

A COUPLING ALGORITHM FOR HIGH ORDER SOLIDS AND LATTICE BOLTZMANN FLUID SOLVERS

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Abstract. *In this paper the implementation of the bidirectional coupling approach for the partitioned Lattice Boltzmann (LB) fluid flow and Finite Element (p-FEM) structural mechanics solver will be discussed. Earlier research on a benchmark configuration [21] showed that the explicit coupling algorithm of the two methods was prone to instabilities for some configurations. For this purpose an implicit coupling algorithm was designed and is presented in this paper.*

1 INTRODUCTION

The validation of fluid structure simulations is an important aspect for the research in the years to come. In addition, efficiency of different coupling approaches (partitioned: strong/weak, monolithic) has not been investigated in sufficient depth. For a partitioned approach it is mandatory that both stand-alone CFD and CSD solvers are highly efficient. For the CFD code *Virtual Fluids* - which is based on the Lattice Boltzmann method - this has been shown in [7]. Efficiency of the CSD solver *AdhoC* [5] in a FSI context is discussed in [17, 18]. First the porous media benchmark described in [7] was successfully validated. For the weakly compressible case, the LB approach has a significant wall clock time advantage compared to these more classical methods, indicating that the LB solver is extremely efficient in the area of weakly compressible flows. The numerical simulation

described in [10] is concerned with fluid structure interaction and was a study of a two dimensional heartflap. While good results were obtained for the displacements, the time history of the drag and lift forces showed the influence of the activation/deactivation of new fluid nodes. These fluctuations also caused problems with the stability for the FSI-benchmark described in [21]. A first remedy to this problem was to integrate the structure with an eigenvalue solver using only the low-frequency modes [6, 19]. The displacements obtained were in very good agreement compared with those published by Hron and Turek [21]. The usage of additional modes in the eigenvalue integrator induced the divergence of the structural deformation. Whether this instability results from the explicit coupling algorithm or is a result of the activation/deactivation of fluid nodes will be investigated in the future. In this context an implicit coupling approach will be studied.

2 LATTICE BOLTZMANN FLUID AND p-FEM STRUCTURE SOLVER

The fluid solver *Virtual Fluids* is based on the Lattice Boltzmann method and on fixed/adaptive cartesian grids. Local grid refinement is achieved by using quadtree and octree type grids. Refinement interface conditions are described in [6]. The CSD solver *AdhoC* [5] provides fully three dimensional spatial discretization based on high order finite elements (p -version) [4, 20]. The time domain is discretized using the generalized- α -method [3]. A time integration scheme based on modal analysis is also available.

As indicated above, fluid nodes have to be created and deleted due to the moving structure. The new fluid nodes have to be initialized with reasonable values. Depending on the geometrical configuration, velocities are linearly inter- or extrapolated to the new nodes. A local Poisson type iteration described in [12] is used at those nodes to compute a consistent pressure and higher order moments.

3 FLUID BOUNDARY CONDITIONS

In the mesoscopic model of the Lattice Boltzmann Method, macroscopic flow quantities can only be set implicitly via the particle distribution functions. A well known and simple way to introduce no-slip walls is the so-called bounce back scheme. As we consider arbitrarily shaped moving objects, we use the modified bounce back scheme developed in [2, 11] for velocity boundary conditions. With this scheme second order accurate results in space are obtained even for curved geometries [7]. In contrast to the simple bounce-back scheme, the use of these interpolation based no-slip boundary conditions results in a notable mass loss across the no-slip lines which can be compensated [1]. The results obtained with the simple bounce-back scheme were inferior, which highlights the importance of a proper geometric resolution of the flow boundaries.

4 FORCE EVALUATION

There are two possibilities to evaluate forces on boundaries using the LB method: (a) the momentum-exchange based method and (b) the pressure/stress integration based

method. A detailed description and comparison of both methods can be found in [14] and additional implementation details in [6]. The boundary of the structure is represented by the interface mesh consisting of flat triangles in 3D or line elements in 2D.

4.0.1 Momentum-exchange

The force acting on moving objects results from the momentum-exchange between the particle distributions and the object [15] at each boundary node k :

$$\mathbf{F}_k = \frac{\Delta x^3}{\Delta t} (\mathbf{e}_i - \mathbf{u}_B)(f_i^t + f_I^{t+1}). \quad (1)$$

Here \mathbf{u}_B is the velocity of the moving boundary and $\mathbf{e}_I = -\mathbf{e}_i$ are the specific microscopic velocities which depend on the discrete velocity model [9]. The forces \mathbf{F}_k - which act on the boundary of the structure and are integrated over all fluid nodes k with a link cut by the boundary - contribute to the resulting force. Note that this method is conserving the total momentum transferred to the structure.

4.0.2 Stress integration

The momentum exchange works well for integral forces on large structural elements, but it is not optimal for the calculation of forces when the extension of the boundary element is in the order of the fluid mesh resolution. Incorrect local forces are introduced if boundary elements have too few or even no matching links. For this case the stress integration method is preferable. One advantage compared to conventional CFD solvers is the locally available stress tensor in the Lattice Boltzmann method. The stresses at the nodes of the boundary are computed by extrapolation as described in [6].

To obtain the load vector \mathbf{F}_{load} for the structure, we multiply the nodal stress tensor with the normal of the boundary and integrate along the boundary Γ

$$\mathbf{F}_{load} = \int_{\Gamma} \mathbf{S} \cdot \mathbf{n}_{boundary} d\Gamma. \quad (2)$$

Note that this method does not conserve the momentum but is preserving the profile of the stresses.

5 COUPLING APPROACH

Since we have different discretizations for the structure (coarse and curved p -elements) and the fluid (in 3D relatively small cubes and a locally refined octree type grid), we introduce an interface mesh for the data transfer between both meshes. The interface mesh is a moving surface mesh consisting of flat triangles. On each node the values for the velocity, the load vector and other physical quantities required for the exchange are stored. The mesh is constructed as follows. The nodes are defined by the Gaussian integration points of the p -FEM solver and build the basis for a quadrilateral mesh which is then triangulated. This interface mesh is exchanged between the p -FEM and the LB-solver. For problems in two dimensions the interface mesh is reduced to a polyline.

Before discussing implementation details of the implicit scheme, the explicit coupling approach (see Figure 1) will be reconsidered. At timestep t the fluid solver computes the load vector on the interface mesh points. The displacements due to these loads are then calculated by the structural solver. Usually the timestep for the fluid solver is smaller than for the structural solver. Internal subcycling is used for integrating the fluid problem from timestep t to $t + 1$, where the geometric position of the interface mesh is obtained by linear interpolation [6].

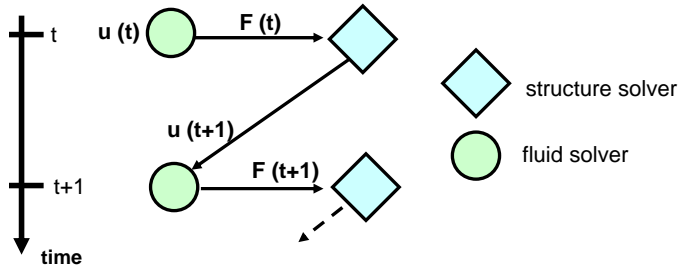


Figure 1: Explicit coupling algorithm

In contrast to the explicit scheme, the implicit scheme iterates over the same time step until either the force or the displacement or both of the last two iterations converge. It is well known, that this type of implicit iteration is similar to a Richardson Iteration [13] with respect to the interface degrees of freedom. Therefore, an underrelaxation may be applied either to the forces or to the displacements to accelerate convergence [13].

In the classical approaches of discretizing the Navier Stokes equations (FEM, FVM), the entire domain usually is recalculated within each substep. As a result, each subiteration of an implicit coupling is as expensive as one explicit step. We expect an advantage by using only a small region of the domain (see Figure 2) within each subiteration of the implicit scheme. This is algorithmically easily accomplished due to the explicit and weakly compressible nature of the LB method and the tree-type data structure used for

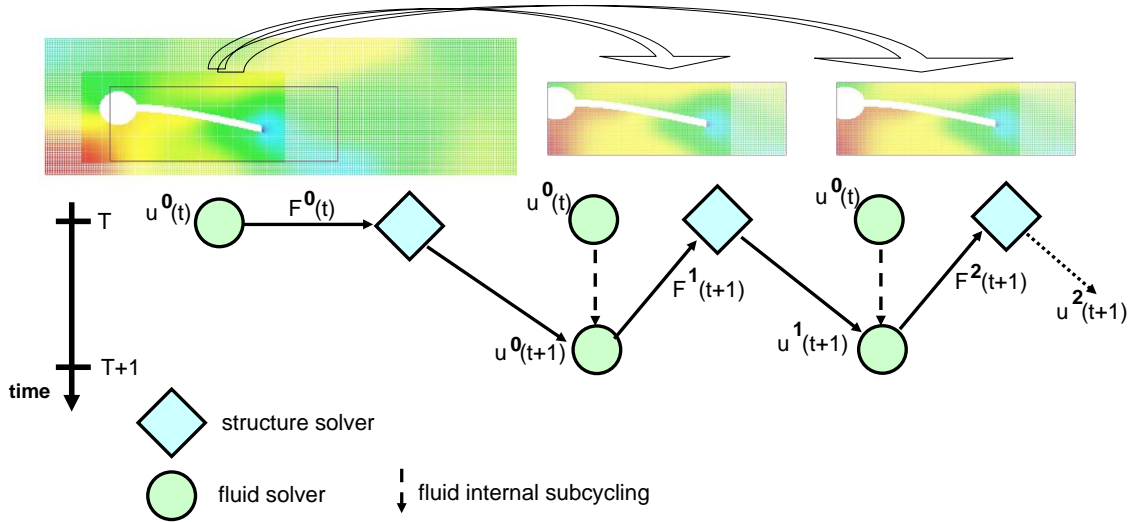


Figure 2: Implicit coupling algorithm

the discretization of the fluid domain. Within this process only this subregion is advanced to the next time step until convergence is achieved. The resulting configuration is then applied to the entire domain and the coupled system is advanced to the next time step. The subiterations within this semi-implicit procedure are thus much cheaper to perform.

6 OUTLOOK

The proposed implicit coupling approach will be tested and the results will be presented at the conference. Additionally we plan to use a more favourable mesh. The surface mesh responsible for the load transfer is presently defined by the Gauss points of the p-FEM solver. This implies that the local size of the triangles is proportional to the inverse square of the degree of the Ansatz functions, resulting in very small sizes for high order elements in the vicinity of the p -element edges which in turn results in localized reduced accuracy with respect to load transfer. The introduction of a coupling mesh independent of the Gauss points in combination with a suitable projection of the loads and a global error minimization with respect to the transfer of the displacements is expected to substantially reduce these effects, leading to improved stability for cases where higher modes become essential for the coupled problem.

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