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A Lagrangian traffic flow model considering lane changing behavior: formulation and numerical implementation

Liang Lu, Fangfang Zheng, Xiaobo Liu* and Yufei Yuan

Abstract—**This paper proposes a multilane traffic flow model based on the notions of conservation laws in Lagrangian coordinates. Both the mathematical formulation and the graphical representation are provided. A logit choice model is applied to describe drivers' lane choice probability. The lane changing rate is estimated by employing the Incremental Transfer (IT) principle and the lane choice probability model. The numerical implementation of the model in the case of two lanes is discussed. The simulation results reveal that the proposed Lagrangian model is able to describe lane changing dynamics of vehicle platoons; while the lane changing equilibrium curve at the macroscopic level is consistent with that from the multilane Eulerian model as well as the observed data on the highway.**

I. INTRODUCTION

The kinematic wave model, also known as the Lighthill-Whitham-Richards (LWR) model [1-3], is used to describe traffic flow dynamics and has been widely applied for traffic estimation and control. Originally this model only considers single pipe (or single lane) traffic and no lane changing is included. For road sections with multiple lanes, lane changing behavior is often observed and has shown significant impact on traffic flow [4] and safety [5]. Thus, traffic flow models which are capable of addressing lane-changing characteristics are necessary for accurate traffic estimation, management and control applications in road networks.

In literature, lane changing characteristics are basically described from two perspectives: microscopic and macroscopic. The microscopic modelling of lane changing behavior focuses on investigating lane changing motive, time, location and maneuver [6-8]. At the macroscopic level, various studies have been carried out to investigate lane changing characteristics including lane changing flow rate [9-11], capacity drop [12], traffic instability (due to lane drops, merges) [13, 14] and oscillation (moving bottlenecks) [15]. Several multilane traffic flow models are proposed by extending from the kinematic wave theory with lane changing behavior [9, 10, 16]. Laval et al. [17] proposed a hybrid lane changing model by considering each lane as a separate kinematic wave stream which is interrupted by lane-changing vehicles computed as particles and were considered as moving bottlenecks. The lane choice probability is calculated as proportional to the speed difference among lanes which could not appropriately represent the lane flow equilibrium curve [18]. To overcome this issue, Shiomi et al. [18] propose a macroscopic lane changing model by applying a logit lane choice model where lane changing is treated as utility defined by the traffic state.

The multilane traffic flow models as discussed above are formulated in traditional Eulerian coordinates which are fixed in space. Recently Leclercq et al. [19] propose an alternative formulation of the LWR model based on Lagrangian coordinates which move with the vehicles. Simulations based on the Lagrangian formulation have shown to be more accurate (less numerical diffusion) and computation efficient [20]. Moreover, due to the 'reduced nonlinearity' of Lagrangian models, they have received wide attention and have been applied in traffic state estimation and control, in particular with data assimilation techniques using vehicle trajectory data [21-23]. However, all these Lagrangian models assume single pipe (lane) of traffic and no lane changing characteristics are considered. Though van Wageningen-Kessels et al. [24] discussed the formulation of multiclass traffic flow in Lagrangian coordinates and later they proposed the formulation of discontinuities (merges, diverges) in Lagrangian coordinates [25], lane changing characteristics are not explicitly analyzed in their models.

To fill this gap, a multilane traffic flow model formulated in Lagrangian coordinates is proposed. The model explicitly considers lane changing dynamics by assuming that each lane follows a separated fundamental diagram. The increment-transfer (IT) principle [26] which predicts flow transfers between origin and target is applied to calculate the feasible solution of the conservation equation in Lagrangian coordinates.

The rest of this paper is organized as follows. Section 2 provides the detailed formulation of the multilane traffic flow model in Lagrangian coordinates. In section 3, lane changing dynamics in Lagrangian coordinates are modeled and numerical implementation is provided. Section 4 provides numerical examples and simulation results. Section 5 concludes the paper and some discussions are provided.

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II. LAGRANGIAN FORMULATION OF MULTILANE LWR MODEL

A. Mathematical Derivation

The multilane Lighthill-Whitham-Richards (LWR) model in the traditional Eulerian coordinates has been developed by Laval and Daganzo [15]. We recall the Eulerian formulation of the conservation law for multilane traffic.

$$\frac{\partial k_l}{\partial t}+\frac{\partial q_l}{\partial x}=g_l(t,x), l=1,2,\cdots,n \tag{1}$$

$$g_l(t,x)=\sum_{l'\neq l} g_{l'\to l}(t,x)-\sum_{l'\neq l} g_{l\to l'}(t,x) \tag{2}$$

Where $g_l(t,x)$ is the net lane changing rate on lane l at the time-space point (t,x), in units of veh/time-distance. $g_{l'\to l}(t,x)$ is the rate at which the vehicle changes lane from l' to l at the point (t,x). The equilibrium relation between the flow q and the density k is given by a fundamental diagram (FD) $q=Q(k)=kV(k)$.

The multilane LWR model in Eulerian coordinates *(t, x)* can be transformed in Lagrangian coordinates *(t, n)*, where the vehicle number *n* is fixed while the position *x* changes over time. Since we have $k=\frac{1}{s}$ and *s* is the spacing between two adjacent vehicles, we can derive $q=kv=\frac{v}{s}$. Thus, Eq. (1) can be rewritten as:

$$\begin{aligned}&\frac{\partial}{\partial t}\left(\frac{1}{s_l}\right)+\frac{\partial}{\partial x}\left(\frac{v_l}{s_l}\right)=g_l(t,x(n))\\ &\Rightarrow -\frac{1}{s_l^2}\frac{\partial s_l}{\partial t}+\frac{1}{s_l}\frac{\partial v_l}{\partial x_l}-\frac{v_l}{s_l^2}\frac{\partial s_l}{\partial x}=g_l(t,x(n))\end{aligned} \tag{3}$$

Where s_l, v_l represent the mean spacing and the mean speed on lane *l*, respectively.

By multiplying $-s_l^2$ on both sides of Eq. (3) and employing the definition $s_l=-\partial x/\partial n$, we can derive

$$\frac{\partial s_l}{\partial t}+\frac{\partial v_l}{\partial n}+v_l\frac{\partial s_l}{\partial x}=-s_l^2 g_l(t,x) \tag{4}$$

Subsequently substituting the Lagrangian time derivative $\frac{D}{Dt}=\frac{\partial}{\partial t}+v_l\frac{\partial}{\partial x}$ into Eq. (4), we can obtain:

$$\frac{Ds_l}{Dt}+\frac{\partial v_l}{\partial n}=-s_l^2 g_l(t,x) \tag{5}$$

Eq. (5) is the conservation law of multilane traffic in Lagrangian coordinates for each lane, with the fundamental diagram describing the relation between vehicle speed and spacing:

$$v=V(s) \tag{6}$$

B. Discretization of the Multilane Lagrangian Model

For the purpose of computer implementation of the proposed model, it is necessary to discretize the model. Thus, vehicles are divided into groups of Δn and time is divided into a step size of Δt. The continuous Lagrangian multilane model of Eq. (5) can be discretized as:

$$\frac{s_l^i(t+1)-s_l^i(t)}{\Delta t}+\frac{v_l^i(t)-v_l^{i-1}(t)}{\Delta n}=-s_l^i(t+1)s_l^i(t)g_l^i(t) \tag{7}$$

By rearrangement of Eq. (7), we can obtain the discretized spacing dynamics:

$$s_l^i(t+1)=\frac{s_l^i(t)+\frac{\Delta t}{\Delta n}\left(v_l^{i-1}(t)-v_l^i(t)\right)}{1+s_l^i(t)g_l^i(t)\Delta t} \tag{8}$$

Where i is the number of the vehicle platoon with $i=1,2,...,N(t)$ and $N(t)$ is the maximum number of the vehicle platoon at time step *t*; $s_l^i(t)$ is the average spacing of platoon i on lane l at time step *t*; $v_l^i(t)$ is the speed of platoon i on lane l at time step t.

The corresponding number of vehicles in the platoon may change due to vehicles' lane changing and thus not equal to a fixed platoon size of Δn. The redivision of vehicles in the platoon as well as the update of the spacing for each platoon after redivision is given by

$$\begin{cases} s_l^i(t+1)=\dfrac{\sum_{z\in Z} s_l^z(t+1)\Delta N_l^z(t+1)}{\Delta n}, & i=1,2\cdots,N(t+1)\text{-}1\\ \sum_{z\in Z}\Delta N_l^z(t+1)=\Delta n \end{cases} \tag{9}$$

$$\begin{cases} s_l^i(t+1)=\dfrac{\sum_{z\in Z} s_l^z(t+1)\Delta N_l^z(t+1)}{\Delta n_{last}(t+1)}, & i=N(t+1)\\ \sum_{z\in Z}\Delta N_l^z(t+1)=\Delta n_{last}(t+1) \end{cases} \tag{10}$$

Where Z is the set of platoons before redivision, which contribute to the calculation of spacing after redivision for platoon *i*. ΔN_l^z is the number of vehicles from platoon *z* before redivision. *N(t+1)* is the maximum number of the vehicle platoon after redivision given by:

$$N(t+1)=\left\lfloor\frac{g_l^{N(t)}(t)s_l^{N(t)}(t)\Delta n_{last}(t)\Delta t+\sum_{i=1}^{N(t)-1} g_l^i(t)s_l^i(t)\Delta n\Delta t}{\Delta n}\right\rfloor+N(t) \tag{11}$$

Where $g_l^{N(t)}(t)s_l^{N(t)}(t)\Delta n_{last}(t)\Delta t+\sum_{i=1}^{N(t)-1} g_l^i(t)s_l^i(t)\Delta n\Delta t$ is the total number of vehicles making lane changes on lane *l* at time step *t*.

$\Delta n_{last}(t+1)$ is the size of platoon $N(t+1)$ (the last platoon) at time step *t+1* after redivision given by:

$$Tot_l^{veh}(t+1)=(N(t)\text{-}1)\Delta n+\Delta n_{last}(t)+ g_l^{N(t)}(t)s_l^{N(t)}(t)\Delta n_{last}(t)\Delta t+\sum_{i=1}^{N(t)-1} g_l^i(t)s_l^i(t)\Delta n\Delta t \tag{12}$$

$$\Delta n_{last}(t+1)=(Tot_l^{veh}(t+1))MOD(\Delta n) \tag{13}$$

Where $Tot_l^{veh}(t+1)$ is the total number of vehicles on lane *l* at time step *t+1*. MOD is the remainder operation.

The stability domain of Eq. (8) is defined by the Courant-Friedrichs-Lewy (CFL) condition [27] as given in Eq. (14). Consequently, the CFL-condition can be interpreted as follows: the distance a vehicle platoon can travel within one timestep should not be longer than the length of one platoon. Thus, Δt is chosen so as to ensure no violation of the CFL condition,

$$max\left|\frac{dv}{ds}\right|\Delta t \le \Delta n \tag{14}$$

As can be observed from Eq. (8), the net lane changing rate $g_l^i(t)$ needs to be determined in order to derive the spacing dynamics.

III. Lane changing Dynamics in Lagrangian Coordinates

A. Assumptions

To model the lane changing dynamics in Lagrangian coordinates, we consider the following assumptions:

(1) The motivation of drivers' lane changing is to improve their driving experience or reduce discomfort, i.e., increase their driving speed. Mandatory lane changes with merges or diverges are not considered.

(2) Drivers obey the keep-right rule, i.e., if the traffic state of all lanes is the same, the driver tends to choose the right lane.

(3) Each platoon on the available lane has a specific utility for each driver, who chooses the lane with the highest utility.

B. Lane choice probability

We consider that the fundamental diagram of each lane is described by Newell's speed-spacing relation [28]:

$$V(s)=\min\left\{v_f,\omega\left(k_j s-1\right)^+\right\} \tag{15}$$

where $(\bullet)^+ \equiv max\{\bullet,0\}$, $v_f>0$ is the free flow speed (in km/hr), k_j is the jam density (in veh/km), and $\omega>0$ is the (absolute) speed of the backward wave speed (in km/hr). Fig. 1 illustrates the speed-spacing relation.

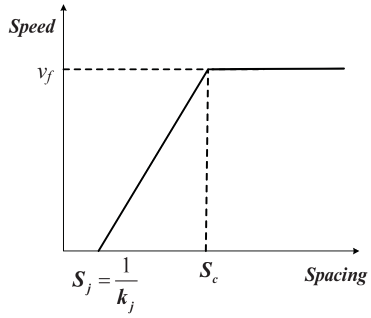


Fig. 1. The fundamental diagram of speed-spacing relation

As described in the assumptions (1) - (3), a driver changes lane to increase his or her driving speed or to follow the keep-right rule. We adopted the cost function proposed in [18]

$$\begin{aligned}C_l(s_l)&=\alpha_l+\beta_l\left\{V_l(s_l)^{-1}\right\}+\varepsilon\\&=c_l(s_l)+\varepsilon\end{aligned} \tag{16}$$

where s_l is the mean spacing on lane l, α_l is the disutility of violating the keep-right rule, β_l is the driver's sensitivity to the travel time for a unit distance, and ε is an error term following Gumbel distribution $G(0,\theta)$, representing the factors that affect utility but are not included in $c_l(s_l)$. The choice probability that a driver selects lane *l* at time *t* can be described using the logit model as:

$$p_l(t)=\frac{exp\left[-\theta\cdot c_l\left(s_l(t)\right)\right]}{\sum_r exp\left[-\theta\cdot c_r\left(s_r(t)\right)\right]} \tag{17}$$

For the case of a road with two lanes, the proportion of drivers who desire to change lanes from platoon i on lane l to platoon j on lane l' at time step t is given by:

$$p_{l\to l'}^{i\to j}(t)=\frac{exp\left[-\theta\cdot c_{l'}^j\left(s_{l'}^j\right)\right]}{exp\left[-\theta\cdot c_{l'}^j\left(s_{l'}^j\right)\right]+exp\left[-\theta\cdot c_l^i\left(s_l^i\right)\right]} \tag{18}$$

C. Lane changing rate

1) Desired number of lane changes

Eq. (7) ensures that vehicles are conserved with lane changing. In order to estimate the number of lane changes, we define the demand $D_l^i(t)$ as the number of vehicles coming from platoon *i* on lane *l* that want to make advancing moves within time step Δt:

$$D_l^i(t)=min\left(\frac{v_l^i(t)}{s_l^i(t)}\Delta t,Q_l\Delta t,\Delta n_l^i(t)\right) \tag{19}$$

Similarly, the supply $R_l^i(t)$ is the number of vehicles that can enter platoon *i* on lane *l* within time step Δt:

$$R_l^i(t) = min(\omega(\frac{1}{s_{j,l}} - \frac{1}{s_l^i(t)})\Delta t, Q_l \Delta t) \quad (20)$$

where Q_l is the capacity of lane *l*, $s_{j,l}$ is the minimum spacing when traffic is stationary on lane *l*.

Based on the lane changing probability as described by Eq. (18), the number of drivers who desire to change lanes from platoon i on lane l to platoon j on lane l' at time step *t* can be calculated as:

$$L_{l\to l'}^{i\to j}(t) = \frac{1}{\tau(t)} D_l^i(t) p_{l\to l'}^{i\to j}(t) \frac{Len_l^{i,j}(t)}{Len_l^i(t)} \quad (21)$$

where $\tau(t)$ is a state-dependent parameter indicating the number of time-steps a driver takes to decide and execute a lane change, and it relates to the gap availability in the target lane. The higher the density of the target lane, the longer the time required to find an available gap. $Len_l^i(t)$ is the length of platoon *i* on lane *l* at time *t*, and $Len_l^{i,j}(t)$ is the overlapping length between platoon i on lane l and platoon j on the adjacent lane l' at time *t*.

The total number of drivers who desire to leave platoon *i* on lane *l* is given by

$$L_l^i(t) = \sum_{l'\in\Omega_{l'}} \sum_{j\in\Omega_{l'}^j} L_{l\to l'}^{i\to j}(t) \quad (22)$$

where $\Omega_{l'}$ is the set of adjacent lanes a vehicle from platoon i on lane l can change to within time step Δt. $\Omega_{l'}^j$ is the set of platoons on lane l' a vehicle from platoon i on lane l can join within time step Δt.

The number of drivers who choose to stay in platoon *i* on lane *l* is given by:

$$K_l^i(t) = \Delta n_l^i(t) - L_l^i(t) \quad (23)$$

2) Lane changing rate estimation

Based on Eqs. (22) and (23), we can calculate the total demand of platoon *i* as:

$$D_{tot,l}^i(t) = K_l^i(t) + \sum_{l'\in\Omega_l} \sum_{j\in\Omega_j} L_{l'\to l}^{j\to i}(t) \quad (24)$$

where $\sum_{l'\in\Omega_l} \sum_{j\in\Omega_j} L_{l'\to l}^{j\to i}(t)$ is the number of vehicles desiring to enter platoon *i* from adjacent lanes at time *t*. According to the Incremental Transfer (IT) principle [26] , when total demand $D_{tot,l}^i(t)$ is less than the available capacity $R_l^i(t)$, all the demands are fulfilled; otherwise $R_l^i(t)$ is assigned to the different origin platoons according to their demands. The proportion of demand able to advance is given by:

$$\gamma_l^i(t) = min\left(1, \frac{R_l^i(t)}{D_{tot,l}^i(t)}\right) \quad (25)$$

Thus, the total number of vehicles entering platoon *i* on lane *l* at time *t* is given by

$$M_l^i(t) = \gamma_l^i(t) \sum_{l'\in\Omega_l} \sum_{j\in\Omega_j} L_{l'\to l}^{j\to i}(t) \quad (26)$$

Considering a road with two lanes, $M_l^i(t)$ can be further specified according to the state of platoons on the current lane and the adjacent lane as follows:

Case 1:

As shown in Fig. 2(a), the number of vehicles desiring to change lanes from platoon *j* on lane *l'* to platoon *i* on lane *l* is given by:

$$L_{l'\to l}^{j\to i}(t) = \frac{1}{\tau(t)} D_{l'}^j(t) p_{l'\to l}^{j\to i}(t) \frac{x_l^i(t) - x_l^{i+1}(t)}{s_{l'}^j(t)\Delta n_{l'}^j(t)} \quad (27)$$

where $x_l^i(t)$ is the position of platoon *i* on lane *l* at time *t*. It evolves according to

$$x_l^i(t) = \begin{cases} x_l^{i-1}(t) - s_l^{i-1}(t)\Delta n_l^{i-1}(t), & i = 2,\cdots,N \\ x_l^i(t-1) + v_l^i(t-1)\Delta t, & i = 1 \end{cases} \quad (28)$$

where *N* represents the maximum number of platoons on lane *l* at time *t*.

Due to the capacity restriction, the number of vehicles actually entering platoon *i* on lane *l* at time *t* is obtained as:

$$M_{l'\to l}^i(t) = \gamma_l^i(t) L_{l'\to l}^{j\to i}(t) \quad (29)$$

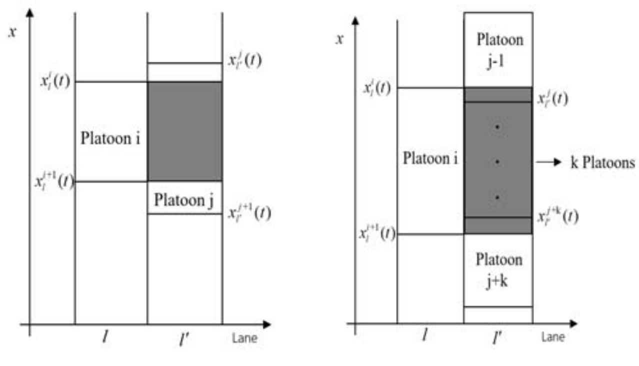
Next, we determine the number of vehicles leaving from platoon *i* on lane *l* to the adjacent lane *l'*. First of all, the number of vehicles desiring to leave from platoon *i* on lane *l* to platoon *j* on the adjacent lane *l'* is given by:

$$L_{l\to l'}^{i\to j}(t) = \frac{1}{\tau(t)} D_l^i(t) p_{l\to l'}^{i\to j}(t) \quad (30)$$

Due to the capacity restriction of lane *l'*, the number of vehicles actually leaving from platoon *i* on lane *l* to lane *l'* is calculated as:

$$LE_{l\to l'}^i(t) = \gamma_{l'}^j(t) L_{l\to l'}^{i\to j}(t) \quad (31)$$

where $\gamma_{l'}^j(t)$ is the proportion of demand able to advance to lane *l'* given by Eq. (25).



(a) (b)

Fig. 2. Relative position between platoons in adjacent lanes (The gray area is the share of vehicles to perform lane changing between platoon *i* on lane *l* and platoons on the adjacent lane *l'*)

Case 2:

As shown in Fig. 2(b), the number of vehicles desiring to change lanes from platoon j on lane l' to platoon i on lane l at time t is calculated as:

$$L_{l'\to l}^{h\to i}(t)=\begin{cases}\frac{1}{\tau(t)}D_{l'}^{h}(t)\cdot p_{l'\to l}^{h\to i}(t)\cdot\frac{x_{l}^{i}(t)-x_{l'}^{h+1}(t)}{s_{l'}^{h}(t)\Delta n_{l'}^{h}(t)}, & h=j-1\\ \frac{1}{\tau(t)}D_{l'}^{h}(t)\cdot p_{l'\to l}^{h\to i}(t), & j-1<h<j+k\\ \frac{1}{\tau(t)}D_{l'}^{h}(t)\cdot p_{l'\to l}^{h\to i}(t)\cdot\frac{x_{l'}^{h}(t)-x_{l}^{i+1}(t)}{s_{l'}^{h}(t)\Delta n_{l'}^{h}(t)}, & h=j+k\end{cases} \quad (32)$$

where $x_{l'}^{j}(t)$ is the position of platoon j on lane l' at time t; h is the platoon number on lane l'.

Due to the capacity restriction, the number of vehicles joining platoon i on lane l at time t is given by:

$$M_{l'\to l}^{i}(t)=\gamma_{l}^{i}(t)\sum_{h=j-1}^{j+k}L_{l'\to l}^{h\to i}(t) \quad (33)$$

Similarly, the number of vehicles desiring to leave from platoon i on lane l to the adjacent lane l' is given by:

$$L_{l\to l'}^{i\to h}(t)=\begin{cases}\frac{1}{\tau(t)}\cdot D_{l}^{i}(t)\cdot p_{l\to l'}^{i\to h}(t)\cdot\frac{x_{l}^{i}(t)-x_{l'}^{h+1}(t)}{s_{l}^{i}(t)\Delta n_{l}^{i}(t)}, & h=j-1\\ \frac{1}{\tau(t)}\cdot D_{l}^{i}(t)\cdot p_{l\to l'}^{i\to h}(t)\cdot\frac{s_{l'}^{h}(t)\Delta n_{l'}^{h}(t)}{s_{l}^{i}(t)\Delta n_{l}^{i}(t)}, & j-1<h<j+k\\ \frac{1}{\tau(t)}\cdot D_{l}^{i}(t)\cdot p_{l\to l'}^{i\to h}(t)\cdot\frac{x_{l'}^{h}(t)-x_{l}^{i+1}(t)}{s_{l}^{i}(t)\Delta n_{l}^{i}(t)}, & h=j+k\end{cases} \quad (34)$$

Considering the capacity restriction of lane l', the number of vehicles actually leaving from platoon i on lane l to lane l' is calculated as:

$$LE_{l\to l'}^{i}(t)=\sum_{h=j-1}^{j+k}\gamma_{l'}^{h}(t)L_{l\to l'}^{i\to h}(t) \quad (35)$$

Thus, the net lane changing rate in the Lagrangian coordinates can be calculated:

$$g_{l}^{i}(t)=g_{l,in}^{i}(t,x)-g_{l,out}^{i}(t,x) =\frac{M_{l'\to l}^{i}(t)}{s_{l}^{i}(t)\Delta n_{l}^{i}(t)\Delta t}-\frac{LE_{l\to l'}^{i}(t)}{s_{l}^{i}(t)\Delta n_{l}^{i}(t)\Delta t} \quad (36)$$

Finally, the spacing of each platoon is updated at every time step by substituting Eq. (36) into Eqs. (8) - (10).

IV. NUMERICAL EXPERIMENTS

We consider a circular road with two lanes and vehicles are assumed to drive counterclockwise. For the convenience of description, the notation $l=1$ represents outer lane and $l=2$ indicates inner lane. The reason for considering a circular road is that a ring road corresponds to a straight road with infinite length and the macroscopic characteristics of the system can be well represented.

Taking the outermost lane as the base line, the parameters of the cost function (Eq. (16)) for the outer lane α_1, β_1 are set as 0 and 1, respectively. This setting is the same as the simulation experiment conducted in [18]. The other parameters of the cost function are set by trial and error. Initial conditions are set as: 1) 20 platoons with each consisting of 5 vehicles; 2) initial spacing of 20 meters per lane. τ increases with the calculation process and is set equal to the time step so as to make lane changing movements settle down gradually. The convergence results are calculated by setting the initial spacing from 10 to 100 (meters) increasing by steps of 2 (meters). Parameters, initial conditions and numerical settings are given in Table 1.

TABLE 1. PARAMETERS, INITIAL CONDITIONS AND NUMERICAL SETTINGS

Parameters	Value
Free flow speed in the outer lane vf_1	$90 km/hr$
Free flow speed in the inner lane vf_2	$100 km/hr$
Minimum safety distance in the l lane $d_{Jl}(l=1,2)$	$8.54m$
The cost parameters for the outer lane, α_1 and β_1	$\alpha_1=0, \beta_1=1$
The cost parameters for the inner lane, α_2 and β_2	$\alpha_2=0.014, \beta_2=0.6$
The parameter of Gumbel distribution $G(0,\theta)$	θ=1000
Time steps	100
Time step size Δt	5s
Number of cars per cell Δn	5
Initial spacing S_1, S_2	$S_1, S_2 \sim N(20,0)$
Critical spacing in the outer lane Sc_1	$Sc_1=1/kc_1=40.99m$
Critical spacing in the inner lane Sc_2	$Sc_2=1/kc_2=31.31m$

The convergence dynamics of lane-flow distribution is shown in Fig. 3 when the initial spacing of platoons is set to 20 (meters) per lane. As we can see from Fig. 3, the fraction of the lane flow is initially the same and begins to oscillate at first. However, increasing by the time step, the fraction of the lane flow gradually stabilizes and eventually converges to equilibrium point at which the traffic volume of the inner lane is higher than that of the outer lane.

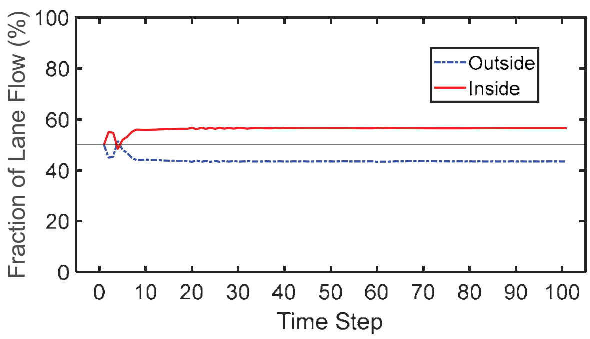
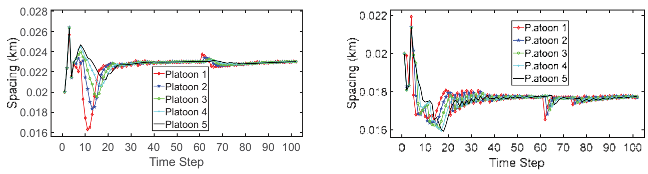


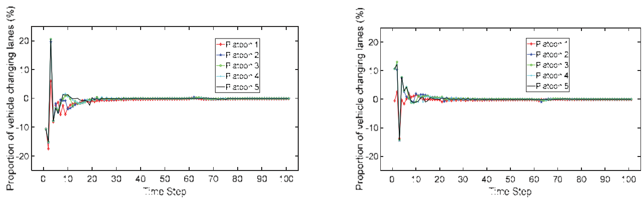
Fig. 3. Convergence process of the fraction of the lane flow

Fig. 4 illustrates spacing dynamics of the first 5 platoons on the outer lane (Fig. 4(a)) and the inner lane (Fig. 4(b)). For each platoon, we can observe spacing oscillations at the beginning of the simulation and becomes more stable as the time step increases. This indicates that the distribution of vehicles per lane tends to be stable and drivers cannot improve their driving experience by changing lanes. The spacing of the inner lane is smaller than that of the outer lane indicating that more vehicles choose to use the inner lane (in this case the left lane). The main reason is that the utility of the inner lane is larger than that of the outer lane. Fig. 5 shows the proportion of vehicles changing lanes over time. Similarly, we can observe that lane changing gradually settles down as the time step increases. The proportion value smaller than zero indicates that vehicles leave the platoon and larger than zero means vehicles enter the platoon.



(a) (b)

Fig. 4. Spacing of the first 5 platoons: (a) Outer lane (b) Inner lane



(a) (b)

Fig. 5. Proportion of lane changing vehicles: (a) Outer lane (b) Inner lane

Fig. 6 shows that the relationship between the mean spacing and the fraction of the lane flow at the equilibrium state. We can observe that more drivers tend to use the inner (left) lane when the spacing is smaller (or the density is higher). With the increase of the spacing (>30 meters), more vehicles choose to drive on the outer (right) lane instead of the inner (left) lane from the inner lane to the outer lane. The results are consistent with those observed from field highways [15] as shown in Fig. 7(a). Fig. 7 (b) shows the simulated results from our proposed model. Fig. 7(a) and 7(b) are consistent in macroscopic relations between the density (the reciprocal of the spacing, veh/km) and the fraction of the flow.

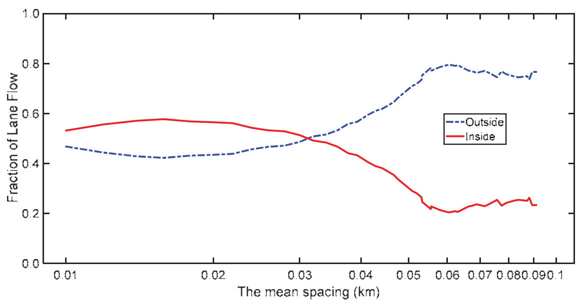
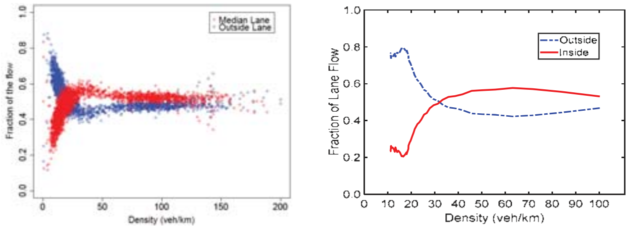
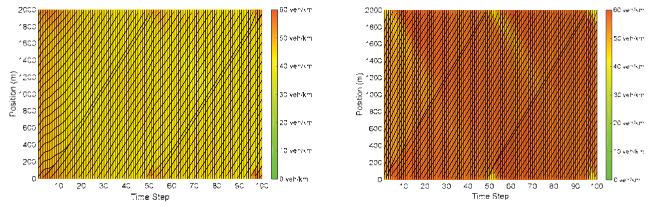


Fig. 6. Computed relation between the fraction of the lane flow and the spacing



(a) (b)

Fig. 7. Lane-flow distribution: (a) Observation (*Shiomi et al. 2013*) (b) Proposed model



(a) (b)

Fig. 8. Density dynamics (in veh/km) with initial spacing of 20m: (a) Outer lane (b) Inner lane

Figs. 8 illustrates platoon dynamics simulated by the proposed multilane Lagrangian model. The initial spacing is set to be 20 meters corresponding to the high density traffic state. As shown in Fig. 8, platoons move forward in space and lane changing dynamics can be clearly observed. With the increase of the time step, the spacing of the platoon on the outer (right) lane is getting larger as shown in Fig.8 (a); whereas for the inner (left) lane, the spacing is getting smaller as the time step increases as illustrated in Fig. 8 (b). The reason is that more vehicles tend to choose the inner (left) lane due to its higher utility.

V. Conclusion

The impact of lane changing on traffic congestion and safety has been received a lot of attention over the past two decades. Most of the multilane traffic flow models are formulated in Eulerian coordinates. According to the authors knowledge, Lagrangian formulation of multilane traffic flow model has not been proposed in the literature, though Wageningen-Kessels et al. [21] proposed a Lagrangian multi-class traffic model which does not consider lane change behavior. In this paper, we propose Lagrangian formulation of the multilane traffic flow model. We provide the mathematical formulation of the Lagrangian conservation equation and further discuss the numerical implementation of the proposed model. A ring road with two lanes (without on/off ramps) is used to verify the proposed model. The simulation results demonstrate that the proposed model can capture lane changing dynamics at the platoon level over space and time. At the macroscopic level, the lane changing equilibrium curve produced by the proposed Lagrangian model is consistent with that observed from the field highway [18].

The proposed model in this paper can be applied to online lane-based traffic estimation and dynamic lane control scheme. In future, we would like to extend the model to a three-lane case (which is more generic in freeway networks) and furthermore to calibrate parameters using real data sets (e.g., NGSIM data).

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