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Linear Aggregate Model for Realizable Dispatch of Homogeneous Energy Storage

Mazen Elsaadany^{id}, *Graduate Student Member, IEEE*, Mads R. Almassalkhi^{id}, *Senior Member, IEEE*, and Simon H. Tindemans^{id}, *Member, IEEE*

Abstract—To optimize the dispatch of batteries, a model is required that can predict the state of energy (SOE) trajectory for a chosen open-loop power schedule to ensure admissibility (i.e., that schedule can be realized). However, battery dispatch optimization is inherently challenging when batteries cannot simultaneously charge and discharge, which begets a non-convex complementarity constraint. In this letter, we develop a novel composition of energy storage elements that can charge or discharge independently and provide a sufficient linear energy storage model of the composite battery. This permits convex optimization of the composite battery dispatch while ensuring the admissibility of the resulting (aggregated) power schedule and its disaggregation to the individual elements.

Index Terms—Battery, energy storage, convex optimization, priority-based control, complementarity constraints.

I. INTRODUCTION

BATTERY energy storage systems (BESS) are increasingly deployed to help facilitate the grid's uptake of variable renewable energy and improve reliability [1], [2], [3]. In many applications, BESS dispatch involves coordinating large numbers of energy storage elements. Examples include battery racks that comprise a single BESS asset, BESS container assets in a utility-scale battery storage facility as shown in Fig. 1, or aggregated BESS units coordinated as a virtual power plant (VPP).

Optimally scheduling the charging and discharging of storage elements requires solving a non-convex optimization problem. Because most storage technologies do not permit simultaneous charging and discharging, *complementarity constraints* are usually required to force the product of the charging and discharging powers to be equal to zero at

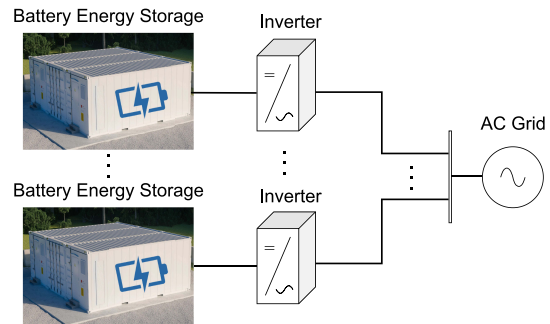


Fig. 1. Example of a composite battery with AC-coupled battery elements.

each time step, and for each element. This nonlinearity is often implemented using binary variables [4], [5]. The non-convexity of the composite BESS dispatch can present a computational challenge when incorporating BESSs into more complex, large-scale power system optimization problems, such as optimal power flow (OPF) and economic dispatch [6].

This motivates the need for a convex BESS model. A common approach is to obtain a convex relaxation by omitting the complementarity constraint, which results in a predicted state of energy (SOE) that is a provable lower bound on the realized SOE and can lead to unexpected SOE saturation and an inadmissible dispatch [7], [8]. Additionally, the relaxation permits solutions that simultaneously charge and discharge [5].

At first glance, such solutions may appear irrational because charging/discharging inefficiencies result in unnecessary dissipation of energy. Nevertheless, there are cases where such solutions are a rational outcome of a battery dispatch schedule, e.g., when energy prices are negative [9], or as a byproduct of minimum up- or downtime constraints in a centrally dispatched system. Any dissipative effects on battery aging can be captured by adding a cycling cost to the objective function, so that the benefit must outweigh the cost. Sufficient conditions under which simultaneously charging and discharging is optimal (i.e., rational) are given in [5].

Conversely, there are cases where simultaneous charging and discharging is guaranteed not to occur. Sufficient *ex-post* conditions for an exact relaxation in economic dispatch are provided in [10]. *Ex-ante* conditions for exactness of the relaxed problem are presented in [11], assuming that battery power limits, as well as initial and maximum energy capacities, satisfy

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certain conditions. Furthermore, [5], [12], [13] show that for specific economic dispatch formulations with positive linear or quadratic cost functions, the relaxation at optimality would be exact with no simultaneous charging and discharging.

Various alternative problem formulations exist to reduce the reliance on the computationally demanding complementarity constraint. For a single time step, tighter relaxations have been proposed [14], [15], which can also heuristically be applied to time series, albeit without realizability guarantees. Authors in [16] avoid the use of the complementarity constraint when applied to DC Optimal Power Flow (DCOPF) but add a penalty term to the objective function to ensure the exactness of the relaxed model. Instead of modifying objective, the authors in [7], [17] present an alternative robust linear battery model which represents a convex restriction of the non-convex formulation that sacrifices optimality for guaranteed admissibility. The robust linear model omits the complementarity constraint but envelopes the actual SOE trajectories to guarantee that any feasible power trajectory is realizable. However, the envelopes beget conservativeness as a function of energy conversion efficiencies and the prediction horizon, which limits practicality. Now, a composite battery made up of identical storage elements can be scheduled with various methods. One way is to model and optimize the schedule of each storage element individually, requiring many complementarity constraints [18]. A simpler and computationally efficient approach is to use a static *equal power sharing* policy where all elements charge or discharge equally [18], [19]. The elements can then be aggregated into a composite battery whose dispatch is optimized, but still requires one non-convex complementarity constraint. In summary, previous work in energy storage optimization that circumvents the complementarity condition either has been application-specific and holds only if specific criteria on the objective function and/or the battery parameters are met, or presents convex restrictions that may yield significantly sub-optimal solutions. Furthermore, previous work has generally (and often, implicitly) assumed an equal power-sharing policy, which limits how elements can be dispatched. This manuscript revisits the modeling of aggregations of storage units by presenting a novel composite model of N identical and independently controllable battery elements, where simultaneous charging and discharging power trajectories at the composite level can be implemented by inhomogeneous disaggregation to the individual elements. Throughout, BESS aggregations are used as an example, but the proposed work can be extended to different energy storage assets. The main contributions of this letter are as follows:

- 1) We present a novel linear model that provides a sufficient representation of aggregations of homogeneous energy storage elements. For large numbers of elements or and small control time steps, the representation asymptotically approaches the true feasibility envelope.
- 2) We develop a priority-based strategy, based on element SOE, to perform composite-to-elements disaggregation.
- 3) We prove that all solutions of the linear aggregate model are realizable using the priority-based strategy, respecting element-wise power limits, energy limits, and complementarity constraints after disaggregation.

Next, we develop the composite battery model in Section II. Section III presents the proposed disaggregation strategy and sufficient conditions for the admissibility of a composite power trajectory. Simulation results that demonstrate the performance of the proposed model and disaggregation strategy make up Section IV. Conclusion and future work are in Section V.

II. COMPOSITE BATTERY MODEL

In this section, we will present element-wise and composite models of BESS systems, composed of N identical and independently controllable elements (e.g., as shown in Fig. 1). For element $i \in \{1, \dots, N\}$, the charge/discharge power is set in *controller time steps* l with duration δt . The charge/discharge power between l and $l+1$, the charge/discharge efficiency and the SOE at timestep l are denoted by $P_{c/d}^i(l)$, $\eta_{c/d}$, and $E^i(l)$, respectively. A realizable BESS dispatch must satisfy power limits ($P_{c/d,\max}$), energy limits (E_{\max}), and complementarity constraints for all battery elements ($\forall i = 1, \dots, N$) and time steps ($\forall l = 0, \dots, L-1$). Note that all elements have identical power and energy limits. This results in (1), where E_0^i is the initial SOE of battery element i :

$$E^i(l+1) = E^i(l) + \delta t \eta_c P_c^i(l) - \delta t \frac{1}{\eta_d} P_d^i(l) \quad (1a)$$

$$E^i(0) = E_0^i \quad (1b)$$

$$0 \leq E^i(l+1) \leq E_{\max} \quad (1c)$$

$$0 \leq P_{c/d}^i(l) \leq P_{c/d,\max} \quad (1d)$$

$$0 = P_c^i(l) P_d^i(l), \quad (1e)$$

where (1c)-(1e) represent energy limits, power limits and, complementarity constraints, respectively. The scheduling of the composite battery is done with (optionally) longer *scheduler time steps* of duration $\Delta t = M\delta t$, $M \in \mathbb{N}^+$. We denote these scheduling time steps with index $[k]$ in square brackets, where $k = 0, \dots, K-1$ and $L = MK$. Constant power in step k implies the following constraint for all l, k with $k = \lfloor \frac{l}{M} \rfloor$:

$$\sum_{i=1}^N P_c^i(l) = P_c[k] \quad \wedge \quad \sum_{i=1}^N P_d^i(l) = P_d[k]. \quad (2)$$

Typically, it is assumed that a composite BESS system is modeled and operated under *balanced* conditions: the dispatcher (e.g., EMS) maintains *equal SOE* ($E_0^i = E_0 \forall i$) and adopts an *equal power sharing policy* across its elements. This results in a composite battery with power and energy limits of $NP_{c/d,\max}$ and NE_{\max} as shown in (3):

$$E[k+1] = E[k] + \Delta t \eta_c P_c[k] - \Delta t \frac{1}{\eta_d} P_d[k] \quad (3a)$$

$$E[0] = \sum_i E_0^i \quad (3b)$$

$$0 \leq E[k+1] \leq NE_{\max} \quad (3c)$$

$$0 \leq P_{c/d}[k] \leq NP_{c/d,\max} \quad (3d)$$

$$0 = P_c[k] P_d[k]. \quad (3e)$$

Under equal power sharing, a complementarity constraint (3e) at the composite battery level is sufficient to satisfy (1e) for

all i and all l . Under balanced operation, a composite dispatch satisfying (3) would satisfy (1) for all i and all l .

The complementarity constraint is often neglected to obtain a convex relaxation of the problem. For example, as

$$(3a)-(3d) \quad (4a)$$

$$\frac{P_c[k]}{NP_{c,\max}} + \frac{P_d[k]}{NP_{d,\max}} \leq 1, \quad (4b)$$

where the cutting plane in (4b) is one of the facets of the convex hull of the feasible set defined by (3), typically used to tighten the relaxation. Even tighter relaxations have been developed [14], [15] to reduce the prevalence of solutions with simultaneous charging and discharging in real-world problems. However, such relaxations can result in an optimal power schedule with simultaneous charging and discharging, which cannot be implemented under equal load sharing.

Such cases *can* be addressed by a composite battery if we omit the equal load sharing requirement. This way, we can realize simultaneous charging and discharging dispatch schedules by appropriately disaggregating them across the N elements while satisfying (1e). This increases the capability of the composite battery at the cost of computational complexity with N complementary constraints.

In the remainder of this letter, we address this challenge by presenting a sufficient (but not unique) priority-based disaggregation strategy that engenders a *linear* set of provably realizable composite power trajectories. These trajectories permit simultaneous charging and discharging of the BESS and approximate the full solution space arbitrarily closely.

III. COMPOSITE TRAJECTORY DISAGGREGATION

A control policy is used to disaggregate a desired piecewise constant composite power inputs $P_{c/d}[k]$ into N individual element charge/discharge inputs, $\{P_{c/d}^i(l)\}_{i=1}^N$. One such controller is the presented priority stack controller discussed below. Sufficient conditions are developed that – in combination with the controller – enable the characterization of $P_c[k]$ and $P_d[k]$ that guarantees (1) is satisfied.

A. Priority Stack Controller (PSC)

For every controller time-step l , the priority stack controller (PSC) sorts all N elements based on the SOE. The element with the lowest SOE has the highest charging priority, while the element with the highest SOE has the highest discharging priority. Note that similar control schemes have been proposed in [20] and investigated in a real-time operational setting in [18], [19].

The priority stack results in N_c elements with lowest SOEs charging and N_d elements with highest SOEs discharging, as follows:

$$N_c(l) := \left\lceil \frac{P_c[k]}{P_{c,\max}} \right\rceil \quad \wedge \quad N_d(l) := \left\lceil \frac{P_d[k]}{P_{d,\max}} \right\rceil. \quad (5)$$

Disaggregating the composite charging trajectory for $P_c[k] > 0$, the PSC sets $P_c^i(l) = P_{c,\max}$ for the $N_c(l) - 1$ elements with the lowest SOE and the next-lowest element is assigned $P_c^i(l) = P_c[k] - (N_c(l) - 1)P_{c,\max}$. Discharging

Algorithm 1 Priority Stack Controller (PSC)

```

1: Given  $P_c[k]$  and  $P_d[k]$ 
2: for each timestep  $l$  do
3:   Sort elements based on  $E^i(l)$  in ascending order
4:   Calculate  $N_{c/d}(l) := \lceil \frac{P_{c/d}[k]}{P_{c/d,\max}} \rceil$ 
5:   Assign  $P_{c/d}^i(l)$  for first  $N_c(l)$ /last  $N_d(l)$  elements
6:   Update SOE for all elements:  $E^i(l) \rightarrow E^i(l+1)$ 
7: end for

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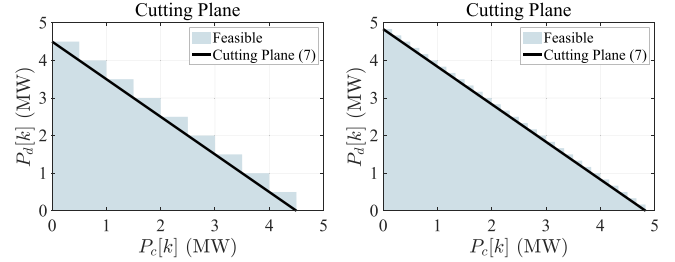


Fig. 2. Feasible region for $N = 10$ and $P_{c/d,\max} = 0.5\text{MW}$ (left) and $N = 30$ and $P_{c/d,\max} = 0.17\text{MW}$ (right). SOE limits are omitted here.

is implemented analogously. The PSC is summarized in Algorithm 1 and begets Property 1.

Property 1: Under PSC, if elements i, j satisfy $E^i(l) \geq E^j(l)$, then $P_c^i(l) \leq P_c^j(l)$ and $P_d^i(l) \geq P_d^j(l)$.

B. Guaranteeing Element Complementarity Constraints

Next, we analyze the effects of the PSC on (1), on complementarity conditions (1e), and on energy limits (1c). To ensure the complementarity constraints are satisfied at the element level, $(P_c^i(l)P_d^i(l) = 0 \quad \forall i, \forall l)$, the priority stack cannot issue a charging and discharging demand to the same battery element i simultaneously. Thus, any overlap in the selection of $N_c(l)$ charging and $N_d(l)$ discharging elements imply a violation of the complementarity constraints, which must, therefore, satisfy

$$N_c(l) + N_d(l) \leq N, \quad \forall l. \quad (6)$$

Proposition 1: Under the PSC, if $P_c[k]$ and $P_d[k]$ satisfy

$$\frac{P_c[k]}{P_{c,\max}} + \frac{P_d[k]}{P_{d,\max}} \leq N - 1, \quad (7)$$

then the resulting $N_c(l)$ and $N_d(l)$ satisfy (6).

Proof: Define $\Gamma_c := \lfloor \frac{P_c[k]}{P_{c,\max}} \rfloor$ and $\Delta_c := \frac{P_c[k]}{P_{c,\max}} - \Gamma_c \in [0, 1)$ to be the integer and fractional components of the quantity $\frac{P_c[k]}{P_{c,\max}}$. Then $\lceil \frac{P_c[k]}{P_{c,\max}} \rceil \leq \Gamma_c + 1$ and, from (7), we get

$$\left\lceil \frac{P_d[k]}{P_{d,\max}} \right\rceil \leq \lceil N - 1 - (\Gamma_c + \Delta_c) \rceil = N - 1 - \Gamma_c. \quad (8)$$

Adding the inequalities, we get $N_c(l) + N_d(l) \leq N \quad \forall l$. ■

Thus, adding (7) to the composite formulation in (3) will satisfy element complementarity constraints under PSC. The feasible regions dictated by (6) and (7) are shown in Fig. 2.

C. Relating Element and Composite Energy Constraints

Similarly, we want to relate element energy limits to those in the composite formulation. Under PSC, the battery elements

are charged/discharged individually every l , which results in differing SOE trajectories. The spread of element SOEs can impact the realizability of the composite dispatch when individual elements hit their SOE limits. Constraint violations can be avoided by quantifying the maximum possible SOE spread under PSC and using that as an energy buffer applied to the composite battery energy limits.

Proposition 2: Under the PSC, a composite dispatch that satisfies

$$N\varepsilon \leq E[k] \leq N(E_{\max} - \varepsilon), \quad (9)$$

with

$$\varepsilon := \delta t \left(\eta_c P_{c,\max} + \frac{P_{d,\max}}{\eta_d} \right), \quad (10)$$

will not violate any individual element's energy limit, provided that (i) $\max_{i,j} \{|E_0^i - E_0^j|\} \leq \varepsilon$ and (ii) the control time step $\delta t = \Delta t/M$ is sufficiently small to ensure $\varepsilon \leq \frac{1}{2}E_{\max}$.

Proof: Define the maximum difference in SOE across all elements as $\Delta E(l) := \max_{i,j} \{|E^i(l) - E^j(l)|\}$ (i.e., the SOE spread). Consider element SOEs $E^i(l)$ and $E^j(l)$ and, without loss of generality, let $E^i(l) \geq E^j(l)$. Using (1a) the following is obtained,

$$\begin{aligned} E^i(l+1) - E^j(l+1) &= (E^i(l) - E^j(l)) \\ &+ \frac{\Delta t}{M} \eta_c (P_c^i(l) - P_c^j(l)) - \frac{\Delta t}{M \eta_d} (P_d^i(l) - P_d^j(l)). \end{aligned} \quad (11)$$

From Property 1 and (1d), we have $-P_{c,\max} \leq P_c^i(l) - P_c^j(l) \leq 0$ and $0 \leq P_d^i(l) - P_d^j(l) \leq P_{d,\max}$, which implies

$$E^i(l+1) - E^j(l+1) \leq E^i(l) - E^j(l) \leq \Delta E(l), \quad (12a)$$

$$E^i(l+1) - E^j(l+1) \geq -\frac{\Delta t}{M} \left(\eta_c P_{c,\max} + \frac{P_{d,\max}}{\eta_d} \right) = -\varepsilon, \quad (12b)$$

where the inequality in (12b) uses $E^i(l) - E^j(l) \geq 0$. Considering all i, j , we can bound $\Delta E(l+1) \leq \max\{\Delta E(l), \varepsilon\}$. Moreover, we see that $\Delta E(l) \leq \varepsilon \Rightarrow \Delta E(l+1) \leq \varepsilon$, so it follows that for a composite battery with initial SOE spread $\Delta E(0) \leq \varepsilon$ (condition (i)), we maintain $\Delta E(l) \leq \varepsilon, \forall l$.

Because energy linearly increases/decreases within each scheduling time step k under PSC, (9) implies its equivalent for controller time steps l : $N\varepsilon \leq E(l) \leq N(E_{\max} - \varepsilon)$. Dividing by N begets $\varepsilon \leq \bar{E}(l) \leq E_{\max} - \varepsilon$, where $\bar{E}(l) := \sum_j E^j(l)/N$ is the average element SOE. Because the SOE spread $\Delta E(l) \leq \varepsilon$, we must have $\bar{E}(l) - \varepsilon \leq E^i(l) \leq \bar{E}(l) + \varepsilon, \forall i$. Thus, (1c) holds for all i and all l .

Lastly, we require condition (ii) ($\varepsilon \leq \frac{1}{2}E_{\max}$) to ensure that (9) does not result in an empty set (infeasibility). This can be achieved with a sufficiently large value of M . ■

D. Realizable Composite Power Trajectories

The following theorem provides a linear formulation for realizable dispatch of a composite battery.

Theorem 1: If $\max_{i,j} \{|E_0^i - E_0^j|\} \leq \varepsilon$, then any feasible composite battery dispatch schedule $\{P_c[k], P_d[k]\}_{k=0}^{K-1}$ satisfying

$$(3a), k = 0, \dots, K-1 \quad (13a)$$

TABLE I
INDIVIDUAL BESS ELEMENT PARAMETERS

Parameters	Value	Unit
Charge/discharge efficiencies, η_c/η_d	0.95	-
Maximum charge/discharge power, P_{\max}	5.00	kW
Maximum energy capacity, E_{\max}	13.5	kWh

$$(3b), \quad (13b)$$

$$\frac{P_c[k]}{NP_{c,\max}} + \frac{P_d[k]}{NP_{d,\max}} \leq \frac{N-1}{N}, k = 0, \dots, K-1 \quad (13c)$$

$$N\varepsilon \leq E[k] \leq N(E_{\max} - \varepsilon), k = 0, \dots, K \quad (13d)$$

$$P_c[k], P_d[k] \geq 0, k = 0, \dots, K-1 \quad (13e)$$

will satisfy (1) for all N elements under PSC and is, thus, realizable.

Proof: Eqs. (1a) and (2) imply (3a) and (1b) implies (3b). From Proposition 1, (13c) ensures satisfaction of (1e). Similarly, with $\max_{i,j} \{|E_0^i - E_0^j|\} \leq \varepsilon$, Proposition 2 guarantees that (1c) holds, and the PSC with (13e) and (2) ensure that (1d) holds for all N elements. Thus, any feasible composite dispatch is realizable. ■

Note that for a fixed composite battery, if N increases, the cutting plane in (13c) approaches the cutting plane in (4b). Furthermore, for $M \gg 1$, ε in (10) tends to zero. That is, with faster control and more granular elements, (13) approaches the relaxed model in (4); however, with guaranteed realizability.

The energy buffer ε in (13d) restricts the set of feasible energy trajectories relative to (3) and (4a). The feasible energy trajectories in the robust linear model presented in [7] are also a subset of the feasible energy trajectories in (3) and (4a). However, the proposed model in (13) has a constant energy buffer ε that is independent of the length of time horizon considered. This is in contrast to the robust model which has an energy buffer that increases with the length of the time horizon which, in the worst case, significantly reduces the feasible set of energy trajectories and can yield sub-optimal solutions. This is illustrated in Fig. 3 where the (worst case) feasible set of energy trajectories for the robust model and the RCB model for $\Delta t = 1$ hr and $M = 1$ ($\delta t = 1$ hr) and $M = 5$ ($\delta t = 12$ min) are compared for the battery parameters of a Tesla Powerwall shown in Table I. Clearly, smaller control time steps result in a smaller ε and thus a larger feasible set of energy trajectories for the proposed model, while the robust model remains unchanged.

Finally, we note that the PSC policy is a greedy sufficient policy that fully activates BESS elements in sequence. More elaborate policies (greedy or not) can be developed that preferentially spread element loading for most aggregate dispatch signals, without violating the conditions of Theorem 1.

IV. SIMULATION RESULTS

We apply the realizable composite battery (RCB) model in (13) to two relevant use cases and compare its performance against the tighter relaxed model given in (4) and the robust model in [7] used under balanced operation. Also, a Mixed-Integer Linear Program (MILP) formulation with both equal

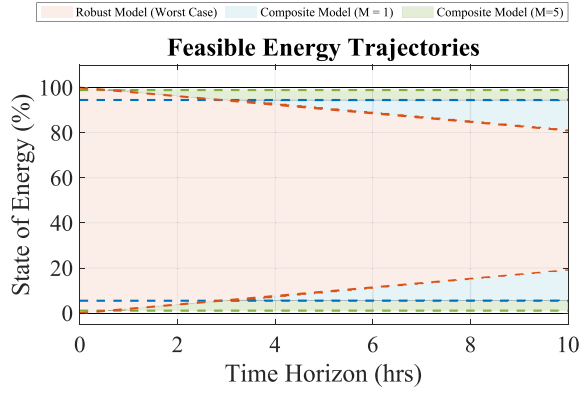


Fig. 3. Feasible set energy trajectories for the relaxed model, the robust model and the proposed realizable composite battery model (RCB).

TABLE II

POWER REFERENCE TRACKING PERFORMANCE METRICS ($N = 100$)

Model	Solver Time	Predicted MSE (kW ²)	Actual MSE (kW ²)
MILP (unequal)	492.60 s	0	0
MILP (equal)	34.37 s	200.0	200.0
Relaxed	45 ms	0	1889
Robust	54 ms	212	212
RCB (M = 1)	50 ms	23.93	23.93
RCB (M = 5)	50 ms	0.231	0.231
RCB (M = 10)	50 ms	0	0

and unequal power-sharing for N elements in (1) used for comparison. The battery element parameters given in Table I were used. The two use-cases are a) power reference tracking and b) revenue maximization. All cases are modeled using JuMP in Julia and were solved using Gurobi v12.0.0. All cases were run on a Macbook Pro with an M1 Pro processor.

A. Power Reference Tracking

In this use case, the goal is to minimize the power tracking error for a given reference signal as shown below,

$$\min_{P_c[k], P_d[k]} \sum_{k=0}^{K-1} ((P_c[k] - P_d[k]) - P_{\text{ref}}[k])^2. \quad (14)$$

A timestep of $\Delta t = 3$ min, along with $N = 100$ was used. The RCB model was used three times with $M = 1$, $M = 5$, and $M = 10$ respectively. The results from using each model are shown in Fig. 4 and the solve times along with the mean squared error (MSE) are shown in Table II. Note that the relaxed model resulted in a dispatch with simultaneous charging and discharging, which cannot be realized under equal power sharing and results in the discrepancy between predicted MSE and actual MSE obtained while realizing the optimal dispatch. The net power trajectory was implemented which caused the BESS to saturate causing the discrepancy between the predicted and realized MSE values.

The MILP model under unequal power sharing was able to track the reference signal with no tracking error. The MILP under equal power sharing couldn't track the reference signal as well, despite both solving to 0% optimality gap,

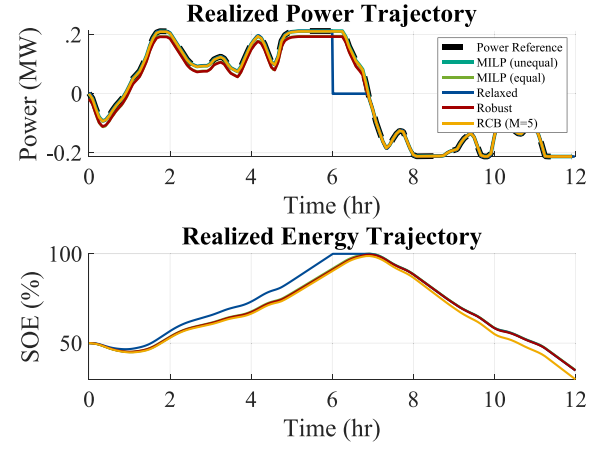


Fig. 4. Open-loop implementation of power optimal reference tracking power schedules using MILP, relaxed, robust and proposed RCB models.

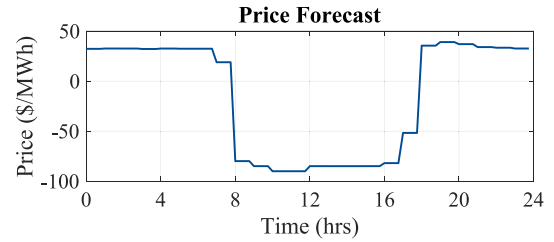


Fig. 5. CAISO day-ahead price forecasts.

indicating that there is a benefit of simultaneously charging and discharging in this use case. The MILP, under both equal and unequal power sharing, came with a significant computational burden compared to the rest of the formulations, attributed to the extra binary variables added to enforce the complementarity constraint. The other formulations are linear and, as a result, have significantly faster solve times. The relaxed model predicts zero tracking error; however, the solution was unrealizable and caused the battery to saturate around $t = 6$ hrs, leading to a large realized tracking error. The robust model yields a realizable yet sub-optimal solution. The RCB model shows improved tracking error with increasing values of M , but even surpassing the robust model with $M = 1$.

B. Revenue Maximization

In this use case, the goal is to maximize the predicted revenue from the composite battery system given a forecasted price signal. The objective function is given by,

$$\max_{P_c[k], P_d[k]} \sum_{k=0}^K C[k](P_d[k] - P_c[k]), \quad (15)$$

where $C[k]$ is the price at time k . The price signal was obtained from the California Independent System Operator (CAISO) day-ahead market prices, plotted in Fig. 5.

A timestep of $\Delta t = 15$ min, along with $N = 100$ was used. The RCB model was tested for $M = 1$, 5, and 10 which correspond to $\delta t = 15$ min, 3 min and 1.5 min respectively. An additional case was added with $M = 900$ ($\delta t = 1$ s) to gain

TABLE III
REVENUE MAXIMIZATION PERFORMANCE METRICS ($N = 100$)

Model	Solver Time	Predicted Revenue (\$)	Actual Revenue (\$)	Optimality Gap (%)
MILP (unequal)	90 mins	2088.99	2088.99	0.02
MILP (equal)	23 mins	2088.97	2088.97	0
Relaxed	1.5 ms	2092.80	1940.35	0
Robust	2.2 ms	1866.42	1866.42	0
RCB ($M = 1$)	1.7 ms	1642.24	1642.24	0
RCB ($M = 5$)	1.9 ms	2000.50	2000.50	0
RCB ($M = 10$)	1.7 ms	2045.24	2045.24	0
RCB ($M = 900$)	1.6 ms	2089.48	2089.48	0

insight on model performance as $M \rightarrow \infty$. The results for the different models are shown in Table III.

The MILP models produced high revenues that could also be realized, but at substantial computational cost, with unequal power sharing taking significantly longer to solve and was not able to solve to 0% optimality gap. The relaxed model predicts the highest revenue but underestimates the SOE, leading to unpredicted battery saturation and a realized performance that is significantly worse. The robust model yields a realizable yet suboptimal solution. The RCB model for small values of M results in a sub-optimal solution which can be attributed to the larger timestep used in this use-case leading to a larger value of ε and, in turn, a more restricted feasible set. However, for $M \geq 5$ ($\delta t \leq 3$ min), the RCB model surpasses all other linear models and for larger values of M the predicted revenue steadily increases, maintaining realizability.

We note that for $M = 900$ the revenue even surpasses the element-wise MILP solution, because individual elements are able to switch from charging to discharging and back at a faster controller timestep when implemented by the PSC. A MILP solution with the control time step δt would again achieve or exceed this result, but at much larger computational cost. The RCB model captures what is feasible at the fast control time step δt without added complexity.

V. CONCLUSION

A new linear dispatch model is presented and analyzed for a composite battery that permits unequal power sharing among its N elements. A priority-based disaggregation strategy is described and used to characterize sufficient conditions on the composite battery dispatch under which element constraints are guaranteed to be satisfied. Previous battery optimization implicitly assumed equal power sharing between all elements. By enabling unequal power sharing within a linear formulation, we have expanded the class of battery models that yield realizable power trajectories. Implementing the proposed controller is of interest and mainly requires communications to/from battery elements faster than δt (similar to VPPs in the field today). Furthermore, embedding the presented linear models into energy system planning problems would inform the impact of battery systems more broadly. Future work

involves exploring other disaggregation strategies and non-linear battery models.

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