

Model-Based Individualization of Human-Like Steering Con- trollers

Parameter Estimation of a Cybernetic Steering Model Us-
ing Global Optimization Technique

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Master of Science Thesis



Model-Based Individualization of Human-Like Steering Controllers

**Parameter Estimation of a Cybernetic Steering Model Using
Global Optimization Technique**

MASTER OF SCIENCE THESIS

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Abstract

Current autonomous steering controllers are designed based on either optimal control strategies or on average human driver steering behavior. Individual drivers may wish to steer differently than those controllers do. Such conflicts can be mitigated by customizing individual steering behavior. This thesis aims to develop a method to individualize human-like steering controllers, using a parameter estimation technique for a human-like steering model based on a global optimization algorithm. With this technique, the accuracy of estimated parameters is investigated in order to help to understand the nature of driver steering behavior.

A linear human-like cybernetic steering model inspired by human physical steering control actions is used in this thesis. In this steering model, seven parameters relate vehicles states to steering activity, of which the parameter values can be interpreted in a physically meaningful way. Preliminary research has demonstrated that this model structure enables parameter estimation of model parameters. But the current implemented parameter estimation technique of the steering model still needs requirements on initial guesses of the parameters and the accuracy of estimated parameters is under-investigated. In this study, parameter estimation of the steering model is investigated by formulating an optimization problem to minimize the prediction error. To deal with the nonlinearity of the optimization problem, a genetic algorithm is complemented with the Levenberg-Marquardt algorithm. This thesis presents the parameter estimation process in two steps. In the first step, the steering model generates a data set by simulating it with known parameters. This data set is used to estimate the steering model parameters and the results are compared with the known parameter set. Accurate parameter estimation results could not be reached in this first step, which is shown to be caused by over-parameterization of the steering model. Two parameters in the neuromuscular system of the steering model are then simplified to mitigate the over-parameterization problem and the other five parameters are remained. These five parameters can be estimated accurately by examining the metrics of VAF and Euclidean distance after the model simplification. In the second step, a simulation data set from a different validated human-vehicle model is used for parameter estimation to demonstrate the algorithm. It is shown that with the simplified steering model, the proposed parameter estimation technique can estimate consistent parameter values with good VAFs. A cross-validation test is also implemented to give the corroborative evidence to indicate over-parameterization of the steering model.

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Preface

This thesis titled “Model-Based Individualization of Human-Like Steering Controllers – Parameter Estimation of a Cybernetic Steering Model Using Global Optimization Technique” is submitted for the degree of Master of Science at Delft University of Technology, Netherlands. The content described in this thesis was supervised by Dr. ir. David A. Abbink from the Department of Biomechanical Engineering and Prof. dr. ir. Bart De Schutter from the Delft Center for System and Control at the Faculty of Mechanical, Maritime and Materials Engineering.

This work is to the best of my knowledge original, except where acknowledgments and references are made to previous work. Neither this, nor any substantially similar thesis has been or being submitted for any other degree, diploma or other qualification at any other university.

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Chapter 1

Introduction

Over the last decades, the automotive industry has achieved important developments in the autonomous vehicle research and the autonomous steering system has been a crucial part of it. Researchers are studying and validating the autonomous steering system due to its huge marketing and research potential [5].

1-1 Background

In autonomous vehicle systems and driver support systems, one perspective is to make the systems behave like a human driver. This is expected to promote trust and understanding, and ultimately driving comfort and safety [6]. One significant issue is to let the autonomous vehicle mimic human driver driving styles [7]. However, the autonomous steering system has not yet been designed under those considerations. Current steering controllers are mainly designed in two different ways. One way is that by comparing the real vehicles states with high-definition mapping data, the steering controllers determine the most optimal track [8, 9, 10]. But these steering controllers usually would not take human-like driving details into account while controlling the car. The other way is that the steering controllers are designed based on average human driver steering behavior [11]. However, one problem of this design mechanism is that the steering controllers do not customize the steering preference of individual human drivers. In reality, individual drivers may wish to steer differently than those steering controllers do. In Fig. 1-1, an illustration is given to show that those two individual drivers have totally different steering behavior compared with the steering controller. Acceptable safety margins are individual and drivers may feel “comfortable when driven by some people, and uncomfortable by others” [12]. Those differences and uncomfot can be regarded as driving conflicts [6, 13]. Driving conflicts influence human drivers’ acceptance of the autonomous system. Most drivers would like to recover control from autonomous systems if they did not like the way those controllers drive [14]. Moreover, conflicts influence automation use, resulting in failures of monitoring or decision making [15]. Since human driver steering behavior and acceptance to the autonomous steering system should be considered appropriately during the

steering controller design process, it is important and necessary to understand and differentiate individual human driver steering behavior. At the same time, only limited research has been done for this purpose and the results are still inconclusive due to their limitations (i.e., the use of a highly simplified model [16]).

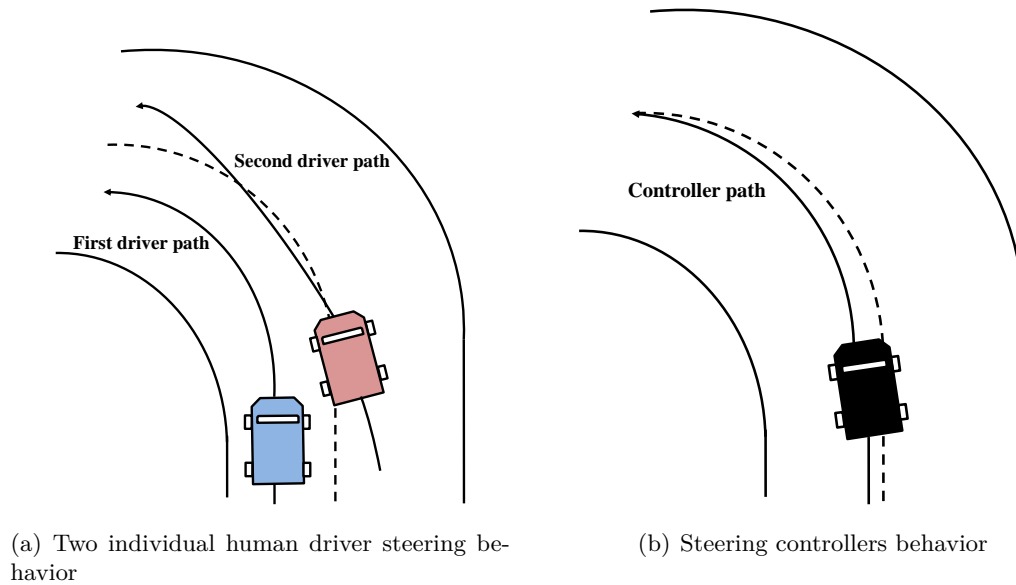


Figure 1-1: Illustration between the steering behavior of individual human driver and the behavior of steering controllers.

1-2 Problem Motivation

The model-based parameter estimation methods can be used to identify different human driver behaviors [17]. A human-like steering model can be used to reflect different modalities of human steering control actions. The steering model parameters can be used to quantify human control effects and describe human steering behavior [4, 18]. Parameter estimation is a tool for estimating model parameters and can be used to understand and individualize human steering control behavior. In the parameter estimation process, a suitable steering model that enables parameter estimation needs to be investigated and a suitable parameter estimation technique needs to be developed to accurately estimate parameters. However, due to limited research, some problems remain open.

A. Individualizing steering controllers only based on highly simplified models

Currently, research on individualizing steering behavior uses simple steering models [16, 19, 20]. These studies only consider steering control actions as a simple control process (i.e., future lateral error correction) and do not consider human characteristics for steering. As a result, human driver steering behavior cannot be described correctly and conflicts occur.

B. Limitations on parameter estimation technique

Frequency domain parameter identification techniques have been employed for steering

system identification [21, 22]. In the frequency domain techniques, nonparametric frequency response functions need to be estimated as the first step. Then parameters of a parametric model are optimized to fit to the frequency response functions. The accuracy of the parameter estimation is affected by the biases that originate from both identification steps [23]. On the other hand, although the time domain identification techniques need less requirements compared with the frequency domain techniques, only a few studies have been done in the literature to apply the time domain parameter estimation techniques for the steering system. Meanwhile, the implemented time domain parameter estimation techniques still have limitations (i.e., they need good initial guesses in the estimated parameters [24]).

C. Inconclusive on accuracy of parameter estimation results

Different degrees of control actions are represented by different parameters values in the human steering mechanism [18]. For example, during the steering process, the visual compensation is always used by human drivers to keep the vehicle in lane and this compensation induces jerky steering actions [25]. A high compensation gain can be explained by the fact that a human driver has more reliance on near visual information and vice versa. Parameter values are hence significant since they can be interpreted in a physically meaningful way. Understanding how drivers use different control modalities is useful for researchers to understand the nature of driver steering behavior. Hence, it is important to estimate those parameters accurately, although under some conditions different parameter combinations may give the same response. This problem is under investigation. The relative parameter estimation studies at this moment do not focus on the accuracy of the estimated parameters, but only focus on the performance of replicating the desired behavior [23, 26].

1-3 Goals of Master Thesis

The conflicts between the individual human drivers and steering controllers influence the acceptance of human drivers on the autonomous steering system and it is important to customize individual steering behavior to mitigate those conflicts. To investigate the way to individualize human driver steering behavior and related proposed research problems, this master thesis aims to develop a parameter estimation technique for the human steering model and evaluate to what extent this technique can estimate individual steering behavior.

A. Model selection

Many researchers have proposed human-like steering models. Based on the previous research and proposed steering model selection criteria [27], this study selects a suitable structured human-vehicle steering model.

B. Parameter estimation technique

This study develops a time domain parameter estimation technique to identify the specified parameters of the selected model. The technique seeks to need less requirements and steps for parameter estimation.

C. Accuracy of parameter estimation

This study investigates to what extent the parameter estimation technique can estimate the true parameter set.

1-4 Research Approach and Process

This study starts with the development of a human-vehicle steering model. A human-like cybernetic steering model which incorporates human steering control actions is used in this thesis [4]. In this model, seven parameters relate vehicle states to steering activity, of which the parameter values can be interpreted in a physically meaningful way to represent the nature of the driver steering behavior. This model structure has been adopted by preliminary research for identification and it is demonstrated to enable parameter estimation of model parameters [21]. A vehicle dynamics model with two degrees of freedom is built [2]. These two models are combined as a whole human-vehicle steering model.

This study is presented in two steps. In the first step, a known parameter set is given as the parameter reference to investigate how accurate the model parameters can be estimated. A time domain data for parameter estimation is generated from the selected steering model with this known parameter set.

A time domain parameter estimation algorithm based on the global optimization technique is used to decrease the parameter estimation bias. The parameter estimation results are compared with the known parameter set. The accuracy of the estimated parameters is investigated both for the full steering model and the simplified steering model to overcome the over-parameterization problem. Global sensitivity analysis is implemented to give a quantitative analysis to indicate parameter interactions.

Simulation data can get rid of uncertainties compared with real experimental data. Therefore in the second step, a different validated human-vehicle model generating data is used for parameter estimation.

The thesis is organized as follows. The human-vehicle steering model selection is introduced along with the data generated in Chapter 2. The parameter estimation technique development and implementation details are introduced in Chapter 3. Parameter retrieval analysis of the known parameters is done as the first step in this chapter. In Chapter 4, parameter estimation based on a simulated data set is intended to validate the theory in Chapter 3 as the second step. Final conclusions and recommendations for future work and final conclusions are given in Chapter 5.

Human-Vehicle Steering Model

The steering task represents a human-machine interface between the driver and the vehicle. The knowledge of the vehicle lateral dynamics and how drivers interact with the vehicle states are essential to design vehicle lateral control strategies. Human drivers use the knowledge of perceptual abilities, physical skills and understanding for steering. In order to capture human drivers' steering behavior and increase human drivers' acceptance of the steering controllers, those factors should be considered in autonomous steering systems and driver support systems design process [28]. Although many researchers have applied classic control methods (i.e., optimal control) to synthesize the steering control process [8, 10], human factors are not considered in those studies which could lead to misuse of automation [29]. As a result, it is important to model the steering controllers in a human-like way. Also, to mimic the steering task, the vehicle lateral dynamics is often complemented to build closed loop simulation scenarios. A typical human-vehicle steering system is depicted in Fig. 2-1. It is shown that the haptic sensation through the steering wheel is the way to represent the interactions between human drivers and vehicle lateral dynamics and human drivers use perceptions as well as the vehicle states feedback to make decisions on steering actions.

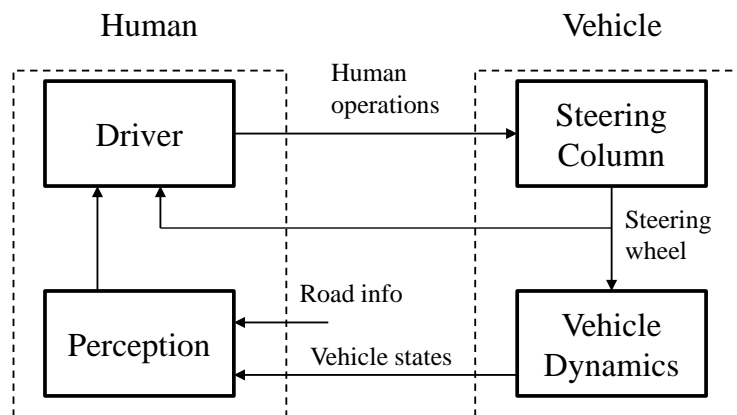


Figure 2-1: Schematic of human-vehicle steering systems [1].

Before studying the parameter estimation technique, a suitable choice of the human-vehicle model is needed to reflect steering process of human drivers. In this chapter, details of the selected human-vehicle steering model are discussed.

2-1 Vehicle Model

2-1-1 Vehicle dynamics model

The vehicle lateral dynamics model is built based on a single track model [2]. Figure 2-2 represents the vehicle dynamics model with two degrees of freedom. The lateral dynamics model of the vehicle is built from the local reference frame on a horizontal plane. The two degrees of freedom are represented by the lateral position of the vehicle and the yaw angle of the vehicle. The lateral position of the vehicle is measured along the lateral axis of the vehicle to the center of rotation of the vehicle. The yaw angle of the vehicle ψ (*rad*) is measured with respect to the global axis X_I . The longitudinal velocity and lateral velocity of the vehicle at the center of gravity are denoted by V_x (*m/s*) and V_y (*m/s*) along with the local coordinate axes of the vehicle X_v and Y_v , respectively. The variables F_{yf} (*N*) and F_{yr} (*N*) are the lateral tire forces of the front and rear tires, respectively. The variable m (*kg*) is the vehicle mass. The distances of the front tire and rear tire to the center gravity of the vehicle are denoted as l_f (*m*) and l_r (*m*), respectively. The notation cog is the center of gravity of the vehicle. The vehicle side slip angle is denoted as β (*rad*). The radius of the road bend is denoted as R_{ref} (*m*). The steering angle of front tire is denoted as δ . The current lateral error is denoted as e (*m*) and defined as the difference between the lateral position of the vehicle and the center of the lane. Then the future lateral error is defined as y_L (*m*). The calculations of e and y_L are given in Section 2-2-2.

The vehicle lateral dynamics is given by Newton's second law of motion along the axis Y_v :

$$ma_y = m(\dot{V}_y + V_x\dot{\psi}) = F_{yf} + F_{yr} \quad (2-1)$$

where a_y (*m/s²*) is the inertial acceleration of the vehicle at the center of gravity.

The moment balance for the yaw dynamics is given by

$$I_z\ddot{\psi} = I_z\dot{r} = l_f F_{yf} - l_r F_{yr} \quad (2-2)$$

where r (*rad/s*) is the yaw rate and I_z (*kg · m²*) is the moment of inertia of the vehicle about the yaw axis.

Experimental results have shown that the lateral force of a tire is proportional to the slip angle when the slip angle is small [30]. These results are used to model the lateral tire forces F_{yf} and F_{yr} . Figure 2-3 gives the illustration of the slip angle of a tire and it is shown to be defined as the angle between the orientation of the tire and the orientation of the tire velocity vector.

Considering the vehicle as a front-wheel-drive vehicle, the slip angles of the front tire and the rear tire are given by

$$\alpha_f = \delta - \theta_{vf} \quad (2-3)$$

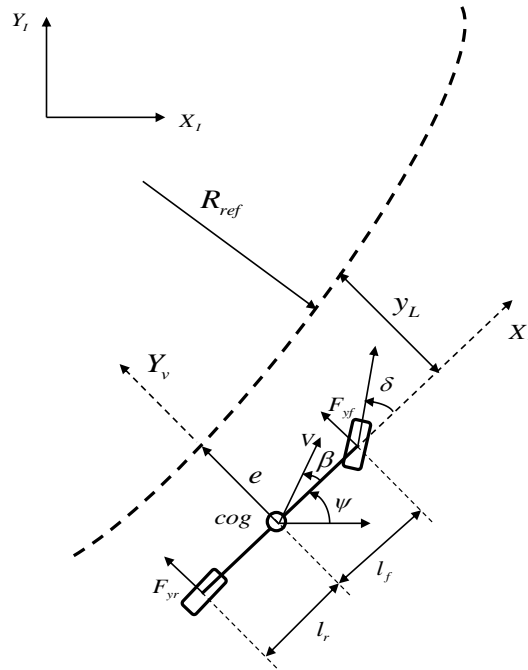


Figure 2-2: Schematic of the single track vehicle model [2].

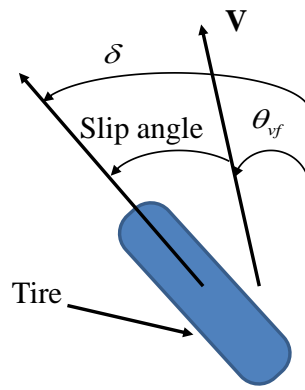


Figure 2-3: Illustration of front tire slip angle [2].

$$\alpha_r = -\theta_{vr} \quad (2-4)$$

where θ_{vr} (rad) and θ_{vf} (rad) are the angles between the tire velocity vector and the longitudinal axis of the vehicle.

The front and rear tire lateral forces are given by

$$F_{yf} = 2C_{\alpha f}\alpha_f \quad (2-5)$$

$$F_{yr} = 2C_{\alpha r}\alpha_r \quad (2-6)$$

where $C_{\alpha f}$ (N/rad) and $C_{\alpha r}$ (N/rad) are the front and rear tire cornering stiffness, respectively.

Using small angle approximation [31], the two angles θ_{vf} and θ_{vr} are given by

$$\theta_{vf} = \frac{V_y + l_f \dot{\psi}}{V_x} \quad (2-7)$$

$$\theta_{vr} = \frac{V_y - l_r \dot{\psi}}{V_x} \quad (2-8)$$

The linearized models which represent vehicle lateral dynamics are consequently given by

$$\dot{V}_y = -\frac{2C_{\alpha f} + 2C_{\alpha r}}{mV_x} V_y + (-V_x - \frac{2C_{\alpha f} l_f}{mV_x} + \frac{2C_{\alpha r} l_r}{mV_x}) r + \frac{2C_{\alpha f}}{mS_r} \delta_{SW} \quad (2-9)$$

$$\dot{r} = \frac{-2l_f C_{\alpha f} + 2l_r C_{\alpha r}}{I_z V_x} V_y + \frac{-2l_f^2 C_{\alpha f} - 2l_r^2 C_{\alpha r}}{I_z V_x} r + \frac{2l_f C_{\alpha f}}{I_z S_r} \delta_{SW} \quad (2-10)$$

where δ_{SW} (*rad*) is the steering wheel angle and S_r is the steering transmission ratio from the steering wheel to the front tire.

2-1-2 Steering column dynamics model

The steering column system plays a significant role to capture interactions between the driver and vehicle. It provides haptic feedback to deliver the information of the vehicle lateral dynamics to the driver. Here the steering column dynamics is modeled as a spring-mass-damper system [11], with the torque being applied by the driver. The steering column dynamics is given by

$$\ddot{\delta}_{sw} = -\frac{K_w}{J_w} \delta_{sw} - \frac{B_w}{J_w} \dot{\delta}_{sw} + \frac{1}{J_w} T \quad (2-11)$$

where T ($N \cdot m$) is the torque that the human driver implements on the steering wheel. The variable J_w ($kg \cdot m^2$) is the steering wheel rotational inertia. The variable K_w ($N \cdot m/rad$) is the steering wheel stiffness and the variable B_w ($N \cdot m \cdot s/rad$) is the damping of the steering wheel.

2-2 Human-Like Steering Control Model

To replace human drivers in steering control tasks, steering controllers should be modeled in the human-like way. Human factors should be considered while designing autonomous steering controllers to make the steering controller behave like real human drivers and increase human drivers' acceptance of the autonomous steering system. Multiple human-like steering models have been proposed in literature. A suitable human-like steering model is selected here based on the criteria as detailed below.

2-2-1 Human-like steering control model selection criteria

Specified human-like steering model selection criteria have been proposed in the literature [27]. The steering model should be evaluated according to the following criteria,

A. Reflection of human characteristics

Important human steering characteristics should be considered appropriately in the steering model. For example, research on human factors indicates that visual compensation and anticipation are commonly observed in the guidance and control level of human steering for vehicle lateral control [32].

B. Demonstrated model performance

The human-like steering model should demonstrate the ability to replicate the steering process of human drivers.

C. Identification feasibility

The model structure should enable system identification and parameter estimation of the model parameters.

The steering process of a human driver can be regarded as a process of multiple tasks. Various schemes have been proposed in literature. McRuer and Krendel [33] used a crossover driver steering model which assumes human drivers take steering actions to correct lateral errors. However, it was shown that the primary sensory channel which is used by drivers for the steering task is visual preview information [34]. A brief and straightforward demonstration of effects of using preview information for the human steering process was introduced in the literature [35]. Donges [32] proposed significant understanding of various visual control strategies in which the steering task is divided into two visual levels. One is the anticipatory control mode and this control mode provides smooth steering control actions for tracking the future curvature which serves the purpose of the guidance. The other control mode is the compensatory control mode and this control mode is used for the lane keeping of the vehicle, which serves the purpose of stabilization of the steering process [36]. The importance of those two levels has been confirmed by several pioneering studies [37, 38]. Modjtahedzadeh and Hess [39] have used the anticipatory module as the steering controller and McRuer and Krendel [40] has used the compensation module as the steering controller. More recently, Salvucci and Gray [16] explored the results of Donges and developed a PI controller to model the two-level control scheme.

However, these models are limited as-

- i) The steering models only consider parts of human preview information and do not combine these two visual control modules together.
- ii) The structures of the steering models give little information on how drivers operate the vehicle.
- iii) The identification feasibility of the steering models has not been investigated.

Mars, Saleh et al. [4] proposed the human-like cybernetic two-point visual steering model with the Neuromuscular System (NMS) and this steering model contains the preview information of both the anticipation module and the compensation module. Moreover, it considers physical operations of drivers. The steering model incorporates road information to present visual scenes of drivers which are known as a far point and a near point. Also, it was demonstrated that this steering model is a prime candidate model for parameter estimation in curve steering tasks [21]. As a result, this steering model, which satisfies the selection criteria and overcomes

disadvantages of previous models, is used for this parameter estimation study. Details of this steering model are given as follows.

2-2-2 Human visual perception and road geometry

The couplings between the vehicle and the human driver are through visual perception and road geometry. The steering model in the literature [4] uses a two-point control scheme. A far point is often at the tangent point. A near point is at a small distance ahead. Several studies have proven that human drivers focus their attention on these two points in their visual field of view [27]. Geometric expressions of visual observations on these two points are given by two angles: θ_{far} and θ_{near} . These two angles can be determined from the road geometry and they are illustrated in Fig. 2-4. The angles θ_{far} and θ_{near} correspond to those two visual points, where θ_{far} is the visual angle between the vehicle heading direction and the tangent point of the road curve and θ_{near} is the visual angle between the near view point and the vehicle heading direction.

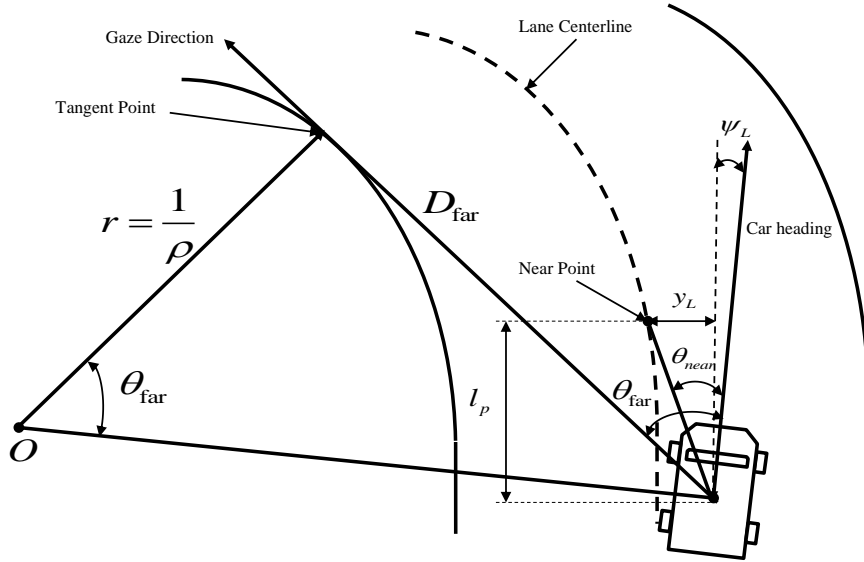


Figure 2-4: Relations between drivers' vision and road geometry [3].

In Fig. 2-4, the variable ψ_L (rad/s) is the yaw angle error and defined as the difference between the vehicle yaw angle and desired orientation of the vehicle. The variable l_p (m) is the look ahead distance for a near point. The variable ρ ($1/m$) is the road curvature, which is defined to be the reciprocal of the road curve radius. The variable D_{far} is the distance between the vehicle and the tangent point.

The change rate of the vehicle desired orientation is defined as [2]:

$$\dot{\varphi}_{des} = \frac{V_x}{R} = V_x \rho \quad (2-12)$$

Using small angle approximation, the future lateral error y_L (m) is given by

$$y_L = e + l_p \psi_L = e + l_p (\psi - \psi_{des}) \quad (2-13)$$

The far angle and near angle are given using small angle approximation by

$$\theta_{far} = \frac{D_{far}}{V_x} \rho \quad (2-14)$$

$$\theta_{near} = \frac{y_L}{l_p} + \psi_L \quad (2-15)$$

2-2-3 Human-like cybernetic steering control model

The steering model in the literature [4] contains four main parts: visual anticipatory module, visual compensatory module, visual processing delay module and NMS module. The related block diagram is depicted in Fig. 2-5. The inputs to the driver model are the two visual angles θ_{far} , θ_{near} and the steering wheel angle δ_{SW} . The steering model uses the anticipatory module and the compensatory module for steering control and uses the haptic feedback to model the NMS which represents the physical operations of drivers to the vehicle. The relative implemented torque is denoted as T_{dr} .

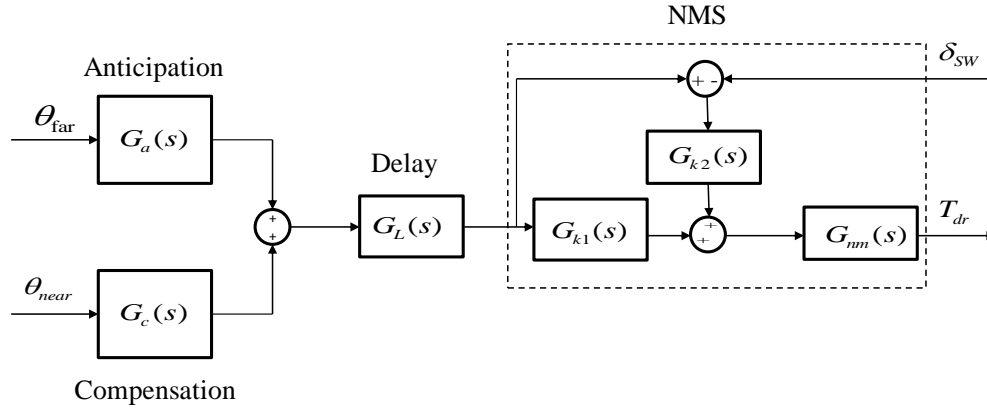


Figure 2-5: Schematic of the human-like cybernetic steering model [4].

In the visual anticipatory module, the model uses a proportional term to track the future reference trajectory:

$$G_a = K_p \quad (2-16)$$

The visual compensatory module is used for correcting instantaneous variations based on the perceived visual near angle and these corrections are related to the vehicle speed. A driver uses the near point information to keep the vehicle on the center lane. The corresponding transfer function is given by

$$G_c = \frac{K_c T_L s + 1}{V_x T_I s + 1} \quad (2-17)$$

where T_L and T_I are the lead and lag time constants, respectively. The variable K_c is the proportional gain for correcting variations.

The human central nervous system and peripheral delay are incorporated in visual processing delay module. This delay is represented by Padé approximation

$$G_L = e^{-\tau_p s} \approx \frac{1 - \frac{\tau_p}{2} s}{1 + \frac{\tau_p}{2} s} \quad (2-18)$$

where τ_p is the delay time.

The NMS is modeled by considering drivers' adaption to the feedback force and it considers that the adaption is through an internal model of steering system compliance. Transfer functions G_{k1} and G_{k2} represent the internal model of steering column stiffness and human haptic feedback control action, respectively. They are given by

$$G_{k1} = K_r \cdot V_x \quad (2-19)$$

$$G_{k2} = K_t \quad (2-20)$$

The NMS of a driver's arm G_{NM} is a first order transfer function as follows:

$$G_{NM} = \frac{1}{T_N s + 1} \quad (2-21)$$

2-2-4 Combined human-vehicle steering model

The vehicle dynamics model and human steering model can be combined as a closed loop model and it is depicted in Fig. 2-6. The inputs of the model come from the road information.

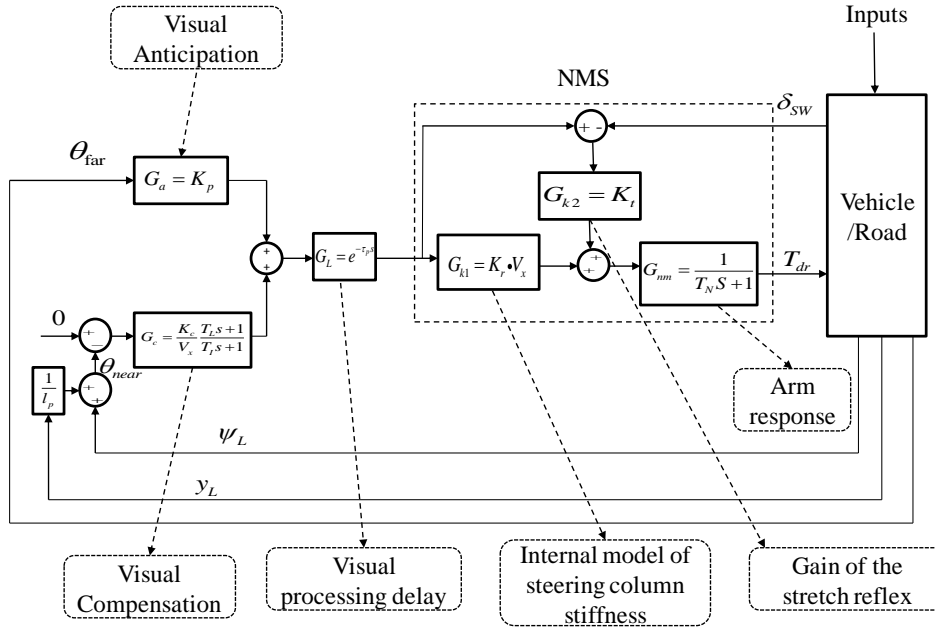


Figure 2-6: Structure of the combined human-vehicle steering model, selected for the current study [4].

The closed loop model depicted in Fig. 2-6 can be written in a state space formulation as follows:

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (2-22)$$

$$y(t) = Cx(t) + Du(t) \quad (2-23)$$

with the state vector $[V_y \ r \ \psi_L \ \delta_{SW} \ \dot{\delta}_{SW} \ x_1 \ x_2 \ x_3]^T$, the inputs road curvature ρ and lateral error e .

Here the states x_1 , x_2 and x_3 are selected and represented into the state space form in the analytical way as follows:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \frac{\theta_{near}}{T_I s + 1} \\ \frac{1}{(\frac{\tau_p}{2} s + 1)} \left[K_p \theta_{far} - \frac{K_c}{V_x} (1 + T_L s) x_1 \right] \\ T_{dr} \end{bmatrix}$$

Details of the mathematical derivations of x_1 , x_2 and x_3 are given in the Appendix A.

The state-space matrices A and B of the continuous model in Eq. 2-22 are given as follows:

$$A = \begin{bmatrix} -2 \frac{C_{\alpha f} + C_{\alpha r}}{m V_x} & -V_x - 2 \frac{C_{\alpha f} l_f - C_{\alpha r} l_r}{m V_x} & 0 & \frac{2 C_{\alpha f}}{m S_r} & 0 & 0 & 0 & 0 & 0 \\ 2 \frac{-l_f C_{\alpha f} + l_r C_{\alpha r}}{I_z V_x} & 2 \frac{-l_f^2 C_{\alpha f} - l_r^2 C_{\alpha r}}{I_z V_x} & 0 & \frac{2 l_f C_{\alpha f}}{I_z S_r} & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{K_w}{J_w} & -\frac{B_w}{J_w} & 0 & 0 & 0 & \frac{1}{J_w} \\ 0 & 0 & \frac{2}{T_I} & 0 & 0 & -\frac{1}{T_I} & 0 & 0 & 0 \\ 0 & 0 & -\frac{4 K_c T_L}{V_x \tau_p T_I} & 0 & 0 & \frac{2 K_c (T_L - T_I)}{V_x \tau_p T_I} & -\frac{2}{\tau_p} & 0 & 0 \\ 0 & 0 & \frac{2 (K_r V_x + K_t) K_c T_L}{T_n V_x T_I} & -\frac{K_t}{T_n} & 0 & -\frac{(K_r V_x + K_t) K_c (T_L - T_I)}{T_n V_x T_I} & -\frac{2 (K_r V_x + K_t)}{T_n} & -\frac{1}{T_n} & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -V_x & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & \frac{1}{l_p} \\ \frac{2 K_p D_{far}}{\tau_p} & -\frac{2 K_c T_L}{\tau_p V_x T_I l_p} \\ -\frac{K_p (K_r V_x + K_t) D_{far}}{T_n} & \frac{K_c (K_r V_x + K_t) T_L}{T_n V_x T_I l_p} \end{bmatrix}$$

The matrix C in Eq. 2-23 are used to output the torque and the steering wheel angle and given as follows:

$$C = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The D matrix is a zero matrix here.

Parameters which reflect human driver perceptual and control modalities are given as follows:

$$\left[K_p \ K_c \ T_I \ T_L \ \tau_p \ K_r \ K_t \ T_N \right]$$

T_N is fixed during the simulations ($T_N = 0.1$), depending on many precedent works that have led to the same value [4]. Another seven parameters are regarded as unknown parameters and they need to be estimated in order to individualize human steering behavior.

2-3 Parameter Estimation Data Choice

In order to investigate to what extent the selected model can be used for parameter estimation on measured data, several possible data sets can be employed:

A. Real vehicle and real human experimental data

Real life data sets contain too many real world uncertainties and are not available for this research.

B. Simulator and real human experimental data

In simulator studies the vehicle dynamics and environment are known exactly, but the data set still captures the variability within and between drivers.

C. Simulation data

A simulation with an assumed driver model and known parameters can also be used to generate a data set. The main benefit is that a ground truth for the parameters is now available.

In this study, the data set C is chosen in order to get rid of uncertainties of the real experimental data and evaluate the parameter estimation algorithm, by comparing the estimated parameters to the ground truth of the known parameters.

In the first step of this study, a known parameter set is used to generate time domain data to investigate the accuracy of estimated parameters. By this means, the estimated parameter set can be compared with the known parameter set. In the second step, a different validated human-vehicle model in the literature [41] is used to generate the time domain data for the parameter estimation and it is chosen because it uses the preview information and does not contain real human uncertainties. The vehicle dynamics parameters are given in Table 2-1 [41]. Road information (near and far point distance) is fixed [24].

Table 2-1: The parameters of the vehicle dynamics model

Parameter	Symbol	Value	Unit
Longitudinal speed	V_x	25	m/s
Vehicle mass	m	1600	kg
Front tire cornering stiffness	$C_{\alpha f}$	30000	N/rad
Rear tire cornering stiffness	$C_{\alpha r}$	30000	N/rad
Vehicle yaw moment of inertia	I_z	3136	$kg \cdot m^3$
Front axis distance	l_f	1.4	m
Rear axis distance	l_r	1.4	m
Steering wheel rotational inertia	J_w	0.2	$kg \cdot m^2$
Steering wheel stiffness	K_w	4.2	N.A
Steering wheel damping	B_w	1	$kg \cdot m$
Steering gear ratio	S_r	15	N.A
Near point distance	l_p	5	m
Far point distance	D_{far}	20	m

The known parameter set is chosen from multiple simulations to guarantee the performance of the generated data by comparing the generated data with the existed data from the validated model in the literature [41]. The reason why the known parameter set is not directly selected

from the literature [24] is that there are a few changes in the model structure. Using those parameters directly induces unstable dynamic response and hence the parameters need to be re-tuned.

Saleh, Chevrel et al. [24] used the torque for parameter estimation and the steering wheel angle for validation. The same strategy is consistently used here as a starting parameter estimation strategy. The known parameter set is given in Table 2-2 and the related generated data is given in Fig. 2-7. The simulation time is 50 seconds. In this study, the positive lateral deviation means that the vehicle is at the left side of the center lane. The positive road curvature means that the vehicle turns left.

Table 2-2: The known parameter set

Parameter	K_p	K_c	T_I	T_L	τ_p	K_r	K_t
Value	0.11	7.78	2.96	1.53	0.001	2.46	6.15

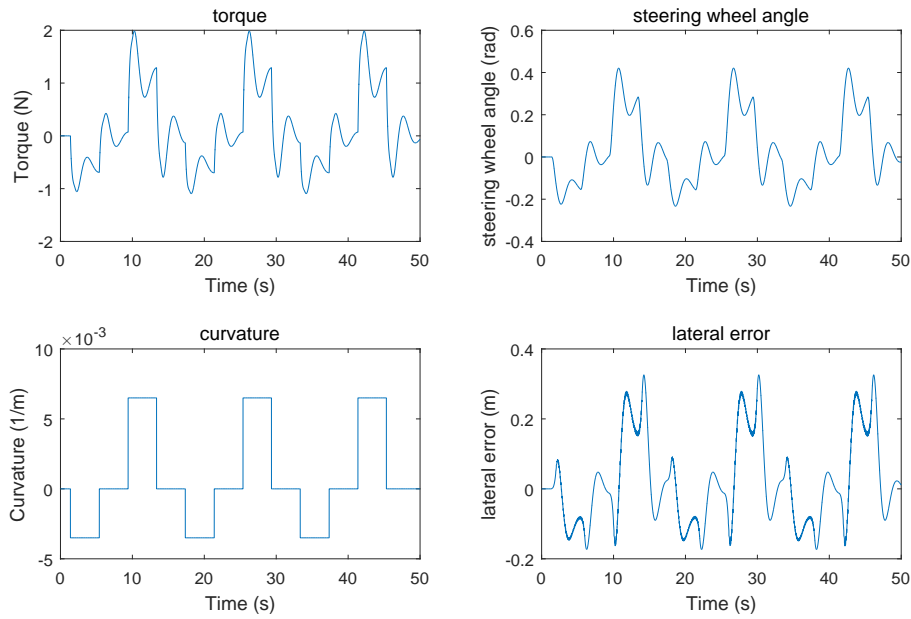


Figure 2-7: The simulation data for parameter estimation from the known parameter set.

To show that the inputs signal are rich enough, the persistently exciting of the inputs signal is examined. It shows that the inputs signal are persistently exciting of order n of the steering model if and only if there exists an integer N such that the matrix

$$\begin{bmatrix} u(0) & u(1) & \cdots & u(N-1) \\ u(1) & u(2) & \cdots & u(N) \\ \vdots & \vdots & \ddots & \vdots \\ u(n-1) & u(n) & \cdots & u(N+n-2) \end{bmatrix}$$

has full rank n [42]. Here it is shown that when the length of N is selected as the length of whole data, the inputs are persistently exciting of the order of the steering model (which is eight). So the inputs signal are shown to be rich enough for parameter estimation.

2-4 Sub-conclusion

This chapter motivates the selection of a human-vehicle steering model, which can serve for our goal to capture differences in individual driving behavior. The human-like cybernetic steering model is chosen, based on the following criteria: 1) It captures driving behavior in meaningful physiological parameters. 2) It is shown in literature to be able to be used for parameterization of average driver behavior. The selected model and its parameters have been explained in detail.

This chapter also motivates the procedure followed to validate the model's capabilities using data generated by driver models in simulation, using two steps. In the first step of this study, the data is initially generated from a known parameter set to examine the accuracy of estimated parameter results. In the second step, simulation data generated from the validated model is used for parameter estimation. The known parameter set and related data for the first step are given in this chapter.

Parameter Estimation Results of Known Parameters

This chapter presents the first step of this study. A parameter estimation technique is developed and implemented to the human-like cybernetic model based on the generated data of the known parameter set. The accuracy of the estimated parameters is investigated by comparing the estimated parameters with the known parameter set.

3-1 Parameter Estimation Technique

Parameter estimation of the selected cybernetic steering model is a nonlinear problem since the parameters of the model have nonlinear interactions. Saleh, Chevrel et al. [24] used this steering model and minimized the prediction error to estimate the parameters, but required good initial guesses of the initial parameter set. This is a severe weakness for a parameter estimation technique intended to capture individual differences.

Many different techniques can be applied for parameter estimation purposes. In general, two groups of parameter estimation can be used: parameter estimation in the frequency domain and time domain [23].

Parameter estimation in the frequency domain can be done in a two-step approach. In the first step, the frequency response functions are identified using Fourier coefficient method or linear time invariant models (i.e., ARMAX) [42]. In the second step, these nonparametric frequency response functions are used to fit a parametric model. One advantage of this technique is that the first step contributes as a data reduction step and the parameter optimization process becomes more efficient [23]. However, this two-step strategy increases the bias and variance in the estimated parameters. On the other hand, this method is always used in the case that a model structure is unknown beforehand [21, 22].

Parameter estimation in the time domain directly fits a parametric model on the time domain data. One disadvantage of this method is that more computational power is required since

time domain data contains more points than the frequency response functions. However, the one-step fitting characteristic decreases the bias and variance of the estimated parameters compared with the frequency domain method [23]. Since one goal in this study is to examine the accuracy of estimated parameters and the steering model structure is known beforehand, a time domain technique is used for this study.

In this study, the parameter estimation problem is formulated as an optimization problem and the objective is to minimize the prediction error. The objective function is given by

$$\min \sum_{k=1}^N (y_{est} - y)^2 \quad (3-1)$$

Here N is the total amount of data. The symbol y_{est} is the estimated output from the steering model and y is the raw data for parameter estimation. The nonlinearity of objective function results in additional local minima.

In the first step, the data set used for parameter estimation comes from the known parameter set. To increase the probability of finding the global optimum of the optimization problem, the Levenberg-Marquardt algorithm used to minimize the prediction error is complemented with a genetic algorithm.

3-1-1 Global optimization technique–Genetic algorithm

Global optimization algorithm can yield promising solutions on finding global optimum [43]. Among the existing algorithms, genetic algorithm is widely used to solve parameter estimation problems in many applications [44, 45]. Unlike gradient-based optimization algorithms, a genetic algorithm searches the optimum by mimicking the process of biological evolution. The algorithm begins with setting a collection of parameters and the parameters need to be coded. Two common used coding methods are binary coding and real coding [46]. The binary coding codes the parameters in the binary representation. It has been shown that the binary representation have difficulties to deal with continuous search spaces when the numerical precision of estimation is required [47]. On the other hand, the real coding represents the genes as real numbers for optimization problems with parameters in continuous domains and it could increase the accuracy of estimated parameters. In this coding method, a chromosome is a vector of floating point numbers, the precision of the numbers depends on the precision of the computer calculating ability [46]. So in this study, real coding method is used for coding the parameters. The real-coded mechanism is shown as follows. Considering that the W_l is the chromosome where $l = 1, \dots, n$ and n is the population size, W_l can be represented as $(w_{l_1}, w_{l_2}, \dots, w_{l_k})$ where w is one of the parameters that need to be estimated and k is total number of parameters. In other words, each gene within a chromosome represents a parameter in the optimization problem. Then the algorithm solves the given minimization task through random genetic functions such as selection, mating, crossover and mutation [43]. These functions cause the population to evolve towards better solutions (i.e, more fit) and the asymptotic convergence of the estimation error has been proved [44].

The genetic algorithm in this study is used to solve the optimization problem in Eq. 3-1 to find the steering model parameters. The algorithm starts by generating a uniformly distributed population within the certain ranges. The population fitness is calculated and iterations are performed in the following way [43].

- “Parents” selection. Two solutions are selected as “parents” through fitness proportionate selection [48]. The fitness is used to associate a probability of selection with each individual solution. If f_i is the fitness of individual i in the population, its probability of being selected is $p_i = \frac{f_i}{\sum_{k=1}^M f_k}$, where M is the population size. These two solutions will mate and produce off-springs.
- After the parents are selected, off-springs are generated using crossover with a certain probability. Here the scattered crossover method is used since the asymptotic convergence of the error has been shown by using this method [49]. This type of crossover creates a random binary vector whose length equals to the number of parameters. Then the genes are selected from the first parent if the element in this vector is a 1, or from the second parent if the element in this vector is a 0. Then the selected genes are combined to form the first child, and vice versa to form the second child. The procedure is illustrated as follows:
consider two chromosomes with four parameters are selected as parents to apply the scattered crossover to them

$$\begin{aligned} \text{Parent 1} \quad P_1 &= (p_1^1, p_2^1, p_3^1, p_4^1) \\ \text{Parent 2} \quad P_2 &= (p_1^2, p_2^2, p_3^2, p_4^2) \end{aligned}$$

with a random vector 1001

$$\begin{aligned} \text{Child 1} \quad C_1 &= (p_1^1, p_2^2, p_3^2, p_4^1) \\ \text{Child 2} \quad C_2 &= (p_1^2, p_2^1, p_3^1, p_4^2) \end{aligned}$$

- Consider that $W_l = (w_{l_1}, \dots, w_{l_i}, \dots, w_{l_k})$ is a chromosome with k parameters and w_{l_i} is in the domain $[c_i, d_i]$. Then w_{l_i} has a certain probability to mutate to a random number in $[c_i, d_i]$ [50]. The mutation provides genetic diversity and enables the genetic algorithm to search a broader space.
- The algorithm will stop if the number of generations reaches the limitation.

A genetic algorithm can avoid converging to local minima using a population that represents many possible solutions and it provides a high probability to approach the global optimum. On the other hand, a genetic algorithm is not deterministic and could give different results every time they are run [23]. A local optimization technique can be used to refine the results of parameter estimation.

3-1-2 Levenberg-Marquardt algorithm

The solutions of the genetic algorithm have a high probability to approach the global optimum. Since the solutions have approached to the global optimal point, the solutions of the genetic algorithm are used as the initial parameter estimations for the Levenberg-Marquardt optimization to refine those solutions. Since the solutions have been close to the global optimum, the parameter refining process is considered here to be a unconstrained optimization problem to find the global optimum. It is concluded that for unconstrained nonlinear non-convex

optimization problems, the Levenberg-Marquardt algorithm is considered as the highest preference since it converge fast to the optimal point [43]. The algorithm is developed from the Newton algorithm with a refined Hessian matrix and given by

$$x_{k+1} = x_k - (\lambda I + H(x_k))^{-1} \nabla f(x_k) \quad (3-2)$$

where λ is selected appropriately to avoid that the inversion of Hessian matrix is numerically ill-conditioned.

The termination of the algorithm is given as follows:

$$\|\nabla f(x_k)\|_2 \leq \varepsilon \quad (3-3)$$

where ε is the stopping tolerance.

3-1-3 Metrics for parameter estimation

Two metrics are used for evaluation of parameter estimation results.

- Variance Accounted For (VAF) [42]. VAF is a criterion for evaluating the data fitness degree and it ranges from 0% to 100%. The higher the VAF is, the lower the prediction error is and the better the data fitting is. The formula is given by

$$VAF(y, y_{est}) = \max(0, (1 - \frac{\frac{1}{N} \sum_{k=1}^N \|y - y_{est}\|_2^2}{\frac{1}{N} \sum_{k=1}^N \|y\|_2^2}) \cdot 100\%) \quad (3-4)$$

- Euclidean distance [51]. In mathematics, Euclidean distance is the distance between two points in Euclidean space. Considering two points $P = (p_1, p_2, \dots, p_n)$ and $Q = (q_1, q_2, \dots, q_n)$ in n dimension of Euclidean space in Cartesian coordinates, the distance between P and Q is given by

$$d(P, Q) = \sqrt{\sum_{i=1}^n (q_i - p_i)^2} \quad (3-5)$$

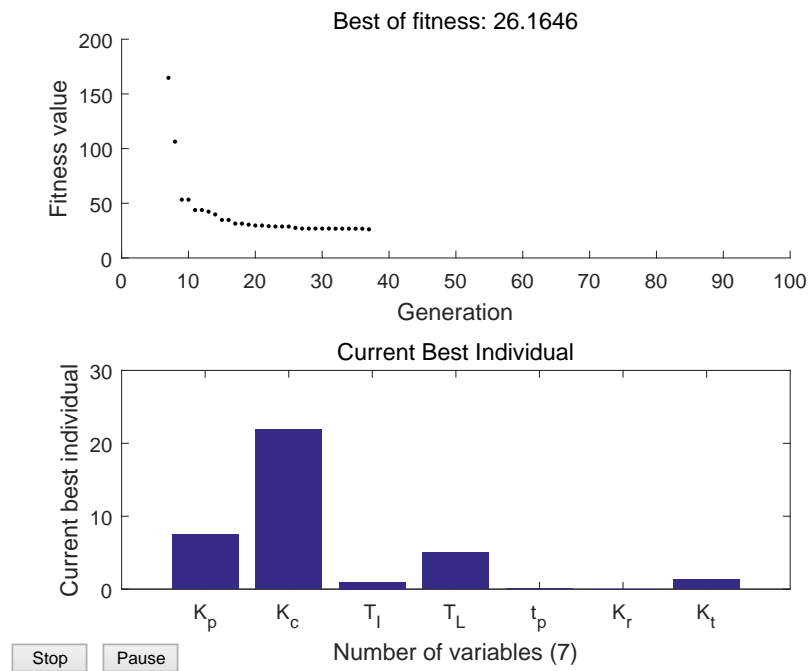
3-2 Parameter Estimation Results of Known Parameters

The data set in Fig. 2-7 is used to estimate the parameters of the cybernetic steering model. The seven parameters of the steering model need to be estimated since they reflect the human drivers steering control actions and operations. The parameters ranges for a genetic algorithm are given in Table 3-1. Here the ranges in literature [4] are softened since the parameters values are sensitive to the vehicle structure and parameters. Since the self-aligning torque is not considered and different vehicle parameters are used in this model, the upper bounds are softened to guarantee those bounds large enough to account for all relevant solutions of the estimation problem and the lower bounds of parameters are set to 0 according to the conclusions in the literature [4].

Table 3-1: Lower and upper parameter bounds used in the genetic algorithm

Parameter	Anti. gain	Comp. gain	Comp. lag time constant	Comp. lead time constant	Processing time delay	Angle to torque coefficient	Neuromuscular reflex gain
Symbol	K_p	K_c	T_l	T_L	τ_p	K_r	K_t
Lower bound	0	0	0	0	0	0	0
Upper bound	30	35	15	15	0.1	30	30

Typically, the population size for the genetic algorithm is 200 and 100 iterations with the genetic algorithm are performed. Those are selected to increase the possibilities that the genetic algorithm could search the solution space thoroughly, thereby reducing the chance that the algorithm returns a local minimum. Crossover probability is set to 0.7 and mutation probability is set to 0.01. Those parameters are selected according to the suggestions in the literature [23]. Then the solution of the genetic algorithm is used as the initial parameter set for the Levenberg-Marquardt optimization. The factor λ and tolerance ε are set to 0.01 and 10^{-6} by trail and error, respectively. 10 repetitions of whole algorithm chain are performed on the data set for estimated parameters comparison. The torque is used for parameter estimation and the steering wheel angle is used for validation. An illustration of one repetition of the genetic algorithm iterations are given in Fig. 3-1. The upper plot gives the parameter estimation progress and the lower plot gives the the values of the estimated parameters.

**Figure 3-1:** An illustration of the genetic algorithm iteration.

It is shown in Figure 3-1 that the prediction error shows asymptotic characteristic of convergence to estimate the parameters and the first several points fitness values are not shown in the plot since they are very large. Details of the data fitting and parameter estimation results are given in Table B-1. Figure 3-2 gives one illustration of the good data fitting results for

torque and steering wheel angle. The satisfactory data fitting results are expected since the data originates from the model.

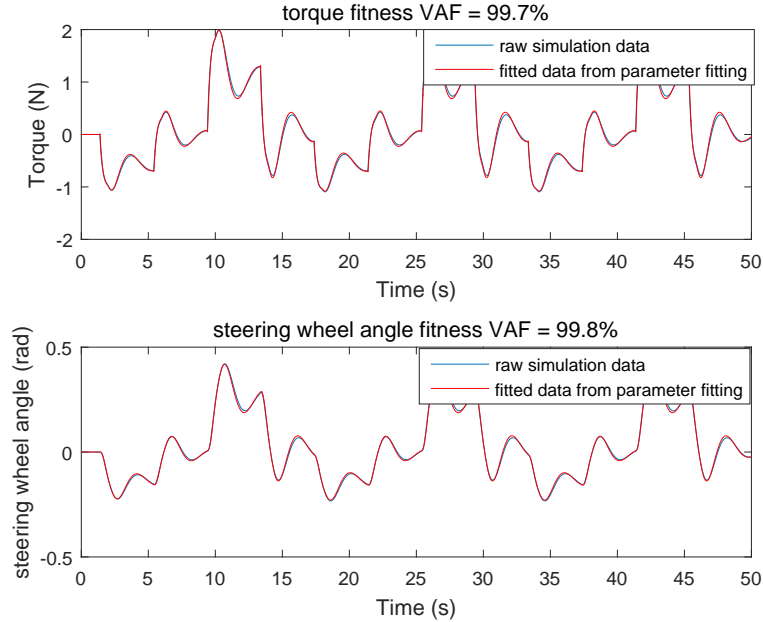


Figure 3-2: The time domain data fitting results of the full steering model.

A straightforward illustration of the distributions of the estimated parameters are given in Fig. 3-3. The plot shows the distributions of the seven estimated parameters from the 10 repetitions. For each individual parameter, 10 sets of the estimated parameters distribute differently compared with the known parameters and large interactions have occurred among those parameters. In Table B-1, the results of Euclidean distance also demonstrate that the estimated parameter sets are not close to the known parameter set if the estimated parameter sets are considered to be the points of the seven dimensional space. It is shown that the parameter estimation results need to be improved since the estimated parameters are not accurate compared with the known parameters.

One suggestion to improve the parameter estimation results is to use the steering wheel angle as the identification data [24]. Another 10 repetitions of parameter estimation are performed. Table B-2 gives the results of estimated parameters and data fitting. Good data fitting performance is shown from the results which give collaborative evidences that the global optimization algorithm has the ability on replicating the output behavior when the model matches the data. On the other hand, the comparisons of Euclidean distance between the torque estimated parameters and the steering wheel angle estimated parameters are given in Fig. 3-5. From the Euclidean distance comparisons, the steering wheel angle estimated parameter sets are getting closer to the known parameter set comparing with the torque estimated parameter sets, if the parameter sets are considered as the points of seven dimensional space. The results indicate that by changing the identified data sets, the accuracy of estimated parameters could be improved. Furthermore, the distributions of estimated parameters are given in Fig. 3-4. It can be seen that for each individual parameter, the

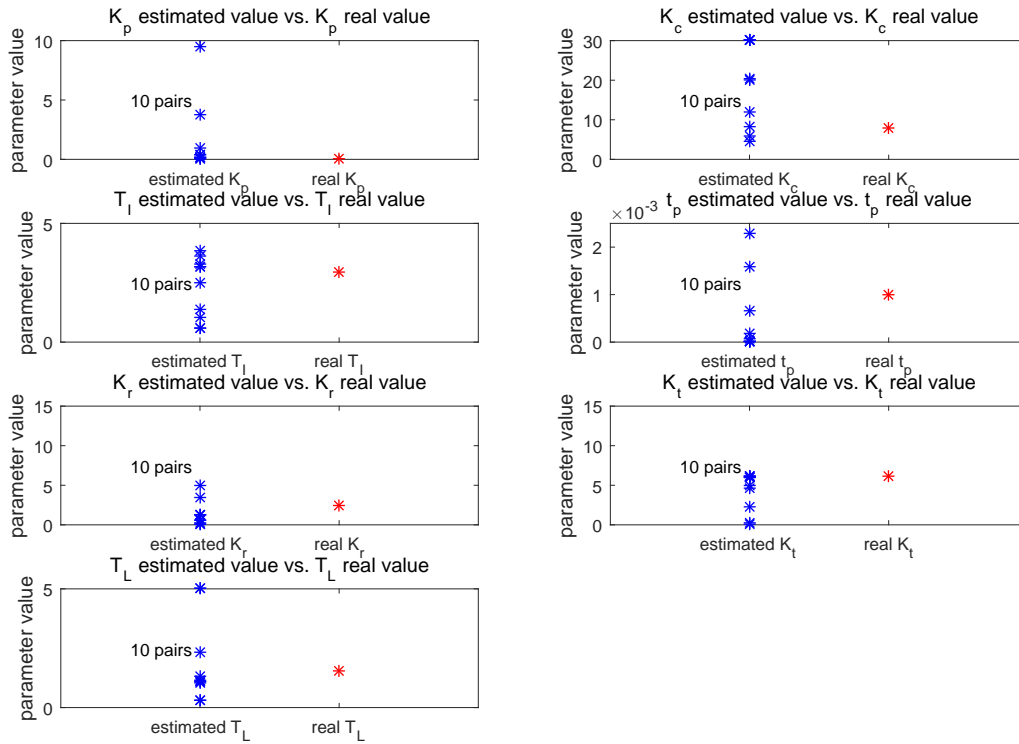


Figure 3-3: The parameter estimation results of full steering model, the torque as identified data.

estimated parameters still distribute differently compared with the known parameter set and the accuracy of the estimated parameters is still problematic. Large interactions among the parameters still exist.

Although the output performance can be replicated, accurate parameter estimation is also important since the parameters values can be interpreted in a physically meaningful way and reflect the nature of how human use different modalities for steering control. Inaccurate parameter estimations lead to misunderstandings of human steering control behavior. Analytical ways need to be investigated to improve the accuracy of estimated parameters.

3-3 Steering Model Parameter Interactions Analysis–Global Sensitivity Analysis

Through the two identification results, parameter interactions are shown and they influence the accuracy of estimated parameters. A tool to analyze the parameter interactions could contribute to the evaluation of the model quality. Global sensitivity analysis is such a tool that can reveal the interactions among the parameters.

Global sensitivity analysis uses variance metric to quantify the contribution of input parameters variability to the total variance of the outputs [52]. Assuming that $y = f(X)$ is the model

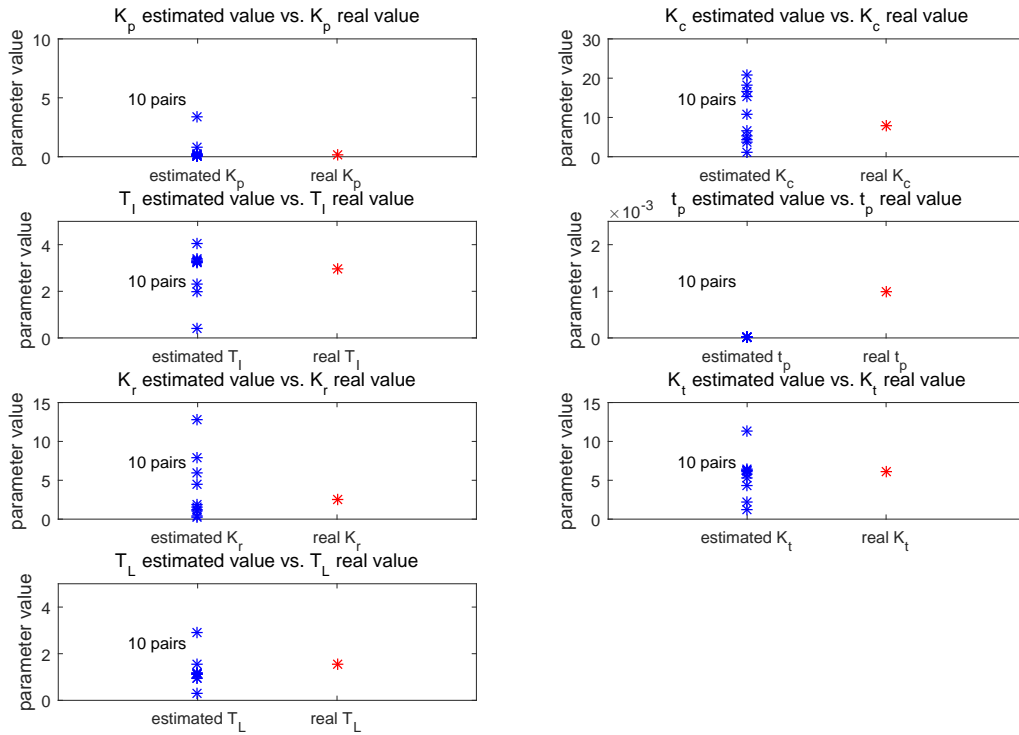


Figure 3-4: The parameter estimation results of the full steering model, the steering wheel angle as identified data.

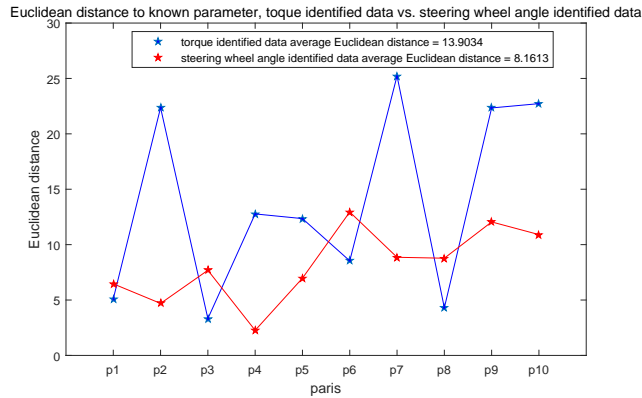


Figure 3-5: The Euclidean distance from the estimated parameters to the known parameter set.

function in which y is the output and $X = (x_1, x_2, \dots, x_n)$ contains n input parameters, each parameter has a uniform probability density function in a certain range. A general method to evaluate the impact of variability of a single input parameter to the output variance is called the first order sensitivity index [53] and the formula is given by

$$S_i = \frac{\text{var}\{E[y|x_i]\}}{\text{var}\{y\}} \quad i = 1, \dots, n \quad (3-6)$$

Here $E[y|x_i]$ is the conditional expectation of all possible x_i . Then the variance of the conditional expectation $E[y|x_i]$ can be calculated. Equation 3-6 normalizes this variance with the unconditional variance of y to get the first order sensitivity index. The index S_i ranges between 0 and 1. The index measures the main contribution of the variability of input x_i to the output variance and does not consider the interactions between x_i and other parameters. If the first order index of the parameters is large, it means that the variance of y mainly comes from the variability of x_i and if the first order index of each parameters is small, the variance of y mainly comes from the interactions among the parameters [52].

3-3-1 Calculations of the first order sensitivity index of parameters

This section aims to give mathematical implementations about how to solve Equation 3-6, in order to calculate the first order index which could be used to indicate the degree of parameter interactions in the steering model [54]. Firstly a decomposition is used to interpret how Equation 3-6 can be calculated mathematically. Then the Monte Carlo method is introduced such that the decomposition can be efficiently computed.

Assuming that I is the unit interval $[0, 1]$, I^n is the n dimensional unit hypercube and $x \in I^n$. All the integrals below are from 0 to 1 for each variable and $dx = dx_1 \dots dx_n$. Considering $y = f(X)$ is square-integrable in the unit hypercube I^n and X is the vector of n inputs variables (x_1, \dots, x_n) . It is possible that $f(X)$ can be written as follows [52]:

$$f(x) = f_0 + \sum_{s=1}^n \sum_{i_1 < \dots < i_s}^n f_{i_1 \dots i_s}(x_{i_1}, \dots, x_{i_s}) \quad (3-7)$$

Here $1 \leq i_1 < i_s \leq n$ and the total number of terms in Eq. 3-7 is 2^n . This formula is called ANalysis Of VAriance (ANOVA) of $f(X)$ [54]:

$$\int f_{i_1 \dots i_s}(x_{i_1}, \dots, x_{i_s}) dx_{i_1} \dots dx_{i_s} = 0 \quad (3-8)$$

In this condition, terms in Eq. 3-7 can be given as integrals of $f(X)$:

$$\int f(X) dx = f_0 \quad (3-9)$$

$$\int f(X) \prod_{k \neq i_j} dx_k = f_0 + f_{i_j}(x_{i_j}) \quad (3-10)$$

$$\int f(X) \prod_{k \neq i_j, i_s} dx_k = f_0 + f_{i_j}(x_{i_j}) + f_{i_s}(x_{i_s}) + f_{i_j i_s}(x_{i_j}, x_{i_s}) \quad (3-11)$$

Here $k = i_1, \dots, i_s$ and $1 \leq j < s$. The variable f_0 is the expected value of the model output. Sobol proposed the idea to calculate the variance of the terms in ANOVA decomposition [54]. Since $f(X)$ is assumed to be square integrable, define

$$D_{i_1 \dots i_s} = \int f_{i_1 \dots i_s}^2(x_{i_1}, \dots, x_{i_s}) dx_{i_1} \dots dx_{i_s} \quad (3-12)$$

By squaring Equation 3-7 and integrating over I^n , the results are given as follows [54]:

$$\int f^2(x) dx - f_0^2 = \sum_{s=1}^n \sum_{i_1 < \dots < i_s}^n D_{i_1 \dots i_s} \quad (3-13)$$

Define the constant D as follows:

$$D = \int f^2(x)dx - f_0^2 \quad (3-14)$$

The variance relation is hence given by

$$D = \sum_{s=1}^n \sum_{i_1 < \dots < i_s} D_{i_1 \dots i_s} \quad (3-15)$$

Then considering Sobol's definition, the first order sensitivity index is given as follows [54]:

$$S_{i_s} = \frac{D_{i_s}}{D} \quad (3-16)$$

It is shown that calculating the first order index of the parameters x_{i_s} depends on calculating the unconditional variance D of y and the conditional expectation D_{i_s} , which need to compute multi-dimensional integrals. The Monte Carlo method uses the probabilistic interpretation to efficiently approximate the multi-dimensional integral calculations [55] and can be constructed in the following way.

Consider a set π_1 of m variables where $1 \leq m \leq n - 1$ to be given by

$$\pi_1 = (x_{i_1}, \dots, x_{i_m}) \quad (3-17)$$

Then π_2 is the complementary set of π_1 and contains $n - m$ variables. Then $X = (\pi_1, \pi_2)$ represent the all the inputs variables.

Define two independent points of X which are uniformly distributed in I^n as follows

$$X_1 = (\pi_1^{(1)}, \pi_2^{(1)}) \quad X_2 = (\pi_1^{(2)}, \pi_2^{(2)}) \quad (3-18)$$

Assuming there are N independent trials of X_1 and X_2 , it has been proven that the following four approximations can be given [54]:

$$\frac{1}{N} \sum_{i=1}^N f(X_{1_i}) \xrightarrow{P} f_0 \quad (3-19)$$

$$\frac{1}{N} \sum_{i=1}^N f^2(X_{1_i}) \xrightarrow{P} D + f_0^2 \quad (3-20)$$

$$\frac{1}{N} \sum_{i=1}^N f(X_{1_i}) f(\pi_{1_i}^{(1)}, \pi_{2_i}^{(2)}) \xrightarrow{P} D(\pi_1) + f_0^2 \quad (3-21)$$

The sign \xrightarrow{P} in these four equations means the stochastic convergence. Then D_{i_s} of Eq. 3-16 can be calculated from Eq. 3-21 when π_1 contains one variable x_{i_s} . The total variance D can be expressed by Eq. 3-20. Then the first order index in Eq. 3-16 can be calculated.

3-3-2 Interactions analysis of steering model

It is proven that the global sensitivity analysis can be used for an arbitrary hyperspace [52, 53, 54]. Then the first order sensitivity index of the seven parameters in the steering model can be analyzed by Eq. 3-16. Here the ranges of the parameters are set to be 80% ~ 120% of the known parameter set (in Table 2-2). The value of the variable N is selected as 2000 to have the stochastic convergence. The first order index of the parameters to the steering wheel angle fitness are investigated. The analysis results are given in Fig. 3-6.

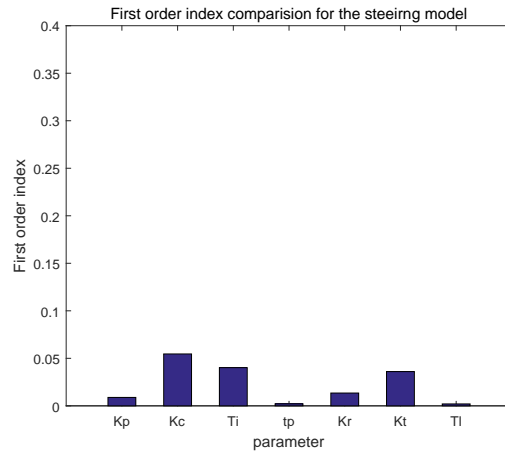


Figure 3-6: The full steering morel parameters first order index, seven parameters.

From Fig. 3-6, it can be seen that the interactions among the parameters are very large since the first order indices are very small. One explanation of the large interactions among those seven parameters could be that excessive numbers of parameters are used in the steering model and make the steering model parametrically inefficient. On the other hand, the accuracy of estimated parameters is influenced by the parameter interactions. It is shown that although the data can be fitted well, the parameters combinations have a high level of uncertainties and that can be one characteristic of model over-parameterization [56]. In short, the large interactions among the parameters could be one explanation to show that the steering model is over-parameterized.

3-4 Mitigating Over-Parameterization–Model Simplification

One possible way to mitigate the over-parameterization problem is to simplify the steering model and make the steering model parametrically efficient. In this human-like cybernetic steering model, the NMS system follows a haptic force feedback law through an internal model of the steering system compliance [4]. Human operations are correlated with the vehicle longitudinal speed V_x and the haptic feedback from the steering wheel. Since the vehicle speed is kept unchanged, a reasonable simplification method is to simplify the NMS system and tune the steering column dynamics such that the steering wheel angle is kept unchanged. As a result, the two parameters K_r and K_t are simplified to mitigate the over-parameterization problem. A schematic of the simplified steering model structure is depicted in Fig. 3-7.

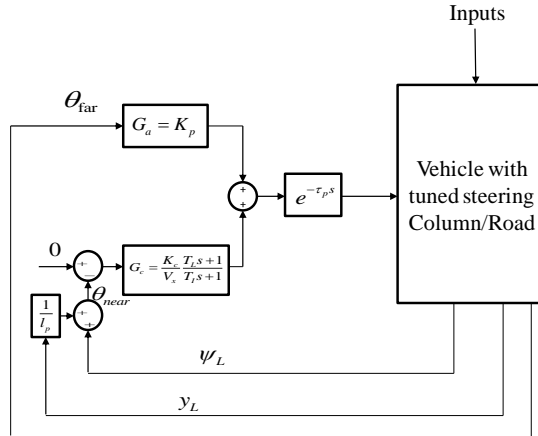


Figure 3-7: Schematic of the simplified steering model structure.

Table 3-2: The initial known parameter set of the simplified steering model, the first parameter set

First set	K_p	K_c	T_I	τ_p	T_L
	0.11	7.78	2.96	0.001	1.53

The steering wheel angle of the simplified steering model approximates the steering wheel angle of the full steering model by tuning the steering column dynamics. The eight states of the steering model $[V_y \ r \ \psi_L \ y_L \ \delta_{SW} \ \dot{\delta}_{SW} \ x_1 \ x_2 \ x_3]$ are compared between the simplified steering model and the full steering model. The comparison results of those eight states are given in Fig. 3-8. It is shown that the eight states of the full steering model and the simplified steering model are almost the same, which demonstrate that the model simplification is reasonable and the steering model can be modeled in a more parametrically efficient way.

After the steering model is simplified, the rest of five parameters that need to be estimated are $[K_p \ K_c \ T_I \ T_L \ \tau_p]$. Starting with the initial known parameter set in Table 2-2, the parameter estimation technique which has been used for the full steering model parameter estimation is repeatedly implemented to the simplified steering model. The steering wheel angle is used for parameter estimation. The initial parameter set is again given in Table 3-2.

The parameter estimation technique is repeated 10 times. Comparisons of the estimated parameters and the known parameters are given in Fig. 3-9(a). An improvement of the accuracy of the estimated parameters has been shown. The estimated parameters converge to the unique value which is near the initial parameter set. It means that when the over-parameterization of the full steering model is mitigated, the left five parameters of the simplified steering model can be estimated accurately with the developed global optimization technique. The Euclidean distance of the estimated parameter sets to the known parameter set is given in Fig. 3-9(b). The Euclidean distance from ten estimated parameter sets to the known parameter set are almost same and it gives collaborative evidence that the estimated parameters are near the initial known parameter set. The details of the estimation results are shown in Table B-3. It can be seen that the data fitting performance is also good with

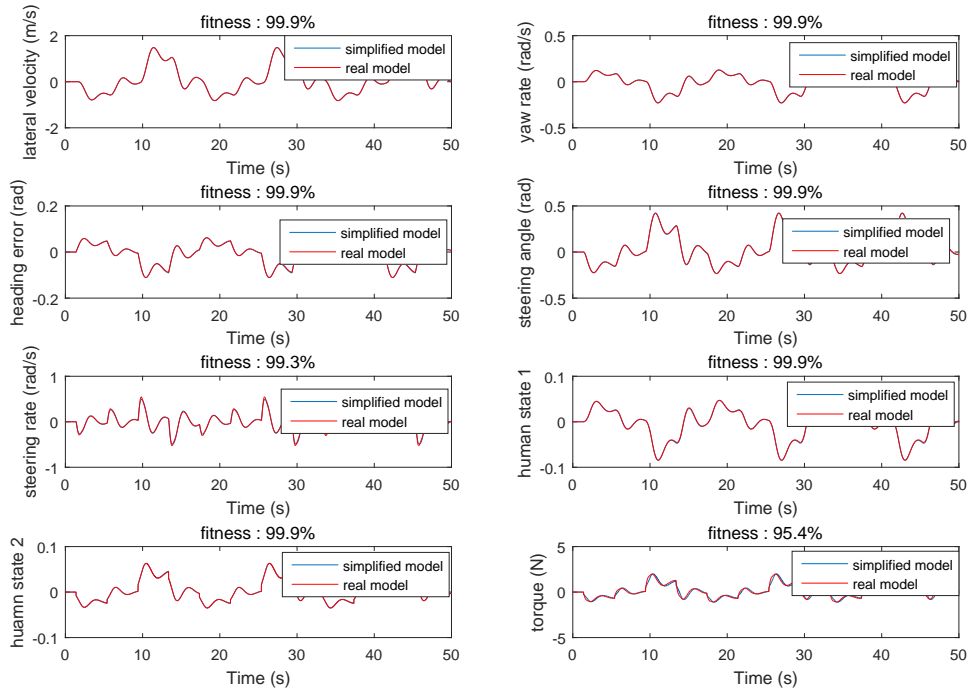


Figure 3-8: Comparisons of the model states between the full steering model and the simplified steering model.

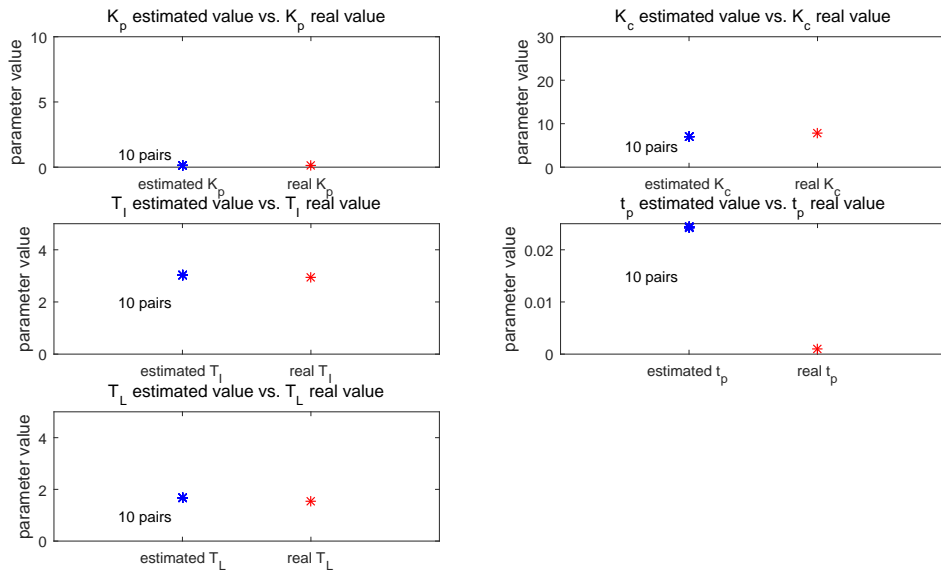
high VAFs which indicate the ability of the algorithm to replicate the output behavior.

To validate the effect of mitigating over-parameterization on parameter estimation, the initial known parameters are tuned to formulate four new parameter sets and these four tuned parameter sets are given in Table 3-3. These four parameter sets can be regarded as another four known parameter sets. The developed parameter estimation technique is implemented to estimate those four parameter sets back. The accuracy of estimated parameters is examined by comparing the results of parameter estimation with those four known parameter sets using the simplified steering model.

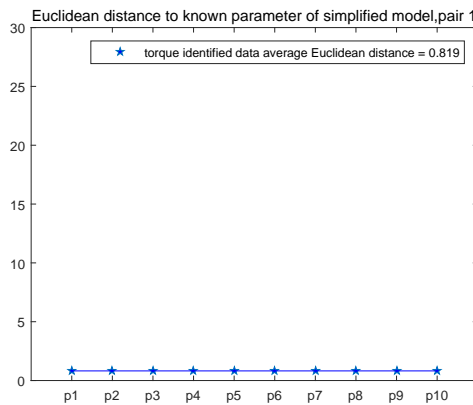
The comparisons of steering model states of these four parameter sets between the full steering model and the simplified steering model are given from Fig. B-1 to Fig. B-4, respectively.

Table 3-3: The tuned parameter sets for the simplified steering model, the first parameter set to the fifth parameter set

Parameter set	Parameters value				
First set	0.11 (K_p)	7.78 (K_c)	2.96 (T_I)	0.001 (τ_p)	1.53 (T_L)
Second set	0.17 ($1.5K_p$)	6.27 ($0.8K_c$)	1.48 ($0.5T_I$)	0.003 (τ_p)	1.22 ($0.8T_L$)
Third set	0.44 ($4K_p$)	3.89 ($0.5K_c$)	5.92 ($2T_I$)	0.01 ($10\tau_p$)	0.76 ($0.5T_L$)
Fourth set	0.09 ($0.8K_p$)	1.68 ($1.5K_c$)	11.84 ($4T_I$)	0.006 ($6\tau_p$)	3.06 ($2T_L$)
Fifth set	1.1 ($10K_p$)	15.56 ($2K_c$)	4.44 ($1.5T_I$)	0.002 ($1.5\tau_p$)	2.29 ($1.5T_L$)



(a) Ten sets of parameters distributions



(b) Euclidean distance from the estimated parameters to the initial known parameter set

Figure 3-9: The parameter estimation results of the simplified steering model, the first parameter set.

From the simplification results, it is clearly that the new four parameter sets represent another four different steering behaviors and the simplification results show that the states are almost the same comparing between the full steering mode and the simplified steering model, which shows that the full steering model can be modeled using fewer parameters.

The developed parameter estimation technique is repeated 10 times for parameter estimation of each parameter set. The comparisons between the estimated parameters and the corresponding known parameter set are given in Figs. 3-10 and 3-11. The second set to the fourth set represent the satisfactory parameter estimation results. The estimated parameter sets are accurate compared with the corresponding known parameter set. For the fifth set, the

parameter estimation results have a little scattering performance. But the estimated parameters are still in a small area near the initial known parameter set. One explanation of the results is that there are still interactions among the parameters. However, the model over-parameterization has been mitigated and the parameter estimation technique can estimate the parameters accurately. Moreover, the results of Euclidean distance from the estimated parameters to the corresponding known parameter set are given in Fig. 3-12. The results also give collaborative evidences that the estimated parameters are near the corresponding initial known parameter set. Details of the estimated parameters of each parameter set are given from Table B-4 to Table B-7. The results have shown that the data fitting results are good as expected. The parameter estimation validations using these four new parameter sets show the effects of model simplification.

One may notice that for the parameter τ_p , there are discrepancies between the estimated results and the known value. This can be explained by a local sensitivity analysis of τ_p . When τ_p varies in a small range (i.e., from 0 to 0.02) and other parameters are kept unchanged, the steering wheel fitting results do not change. An example of the local sensitivity analysis of τ_p with the first parameter set has been shown in Fig. B-5. The problem can be regarded as a limitation of current technique: when there are multiple global optimal points, the technique can only converge to a random one.

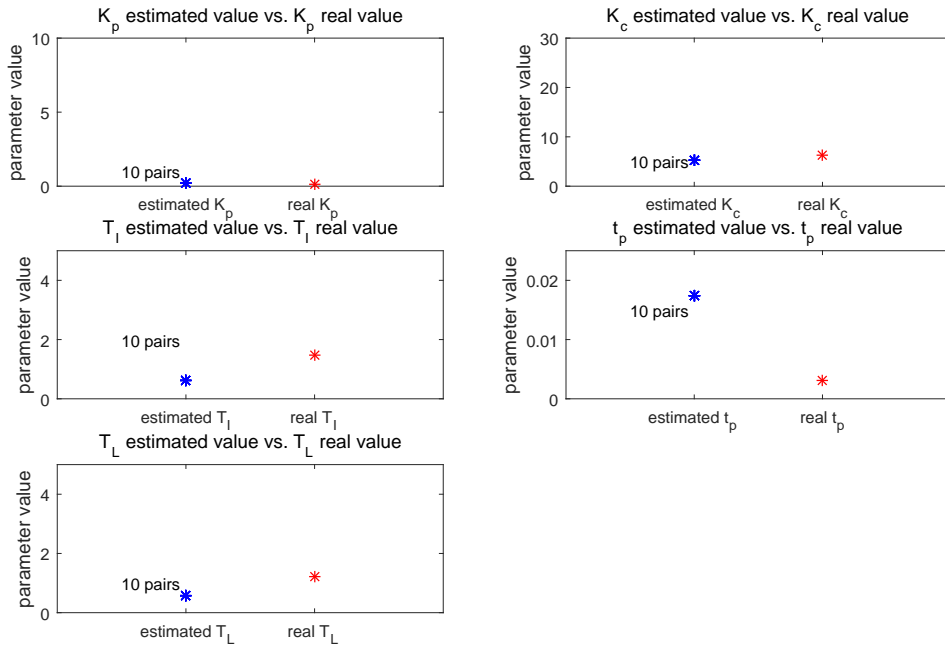
Since the over-parametrization of the full steering model has been mitigated, a global sensitivity analysis is implemented again to the simplified steering model to investigate the degree of interactions. The result is given in Fig. 3-13. The first order index of the parameters of the simplified steering model are compared with the parameters of the full steering model. It shows that interactions among the parameters have decreased.

3-5 Sub-conclusion

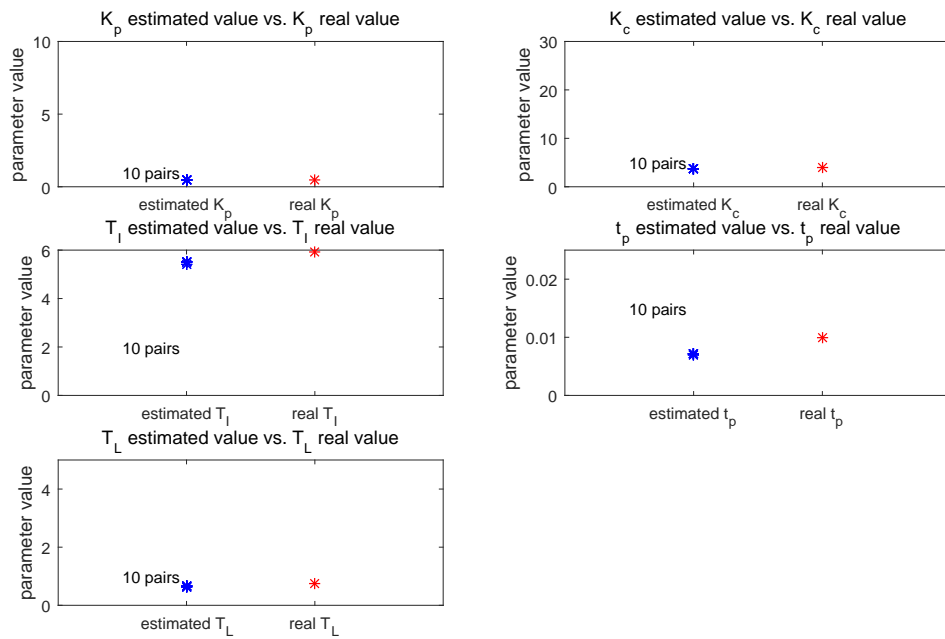
This chapter takes the first step to estimate the proposed steering model parameters which are set to be known in advance. A time domain parameter estimation technique is developed to increase parameter estimation accuracy. A global optimization technique (a genetic algorithm complemented with the Levenberg-Marquardt method) is explained in detail and implemented to estimate the steering model parameters.

The parameter estimation results are examined using the metrics VAF and Euclidean distance. The data performance can be replicated well. But the estimated parameters are inaccurate and present large interactions among the parameters which could be from the over-parameterization of the steering model. Those interactions are analyzed through global sensitivity analysis.

Model simplification is proposed to solve the over-parameterization problem of the steering model. By simplifying two parameters in NMS and tuning the steering column dynamics, the model can be simplified while keeping the steering wheel angle unchanged. Under this condition, several simulations show that the parameters can be estimated accurately.

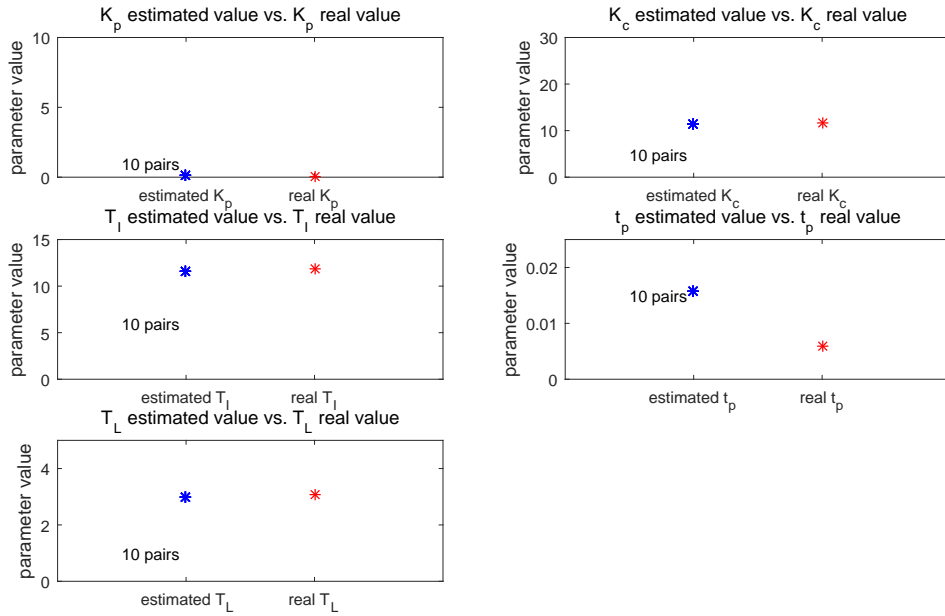


(a) The second parameter set

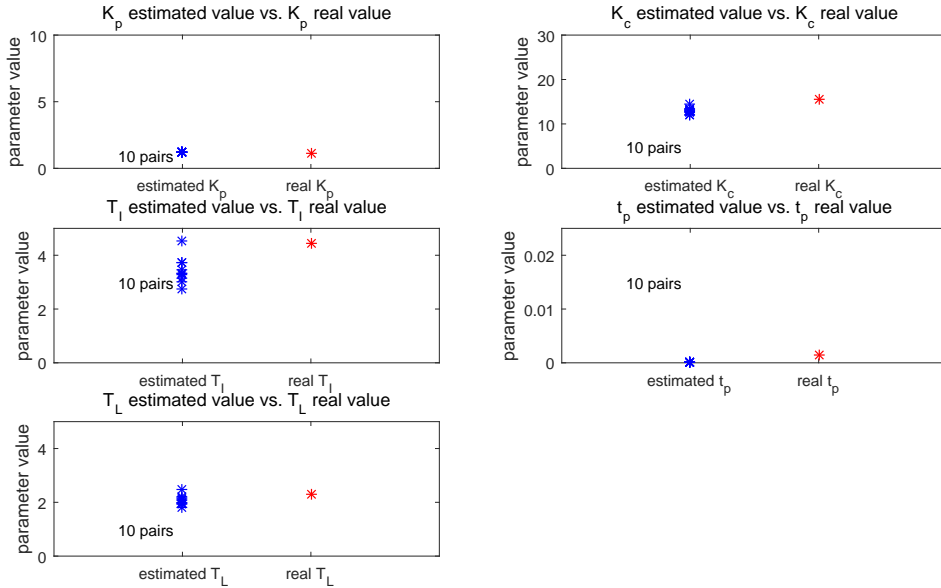


(b) The third parameter set

Figure 3-10: The parameter estimation results of the simplified steering model, the second parameter set and the third parameter set.

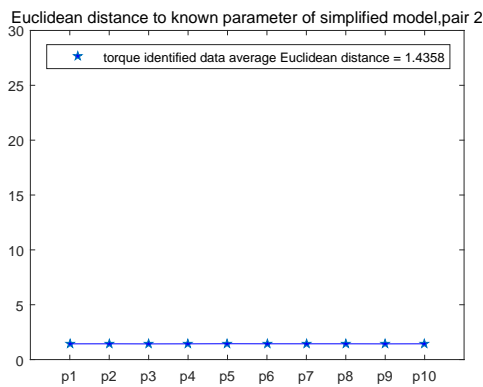


(a) The fourth parameter set

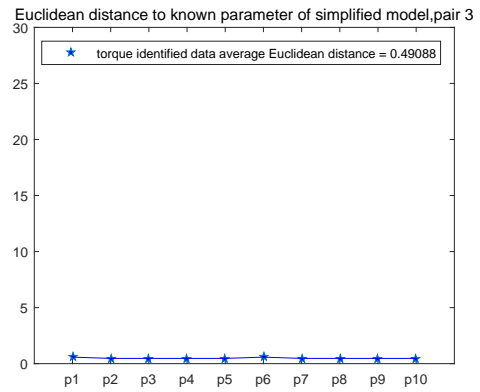


(b) The fifth parameter set

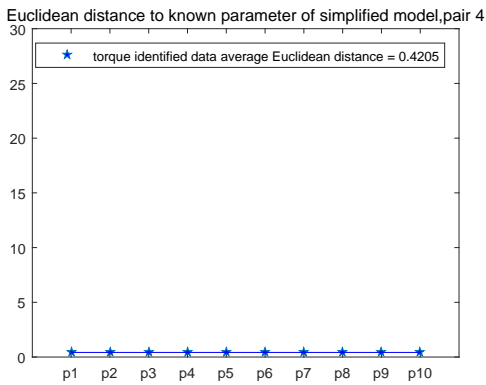
Figure 3-11: The parameter estimation results of the simplified steering model, the fourth parameter set and the fifth parameter set.



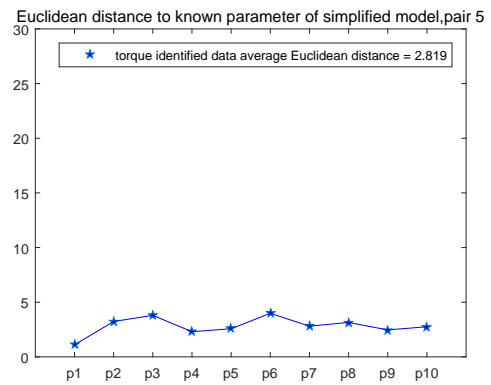
(a) The second parameter set



(b) The third parameter set



(c) The fourth parameter set



(d) The fifth parameter set

Figure 3-12: The Euclidean distance of the simplified steering model from the estimated parameters to the corresponding known parameter set.

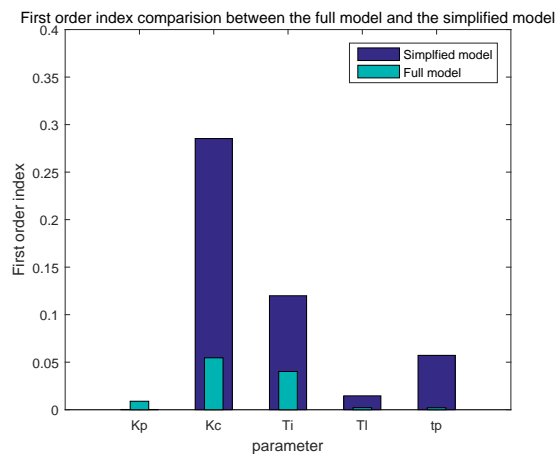


Figure 3-13: Comparison between the first order index of the simplified steering model parameters and the full steering model parameters, five parameters.

Parameter Estimation of Unknown Parameters

The time domain parameter estimation algorithm has been proposed to solve the steering model parameter estimation problem and the estimated parameters have been compared with the known parameters as the first step. In the second step of this study, the parameter estimation algorithm is used to identify a parameter set by using a data set which is simulated from another validated human-vehicle steering control model.

4-1 Parameter Estimation Based on Simulation Data

A new simulated data set is generated to validate the parameter estimation algorithm as the second step of this study. The data set is generated from the validated human-vehicle steering control model [41]. The model uses the human future preview information for the vehicle lateral control and the generated data is considered to represent one category of human drivers who have similar steering preferences. The collected data set is given in Fig. 4-1. The same category of data sets is used here which has been used in the previous chapter. The inputs signal here are also persistently exciting of the steering model order using the same method in the Chapter 2 and can be used for parameter estimation.

Parameter estimation is started with the full steering model. The torque is used for parameter estimation and the steering wheel angle is used for validation. The genetic algorithm complemented with the Levenberg-Marquardt algorithm is implemented to minimize the prediction error. 10 repetitions of the developed algorithm are performed for the parameter estimation.

One illustration of data fitting is given in Fig. 4-2. Satisfactory data fitting results are obtained from the parameter estimation. One may notice that the high frequency components in the torque could not be captured. This can be regarded as one limitation of the cybernetic steering model and it shows that the steering model does not have the ability to capture the high frequency components and the steering model structure should be refined to incorporate those components before implementing the parameter estimation algorithm to the real

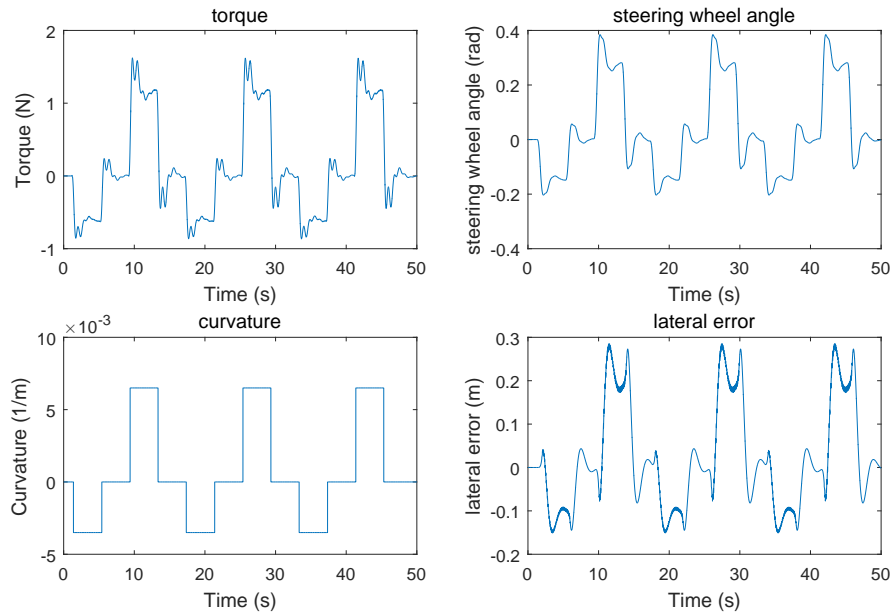


Figure 4-1: Collection of the simulation data for parameter estimation.

experimental data set. Real human drivers always implement high frequency operations to the steering wheel. For example, Kolekar [11] has explained some high frequency components that need to be considered in the human-like steering model design.

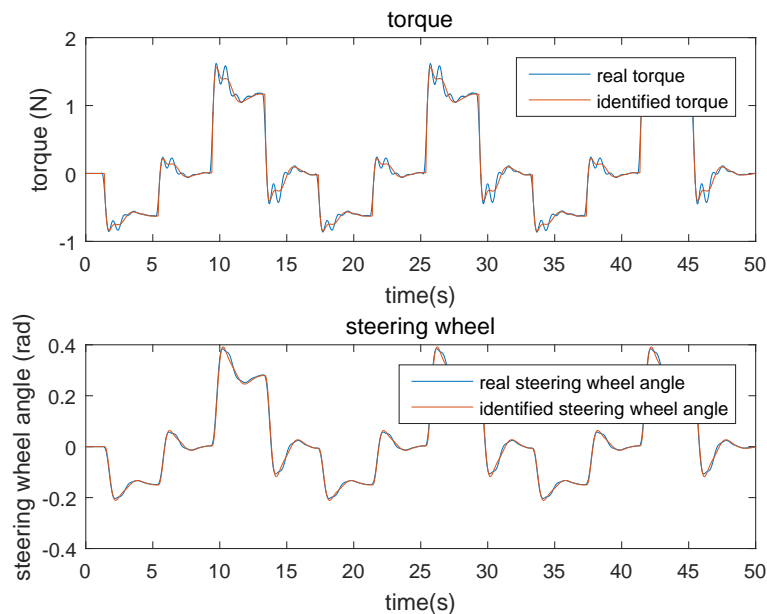


Figure 4-2: Illustration of the simulation data fitting.

The illustration of the estimated parameters of the full steering model is given in Fig. 4-

3. Details of the results are given in Table B-8. The results show that the data fitting performance is good and the steering model has the ability to capture most of the information in this simulation data set. However, the estimated parameters of each individual parameter have a scattering distribution and do not converge to the unique value. Large parameter interactions are shown among the estimated parameters. These results are expected since these large parameter interactions are induced from the steering model over-parameterization from the analysis in Chapter 3 and the steering model should be simplified to mitigate the over-parameterization and make the steering model parametrically efficient.

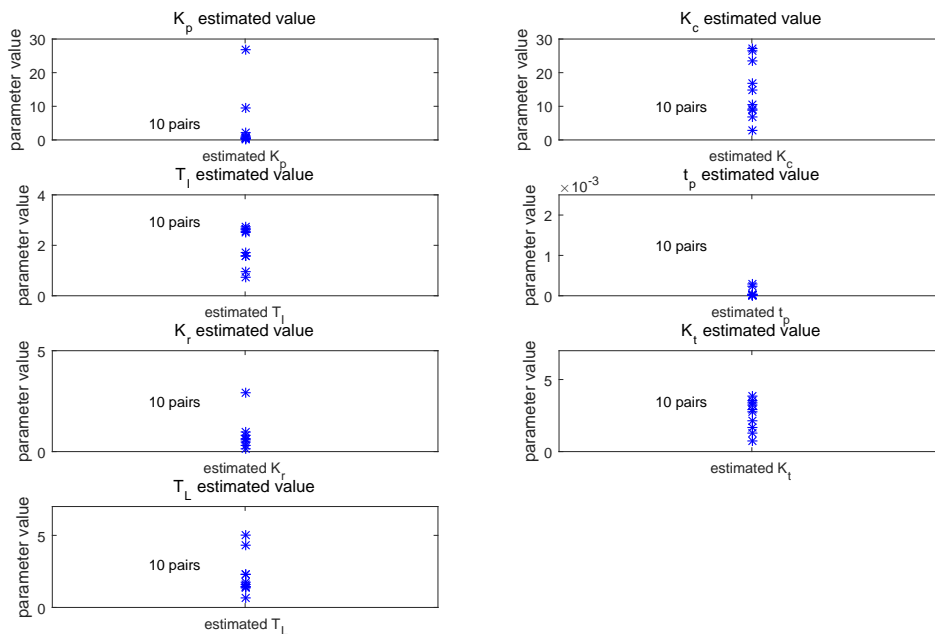


Figure 4-3: The parameter estimation results of the full steering model based on the simulation data.

To mitigate the steering model over-parameterization, the method which is indicated in Chapter 3 is used and the NMS system is simplified by approximations. The steering column dynamics is tuned through a couple of simulations to adapt to the model simplification. For parameter estimation problem on real experimental data, one challenge could be simplification of the NMS. One comparison of the simplification results between the full steering model and the simplified steering model is given in Fig. B-6. It is shown that the simplification results are satisfactory that the steering wheel angle of the simplified steering model is almost same compared with the steering wheel angle of the full steering model.

Parameter estimation results for the simplified steering model are given in Fig. 4-4. Details of the results are given in Table B-9. The results show that the data fitting results are good as expected and the parameter estimation results indicate that the estimated parameters for each parameter are in a small region which has a high possibility to be the place of the real parameter from the analysis results in Chapter 3. They converge to the unique values and those results prove again that the model over-parameterization has been mitigated. However,

one important notice is that the accuracy of estimated parameters depends on the accuracy of NMS approximation. On the other hand, the parameter estimation results can be influenced by whether the model structure matches the data or not.

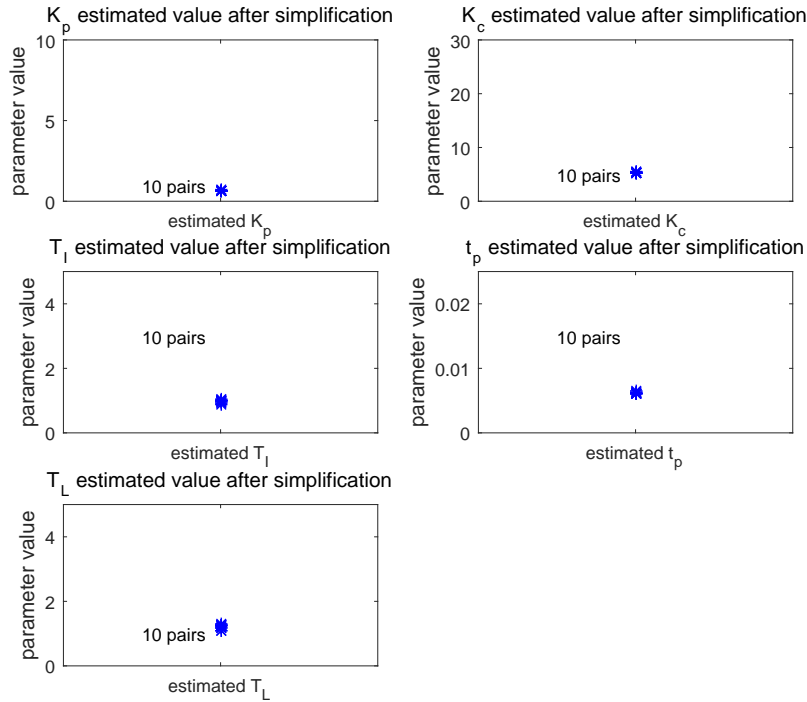


Figure 4-4: Parameter estimation illustration of the simplified model based on the simulation data.

A cross-validation test is implemented for model validation. The inputs of the steering model are the curvature and lateral error. The identification data batch is divided into two parts: the first 2/3 of the total samples is used for the parameter estimation and the final 1/3 is used for assessing the quality of the model. The cross-validation is implemented for both the full steering model and the simplified steering model. Results are given in Fig. 4-5. The results indicate that the simplified steering model has a better fitting than the full steering model. With less data information, the simplified steering model can replicate the data information better than the full steering model, which indicates the full steering model is over-parameterized.

4-2 Sub-conclusion

In this chapter, the simulation data set is collected from the validated model to test the parameter estimation algorithm. By implementing the parameter estimation technique to the full steering model, parameters cannot be estimated accurately. The simplified steering model with an approximation of NMS system is implemented for parameter estimation to mitigate the over-parameterization of the full steering model. The estimated parameters

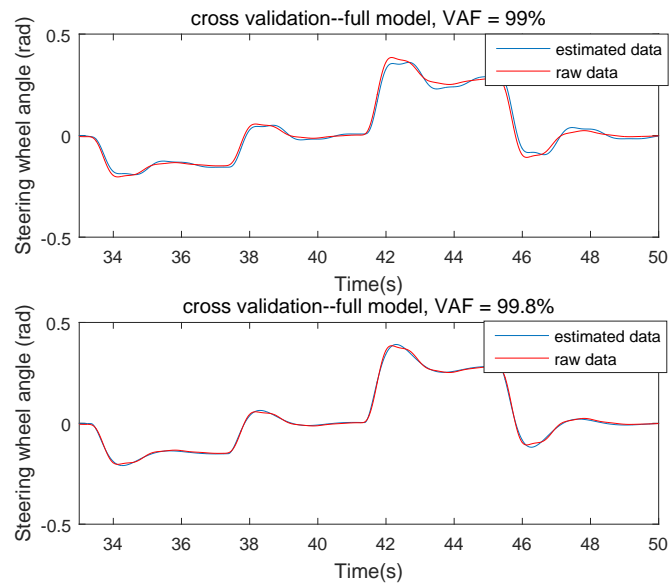


Figure 4-5: The cross-validation with the full steering model and the simplified steering model.

are located in an area where real parameters have a high probability to be. The parameter estimation results depend on the approximation accuracy of NMS. On the other hand, whether the model structure matches the data or not also influences the parameter estimation results. If the steering model lacks the ability on capturing some parts of human characteristics, the results of estimated parameters could be inaccurate.

Conclusions and Recommendations

5-1 Conclusions

The contribution of this study is that this study developed a versatile parameter estimation technique to estimate a well-structured human-like steering model parameters in order to help to individualize human steering control behavior. The general literatures on this subject have limitations on developed parameter estimation techniques. Those literatures are also inconclusive on whether true parameters are estimated accurately. The developed technique in this work softens the stringent requirements that current implemented parameter estimation techniques need for estimating the steering model parameters. This study also investigated how accurate the steering model parameters can be estimated since the parameters of the human-like steering model have definite physical meanings reflecting the nature of human steering control mechanism. The main workload of this thesis is on building the steering model and related parameter estimation algorithm in MATLAB and doing parameter estimation simulations based on different types of data.

The main conclusions of this study are:

- When the developed parameter estimation technique was implemented on the simulation data generated by the full cybernetic steering model itself, it could estimate parameters from the steering model with high VAFs (over 99%) for steering torque and steering wheel angle. However, the steering model is over-parameterized and therefore the estimated parameters could not converge to the true parameters using this developed technique.
- A simplified steering model structure mitigated the over-parameterization problem and allowed the technique to estimate the true parameters with high fitting (VAFs over 99.9%) for the steering wheel angle.
- A second and more strict validation method was used, where parameter estimation of the simplified model was implemented on the data generated by a different steering model,

thereby essentially making it less possible for the model to capture data information perfectly. Again, the developed method yielded consistent parameters values, with high VAFs (over 99%) for the torque and steering wheel angle fitting. A cross-validation was also implemented to show that the simplified steering model yielded better VAF (99.8%) for the steering wheel angle than the full cybernetic steering model (VAF 99%).

In conclusion, the developed approach was promising for the tested model-generated data. The technique can be used to differentiate different human driver steering characteristics if the used steering model is not over-parameterized.

5-2 Recommendations

The future work will focus on the human-in-the-loop experimental test. The developed technique has several main parts that need to be considered before the technique could be applied to customize steering controllers to drivers on real roads:

A. Model revisions

- A more accurate vehicle dynamics model can be investigated to match the data obtained by human-in-the-loop experiments instead of using a single track model.
- This steering model assumes the vehicle is operated under a constant longitudinal velocity. The acceleration and deceleration cases can be considered for human-in-the-loop experiments.
- High frequency components of human operations and intra-individual differences (i.e., noise in personal characteristics) can to be considered to refine the steering model before testing the human-in-the-loop experiments data. For instance, Kolekar [11] proposed those parts which should be considered in the human-like steering controllers design.

B. Experimental data validation

Parameter estimation of the data obtained by human-in-the-loop experiments should be investigated to study real human characteristics with a revised model to individualize human driver steering behavior.

C. Parameter estimation technique

The parameter estimation technique in this study could not solve the parameter estimation problem with the over-parameterized steering model. A global multi-objective optimization problem can be formulated to estimate parameters. Furthermore, maximum likelihood estimation has been proved to have good parameter estimation performance [57] and could be considered as a method to overcome the over-parametrization problem.

Appendix A

Mathematical Derivations in the Thesis

This Appendix give mathematical derivations of the differential equations of the steering model states (see Equations 2-22 and 2-23). The three states x_1 , x_2 and x_3 are selected in the literature [4].

The state x_1 is shown in Chapter 2 and the differential equation of x_1 is shown as follows:

$$x_1 = \frac{1}{T_I s + 1} \theta_{near} \quad (\text{A-1})$$

$$T_I \dot{x}_1 + x_1 = \theta_{near} \quad (\text{A-2})$$

$$\dot{x}_1 = -\frac{1}{T_I} x_1 + \frac{1}{T_I} \theta_{near} \quad (\text{A-3})$$

The state x_2 is shown in Chapter 2 and the differential equation of x_2 is shown as follows:

$$x_2 = \frac{1}{1 + \frac{\tau_p}{2} s} [K_p \theta_{far} - \frac{K_c}{V_x} (1 + T_L s) x_1] \quad (\text{A-4})$$

$$\frac{\tau_p}{2} \dot{x}_2 + x_2 = K_p \theta_{far} - \frac{K_c}{V_x} (1 + T_L s) x_1 \quad (\text{A-5})$$

$$\frac{\tau_p}{2} \dot{x}_2 + x_2 = K_p \theta_{far} + \frac{K_c}{V_x} (-1 - T_L s + \frac{T_L}{T_I} - \frac{T_L}{T_I}) x_1 \quad (\text{A-6})$$

$$\frac{\tau_p}{2} \dot{x}_2 + x_2 = K_p \theta_{far} + \frac{K_c}{V_x} [(\frac{T_L}{T_I} - 1) x_1 - \frac{T_L}{T_I} (1 + T_L s) x_1] \quad (\text{A-7})$$

$$\frac{\tau_p}{2} \dot{x}_2 + x_2 = K_p \theta_{far} + \frac{K_c}{V_x} (\frac{T_L}{T_I} - 1) x_1 - \frac{K_c T_L}{V_x T_I} \theta_{near} \quad (\text{A-8})$$

$$\dot{x}_2 = \frac{2}{\tau_p} [K_p \theta_{far} + \frac{K_c}{V_x} (\frac{T_L}{T_I} - 1) x_1 - \frac{K_c T_L}{V_x T_I} \theta_{near} - x_2] \quad (\text{A-9})$$

The state x_3 is shown in Chapter 2 and the differential equation of x_3 is shown as follows:

$$T_{dr} = x_3 = \frac{1}{T_n s + 1} \{K_r v (1 - \frac{\tau_p}{2} s)x_2 + K_t [(1 - \frac{\tau_p}{2} s)x_2 - \delta_{sw}]\} \quad (\text{A-10})$$

$$T_n \dot{x}_3 + x_3 = (K_r V_x + K_t) (1 - \frac{\tau_p}{2} s)x_2 - K_t \delta_{sw} \quad (\text{A-11})$$

$$T_n \dot{x}_3 + x_3 = (K_r V_x + K_t) \frac{(1 - \frac{\tau_p}{2} s)}{1 + \frac{\tau_p}{2} s} [K_p \theta_{far} - \frac{K_c}{V_x} (1 + T_L s)x_1] - K_t \delta_{sw} \quad (\text{A-12})$$

$$T_n \dot{x}_3 + x_3 = (K_r V_x + K_t) [(\frac{2}{1 + \frac{\tau_p}{2} s} - 1) K_p \theta_{far} + (1 - \frac{2}{1 + \frac{\tau_p}{2} s}) \frac{K_c}{V_x} (1 + T_L s)x_1] - K_t \delta_{sw} \quad (\text{A-13})$$

$$T_n \dot{x}_3 + x_3 = (K_r V_x + K_t) [\frac{2K_p \theta_{far}}{1 + \frac{\tau_p}{2} s} + \frac{K_c}{V_x} (1 + T_L s)x_1 - \frac{2K_c x_1 (1 + T_L s)}{(1 + \frac{\tau_p}{2} s)V_x}] - K_t \delta_{sw} - K_p (K_r V_x + K_t) \theta_{far} \quad (\text{A-14})$$

$$T_n \dot{x}_3 + x_3 = (K_r V_x + K_t) [2x_2 + \frac{K_c}{V_x} (1 + T_L s)x_1] - K_t \delta_{sw} - K_p (K_r V_x + K_t) \theta_{far} \quad (\text{A-15})$$

$$T_n \dot{x}_3 + x_3 = 2(K_r V_x + K_t)x_2 + (K_r V_x + K_t) \frac{K_c}{V_x} \{[(1 + T_L s) \frac{T_L}{T_I} - \frac{T_L}{T_I} + 1]x_1\} - K_t \delta_{sw} - K_p (K_r V_x + K_t) \theta_{far} \quad (\text{A-16})$$

$$T_n \dot{x}_3 + x_3 = 2(K_r V_x + K_t)x_2 + (K_r V_x + K_t) \frac{K_c}{V_x} [\theta_{near} \frac{T_L}{T_I} - (\frac{T_L}{T_I} - 1)x_1] - K_t \delta_{sw} - K_p (K_r V_x + K_t) \theta_{far} \quad (\text{A-17})$$

$$\dot{x}_3 = \frac{1}{T_n} \{2(K_r V_x + K_t)x_2 + (K_r V_x + K_t) \frac{K_c}{V_x} [\theta_{near} \frac{T_L}{T_I} - (\frac{T_L}{T_I} - 1)x_1] - K_t \delta_{sw} - K_p (K_r V_x + K_t) \theta_{far} - x_3\} \quad (\text{A-18})$$

Appendix B

Details of the Parameter Estimation Results

Table B-1: The parameter estimation results (known parameters) of the full steering model using the torque as identified data

Repetition	K_p	K_c	T_I	τ_p	K_r	K_t	T_l	Fit steering VAF(%)	Euclidean distance
1	0.224	11.9553	0.5892	0.0016	1.2328	6.0592	0.2953	99.8	5.1
2	0.3802	30.0097	2.7603	$1.90 * 10^{-4}$	0.5726	5.0026	1.096	99.9	22.3
3	0.0586	5.7414	3.282	$6.50 * 10^{-4}$	5.0293	6.0728	1.149	99.9	3.3
4	0.2002	20.4889	3.1883	$1.10 * 10^{-2}$	1.2494	6.1451	1.1047	99.9	12.7
5	0.1941	20.0679	3.1727	$8.23 * 10^{-7}$	1.2809	6.1484	1.0958	99.9	12.3
6	3.8236	8.3917	1.0524	$2.10 * 10^{-10}$	0.0829	0.0431	5.2535	99.3	8.6
7	9.5074	30.1504	1.3862	$2.24 * 10^{-9}$	0.023	0.2723	5.139	99.3	25.3
8	0.0908	4.5888	0.5851	0.0023	3.4902	5.9086	0.2983	99.3	4.2
9	0.366	30.0907	3.6377	$1.20 * 10^{-8}$	0.6295	4.9066	1.2977	99.9	22.4
10	1.0184	30.0837	3.8129	$1.32 * 10^{-4}$	0.2339	2.1819	2.3233	99.6	22.8

Table B-2: The parameter estimation (known parameters) results of the full steering model using the steering wheel angle as identified data

Repetition	K_p	K_c	T_I	τ_p	K_r	K_t	T_L	Fit steering VAF(%)	Euclidean distance
1	0.8575	10.8534	2.3278	$1.94 * 10^{-4}$	0.3202	1.1558	2.8849	99.1	6.5
2	3.3540	5.4102	1.9774	$1.8 * 10^{-8}$	0.4838	5.2938	0.9687	99.3	4.7
3	0.1468	15.4208	3.2188	$1.29 * 10^{-6}$	1.8236	6.4457	1.1078	99.9	7.7
4	0.0657	6.7684	3.3214	$1.93 * 10^{-9}$	4.4083	6.2801	1.1431	99.9	2.3
5	0.0379	3.5507	3.2904	$8.78 * 10^{-11}$	7.9678	5.8508	1.1654	99.3	7.0
6	0.2017	20.6636	3.2139	$5.85 * 10^{-8}$	1.2708	6.3361	1.1137	99.9	12.9
7	0.0554	1.1872	0.4203	$3.21 * 10^{-10}$	5.8913	2.2451	0.3239	99.5	8.9
8	0.1667	16.5025	3.2752	$9.2 * 10^{-11}$	1.6056	6.1212	1.1426	99.9	8.8
9	0.0184	4.4384	3.4231	$4.19 * 10^{-10}$	12.7921	11.3123	0.9482	99.5	12.0
10	0.2639	18.3702	4.0692	$6.75 * 10^{-9}$	1.0219	4.2661	1.5573	99.9	10.9

Table B-3: The parameter estimation (known parameters) results of the simplified steering model using the steering wheel angle as identified data, the first parameter set

Repetition	K_p	K_c	T_I	τ_p	T_L	Fit steering VAF(%)	Euclidean distance
1	0.1374	6.9839	3.0471	0.0243	1.6758	99.9	0.82
2	0.1374	6.9839	3.0471	0.0243	1.6758	99.9	0.82
3	0.1374	6.9839	3.0471	0.0243	1.6758	99.9	0.82
4	0.1374	6.9839	3.0471	0.0243	1.6758	99.9	0.82
5	0.1374	6.9839	3.0471	0.0243	1.6758	99.9	0.82
6	0.1374	6.9839	3.0471	0.0243	1.6758	99.9	0.82
7	0.1374	6.9839	3.0471	0.0243	1.6758	99.9	0.82
8	0.1374	6.9839	3.0471	0.0243	1.6758	99.9	0.82
9	0.1374	6.9839	3.0471	0.0243	1.6758	99.9	0.82
10	0.1374	6.9839	3.0471	0.0243	1.6758	99.9	0.82

Table B-4: The parameter estimation (known parameters) results of the simplified steering model using the steering wheel angle as identified data, the second parameter set

Repetition	K_p	K_c	T_I	τ_p	T_L	Fit steering VAF(%)	Euclidean distance
1	0.1977	5.2686	0.6311	0.0174	0.5762	99.9	1.43
2	0.1977	5.2676	0.6286	0.0174	0.5739	99.9	1.44
3	0.1976	5.2702	0.6339	0.0174	0.5787	99.9	1.43
4	0.1977	5.2683	0.6307	0.0174	0.5758	99.9	1.43
5	0.1977	5.2653	0.6237	0.0174	0.5695	99.9	1.44
6	0.1977	5.2672	0.6277	0.0174	0.5731	99.9	1.44
7	0.1977	5.2675	0.6283	0.0174	0.5737	99.9	1.44
8	0.1977	5.2676	0.6285	0.0174	0.5738	99.9	1.44
9	0.1977	5.2696	0.6326	0.0174	0.5775	99.9	1.43
10	0.1977	5.2676	0.6287	0.0174	0.5741	99.9	1.44

Table B-5: The parameter estimation (known parameters) results of the simplified steering model using the steering wheel angle as identified data, the third parameter set

Repetition	K_p	K_c	T_I	τ_p	T_L	Fit steering VAF(%)	Euclidean distance
1	0.456	3.6708	5.3997	0.007	0.6295	99.9	0.58
2	0.4547	3.7105	5.5046	0.0071	0.65	99.9	0.47
3	0.4546	3.7105	5.5047	0.0071	0.65	99.9	0.47
4	0.4547	3.7104	5.5045	0.0071	0.65	99.9	0.47
5	0.4547	3.7105	5.5045	0.0071	0.65	99.9	0.47
6	0.456	3.6711	5.4003	0.007	0.6297	99.9	0.58
7	0.4547	3.7105	5.5045	0.0071	0.65	99.9	0.47
8	0.4547	3.7105	5.5046	0.0071	0.65	99.9	0.47
9	0.4547	3.7104	5.5043	0.0071	0.6499	99.9	0.47
10	0.4547	3.7104	5.5043	0.0071	0.6499	99.9	0.47

Table B-6: The parameter estimation (known parameters) results of the simplified steering model using the steering wheel angle as identified data, the fourth parameter set

Repetition	K_p	K_c	T_I	τ_p	T_L	Fit steering VAF(%)	Euclidean distance
1	0.1016	11.3582	11.5797	0.0158	2.9671	99.9	0.42
2	0.1016	11.3582	11.5797	0.0158	2.9672	99.9	0.42
3	0.1016	11.3582	11.5796	0.0158	2.9671	99.9	0.42
4	0.1016	11.3582	11.5797	0.0158	2.9671	99.9	0.42
5	0.1016	11.3582	11.5796	0.0158	2.9671	99.9	0.42
6	0.1016	11.3582	11.5797	0.0158	2.9671	99.9	0.42
7	0.1016	11.3582	11.5797	0.0158	2.9671	99.9	0.42
8	0.1016	11.3582	11.5797	0.0158	2.9671	99.9	0.42
9	0.1016	11.3582	11.5797	0.0158	2.9671	99.9	0.42
10	0.1016	11.3582	11.5797	0.0158	2.9672	99.9	0.42

Table B-7: The parameter estimation (known parameters) results of the simplified steering model using the steering wheel angle as identified data, the fifth parameter set

Repetition	K_p	K_c	T_I	τ_p	T_L	Fit steering VAF(%)	Euclidean distance
1	1.1938	14.484	4.5151	10^{-9}	2.4718	99.8	1.11
2	1.2165	12.5291	3.3474	$2.5 * 10^{-9}$	2.0858	99.8	3.24
3	1.2149	12.0547	3.0271	$9.9 * 10^{-7}$	1.9678	99.8	3.80
4	1.1987	13.3859	3.7173	$1.1 * 10^{-9}$	2.1818	99.8	2.30
5	1.1937	13.2792	3.3174	$4.32 * 10^{-10}$	1.9305	99.6	2.57
6	1.1973	11.9883	2.7542	$1.16 * 10^{-9}$	1.7998	99.8	3.99
7	1.2035	12.9343	3.4773	$6.02 * 10^{-6}$	2.1037	99.8	2.81
8	1.1962	12.7107	3.1546	$9.23 * 10^{-10}$	1.9475	99.8	3.15
9	1.2193	13.2022	3.7463	$4.43 * 10^{-9}$	2.2226	99.8	2.47
10	1.2018	13.0786	3.299	$3.33 * 10^{-9}$	1.9703	99.8	2.76

Table B-8: The parameter estimation (unknown parameters) results of the full steering model using the torque as identified data

Iteration	K_p	K_c	T_I	τ_p	K_r	K_t	T_L	Fit steering VAF(%)
1	9.3474	26.5661	1.5875	$4.8 * 10^{-8}$	0.0061	1.3134	4.3883	99.6
2	0.3882	8.7012	2.7247	$3.9 * 10^{-10}$	0.9964	3.2585	1.6318	99.8
3	0.5856	16.7655	2.6391	$2.4 * 10^{-4}$	0.5786	3.8583	1.4389	99.7
4	0.7249	14.8345	0.9746	$3.3 * 10^{-7}$	0.458	3.5213	0.6603	99.8
5	26.8434	27.0121	0.7491	$5.1 * 10^{-5}$	0.0128	0.7151	5.0371	99.5
6	0.4484	10.6551	2.4999	$1.9 * 10^{-11}$	0.8259	3.3588	1.4614	99.8
7	0.6031	9.061	1.7231	$8 * 10^{-7}$	0.6214	2.7819	1.3359	99.8
8	2.146	23.6069	2.5367	$3.8 * 10^{-10}$	0.1185	2.158	2.2836	99.8
9	1.1724	6.9188	1.5838	$2.9 * 10^{-4}$	0.3069	1.6805	2.3061	99.7
10	0.1441	2.7618	2.644	$4.7 * 10^{-10}$	2.9028	2.9201	1.7124	99.8

Table B-9: The parameter estimation (unknown parameters) results of the simplified steering model using the steering wheel angle as identified data

Repetition	K_p	K_c	T_I	τ_p	T_L	Fit steering VAF(%)
1	0.6833	5.3312	1.0132	0.0062	1.2414	99.9
2	0.6833	5.3314	1.0215	0.0062	1.2506	99.9
3	0.6829	5.4138	0.887	0.0062	1.0741	99.9
4	0.6845	5.3072	0.97	0.0066	1.1634	99.9
5	0.6833	5.3359	1.0152	0.0062	1.2438	99.9
6	0.6831	5.3662	0.9693	0.0062	1.1817	99.9
7	0.6835	5.3218	1.0133	0.0062	1.239	99.9
8	0.6835	5.3218	1.0133	0.0062	1.239	99.9
9	0.683	5.4065	0.9072	0.0062	1.1	99.9
10	0.6833	5.3304	1.0198	0.0062	1.2497	99.9

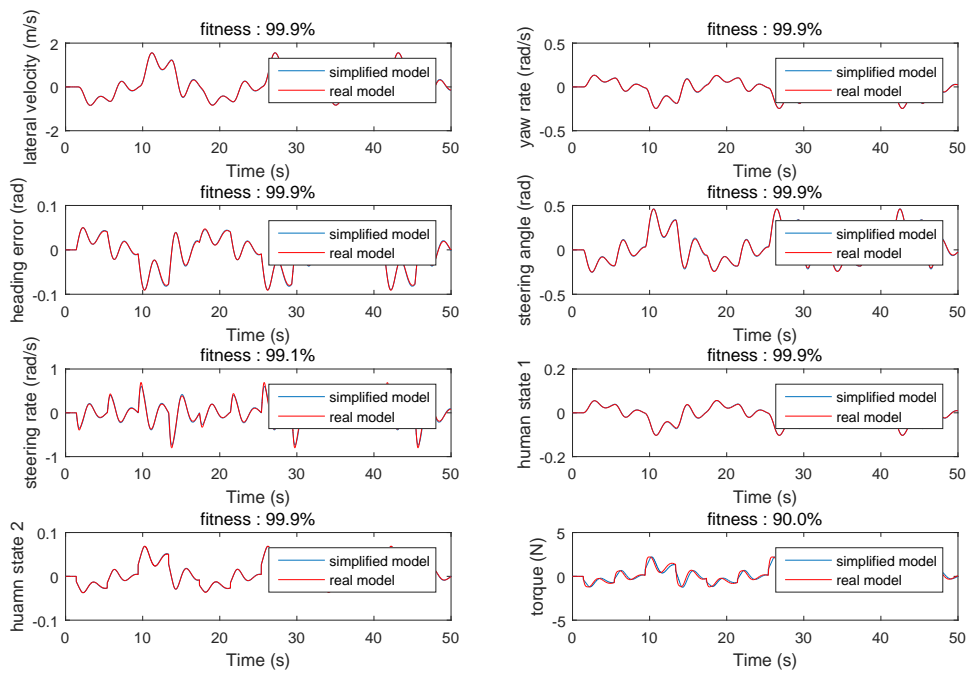


Figure B-1: States comparisons of the simplified steering model, the second parameter set.

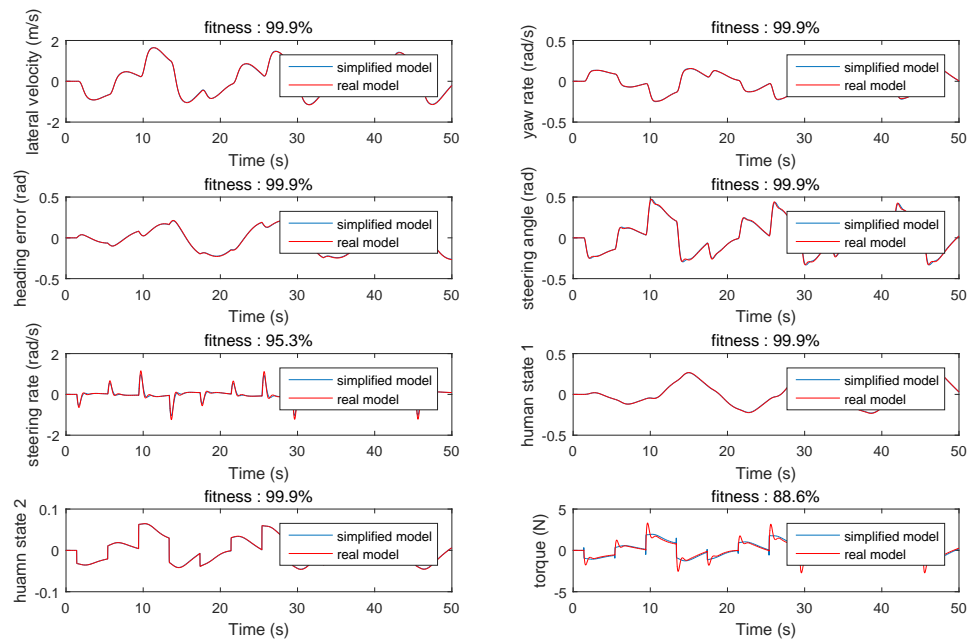


Figure B-2: States comparisons of the simplified steering model, the third parameter set.

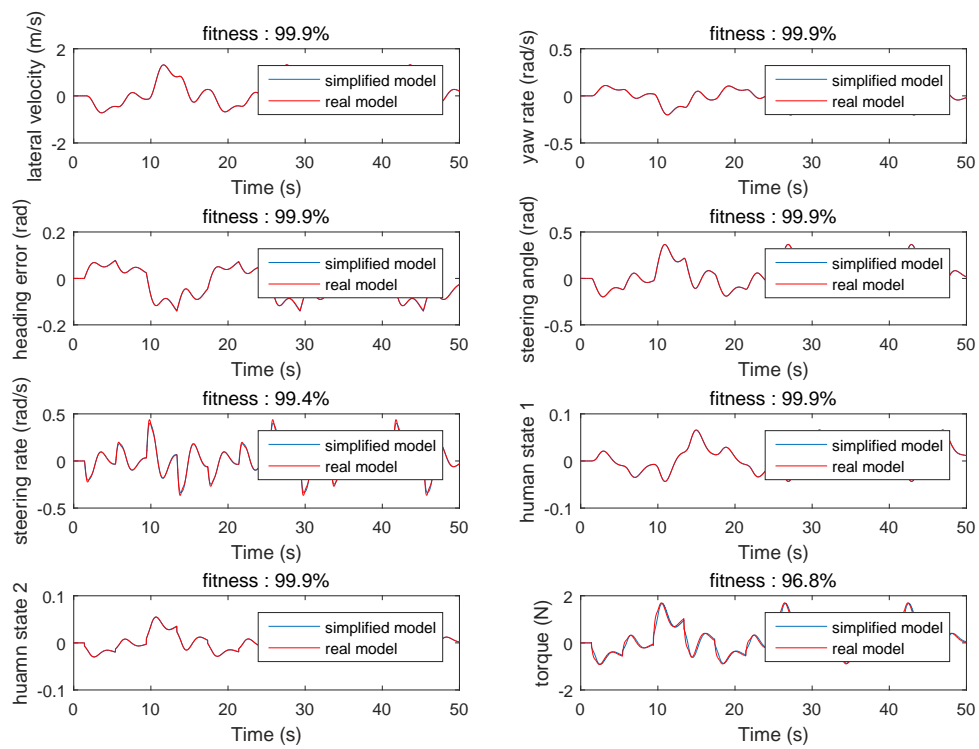


Figure B-3: States comparisons of the simplified steering model, the fourth parameter set.

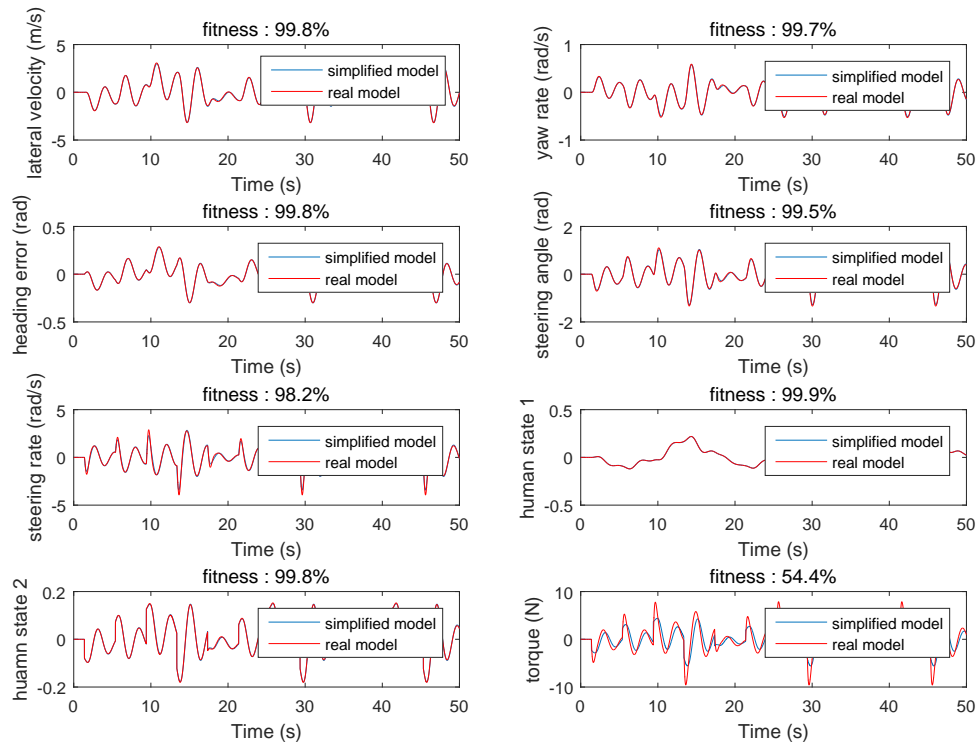


Figure B-4: States comparisons of the simplified steering model, the fifth parameter set.

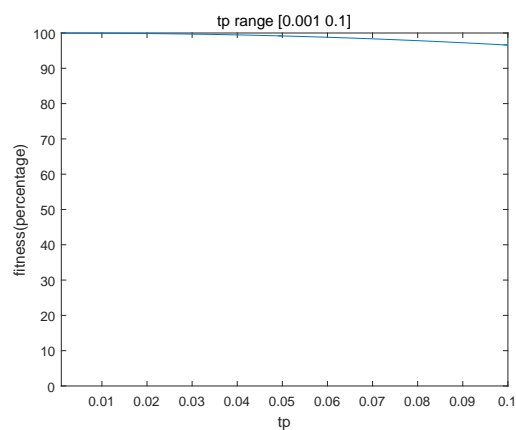


Figure B-5: The parameter τ_p local sensitivity analysis (the first parameter set).

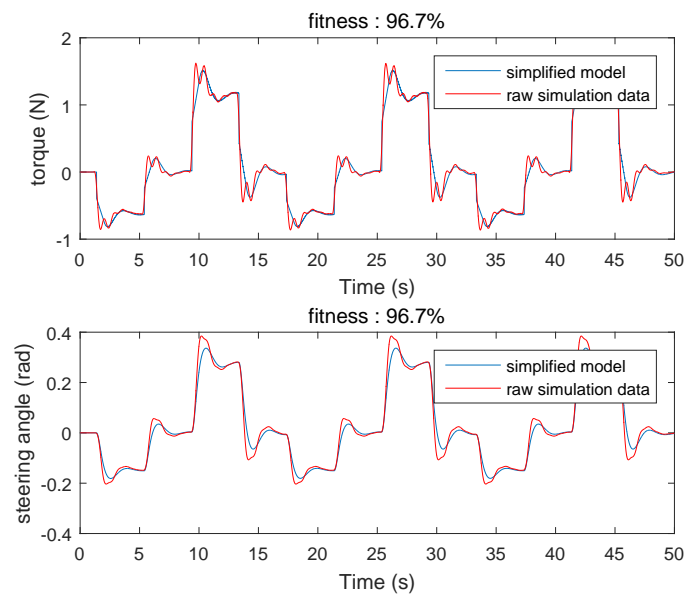


Figure B-6: Illustration of the comparisons of the steering wheel angle and the torque between the full steering model and the simplified steering model.

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Glossary

List of Acronyms

3mE	Mechanical, Maritime and Materials Engineering
DCSC	Delft Center for Systems and Control
TU Delft	Delft University of Technology
NMS	Neuromuscular System
ARMAX	Auto-Regressive Moving Average model with eXogenous inputs
VAF	Variance Accounted For
ANOVA	ANalysis Of VAriance
PI	Proportional-Integral

