stochastic programming, disjunctive constraint, Modified Benders decomposition approach**

NOMENCLATURE

The main notation is presented below for a quick reference. Additional symbols are introduced throughout the text.

Indices

i	Generating units
t	Time periods
l	Transmission lines
n	System nodes
k	Bid alternatives
s	Bidding strategies
w	Stochastic scenarios
j	Benders iterations

Sets

I	Generating units
I^h	Strategic hydropower units, $I^h \subset I$
I^{up},I^{dn}	Upstream and downstream units, I^{up},$I^{dn} \subset I^h$

Parameters (upper-case letters)

M_i^0	Initial water level of unit $i \in I^h$, m^3
M_i	Maximum water level of unit $i \in I^h$, m^3
V_{itw}	Inflow to the reservoir of unit $i \in I^h$ in t, m^3
Γ_i	Production equivalent of unit $i \in I^h$, m^3/MWh
Q_i	Maximum generation of unit i, MW
R_i	Maximum ramp rate of unit i, MW/h
C_i	Maximum price bid of unit $i \in I^h$, \$/MWh
C_i^M	Marginal cost of unit i, \$/MWh
Λ_i^f	Future price for unit $i \in I^h$, \$/MWh
P_w	Probability of scenario w
H_{ln}	Power Transfer Distribution Factor (PTDF)
L_i	Time lag of downstream water relocation, h
F_l	Power flow limit on line l, MW
D_{ntw}	Demand at node n in t, MWh

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Variables (lower-case letters)

π	Profit of strategic hydropower producer, \$
q_{itw}	Dispatched generation of unit i in t, MWh
s_{itw}	Spillage of unit $i \in I^h$ in t, m^3
m_{itw}	Water level of unit $i \in I^h$ in t, m^3
x_{itk}^q	Binary variable of quantity bidding decision
x_{ik}^r	Binary variable of ramp-rate bidding decision
x_{itk}^c	Binary variable of price bidding decision
$\hat{q}_{it}$	Quantity bid of unit i in t, MW
$\hat{r}_i$	Ramp-rate bid of unit i, MW/h
$\hat{c}_{it}$	Price bid of unit i in t, \$/MWh
ϵ_i	Deviation of $i \in I^h$ from Nash equilibrium, \$/MWh

Lagrange multipliers (LM)

$\mu_{itw}^{A_1}$, $\nu_{itw}^{A_1}$	LM of unit i lower capacity limit constraint
$\mu_{itw}^{A_2}$, $\nu_{itw}^{A_2}$	LM of unit i upper capacity limit constraint
λ_{tw}^B, ϕ_{tw}^B	LM of energy balance constraint
μ_{ltw}^C, ν_{ltw}^C	LM of the flow constraint of line l
$\mu_{itw}^{D_1}$, $\nu_{itw}^{D_1}$	LM of unit i ramp-up constraint
$\mu_{itw}^{D_2}$, $\nu_{itw}^{D_2}$	LM of unit i ramp-down constraint
μ_{itw}^E, ν_{itw}^E	LM of spillage nonnegativity constraint
$\mu_{itw}^{F_1}$, $\nu_{itw}^{F_1}$	LM of lower water-level-limit constraint
$\mu_{itw}^{F_2}$, $\nu_{itw}^{F_2}$	LM of upper water-level-limit constraint
λ_{itw}^G, ϕ_{itw}^G	LM of water balance constraint

I. INTRODUCTION

HYDROPOWER is a renewable energy source that is economically attractive, provides high security of supply and zero operational emissions of CO_2. It is by far the leading renewable energy source in Europe – hydropower accounts for 95-99% of electricity generation in Norway, 70% in Austria, and 42% in Sweden [1]. The balancing capabilities of hydropower producers gained a special importance in recent time with a massive introduction of variable renewable sources, such as wind, to the power system. The critical importance of hydropower, its low marginal cost, and high market share in some countries all contribute to the strategic position of hydropower producers relative to other generating companies. For example, extraordinary high prices, which were observed in the Nordic power market in winter 2002-2003, were attributed to the strategic behavior of hydropower producers by some of the observers. Hydropower producers have used "too much" water in summer, creating scarcity and high prices in winter [2]. In order to predict such cases, strategic interaction in the market has to be mathematically modeled.

Equilibrium problems with equilibrium constraints (EPECs) have been widely used for modeling the strategic interaction in electricity markets, [3], [4]. Such models allow representing the bilevel structure of the market – profit-maximizing strategic generators decide on their bids and submit them to the power system operator. The operator orders the bids to minimize the dispatch cost, subject to system constraints. EPECs also take into account the interaction between multiple strategic producers.An EPEC solution aims at finding a Nash equilibrium – a point, at which no player wants to change the strategy (strategic bid) unilaterally.

However, solving EPEC is a computationally and methodologically challenging task [5]. Depending on a case study, there might be multiple Nash equilibria, or no equilibria at all. Finding each equilibrium usually requires solving a non-convex non-regular problem. Uncertainty additionally hinders finding a solution. The applications of EPECs to the power systems are often stylized, due to the aforementioned difficulties. The case studies are often limited to 3 nodes, or greatly simplify modeling of generators [6].

Modeling the hydro-dominated systems requires accounting for hydro-specific constraints. The few studies that have looked at market power assessment, involving the hydropower producers, have significantly simplified the modeling. Usually only a single profit-maximizing hydropower producer is considered, and the optimal dispatch conditions are simplified as a residual demand curve [7], [8]. In reference [9] the authors consider the market power assessment in a hydrothermal power system, but employ a simplifying residual demand approach, expressing price as a function of price-makers' production, rather than as a result of the optimal dispatch. The authors in [10] include the equilibrium constraints, but their model neglects the uncertainty in reservoir inflows and contains nonlinearities.

This paper proposes an EPEC formulation for modeling the Bayesian and robust Nash equilibria in hydro-dominated power systems. Price-making hydropower producers bid on price, quantity, and ramp rate. The bids are considered in the optimal dispatch problem, which results in determining the market prices. Also, hydro-specific constraints (hydrological coupling of the reservoirs with a delay in water relocation, spillage, and water level constraints) are fully considered. The stochasticity of demand, inflows, and wind generation is modeled through moment-matching scenario generation technique [11]. The proposed EPEC is reformulated as a stochastic mixed-integer linear program (MILP) with disjunctive constraints. We discuss two types of Nash equilibria under uncertainty: Bayesian and robust Nash equilibria [12], [13]. While Bayesian Nash equilibrium is one of the most wide-spread definitions of Nash equilibrium under uncertainty, robust Nash equilibrium has an important practical meaning. Robust Nash equilibrium does not require a prior knowledge regarding the probability distribution function of the uncertain parameter. This is particularly useful when the event has no observed history, for example political decisions or security considerations.

The authors in [20] propose a special version of Benders decomposition approach in order to solve single-level optimization problems with disjunctive constraints. In our previous work we have applied modified Benders decomposition approach (MBDA) to solve mathematical problems with equilibrium constraints (MPECs). In this paper we use MBDA to solve EPEC, reformulated as a stochastic MILP model. The MBDA exploits the disjunctive structure of the stochastic MILP model and does not require optimal tuning of the disjunctive parameters. Through an illustrative 3-node example, we carefully study the market aspects of strategic bidding in a hydro-dominated power system. The IEEE 24-node, 118-node, and 300-node systems are used to show the computational efficiency and scalability of the proposed application of MBDA. We show through the numerical results that MBDA outperforms the standard Benders algorithm for large case studies. We also compare MBDA to the monolith (non-decomposed) problem solution using CPLEX.

The main contribution of this work is threefold: **(1)** We propose an EPEC model representing the strategic interaction of price-making hydropower producers. We model the uncertainty, typically observed in hydro-dominated power systems: wind power production, demand levels, and inflow uncertainty. We reformulate the problem as a mixed-integer linear program (MILP), which can be solved to a global optimum. **(2)** We discuss Bayesian and robust formulations for modeling the Nash equilibrium under uncertainty. We point out the observed difference between them in terms of strategic bidding and market results using an illustrative case study. **(3)** We solve our EPEC model using a modified Benders decomposition approach, which removes the disjunctive parameters from the formulation and provides faster solution for large case studies.

The paper is organized as follows. Section II presents the EPEC model and its reformulation as a stochastic MILP. Section III explains how MBDA can be applied to the proposed formulation. Section IV illustrates the proposed model and provides a comparison of different types of Nash equilibria. The computational properties of the MBDA are detailed in Section V. Finally, Section VI concludes the paper.

II. Mathematical modeling

Strategic interaction in hydro-dominated systems can be modeled as an EPEC: profit-maximizing hydropower producers compete in electricity market by submitting strategic bids. The bids are received by the system operator, which orders them and determines the system price. This structure is illustrated in Fig. 1. Solving EPEC results in finding a Nash equilibrium – a stable set of strategies, from which no generator wants to deviate unilaterally. In this section we present two definitions for the Nash equilibrium under uncertainty and discuss how to model finding a Nash equilibrium as an optimization problem. We then formulate the profit of the strategic hydropower producer and conclude the section by formulating the whole EPEC as a MILP.

A. *Nash equilibrium under uncertainty*

Nash equilibrium is a point at which each player's strategy maximizes its payoff if the strategies of the competitors are

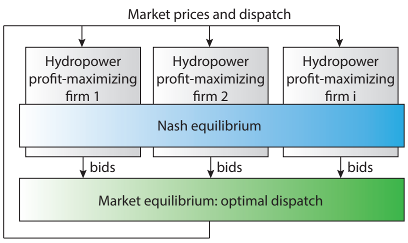

Fig. 1. Structure of the proposed EPEC.

held fixed. Thus each player's strategy is optimal against those of the others [14]. This can be expressed as follows:

$$\pi_i(s_i^*, s_{-i}^*) \geq \pi_i(s_i, s_{-i}^*), \quad \forall i{\in}I^h. \tag{1}$$

This constraint guarantees that for each strategic hydropower producer $i{\in}I^h$ the profit in the candidate strategy combination s_i^* must be greater or equal than the profit under alternative choice of strategy s_i, while the strategies of the competitors s_{-i}^* are held fixed. We can formulate Nash equilibrium as an optimization problem:

$$\underset{\Omega^{\text{MILP}}}{\text{minimize}}\ \Delta\pi = \sum_{i\in I^h} \epsilon_i \tag{2a}$$

$$\text{subject to:}\quad \pi_i(s_i^*, s_{-i}^*) + \epsilon_i \geq \pi_i(s_i, s_{-i}^*), \quad \forall i{\in}I^h, \tag{2b}$$

$$\epsilon_i \geq 0, \quad \forall i{\in}I^h \tag{2c}$$

At a Nash equilibrium point the deviation ϵ_i is zero for all players. Under uncertainty the profit π_i can be different in different scenarios. There are several ways of how to extend the definition of Nash equilibrium in the case of uncertainty.

1) Bayesian Nash equilibrium: Each player is assumed to have a subjective uncertainty probability distribution function [12]. This assumption is applicable to the most of the uncertainties observed in the power system, such as wind, reservoir inflows, demand uncertainty. When introducing scenarios w describing the uncertainty, the Nash equilibrium (2b) becomes:

$$E_w[\pi_{iw}(s_i^*, s_{-i}^*)] + \epsilon_i \geq E_w[\pi_{iw}(s_i, s_{-i}^*)], \quad \forall i{\in}I^h. \tag{3}$$

2) Robust Nash equilibrium: Finding robust Nash equilibrium does not require the prior knowledge of probability distribution function for the incomplete information [13]. This is very useful, when certain scenarios have no historic data or when probabilities of the scenarios are difficult to compute. Robust Nash equilibrium uses the worst-case approach, where (2b) is reformulated as follows:

$$\min_w[\pi_{iw}(s_i^*, s_{-i}^*)] + \epsilon_i \geq \min_w[\pi_{iw}(s_i, s_{-i}^*)], \quad \forall i{\in}I^h. \tag{4}$$

B. Profit of a hydropower producer

A profit function of a price-making hydropower producer in scenario w is a function of strategic bid $\hat{q}_{it}, \hat{r}_i, \hat{c}_{it}$ and can be formulated as follows:

$$\pi_{iw} = \sum_t (\lambda_{ntw} - C_i^M) q_{itw} + m_{iTw}\Lambda_i^f \frac{1}{\Gamma_i}. \tag{5}$$

Here λ_{ntw} is a Locational Marginal Price (LMP). The term $m_{iTw}\Lambda_i^f \frac{1}{\Gamma_i}$ describes the future value of water left in the reservoir by the end of the modeling horizon (at time step T). Strategic hourly bid of a hydropower producer is limited by the constraints:

$$0 \leq \hat{q}_{it} \leq Q_i,\ 0 \leq \hat{r}_i \leq R_i,\ 0 \leq \hat{c}_{it} \leq C_i. \tag{6}$$

LMP can be expressed as a result of the optimal system dispatch. The dispatch problem can be formulated as follows:

$$\underset{q_{itw}, s_{itw}, m_{itw}}{\text{minimize}} \sum_{i,t,w} P_w \hat{c}_{it} q_{itw} \tag{7a}$$

$$\text{subject to:}\quad 0 \leq q_{itw} \leq \hat{q}_{it} \quad : \mu_{itw}^{A_1}, \mu_{itw}^{A_2}, \quad \forall it, \tag{7b}$$

$$\sum_i q_{itw} = \sum_n D_{ntw} \quad : \lambda_{tw}^B, \quad \forall tw, \tag{7c}$$

$$\sum_n H_{ln}(\sum_{i:n} q_{itw} - D_{ntw}) \leq F_l \ : \mu_{ltw}^C, \quad \forall ltw, \tag{7d}$$

$$-\hat{r}_i \leq q_{i(t-1)w} - q_{itw} \leq \hat{r}_i : \mu_{itw}^{D_1}, \mu_{itw}^{D_2}, \quad \forall itw, \tag{7e}$$

$$0 \leq s_{itw} \quad : \mu_{itw}^E, \quad \forall (i \in I^h)tw, \tag{7f}$$

$$0 \leq m_{itw} \leq M_i \quad : \mu_{itw}^{F_1}, \mu_{itw}^{F_2}, \quad \forall (i \in I^h)tw, \tag{7g}$$

$$m_{itw} - m_{i(t-1)w} = M_i^0 + V_{itw} - \Gamma_i q_{itw} - s_{itw} + \sum_{i\in I^{up}} (\Gamma_i q_{i(t-L_i)w} + s_{i(t-L_i)w}) : \lambda_{itw}^G, \forall (i{\in}I^h)tw. \tag{7h}$$

Here (7a) expresses the system's dispatch cost. Expressions (7b) are the generation constraints. Expression (7c) represents the energy balance constraint. Constraint (7d) accounts for the network representation and (7e) is setting the constraints on ramp rate. A system of connected reservoirs is considered for the hydropower producers. We model the constraints on spillage (7f), water level (7g), and hydrological balance condition (7h): we model connected reservoirs and time delay L_i for the water relocated from the upstream to the downstream reservoir. In hydropower plants with a large storage capacity, head variation has negligible influence on operating efficiency in the short-term. Therefore we assume a constant production equivalent Γ_i [15].

The optimal dispatch problem is linear in its decision variables. We can equivalently write it as KKT conditions. The stationary conditions of (7) are:

$$\frac{dL}{dq_{itw}} = -P_w \hat{c}_{it} + \mu_{itw}^{A_1} - \mu_{itw}^{A_2} + \lambda_{tw}^B - \sum_{n,l} \mu_{ltw}^C H_{ln} - \mu_{itw}^{D_1} + \mu_{itw}^{D_2} + \mu_{i(t+1)w}^{D_1} - \mu_{i(t+1)w}^{D_2} + \Gamma_i \lambda_{itw}^G - \Gamma_i \sum_{i\in I^{dn}} \lambda_{itw}^G = 0 \ : \phi_{itw}^{stq}, \ \forall itw, \tag{8a}$$

$$\frac{dL}{ds_{itw}} = \mu_{itw}^E + \lambda_{itw}^G - \sum_{i\in I^{dn}} \lambda_{itw}^G = 0 : \phi_{itw}^{sts}, \forall (i{\in}I^h)tw, \tag{8b}$$

$$\frac{dL}{dm_{itw}} = \mu_{itw}^{F_1} - \mu_{itw}^{F_2} + \lambda_{itw}^G - \lambda_{i(t+1)w}^G = 0 : \phi_{itw}^{stm}, \ \forall (i{\in}I^h)tw. \tag{8c}$$

The complementary slackness conditions are equivalent to the result of the strong duality theorem:

$$\sum_{i,t,w} P_w \hat{c}_{it} q_{itw} = -\sum_{i,t,w} \mu_{itw}^{A_2} \hat{q}_{itw} + \sum_{t,w} \lambda_{tw}^B \sum_n D_{ntw} - \sum_{l,t,w} \mu_{ltw}^C (F_l + \sum_n H_{ln} D_{ntw}) - \sum_{i,t,w} \mu_{itw}^{D_1} \hat{r}_i - \sum_{i,t,w} \mu_{itw}^{D_2} \hat{r}_i$$

$$-\sum_{(i\in I^h),t,w}\mu^{F_2}_{itw}M_i+\sum_{(i\in I^h),t,w}\lambda^G_{tw}(M^0_i+V_{itw}):\phi^{dual}_{itw}. \quad (9)$$

Since Lagrange multiplier of system balance λ^B_{tw} carries the information regarding the system price, LMP can be expressed as $\lambda_{ntw}=\lambda^B_{tw}-\sum_{l,n}\mu^C_{ltw}H_{ln}$. We can reformulate the profit using stationary conditions (8), complementary slackness, and hydropower balance equation [16]:

$$\pi_{iw}=\sum_t(\lambda^B_{tw}-\sum_{l,n}\mu^C_{ltw}H_{ln}-C^M_i)q_{itw}+m_{iTw}\Lambda^f_i\frac{1}{\Gamma_i}\overset{(8a)}{=}$$
$$\sum_t(P_w\hat{c}_{it}-\mu^{A_1}_{itw}+\mu^{A_2}_{itw}+\mu^{D_1}_{itw}-\mu^{D_2}_{itw}-\mu^{D_1}_{i(t+1)w}+\mu^{D_2}_{i(t+1)w}$$
$$-\Gamma_i\lambda^G_{itw}+\Gamma_i\sum_{i\in I^{dn}}\lambda^G_{i(t+L_i)w}-C^M_i)q_{itw}+m_{iTw}\Lambda^f_i\frac{1}{\Gamma_i}$$
$$=\sum_t\Big(\mu^{A_2}_{itw}\hat{q}_{it}+(\mu^{D_1}_{itw}+\mu^{D_2}_{itw})\hat{r}_i+P_w\hat{c}_{it}q_{itw}-C^M_iq_{itw}$$
$$+\mu^{F_2}_{it}M_i-\lambda^G_{itw}(M^0_i+V_{it})\Big)+m_{iTw}\Lambda^f_i\frac{1}{\Gamma_i}. \quad (10)$$

In the next section we show how we can combine the profit formulation and the Nash equilibrium definition.

C. EPEC as a MILP

To formulate the whole EPEC as a MILP we combine the Nash equilibrium definition (2), profit definition for each of the players (10), and the KKT conditions of the dispatch problem (7) for the candidate strategy s^*_i and alternative strategies s_i [17]. The variables of one-level optimization include primal and dual variables of (7), and variables of the upper-level problem: Ω^{NLP}={q_{itw}, s_{itw}, m_{itw}, $\mu^{A_1}_{itw}$, $\mu^{A_2}_{itw}$, $\mu^{D_1}_{itw}$, $\mu^{D_2}_{itw}$, μ^{E}_{itw}, $\mu^{F_1}_{itw}$, $\mu^{F_2}_{itw}$, μ^{C}_{ltw}, λ^B_{tw}, λ^G_{itw}, $\hat{q}_{it}$, $\hat{r}_i$, $\hat{c}_{it}$}. The final profit expression (10) and strong duality condition (9) contain bilinear terms $\mu^{A_2}_{itw}\hat{q}_{it}$, $\mu^{D_1}_{itw}\hat{r}_i$, $\mu^{D_2}_{itw}\hat{r}_i$ and $\hat{c}_{it}q_{itw}$. For the non-strategic units we assume: $\hat{q}_{it}=Q_i$, $\hat{r}_i=R_i$, $\hat{c}_{it}=C^M_i$, $\forall i\in(I\backslash I^h)$. For the strategic units these terms need to be linearized.

We approximate the generation capacity Q_i by a pre-defined number of discrete capacities $\hat{Q}_{ik}$. Therefore: $\mu^{A_2}_{itw}\hat{q}_{it}=\mu^{A_2}_{itw}\sum_k x^q_{itk}\hat{Q}_{ik}$=$\sum_k z^{A_2}_{itwk}$, where $\sum_k x^q_{itk}=1$. Variable $z^{A_2}_{itwk}$ can be linearized for all $(i\in I^h)twk$ as follows [18]:

$$-\bar{K}^q(1-x^q_{itk})\le z^{A_2}_{itwk}-\hat{Q}_{ik}\mu^{A_2}_{itw}\le\bar{K}^q(1-x^q_{itk}):\overline{\alpha}^{A_2}_{itwk},\underline{\alpha}^{A_2}_{itwk}, \quad (11a)$$
$$-\bar{K}^qx^q_{itk}\le z^{A_2}_{itwk}\le\bar{K}^qx^q_{itk}:\ \overline{\beta}^{A_2}_{itwk},\underline{\beta}^{A_2}_{itwk}. \quad (11b)$$

In the constraints above $\bar{K}^q$ is a suitably large constant, not too high to create computational instabilities, and not too low to put extra bounds on the variables [19]. Introducing $\bar{K}^r$, we reformulate (7b) and (7e) for all $(i\in I^h)twk$ as follows:

$$q_{itw}\le\hat{Q}_{ik}+\bar{K}^q(1-x^q_{itk}):\nu^{A_2}_{itwk}, \quad (12a)$$
$$q_{i(t-1)w}-q_{itw}\le\hat{R}_{ik}+\bar{K}^r(1-x^r_{ik}):\nu^{D_1}_{itwk}, \quad (12b)$$
$$q_{itw}-q_{i(t-1)w}\le\hat{R}_{ik}+\bar{K}^r(1-x^r_{ik}):\nu^{D_2}_{itwk}. \quad (12c)$$

We repeat the linearization steps for other bilinear terms, introducing $z^{D_1}_{itwk}=\mu^{D_1}_{itw}\sum_k x^r_{ik}\hat{R}_{ik}$, $z^{D_2}_{itwk}=\mu^{D_2}_{itw}\sum_k x^r_{ik}\hat{R}_{ik}$, $z^{obj}_{itwk}=q_{itw}\sum_k x^c_{itk}\hat{C}_{ik}$. We form a new set of variables Ω^{MILP} by adding x^q_{itk}, x^r_{ik}, x^c_{itk}, $z^{A_2}_{itwk}$, $z^{D_1}_{itwk}$, $z^{D_2}_{itwk}$, z^{obj}_{itwk} to Ω^{NLP}. Additionally, we introduce π^{min}_w to formulte a mixed-integer reformulation of (4):

$$\pi^{min}_i(s^*_i,s^*_{-i})+\epsilon_i\ge\pi^{min}_i(s_i,s^*_{-i}),\ \ \forall i\in I^h \quad (13)$$

Introducing binary variables x^{rob}_{iw} we ensure that π^{min}_i takes the smallest value in scenarios $\min_w[\pi_{iw}]$:

$$\pi^{min}_i\le\pi_{iw},\forall(i\in I^h)ws \quad (14a)$$
$$\pi^{min}_i\ge\pi_{iw}-\bar{K}(1-x^{rob}_{iw}),\forall(i\in I^h)ws, \quad (14b)$$
$$\sum_w x^{rob}_{iw}=1. \quad (14c)$$

Proposed stochastic MILP model is set out in (15).

$$\underset{\Omega^{\text{MILP}}}{\text{minimize}}\ \Delta\pi=\sum_{i\in I^h}\epsilon_i \quad (15a)$$

subject to: NE definition (3), or (13)-(14), $\epsilon_i\ge0$,

For s^*_i and (s_i,s^*_{-i}), $\forall i\in I^h$:

$$\{\pi_{iw}=\sum_t(\sum_k(z^{A_2}_{itwk}+z^{D_1}_{itwk}+z^{D_2}_{itwk}+P_wz^{obj}_{itwk})-C^M_iq_{itw}+$$
$$\mu^{F_2}_{itw}M_i-\lambda^G_{itw}(M^0_i+V_{it}))+m_{iTw}\Lambda^f_i\frac{1}{\Gamma_i},\forall(i\in I^h)w, \quad (15b)$$

(7c), (7d), (7f)-(7h), (8), (11), (12),

(7b), (7e), $\forall i\in I\backslash I^h$,

$\mu^{A_1}_{itw},\mu^{A_2}_{itw},\mu^{C}_{ltw},\mu^{D_1}_{itw},\mu^{D_2}_{itw},\mu^{E}_{itw},\mu^{F_1}_{itw},\mu^{F_2}_{itw}\ge0$,

Linearization of $z^{D_1}_{itwk}$, $z^{D_2}_{itwk}$, z^{obj}_{itwk} as in (11), $\forall i\in I^h$,

Linearization of (9) as in (11)}.

III. Solution approach: Modified Benders Decomposition

Our proposed EPEC includes constraints of all strategic generators for all strategies and scenarios. This makes it very hard to solve directly using the commercial MILP solvers. In this section we apply MBDA [20] to our EPEC formulation, which allows us to decompose the initial MILP into a small coordinating master problem and an LP subproblem, which is easier to solve. MBDA removes the disjunctive parameter, resulting in the improved computational properties[1].

A. Benders decomposition for disjunctive programming

A general linear disjunctive program has a form:

$$\underset{y,x_k}{\text{minimize}}\ cy \quad (16)$$
$$\text{subject to: } Ay\ge b,\quad \tilde{A}y\ge\tilde{b}-\bar{K}(1-x_k),\forall k$$
$$\sum_k x_k=1,\quad x_k\in\{0,1\},\quad y\ge0.$$

We employ the standard Benders decomposition. Given a candidate vector $\check{x}_k$, we can formulate a linear program and find its dual:

Primal subproblem $\rightarrow$ *Dual subproblem* (17)

$$\underset{y}{\text{minimize}}\ cy \qquad \underset{u,\tilde{u}}{\text{maximize}}\ ub+\sum_k\tilde{u}_k(\tilde{b}$$

[1]For conciseness we show Benders decomposition algorithm applied to Bayesian Nash equilibrium.

$$
\begin{aligned}
&\text{subject to: } Ay \geq b, && -\bar{K}(1-\check{x}_k))\\
&\tilde{A}y \geq \tilde{b} - \bar{K}(1-\check{x}_k), \forall k && \text{subject to: } uA + \tilde{u}\tilde{A} \leq c,\\
&y \geq 0. && u, \tilde{u} \geq 0.
\end{aligned}
$$

Note that the feasible region of the dual problem is independent from x_k and $\bar{K}$. The master problem is stated as:

$$
\begin{aligned}
&\underset{y, x_k}{\text{minimize}}\ z && (18)\\
&\text{subject to: } z \geq u_j b + \sum_k \tilde{u}_{kj}(\tilde{b} - \bar{K}(1-x_k)),\\
&\sum_k x_k = 1, \quad x_k \in \{0,1\}.
\end{aligned}
$$

Using the special disjunctive structure of (16) we can restate the master problem as a set-partitioning problem [20]:

$$
\begin{aligned}
&\underset{x_k, \theta_j}{\text{minimize}} \quad z = K_0\theta_0 + \sum_j K_j \theta_j && (19)\\
&\text{subject to: } \sum_{k \in \Omega_j^p} (1-x_k) \leq |\Omega_j^p| - 1 + \sum_{j' \in J'} \theta_{j'} \mathbf{I}(K_j \leq K_{j'}), \forall j,\\
&\sum_k x_k = 1, \quad \theta_0 + \sum_j \theta_j = 1, \quad \{x_k, \theta_j\} \in \{0,1\}.
\end{aligned}
$$

Here parameter $K_j = u_j b + \sum_k \tilde{u}_{kj}(\tilde{b} - \bar{K}(1-x_k))$ is the calculated objective function of the subproblem and $K_0 = -\infty$ is the lower bound of the master problem. θ_j is an auxiliary binary variable created at every iteration. J' is the index set of previous iterations and j is the index of current iteration ($\mathbf{I}$ is an "if" operator). Ω_j^p is a set of positive Lagrange multipliers $\tilde{u}_j$ corresponding to the disjunctive constraints in j, and $|\Omega_j^p|$ is the cardinality of this set.

B. MBDA for EPEC

The disjunctive nature of (17) leads to an important observation regarding the term with disjunctive constraint in the objective function of the dual subproblem: $\sum_k \bar{K}\tilde{u}_k(1-\check{x}_k)$. If $\check{x}_k=1$, the term will be equal to zero, as $1-\check{x}_k=0$. Also, if $\check{x}_k=0$ and $\bar{K}$ is properly tuned, the inequality $\tilde{A}y \geq \tilde{b} - \bar{K}(1-\check{x}_k)$ in the primal subproblem will be non-binding, and therefore corresponding Lagrange multiplier $\tilde{u}_k=0$. Taking into account that $\bar{K}$ is a positive parameter, we can divide the term by $\bar{K}$ and move it to the constraint set, as equal to zero: $\sum_k \tilde{u}_k(1-\check{x}_k)=0$.

For a fixed strategic bid $(\check{x}_{itk}^q, \check{x}_{ik}^r, \check{x}_{itk}^c)$ optimization problem (2) becomes an LP. Similarly to the logic in Section III-A by applying the modification described in the beginning of this section, we can derive a dual of this problem:

$$
\begin{aligned}
&\underset{\Omega^{\text{SP}}}{\text{maximize}}\ \Delta\pi^{dual} = \sum_{wk} P_w \sum_t \Big(\phi_{tw}^B \sum_n D_{ntw} - \sum_l \nu_{ltw}^C (F_l +\\
&\sum_n H_{ln} D_{ntw}) - \sum_{i \in I \setminus I^h} \phi_{itw}^{stq} C_i - \sum_{i \in I^h} \phi_{itw}^{stq} \sum_k C_{ik} \check{x}_{ik}^c - \sum_{i \in I \setminus I^h} \phi_{itw}^{A_2} Q_i\\
&- \sum_{i \in I^h} \phi_{itw}^{A_2} \sum_k \hat{Q}_{ik} - \sum_{i \in I \setminus I^h} (\phi_{itw}^{D_1} + \phi_{itw}^{D_2}) R_i - \sum_{i \in I^h} (\phi_{itw}^{D_1} +\\
&\phi_{itw}^{D_2}) \sum_k \hat{R}_{ik} - \sum_{i \in I^h} \phi_{itw}^{F_2} M_i + \sum_{i \in I^h} \phi_{itw}^G (M_i^0 + V_{itw})\Big), && (20a)\\
&\text{subject to: } \Omega^{\text{SP}} \in X, && (20b)\\
&(1-\check{x}_{itk}^q)(\nu_{itwk}^{A_2} + \overline{\alpha}_{itwk}^{A_2} + \underline{\alpha}_{itwk}^{A_2}) \check{x}_{itk}^q (\overline{\beta}_{itwk}^{A_2} + \underline{\beta}_{itwk}^{A_2})\\
&+ (1-\check{x}_{ik}^r)(\nu_{itwk}^{D_1} + \overline{\alpha}_{itwk}^{D_1} + \underline{\alpha}_{itwk}^{D_1}) + \check{x}_{ik}^r (\overline{\beta}_{itwk}^{D_1} + \underline{\beta}_{itwk}^{D_1})\\
&+ (1-\check{x}_{ik}^r)(\nu_{itwk}^{D_2} + \overline{\alpha}_{itwk}^{D_2} + \underline{\alpha}_{itwk}^{D_2}) + \check{x}_{ik}^r (\overline{\beta}_{itwk}^{D_2} + \underline{\beta}_{itwk}^{D_2})\\
&+ (1-\check{x}_{itk}^c)(\overline{\alpha}_{itwk}^{obj} + \underline{\alpha}_{itwk}^{obj}) + \check{x}_{itk}^c (\overline{\beta}_{itwk}^{obj} + \underline{\beta}_{itwk}^{obj}) = 0,\\
&\forall (i \in I^h) twk. && (20c)
\end{aligned}
$$

Algorithm 1: Modified Benders Decomposition Approach (MBDA)

Data: $\{\check{x}_{itk}^q, \check{x}_{ik}^r, \check{x}_{itk}^c\} \leftarrow$initialize, $j \leftarrow 1$, $z^{up} \leftarrow +\infty$, $z^{lo} \leftarrow -\infty$

while $z^{up} - z^{lo} \geq 0$ **do**
  `solve` subproblem;
  $\Omega_j^p \leftarrow \Omega^p$, $z^{up} \leftarrow \max\{\Delta\pi^{dual}, z^{up}\}$, $K_j \leftarrow \Delta\pi^{dual}$;
  `solve` master problem `use` **tree-search algorithm**:
  begin
    $\theta_0 \leftarrow 1$, $J' \leftarrow J$;
    while master problem `infeasible` **do**
      $\theta_{j'} \leftarrow 1\mathbf{I}(K_{j'} \leq K_{j \in J'})$;
      $J' \setminus \{j'\}$;
    end
  end
  $\check{x}_{itk}^q \leftarrow x_{itk}^q$, $\check{x}_{ik}^r \leftarrow x_{ik}^r$, $\check{x}_{itk}^c \leftarrow x_{itk}^c$, $j \leftarrow j+1$, $z^{lo} \leftarrow z$;
end

Here Ω^{SP} is the set of decision variables in (20): Ω^{SP2} = $\{\underline{\alpha}_{itwk}^{A_2}, \overline{\alpha}_{itwk}^{A_2}, \underline{\alpha}_{itwk}^{D_1}, \overline{\alpha}_{itwk}^{D_2}, \underline{\alpha}_{itwk}^{obj}, \overline{\alpha}_{itwk}^{obj}, \underline{\beta}_{itwk}^{A_2}, \overline{\beta}_{itwk}^{A_2}, \underline{\beta}_{itwk}^{D_1}, \overline{\beta}_{itwk}^{D_2}, \underline{\beta}_{itwk}^{obj}, \overline{\beta}_{itwk}^{obj}, \nu_{itw}^{A1}, \nu_{itw}^{A2}, \nu_{itw}^{C}, \nu_{itw}^{D1}, \nu_{itw}^{D2}, \nu_{itw}^{E}, \nu_{itw}^{F1}, \nu_{itw}^{F2}, \phi_{tw}^{B}, \phi_{itw}^{G}, \phi_{itw}^{stq}, \phi_{itw}^{sts}, \phi_{itw}^{stm}, \phi^{dual}, \nu_{itw}^{A_1^+}, \nu_{itw}^{A_2^+}, \nu_{ltw}^{C^+}, \nu_{itw}^{D1^+}, \nu_{itw}^{D_2^+}, \nu_{itw}^{E^+}, \nu_{itw}^{F_1^+}, \nu_{itw}^{F_2^+}\}$. Variables with ν and + are the Lagrange multipliers of dual feasibility constraints in (15). For example, $\nu_{itw}^{A_1^+}$ is the Lagrange multiplier of $\mu_{itw}^{A_1} \geq 0$. The feasible region X is represented by the corresponding stationary, primal feasibility, and dual feasibility conditions of optimization problem (2), when x_{itk}^q, x_{ik}^r and x_{itk}^c are fixed to $\check{x}_{itk}^q$, $\check{x}_{ik}^r$ and $\check{x}_{itk}^c$. There are many constraints representing these conditions, which we did not include here for the sake of conciseness. The master problem can be formulated as follows:

$$
\begin{aligned}
&\underset{x_{itk}^q, x_{ik}^r, x_{itk}^c, \theta_j}{\text{minimize}} \quad z = K_0\theta_0 + \sum_j K_j \theta_j && (21a)\\
&\text{subject to:} \sum_{i,k \in \Omega_j^p} \Big(\sum_t ((1-x_{itk}^q) + (1-x_{itk}^c)) + (1-x_{ik}^r) \Big)\\
&\qquad \leq |\Omega_j^p| - 1 + \sum_{j' \in J'} \theta_{j'} \mathbf{I}(K_j \leq K_{j'}), \forall j, && (21b)\\
&\sum_k x_{itk}^q = 1, \sum_k x_{ik}^r = 1, \sum_k x_{itk}^c = 1, \forall (i \in I^h) t, && (21c)\\
&\theta_0 + \sum_j \theta_j = 1, \quad \{x_{itk}^q, x_{ik}^r, x_{itk}^c, \theta_j\} \in \{0,1\}. && (21d)
\end{aligned}
$$

In optimization problem (21) parameter $K_j = \Delta\pi_j^{dual}$ and $\Omega = \{\overline{\alpha}_{itwk}^{A_2}, \underline{\alpha}_{itwk}^{A_2}, \overline{\beta}_{itwk}^{A_2}, \underline{\beta}_{itwk}^{A_2}, \overline{\alpha}_{itwk}^{D_1}, \underline{\alpha}_{itwk}^{D_1}, \overline{\beta}_{itwk}^{D_1}, \underline{\beta}_{itwk}^{D_1}, \overline{\alpha}_{itwk}^{D_2}, \underline{\alpha}_{itwk}^{D_2}, \overline{\beta}_{itwk}^{D_2}, \underline{\beta}_{itwk}^{D_2}, \overline{\alpha}_{itwk}^{obj}, \underline{\alpha}_{itwk}^{obj}, \overline{\beta}_{itwk}^{obj}, \underline{\beta}_{itwk}^{obj}, \nu_{itwk}^{A_2}, \nu_{itwk}^{D_1}, \nu_{itwk}^{D_2}\}$. Ω_j^p is the index set of strictly positive variables of Ω in iteration j.

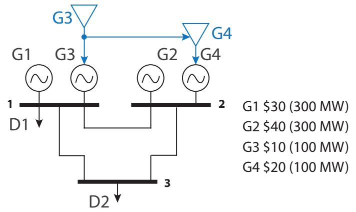

Fig. 2. 3-node case study

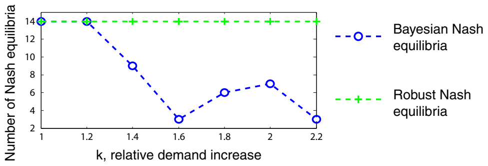

Fig. 3. Number of Nash equilibria under increasing uncertainty. Relative demand increase: $D_{ntw_2} = kD_{ntw_1}$.

This master problem can be efficiently solved using the tree-search algorithm [20]. Using this algorithm we set θ_0=1 and find feasible $x^q_{itk}, x^r_{ik}, x^c_{itk}$. If the master problem turns out infeasible we set θ_j, corresponding to the best previously obtained K_j, to 1 and search for a feasible solution again repeating until the master problem is feasible. The whole procedure of MBDA is outlined in Algorithm 1.

IV. Illustrative Case Study

In this section we illustrate the mathematical model developed in Section II. We use a 3-node case study with 4 generators, of which $G3$ and $G4$ are strategic hydropower producers. The reservoirs of hydropower producers are connected by a waterway. For our case study we consider a time horizon of two time period. The demand values are: $D_{n_1t_1w}$=$D_{n_1t_2w}$=50 MW, $D_{n_2t_1w}$=40 MW, $D_{n_2t_2w}$=60 MW. For the ease of interpretation we assume that strategic generators bid the same price-quantity in each of two time periods. Data for the system are presented in Fig. 2. The electric network is shown in black and the waterway system in blue. The ramp rates of all units are assumed to be 0.1 of the generation capacity.

A. Number of Nash equilibria under uncertainty

EPEC has a non-convex feasible region, resulting in a multitude of possible solutions (or no solutions in some cases). It is logical that a higher number of strategic actions available to the generators makes the number of Nash equilibria higher. In this section we check how uncertainty affects the number of Bayesian and robust Nash equilibria.

We assume that the only source of uncertainty in the system is demand and introduce a coefficient k, showing the relation between demand levels in two different scenarios. In Fig. 3 we plot the number of Nash equilibria obtained for different values of k. After each new equilibrium we add an integer cut to ensure that the same equilibrium is not obtained again [21].

TABLE I
Comparison of Bayesian and robust best and worst Nash equilibria. DC: dispatch cost.

		Bayesian		Robust	
		best	worst	best	worst
Capacity bid, $\hat{q}$, MW	G3	100	100	100	**75**
	G4	**25**	**50**	**75**	100
Ramp-rate bid, $\hat{r}$, $\frac{MW}{\Delta t}$	G3	10	10	10	**2.5**
	G4	10	**7.5**	10	**5**
Price bid, $\hat{c}$, $\frac{\$}{MWh}$	G3	**20**	**20**	**15**	**20**
	G4	20	**35**	20	**35**
Profit, $\sum_w \pi_{iw}$, $	G3	4637.5	5400	2845	4500
	G4	287.5	287.5	100	250
DC, $		4950	5250	3890	5700

Our first observation is that the number of Bayesian and robust Nash equilibria is the same, when scenarios are the same, i.e. when there is no uncertainty. This occurs because (3) and (4) are simplified to the same expression under the assumption of no uncertainty.

Second, the number of Bayesian Nash equilibria decreases with greater uncertainty. In general it is harder for the strategic producers to offer a bid, which will exploit the market conditions. Therefore there are fewer possibilities for strategic behavior and less Nash equilibria correspondingly. As robust Nash equilibrium is only taking into account the worst-case scenario, changing the more profitable scenario does not affect the amount of Nash equilbria.

B. Comparison of Nash equilibria

Since a multitude of solutions is difficult in interpretation, the policy makers and market participants would often prefer to have a single Nash equilibrium reference that can characterize the market situation. This is sometimes implemented with a concept of the best/worst Nash equilibrium [17]. We implement this by extending the objective function (15a) by minimization/maximization of the total social costs in the system correspondingly. In this section we analyze the strategic behavior, as different concepts of Nash equilibrium introduced in Section II are applied.

We compare the results from the best and worst Nash equilibria in Table I: we provide the bids offered by the strategic hydropower producers to the market operator (bold values show withholding), profits, and dispatch cost corresponding to the different Nash equilibria. The following observations can be made:

- **Bids:** Withholding occurs in both types of Nash equilibria. There is more withholding if we consider the worst Nash equilibria, resulting in higher profits for the strategic generators. Worst Nash equilibria (high social costs) – is usually the result from high market prices.
- **Profits of strategic generators:** The profits in Bayesian Nash equilibrium are higher, than in the robust Nash equilibrium. In robust Nash equilibrium strategic bid is optimal for the worst-case scenario, while for Bayesian Nash eqilibrium all scenarios are taken into account.
- **Social costs span:** The difference between the worst and the best Nash equilibria is the largest for the robust Nash equilibria. This can be explained by the nature of

TABLE II
EFFECT OF THE FUTURE WATER VALUE AND HYDROLOGICAL COUPLING ON HYDROPOWER PRODUCERS' STRATEGIES. DC: DISPATCH COST.

		Future water value		Waterway	
		$\lambda_i^f = 0$	$\lambda_i^f = 10$	no	yes
Capacity bid, $\hat{q}$, MW	G3	100	**50**	100	**25**
	G4	100	100	100	100
Ramp-rate bid, $\hat{r}$, $\frac{MW}{\Delta t}$	G3	10	**5**	10	10
	G4	**2.5**	**5**	10	**2.5**
Price bid, $\hat{c}$, $\frac{\$}{MWh}$	G3	**20**	**20**	**20**	**20**
	G4	**30**	**40**	**40**	**30**
Profit, $\sum_w \pi_{iw}$, $	G3	5400	4950	300	325
	G4	287.5	800	150	281.5
DC, $		5250	6200	7265	7209

robust Nash equilibrium. For this type of equilibrium we only consider the worst-case scenario, therefore we do not consider if the chosen strategy holds as a Nash equilibrium for another scenario. This provides more flexibility for the strategic generator when choosing its strategy.

Different types of Nash equilibrium can be used depending on the modeling goals. We can interpret from the results that Bayesian Nash equilibrium is more consistent with realistic behavior, assuming that the probability distributions of the uncertain parameters are known. Robust Nash equilibrium can be applied, when the probability distribution functions of the uncertain parameters are impossible to obtain (e.g., if the event does not have a historical data).

C. Strategic hydropower behavior

In the previous section we have observed that all parts of the strategic bid – production, ramp, price – have an effect on the profits of the strategic generators. Hydropower producers have additional specific characteristics that make the analysis of their strategic bidding different from thermal generators.

Two main aspects that differentiate hydropower producers from thermal generators are the future water value and the waterway coupling of the reservoirs in a congested network. In Table II we study the effect of these aspects on the worst Bayesian Nash equilibrium.

Higher future water value has an important effect on the dispatch cost of the system. Since $G4$ is a more expensive generator, it has less advantageous position in the optimal dispatch. Generally its profit is lower than the profit of $G3$. However, with the increased future water value $G4$ withholds most of its production from the market led by the assumption that it can sell energy generated from water for the higher price later. The dispatch cost increases considerably due to this behavior.

To study the effect of waterway coupling we decrease the flow limits on the power lines. In these conditions hydropower units can take advantage of relocating water between the nodes using the waterways instead of relocating energy in the network. In case of waterway coupling we can obtain lower dispatch cost, and higher profits for the generators.

V. NUMERICAL RESULTS

This section describes the relative computational performance of MBDA. We implement and test MBDA, primal, and dual Benders approaches, as described below.

TABLE III
EFFECT OF DISJUNCTIVE PARAMETER ON THE NUMBER OF ITERATIONS OF BENDERS PROCEDURE. INFEAS: THE PROBLEM IS INFEASIBLE.

$\bar{K} =$	10^3	10^4	10^6	10^8	10^{12}
	number of iterations until convergence				
Primal Benders	infeas	5	9	16	infeas
Dual Benders	infeas	5	5	5	infeas
MBDA	5	5	5	5	5

In primal Benders approach, as implemented in e.g. [22], binary variables $x^q_{itk}, x^r_{ik}, x^c_{itk}$ are taken as continuous variables. Constraints are added to fix them to the values calculated from the master problem. The marginal values of these constraints are the sensitivities, which are used to create the cuts in the master problem [23].

The dual Benders approach, as presented in e.g. [24], is a non-modified version of Benders: the subproblem has the form of the dual problem in (16), while master problem has a form as in (18). In the dual Benders approach the disjunctive parameter moves from the constraint set to the objective function. In contrast, MBDA is completely free from the disjunctive parameter. We will first discuss the effect of the dsjunctive parameter on the convergence of Benders procedure. Second, we will provide the computational results for realistically-sized case studies.

A. Mitigating the disjunctive parameter effect

Applying the MBDA allows us to formulate the problem without the disjunctive parameter. In comparison, primal and dual Benders methods are affected by the value of the disjunctive parameter. Table III presents the convergence procedure of Benders algorithm for primal, dual Benders methods, and MBDA for a 3-node case study. It can be seen that even for a very small case study both primal and dual methods are affected by the value of the disjunctive parameter, while MBDA does not contain it in the formulation.

B. Performance of MBDA and standard Benders on realistic-size networks

EPEC problems are known to be computationally challenging. In this section we demonstrate the computational effort needed for finding the Nash equilibria. All simulations are performed on a computer with 18 cores with hyper-threading Intel Xeon E5-2699 CPU and 128 GB of RAM. We solve linear subproblems using barrier method in CPLEX, as primal and dual simplex methods appear to be less stable for large case studies. We also avoid matrix scaling. While in the 24-node case study the subproblem is more efficiently solved as one block, for larger case studies we make use of the parallelizable robust Nash equilibrium structure. We separate the calculation of profit for different strategies as a separate problem and parallelize it by scenario using grid computing to improve the numerical efficiency [25]. In this way we also avoid using the binary variables x^{rob}_{iw}. Bayesian Nash equilibrium is computed using the internal CPLEX parallelization function and 4 computing threads.

TABLE IV
BENDERS PERFORMANCE ON REALISTIC CASE STUDIES. *: NO RESULTS OBTAINED AFTER 10 HOURS.

Benders algorithm	24-node, $G1, G2 \in I^h$				118-node, $G1, G2 \in I^h$				300-node, $G5, G6 \in I^h$			
	Bayesian		Robust		Bayesian		Robust		Bayesian		Robust	
	time, s	size, MB	time, s	size, MB	time, s	size, MB	time, s	size, MB	time, s	size, MB	time, s	size, MB
1) Standard Benders: primal subproblem	221	1027	174	972	*	*	2122	655	*	*	*	*
2) Standard Benders: dual subproblem	158	1023	134	972	2334	13706	*	*	*	*	*	*
3) Modified Benders approach (MBDA)	179	1023	133	972	2629	13408	1368	655	18058	62298	13172	4397
CPLEX MILP	191	1027	174	1027	*	*	*	*	*	*	*	*

We assume 4 time periods and 2 strategic hydropower producers. The total number of combinations for bidding is 262144. We represent the uncertainty in the system by 20 scenarios, obtained by applying the moment-matching technique described in [11] to the real Nord Pool data [26]. The number of constraints and variables raises the memory consumption of (15) up to 62 GB.

The results of simulations are presented in Table IV. The time needed for convergence and the maximum size of the problems are shown. The results show that standard Benders with a dual subproblem and MBDA are clearly the most efficient methods. For small case study both methods are providing results in a similar computation time. However, standard Benders is affected by the disjunctive parameter and in some cases does not converge even after 10 hours. This is due to a large coefficient matrix and a very large number of variables. For a 300-node case study the number of discrete variables in the case of robust Nash equilibrium reaches 4090. This makes tuning of the disjunctive parameter complicated.

We have shown that MBDA can reliably solve large case studies. The performance of the method can be attributed to the absence of the disjunctive parameter and to a decomposition of a large MILP problem into a large LP subproblem and a small MILP master problem, which can be efficiently solved by a tree-search algorithm.

VI. CONCLUSION

In this paper we propose an EPEC formulation to model Bayesian and robust Nash equilibria in a hydro-dominated power system under uncertainty. The problem can be reformulated as a MILP. We compare two definitions of Nash equilibrium under uncertainty, using an illustrative case study. For large case studies the problem can be efficiently solved by applying the Benders decomposition method. The application of the solution algorithm – MBDA – to EPEC problem reformulated as a MILP does not require the optimal tuning of the disjunctive parameter.

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