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Reliability of Fatigue Strength Curves for Riveted Connections Using Normal and Weibull Distribution Functions

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Abstract: In North America and Europe, there is a significant number of centenary metallic bridges that require maintenance and strengthening operations. These structures need to be adapted to increasing traffic intensities, and their structural integrity should be constantly evaluated. Fatigue damages were not considered in the original design of old metallic bridges, and riveted connections are one of the most frequent sources of fatigue damages on these structures. This paper intends to be a contribution for the reliable assessment of the fatigue behavior of riveted connections by proposing S-N curves for these structural details. Experimental fatigue data is analyzed from multiple bridges across Europe, and different statistical methods were implemented. A normal distribution function was implemented following the ISO 12107 standard, and the results were compared with the implementation of the two-parameter Weibull distribution function. Different estimation methodologies were implemented to determine the parameters of the Weibull distribution. S-N curves obtained by the statistical analysis were then compared with design recommendations from North American and European standards. **DOI: 10.1061/AJRUA6.0001081.** *© 2020 American Society of Civil Engineers.*

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Introduction

Metallic bridges started to be constructed all over the world, mainly in the second half of the 19th century, which was the period when iron became available to be used as construction material. A significant part of these structures is still in service either because maintenance and strengthening operations are easily affordable compared with constructing a new bridge or due to the architecture and cultural importance of these structures for society. From the period that these structures were designed until now, the differences in the traffic intensity, not only in terms of their frequency but also in terms of their weight, might have increased significantly. In this sense, it is essential to monitor their structural integrity and evaluate their performance in order to accurately define what measures should be taken to maintain and prolong their service life.

There is a specific phenomenon, relevant for structures with long service periods, that can lead generally to noncollapsed failures but also in some cases to total failure: fatigue cracks. This is especially important because old metallic riveted bridges were not originally designed with this phenomenon in mind (Akesson 2010; Mohammad 2002). Only recently, scientific studies have been conducted to evaluate the fatigue behavior of materials used in these structures (wrought-iron, puddle iron, or mild steels) (De Jesus et al. 2015, 2011; Lesiuk et al. 2019; Pedrosa et al. 2019). The most feasible method to assess the fatigue strength of materials and components is to perform experimental fatigue tests on structural details extracted from old metallic bridges. North American and European standards used to design metallic bridges for the fatigue phenomenon are based on experimental evidence using modern metallic materials, and therefore, they might not be adequate for these centenary bridges. Moreover, these standards do not describe how to design structural details, such as riveted connections. Kulak (2000) and Di Battista et al. (1998) analyzed a significant amount of fatigue data from experimental tests on riveted components and proposed the use of Detail category 71 from Eurocode 3 Part 1-9 (EC3-1-9) (CEN 2005) and Category D from AASHTO (2012). These design curves are very similar because they have the same inverse slope (3) and detail category (71 MPa, 2 million cycles). They only differ in the value of the constant amplitude fatigue limit (52 MPa in the case of EC3-1-9 and 48 MPa in the case of AASHTO Category D).

The computation of reliable design S-N curves for metallic components has been conducted using several approaches. Zhao et al. (2009) implemented a log-normal distribution to determine probabilistic S-N fields for long-term regimes. Schijve (2005) studied the implementation of three statistical distribution functions on S-N fields. Those functions were normal distribution, Weibull distribution, and log-normal distribution. It was found that both functions had a good correlation with the experimental data. Furthermore, scientific investigations performed by Correia et al. (2010) and Sanches et al. (2015) focused on the evaluation of the fatigue strength of riveted joints from old bridges using a probabilistic model. Their work was to consider the adequate probabilistic distribution functions on individual parameters/properties of existing deterministic fatigue models.

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Table 1. Details of the experimental data used in the analysis

Type of connection	Bridge	Year	Stress ratio	No. of specimens
Single shear	Luiz I (Pedrosa et al. 2019)	1886	0.1	7
	Pinhão (Pedrosa et al. 2019)	1906	0.1	7
	Trezoi (Pedrosa et al. 2019)	1956	0.1	8
	Taras and Greiner (2010a)	1975–1995	Variable	13
Double shear	Eiffel (Pedrosa et al. 2019)	1878	0.1	14
	Fão (Pedrosa et al. 2019)	1892	0.01	15
	Adour (Mayorga et al. 2016)	1864	0.0	10
	Taras and Greiner (2010a)	1935–1941	Variable	107

This paper presents experimental fatigue data from different scientific investigations on single and double riveted connections. Specimens are composed of metallic materials from centenary bridges. Different statistical methods were implemented to analyze the studied data and to elaborate reliable design S-N curves. The statistical analysis proposed in the Eurocode to define design curves were implemented, and a comparison was established with the implementation of the Weibull distribution. This alternative is widely used in the literature (Sakin and Ay 2008; Zhang et al. 2008) to analyze fatigue experimental data, and it is characterized by its flexibility to establish reliable assessments. Several scientific investigations can also be found in literature (Castillo et al. 2006; Castillo and Fernández-Canteli 2009), recommending that the two-parameter Weibull distribution does correlate with fatigue data. Therefore, this Weibull distribution depends on both shape and scale parameter whose estimation can be made by several methods: the maximum likelihood method (MLM), method of moments (MM), linear least squares method (LLSM), and weighted linear least squares method (WLLSM). All these methods were implemented, and the evaluation of each method was assessed by three different goodness-of-fit statistics tests: the Kolmogorov-Smirnov, Anderson-Darling, and χ-squared tests.

Fatigue Experimental Data

A set of experimental fatigue data from different scientific investigations (Mayorga et al. 2016; Pedrosa et al. 2019; Taras and Greiner 2010b) on riveted connections using material from European centenary bridges were used in this paper. For single shear riveted connections, a total of 35 specimens were analyzed. Some of them were extracted from structural elements of Portuguese centenary bridges, namely, seven specimens from the Luiz I bridge (located in Porto, Portugal, and designed in 1886), seven specimens from the Pinhão bridge (located in Esposende, Portugal, and designed in 1906), and eight specimens from the Trezoi bridge (located in Mortágua, Portugal, and designed in 1956). For double shear riveted connections, a total of 146 specimens were used. Part of them were also extracted from structural elements of Portuguese centenary bridges, namely, 14 specimens from the Eiffel bridge (located in Viana do Castelo, Portugal, and designed in 1878) and 15 specimens fabricated with metallic plates extracted from structural elements of the Fão bridge (located in Esposende, Portugal, and designed in 1892) (Pedrosa et al. 2019). Double shear specimens were complemented with experimental data from fatigue tests with 10 fabricated specimens composed of puddle iron plates extracted from structural elements of the bridge over the river Adour built in France in 1864 (Mayorga et al. 2016). Experimental data collected by Taras and Greiner (2010b) of fatigue tests for both single (13 specimens) and double (107 specimens) shear riveted connections were also added to the present analysis. A summary is presented in Table 1 with the details of the experimental data.

It is necessary to take into account mean stress effects when using fatigue data tested under different conditions, and therefore, experimental data used in this paper was normalized by computing the normalized stress range, $\Delta\sigma_{norm}$, as shown in Eq. (1). This methodology was proposed by Taras and Greiner (2010b)

$$\Delta\sigma_{norm} = \frac{\Delta\sigma}{f(R_\sigma)} \tag{1}$$

where $\Delta\sigma$ = stress range applied during the experimental test; and $f(R_\sigma)$ = normalization function to consider the stress ratio effects. This function depends on the year that the bridge was built. For wrought iron and mild steel manufactured before 1900, $f(R_\sigma)$ is defined

$$f(R_\sigma) = \frac{1-R_\sigma}{1-0.7\cdot R_\sigma} \Leftarrow -1 \leq R_\sigma \leq 0$$
$$f(R_\sigma) = \frac{1-R_\sigma}{1-0.75\cdot R_\sigma} \Leftarrow R_\sigma > 0 \tag{2}$$

For mild steel after 1900 (St37, St48, and St52, corresponding to S235, S275 and S355, respectively) the following normalization function is proposed:

$$f(R_\sigma) = \frac{1-R_\sigma}{1-0.4\cdot R_\sigma} \Leftarrow -1 \leq R_\sigma \leq 0$$
$$f(R_\sigma) = \frac{1-R_\sigma}{1-0.6\cdot R_\sigma} \Leftarrow R_\sigma > 0 \tag{3}$$

Statistical Analysis

There are multiple sources of scatter in fatigue, not only on laboratory test series but also on structures in service. Within the aspects to be consider on laboratory experimental campaigns, there is the material source (single batch of material or different batches), the production (specimen production and surface treatment), and the load (accuracy of the test equipment). However, statistical methods should be used to overcome this matter and establish practical solutions to analyze the obtained data. Unfortunately, the distribution function cannot be derived on the basis of physical arguments. In general, it is simply assumed or adjusted to the experimental data of a large test series. Two popular distributions are the normal or Gaussian distribution and the Weibull distribution (Schijve 2004).

ISO 12107

Usually, fatigue experimental data is represented with the applied stress range, $\Delta\sigma$, and number of cycles, N, using a logarithmic scale. This methodology allows one to define a linear relation (Basquin relation) between those parameters, as described in the following equation:

$$\log N = \log C + m \log \Delta\sigma \tag{4}$$

where m = inverse slope; and $\log C$ = intersection with the axis $\log \Delta\sigma$. Thereby, a mean S-N curve can be defined for the obtained results using a linear regression based on the least squares estimation method. Moreover, a characteristic S-N curve can be established as the lower limit corresponding to a probability of failure p_n for the population at a confidence level $(1-\alpha_n)$ and for a sample size n using the following equation:

$$\log N = \log C + m \log \Delta\sigma - k_{(1-\alpha_n),p_n,n} s \sqrt{1 + \frac{1}{n} + \frac{(\log \Delta\sigma - \overline{\log \Delta\sigma})^2}{\sum_{i=1}^{n} (\log \Delta\sigma_i - \overline{\log \Delta\sigma})^2}} \tag{5}$$

where $k_{(1-\alpha_n),p_n,n}$ = one-sided tolerance limit for a normal distribution, which depends on the confidence level, $(1-\alpha_n)$, probability of failure, p_n, and the sample size, n; s = standard deviation; and $\overline{\log \Delta\sigma}$ represents the mean value of applied stress ranges. The term inside the root sign is a correction to the estimated standard deviation. It is stated in the ISO 12107 standard (ISO 2012) that it can be neglected when the number and range of tests are large enough. Because S-N curves proposed in Eurocode 3 consider a constant value of standard deviation (straight line), this correction term was neglected in this study.

Two-Parameter Weibull Distribution and Estimation of Its Parameters

The Weibull distribution function is a statistical distribution function of wide applicability (Weibull 1951). The description of the cumulative probability function with two parameters is given in Eq. (6)

$$P(N) = 1 - e^{-\left[\frac{N}{\alpha_w}\right]^{\beta_w}} \tag{6}$$

where N = number of cycles at failure; α_w = scale parameter; and β_w = shape parameter. These parameters can be estimated with different methods. In this paper, four different estimation methods will be applied in order to determine their most accurate values using fatigue data from the experimental campaign: the MLM, MMM, LLSM, and WLLSM. The evaluation of each method is performed by computing mean-squared errors (MSE) between the estimative values for each method and the estimative values using Bernard's median rank (Fothergill 1990), which is presented in Eq. (7)

$$P(N_i) = \frac{i - 0.3}{n + 0.4} \tag{7}$$

where i = order number of failures; and n = sample size.

Maximum Likelihood Method

This method is popular within the scientific community mainly because it is versatile and reliable. The likelihood function of the Weibull distribution (Goglio and Rossetto 2004) is given by Eq. (8), where n is the sample size

$$\begin{aligned} L(\alpha_w, \beta_w | N) &= \prod_{i=1}^{n} f(N_i | \alpha_w, \beta_w) \\ &= \prod_{i=1}^{n} \left\{ \frac{\beta_w}{\alpha_w \beta_w} N_i [\beta_w - 1] \exp\left[-\left(\frac{N_i}{\alpha_w} \right)^{\beta_w} \right] \right\} \\ &= \left(\frac{\beta_w}{\alpha_w \beta_w} \right)^n \prod_{i=1}^{n} \left\{ N_i [\beta_w - 1] \exp\left[-\left(\frac{\sum N_i}{\alpha_w} \right)^{\beta_w} \right] \right\} \end{aligned} \tag{8}$$

The estimation of the Weibull parameters is obtained with the log-likelihood function. They are computed by maximizing the logarithmic function by means of interactive numerical methods.

Method of Moments

The moment method is one of the oldest estimation methods (Bolfarine and Sandoval 2001). The estimation of distribution parameters relies on matching sampling moments to the experimental data. Sampling moments are determined using Eq. (9)

$$\mu_k = \frac{1}{n} \sum_{i=1}^{n} N_i^k \tag{9}$$

For N as an independent and identically distributed variable, the first two population moments (M_1, M_2) are enough to estimate Weibull distribution parameters. These moments are computed with Eqs. (10) and (11)

$$M_1 = \alpha_w \Gamma\left(1 + \frac{1}{\beta_w}\right) \tag{10}$$

$$M_2 = \alpha_w^2 \Gamma\left(1 + \frac{2}{\beta_w}\right) \tag{11}$$

where Γ = gamma function. Using mathematical operations, it is possible to relate these equations resulting in Eq. (12). It corresponds to the coefficient of variation of the sample, and it is dependent only on the shape parameter, β_w. The solution can be found using the Newton-Raphson method. Ben-Israel (1966) detailed the application of this method

$$\frac{\mu_2}{\mu_1^2} = \frac{\Gamma(1 + \frac{2}{\beta_w})}{\Gamma^2(1 + \frac{1}{\beta_w})} \tag{12}$$

Linear Least Squares Method

The application of a logarithm in Eq. (6) allows one to establish a linear model as presented in Eq. (13) in which $X = \ln(N_i)$ and $Y = \ln(-\ln(1 - P(N_i)))$

$$\ln(-\ln(1 - P(N_i))) = \alpha_w \ln(N_i) - \alpha_w \ln(\beta_w) \tag{13}$$

The estimation of Weibull parameters is then computed with a simple linear regression (Barbosa et al. 2018). The optimization method is applied using the following equation:

$$\min QQ = \sum_{i=1}^{n} [Y_i - (\alpha_w X_i - \alpha_w \ln(\beta_w))]^2 \tag{14}$$

Finally, the estimation of the Weibull parameters is performed with the partial derivatives of QQ, as presented subsequently

$$\alpha_w = \frac{n \sum_{i=1}^{n} X_i Y_i - \sum_{i=1}^{n} X_i \sum_{i=1}^{n} Y_i}{n \sum_{i=1}^{n} X_i^2 - (\sum_{i=1}^{n} X_i)^2} \tag{15}$$

$$\beta_w = \exp\left(\frac{\sum_{i=1}^{n} Y_i - \alpha_w \sum_{i=1}^{n} X_i}{n\alpha_w}\right) \tag{16}$$

Weighted Linear Least Squares Method

The weighted linear least squares estimation method assigns different relevance for each element of the data set (Zhang et al. 2008). The approximation of weights, w_i, is modeled with a polynomial function dependent on the estimated values of the accumulated probability function $P(N_i)$, as presented in Eq. (17) (Barbosa et al. 2018)

$$w_i = -0.076 + 3.610P(N_i) - 6.867P(N_i)^2 + 13.54P(N_i)^3 - 9.231P(N_i)^4 \tag{17}$$

The optimization method follows the same strategy as in the linear least square method [Eq. (14)]. In this case, the values of scale and shape parameters of the Weibull distribution are obtained with Eqs. (18) and (19), respectively

$$\alpha_w = \frac{\sum_{i=1}^{n} w_i \sum_{i=1}^{n} w_i X_i Y_i - \sum_{i=1}^{n} w_i X_i \sum_{i=1}^{n} w_i Y_i}{\sum_{i=1}^{n} w_i \sum_{i=1}^{n} w_i X_i^2 - (\sum_{i=1}^{n} w_i X_i)^2} \tag{18}$$

$$\beta_w = \exp\left(\frac{\sum_{i=1}^{n} w_i Y_i - \alpha_w \sum_{i=1}^{n} w_i X_i}{\alpha_w \sum_{i=1}^{n} w_i}\right) \tag{19}$$

Probabilistic S-N Curves

For the computation of S-N curves based on the two-parameter Weibull distribution, it is necessary to consider that the values of the scale parameter and the shape parameter are constants for all stress levels of a S-N curve. Therefore, the number of cycles at failure need to be normalized by dividing the number of cycles obtained experimentally by the number of cycles obtained with the mean S-N curve. This is a commonly-used strategy (Júnior and Belísio 2014). After the determination of the Weibull distribution parameters, probabilistic S-N curves were computed using the following equation:

$$\log \Delta\sigma = \frac{\log\left(\frac{N}{\beta_w[-\ln(1-p)]^{\alpha_w}}\right)}{m} - \frac{\log C}{m} \tag{20}$$

where p = probability of failure.

Goodness-of-Fit Statistic Tests

The evaluation of the results from each estimation method was performed by using goodness-of-fit statistics. Three different tests were implemented for the Weibull distribution adherence to the experimental data: the Kolmogorov-Smirnov (KS) test, described in Eq. (21); the Anderson-Darling (AD) test, described in Eq. (22); and the χ-squared test, described in Eq. (23)

$$\mathrm{KS} = \sup |P(N_i) - P(N_i)_{\text{est·method}}| \tag{21}$$

$$\mathrm{AD} = -n - \frac{1}{n}\sum_{i=1}^{n}\{(2i-1)(\log(P(N_i)_{\text{est·method}}) + \log(1 - P(N_{n+1-i})_{\text{est·method}}))\} \tag{22}$$

$$\chi^2 = \frac{(P(N_i)_{\text{est·method}} - P(N_i))^2}{P(N_i)} \tag{23}$$

This evaluation enables one to determine the most accurate estimation method for each study case (single and double shear specimens) and choose the values for the Weibull distribution function.

Single Shear

Experimental data from single shear riveted specimens was analyzed and normalized, as presented in Fig. 1. The statistical analysis was implemented using the least square method to find the values for the inverse slope, m, and the intersection with the vertical axis, $\log c$, of the mean S-N curve. It was found that the optimal mean S-N curve has an inverse slope with a value of 2.9 and a coefficient of determination with a value of 0.54. In order to determine a mean S-N curve whose value of the inverse slope is a natural number, a new mean S-N curve was defined, and in this case, imposing that the value of the inverse slope is the closest natural value of 2.9, which is 3. This value imposed to the inverse slope of the curve is the value used in the standards [EC3-1-9 (CEN 2005) and AASHTO (2012)]. It was found that this new curve has the same value of the coefficient of determination. In this sense, in the following analysis, only the curve with an inverse slope equal to 3 is proposed to be used.

ISO 12107

Taking into account the analysis made with mean S-N curves, the determination of the characteristic curves using the standard ISO 12107 (ISO 2012) was conducted only for the curve with an inverse slope equal to 3. There are two parameters that need to be defined: the confidence level $(1-\alpha_n)$ and the probability of failure (p_n). Eurocode 3 Part 1-9 (CEN 2005) recommends the use of $(1-\alpha_n) = 75\%$ and $p_n = 95\%$; however, the American Institute of Steel Construction (Fisher 1981) used a different value for the confidence level—95% in this case. Fig. 2 presents the characteristic curves for both scenarios. In this case, it is possible to observe that both curves represent a good design approach for the presented data.

The design curve defined by the Detail category 71 and inverse slope equal to 5 was proposed by Taras and Greiner (2010b). It can be observed that this curve is not able to represent the fatigue strength of single shear riveted connections, especially for lower levels of an applied stress range. Furthermore, the Detail category 71 from Eurocode 3 (CEN 2005) and the Category D from AASHTO (2012) for riveted connections do not represent a design

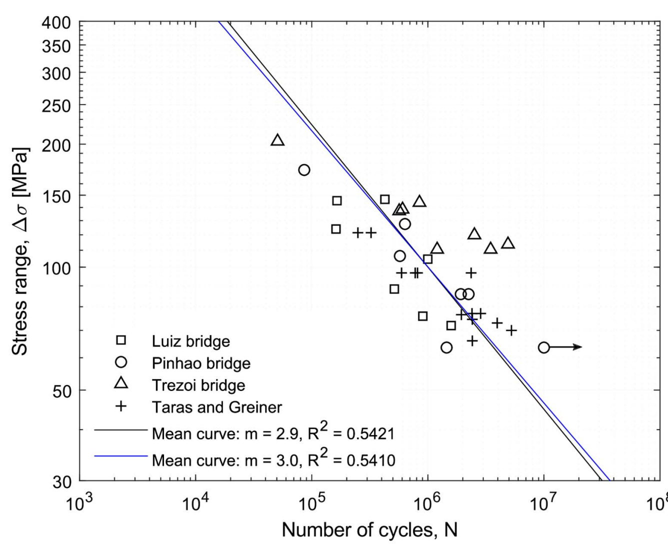

Fig. 1. Experimental data for single riveted connections: mean S-N curves.

safe criterion also. In fact, as previously discussed by several authors (De Jesus et al. 2015; Pedrosa et al. 2019), S-N curves proposed in Eurocode 3 (CEN 2005) and AASHTO (2012) are not able to represent the fatigue behavior of structural details composed by old metallic materials, such as puddle iron.

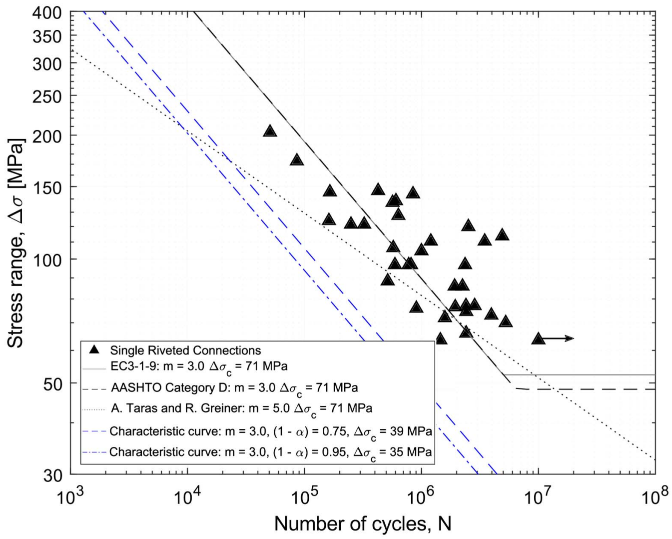

Fig. 2. Experimental data for single riveted connections: design S-N curves.

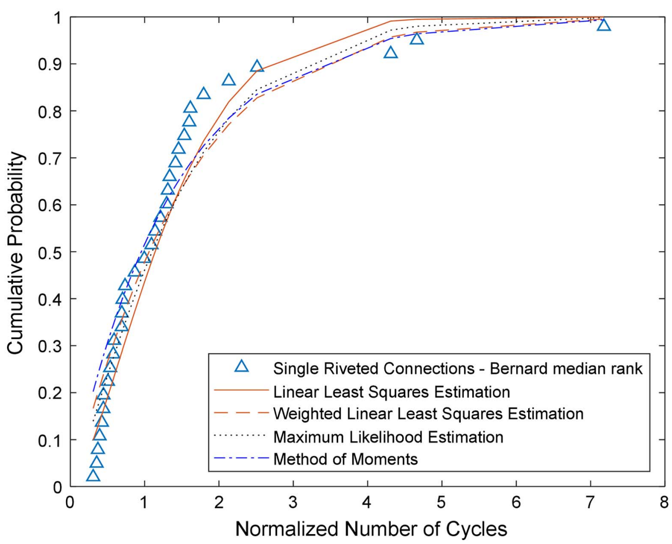

Fig. 3. Cumulative Weibull distribution function: single riveted connections.

Two-Parameter Weibull Distribution

The two-parameter Weibull distribution was applied to the fatigue data of single shear riveted specimens. The relation between the cumulative probability obtained for each data point with the Bernard median rank and the cumulative probability obtained with the Weibull distribution using four different estimation methods is presented in Fig. 3. The values of the Weibull distribution parameters are presented in Fig. 4, as well as the values for each goodness-of-fit statistic test for each estimation method. It is verified that LLSM is the estimation method that led to more accurate values because two out of three tests indicate this method as the best in relation to the Weibull distribution function (shaded cells in Fig. 4).

In Fig. 5 are presented S-N curves computed with the Weibull distribution function for 50%, 95%, and 97.5% probability of failure as well as the design (characteristic) curve obtained with ISO 12107 (ISO 2012). It is possible to observe that the curve with the 95% probability of failure for the Weibull distribution can be considered as a good design criterion. It has a detail category of 45 MPa. This curve is less conservative when compared to the design curve defined with ISO 12107 (ISO 2012), but it represents a safe design criterion for the presented fatigue data.

Comparison

The statistical analysis implemented previously enable one to establish fatigue design curves using different approaches. In Table 2, the main characteristics of these curves are summarized for both normal and Weibull distribution functions for single shear experimental fatigue data.

Double Shear

In Fig. 6 is presented the experimental data from double shear riveted specimens. The statistical analysis using the least square method enable the definition of a mean S-N curve with two degrees of freedom (inverse slope, m, and intersection with vertical axis, $\log c$) resulting in an inverse slope of 4.1 and a coefficient of determination of 0.52. In order to define a mean S-N curve with an inverse slope with a natural number, a new curve was establish imposing an inverse slope of 4.0 (the closest natural number to 4.1). This new curve has the same value of the coefficient of determination as the previous one. In the following analysis, only the curve with an inverse slope equal to 4 is proposed to be used.

ISO 12107

Taking into account the analysis made with mean S-N curves, the determination of the characteristic curves using the ISO 12107 standard (ISO 2012) was conducted only for the curve with an inverse slope equal to 4. The confidence level $(1-\alpha_n)$ and probability of failure (p_n) were defined as Eurocode 3 Part 1-9 (CEN 2005) recommends: $(1-\alpha_n) = 75\%$ and $p_n = 95\%$. The possibility of using a confidence level equal to 95% was analyzed. In Fig. 7 are

Method		LLSM	WLLSM	MLM	MM
α_w		1.4424	1.0782	1.1996	0.9881
β_w		1.4696	1.4875	1.4921	1.3816
Goodness-of-fit	KS	0.1234	0.1480	0.1384	0.1820
	AD	1.3486	1.3675	1.2318	1.6717
	χ^2	1.0592	2.2452	1.6265	3.4251

Fig. 4. Parameters of Weibull distribution and goodness-of-fit statistics: single riveted connections.

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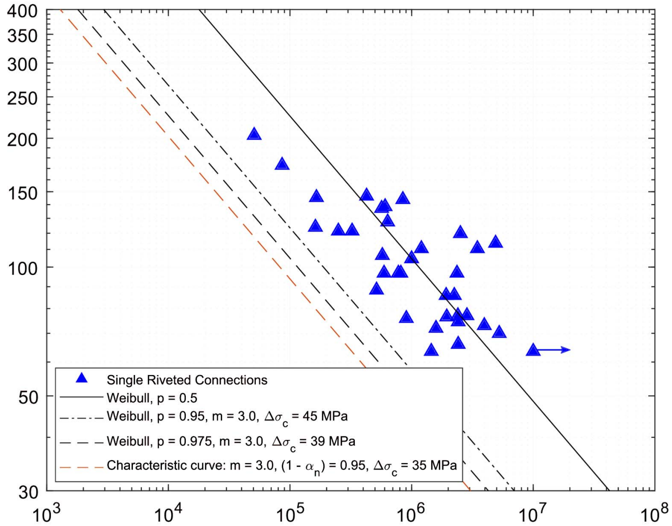

Fig. 5. Experimental data for single shear riveted connections: normal and Weibull distributions.

Table 2. Summary of proposed design curves for single shear riveted connections

Statistical distribution	Connection type	Single shear
	Inverse slope, m	3
Normal	Confidence level	0.95
	Probability of failure (%)	95
	Detail category, $\Delta\sigma_c$ (MPa)	35
Weibull	Estimation method	LLSM
	α_w	1.4424
	β_w	1.4696
	Probability of failure (%)	95
	Detail category, $\Delta\sigma_c$ (MPa)	45

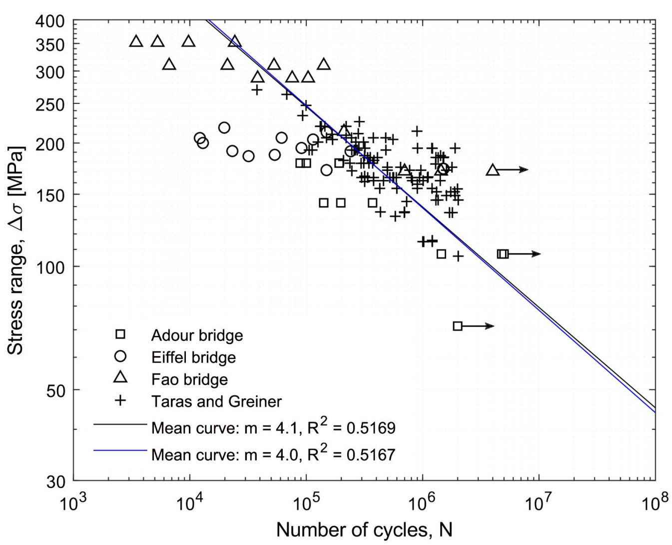

Fig. 6. Experimental data for double riveted connections: mean S-N curves.

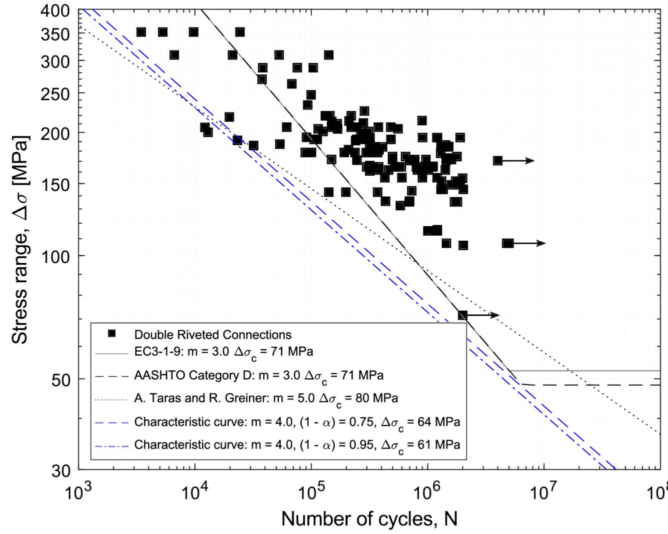

Fig. 7. Experimental data for double riveted connections: design S-N curves.

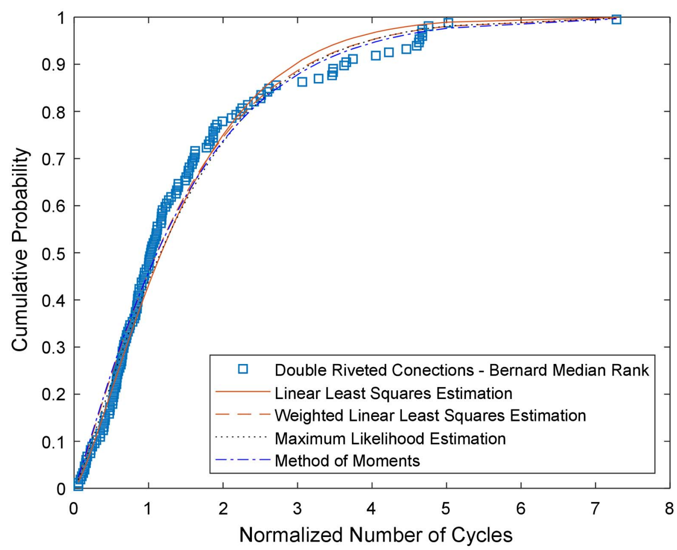

Fig. 8. Cumulative Weibull distribution function: double riveted connections.

presented the characteristic curves for both scenarios. In this case, it is possible to observe that the curve with a higher confidence level should be used.

Taras and Greiner (2010b) proposed a design curve with a detail category of 80 and an inverse slope equal to 5 for this detail. It can be observed that this curve represents fewer conservative predictions compared to the curves obtained with the statistical analysis. Furthermore, the Detail category 71 from Eurocode 3 (CEN 2005) and the Category D from AASHTO (2012) for riveted connections do not represent a design safe criterion also. The inadaptability of current standards to predict the fatigue strength of structural details with old metallic materials is evident.

Two-Parameter Weibull Distribution

The two-parameter Weibull distribution was applied to the fatigue data of double shear riveted specimens. The relation between the

Method		LLSM	WLLSM	MLM	MM
α_w		1.2941	1.1545	1.1943	1.1233
β_w		1.5537	1.5251	1.5770	1.5435
Goodness-of-fit	KS	0.0843	0.0689	0.0811	0.0733
	AD	1.5266	1.2562	1.2827	1.3459
	χ^2	0.4317	0.7646	0.5825	0.9198

Fig. 9. Parameters of Weibull distribution and goodness-of-fit statistics: double riveted connections.

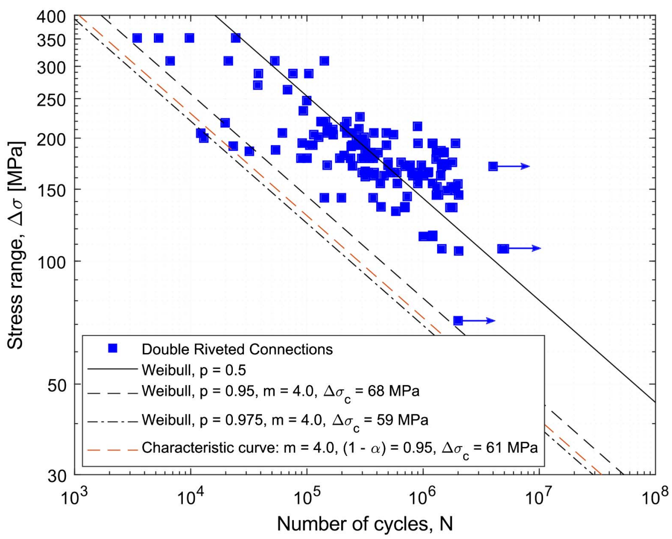

Fig. 10. Experimental data for double shear riveted connections: normal and Weibull distributions.

Table 3. Summary of proposed design curves for double shear riveted connections

Statistical distribution	Connection type	Double shear
	Inverse slope, m	4
Normal	Confidence level	0.95
	Probability of failure (%)	95
	Detail category, $\Delta\sigma_c$ (MPa)	61
Weibull	Estimation method	WLLSM
	α_w	1.1545
	β_w	1.5251
	Probability of failure (%)	97.5
	Detail category, $\Delta\sigma_c$ (MPa)	59

cumulative probability obtained for each data point with the Bernard median rank and the cumulative probability obtained with the Weibull distribution using four different estimation methods for its parameters is present in Fig. 8. The values of the Weibull distribution parameters are presented in Fig. 9, as well as the values for each goodness-of-fit statistic test for each estimation method. For double shear riveted connections, the estimation method that leads to more accurate results is the WLLSM because two out of three tests indicate this method as the best in relation to the Weibull distribution function (shaded cells in Fig. 9).

In Fig. 10 are presented S-N curves computed with the Weibull distribution function for 50%, 95%, and 97.5% probability of failure as well as the design (characteristic) curve obtained with ISO 12107 (ISO 2012). It is possible to observe that the curve with a 97.5% probability of failure for the Weibull distribution can be considered as a good design criterion. It has a detail category of 73 MPa.

Comparison

The statistical analysis implemented previously enable one to establish fatigue design curves using different approaches. In Table 3 the main characteristics of these curves are summarized for both normal and Weibull distribution functions for double shear riveted connections.

Concluding Remarks

The scientific work developed within this paper enable one to analyze fatigue experimental data from fatigue tests with riveted connections from different sources and to establish reliable proposals for design S-N curves.

It was found that a S-N curve with an inverse slope of 3 has a good correlation with the experimental data for single shear riveted connections. It was observed that design curves proposed in North American and European standards are not able to represent riveted connections made with old metallic materials. For this structural detail, the curve proposed by Taras and Greiner (2010b) was not a good design approach also. Parameters of the Weibull distribution function, α_w and β_w, were estimated, and the method that leads to better goodness-of-fit results is the linear least squared method. Characteristic curves computed with the Weibull distribution function showed that a probability of failure equal to 95% can be used.

For double shear riveted connections, a S-N curve with an inverse slope of 4 was found to have a good correlation with the experimental data. The inadaptability of the current standards was also verified for this detail. The estimation of Weibull distribution parameters allowed the authors to understand that the weighted linear least squared method leads to more accurate results. It was found that a design curve using the Weibull distribution function with a probability of failure equal to 97.5% can be considered.

Additionally, it was found that a normal distribution with a 95% probability of failure (using either a 95% or 75% confidence level) leads to more conservative results compared to a Weibull distribution with a 95% probability of failure. It should be stated that alternative approaches should be implemented in future investigations in order to improve the reliability of design S-N curves, namely, using numerical approaches by computing initiation and propagation fatigue phases. It is also important to study the implementation of alternative fatigue models, which are capable to predict the fatigue strength in all regimes (low to high cycle regimes).

Data Availability Statement

Some or all data, models, or code that support the findings of this study are available from the corresponding author upon reasonable request.

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