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**Publication date**
2018

**Document Version**
Final published version

**Published in**
Cone Penetration Testing 2018

PROCEEDINGS OF THE 4TH INTERNATIONAL SYMPOSIUM ON CONE PENETRATION TESTING (CPT'18), DELFT, THE NETHERLANDS, 21–22 JUNE 2018

# Cone Penetration Testing 2018

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CRC Press
Taylor & Francis Group
Boca Raton London New York Leiden

CRC Press is an imprint of the
Taylor & Francis Group, an **informa** business
A BALKEMA BOOK

*Cone Penetration Testing 2018 – Hicks, Pisanò & Peuchen (Eds)*
*© 2018 Delft University of Technology, The Netherlands, ISBN 978-1-138-58449-5*

# Detection of soil variability using CPTs

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ABSTRACT: The variability of soil is well known to affect the geotechnical performance of structures. As probabilistic design methods become more commonly used, the ability to measure the variability of soil becomes more important. However, by using only the point statistics of soil parameters in design (e.g. the mean and standard deviation), typically an over-estimation of failure probabilities occurs, leading to over-conservative designs. By looking at the spatial correlation (e.g. scales of fluctuation) a more accurate representation can be achieved. This paper presents a method to use vertical Cone Penetration Tests (CPTs) to detect both the vertical and horizontal scales of fluctuation. An extensive numerical and experimental investigation has been undertaken to understand how spatial variation can be estimated and to quantify the accuracy in that estimation. The impact of being able to quantify the uncertainty is illustrated via a simple slope stability example.

## 1 INTRODUCTION

Soil properties are intrinsically variable and addressing these variations in design is one of the main challenges in geotechnical engineering (Honjo 2011). The impact of soil variability has been shown to be significant in many types of geotechnical analyses, including: shallow, strip and pile foundations (Jaksa et al. 2005, Naghibi et al. 2016); retaining walls (Sert et al. 2016); liquefaction of hydraulic sand fills (Popescu et al. 1997, Wong 2004, Hicks & Onisiphorou 2005); and slope stability (Griffiths & Fenton 2000, Spencer & Hicks 2007, Hicks & Spencer 2010, Li & Hicks 2014, Vardon et al. 2016). Using embankments as an example, it has been shown that the spatial variation of material properties in combination with the problem geometry plays an important role in the slope stability and failure mode (Hicks & Samy 2002, Hicks et al. 2014, Li et al. 2015, 2016).

Comprehensive theoretical overviews on the quantification of the spatial variation in soils are given by Vanmarcke (1977a), Campanella et al. (1987) and Wickremesinghe & Campanella (1993), and later discussed by Fenton (1999a, 1999b) and Griffiths et al. (2007). The scale of fluctuation can be estimated using a range of techniques and, in particular, by using an auto-correlation function (e.g. de Gast et al. (2017)).

However, little experimental evidence that considers the scale of fluctuation, especially in the horizontal direction, is available. In this paper, the auto-correlation function is reviewed for its accuracy in obtaining the spatial correlation in synthetic data and proposes a method to estimate the accuracy in measuring the spatial correlation. In this way a CPT campaign can be designed to measure horizontal scales of fluctuation using limited CPT data.

This paper summarises the method proposed by de Gast et al. (2018) and gives an example of how to apply it.

## 2 THEORETICAL BACKGROUND

Soil properties are variable, although they are generally correlated to the properties of material in close proximity. A convenient measure of the spatial variability is the auto-correlation length $\theta$, often referred to as the scale of fluctuation (SoF). Loosely speaking, it is the distance within which material properties are significantly correlated. Conversely, the properties at two points separated by a distance greater than $\theta$ will be largely uncorrelated (Griffiths & Fenton 1997). The scale of fluctuation has been defined by Vanmarcke et al. (1986) as

$$\theta = 2\int_0^\infty \rho(\tau)\, d\tau \qquad (1)$$

where $\rho(\tau)$ is the auto-correlation function describing the spatial auto-correlation structure and $\tau$ is the lag distance, i.e. the distance separating two points. Hence, $\theta$ is the area under the auto-correlation function over the range $-\infty \le \tau \le \infty$, and, while it can have different orientations, for soils it is commonly considered to be different in the vertical and horizontal directions due to deposition processes.

An experimental auto-correlation function can be obtained from

$$\hat{\rho}(\tau)=\frac{\hat{\gamma}(\tau)}{\hat{\gamma}(0)} \tag{2}$$

where $\hat{\gamma}(\tau)$ is the experimental covariance function. This is given by Vanmarcke (1983) as

$$\hat{\gamma}(\tau)=\frac{1}{(t-1)}\sum_{j=1}^{t}\left(y_j-\hat{\mu}\right)\left(y_{j+\Delta j}-\hat{\mu}\right) \tag{3}$$

where $\hat{\mu}$ is the estimated mean (or trend) of the dataset, $j$ is a counter representing the first of a data pair at lag distance $\tau$, $j+\Delta j$ represents the second of the data pair, and $t$ is the number of pairs at lag distance $\tau$. By using, for example, the following Markov theoretical auto-correlation function,

$$\rho(\tau)=e^{\frac{-2|\tau|}{\theta}} \tag{4}$$

and finding the minimum of the error given by

$$E(\theta)=\sum\left(\rho(\tau)-\hat{\rho}(\tau)\right)^2 \tag{5}$$

an estimate for the scale of fluctuation may be obtained.

As with any sampled data, a sample from a population is taken and the accuracy of the method depends on the amount and r epresentativeness of the data available. It has been observed that as more data (CPTs) are considered, the better the mean auto-correlation function is (Lloret-Cabot et al. 2014), especially at larger lag lengths. This feature is investigated in detail in the following section.

## 3 INVESTIGATION USING SYNTHETIC DATA

The effectiveness of the experimental auto-correlation function for a set of data was investigated using computer generated data, i.e. data where the scale of fluctuation and auto-correlation were known a priori. 1D strings of data of varying length, data spacing and correlation length, representing CPT profiles, were generated using covariance matrix decomposition (Davis 1987, van den Eijnden & Hicks 2017). The data were generated using a mean of zero, a standard deviation of unity and a Markov auto-correlation function with a scale of fluctuation of 5 (units of length).

In order to test the effectiveness of estimating the auto-correlation function from equations (2)-(5), different variables were investigated: (1) the number of datasets—which is, in the vertical direction, analogous to the number the number of CPT profiles; (2) the number of data points used per dataset—which is analogous to the total number of data points in single CPT profile; (3) the value of θ; (4) the effect of grouping data at larger intervals—in the horizontal direction, this is analogous to having several CPT profiles in groups with a significant space between groups, or, in the vertical direction, it is analogous to having a data gap in the CPT profile.

Multiple datasets were generated representing different combinations of the four variables, and these are presented in detail in de Gast et al. (2018). Figure 1 shows the impact of increasing the number of datasets on the estimated $\theta$, which has the largest impact on the coefficient of variation (COV) of $\theta$. By increasing the number of datasets, the COV decreases rapidly as indicated by the broken line.

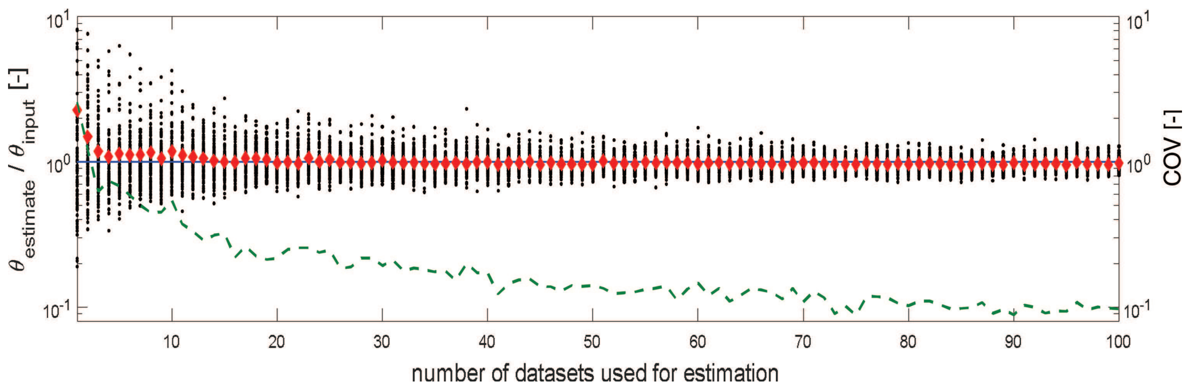


Figure 1. Example analysis investigating the accuracy in calculating $\theta$ as a function of the number of datasets. Each dot is a single estimation of $\theta$, normalized by the input $\theta=\theta_{input}$; the horizontal line equal to 1 is the normalized input $\theta$; the red diamonds are the calculated average from the individual estimates; and the broken line is the calculated coefficient of variation (COV = σ/μ) of the estimated $\theta$.

## 4 ESTIMATING THE VARIATION

It has been found that the coefficient of variation of the measured horizontal or vertical scale of fluctuation is related to the actual scale of fluctuation, the number of datasets, the domain size and the distribution (i.e. spacing) of CPT profiles. The following equation has been proposed to predict the COV of $\theta$ (de Gast et al. 2018):

$$COV = \underbrace{1.1}_{\substack{fitting\\coefficient}} \times \underbrace{\tan^{-1}\left(\frac{5\theta}{D_g}\right)}_{factor\ for\ \theta<D_g} \times \underbrace{\frac{1}{\sqrt{nf}}}_{factor\ for\ no.\ of\ datasets} \times \underbrace{\left(1+\frac{in}{ng\times\theta}\right)}_{factor\ for\ spacing} + \underbrace{\frac{\theta}{5nf\times D_t}}_{factor\ for\ \theta>D_t} \tag{6}$$

where:

$$nf = \begin{cases} \dfrac{D_p}{\theta_p}; & D_p > \theta_p \\ 1; & D_p \le \theta_p \end{cases} \tag{7}$$

and in which $\theta$ is the scale of fluctuation, $nf$ is the number of datasets (with a minimum of 1), $in$ is the space interval between the groups, $ng$ is the number of groups (if the data have different intervals), $D_t$ is the total domain length (the length over which the data points are obtained), $D_g$ is the domain length of the groups (if $D_t = D_g$, $in$ is the interval between data points), $D_p$ is the domain length perpendicular to the investigated direction and $\theta_p$ is the scale of fluctuation perpendicular to the investigated direction.

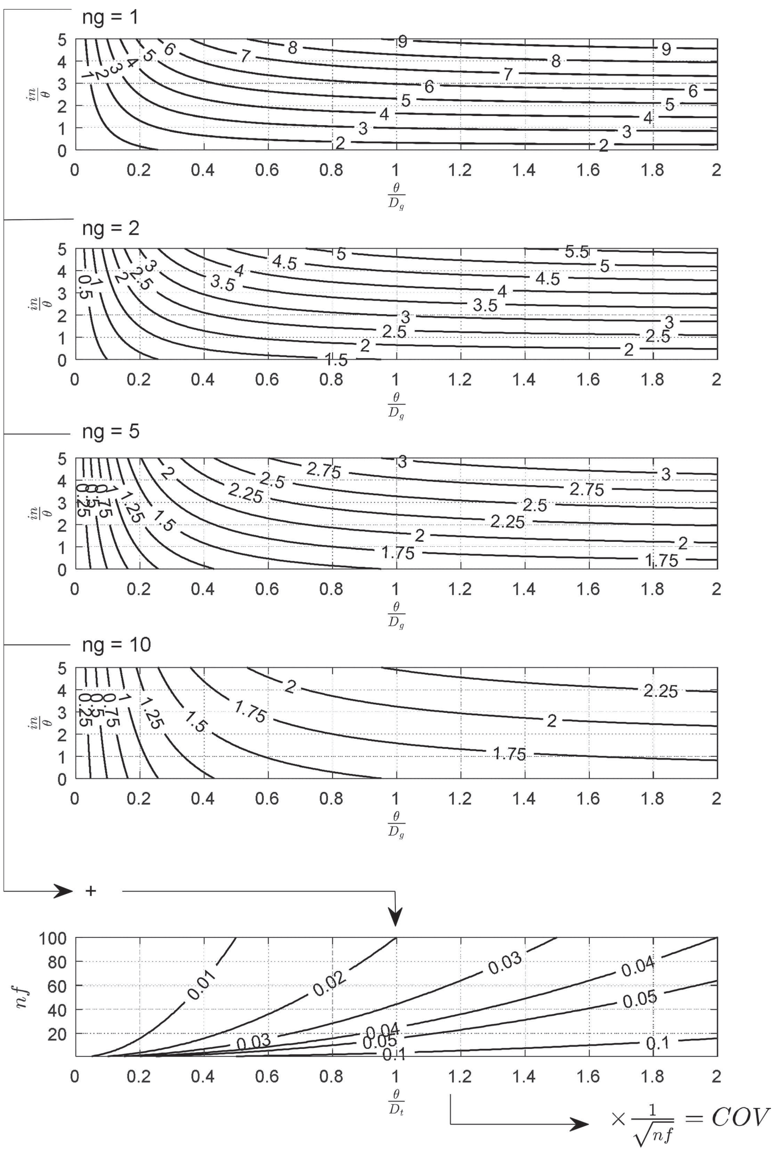


Figure 2. Method of estimating COV of the scale of fluctuation using groups of data.

Equation (6) can be rearranged to aid graphical interpretation as:

$$COV = \frac{1}{\sqrt{nf}}\left(\frac{\theta}{5\sqrt{nf}\times D_t}+1.1\left(1+\frac{in}{ng\times\theta}\right)\tan^{-1}\left(\frac{5\theta}{D_g}\right)\right) \quad (8)$$

and is graphically presented in Figure 2, which can be used after first calculating the scale of fluctuation using equations (2)-(5). In the first four subfigures in Figure 2, which represent the last term in equation (8) with different values of *ng*, the contour lines can be assumed to be horizontal at the right-hand side, i.e. if $\theta / D_g > 2$, the value can be evaluated at $\theta / D_g = 2$.

For example, consider 10 CPTs grouped in $ng = 5$ groups of 2 CPTs, with a spacing of $D_g = 2.5$ m between CPTs within each group and a spacing between groups of $in = 25$ m, giving a total domain of $D_t = 112.5$ m. If the horizontal scale of fluctuation is estimated to be $\theta = 50$ m, the first part of the figure gives a contour value of between 1.75 and 2.0, and the second part of the figure gives a contour value below 0.05; adding these numbers together gives a value between 1.8 and 2.05, and dividing by the square root of $nf = 20$ ($D_p = 5$ m, $\theta_p = 0.25$ m) yields an estimated COV of 0.40–0.46; using equation (6) yields an estimated COV of 0.43.

## 5 EFFECT ON DYKE STABILITY

Considering a hypothetical study of the stability of a dyke, the impact of the sampling has been investigated. The analytical model of Vanmarcke (1977b) is used, which considers 3D effects and has been examined in detail by Li et al. (2015) and Varkey et al. (2017). It is summarised by the following equations:

$$F_{b,\mu} = F\left(1+\frac{d}{b}\right) \quad (9)$$

$$F_{b,\sigma} = F\Gamma(L_a)\Gamma(b)V_s \quad (10)$$

$$\Gamma(b) = \begin{cases}\sqrt{\frac{\theta_h}{b_c}}; & b_c > \theta_h \\ 1; & b_c \le \theta_h\end{cases} \quad (11)$$

$$\Gamma(L_a) = \begin{cases}\sqrt{\frac{\theta_e}{L_a}}; & L_a > \theta_e \\ 1; & L_a \le \theta_e\end{cases} \quad (12)$$

$$b_c = \begin{cases}\frac{F}{F-1}d; & b_c > \theta_h \\ \theta_h; & b_c \le \theta_h\end{cases} \quad (13)$$

$$d = 2\frac{A}{L_a} \quad (14)$$

where $F$ is the mean 2D factor of safety, $F_{b,\mu}$ is the corrected mean for 3D end effects, $d$ is related to the cross-sectional sliding area $A$ and failure arc length $L_a$, $b_c$ is the critical failure length in the third dimension, $F_{b,\sigma}$ is the standard deviation of the 3D safety factor, $\Gamma(L_a)$ and $\Gamma(b)$ are variance reduction factors depending on $\theta$, $V_s$ is the coefficient of variation of the strength point statistics, $\theta_h$ is the horizontal scale of fluctuation, and $\theta_e$ is the equivalent scale of fluctuation obtained by a weighted average of the horizontal and vertical components of the scale of fluctuation along the 2D slip circle.

In this example analysis, it is assumed that a budget for 10 CPTs is available to assess the stability of a 5 m high dyke with a length of 150 m. It is also assumed that the following are calculated: $F = 1.5$, $V_s = 0.3$, $L_a = 11.25$ m, $A = 16$ m$^2$ and $\theta_v = 0.25$ m. $\theta_v$ is measured using an interval of 0.01 m between measurements down to 5 m depth for all CPTs. The CPTs are either evenly distributed (i.e. ungrouped) over the length of the dyke at 16.7 m spacing, or in five groups of 2 CPTs, with a distance of 2.5 m between the 2 CPTs in each group and a spacing of 34.4 m between each group.

To illustrate the effect of CPT positioning and the corresponding uncertainties obtained from equation (6), two scenarios have been considered, where the horizontal scales of fluctuations are (a) $\theta_h = 50$ m and (b) $\theta_h = 5$ m. Using the COV obtained from equation (6), three likely outcomes of $\theta_e$ have been calculated (following the approach of Li et al. (2015)) for the following combinations of $(\theta_h+\sigma, \theta_v+\sigma)$, $(\theta_h, \theta_v)$ and $(\theta_h-\sigma, \theta_v-\sigma)$. For each combination, the five percentile factor of safety $F_{b,5\%}$ has been calculated, following the procedure of equations (9)-(14).

In Table 1 the results of the different scenarios and CPT groupings are presented. For a large $\theta_h$ (scenario 1), the difference between the grouped and ungrouped data is not apparent as they yield almost the same $F_{b,5\%}$. For a small $\theta_h$ (scenario 2), there is a clear advantage in grouping the CPTs; this is because the small scale of fluctuation can then be measured, whereas, for the ungrouped data, this is not the case and the calculated $\theta_h$ has a minimum value equal to the CPT spacing (16.7 m). Table 1 shows that based on the same point statistics, a large range of $F_{b,5\%}$ can be found, from 1.24 to 1.90, depending on the value of $\theta_h$ and the distance between the CPT locations.

As the scale of fluctuation is not generally known a priori, it is more useful to use grouped CPTs, as then both large and small scales of fluctuation can be estimated using the same number of CPTs. Comparing any pair of calculated safety

Table 1. Effect of CPT positioning (ungrouped or grouped) and $\theta_h$ (scenario 1 or 2) on the 5 percentile factor of safety.

| Scenario/ positioning | Mean calculated $\theta_h$ [m] | COV $\theta_h$ | $F_{b,5\%}$ | | |
|---|---|---|---|---|---|
| | | | $\theta_e + \sigma$ | $\theta_e$ | $\theta_e - \sigma$ |
| 1.Ungrouped | 50 | 0.34 | 1.26 | 1.32 | 1.40 |
| 1.Grouped | 50 | 0.44 | 1.24 | 1.32 | 1.44 |
| 2.Ungrouped | 16.7* | 0.25 | **1.57** | 1.60 | 1.64 |
| 2.Grouped | 5 | 0.86 | **1.72** | 1.78 | 1.90 |

*the minimum distance between the CPTs in the horizontal direction equals 16.7 m; as this is therefore the smallest $\theta_h$ that can be found, CH036_118-E035.eps m is used

factors (for the same scenario and same calculated $\theta_e$), e.g. the data in bold text, it is possible to calculate up to an 10% increase in the five percentile factor of safety.

## 6 CONCLUSIONS

A method is presented to quantify the uncertainty in the measured values of the spatial scale of fluctuation (which characterises the soil heterogeneity). In a simple example used to illustrate the calculation process, it has been demonstrated that, by careful design of the site investigation, the factor of safety of an embankment may be increased by ~10%.

## ACKNOWLEDGEMENTS

This research was supported under project number 13864 by the Dutch Technology Foundation STW, which is part of the Netherlands Organisation for Scientific Research (NWO), and which is partly funded by the Ministry of Economic Affairs.

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