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Robust randomized model predictive control for energy balance in smart thermal grids

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Abstract— This paper presents a stochastic model predictive control approach for a thermal grid with uncertainties in the consumer demand profiles. This approach leads to a finite-horizon chance-constrained mixed-integer linear optimization problem at each sampling time, which is in general non-convex and hard to solve. Earlier approaches for such problems are either suboptimal ad-hoc methodologies, or computationally intractable formulations. We provide a unified framework to deal with production planning problems for uncertain systems, while providing a-priori probabilistic certificates for the robustness properties of the resulting solutions. Our methodology is based on solving a random convex optimization problem to compute the uncertainty bounds using the so-called scenario approach and then, solving a robust mixed-integer optimization problem with the computed randomized uncertainty bounds at each sampling time. Using a tractable approximation of uncertainty bounds, the proposed problem formulation retains the complexity of the problem without chance constraints. In the presented thermal grid application this implies that a robust mixed-integer program is solved to provide a day-ahead prediction for the thermal energy production plan in the grid. The performance of the proposed methodology is illustrated using Monte Carlo simulations and employing two different problem formulations: optimization over input sequences (open-loop MPC) and optimization over affine feedback policies (closed-loop MPC).

I. INTRODUCTION

Smart Thermal Grids (STGs) represent a new concept in the energy sector that involves the use of the smart grid concept in thermal energy networks connecting several households and greenhouses (agents) to each other via a transport line of thermal energy. One of the major challenges in sustainable energy systems is to improve the efficiency, reliability, and sustainability of the production and the distribution of energy. STGs can contribute to obtaining sustainable energy systems by introducing a reliable production plan using renewable energy sources such as solar or geothermal energy and provide efficient large-capacity storage options. This results in a reduction of carbon dioxide (CO_2) emissions, improved energy efficiency, and the implementation of renewable energy systems [1].

In an STG setting, the agents have a potential to contribute to the overall energy balance. Every agent fulfills the role of a consumer when it demands more energy than it produces with its production units (e.g. micro-combined heat and power), and fulfills the role of a producer when the

demand is less than the production of its production units [2]. Since the major energy consumption is typically used for thermal purposes, the motivation for STGs can be both economical and environmental. A better price is achieved with less energy transport when the resources are used more efficiently, while the thermal energy losses are reduced.

We therefore foresee a shift towards a situation where a large number of small scale agents (e.g. utility companies and independent users) have more impact on the energy balance of the grid, while their optimal decisions are made by considering the thermal demand profiles, which are uncertain. The planning of thermal energy production to match supply and demand is challenging since predictions on the thermal energy demand are not perfect. This highlights the necessity of formulating stochastic variants of standard day-ahead planning problems in the grid, while providing probabilistic guarantees regarding the satisfaction of smart grid system constraints.

Model predictive control (MPC) is one of the most widely used advanced control design method that can handle constraints on both inputs and states, and can obtain an optimal control sequence that minimizes a given objective function subject to the model and operational constraints in a receding horizon fashion. One way to treat uncertainty is to use a robust MPC formulation [3], [4], [5], which provides a control law that satisfies the problem constraints for all admissible uncertain variables by assuming that the uncertainty is bounded. However, the resulting solution tends to be conservative since all uncertainty realizations are treated equally. Stochastic MPC offers an alternative approach to achieve a less conservative solution, thereby the system constraints are treated in a probabilistic sense (chance constraints), meaning that the constraints need to be satisfied only probabilistically up to a pre-assigned level to reduce the conservatism of robust MPC. An effective solution to address such problems is to employ randomized algorithms that require substituting the chance constraint with a finite number of hard constraints corresponding to samples of the uncertainty set. Randomized MPC approximates stochastic MPC via the so-called scenario approach (see [6] and the references therein), and if the underlying optimization problem is convex with respect to the decision variables, finite sample guarantees can be provided for a desired confidence level of constraint fulfillment.

In this paper we cannot employ the well-known scenario approach due to the fact that the underlying problem is not convex (mixed-integer program). The main challenge here is in the presence of the uncertain thermal energy demands to

compute a discrete (binary) variable vector that corresponds to the on-off status of the generating units, and a continuous variable vector that is related to the amount of thermal energy that each unit should produce to satisfy a given demand level at each sampling time. Instead, we propose a-two-step procedure that is based on a mixture of randomized and robust optimization [7]. We first determine a probabilistic bounded set of uncertainties that is guaranteed to include a given percentage of uncertainty realizations. Then, we use the obtained set in a deterministic robust MPC approach. Note that the first step leads to a convex sub-problem even if the original problem contains binary variables. In this way we can have similar results in terms of confidence level of constraint fulfillment as in the standard scenario approach. Using a tractable approximation of uncertainty bounds, the deterministic robust formulation leads to a tractable problem for each sampling time. A framework for stochastic linear systems using a combination of randomization and robust optimization was introduced in [8]. In this paper instead we introduce a new framework for stochastic hybrid linear systems that leads to stochastic mixed-integer optimization problems.

The main contributions of this paper are as follows:

- A technical description of smart thermal grids with uncertainties in the consumer demand profiles as an optimization problem formulation.
- The problem formulation leads to a finite-horizon chance-constrained mixed-integer linear program at each sampling time. To solve this problem, we first formulate an auxiliary problem to obtain a bounded set for the uncertainty. Using the scenario approach, the result of this sub-problem is a subset of the uncertainty space that contains a portion of the probability mass of the uncertainty with high confidence level. We then solve a robust version of the initial problem subject to the uncertainty confined in the obtained set. Note that our method does not restrict the underlying probability distribution of uncertainty as in robust optimization methods and it is only assumed that the uncertainties are independent and identically distributed.
- To guarantee that the resulting problem is solvable, we develop a tractable scheme based on the dependency of the constraint functions on the uncertainty sets.
- Both the open-loop stochastic MPC formulation and the closed-loop affine feedback policies of stochastic MPC formulation are described and used to illustrate a performance of the proposed methodology using Monte Carlo simulations.

The layout of this paper is as follows: Section II provides a general stochastic MPC framework for the problem of uncertain smart thermal grids. In Section III a tractable methodology is developed and probabilistic performance guarantees are provided. In Section IV, we demonstrate the efficiency of the proposed methodology through a numerical example. Finally, Section V provides some concluding remarks and directions for future work.

II. PROBLEM FORMULATION

This section provides a brief description of smart thermal grids with multiple agents that can be producers and consumers of power and heat in a smart grid setting. The goal of the agents is to match the local consumption and production to avoid transport losses in the network and improve energy efficiency.

A. System Description

Consider a regional thermal grid consisting of N agents (households, greenhouses). We describe the model of a single agent that is facilitated with a micro-combined heat and power plant (micro-CHP), a boiler, and a heat storage. Each agent can be both producer and consumer which is known as the prosumer concept. This model introduces the technical constraints of each agent and the coupling between such agents in the network. Moreover, for every transaction of thermal energy in the smart grid, there are several heat exchangers located near the corresponding agents and we assume that the heat exchangers do not add additional costs to the heat production for the sake of simplicity.

For a day-ahead planning production problem of each agent, we consider a finite horizon $N_t = 24$ problem with hourly steps, and introduce the subscript t in our notation to characterize the value of the quantities for a given time instance $t \in \{0, 1, \cdots, N_t - 1\}$. For each sampling time t of the problem for all agents $i \in \{1, 2, \cdots, N\}$, we define the main vector of control decision variables to be

$$u_{i,t}^m := [p_{\mathbf{g},t}, p_{\mathbf{ug},t}, p_{\mathbf{dg},t}, h_{\mathbf{g},t}, h_{\mathbf{b},t}, h_{\mathbf{im},t}, c_{\mathbf{g},t}^{\mathbf{su}}, c_{\mathbf{b},t}^{\mathbf{su}}, z_{\mathbf{g},t}, z_{\mathbf{b},t}]^\top \in \mathbb{R}^{10},$$

where $p_{\mathbf{g},t}, h_{\mathbf{g},t}$ denote the electrical power and heat production by the micro-CHP, $p_{\mathbf{ug},t}, p_{\mathbf{dg},t}$ relate to the up and down spinning of electrical power by the micro-CHP, $h_{\mathbf{b},t}, h_{\mathbf{im},t}$ correspond to the heat provided by the boiler and imported heat from external parties during period of high heat demand, $c_{i,t}^{\mathbf{su}} := [c_{\mathbf{g},t}^{\mathbf{su}}, c_{\mathbf{b},t}^{\mathbf{su}}]^\top$ is a vector that contains the startup cost of micro-CHP and boiler, and $z_{i,t} := [z_{\mathbf{g},t}, z_{\mathbf{b},t}]^\top$ are auxiliary variables needed to model the minimum up and down times of each micro-CHP and boiler, respectively. Moreover, $v_{i,t} : [v_{\mathbf{g},t}, v_{\mathbf{b},t}]^\top \in \{0,1\} \times \{0,1\}$ is a binary vector of dimension 2 and denotes the on-off status of each micro-CHP and boiler for each agent i at step t. We call the difference between the level of heat storage and the forecast of heat demand $h_{\mathbf{d},t}^{\mathbf{f}}$ the imbalance error $x_{i,t} \in \mathbb{R}$ at agent i, defined as

$$x_{i,t} = h_{\mathbf{s},t} - h_{\mathbf{d},t}^{\mathbf{f}} , \tag{1}$$

where $h_{\mathbf{s},t}$ represents the heat storage level (assuming there are no thermal losses in the conversion and storage system). The heat storage level has the following dynamics:

$$h_{\mathbf{s},t+1} = \eta_{\mathbf{s}} x_{i,t} + \eta_{\mathbf{s}} \left(h_{\mathbf{g},t} + h_{\mathbf{b},t} + h_{\mathbf{im},t} + \sum_{j \in N_{-i}} (1 - \alpha_{ij}) h_{\mathbf{xc},t}^{ij} \right),$$

where $\eta_{\mathbf{s}} \in (0,1)$ and $\alpha_{ij} \in (0,1)$ denote the efficiency of storage and the heat loss coefficient due to transportation

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between agent i and j, respectively. N_{-i} is the set of neighbors of agent i and is given by

$$N_{-i} \subseteq \{1, 2, \cdots, N\} \setminus \{i\} .$$

In order for an agent to contribute to the local balancing of heat by exchanging heat with neighbors, we define an auxiliary control variable vector $u_{i,t}^a \in \mathbb{R}^{|N_{-i}|}$ with elements $h_{\mathbf{xc},t}^{ij}$ denoting the exchanged heat between agent i and other adjacent agents $j \in N_{-i}$. Notice that $h_{\mathbf{xc},t}^{ij}$ can have either positive or negative values depending on if agent i imports or exports heat from or to agent j, respectively. By substituting $h_{\mathbf{s},t}$ in (1), one can derive the dynamical behavior of imbalance $x_{i,t}$ that is given by

$$x_{i,t+1} = A_i x_{i,t} + B_i u_{i,t} + w_{i,t} , \qquad (2)$$

where $B_i = \eta_{\mathbf{s}}[b_1^\top, b_2^\top]^\top$, with $b_1 = [0,0,0,1,1,1,0,0,0,0] \in \mathbb{R}^{10}$, $b_2 \in \mathbb{R}^{|N_{-i}|}$ containing elements $(1 - \alpha_{ij})$, and $A_i = \eta_{\mathbf{s}}$. The complete vector of control decision variables is $u_{i,t} = [u_{i,t}^{m\top}, u_{i,t}^{a\top}]^\top \in \mathbb{R}^{N_u}$ with $N_u = 10 + |N_{-i}|$ for every agent i at each step t. By definition (1), $w_{i,t} := -h_{\mathbf{d},t+1}^{\mathbf{f}} \in \mathbb{R}$ corresponds to the forecast heat demand in the next time step. We now consider that the only uncertainty is due to the deviation of the actual heat demand from its forecast value and therefore, $w_{i,t}$ represents an uncertain parameter for every hour and for each agent.

Our goal is to find the control input $u_{i,t}$ for each agent i such that the imbalance error stays as a small positive value for all steps $t \in \{0, 1, \cdots, N_t - 1\}$ at minimal production cost and satisfying physical constraints. We associate an economical linear cost function with each agent i at step t as $J_i(u_{i,t}) = c_i^\top u_{i,t}^m$, where c_i is a cost vector and is defined as

$$c_i := [c_{\mathbf{gas}}\eta_{\mathbf{CHP}}^{-1}, c_{\mathbf{up}}, c_{\mathbf{dp}}, 0, c_{\mathbf{b}}\eta_{\mathbf{b}}^{-1}, c_{\mathbf{im}}, 1, 1, 0, 0]^\top \in \mathbb{R}^{10}.$$

$c_{\mathbf{gas}}$ relates to the cost of natural gas that is used by micro-CHP and $\eta_{\mathbf{CHP}}, \eta_{\mathbf{b}} \in (0,1)$ are the efficiency of a micro-CHP and a boiler at each step, respectively. $c_{\mathbf{up}}, c_{\mathbf{dp}}$ denote the cost of up and down spinning electrical power production by micro-CHP, respectively. We define $p_{\mathbf{ug},t}, p_{\mathbf{dg},t}$ to be up and down spinning variables that are related to the amount of surplus and needed electrical power, respectively, in each agent at each step with respect to the local power demand. The cost of heat generated by a boiler is $c_{\mathbf{b}}$, and the cost of imported heat from an external party is $c_{\mathbf{im}}$. The seventh and eighth entry of c_i represent the start-up costs. The cost associated to the thermal energy produced by the micro-CHP is considered to be zero due to the fact that the electrical power and thermal energy generated by a micro-CHP are coupled by $h_{\mathbf{g},t} = \frac{\eta_{\mathbf{h}}}{\eta_{\mathbf{p}}} p_{\mathbf{g},t}$ where $\eta_{\mathbf{h}}, \eta_{\mathbf{p}} \in (0,1)$ are the efficiency of a micro-CHP for production of electrical power and thermal energies, respectively.

The resulting optimization problem for an agent i is given by

$$\min_{\{u_{i,t}, v_{i,t}\}_{t=0}^{N_t-1}} \sum_{t=0}^{N_t-1} J_i(u_{i,t}) \qquad (3a)$$

subject to:

1) Startup cost constraints for $t = 0, 1, \cdots, N_t - 1$:

$$c_{i,t}^{su} \geq \Lambda^{su}(v_{i,t} - v_{i,t-1}) , \ c_{i,t}^{su} \geq 0 , \qquad (3b)$$

where Λ^{su} is a diagonal matrix including the startup costs of each micro-CHP and boiler.

2) Production and transportation capacity constraints for $t = 0, 1, \cdots, N_t - 1$:

$$v_{\mathbf{g},t} p_{\mathbf{g}}^{\min} \leq p_{\mathbf{g},t} \leq p_{\mathbf{g}}^{\max} v_{\mathbf{g},t} , \qquad (3c)$$

$$v_{\mathbf{g},t} h_{\mathbf{g}}^{\min} \leq h_{\mathbf{g},t} \leq h_{\mathbf{g}}^{\max} v_{\mathbf{g},t} , \qquad (3d)$$

$$v_{\mathbf{b},t} h_{\mathbf{b}}^{\min} \leq h_{\mathbf{b},t} \leq h_{\mathbf{b}}^{\max} v_{\mathbf{b},t} , \qquad (3e)$$

$$h_{\mathbf{im},t}^{\min} \leq h_{\mathbf{im},t} \leq h_{\mathbf{im},t}^{\max} , \qquad (3f)$$

$$h_{\mathbf{xc},t}^{\min} \leq h_{\mathbf{xc},t}^{ij} \leq h_{\mathbf{xc},t}^{\max} , \ \forall i, j \in N_{-i} \qquad (3g)$$

where $p_{\mathbf{g}}^{\min}, p_{\mathbf{g}}^{\max}$ denote the minimum and maximum electrical power production capacities of each micro-CHP, $h_{\mathbf{g}}^{\min}, h_{\mathbf{g}}^{\max}$ relate to the minimum and maximum heat production capacities of each micro-CHP, $h_{\mathbf{b}}^{\min}, h_{\mathbf{b}}^{\max}$ are the minimum and maximum heat production capacities of each boiler, $h_{\mathbf{im}}^{\min}, h_{\mathbf{im}}^{\max}$ are the minimum and maximum available heat capacities of each external party, $h_{\mathbf{xc}}^{\min}, h_{\mathbf{xc}}^{\max}$ represent the minimum and maximum transportation capacities of neighbor.

3) Balance constraints for heat exchanged with neighbors for $t = 0, 1, \cdots, N_t - 1$:

$$h_{\mathbf{xc},t}^{ij} + h_{\mathbf{xc},t}^{ji} = 0 . \ \forall i, j \in N_{-i} \qquad (3h)$$

4) Up and down spinning electrical power constraints for $t = 0, 1, \cdots, N_t - 1$:

$$-p_{\mathbf{dg},t} \leq p_{\mathbf{g},t} - p_{\mathbf{d},t} \leq p_{\mathbf{ug},t}, \ p_{\mathbf{ug},t} \geq 0, p_{\mathbf{dg},t} \geq 0, \quad (3i)$$

where $p_{\mathbf{d},t}$ is a local electrical power demand for each agent $i \in \{1, \cdots, N\}$.

5) Ramping capacity constraint for all $t = 0, 1, \cdots, N_t - 1$:

$$-p_{\mathbf{g}}^{\mathbf{down}} \leq p_{\mathbf{g},t} - p_{\mathbf{g},t-1} \leq p_{\mathbf{g}}^{\mathbf{up}} , \qquad (3j)$$

where $p_{\mathbf{g}}^{\mathbf{down}}, p_{\mathbf{g}}^{\mathbf{up}}$ denote the down and up capacity of decreasing and increasing electrical power of a micro-CHP within two consecutive periods, respectively. Note that this constraint is considered just for the electrical power of micro-CHP due to the fact that heat can be produced within each step.

6) Status change constraints for $t = 0, 1, \cdots, N_t - 1$:

$$z_{i,t} \geq v_{i,t} - v_{i,t-1} , \ z_{i,t} \geq 0 ,$$

$$\sum_{k=t+1-\Delta t_{\mathbf{up}}}^{t} z_{i,k} \leq v_{i,t} , \forall t \in \{\Delta t_{\mathbf{up}}, \cdots, N_t - 1\} ,$$

$$\sum_{k=t+1}^{t+\Delta t_{\mathbf{down}}} z_{i,k} \leq 1 - v_{i,t} , \forall t \in \{1, \cdots, N_t - 1 - \Delta t_{\mathbf{down}}\} , \qquad (3k)$$

where $\Delta t_{\mathbf{up}}, \Delta t_{\mathbf{down}} \in \mathbb{R}_+$ denote the minimum time an agent needs to change status of the micro-CHP and the boiler.

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7) Probabilistic constraint:

$$\mathbb{P}(x_{i,t+1} \geq 0 \ , \ \forall t \in \{0,1,\cdots,N_t-1\}) \geq 1-\varepsilon \ , \quad (3l)$$

where $\varepsilon \in (0,1)$ is the admissible constraint violation parameter. This constraint implies that the imbalance error should be a positive value at minimum production cost for all heat demand realizations except for a set of probability at most ε.

The proposed optimization problem (3) is a finite-horizon multi-stage, chance-constrained mixed-integer linear program, whose stages are coupled by the binaries (3b), ramping (3j), status change (3k) and imbalance error (3l) constraints.

B. Open-loop Stochastic MPC

In order to formulate a stochastic MPC for the overall smart thermal grid imbalance problem, we first extend the optimization problem (3) for all agents in the grid. Let us define $X_t := [x_{1,t}^\top,\cdots,x_{N,t}^\top]^\top \in \mathbb{R}^N$, $U_t := [u_{1,t}^\top,\cdots,u_{N,t}^\top]^\top \in \mathbb{R}^{N_u N}$ and $V_t := [v_{1,t}^\top,\cdots,v_{N,t}^\top]^\top \in \mathbb{R}^{2N}$ to be the state, control input and binary variables of the grid, respectively. We also define $W_t := [w_{1,t}^\top,\cdots,w_{N,t}^\top]^\top \in \mathbb{R}^N$ to be an uncertainty vector of all the agents. The grid cost $J(U_t)$ at step t is assumed to be the sum of the individual costs for all agents

$$J(U_t) = C^\top U_t = \sum_{i=1}^N J_i(u_{i,t}) \ ,$$

where $C := [c_1^\top,\cdots,c_N^\top]^\top$. The dynamics of the imbalance error of all agents in the grid can be expressed as

$$X_{t+1} = AX_t + BU_t + W_t \ , \quad (4)$$

where $A = \mathrm{diag}(A_1,\cdots,A_N)$ and $B = \mathrm{diag}(B_1,\cdots,B_N)$. The uncertain variable vector $W_t \in \mathbb{R}^N$ is defined on a probability space Δ. It is assumed that Δ is endowed with the Borel $\sigma-$algebra and $\mathbb{P}$ is a probability measure defined over Δ. It is important to note that for our study we only need a finite number of instances of W_t, and we do not require the probability space Δ and the probability measure $\mathbb{P}$ to be known explicitly.

To illustrate the advantages obtained by adopting the policies that were discussed at the end of the preceding section, we first need to introduce some compact notations for the overall system dynamics evolution along the finite time horizon. Consider the following vectors of state, control input, binary variables, and uncertainty parameter matrices.

$$\mathbf{X} = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_{N_t} \end{bmatrix}, \ \mathbf{U} = \begin{bmatrix} U_0 \\ U_1 \\ \vdots \\ U_{N_t-1} \end{bmatrix}, \ \mathbf{V} = \begin{bmatrix} V_0 \\ V_1 \\ \vdots \\ V_{N_t-1} \end{bmatrix}, \ \mathbf{W} = \begin{bmatrix} W_0 \\ W_1 \\ \vdots \\ W_{N_t-1} \end{bmatrix}.$$

The imbalance error dynamics for all agents over the prediction horizon can be now written as

$$\mathbf{X} = \mathbf{A}X_0 + \mathbf{B}\mathbf{U} + \mathbf{H}\mathbf{W} \ .$$

The exact form of $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{H}$ matrices are omitted in the interest of space and can be found in [9]. The initial state values are defined by $X_0 := [x_{1,0}^\top,\cdots,x_{N,0}^\top]^\top \in \mathbb{R}^N$ and

it is assumed that the full information of each agent, such as the production plan, the demand request for the next day stepwise, is available to the whole grid. The objective function can be expressed by $\mathbf{J}(\mathbf{U}) = \mathbf{C}^\top\mathbf{U}$, where $\mathbf{C} = \mathbf{1}^{N_t}\otimes C$ using the Kronecker product.

We are now in a position to define the optimization problem for the overall smart thermal grid as follows:

$$\min_{\mathbf{U}} \quad \mathbf{J}(\mathbf{U}) \quad (5a)$$

$$\text{s.t.} \quad \mathbb{P}_{\mathbf{W}}(\mathbf{A}X_0 + \mathbf{B}\mathbf{U} + \mathbf{H}\mathbf{W} \geq 0) \geq 1-\varepsilon \ , \quad (5b)$$

$$\mathbf{E}\mathbf{U} + \mathbf{F}\mathbf{V} + \mathbf{P} \leq 0 \ , \quad (5c)$$

where $\mathbf{E}$, $\mathbf{F}$ and $\mathbf{P}$ are matrices of appropriate dimensions. Notice that $\mathbb{P}_{\mathbf{W}}$ depends on the string of uncertain scenario realizations. The solution of (5) is the optimal planned input sequence $\{U_0^*,V_0^*,\cdots,U_{N_t-1}^*,V_{N_t-1}^*\}$. Based on the MPC paradigm the current input is set to $\{U_t,V_t\} := \{U_0^*,V_0^*\}$ and we proceed in a receding horizon fashion. This means (5) is solved at each step t by using the current measurement of the state X_t. Due to the presence of chance constraints, the feasible set is in general non-convex and hard to determine explicitly. We describe a tractable formulation to solve (5) by using robust randomization techniques in Section III.

C. Closed-loop Stochastic MPC

In the presence of uncertainty, the problem of finding the optimal state feedback policy becomes quite challenging. One way to tackle this problem is to look for a sub-optimal solution by parameterizing the control input variables. As a first approach, one can directly parameterize the control input variables as an affine function of the uncertainty

$$U_t = \overline{U}_t + \sum_{j=0}^{t-1} \theta_{t,j} W_j \ ,$$

where $\overline{U}_t$ and $\theta_{t,j}$ are optimization variables. In this way the designed closed-loop control system is equivalent to an open-loop control system with a feedforward uncertainty compensator [9], [10]. Consider the following finite-horizon chance-constrained mixed-integer linear program by adopting a feedback control policy (6d) that is affine in the uncertainty samples.

$$\min_{\overline{\mathbf{U}},\mathbf{G},\mathbf{V}} \quad \mathbf{J}(\mathbf{U}) \quad (6a)$$

$$\text{s.t.} \quad \mathbb{P}_{\mathbf{W}}(\mathbf{A}X_0 + \mathbf{B}\mathbf{U} + \mathbf{H}\mathbf{W} \geq 0) \geq 1-\varepsilon \ , \quad (6b)$$

$$\mathbf{E}\mathbf{U} + \mathbf{F}\mathbf{V} + \mathbf{P} \leq 0 \ , \quad (6c)$$

$$\mathbf{U} = \overline{\mathbf{U}} + \mathbf{G}\mathbf{W} \ , \quad (6d)$$

where matrices $\overline{\mathbf{U}}$ and $\mathbf{G}$ are given by

$$\overline{\mathbf{U}} = \begin{bmatrix} \overline{U}_0 \\ \overline{U}_1 \\ \vdots \\ \overline{U}_{N_t-1} \end{bmatrix}, \ \mathbf{G} = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ \theta_{1,0} & 0 & \ddots & 0 \\ \vdots & \ddots & \ddots & 0 \\ \theta_{N_t-1,0} & \cdots & \theta_{N_t-1,N_t-1} & 0 \end{bmatrix}.$$

Notice that each element of $\mathbf{G}$ has dimension $\mathbb{R}^{N_u N\times N}$.

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To solve (5) and (6), we have to transform the chance-constrained problem to a tractable one without introducing any assumptions on $\mathbb{P}$ and its moments. Hence, we follow a randomization-based approach. The proposed procedure in [6] is called scenario approach that allows to substitute the chance constraints with a finite number of hard constraints corresponding to scenarios of the uncertainty and provides a probabilistic guarantee, if the underlying problem is convex with respect to the decision variables. The number of scenarios of the uncertainty realizations N_s that needs to be extracted must satisfy

$$N_s \geq \frac{2}{\varepsilon}\left(d + \ln \frac{1}{\beta}\right), \tag{7}$$

where $\varepsilon \in (0,1)$ is a desired level of constraint violations, d is the number of decision variables and $\beta \in (0,1)$ is a desired confidence level with which the drawn scenarios lead to a feasible solution. Unfortunately, we cannot follow this approach, due to the binary vector $\mathbf{V}$. Even if the convexity condition were satisfied, the number of scenarios that one needs to generate grows linearly with the number of decision variables, thus hampering the applicability of the method to large-scale problems [11]. For example, the number of decision variables in the proposed open-loop stochastic MPC formulation (5) is $d = (N_u + N_v)N_t N$, and the number of decision variables in the proposed closed-loop affine uncertainty feedback policy stochastic MPC formulation (6) is $d + d_G$, where $d_G = N_u N_w \frac{(N_t - 1)N_t}{2}$. Due to the high dimension of decision space, we cannot even employ the extended results in [12]. To overcome this difficulty, we propose a tractable methodology based on the results of [7] in the Section III.

III. TRACTABLE METHODOLOGY

In this section, we use the results in [7] to approximate the chance constraints that appear in the proposed formulations (5) and (6). We then develop a tractable methodology to reformulate the proposed robust formulations. The approximation is done in a way to provide a feasible solution for all scenarios of the uncertainty realizations with probabilistic guarantees. In the first step, a bounded set that contains the uncertainty realizations with a specific probability of violations is constructed. We then formulate a robust optimization problem with respect to that set and show that the solution is guaranteed to be feasible for the initial chance constrained problems (5) and (6) with the desired level of confidence.

A. Randomization-Based Reformulation

Define $\mathcal{B}_i(\gamma)$ to be a bounded set of uncertainty realizations and we assume that it is an axis-aligned hyper-rectangle for each agent i. Note that the choice of a hyper-rectangle is not restrictive and any convex set with convex volume could have been chosen instead [7]. We parametrize $\mathcal{B}_i(\gamma) := \times_{k=0}^{N_t-1}[\underline{\gamma}_k, \overline{\gamma}_k]$ by $\gamma = (\underline{\gamma}, \overline{\gamma}) \in \mathbb{R}^{2N_t}$, where $\underline{\gamma} = (\underline{\gamma}_0, \cdots, \underline{\gamma}_{N_t-1}) \in \mathbb{R}^{N_t}$ and $\overline{\gamma} = (\overline{\gamma}_0, \cdots, \overline{\gamma}_{N_t-1}) \in \mathbb{R}^{N_t}$. Consider now the following chance-constrained optimization problem

$$\min_{\gamma} \quad \sum_{k=0}^{N_t-1} \overline{\gamma}_k - \underline{\gamma}_k \tag{8}$$
$$\text{s.t.} \quad \mathbb{P}(w_i \in \Delta \mid w_{i,k} \in [\underline{\gamma}_k, \overline{\gamma}_k], \ \forall k) \geq 1 - \varepsilon.$$

By construction the problem (8) is a convex program and we can apply the standard scenario approach to obtain a solution as follows.

$$\min_{\gamma} \quad \sum_{k=0}^{N_t-1} \overline{\gamma}_k - \underline{\gamma}_k \tag{9}$$
$$\text{s.t.} \quad w_{i,k}^j \in [\underline{\gamma}_k, \overline{\gamma}_k] \ , \quad \begin{cases} \forall k \in \{0, \cdots, N_t - 1\} \\ \forall j \in \{1, \cdots, N_s\} \end{cases},$$

where N_s is the required number of scenarios (7) for each agent i with $d = N_t N$. The optimal solution of (9) γ^* is a feasible solution for the problem (8) with confidence $1 - \beta$.

We now establish $\mathcal{B}_i(\gamma^*)$ for all $i \in \{1, \cdots, N\}$ and define $\mathfrak{B}^* := \{\mathcal{B}_1(\gamma^*), \cdots, \mathcal{B}_N(\gamma^*)\}$ and pose the robust counterpart of the problems (5)-(6) where $\mathbf{W} \in \mathfrak{B}^* \cap \Delta^N$. Note that the robust counterparts of (5)-(6) are not randomized programs and instead, they are finite-horizon robust mixed-integer linear problems where the constraints have to be satisfied for all values of the uncertainty inside $\mathfrak{B}^* \cap \Delta^N$. It is worth to mention that any feasible solution of the robust counterparts of (5)-(6) is a feasible solution for the problems (5) and (6) with at least confidence of $1 - \beta$. The robust counterpart problems are tractable and equivalent to mixed-integer linear programs, since the uncertainty is bounded in a convex set [13]. It is shown in [13] that the robust problems are tractable and remain in the same class as the original problems, e.g. robust mixed-integer programs remain mixed-integer programs, for a certain class of uncertainty sets. This is achieved under the assumptions that the constraint functions are linear and homogeneous with respect to the uncertainty vector. In the sequel, we describe a tractable scheme for the robust counterparts of (5)-(6).

B. Tractable Robust Reformulation

Following the methodology outlined in the previous section, define $\boldsymbol{\gamma^o} := [\gamma_0^o, \gamma_1^o, \cdots, \gamma_{N_t-1}^o] \in \mathbb{R}^{NN_t}$ to be a vector whose elements are the middle points of the hyper-rectangle $\mathfrak{B}^*$ and is defined as $\boldsymbol{\gamma^o} = 0.5(\overline{\boldsymbol{\gamma}}^* + \underline{\boldsymbol{\gamma}}^*)$ and each element of $\boldsymbol{\gamma^o}$ represents a vector for all agents $i \in \{1, 2, \cdots, N\}$. Consider now the following tractable reformulations of the proposed robust counterpart of problems (5) and (6).

$$\min_{\mathbf{U}, \mathbf{V}} \quad \mathbf{J}(\mathbf{U}) \tag{10a}$$
$$\text{s.t.} \quad \mathbf{A}X_0 + \mathbf{B}\mathbf{U} + \mathbf{H}\boldsymbol{\gamma^o} + \boldsymbol{\eta} \geq 0 \ , \tag{10b}$$
$$\mathbf{E}\mathbf{U} + \mathbf{F}\mathbf{V} + \mathbf{P} \leq 0 \ , \tag{10c}$$

where $\boldsymbol{\eta} := [\eta_0, \eta_1, \cdots, \eta_{N_t-1}] \in \mathbb{R}^{NN_t}$ is a vector with each element $\eta_t \in \mathbb{R}^N$ denoting a bound for the worst-case uncertainty realizations at step t for all agents $i \in \{1, 2, \cdots, N\}$. We refer to Proposition 1 below that shows how to achieve

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this bound. We next present a tractable reformulation of the proposed robust counterpart of problem (6).

$$\min_{\mathbf{U},\mathbf{G},\mathbf{V}} \quad \mathbf{J}(\mathbf{U}) \tag{11a}$$

$$\text{s.t. } \mathbf{A}X_0 + \mathbf{B}\mathbf{U} + \mathbf{H}\boldsymbol{\gamma}^o + \boldsymbol{\eta} \geq 0 \ , \tag{11b}$$

$$\mathbf{E}\mathbf{U} + \mathbf{F}\mathbf{V} + \mathbf{P} \leq 0 \ , \tag{11c}$$

$$\mathbf{U} = \overline{\mathbf{U}} + \mathbf{G}\boldsymbol{\gamma}^o + \boldsymbol{\eta}_{\boldsymbol{g}} \ , \tag{11d}$$

$$\eta_{g,t} \leq [\mathbf{G}(\overline{\boldsymbol{\gamma}}^* - \boldsymbol{\gamma}^o)]_t \ , \forall t \in \{0,1,\cdots,N_t-1\}, \tag{11e}$$

$$\eta_{g,t} \leq [\mathbf{G}(\underline{\boldsymbol{\gamma}}^* - \boldsymbol{\gamma}^o)]_t \ , \forall t \in \{0,1,\cdots,N_t-1\}, \tag{11f}$$

where $\boldsymbol{\eta}_{\boldsymbol{g}} := [\eta_{g,0}, \eta_{g,1}, \cdots, \eta_{g,N_t-1}] \in \mathbb{R}^{NN_t}$ is a vector with each element $\eta_{g,t} \in \mathbb{R}^N$. The following proposition shows the link between the tractable problems (10) and (11), and the proposed formulations (5) and (6), respectively.

Proposition 1: If the tractable problems (10) and (11) have an optimal solution, where $\boldsymbol{\eta}$ is obtained by solving the following problem

$$\max_{\boldsymbol{\eta} \in \mathbb{R}^{NN_t}} \boldsymbol{\eta}$$
$$\text{s.t. } \eta_t \leq [\mathbf{H}(\overline{\boldsymbol{\gamma}}^* - \boldsymbol{\gamma}^o)]_t \ , \forall t \in \{0,1,\cdots,N_t-1\}, \tag{12}$$
$$\eta_t \leq [\mathbf{H}(\underline{\boldsymbol{\gamma}}^* - \boldsymbol{\gamma}^o)]_t \ , \forall t \in \{0,1,\cdots,N_t-1\},$$

then it is a feasible solution for the chance-constrained problems (5) and (6) with at least $1-\beta$ confidence level, respectively.

Proof: It is shown in [14, Proposition 1] that any feasible solution of the robust counterparts of (5)-(6) is a feasible solution of the initial chance-constrained problems (5) and (6), respectively. Therefore, we have to show that the proposed tractable problems (10) and (11) are equivalent with the robust counterparts of (5)-(6). Consider the following robust constraint,

$$0 \leq \mathbf{A}X_0 + \mathbf{B}\mathbf{U} + \mathbf{H}\mathbf{W} \ , \ \forall \mathbf{W} \in \mathfrak{B}^* \cap \Delta^N \ ,$$

that can be written in an equivalent format using the linearity and homogeneity assumption of the constraint with respect to the uncertainty, leads to

$$0 \leq \mathbf{A}X_0 + \mathbf{B}\mathbf{U} + \mathbf{H}(\boldsymbol{\gamma}^o + \Delta\boldsymbol{\gamma}) =$$
$$\mathbf{A}X_0 + \mathbf{B}\mathbf{U} + \mathbf{H}\boldsymbol{\gamma}^o + \mathbf{H}\Delta\boldsymbol{\gamma} \ , \ \forall \Delta\boldsymbol{\gamma} \in [\underline{\boldsymbol{\gamma}}^* - \boldsymbol{\gamma}^o, \boldsymbol{\gamma}^o - \overline{\boldsymbol{\gamma}}^*] \ .$$

We need to introduce the vector $\boldsymbol{\eta} := [\eta_0, \eta_1, \cdots, \eta_{N_t-1}] \in \mathbb{R}^{NN_t}$ with each element $\eta_t \in \mathbb{R}^N$ representing a bound for $\mathbf{H}\Delta\boldsymbol{\gamma}$. Consider now the worst-case uncertainty realizations to be $(\overline{\boldsymbol{\gamma}}^* - \boldsymbol{\gamma}^o)$ and $(\underline{\boldsymbol{\gamma}}^* - \boldsymbol{\gamma}^o)$. We pose the problem (12) to find bound $\boldsymbol{\eta}$, using this bound leads to

$$0 \leq \mathbf{A}X_0 + \mathbf{B}\mathbf{U} + \mathbf{H}\boldsymbol{\gamma}^o + \boldsymbol{\eta}$$
$$\leq \mathbf{A}X_0 + \mathbf{B}\mathbf{U} + \mathbf{H}\boldsymbol{\gamma}^o + \mathbf{H}\Delta\boldsymbol{\gamma} = \mathbf{A}X_0 + \mathbf{B}\mathbf{U} + \mathbf{H}(\boldsymbol{\gamma}^o + \Delta\boldsymbol{\gamma}).$$

∎

Remark 2: Note that we can use the same approach by introducing $\eta_{g,t}$ to be the worst-case superposition of the uncertainty realizations with the following constraints:

$$\eta_{g,t} \leq \sum_{j=0}^{t-1} \theta_{t,j}[(\overline{\boldsymbol{\gamma}}^* - \boldsymbol{\gamma}^o)]_j \ , \ \eta_{g,t} \leq \sum_{j=0}^{t-1} \theta_{t,j}[(\underline{\boldsymbol{\gamma}}^* - \boldsymbol{\gamma}^o)]_j \ .$$

C. Robust Randomized Model Predictive Control

The proposed procedure of a robust randomized MPC is summarized in Algorithm 1. We compare our proposed methodology to illustrate its performance against a hybrid approach as a benchmark, where the generating unit status problem is solved deterministically meaning that we initialize $\boldsymbol{\gamma}^o \equiv \mathbf{W}^{\text{forecast}}$, $\boldsymbol{\eta} \equiv 0$ in (10), (11) with the forecast value of the energy demand and solve the deterministic variant of the problems. At the next step, we fix the on-off status of the generating units (and also the startup cost and auxiliary variables) to the binary vector computed by the previous deterministic program, and formulate a stochastic production planning problem. We refer to this as the Benchmark approach and the steps are summarized in Algorithm 2.

Algorithm 1 Robust Randomized MPC

1: Fix $X_0 \in \mathbb{R}^N, V_0 \in \{0,1\}^{2N}$ ▷ initial state and status of the generating units, respectively.
2: Fix $\varepsilon \in (0,1)$, $\beta \in (0,1)$ ▷ level of constraint violations and confidence level, respectively.
3: Generate N_s scenarios (7) with $d = 2N_tN$ and establish $\mathfrak{B}^*$ by solving the optimization problem (9).
   Open-loop
4: Solve (10) and determine an optimal solution $\mathbf{U}^*, \mathbf{V}^*$. Apply the first optimal solution $U_t := U_0^\star, V_t := V_0^\star$ to the smart thermal grid agents.
   Affine Uncertainty Feedback
5: Solve (11) and determine an optimal solution $\overline{\mathbf{U}}^*, \mathbf{G}^*, \mathbf{V}^*$. Apply the first optimal solution $U_t := U_0^\star + [\mathbf{G}^*\boldsymbol{\gamma}^o + \boldsymbol{\eta}_{\boldsymbol{g}}]_0, V_t := V_0^\star$ to the smart thermal grid agents.
6: Go to step 2.

Remark 3: The proposed framework solves a stochastic mixed-integer program and it does not necessarily lead to a less conservative approach due to the fact that the number of required scenarios (7) is a function of the dimension of the decision variables. The decision size in our framework is proportional to the uncertainty dimension and in case of the high uncertainty size, the advantage of our solution comes at the expense of a more conservative performance.

IV. NUMERICAL STUDY

We carried out Monte Carlo simulations and a comparison with the Benchmark Algorithm 2 to illustrate the performance of our proposed Algorithm 1 (robust randomized MPC) for both open-loop and closed-loop formulations. All optimization problems were solved using the solver BNB via the MATLAB interface YALMIP [15].

A. Simulation Setup

In this simulation study we consider a small thermal grid with three agents as an example. Figure 1 depicts the connections between each agent and their local components. Each agent has a micro-CHP, a boiler and a thermal storage. In the proposed model the difference between the level of thermal storage and local thermal energy demand (imbalance

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Algorithm 2 Benchmark Approach

    Deterministic Generating Unit Status
1: Fix $X_0 \in \mathbb{R}^N, V_0 \in \{0,1\}^{2N}$ and $\boldsymbol{\gamma}^o \equiv \mathbf{W}^{\text{forecast}}$, $\boldsymbol{\eta} \equiv 0$ ▷ initial state and status of the generating units, respectively, and no heat demand prediction error.
    Open-loop
2: Solve (10) and determine an optimal solution $\mathbf{V}^*_{\text{OLP}}$.
    Affine Uncertainty Feedback
3: Solve (11) and determine an optimal solution $\mathbf{V}^*_{\text{AUF}}$.
    Stochastic Production Planning
4: Fix $\varepsilon \in (0,1)$, $\beta \in (0,1)$ and consider $\boldsymbol{\gamma}^o \equiv \mathbf{W}^{(j)}$, $\boldsymbol{\eta} \equiv 0$, $j = 1, \cdots, N_s$ ▷ level of constraint violations and confidence, respectively.
    Open-loop
5: Generate N_s scenarios (7) with $d = N_u N_t N$.
6: Fix $V_t = V^*_{\text{OLP},0} \to$ Solve (10) and determine an optimal solution $\mathbf{U}^*$. Apply the first optimal solution $U_t := U_0^\star$ to the STGs agents. Go to step 2.
    Affine Uncertainty Feedback
7: Generate N_s scenarios (7) with $d = N_u N_t (N + N_w (N_t - 1)/2)$.
8: Fix $V_t = V^*_{\text{AUF},0}$ and solve (11) and determine an optimal solution $\overline{\mathbf{U}}^*, \mathbf{G}^*$. Apply the first optimal solution $U_t := \overline{U}_0^\star + [G^* \boldsymbol{\gamma}^o + \boldsymbol{\eta}_g]_0$ to the STGs agents. Go to step 3.

Parameter	Value	Unit
$p_{\mathbf{g}}^{\max}, h_{\mathbf{g}}^{\max}, p_{\mathbf{g}}^{\min}, h_{\mathbf{g}}^{\min}$	$120.0, 120.0, 0.0, 0.0$	[KW]
$h_{\mathbf{b}}^{\max}, h_{\mathbf{im}}^{\max}, h_{\mathbf{b}}^{\min}, h_{\mathbf{im}}^{\min}$	$120.0, 120.0, 0.0, 0.0$	[KW]
$h_{\mathbf{xc}}^{\max}, h_{\mathbf{xc}}^{\min}, h_{\mathbf{s},0}^{i}$	$20.0, -20.0, 10.0$	[KW]
$\eta_{\mathbf{CHP}}, \eta_{\mathbf{h}}, \eta_{\mathbf{p}}$	$0.25, 0.7, 0.3$	-
$\eta_{\mathbf{s}}, \eta_{\mathbf{b}}, \alpha_{ij}$	$0.85, 1.0, 0.25$	-
$c_{\mathbf{gas}}, c_{\mathbf{up}}, c_{\mathbf{dp}}, c_{\mathbf{b}}, c_{\mathbf{im}}$	$45.0, 0.0, 100.0, 45.0, 300.0$	€
$\Lambda^{\mathbf{su}}$ (micro-CHP, boiler)	diag$(60.0, 120.0)$	€
ε, β	$0.1, 0.0001$	-

TABLE I

PARAMETERS WITH THEIR SYMBOLS AND VALUES.

errors) is defined as a state of the local agent. The thermal storage level of agent one, two and three are presented in Figure 1 using h_s^1, h_s^2, h_s^3, respectively. There are also three lines between agents indicating that thermal energy exchange is possible. It is considered to have an external party available for all agents to provide thermal energy.

The proposed Algorithm 1 and the Benchmark approach are applied to the example provided in Figure 1 with $N = 3$. We solve a day-ahead production planning problem for an uncertain thermal grid with $N_t = 24$ and hourly steps. It is assumed that the up and down capacity of decreasing and increasing electrical power are $p_{\mathbf{g}}^{\mathbf{up}} = p_{\mathbf{g}}^{\mathbf{down}} = p_{\mathbf{g}}^{\max}/3$ and the minimum time for a change of production unit status $(\Delta t_{\mathbf{up}}, \Delta t_{\mathbf{down}})$ is 2 hours. Table I contains all parameters that are considered for the example in Figure 1. In order to generate scenarios for the thermal energy demand error, we used a Markov chain based model (we refer the reader to [16] for more details). Moreover, we consider to have different forecast of energy demand profiles for each agent. We construct scenarios of uncertain demand profiles assuming that the realization changes randomly to represent historical uncertain demand data.
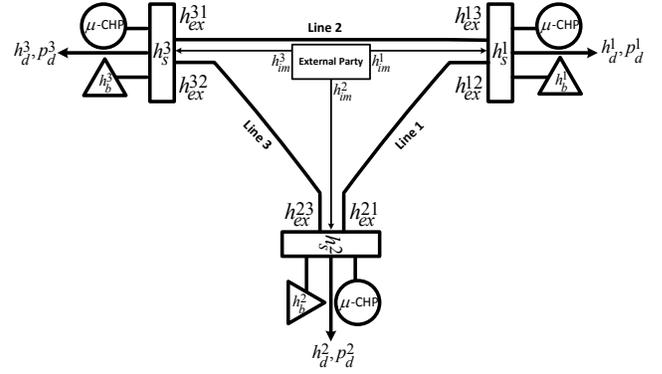

Fig. 1. Three-agent (households, greenhouses) thermal grid example. Each agent has a μ−CHP, a boiler and a thermal storage. h_s^1, h_s^2, h_s^3 are related to the local thermal storage in agent one, two and three, respectively. There are also three lines between agents indicating that thermal energy exchange is possible. It is considered to have an external party available for all agents to provide thermal energy.

B. Simulation Results

Figure 2 shows imbalance error trajectories for different agents. Due to the definition of the imbalance error in equation (1), our goal is to minimize these errors. This means that the requested thermal energy demand is provided for each agent at each step with the desired level of violation ε as in equation (3l). The initial value for the storage level in each agent is considered to be 10 [KW]. In Figure 2 the 'blue', 'red' and 'green' lines are related to the imbalance error profiles (x_1, x_2, x_3) in the first, second and third agent. The top sub-figure shows the result of optimizing directly over input sequences in each sampling time (open-loop MPC), the middle depicts the result of closed-loop MPC considering affine uncertainty feedback, and the last sub-figure shows the result of the Benchmark approach. In Figure 3 the ON/OFF status of boilers at each sampling time using blue for the first, second and third agent are shown, respectively. The left-hand-side sub-figures are related to the closed-loop MPC considering affine uncertainty feedback, and the-right-hand side ones shows the results of Benchmark approach. In Figure 4 the relative cost improvement between the cost generated by the closed-loop MPC considering affine uncertainty feedback and the cost generated by the Benchmark approach in each sampling time is shown.

The proposed Algorithm 1 offers a better working plan for production units as well as an hourly-based production cost improvement compared to the Benchmark approach. The improvement in terms of cost is due to the scheduling flexibility offered by the proposed algorithm, where the binary variables are solved together with the production planning problem, allowing us to identify more optimal working status for production units.

V. CONCLUDING REMARKS

This paper formulated an optimization problem for a day-ahead prediction plan of smart thermal grids with uncertain local demands. Smart thermal grids refer to energy networks whose main goal is to provide and distribute thermal energy
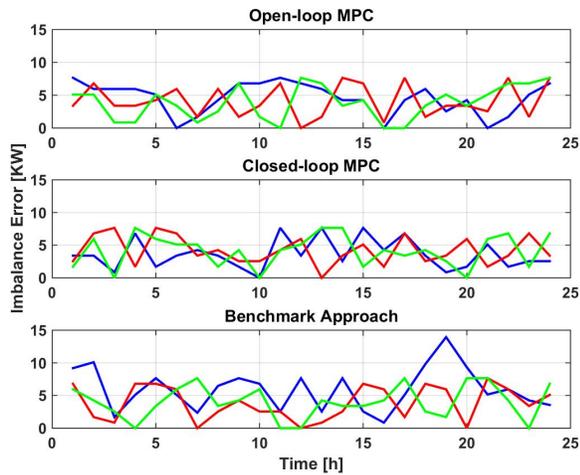
Fig. 2. Imbalance error trajectories. 'Blue' lines show to the imbalance error x_1 in the first agent and x_2 the imbalance error in the second agent is shown by 'Red' lines. 'Green' lines represent the imbalance error x_3 in the third agent. The first, second and third sub-figures are related to the results of open-loop MPC, closed-loop MPC considering affine uncertainty feedback, and the Benchmark approach, respectively.
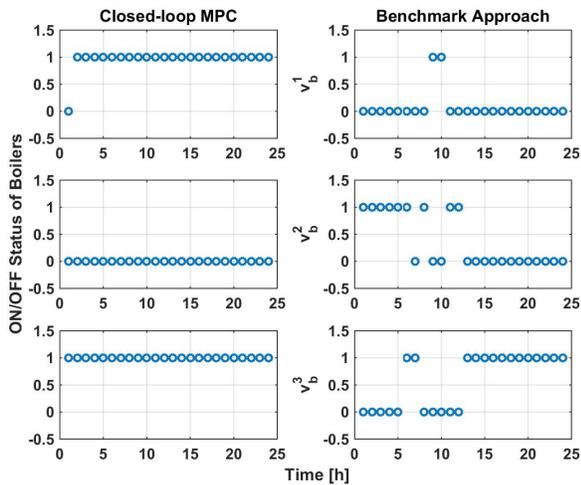

Fig. 3. ON/OFF status of boilers. The first, second and third sub-figures are related to agent 1, 2, and 3, respectively. The sub-figures on the left are the results of closed-loop MPC considering affine uncertainty feedback, and the sub-figures on the right are the results of the Benchmark approach.

among their local agents. This formulation leads to a multi-stage chance-constrained mixed-integer linear program. We proposed a unified framework, namely a robust randomized MPC approach to solve such a problem, while providing a-priori guarantees for the chance constraint fulfillment. Our current work focuses on incorporating aquifer thermal energy storage (ATES) systems in the developed framework and distributing the computations in the developed scheme among the agents.

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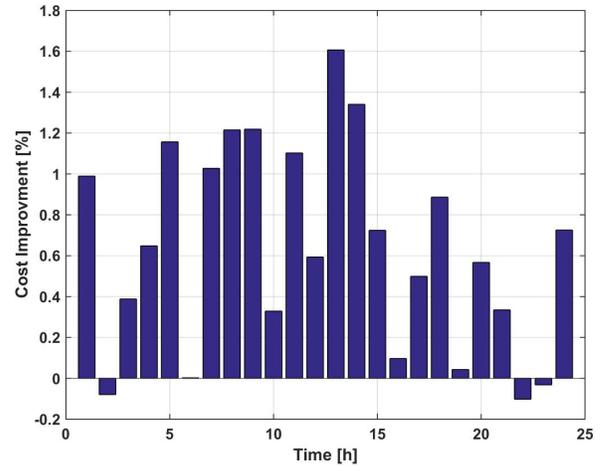
Fig. 4. Normalized relative cost improvement of the closed-loop MPC approach (affine uncertainty feedback) over the Benchmark approach.

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