

Coupled Cross-Flow and In-Line VIV Analysis of a Lazy Wave Dynamic Power Cable

Single Wake Oscillator and OrcaFlex Approach with Experimental Validation

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by

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Summary

Floating offshore wind deployment requires dynamic power cables, as part of the system that transmits the generated electricity from the floating platform to the onshore grid. These cables are loaded continuously by waves and current and follow the platform motions, so they experience cyclic loading throughout their design life. Vortex-induced vibration adds a further dynamic contribution, which makes fatigue one of the governing limit states in their design. The industry prediction tools, however, are based mostly on rigid or straight cylinders, and field data, design standards and operational experience remain scarce, so their design relies on large safety margins.

To address this gap, a coupled cross-flow and in-line wake oscillator model was implemented to assess whether it predicts the response of a lazy wave power cable more accurately than the single-degree-of-freedom models common in industry. The model of Qu and Metrikine (2021) was adopted, in which a single wake oscillator drives both directions, forced by the cross-flow acceleration and by the in-line motion through a parametric term, so that the lift and the fluctuating drag originate from the same wake. In-line amplitudes are typically smaller than cross-flow ones, but for long flexible cylinders neglecting the in-line motion under-predicts the fatigue damage, which motivates the coupled formulation.

The model was implemented in a nonlinear finite element solver in Python and as an external function in OrcaFlex, the latter placing it within an industrial framework. The two implementations were verified against each other and validated against a tensioned vertical riser, where they reproduce the measured cross-flow and in-line response and recover the in-line motion that cross-flow-only models cannot capture. The model was then applied to an experimental lazy wave power cable in OrcaFlex and compared with the industry models in the same framework.

The coupled model reproduces the measured frequency content and the main responding modes, and recovers the shared in-line and cross-flow frequency that arises from the curved geometry. It over-predicts the in-line content at twice the cross-flow frequency, which appears only weakly in the experiment. Its Strouhal number can be set for each case, whereas the industry models are fixed at a single value above the measured one. The cross-flow amplitude is over-predicted by about a factor of two, mainly as a result of the selected tuning parameters, an over-prediction shared by the industry models. Overall, the coupled model predicts the response more accurately than the industry models, while the amplitude remains conservative.

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Contents

Summary	ii
Acknowledgements	iv
Nomenclature	xiv
I Theoretical Background	2
1 Introduction	3
1.1 Dynamic Power Cables for Floating Offshore Wind.	4
1.1.1 Lazy-Wave Power Cables.	4
1.1.2 Vortex-Induced Vibration of Dynamic Cables	5
1.2 Problem Statement	6
1.3 Research Objective	6
1.4 Methodological Approach	7
1.5 Thesis Outline	8
2 Vortex-Induced Vibration	10
2.1 Flow Around a Circular Cylinder	10
2.1.1 Flow Separation and Wake Formation	10
2.1.2 Flow Regimes	11
2.1.3 Mechanism of Vortex Shedding	12
2.1.4 Vortex Shedding Frequency	12
2.1.5 Drag and Lift Force	13
2.2 VIV Mechanism	14
2.2.1 Cross-flow Oscillations	15
2.2.2 In-line Oscillations	16
2.3 Experimental Analysis of VIV	16
2.3.1 Stationary Cylinder Experiments.	16

2.3.2 Free Vibration Experiments	17
2.3.3 Forced Vibration Experiments	19
2.4 VIV in Flexible Cylinders	20
2.4.1 Multi-mode and Coupled Response	20
2.4.2 VIV in Lazy-Wave Dynamic Cables	21
2.5 Conclusion.	22
3 Coupled Wake Oscillator Model	23
3.1 Introduction	23
3.2 Structural Model.	23
3.3 Van der Pol Wake Oscillator Model	25
3.4 Single Wake Oscillator Model for Coupled VIV	26
3.4.1 Rigid-Cylinder Coupled Model	27
3.4.2 Extension to a Flexible Cylinder	30
3.4.3 Coupled System	33
3.4.4 Conclusion	33
 II Implementation & Results	 34
4 Numerical Implementation	35
4.1 Overview	35
4.2 Full Coupled Implementation in Python	35
4.3 OrcaFlex - Python Coupling Implementation	37
4.3.1 Overview	37
4.3.2 External Function Implementation.	38
4.4 Model Validation.	41
4.4.1 Overview	41
4.4.2 Python Model Description.	42
4.4.3 OrcaFlex Model Description	43
4.4.4 Natural Frequencies	44
4.4.5 VIV Response	45
4.5 Conclusion.	49

5	Lazy Wave Cable VIV Analysis	50
5.1	Overview	50
5.2	OrcaFlex Model Description	51
5.3	Natural Frequencies	52
5.4	VIV Response Results	54
5.4.1	Primary Validation Case $\theta = 90^\circ$, $U = 0.1$ m/s	55
5.4.2	Sensitivity Analysis	59
5.4.3	Model Comparison	64
5.4.4	Effect of Current Direction.	66
5.5	Vertical Cable: Isolating the Low Mass Ratio	68
5.5.1	Response	68
5.6	Conclusion.	69
6	Conclusions & Recommendations	70
6.1	Conclusions.	70
6.2	Recommendations	72
	References	73
A	Nonlinear FEM Implementation	81
A.1	Beam Element Formulation.	81
A.1.1	Discretisation and Degrees of Freedom	81
A.1.2	Shape Functions	82
A.1.3	Elemental Matrices	82
A.1.4	Gauss Quadrature	84
A.2	Wake Element Formulation.	84
A.2.1	Discretisation and Shape Functions.	84
A.2.2	Elemental Matrices.	85
A.3	Assembly and Boundary Conditions	85
A.3.1	Global Assembly	85
A.3.2	Boundary Conditions and Applied Loads.	86
A.4	Static Equilibrium and Local Coordinate Frame	86
A.4.1	Static Equilibrium	86
A.4.2	Local Coordinate Frame.	87

A.5 Hydrodynamic Force and Wake Forcing	87
A.5.1 Hydrodynamic Force on the Structure	87
A.5.2 Morison Drag Outside the Shedding Region	88
A.5.3 Wake Forcing	88
A.6 Newmark-Newton Time Integration	88
A.6.1 Predictors	89
A.6.2 Newton Iteration	89
B OrcaFlex VIV Toolbox	91
B.1 Milan Wake Oscillator	91
B.2 Iwan and Blevins Wake Oscillator	93
C External Function Integration	95
D Delta Flume: Additional Cases	96
D.1 Higher Tension (810 N)	96
D.2 Lower Tension (389 N)	99
E Vertical Cable Results	101
E.1 OrcaFlex Model Description	101
E.2 Natural Frequencies	102
E.3 Response at Stations 3L/4 and L	103
E.4 Frequency Content	104
E.5 Lock-in Position and Over-prediction	106
E.6 Spatial Distribution	107
F Modal Decomposition Method	108
F.1 Classical Formulation	108
F.2 Adaptation for Curved Geometry	109
F.2.1 Local Directions and Projection	109
F.2.2 Joint Fit	110
F.2.3 Number of Retained Modes	110
F.2.4 Application to the Model	111

List of Figures

1.1	Dynamic power cable layout [7]	4
1.2	Components of a lazy-wave system [7]	5
1.3	Overview of the research methodology and simulation framework	9
2.1	Flow around a circular cylinder [40] (a) and reverse flow schematic [41] (b) . . .	10
2.2	Flow around a cylinder as a function of the Reynolds number [42]	11
2.3	Laminar and turbulent boundary layer separation [43]	11
2.4	Shear layer development (a) and vortex formation sequences (b–c) [42]	12
2.5	Strouhal - Reynolds number relationship for a smooth circular cylinder [46] . . .	13
2.6	Mean drag and RMS lift coefficients as a function of Reynolds number	14
2.7	IL and CF vibrations [50]	14
2.8	CF response of a flexibly mounted circular cylinder, obtained from Feng's experiment [48]	15
2.9	In-line vibration mechanisms and shedding patterns	16
2.10	Free vibration response characteristics	18
2.11	Out-of-phase and in-phase force coefficients, adapted from Sarpkaya (1979) [45] .	19
2.12	Wake transitions in forced vibrations of a circular cylinder [57]	20
2.13	VIV response characteristics of flexible and two-degree-of-freedom cylinders . .	21
3.1	Schematic representation of a flexible riser subjected to uniform current and VIV [63]	24
3.2	Time histories of the unforced van der Pol oscillator	26
3.3	Decomposition of the vortex force [28]	28
3.4	Local coordinate frame [28]	31
3.5	Force decomposition in the $\mathbf{b}^{\text{eq}}\text{--}\mathbf{n}^{\text{eq}}$ plane [28]	32
4.1	ANCF discretisation of the cable [69]	36

4.2	Solution procedure of the fully coupled ANCF wake oscillator model	37
4.3	OrcaFlex line discretisation [70]	38
4.4	Evaluation of external VIV loads within the OrcaFlex integration scheme	40
4.5	Delta Flume experimental setup and corresponding OrcaFlex model	44
4.6	Cross-flow (top) and in-line (bottom) displacement envelopes at $V = 0.49$ m/s . .	46
4.7	Cross-flow (top) and in-line (bottom) displacement envelopes at $V = 0.85$ m/s . .	46
4.8	Displacement envelopes for Case 1, $V = 0.16$ m/s ($T_t = 405$ N)	48
4.9	Displacement envelopes for Case 3, $V = 0.31$ m/s ($T_t = 457$ N)	48
5.1	Still-water lazy wave profile and corresponding OrcaFlex model	51
5.2	Mode shapes in X , Y and Z corresponding to modes 1 to 10, based on the OrcaFlex model	53
5.3	Depth profile of the lazy wave cable as a function of normalised arc length	55
5.4	Global components of the local frame unit vectors	56
5.5	Mean power spectral density, $\theta = 90^\circ$, $U = 0.1$ m/s	56
5.6	Cross-flow response, $\theta = 90^\circ$, $U = 0.1$ m/s	57
5.7	In-line response, $\theta = 90^\circ$, $U = 0.1$ m/s	59
5.8	Effect of damping ratio ζ on the RMS displacement envelopes at $\theta = 90^\circ$, $U = 0.1$ m/s	60
5.9	Effect of lift coefficient C_{L0} on the RMS displacement envelopes at $\theta = 90^\circ$, $U = 0.1$ m/s	61
5.10	Effect of Strouhal number St on the RMS displacement envelopes at $\theta = 90^\circ$, $U = 0.1$ m/s	63
5.11	Effect of Strouhal number St on the mean PSD at $\theta = 90^\circ$, $U = 0.1$ m/s	63
5.12	Mean PSD, model comparison at $\theta = 90^\circ$, $U = 0.1$ m/s	64
5.13	RMS envelopes, model comparison at $\theta = 90^\circ$, $U = 0.1$ m/s	65
5.14	Normal velocity ratio U_n/U along the cable for all current directions, $U = 0.1$ m/s	66
5.15	Model and experiment RMS envelopes at $\theta = 90^\circ$ and $\theta = 0^\circ$, $U = 0.1$ m/s	67
5.16	In-line and cross-flow response at station $L/2$ against reduced velocity	69
B.1	Milan wake oscillator model [83]	92

B.2	Iwan and Blevins wake oscillator model [84]	93
D.1	Cross-flow (top) and in-line (bottom) displacement envelopes at $V = 0.63$ m/s . .	98
D.2	Cross-flow (top) and in-line (bottom) displacement envelopes at $V = 0.75$ m/s . .	98
D.3	Cross-flow time series at the transitional flow velocities	99
D.4	Displacement envelopes for $V = 0.60$ m/s, measured $\bar{T}_t = 670$ N	100
D.5	Displacement envelopes for $V = 0.95$ m/s, measured $\bar{T}_t = 1002$ N	100
E.1	Vertical cable test setup and corresponding OrcaFlex model	102
E.2	First four mode shapes based on the OrcaFlex model	103
E.3	In-line and cross-flow response at station $3L/4$ against reduced velocity	104
E.4	In-line and cross-flow response at station L against reduced velocity	104
E.5	Cross-flow frequency content at station $L/2$	105
E.6	In-line PSD at station $L/2$	105
E.7	RMS displacement envelopes along the cable	107
F.1	Condition number and reconstruction residual against the number of retained modes, primary case $\theta = 90^\circ$, $U = 0.1$ m s $^{-1}$	111

List of Tables

2.1	Lift and drag force expressions and coefficients	13
2.2	Key non-dimensional parameters governing VIV response	17
4.1	Structural properties of the Delta Flume riser model [30]	42
4.2	Hydrodynamic and wake oscillator parameters	42
4.3	First seven natural frequencies (Hz) in still water at $T_{t0} = 813 \text{ N}$	45
5.1	Structural properties of the lazy wave cable model	52
5.2	Line sections and segmentation of the OrcaFlex model	52
5.3	First 15 natural frequencies of the lazy wave cable in still water	53
B.1	Milan coefficients [83]	93
B.2	Iwan and Blevins coefficients [84]	94
D.1	Mean tensions and reduced velocities for the OrcaFlex and ANCF implementations	96
D.2	VIV response comparison across all sources	97
E.1	Line types and segmentation of the OrcaFlex model	102
E.2	First four natural frequencies of the vertical cable in still water	103

Nomenclature

A	Cross-flow tuning parameter of the wake oscillator	–
C_a	Added mass coefficient	–
C_D	Drag coefficient	–
C_L	Lift coefficient	–
C_{L0}	Amplitude of the fluctuating lift coefficient	–
D	Characteristic (outer) diameter of the cylinder	m
EI	Bending stiffness	N·m ²
f_{CF}	Cross-flow response frequency	Hz
f_{IL}	In-line response frequency	Hz
f_s	Vortex-shedding frequency	Hz
L	Length of the cylinder	m
m	Structural mass per unit length	kg/m
m^*	Mass ratio	–
q	Wake variable	–
Re	Reynolds number	–
St	Strouhal number	–
T_t	Top tension	N
U	Incoming flow velocity	m/s
U_r	Reduced velocity	–
V_N	Flow velocity normal to the cable axis	m/s
x	In-line displacement	m
y	Cross-flow displacement	m
ε	Damping tuning parameter of the wake oscillator	–
ζ	Structural damping ratio	–
θ	Current direction relative to the cylinder	deg
κ	In-line coupling parameter of the wake oscillator	–
ν	Kinematic viscosity of the fluid	m ² /s
ρ	Fluid density	kg/m ³
ω_n	Natural angular frequency	rad/s

Part I

Theoretical Background

1 Introduction

Floating offshore wind technology enables access to the deep-water sites where approximately 80% of global wind resources are located [1], overcoming the depth limitations of fixed-bottom installations restricted to waters shallower than 50-60 m [2, 3]. With a theoretical potential exceeding 13 TW worldwide [4] and full commercial deployment expected by the end of the decade [5], it is emerging as a key contributor to offshore renewable energy capacity and to the global responses to energy security and the climate crisis [5, 6].

As the sector evolves toward larger turbine ratings and installation in deeper waters, the transmission of electricity from offshore installations to the onshore grid is an increasingly significant engineering challenge. Floating wind farms require long sections of dynamic cables that extend from the floating structure to the seabed, which must adapt to the platform's continuous motions and withstand fatigue accumulation over the design life [7]. The configuration of the mooring system also affects their layout and design, creating an interdependence that is largely absent in fixed-bottom systems [8], where cables are primarily static and buried.

Although certain design practices have been adapted from the offshore oil and gas sector, these assets were not engineered for the dynamic environments and higher voltage requirements characteristic of floating wind installations [7, 8]. As a result, no standardised design framework or widely accepted assessment criteria have yet been established to address the full range of water depths, metocean conditions, and loading demands involved [9, 10, 11].

The scale of this challenge is substantial, as Europe alone is expected to require approximately 6,000 km of subsea power cables annually to support this growth [12], while global projections indicate that more than 63,200 km of array cables, connecting turbines within each farm, and nearly 40,000 km of export cables, transmitting power to shore, will be installed by 2030 [12]. Meanwhile, although they account for only 5-10% of total project capital expenditure [7, 10, 13, 14], subsea power cables are consistently identified as the largest contributor to offshore wind farm downtime, accounting for approximately 80% of offshore wind-related financial losses and insurance claims [10].

At the same time, repair campaigns are both costly and time-consuming, with typical downtimes of approximately 40 days for inter-array cables and 60 days for export cables, and associated financial losses ranging from USD 1.8 to 30 million per incident [7, 10, 13]. However, operational experience with dynamic power cables remains limited, and failure databases are scarce and lack standardisation [10, 11]. The elevated failure rates already reported for conventional subsea cables in offshore applications also raise concerns that dynamic cables may encounter similar reliability challenges as their deployment continues to expand [15].

1.1 Dynamic Power Cables for Floating Offshore Wind

A floating wind farm connects its turbines through array cables that form strings feeding an offshore substation, from which export cables transmit the combined power to the onshore grid at a higher voltage [16, 17]. Both cable types also supply auxiliary power to the turbines when they are not generating and carry the fibre optic links across the farm [17]. A typical layout is shown in Figure 1.1.

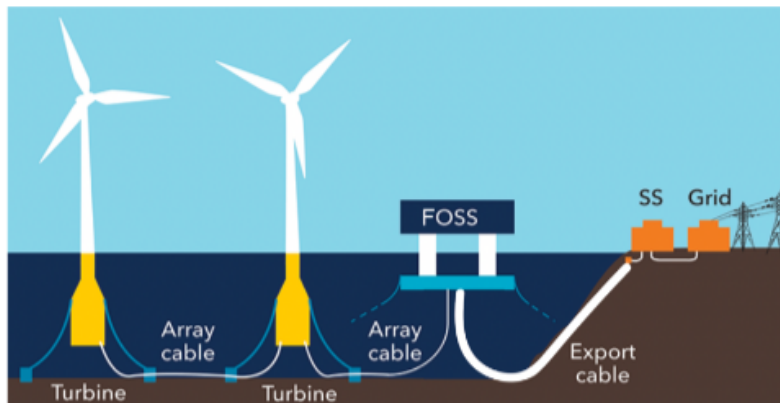


Figure 1.1. Dynamic power cable layout [7]

In a fixed-bottom farm the cables are largely static, buried or laid along the seabed, with only a short protected length exposed near the base of each structure [7]. A floating farm allows no such arrangement. Because it moves continuously under the environmental loads, the cable requires a long suspended section that extends from the platform to the seabed touchdown point and accommodates this motion throughout its design life [7]. The section is therefore subjected to cyclic loading from the waves and the current, which raises the tension and bending it must sustain [7]. Fatigue from this repeated loading is the limit state that most often governs the design of a floating-wind cable [8, 18].

The suspended section can take several shapes. The most common are the simple catenary, the lazy wave and the steep wave, and the choice follows from the compliance needed for the platform motions, the water depth and the installation cost [8, 16]. The catenary hangs in a single curve under its own weight. It is the cheapest to make and install, but its low compliance raises the tension and the bending at the ends, which limits it to benign sites [8, 19]. The steep wave meets the seabed at a near-perpendicular angle through a subsea base, which suits high-energy seabeds but adds equipment and cost [8, 16]. The lazy wave lies between these two, offering more compliance than the catenary at a lower cost than the steep wave.

1.1.1 Lazy-Wave Power Cables

The lazy wave resembles the catenary but adds mid-water support from distributed buoyancy modules along part of its length [8, 20]. These modules lift the cable into an arch and give it its characteristic shape. The name follows from this geometry. The cable centreline runs almost parallel to the seabed at the contact, which gives the lazy part of the name, in contrast

to the near-perpendicular touchdown of the steep wave, while the curve held by the buoyancy modules gives the wave part [8, 20].

The lazy wave configuration has two regions of high curvature. The sag bend is the lower trough, where the cable dips and then curves back upward toward the buoyancy section. The hog bend is the upper crest, the arch formed over the buoyancy modules.

Past the hog bend the cable descends again to the touchdown point, where it first meets the seabed. At the upper end, the hang-off is the point where the cable leaves the floater, and a bend stiffener there limits the curvature and reduces the bending stress at this interface [7]. These regions are shown in Figure 1.2, and they are where the curvature and the fatigue concentrate along the cable.

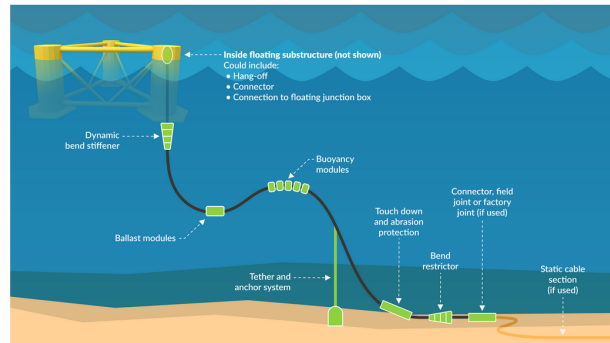


Figure 1.2. Components of a lazy-wave system [7]

The placement of the buoyancy modules sets the tension distribution and the natural frequencies of the cable [8]. With the modules in the right position, the wave shape absorbs the platform excursions in the upper part of the cable before they reach the touchdown point, so the seabed contact region is decoupled from the longer-period platform motions [8]. This decoupling lowers the dynamic tension ranges and improves the fatigue life at the touchdown region [21, 22, 23]. For these reasons the lazy wave is the favoured configuration for floating wind, and it is the case considered in the rest of this work.

1.1.2 Vortex-Induced Vibration of Dynamic Cables

A dynamic cable spends its life submerged in a current, so it experiences vortex-induced vibration in the same way as a marine riser [24, 25]. As the current passes the cable, vortices form and shed from its two sides in turn. The shedding sets up an alternating pressure imbalance that drives the cable both across the flow and in line with it [26]. The motion is largest during lock-in, when the shedding frequency meets a natural frequency of the cable and the two synchronise [26]. The cyclic bending that this motion produces adds to the fatigue damage along the cable [8], which makes VIV one of the loads the design has to account for.

The response of a power cable still differs in important ways. Its lower mass ratio is associated with a wider lock-in range and larger amplitudes, and in the lazy wave the curved, spatially varying geometry changes how the cross-flow and in-line motions couple compared with a straight riser. The strakes and fairings used to suppress VIV on risers cannot be fitted on the buoyant section of a lazy-wave cable [27], so the response there is left unmitigated.

1.2 Problem Statement

Vortex-induced vibration in dynamic power cables for floating offshore wind remains inadequately characterised, particularly for lazy-wave configurations. Most existing VIV prediction tools derive from experiments on rigid or straight cylinders and have limited applicability to lazy-wave dynamic cables [24, 25]. The scarcity of full-scale experimental data for such systems further constrains reliable model calibration and validation, and the limited operational experience accumulated to date restricts the ability to improve predictions from field observations [10, 11].

The VIV response of a flexible cylinder differs from that of a rigid one. Capturing it requires a model that resolves the coupled cross-flow and in-line dynamics, since these govern the response pattern, the force distribution and the fatigue damage along the structure [28, 29]. A lazy-wave cable adds the difficulty of a curved profile and properties that vary along its length, which together produce a multi-mode response across both directions.

Current practice manages this uncertainty with high safety factors and conservative margins. This has kept damage-related failures rare, but it raises production costs and leads to overdimensioned designs and maintenance that may not be needed [24]. As floating offshore wind scales up in size, power rating and economic exposure, this conservatism becomes harder to justify and restricts the development of more efficient cable designs. A more accurate and realistic prediction of the VIV response is therefore needed, both to size these cables closer to their real characteristic limits and to assess their fatigue life with greater confidence [24].

1.3 Research Objective

In response to the problem defined above, this thesis aims to implement a physics-based mathematical model for vortex-induced vibration of flexible cylinders for use in industrial dynamic analysis software. The focus is on evaluating whether a coupled cross-flow and in-line (CF-IL) wake-oscillator model can reduce modelling uncertainty and reliance on conservative assumptions compared with commonly used industry approaches, by applying it to a lazy-wave dynamic power cable configuration. This is achieved through computational implementation of a selected wake-oscillator model and systematic comparison with experimental data and industry-standard approaches.

Therefore, the main research objective of this thesis can be described as follows:

“To evaluate the capability of a physics-based, coupled cross-flow and in-line wake-oscillator model to improve the prediction of vortex-induced vibration response in lazy-wave dynamic power cables, through implementation within an industrial dynamic analysis framework and comparison with experimental observations.”

To frame this objective, the following research questions are posed.

- RQ1: How can the coupled CF-IL wake-oscillator model be implemented within an industrial dynamic analysis solver and validated against a reference experiment before its application to a lazy-wave power cable?
- RQ2: How accurately does the coupled model reproduce the measured VIV response of a low-mass-ratio lazy-wave power cable, in terms of amplitude, frequency content and modal pattern?
- RQ3: How do the coupled model's predictions compare with those of the industry-standard single-degree-of-freedom wake-oscillator models for the same cable, and in which response characteristics does the coupling change the outcome?

1.4 Methodological Approach

The overview of the research approach is presented in Figure 1.3.

i. Literature Review

The first step was an integrated literature review on vortex-induced vibration, covering the underlying hydrodynamic mechanisms, the experimental investigation methods, and the modelling approaches used in both research and industry, with particular focus on flexible structures.

ii. Selection of the wake-oscillator model

Among the available approaches, the coupled cross-flow and in-line single wake-oscillator model of Qu and Metrikine (2021) [28] was selected for three reasons. First, its formulation assumes that the lift and the fluctuating drag arise from the same wake dynamics and so represents them with a single oscillator, in contrast to double-oscillator models that assign a separate oscillator to each direction. Second, wake-oscillator models are computationally efficient while still giving reasonable accuracy against higher-fidelity methods such as computational fluid dynamics. Third, the coupled formulation captures the interaction between the cross-flow and in-line responses, which is particularly relevant for flexible structures.

iii. Selection of the experimental data

The implementation was validated against the VIV measurements of a vertical tension riser in a stepped current by Huera-Huarte (2006) [30] in the Delta Flume. Qu and Metrikine (2021) [28] validated their model against the same measurements, which gives a second basis for comparison. The model was then applied to the lazy-wave cable experiments of Moideen et al. (2025) [26], which use a distorted-scale model with an increased diameter to keep the Reynolds number representative of offshore conditions. The authors report that no comparable dataset for lazy-wave power cables at this Reynolds number had been published before. A vertical cable from the FLOTANT project [31] was added as a supplementary case, a scaled inter-array power cable suspended vertically and towed in uniform flow, used to separate the effect of the low mass ratio from that of the lazy-wave geometry.

iv. Implementation

The model was implemented in two ways. The first is a dedicated finite element solver in Python, with the wake oscillator distributed along the structure. The second is a Python external function coupled to OrcaFlex through its external function interface, which applies the same wake-oscillator forcing inside an industrial dynamic analysis environment. The first implementation serves as an independent reference against which the OrcaFlex coupling is assessed.

v. Verification and validation

Both implementations were validated against the Delta Flume riser measurements and the published predictions of Qu and Metrikine (2021) [28], and verified against each other on the same configuration. A tension below the value validated by Qu and Metrikine (2021) was also examined, and the results were compared with the earlier cross-flow only implementations of Ntounmpis (2017) [32] and Slingsby (2015) [33] on the same cases. This step confirms that the two implementations reproduce a known VIV response, including the in-line response that the cross-flow only models could not capture.

vi. Application and comparative evaluation

The validated model was applied to the lazy-wave cable of Moideen et al. (2025) [26], the predicted response was compared with the experimental measurements and with the industry-standard single-degree-of-freedom wake-oscillator models, the Milan and the Iwan and Blevins formulations, applied to the same configuration. This comparison assesses how well the coupled model reproduces the measurements and whether it improves on current industry practice.

1.5 Thesis Outline

The thesis is organised in two parts. The first presents the theory and the model description, and the second the implementation and the results. In more detail:

PART I – Theory & Background

Chapter 2 presents the physics of vortex-induced vibration, the experimental characterisation of the response, and its extension to flexible cylinders and lazy-wave cables.

Chapter 3 describes the coupled cross-flow and in-line wake-oscillator model and its governing equations.

PART II – Implementation & Results

Chapter 4 presents the two implementations, the dedicated finite element solver and the OrcaFlex coupling, and validates both against the Delta Flume riser experiment.

Chapter 5 applies the model to the lazy-wave cable, compares it with the experiment and the industry-standard models, and examines the vertical cable as a supplementary case.

Chapter 6 gives the conclusions and recommendations for further work.

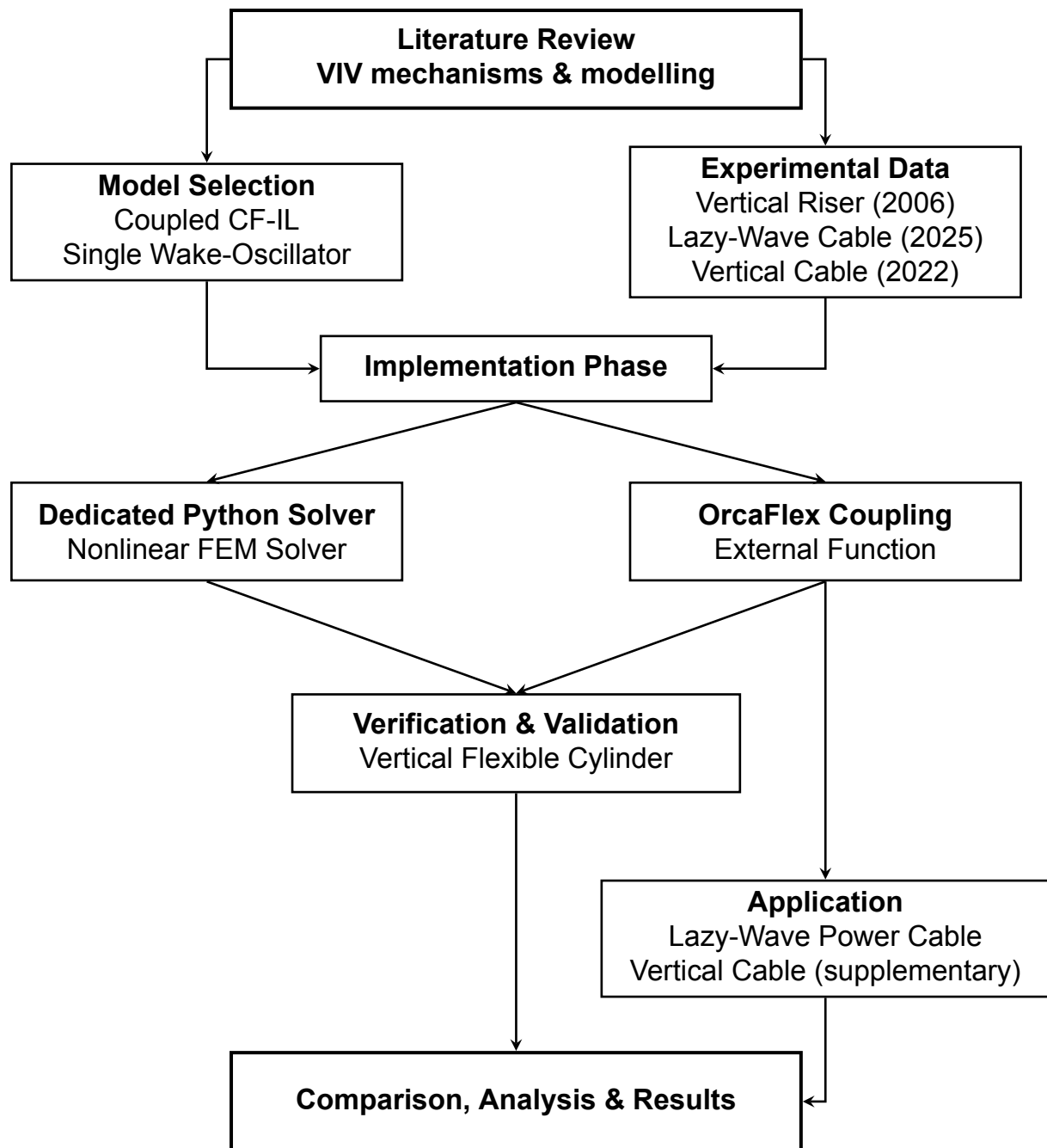


Figure 1.3. Overview of the research methodology and simulation framework

2 | Vortex-Induced Vibration

This chapter synthesises the key concepts of vortex-induced vibration, covering the underlying hydrodynamic mechanisms, experimental investigation methods, and how these extend to flexible cylinders and lazy-wave power cables.

2.1 Flow Around a Circular Cylinder

2.1.1 Flow Separation and Wake Formation

The external viscous flow field around blunt bodies immersed in a fluid stream can be divided into two general regions: a region adjacent to the body surface where viscous (shear and no-slip) effects are present, and an outer, irrotational and essentially inviscid flow region where velocity gradients, and hence shear stresses, are zero [34, 35, 36]. According to Zdravkovich (1997) [37], the described flow field can be divided into four sub-regions, as shown in Figure 2.1a.

The first zone is a narrow region of retarded flow, where the flow decelerates as it approaches the body, reaching zero velocity at the stagnation point while pressure rises to its maximum value. The second area consists of two thin boundary layers covering the front side of the obstacle under a favourable pressure gradient. In the rear, the boundary layer encounters an adverse pressure gradient and separates, forming two free shear layers that trail aft in the flow and bound a pulsating low-pressure region called the wake [34, 35, 38]. Once separated, the adverse pressure gradient produces backflow near the wall, preventing full pressure recovery and increasing drag [39], such that the greater the size of the wake, the greater the pressure drag [35].

Form drag, also known as pressure drag, arises from the net unbalance of pressure forces between the upstream and downstream sides of the body and is the main component of the total drag force on a bluff body [35]. Although the viscous drag component is comparatively lower, it is nevertheless necessary for the occurrence of flow separation and thus for the differential pressure responsible for form drag development.

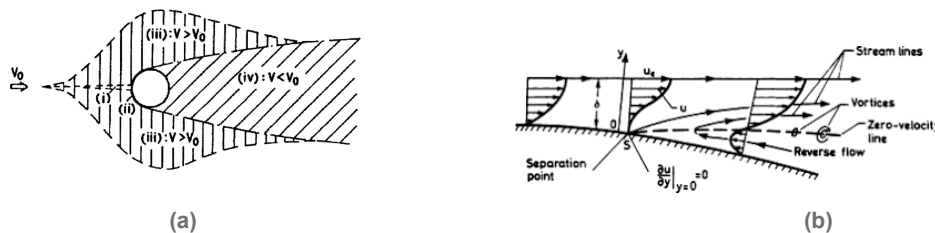


Figure 2.1. Flow around a circular cylinder [40] (a) and reverse flow schematic [41] (b)

2.1.2 Flow Regimes

The non-dimensional quantities describing the flow around a smooth circular cylinder depend on the Reynolds number, which represents the ratio between inertia and viscous forces [42]:

$$Re = \frac{DU}{\nu} \quad (2.1)$$

in which D is the diameter of the cylinder, U is the flow velocity, and ν is the kinematic viscosity. As the Reynolds number increases, the wake structure undergoes significant changes, evolving from steady attached vortices to periodic vortex shedding, with corresponding modifications in the drag force on the body, as shown in Figure 2.2.

For very low Reynolds numbers ($Re \approx 1$), viscous forces govern and the flow remains symmetric and attached to the cylinder surface. When the Reynolds number increases ($5 < Re < 40$), flow separation begins and two steady symmetric vortices form behind the cylinder. At higher Reynolds numbers, the wake becomes unstable and periodic vortex shedding occurs, forming the von Kármán vortex street [38]. Up to approximately $Re \approx 200$, shedding remains two-dimensional, while beyond this the wake becomes three-dimensional as turbulence sets in.

For $300 \leq Re \leq 3 \times 10^5$, the subcritical regime, the wake is turbulent but the boundary layer remains laminar at separation, and vortex emission is strong and periodic. Around $Re \approx 3 \times 10^5$, the boundary layer transitions to turbulence before separation, leading to a narrower separation zone and smaller differential pressure between the upstream and downstream sides of the cylinder, a sharp reduction in drag coefficient known as the drag crisis [42]. In the supercritical and transcritical regimes the boundary layer remains largely turbulent, shifting the separation angle from approximately $\pm 80^\circ$ to about $\pm 120^\circ$ [42], as shown in Figure 2.3.

a)		No separation. Creeping flow	$Re < 5$
b)		A fixed pair of symmetric vortices	$5 < Re < 40$
c)		Laminar vortex street	$40 < Re < 200$
d)		Transition to turbulence in the wake	$200 < Re < 300$
e)		Wake completely turbulent. A: Laminar boundary layer separation	$300 < Re < 3 \times 10^5$ Subcritical
f)		A: Laminar boundary layer separation B: Turbulent boundary layer separation, but boundary layer laminar	$3 \times 10^5 < Re < 3.5 \times 10^5$ Critical (Lower transition)
g)		B: Turbulent boundary layer separation; the boundary layer partly laminar partly turbulent	$3.5 \times 10^5 < Re < 1.5 \times 10^6$ Supercritical
h)		C: Boundary layer com- pletely turbulent at one side	$1.5 \times 10^6 < Re < 4 \times 10^6$ Upper transition
i)		C: Boundary layer com- pletely turbulent at two sides	$4 \times 10^6 < Re$ Transcritical

Figure 2.2. Flow around a cylinder as a function of the Reynolds number [42]

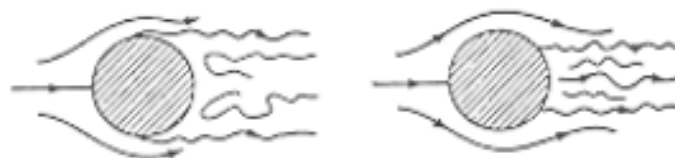


Figure 2.3. Laminar and turbulent boundary layer separation [43]

2.1.3 Mechanism of Vortex Shedding

The formation of the vortex street results from the interaction of the two shear layers that separate from the cylinder surface. Each separated layer releases circulation that concentrates into a free vortex filament which, once no longer constrained by the cylinder, rolls up into a concentrated vorticity core that develops into a vortex [44]. The development of the separated shear layers is shown in Figure 2.4a, while Figures 2.4b and 2.4c show the subsequent vortex formation mechanism described by Sumer and Fredsøe (2006) [42]. The roll-up behaviour occurs because the inner portion of each shear layer convects more slowly than the outer region exposed to the free stream, promoting the formation of discrete vortices and the regular wake pattern [38].

Vortex formation begins when a vortex strengthens on one side of the cylinder and induces a rotational velocity field that draws the free filament released by the opposite shear layer toward it. As opposite-signed vorticity approaches, the supply of circulation from the boundary layer is interrupted, causing the vortex to detach and convect downstream. A new vortex then forms on the opposite side, leading to alternating vortex shedding [42]. Sarpkaya (1979) [45] additionally notes that this alternation is also influenced by variations in base pressure and by diffusion and dissipation of vorticity, which further weaken the feeding shear layer and reinforce the periodic shedding pattern.

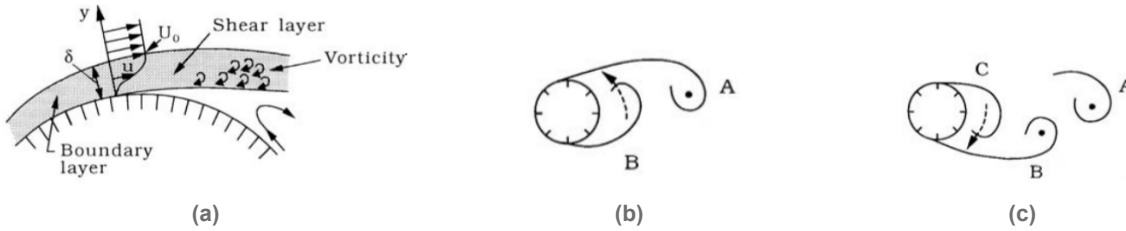


Figure 2.4. Shear layer development (a) and vortex formation sequences (b–c) [42]

2.1.4 Vortex Shedding Frequency

The normalised vortex shedding frequency is expressed by the Strouhal number St [42]:

$$St = \frac{f_v D}{U} \quad (2.2)$$

where f_v is the shedding frequency, D the cylinder diameter, and U the flow velocity. The relation between the Reynolds number and the Strouhal number is presented in Figure 2.5 for circular cylinders. Vortex shedding begins at $Re \approx 40$ with $St \approx 0.1$ and gradually increases to $St \approx 0.2$ at $Re = 300$, remaining nearly constant across the subcritical regime [42]. In this range, shedding occurs in a regular pattern with a narrow frequency spectrum [42], although a vortex street does not always appear [46].

In critical and supercritical regimes, the shedding frequency spectrum widens. The sharp increase in St observed between $Re = 2 \times 10^5$ and $Re = 0.8 \times 10^6$ results from turbulent boundary layers delaying separation downstream, bringing vortices closer together and increasing

the shedding rate [42]. Vortices become irregular and weaker [47], and for $Re > 2 \times 10^6$, St increases again to values of 0.25-0.3 [47].

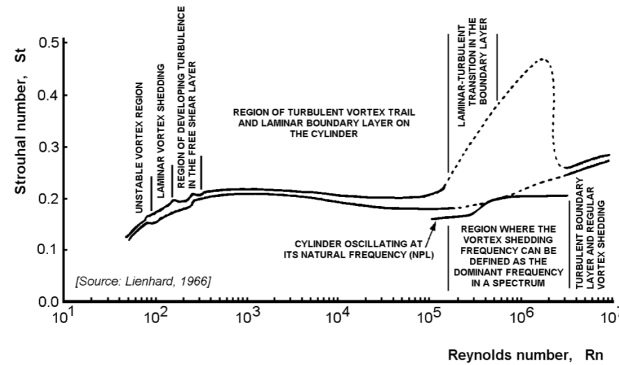


Figure 2.5. Strouhal - Reynolds number relationship for a smooth circular cylinder [46]

2.1.5 Drag and Lift Force

As the vortices shed alternately from each side, upper and lower parts, of the rear of the cylinder, cyclic pressure variations occur in the wake. At the location where a vortex passes closest to the cylinder, local velocities are highest and pressures lowest, generating a periodic force that decomposes into a drag component in the flow direction and a lift component perpendicular to it [46]. For a single vortex-shedding frequency, the lift and drag forces on a stationary cylinder and the corresponding force coefficients are given in Table 2.1 [46].

Table 2.1. Lift and drag force expressions and coefficients

Force	Expression
Lift force	$F_L(t) = F_L \cos(2\pi f_v t + \alpha)$
Drag force	$F_D(t) = \bar{F}_D + A_D \cos(4\pi f_v t + \beta)$
Lift coefficient	$C_L = \frac{ F_L }{\frac{1}{2}\rho U_\infty^2 D}$
Drag coefficient	$C_D = \frac{\bar{F}_D}{\frac{1}{2}\rho U_\infty^2 D}$

From the expressions in Table 2.1, the drag force oscillates around a mean value, making it convenient to present the mean drag coefficient as a function of the Reynolds number [48]. As shown in Figure 2.6a, it remains practically constant at approximately 1.2 throughout the subcritical regime, drops abruptly to around 0.25 at the start of the supercritical regime due to the drag crisis, before recovering to approximately 0.5 in the transcritical regime [48].

The oscillatory lift force varies irregularly from cycle to cycle and is therefore commonly characterised by its root-mean-square value [42]. As shown in Figure 2.6b, the RMS lift coefficient shows a sharp change in the critical flow regime, consistent with the boundary layer transition and drag crisis described earlier, before dropping to low levels in the supercritical regime where vortex interaction weakens considerably [42].

The oscillatory drag component A_D varies only slightly, typically around 20% of $\bar{F}_D$ [46]. Since drag always acts in the flow direction regardless of which side the vortex originates from, drag forces on all cylinder segments add up when integrated over the structure, giving a significant total drag force [46, 47]. In contrast, the lift force depends on the side from which the vortex is shed, and because vortices are distributed along the wake in an uncorrelated manner, lift forces on adjacent segments may act in opposite directions, leading to cancellation when integrated along the cylinder and resulting in a much smaller net lift force [46, 47].

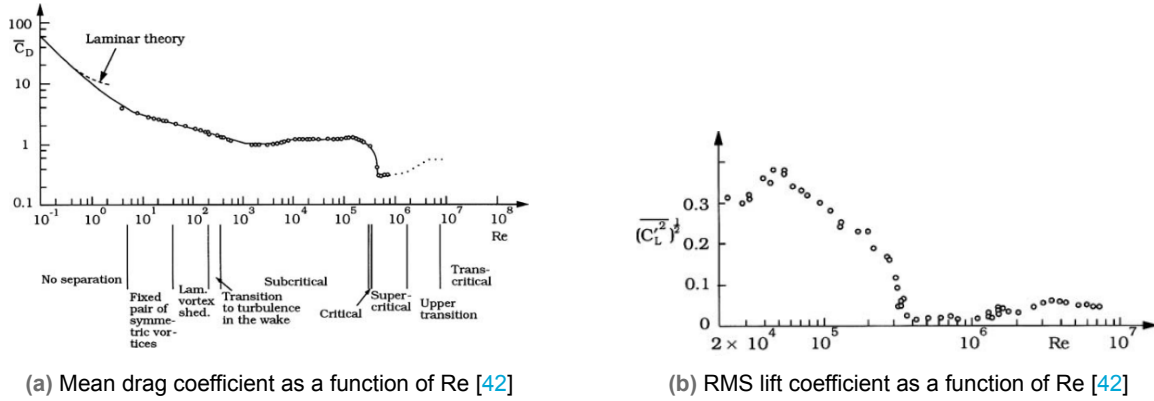


Figure 2.6. Mean drag and RMS lift coefficients as a function of Reynolds number

2.2 VIV Mechanism

Vortex-induced vibrations belong to the broader class of flow-induced vibrations. Unlike unstable phenomena such as galloping, VIV is self-limiting, oscillation amplitudes stabilise once excitation forces and fluid damping reach equilibrium [49], producing moderate response amplitudes that accumulate fatigue damage through prolonged cyclic loading rather than causing immediate failure.

As shown in Figure 2.7, when acting on a flexibly mounted cylinder [48]:

- Lift forces induce **cross-flow vibrations** (CF) perpendicular to the flow direction
- Drag forces induce **in-line vibrations** (IL) parallel to the flow direction

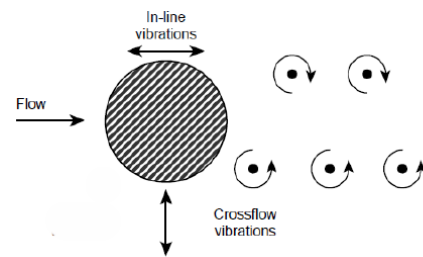


Figure 2.7. IL and CF vibrations [50]

The distinct frequencies of lift and drag oscillation directly determine the corresponding structural response modes. Since vortices alternate between opposite sides, the lift force oscillates at the vortex shedding frequency, while the drag force oscillates at twice this frequency. Consequently, when the shedding frequency approaches the structural natural frequency, cross-flow vibrations can reach amplitudes on the order of one cylinder diameter, whereas in-line vibrations typically remain below 0.5 diameters [49]. This amplitude disparity explains why early experimental investigations focused primarily on cross-flow response, though the importance of in-line motion for fatigue accumulation has since been well established [49].

Resonance occurs when the vortex-shedding frequency f_v approaches the structural natural frequency f_n [47]. A convenient indicator is the dimensionless reduced velocity:

$$U_r = \frac{U}{f_n D} \quad (2.3)$$

For $St = 0.2$, this corresponds to $U_r \approx 5$.

2.2.1 Cross-flow Oscillations

The lock-in phenomenon is captured by the classic experiment of Feng (1968), shown in Figure 2.8, where the vortex-shedding frequency f_v , the vibration frequency f , and the stationary-cylinder Strouhal frequency are presented [48]. As the flow velocity increases from zero, no vibration is observed until $V_r \approx 4$. Beyond this point, small-amplitude vibrations appear at the natural frequency while vortex shedding still follows the Strouhal relationship. As V_r increases further, the shedding frequency deviates from the Strouhal trend and aligns with the natural frequency of the system (the horizontal line $f/f_n = 1$), locking onto it at $V_r \approx 5$ and transferring energy efficiently into the structure, producing large vibration amplitudes [38].

Within the lock-in interval, three frequencies coincide: the vibration frequency, the vortex-shedding frequency, and the natural frequency ($f = f_v = f_n$), resulting in very large vibration amplitudes [48]. As the cylinder oscillates, vortices become more aligned along the span and lift forces act more coherently, reinforcing the mechanism until approximately $V_r = 7$ [47]. Beyond this range, the lock-in condition breaks down and the shedding frequency returns to the Strouhal value. Sarpkaya (1979) [45] notes that although linear resonance conditions are reached during lock-in, the cylinder motion introduces additional fluid-structure interaction nonlinearities beyond any inherent structural nonlinearities. Systems with lower structural damping experience larger amplitudes and a wider lock-in interval [48].

A secondary effect of cross-flow oscillations is an increase in drag, as the widening wake produces an effectively larger diameter [47], expressed by Faltinsen (1990) [46] as an apparent projected width of $D + 2A$, where A is the oscillation amplitude. The transverse motion of the cylinder further increases the effective flow velocity, adding to the drag [46].

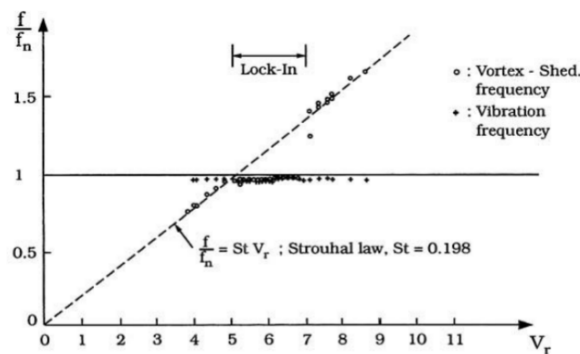


Figure 2.8. CF response of a flexibly mounted circular cylinder, obtained from Feng's experiment [48]

2.2.2 In-line Oscillations

In-line vibrations occur at lower reduced velocities than cross-flow vibrations, with amplitudes typically an order of magnitude smaller [45, 48]. Sumer and Fredsøe (2006) [48] identify three instability regions, shown in Figure 2.9.

The first, around $V_r \approx 1.7$, results from the interaction between normal alternating shedding and a secondary symmetric shedding mode generated by the in-line motion of the cylinder, shown in Figure 2.9b. The symmetric mode generates two vortices per oscillation cycle, and when combined with the normal shedding frequency, the resulting in-line forcing occurs at approximately three times the Strouhal frequency ($f = 3StU/D$). A resonance develops when this combined forcing frequency approaches the natural frequency f_n . As V_r increases, the interaction weakens, normal shedding is restored, and the drag force oscillates at $f = 2StU/D$, leading to a second in-line resonance near $V_r \approx 2.5$. A third instability region appears only when the cylinder is free to move in both directions, where strong cross-flow oscillations increase the coherence and strength of the shed vortices [48].

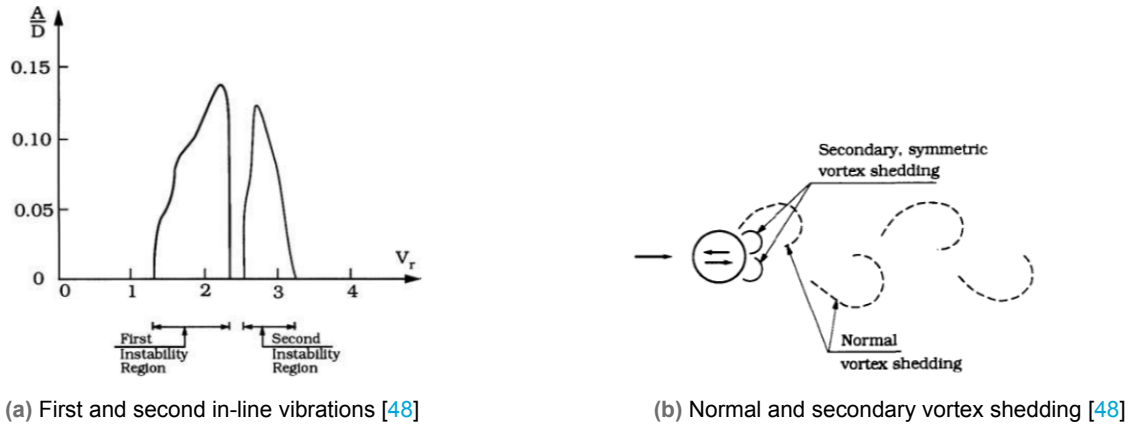


Figure 2.9. In-line vibration mechanisms and shedding patterns

2.3 Experimental Analysis of VIV

The understanding of VIV presented above has been shaped primarily through physical experiments, given the inherent complexity of near-wake flow dynamics and fluid-structure interaction [51]. Experimental studies typically involve a rigid circular cylinder in steady flow and can be classified into three categories: stationary experiments, free-vibration experiments, and forced-vibration experiments [51, 52], each providing distinct and complementary insights into the VIV phenomenon, as discussed below.

2.3.1 Stationary Cylinder Experiments

In stationary experiments, the cylinder is rigidly held while the flow passes around it, and the resulting forces are measured directly. The Strouhal relationship, drag and lift coefficients, and vortex shedding patterns described in the previous sections are all outcomes of this type of experiment [52]. This setup is particularly useful for stiff structures where flow-induced loads

must be determined without structural motion. Notably, the rigid structure itself is not the direct trigger for vortex creation. Rather, it alters the fluid dynamics by enabling a feedback mechanism between the downstream wake and the points of flow separation [52].

An important finding from stationary experiments is the lack of strict two-dimensionality in vortex shedding. The spanwise coupling between the two separating shear layers is generally weak, meaning that unsteady quantities such as surface pressure and fluctuating lift are not uniform along the cylinder span [52]. Regions along the span where shedding remains similar are described in terms of correlation lengths, and even small departures from uniformity, such as cylinder taper or shear in the approaching flow, can significantly reduce this correlation length [52]. VIV is therefore both a temporal and a spatial phenomenon, since spanwise coherence strongly influences vibration response.

Many stationary-cylinder experiments have been carried out at high Reynolds numbers, including studies by Roshko (1961), Achenbach (1968), and Schewe (1983), though significant discrepancies exist among datasets regarding hydrodynamic quantities such as lift and drag coefficients and Strouhal number [53].

2.3.2 Free Vibration Experiments

In free vibration experiments, the structure is permitted to oscillate in response to fluid forces, showing the two-way interaction between the cylinder and its wake. As flow velocity increases and shedding departs from the Strouhal frequency, these experiments show both the lock-in phenomenon and the nonlinear modification of the flow field by the moving structure [52, 45]. Notably, a freely vibrating cylinder experiences significantly stronger lift and drag forces than a stationary one, reflecting the energy exchange between the structure and the wake [52]. Since the response is governed by the specific mass and damping of the tested system, each dataset is applicable only to conditions that closely match those of the experiment [51].

The VIV response depends on two key non-dimensional parameters, defined in Table 2.2 [54, 52].

Table 2.2. Key non-dimensional parameters governing VIV response

Parameter	Definition
Mass ratio	$m^* = \frac{M}{\frac{\pi D^2}{4} L \rho}$
Damping ratio	$\zeta = \frac{c}{2\omega_n(m + m_a)}$
Scruton number	$Sc = \frac{4\pi m \zeta}{\rho D^2}$

where M is the structural mass, D the cylinder diameter, L the spanwise length, ρ the fluid density, c the damping coefficient, m_a the added mass, and ω_n the natural circular frequency. Offshore structures typically fall within the low mass-ratio regime ($m^* \lesssim 3$) with very small

structural damping ($\zeta < 0.05$), making added-mass effects essential for VIV prediction [49]. These parameters are often combined into the mass-damping parameter $m^*\zeta$, also expressed through the Scruton number [52].

The role of these parameters becomes clear when revisiting Feng's experiment discussed earlier. Conducted in air, where mass and damping are high, the vibration frequency remained close to the still-air natural frequency throughout lock-in. When an equivalent configuration is tested in water, as shown by Khalak and Williamson (1997) [55] at very low mass-damping values, the added inertia of the surrounding fluid causes the vibration frequency to rise steadily with reduced velocity, while amplitudes become considerably larger [48].

For lightly damped, low mass-ratio cylinders, they identified three response branches, initial, upper, and lower, shown in Figure 2.10a. The initial-to-upper transition is hysteretic, while the upper-to-lower transition is characterised by intermittent switching and a 180° phase jump in fluid forcing [54, 56]. At high mass-damping, the upper branch disappears and only the initial and lower branches remain. Peak amplitude is governed primarily by $m^*\zeta$, while the width of the synchronisation region depends on m^* [56]. A notable consequence of very low mass ratios is the existence of a critical threshold below which the lower response branch disappears entirely. When the structural mass falls below approximately $m^* \approx 0.54$, the lock-in condition never breaks down, meaning the structure remains in resonance indefinitely with increasing flow velocity [54].

For low mass-ratio cylinders the oscillation frequency within lock-in rises well above the natural frequency, reaching $f^* \approx 1.4$ – 1.5 , suggesting that synchronisation is better understood as a matching between the wake vortex mode and the structural oscillation frequency rather than a strict $f/f_n = 1$ condition [55]. As shown in Figure 2.10b, increasing damping reduces peak amplitude and narrows the lock-in band [52, 56].

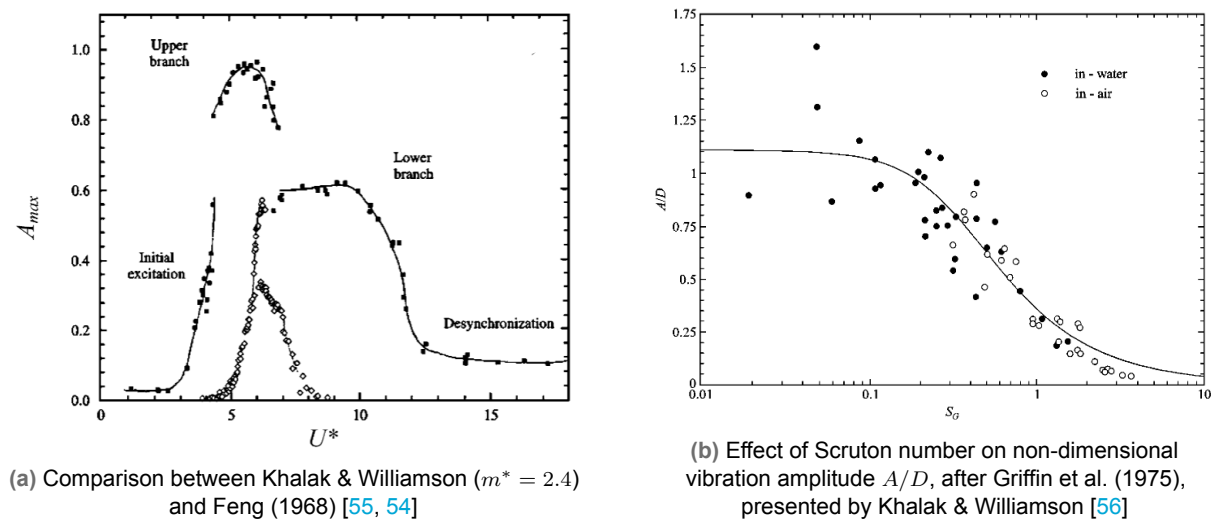


Figure 2.10. Free vibration response characteristics

2.3.3 Forced Vibration Experiments

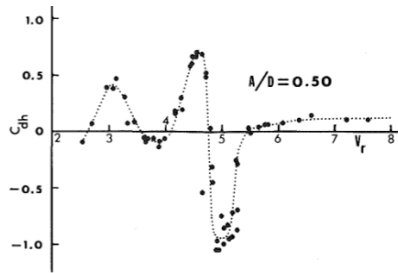
In forced vibration experiments, the cylinder is oscillated with a prescribed amplitude and frequency while hydrodynamic forces and wake structure are measured [29]. The lift force is well represented by a sinusoid at the excitation frequency [49]. Writing the motion as $y = A_y \sin(\omega t)$ and the lift force as $L = F_L \sin(\omega t + \phi)$, the force separates into two components [49]:

$$C_{Lv} = C_L \sin(\phi) \quad (2.4)$$

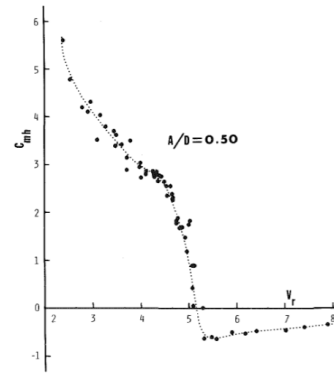
$$C_{La} = C_L \cos(\phi) \quad (2.5)$$

corresponding to hydrodynamic damping and added mass respectively. By oscillating the cylinder over a range of amplitudes and frequencies, these coefficients can be quantified and used to predict the steady-state response of a freely vibrating cylinder [51].

Sarpkaya (1979) [45] identified two mechanisms central to VIV, shown in Figure 2.11a and Figure 2.11b. Within the lock-in region near $V_r \approx 5$, the damping coefficient C_{dh} becomes negative, meaning the fluid force acts in the same direction as the motion and transfers energy to the structure. Simultaneously, the added-mass coefficient C_{mh} also becomes negative, reducing the effective mass of the cylinder and causing it to behave as if lighter. When the out-of-phase force component eventually changes sign at larger amplitudes, the motion becomes self-limiting, preventing further growth [45].



(a) C_{dh} versus V_r for $A/D = 0.50$ [45]

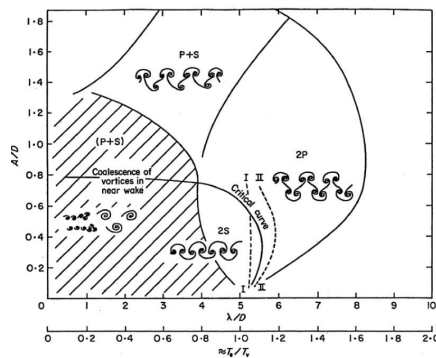


(b) C_{mh} versus V_r for $A/D = 0.50$ [45]

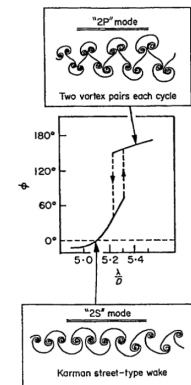
Figure 2.11. Out-of-phase and in-phase force coefficients, adapted from Sarpkaya (1979) [45]

Williamson and Roshko (1988) [57] showed that the wake structure depends on the amplitude ratio A/D and the trajectory wavelength λ/D . Two main shedding modes were identified: the 2S mode, where one vortex sheds from each side per cycle, and the 2P mode, where two vortex pairs shed per cycle, as shown in Figure 2.12a. The transition between these modes sets the phase of the lift force and determines the direction of energy transfer between flow and structure [52]. At the critical transition wavelength, vorticity becomes highly concentrated and lift reaches its maximum, a condition termed resonant synchronisation [57]. The corresponding phase jump in lift is shown in Figure 2.12b.

Free and forced vibration experiments differ mainly in how motion parameters are controlled. In forced tests, reduced velocity and amplitude can be prescribed independently, whereas in free vibration both are coupled. Only a limited range of forced vibration conditions matches free vibration response, though similar flow fields can be produced when the detailed motion history is not critical [52].



(a) Map of vortex synchronisation [57]



(b) Lift force phase angle vs wavelength ratio [57]

Figure 2.12. Wake transitions in forced vibrations of a circular cylinder [57]

2.4 VIV in Flexible Cylinders

2.4.1 Multi-mode and Coupled Response

The experimental approaches described above are adequate for simple configurations but are generally limited to rigid cylinders vibrating in a single mode. In offshore engineering, however, many critical components, risers, power cables, mooring lines, umbilicals, are long, slender and flexible, continuously exposed to environmental loads [30, 28]. These structures possess multiple natural frequencies and mode shapes and are free to respond in multiple directions [58, 29], meaning resonance may occur simultaneously in both cross-flow and in-line directions, as shown in Figure 2.13a.

Although in-line amplitudes are typically half those of cross-flow response, neglecting in-line motion leads to significant errors in fatigue prediction for long flexible cylinders [49, 29, 58]. When the in-line natural frequency approaches twice the transverse natural frequency, perfect vortex shedding resonance occurs, producing stable figure-eight trajectories and multivortex wake patterns that generate high-amplitude odd harmonics in lift force [49]. According to Zheng (2014) [59], incorporating these higher harmonics into fatigue calculations can increase predicted damage by an order of magnitude, with field experiments reporting fatigue rates up to forty times higher when third harmonic contributions are included [60].

Investigations of two-degree-of-freedom systems by Jauvtis and Williamson (2003) [61] show that at high mass ratios (above $m^* = 6$), the addition of the streamwise degree of freedom has little effect, yielding the same response branches and 2S/2P vortex modes as transverse-only experiments. However, when the mass ratio drops below $m^* = 6$, the behaviour changes

substantially. A new response branch, termed the super-upper branch, emerges with significant streamwise motion and cross-flow amplitudes reaching up to 1.5 diameters [61]. This regime is driven by a triplet vortex-shedding pattern (2T mode), where three vortices form per half-cycle, with the third vortex providing the energy transfer that sustains these large amplitudes [54, 61].

The overall response divides into three regions: in-line governed ($U^* < 4$), strongly coupled ($4 < U^* < 8$), and cross-flow governed ($U^* > 8$) [54, 62], as shown in Figure 2.13b.

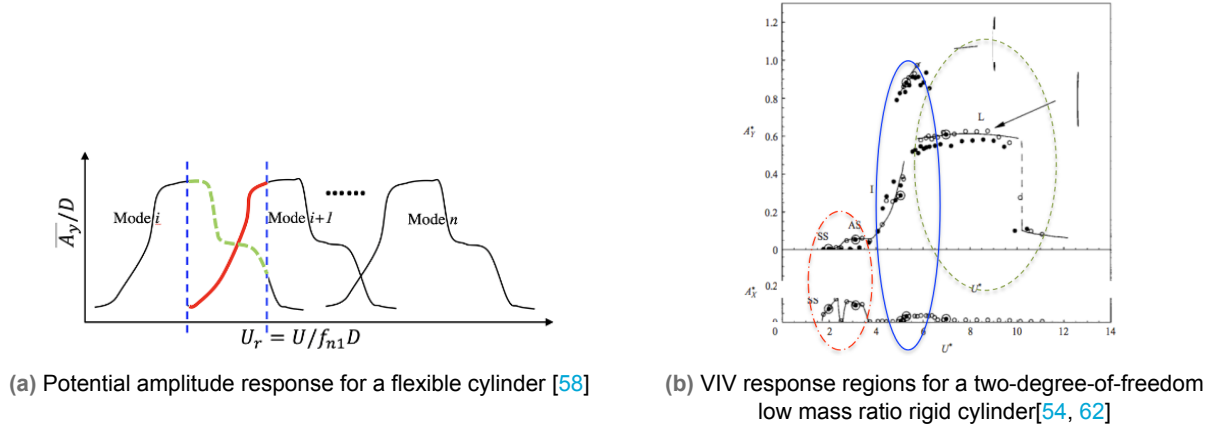


Figure 2.13. VIV response characteristics of flexible and two-degree-of-freedom cylinders

2.4.2 VIV in Lazy-Wave Dynamic Cables

The VIV behaviour of lazy-wave cables differs from that of straight or uniformly tensioned cylinders. The curved geometry makes the incident flow angle vary along the cable, so the normal velocity component and the local VIV response change from point to point [27]. The buoyancy modules also break the spanwise coherence of vortex shedding. Each module leaves a localised wake, and the cable sections downstream of it meet a reduced effective flow velocity [27]. Because the modules are small relative to the full span, the resulting vibration amplitudes stay below those of long uniform cylinders [27].

Marine power cables also differ from the steel risers used in offshore oil and gas systems. They carry no internal fluid, have lower bending stiffness, and are more flexible [26]. The lower bending stiffness gives them lower, more closely spaced natural frequencies and changes their lock-in behaviour, with a strong sensitivity to the mass ratio [26]. Results obtained for risers therefore do not transfer directly, and the configuration needs dedicated study [26].

Experimental data on VIV of lazy-wave power cables remain scarce. The measurements of Moideen et al. (2025) [26] show several features that separate this configuration from a tensioned riser. Although VIV appears in both directions, the in-line and cross-flow motions share mostly one frequency rather than the classic two-to-one ratio. The figure-eight orbits tied to that ratio therefore do not form, and the trajectories stay highly non-stationary and irregular [26]. The high structural compliance also prevents a distinct, persistent lock-in, so any lock-in is intermittent, the flow excites a broad band of frequencies rather than a single mode, and the overall response changes from one flow condition to the next [26].

2.5 Conclusion

This chapter reviewed the physics of vortex-induced vibration used in the rest of the thesis. Vortex shedding produces an oscillating lift at the shedding frequency and a drag fluctuation at twice that frequency, and the structure locks in when the shedding frequency nears a natural frequency. At the low mass ratios typical of offshore structures the cross-flow and in-line motions couple, the lock-in band widens, and the response frequency rises above the still-water value.

Flexible cylinders respond in several modes and in both directions, so in-line motion and its higher harmonics contribute significantly to fatigue damage. Lazy-wave cables add curved geometry and buoyancy modules, and the limited measurements available show a non-stationary, multi-modal response rather than a distinct lock-in. Most established VIV tools were calibrated on rigid or straight cylinders and do not capture this coupled, multi-modal behaviour, which the selected model presented in Chapter 3 is designed to address.

3 Coupled Wake Oscillator Model

This chapter presents the theoretical framework of the coupled cross-flow and in-line wake oscillator model proposed by Qu and Metrikine (2021) [28], which forms the basis of the numerical implementation described in Chapter 4. The model is introduced from its van der Pol basis, extended to flexible cylindrical structures through a local coordinate frame, and the complete coupled system is assembled.

3.1 Introduction

Among the available numerical approaches for VIV prediction, computational fluid dynamics (CFD) methods have become increasingly accurate, but remain constrained by significant computational cost, making them prohibitive for industrial applications involving long flexible cylinders [63]. Semi-empirical methods offer a more practical alternative. Among these, force decomposition methods dominate current industry practice but are inherently limited by their frequency-domain formulation and reliance on empirical hydrodynamic coefficient databases, which do not readily capture the coupled cross-flow and in-line dynamic response [64, 28]. More recently, artificial intelligence and machine learning approaches have shown promise in extracting key VIV characteristics and optimising hydrodynamic coefficient databases, though their integration with physics-based models remains an active area of research [58].

Wake oscillator models address these limitations by representing the near-wake dynamics phenomenologically through a nonlinear oscillator coupled to the structural equation of motion. Rather than resolving the flow field, the fluctuating hydrodynamic force is described through the oscillator equation, with vibration amplitude and frequency determined as part of the solution [63, 64, 65]. Operating in the time domain, they account for structural nonlinearities and are capable of capturing multimode responses of long flexible cylinders, making them a practical physics-based option for VIV analysis [64].

3.2 Structural Model

The structural model adopted in this thesis is described in detail in Section 4.2. It is formulated in terms of the full three-dimensional position vector $\mathbf{r}(p, t)$ and handles both straight and curved flexible configurations. A simplified case is presented here to introduce the structural side of the model. The coupling to the wake oscillator is established in the following sections. A schematic of the system considered is shown in Figure 3.1.

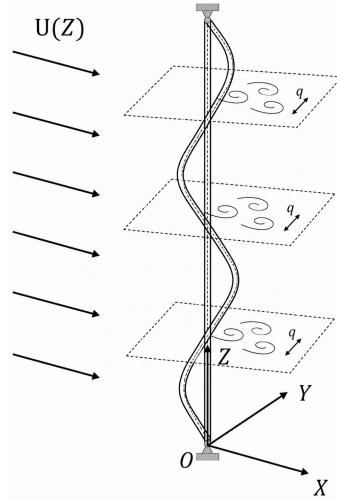


Figure 3.1. Schematic representation of a flexible riser subjected to uniform current and VIV [63]

For slender flexible cylinders such as risers and cables, where the diameter is small compared to the total length, the Euler-Bernoulli beam assumption is applicable. Under this assumption, cross-sections are assumed to remain perpendicular to the beam axis and shear and torsional deformations are considered negligible relative to bending and axial deformations [29, 66]. Assuming that the riser is nearly vertical and that the VIV displacements are small compared to the total length, geometric nonlinearities become negligible and the motions in the cross-flow and in-line planes can be treated independently [30]. Under these assumptions, the structural dynamics of a tensioned flexible riser with both ends pinned can be expressed as two decoupled equations [63]:

$$M \frac{\partial^2 y(z, t)}{\partial t^2} + R(z) \frac{\partial y(z, t)}{\partial t} + EI \frac{\partial^4 y(z, t)}{\partial z^4} - \frac{\partial}{\partial z} \left(T_e(z, t) \frac{\partial y(z, t)}{\partial z} \right) = F_y(z, t) \quad (3.1)$$

$$M \frac{\partial^2 x(z, t)}{\partial t^2} + R(z) \frac{\partial x(z, t)}{\partial t} + EI \frac{\partial^4 x(z, t)}{\partial z^4} - \frac{\partial}{\partial z} \left(T_e(z, t) \frac{\partial x(z, t)}{\partial z} \right) = F_x(z, t) \quad (3.2)$$

where:

- $y(z, t)$ is the cross-flow displacement [m]
- $x(z, t)$ is the in-line displacement [m]
- EI is the bending stiffness [$N \cdot m^2$]
- z is the axial coordinate along the riser [m]
- M is the total mass per unit length, $M = M_s + M_f$ [kg/m]
- M_s is the structural mass per unit length [kg/m]
- M_f is the added mass per unit length, $\rho \pi D^2 C_a / 4$ [kg/m]
- $R(z)$ is the total damping, $R_s + R_f(z)$ [kg/(m · s)]
- R_s is the structural damping [kg/(m · s)]
- $R_f(z)$ is the fluid damping, $\gamma \Omega_s(z) \rho D^2$ [kg/(m · s)]
- γ is the empirical damping parameter, $C_D / (4\pi St)$ [–]

The effective tension along the riser is [63]:

$$T_e(z, t) = T_{top} - W_e(L - z) + \Delta T_e(z, t) \quad (3.3)$$

where: T_{top} is the top pretension [N]

W_e is the wet weight per unit length, $(M_s - \rho\pi D^2/4)g$ [N/m]

ΔT_e is the additional tension induced by bending deformation [N]

The external forces $F_y(z, t)$ and $F_x(z, t)$ are the hydrodynamic cross-flow and in-line forces respectively, arising from vortex shedding. As shown in the following sections, these forces are expressed in terms of the wake variable q , establishing the coupling between the structural equations and the wake oscillator.

3.3 Van der Pol Wake Oscillator Model

As a structure oscillates, it alters the surrounding wake and thereby influences the hydrodynamic lift. When the shedding frequency aligns with the natural frequency of the structure, vibration amplitudes grow until excitation forces reach equilibrium with fluid damping. This behaviour is therefore self-excited by the fluid flow yet inherently self-limiting [52, 59, 64]. Since nonlinear oscillators producing stable limit cycles share these same self-regulating characteristics, any such oscillator can in principle be used to model the vortex shedding process [64].

The van der Pol equation was originally formulated by the Dutch physicist Balthasar van der Pol in the 1920s to describe nonlinear oscillations in electrical circuits [67], and has since been extensively adopted in wake oscillator formulations to model the fluctuating cross-flow force [65]. In its unforced form, the governing equation reads:

$$\ddot{q} + \varepsilon\omega_s (q^2 - 1) \dot{q} + \omega_s^2 q = 0 \quad (3.4)$$

where $q(t)$ is the scalar wake variable, often associated with the fluctuating lift coefficient, $\varepsilon > 0$ is a tuning parameter controlling the strength of the nonlinear damping, and $\omega_s = 2\pi StV/D$ is the Strouhal shedding frequency [64, 68]. For $\varepsilon = 0$, the nonlinear damping vanishes and the equation reduces to a linear harmonic oscillator. The nonlinear damping behaviour can be understood by inspecting the sign of the term $(q^2 - 1)$:

- When $|q| > 1$ the damping is positive and dissipates energy
- When $|q| < 1$ it becomes negative, injecting energy into the system and driving amplitude growth

This self-regulating mechanism gives rise to a stable limit cycle, towards which solutions converge regardless of initial conditions [68].

For small values of ε , the oscillator produces a quasi-harmonic oscillation of finite amplitude $q_0 = 2$ at the angular frequency ω_s [68]. Higher values of ε generate stronger nonlinear damping and faster convergence to the limit cycle, as shown in Figure 3.2.

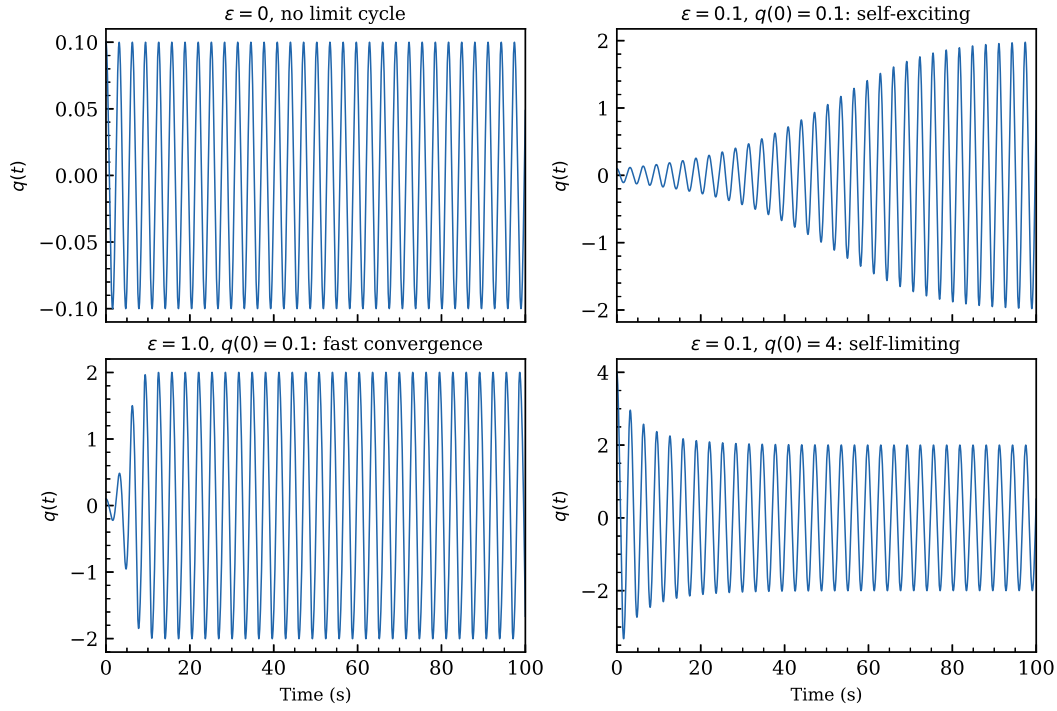


Figure 3.2. Time histories of the unforced van der Pol oscillator

To capture the effect of cylinder motion on the wake, a forcing term is introduced on the right-hand side of Eq. (3.4). Following Facchinetti et al. (2004) [68], acceleration coupling is adopted, and the wake equation reads:

$$\ddot{q} + \varepsilon \omega_s (q^2 - 1) \dot{q} + \omega_s^2 q = \frac{A}{D} \ddot{y} \quad (3.5)$$

where A is a tuning parameter controlling the magnitude of the fluid-structure coupling, with larger values producing larger steady-state vibration amplitudes. Increasing either A or ε broadens the lock-in range [68], while both are empirical and must be calibrated against experimental data [52, 65].

3.4 Single Wake Oscillator Model for Coupled VIV

As the importance of in-line motion on wake dynamics has become increasingly recognised, coupled cross-flow and in-line wake oscillator models have attracted growing research interest [64]. Two approaches appear in the literature, introducing a second wake oscillator for fluctuating drag, or coupling a single oscillator to both motions.

Qu and Metrikine (2020) [29] proposed a single oscillator formulation with classical acceleration coupling for cross-flow motion and a nonlinear parametric term for in-line motion, arguing that this is physically more consistent since both forces share the same wake origin. The model was first validated against rigid cylinder experiments [29]. It was subsequently extended to flexible structures by distributing the wake oscillator along the span through a local coordinate frame at each cross-section [28].

The coupling runs in both directions. The structural acceleration drives the wake oscillator, the wake variable q sets the fluctuating hydrodynamic force, and that force in turn drives the structure. The rigid-cylinder form of the coupled model is presented first, and the extension to a flexible cylinder follows.

3.4.1 Rigid-Cylinder Coupled Model

Wake Oscillator Equation

For a rigid cylinder in uniform cross-flow, the wake dynamics are governed by a van der Pol equation in which the cross-flow acceleration coupling term is retained and a nonlinear parametric in-line coupling term is added to account for the influence of in-line motion on the wake [28]. The wake oscillator equation reads [28]:

$$\ddot{q} + \varepsilon \omega_s (q^2 - 1) \dot{q} + \omega_s^2 q - \kappa \frac{\omega_s^4 D \ddot{x}}{\omega_s^4 D^2 + \ddot{x}^2} q = \frac{A}{D} \ddot{y} \quad (3.6)$$

where q is the dimensionless wake variable, A , ε and κ are tuning parameters, ω_s is the Strouhal frequency, and $\ddot{x}$, $\ddot{y}$ are the in-line and cross-flow structural accelerations respectively. The Strouhal frequency is set by the normal flow velocity [28]:

$$\omega_s = \frac{2\pi \text{St} V}{D} \quad (3.7)$$

where St is the Strouhal number and V is the free-stream velocity normal to the cylinder axis. The wake variable is related to the lift coefficient through [28, 68]:

$$C_{VL} = \frac{q}{2} C_{L0} \quad (3.8)$$

where the factor $\frac{1}{2}$ ensures that the amplitude of C_{VL} equals C_{L0} for a stationary cylinder, since the wake variable q reaches a limit cycle of amplitude 2 in the absence of structural motion, as discussed in Section 3.3.

The tuning parameters are established from rigid cylinder experiments, following Qu and Metrikine (2021) [28], and consistent with the independence principle, which makes two-dimensional rigid cylinder results applicable locally along a flexible structure. Among the parameter sets explored, those corresponding to the upper response branch of a rigid cylinder [29], referred to as Case U, best capture the characteristic features of flexible cylinder VIV. The adopted values are $A = 8$, $\varepsilon = 0.08$ and $\kappa = 5$ [28]. This set was originally calibrated against a rigid cylinder with a mass ratio of $m^* = 7$, and was later shown to reproduce the coupled cross-flow and in-line response of a rigid cylinder at a lower mass ratio of $m^* = 2.6$ [29]. Qu and Metrikine (2021) [28] adopted the same set for their flexible riser model at a mass ratio of $m^* = 3$, the Delta Flume riser used for validation in Chapter 4. Its use here for low-mass-ratio cables follows the same reasoning.

Hydrodynamic Force Model

The hydrodynamic force model is introduced for a rigid cylinder in a flow with undisturbed velocity V perpendicular to its axis. For a stationary cylinder, the drag force acts purely in the flow direction and the lift force oscillates perpendicular to it due to vortex shedding. The cross-flow and in-line hydrodynamic forces per unit length are simply:

$$F_{VY} = \frac{1}{2}\rho DV^2 C_{VY} \quad (3.9)$$

$$F_{VX} = \frac{1}{2}\rho DV^2 C_{VX} \quad (3.10)$$

When the cylinder oscillates, however, the in-line force F_{VX} is no longer equivalent to the drag force F_{VD} , and the cross-flow force F_{VY} is no longer equivalent to the lift force F_{VL} , as shown in Figure 3.3. This is because drag always acts aligned with the relative flow velocity U , and lift always acts perpendicular to it. As the cylinder moves, U deviates from the undisturbed flow V , so a decomposition is required to express the in-line and cross-flow forces in terms of the drag and lift components.

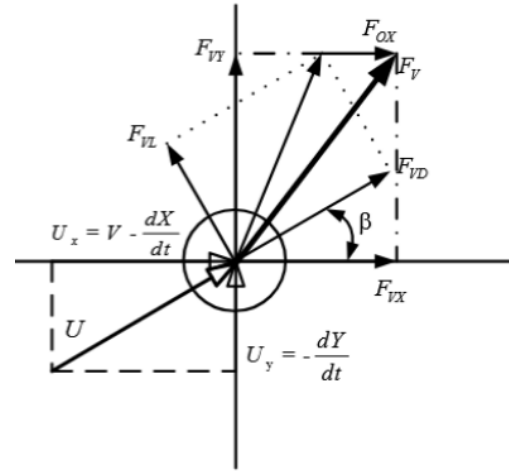


Figure 3.3. Decomposition of the vortex force [28]

The relative flow velocity accounts for the motion of the cylinder in both directions. Assuming the undisturbed flow velocity V acts horizontally in the in-line direction, the relative flow velocity components are defined as the vector difference between the undisturbed flow and the cylinder velocity [28]:

$$U_x = V - \frac{dx}{dt}, \quad U_y = -\frac{dy}{dt} \quad (3.11)$$

$$U = \sqrt{U_x^2 + U_y^2} \quad (3.12)$$

The magnitudes of the lift and drag forces per unit length are related to the relative flow velocity U as [28]:

$$F_{VL} = \frac{1}{2}\rho DU^2 C_{VL} \quad (3.13)$$

$$F_{VD} = \frac{1}{2}\rho DU^2 C_{VD} \quad (3.14)$$

Introducing β as the angle between the relative flow direction and the undisturbed flow direction V , the sine and cosine of β are given by [28]:

$$\sin(\beta) = \frac{U_y}{U} = -\frac{dy/dt}{U}, \quad \cos(\beta) = \frac{U_x}{U} = \frac{V - dx/dt}{U} \quad (3.15)$$

The total vortex force consists of a mean drag component and a fluctuating excitation from vortex shedding. While $C_{VD} = C_{DM}$ accounts for the steady mean drag, the oscillatory in-line contribution cannot be captured by a constant coefficient alone. A separate force F_{OX} , coupled to the lift force, is therefore introduced [28, 29]:

$$F_{OX} = \frac{1}{2}\alpha C_{VL}^2 \rho D |U_x| U_x \quad (3.16)$$

where α is an empirical parameter. Decomposing the forces in the x and y directions [28]:

$$F_{VY} = F_{VD} \sin \beta + F_{VL} \cos \beta \quad (3.17)$$

$$F_{VX} = F_{VD} \cos \beta - F_{VL} \sin \beta + F_{OX} \quad (3.18)$$

Substituting the force expressions yields [29]:

$$\frac{1}{2}\rho D L C_{VY} V^2 = \frac{1}{2}\rho D L C_{VL} U^2 \cos \beta + \frac{1}{2}\rho D L C_{VD} U^2 \sin \beta \quad (3.19)$$

$$\frac{1}{2}\rho D L C_{VX} V^2 = -\frac{1}{2}\rho D L C_{VL} U^2 \sin \beta + \frac{1}{2}\rho D L C_{DM} U^2 \cos \beta + \frac{1}{2}\alpha C_{VL}^2 \rho D L U_x |U_x| \quad (3.20)$$

Rearranging for the force coefficients [29]:

$$C_{VY} = C_{VL} \frac{U}{V^2} \left(V - \frac{dx}{dt} \right) - C_{VD} \frac{U}{V^2} \frac{dy}{dt} \quad (3.21)$$

$$C_{VX} = -C_{VL} \frac{U}{V^2} \frac{dy}{dt} + C_{DM} \frac{U}{V^2} \left(V - \frac{dx}{dt} \right) + \alpha C_{VL}^2 \left(1 - \frac{1}{V} \frac{dx}{dt} \right) \left| 1 - \frac{1}{V} \frac{dx}{dt} \right| \quad (3.22)$$

Substituting into the force formula gives [29, 28]:

$$F_{VY} = \frac{1}{2}\rho D L U^2 (C_{VL} \cos \beta + C_{VD} \sin \beta) \quad (3.23)$$

$$F_{VX} = \frac{1}{2}\rho D L U^2 (-C_{VL} \sin \beta + C_{DM} \cos \beta) + \frac{1}{2}\alpha C_{VL}^2 \rho D L U_x |U_x| \quad (3.24)$$

The cross-flow and in-line forces have now been expressed in terms of the drag and lift coefficients C_{VD} and C_{VL} , which can be determined from experiments, rather than in terms of the unknown lateral and vertical force coefficients C_{VX} and C_{VY} [28]. To satisfy the hydrodynamic forces measured on a fixed cylinder, the values of C_{VD} and α are determined by comparing the mean and fluctuating components of C_{VX} with experimental measurements [28]:

$$C_{VD} = \bar{C}_D - \frac{1}{2}\alpha C_{L0}^2 \quad (3.25)$$

$$\alpha = 2 \frac{C_{D0}}{C_{L0}^2} \quad (3.26)$$

where $\bar{C}_D$ is the mean drag coefficient and C_{D0} is the amplitude of the fluctuating drag coefficient measured on a fixed cylinder [28].

3.4.2 Extension to a Flexible Cylinder

Local Coordinate Frame

The flexible cylinder is modelled as a one-dimensional structural element embedded in three-dimensional space, parameterised by the arc-length coordinate $p \in [0, L]$ along the beam axis, where L is the total length of the undeformed beam. Following a Lagrangian description, p is a material coordinate that labels a fixed material point on the beam and tracks its position in both space and time as the structure deforms, unlike the Eulerian approach which monitors fixed points in space.

The position of the material point p in three-dimensional space at time t is given by:

$$\mathbf{r}(p, t) = [x(p, t), y(p, t), z(p, t)]^\top \quad (3.27)$$

As the beam deforms and vibrates, the spatial coordinates x, y, z evolve with time, whilst the material label p remains fixed. In the undeformed configuration of a vertical cylinder, $z(p, 0) = p$ and the beam axis coincides with the z -axis.

Implementing the wake oscillator model on flexible or curved structures requires the definition of a distinct local coordinate frame at every cross-section to account for flows that are non-uniform or oblique to the beam axis, as shown in Figure 3.4. Following the independence principle, only the flow velocity component normal to the cylinder axis drives the vortex shedding process [28]. The frame is defined at the equilibrium position and held fixed throughout the simulation, which is justified since VIV amplitudes are small relative to the overall structural length and the equilibrium flow conditions remain a valid local approximation [28].

The tangential unit vector $\mathbf{t}^{\text{eq}}$ is defined as the normalised derivative of the equilibrium position vector with respect to the arc-length coordinate p , representing the local slope of the beam axis at each cross-section [28]:

$$\mathbf{t}^{\text{eq}} = \frac{\mathbf{r}_{,p}^{\text{eq}}}{|\mathbf{r}_{,p}^{\text{eq}}|} \quad (3.28)$$

The axial component of the free-stream velocity $\mathbf{V}^{\text{eq}}$ is given by $((\mathbf{V}^{\text{eq}})^\top \mathbf{t}^{\text{eq}}) \mathbf{t}^{\text{eq}}$. Subtracting this from the total velocity yields $\mathbf{V}_N^{\text{eq}}$, the component lying in the plane perpendicular to the beam axis [28]:

$$\mathbf{V}_N^{\text{eq}} = \mathbf{V}^{\text{eq}} - ((\mathbf{V}^{\text{eq}})^\top \mathbf{t}^{\text{eq}}) \mathbf{t}^{\text{eq}} \quad (3.29)$$

Since the projection onto a plane is unique, $\mathbf{V}_N^{\text{eq}}$ is uniquely determined. However, the choice of in-plane axes is not, as there are infinitely many orthogonal pairs that span the plane perpendicular to $\mathbf{t}^{\text{eq}}$. Following Qu and Metrikine (2021) [28], the in-line unit vector $\mathbf{n}^{\text{eq}}$ is chosen in the direction of $\mathbf{V}_N^{\text{eq}}$, which is physically natural as it aligns the in-line direction with the flow:

$$\mathbf{n}^{\text{eq}} = \frac{\mathbf{V}_N^{\text{eq}}}{|\mathbf{V}_N^{\text{eq}}|} \quad (3.30)$$

Given that $\mathbf{t}^{\text{eq}}$ and $\mathbf{n}^{\text{eq}}$ are mutually orthogonal unit vectors, their cross product is uniquely defined and automatically yields a unit vector. The cross-flow unit vector $\mathbf{b}^{\text{eq}}$ is therefore uniquely determined as [28]:

$$\mathbf{b}^{\text{eq}} = \mathbf{t}^{\text{eq}} \times \mathbf{n}^{\text{eq}} \quad (3.31)$$

The three vectors form a right-handed orthogonal coordinate system at each cross-section, with $\mathbf{b}^{\text{eq}}$ corresponding to the cross-flow direction and $\mathbf{n}^{\text{eq}}$ to the in-line direction. Within the $\mathbf{b}^{\text{eq}}\text{-}\mathbf{n}^{\text{eq}}$ plane, the wake oscillator formulation reduces to that of a rigid cylinder, justifying the local application of the rigid cylinder model [29] to a flexible structure [28].

- $\mathbf{t}^{\text{eq}}$: unit tangent to the beam axis
- $\mathbf{n}^{\text{eq}}$: in-line unit vector, aligned with $\mathbf{V}_N^{\text{eq}}$
- $\mathbf{b}^{\text{eq}}$: cross-flow unit vector, normal to both

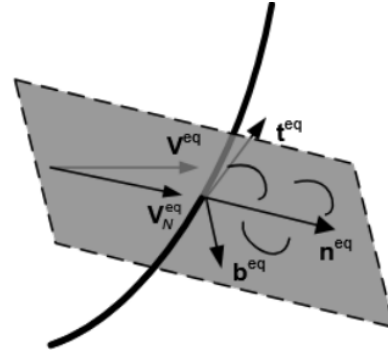


Figure 3.4. Local coordinate frame [28]

Wake Oscillator Equation

Extending the rigid-cylinder wake equation, Eq. (3.6), to a flexible cylinder using the local coordinate frame, the structural accelerations are replaced by their projections onto the local in-line and cross-flow directions [28]:

$$\ddot{q} + \varepsilon\omega_s (q^2 - 1) \dot{q} + \omega_s^2 q - \kappa \frac{\omega_s^4 D \ddot{\mathbf{r}}^{\text{T}} \mathbf{n}^{\text{eq}}}{\omega_s^4 D^2 + (\ddot{\mathbf{r}}^{\text{T}} \mathbf{n}^{\text{eq}})^2} q = \frac{A}{D} \ddot{\mathbf{r}}^{\text{T}} \mathbf{b}^{\text{eq}} \quad (3.32)$$

where $\ddot{\mathbf{r}}^{\text{T}} \mathbf{b}^{\text{eq}}$ and $\ddot{\mathbf{r}}^{\text{T}} \mathbf{n}^{\text{eq}}$ are the structural accelerations in the cross-flow and in-line directions respectively, corresponding to the local accelerations $\ddot{y}$ and $\ddot{x}$. For the flexible cylinder, the Strouhal frequency is set by the local normal flow velocity [28]:

$$\omega_s = \frac{2\pi \text{St} |\mathbf{V}_N^{\text{eq}}|}{D} \quad (3.33)$$

where $|\mathbf{V}_N^{\text{eq}}|$ is the magnitude of the normal flow velocity defined in Eq. (3.29).

Hydrodynamic Force Model

To extend the hydrodynamic force model to a flexible cylinder, the force decomposition is applied locally at each cross-section within the $\mathbf{b}^{\text{eq}}\text{-}\mathbf{n}^{\text{eq}}$ plane, as shown in Figure 3.5.

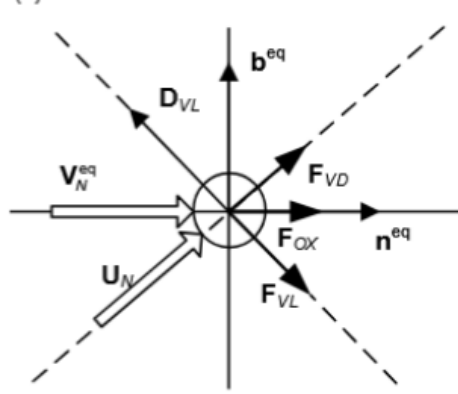


Figure 3.5. Force decomposition in the $\mathbf{b}^{\text{eq}}\text{-}\mathbf{n}^{\text{eq}}$ plane [28]

The relative flow velocity vector within the $\mathbf{b}^{\text{eq}}\text{-}\mathbf{n}^{\text{eq}}$ plane is [28]:

$$\mathbf{U}_N = \mathbf{V}_N^{\text{eq}} - (\dot{\mathbf{r}} - (\dot{\mathbf{r}}^T \mathbf{t}^{\text{eq}}) \mathbf{t}^{\text{eq}}) \quad (3.34)$$

with in-line and cross-flow components:

$$U_{Nx} = \mathbf{U}_N^T \mathbf{n}^{\text{eq}} = |\mathbf{V}_N^{\text{eq}}| - \dot{\mathbf{r}}^T \mathbf{n}^{\text{eq}}, \quad U_{Ny} = \mathbf{U}_N^T \mathbf{b}^{\text{eq}} = -\dot{\mathbf{r}}^T \mathbf{b}^{\text{eq}} \quad (3.35)$$

$$|\mathbf{U}_N| = \sqrt{U_{Nx}^2 + U_{Ny}^2} \quad (3.36)$$

The unit vector $\mathbf{D}_{VL}$ perpendicular to $\mathbf{U}_N$ in the $\mathbf{b}^{\text{eq}}\text{-}\mathbf{n}^{\text{eq}}$ plane, representing the lift direction, is defined as [28]:

$$\mathbf{D}_{VL} = \frac{\mathbf{t}^{\text{eq}} \times \mathbf{U}_N}{|\mathbf{t}^{\text{eq}} \times \mathbf{U}_N|} \quad (3.37)$$

The instantaneous lift and drag forces per unit length are [28]:

$$\mathbf{F}_{VL} = \frac{1}{2} \rho D \frac{C_{L0}}{2} q |\mathbf{U}_N|^2 \mathbf{D}_{VL} \quad (3.38)$$

$$\mathbf{F}_{VD} = \frac{1}{2} \rho D C_{VD} |\mathbf{U}_N| \mathbf{U}_N \quad (3.39)$$

The fluctuating in-line force vector is [28]:

$$\mathbf{F}_{OX} = \frac{1}{2} \rho D \alpha \frac{C_{L0}^2}{4} q^2 |\mathbf{U}_{NX}| \mathbf{U}_{NX} \quad (3.40)$$

The total hydrodynamic force vector per unit length acting on the flexible cylinder is then [28]:

$$\mathbf{F} = \mathbf{F}_{VL} + \mathbf{F}_{VD} + \mathbf{F}_{OX} \quad (3.41)$$

3.4.3 Coupled System

The complete coupled system is formed by combining the structural equations of motion, Eqs. (3.1) and (3.2), with the wake oscillator equation, Eq. (3.32). The three equations read:

$$M \frac{\partial^2 y}{\partial t^2} + R(z) \frac{\partial y}{\partial t} + EI \frac{\partial^4 y}{\partial z^4} - \frac{\partial}{\partial z} \left(T_e \frac{\partial y}{\partial z} \right) = F_y(z, t, q) \quad (3.42)$$

$$M \frac{\partial^2 x}{\partial t^2} + R(z) \frac{\partial x}{\partial t} + EI \frac{\partial^4 x}{\partial z^4} - \frac{\partial}{\partial z} \left(T_e \frac{\partial x}{\partial z} \right) = F_x(z, t, q) \quad (3.43)$$

$$\ddot{q} + \varepsilon \omega_s (q^2 - 1) \dot{q} + \omega_s^2 q - \kappa \frac{\omega_s^4 D \ddot{x}}{\omega_s^4 D^2 + \ddot{x}^2} q = \frac{A}{D} \ddot{y} \quad (3.44)$$

The structural equations are driven by the hydrodynamic forces F_y and F_x , which depend on the wake variable q through the force model of Section 3.4.1. The wake oscillator equation is in turn driven by the structural accelerations $\ddot{x}$ and $\ddot{y}$, establishing a bidirectional coupling between the structural and wake dynamics. The system must therefore be solved simultaneously.

3.4.4 Conclusion

This chapter presented the coupled cross-flow and in-line wake oscillator model of Qu and Metrikine (2021) [28]. A single van der Pol oscillator carries both the cross-flow and the in-line forcing, on the assumption that lift and fluctuating drag force share the same wake origin. For a rigid cylinder, the wake variable drives a hydrodynamic force that is decomposed into lift and drag in the relative flow frame, with a separate fluctuating term for the in-line force.

For a flexible cylinder, the rigid formulation is applied locally at each cross-section. A local coordinate frame is built at the equilibrium position, the free-stream velocity is projected onto the plane normal to the beam axis, and the wake oscillator and force model act within that plane. The flexible model is therefore a spanwise distribution of the local rigid form, which is consistent with the independence principle and with the small VIV amplitudes relative to the configuration length. The assembled system couples the two structural equations to the wake oscillator, and its numerical solution is described in Chapter 4.

Part II

Implementation & Results

4 Numerical Implementation

This chapter describes the numerical implementation of the coupled cross-flow and in-line wake oscillator model presented in Chapter 3. The model is developed in two ways, one fully implemented in Python and one coupled to the commercial dynamic analysis software OrcaFlex. Each implementation is verified against experimental data from a tensioned vertical riser before the model is applied to the lazy wave power cable cases presented in Chapter 5.

4.1 Overview

The implementation follows two distinct workflows that share the same wake oscillator and differ in how the structural response is computed. The first solves the full coupled model of Qu and Metrikine (2021) [28] in Python, where the structural response and the wake oscillator are solved together within a single solver, both discretised by a nonlinear finite element method. The second is a Python external function that solves only the wake oscillator, coupled to OrcaFlex through its Python interface, with OrcaFlex handling the structural dynamics.

Both workflows are verified first on a rigid cylinder, with a reduced form of the model, and then on the flexible vertical riser of the Delta Flume experiment [30]. The riser response is compared with the experimental data and with previous numerical studies, at a higher and a lower top tension. This establishes that each implementation reproduces the known VIV behaviour of the riser before the verified OrcaFlex coupling is applied in Chapter 5 to the power cable cases.

4.2 Full Coupled Implementation in Python

The main idea is to solve the coupled dynamics of the structure and the wake with the finite element method. Before discretisation, the structure and the wake are governed by partial differential equations in space and time, the wake by the oscillator equation of Chapter 3, Eq. (3.32). The structure here is represented with the Absolute Nodal Coordinate Formulation (ANCF), a representation of flexible components in three dimensions in which the degrees of freedom are the nodal position vectors and slope vectors, allowing larger deformation to be described than the small deformation of a linear formulation.

For this configuration, only axial and bending deformation is considered, and cross-sections are assumed to remain perpendicular to the beam axis, so an Euler-Bernoulli beam is used [28]. The beam element can therefore be described as a lower-order ANCF cable element [66], with two nodes and 6 degrees of freedom per node, giving 12 per element. Each node i carries a position vector $\mathbf{r}_i$ and its gradient $\mathbf{r}_{i,p}$ along the beam axis p , so its coordinates are $\mathbf{u}_i(t) = [\mathbf{r}_i^\top \ \mathbf{r}_{i,p}^\top]^\top$.

The wake oscillators are discretised consistently on the same mesh, with 2 degrees of freedom per node and 4 per element, so the wake coordinates of node i are $\mathbf{q}_i(t) = [q_i \ q_{i,p}]^\top$, the wake variable q_i and its gradient $q_{i,p}$ along the beam axis.

Figure 4.1 shows the discretised cable, with the nodes P_1 to P_n , the position vectors from the global origin, the slope vectors along the centreline, and the cable elements between consecutive nodes. The displacement of an arbitrary point within an element, originally at p on the undeformed beam axis, is then $\mathbf{r}(p, t)$, a function of p and t in the three directions.

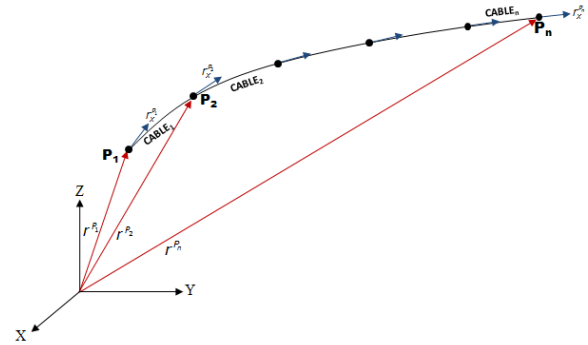


Figure 4.1. ANCF discretisation of the cable [69]

The implementation followed the classical finite element procedure. After discretisation of the domain, the structural and wake responses within an element are represented as the product of the nodal coordinates and the same Hermite shape functions [28],

$$\mathbf{r}(p, t) = \mathbf{S}(p) \mathbf{u}_e(t), \quad q(p, t) = \mathbf{S}(p) \mathbf{q}_e(t), \quad (4.1)$$

for the structure and the wake respectively.

The elemental mass, damping and stiffness matrices of both equations of motion, together with the hydrodynamic force and the external nodal force of the wake oscillator, are evaluated with 3-point Gauss quadrature as the numerical integration method. The global matrices are then assembled and the boundary conditions applied, giving the coupled system as ordinary differential equations in time [28],

$$\mathbf{M}_s \ddot{\mathbf{u}} + \mathbf{C}_s \dot{\mathbf{u}} + \mathbf{K}_s(\mathbf{u}) \mathbf{u} = \mathbf{F}^e(\dot{\mathbf{u}}, \mathbf{q}), \quad (4.2)$$

$$\mathbf{M}_q \ddot{\mathbf{q}} + \mathbf{C}_q(\mathbf{q}) \dot{\mathbf{q}} + \mathbf{K}_q(\ddot{\mathbf{u}}) \mathbf{q} = \mathbf{R}^e(\ddot{\mathbf{u}}), \quad (4.3)$$

where $\mathbf{u}$ and $\mathbf{q}$ are the structural and wake degrees of freedom.

The wake matrices, the hydrodynamic force and the wake forcing require a local coordinate frame, defined once from the static equilibrium of the structure. The equilibrium is solved under gravity, buoyancy and the top tension by Newton-Raphson. From this configuration and the prescribed flow velocity vector, the frame $\mathbf{t}^{eq}$, $\mathbf{n}^{eq}$, $\mathbf{b}^{eq}$ and the Strouhal frequency ω_s are computed at each Gauss point and kept fixed for the whole simulation.

The coupled system is advanced with the implicit Newmark method, and as both equations are nonlinear, since the structural stiffness depends on the configuration and the wake matrices depend on the structural acceleration, each Newmark step is solved by Newton iteration. The full procedure is given in Appendix A and shown in Figure 4.2.

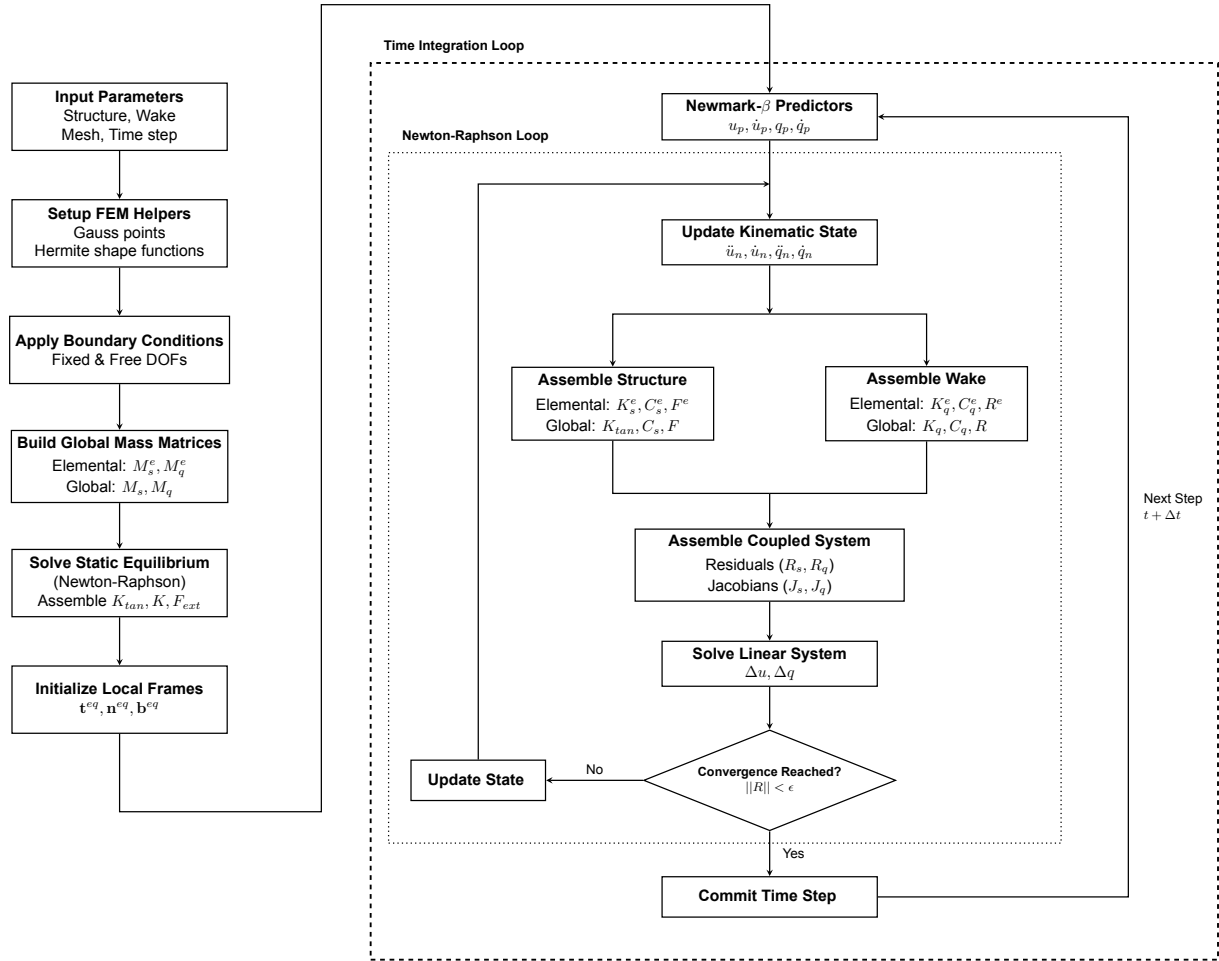


Figure 4.2. Solution procedure of the fully coupled ANCF wake oscillator model

4.3 OrcaFlex - Python Coupling Implementation

4.3.1 Overview

OrcaFlex (www.orcina.com) is a commercial finite element analysis tool widely used in the off-shore engineering sector. Regarding VIV, it offers semi-empirical options in both the frequency and time domain [70]. The frequency domain capabilities use the force decomposition method, with interfaces to external tools such as SHEAR7 and VIVA. For time domain analysis, four options are available, two wake oscillator models, the Milan Wake Oscillator (MWO) and the Iwan & Blevins Wake Oscillator (IBWO), detailed in Appendix B, and two vortex tracking models, VT1 and VT2 [70].

However, they are not considered a mature technology, as these are single degree-of-freedom formulations that solve only the transverse direction and are closely tied to the dataset used for their calibration [70]. The in-line direction enters only through the velocity input to the wake oscillator and the decomposition of the vortex force, without contributing to the structural response, as no in-line equation of motion is solved.

Nevertheless, OrcaFlex provides the OrcFxAPI [71], a Python interface that enables external functions. At each time step OrcaFlex passes the model and the current simulation state to an external Python script, which returns computed values for use in the simulation, through a shared object updated in both directions [71]. This allows the coupled flexible-cylinder force model to be implemented as an external function that handles the wake part of the VIV. The external function distributes the wake oscillator over the line segments and returns the resulting forces to the line nodes, so the discretisation is set out first.

In OrcaFlex, the line object is divided into a series of line segments modelled as straight massless elements with a node at each end. Each node represents a short straight rod made up of the two half-segments either side of it, as shown in Figure 4.3. The line properties are set by a line type, which the nodes and segments inherit. Following the discretisation adopted, the line is divided into n segments of equal length l_e , with $n + 1$ nodes.

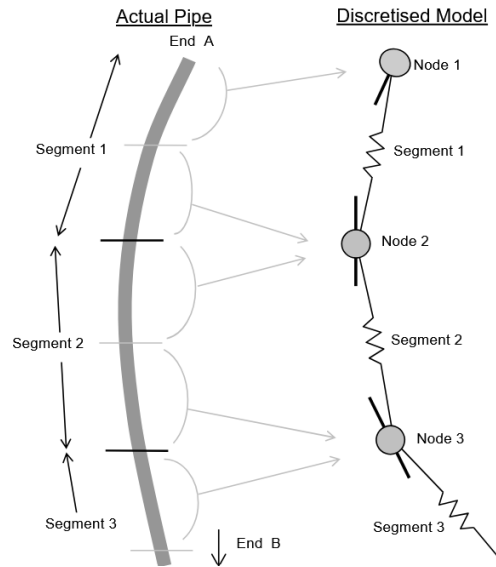


Figure 4.3. OrcaFlex line discretisation [70]

4.3.2 External Function Implementation

The external function is written as a class, so the wake variable q can be carried from one step to the next rather than recomputed each time. The class stores q and its derivative $\dot{q}$, updating them at each step so the next step continues from the current wake state. It is built around two main functions, “Initialise” and “Calculate”. Initialise runs once at the start. It reads the structural and mesh parameters, sets the wake oscillator parameters, and allocates q and $\dot{q}$ with their initial values. Calculate runs at every time step and returns the VIV force to OrcaFlex.

The steps in Calculate are organised into a set of helper routines, presented in the following subsections in the order they run at each time step. These build the local coordinate frame on the first step, read the cable motion and advance the wake oscillator, and assemble the VIV force returned to OrcaFlex, as shown in Figure 4.4.

1. Local coordinate frame

On the first dynamic step, the node positions are read immediately after the static analysis has completed, so they correspond to the drag-deflected post-statics configuration. The node positions $\mathbf{r}(p_i) = [x_i, y_i, z_i]^T$ at arc length $p_i = i l_e$ are read, and from these a local frame is built at each segment midpoint. The midpoint arc-length position is therefore:

$$p_{mid,i} = \left(i + \frac{1}{2}\right) l_e, \quad i = 0, 1, \dots, n-1 \quad (4.4)$$

and the tangent vector at each midpoint is the normalised difference of its two adjacent node positions,

$$\mathbf{t}_{mid,i}^{eq} = \frac{\mathbf{r}(p_{i+1}) - \mathbf{r}(p_i)}{|\mathbf{r}(p_{i+1}) - \mathbf{r}(p_i)|}. \quad (4.5)$$

Using the normal component of the flow velocity vector, the in-line and cross-flow unit vectors $\mathbf{n}_{mid,i}^{eq}$ and $\mathbf{b}_{mid,i}^{eq}$ then follow from Eqs. (3.29)-(3.31), and the shedding frequency ω_s from Eq. (3.33). These quantities are computed once and held fixed for the whole simulation, as justified in Section 3.4.2. Outside the current zone there is no shedding, so $\omega_s = 0$ and the wake oscillator produces no force there. The current zone is identified per segment from its position, as the part of the structure exposed to the current.

2. Cable motion and wake update

At each time step the nodal accelerations are read from OrcaFlex and averaged to the segment midpoints,

$$\ddot{\mathbf{r}}_{mid,i} = \frac{1}{2} (\ddot{\mathbf{r}}_i + \ddot{\mathbf{r}}_{i+1}), \quad i = 0, 1, \dots, n-1 \quad (4.6)$$

The midpoint acceleration is projected onto the local frame to obtain its cross-flow and in-line components,

$$\ddot{r}_b = \ddot{\mathbf{r}}_{mid,i}^\top \mathbf{b}_i^{eq}, \quad \ddot{r}_n = \ddot{\mathbf{r}}_{mid,i}^\top \mathbf{n}_i^{eq} \quad (4.7)$$

These components enter the wake oscillator equation as the structural forcing, in place of $\ddot{\mathbf{r}}^\top \mathbf{b}^{eq}$ and $\ddot{\mathbf{r}}^\top \mathbf{n}^{eq}$ in Eq. (3.32). The equation is advanced over the time step Δt using the explicit Runge-Kutta method of order 5(4) (RK45), as described in Appendix C, updating the wake variable from q_n to q_{n+1} . The wake is initialised with $q_0 = 0.1$ and $\dot{q}_0 = 0$ at the start of the simulation.

3. Hydrodynamic force

With the wake variable updated, the hydrodynamic VIV force at each midpoint follows from the wake state and the local relative flow. OrcaFlex computes the mean drag internally through its Morison formulation with $C_D = 1.2$, so the mean drag term $\mathbf{F}_{VD}$ is left out of the external function to avoid counting it twice. The external function therefore returns only the oscillating lift and the oscillating in-line force.

The force depends on the wake variable and the local relative flow. The relative-velocity quantities $\mathbf{U}_N$ and $\mathbf{U}_{NX}$ are formed following Eqs. (3.34)-(3.35), as the flow velocity minus the cable velocity. Unlike the fixed frame velocity $\mathbf{V}_{Neq}$, they are recomputed each step, so the cable motion enters the response. The oscillating lift $\mathbf{F}_{VL}$ then follows from Eq. (3.38) and the oscillating

in-line force $\mathbf{F}_{OX}$ from Eq. (3.40), each expressed per unit length and scaled by the segment length l_e to give the total force on that segment and returned in kN:

$$\mathbf{F}_{total,i} = (\mathbf{F}_{VL,i} + \mathbf{F}_{OX,i}) l_e \quad (4.8)$$

The force is returned to OrcaFlex in global axes. The wake oscillator is evaluated at the segment midpoints, while OrcaFlex applies external loads at its nodes, so each midpoint force is returned as the applied load at the node of that segment, where OrcaFlex applies it to the line. Only the midpoints in the current zone contribute. The number of components returned follows the response. For a straight axis the response stays in the plane normal to it and two force components are returned, while for a curved axis the normal plane varies along the span and all three are returned.

OrcaFlex solves each time step iteratively, so as noted above Calculate may be called several times within a single step. The wake oscillator is advanced only on the first of these calls, using the structural acceleration from the previous converged step, since the interface exposes only the converged nodal accelerations and not the trial values formed during the iteration. On the later calls the wake variable is held and the cached force returned. The force applied at step $n + 1$ therefore rests on the structural state at step n , which leaves a one time step lag between the structural and wake solutions.

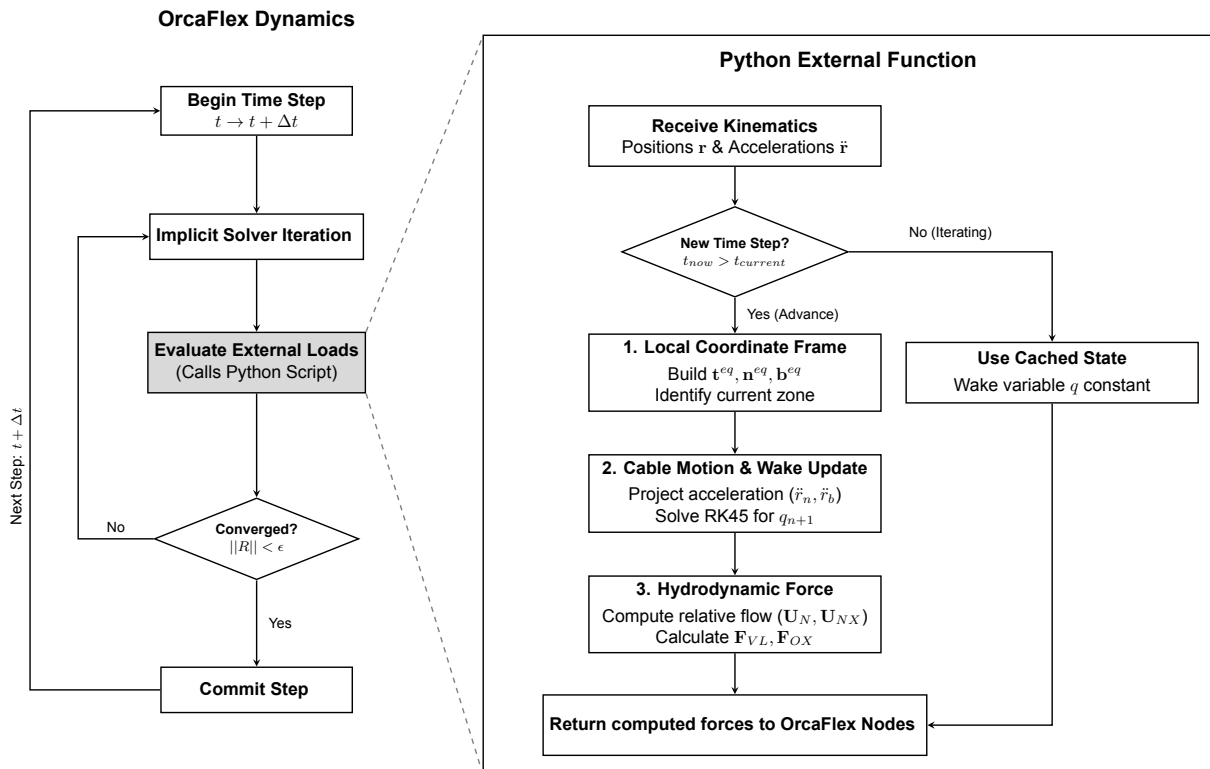


Figure 4.4. Evaluation of external VIV loads within the OrcaFlex integration scheme

4.4 Model Validation

4.4.1 Overview

Both implementation approaches are verified against experiment to confirm their performance before the model is applied to the lazy wave power cable. The reference dataset is the VIV measurements of a vertical tension riser in a stepped current, conducted by Huera-Huarte (2006) [30] at the Delft Hydraulics Water Laboratory in the Delta Flume. Qu and Metrikine (2021) [28] also used these measurements to validate their model, which gives a second basis for comparison, as the present results can be assessed against both the experimental data and the published numerical predictions.

The experiments used a flexible riser model of total length $L = 13.12$ m and outer diameter $D = 0.028$ m, giving an aspect ratio of $L/D = 470$. The model was mounted on a towing carriage that moved along the flume, exposing only the lower 45% of the riser to a uniform relative current with speeds up to 1 m/s, while the upper 55% remained enclosed in a sealed tank of still water fixed to the carriage, simulating a stepped current profile. Universal joints at both ends allowed free bending while preventing torsional motion. The top tension was applied through a spring system, with initial static tensions ranging from 350 N to 2000 N. As the riser deflected under hydrodynamic drag, the spring allowed the top node to move axially, causing the tension to increase with deflection. The experimental layout is shown in Figure 4.5 [30, 72].

The verification is carried out at two top tension levels. The higher tension of 810 N is the tension Qu and Metrikine (2021) [28] used for their original validation, and the one for which Huera-Huarte (2006) [30] reports explicit displacement envelopes. The four flow velocities, $V = 0.49, 0.63, 0.75$ and 0.85 m/s, are compared against both the experiment and the published numerical results. This set confirms that the two implementations reproduce the known VIV response of the riser, including the in-line deflection.

The lower tension of 389 N is not covered by the validation of Qu and Metrikine (2021) [28], so it also extends their comparison. The four cases correspond to Cases 1, 3, 6 and 9 of Chaplin et al. (2005) [72], at $V = 0.16, 0.31, 0.60$ and 0.95 m/s. They report the mean top tension during each run and not the initial still-water value. The initial tension was therefore identified as $T_{t0} = 389$ N by matching these tests to the corresponding test series reported by Huera-Huarte (2006) [30].

The same cases were used by Ntroupis (2017) [32] and Slingsby (2015) [33], allowing direct comparison with their independent implementations. Both employed the cross-flow only wake oscillator of Ogink and Metrikine (2010) [73], and neither modelled the spring compliance of the tensioning system. Ntroupis (2017) applied the top tension through an OrcaFlex winch, while Slingsby (2015) prescribed it directly with clamped ends. The present study instead adopts the coupled formulation of Qu and Metrikine (2021) [28] and models the spring explicitly, so the top tension evolves from its still-water value as the riser deflects. This set therefore shows the in-line response the cross-flow only studies could not, together with the cross-flow under-prediction discussed below.

For each tension, two representative velocities are presented in the main text as full cross-flow and in-line envelopes. The complete sets are given in Appendix D. All simulations used $n_e = 50$ elements and a time step of $\Delta t = 0.005$ s, run for a total duration of 60 s consistent with the experimental test duration. The steady-state analysis was performed over a window of at least 50 shedding cycles, determined from the Strouhal relationship, consistent with the approach of Qu and Metrikine (2021) [28]. The structural properties and wake oscillator parameters are summarised in Tables 4.1 and 4.2.

Table 4.1. Structural properties of the Delta Flume riser model [30]

Parameter	Sym.	Value	Unit	Parameter	Sym.	Value	Unit
Total length	L	13.12	m	Spring stiffness	K_s	38100	N/m
Outer diameter	D	0.028	m	Bending stiffness	EI	29.88	N m ²
Aspect ratio	L/D	470	–	Axial stiffness	EA	5.88	MN
Exposed length	L_{sub}	5.94	m	Mass per unit length	m_0	1.845	kg/m
Mass ratio	m^*	3.0	–	Added mass coeff.	C_a	1.0	–
Fluid density	ρ_f	1000	kg/m ³	Rayleigh mass	α	0.0555	–
Initial top tension	T_{t0}	810/389	N	Rayleigh stiffness	β	0.0002	s

Table 4.2. Hydrodynamic and wake oscillator parameters

Parameter	Sym.	Value	Unit	Parameter	Sym.	Value	Unit
Strouhal number	St	0.17	–	Cross-flow coupling	A	8.0	–
Lift coeff. amp.	C_{L0}	0.3	–	Van der Pol damping	ε	0.08	–
Fluct. drag amp.	C_{D0}	0.1	–	In-line coupling	κ	5.0	–
Mean drag coeff.	C_{Dm}	1.2	–	Initial wake var.	q_0	0.1	–

4.4.2 Python Model Description

The vertical tension riser was modelled in the full Python implementation using the ANCF discretisation described in Section 4.2, with the structural properties listed in Table 4.1. The bottom end is pinned in all three translational directions, replicating the universal joint of the experiment, which prevents translation while allowing free rotation. The top end is pinned in the two lateral directions only, with the axial degree of freedom left free to allow vertical movement.

The accurate modelling of the spring boundary condition is critical. As noted by Huera-Huarte (2006) [30], small vertical displacements of the top node can result in significant changes in the mean in-line deflection, due to the nonlinear variation of axial stiffness with tension. As the riser deflects under mean drag, the top node descends and the spring tension increases above its still-water value. This effect was confirmed during the present study, as prescribing a constant initial top tension instead of the spring system resulted in mean in-line deflections significantly larger than the experimental values, approaching double, for otherwise identical structural and wake parameters.

The spring force is therefore incorporated on the right-hand side of the discretised equation of

motion as a concentrated nodal force acting on the axial degree of freedom of the top node,

$$F_{\text{spring}}(z_{\text{top}}) = T_{t0} + K_s(z_{\text{ref}} - z_{\text{top}}) \quad (4.9)$$

where T_{t0} is the prescribed initial top tension, K_s is the spring stiffness, and z_{ref} is a reference position. The reference position is calibrated at initialisation through a short iterative procedure, so that the spring tension at static equilibrium in still water equals T_{t0} . This follows the experimental procedure, in which the reference tension was set before each run [30], after which the carriage accelerated to the target speed and the top tension rose above it under drag. During the simulation z_{top} is a free degree of freedom, and the spring tension evolves within the Newton-Raphson solver at each time step as the riser deflects. The assembly of the spring force and its contribution to the tangent stiffness matrix are given in Appendix A. Following Qu and Metrikine (2021) [28], the initial vertical configuration was adopted as the equilibrium reference, since the lateral displacements are small relative to the riser length, which avoids the iterative procedure needed to determine the drag-deflected static shape within the ANCF framework.

The remaining forcing terms in the residual are gravity and buoyancy, the VIV forces, and a Morison drag on the upper section. Gravity and buoyancy are assembled as lumped nodal loads over the full length, since the entire riser is submerged. The VIV forces from the wake oscillator are the oscillating lift $\mathbf{F}_{VL}$ (Eq. 3.38), the mean drag $\mathbf{F}_{VD}$ (Eq. 3.39) and the fluctuating in-line force $\mathbf{F}_{OX}$ (Eq. 3.40). These act only on the lower 45% of the riser exposed to the current, the 5.94 m nearest the bottom end, imposed by setting $\omega_s = 0$ at all Gauss points above this zone. On the upper 55%, a relative-velocity Morison drag acts in the plane normal to the riser axis, providing damping from the surrounding still water.

4.4.3 OrcaFlex Model Description

The vertical tension riser was modelled in OrcaFlex as a line object with the structural properties listed in Table 4.1. The bottom end (End B) was anchored with zero bending and torsional connection stiffnesses, allowing free rotation while constraining all translational degrees of freedom. The top end (End A) was connected to a constraint with all translational and rotational degrees of freedom fixed, except the axial direction, which was left free to allow vertical movement.

The spring system of the experiment was replicated using an OrcaFlex link element, connected at one end to End A and at the other end to a fixed point above, defining the reference position. OrcaFlex models a linear spring as

$$T = k(l - l_0) \quad (4.10)$$

where k is the linear stiffness and l_0 is the unstretched length. For each validation case, the unstretched length was determined through an iterative process to produce the corresponding initial top tension after static analysis in still water.

OrcaFlex can resolve the static equilibrium under the current as part of its standard solution

procedure, so the drag-deflected configuration was adopted as the initial condition. This is in contrast to the vertical reference used in the Python implementation. Comparing the two implementations therefore tests the influence of the equilibrium reference state on the predicted VIV response, which is expected to be small given the lateral displacements relative to the riser length.

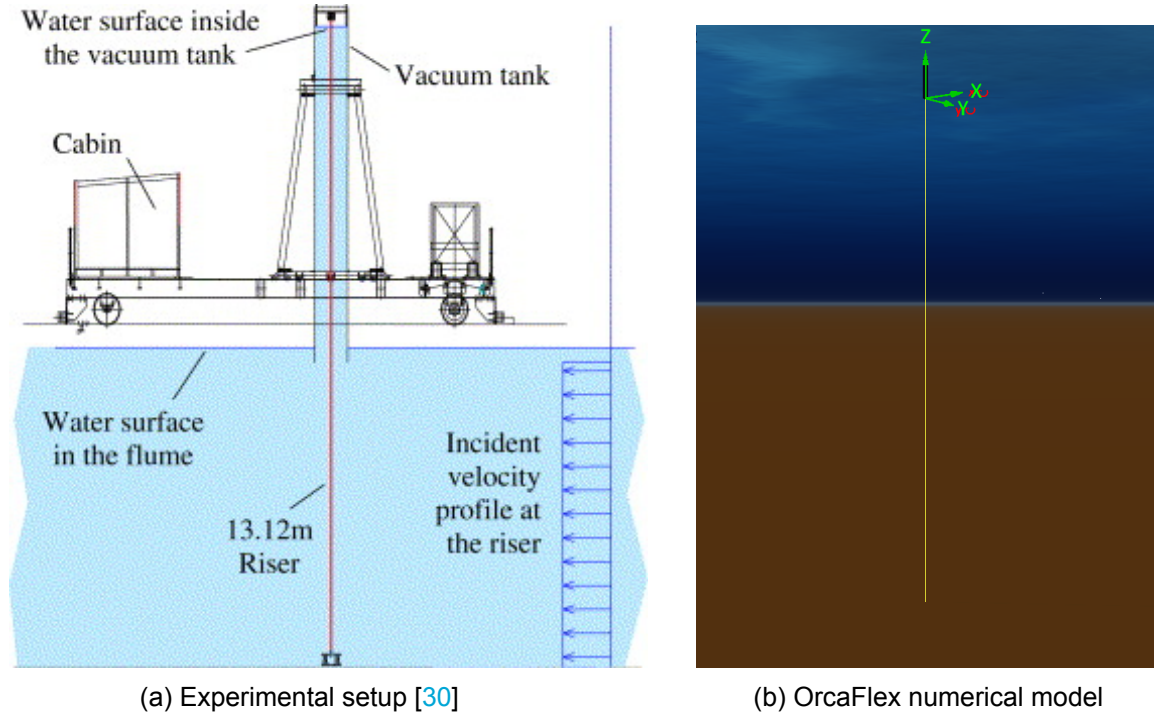


Figure 4.5. Delta Flume experimental setup and corresponding OrcaFlex model

4.4.4 Natural Frequencies

A modal analysis was carried out to verify that the numerical models reproduce the dynamic properties of the experimental riser before the VIV response comparison. The first seven natural frequencies in still water were compared against the experimental measurements at a top tension of 813 N [72], the tension at which frequency data are reported and the reference used by Nttroumpis (2017) [32] and Slingsby (2015) [33].

The results are presented in Table 4.3, together with the percentage differences with respect to the experimental values. Both the OrcaFlex and ANCF models show good agreement with the experimental frequencies, with a maximum deviation of 3.95% at Mode 1 and average deviations of 1.78% and 2.11% for the OrcaFlex and ANCF models respectively. The two implementations are mutually consistent, with differences not exceeding 0.1 Hz across all modes. The present OrcaFlex and Nttroumpis (2017) results are in close agreement, with differences not exceeding 0.5%. Slingsby (2015) obtains better overall agreement with the experimental values for the higher modes, which is attributable to the different modelling choices adopted in that study, notably the use of clamped boundary conditions and an added mass coefficient of $C_a = 1.2$ to tune the natural frequencies. Applying the same value in the present OrcaFlex model yields frequencies of [0.642, 1.288, 1.941, 2.606, 3.286, 3.982, 4.699] Hz, with an average

Table 4.3. First seven natural frequencies (Hz) in still water at $T_{t0} = 813$ N

Mode	Experimental [72]	OrcaFlex (present)	ANCF (present)	Ntroumpis [32]	Slingsby [33]
1	0.633	0.658 (+3.9%)	0.658 (+3.9%)	0.655 (+3.5%)	0.646 (+2.1%)
2	1.329	1.320 (-0.7%)	1.320 (-0.7%)	1.314 (-1.1%)	1.296 (-2.5%)
3	1.954	1.989 (+1.8%)	1.992 (+1.9%)	1.982 (+1.4%)	1.955 (+0.1%)
4	2.625	2.671 (+1.8%)	2.678 (+2.0%)	2.662 (+1.4%)	2.626 (+0.0%)
5	3.312	3.367 (+1.7%)	3.381 (+2.1%)	3.359 (+1.4%)	3.313 (+0.0%)
6	4.028	4.081 (+1.3%)	4.107 (+2.0%)	4.075 (+1.2%)	4.020 (-0.2%)
7	4.753	4.816 (+1.3%)	4.856 (+2.2%)	4.814 (+1.3%)	4.750 (-0.1%)

deviation of 1.28% from the experimental values, representing an improvement for all modes except Mode 2 and bringing the results into closer agreement with those of Slingsby (2015).

A value of $C_a = 1.0$ was however retained, consistent with the formulation of Qu and Metrikine (2021) [28], to keep the structural model consistent with the wake oscillator rather than tuning it to match the natural frequencies. The good agreement confirms that both structural models reproduce the dynamic properties of the riser, and the VIV response comparison can proceed.

4.4.5 VIV Response

The VIV response is presented as cross-flow and in-line displacement envelopes along the riser, for both the OrcaFlex and ANCF implementations. Each envelope is formed from instantaneous line profiles taken every 0.25 s within the steady-state window. The outer edge of the envelope gives the maximum excursion reached at each point along the span.

Higher Tension (810 N)

As mentioned above, at 810 N the response is examined against the experiment and the published numerical results of Qu and Metrikine (2021) [28], at the tension they used for their own validation. Two velocity cases are shown here, $V = 0.49$ and 0.85 m/s (Figures 4.6 and 4.7), and the full set is given in Appendix D.1, together with the per-implementation tensions and reduced velocities, and the full comparison against both references.

Both implementations show good overall agreement with each other and with the reference results. They predict the correct dominant cross-flow and in-line mode numbers, in agreement with [28] and the experiment [30]. The dominant frequency ratios f_y/f_1 and f_x/f_1 , formed with the fundamental natural frequency f_1 in still water at the mean top tension, match both references. The mean of the maximum in-line deflection is captured at both velocities shown. It sits slightly above the experimental quadratic fit [30], consistent with the over-prediction reported by Qu and Metrikine (2021) [28]. The in-line response is reproduced at this tension, which the cross-flow only studies could not provide accurately.

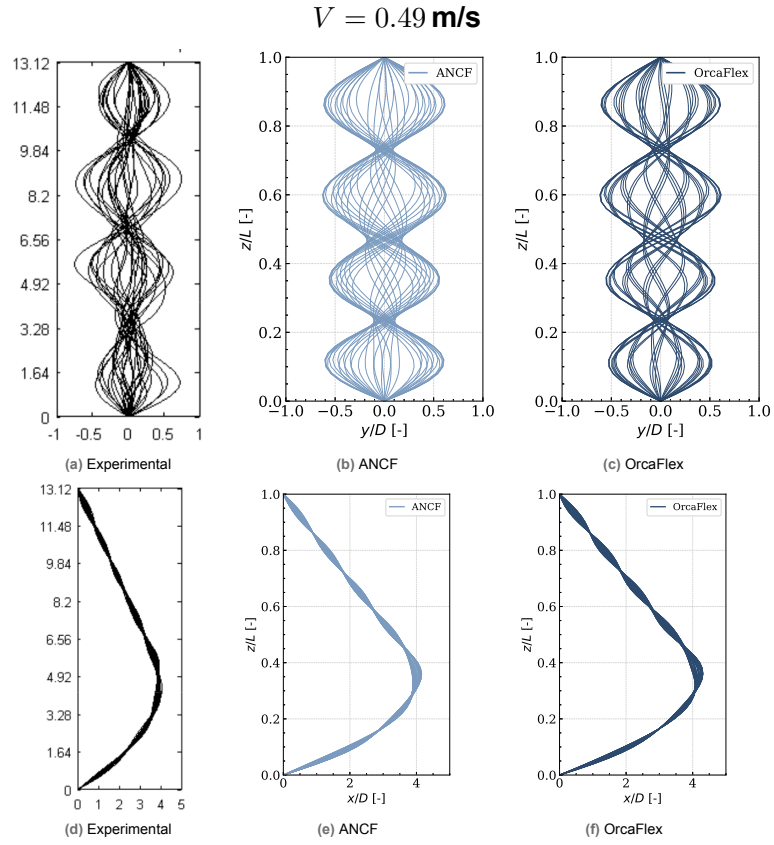


Figure 4.6. Cross-flow (top) and in-line (bottom) displacement envelopes at $V = 0.49 \text{ m/s}$

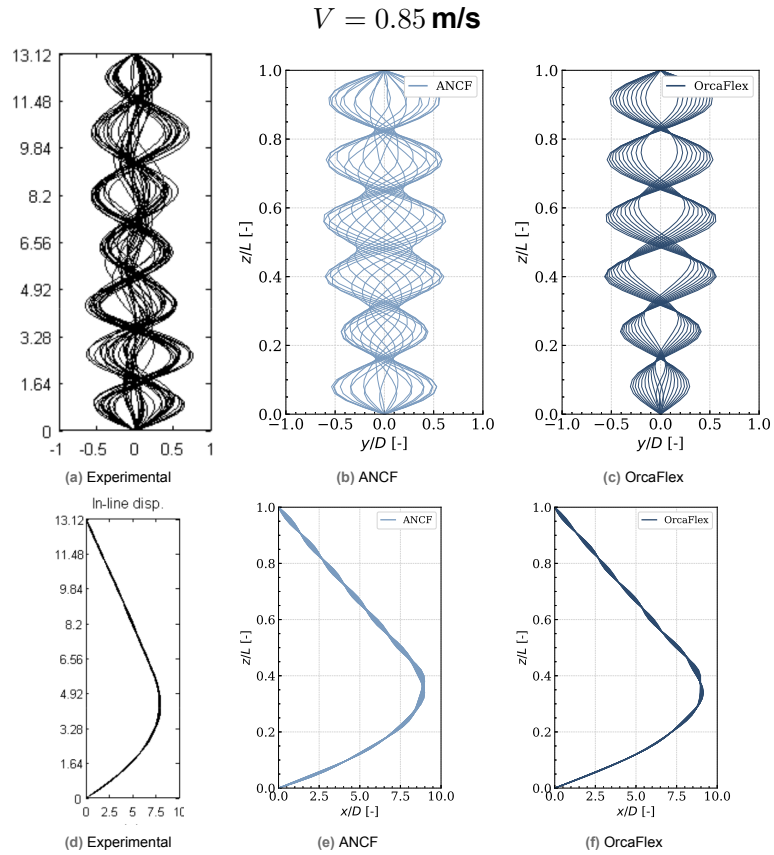


Figure 4.7. Cross-flow (top) and in-line (bottom) displacement envelopes at $V = 0.85 \text{ m/s}$

The envelopes at both velocities follow the experimental shape, mode 4 in cross-flow and mode 7 in-line at 0.49 m/s, and mode 6 and mode 11 at 0.85 m/s. As noted above, the two implementations agree closely at these steady velocities, which confirms that the equilibrium reference state, vertical in the Python ANCF model and drag-deflected in OrcaFlex, has negligible influence on the response outside the transitional regions.

The two implementations compute slightly different mean top tensions, since they use different equilibrium references and different solvers. The difference stays small at $V = 0.49$ and 0.85 m/s, with mean tensions of 920 N for ANCF and 925 N for OrcaFlex at 0.49 m/s, and 1295 N and 1301 N at 0.85 m/s respectively, both within about 0.5%.

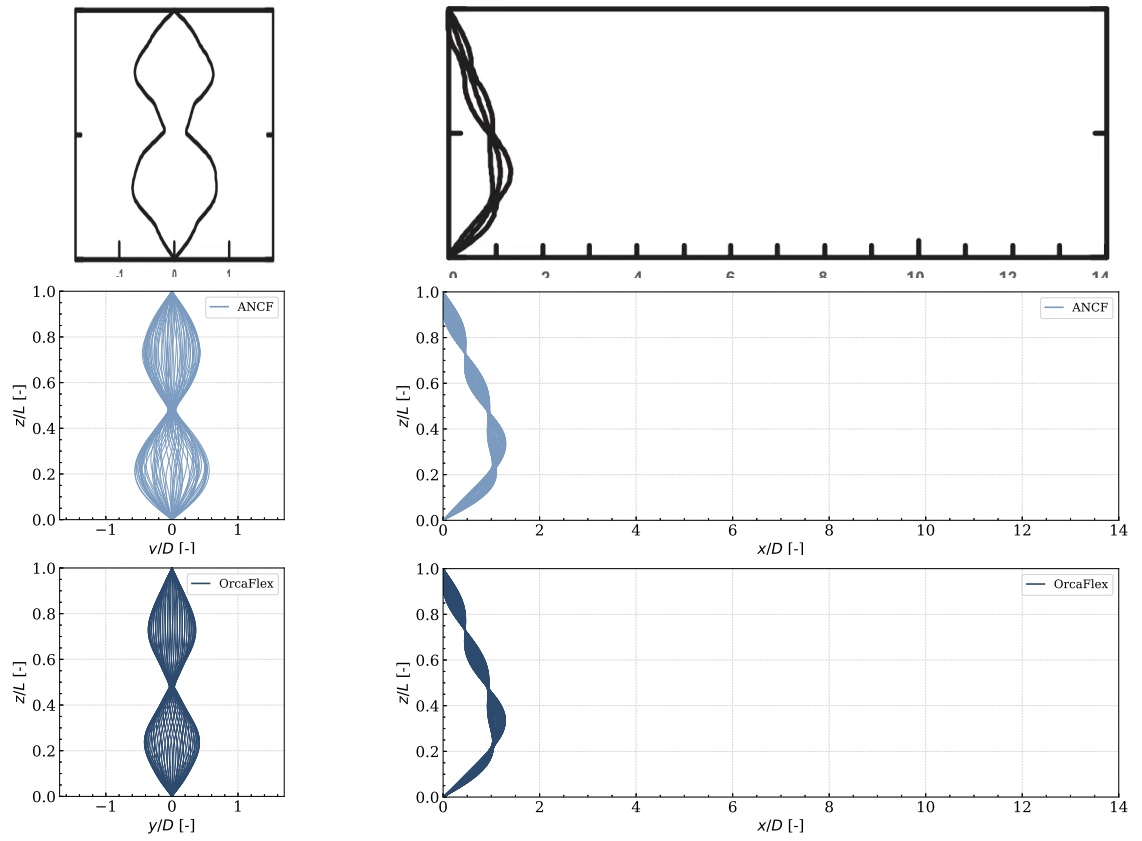
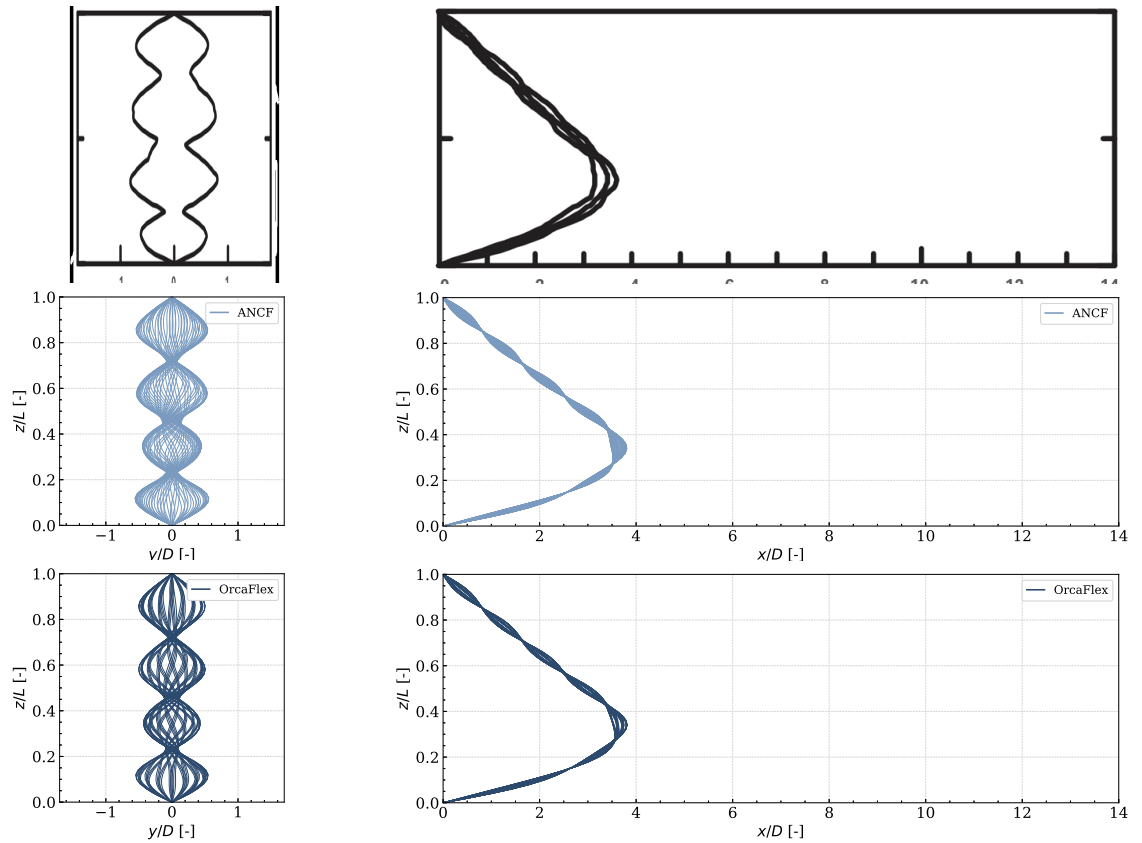
The mean tension sets the effective stiffness of the riser, and through it the natural frequencies. The response locks in at the velocity where a natural frequency is close to the shedding frequency, so a small change in mean tension moves the lock-in. At $V = 0.49$ and 0.85 m/s the response sits inside a single lock-in branch, and the small tension difference between the implementations has no effect on it. At $V = 0.63$ and 0.75 m/s the response is transitional and sits between two adjacent branches, where the experiment showed competing modes with continually changing amplitudes. Here the same small tension difference is enough to move the lock-in from one branch to the other, so the two implementations settle on different envelopes at the same flow velocity.

Lower Tension (389 N)

As mentioned above, the lower tension of 389 N was not part of the validation of Qu and Metrikine (2021) [28], and is the regime examined by the cross-flow only studies of Slingsby (2015) [33] and Ntroupis (2017) [32]. Two velocities are shown here, $V = 0.16$ and 0.31 m/s, in Figures 4.8 and 4.9, where both implementations agree closely. The full set is given in Appendix D.2.

The experimental tensioning system used different spring sets depending on the specific test case, with stiffnesses of either 38.1 kN/m or 26.7 kN/m available [30]. Since the exact spring stiffness used for each case is not reported in Chaplin et al. (2005) [72], both values were simulated. The 26.7 kN/m stiffness produced mean top tensions in closer agreement with the experimentally measured values across all four cases, and was therefore adopted for the results below. With this stiffness the two implementations gave matching mean tensions at the two velocities shown, 397 N against the measured 405 N at 0.16 m/s and 456 N against 457 N at 0.31 m/s.

The in-line response is well captured in the present model, in contrast to Slingsby (2015) [33] and Ntroupis (2017) [32], who both under-predict it significantly. Chaplin et al. (2005) [72] presented results from several numerical codes applied to the same configuration, of which only two predicted mean in-line displacements in reasonable agreement with the measurements, both relying on drag amplification through the Morison equation rather than on vortex-induced forcing. The coupled formulation reproduces the mean in-line deflection directly, and additionally models the spring boundary condition explicitly, capturing the tension increase that drives it.

Case 1: $V = 0.16$ m/s, $T_t = 405$ N**Figure 4.8.** Displacement envelopes for Case 1, $V = 0.16$ m/s ($T_t = 405$ N)**Case 3: $V = 0.31$ m/s, $T_t = 457$ N****Figure 4.9.** Displacement envelopes for Case 3, $V = 0.31$ m/s ($T_t = 457$ N)

The cross-flow envelopes are under-predicted relative to the experiment across all four cases. This is most likely due to a combination of effects. The first is the wake parameters $A = 8$, $\varepsilon = 0.08$ and $\kappa = 5$, calibrated against rigid cylinder data. At 810 N the stiff riser deflects little and stays close to those calibration conditions, whereas at 389 N it is less stiff, deflects further under drag, and its self-weight lowers the effective tension over the exposed lower span [30], so the same parameters are less representative. The values $A = 10$ and $\varepsilon = 0.07$ used by Slingsby (2015) [33] and Ntroumpis (2017) [32] give closer cross-flow amplitudes in their models, but did not improve the result when tested here. The κ in-line coupling adds a tuning parameter absent from the cross-flow only formulations, so a direct comparison of A and ε alone is not that straightforward.

A second effect is the fixed added mass coefficient $C_a = 1.0$. Liu et al. (2020) [74] simulated the same experiment at low tension and matched the measured response only by letting the added mass coefficient vary, reaching values well below 1.0, while a fixed value underestimated the VIV. Their model differs from the present one, so the comparison is not direct, but points to the added mass as a likely contributor. The added mass is the water carried with the riser as it moves, and it adds to its effective mass. A value higher than the true one makes the riser behave as heavier than it is, which lowers its natural frequencies, shifts the lock-in flow speed, and raises the mass ratio, which lowers the amplitude [55]. The added mass enters the inertia force which is proportional to the riser's acceleration, so it stays small at 810 N where the stiff riser barely moves, and becomes significant at 389 N where the flexible riser responds with large amplitude.

A further limitation is the spanwise coherence of the shedding, which Huera-Huarte (2006) [30] identified as an additional source of force amplification at lower tension. It is absent from the model, as the sections are coupled only through the structure and not through the wake equation.

At the higher velocities $V = 0.60$ and 0.95 m/s the tension rises well above its initial still-water value. The two implementations diverge here onto different dominant modes, as their mean top tensions deviate more from the measured values and from each other, shifting the response onto different lock-in branches. These cases are shown in full in Appendix D.2.

4.5 Conclusion

The two implementations and the validation show that the model resolves the in-line motion directly through the coupled wake oscillator model, and that the explicit spring system reproduces the tension that evolves as the riser deflects. They match each other and the experiment well in both the cross-flow and in-line response, which the earlier cross-flow only studies could not provide. Two limitations were identified. At the transitional velocities the response is sensitive to the mean tension, so a small deviation from the experimental value moves the response onto a different mode. At the lower tension, where the riser is more flexible, the cross-flow amplitude is under-predicted, most likely from the rigid cylinder calibration of the wake parameters.

5

Lazy Wave Cable VIV Analysis

This chapter applies the coupled cross-flow and in-line wake oscillator model to an experimentally tested lazy wave power cable to assess its ability to capture the more complex response of this geometry, by means of its frequency content and amplitude, and compares it with industry-standard wake oscillator models.

5.1 Overview

To investigate the ability of the developed model to capture and predict VIV of a lazy wave power cable, the experimental study of Moideen et al. (2025) [26] is used as the validation reference. The experiments were conducted at the FloWave Ocean Energy Research Facility at the University of Edinburgh, a circular basin of 25 m diameter and 2 m water depth capable of generating a current in any direction.

The cable model was designed to represent a dynamic marine power cable for offshore renewable energy transmission. A 1:50 Froude scaling factor was applied to the structural parameters and current velocities, with the exception of the outer diameter, which was intentionally increased to 31 mm to maintain Reynolds numbers in the subcritical regime (10^3 to 10^4). The total cable length was 5 m, with an effective submerged length of 4.93 m after accounting for the part of the cable inserted into the end connections. The mass ratio of the cable is low, at $m^* = 1.22$. Five discrete buoyancy modules were attached along the midsection to elevate the cable into the characteristic hogging curve of the lazy wave profile. Each module had an outer diameter of 70 mm and a length of 40 mm along the cable axis. Current speeds ranged from 0.1 to 0.9 m/s in increments of 0.1 m/s.

The structural damping ratio was measured from free decay tests in air, yielding a value of 0.0048, while the in-water tests in the lazy wave configuration showed much stronger damping that could not be quantified.

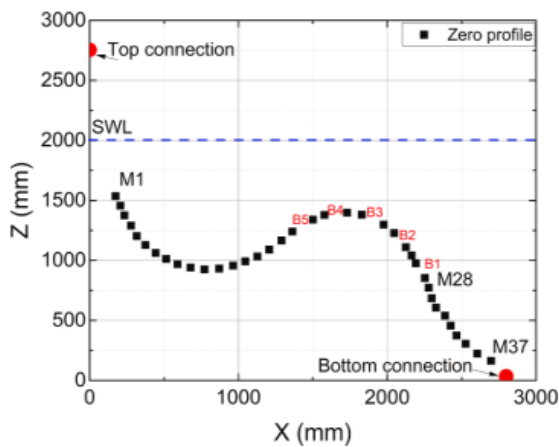
The experimental results [26] show that the cable response changes with current speed. At low reduced velocities, $U = 0.1$ and 0.2 m/s ($U_r = 5.28$ and 10.57 , based on the natural frequency in air), the cable shows clear single-frequency oscillations consistent with organised vortex-induced vibration. The authors identify $U_r = 5.28$ as the point at which a marked increase in cross-flow displacement first occurs, and note that if classical lock-in takes place at all, it is most likely within this low reduced velocity range. Beyond $U = 0.2$ m/s, the response transitions to broad-banded, multi-frequency motion as increasing drag deflection drives the cable out of its lazy wave shape and structural modal excitation becomes the main mechanism. For this reason, the case $U = 0.1$ m/s is used as the primary reference for the model comparison in this chapter.

5.2 OrcaFlex Model Description

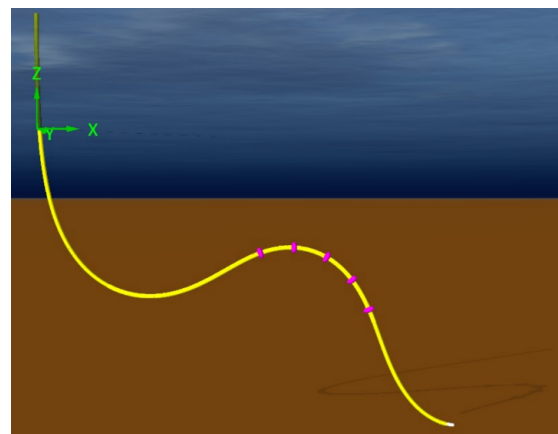
The lazy wave power cable was modelled in OrcaFlex as a line object, with structural properties and segmentation listed in Tables 5.1 and 5.2. The five buoyancy modules were represented as discrete clump attachments along the buoyancy section, each assigned the physical properties listed in Table 5.1. The exact spanwise positions of the buoyancy modules were not reported. These positions affect the cable profile and the natural frequencies, so the module locations and the corresponding section lengths were set by matching the OrcaFlex static profile, shown in Figure 5.1b, to the still-water shape recorded by the Qualisys motion capture system, which tracked 37 reflective markers along the span, shown in Figure 5.1a. Due to the cable's low mass ratio, its static configuration and dynamic response are highly sensitive to geometric and structural parameters, including the buoyancy module positions, section sub-lengths, cable stiffness properties, and boundary conditions.

End A was modelled as a fixed connection at $(0, 0, 0.77)$ m, corresponding to the top connection point above the still water level, while End B was anchored to the seabed at $(2.8, 0, -2)$ m, determined through the same profile-matching procedure. A finite bending connection stiffness of 10×10^{-6} kNm/deg and a torsional stiffness of 100×10^{-6} kNm/deg were applied at both ends, as these values best reproduced the still-water cable profile and the natural frequencies reported by Moideen et al. (2025) [26]. The cable was discretised into 109 segments with a target segment length of 0.046 m, ensuring nodes at all module attachment points and at least four segments between adjacent modules, consistent with Orcina guidance [20].

The wake oscillator parameters were taken as the same values used for the flexible riser validation in Table 4.2, with the exception of the Strouhal number, which was set to $St = 0.15$, the upper bound of the values calculated at this current direction [26].



(a) Qualisys marker profile [26]



(b) OrcaFlex numerical model

Figure 5.1. Still-water lazy wave profile and corresponding OrcaFlex model

Table 5.1. Structural properties of the lazy wave cable model

Parameter	Sym.	Value	Unit	Parameter	Sym.	Value	Unit
Total length	L	5.00	m	Bending stiffness	EI	2.17×10^{-4}	kNm ²
Outer diameter	D	0.031	m	Axial stiffness	EA	495	kN
Mass/length	m_0	0.919	kg/m	Torsional stiffness	GJ	158×10^{-6}	kNm ²
Mass ratio	m^*	1.22	–	Added mass coeff.	C_a	1.0	–
Fluid density	ρ_f	1000	kg/m ³	Struct. damping	ζ	0.0048	–
Buoyancy modules							
No. of modules	N_b	5	–	First module arc	$s_{b,1}$	3.059	m
Module density	ρ_b	100	kg/m ³	Module spacing	Δs	0.230	m
Module mass	m_b	12.4	g	Module height	h_b	0.040	m
Module volume	V_b	1.24×10^{-4}	m ³	Added mass coeff.	$C_{a,b}$	1.0	–

Table 5.2. Line sections and segmentation of the OrcaFlex model

Section	Length (m)	Segments	Target length (m)
Above-water section	0.770	17	0.046
Upper catenary and sag bend	2.289	50	0.046
Buoyancy section	0.920	20	0.046
Lower catenary	1.021	22	0.046
Total	5.00	109	–

5.3 Natural Frequencies

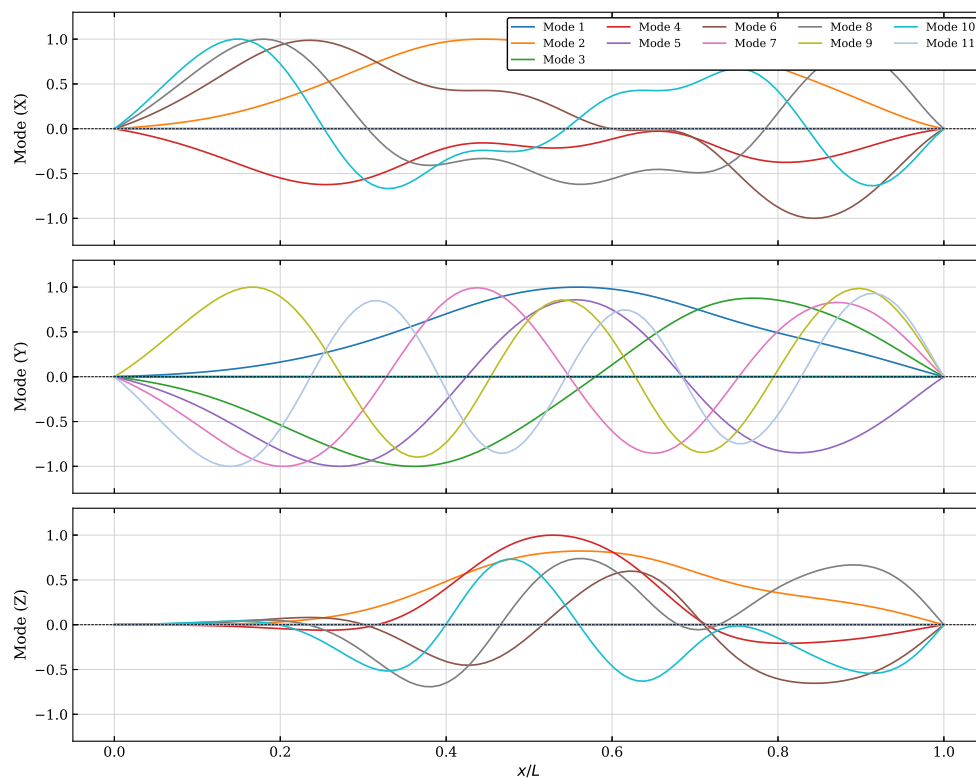
As for the flexible riser validation, a modal analysis was carried out to verify that the OrcaFlex model reproduces the dynamic properties of the cable before proceeding to the VIV response simulations. In-water free decay tests on the submerged cable in its lazy wave profile yielded natural frequencies of 0.125 Hz (in-plane) and 0.141 Hz (out-of-plane). The response in water showed very strong damping, which prevented further analysis of the decay signals, so the modal frequencies used for comparison were obtained from the OrcaFlex numerical model developed by Moideen et al. [23, 26] rather than directly from the measurements.

The present model matches these values within 5.9 % at worst, at mode 6, and 1.9 % on average across all 15 modes, as listed in Table 5.3. Due to the low bending stiffness and the absence of tension, the natural frequencies are much lower and more closely spaced than those of the flexible riser [26]. Improving the frequency agreement further moved the static shape away from the measured profile, so the end stiffness was held at the compromise value set in Section 5.2.

Table 5.3. First 15 natural frequencies of the lazy wave cable in still water

Mode	Moideen et al. (Hz) [26]	Present OrcaFlex (Hz)	Plane
1	0.089	0.085 (4.8%)	Y-plane
2	0.127	0.122 (4.2%)	X-plane
3	0.186	0.183 (1.7%)	Y-plane
4	0.271	0.274 (1.2%)	X-plane
5	0.303	0.302 (0.2%)	Y-plane
6	0.400	0.423 (5.9%)	X-plane
7	0.517	0.504 (2.6%)	Y-plane
8	0.644	0.637 (1.0%)	X-plane
9	0.726	0.731 (0.7%)	Y-plane
10	0.911	0.887 (2.6%)	X-plane
11	0.990	0.989 (0.1%)	Y-plane
12	1.225	1.218 (0.6%)	X-plane
13	1.291	1.305 (1.1%)	Y-plane
14	1.576	1.561 (0.9%)	X-plane
15	1.642	1.647 (0.3%)	Y-plane

The corresponding mode shapes for the first ten modes are shown in Figure 5.2. Unlike the straight riser, where each mode was a simple shape set by its number of nodes, the curved static profile spreads the modes across more than one global axis.

**Figure 5.2.** Mode shapes in X , Y and Z corresponding to modes 1 to 10, based on the OrcaFlex model

5.4 VIV Response Results

All results are presented as a function of normalised arc length x/L , from the hang-off point (End A, $x/L = 0$) to the touchdown point (End B, $x/L = 1$). The 37 Qualisys markers were mapped onto this coordinate from the still-water static profile. The instrumented region spans $x/L = 0.277$ to 0.970 , marked by the grey dashed lines in the figures that follow. Model output outside this range is shown for completeness but lies beyond the region where direct comparison with the experiment is possible.

Global displacements from both the Qualisys motion capture system and the OrcaFlex model are projected onto local in-line and cross-flow directions following [23]. For each segment, the local tangent vector $\mathbf{e}_c$ is computed from the time-mean positions of the two adjacent markers or nodes under current. Together with the current direction vector $\mathbf{e}_{flow} = (\cos \theta, \sin \theta, 0)$, where θ is the current incidence angle, this defines a local coordinate frame with normalised unit vectors $\mathbf{e}_{CF} = \mathbf{e}_c \times \mathbf{e}_{flow}$ and $\mathbf{e}_{IL} = \mathbf{e}_c \times \mathbf{e}_{CF}$. The in-line and cross-flow components are obtained by projecting the oscillating part of the displacement, averaged over the two adjacent nodes, onto these unit vectors.

A high-pass filter with a cutoff frequency of 0.2 Hz is applied to both datasets to remove slow current oscillations in the experiment [23] and residual low-frequency content in the model. The OrcaFlex simulations are run for a total duration of 120 s, of which the first 80 s are discarded as transient, leaving a 40 s steady-state window for analysis. The same window is applied to the experimental data.

The modes that carry the VIV response are identified by decomposing the filtered displacement onto the mode shapes of the cable, which is an established reconstruction method for flexible-cylinder VIV [75, 76]. The response is written as a sum of mode shapes, each scaled by a time-varying generalised weight,

$$\mathbf{r}(x/L, t) = \sum_{n=1}^N q_n(t) \phi_n(x/L), \quad (5.1)$$

where $\phi_n(x/L)$ is the shape of mode n along the normalised arc length, $q_n(t)$ is its weight at time t , and $N = 14$ is the number of modes retained. Because the cable is curved, each mode carries motion in both the in-line and cross-flow directions, so a single weight $q_n(t)$ is fitted to the two directions together rather than to each on its own. The mode shapes come from an OrcaFlex modal analysis about the static equilibrium under the current. The full procedure is set out in Appendix F.

The decomposition is exact only for a linear system. The cable response is nonlinear, so the weights are a best linear fit rather than the true modal participation. The root-mean-square of each weight then shows which modes carry the response along the span.

5.4.1 Primary Validation Case $\theta = 90^\circ$, $U = 0.1 \text{ m/s}$

The current is applied normal to the cable at the lowest tested speed of 0.1 m/s, the case where the experiment shows clear single-frequency vortex-induced vibration. The response is examined through its frequency content and the cross-flow and in-line amplitude along the span. The model reproduces the cross-flow mode that carries the response, and the single primary frequency shared by both directions, which follows from the geometrical coupling of the curved cable, while the amplitude is over-predicted.

The depth profile of the cable against normalised arc length is shown in Figure 5.3. It is included so that the cross-flow and in-line RMS distributions presented below can be read and interpreted directly against the characteristic geometric features of the lazy wave profile.

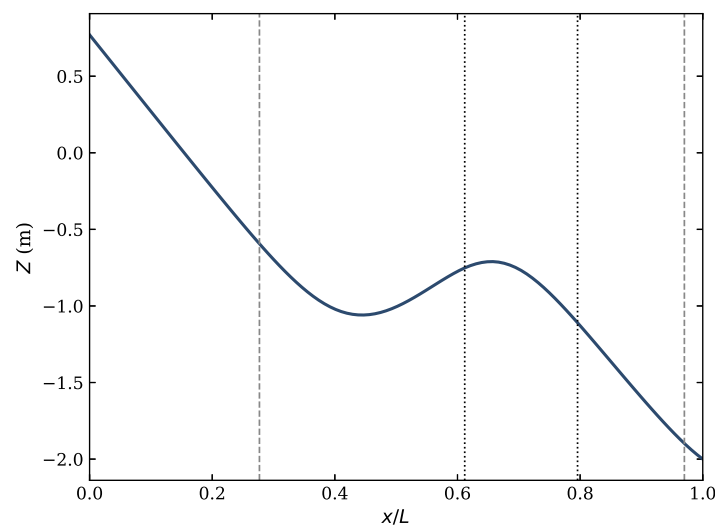


Figure 5.3. Depth profile of the lazy wave cable as a function of normalised arc length

The current is applied at $\theta = 90^\circ$, normal to the cable. For a straight cylinder in this condition the cross-flow direction is the global X -axis and the in-line direction is the global Y -axis, so the VIV response maps directly onto the structural modes in those two planes. However, in a lazy wave cable the tangent varies along the span, so the local cross-flow and in-line directions rotate away from the global axes and this fixed correspondence cannot be assumed.

The orientation of the two local directions along the span is shown in Figures 5.4a and 5.4b, through the global X , Y and Z components of each unit vector. The in-line direction stays close to the global Y -axis, but its X and Z components remain non-zero over almost the entire span and reach 30 to 40% at several locations. The cross-flow direction rotates from the global X -axis toward Z and is aligned almost entirely with Z at the sag bend, so the lift force there acts vertically rather than laterally. The structural response at a given location therefore draws on modes from more than one global plane, and the relative contribution of each shapes the response, which a pure global cross-flow and in-line view would miss.

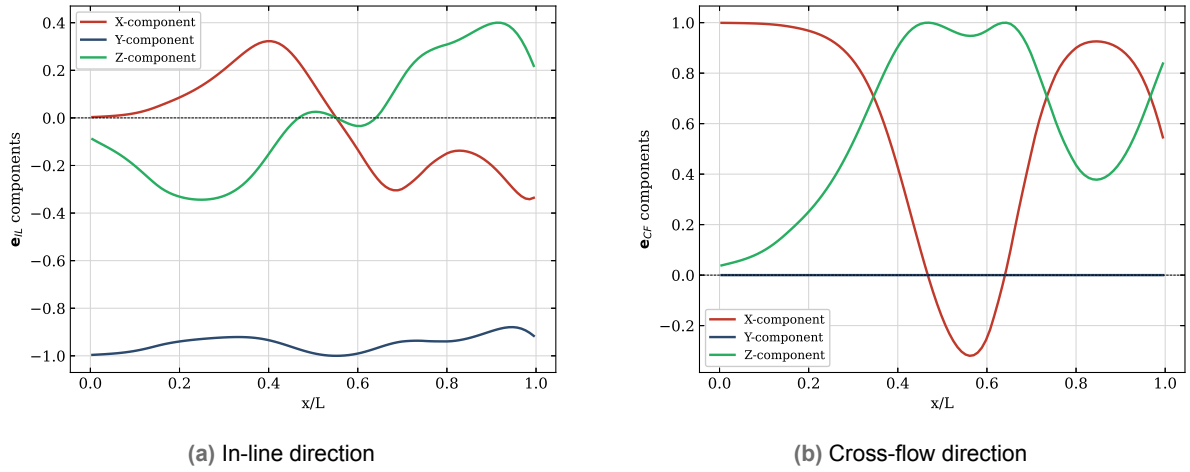


Figure 5.4. Global components of the local frame unit vectors

Frequency Content

The in-line and cross-flow responses are coupled in the model, and the cross-flow frequency f_{CF} sets the harmonics at which both directions respond. The fluctuating in-line drag is proportional to the square of the lift coefficient. This quadratic dependence doubles the cross-flow frequency and places the in-line forcing on the even harmonics, principally $2f_{CF}$. The cross-flow response is governed by the self-excited wake oscillator, which sustains f_{CF} and its odd harmonics. The parametric κ term feeds the $2f_{CF}$ in-line motion back into the wake, adding energy at $3f_{CF}$. The cross-flow response therefore carries the odd harmonics of f_{CF} and the in-line forcing the even harmonics.

The power spectral density of the in-line and cross-flow displacement is shown in Figure 5.5. Following Moideen et al. [26], each spectrum is plotted against the normalised frequency f/f_m , where f_m is a modal frequency of the OrcaFlex model. The in-line spectrum is scaled by $f_{m2} = 0.122$ Hz, the second mode (X-plane), and the cross-flow spectrum by $f_{m3} = 0.183$ Hz, the third mode (Y-plane). Each direction is scaled by a different modal frequency, so the same physical frequency appears at a different position on the two axes.

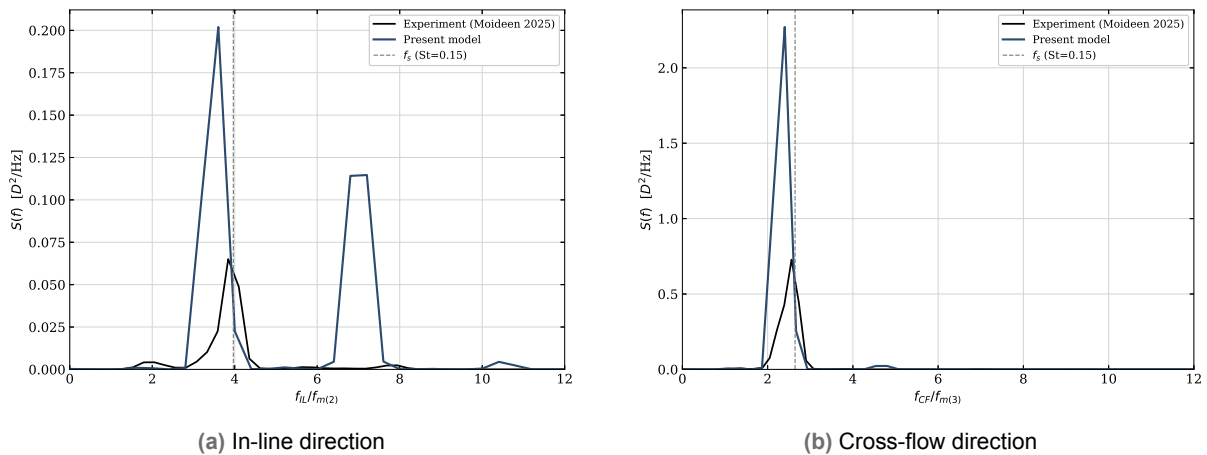


Figure 5.5. Mean power spectral density, $\theta = 90^\circ$, $U = 0.1$ m/s

Both directions peak at the same physical frequency, 0.440 Hz in the model and 0.469 Hz in the experiment. This is the cross-flow frequency f_{CF} , close to the sixth-mode frequency of the cable at 0.423 Hz. On the normalised axes this single frequency appears at $f/f_{m2} = 3.60$ in-line and $f/f_{m3} = 2.40$ cross-flow in the model, and at 3.84 and 2.56 in the experiment. The in-line spectrum has a second peak at twice f_{CF} , the even-harmonic forcing. Measured against the primary peak of the same spectrum, this is about 4% in the experiment and 57% in the model, so the model over-predicts the $2f_{CF}$ in-line content. Moideen et al. (2025) [26] report the same shared primary frequency in both directions.

On a straight cylinder the even-harmonic forcing would drive the in-line response at $2f_{CF}$. Here the in-line peak sits at f_{CF} instead. This follows from the curved shape of the cable. At the sag bend the local cross-flow direction is almost vertical (Figure 5.4b), so the cross-flow lift force at f_{CF} heaves this section vertically at f_{CF} . The cable is continuous and curved, and the local in-line direction keeps non-zero X and Z components over most of the span (Figure 5.4a). The vertical motion at the sag bend therefore projects onto the local in-line direction, so the in-line response carries f_{CF} . Coupling between the two directions is expected in curved slender structures [64], and the experiments on the same cable report bending modes acting together [26]. The $f_{IL} = f_{CF}$ behaviour is therefore recovered, since the wake-oscillator forcing is coupled to the full three-dimensional structure in OrcaFlex, while the $2f_{CF}$ component remains present but over-predicted.

Cross-flow Response

The cross-flow RMS envelope in Figure 5.6a shows that the model and the experiment place the amplitude differently along the span. The modal decomposition in Figure 5.6b sets both responses in the same basis and shows how the single frequency f_{CF} identified above is distributed over the modes. In the model the cross-flow energy sits almost entirely in mode 6, the X-plane mode nearest f_{CF} . In the experiment it is spread across the neighbouring X-plane modes 4, 6 and 8, with the largest weight on mode 4.

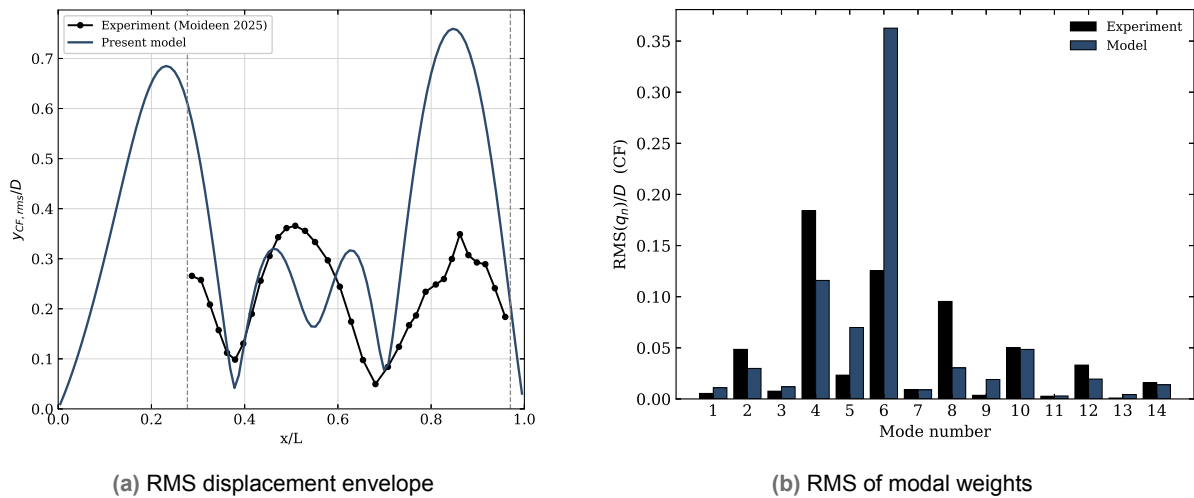


Figure 5.6. Cross-flow response, $\theta = 90^\circ$, $U = 0.1$ m/s

The response frequency and the mode number therefore do not coincide. The cross-flow response locks near f_{CF} , in the band of the sixth mode (Figure 5.5), yet the modal weights place the largest amplitude on mode 4 (Figure 5.6b). Part of this follows from the closely spaced X-plane modes, whose shapes are similar over the measured span and cannot be fully separated, so a single response near f_{CF} is distributed over neighbouring modes. It is also consistent with low-mass-ratio flexible cylinders, where the lock-in frequency and the dominant mode number need not coincide [75], and with the reduced lock-in reported for the lazy wave at this speed [26]. The model motion is close to a single mode, so the decomposition returns mode 6 alone.

The two envelopes have their nodes at the same positions, near $x/L \approx 0.38$ and $x/L \approx 0.70$. Mode 6 has the largest natural-frequency difference between the model and the experiment in Table 5.3, at 5.9%, but its shape matches the one reported by Moideen et al. [26], so the mode shape is reproduced and the difference lies in the amplitude carried by each antinode.

The experiment spreads the cross-flow amplitude across the span, with a strong antinode at mid-span ($y_{CF}/D \approx 0.36$ at $x/L \approx 0.50$) and a comparable one toward the lower end (≈ 0.35 at $x/L \approx 0.86$). The model instead concentrates the amplitude near the ends, reaching ≈ 0.69 at $x/L \approx 0.23$ and ≈ 0.76 at $x/L \approx 0.85$, with a reduced mid-span response.

This difference in distribution most likely follows from a mechanism the model lacks. In the implemented model the wake oscillators act independently at each section, driven only by the local structural motion, with no coupling between adjacent sections through the fluid [64], so the cross-flow energy follows the structural mode shape and its end antinodes. In the experiment the wake is coupled along the span, with vibration in one section changing the inflow seen by its neighbours and reducing their response, and the buoyancy modules disrupting and damping the shedding [26]. This spanwise coupling spreads the energy along the cable and lowers the response near the buoyancy section, and it is not represented in the model.

The buoyancy modules themselves are included in the model, both as structural properties and through their larger diameter in the forcing. Adding this diameter lowers the cross-flow amplitude near the buoyancy section, but not enough to bring the response close to the experiment, and it leaves the spatial pattern unchanged. The distribution difference therefore arises mainly from the missing spanwise coupling rather than the local geometry. The amplitude over-prediction is also linked to the selected wake parameters A , ε and κ , calibrated on straight, rigid cylinders and unlikely to transfer to a highly flexible, low-mass-ratio cable.

In-line Response

The in-line RMS envelope in Figure 5.7a shows that the model over-predicts the in-line response, with the largest over-prediction at mid-span. The model reaches about $0.28 D$ at $x/L \approx 0.53$, while the experiment there is only about $0.065 D$. The experimental response is lowest near mid-span and higher toward the upper and lower sections of the cable [26], reaching about $0.12 D$ in the buoyancy region at $x/L \approx 0.77$. The model therefore places its largest in-line response where the experiment is smallest.

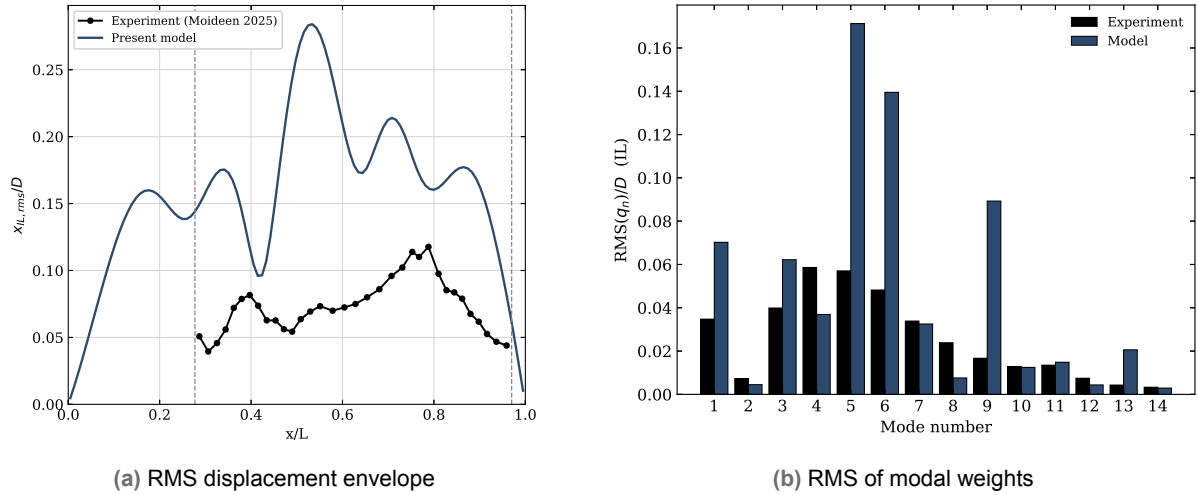


Figure 5.7. In-line response, $\theta = 90^\circ$, $U = 0.1$ m/s

The modal decomposition in Figure 5.7b shows the same contrast as the cross-flow response. The experiment spreads the in-line energy across modes 3 to 7 with no single leading mode, while the model concentrates it in modes 5, 6 and 9. In the model, mode 5, a Y-plane mode at 0.302 Hz, carries the response at f_{CF} , and mode 9, a Y-plane mode at 0.731 Hz, carries the $2f_{CF}$ component that the experiment does not show. Mode 6 is the X-plane cross-flow mode, and its in-line weight is the geometric projection of the cross-flow motion onto the local in-line direction. This projection oscillates at f_{CF} , in step with the cross-flow motion, so the curved geometry places the in-line response at f_{CF} rather than at the even harmonic $2f_{CF}$.

Global mode 1 also takes a large share of the weight in the model, but the spectrum shows no energy at its frequency, so this is not a physical excitation. Its broad bow (Figure 5.2, Y panel) matches the mid-span deflection, which is why the fit assigns weight to it. Modes 5 and 9 both carry their amplitude in the central region, which produces the over-predicted peak at mid-span and leaves the response toward the ends under-predicted.

5.4.2 Sensitivity Analysis

The wake tuning parameters A , ε and κ are held at the values reported by [28] and are not adjusted to the lazy wave case. The model also depends on physical and numerical inputs that are not tuned to the measurements. This section varies each in turn and tests whether any of them removes the over-prediction found in the primary case. The structural damping, the lift coefficient and the Strouhal number are varied around their base values while all other inputs are kept fixed, under the primary validation condition of Section 5.4.1, $\theta = 90^\circ$ and $U = 0.1$ m/s. A comparable study was reported by Elrick (2025) [24] for dynamic power cables, using the frequency-domain model SHEAR7 rather than a time-domain wake oscillator. The two methods differ, but the parameters examined are common to both, and the reported trends are compared below where relevant.

Structural Damping

Damping removes energy from the motion over each cycle, so a higher damping level gives a smaller amplitude. Elrick (2025) [24] reported this for dynamic lazy wave power cables, with the predicted amplitude roughly doubling at zero damping and roughly halving at very high damping relative to the calibrated value, while the shape of the displacement profile and the positions of its peaks remained almost unchanged.

Two contributions act on the cable. Structural damping comes from within the cable, from stick-slip and the interaction of its internal components [24], and is applied through the bending term of the separated Rayleigh damping in OrcaFlex, which damps the transverse bending in both the cross-flow and in-line directions. This is the contribution varied here. Hydrodynamic damping comes from the fluid, defined as the reduction of amplitude by forces in anti-phase with the velocity [77]. In the applied model the cable velocity relative to the water enters the fluid loading, and the drag acting on this relative velocity opposes the motion and supplies the hydrodynamic damping. It is intrinsic to the force model and is not varied.

The structural damping ratio is varied from the base value of $\zeta = 0.48\%$, reported by the experimental report [26], up to $\zeta = 7.5\%$. This range starts at the measured base and extends past the value of about 2.8% that [24] used for an umbilical, so the high damping end is also covered. The choice is supported by the experimental observation of strong hydrodynamic damping in water during free decay that could not be quantified, and by the lazy wave shape itself, which adds damping through its slack sections and varying curvature [26]. Figures 5.8a and 5.8b show the in-line and cross-flow RMS envelopes for the five damping levels.

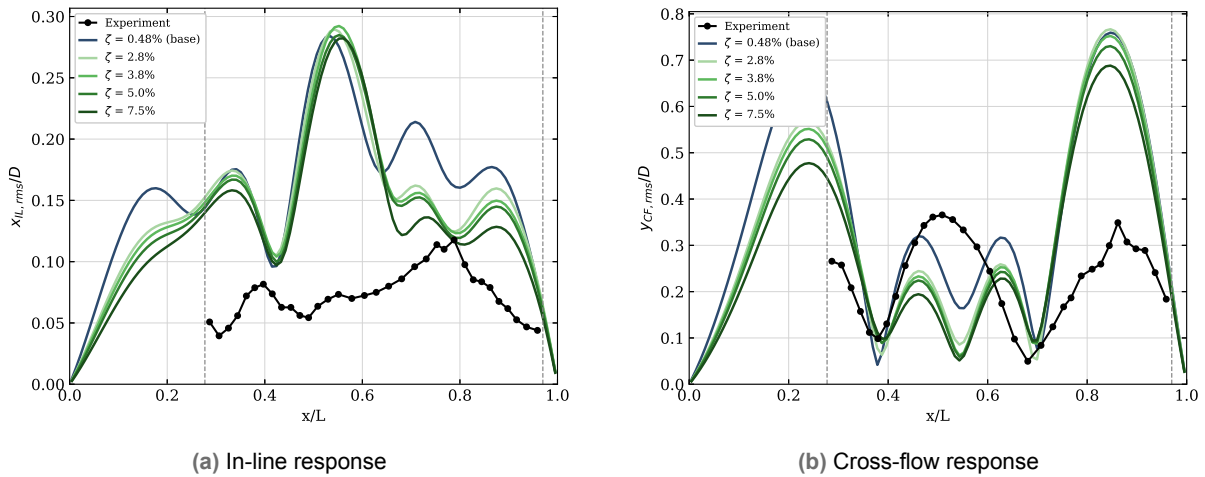


Figure 5.8. Effect of damping ratio ζ on the RMS displacement envelopes at $\theta = 90^\circ$, $U = 0.1$ m/s

In the cross-flow direction the two main antinodes fall monotonically as the damping increases, from about 0.69 to 0.48 near $x/L \approx 0.23$ and from about 0.76 to 0.69 near $x/L \approx 0.85$, while the peak and node positions do not move. The in-line response behaves in the same way, with the secondary peaks decreasing and the main mid-span peak near 0.28 almost unaffected. The over-prediction of the primary validation case is not removed at any damping level. Even at $\zeta = 7.5\%$, which is above any defensible structural value for this cable, the antinodes stay far

above the measured envelope, and the mid-span region is matched more closely by the base case than by the damped cases.

Damping therefore affects the response and has to be accounted for, but it does not remove the over-prediction. A damping large enough to lower the over-predicted antinodes also pushes the better-matching mid-span region below the measurement, so no single value improves the agreement along the whole span.

Lift Coefficient

In the applied model the lift coefficient C_{L0} is the amplitude of the fluctuating lift on a stationary cylinder [64], entering through $C_{VL} = (C_{L0}/2)q$. On a fixed cylinder the wake variable settles at its limit cycle amplitude $q = 2$, so the peak lift coefficient equals C_{L0} , and the value used is $C_{L0} = 0.3$. Once the cylinder moves the wake variable is driven above $q = 2$ inside lock-in and the peak lift rises above C_{L0} . The lift on a vibrating cylinder is not the stationary value. It depends on the amplitude ratio A/D and the Reynolds number and is normally taken from forced-oscillation tests [77], so C_{L0} sets only a starting value for the sensitivity study.

The lift coefficient depends on the Reynolds number. For the present case, with $U = 0.1$ m/s and $D = 0.031$ m, the Reynolds number is $Re \approx 3100$. Norberg (2003) [78], reports a sectional RMS lift coefficient over a wide Reynolds range, and at $Re \approx 3100$ the data give a value between about 0.05 and 0.15. Scaled by $\sqrt{2}$ for the near-sinusoidal lift signal reported there, this is a lift amplitude of about 0.07 to 0.21, all below the assumed 0.3, which motivates testing lower values. On this basis the lift coefficient is varied from the baseline $C_{L0} = 0.3$ down to 0.15, toward the range indicated by [78]. The corresponding in-line and cross-flow RMS displacement envelopes are shown in Figures 5.9a and 5.9b.

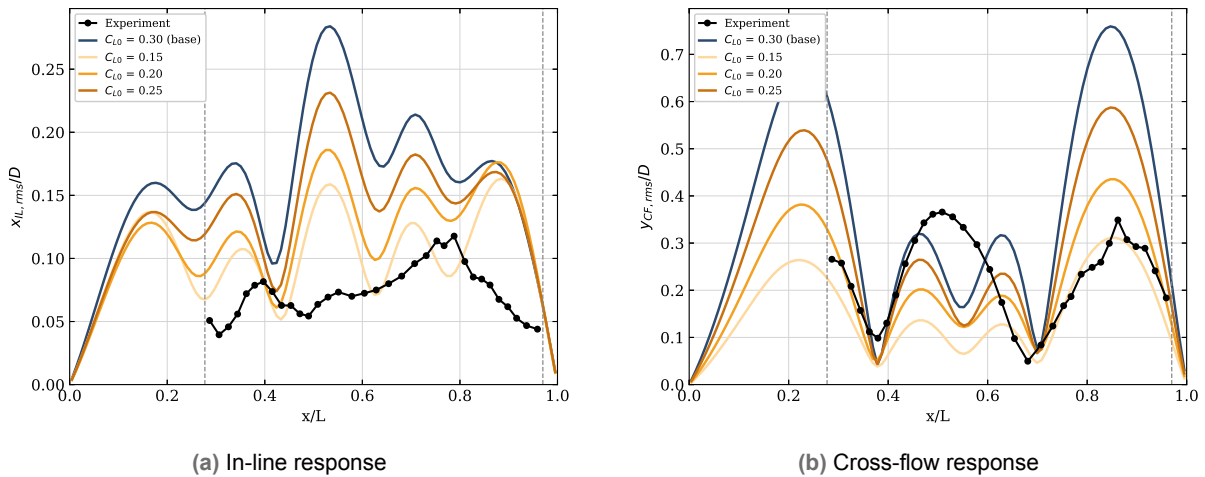


Figure 5.9. Effect of lift coefficient C_{L0} on the RMS displacement envelopes at $\theta = 90^\circ$, $U = 0.1$ m/s

The effect of C_{L0} on the amplitude is larger than that of the damping. The excited frequency does not change with C_{L0} . The lift coefficient therefore only changes the response magnitude at that frequency. In the cross-flow direction a lower C_{L0} lowers the antinodes strongly. At $C_{L0} = 0.15$ the antinode near $x/L \approx 0.85$ falls to about 0.31, close to the measured 0.35, while the mid-span near $x/L \approx 0.5$ falls to about 0.13, below the measured 0.36. The base value

matches the mid-span but overstates the antinodes, so no single value fits the whole span. The in-line response falls as well, following the cross-flow, with the mid-span peak near $x/L \approx 0.53$ dropping from about 0.28 to 0.16 but staying above the experiment.

The lift coefficient therefore has a first-order effect on the response, larger than the damping, and the lower values consistent with the Reynolds number reduce the over-prediction at the antinodes. A value below the tested range would only reduce the response further and deepen the mid-span under-prediction, so no single value reconciles the model with the experiment along the span. The strong amplitude sensitivity to the lift coefficient is consistent with [24].

Strouhal Number

The Strouhal number sets the shedding frequency through $\omega_s = 2\pi St V_N/D$, so unlike the damping and the lift coefficient it changes the frequency of the response, not only its magnitude. Sarpkaya (2010) [77] defines the parameter as $St = f_{st}D/U$, in which f_{st} is the shedding frequency of a cylinder at rest, and the standard value of 0.2 belongs to that case.

A freely vibrating cylinder has no such fixed frequency. Its motion changes the wake, so the shedding and the body lock to a common frequency that sits below f_{st} and shifts with the added mass [77]. Because a vibrating cylinder has no f_{st} , a Strouhal number reported for one is its response frequency divided by U/D , and the two are used interchangeably [79]. Potts et al. (2022) [79] compiled test data for elastically mounted cylinders on this basis and sorted it by degree of freedom. Cylinders free in cross-flow only give a mean of 0.185 to 0.2, matching fixed cylinders, while cylinders free in both cross-flow and in-line give about 0.14 to 0.15, with a spread of roughly $\pm 5\%$ about each mean. Design practice sits above the values for two degrees of freedom, since the recommended practice for riser VIV fatigue analysis (DNVGL-RP-F204) gives an effective range of 0.17 to 0.25 for oscillating cylinders [79].

The Strouhal number of this cable was reported by Moideen et al. (2025) from both the bending strain [23] and the cable displacement [26]. The displacement result is the closer comparison here, since the model output is displacement, and the strain result gives an independent check on the range. From the strain, the curvature peak frequency $\hat{f}_p = f_p D/U$ falls between 0.127 and 0.145 over subcritical Reynolds numbers of 3100 to 24,800, in agreement with Potts et al. (2022) [79], and at the direction and speed used here the sag-bend peak gives $St \approx 0.13$ [23].

The displacement spectra separate the two directions, with shedding curves at $St = 0.1, 0.15$ and 0.2 superimposed on the peak frequency ratios [26]. At $\theta = 90^\circ$ the peaks follow $St \approx 0.1$ to 0.15 as the reduced velocity rises, and no component exceeds 0.15 [26]. At $U = 0.1$ m/s both spectra show a single sharp peak across all markers, at $f_{IL}/f_{m(2)} = 3.61$ and $f_{CF}/f_{m(3)} = 2.52$, which give $St = 0.142$ in-line and 0.145 cross-flow [26]. The model takes that upper bound as its base, $St = 0.15$. It is varied to 0.14 and 0.13 inside the range measured for this cable, to 0.12 at the lower end of the displacement range, and to 0.18 and 0.20 above it toward the fixed-cylinder and design values. Figure 5.10 and Figure 5.11 show the RMS envelopes and the power spectra.

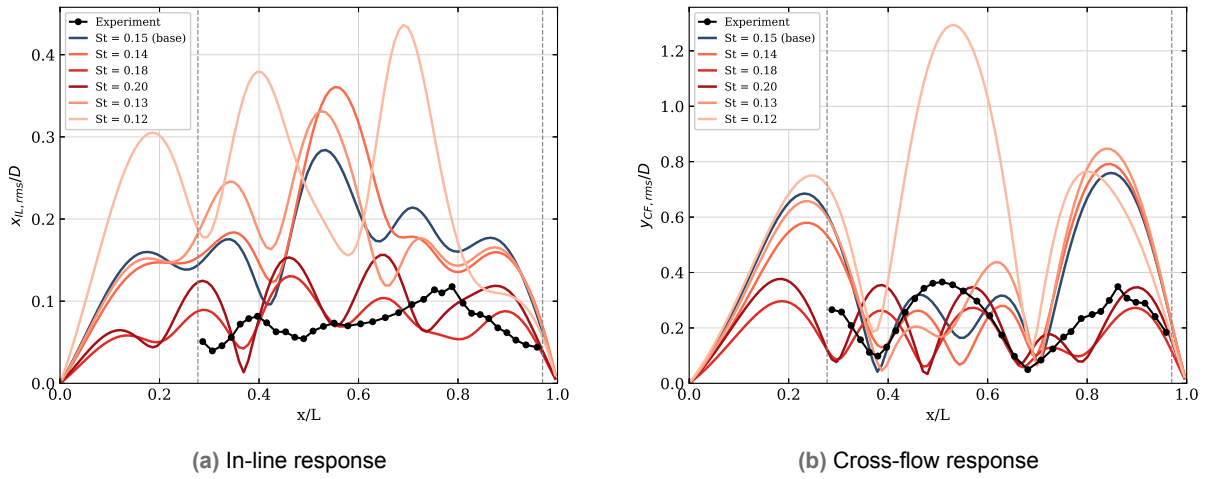


Figure 5.10. Effect of Strouhal number St on the RMS displacement envelopes at $\theta = 90^\circ$, $U = 0.1$ m/s

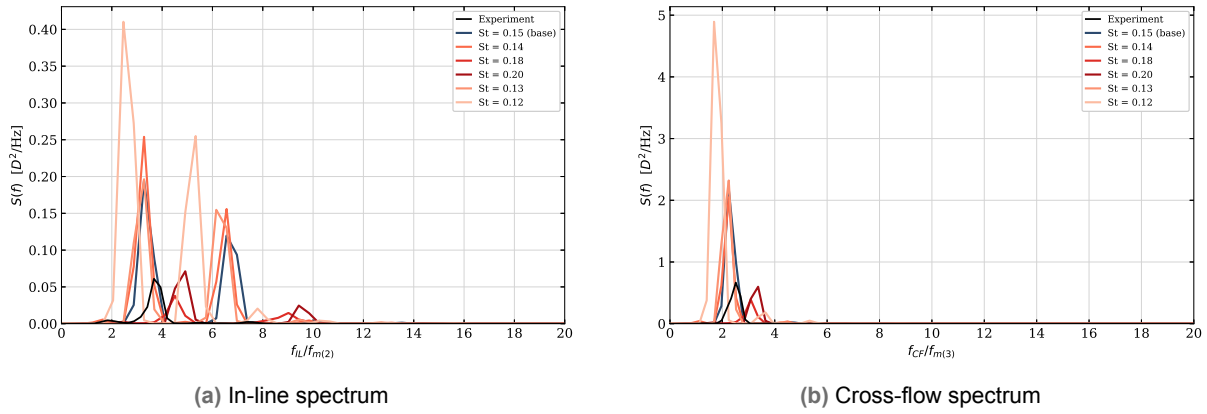


Figure 5.11. Effect of Strouhal number St on the mean PSD at $\theta = 90^\circ$, $U = 0.1$ m/s

The excited frequency moves with St , and the values fall into three groups set by the responding mode. At $St = 0.13$, 0.14 and 0.15 the cross-flow peak sits closest to the measured one, and the envelope keeps the shape closest to the experiment. At $St = 0.18$ and 0.20 the higher shedding frequency shifts the peak above the measurement and onto a higher mode, with more antinodes along the span. At $St = 0.12$ the lower shedding frequency shifts the peak below the measurement and onto a lower mode, with a single large antinode near mid-span.

The Strouhal number therefore has the strongest effect on the spatial response of the three inputs, since it is the only one that selects the responding mode rather than scaling the response. Its value is not known precisely for this cable, since it depends on the Reynolds number and on the lazy wave configuration [79, 23], as the natural frequencies of this cable are closely spaced, so a small change in St is enough to reach an adjacent mode. The same strong influence of St was reported by Elrick (2025) [24].

Across the three inputs, none removes the over-prediction. The response is more sensitive to the lift coefficient than to the damping, and both scale the amplitude down, so reducing the over-predicted antinodes near the ends also drops the mid-span below the measurement.

5.4.3 Model Comparison

The implemented model is compared with two established wake oscillator models available in OrcaFlex, the Milan model and the Iwan and Blevins model [80], under the primary validation condition $\theta = 90^\circ$ and $U = 0.1$ m/s. Both are single degree of freedom transverse oscillators, so the wake force acts in the cross-flow direction only, with no coupling to the in-line force. Both are also fixed at a Strouhal number of 0.2, the stationary, cross-flow-only value, and the OrcaFlex documentation ties their empirical parameters to it so that they cannot be retuned independently [80]. This value does not describe the present cable. As established in Section 5.4.2, the combined cross-flow and in-line motion lowers the effective Strouhal number to the measured 0.14 to 0.15. The implemented model is set to this value, while the two comparison models remain at 0.2 and therefore shed at a higher frequency.

This higher shedding frequency is visible in the cross-flow spectra of Figure 5.12. The implemented model places the peak at $f/f_{m3} \approx 2.40$, closest to the measured 2.56, while the Milan and Iwan and Blevins models place it at about 3.1 and 3.4. Because the natural frequencies of this cable are closely spaced, the raised frequency moves the two comparison models onto a higher structural mode, with more half waves than the experiment, while the implemented model holds the measured mode.

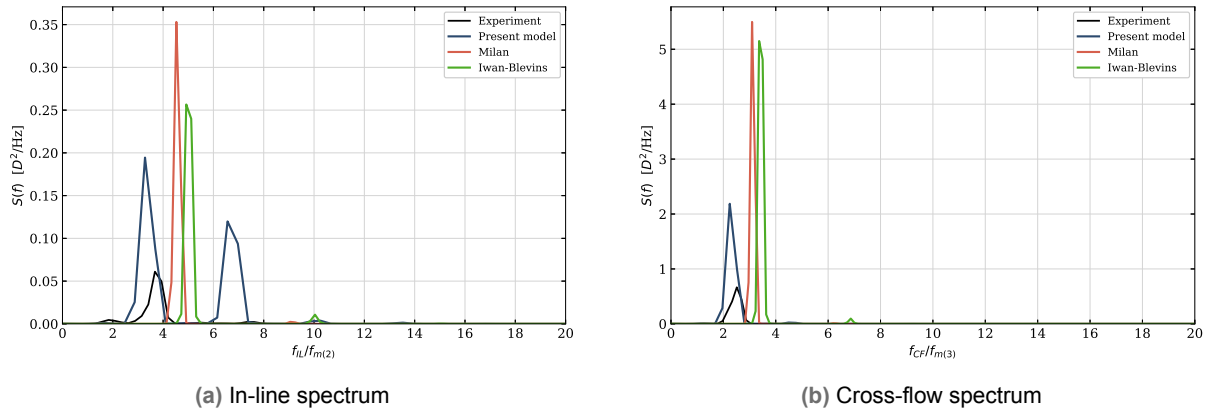


Figure 5.12. Mean PSD, model comparison at $\theta = 90^\circ$, $U = 0.1$ m/s

The same mode selection carries into the cross-flow RMS envelopes of Figure 5.13. All three models over-predict the antinode amplitude, with peaks near 0.76 for the implemented model, 0.80 for Iwan and Blevins and 0.73 for Milan, against a measured 0.35, so the over-prediction is common to the three rather than specific to the implemented model. The distinction is again the spatial pattern. The implemented model reproduces the rounded mid-span of the experiment between $x/L \approx 0.4$ and 0.7, while the two comparison models place a node there and raise antinodes on either side.

In the in-line direction the three models look alike in the spectra of Figure 5.12, where all show a peak at f_{CF} . This is a geometric effect, since the curved cable projects the cross-flow lift onto the in-line direction, so the $f_{IL} = f_{CF}$ behaviour of the experiment is reproduced by all three through the projection and not by the implemented model alone. The forcing behind the peak is what differs. The implemented model adds a coupled in-line forcing, making it the only one of the three with a modelled in-line mechanism, although this overshoots the experiment near mid-span.

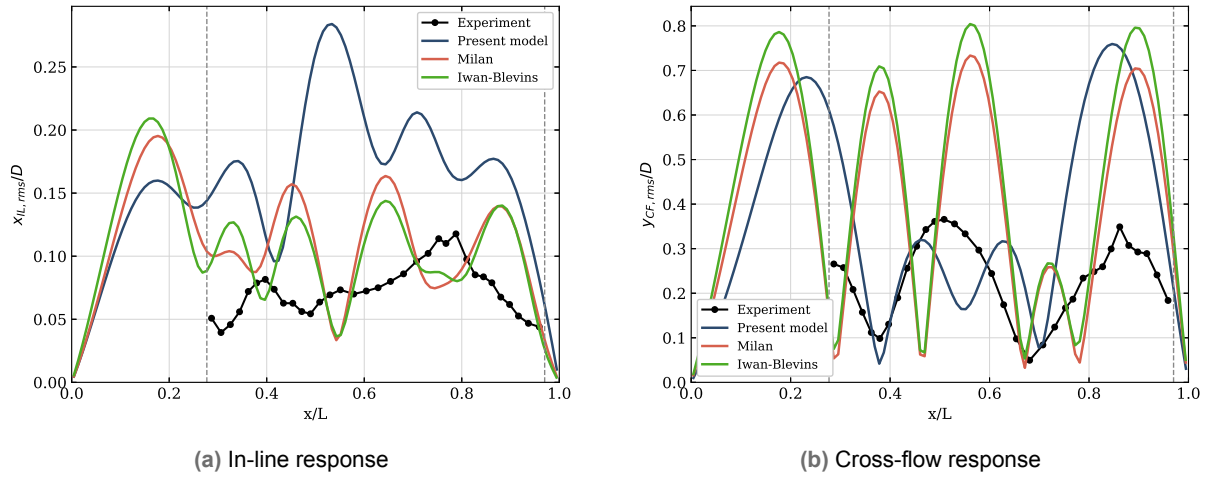


Figure 5.13. RMS envelopes, model comparison at $\theta = 90^\circ$, $U = 0.1$ m/s

Taken together, the implemented model is the most suitable of the three for a lazy wave cable without test data. It reproduces the measured frequency and mode, and it is the only one that represents the in-line response through a physical forcing. Its Strouhal number can also be set for each case, which for this cable is the measured 0.14 to 0.15 rather than the fixed 0.2 of the comparison models.

This is directly related to design. A field cable operates at a Reynolds number of order 10^5 , within the 10^4 to 10^6 range of offshore conditions and beyond what the laboratory can reach [26], so the shedding frequency of an untested cable is not known in advance. Being able to set the Strouhal number to the expected shedding is therefore what makes the model usable for such a cable. Since lazy wave power cables have closely spaced natural frequencies, the recommended use is to sweep St across a plausible band and take the envelope of the response, which conservatively bounds the mode uncertainty.

The limitation, shared by all three models, is the amplitude. The cross-flow antinodes are over-predicted by about a factor of two against the measured 0.35, so the model gives the frequency, the mode and the in-line behaviour reliably but the amplitude only as a conservative bound. Closing this gap most likely requires recalibrating the wake parameters away from their rigid-cylinder values and adding a spanwise correlation to couple the wake. More experiments on lazy wave power cables are needed to characterise their response and support this recalibration.

5.4.4 Effect of Current Direction

For a straight vertical riser the flow stays normal to the axis along the whole span whatever its horizontal direction, so the current orientation has little effect on the response. A lazy wave cable curves through a vertical plane, so its inclination to the flow varies along the span and changes with the current direction. The motion and the local forcing are therefore more sensitive to direction than for a straight riser. This section examines that sensitivity at $U = 0.1$ m/s, the primary validation speed, and tests the limits of the normal-only forcing across current directions.

Normal Velocity Distribution

The hydrodynamic forcing is defined in a local frame at each cross section, with the wake oscillator acting in the plane normal to the cable axis, following the independence principle introduced in Section 3.4.2. The principle retains only the component of the free stream normal to the axis and treats the aligned component as negligible, so the normal velocity ratio $U_n/U = \cos \beta$ measures how much of the current is effective at each point, where β is the angle between the incoming flow and the plane perpendicular to the cable axis. The principle holds only up to about $\beta = 65^\circ$ for a cylinder free to move in the cross-flow direction [64], which corresponds to $U_n/U \approx 0.42$, taken here as the threshold below which the normal-only description no longer holds.

The mean deflected shape sets the local frame and through it the normal velocity. The current resolves into an in-plane component that scales with $\cos \theta$ and a transverse component that scales with $\sin \theta$, so both the deflected shape and the normal velocity vary with direction.

Figure 5.14 gives U_n/U along the span. At $\theta = 90^\circ$ the current is normal to the cable plane and the ratio stays high across the whole span. At $\theta = 0^\circ$ and 180° the current lies in the cable plane, and the ratio falls to near zero at the sag trough ($x/L \approx 0.44$) and the buoyancy hump ($x/L \approx 0.65$), the turning points where the tangent runs along the flow. The intermediate directions lie between these limits, following their in-plane component.

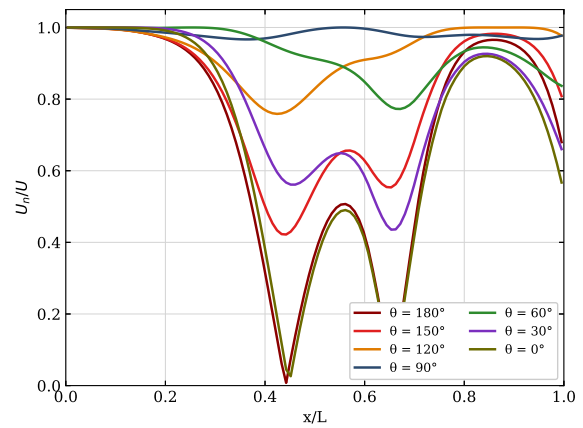


Figure 5.14. Normal velocity ratio U_n/U along the cable for all current directions, $U = 0.1$ m/s

Where the ratio falls to near zero the model applies almost no local forcing, since the normal velocity sets the shedding frequency and the hydrodynamic forces are built within the normal plane. The aligned directions are therefore where the normal-only description is least valid.

Response

Figure 5.15 compares the model and the experiment at $\theta = 90^\circ$ and $\theta = 0^\circ$, in the in-line direction on the left and the cross-flow direction on the right. At $\theta = 90^\circ$, where the normal velocity stays close to U , the model reproduces a strong cross-flow response and over-predicts the amplitude by about a factor of two, as in the primary case. At $\theta = 0^\circ$ the model concentrates the response into lobes with deep nodes at the sag trough and the buoyancy hump, where the normal velocity vanishes. The experiment behaves in the opposite way, with the smallest and flattest response of all directions, staying near 0.15 to 0.25 and not dropping at the turning points. The model therefore makes $\theta = 0^\circ$ its largest case while the experiment makes it the smallest.

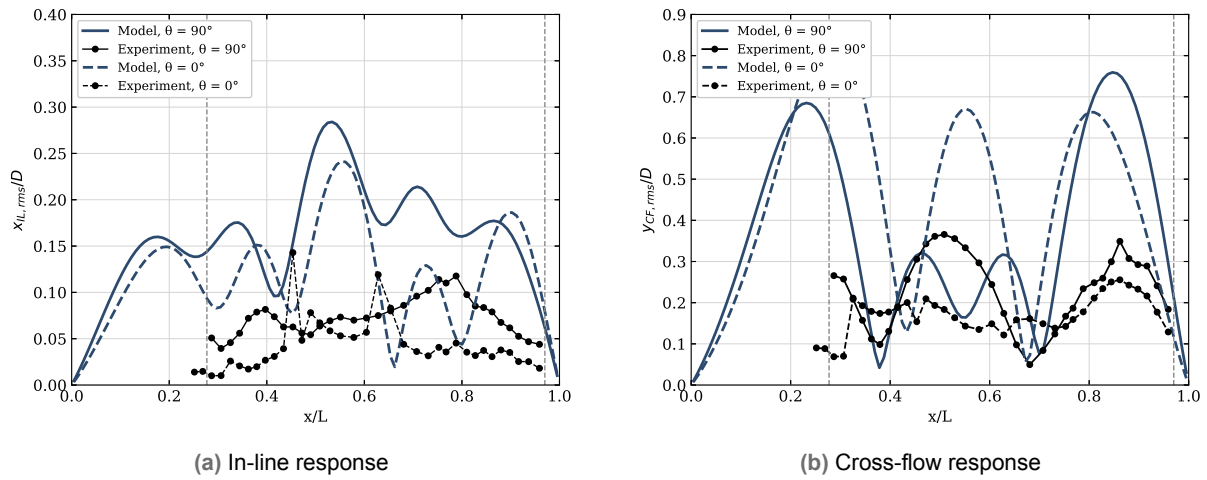


Figure 5.15. Model and experiment RMS envelopes at $\theta = 90^\circ$ and $\theta = 0^\circ$, $U = 0.1$ m/s

This contrast is most likely linked to the normal-only forcing. At the aligned direction the model removes the forcing at the high-inclination turning points and produces nodes, while the experiment keeps a steady response there, consistent with the known breakdown of the independence principle beyond $\beta = 65^\circ$, where the near-axial flow still drives shedding that the normal-only forcing, set by U_n alone, predicts as almost zero. The responding mode also changes with direction, from mode 6 at $\theta = 90^\circ$ to mode 5 at the aligned directions, as the lift rotates with the local frame. The in-line envelopes show the same pattern, with the experiment staying low while the model over-predicts and drops to troughs at the turning points that the experiment does not have.

The normal-only forcing is therefore reliable while the flow stays close to normal and reaches its limit at the aligned in-plane directions, where the inclination is highest. This must be accounted for when the model is applied to a lazy wave cable, whose inclination varies along the span, unlike a near-vertical configuration, which stays close to normal to a horizontal current at any heading.

5.5 Vertical Cable: Isolating the Low Mass Ratio

The lazy wave case showed that the model over-predicts both the in-line and the cross-flow response of a power cable with a mass ratio of $m^* = 1.22$, below the lowest mass ratio of $m^* = 2.6$ at which the parameter set had been tested [29]. In that configuration the curvature of the cable couples the two response directions and places part of the in-line response at the cross-flow frequency, so the over-prediction and the geometric coupling act together. To separate the effect of the low mass ratio from that of the geometry, the model is applied here to a vertical cable in uniform flow, which has a straight static shape.

The validation reference is the FLOTANT project, an experimental study of the vortex-induced vibration of a submarine inter-array power cable for floating offshore wind [31]. The cable is a 1:3 scaled model with an outer diameter of 48 mm and a tested length of 6.75 m, suspended vertically from a towing carriage with a 200 kg clump weight at the lower end [81, 31]. The carriage towed the cable at constant speed, giving a uniform inflow normal to the cable, with tow speeds from 0.025 to 1.0 m/s. The mass ratio is $m^* = 1.50$, again low and below the calibration range, comparable to the lazy wave cable. The in-line and cross-flow response are reported as A_{rms}/D at the $L/2$, $3L/4$ and L stations [81, 31]. The OrcaFlex model, the natural frequencies and the full set of station results are given in Appendix E.

5.5.1 Response

Figure 5.16 shows the in-line and cross-flow response at station $L/2$ against the reduced velocity $U_r = V/(f_1 D)$, with $f_1 = 1.77$ Hz as in the report [31]. The cross-flow response forms two branches, the first on mode 2 and the second on mode 3. The model over-predicts both. The first peak reaches $A_{\text{rms}}/D \approx 1.1$ against a measured 0.78, and the second branch rises to ≈ 1.55 , above its own first peak, while in the experiment the second peak falls below the first at 0.56. The over-prediction is present on both branches, and the second branch is the more strongly over-predicted of the two. The in-line response is smaller and follows the same two branches. The model matches the first in-line peak, about 0.24 against a measured 0.23, but over-predicts the second branch, about 0.23 against 0.11.

The model places the first peak at a higher reduced velocity than the experiment, at $U_r \approx 7.4$ against a measured 5.7. Two effects most likely contribute. The Strouhal number sets where the synchronisation band falls on the reduced-velocity axis. The low mass ratio then widens that band, so the response stays locked past the shedding crossing and reaches its largest amplitude at the upper edge rather than near the crossing. Both are examined in Appendix E.

The model in-line response also shows a small early peak near $U_r = 2$ to 3, where the model lifts off in both directions. The cross-flow frequency there sits near half of f_1 , so its even harmonic falls on the mode-3 frequency and raises this small in-line resonance, which is absent in the experiment.

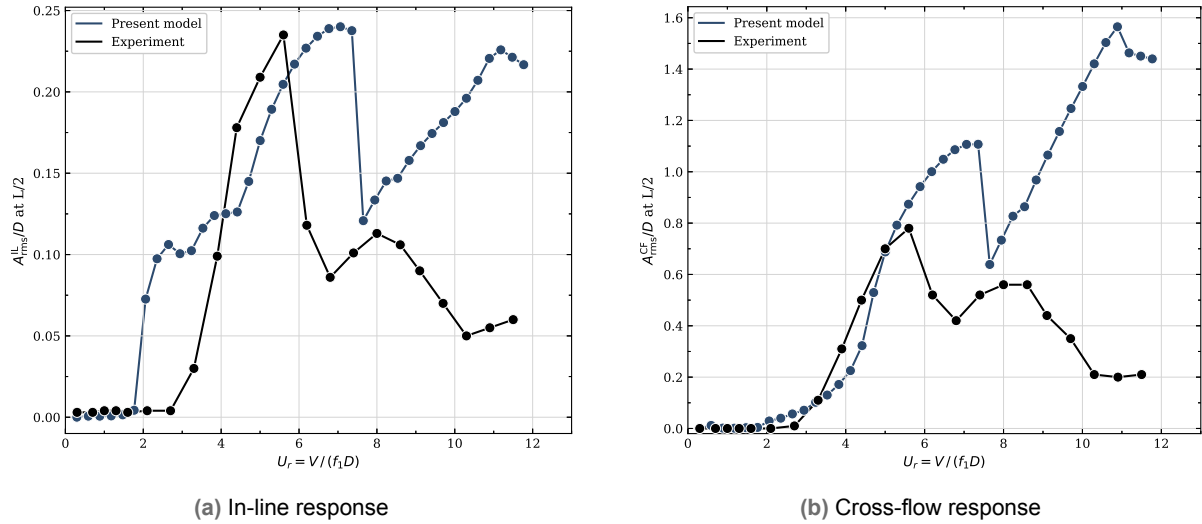


Figure 5.16. In-line and cross-flow response at station $L/2$ against reduced velocity

The straight cable confirms that removing the curvature does not remove the over-prediction. With no curvature to couple the directions, the in-line response sits at twice the cross-flow frequency, the expected 2:1 ratio, unlike the lazy wave cable where the curvature placed it at the cross-flow frequency. The over-prediction persists all the same, so it follows from the low mass ratio and the wake parameters A , ε and κ calibrated on rigid cylinders well above this range.

This is consistent with the comparison models on the same cable. Elrick (2025) [24] applied the Milan and Iwan and Blevins models to the FLOTANT cable, both at $St = 0.20$. They place the first peak closer to the measured position than the present model, at $U_r \approx 5.8$ and 4.9 against the measured 5.7 , but $St = 0.2$ is the stationary smooth-cylinder value and is not representative of this cable in combined motion at subcritical Reynolds number, where the effective Strouhal number is lower. Their closer peak position therefore follows from a Strouhal value that places the band correctly rather than from a better description of the shedding. On amplitude all three over-predict the first cross-flow peak, by about 40% for the present model, 70% for the Milan model and more than 100% for the Iwan and Blevins model, so the coupled model is the least over-predicting of the three.

5.6 Conclusion

The coupled model reproduces the frequency content and the leading modes of the lazy wave cable, concentrating the response in a single mode where the experiment spreads it across neighbours, and it recovers the in-line response at the cross-flow frequency produced by the curved geometry, though it places more energy in the $2f_{CF}$ harmonic than the experiment. The response is most sensitive to the Strouhal number and the lift coefficient, and the vertical cable showed the over-prediction is not only geometric, so the wake coefficients A , ε and κ , calibrated on rigid higher-mass-ratio cylinders, play an important part. The current direction study marks the limit of normal-only forcing at high inclination, which must be accounted for in a lazy wave cable, whose inclination varies along the span. Against the Milan and Iwan and Blevins models, the coupled model predicts the dominant frequency and mode more closely and is the only one with a physical in-line mechanism, yet all three over-predict the amplitude.

6

Conclusions & Recommendations

6.1 Conclusions

The primary objective of this thesis was to evaluate whether a coupled cross-flow and in-line wake oscillator model, implemented via an external function in OrcaFlex, can improve the prediction of VIV for a lazy wave dynamic power cable. The answer is given through the three research questions, addressed in turn below.

RQ1: Implementation and Validation

How can the coupled CF-IL wake-oscillator model be implemented within an industrial dynamic analysis solver and validated against a reference experiment before its application to a lazy-wave power cable?

- The model was implemented in two ways. Within the industrial solver it was added as a Python external function coupled to OrcaFlex, exchanging the node kinematics and the hydrodynamic forces at each time step. An independent reference was built as a full Python code that solves the wake and the structure simultaneously. The OrcaFlex coupling runs at a lower computational cost than the full Python solver. Both model the top spring explicitly, so the tension grows from its still-water value as the structure deflects.
- The two implementations were verified against each other and validated against a tensioned vertical riser [30] at two top tensions. At 810 N, the value used by Qu and Metrikine (2021) [28], they agree with each other, with the experiment and with the published predictions, in both directions.
- At the lower tension of 389 N, not covered by that earlier validation, the cross-flow amplitude is under-predicted, while the in-line response is captured better than in the single-degree-of-freedom cross-flow only studies of Slingsby (2015) [33] and Ntroumpis (2017) [32]. The coupled formulation recovers it directly, through an in-line forcing those models lack and the explicit spring that reproduces the tension growth driving the mean deflection. The cross-flow under-prediction is most likely linked to the wake parameters A , ε and κ calibrated on rigid cylinders and to the fixed added mass coefficient $C_a = 1$.
- The response is sensitive to the mean tension at the transitional velocities, where a small deviation moves it onto a different mode and the two implementations diverge. This grows at the higher velocities of the lower tension case, where the larger deflection most likely widens the difference between the two static equilibria the implementations are built about.
- With these limitations identified, the model qualifies as a verified and validated tool for the VIV response of flexible risers.

RQ2: Accuracy on the Lazy Wave Cable

How accurately does the coupled model reproduce the measured VIV response of a low-mass-ratio lazy-wave power cable, in terms of amplitude, frequency content and modal pattern?

- The model reproduces the single primary frequency shared by both directions, which follows from the geometric coupling of the curved cable and matches the experiment, with the in-line response carrying this cross-flow frequency through the projection of the curved geometry.
- The model also places significant in-line content at $2f_{CF}$, a feature mainly of straight flexible risers that the experiment does not show, which over-predicts the in-line response and places it at mid-span rather than where the experiment responds. Conversely, on the straight vertical cable the in-line sits at $2f_{CF}$, the expected 2:1 ratio, and there the in-line amplitude is reproduced more closely, with the first in-line peak matched.
- The cross-flow mode that carries the response is reproduced, with the nodes at the same positions and the shape matching the experiment. The difference is in the amplitude. Its magnitude is over-predicted by about a factor of two, most likely from the wake parameters A , ε and κ calibrated on rigid higher-mass-ratio cylinders. Its distribution also differs, concentrated near the ends in the model while the experiment spreads it with a mid-span antinode, most likely from the missing spanwise coupling of the wake.
- The vertical cable isolates the low mass ratio. Removing the curvature does not remove the over-prediction, so it follows from the low mass ratio together with these rigid-cylinder parameters and not from the geometry alone.
- A sensitivity study showed the over-prediction is multi-factorial. The structural damping, the lift coefficient and the Strouhal number each change the response but none removes the over-prediction cleanly. The lift coefficient has a stronger effect than the damping, both scaling the amplitude, while the Strouhal number selects the responding mode.
- The current direction study marks the limit of the normal-only forcing at high inclination. This must be taken into account for a lazy wave cable, whose inclination varies along the span, unlike a near-vertical riser in a horizontal current.

RQ3: Comparison with the Industry-Standard Models

How do the coupled model's predictions compare with those of the industry-standard single-degree-of-freedom wake-oscillator models for the same cable, and in which response characteristics does the coupling change the outcome?

- Against the Milan and the Iwan and Blevins models, the coupled model is the most suitable of the three for a lazy wave cable without test data. It places the response on the measured frequency and mode, while the two comparison models, fixed at $St = 0.2$, shed at a higher frequency and move onto a higher structural mode. Its Strouhal number can instead be

set for each case. This matters under real offshore conditions, where the combined in-line and cross-flow motion and the Reynolds regime lower the effective value well below 0.2, to about 0.13 to 0.15 for this cable, so the shedding response of an untested field cable can be represented while the fixed value of the comparison models cannot.

- It is the only one of the three with a physical in-line forcing. The in-line peak at f_{CF} is recovered by all three through the geometric projection of the curved cable, but only the coupled model adds a modelled in-line mechanism on top of it.
- Each case runs at a higher computational cost than the two comparison models, but within a time acceptable for design use.
- All three over-predict the cross-flow amplitude by about a factor of two, so the amplitude is the one characteristic the coupling does not improve.

6.2 Recommendations

The findings above point to several directions for further work.

- More experiments on lazy wave and catenary power cables are needed. The comparison here is based on a single speed and a single cable, the only case with clear single-frequency vortex-induced vibration. Tests at representative Reynolds number across a wider range of speeds would allow a broader assessment of the model.
- The wake parameters A , ε and κ should be recalibrated for low-mass-ratio cables, as they were calibrated on rigid higher-mass-ratio cylinders. Fitting them against measured low-mass-ratio response would address this, which links this recommendation to the experiments above.
- A direct coupling of the wake along the span could be added. In the present model the wake oscillators are linked only through the structure, not through the fluid. This addition would be expected to represent the response better, in particular for a curved cable, where the inclination changes along the span and neighbouring sections are subjected to a different flow.
- The normal-only forcing could be replaced by a formulation valid at high inclination, as the current direction study showed the independence principle reaches its limit where the flow aligns with parts of the cable. A lazy wave cable spans a range of inclinations, so it is more exposed to this limit than a near-vertical configuration.
- The extension to combined wave and current loading could be investigated. OrcaFlex already carries the wave kinematics, so the external function could include the wave-induced relative velocity in the forcing. This extends the experimental work recommended above.

The coupled model is a step towards more accurate VIV prediction for floating offshore wind, setting the response on the right frequency and mode and recovering the in-line motion. It still over-predicts the amplitude, so it does not yet remove the conservative margins of current practice.

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This appendix gives the full finite element implementation of the coupled structure and wake oscillator system summarised in Section 4.2, following the formulation of Qu and Metrikine (2021) [28] and the finite element solution structure of Colomés et al. (2024) [82]. It provides the detail needed to reproduce the full Python solver, in the order the solver operates, the beam and wake element formulations (Sections A.1, A.2), the global assembly and boundary conditions (Section A.3), the static equilibrium and local coordinate frame (Section A.4), the hydrodynamic force and wake forcing (Section A.5), and the Newmark-Newton time integration (Section A.6).

A.1 Beam Element Formulation

A.1.1 Discretisation and Degrees of Freedom

The flexible cylinder is modelled as a continuous body that deforms in three-dimensional space. A material coordinate $p \in [0, L]$ is introduced along the beam axis as a fixed label that identifies each point along the beam and remains unchanged during deformation. The spatial configuration of the beam is described by the centreline position vector:

$$\mathbf{r}(p) = \begin{bmatrix} x(p) & y(p) & z(p) \end{bmatrix}^\top \quad (\text{A.1})$$

which specifies the three-dimensional location of the material point p . The beam is a one-dimensional element parameterised by p , embedded in three-dimensional space, and can deform freely in all directions. In the undeformed configuration it is aligned with the z -axis, so that $\mathbf{r}(p) = [0, 0, p]^\top$. After deformation the coordinates x, y, z change with time while p continues to identify the same physical point, so the beam is represented as a continuous parametric curve in three-dimensional space.

Following the Absolute Nodal Coordinate Formulation with the lower-order cable element [66], the beam is discretised into n_e elements of equal length:

$$l_e = \frac{L}{n_e}. \quad (\text{A.2})$$

Each element connects two nodes, and each node carries 6 degrees of freedom, the nodal position vector and the slope vector:

$$\mathbf{u}_i = \begin{bmatrix} \mathbf{r}_i \\ \mathbf{r}_{i,p} \end{bmatrix} = \begin{bmatrix} x_i, y_i, z_i, x_{i,p}, y_{i,p}, z_{i,p} \end{bmatrix}^\top. \quad (\text{A.3})$$

This gives 12 degrees of freedom per element, collected in the element displacement vector $\mathbf{u}_e \in \mathbb{R}^{12}$:

$$\mathbf{u}_e = \begin{bmatrix} r_x^A, r_y^A, r_z^A, r_{p,x}^A, r_{p,y}^A, r_{p,z}^A, r_x^B, r_y^B, r_z^B, r_{p,x}^B, r_{p,y}^B, r_{p,z}^B \end{bmatrix}^\top. \quad (\text{A.4})$$

A.1.2 Shape Functions

In the nonlinear beam formulation the unknown is not a scalar displacement field but the position vector of the beam centreline, so the geometry itself is the primary unknown. The finite element approximation takes the form of:

$$\mathbf{r}(p) \approx \mathbf{S}(p) \mathbf{u}_e, \quad (\text{A.5})$$

where $\mathbf{S}(p)$ is the (3×12) shape function matrix that maps the 12 element degrees of freedom to the position vector. With the normalised coordinate $\xi = p/l_e \in [0, 1]$, the four scalar cubic Hermite functions are:

$$\begin{aligned} S_1(\xi) &= 2\xi^3 - 3\xi^2 + 1, \\ S_2(\xi) &= l_e(\xi^3 - 2\xi^2 + \xi), \\ S_3(\xi) &= -2\xi^3 + 3\xi^2, \\ S_4(\xi) &= l_e(\xi^3 - \xi^2). \end{aligned} \quad (\text{A.6})$$

These satisfy the Kronecker-delta property for position and slope at the element nodes, so at $p = 0$ the interpolation recovers the degrees of freedom of the first node and at $p = l_e$ those of the second. Between the nodes $\mathbf{S}(p)$ gives a smooth cubic interpolation of the deformed configuration. The first and second derivatives $\mathbf{S}_{,p}$ and $\mathbf{S}_{,pp}$ are also required, and follow from differentiating S_1 to S_4 with respect to p . The axial stretch is computed from $\mathbf{S}_{,p}$ and the bending curvature from $\mathbf{S}_{,pp}$, and both enter the elemental stiffness matrix.

A.1.3 Elemental Matrices

Following the derivations of Qu and Metrikine (2021) [28], the elemental mass, stiffness, tangent stiffness and damping matrices are obtained as follows.

Mass matrix

The elemental mass matrix is:

$$\mathbf{M}_s^e = \int_0^{l_e} (m_0 + m_a) \mathbf{S}^\top \mathbf{S} dp, \quad (\text{A.7})$$

where

- m_0 is the structural mass per unit length,
- $m_a = C_a \rho_f \pi D^2/4$ is the potential added mass per unit length, with $C_a = 1$, acting on the submerged length of the structure.

Since $\mathbf{S}$ depends only on p and l_e , the mass matrix is constant and computed once before the time loop.

Stiffness matrix

The elemental stiffness matrix is nonlinear and reads:

$$\mathbf{K}_s^e = \int_0^{l_e} \left(EI \gamma^2 \mathbf{Q}_1 - EI \gamma^4 \mathbf{Q}_3 \mathbf{u} \mathbf{u}^\top \mathbf{Q}_1 - EI \gamma^4 (\mathbf{Q}_2^\top + \mathbf{Q}_2) \mathbf{u} \mathbf{u}^\top \mathbf{Q}_2 + 2EI \gamma^6 (\mathbf{u}^\top \mathbf{Q}_2 \mathbf{u})^2 \mathbf{Q}_3 + EA (1 - \gamma^1) \mathbf{Q}_3 \right) dp, \quad (\text{A.8})$$

where EI is the bending stiffness and EA the axial stiffness of the cable, and the auxiliary matrices are:

$$\mathbf{Q}_1 = \mathbf{S}_{,pp}^\top \mathbf{S}_{,pp}, \quad \mathbf{Q}_2 = \mathbf{S}_{,pp}^\top \mathbf{S}_{,p}, \quad \mathbf{Q}_3 = \mathbf{S}_{,p}^\top \mathbf{S}_{,p}. \quad (\text{A.9})$$

The stretch factor γ^n depends on the axial stretch $|\mathbf{S}_{,p} \mathbf{u}|$:

$$\gamma^n \approx (n+1) - n \sqrt{(\mathbf{S}_{,p} \mathbf{u})^\top (\mathbf{S}_{,p} \mathbf{u})}. \quad (\text{A.10})$$

The terms of the matrix represent the linear bending, the nonlinear bending contributions, and axial stretching, respectively. Unlike the mass matrix, $\mathbf{K}_s^e$ depends on the current displacement $\mathbf{u}$ through γ^n and the quadratic terms, so it is reassembled at each Newton iteration. The elemental internal force is $\mathbf{F}_{int}^e = \mathbf{K}_s^e \mathbf{u}$, which enters the structural residual during the time integration.

Tangent stiffness

The tangent stiffness matrix, required for the Newton-Raphson iteration, is the analytical derivative $\mathbf{J}_s^e = \partial(\mathbf{K}_s^e \mathbf{u})/\partial \mathbf{u}$:

$$\begin{aligned} \mathbf{J}_s^e = \int_0^{l_e} EI \bigg(& \gamma^2 \mathbf{Q}_1 - 2\gamma^4 \mathbf{Q}_1 \mathbf{u} \mathbf{u}^\top \mathbf{Q}_3 - \gamma^4 \left[(\mathbf{u}^\top \mathbf{Q}_1 \mathbf{u}) \mathbf{Q}_3 + \mathbf{Q}_3 \mathbf{u} \mathbf{u}^\top (\mathbf{Q}_1 + \mathbf{Q}_1^\top) \right] \\ & + 4\gamma^6 \mathbf{Q}_3 \mathbf{u} \mathbf{u}^\top \mathbf{Q}_1 \mathbf{u} \mathbf{u}^\top \mathbf{Q}_3 - \gamma^4 \left[(\mathbf{u}^\top \mathbf{Q}_2 \mathbf{u}) (\mathbf{Q}_2 + \mathbf{Q}_2^\top) + (\mathbf{Q}_2 + \mathbf{Q}_2^\top) \mathbf{u} \mathbf{u}^\top (\mathbf{Q}_2 + \mathbf{Q}_2^\top) \right] \\ & + 4\gamma^6 (\mathbf{Q}_2 + \mathbf{Q}_2^\top) \mathbf{u} \mathbf{u}^\top \mathbf{Q}_2 \mathbf{u} \mathbf{u}^\top \mathbf{Q}_3 + 2\gamma^6 \left[2\mathbf{Q}_3 \mathbf{u} \mathbf{u}^\top \mathbf{Q}_2^\top \mathbf{u} \mathbf{u}^\top (\mathbf{Q}_2 + \mathbf{Q}_2^\top) + (\mathbf{u}^\top \mathbf{Q}_2^\top \mathbf{u})^2 \mathbf{Q}_3 \right] \\ & - 12\gamma^8 (\mathbf{u}^\top \mathbf{Q}_2^\top \mathbf{u})^2 \mathbf{Q}_3 \mathbf{u} \mathbf{u}^\top \mathbf{Q}_3 \bigg) + EA \left[(1 - \gamma^1) \mathbf{Q}_3 + \gamma^3 \mathbf{Q}_3 \mathbf{u} \mathbf{u}^\top \mathbf{Q}_3 \right] dp. \end{aligned} \quad (\text{A.11})$$

Damping matrix

The elemental damping uses the Rayleigh model:

$$\mathbf{C}_s^e = \alpha \mathbf{M}_s^e + \beta \mathbf{J}_s^{e,eq}, \quad (\text{A.12})$$

where α and β are the Rayleigh coefficients and $\mathbf{J}_s^{e,eq}$ is the tangent stiffness above, evaluated at the static equilibrium configuration. The damping matrix is therefore computed once at equilibrium and kept fixed, while the full nonlinear tangent stiffness is used inside the Newton loop during the time integration.

A.1.4 Gauss Quadrature

Because the stretch factor γ depends on the element coordinates $\mathbf{u}$ inside the integrals, $\mathbf{u}$ cannot be factored out and the stiffness and tangent matrices cannot be integrated analytically [28]. Analytical integration of the remaining matrices is expensive, so the integrals are all evaluated numerically with 3-point Gauss quadrature. The three Gauss point locations within an element of length l_e are:

$$p_1 = \frac{l_e}{2} \left(1 - \sqrt{\frac{3}{5}} \right), \quad p_2 = \frac{l_e}{2}, \quad p_3 = \frac{l_e}{2} \left(1 + \sqrt{\frac{3}{5}} \right), \quad (\text{A.13})$$

with weights:

$$w_1 = \frac{5}{18}l_e, \quad w_2 = \frac{8}{18}l_e, \quad w_3 = \frac{5}{18}l_e. \quad (\text{A.14})$$

A generic elemental matrix is then approximated as:

$$\mathbf{A}^e \approx \sum_{g=1}^3 w_g f(p_g). \quad (\text{A.15})$$

A.2 Wake Element Formulation

A.2.1 Discretisation and Shape Functions

The wake variable $q(p, t)$ is discretised on the beam mesh with the same Hermite shape functions as the structure [28]:

$$q(p, t) = \mathbf{S}_q(p) \mathbf{q}^e, \quad (\text{A.16})$$

where

- $\mathbf{q}^e = [q_A, q_{A,p}, q_B, q_{B,p}]^\top$ is the vector of wake degrees of freedom at the element nodes,
- $\mathbf{S}_q = [S_1 \ S_2 \ S_3 \ S_4]$ is the (1×4) row vector of shape functions

Each node carries 2 wake degrees of freedom, the wake variable and its spatial derivative, giving 4 per element.

A.2.2 Elemental Matrices

Following the derivations of Qu and Metrikine (2021) [28], the elemental wake mass, damping and stiffness matrices are obtained as follows.

Mass matrix

The elemental wake mass matrix is:

$$\mathbf{M}_q^e = \int_0^{l_e} \mathbf{S}_q^\top \mathbf{S}_q dp. \quad (\text{A.17})$$

Since $\mathbf{S}_q$ depends only on p and l_e and the mesh is uniform, $\mathbf{M}_q^e$ is identical for all elements and precomputed once.

Damping matrix

The elemental wake damping matrix is:

$$\mathbf{C}_q^e = \int_0^{l_e} \varepsilon \omega_s \left(\mathbf{q}^{e\top} \mathbf{S}_q^\top \mathbf{S}_q \mathbf{q}^e - 1 \right) \mathbf{S}_q^\top \mathbf{S}_q dp. \quad (\text{A.18})$$

Since $\mathbf{C}_q^e$ depends on the current wake variable q , which changes at every Newton iteration, it is reassembled each time.

Stiffness matrix

The elemental wake stiffness matrix is:

$$\mathbf{K}_q^e = \int_0^{l_e} \omega_s^2 \mathbf{S}_q^\top \mathbf{S}_q dp - \kappa \int_0^{l_e} \frac{\omega_s^4 D \ddot{r}_n}{\omega_s^4 D^2 + \ddot{r}_n^2} \mathbf{S}_q^\top \mathbf{S}_q dp, \quad (\text{A.19})$$

where $\ddot{r}_n = (\mathbf{S} \ddot{\mathbf{u}}_e) \cdot \mathbf{n}^{eq}$ is the structural acceleration in the in-line direction and $\mathbf{n}^{eq}$ is the in-line frame vector defined in Section A.4. The first term depends only on ω_s and is linear. The second is nonlinear, introducing the parametric coupling with the structural motion, representing how the in-line acceleration of the cylinder modifies the wake. Since $\mathbf{K}_q^e$ depends on the structural acceleration, it is reassembled at each Newton iteration.

A.3 Assembly and Boundary Conditions

A.3.1 Global Assembly

The elemental matrices are combined into the global system following the standard finite element assembly procedure [82]. Each element contributes its local matrices to the global rows and columns of its two nodes, and at the node shared by two adjacent elements these contributions add, which couples the elements into one continuous system. The global mass, damping and stiffness matrices are assembled from their elemental contributions $\mathbf{M}_s^e$, $\mathbf{C}_s^e$ and $\mathbf{K}_s^e$, and the global internal force from the elemental products $\mathbf{F}_{int}^e = \mathbf{K}_s^e \mathbf{u}_e$, and likewise for the wake. After

assembly the material coordinate p is no longer needed, since it served only to construct the elemental matrices through the Gauss integration. The global displacement vector $\mathbf{u} \in \mathbb{R}^{6n_n}$, with n_n the number of nodes, holds three translations and three slope components at every node.

A.3.2 Boundary Conditions and Applied Loads

Boundary conditions are applied by identifying the fixed and free degrees of freedom and reducing the global system to the free degrees of freedom, eliminating the corresponding rows and columns. Applied loads enter through the external force vector. A load that depends on the configuration also contributes to the tangent stiffness through its derivative with respect to $\mathbf{u}$, and is recomputed at each Newton iteration. Loads that are constant, such as gravity and buoyancy, are added to the external force vector once. No conditions are imposed on the wake degrees of freedom.

A configuration-dependent load arises, for example, when the top tension is applied through a spring, so that the tension increases as the top node moves under deflection. The spring force at the top node is:

$$F_{spring}^z = T_{t0} + K_s (z_{ref} - z_{top}), \quad (\text{A.20})$$

where T_{t0} is the pre-tension, K_s the spring stiffness, z_{ref} the reference position and z_{top} the current axial position of the top node. The constant part $T_{t0} + K_s z_{ref}$ is added to the external force vector once, and the configuration-dependent part $-K_s z_{top}$ is recomputed each Newton iteration, its derivative adding $+K_s$ to the diagonal of the tangent stiffness at the top axial degree of freedom.

A.4 Static Equilibrium and Local Coordinate Frame

A.4.1 Static Equilibrium

Before the time integration, the static equilibrium of the structure is solved on the assembled and constrained system of Section A.3. With the inertia and damping terms absent, equilibrium requires the internal and external forces to balance:

$$\mathbf{F}_{int}(\mathbf{u}) - \mathbf{F}_{ext}(\mathbf{u}) = \mathbf{g}(\mathbf{u}) = \mathbf{0}, \quad (\text{A.21})$$

where $\mathbf{g}$ is the residual. It is found by Newton iteration on the free degrees of freedom. At iteration k the increment $\Delta \mathbf{u}^{(k)}$ follows from the global tangent stiffness $\mathbf{J}_s$, assembled from the elemental $\mathbf{J}_s^e$ of Section A.1:

$$\mathbf{J}_s \Delta \mathbf{u}^{(k)} = -\mathbf{g}(\mathbf{u}^{(k)}), \quad (\text{A.22})$$

and the configuration is updated as:

$$\mathbf{u}^{(k+1)} = \mathbf{u}^{(k)} + \Delta \mathbf{u}^{(k)}, \quad (\text{A.23})$$

repeated until the residual falls below tolerance. Since the loads and the stiffness depend on the configuration, the residual and the tangent are reassembled at each iteration. The converged configuration $\mathbf{u}^{eq}$ is the starting point for the time integration and the configuration on which the local frame is built.

A.4.2 Local Coordinate Frame

The local coordinate frame is defined at the equilibrium configuration $\mathbf{u}^{eq}$ at each Gauss point. At Gauss point g of element e the tangent vector follows from:

$$\mathbf{r}_{,p}^{eq}(p_g) = \mathbf{S}_{,p}(p_g) \mathbf{u}_e^{eq}, \quad (\text{A.24})$$

and is normalised to give $\mathbf{t}^{eq}$. The flow velocity is:

$$\mathbf{V}^{eq} = V \hat{\mathbf{v}}, \quad (\text{A.25})$$

where V is the flow speed and $\hat{\mathbf{v}}$ the unit vector of the flow direction, which can point anywhere in three-dimensional space. The normal direction $\mathbf{n}^{eq}$ is the component of $\mathbf{V}^{eq}$ in the plane normal to $\mathbf{t}^{eq}$, normalised, and the binormal is $\mathbf{b}^{eq} = \mathbf{t}^{eq} \times \mathbf{n}^{eq}$. The Strouhal frequency ω_s follows from the normal velocity. Since the frame depends on $\mathbf{V}^{eq}$, a different flow speed or direction requires recomputing it before the time integration. The frame is computed once from the equilibrium configuration and frozen for the whole simulation, then reused at every time step and Newton iteration when assembling the wake matrices and the wake forcing.

A.5 Hydrodynamic Force and Wake Forcing

A.5.1 Hydrodynamic Force on the Structure

The hydrodynamic force on the structure has three contributions, the cross-flow lift force, the mean drag force, and the in-line oscillating force. At each Gauss point, three velocity quantities are computed from the current nodal velocities $\dot{\mathbf{u}}_e$. The relative flow velocity in the plane normal to the beam axis is:

$$\mathbf{U}_N = \mathbf{V}_N^{eq} - \left(\mathbf{S} \dot{\mathbf{u}} - (\dot{\mathbf{u}}^\top \mathbf{S}^\top \mathbf{t}^{eq}) \mathbf{t}^{eq} \right), \quad (\text{A.26})$$

the in-line component of the relative velocity is:

$$\mathbf{U}_{NX} = \mathbf{V}_N^{eq} - \left((\mathbf{n}^{eq})^\top \mathbf{S} \dot{\mathbf{u}} \right) \mathbf{n}^{eq}, \quad (\text{A.27})$$

and the lift direction unit vector is:

$$\mathbf{D}_{VL} = \frac{\mathbf{t}^{eq} \times \mathbf{U}_N}{|\mathbf{t}^{eq} \times \mathbf{U}_N|}. \quad (\text{A.28})$$

The frame vectors $\mathbf{t}^{eq}$, $\mathbf{n}^{eq}$, $\mathbf{V}_N^{eq}$ are frozen at equilibrium, so only $\mathbf{S}\dot{\mathbf{u}}$ changes at each time step and Newton iteration. The elemental hydrodynamic force vector $\mathbf{F}^e \in \mathbb{R}^{12}$ follows from:

$$\begin{aligned} \mathbf{F}^e = & \frac{1}{2} \rho D \frac{C_{L0}}{2} \int_0^{l_e} |\mathbf{U}_N|^2 \mathbf{S}^\top \mathbf{D}_{VL} \mathbf{S}_q \mathbf{q}_e dp \\ & + \frac{1}{2} \rho D C_{DM} \int_0^{l_e} |\mathbf{U}_N| \mathbf{S}^\top \mathbf{U}_N dp \\ & + \frac{1}{2} \rho D \alpha \frac{C_{L0}^2}{4} \int_0^{l_e} (\mathbf{S}_q \mathbf{q}_e)^2 |\mathbf{U}_{NX}| \mathbf{S}^\top \mathbf{U}_{NX} dp. \end{aligned} \quad (\text{A.29})$$

The assembled global hydrodynamic force enters the structural residual as an external force.

A.5.2 Morison Drag Outside the Shedding Region

The vortex force applies where vortex shedding is active. Where the shedding frequency is zero, the vortex force is replaced by a Morison drag. This occurs on parts of the structure that are submerged but see no local current, where there is no shedding but the surrounding still water still resists the structural motion. The drag captures this resistance through:

$$\mathbf{F}_{drag}^e = \frac{1}{2} \rho D C_{DM} \int_0^{l_e} |\mathbf{U}_N| \mathbf{S}^\top \mathbf{U}_N dp. \quad (\text{A.30})$$

A.5.3 Wake Forcing

The forcing of the wake oscillator is:

$$\mathbf{R}^e = \frac{A}{D} \int_0^{l_e} \mathbf{S}_q^\top \ddot{\mathbf{u}}^\top \mathbf{S}^\top \mathbf{b}^{eq} dp. \quad (\text{A.31})$$

This is the structural acceleration projected onto the cross-flow direction $\mathbf{b}^{eq}$, weighted by the wake shape functions, and is the forcing on the right-hand side of the wake equation.

These three force vectors are evaluated by the same 3-point Gauss quadrature as the elemental matrices, and reassembled at each Newton iteration since they depend on the structural velocity or acceleration.

A.6 Newmark-Newton Time Integration

After assembly, the coupled discretised system is:

$$\mathbf{M}_s \ddot{\mathbf{u}} + \mathbf{C}_s \dot{\mathbf{u}} + \mathbf{K}_s(\mathbf{u}) \mathbf{u} = \mathbf{F}_{ext}(\mathbf{u}) + \mathbf{F}^e(\mathbf{u}, \dot{\mathbf{u}}, \mathbf{q}), \quad (\text{A.32})$$

$$\mathbf{M}_q \ddot{\mathbf{q}} + \mathbf{C}_q(\mathbf{q}) \dot{\mathbf{q}} + \mathbf{K}_q(\ddot{\mathbf{u}}) \mathbf{q} = \mathbf{R}^e(\ddot{\mathbf{u}}). \quad (\text{A.33})$$

Both equations are nonlinear and coupled. The system is advanced with the implicit Newmark method, $\beta = 1/4$ and $\gamma = 1/2$, with the structure and wake solved together at each step. The equilibrium configuration, the local frame, and the constant mass and damping matrices from the earlier sections are computed once before the time loop and reused throughout.

A.6.1 Predictors

At each step $n \rightarrow n + 1$ the Newmark predictors are computed first, the estimate of the new state at zero acceleration increment:

$$\mathbf{u}^{pred} = \mathbf{u}_n + \Delta t \dot{\mathbf{u}}_n + \Delta t^2 \left(\frac{1}{2} - \beta \right) \ddot{\mathbf{u}}_n, \quad (\text{A.34})$$

$$\dot{\mathbf{u}}^{pred} = \dot{\mathbf{u}}_n + \Delta t(1 - \gamma)\ddot{\mathbf{u}}_n, \quad (\text{A.35})$$

and likewise for $\mathbf{q}$. The Newton iteration starts from $\mathbf{u}_{n+1}^{(0)} = \mathbf{u}^{pred}$ and $\mathbf{q}_{n+1}^{(0)} = \mathbf{q}^{pred}$.

A.6.2 Newton Iteration

At iteration k the Newmark correctors give the velocity and acceleration consistent with the current iterate:

$$\ddot{\mathbf{u}}_{n+1} = a_1 \left(\mathbf{u}_{n+1}^{(k)} - \mathbf{u}^{pred} \right), \quad a_1 = \frac{1}{\beta \Delta t^2}, \quad (\text{A.36})$$

$$\dot{\mathbf{u}}_{n+1} = \dot{\mathbf{u}}^{pred} + b_1 \left(\mathbf{u}_{n+1}^{(k)} - \mathbf{u}^{pred} \right), \quad b_1 = \frac{\gamma}{\beta \Delta t}, \quad (\text{A.37})$$

and likewise for $\mathbf{q}$. The state-dependent matrices and forces are assembled and the residuals computed:

$$\mathbf{R}_s = \mathbf{M}_s \ddot{\mathbf{u}}_{n+1} + \mathbf{C}_s \dot{\mathbf{u}}_{n+1} + \mathbf{F}_{int}(\mathbf{u}_{n+1}^{(k)}) - \mathbf{F}_{ext}(\mathbf{u}_{n+1}^{(k)}) - \mathbf{F}^e(\mathbf{u}_{n+1}^{(k)}, \dot{\mathbf{u}}_{n+1}, \mathbf{q}_{n+1}^{(k)}), \quad (\text{A.38})$$

$$\mathbf{R}_q = \mathbf{M}_q \ddot{\mathbf{q}}_{n+1} + \mathbf{C}_q(\mathbf{q}_{n+1}^{(k)}) \dot{\mathbf{q}}_{n+1} + \mathbf{K}_q(\ddot{\mathbf{u}}_{n+1}) \mathbf{q}_{n+1}^{(k)} - \mathbf{R}^e(\ddot{\mathbf{u}}_{n+1}). \quad (\text{A.39})$$

The step is accepted when both residuals fall below tolerance. Otherwise the correction follows from the Jacobian of the coupled residual:

$$\mathbf{J} = \begin{bmatrix} \mathbf{J}_{s,u} & \mathbf{J}_{s,q} \\ \mathbf{J}_{q,u} & \mathbf{J}_{q,q} \end{bmatrix}, \quad (\text{A.40})$$

whose blocks, with the Newmark chain rules $\partial \ddot{\mathbf{u}} / \partial \mathbf{u} = a_1$ and $\partial \dot{\mathbf{u}} / \partial \mathbf{u} = b_1$, are:

$$\mathbf{J}_{s,u} = a_1 \mathbf{M}_s + b_1 \mathbf{C}_s + \mathbf{J}_s^{tan} - \frac{\partial \mathbf{F}^e}{\partial \mathbf{u}}, \quad (\text{A.41})$$

$$\mathbf{J}_{s,q} = -\frac{\partial \mathbf{F}^e}{\partial \mathbf{q}}. \quad (\text{A.42})$$

$$\mathbf{J}_{q,u} = a_1 \left(\frac{\partial (\mathbf{K}_q \mathbf{q})}{\partial \ddot{\mathbf{u}}} - \frac{\partial \mathbf{R}^e}{\partial \ddot{\mathbf{u}}} \right), \quad (\text{A.43})$$

$$\mathbf{J}_{q,q} = a_1 \mathbf{M}_q + b_1 \mathbf{C}_q + \mathbf{K}_q + \frac{\partial \mathbf{C}_q}{\partial \mathbf{q}} \dot{\mathbf{q}}_{n+1}. \quad (\text{A.44})$$

The correction is obtained from:

$$\mathbf{J} \begin{bmatrix} \delta \mathbf{u} \\ \delta \mathbf{q} \end{bmatrix} = - \begin{bmatrix} \mathbf{R}_s \\ \mathbf{R}_q \end{bmatrix}, \quad (\text{A.45})$$

and the iterates are updated as $\mathbf{u}_{n+1}^{(k+1)} = \mathbf{u}_{n+1}^{(k)} + \delta \mathbf{u}$ and $\mathbf{q}_{n+1}^{(k+1)} = \mathbf{q}_{n+1}^{(k)} + \delta \mathbf{q}$, with the boundary conditions re-enforced after each update. The iteration repeats until both residuals converge, and the solution is then advanced to the next step.

The off-diagonal blocks $\mathbf{J}_{s,q}$ and $\mathbf{J}_{q,u}$ are the structure-wake coupling of the Jacobian. Assembling them requires the cross derivatives of the hydrodynamic force and the wake forcing at each iteration, so to reduce this cost they are dropped and only the diagonal blocks $\mathbf{J}_{s,u}$ and $\mathbf{J}_{q,q}$ are retained, and the structure and wake corrections are solved separately. The coupling remains in the residuals $\mathbf{R}_s$ and $\mathbf{R}_q$, which are driven to zero in full, so the converged solution is unchanged and only the rate of convergence is affected.

B | OrcaFlex VIV Toolbox

The model comparison in Chapter 5 set the coupled model against two wake oscillator models, the Milan model and the Iwan and Blevins model, both taken from the OrcaFlex VIV Toolbox [80]. This appendix sets out their formulations and the parameter values used. In OrcaFlex the Milan model has been validated against the same Delta Flume riser tests used for the validation in Chapter 4, where it predicted the cross-flow response with good accuracy across the speed range and was at least as accurate as the other methods, which included CFD and frequency-domain codes [80].

Both are single-degree-of-freedom wake oscillator models that run in the time domain. A single variable represents the wake behind the cylinder and follows a near-linear second-order equation tuned to reproduce the features of vortex shedding [80]. The wake variable drives a lift force normal to the cylinder axis and to the flow, and the cylinder motion feeds back into the wake equation, so the two form a coupled nonlinear system. The parameters carry no direct physical meaning and are fitted to measured response, an inverse method rather than a derivation from first principles [80].

Two features are relevant for the comparison in this thesis. First, both models act in the transverse direction only. They give the lift force only and leave the effect of vortex shedding on the drag to the standard Morison force [80]. Second, OrcaFlex attaches one oscillator to each node and links them only through the structure, not through the fluid [80]. The interaction between vortex shedding at different points along the line is therefore taken to act through the structure alone, the same assumption made in the coupled model of this thesis. Both models were calibrated for steady flow with a default Strouhal number of 0.2 [80].

B.1 Milan Wake Oscillator

The Milan model follows the formulation of Falco, Fossati and Resta (1999), implemented in OrcaFlex [80]. The equations and schematics below follow the presentation of Alvino (2018) [83]. At each node the wake is represented by an oscillator of mass m_o , connected to the cylinder through a nonlinear spring and damper, as in Figure B.1. The cylinder of sectional mass m_s carries its own linear stiffness k and damping c , and moves in both the transverse and the in-line direction.

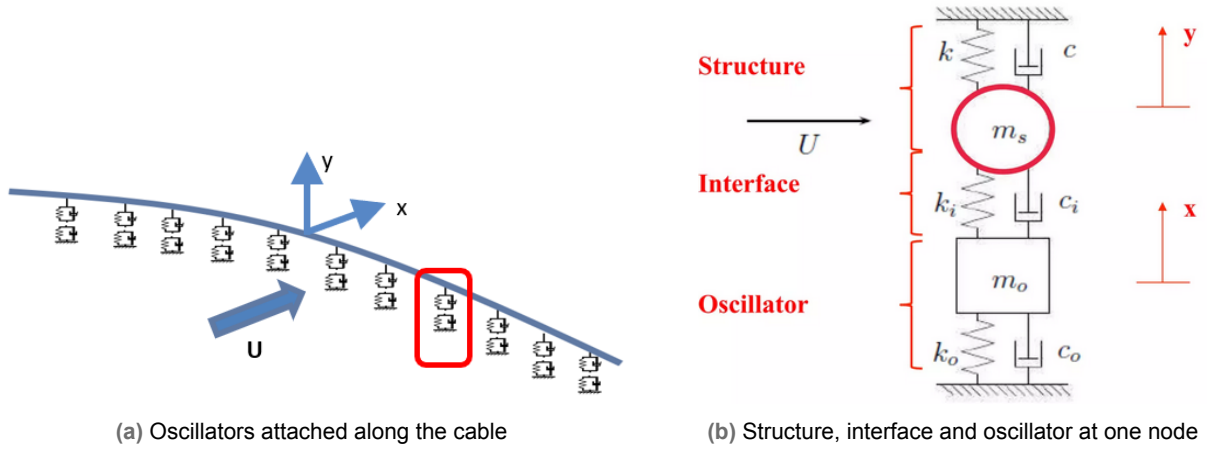


Figure B.1. Milan wake oscillator model [83]

The wake force F_i enters the transverse equation only, and the in-line equation is driven by the Morison force alone:

$$m_s \ddot{y}_c + c \dot{y}_c + k y_c = F_i + F_{my}, \quad (\text{B.1})$$

$$m_s \ddot{x}_c + c \dot{x}_c + k x_c = F_{mx}, \quad (\text{B.2})$$

with y_c and x_c the cylinder displacement in the transverse and in-line directions. The Morison force in each direction uses the relative velocity between the flow and the cylinder:

$$F_{my} = -C_A \rho \frac{\pi D^2}{4} \ddot{y}_c - C_D \rho \frac{D}{2} \sqrt{(U + \dot{x}_c)^2 + \dot{y}_c^2} \dot{y}_c, \quad (\text{B.3})$$

$$F_{mx} = -C_A \rho \frac{\pi D^2}{4} \ddot{x}_c - C_D \rho \frac{D}{2} \sqrt{(U + \dot{x}_c)^2 + \dot{y}_c^2} \dot{x}_c, \quad (\text{B.4})$$

with C_A the added mass coefficient, C_D the drag coefficient, ρ the fluid density, D the diameter and U the combined current and wave velocity. The wake oscillator of displacement y_o and mass m_o follows:

$$m_o \ddot{y}_o = F_o - F_i, \quad (\text{B.5})$$

with the oscillator and interface forces carrying the nonlinearity as a third-order model in the stiffness and damping:

$$F_o = -k_{ol} y_o + k_{on} y_o^3 + c_{ol} \dot{y}_o - c_{on} \dot{y}_o^3, \quad (\text{B.6})$$

$$F_i = -k_{il} y_{rel} + k_{in} y_{rel}^3 - c_{il} \dot{y}_{rel} + c_{in} \dot{y}_{rel}^3, \quad (\text{B.7})$$

where $y_{rel} = y_c - y_o$ is the relative displacement, the subscripts l and n mark the linear and nonlinear terms, and o and i the oscillator and interface springs and dampers. The shedding frequency and the oscillator mass are:

$$\omega_s = 2\pi S \frac{U}{D}, \quad m_o = \frac{k_{ol} + k_{il}}{\omega_s^2}, \quad (\text{B.8})$$

with S the Strouhal number, so that the oscillator frequency matches the shedding frequency when the cylinder is held still.

The stiffness and damping coefficients are made non-dimensional through the relations on the left below, with the calibrated values on the right [83]. They were fitted to forced-vibration tests of a transversely vibrating linearly supported cylinder.

$$\begin{aligned} C_{k_{ol}} &= \frac{k_{ol} D}{\frac{1}{2} \rho U^2 D}, & C_{k_{il}} &= \frac{k_{il} D}{\frac{1}{2} \rho U^2 D}, \\ C_{c_{ol}} &= \frac{c_{ol} \omega_s D}{\frac{1}{2} \rho U^2 D}, & C_{c_{il}} &= \frac{c_{il} \omega_s D}{\frac{1}{2} \rho U^2 D}, \\ C_{k_{on}} &= \frac{k_{on} D^3}{\frac{1}{2} \rho U^2 D}, & C_{k_{in}} &= \frac{k_{in} D^3}{\frac{1}{2} \rho U^2 D}, \\ C_{c_{on}} &= \frac{c_{on} (\omega_s D)^3}{\frac{1}{2} \rho U^2 D}, & C_{c_{in}} &= \frac{c_{in} (\omega_s D)^3}{\frac{1}{2} \rho U^2 D}. \end{aligned}$$

Table B.1. Milan coefficients [83]

Oscillator		Interface	
$C_{k_{ol}}$	3.1	$C_{k_{il}}$	1.364
$C_{k_{on}}$	0.05	$C_{k_{in}}$	0.2
$C_{c_{ol}}$	2.15	$C_{c_{il}}$	5.4
$C_{c_{on}}$	0.45	$C_{c_{in}}$	0.0

In the OrcaFlex implementation the transverse Morison drag is added to the wake force rather than replaced by it, so the response depends on the transverse drag coefficient [80]. The coefficients above are the default values in OrcaFlex, recommended unless experimental or other data are available for calibration [80].

B.2 Iwan and Blevins Wake Oscillator

This model is based on the paper of Iwan and Blevins (1974) [84] and represents the wake through a van der Pol equation. The main difference from the Milan model is that its vortex force already carries the transverse Morison drag [80]. The force on the cylinder depends on the velocity and the acceleration of the flow relative to the cylinder, and enters as a linear function of the wake variable z . The model acts in the transverse direction only and is shown in Figure B.2.

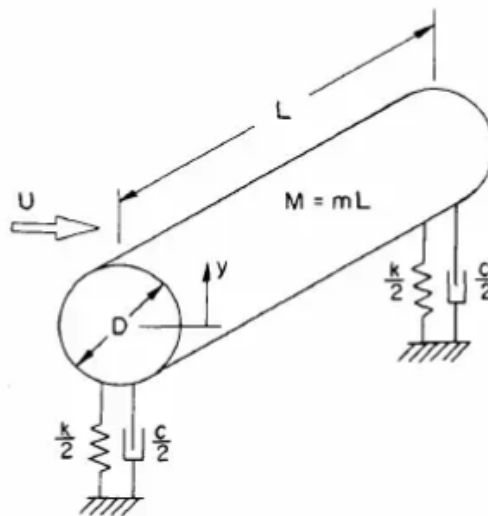


Figure B.2. Iwan and Blevins wake oscillator model [84]

At each node i the wake is represented by a transverse variable z_i , coupled to the transverse cylinder motion y_{ci} . The cylinder follows:

$$m_{ci} \ddot{y}_{ci} + c \dot{y}_{ci} + k y_{ci} = F_{vyi}, \quad (\text{B.9})$$

with the vortex force from the wake variable:

$$F_{vyi} = a_3 \rho D^2 dL \ddot{z}_i + a_4 \rho U D dL \dot{z}_i, \quad (\text{B.10})$$

where dL is the segment length. The nodal mass and damping are:

$$m_{ci} = m_i + a_3 \rho D^2 dL, \quad c = 2\zeta_T \omega_n dL, \quad (\text{B.11})$$

$$\zeta_T = \zeta_{cr} + \zeta_f, \quad \zeta_f = \frac{a_4 \rho D U}{2m_i \omega_n}, \quad (\text{B.12})$$

with m_i the cylinder mass per unit length, ω_n the natural frequency, ζ_{cr} the structural damping ratio and ζ_f the fluid damping. The wake variable follows a van der Pol equation:

$$\ddot{z}_i + \omega_s^2 z_i = (a'_1 - a'_4) \frac{U}{D} \dot{z}_i - a'_2 \frac{\dot{z}_i^3}{UD} + a'_3 \ddot{y}_{ci} + a'_4 \frac{U}{D} \dot{y}_{ci}, \quad (\text{B.13})$$

with the reduced coefficients and the shedding frequency:

$$a'_i = \frac{a_i}{a_0 + a_3}, \quad i = 1, 2, 3, 4, \quad \omega_s = 2\pi S \frac{U}{D}, \quad (\text{B.14})$$

where S is the Strouhal number. The calibrated values are given in Table B.2.

Table B.2. Iwan and Blevins coefficients [84]

a_0	a_1	a_2	a_3	a_4
0.48	0.44	0.20	0	0.38

The values above are the default values in OrcaFlex, recommended unless experimental or other data are available for calibration [80].

C

External Function Integration

This appendix describes how the wake oscillator is integrated in time inside the OrcaFlex external function of Section 4.3.2. In that coupling OrcaFlex solves the structure and the external function solves the wake, so the only equation integrated here is the wake oscillator, advanced on its own with the explicit adaptive Runge-Kutta 5(4).

In the external function implementation, the wake oscillator equation of motion of Eq. (3.9) must be solved at each time step, repeated here for completeness:

$$\ddot{q} + \varepsilon\omega_s(q^2 - 1)\dot{q} + \omega_s^2 q - \kappa \frac{\omega_s^4 D \ddot{r}_n}{\omega_s^4 D^2 + \ddot{r}_n^2} q = \frac{A}{D} \ddot{r}_b. \quad (\text{C.1})$$

This is a second-order ODE in the wake variable $q(t)$, where the only independent variable is time. To apply a standard ODE solver, it is first transformed into a system of two coupled first-order ODEs. Given a second-order ODE of the form $\ddot{q} = f(t, q, \dot{q})$, the substitution $u = \dot{q}$ gives:

$$\frac{dq}{dt} = u, \quad (\text{C.2})$$

$$\frac{du}{dt} = f(t, q, u) = -\varepsilon\omega_s(q^2 - 1)u - \omega_s^2 q + \kappa \frac{\omega_s^4 D \ddot{r}_n}{\omega_s^4 D^2 + \ddot{r}_n^2} q + \frac{A}{D} \ddot{r}_b. \quad (\text{C.3})$$

The state vector is $\mathbf{y} = [q, u]^\top$, so the system reads $\dot{\mathbf{y}} = \mathbf{f}(t, \mathbf{y})$, the standard form required by the ODE solver. In practice this is a function that, given a time t and a state $\mathbf{y}$, returns the derivative $\dot{\mathbf{y}}$.

The system is integrated with the explicit Runge-Kutta 5(4) method of Dormand-Prince (RK45), implemented in `scipy.integrate.solve_ivp` [85]. The notation 5(4) denotes an embedded pair, a fifth-order formula that advances the solution and a fourth-order formula that estimates the error from the same stage evaluations. The difference between the two estimates the local truncation error and is used to adapt the step size, so the method keeps accuracy at low cost. The integration uses relative and absolute tolerances of 10^{-6} and 10^{-8} .

D Delta Flume: Additional Cases

This appendix gives the remaining validation velocities at both tension levels, 810 N and 389 N, completing the VIV response presented in Section 4.4.5.

D.1 Higher Tension (810 N)

This section gives the quantitative comparison for the 810 N set and the two transitional cases, $V = 0.63$ and 0.75 m/s. Following Huera-Huarte (2006) [30], the fundamental natural frequency f_1 at the mean top tension $\bar{T}_t$ during the steady-state window is the reference parameter, with the reduced velocity:

$$V_1 = \frac{V}{f_1 D} \quad (\text{D.1})$$

The V_1 values were used to read the dominant mode numbers, mean in-line deflection, and frequency ratios from the published figures of both references at the matching reduced velocity. These values are approximate, as the published results span a continuous range of V_1 . The two implementations differ in their FEM formulation, time integration, and equilibrium reference state, so they produce different $\bar{T}_t$ and different V_1 at the same flow velocity, as seen in Table D.1.

Table D.1. Mean tensions and reduced velocities for the OrcaFlex and ANCF implementations

V (m/s)	Source	f_1 (Hz)	$\bar{T}_t$ (N)	V_1	$\hat{y}/D$	$\hat{x}/D$
0.49	OrcaFlex	0.706	925	24.8	0.618	0.176
	ANCF	0.704	920	24.8	0.629	0.177
0.63	OrcaFlex	0.771	1087	29.2	0.830	0.241
	ANCF	0.755	1045	29.8	0.887*	0.267*
0.75	OrcaFlex	0.797	1155	33.6	0.648	0.569**
	ANCF	0.794	1146	33.8	0.527	0.173
0.85	OrcaFlex	0.849	1301	35.8	0.592	0.207
	ANCF	0.847	1295	35.8	0.623	0.196

* Extracted from stable sub-window due to mode transition ** Reflects multi-frequency IL response at transitional V_1

Table D.2. VIV response comparison across all sources

V (m/s)	Source	n_y	n_x	$\tilde{x}_{\max}/D$	f_y/f_1	f_x/f_1
0.49	OrcaFlex	4	7	4.17	4.05	7.27
	ANCF	4	7	4.01	4.06	7.90
	Qu & Metrikine	4	7	4.19	4.0	8.0
	Experiment	4	7	3.81	3.9	7.9
0.63	OrcaFlex	4	7	6.64	4.04	7.22
	ANCF	4	7	5.99	4.05	7.29
	Qu & Metrikine	4–5	7–9	6.03	4–5	8–10
	Experiment	4–5	8–9	5.49	4.2–5	8.2–10
0.75	OrcaFlex	6	11	7.52	6.13	11.74
	ANCF	6	11	7.32	6.17	12.02
	Qu & Metrikine	6	11	7.75	6.2	12.0
	Experiment	5–6	10–11	7.05	5.0	10.0
0.85	OrcaFlex	6	11	9.05	6.11	11.63
	ANCF	6	11	8.90	6.15	11.90
	Qu & Metrikine	6	11	8.68	6.2	12.0
	Experiment	6	11	7.91	6.1	12.1

The experiment showed that modal competition occurs when closely spaced natural frequencies cause adjacent lock-in branches to overlap, producing continually changing modal amplitudes and momentary large excursions during mode switching [30, 72]. The cases $V = 0.63$ and 0.75 m/s fall within such transitional regimes, shown in the cross-flow time series of Figure D.3.

At the two steady velocities both implementations predict the same dominant mode numbers as Qu and Metrikine (2021) [28] and the experiment, mode 4 in cross-flow and mode 7 in-line at 0.49 m/s, and mode 6 and mode 11 at 0.85 m/s (Table D.2). The in-line deflection $\tilde{x}_{\max}/D$ is slightly above the experiment at both velocities, 4.17 against 3.81 at 0.49 m/s and 9.05 against 7.91 at 0.85 m/s for OrcaFlex, the same over-prediction Qu and Metrikine (2021) [28] report. The frequency ratios f_y/f_1 and f_x/f_1 sit within a few percent of both references, the in-line response at roughly twice the cross-flow frequency, as expected for the coupled VIV.

At the transitional velocities $V = 0.63$ and 0.75 m/s the experiment does not lock to a single mode but drifts across a range, $n_y = 4$ -5 and 5 -6 respectively, as adjacent branches overlap. The present implementations settle within these ranges. At 0.63 m/s both reach mode 4 and agree, below the mode 5 shown in the experimental envelope of Figure D.1, with Qu and Metrikine (2021) [28] reporting 4-5. At 0.75 m/s both reach mode 6, matching the same reference, but the OrcaFlex response stays modulated rather than resolving to a single mode (Figure D.2). The settled mode depends on where the mean tension places the natural frequencies relative to the shedding frequency, which is also why the implementations can respond differently.

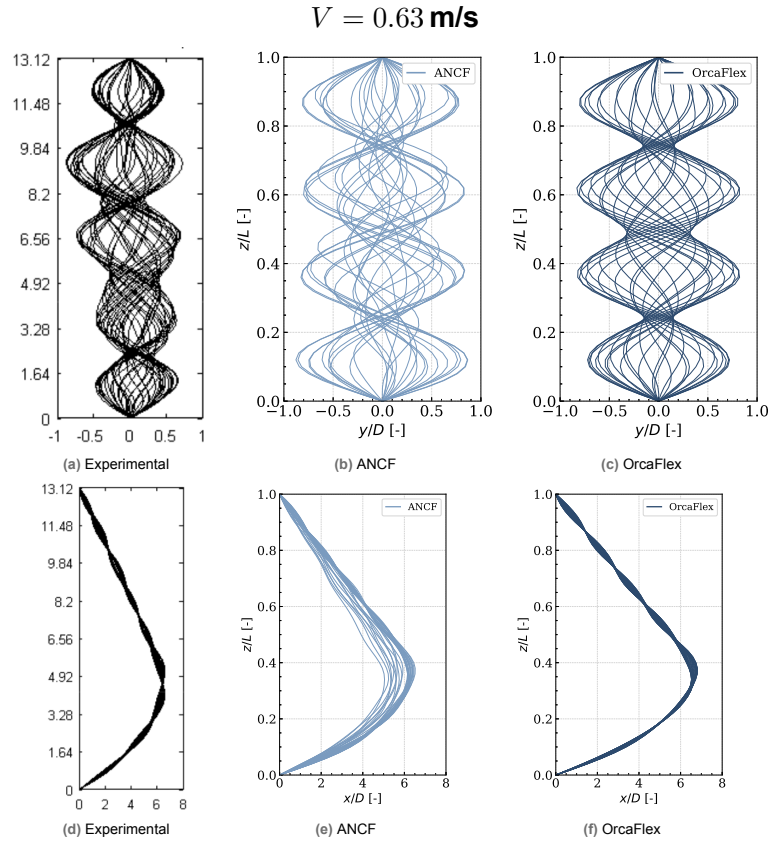


Figure D.1. Cross-flow (top) and in-line (bottom) displacement envelopes at $V = 0.63 \text{ m/s}$

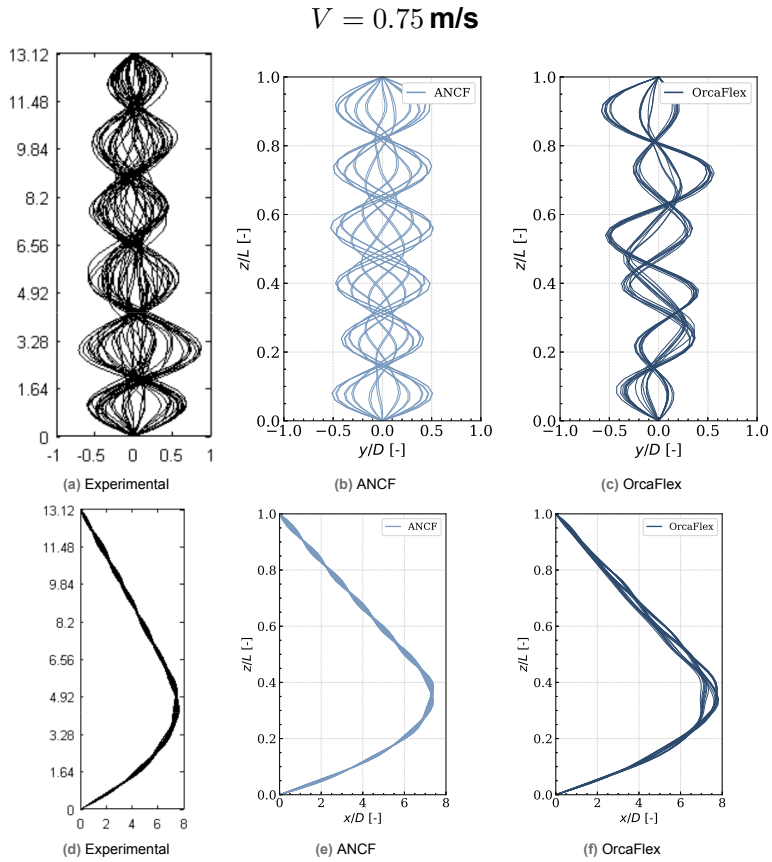


Figure D.2. Cross-flow (top) and in-line (bottom) displacement envelopes at $V = 0.75 \text{ m/s}$

At $V = 0.63$ m/s the ANCF cross-flow response grows gradually from $t = 0$ s with a modulated envelope, indicating competition between modes 4 and 5. At about $t = 50$ s the competition resolves and the response steps into a higher amplitude regime, with a brief excursion reaching $y/D \approx \pm 1.0$, consistent with the momentary large amplitudes during mode switching reported by Chaplin et al. (2005) [72]. The OrcaFlex model, from a drag-deflected equilibrium with a slightly different mean tension, resolves the competition earlier and settles before $t = 50$ s.

At $V = 0.75$ m/s the OrcaFlex cross-flow response, where the competition is between modes 5 and 6, does not resolve into a single dominant mode. The amplitude is continuously modulated, characteristic of a multi-frequency response when the reduced velocity falls in the overlap of two adjacent lock-in branches, so multiple frequency components coexist and the competition is never resolved [28]. The ANCF model, at a slightly lower mean tension and reduced velocity, settles into a single dominant mode and a stationary envelope. The difference at this velocity follows from the sensitivity of the transitional response to the mean top tension.

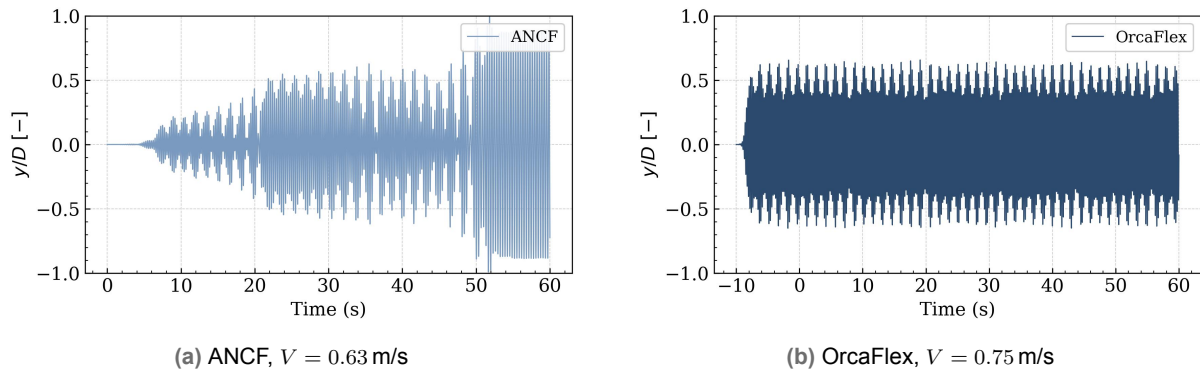


Figure D.3. Cross-flow time series at the transitional flow velocities

D.2 Lower Tension (389 N)

This section gives the two higher-velocity cases at 389 N, $V = 0.60$ and 0.95 m/s, in Figures D.4 and D.5. At these velocities the riser deflects far more under drag, and the mean top tension rises well above its still-water value, by different amounts in the two models, 718 N against 643 N at 0.60 m/s and 1179 N against 1115 N at 0.95 m/s for OrcaFlex and ANCF respectively, against measured mean tensions of 670 N and 1002 N.

The cross-flow under-prediction seen at the lower velocities in Section 4.4.5 persists here, with both implementations below the experimental cross-flow amplitude. The two also settle on different dominant modes that bracket the experiment, ANCF one mode above and OrcaFlex one below, mode 7 and mode 5 at 0.60 m/s against the experimental mode 6, and mode 8 and mode 6 at 0.95 m/s against roughly mode 7. This separation follows from the large drag deflection, which sets the vertical reference of the ANCF model apart from the drag-deflected equilibrium of OrcaFlex, so the two reach different mean tensions and lock in at different modes.

$V = 0.60 \text{ m/s}, T_t = 670 \text{ N}$

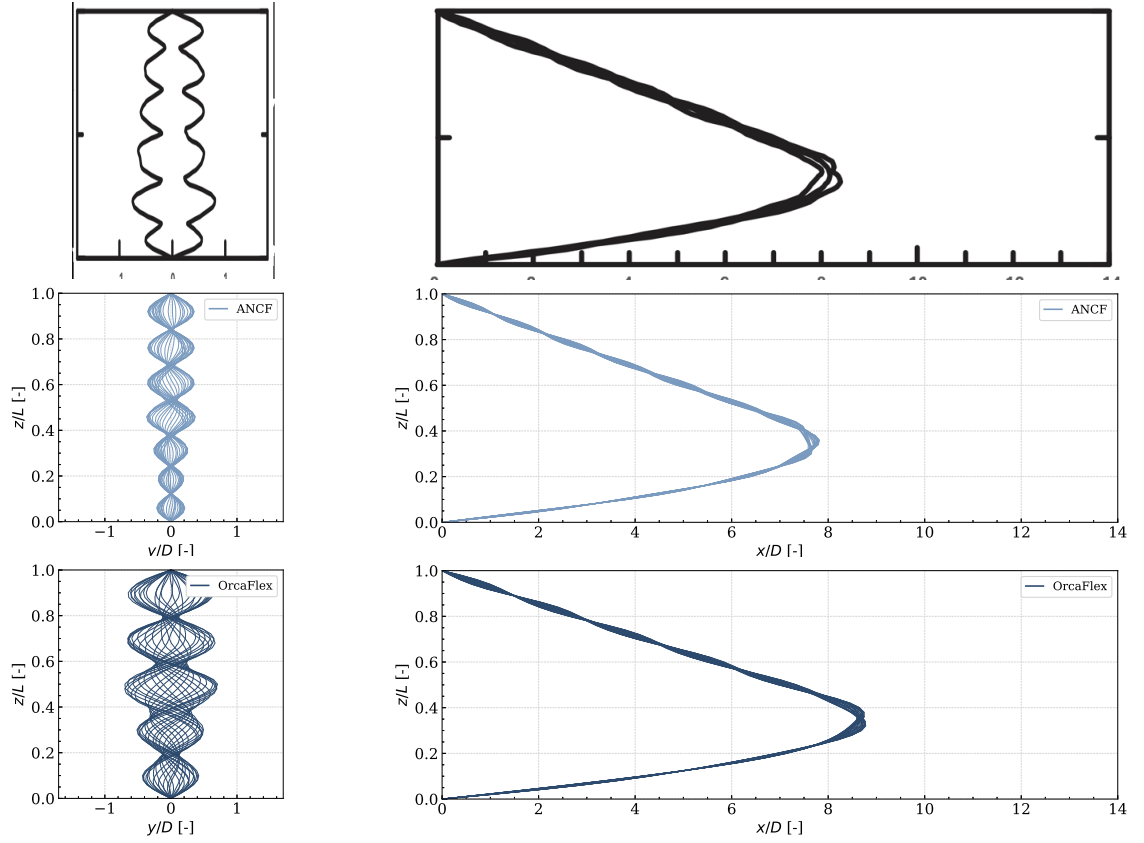


Figure D.4. Displacement envelopes for $V = 0.60 \text{ m/s}$, measured $\bar{T}_t = 670 \text{ N}$

$V = 0.95 \text{ m/s}, T_t = 1002 \text{ N}$

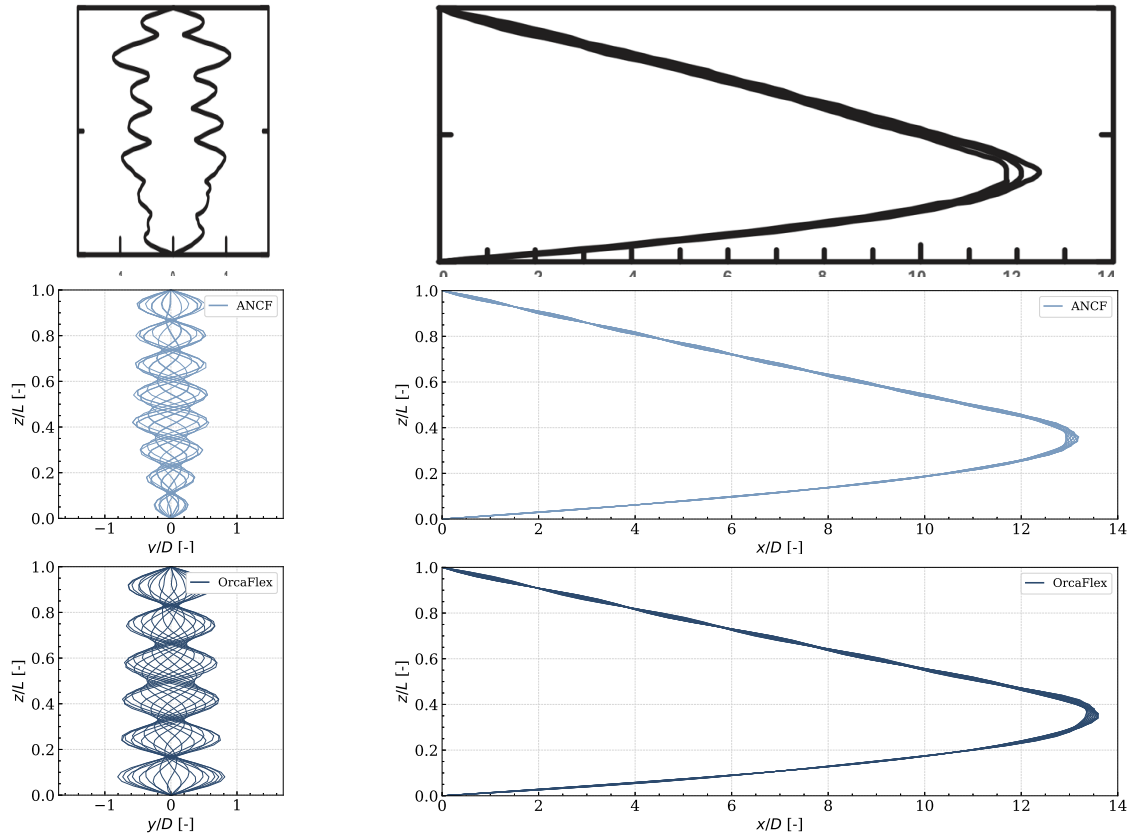


Figure D.5. Displacement envelopes for $V = 0.95 \text{ m/s}$, measured $\bar{T}_t = 1002 \text{ N}$

E

Vertical Cable Results

This appendix gives the full detail of the vertical cable case introduced in Section 5.5. It covers the OrcaFlex model, the natural frequencies, the response at the $3L/4$ and L stations, the frequency content, and the analysis of the peak position and the second branch.

E.1 OrcaFlex Model Description

The vertical cable was modelled in OrcaFlex as a line object, following the model proposed by Delizisis et al. (2022) [81], with structural properties and segmentation in Table E.1. The model represents the full suspended arrangement of the towing test rather than the cable alone. It is built from five line-type sections, an upper hinge, the cable, a lower hinge, a chain, and a weight clamp [81]. The cable section carries the structural properties of the 48 mm research cable, and the upper and lower hinges represent the end fittings that connect the cable to the carriage at the top and to the 200 kg clump weight at the bottom [81, 31].

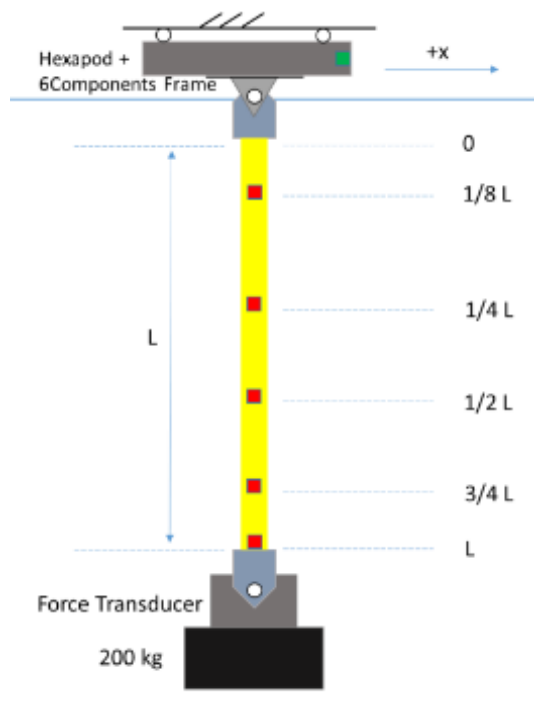
End A was modelled as a fixed connection at the top hang-off point, and End B was left free at the bottom, so the tension develops from the self-weight of the suspended sections and the clump weight, as in the experiment. The cable section was discretised into 88 segments with a target length of 0.077 m, within a full model of 100 segments over 7.842 m [81].

Delizisis et al. (2022) [81] do not report the axial stiffness of the cable and chain sections, which were set to 100 MN here, with all other properties taken from the same source. The wake oscillator parameters were the same as for the lazy wave and flexible riser cases, with $St = 0.14$ and an added mass coefficient $C_a = 1.0$. The VIV forcing was applied along the cable section only. The structural damping ratio was set to $\zeta = 2.8\%$ [81], applied through a bending Rayleigh coefficient of 0.00638. The simulations were run for 60 s with a time step of 0.001 s. For each simulation the first 20 s were discarded as the transient, and the steady-state response was taken from the remaining window.

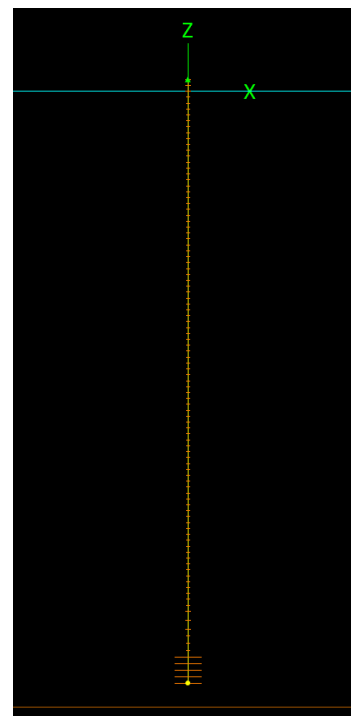
The test arrangement and the OrcaFlex model are shown in Figure E.1. The platform at the top imposed forced oscillations in a separate set of tests and was not active for the VIV measurements, where the carriage moved the cable along the basin to impose a uniform current [31].

Table E.1. Line types and segmentation of the OrcaFlex model

Section	L (m)	M (kg)	D (mm)	EA (MN)	EI (kNm ²)	Segments	Target (m)
Upper hinge	0.152	5.3	68	7.26×10^2	210	2	0.100
Cable	6.750	18.8	48	100	3.0	88	0.077
Lower hinge	0.230	8.0	68	7.26×10^2	210	2	0.100
Chain	0.370	11.6	40	100	0	4	0.100
Weight clamp	0.340	200	340	1.82×10^4	1.31×10^5	4	0.077
Total	7.842	–	–	–	–	100	–



(a) Experimental setup [31]



(b) OrcaFlex numerical model

Figure E.1. Vertical cable test setup and corresponding OrcaFlex model

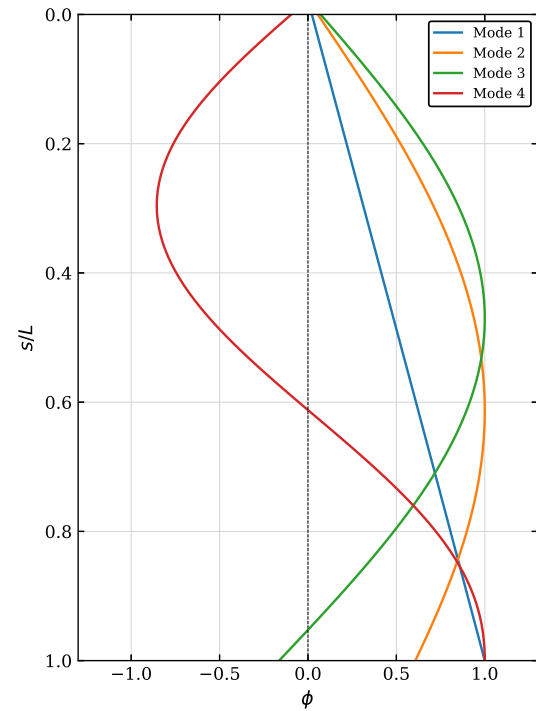
E.2 Natural Frequencies

A modal analysis was carried out about the static equilibrium to verify the dynamic properties before the VIV simulations. The first four frequencies are compared in Table E.2 against the numerical and experimental values of [81], with the mode shapes in Figure E.2. The model reproduces the first four frequencies within 4.2%, with the closest agreement at the lower modes. Delizisis et al. (2022) [81] note that their fourth numerical value is approximate and that the true fourth mode is likely above 3.07 Hz, and the present 3.20 Hz is on that side. The first mode is the pendulum mode of the suspended cable, and modes 2 to 4 are the first three elastic modes [81].

Table E.2. First four natural frequencies of the vertical cable in still water

Mode	Present OrcaFlex (Hz)	Delizisis et al. num. (Hz) [81]	Delizisis et al. exp. (Hz) [81]
1	0.15 (0.0%)	0.15	0.20
2	1.31 (0.8%)	1.32	1.30
3	1.77 (1.7%)	1.80	1.91
4	3.20 (4.2%)	3.07	> 3

The FLOTANT report [31] used a reference natural frequency of 1.77 Hz to compute the reduced velocity of the towing tests, and described the dominant cross-flow response as a half-sine mode with the lower end nearly stationary because of the large inertia of the clump weight. This value coincides with the present mode 3 at 1.77 Hz, which is a half-sine along the cable with a node at $s/L \approx 0.95$ and a short reversed segment over the lowest part. The lower end of the cable is not pinned, since the lower hinge, chain, and weight clamp hang below it, so the mode shape is not forced to zero there. The report does not provide exact mode shapes or natural frequencies and notes that a closer evaluation of the mode shape would require further analysis [31].

**Figure E.2.** First four mode shapes based on the OrcaFlex model

E.3 Response at Stations 3L/4 and L

Section 5.5 gave the response at station $L/2$. The $3L/4$ and L stations are given here in Figures E.3 and E.4, against the reduced velocity $U_r = V/(f_1 D)$ with $f_1 = 1.77$ Hz. The response is reported as A_{rms}/D , the standard deviation σ of the displacement normalised by the diameter, following the definition in the report [31].

At station $3L/4$ the cross-flow response forms two branches. The first peaks near $U_r = 7$ at $A_{\text{rms}}/D \approx 1.14$, and the second rises to ≈ 1.15 near $U_r = 11$. The experiment has a first peak of about 0.76 near $U_r = 5.5$ and a lower second peak of about 0.43 near $U_r = 8$. The model over-predicts both branches and places both peaks higher on the axis. The two model branches reach about the same level, while in the experiment the second falls below the first. The in-line first peak is about 0.22 near $U_r = 7$, close to the measured 0.22, and the in-line second peak is about 0.16 near $U_r = 9$ against a measured 0.11.

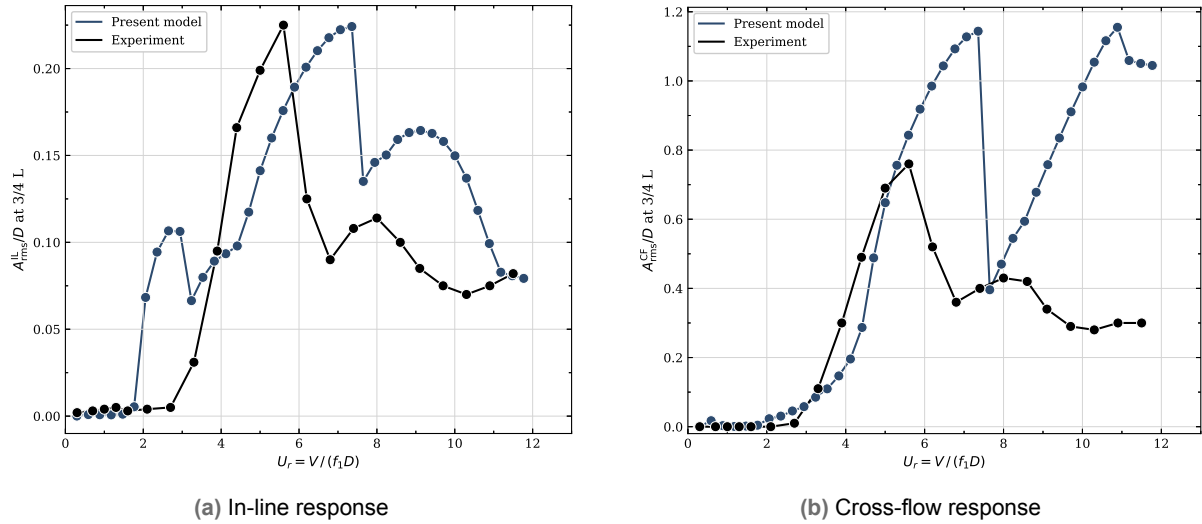


Figure E.3. In-line and cross-flow response at station $3L/4$ against reduced velocity

At station L the cross-flow response forms two branches with the second below the first. The first peaks near $U_r = 7$ at $A_{rms}/D \approx 0.83$, and the second reaches about 0.33 near $U_r = 8.8$. The experiment has a first peak of about 0.51 near $U_r = 5.5$ and a lower second peak of about 0.32 near $U_r = 11$. As at the upper stations, the model over-predicts the first peak and places it higher on the axis. The second branch is below the first in both model and experiment here, unlike at $L/2$. The in-line response at L rises gradually to about 0.26 near $U_r = 10$, while the experiment shows two small in-line peaks of about 0.08.

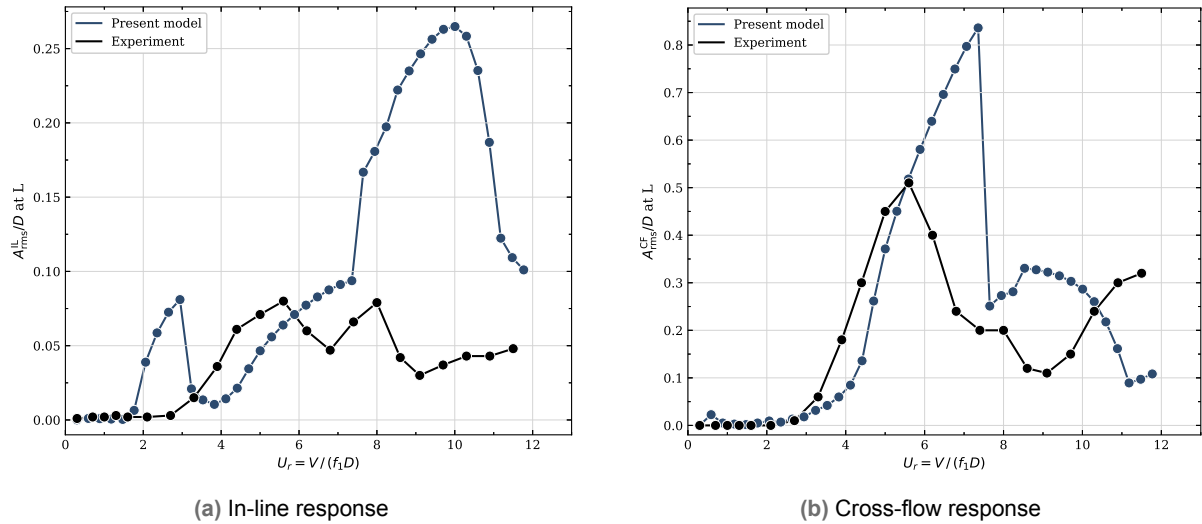


Figure E.4. In-line and cross-flow response at station L against reduced velocity

E.4 Frequency Content

The frequency content identifies which modes carry the response and relates the two amplitude branches to the natural frequencies. The spectra are shown at station $L/2$, representative of the response. A set of velocities was selected from the amplitude curves to give a spread of points along both branches, so the frequency content can be followed as the response moves from the first branch to the second. The spatial distribution is shown along the full cable length.

Cross-flow Frequency

Figure E.5 shows the cross-flow PSD and the peak cross-flow frequency against reduced velocity, with the frequency axis normalised by $f_1 = 1.77$ Hz. The still-water modes are marked at $f_2/f_1 = 0.74$ and $f_3/f_1 = 1.00$. At the lower speeds the spectrum has a single peak near f_2/f_1 . As the speed increases the peak moves to f_3/f_1 and then slightly above it, consistent with the response reported by [81]. Up to $U_r \approx 4$ the response follows the shedding line at $St = 0.14$, then locks to mode 2 near f_2 , before stepping toward mode 3.

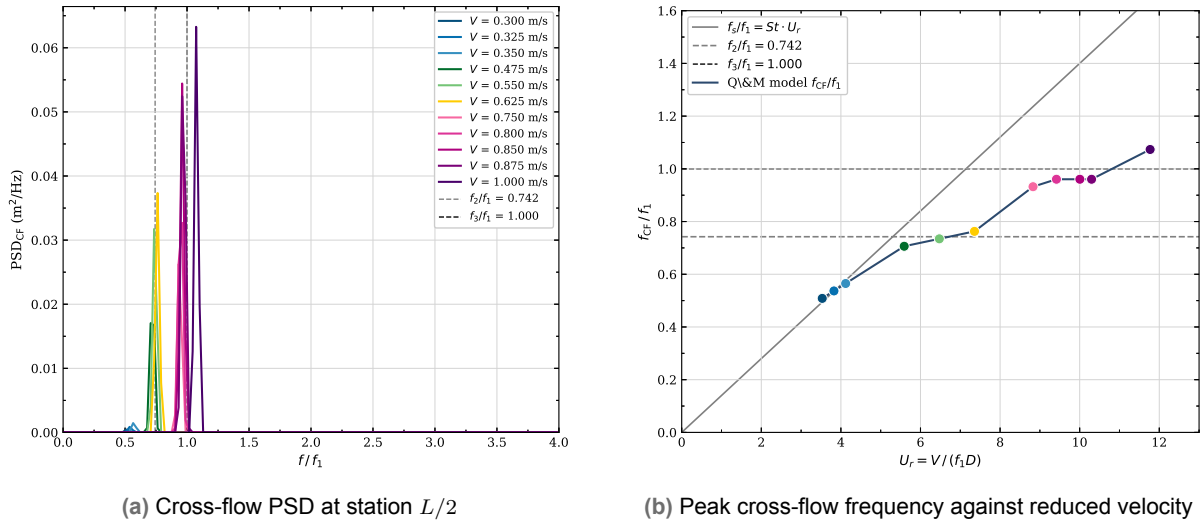


Figure E.5. Cross-flow frequency content at station $L/2$

In-line Frequency

Figure E.6 shows the in-line PSD at the same station. The in-line peak sits at twice the cross-flow frequency at all speeds, at $f/f_1 \approx 1.45$ to 1.5 on the first branch and ≈ 1.9 to 2.15 on the second. This is the 2:1 ratio of Section 5.5, the even-harmonic in-line force acting at twice the cross-flow shedding frequency on the straight cable. The small early peak near $f/f_1 = 1$ reaches $A_{rms}/D \approx 0.10$ at $U_r \approx 2.3$ against about 0.005 in the experiment, sitting at the mode-3 frequency where the even harmonic of the early cross-flow response at $\approx 0.5 f_1$ falls.

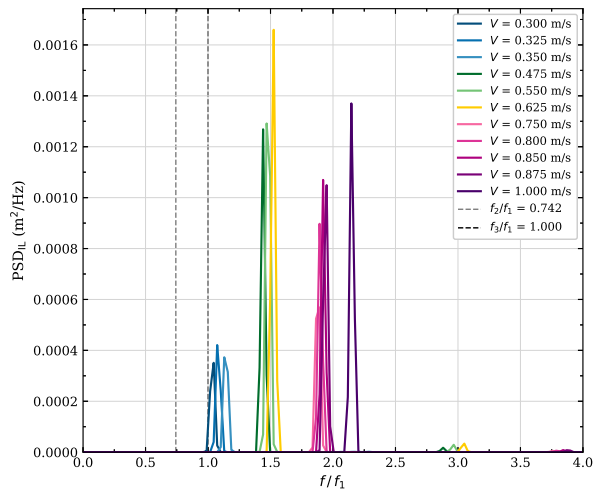


Figure E.6. In-line PSD at station $L/2$

E.5 Lock-in Position and Over-prediction

Section 5.5 noted that the model places the first cross-flow peak higher on the reduced-velocity axis than the experiment, at $U_r \approx 7.4$ against a measured 5.7. Two effects set this position. The first is the Strouhal number, which fixes where the synchronisation branch starts. The shedding line at $St = 0.14$ meets f_2 at $U_r = 5.3$ and f_3 at $U_r = 7.1$. At $St = 0.2$ the same crossings move down to approximately $U_r = 3.7$ and 5.0, so the branch starts lower on the axis and the peak comes earlier. This is what the Milan and Iwan and Blevins models show, both run at $St = 0.20$ by Elrick (2025) [24], with their first peaks closer to the measured one.

The second is the low mass ratio, which widens the branch. A low mass ratio is associated with a wider lock-in range, not seen in high-mass-ratio systems [55]. In the model this is expected to act through the acceleration coupling, since a lighter structure accelerates more for the same hydrodynamic force, which widens the entrainment range of the wake oscillator. Across this wide branch the response keeps growing and peaks near its upper edge, so the first peak sits at $U_r = 7.4$ rather than at the crossing at $U_r = 5.3$.

The response forms two branches, mode 2 then mode 3, separated by a dip where it desynchronises, at $U_r \approx 6.8$ in the experiment and ≈ 7.5 in the model. On the first branch the model follows the measured rise, and its first peak of $A_{\text{rms}}/D \approx 1.1$ is above the measured 0.78. The branches then diverge. The measured second branch is weaker than its first and decays toward the top speed, reaching about 0.56, while the model re-locks on mode 3 and climbs to ≈ 1.55 , above its own first peak and still rising at the highest speed tested. The over-prediction is largest here, where the model re-synchronises on the second branch far more strongly than the experiment.

The fixed added mass does not account for the offset in peak position. At low mass ratio the effective added mass drops within lock-in, raising the response frequency to about 1.4 to 1.5 times the still-water value [56]. The present model fixes $C_a = 1$ and does not reproduce this rise. A varying added mass would lower the effective inertia and raise the response frequency, and synchronisation at that higher frequency would need a higher flow speed, moving the peak further up the reduced-velocity axis rather than toward the experiment. The fixed $C_a = 1$ therefore does not explain the offset, which is also seen in the Milan and Iwan and Blevins models that share the same fixed added mass yet peak near the measurement at $St = 0.2$ [24].

A similar shift appears in Qu (2019) [64], where a cross-flow-only model, different from the coupled formulation used here, is validated against the free-vibration measurements of Khalak and Williamson (1997) [55] and places its peak at $V_n \approx 6$ against a measured $V_n \approx 5$. The shift is attributed there to the drag, lift and Strouhal parameters being calibrated on forced-vibration tests under different conditions. The model and calibration differ from those used here, but the parallel suggests the selected parameters may contribute to the offset in the present case as well.

The response is therefore sensitive to several parameters together, the Strouhal number, the wake coefficients A , ε and κ , the lift and drag coefficients, and the mass ratio, which set both the reduced velocity of the peak and its amplitude. No single adjustment aligns the response, which over-predicts the amplitude throughout. The measured response frequency is not available, so the comparison rests on the peak position and the amplitude shape.

E.6 Spatial Distribution

Figure E.7 shows the cross-flow and in-line RMS envelopes along the cable, with $x/L = 0$ at the hang-off and $x/L = 1$ at the lower end. The pendulum mode was not excited, so the response is carried by the elastic modes.

In the cross-flow direction the envelope forms a single antinode as the speed increases. At $V = 0.625$ m/s it takes the shape of mode 2, with its antinode at $x/L \approx 0.65$ and a peak of $A_{\text{rms}}/D \approx 1.17$. At $V = 1.00$ m/s it takes the shape of mode 3, with its antinode at $x/L \approx 0.45$, a peak of ≈ 1.45 , and a node near $x/L \approx 0.9$ with a short reversed segment below it. The lower end does not go to zero, consistent with the end not being pinned. The in-line envelope carries one more node than the cross-flow envelope, consistent with the in-line response sitting at twice the cross-flow frequency, and its peak grows from ≈ 0.12 at the lowest speeds to ≈ 0.26 at the highest.

The mode-3 shape sets the second-branch response at the three stations. Station $L/2$ sits near the antinode and carries the largest response, $3L/4$ lies between the antinode and the node, and L sits below the node in the reversed segment, where the response is smallest. The shedding frequency meets mode 4 at $V \approx 1.1$ m/s, above the top tested speed, so the response stays on modes 2 and 3.

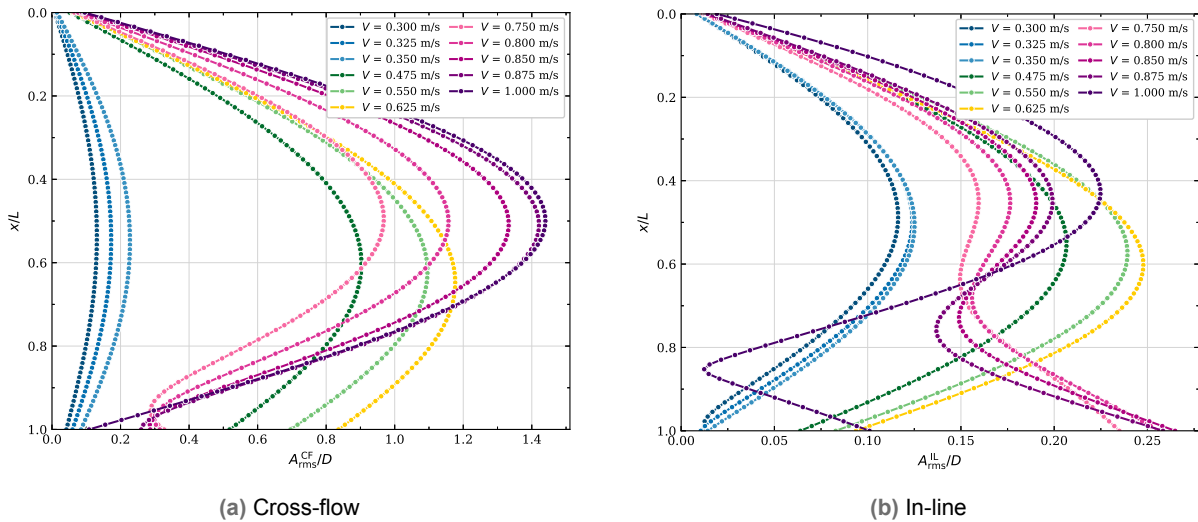


Figure E.7. RMS displacement envelopes along the cable

F

Modal Decomposition Method

This appendix sets out the modal decomposition used to identify the modal content of the response in Section 5.4.1, in both the experimental data and the model predictions, on equal terms. The method is the classical modal decomposition used for flexible risers, applied to the vertical riser in the Delta Flume experiment by Huera-Huarte (2006) [30] and more recently by Zhu et al. (2023) [76]. It is adapted here for a lazy wave cable, where the geometry is curved and only part of the span is instrumented.

F.1 Classical Formulation

The dynamic displacement of the cable is written as a weighted sum of a finite set of structural mode shapes. Each mode shape is a fixed spatial pattern and each weight is a function of time:

$$\mathbf{r}(s, t) = \sum_{n=1}^N q_n(t) \phi_n(s) \quad (\text{F.1})$$

Here s is the arc length along the cable, $\phi_n(s)$ is the shape of mode n , $q_n(t)$ is its weight at time t , and N is the number of retained modes. The representation is exact only for a linear system. The cable response and the underlying system are nonlinear, so the weights $q_n(t)$ are a best linear fit rather than the true modal participation.

For a straight vertical riser the cross-flow and in-line directions coincide with two fixed global axes along the whole span, so each direction is decomposed on its own. Sampling (F.1) at M sensor positions $z_1, \dots, z_M$ along the riser gives a linear system in the modal weights. For the cross-flow case [76]:

$$\begin{bmatrix} y(z_1, t) \\ y(z_2, t) \\ \vdots \\ y(z_M, t) \end{bmatrix} = \begin{bmatrix} \phi_1(z_1) & \phi_2(z_1) & \cdots & \phi_N(z_1) \\ \phi_1(z_2) & \phi_2(z_2) & \cdots & \phi_N(z_2) \\ \vdots & \vdots & & \vdots \\ \phi_1(z_M) & \phi_2(z_M) & \cdots & \phi_N(z_M) \end{bmatrix} \begin{bmatrix} q_1(t) \\ q_2(t) \\ \vdots \\ q_N(t) \end{bmatrix} \quad (\text{F.2})$$

which in compact form reads:

$$\mathbf{Y} = \Phi \mathbf{q}(t) \quad (\text{F.3})$$

where $\mathbf{Y}$ is the measured cross-flow displacement at the M sensors, Φ is the matrix of mode shapes sampled at the sensors, and $\mathbf{q}(t)$ is the vector of modal weights. The number of sensors M must be at least the number of modes N , so $M \geq N$. When $M = N$ the matrix Φ is square and the weights follow by direct inversion [76]:

$$\mathbf{q}(t) = \Phi^{-1} \mathbf{Y} \quad (\text{F.4})$$

When $M > N$ the system has more equations than unknowns and Φ is not square, so it cannot be inverted directly. The weights are found in the least-squares sense, by multiplying both sides by Φ^T to form a square $N \times N$ system that is then inverted [76]:

$$\mathbf{q}(t) = (\Phi^T \Phi)^{-1} \Phi^T \mathbf{Y} \quad (\text{F.5})$$

The in-line direction is treated the same way with its own measured motion.

F.2 Adaptation for Curved Geometry

The classical procedure above assumes a straight riser, where the cross-flow and in-line directions are fixed global axes. Two features of the present case differ. The geometry is curved, so the local directions rotate along the span and the fit cannot be applied to each global axis on its own. The cable is also only partly instrumented, so the modes are sampled over a limited span, and their separability over that span must be examined rather than assumed. The adapted procedure follows below.

F.2.1 Local Directions and Projection

Each OrcaFlex mode shape is a three-dimensional vector at every node. At each arc position the local cross-flow and in-line unit vectors follow from the unit tangent $\mathbf{t}(s)$ and the flow direction $\mathbf{e}_{flow} = (\cos \theta, \sin \theta, 0)$, where θ is the current incidence angle:

$$\mathbf{e}_{CF}(s) = \frac{\mathbf{t}(s) \times \mathbf{e}_{flow}}{\|\mathbf{t}(s) \times \mathbf{e}_{flow}\|}, \quad \mathbf{e}_{IL}(s) = \mathbf{e}_{CF}(s) \times \mathbf{t}(s) \quad (\text{F.6})$$

Each mode shape is projected onto these directions to give two scalar shapes per mode:

$$\phi_n^{CF}(s) = \phi_n(s) \cdot \mathbf{e}_{CF}(s), \quad \phi_n^{IL}(s) = \phi_n(s) \cdot \mathbf{e}_{IL}(s) \quad (\text{F.7})$$

and the measured marker motion is projected the same way to give the scalar cross-flow and in-line motion:

$$y(s, t) = \mathbf{r}_{meas}(s, t) \cdot \mathbf{e}_{CF}(s), \quad x(s, t) = \mathbf{r}_{meas}(s, t) \cdot \mathbf{e}_{IL}(s) \quad (\text{F.8})$$

The measured motion and the mode shapes are then expressed in the same local directions. The mode shapes are known at the OrcaFlex nodes and interpolated onto the marker arcs. The measured span is $s/L \in [0.283, 0.959]$ under the current at $U = 0.1$ m/s, $\theta = 90^\circ$, since the cable deflects and the marker arc positions shift slightly from their still-water values. The markers define 36 segments along the submerged part of the cable, of which $M = 34$ provide tracking over the analysis window and are used in the fit. The region above the first marker, the hang-off, is not instrumented.

F.2.2 Joint Fit

A structural mode has a single weight $q_n(t)$ that drives both directions, through the two scalar shapes of (F.7):

$$y(s_m, t) = \sum_{n=1}^N q_n(t) \phi_n^{\text{CF}}(s_m), \quad x(s_m, t) = \sum_{n=1}^N q_n(t) \phi_n^{\text{IL}}(s_m) \quad (\text{F.9})$$

with the same $q_n(t)$ in both. Fitting each direction on its own could return a different weight for the same mode, which is not physical. The two directions are therefore fitted together in one system with a shared weight vector:

$$\mathbf{U}(t) = \begin{bmatrix} \mathbf{y}(t) \\ \mathbf{x}(t) \end{bmatrix} = \mathbf{\Phi} \mathbf{q}(t), \quad \mathbf{\Phi} = \begin{bmatrix} \mathbf{\Phi}^{\text{CF}} \\ \mathbf{\Phi}^{\text{IL}} \end{bmatrix} \quad (\text{F.10})$$

where $\mathbf{\Phi}^{\text{CF}}$ and $\mathbf{\Phi}^{\text{IL}}$ are the scalar mode shapes of (F.7) sampled at the markers. One weight vector now reproduces both directions at once. The weights are found at each time step by minimising the difference between the measured motion and the modal reconstruction, which gives a time history $q_n(t)$ for each mode.

F.2.3 Number of Retained Modes

The matrix $\mathbf{\Phi}$ depends on the marker placement and can be ill-conditioned. Over the full cable the mode shapes are close to orthogonal, but over the measured span two modes can become close in shape, which makes the fit sensitive to measurement error. The conditioning is measured by the condition number, the ratio of the largest to the smallest singular value of the matrix, $\sigma_{\max}$ and $\sigma_{\min}$ [76]:

$$\text{cond}(\mathbf{\Phi}) = \frac{\sigma_{\max}(\mathbf{\Phi})}{\sigma_{\min}(\mathbf{\Phi})} \quad (\text{F.11})$$

A value near one means the fit is stable. A large value means small measurement errors are strongly amplified in the weights. Here the condition number of the joint basis stays near 8 up to fourteen modes and rises steeply beyond it (Figure F.1), so higher modes cannot be separated over the measured span.

The number of retained modes is also set from the reconstruction residual, the part of the measured motion the modes fail to reproduce. It is the norm of the difference between the measured motion and the modal reconstruction, relative to the norm of the measured motion:

$$r(N) = \frac{\|\mathbf{U} - \mathbf{\Phi}_N \mathbf{q}_N\|}{\|\mathbf{U}\|} \quad (\text{F.12})$$

where $\mathbf{U}$ is the stacked measured motion of (F.10), $\mathbf{\Phi}_N$ keeps the first N modes, and $\mathbf{q}_N$ are the corresponding weights. The residual flattens by about fourteen modes, reaching 0.08 for the experiment and 0.02 for the model. Therefore, fourteen modes are retained, and the root-mean-square of each weight then shows which modes carry the response.

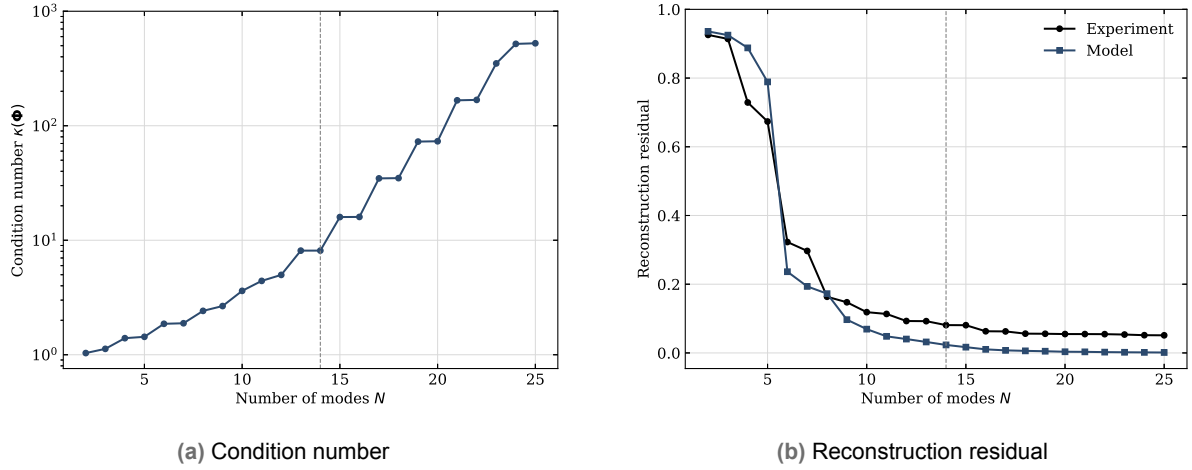


Figure F.1. Condition number and reconstruction residual against the number of retained modes, primary case $\theta = 90^\circ$, $U = 0.1 \text{ m s}^{-1}$

F.2.4 Application to the Model

The same procedure is applied to the model predictions. The OrcaFlex displacement is projected onto the same local cross-flow and in-line directions, sampled at the same 34 active segments, and decomposed with the same set of mode shapes, so the only difference between the two cases is the input motion. The much smaller model residual shows that the retained modes reproduce the model motion almost exactly, so the larger experimental residual points to measurement noise rather than the limited span. The experimental and model modal amplitudes are shown in Figures 5.6b and 5.7b.

