BULLETIN DE L'ACADÉMIE
POLONAISE DES SCIENCES
Série des sciences techniques
Volume XXIII, No. 12 — 1975

V A R I A
(COMPUTER SCIENCE)

The Structure of Safety Regulations

by

J. S. PAWŁOWSKI

Presented by J. WIĘCKOWSKI on September 25, 1975

Summary. The term "safety regulations" used in this paper denotes the regulations restricting the parameters of mechanical devices (such as ships, for example) in order to assure that their operations will be safe. The author attempts to present the structure of the regulations, used more or less intuitively in their composition, in the terms of the fundamental notions of set theory. Such a presentation visualizes several assumptions made tacitly in the intuitive approach. One of those assumptions, which seems to be disputable in some cases at least, and is fundamental for the whole structure, is discussed more closely at the end of the paper.

1. Introduction and fundamentals. The term "safety regulations" used in the title denotes the regulations restricting the parameters of mechanical devices (such as ships, for example) in order to assure that their operations will be safe. The author holds that all those regulations are of the same nature and their internal structures are the same. Hence one can write "the regulation", instead "the regulations" when referring to their structure, and that convention is adopted in what follows. The structure of the regulation presented in this note is generally used more or less intuitively. The notions of set theory employed in the presentation make possible the visualization of several assumptions tacitly involved in the intuitive approach.

Let $\mathbf{X}, \mathbf{Y}, \mathbf{Z}$ be pairwise disjoint, nonempty sets and $\mathbf{Z}_o$ be a proper subset of $\mathbf{Z}$. Moreover, let f be a mapping $f: \mathbf{X} \times \mathbf{Y} \to \mathbf{Z}$ and $\mathbf{X}_o$ be defined as:

$$\mathbf{X}_o = \{x \in \mathbf{X}: \bigwedge_{y \in \mathbf{Y}} (f(x, y) \in \mathbf{Z}_o)\}. \tag{1.1}$$

It is assumed that

$$\bigvee_{x} (x \in \mathbf{X}_o). \tag{1.2}$$

The sets and the mapping mentioned above are regarded as representing, respectively: $\mathbf{X}$ — the space of the devices subjected to the regulation, $\mathbf{Y}$ — the space of the external conditions, under which the devices are intended to operate, $\mathbf{Z}$ — the space of the physical states of the devices likely to be experienced in operation, $\mathbf{Z}_o$ — the physical states considered to be safe, f — the cause-effect relation between

the members of $\mathbf{X}\times\mathbf{Y}$ and the members of $\mathbf{Z}$, $\mathbf{X}_o$ — the devices which are safe under the conditions $\mathbf{Y}$. The assumption (1.2) states the existence of a device being safe under the conditions $\mathbf{Y}$. The notions mentioned above, even if not explicitly expressed in the formula of the regulation, are fundamental to it. As an example ships may constitute $\mathbf{X}$, weather conditions (waves and winds), occurring on different areas of seas and oceans, may form $\mathbf{Y}$, and then all possible performances of the ships sailing in the conditions constitute $\mathbf{Z}$.

2. The criterion of safety. Besides the assumption (1.2) the existence of mappings $k_1 : \mathbf{X}\to\mathbf{R}^k$, $k_2 : \mathbf{Y}\to\mathbf{R}^m$, $k_3 : \mathbf{Z}\to\mathbf{R}^n$ is postulated with $\mathbf{R}$ denoting the set of real numbers. An order relation in $\mathbf{R}^n$ is introduced in a natural way:

$$\bigwedge_{x,y\in\mathbf{R}^n}\left[(x\leqslant y)\Leftrightarrow\bigwedge_{i=1,\ldots,n}\left(\pi^i(x)\leqslant\pi^i(y)\right)\right], \tag{2.1}$$

where $\pi^i(y)$ is the projection of $y\in\mathbf{R}^n$ on the i-th factor.
It is assumed that:

$$\bigvee_{c\in R^n}\bigwedge_{x\in k_3(\mathbf{Z})}\left(x\leqslant c\Rightarrow k_3^{-1}(x)\subset\mathbf{Z}_0\right), \tag{2.2}$$

c need not be unique but for the construction of the regulation only one element of $\mathbf{R}^n$, for which the proposition of (2.2) is valid, must be chosen. That element may be called "the criterion of safety". The proper choice of the criterion of safety is not intended to be discussed here and "c" will be used to denote the criterion of safety in the sequel.

Some further remarks are introduced to explain the properties of the criterion of safety. Let the mappings defined so far be exemplified in the form of a diagram shown below. In order to simplify notation $\mathbf{X}\times\mathbf{Y}$ is denoted by $\mathbf{W}$, $\mathbf{R}^k\times\mathbf{R}^m$ by $\mathbf{R}^{k+m}$ and $k_1\times k_2$ by g.

$$\begin{array}{ccc} \mathbf{W} & \xrightarrow{f} & \mathbf{Z} \\ {\scriptstyle g}\downarrow & & \downarrow{\scriptstyle k_3} \\ \mathbf{R}^{k+m} & & \mathbf{R}^n \end{array}$$

It can easily be verified that:

$$\bigwedge_{x\in k_1(\mathbf{X})}\Big[\Big(\bigwedge_{y\in k_2(\mathbf{Y})}\bigwedge_{u\in g^{-1}(x,y)}\left((k_3\circ f)(u)\leqslant c\right)\Rightarrow\left(k_1^{-1}(x)\subset X_0\right)\Big], \tag{2.3}$$

later on the antecedent of the implication in the brackets will be called "the condition of safety", for any $x\in k_1(\mathbf{X})$.

If $h : \mathbf{R}^{k+m}\to\mathbf{R}^n$ exists which closes the diagram in such a way that it becomes commutative, that is:

$$\bigwedge_{u\in\mathbf{W}}\left((h\circ g)(u)=(k_3\circ f)(u)\right), \tag{2.4}$$

then the condition of safety can be written in an equivalent form:

$$\bigwedge_{y\in k_2(\mathbf{Y})}\left(h(x,y)\leqslant c\right), \tag{2.5}$$

for $x\in k_1(\mathbf{X})$. How
and sufficient that:

(2.6)

(see, e.g. [1]). In th
of safety can be fo

(2.7)

where the expressio
second internal bra
the condition $u\in g$
the probabilistic co
bility of the event
and for $x\in k_1(\mathbf{X})$.

3. The structur
shown which arises
follows the structure
tions fundamental to
their compatibility w
is not always obviou
part of the paper.

Let l be a natur
Therefore an equiva

(3.1)

with $pr_i(x)$ denoting
over, let $\mathbf{R}^l$ be ord

(3.2)

If (2.6) is valid, fo

(3.3) B

In the opposite case
be replaced by (2.7
cases. For further
below*) in $\mathbf{R}^l$. Her
by "b_x", $\pi^l(b_x)$ is

(3.4)

*) An equivalent
of greatest lower boun

for $x \in k_1(\mathbf{X})$. However, it should be recalled that for the existence of h it is necessary and sufficient that:

$$\bigwedge_{u,z \in \mathbf{W}} [(g(u)=g(z)) \Rightarrow ((k_3 \circ f)(u)=(k_3 \circ f)(z))], \tag{2.6}$$

(see, e.g. [1]). In the case when (2.6) is not true another probabilistic conditions of safety can be formulated as follows:

$$\bigwedge_{y \in k_2(\mathbf{Y})} [\text{Prob}\{(k_3 \circ f)(u) \leqslant c \mid u \in g^{-1}(x, y)\} \geqslant \eta], \tag{2.7}$$

where the expression on the left-hand side of the inequality sign outside the second internal brackets denotes the probability of the event $(k_3 \circ f)(u) \leqslant c$, under the condition $u \in g^{-1}(x, y)$, where $x \in k_1(\mathbf{X})$ and $\eta \in \langle 0, 1 \rangle$. With the use of the probabilistic condition of safety it can be said, if it is satisfied, that the probability of the event $f(u) \in \mathbf{Z}_o$ is at least η for any randomly chosen $u \in k_I^{-1}(x) \times \mathbf{Y}$ and for $x \in k_I(\mathbf{X})$.

3. The structure of the regulation. Up to now the general situation has been shown which arises when one attempts to construct the safety regulation. In what follows the structure of the regulation generally used will be presented. The assumptions fundamental to it are important despite of their mathematical simplicity because their compatibility with the physical nature of the objects subjected to the regulation is not always obvious. That situation is illustrated by two examples set in the final part of the paper.

Let l be a natural number and $l<k$, then $\mathbf{R}^k$ can be considered as $\mathbf{R}^l \times \mathbf{R}^{k-l}$. Therefore an equivalence relation ρ can be introduced in $\mathbf{R}^k$ as follows:

$$\bigwedge_{x,y \in \mathbf{R}^k} [(x \rho y) \Leftrightarrow (pr_2(x)=pr_2(y))], \tag{3.1}$$

with $pr_i(x)$ denoting the projection of $x \in \mathbf{R}^l \times \mathbf{R}^{k-l}$ on the i-th factor, $i \leqslant 2$. Moreover, let $\mathbf{R}^l$ be ordered by a relation analogous to (2.1) and

$$\mathbf{C}_x = \{y \in \mathbf{R}^k : y \rho x\}. \tag{3.2}$$

If (2.6) is valid, for every $x \in k_I(\mathbf{X})$ the set $\mathbf{B}_x$ is defined as:

$$\mathbf{B}_x = \{pr_I(y) \in \mathbf{R}^l : (y \in \mathbf{C}_x) \wedge \bigwedge_{z \in k_2(\mathbf{Y})} (h(y, z) \leqslant c)\}. \tag{3.3}$$

In the opposite case the second component of the conjuction in the brackets should be replaced by (2.7). In the sequel no distinction will be made between the two cases. For further consideration it is assumed that $\mathbf{B}_x$ is nonempty and bounded below*) in $\mathbf{R}^l$. Hence there exists the greatest lower bound of $\mathbf{B}_x$ in $\mathbf{R}^l$ denoted by "b_x", $\pi^l(b_x)$ is simply the greatest lower bound of the set:

$$\mathbf{D}_i = \{\zeta \in \mathbf{R} : \bigvee_{y \in \mathbf{B}_x} (\pi^l(y)=\zeta)\}, \tag{3.4}$$

*) An equivalent assumption would be that $\mathbf{B}_x$ is bounded above with the resulting change of greatest lower bounds to the least upper bounds and vice versa in the sequel.

$i \leqslant l$. From the definition it follows that:

(3.5) $$\bigwedge_{y \in \mathbf{B}_x} (y \geqslant b_x)$$

and

(3.6) $$\bigwedge_{y \in \mathbf{R}^l} \left((y > b_x) \Rightarrow \bigvee_{z \in \mathbf{B}_x} \neg (z \geqslant y)\right).$$

Let for a $\mathbf{P} \subset k_l(\mathbf{X})$, $\mathbf{P} \neq \emptyset$, the set $\mathbf{B}_\mathbf{p}$ be defined as:

(3.7) $$\mathbf{B}_\mathbf{p} = \{b_x : x \in P\}.$$

The assumption is made that $\mathbf{B}_\mathbf{p}$ is bounded above in $\mathbf{R}^l$. Therefore there exists the least upper bound of $\mathbf{B}_\mathbf{p}$ in $\mathbf{R}^l$ denoted by "$b_\mathbf{p}$". From now on every element $y \in \mathbf{R}^l$ will be called safe for an $x \in k_l(\mathbf{X})$, if and only if $y \in \mathbf{B}_x$. A sufficient condition for $b_\mathbf{p}$ to be safe for every $x \in \mathbf{P}$ is:

(3.8) $$\bigwedge_{x \in \mathbf{P}} \bigwedge_{y \in \mathbf{R}^l} (y \geqslant b_x \Rightarrow y \in \mathbf{B}_x).$$

In the case when $\bigvee_{x \in k_l(\mathbf{X})} (\mathbf{P} = \mathbf{C}_x)$ there is:

(3.9) $$b_\mathbf{p} = b_x,$$

and then

(3.10) $$b_x \in \mathbf{B}_x$$

is the sufficient and necessary condition for $b_\mathbf{p}$ to be safe for every $x \in \mathbf{P}$.

The regulation of safety assigns $b_\mathbf{p}$ to a set $\mathbf{P}$ as the restriction from below for the regulated parameters $y \in \mathbf{R}^l$ ($\pi^i(y)$ is simply the i-th regulated parameter of an object of the regulation). Therefore the validity od (3.8) constitutes an assumption which is fundamental to the structure of the regulation. In the author's opinion cases in which (3.8) is not satisfied are possible in practice. An illustration of such a situation is given below as an example.

Some additional remarks can be made concerning the assumptions that $\mathbf{B}_x$ and $\mathbf{B}_\mathbf{p}$ are bounded. Excluding the necessity of restricting the regulated parameters from above and below at the same time, and taking into account that every parameter restricted from above becomes restricted from below after reversing its sign, the restriction of the parameters can be reduced to the restriction from below (cf. the footnote on the page 99 [1035]). On the other hand, if some parameters need to be restricted from both sides, restriction from above can be treated separately from that from below which leads to another regulation of analogous structure.

Subsequently the case of restriction from below is sufficiently general. The admission that $\mathbf{B}_x$ is unbounded below leads to the conclusion that there exists a natural number $i \leqslant l$ and a sequence (y_n), $y_n \in \mathbf{B}_x$, such as that:

(3.11) $$\lim_{n \to \infty} \pi^i(y_n) = -\infty.$$

In practice it means that the i-th parameter need not be restricted at all by the regulation. Therefore the dimenison of $\mathbf{R}^l$ is reduced by one after eliminating the i-th

parameter. Such a
for practical reason
ments apply to the
above there exists
as that:

(3.12)

In practice it mean
made, there is alw
parameter should b
(x_n) having the pro

4. Examples. Le
deck of the vessel
Moreover, let the w
of the frequency o
safety. A safe freebo
The heights of the
If the frequency of
the height of the de
stations, then the co
to establish the rest
required by the reg
a station depends n
of the deck at the ot
For example, let π
$\to \{0, 1\}$, $x \in k_l(\mathbf{X})$

(4.1) φ_x

where d_x is a real

(4.2)

and d_i, $i = 1, \ldots, l$,

(4.3)

and therefore:

(4.4)

for any $y \in \mathbf{R}^l$.

parameter. Such a process of elimination shows that $\mathbf{B}_x$ should be bounded below for practical reasons or otherwise there is no need for the regulation. Similar arguments apply to the assumption that $\mathbf{B}_p$ is bounded above. If $\mathbf{B}_p$ is not bounded above there exists a sequence (x_n), $x_n \in k_l(\mathbf{X})$, and a natural number $i \leqslant l$ such as that:

$$\lim_{n\to\infty} \pi^i(b_{x_n}) = \infty . \tag{3.12}$$

In practice it means that no matter how large the value of the i-th parameter is made, there is always an object of the regulation, for which the value of the parameter should be larger in order to ensure its safety. In fact the existence of (x_n) having the property (3.12) seems most unlikely in practice.

4. Examples. Let the freeboard of a vessel be considered as the height of the deck of the vessel above still waterplane specified at l stations along the vessel. Moreover, let the wetness of the deck be restricted, then the largest permitted values of the frequency of wetness at the respective stations constitute the criterion of safety. A safe freeboard is the freeboard, for which the condition of safety is satisfied. The heights of the deck at the stations $i=1, ..., l$ constitute an element $y \in \mathbf{R}^l$. If the frequency of the wetness at a station is a continuous decreasing function of the height of the deck at the station and is independent of the heights at the other stations, then the condition (3.8) is satisfied, and one can look for b_x or b_p, in order to establish the restriction of freeboard height, for a vessel or a group of vessels, required by the regulation. On the other hand, if the frequency of the wetness at a station depends not only on the height of the deck at the station, but on the heights of the deck at the other stations as well, then the condition (3.8) need not be satisfied. For example, let $\pi^i(y)=\zeta_i$ for $y \in \mathbf{R}^l$, and let the family of functions $\varphi_x : \mathbf{R}^l \to \{0, 1\}$, $x \in k_l(\mathbf{X})$ be defined as follows:

$$\varphi_x(y) = \begin{cases} 1 & \text{if} \quad \left(\sum_{i=1}^{l} \zeta_i d_i \geqslant d_x\right) \wedge (y \geqslant b_x), \\ 0 & \text{if} \quad \left(\sum_{i=1}^{l} \zeta_i d_i < d_x\right) \wedge \neg (y \geqslant b_x), \end{cases} \tag{4.1}$$

where d_x is a real number such that:

$$\bigwedge_{x,y \in k_l(\mathbf{X})} \left((x \rho y) \Leftrightarrow (d_x = d_y)\right), \tag{4.2}$$

and d_i, $i=1, ..., l$, are nonnegative real numbers. Moreover, let:

$$(\varphi_x(y)=1) \Leftrightarrow (y \in B_x), \tag{4.3}$$

and therefore:

$$(\varphi_x(y)=0) \Leftrightarrow (y \notin B_x), \tag{4.4}$$

for any $y \in \mathbf{R}^l$.

Then, if for some $x \in k_l(\mathbf{X})$:

$$\sum_{i=1}^{l} \pi^i(b_x)\, d_i < d_x\,, \tag{4.5}$$

it is easy to see that $\varphi_x(b_x)=0$, and consequently $b_x \notin \mathbf{B}_x$. It is also possible that for $\mathbf{P} \subset k_l(\mathbf{X})$:

$$\bigvee_{x \in \mathbf{P}} \left(\sum_{i=1}^{l} (\pi^i(b_{\mathbf{P}})\, d_i < d_x \right), \tag{4.6}$$

and then (3.8) becomes not true.

The last example shows that such situations to which the structure of the regulation presented in this paper is not applicable are possible. Besides, it shows that the understanding of the structure may be essential in practice.

SHIP RESEARCH INSTITUTE, TECHNICAL UNIVERSITY, MAJAKOWSKIEGO 11/12, 80-952 GDAŃSK
(INSTYTUT OKRĘTOWY, POLITECHNIKA GDAŃSKA)

REFERENCES

[1] K. Maurin, *Analiza*, Część I, PWN, Warszawa, 1971.
[2] K. Kuratowski, *Wstęp do teorii mnogości i topologii*, PWN, Warszawa, 1972.

Я. С. Павловски, Структура правил безопасности

Содержание. В представленной работе под правилами безопасности подразумеваются правила налагающие, по соображениям безопасности, ограничения на параметры механических конструкций, таких как напр. самолеты либо корабли. Настоящая работа представляет собой попытку построения, с использованием понятий из теорий множеств, общей схемы, применимой ко всем видам правил безопасности.

BULLETIN DE L'A
POLONAISE DES S
8. Série des sciences
Volume XXIII, No.

Summary. The term "s
parameters of mechanic
will be safe. The auth
intuitively in their com
sentation visualizes se
assumptions, which see
structure, is discussed

1. Introduction

title denotes the re
as ships, for examp
author holds that
structures are the s
tions" when referr
follows. The struct
more or less intuit
make possible the
intuitive approach.

Let X, Y, Z be
of Z. Moreover, le

(1.1)

It is assumed tha

(1.2)

The sets and the n
vely: X — the spa
the external condit
space of the physi
$\mathbf{Z}_o$ — the physical