Department of Precision and Microsystems Engineering

Dynamic balance principles based on a flexible beam for the synthesis of dynamically balanced compliant mechanisms

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Preface

Looking back on my childhood, my passion for technology started at a young age. Playing Lego developed in building papercraft, repairing my sailing boat, and studying mechanical engineering. The personality of this curious, cheerful little girl never disappeared. During my college days in Delft, I enjoyed the study courses and loved the motivating atmosphere. The extracurricular activities inspired me to continue in this field!

At the start of my master's, I was impressed by the number of intelligent fellow students and the connections between all the new knowledge I obtained. The great atmosphere at the department, the study sessions with my study friends, and lunch breaks with my fellow Taylor board members made sure I understood the study material very well and was impressed by my own abilities. For my graduation project, Just Herder introduced me to Volkert van der Wijk. His so 'simple' but smart solution to achieve dynamic balance amazed me. While working on this thesis, I started with the challenge of an unknown result. Later, this insecurity developed into curiosity and enthusiasm. I owe this development to a large extent to the help of my supervisor Volkert van der Wijk. His positivity that 'we will find something' and sharing his view about what defines research motivated me. The critical discussions, the funny brainstorm sessions, and the 'balance' in his guidance of giving me the freedom to discover it myself and supporting me when needed improved this thesis and gave me a taste of being a researcher. Next to my supervisor, I would like to thanks Jaap Meijaard for answering all the questions in my long emails to him, sharing his thought in our Zoom meetings, and sharing his passion for research.

Next to the university supervision, this thesis would not be here without the help of friends and family. First, a big thanks to my parents for offering me a 'home office' at their house, the motivating conversations, and the in-depth discussions. Another word of thanks to my brother, sister, housemates, and other friends for their mental support and interest! Last but not least, I would like to say to you as a reader, thanks for reading but most important, enjoy!

Lisanne Nijdam
The Hague, January 2021

Contents

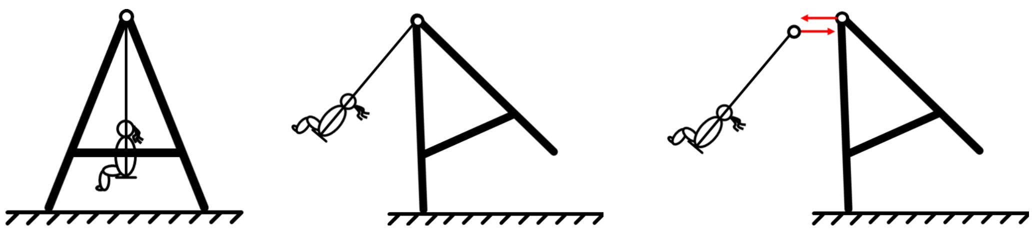
1

Introduction

In the ever developing world that we live in, machines perform their task faster, better, more precise, cheaper, etc. every year. The improvements in a lot of mechanisms are inspired by the natural behavior of animals and humans. While falling off a balance beam, the human instinct let an arm or leg accelerate in the opposite direction of the fall to find balance. The principle that the vestibular system and muscles work together to keep the body static and dynamic actively balanced works quite well. Although, making a coloring page on a roller coaster is a challenge for the control system of the human body. Nowadays, mechanisms in the world around us are desired to perform even better than the human is doing.

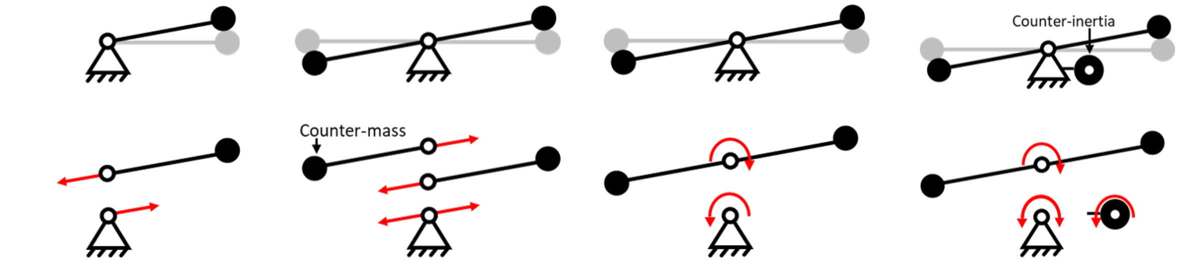
1.1. Principle of dynamic balance

In Figure 1.1a, a kid at a swing is shown to explain the dynamic balance principle. In this example, the swinging girl rotates around the pivot at the top of the swing. A mechanism is defined as dynamically balanced when the sum of both the dynamic forces and moments are zero on its base. The swing, that is not connected to the ground, falls over due to the generated dynamic reactions (Figure 1.1b). This non-zero resultant force defines the system of the swing as unbalanced (Figure 1.1c). A simplified version of the swing and its dynamic reaction forces is shown in Figure 1.2a. By adding a counter-mass as shown in Figure 1.2b, the forces on the base have a zero resultant sum so this system is dynamically force balanced. A requirement for this counter-mass is that is it attached at the opposite side of the base pivot compared to the other masss and the weight times the length to the base pivot equals the weight times the length of the actuated mass. The rotation of these masses introduces a resulting moment at the base pivot. To eliminate the resulting moment (Figure 1.2c), a counter-inertia in opposite direction with equal value is added to the system. This both force and moment balanced mechanism (in Figure 1.2d) is named dynamically balanced. When no forces or moments are interacting between a mechanism and the base, some system characteristics follow. First, when no forces are interacting between the system and the base, the linear momentum is constant. Furthermore, no resulting interacting moments results in a constant angular momentum with respect to the base for all motions of the mechanism [10]. Another characteristic of dynamic balance is a constant speed of the common center of mass (CoM) of the system. Forces that interact between the common CoM and base are the only way to let this CoM change its speed. While stated that the resultant force is zero, the common CoM must or be stationary



(a) (b) (c)

Figure 1.1: (a) A swinging kid who let (b) the swing fall over due to (c) the generated dynamic reactions

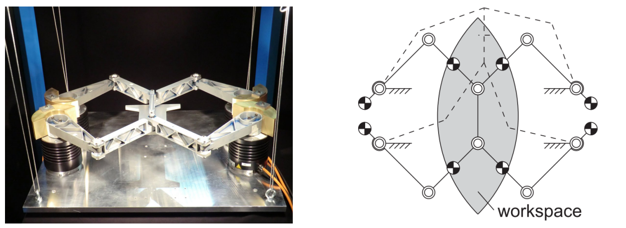


(a) Not force balanced (b) Force balanced (c) Not moment balanced (d) Force and moment balanced

Figure 1.2: (a) A rotatable beam with a lumped mass about a base pivot and its free body diagram. The introduced (b) counter-mass ensures force balance. The (c) resulting moment is eliminated by the introduced (d) counter-inertia which results in force and moment balanced mechanism. The grey lines indicate the initial position and the black lines the position after some rotation.

(a) Jigsaw [2] (b) Camera with stabilizer [1]

Figure 1.3: Two dynamically force balanced mechanisms



(a) The prototype (b) The configuration

Figure 1.4: An dynamically balanced system, the DUAL-V manipulator [11]

or have a constant velocity relative to its base for force balanced mechanisms. Having a constant CoM is more usual for force balanced mechanism, although a constant velocity is not impossible [9]. A last characteristic is given by the fact that a stationary common CoM of the system proves a dynamically balanced mechanism is statically balanced also.

To clarify the term static balance and avoid confusion with dynamic balance, static balance is achieved when the energy in the system is constant. Magnetic, spring, and gravitational forces influence the energy of the system [10]. Another term not to mix up with dynamic balance is vibration isolation, which is about reducing the vibrations instead of eliminating or removing them. Reducing them can be done by increasing the base mass or damping the vibrations by springs or a complex controller [10]. For instance, the springs in a car damp the impact of a speed bump and so provide a comfortable ride, although the impact is not removed.

1.2. Motivations of dynamic balance

The example of the kid at the swing shows that the dynamic reactions influence the behavior of the mechanism. A possibility to prevent the swing to fall over would be to have another kid swinging in the opposite direction and so balance the swing. In many more systems, this changing behavior influences performance. A classic alarm clock that vibrates off your bedside table when ringing is annoying but does not cause any unpleasant consequences. However, while using a high-speed hand-held tool like a jigsaw (Figure 1.3a), without a balancing system or physical exertion, vibrations make it impossible to follow your planned pattern. To ensure a smooth operation, the jigsaw is designed force balanced by counterbalancing the vibrating saw [2]. The balanced solution does not disturb its base anymore, in this case, the human hand. This principle acts in a contradictory direction too, meaning that disturbing the base of a balanced system does not influence the behavior of the mechanism. The indicated principle is applied in the force balanced camera in Figure 1.3b [1]. By translating the base of the mechanism, the human hand, in this case, the camera itself does not change its position relative to the hand. Evaluating this mechanism, it can be compared with the system in Figure 1.2b where the camera equals the right mass, the hand the base, and the mass at bottom of the blue stick the left counter-mass. An application of a force and moment balanced mechanism is the DUAL-V manipulator (Figure 1.4a). The manipulator can translate in the workspace (Figure 1.4b) and the counter-masses ensure dynamic force and moment balance. Compared to an unbalanced variant of the manipulator, experiments of

Figure 1.5: Unbalanced compliant variant of an otherwise theoretical balanced rigid mechanism after some rotation, blue indicates the translated common CoM

the balanced variant show 97% lower dynamic forces and 96% lower dynamic moments [11]. These examples show the influence of dynamic balance on the performance of a mechanism. Therefore, the implementation of dynamic balance proved to be essential in the high precision industry.

1.3. Problem statement

Literature gives several solutions to dynamically balance a mechanism. Although, most of these methods assume the system is rigid. Nowadays, to simplify the fabrication process and reduce weight, part-count, and assembly time, compliant mechanisms are used in the industry more often [4, 6]. A mechanism is named compliant if its motion is accomplished by its flexibility, which is a result of elastic deformation in the material. This is in contrast with rigid body kinematics where motion is accomplished by the movement of different parts relative to each other [4]. A load of advantages of compliant mechanisms has led to an increase in the study and application of compliant mechanisms in recent years [7]. Research for the structural and kinematic analysis of compliant mechanism that is done by Yu et al. give more insight into the behavior of these mechanisms [13–15].

When changing the rigid beam of the mechanism in Figure 1.2b to a flexible beam, the system in Figure 1.5 arises. Due to the flexibility of the beam, indicated in the illustration, the CoM is not stationary anymore and so this system is unbalanced. The limitation of the available literature of balancing of flexible mechanisms gives the impression that the combination of these fields is relatively new. Literature survey exposes different methods however, these methods have limitations and do not ensure both force and moment balance. For instance, the research of Weeke et al. presents a successful force balanced prismatic oscillator, although it is not moment balanced and only force balanced for small rotations [12]. Next, this oscillator contains a mirror-copy to obtain balance, no new balance principles are discovered. Another research of Kalas et al. analyzed the behavior of a compliant link while applying the rigid body balancing principles. This led to undesired eigenfrequencies and so unbalance of an otherwise perfectly balanced mechanism [3, 5]. Finally, Martínez et al. discovered a set of stiffness related balance principles for a compliant link. Experiments showed that these design principles resulted in a reduction of 93% of the dynamic forces. However, this is not perfect force balance and shown for one specific prototype only. To conclude, the design principles to achieve perfect force and moment balance, and besides, the implementation of these principles in the design process are missing.

1.4. Research objective & approach

The objective of this thesis is to derive dynamic balance principles for the synthesis of dynamic balanced compliant mechanisms. To reach the goal of this thesis, the following subgoals were established:

1. Compare the existing methods and investigate their potential to dynamically balance a compliant mechanism.
2. Derive the dynamic balance principles of a flexible beam and to use the flexible beam as a building block in the synthesis of dynamically balanced compliant mechanisms.

The first subgoal is the focus of the paper presented in chapter 2. Here, a categorization of existing methods to dynamically balance a compliant mechanism is given. The balancing methods with the most potential are a base for the research presented in the second paper. This paper is presented in chapter 3 and focuses on the second subgoal. A theoretical approach is arranged to investigate the design principles for a flexible beam. These outcomes are validated by simulations and the flexible beam is applied as a building block for the synthesize of dynamically balanced compliant mechanisms. After investigating the sub-goals, this thesis will end with a discussion and conclusion in chapter 4 and chapter 5 respectively.

2

The potential of the existing methods to dynamically balance a compliant mechanism

The potential of the existing methods to dynamically balance a compliant mechanism

Lisanne Nijdam

***Abstract*— Dynamic balancing of compliant mechanisms is a relatively new and undiscovered field but urgently needed in the high precision industry. This literature survey compares the existing methods and investigates their potential to dynamically balance a compliant mechanism. The passive balancing methods are categorized into three groups. First, rigid body balancing principles are applied to both lumped and secondly distributed compliant mechanisms. Thirdly, modal balancing is discovered as a balancing method. Of these approaches, modal balancing has the highest potential to dynamically balance a mechanism for a large range of motion. Although, no implementations of this method in a design are researched yet.**

I. Introduction

During the design process of systems, the static and dynamic balance are considered to achieve the system its function. Dynamic balance is achieved when during motion of the mechanism the sum of the dynamic forces and dynamic moments is zero at the base, see Chapter 1 Introduction. Static balance is achieved when the energy in the system is constant. Magnetic, spring, and gravitational forces influence the energy of the system [15].

I-A. Rigid body balancing applications and principles

Applications of statically balanced rigid mechanisms are sufficiently available. An example is a hexapod that is statically balanced by counterweights in the research of Russo et al. [11]. Furthermore, a standard IKEA desk lamp is statically balanced, the springs and mass of the lamp ensure constant energy.

Besides static balancing of rigid bodies, the research about dynamic balance started more than two decades ago. Xi and Sinatra investigated the effect of dynamic balance of vibrating a four-bar-linkage mechanism. The moment of inertia of the counterweight attached to the input link resulted to be the dominant factor for the intensity of the vibrations [19]. Another strategy was used in the research of Wu and Gosselin. Here, the author replaced a moving platform by point masses and found an effective algorithm for dynamic balance. Although, the balancing condition for point masses is more restrictive than for an equivalent platform that is balanced globally [18]. The third research presents a model-based method, called Dynamic Balance Force Control, that determines the full-body joint torques. The principle used to achieve balance is that the motion of the center of mass (CoM) is effected by the contact forces [14]. Besides, the thesis of Van der Wijk shows different inherent dynamic force balanced solutions for mechanisms. These solutions are based on the linear momentum equations [15]. Later, van der Wijk et al. summarized the existing balancing principles for adding mass or inertia and compared the performances. The result was that the system can be either duplicated or, counter-masses or a counter-rotation can be used to achieve the dynamic balance [16]. Kalas et al. shows in his thesis a similar result. The derivation from a constant linear momentum results in two cases to achieve shaking force balancing; using a counter mass or mirroring the mechanism about the base [5].

I-B. Static balancing of compliant mechanisms

Compliant mechanisms are mechanisms that accomplish motions due to their flexibility, the material deforms elastically during the motion. In contrast to rigid body kinematics where motion is accomplished by the movement of different parts relative to each other [4]. Compliant mechanisms have advantages compared to rigid body mechanisms, such as reduction of the wear, weight, part-count, and assembly time, furthermore the increase of the precision and simplification of the manufacturing progresses [4, 8]. These benefits, especially the increase in precision and accuracy, make them attractive for applications such as positional stages and motion transmissions.

When combining the field of compliant mechanisms with balancing techniques, research is found about the static balancing of compliant mechanisms. Chen and Zhang found new concepts for designing fully-compliant statically balanced mechanisms without prestressing assembly. One of the concepts combines two multi-stable mechanisms to achieve a near-zero-stiffness mechanism. The other concept employs a constant-force compliant mechanism containing a weight compensator. These concepts can, because of their static balance, be used in mechanisms where a low and accurate actuation force or precise force feedback are main concerns [1]. The second example of a statically force balanced compliant mechanism is the compliant power transmission of Machekposhti et al.. This compliant transmission design is based on the Pseudo-Rigid-Body model (PRBM) of the Oldham coupling. This design removes the internal stiffness by static balancing it so a high efficiency can be achieved [7]. The same researcher came up with a power transmission coupling. This fully compliant and potentially monolithic coupling can accommodate high misalignment angles between the input and the output rotational axes. The design is nearly statically balanced because of the number of compliant sets around the rotational axes [8]. Finally, an example of a haptic interface of a compliant material utilizes static balance to improve the feeling of the user by reducing the friction and inertia. Again, the PRBM is the approach to derive the mathematical model for the static balanced design [6].

I-C. Problem statement

In Table I an overview of the literature in the different groups discussed above is given. Although, information about the dynamic balancing of compliant mechanisms is leaking and so gives the impression that this a relatively new and undiscovered field. The motivation to find dynamically balanced solutions for compliant mechanisms can be found in the example of a robotic manipulator in high-speed acceleration applications. Because of the high speed and high inertia of a robot arm, high reaction forces and moments occur, which decrease the dynamic performance. Next to that, these unwanted vibrations affect the components and so reduce the lifetime [5]. This unavoidable flexibility in links emphasizes the need for a dynamic balance solution. An effective way of balancing could be active balancing. The redundant drives that are used in the researches of Yu and Lin show the success of active control of flexible mechanisms [20–22]. Although, for active control, actuators and sensors are needed. A passive balancing system could also be used for balancing compliant mechanisms in the precision engineering industry where simplicity plays a role.

Table I: Discussed literature overview

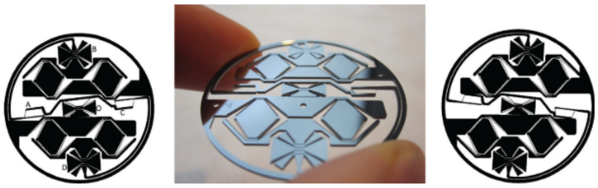
	Rigid-body mechanisms	**Compliant mechanisms** [4, 23–25]
Static balance [15]	[11]	[1, 6–8, 15]
Dynamic balance [15]	[5, 14, 16, 18, 19]	Focus of this paper

II. Research objective & approach

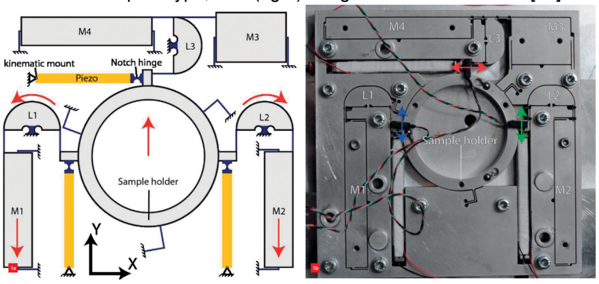
The objective of this literature review is to compare the existing methods and investigate their potential to dynamically balance a compliant mechanism. The existing passive balancing methods are categorized into three groups. First, two groups that apply the rigid body balancing principles for mechanisms containing either lumped compliant or distributed compliant parts, are presented in Sections III and IV respectively. Next, the third method is presented in section V, the modal balancing strategy. This paper will end with a discussion and the conclusion in section VI.

III. Rigid body balancing principles & lumped compliance parts

Lumped compliant is one type in the classification of compliant mechanisms. The deformation in lumped compliant mechanisms takes place in a concentrated part of the element [3]. The compliant force balanced mechanism based on the rectilinear motion that is presented by Weeke et al. is an example of a design that applies rigid body balancing techniques and includes lumped compliant joints. An image this dynamically force-balanced mechanism is given in Figure 1a. This mechanism enables the usage of prismatic oscillators in translational accelerating environments without accuracy loss. The design is a combination of the most optimal of the kinematics, opposite movement principle, combined with the most optimal prismatic and revolute joints. Three different measurements to evaluate the performance had successful results; the stationary center of mass, zero resultant inertial force, and dy-



(a) Prismatic oscillator, displayed (left) in left rotated orientation, (middel) the 1:1 scaled prototype, and (right) in right rotated orientation [17]



(b) Rectilinear scanning mechanism, displayed (left) the schematic representation and (right) the prototype [12, 13]

Figure 1: Two lumped compliant mechanisms that are dynamic force balanced by rigid body balancing principles

Figure 2: An attempt to dynamically balance a distributed compliant mechanism (robot arm) by rigid body balancing principles [2]

namic decoupling. Although, no moment balance and only force balance for small rotations were achieved [17]. Another mechanism that equals above conclusion, is the scanning mechanism of Sebek et al. [13]. In this mechanism, the ring at which the sample will be mounded is attached with flexible hinges to counter masses. These passively actuated counter-masses ensure a constant momentum of the mechanism [12, 13]. An image of the scanning mechanism is given in Figure 1b.

IV. Rigid body balancing principles & distributed compliant parts

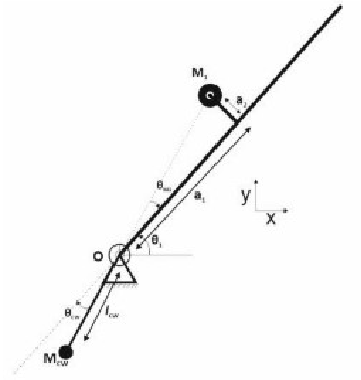
In a distributed compliant mechanism the deformation occurs along a broader part on the element [3]. Whereas in lumped compliant, a concentrated part of the element will deform. De Jong et al. researched counter-masses on flexible links. The reason for his research were the undesirable vibrations and loss of accuracy during the high accelerations of fast-moving robots that cause high reaction forces and moments. Adding counter-masses was found as a technique to dynamically balance the mechanism and reduces or even eliminate the fluctuations. The consequences of the flexibility of the linkages lead to undesirable eigenfrequencies and unbalance of an otherwise perfectly balanced mechanism [2]. An image of the distributed compliant balanced mechanism described is shown in Figure 2.

The same technique was used in Kalas et al. his research. He described two balancing techniques. The first case included a counter-mass to force balance a compliant link, see Figure 3a. A second case included a duplication of the mechanism, see Figure 3b. In this research, the normally rigid links were replaced by compliant links. As a result, the first technique with a counter-mass was not shaking force balanced anymore. The second case with the same link at the other side of the base did not lose its shaking force balance quality. Kalas et al. tried the same technique, replacing a rigid link with a distributed compliant link, for a link that was pinned at two sides.

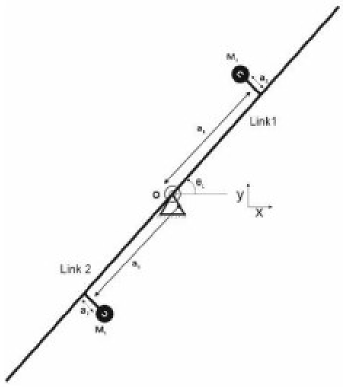
Figure 4a shows another research of Kalas et al. where he is balancing a rigid link by counter-masses. The same masses are used in Figure 4b to balance a compliant link. Shaking force balancing the mechanism remained a challenge. Although, during an attempt to balance the first mode of deflection, the shaking force magnitude could be reduced by increasing the overhang length of the counter-masses and decreasing the mass of the counter-mass [5]. This motivates us to look for balancing strategies that make use of modal balancing techniques.

V. Modal balancing

The third category of dynamic balancing strategies for compliant mechanisms is to approach the mechanism or linkages as 100% elastic and balance it modal. Modal balancing is the balancing method in the research of Meijaard and van der Wijk. He researched the shaking force and moment balance for different placements of the support points of a single flexible beam. The placement of the support points proved to be of great influence on the force balancing of the beam. Supporting at the end compared to supporting at reciprocal points gives a huge reduction in the shaking forces. Balancing the shaking moments turned out to be more difficult. Supporting in the two nodes of the first bending natural mode also gave some reduction of the moments and forces, again not so much for the moments [10]. An image of the research is shown in Figure 5. The flexible beam in different modes

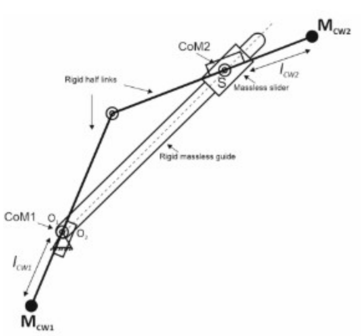


(a) Adding a counter-mass results in unbalance [5]

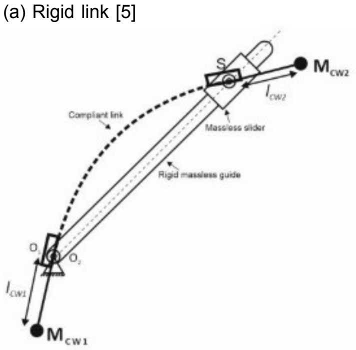


(b) Duplication of the link results in balance [5]

Figure 3: Two attempts to dynamically balance a flexible link



(a) Rigid link [5]



(b) Compliant link [5]

Figure 4: Balancing an at two sides pinned rigid and compliant link by addition of counter-masses [5]

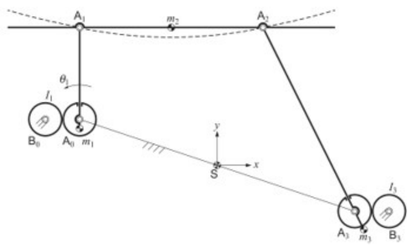


Figure 5: Modal balanced flexible beam [10]

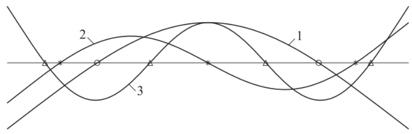


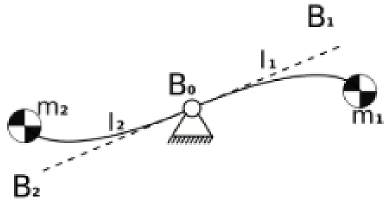
Figure 6: The different modes of a flexible beam and its support points [10]

with his support point is shown in Figure 6.

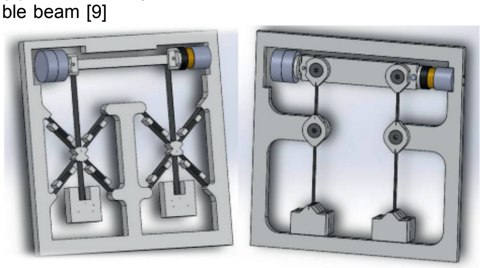
Another research about modal balancing was the research of Martínez et al. [9]. He found design principles for shaking force balance for compliant mechanisms. He concluded that there are two stiffness related balance conditions in addition to the balance condition known for a rigid link. In Figure 7a a flexible link is shown which is force balanced. An image of the CAD drawing of the planar parallelogram mechanism that contains twice the flexible force balanced link is shown in Figure 7b. The simulations and experiments are done for this parallelogram mechanism and show almost perfect balance for a model with revolute joints and a model with compliant joints. These discovered design principles correspond to modal balancing techniques and are an addition to the classical kinematics [9]. This research raises questions, like if next to shaking force balance, also shaking moment balance could be achieved.

VI. Discussion & Conclusion

The limitation of the examples that are shown in Sections III to V gives the impression that the field of dynamic balancing of compliant mechanisms is relatively new. The three methods present the existing methods to passively dynamically balance a compliant mechanism. For mechanisms containing lumped compliant parts that are balanced following the rigid body principles, force balance can be achieved successfully. Although, this is only possible for small rotations. For mechanisms containing distributed compliant parts, balancing following the rigid body principles results in undesired eigenfrequencies. No success is achieved. Modal balancing shows more successful results. The force balance is achieved for even big rota-



(a) Schematic representation of one force balanced flexible beam [9]



(b) CAD of the planar parallelogram mechanism [9]

Figure 7: Dynamically force balanced parallelogram mechanism containing two flexible beams [9]

tions. Although, no research is found where this strategy is implemented in a successful design.

To conclude, modal balancing has the most potential to balance a compliant mechanism dynamically. No examples are found where this method is implemented in the design of a mechanism so more research is needed. Also the possibility to achieve, next to dynamic force balance, dynamic moment balance based on modal balancing, forms a basis for new research.

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3

The dynamic balance principles of a flexible beam and its application as a building block in the synthesis of dynamically balanced compliant mechanisms

The dynamic balance principles of a flexible beam and its application as a building block in the synthesis of dynamically balanced compliant mechanisms

Lisanne Nijdam

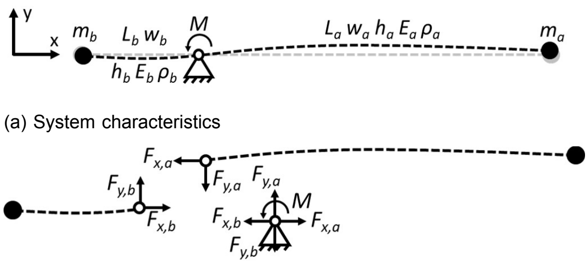
***Abstract*— In this paper, the dynamic balance principles of a flexible beam are derived and applied as a building block in the synthesis of dynamically balanced compliant mechanisms. A theoretical approach is arranged to investigate the design principles for a force and moment balanced flexible beam. Dynamic simulations are used to validate the principles. Combining these flexible balanced beams resulted in the synthesis of two dynamically balanced compliant mechanisms, a force balanced compliant watch and the design of a compliant variant of the dynamically balanced DUAL-V manipulator.**

I. Introduction

Dynamic balance is considered in the design process of rigid body mechanisms like robotic manipulators to prevent, even during high accelerations, vibrations. Besides, the use of compliant mechanisms in the industry is increasing compared to rigid body mechanisms. This is because of their reduction of wear, weight, part-count, backlash, and assembly time, furthermore, the increase of the precision and simplification of the manufacturing progresses [4, 9]. A mechanism is compliant if its motion is accomplished by its flexibility, which is a result of elastic deformation in the material. This is in contrast with rigid body kinematics where motion is accomplished by the movement of different parts relative to each other [4].

From literature, different approaches for balancing of compliant mechanisms can be found. An effective way of balancing could be active balancing. Although, for active control, actuators and sensors are needed [19–21]. The passive balancing methods are categorized into three groups. First, the method where rigid body balancing principles are applied to lumped compliant mechanisms. The deformation in lumped compliant mechanisms takes place in a concentrated part of the element [2]. For these mechanisms, force balance can be achieved successfully. Although, this is only possible for small rotations [12, 13, 18]. Secondly, the method where rigid body balancing principles are applied at distributed compliant mechanisms, a mechanism where the deformation occurs along a broader part on the element [2]. This method results in vibrations and so an unbalanced system. The unwanted vibrations disturb the otherwise balanced mechanism [1]. Next to that, the undesirable vibrations affect the components and so reduce the lifetime of mechanisms [6]. Finally, the method with most potential to achieve balance, modal balancing. One modal balancing study shows some stiffness related principles to reduce the dynamic forces of a flexible rotatable beam [10]. Another study presents that the placement of the support points of a flexible beam proved to be of great influence on the force balancing of the beam [11]. However, principles to achieve a perfect dynamic force and moment balance for a compliant/flexible link/beam and besides, to synthesize a dynamically balanced compliant system/mechanism, are missing.

In this work, the dynamic balance principles of flexible beams are derived in order to apply the flexible beam as a building block in the synthesis of dynamically balanced compliant mechanisms. The theoretical derivation of the balance principles is presented in section II. After determining the methodology for validation in section III, simulations of the flexible beam are performed in section IV. The application of these beams as a building block for the synthesis of dynamically balanced compliant mechanisms is presented in (section V). These are applied in the redesign of present mechanisms. This results in a redesign of



(a) System characteristics

(b) Free body diagram including the dynamic forces

Figure 1: A flexible rotatable beam divided in an a and b beam part that is actuated by a moment M at its base pivot

a force balanced compliant watch and a compliant variant of the dynamically balanced DUAL-V manipulator in Sections VI and VII respectively. This paper ends with a discussion in section VIII and the conclusion in section IX.

II. Dynamic balance conditions for a flexible beam

Figure 1a shows a flexible rotatable beam that is divided in two parts, the right side will be referred as part a and the left side as part b. These beam parts have a length of L_a and L_b, for side a and b respectively. At the tips of the beam, a mass of m_a and m_b is attached respectively. Other characteristics of these parts are; a width of w_a and w_b, a height of h_a and h_b, a Young's modulus of E_a and E_b, and a density ρ_a and ρ_b.

When an actuation moment M is applied on the beam in the base pivot, for instance by a motor, then the beam will rotate around its pivot. The acceleration and deceleration of the point tip masses and distributed beam masses, induce dynamic forces and moments at the base. A free body diagram including the dynamic forces is presented in Figure 1b. For dynamic force balance, the sum of the reaction forces at the base pivot must be zero, meaning $F_{x,a} = F_{x,b}$ and $F_{y,a} = F_{y,b}$.

When the beam in Figure 1 would be rigid,

$$m_a L_a + \frac{1}{2}\rho_a A_a L_a^2 = m_b L_b + \frac{1}{2}\rho_b A_b L_b^2 \quad (1)$$

should be true to ensure dynamic force balance. Here, A is denoted as the cross-section area of the beam. The first coefficient of each side of Equation 1 relates to the balance of the point masses. The second coefficient relates to the balance of the distributed beam masses. When the beam in Figure 1 is modelled flexible, it will show bending. Because of this, a delay in transmitting the behavior of the system to the base pivot is introduced. To find the additional conditions for which the flexible beam is balanced too, the scaled shape of, and wave speed in, the beam need to be considered. These aspects are investigated in the next sections, subsection II-A and subsection II-B respectively. The resulting relation of the design parameters of beam part a and b to ensure dynamic balance will follow in subsection II-C.

II-A. Beam shape

To have both sides of the flexible beam generate equal and opposite reaction forces to obtain force balance, their dynamics need to be and remain synchronous. This property was already found by Martínez et al. as

$$f_a = f_b, \quad (2)$$

where f is the eigenfrequency [10]. The eigenfrequency of a beam depends on its boundary conditions and design parameters. Possible boundary conditions are clamped, sliding, simply supported, or free. In literature, different approaches are used to define the eigenfrequency of different cases [15]. The cases can be classified into two groups. First, systems containing a concentrated mass at the tip only, without a distributed beam mass, are evaluated. These systems hold an eigenfrequency of

$$f_{ConcentratedMass} = C_1 \sqrt{\frac{EI}{mL^3}}. \quad (3)$$

Here, the moment of inertia is indicated by I and the dimensionless value C_1 depends on the boundary conditions of the beam. For instance, the frequency of a one end clamped massless beam with a point mass at the free end is calculated by Gere and Timoshenko as $f = \frac{1}{2\pi}\sqrt{\frac{3EI}{mL^3}}$ [3].

The second group contains systems with an equally distributed mass over the beam only, so without a concentrated mass at its tip. This group has an eigenfrequency of

$$f_{DistributedMass} = C_2 \sqrt{\frac{EI}{\rho A L^4}}. \quad (4)$$

Again, constant C_2 depends on the boundary conditions. For instance, in literature, it is found that the frequency of a system with one fixed side and one free end can be calculated by taking $C_2 = 0.56$ [14, 17].

Since the flexible beam has both concentrated masses and distributed beam masses, both equations must hold, meaning, $C_{1_b}\sqrt{\frac{E_b I_b}{m_b L_b^3}} =$

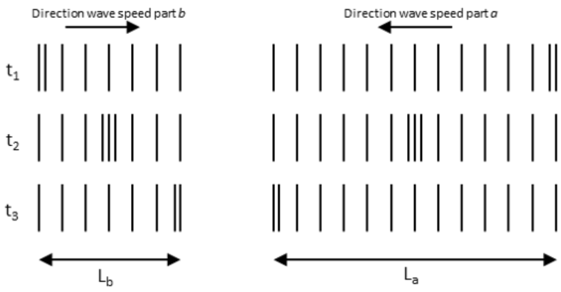


Figure 2: Development of the longitudinal wave in beam part a and b, displayed just after a distribution (t_1), half way (t_2) and when arriving at the base (t_3)

$C_{1_a}\sqrt{\frac{E_aI_a}{m_aL_a^3}}$ and $C_{2_b}\sqrt{\frac{E_bI_b}{\rho_bA_bL_b^4}} = C_{2_a}\sqrt{\frac{E_aI_a}{\rho_aA_aL_a^4}}$. Setting as requirement equal boundary conditions for both parts, so $C_{1_b} = C_{1_a}$ and $C_{2_b} = C_{2_a}$, the condition

$$\sqrt{\frac{E_bI_b}{L_b^2}} = \sqrt{\frac{E_aI_a}{L_a^2}} \tag{5}$$

applies, and it follows that

$$m_bL_b = m_aL_a \text{ and } \rho_bA_bL_b^2 = \rho_aA_aL_a^2. \tag{6}$$

These three conditions were also found by Martínez et al. [10]. In addition, from Equation (6) follows that Equation (1) is valid for flexible beams too and the center of mass of the system is stationary for all motions.

II-B. Wave speed

A fourth condition follows from the physical behaviour that a disturbance causes a pressure wave in the medium of the beam. An external source can disturb a part of the beam which exerts a force on the adjacent beam particles and so transports a longitudinal wave through the beam. For instance, when exerting a force at both beam tips (t_1 in Figure 2), a wave in both beam parts a and b arises. This wave is know as the speed of sound and for a thin beam its value is defined as the square-root of the Young's modulus over the density, $\sqrt{\frac{E}{\rho}}$ in $\frac{\text{m}}{\text{sec}}$ [7]. To ensure balance, the waves should disturb the base at the same time and in opposite direction. Meaning, the time it takes for the waves to arrive at the pivot (t_3 in Figure 2) should be equal for the a and b part and so,

$$\frac{1}{L_a}\sqrt{\frac{E_a}{\rho_a}} = \frac{1}{L_b}\sqrt{\frac{E_b}{\rho_b}} \tag{7}$$

must satisfy.

Table I: Four cases of possible relations of the design parameters in between beam part a and b

Case 1: Equal width	Case 2: Equal densities	Case 3: Equal Young's modulus	Case 4: Intermediate
$L_b = \frac{L_a}{n}$	$L_b = \frac{L_a}{n}$	$L_b = \frac{L_a}{n}$	$L_b = \frac{L_a}{n}$
$m_b = m_an$	$m_b = m_an$	$m_b = m_an$	$m_b = m_an$
$w_b = w_a$	$w_b = w_an^3$	$w_b = w_an$	$w_b = w_an^2$
$h_b = \frac{h_a}{n}$	$h_b = \frac{h_a}{n}$	$h_b = \frac{h_a}{n}$	$h_b = \frac{h_a}{n}$
$E_b = E_an$	$E_b = \frac{E_a}{n^2}$	$E_b = E_a$	$E_b = \frac{E_a}{n}$
$\rho_b = \rho_an^3$	$\rho_b = \rho_a$	$\rho_b = \rho_an^2$	$\rho_b = \rho_an$

II-C. Relation design parameters

In summary, the four balance conditions for a flexible beam are

$$\frac{1}{L_a}\sqrt{\frac{E_a}{\rho_a}} = \frac{1}{L_b}\sqrt{\frac{E_b}{\rho_b}} \quad , \quad \sqrt{\frac{E_bI_b}{L_b^2}} = \sqrt{\frac{E_aI_a}{L_a^2}} \quad , \tag{8}$$
$$m_bL_b = m_aL_a \quad , \quad \rho_bA_bL_b^2 = \rho_aA_aL_a^2.$$

These valid conditions present the relations between the design parameters of beam parts a and b. The six design parameters of part a (L_a, m_a, w_a, h_a, E_a and ρ_a) can be set at any realistic value by the user. Since there are 4 relations presented (Equation 8) and 6 unknowns design parameters of beam part b (L_b, m_b, w_b, h_b, E_b and ρ_b), 2 additional equations are needed to obtain a determined system of equations. By setting a ratio n between the two lengths, $n = \frac{L_a}{L_b}$, and making one last design choice, the system is determined and so the relations can be analysed. This last design decision could depend on the intended application. Four possible cases are given in Table I. The first column shows a case were the width of part a and b are equal. Note that the density scales here with n^3 which could be impossible to design in some applications. The other extreme is setting the density equal for beam part a and b, as shown in the second column of Table I, which causes the width to scale with n^3. An option in between the extremes is shown in the third column, where the Young's modulus is constant. Another intermediate option is shown in the fourth column. Next to the four shown cases, other cases for the intended application are possible as long as the design parameters meet Equation 8 and $n = \frac{L_a}{L_b}$. To visualize the effect of these relations, in case the length of beam b is smaller than beam a, beam b should be easier to bend and heavier in total weight than beam a.

III. Methodology for validation

This section provides the methodology used for the validation of the balance conditions. Sim-

Table II: Design parameters of beam part a and b when $n = 3$ and the case of an equal Young's modulus is chosen

Parameter	Meaning	Beam part a	Beam part b
L (m)	Length beam	0.1	0.033
m (kg)	Mass at tip	0.05	0.15
w (m)	Width beam	0.01	0.03
h (m)	Height beam	0.002	6.67×10^{-4}
E (Pa)	E modulus	5×10^{9}	5×10^{9}
ρ (kg/m^3)	Density	500	4500

ulations are performed for a flexible beam of which the design parameters are given in subsection III-A. The manner and size of actuating the flexible beam are given in subsection III-B. Finally, in subsection III-C, details about the simulation program SPACAR are provided.

III-A. Design parameters

The design parameters of beam part a and b are listed in the third and fourth column of Table II. As relation between the design parameters of the beam parts, an equal Young's modulus is chosen, case 3 in Table I. Next, n is set at 3 and the beam has a rectangular cross-section, where part b is solid and part a is, to maintain a lower density, nor solid or made of another material.

III-B. Actuation

There are four different ways to actuate the flexible beam which will be evaluated. The first way is exerting a force F at the tip of beam part a only. This situation is illustrated in Figure 3a. The size of F is a half sine during a time of 0.1s where the amplitude of the sine is 0.1N. In formula form written as $F = 0.1\sin(10\pi t)$. After this tenth second, no actuation is applied.

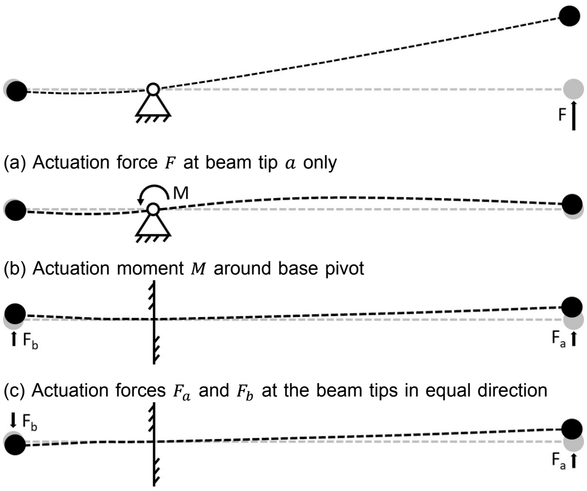
The second actuation way is actuating the flexible beam by a moment M around the base pivot as shown in Figure 3b. This actuation is a half sine during a time of 0.1s, where the amplitude of the sine is 0.01N m, based on $M = FL_a$.

Thirdly, two actuation forces in equal direction are exercised on both beam tips (Figure 3c). In this case, the beam parts are attached to the base fixed to prevent the beam from rotating. These forces have a value of $F_a = F_b = \frac{F}{2}$.

The fourth and last way to actuate the flexible beam is by exerting a force at both beam tips in opposite direction (Figure 3d). Again, these forces have a value of $F_a = F_b = \frac{F}{2}$.

III-C. Spacar

Simulations were performed with the dynamic simulation software SPACAR which is based on the non-linear finite element theory for multi-degree of freedom mechanisms [5]. The flexible



(a) Actuation force F at beam tip a only

(b) Actuation moment M around base pivot

(c) Actuation forces F_a and F_b at the beam tips in equal direction

(d) Actuation forces F_a and F_b at the beam tips in opposite direction

Figure 3: Four ways to actuate the flexible beam where the dashed black lines are the deformed shapes and the grey lines the initial shapes of the flexible beam

beam parts are modelled as 5 beam elements with equal distributed mass. The step size for this numerical model is variable, a minimal of 1×10^{-3} and maximum of 1×10^{-6}. The solver was set at ODE45 (Dromand-Prince) and the step size relative tolerance is 1×10^{-5}. The initial step size is 1×10^{-5}.

IV. Simulation result of the flexible beam

For all four types of actuation, a simulation was made. The first way of actuating was shown in Figure 3a and the forces during this simulation in the horizontal and vertical direction at the base pivot are plot in Figure 4. Due to the given actuation, the beam starts to rotate around its base pivot and the beam parts start to vibrate individually. When the beam is $\frac{1}{2}\pi$rad rotated, the x-force shows its maximum and the y-force its minimum. The time it takes for the beam to make one cycle, equals the cycle time of the sinus formed by the maximums and minima of the forces. So, the relation between the x- and y-force is explainable by the rotation of the entire beam. The vibration of beam part b is caused by the vibration in beam part a. Although, because only part a is actuated and so the vibration of part b lags behind, there is a phase shift between the two vibrating beams. The resulting forces appear because of this asynchronous behavior. Both beam parts have the same eigenfrequencies of 7.1Hz which was dependent on their design parameters. Because the beams vibrate with this same frequency, the phase shift remains equal

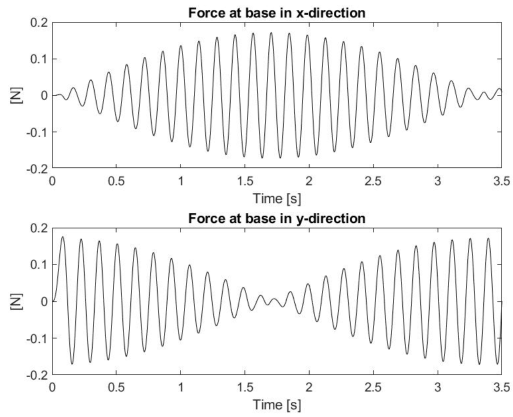


Figure 4: The resulting horizontal and vertical force at the base pivot during the simulation of the flexible beam actuated by force F

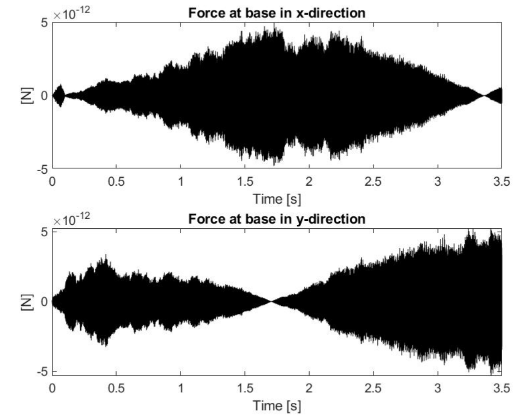


Figure 5: The resulting horizontal and vertical force at the base pivot during the simulation of the flexible beam actuated by moment M

and so the pattern of the resulting forces is stable.

The actuation moment M exercised at the flexible beam and illustrated in Figure 3b causes the horizontal and vertical forces plotted in Figure 5. The shown resulting forces are noise caused by integration errors of the simulation program. Next to the noise, no resulting forces are visible so this system is dynamically force balanced. Because this beam rotates around its base pivot, the system is not moment balanced. This rotation has the same angular velocity as the rotation caused by actuation force F. This is explainable by the fact that the actuation moment is set at $M = FL_a$.

The resulting forces due to the third actuation way, two actuation forces F_a and F_b in equal direction at both beam tips, are presented in Figure 6. Because of the equal actuation direction of both beam parts, they start vibrating synchronously up

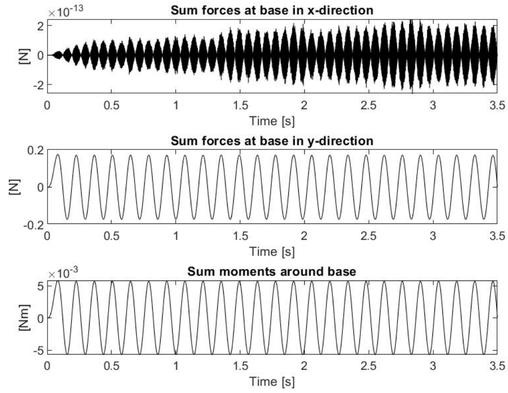


Figure 6: The resulting horizontal and vertical force and the resulting moment at the base during the simulation of the flexible beam actuated by equal force F_a and F_b in equal direction

and down. The results show noise and a frequency with a very small amplitude. The resulting forces at the base in the vertical direction are a result of that the beams are moving up and down at the same time so their resulting forces have the same sign. The frequency of these resulting forces equals the frequency the beams are vibrating in, 7.1Hz. The amplitude of these forces depends on the design properties of the beam. The shown resulting moment during the simulation is a result of the actuation forces and the beam parts lengths. At both tips, an equal actuation force is applied while the beam parts do not have the same length. The amplitude of the resulting moment equals the difference of the beam length times the resulting force. This system could be moment balanced by actuating the beam parts with a scaling force, so $F_b = nF_a$. Although in that case, the horizontal forces are not as low as they were for $F_b = F_a$.

The flexible beam parts actuated by two actuation forces F_a and F_b in opposite direction (Figure 3d), result in the simulation results plot in Figure 7. As before, because of the equal eigenfrequency of the beam parts and that they are actuated the same, the horizontal forces are simulation noise and a frequency with very small amplitude only. Because the forces are in opposite directions, the vertical forces show forces with a very small amplitude too so this system is dynamically force balanced. However, the resulting moment at the base is increased due to these opposite forces compared to the last system. The forces cause a moment in the same direction so the resulting moment remains.

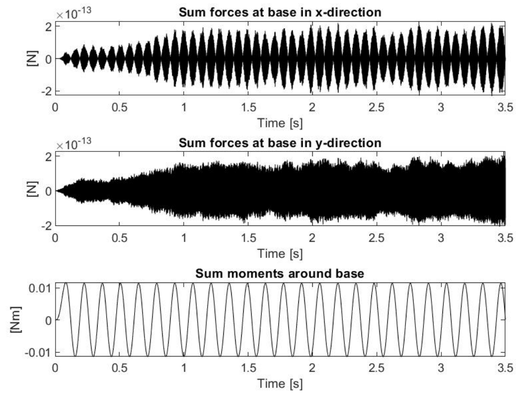
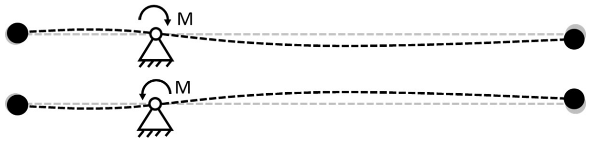
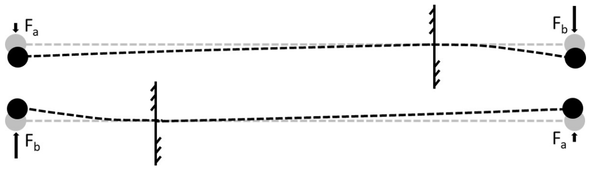


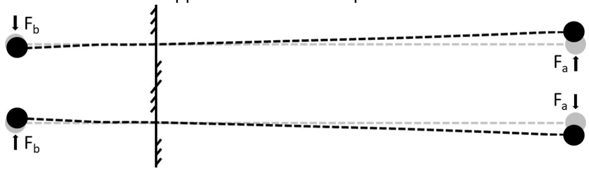
Figure 7: The resulting horizontal and vertical force and the resulting moment at the base during the simulation of the flexible beam actuated by eqaul force F_a and F_b in opposite direction



(a) Two flexible beams actuated by actuation moment M in opposite direction



(b) Two flexible beams both actuated by actuation forces F_a and F_b at the beam tips, where $F_b = nF_a$ and the forces are in equal direction at each beam and opposite direction compared to the other beam



(c) Two flexible beams both actuated by equal actuation forces F_a and F_b at the beam tips, in opposite direction at each beam and also opposite direction compared to the other beam

Figure 8: Three combinations of two flexible beams with different actuation types to achieve force and moment balance

V. Synthesis approach for dynamically balanced compliant mechanisms

Two flexible beams can be combined to achieve force and moment balance. The different actuation types lead to various options to achieve this of which three will be presented in this section.

The first synthesized system contains two flexible beams which are both actuated by an actuation moment M at their base pivot (Figure 8a). Because the direction of these actuation moments is opposite, the resulting moment of a single force balanced flexible beam is compensated by the opposite rotation of the second beam and so this system is dynamically force and moment balanced.

The second system, shown in Figure 8b, contains two flexible beams that are actuated by actuation forces F_a and F_b at the beam tips. The forces are in equal direction for the beam itself and in opposite direction compared to the other beam. The value of the forces scales with the beam length, thus $F_b = nF_a$ so the single flexible beam is moment balanced. The combination of these both moment balanced flexible beams leads to dynamic force balance too.

The third and last system is a combination of two flexible beams actuated by equal actuation force F_a and F_b at the beam tips in opposite direction (Figure 8c). The forces of the second beam are mirrored compared to the forces applied at the first beam. This results in opposite resulting forces of the two beams. This ensures, next to the force balance of each single beam, moment force balance for the combination too.

VI. Compliant Watch

This section shows how the force balanced flexible beam of Figure 3d can be applied for the synthesis of a compliant watch. First, the working principle, called the positioner system is explained. Next, the resulting forces during a simulation of the positioner system are presented. Lastly, the redesign of the compliant watch is shown.

VI-A. Positioner system

The positioner system is shown in Figure 9 and inspired by Lantz et al. his vibration resistant nanopositioner [8]. The positioner system contains four flexible beams, two at side a and two at side b, all illustrated by dashed lines in the illustration. The beams have the same design parameters as the values of the beam parts in Table II. The horizontal rigid beam (R_{hor}) in this system has a length of L_b and L_a for the left $R_{hor,b}$ and right $R_{hor,a}$ side respectively. This rigid beam is attached to the base pivot and actuated by an actuation moment M. This actuation introduces a translation of the vertical rigid beams, $R_{vert,b}$ and $R_{vert,a}$, which have both a length of 0.03m. These vertical beams are connected with revolute joints to the horizontal rigid beam and fixed to the tips of the flexible beams.

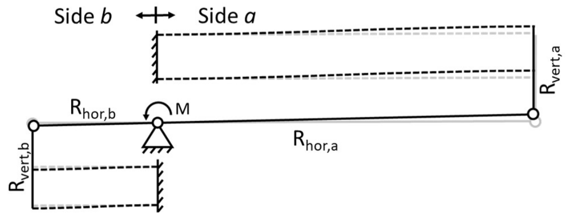


Figure 9: The positioner system actuated by actuation moment M where the dashed lines are flexible beams and the continuous lines are rigid beams

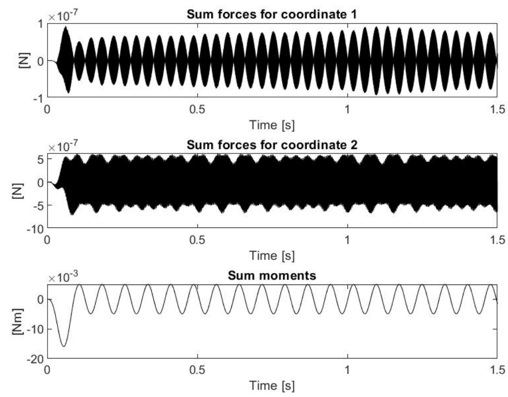


Figure 10: The resulting forces and moment at the base during the simulation of the positioner system that is actuated by actuation moment M

VI-B. Simulation of the positioner system

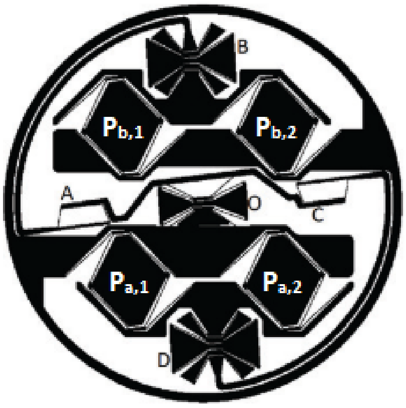
The actuation moment M has a value of $M = 0.02\sin(10\pi t)$ over the time until 0.1s and remains zero after this tenth second. The actuation moment introduces an equal force at each compliant beam tip of $F_a = F_b = 0.05\sin(10\pi t)$ overtime for the first tenth second, which equals the situation shown in Figure 3d. A plot of the resulting forces and moment at the base during the simulation is given in Figure 10. The resulting forces show a frequency and noise. Because the small amplitude of this frequency, this system is recognized as dynamically force balanced. The system is not dynamically moment balanced for the same reason as the flexible beam of Figure 3d is not balanced. The frequency of this resulting moment equals the eigenfrequency of the system. Compared to the system of Lantz et al. that has equal design parameters for sides, the new positioner system has scaled design parameters, so $n \neq 1$ [8]. The results show that even with these scaling design parameters, the system is force balanced.

Note, the positioner system can be adjusted for its application. For instance, changing the boundary conditions of the beam parts to one fixed and one jointed side, instead of two fixed sides as in the positioner system described in this section. Furthermore, masses can be added at both beam part tips. As long as the balance conditions are guaranteed, the positioner system is dynamic force balanced.

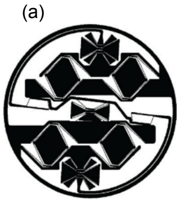
VI-C. Redesign

In literature, a force-balanced oscillator of Weeke et al. is found [18]. This oscillator is applied in the design of a compliant watch, shown in Figure 11. This system is force balanced successfully following the rigid body balance principles. The deformation in this system occurs in the lumped compliant parts, a concentrated part of the element [2]. The biggest disadvantage for applying the rigid body balancing principles for mechanisms containing lumped compliant parts is that force balance is possible for small rotations only [12, 13, 18]. When the watch is redesigned using the balanced flexible beams as building block, then the mechanism in Figure 12 can be obtained. This redesign is flexible and balanced but not limited for small rotations. The original design contains a revolute joint at location O, A, B, C, and D, as indicated in Figure 11. Next to these, the system includes four prismatic joints ($P_{b,1}$, $P_{b,2}$, $P_{a,1}$ and $P_{a,2}$). By changing the prismatic joints to flexible beams, the design shown in Figure 12 came up. The bottom half of this design shows the a side of the positioner system and the top half the b side. The four flexible beams are drawn with dashed lines where the rigid beams are drawn as continuous lines. Joint O, B, and D are the original compliant butterfly joints. As optimization of the design, joint A and C could be implemented too.

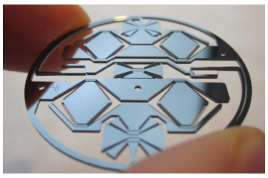
This new design has advantages. First, the system is simplified so the manufacturing process will be easier. Secondly, the top side of the shown design, equally to part b of the positioner, can be scaled by the balance conditions. By scaling the system, based on the design requirements, the design can be constructed smaller but heavier, or bigger but lighter. Finally, as already mentioned, even for bigger rotations the system holds its dynamic force balance. As an advantage of the original design, one could argue that the original prismatic joints have the option to implement some weight. This extra weight can lower the eigenfrequency. The adjusted version does not have that much space for this weight. Although, it depends on the application if this lower eigenfrequency is a requirement.



(a)



(b)



(c)

Figure 11: Original design of the force balanced oscillator of Weeke et al. where (a) is the oscillator in left rotated orientation, (b) the right rotated orientated oscillator and (c) the 1:1 scaled prototype [18]

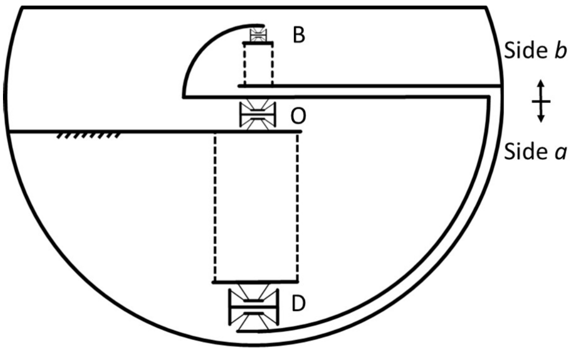
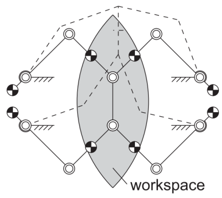


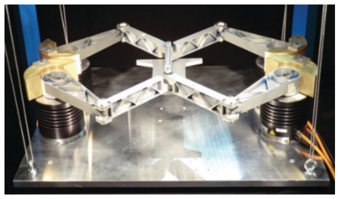
Figure 12: Redesign of the complaint watch, where the dashed lines are flexible beams and the continuous lines rigid beams that are connected by three compliant joints, B, O and D. The top halve contains two flexible beams that are positioner side b and the bottom equals positioner side a.

VII. DUAL-V-flex

The dynamically balanced system containing two flexible beams, actuated by an actuation moment M in opposite direction, described in section V and illustrated in Figure 8a, is applied for the design of a pick and place manipulator. The DUAL-V manipulator is taken as a starting point and adjusted. First, the working principle of the rigid body DUAL-V manipulator will be explained, later, the compliant redesign, referred as the DUAL-V-flex, and the performance results are shown.



(a) Representation with the indicated workspace of the vertical beam [16]



(b) The prototype of the experiments of van der Wijk et al. [16]

Figure 13: The original rigid body DUAL-V manipulator [16]

VII-A. Working principle of the DUAL-V manipulator

The research of van der Wijk et al. displays a dynamically balanced redundant planar 4-RRR parallel manipulator, mentioned as the DUAL-V manipulator [16]. A representation of this system is shown in Figure 13a. A photo of his prototype for his experiments is shown in Figure 13b. By rotating the four base pivots of this system, the vertical beam can be positioned everywhere in the workspace (indicated in grey in Figure 13a). For a non rotating platform and motion along the orthogonal axes, the system is dynamically balanced due to the added counter masses that cause opposite forces and moments. The experiments of the balanced DUAL-V manipulator of van der Wijk et al. showed 97% lower dynamic forces and 96% lower dynamic moments compared to an unbalanced variant.

VII-B. Redesign DUAL-V manipulator

Combining this current manipulator with the dynamically balanced flexible beams in Figure 8a, the DUAL-V manipulator could be designed to be compliant. An illustration of this redesign is shown in Figure 14. In this design, the compliant beam parts a (C_a) and b (C_b) appear four times each (*tr, tl, br, bl*) and equal the design requirements given in Table II. When applying an actuation moment at the four base pivots, the flexible beams start to rotate. This rotation causes a translation of the rigid V (*R-V*) and rigid center (*R-Center*) beams. For instance, if the two left pivots rotate clockwise and the two right pivots rotate counter-clockwise, the rigid center beam translates in the negative y-direction. After some translation, the two top rigid V beams are pushing stronger at the rigid center beam than the bottom rigid V beams are pulling. This behavior occurs because the top beams are aligning with the center beam while the bottom beams are increasing the angle compared to the center beam. This intern force in the rigid center beam induces a force at the compliant beam parts a. Because this force is not exerted at the compliant b beam parts, the behavior of the beam parts a and b is not synchronous anymore. As seen in the simulation of a force at one tip of the flexible beam (Figure 4), this introduces unbalance. In the original design of the DUAL-V manipulator, this phenomenon does not happen because rigid beams move synchronously always. Another case is when, instead of a resulting translation, the center beam rotates. For instance by giving the top left and bottom right pivot a clockwise actuation moment and the two others a counter-clockwise actuation moment. While the center beam rotates clockwise, this redesigned DUAL-V manipulator is dynamically force balanced. This is caused by the mirrored effect of the movement of the compliant beams, the dynamic forces remain zero, although a resulting moment is introduced.

To ensure these flexible beams behave synchronously, a design modification is proposed in Figure 15 and will be referred to as the DUAL-V-flex. In this redesign, an outer rigid square is introduced. This square is connected by the four rigid beams (*R-V-out*) to the tips of the compliant b beam parts (C_b). While the center beam translates in the negative direction due the actuation moments, the outer square moves in the positive y-direction. This ensures synchronous behaviour. The design parameters of the rigid beams are as follow. The length of the outer rigid beams (R_{outer}) are 0.8m, the length of the rigid inner (*R-V*) and outer V (*R-V-out*) beams is $\sqrt{2}$m and the length of the rigid center beam (*R-Center*) is 0.4m. The rigid beams are simulated stiff and massless. In the center of the rigid center beam is a point mass of $4m_a$ attached and at each corner of the outer rigid square a mass of m_b. No other point masses are located in this system.

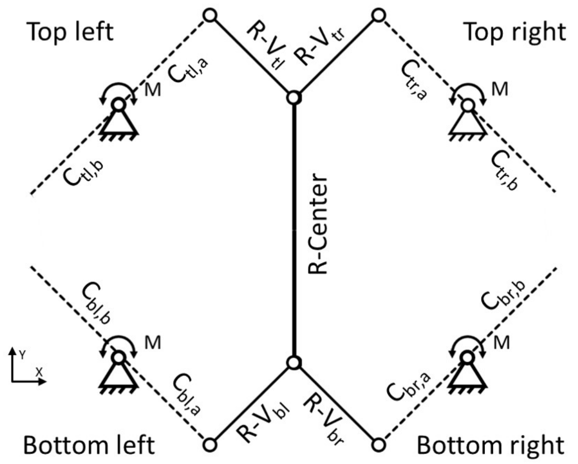


Figure 14: A first redesign of the rigid body DUAL-V manipulator where the actuated rigid beams are replaced by flexible beams

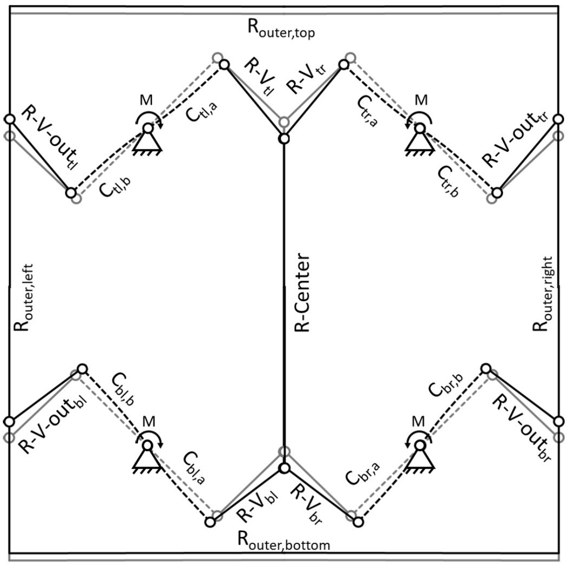


Figure 15: The DUAL-V-flex manipulator where grey shows the starting position and black the translated position

VII-C. Simulation DUAL-V-flex manipulator

As actuation, an actuation moment of $M = 0.01 * sin(10\pi t)$ is given for the first tenth second, in the clockwise direction for the two left pivots and counter-clockwise for the two right pivots. After 0.1s the actuation is zero and the mechanism continues its translation with constant speed. The simulation results show the forces and moment at the base of the DUAL-V-flex manipulator (Figure 16). The results show for the first 0.5s for resulting horizontal force and resulting moment noise only. For the resulting vertical forces, next to noise, a pattern can be recognized. Although, the amplitude is small of this resulting force so this mechanism is defined as dynamically balanced. After 0.5s the resulting forces and moment grow and so the system is unbalanced. It is questionable if this stadium will be reached when commissioning the system. In this simulation, at 0.7s the center beam has translated 2cm, this displacement could be enough for its application.

To modify the design to a monolithic compliant DUAL-V manipulator, the joints could be changed to compliant joints. Next, some design freedom is possible by scaling the beam parts b. While the balance conditions are guaranteed, many designs are possible for this dynamically balanced DUAL-V-flex manipulator.

VIII. Discussion

For its application as a building block in the synthesis of dynamically balanced compliant mech-

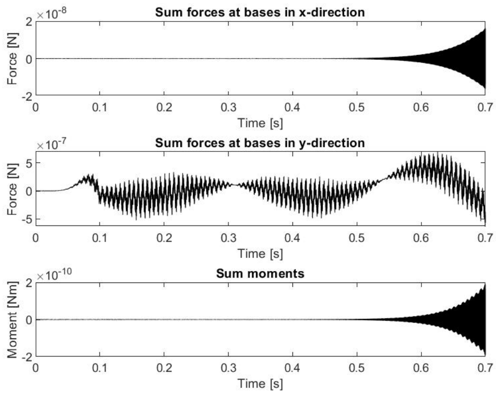


Figure 16: Resulting forces and moment at the base during the simulation of the DUAL-V-flex manipulator

anisms, the balance principles of a flexible beam were derived. The presented conditions followed from the principles and relate the design parameters of the a and b part of the flexible beam. The design parameters for the simulations of this paper are chosen in a way that the simulation time is decreased. A more realistic value of the Young's modulus and density should be chosen for the design of a prototype. Although, for the purpose of this paper, these unrealistic design parameters do not affect the conclusions, if the system is balanced or unbalanced. The use of metamaterials could be a solution to decrease the density while keeping a constant Young's modulus while building a prototype.

The design opportunities are limited by the requirement of equal boundary conditions of the beam parts, $C_{1_b} = C_{1_a}$ and $C_{2_b} = C_{2_a}$. For extra design freedom, the value of C_1 and C_2 for different boundary conditions must be calculated. To derive these, more research about the eigenfrequency of a beam is needed.

The redesigns were the flexible balanced beam is applied as a building block for the synthesis of dynamically balanced compliant mechanisms, present the opportunities that the new balance conditions offer. Multiple mechanisms could be redesigned with this new knowledge. During performing the simulations of the given redesigns, some extra DoFs were given to prevent over constraints. The behavior of the beam elements due to this given extra DoF was evaluated precisely. For long simulations that cause large displacements, the behavior of the mechanism was influenced by these DoFs. For instance, during the simulation of the DUAL-V-flex manipulator, the outer top and bottom rigid beam have a DoF in their length direction. While continuing the actuation moment for a longer time, these beams started to stretch. This gives an unrealistic behavior and thus unreliable results, nevertheless it could be the explanation of the growing forces in the simulation results after 0.5s. An experiment should be performed to validate this conclusion. Another solution to prevent the internal forces in the rigid center beam of the DUAL-V manipulator is by actively controlling the actuation moments. New research focusing on controlling this actuation should be set up to investigate this expectation.

IX. Conclusion

This paper its objective was to derive the dynamic balance principles of a flexible beam in order to use the flexible beam as a building block in the synthesis of dynamically balanced compliant mechanisms. The balance conditions resulted of two principles. First, the beam parts on each side of the base pivot should behave equally at any time. The given actuation should not disturb the equal behavior of the beam parts. Secondly, the wave speed in the beam parts should scale with the length ratio of beam parts. These principles and their balance conditions were validated successfully.

By combining force and moment balanced flexible beams, two dynamically balanced systems were synthesized. First, a positioner system was redesigned with balanced flexible beams and applied in the design of a compliant force balanced oscillator. This oscillator is a part of a compliant watch and due to these improvements, the watch could be fabricated easier and has more design freedom while it contains its force balance. Next, rigid beams in the DUAL-V manipulator were replaced by flexible balanced beams. After adding a connection between the tips of the compliant beams in the system, the, denoted as DUAL-V-flex manipulator, was dynamic balanced. This concludes that mechanisms containing compliant parts can be balanced dynamically successfully by the use of the presented balance principles for a flexible beam. The performance of these dynamically balanced compliant mechanisms is improved compared to the rigid and unbalanced variant.

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4 Discussion

In this thesis, dynamic balance principles based on a flexible beam have been derived for the synthesis of dynamic balanced compliant mechanisms. Two current mechanisms were redesigned to demonstrate the new opportunities. In this chapter, the advantages of the developed balance principles and the shown redesigns are listed and evaluated. Next, a numeration of research opportunities for these balance principles of the flexible beam are given. Lastly, a discussion about other ways, that are not based on a flexible beam, to design a dynamically balanced compliant mechanism is given.

4.1. Advantages of the balance principles of a flexible beam

The advantages of the balance principles of a flexible beam are listed and discussed.

- **Broad applicability** - From the balance principles, balance conditions for the design parameters of the beam parts follow. Nowadays, in most designs, the assumption is made the material is stiff. In the real world, nothing is 100% stiff, so considering the deformation of the material has advantages for every design.
- **Design freedom** - The balance conditions give freedom during the design phase. For instance, while comparing the initial positioner system with the redesign of the compliant watch, there is the freedom to choose for instance either the weight or size of the b beam part. This allows the designer to design a dynamic balanced compliant mechanism that fulfills his wishes.
- **More redesigns** - The first paper in chapter 2 displays several compliant mechanisms that are balanced unsuccessfully. The systems that contain distributed compliant parts were presented as unsuccessfully balanced due to only applying the rigid body balancing principles. Applying the balanced principles of a flexible beam presented in this thesis can solve this unbalance. For instance, the scanning mechanism of Sebek et al. and the flexible links of Kalas et al., can be redesigned dynamically balanced by applying the discovered balance principles for a flexible beam [5, 8]. This results in the conclusion that soon more mechanisms could be redesigned while considering the advantages of dynamic balance for their application.

4.2. Research opportunities for the balanced flexible beam

Some interesting aspects that were encountered during the research are listed as inspiration for continuing this work.

- **3D** - Current research is performed for 2D systems. Validating the expectation that the results agree for 3D mechanisms is an interesting investigation.
- **Gravity** - The assumption is made that the system is not influenced by gravity, for instance by a translation of the system in an orthogonal direction compared to the direction the gravity is pointing. If this is not the case and the gravity influences the mechanism, it is expected that the system is still balanced for most motions. Both parts of the mechanism are affected and deform

due to the same gravity force so this extra input does not let the system behave asynchronous. Despite the common CoM of the system is not in the base any more, it can still be stationary.

- **Monolithic** - In the shown simulations, the joints and some beams are not compliant. Interesting research can be considering the behavior of a compliant joint and replacing more rigid beams with flexible beams. This can cause extra behavior of the mechanism that should be considered during the design process but gives a more realistic view of the real working principle of the mechanism and design opportunities.
- **Experiments** - The simulations should be validated by experiments with a prototype.
- **Unequal boundary conditions** A requirement to use the balance conditions of a flexible beam is that the boundary conditions for both beam parts are equal. To find balance conditions that are valid even with unequal boundary conditions of the sides, more research calculate the eigenfrequency of a beam is needed.

4.3. Dynamic balance principles not based on the flexible beam

The literature survey in chapter 2 shows modal balancing of a flexible beam as the most potential method to dynamically balance a compliant mechanism. Although, next to the flexible beam, other structure like a combination of flexible beams, a flexible square or a flexible triangle could be used as a building block for the synthesis of a balanced compliant mechanisms too. This can be done by using a mirrored copy of the structure to balance the structure. Although, this limits the design freedom. The expectation is that the same balance principles as for a flexible beam should be considered to determine the balance conditions for other structures. From the balance principles, the scaled shape of and the wave speed in the structure, the structure depending balance conditions follow. These can be determined by the same theoretical approach that is used to obtain the conditions for the flexible beam.

5

Conclusion

The objective of this thesis was to derive dynamic balance principles for the synthesis of dynamically balanced compliant mechanisms. To do this, first, the potential of existing dynamically balance methods was investigated. Modal balancing of a flexible beam resulted as the method with the most potential for the synthesis of dynamic balanced compliant mechanisms because of force balanced was achieved for even big rotations.

Subsequently to complete set of balancing principles to balance a flexible beam were derived. The principle of synchronous opposite behavior of the beam parts resulted in three conditions that relate to the design parameters of the beam parts. In other words, they ensure an equal eigenfrequency of the beam parts. A requirement for these conditions is an equal boundary for both beam parts. A second principle followed from the fact that a disturbance should disturb its base at the same time in the opposite direction. This principle resulted in a fourth balance condition for the flexible beam. Taking into account these conditions while setting the design parameters, a flexible beam was designed for validation of these principles. The simulation results presented a perfect force balanced flexible beam when both beam parts were actuated equally. By scaling the actuation of the beam parts by the same scale as the length, the flexible beam was perfect moment balanced.

These flexible balanced beams were applied as a building block for the synthesis dynamically balanced compliant mechanisms. Two redesigns of existing mechanisms were presented. First, a known positioner system was redesigned to include the flexible balanced beams and applied in the design of a compliant force balanced oscillator. This oscillator is a part of a compliant watch and due to these improvements, the watch has the potential to be fabricated easier with more design freedom while maintaining its force balance. As a second example, rigid beams in know DUAL-V manipulator were replaced by flexible balanced beams resulting in the new flexible DUAL-V-flex manipulator. After adding a connection between the outer beam parts of the system, this system was dynamic balanced.

These examples have shown that dynamically balanced compliant mechanisms can be synthesized by the use of the balance principles of a flexible beam. Therefore, a solid foundation has been laid for designing a variety of dynamically balanced compliant mechanisms and hence for, improving the performance of the mechanisms in the high precision industry.

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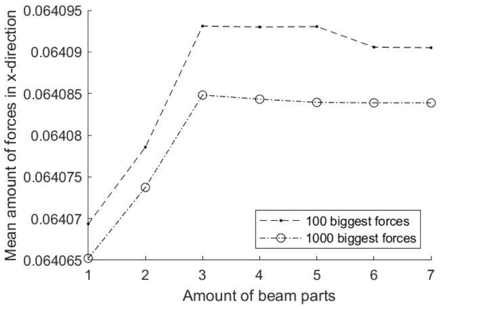
A

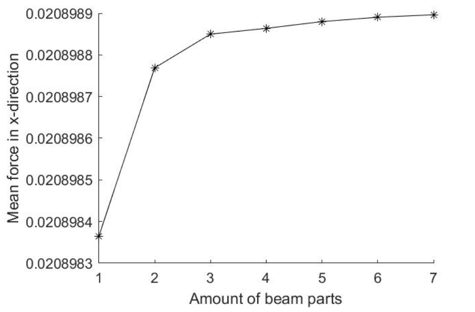
SPACAR linearization

The simulation program SPACAR linearises every beam element and calculates its behavior. The amount of element a beam is split in influence the result. In the presented results, the compliant beams are divided into 5 elements with equal length. Figure A.1 shows the development of the mean forces at the base pivot for a different amount of element. As seen, from five on the result is stable.

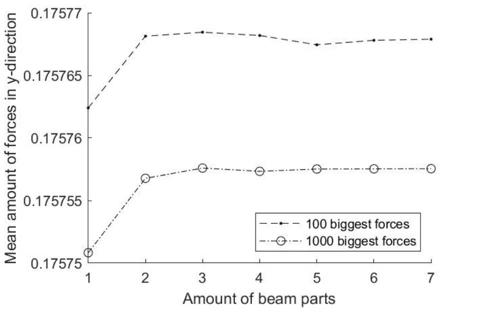
To conclude, the results containing 5 elements for their compliant beams give reliable results. Taking more elements per beam gives equal results but takes more calculation time. Splitting the beam in way fewer elements can give less exact results due to linearization errors.



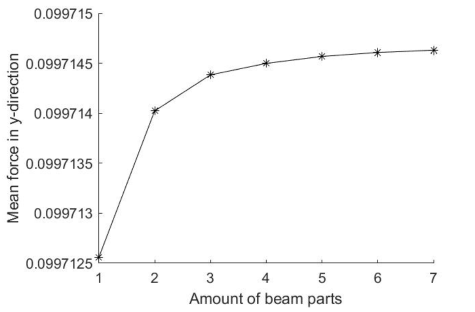
(a) Relation between the amount of beam elements and the mean of a certain amount of biggest resulting forces in x-direction



(b) Relation between the amount of beam elements and the mean of the resulting forces in x-direction



(c) Relation between the amount of beam elements and the mean of a certain amount of biggist resulting forces in y-direction



(d) Relation between the amount of beam elements and the mean of the resulting forces in y-direction

Figure A.1: Development of the resulting forces at the base while changing the amount of beam parts for the flexible beam that due to a force at the tip of one beam part only is unbalanced