Quantification and comparison of hierarchy in Public Transport Networks

Wang, Ziyulong; Huang, Ketong; Massobrio, Renzo; Bombelli, Alessandro; Cats, Oded

DOI
10.1016/j.physa.2023.129479

Publication date
2023

Document Version
Final published version

Published in
Physica A: Statistical Mechanics and its Applications

Physica A 634 (2024) 129479

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Quantification and comparison of hierarchy in Public Transport Networks

Ziyulong Wang [a,*], Ketong Huang [a,b], Renzo Massobrio [a,c], Alessandro Bombelli [d], Oded Cats [a]

[a] Department of Transport and Planning, Delft University of Technology, Delft, The Netherlands
[b] Strategic & Policy Department, Beijing Transport Institute, Beijing, China
[c] Escuela Superior de Ingeniería, Universidad de Cádiz, Cádiz, Spain
[d] Section of Air Transport and Operations, Delft University of Technology, Delft, The Netherlands

ARTICLE INFO

Keywords:
Public transport networks
Hierarchy
Network science
Topology
General Transit Feed Specification (GTFS)

ABSTRACT

Network hierarchy describes the relative arrangement of network elements and reflects its fundamental structure. We propose a multi-dimensional topology-based method for quantifying and comparing the extent to which different Public Transport Networks (PTNs) exhibit a hierarchical structure. The proposed method considers the uneven distribution of node importance with different definitions (e.g., degree centrality and betweenness centrality) in a PTN, the clustering of nodes and the node connection patterns. We apply the developed method on 63 high-capacity PTNs worldwide using General Transit Feed Specification (GTFS) data. In addition to global indicators, we use the goodness-of-fit between the probability density function of local indicators and a skew-normal distribution to quantify the extent of PTN hierarchy. Results show that the scale-free network structure and preferential attachment do not vary much across PTNs. In contrast, stop accessibility and traffic intermediacy vary considerably across PTNs as reflected by the closeness centrality and betweenness centrality distributions. Lastly, metro systems exhibit a more hierarchical structure than their tram and Bus Rapid Transit (BRT) counterparts. This work makes a first step towards a better mapping and comparison of different PTNs, which can assist academics and practitioners in better (re)designing and planning the PTNs of the future.

1. Introduction

Hierarchy is an arrangement based on the relative status of elements of an organisation, a society or a system. Investigating hierarchy is one of the fundamental approaches to capturing the structure and dynamics of complex systems, such as neural networks [1], social networks [2], airline networks [3] and, in particular, Public Transport Networks (PTNs) [4]. Public transport planners are interested in understanding the PTN hierarchy, which is a first step towards a better assessment to improve service design, performance, scalability and cost-efficiency [5,6]. This is of paramount importance for high-capacity PTNs, which undertake the majority of people's mobility in a city with an exclusive right of way, including metro, tram and Bus Rapid Transit (BRT).

Multiple definitions and methods appear in the previous works that endeavour to identify the PTN hierarchy, which correspondingly shows the complexity emerging from the extensive relationships among the elements of the public transport system. One typical PTN hierarchy identification method relies on the attributes of a transportation network pertaining to mode, infrastructure

* Corresponding author.
E-mail addresses: z.wang-19@tudelft.nl (Z. Wang), kelton0792@outlook.com (K. Huang), r.m.massobrio@tudelft.nl (R. Massobrio), a.bombelli@tudelft.nl (A. Bombelli), o.cats@tudelft.nl (O. Cats).

https://doi.org/10.1016/j.physa.2023.129479
Received 25 July 2023; Received in revised form 2 December 2023
Available online 27 December 2023

function and trip distance. Gallotti and Barthelemy [7] and Aleta et al. [8] aggregate all the lines of the same mode (i.e., bus, metro and tram) in a superlayer to study the interdependency among transport modes in a city. Similarly, the PTN hierarchy can be distinguished according to the infrastructure function determined by the service mode, purpose and distance. Referring to two different trip purposes, urban and interurban systems can be regarded as two interconnected subsystems in a PTN [4]. Following the same stream, a multi-level PTN can be divided into (inter)regional (train) level, agglomeration (metro/light rail) level and urban (tram) level [9]. Given the hierarchy from the infrastructure function, Gao et al. [10] further rank the links in a PTN using the residential trip distance. All the above-mentioned studies assume that each public transport mode constitutes a level in the network hierarchy as opposed to directly measuring and quantifying the degree of hierarchy.

An alternative and emerging approach pertains to the topological features of PTNs. A possible interpretation of a hierarchical network arises with a combination of power-law degree distribution in various scale-free topologies and a high degree of clustering [11,12]. Whereas scale-free networks are rare in reality [13], many commonly observed topological properties of networks follow a skew-normal distribution [14]. To unfold the topological characteristics of PTNs, the essential features of PTNs are established by meaningful graph representations, including infrastructure and service dimensions. These two most deployed types of graph representations are called the L-space topology and P-space topology, respectively [15]. In L-space, each node denotes a stop and a link is formed between two stops if these two stops are consecutively connected by an infrastructure segment [16–18]. In P-space, each node also denotes a stop while nodes are connected if at least one common route traverses them [16,17,19]. Generally, the scale-free structure is prevailing in L-space public transport systems while the small-world property is more visible in P-space than in L-space [20,21].

Most literature associates network hierarchy only with the distribution of degree centrality, which can merely explain the local organisation of spatial importance by node connection. However, other relevant local network indicators may reflect different node importance aspects. For instance, betweenness centrality can manifest the ability of node forming connections as a bridge [17,22,23] and is frequently used to identify critical links in a network [24–26]. Therefore, the distribution of betweenness centrality can describe the hierarchy of intermediacy – and therefore potential traversing and interchanging passenger flows – in a network. Likewise, the distribution of travel time impedance [27] or closeness centrality [28] can characterise the difficulty of reaching a node from others in a network, representing the hierarchy of accessibility. Other studies aim to utilise global indicators for determining the importance of a cluster of nodes as nodes at the same hierarchy level have denser inner-level interactions and sparse inter-level interactions with nodes at different hierarchy levels [29]. Indicators, such as eigenvector centrality [30], modularity [31], clique [32], assortativity [33] and PageRank [34], are hence proposed to seek the strongly connected node clusters in the PTN.

Recent studies applied data-driven methods to uncover the PTN hierarchy, benefiting from the ever-enriching passenger flow data sources. Bassolas et al. [35] utilise census and location history data to study the hierarchical organisation of urban mobility and its connection with key urban livability indicators. Yap et al. [36] use the extensively equipped Automatic Fare Collection data (i.e., smart card data) to identify and cluster significant transfer hubs in a PTN. Gallotti et al. [37] unravel that cities with a low flow hierarchy have more direct connections between central and marginal areas using check-ins data. There are also approaches that fuse data-driven and topology-based methods to simultaneously consider the dynamics of passengers and the intrinsic structure of a PTN. Wang et al. [31] identify a multi-level time-dependent PTN hierarchy using passenger transfer flow data as link weight and modularity indicator. Moreover, an integrated topology- and flow-based hierarchy metric is developed to quantify the importance of stations in the PTN [38].

None of the aforementioned studies has proposed a PTN hierarchy quantification method which is independent of the demand data and capable of capturing various aspects of element importance. To fill in such a research gap, we propose a multi-dimensional topology-based unimodal PTN hierarchy quantification method to capture various aspects of importance in a PTN. This method can reveal the network hierarchy embedded in the planning rather than in the usage of the system and can enable cross-system comparison for which demand data are often unavailable. This method considers not only the scale-free network topology and clustering of nodes as one critical definition of network hierarchy but also other unique essential features of PTN, including accessibility and intermediacy. We further apply the proposed method to 63 unimodal PTNs worldwide with several public transport modes to compare the cross-system network hierarchy. The specific contributions of our paper are as follows:

- A definition of PTN hierarchy based on multiple aspects of node importance.
- A unimodal PTN hierarchy quantification method from 6 essential topological indicators, which directly allows for a cross-system comparison.
- An analysis of PTN hierarchies with 63 cities worldwide and a variety of public transport modes.

The remainder of this study is structured as follows. We present our multi-dimensional topology-based PTN hierarchy quantification and comparison method in Section 2. Then, Section 3 introduces the case studies of 63 urban regions worldwide. Lastly, we end our paper with conclusions and further research avenues in Section 4.

2. Method

In this section, we start by describing the definition of PTN hierarchy in this study (Section 2.1). Following data collection and processing steps, we construct the graph representation based on the acquired data as detailed in Section 2.2. Next, we quantify the PTN hierarchy through several topological indicators on the node scale and network scale in Sections 2.3 and 2.4. To enable the comparison of hierarchy across PTNs, the probability distribution of node-based indicators is used to fit a skew-normal distribution. Eventually, the multi-dimension topology-based hierarchy quantification is encapsulated in a radar chart to compare PTN hierarchies across different cities worldwide in Section 2.5. A summary of the proposed methodology is illustrated in Fig. 1.

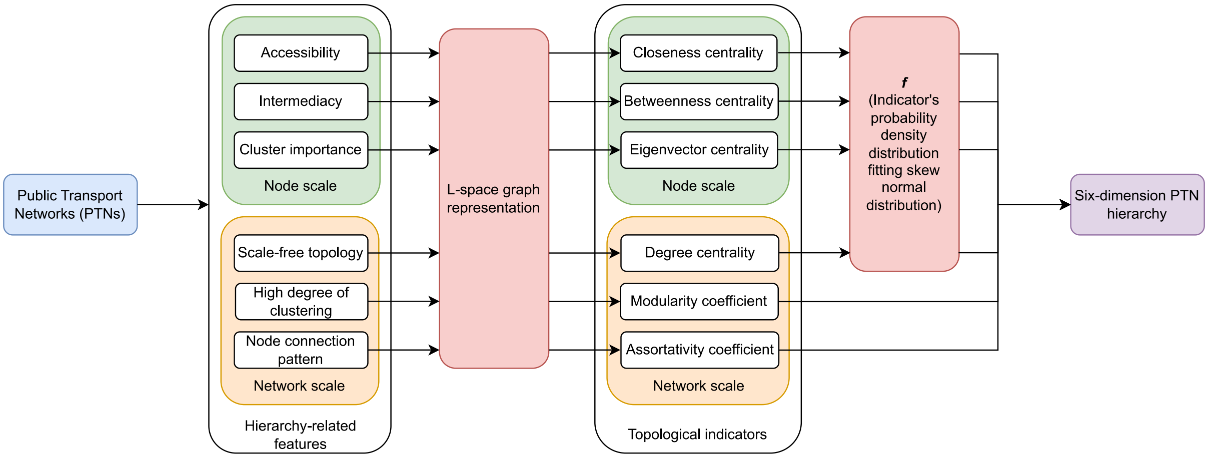

Fig. 1. Overall workflow for quantifying and comparing PTN hierarchy using multi-dimensional topology-based indicators.

2.1. PTN hierarchy

We conceptualise the PTN hierarchy as a network property that defines the relative organisation of elements by importance. Specifically, this organisation leads to a high heterogeneity of PTN element importance, and the number of elements within each level of importance should gradually descend with the importance ascending. Moreover, the element importance can be reflected by multiple aspects, proxying the various fundamental properties of a PTN. In this study, we consider the scale-free topology, node accessibility, node intermediacy, node clusters, high degree of clustering and the node connection pattern. Therefore, each element importance can shed light on the PTN hierarchy, forming one out of several dimensions of the PTN hierarchy.

2.2. PTN representation

We choose directed and weighted L-space for representing the physical PTN networks $G = (V, E, W)$. The node set V denotes stops (i.e., bus stops or rail stations), and the link set $E \subseteq V \times V$ corresponds to the connection between two adjacent stops on an infrastructure segment. In addition, there is at most one link between nodes. Further enrichment of the graph considers the weight of a link w_{ij} which is specified as the impedance of connections represented by the average travel time t_{ij} or the efficiency of connections which is regarded in this work as the inverse of the average travel time $\frac{1}{t_{ij}}$. Finally, A is the adjacency matrix of the weighted graph G in which each entry a_{ij} in the matrix equals w_{ij} if there is a link connecting between two nodes $i, j \in V$ and equals zero otherwise.

2.3. Topological indicators on node scale

2.3.1. Degree centrality

The first used topological indicator is the degree centrality, which represents the number of links a node connects to. It is one of the most basic indicators in network topology, describing node connectivity [39]. In a directed network, the node degree equals the sum of both in-degree and out-degree. Although degree centrality is on the node scale, the degree distribution of a PTN can characterise the network structure. If the degree distribution follows a skew-normal or power-law distribution, it implies a scale-free network topology. We normalise the degree centrality by dividing the maximum possible degree of a network to exclude the differences in the number of nodes among various PTNs. The normalised degree centrality is thus calculated as follows:

$$k'_i = \frac{\sum_{j\in V} a_{ij} + \sum_{j\in V} a_{ji}}{k_{max}} \quad \forall i \in V, \tag{1}$$

where k'_i is the normalised degree of node i while k_{max} is the maximum degree of the network.

2.3.2. Closeness centrality

The closeness centrality originates from the concept that two nodes in a network are topologically close if they share a direct connection whereas they are distant if tied indirectly through many intermediate nodes in a topological fashion [40]. The topological distance between two nodes can therefore be defined as the number of links on the shortest path d_{ij} between them. More formally, the shortest path minimises the sum of the weights of its constituent links from a set of paths $P_{ij} \in V \times V \times \cdots \times V$ that contains all possible sequences of adjacent nodes between two nodes i and j. As a result, a more central node can interact with many network elements via only a few links, i.e., having a short average path length and better accessibility. In this sense, the closeness centrality c_i of node i can be defined as the reciprocal of its average shortest path length. In this work, we consider the impedance of connections between two nodes as the weight to compute closeness centrality. The average shortest path results in a normalised closeness centrality and enables the comparison of nodes in PTNs of different sizes:

$$c_i' = \frac{N-1}{\sum_{i \neq j \in V} d_{ij}} \quad \forall i \in V, \tag{2}$$

where c_i' is the normalised closeness centrality of node i and N is the number of nodes in the network. The distribution of node closeness centrality over the entire PTN can reflect the hierarchy of network accessibility and efficiency as it captures the passed intermediate stops and induced travel costs.

2.3.3. Betweenness centrality

Unlike degree centrality and closeness centrality which try to quantify the importance of a node itself, betweenness centrality b_i measures the proportion of the shortest paths a node has been traversed [41]. Suppose that services or passengers flow through a PTN via the shortest path, nodes that lie on more shortest path undertake a higher proportion of traffic and become more central in the network. Betweenness centrality can be particularly useful to apprehend the role of a bridge in a PTN, for instance, a peripheral stop connecting two sub-networks. This bridging node may be regarded as a terminal hub or a passage in a PTN, having significant intermediacy. Since the value of betweenness centrality depends inherently on the number of nodes, we divide it by the number of pairs of nodes, not including the considered node i for normalisation. This division is done by $(N-1)(N-2)$ for the directed graph in our study. When acquiring the shortest path, the impedance of connection is included as weight. The computation of normalised betweenness centrality is hence presented as follows:

$$b_i' = \frac{1}{(N-1)(N-2)} \sum_{j \neq k \in V} \frac{\sigma_{jk}(i)}{\sigma_{jk}} \quad \forall i \in V, \tag{3}$$

where b_i' is the normalised betweenness centrality of node i. σ_{jk} is the total number of weighted shortest paths from node j to node k, and $\sigma_{jk}(i)$ is the number of those paths that travel through node i.

2.3.4. Eigenvector centrality

One natural extension of degree centrality is eigenvector centrality, which accounts for both the centrality of the considered node and the centrality of adjacent nodes that it connects to [39]. Often, the importance of a node is increased by connecting to other critical nodes in the network. Instead of assigning the same point to every neighbour like degree centrality, eigenvector centrality offers each node a score proportional to the total score of its neighbours. Consequently, a higher eigenvector centrality entails a higher importance level of network-wide connectivity via a highly connected node cluster. This high-degree cluster concentrated in a small area implies a mono-centric structure of a PTN.

The determination of eigenvector centrality is an iterative process as the value of the immediate neighbours of each node influences it. In the first step, all nodes are evenly assigned the same value. Then, the total eigenvector centrality is reassigned based on the adjacency matrix A until reaching the convergence or a chosen number of iterations. Furthermore, the efficiency of connections is taken as the link weight and the eigenvector centrality is already a normalised indicator with the following expressions:

$$e_i = \frac{1}{\lambda} \sum_{i \neq j \in V} a_{ij} \frac{1}{t_{ij}} e_j \quad \forall i \in V, \tag{4}$$

$$\lambda \mathbf{e} = A\mathbf{e}, \tag{5}$$

where λ is a constant and $\mathbf{e}$ is the eigenvector that contains the eigenvector centrality values such as e_i and e_j. For directed networks like the ones addressed in this study, the calculation of eigenvector centrality only considers the in-links of nodes to avoid duplication.

Given the definition of eigenvector centrality, we can assume that a relatively higher eigenvector centrality represents a preferred transfer stop in a PTN. All else being equal, stops with a higher eigenvector centrality are expected to be a more essential node cluster served as a centre in a network, benefiting from their connections with highly connected neighbours.

2.3.5. Goodness-of-fit

For all the aforementioned local topological indicators on the node scale, we derive their probability density distributions and compare how well these distributions that represent one aspect of the four node organisations follow the nature of a hierarchical network, namely how well these distributions fit a skew-normal distribution. The underlying reason is to seek a single value to quantify the degree of hierarchy from one aspect of importance and become comparable across different networks. Additionally, we choose the skew-normal distribution to estimate and test as it represents the nature of hierarchy: the presence of hubs and large numbers of nodes with few connections. We prefer skew-normal distribution over power-law distribution as the real-world networks are mostly finite, and thus a power-law network is hardly observed [13].

We first categorise the values of each indicator in a network into bins, following Scott's rule that considers the size and variance of data [42]:

$$\delta = \frac{3.49\sigma}{\sqrt[3]{n}}, \tag{6}$$

where δ indicates the bin width, σ is the standard deviation of the data, and n is the size of the data. Using Scott's rule, the number of bins is proportional to an estimate of the standard deviation and inversely proportional to the cube root of the sample size. As a result, a larger network will be penalised for its size. Second, we use the midpoints of bins as the independent variable x and the height of the bins as the dependent variable y to derive the skew-normal probability density function as follows [43]:

$$\phi(x) = \frac{1}{\omega\pi} e^{-\frac{(x-\xi)^2}{2\omega^2}} \int_{-\infty}^{\alpha(\frac{x-\xi}{\omega})} e^{-\frac{t^2}{2}} dt, \tag{7}$$

where x ranges from negative infinity to positive infinity and real numbers ξ, ω and α denote the location, scale and shape of the distribution, respectively. In the end, we adopt the coefficient of determination as a goodness-of-fit measure to compare the regressed value and the observed value (i.e., the ground truth). Rather than comparing the distribution parameters, the goodness of fit can deal with PTNs of different sizes and facilitate the comparison in a relative way. A larger fitness value f signifies a better fit of the skew-normal distribution and therefore a more hierarchical organisation of the PTN element along the tested dimension. The fitness f value lies in the ranges from negative infinity to 1, which is calculated as:

$$f = 1 - \frac{\sum_i^n (y_i - \tilde{y}_i)^2}{\sum_i^n (y_i - \bar{y})^2}, \tag{8}$$

where the set of observed values has n values marked as y_i, each associated with a fitted value $\tilde{y}_i$ and the average value of the observation is $\bar{y}$.

2.4. Topological indicators on network scale

2.4.1. Network modularity

A graph can be partitioned into communities of nodes. These communities exhibit different strengths of connections inside and outside, which are quantified by modularity. Networks with high modularity have denser connections between nodes of the same group, while sparse connections between nodes in different communities. Such a phenomenon can be leveraged to measure the degree of clustering, the inter- and inner-connections among formed communities of nodes of a PTN. This community structure has substantial importance in understanding the dynamics of the PTN. A more hierarchical PTN should comprise a closely connected community, having a faster rate of transporting passengers than a loosely connected community. In particular, a large number of widely distributed high-degree nodes increase the value of optimal network modularity, which tends to manifest a multi-centric structure.

Modularity is the fraction of the links that belong to the same communities minus the expected fraction if links were distributed randomly, and is shown as follows:

$$Q = \frac{1}{2m} \sum_{ij} (a_{ij} - \frac{k_i k_j}{2m}) \delta(c_i, c_j) \quad \forall i, j \in V, \tag{9}$$

where Q is the modularity and k_i and k_j are the degree centrality of nodes i and j. m is the total number of links in the network and hence $2m$ is the number of endpoints of all the links. c_i and c_j represent the community to which nodes i and j belong to, respectively. $\delta(c_i, c_j)$ is the Kronecker delta of the communities of c_i and c_j. Specifically, $\delta(c_i, c_j)$ equals 1 if c_i and c_j are the same community and equals 0 otherwise. The value of modularity ranges from $-\frac{1}{2}$ to 1 and is influenced mainly by the compositions of communities. We apply the Leiden algorithm, a heuristic method to seek the optimal community composition in directed networks [44]. This algorithm starts by assigning each node in the network to a community and then these communities are aggregated or reassigned to new communities until they converge to the maximum modularity. Finally, the optimal modularity of a PTN is used to investigate the ability to form communities and their interactions in our work.

2.4.2. Network assortativity

Different from modularity which assesses the connection pattern of a network via communities, network assortativity indicates the connection pattern that is based on the node characteristic. Assortativity describes a preference for the nodes in a network to attach to others that are similar in some way, mostly through degree centrality. Positive assortativity refers to the tendency of nodes to connect with similar nodes, namely, high-degree nodes connecting to high-degree nodes and low-degree nodes connecting to low-degree nodes. This appears in a network with a dense core of high-degree nodes surrounded by a periphery of lower-degree ones. In contrast, negative assortativity means disassortativity and dissimilarity where the high-degree nodes tend to connect with low-degree nodes similar to a star-like structure. As proposed by [39], degree assortativity corresponds to a Pearson correlation coefficient of the node degree:

$$r = \frac{\sum_{ij} (\frac{a_{ij} - k_i k_j}{2m}) k_i k_j}{\sum_{ij} (\frac{k_i \delta_{ij} - k_i k_j}{2m}) k_i k_j} \quad \forall i, j \in V, \tag{10}$$

where r is the assortativity and ranges from −1 to 1. The Kronecker delta here designates whether the degrees of two connected nodes are the same. Essentially, assortativity has the covariance between nodes i and j as the numerator and the product of their variances as the denominator. A higher assortativity is obtained when the connected nodes are similar in their degree values.

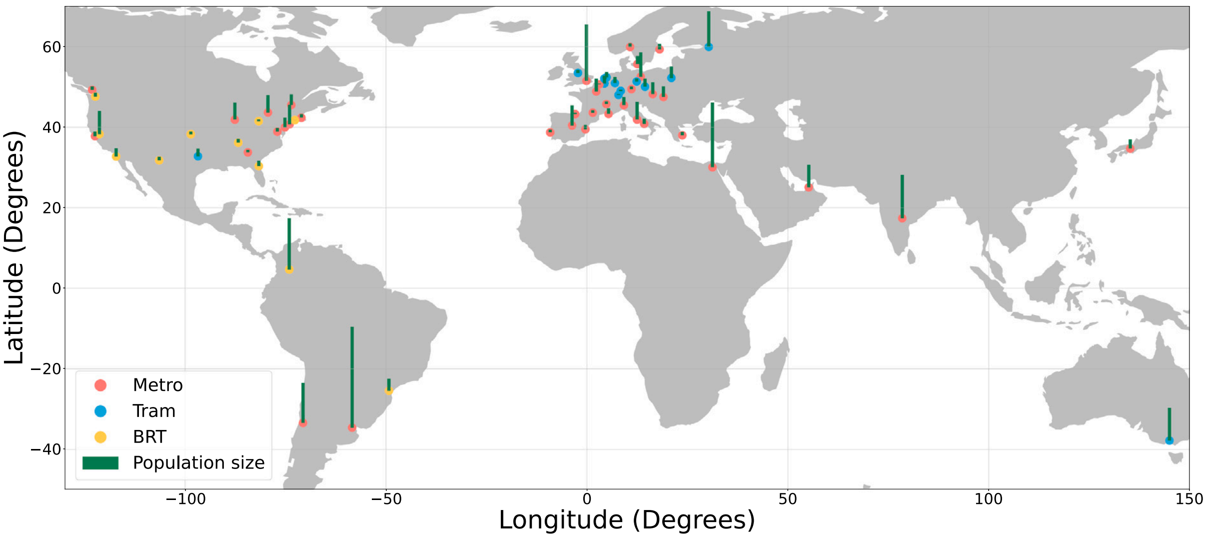

Fig. 2. Map displaying the selected high-capacity PTNs with a variety of modes and population size.

2.5. Multi-dimension PTN hierarchy

In the last step, we encapsulate all the aforementioned dimensions into a radar chart to represent the multi-dimensional property of the PTN hierarchy. This multi-dimension comparison of the PTN hierarchy leads to a comprehensive assessment. Since the selected indicators have different ranges across different PTNs along the same dimension, we use the min–max normalisation to re-scale them between 0 and 1 for avoiding biases in interpretation and facilitating the relative comparison:

$$H' = \frac{H - \min(H)}{\max(H) - \min(H)}, \tag{11}$$

where the original hierarchy value of the chosen dimension is H, the normalised PTN hierarchy value is H', $\max(H)$ and $\min(H)$ represent the maximum and minimum values of the PTN hierarchy in each dimension, respectively. To this end, each axis of the radar chart represents one PTN hierarchy dimension out of the several chosen aspects. The size of the area enclosed by the axes denotes the overall multi-dimension hierarchy of a PTN. When comparing goodness-of-fit related to a specific local indicator or a selected global indicator of two PTNs, we label the PTN with the higher value as more hierarchical than the other one. In other words, we directly map goodness-of-fit and global indicator values into hierarchical relationships across PTNs.

3. Results

The method presented in the previous section is applied to selected case study PTNs in this section. After describing these case studies and the implementation process in Section 3.1, we report the PTN hierarchy radar charts of the case study PTNs and compare them per city (Section 3.2), dimension (Section 3.3), and mode (Section 3.4).

3.1. Case study and implementation description

We select 63 high-capacity PTNs worldwide for applying the proposed method due to data availability and quality. These PTNs comprise 37 metro, 15 tram and 11 BRT systems, covering 6 continents. Except for Berlin, which has both tram and metro systems with approximately the same usage, the unimodal PTN with the highest annual ridership is investigated for each of the remaining 61 urban regions. Fig. 2 displays a map showing the locations of the selected PTNs and a relative indication of the population size. The largest urban region is Buenos Aires with more than 15 million people, while the smallest is Hartford with approximately 120,000 people. The population median of the case study areas is about 1 million.

We use GTFS data mostly from the second half of 2022 or the latest date possible in this study, which are generated during the public transport operations and collected by either OpenMobilityData[1] or service providers. The raw GTFS data usually include the transit information of all modes in the city, such as stop coordinates, service frequency, and arrival and departure times at stops. We filter the data to obtain the relevant information for the unimodal PTN in question using mode ID for metro and tram while turning to route ID or operator ID for BRT. We refer interested readers in the L-space of metro networks to the curated dataset.[2] Furthermore,

[1] https://transitfeeds.com/
[2] https://data.4tu.nl/articles/dataset/A_curated_data_set_of_L-space_representations_for_51_metro_networks_worldwide/21316824

we reduce the data size by employing the most representative date closest to the data release date that contains over 90% of the maximum single-day trips. The operating hours of this date are from 5 AM to midnight, thereby excluding public transport night services.

With the filtered data, we obtain the temporal network through a Python package called GTFSPY specialised in analysing public transport timetable data provided in the GTFS format [45]. Additional processing has been done for unrecognised or duplicated stops and missing link information. The processed resulting networks are topologically analysed using the Python extension module of the IGRAPH library [46].

3.2. City-wise multi-dimensional PTN hierarchy discussion

Fig. 3 summarises the results of the proposed multi-dimensional PTN hierarchy quantification and cross-system comparison in the form of a radar chart per case study PTN. It is worth noting that the first subplot is an empty radar chart as a legend. When comparing the PTN hierarchy of two instances, higher values along the node degree dimension depict a more significant scale-free network structure as the degree distribution fits well with the skew-normal distribution. A higher network assortativity dimension reflects a more pronounced rich-getting-richer effect since a node connection pattern in which high-degree nodes tend to connect can be observed. For example, the metro network in Athens is more hierarchical than the tram network in Amsterdam with respect to both a scale-free network structure and a rich-getting-richer effect as can be seen in the top-left corner of Fig. 3. A larger fitness f value in terms of closeness centrality and betweenness centrality indicates a high heterogeneity of importance in node accessibility and traffic load intermediacy, respectively. For instance, the metro network in Naples is more hierarchical than the BRT network in Nashville regarding stop accessibility and potential traversing and interchanging passenger flow as displayed in the sixth row of Fig. 3. Hence, the metro network in Naples has a clearer ranking of the easily-reached stops and the transfer stops than the BRT network in Nashville. This could possibly affect the passenger flow distribution and the resulting public transport operation performance since both closeness and betweenness are related to paths, which can influence the route choice of passengers. As for eigenvector centrality and network modularity dimensions, a higher value based on these two dimensions implies a powerful centre and a high degree of clustering. The tram network of Saint Petersburg has a smaller f for eigenvector centrality but larger modularity than the BRT network of San Diego which appears to its side in Fig. 3. Consequently, a multi-centric network and a mono-centric network can be expected from Saint Petersburg and San Diego, respectively.

Moreover, we identify two interesting clusters: The metro networks of Athens, Cairo, Oslo, Rome and Vienna and the tram network of Manchester show high f values in closeness centrality, betweenness centrality and eigenvector centrality dimensions. These radar charts display values close to 1 on the left side. On the other hand, they present considerably lower values on the right side for assortativity and modularity. This cluster arguably stems from three factors that include a ring at the centre of the network with a few concentrated high-degree nodes in the central area, limited direct connections between high-degree nodes within the ring, and a few branch lines developed from the mono-centre. These networks are planned to develop one leading urban pole with related suburban areas in the vicinity of this core, yielding the benefits of agglomeration. Another imbalanced shape towards the bottom-right corner of the radar chart with high values in closeness centrality and network modularity dimensions can be observed for the Amsterdam tram network, Hartford BRT network, and London metro networks. These highly inner-connected multi-centric networks have widely distributed high-degree nodes and a hierarchical organisation of stop accessibility such that passengers can access stops with less detouring. The large number of end nodes on the long branch lines incorporate a large number of low-degree and low-intermediacy nodes. Accordingly, these nodes are less hierarchical in terms of their degree centrality, betweenness centrality and assortativity dimensions. The networks belonging to this second cluster focus on building multi-urban poles.

Lastly, PTNs in North America have similar degree centrality and network assortativity dimensions compared with their European counterparts. They generally show a higher f value in the eigenvector centrality dimension but lower values in the other dimensions. We present a radar chart of the median of each hierarchy dimension for these two continents in Fig. 4. This result is likely coupled with the modal composition of these two continents where the case studies include 10 metros, 9 BRTs and 1 tram from North America versus 21 metros and 13 trams from Europe. The preferred rail-bound services in the European PTNs lead to a more hierarchical organisation of stop cluster, stop accessibility and traffic intermediacy. Therefore, the European PTNs exhibit a clearer structure and their nodes perform distinctive functions, such as centres, hubs and bridges. They generally show an urban development towards a multi-centric network design instead of a strong and dense centre.

3.3. Dimension-wise PTN hierarchy comparison

In this subsection, the computational results are presented and discussed per dimension. In Fig. 5, each sub-figure displays the distribution of one of the several chosen PTN hierarchy dimension indicators amongst the 63 PTNs included in our analysis. The distributions of f based on node degree centrality and assortativity coefficient exhibit a right-skewed distribution. The mean value and median values of f based on node degree centrality are 0.386 and 0.373 respectively. Most of the f values are between 0.25 to 0.45 and therefore more PTNs tend to have a relatively low f value along the degree centrality dimension. The assortativity coefficient ranges from −0.184 to 0.551 with median and mean values of 0.041 and 0.066, respectively. One-third of the selected PTNs have a negative assortativity coefficient and three-quarters of the chosen PTNs have an assortativity coefficient between −0.1 to 0.2. In contrast to findings reported in the literature [34], a large number of low-degree nodes and rare high-degree nodes in the PTNs lead to low heterogeneity in the node connectivity and connection pattern. Therefore, a scale-free network that relies on the degree distribution following a power law or skew-normal distribution and a rich-club effect where nodes with high degrees tend

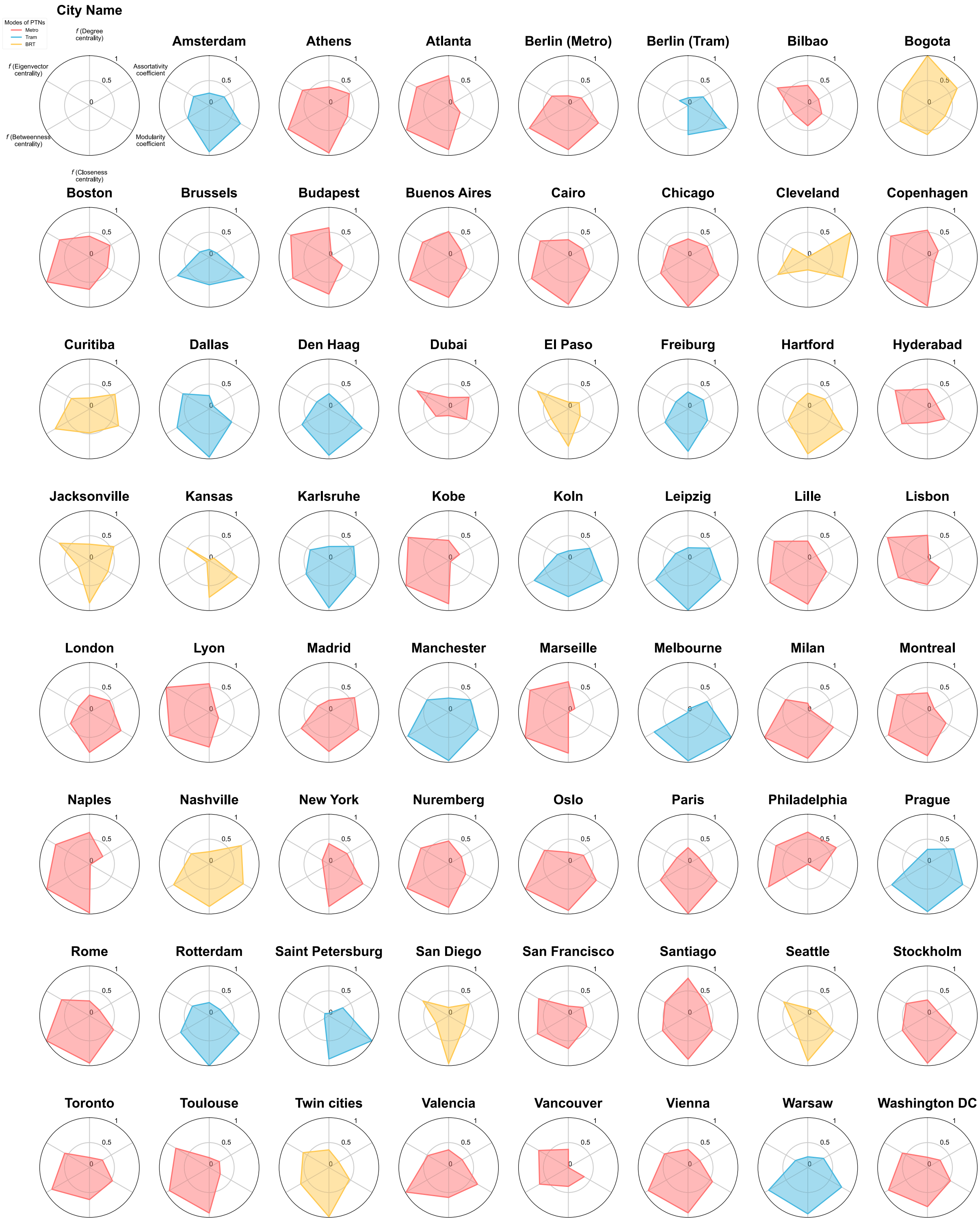

Fig. 3. Radar charts of considered PTNs with all chosen hierarchy quantification dimensions.

to connect to each other are not found to be prevalent among PTNs. The traditional way of assessing network hierarchy may not be applicable in PTNs since the vast majority of nodes in most PTNs are planned to provide nearly the same number of connections instead of a few important nodes having a huge number of connections.

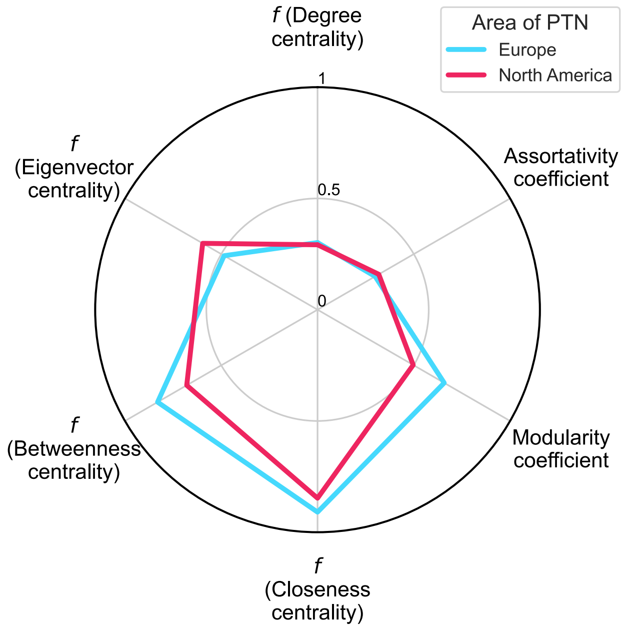

Fig. 4. Radar chart of the median from each PTN hierarchy dimension in Europe and North America.

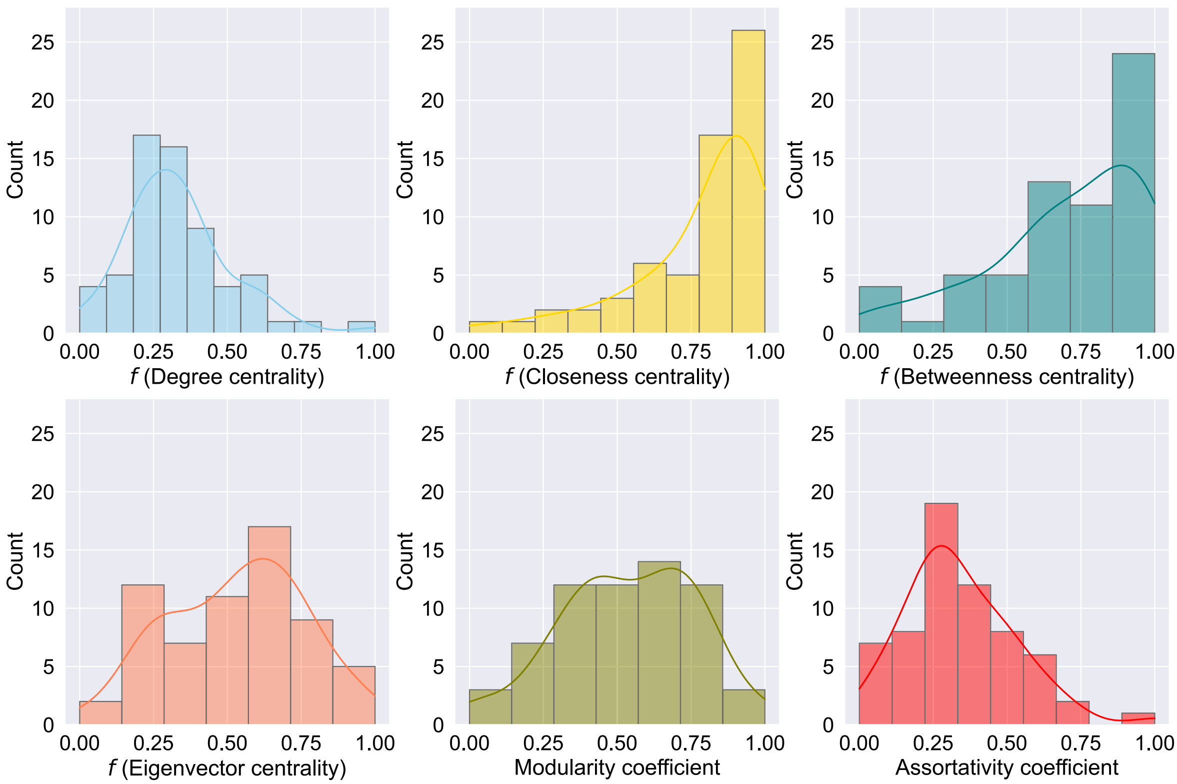

Fig. 5. Distribution of the chosen PTN hierarchy dimension indicators for 63 case studies.

Conversely, the distributions of f based on closeness centrality and betweenness centrality are left-skewed. Both indicators are computed using the shortest paths between node pairs. A wide distribution ranging from −1.94 to 0.948 can be observed for the closeness centrality dimension with a median of 0.564 and a mean of 0.32. Nearly 20% of the PTNs have a negative value as

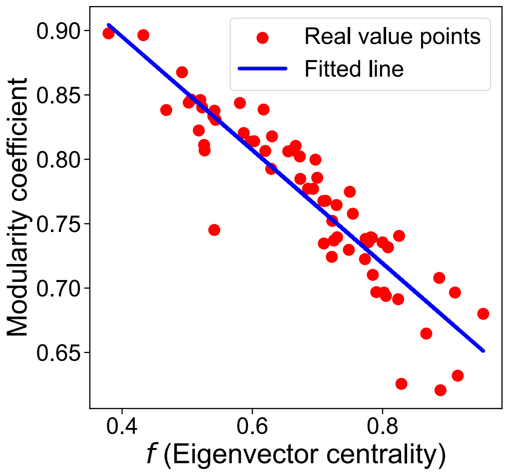

Fig. 6. Scatter plot of f based on eigenvector centrality distribution and modularity coefficient with the fitted line.

small-size PTNs tend to have few transfer nodes and a large proportion of nodes with similar accessibility on branch lines without transfer stops. This results in a small number of categorised bins and a poor fitting with the skew-normal distribution. Nonetheless, more than half of the studied PTNs have an f above 0.5 based on closeness centrality, which implies a hierarchical organisation of stop accessibility is common in PTNs. In terms of betweenness centrality, the distribution of f spans between 0.57 and 0.99 with a median of 0.886 and a mean of 0.866. The difference in the f of betweenness centrality also owes to the number of branch lines without transfer stops and the number of end nodes with a low betweenness centrality, leading to a relatively worse fitting with the skew-normal distribution. Almost 80% of the PTNs have an f higher than 0.8, indicating an obvious hierarchy concerning the traffic load intermediacy of PTNs. Thus, most of the PTNs present a hierarchical structure with respect to closeness centrality and, in particular, betweenness centrality. Hubs in PTNs appear to have significant accessibility or traffic intermediacy, and both indicators could influence passenger flow distribution [17].

We further observe an approximately normal distribution for the f based on eigenvector centrality and network modularity. The f of eigenvector centrality distribution takes a median of 0.7 and a mean of 0.685, ranging from 0.379 to 0.955. The majority of the PTNs have a value ranging between 0.5 and 0.9, revealing a hierarchical organisation of the node clusters and therefore a mono-centric network structure. Intriguingly, we find that large-size PTNs with abundant and dispersed high-degree nodes have a large range of eigenvector centrality, however, the f based on the eigenvector centrality distribution is not significant and therefore does not follow a hierarchical organisation. On the other hand, PTNs with a few but concentrated high-degree nodes form a higher level of importance while the rest are gradually less important. This results in a clearer hierarchy. Regarding modularity, it varies between 0.621 and 0.898 with a median of 0.775 and a mean of 0.77. About 80% of the PTNs have optimal modularity between 0.7 and 0.85, signifying a high-clustering network structure. Given the modularity definition, the number and variance of high-degree nodes in a PTN influence the inner-similarity and inter-difference and determine the clusters. Opposite to the findings on eigenvector centrality, higher modularity is realised when a large number of high-degree nodes (e.g., transfer hubs) is widely distributed and presents a multi-centric network structure.

The non-trivial contrasting relationship between the f for eigenvector centrality and modularity is further investigated using a scatter plot in Fig. 6. This figure demonstrates a negative relationship between these two indicators with an f of 0.832. The eigenvector centrality dimension reflects a mono-centric network structure with a small number and concentrated high-degree nodes while the modularity dimension denotes a multi-centric network structure with a large number and widely distributed high-degree nodes, respectively. Hence, public transport planners can utilise this information to identify centres for regional planning and make a trade-off between enhancing the influence of a powerful centre and adding strategic nodes along the lines between centres to develop and connect sub-centres.

3.4. Mode-wise PTN hierarchy comparison

This subsection is dedicated to analysing the modal effect on the multi-dimensional PTN hierarchy. Since the dataset size of each mode is uneven, we list the median values of each chosen indicator in Table 1. It can be seen from the table that no transport mode has the highest value in all hierarchy dimensions and the differences between modes are relatively small. Nevertheless, we find that the metro networks have the highest hierarchy in the dimensions of degree centrality, betweenness centrality and eigenvector centrality while the tram networks are more hierarchically organised in the closeness centrality and network modularity dimensions. Conversely, the BRT networks show a higher hierarchy along the network assortativity dimension than the other two modes. Overall, the metro demonstrates a higher PTN hierarchy than the tram and the BRT PTNs.

Table 1
Table of medians of PTN hierarchy in each dimension for the three public transport modes.

	f (Degree centrality)	f (Closeness centrality)	f (Betweenness centrality)	f (Eigenvector centrality)	Modularity coefficient	Assortativity coefficient
BRT	0.221	0.846	0.452	0.557	0.595	**0.477**
Metro	**0.373**	0.862	**0.901**	**0.652**	0.427	0.286
Tram	0.259	**0.927**	0.733	0.282	**0.768**	0.355

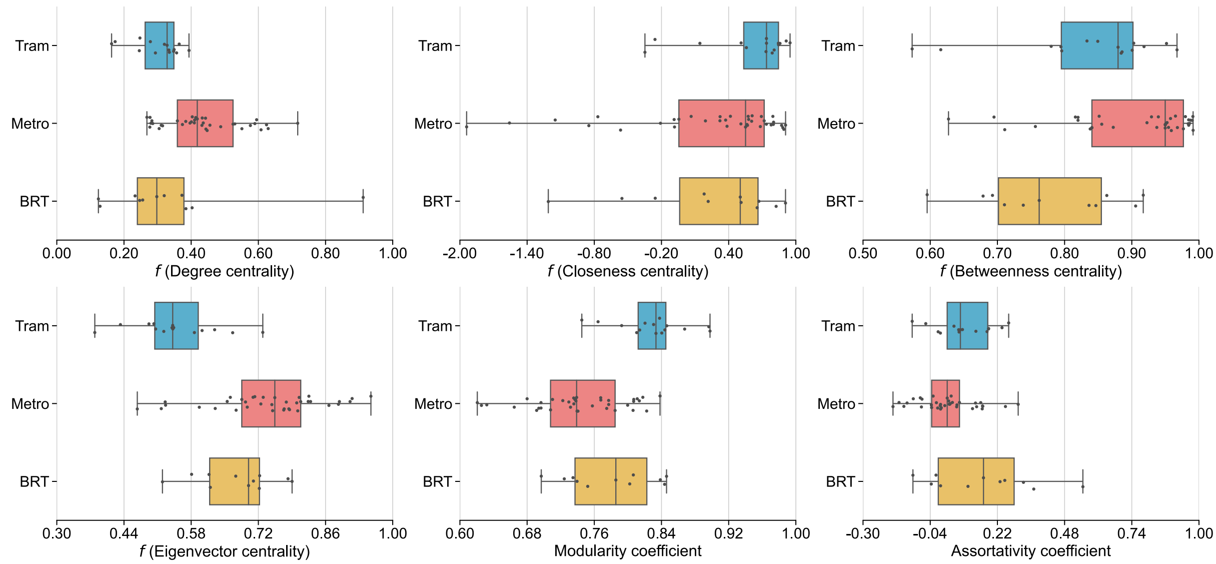

Fig. 7. Box plots of mode-wise PTN hierarchy comparison across the chosen dimensions.

Fig. 7 presents the detailed mode-wise PTN hierarchy comparison per chosen dimension. Starting from the degree centrality, the metro networks generally have the highest hierarchy among all three modes in line with the median comparison. The distribution of the f for degree centrality of tram networks is concentrated between 0.2 and 0.4 while more spreading is seen for BRT networks. Even though one BRT network has the highest f based on degree centrality distribution (i.e., Bogota), most of the BRT networks have relatively low values. This originates from the large number of low-degree nodes located on the branch lines without intersecting with other nodes and the lack of high-degree nodes. In contrast, the low-degree nodes of the metro systems on these branch lines are due to a larger stop spacing. Furthermore, a high-degree node in a network is often a transfer hub for multiple lines, leading to a high degree centrality at these stops. On the contrary, for tram and BRT networks, the number of lines crossing at the same stop is limited.

In terms of closeness centrality, the highest f value among all modes is similar while the distribution range of the metro networks is the largest among the three public transport modes, nearly two times larger than that of tram networks. Given the differences in stop and line spacings, tram networks are more likely to form connections between lines such that only a few tram networks have poor accessibility. Conversely, the metro and BRT networks have connections only around the central areas, resulting in a division of high- and low-accessibility without a gradual hierarchical structure.

Metro networks are characterised by a high traffic intermediacy with a median that ranges from 0.9 to 1, higher than for the other modes. As the stop and line spacings of the metro are generally wider, the density of nodes is accordingly smaller. This contributes to a higher intermediacy around the central areas, which gradually decreases in the suburbs. Different from rail-bound networks, BRT lines are relatively more independent from each other and have fewer transfer stops between lines, resulting in a large number of low-intermediacy nodes that are not traversed by traffic. Therefore, BRTs show a relatively low f value in the betweenness centrality dimension.

Over a quarter of the metro networks included in our analysis have a higher hierarchy than all tram and BRT networks with respect to eigenvector centrality. The reason is that the number of high-degree nodes is relatively small mostly around the centre used for forming a mono-centric structure. Besides, the operating speed also influences the PTN hierarchy based on eigenvector centrality. A higher operating speed reduces the average travel times on links and increases the importance of the node clusters. Conversely, the small line and stop spacings increase the number of relatively high-degree nodes widely spread in a tram network, which shows high-clustering and multi-centric structures reflected by the highest modularity coefficient among all modes.

As for the network assortativity dimension, rail-bound systems exhibit the lowest hierarchy while the BRT system shows the highest hierarchy with the widest range from negative values to over 0.5. A high-degree transfer hub usually covers a wider area and is thus distant from other high-degree nodes, which reduces the rich-club phenomenon. On the other hand, the flexible line planning of BRT networks is more inclined to offer high-degree connection patterns since the stops are flexible during different periods of the day, which is quite often seen in our dataset.

The results pertaining to the difference in PTN hierarchy dimensions can be concluded as follows: First, the stop spacing differs among these modes, leading to various shortest path values. In practice, the average stop spacing of metro networks is larger than the other two modes and the tram networks have the shortest stop spacing. Second, the typical operating speed of the metro is usually higher than the tram and BRT with the latter two having a similar speed. Therefore, the travel time on the links varies among these modes. Third, the stop service coverage of the metro system is commonly larger than the other two. Correspondingly, it allows the metro transfer stop to have more line intersections and thus form a hub. Lastly, it is more likely that the BRT systems have flexible line planning than the metro and tram systems. Using the same infrastructure, BRT systems tend to have a range of lines with different stop spacings while instead, trams often serve at every stop, providing a more local accessibility.

As different public transport modes show distinctive characteristics in their network hierarchy, PTN planners can leverage these patterns to decide the complementary mode or intermodal hub in an evolving multi-modal PTN. For instance, a tram with a multi-centric network and a hierarchical organisation of stop accessibility can be supplementary to a metro network that presents a more centralised network and a clear hierarchy of potential passenger flow. At the same time, planners can also choose to build an intermodal hub based on the node with high closeness centrality or high betweenness centrality, which has a high hierarchy in a network.

4. Conclusion

We propose a multi-dimensional topology-based PTN hierarchy quantification method that enables cross-system comparison. The method is independent of passenger demand data which are often unavailable and unravels the network hierarchy as embedded in the planning. It considers both the scale-free network topology with clustering of nodes and the hierarchical organisation of nodes with multiple importance-related dimensions. These dimensions correspond to connectivity, accessibility, intermediacy, mono- or multi-centric network structure and node connection pattern as measured by degree centrality, closeness centrality, betweenness centrality, eigenvector centrality, modularity coefficient and assortativity coefficient, respectively. The goodness-of-fit value between the probability density function of the local indicators and the regressed skew-normal distribution is used for quantifying the network-wide hierarchy. With the infrastructure graph representation of PTNs that explicitly incorporates travel times or the connection efficiency according to the planned services contained in GTFS data, we are able to compare the hierarchy of 63 high-capacity highly-used PTNs with a variety of modes worldwide.

Our main findings from comparing 63 PTNs worldwide are summarised as follows: The scale-free network structure and the preferential attachment are relatively insignificant for most PTNs in the study because of the large number of low-degree nodes, the lack of high-degree nodes and the limited connection between high-degree nodes. Therefore, the assumption of network hierarchy based on the skew-normal distribution of degree centrality may not be applicable in PTNs. In contrast, the goodness-of-fit value based on closeness centrality and betweenness centrality reveals a relatively high hierarchy along these two dimensions. PTNs tend to present a hierarchical organisation of node closeness centrality and intermediacy. Hubs of a PTN can also be determined from nodes with high closeness or betweenness centrality since they are positioned at a higher level of importance with respect to stop accessibility and traffic intermediacy. In addition, a negative relationship is found between the goodness-of-fit value based on eigenvector centrality and optimal modularity. Planners can utilise this information to plan the growth of a network towards a mono-centric network with a powerful centre or a multi-centric network with strongly connected node clusters. Among the chosen three high-capacity public transport modes, metro networks exhibit an overall more hierarchical structure across the chosen dimensions of hierarchy, however, no single mode has the highest PTN hierarchy in every single dimension. Such differences in the PTN hierarchy among modes pertain to the number of nodes, the connection between nodes and the travel time on links. These factors are influenced by stop spacing, line spacing, stop infrastructure, operating speed and line planning of different modes. Nevertheless, different modes can be complementary to each other in a multimodal PTN development due to their unique hierarchy characteristics.

The network hierarchy assessment proposed in this study can provide a quantitative point of reference for network planners in identifying pros and cons across different PTNs and a variety of modes. It supports public transport authorities and operators in defining where to invest in the network from a specific or multi-dimensional aspect of importance. The resulting method offers efficient scanning of the PTN hierarchy without the need for ex-post empirical passenger flow data or computationally expensive transport modelling tools.

This study opens avenues for further research along the following directions. First, disruption data from PTNs can be incorporated to correlate the vulnerability and the hierarchy to extend the practical implication of this study. Second, the proposed method can be applied to multi-modal PTN systems by choosing suitable modifications and adopting multi-layer network analysis principles. Finally, adding passenger demand data to the analysis can quantify the PTN hierarchy in system usage and correlate with the proposed method that quantifies the PTN hierarchy embedded in the planning.

CRediT authorship contribution statement

Ziyulong Wang: Conceptualization, Formal analysis, Investigation, Methodology, Visualization, Writing – original draft, Writing – review & editing. **Ketong Huang:** Conceptualization, Data curation, Investigation, Methodology, Visualization, Writing – review & editing. **Renzo Massobrio:** Conceptualization, Methodology, Software, Supervision, Writing – review & editing. **Alessandro Bombelli:** Methodology, Supervision, Writing – review & editing. **Oded Cats:** Conceptualization, Methodology, Project administration, Supervision, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

Acknowledgments

The work of R. Massobrio was funded by call UCA/R155REC/2021 (European Union - NextGenerationEU), project TED2021-131880B-I00 funded by MCIN/AEI/10.13039/501100011033 and the European Union ‘‘NextGenerationEU’’/PRTR, and Project eMob (PID2022-137858OB-I00) funded by Spanish MCIN /AEI /10.13039/501100011033 / FEDER, UE.

References

[1] Claus C. Hilgetag, Alexandros Goulas, ‘Hierarchy’ in the organization of brain networks, Philos. Trans. R. Soc. B 375 (1796) (2020) 20190319.
[2] Stephen D. Reese, Pamela J. Shoemaker, A media sociology for the networked public sphere: The hierarchy of influences model, Mass Commun. Soc. 19 (4) (2016) 389–410.
[3] Vicente Aprigliano Fernandes, Ricardo Rodrigues Pacheco, Elton Fernandes, William Ribeiro da Silva, Regional change in the hierarchy of Brazilian airports 2007–2016, J. Transp. Geogr. 79 (2019) 102467.
[4] Rob van Nes, Multilevel network optimization for public transport networks, Transp. Res. Rec. 1799 (1) (2002) 50–57.
[5] Rob van Nes, Design of Multimodal Transport Networks: A Hierarchical Approach (Ph.D. thesis), Delft University of Technology, 2002.
[6] Bhanu M. Yerra, David M. Levinson, The emergence of hierarchy in transportation networks, Ann. Reg. Sci. 39 (2005) 541–553.
[7] Riccardo Gallotti, Marc Barthelemy, The multilayer temporal network of public transport in Great Britain, Sci. Data 2 (2015) 140056.
[8] Alberto Aleta, Sandro Meloni, Yamir Moreno, A multilayer perspective for the analysis of urban transportation systems, Sci. Rep. 7 (2017) 44359.
[9] Menno D. Yap, Niels van Oort, Rob van Nes, Bart van Arem, Identification and quantification of link vulnerability in multi-level public transport networks: a passenger perspective, Transportation 45 (2018) 1161–1180.
[10] Jian Gao, Peng Zhao, Chengxiang Zhuge, Hui Zhang, Research on public transit network hierarchy based on residential transit trip distance, Discrete Dyn. Nat. Soc. 2012 (2012) 390128.
[11] Albert-László Barabási, Zoltán Dezső, Erzsébet Ravasz, Soon-Hyung Yook, Zoltán Oltvai, Scale-free and hierarchical structures in complex networks, in: AIP Conference Proceedings, Vol. 661, American Institute of Physics, 2003, pp. 1–16.
[12] Enys Mones, Lilla Vicsek, Tamás Vicsek, Hierarchy measure for complex networks, PLoS One 7 (3) (2012) e33799.
[13] Anna D. Broido, Aaron Clauset, Scale-free networks are rare, Nature Commun. 10 (2019) 1017.
[14] Aaron Clauset, Cristopher Moore, M.E.J. Newman, Hierarchical structure and the prediction of missing links in networks, Nature 453 (2008) 98–101.
[15] Christian von Ferber, Taras Holovatch, Yurij Holovatch, Vasyl Palchykov, Public transport networks: empirical analysis and modeling, Eur. Phys. J. B 68 (2009) 261–275.
[16] Ding Luo, Oded Cats, Hans van Lint, Graham Currie, Integrating network science and public transport accessibility analysis for comparative assessment, J. Transp. Geogr. 80 (2019) 102505.
[17] Ding Luo, Oded Cats, Hans van Lint, Can passenger flow distribution be estimated solely based on network properties in public transport systems? Transportation 47 (2020) 2757–2776.
[18] Ho-Yin Chan, Anthony Chen, Guoyuan Li, Xiangdong Xu, William Lam, Evaluating the value of new metro lines using route diversity measures: The case of Hong Kong’s Mass Transit Railway system, J. Transp. Geogr. 91 (2021) 102945.
[19] Sheng Wei, Shuqing N. Teng, Hui-Jia Li, Jiangang Xu, Haitao Ma, Xia-li Luan, Xuejiao Yang, Da Shen, Maosong Liu, Zheng Y.X. Huang, Chi Xu, Hierarchical structure in the world’s largest high-speed rail network, PLoS One 14 (2) (2019) e0211052.
[20] Christian von Ferber, Taras Holovatch, Yurij Holovatch, Vasyl Palchykov, Network harness: Metropolis public transport, Physica A 380 (2007) 585–591.
[21] Jingyi Lin, Yifang Ban, Complex network topology of transportation systems, Transp. Rev. 33 (6) (2013) 658–685.
[22] Sybil Derrible, Network centrality of metro systems, PLoS One 7 (7) (2012) e40575.
[23] Jianhua Zhang, Mingwei Zhao, Haikuan Liu, Xiaoming Xu, Networked characteristics of the urban rail transit networks, Physica A 392 (6) (2013) 1538–1546.
[24] Oded Cats, Panchamy Krishnakumari, Metropolitan rail network robustness, Physica A 549 (2020) 124317.
[25] Hana Takhtfiroozeh, Mihalis Golias, Sabyasachee Mishra, Topological-based measures with flow attributes to identify critical links in a transportation network, Transp. Res. Rec. 2675 (10) (2021) 863–875.
[26] Xian Bin Wee, Manuel Herrera, Georgios M. Hadjidemetriou, Ajith Kumar Parlikad, Simulation and criticality assessment of urban rail and interdependent infrastructure networks, Transp. Res. Rec. 2677 (1) (2023) 1181–1196.
[27] Edward A. Beimborn, Michael J. Greenwald, Xia Jin, Accessibility, connectivity, and captivity: Impacts on transit choice, Transp. Res. Rec. 1835 (1) (2003) 1–9.
[28] Matthew Bediako Okrah, Evaluation of methods for estimation of intrazonal travel impedances, Transp. Res. Rec. 2563 (1) (2016) 114–121.
[29] M.E.J. Newman, Detecting community structure in networks, Eur. Phys. J. B 38 (2004) 321–330.
[30] Hiroe Ando, Michael Bell, Fumitaka Kurauchi, K.I. Wong, Kam-Fung Cheung, Connectivity evaluation of large road network by capacity-weighted eigenvector centrality analysis, Transp. A Transp. Sci. 17 (4) (2021) 648–674.
[31] Ziyulong Wang, Ding Luo, Oded Cats, Trivik Verma, Unraveling the hierarchy of public transport networks, in: 2020 IEEE 23rd International Conference on Intelligent Transportation Systems, ITSC, IEEE, 2020, pp. 1–6.
[32] László Hajdu, András Bóta, Miklós Krész, Alireza Khani, Lauren M. Gardner, Discovering the hidden community structure of public transportation networks, Netw. Spat. Econ. 20 (2020) 209–231.
[33] Harold Soh, Sonja Lim, Tianyou Zhang, Xiuju Fu, Gary Kee Khoon Lee, Terence Gih Guang Hung, Pan Di, Silvester Prakasam, Limsoon Wong, Weighted complex network analysis of travel routes on the Singapore public transportation system, Physica A 389 (24) (2010) 5852–5863.
[34] Mary Luz Mouronte-López, Analysing the vulnerability of public transport networks, J. Adv. Transp. 2021 (2021) 5513311.

[35] Aleix Bassolas, Hugo Barbosa-Filho, Brian Dickinson, Xerxes Dotiwalla, Paul Eastham, Riccardo Gallotti, Gourab Ghoshal, Bryant Gipson, Surendra A. Hazarie, Henry Kautz, Onur Kucuktunc, Allison Lieber, Adam Sadilek, José J. Ramasco, Hierarchical organization of urban mobility and its connection with city livability, Nature Commun. 10 (2019) 4817.

[36] Menno D. Yap, Ding Luo, Oded Cats, Niels van Oort, Serge Hoogendoorn, Where shall we sync? Clustering passenger flows to identify urban public transport hubs and their key synchronization priorities, Transp. Res. C 98 (2019) 433–448.

[37] Riccardo Gallotti, Giulia Bertagnolli, Manlio De Domenico, Unraveling the hidden organisation of urban systems and their mobility flows, EPJ Data Sci. 10 (1) (2021) 3.

[38] Abel Buijtenweg, Trivik Verma, Oded Cats, Barth Donners, Huijuan Wang, Quantifying the hierarchy of public transport networks, in: 2021 7th International Conference on Models and Technologies for Intelligent Transportation Systems, MT-ITS, IEEE, 2021, pp. 1–6.

[39] M.E.J. Newman, Networks: An Introduction, Oxford University Press, Oxford, 2010.

[40] Alex Bavelas, Communication patterns in task-oriented groups, J. Acoust. Soc. Am. 22 (6) (1950) 725–730.

[41] Linton C. Freeman, A set of measures of centrality based on betweenness, Sociometry (1977) 35–41.

[42] David W. Scott, On optimal and data-based histograms, Biometrika 66 (3) (1979) 605–610.

[43] Adelchi Azzalini, Alessandra Dalla Valle, The multivariate skew-normal distribution, Biometrika 83 (4) (1996) 715–726.

[44] Vincent A. Traag, Ludo Waltman, Nees Jan van Eck, From Louvain to Leiden: guaranteeing well-connected communities, Sci. Rep. 9 (1) (2019) 1–12.

[45] Rainer Kujala, Christoffer Weckström, Miloš N. Mladenović, Jari Saramäki, Travel times and transfers in public transport: Comprehensive accessibility analysis based on Pareto-optimal journeys, Comput. Environ. Urban Syst. 67 (2018) 41–54.

[46] Gabor Csardi, Tamas Nepusz, The igraph software package for complex network research, InterJournal Complex Syst. (2006) 1695.