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# Predictive maintenance for multi-component systems of repairables with Remaining-Useful-Life prognostics and a limited stock of spare components

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# ABSTRACT

Keywords: Aircraft predictive maintenance of repairables RUL prognostics Aircraft Cooling Units Management of spare components Multiple multi-component systems Aircraft maintenance is undergoing a paradigm shift towards predictive maintenance, where the use of sensor data and Remaining-Useful-Life prognostics are central. This paper proposes an integrated approach for predictive aircraft maintenance planning for multiple multi-component systems, where the components are repairables. First, model-based Remaining-Useful-Life prognostics are developed. These prognostics are updated over time, as more sensor data become available. Then, a rolling horizon integer linear program is developed for the maintenance planning of multiple multi-component systems. This model integrates the Remaining-Useful-Life prognostics with the management of a limited stock of spare repairable components. The maintenance of the multiple systems is linked through the availability of spare components and shared maintenance time slots. Our approach is illustrated for a fleet of aircraft, each equipped with a Cooling System consisting of four Cooling Units. For an aircraft to be operational, a minimum of two Cooling Units out of the four need to be operational. The maintenance planning results show that our integrated approach reduces the costs with maintenance by 48% relative to a corrective maintenance strategy and by 30% relative to a preventive maintenance strategy. Moreover, using predictive maintenance, components are replaced in anticipation of failure without wasting their useful life. In general, our approach provides a roadmap from Remaining-Useful-Life prognostics to maintenance planning for multiple multi-component systems of repairables with a limited stock of spares.

# 1. Introduction

Aircraft maintenance is key for safe and efficient airline operations, with airlines spending approximately 9% of their total operation costs on Maintenance, Repair and Overhaul, which, in 2018, was estimated to be 69 billion dollars [1]. Striving for cost savings, aircraft maintenance is currently shifting from corrective or preventive maintenance towards predictive maintenance. For predictive maintenance, sensors are continuously monitoring the health of components and systems, algorithms are generating Remaining-Useful-Life (RUL) prognostics, and maintenance is performed based on these prognostics in anticipation of failures [2]. One of the main challenges of predictive maintenance is to obtain Remaining-Useful-Life (RUL) prognostics for systems and components. RUL prognostics support a high exploitation time of the systems and components, while limiting Aircraft-On-Ground events due to unexpected failures. Equally challenging is to integrate RUL prognostics into the aircraft maintenance planning, while the entire complexity of this process is taken into account: the management of spare components, the availability of maintenance slots during which the aircraft can be maintained, and system reliability requirements.

Most studies focus solely on developing RUL prognostics using either a model-based, a data-driven or a hybrid approach [3]. Modelbased approaches are proposed in, for instance, [4,5]. In [4] a twofactor state-space model of the degradation is used to develop RUL prognostics, with an application to rolling element bearings. In [5], particle filtering is combined with a support vector data description to obtain RUL predictions for engines. In this paper, we also propose a model-based approach to obtain RUL prognostics for Cooling Units (CUs) of wide-body aircraft. However, our focus does not lie on developing RUL prognostics only, but also on proposing a maintenance planning model that integrates such prognostics.

For predictive maintenance planning, threshold-based maintenance policies are frequently used [6], i.e., as soon as the degradation of a component exceeds a threshold, a maintenance action is planned [7–13]. Optimal moments for such maintenance actions and degradation thresholds are determined using Monte Carlo simulation [8,13, 14], semi-regenerative processes [7,11], Bayesian networks [9], or heuristics [10,12].

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Received 1 October 2020; Received in revised form 19 April 2021; Accepted 4 May 2021 Available online 12 May 2021 0951-8320/© 2021 The Authors. Published by Elsevier Ltd. This is an open access article under the CC BY-NC-ND license (http://reativecommons.org/licenses/by-nc-nd/4.0/). Other frequently used maintenance planning approaches focus on a non-restrictive policy search space using Markov Decision Processes (MDPs) [15,16] and Partially Observable Markov Decision Processes (POMDPs) [6,17–19]. In [16] an MDP is formulated for the maintenance optimization of a system subject to both failures due to gradual deterioration and to abrupt, sudden failures. In [17] POMDPs are proposed to model predictive maintenance planning, with a focus on civil engineering structures. This methodology is further applied to obtain an optimal maintenance planning for concrete structures in [18]. Also in [19] a continuous-state POMDP formulation is proposed for the maintenance of civil structures. One of the challenges for (PO)MDPs is the large computational time needed [15,17]. To address this issue, [17] propose a point-based algorithm, while [6] develop a deep reinforcement learning algorithm with applications to the maintenance of steel bridge structures.

Only a few studies, however, develop prognostics models and integrate them in the maintenance planning. In [20], the RUL of a rolling element bearing is predicted with a feedforward Neural Network. Based on these prognostics, maintenance is planned using a search algorithm. In [21], an exponential model is developed to predict the RUL of a rolling element bearing, and maintenance is planned just before the predicted failure time. In [22], an exponential model is also used to predict the RUL of a rolling element bearing. With this, optimal maintenance moments and ordering times of spare components are determined. In [23], the RUL of an aircraft component is predicted using a Short Long-Term Neural Network. This is used to determine optimal times to order new spare parts and plan maintenance as well. In [24], an extended Kalman filter is developed to predict the crack growth in an airframe of an aircraft. Using these predictions, maintenance for the airframe is planned. However, all these studies consider the maintenance planning of only one component, while in this study we consider multiple multi-component systems.

In [25], RUL prognostics for an aircraft hydraulic system, consisting of multiple sub-systems, are developed using a Kalman filter. With this, a maintenance planning for a single aircraft is proposed using an exhaustive search strategy. In contrast, we plan maintenance for multiple aircraft, i.e., multiple multi-component systems, that are linked through the availability of spare components and shared maintenance opportunities.

Last, but not least, the consideration of spare parts for predictive and condition-based maintenance (with or without integrated RUL prognostics) is crucial. One cannot execute a maintenance replacement without having a spare component to perform the replacement with. Many studies determine an optimal component replacement time and assume that a spare component is always available at these times [20,21,26]. Other studies determine optimal times to order one-time-use, non-repairable components [10,22,23]. For aircraft, however, many components are repairables, i.e., a failed component is sent to a repair shop to be repaired (overhauling [27]). Ordering repairable components is either expensive and/or it takes a long time to receive these components from the manufacturer. In general, the airlines repair and reuse components or, if really necessary, lease a new component. The lease is ended as soon as an own spare component is repaired. Our approach proposes a predictive maintenance planning model for repairables. To the best of our knowledge, this is the first study that considers predictive maintenance planning for repairable components of multi-component systems [27]. While this is relevant for aircraft maintenance, a similar approach can also be used for the maintenance planning of repairable components for other systems and domains.

In this paper we propose a rolling horizon maintenance planning model for multiple multi-component systems of repairable components. This rolling horizon maintenance planning model integrates (i) modelbased RUL prognostics for the components, (ii) the availability of spare components and, (iii) available maintenance time slots when an aircraft could be maintained (see Fig. 1). Moreover, the planning model incorporates a reliability constraint for each multi-component system.



Fig. 1. An integrated maintenance planning approach with Remaining-Useful-Life prognostics for components, the management of spare components and fixed maintenance opportunities..

The RUL prognostics are generated using a model-based approach with a particle filtering algorithm. Over time, as more sensor data becomes available, these prognostics are updated. The updated RUL prognostics are then used in each time window of the rolling horizon maintenance planning model to decide which components to replace. A linear integer program is proposed to solve the maintenance planning problem.

To illustrate our approach, a case study with a fleet of 13 widebody aircraft, each equipped with a multi-component system of Cooling Units (CUs), is considered. An optimal maintenance planning for CU replacements for the fleet of aircraft is obtained using a rolling horizon approach. The performance of this planning in terms of maintenance costs, number of replacements and number of system failures is analyzed. Lastly, the long-term performance of our prognostic-based maintenance planning model is compared against a corrective and a preventive maintenance strategy. The results show that our model outperforms these two strategies with respect to maintenance costs and the number of Aircraft-On-Ground events.

The main contributions of this paper are as follows:

- An integrated, rolling horizon maintenance planning model for a *fleet* of aircraft, each equipped with a system of multiple *repairable* components, is developed. This maintenance planning integrates model-based Remaining-Useful-Life prognostics with the management of a *limited stock of spare* repairable components.
- A realistic maintenance setting is considered, where aircraft maintenance can only be performed during pre-defined time slots, during which the aircraft is on ground and can undergo maintenance.
- The overhauling of repairable components is considered, i.e. a limited total number of spare components is assumed to be available. Upon failure, a component is sent to a repair-shop. Once repaired, the component is returned to the pool of spares components. The overhauling of repairable components has been identified as a research gap in [27].
- A predictive maintenance planning model is developed for *multiple multi-component* systems. The maintenance of multiple systems is linked through the availability of spare repairable components and shared maintenance opportunities.

The remainder of this paper is structured as follows. In Section 2 we provide the problem description and introduce the main notations. We then develop model-based RUL prognostics for aircraft Cooling Units in Section 3. In Section 4 we develop an integrated maintenance planning model for a fleet of aircraft, each equipped with a multi-component system of repairable components. This model integrates the RUL prognostics, the management of a limited stock of spare components, and the available maintenance slots. In Section 5 we illustrate our model for a fleet of wide-body aircraft, each equipped with a multi-component

system of Cooling Units. The performance of our prognostics-based maintenance planning model is compared against a corrective and a preventive maintenance strategy in Section 6. Lastly, Section 7 provides conclusions and recommendations for future research.

#### 2. Problem description

We consider a discrete time model, where every day d there are decisions made regarding the maintenance planning of the aircraft. These decisions are based on the Remaining-Useful-Life prognostics of the aircraft components, the available spare components and the available time slots in which maintenance can be performed.

### 2.1. Multi-component aircraft system

Let A denote a fleet of aircraft. Each aircraft has a system of multiple, identical repairable components. Let  $C_a$ ,  $a \in A$ , denote the set of components of this system in aircraft  $a \in A$ . Each component is assumed to fail independently of the other components. When a component fails, it is replaced with an as-good-as-new one. A replacement can also be triggered by the Remaining-Useful-Life prognostic of this component, in anticipation of a failure. The installation day of the as-good-as-new component is denoted by  $d_{ac}^{install}$ ,  $a \in A$ ,  $c \in C_a$ . At the same time, the removed component is sent for repair.

The aircraft is said to be in an *Aircraft-On-Ground* (AOG) condition and, thus, can no longer fly, if this multi-component system fails. A system is considered to be failed when the number of failed components exceeds the number of minimum allowed component failures, as specified by the Minimum Equipment List (MEL) [28].

### 2.2. Maintenance slots

A maintenance slot is a time interval during which maintenance on an aircraft can be performed [24,25]. Over time, there is a sequence of slots *S*. Each slot  $s \in S$  has a capacity  $m_s$  specifying the number of aircraft that can be simultaneously maintained during this slot. There is no limit on the number of components that can be replaced per aircraft within a maintenance slot. For a specific aircraft  $a \in A$ , the set  $S_a \subseteq S$ specifies the slots in which aircraft *a* can be maintained. A slot *s* starts during day  $d_s$ . The cost of maintaining an aircraft in slot *s* is  $c_s$ .

### 2.3. Repairable components

We plan maintenance for repairable components, i.e., after removal the component undergoes a repair process such that it can be used again instead of being discarded [27,29]. When a component fails, it is removed from the aircraft while a spare, as-good-as-new component from the stock is installed instead. The faulty component is repaired. This repair takes  $\Delta$  days. Once repaired, the component is added to the stock. We assume that a component is in an as-good-as-new condition once repaired. There is a limited amount of spare components (limited stock). A component is leased from an external supplier if there are no spares in stock when a component is replaced. We assume that a leased component is immediately available for installment. For the prognostics and the case study, we consider the repairable aircraft Cooling Units.

There is a fixed cost  $c^{\text{Lf}}$  for leasing a component. Additionally, a cost  $c^{\text{Ld}}$  is incurred for every day the component is leased. Lastly,  $c^{\text{fix}}$  denotes the cost of repairing a component that is not failed but for which the RUL prognostic indicates a failure in the near-future. If, however, the component is already failed at the time of replacement, then a cost  $c^{\text{fix}} + c^{\text{ex}}$  is incurred to repair the component. It is thus more costly to replace a failed component than a non-failed component with a predicted failure in the near-future.

# 2.4. Remaining-Useful-Life (RUL) prognostics

Each component  $c \in C_a$  of aircraft *a* is monitored by sensors. Based on the available sensor measurements, at current day  $d_0$ , a RUL prognostic for each component is made. Based on these RUL prognostics, we determine  $P_{acd}^{fail}$ , the probability that component *c* of aircraft *a* fails by the beginning of day  $d > d_0$ . The RUL prognostic model and  $P_{acd}^{fail}$  are discussed in detail in Section 3.

model and  $P_{acd}^{fail}$  are discussed in detail in Section 3. Based on  $P_{acd}^{fail}$ , the probability of a system failure at the beginning of day  $d > d_0$ , or equivalently, the probability of the aircraft being in an AOG-condition at the beginning of day d, denoted by  $P_{ad}^{AOG}$ , is determined.

#### 2.5. Maintenance planning objective

Taking into account i) the maintenance slots available for each aircraft to undergo maintenance, ii) the RUL prognostic of each aircraft component and iii) the available spare components, we are interested in assigning the aircraft to maintenance slots, such that the total cost of the maintenance planning is minimized. Furthermore, for each aircraft assigned to a maintenance slot it is determined which components of this aircraft are replaced.

#### 2.6. Rolling horizon maintenance planning

We determine a maintenance planning using a rolling horizon approach [12,30,31]. In each iteration of the rolling horizon approach, we optimize the maintenance planning for a time window of *PH* days, that starts at day  $d_0$ . At the beginning of this time window, we have: i) all the maintenance slots available during this time window, given by the set *S*, ii) the RUL prognostics for each component and for each day  $d \in [d_0, d_0 + PH)$  (i.e.,  $P_{acd}^{fail}$  is specified for each day *d* within the time window, and for each component  $c \in C_a$  of each aircraft  $a \in A$ ), and (iii) the number of spare components initially available at the beginning of each day  $d \in [d_0, d_0 + PH]$ , denoted by  $S_d^{\text{begin}}$ . If initially, components are leased at the beginning of day *d*, then  $S_d^{\text{begin}}$  is negative. For the first time window, a maintenance planning are then fixed, and the time window is moved forward  $\tau$  days. Here,  $\tau \leq PH$ . Next, a new maintenance planning is created for several successive time windows.

An example of the rolling horizon approach is given in Fig. 2. Here, there are three iterations of the rolling horizon procedure, with a time window of PH = 15 days that moves forward  $\tau = 5$  days each iteration. The first iteration (Fig. 2(a)) starts at day  $d_0 = 120$ . All decisions regarding the maintenance planning before day  $d_0 = 120$  and day  $d_0 + PH = 135$  is under optimization. Then, the decisions of the first  $\tau = 5$  days of this maintenance planning are fixed and the time window is moved  $\tau = 5$  days forwards. In the next iteration (Fig. 2(b)), the maintenance planning is optimized between day  $d_0 = 125$  and day  $d_0 + PH = 140$ . This is repeated for the last iteration as well (Fig. 2(c)). Also, at the beginning of each iteration the RUL prognostics of the components are updated. This is illustrated for a component  $c \in C_a$ of an aircraft  $a \in A$ .

#### 3. Remaining-Useful-Life prognostics for aircraft Cooling Units

In this section, using sensor measurements, we determine modelbased Remaining-Useful-Life prognostics for aircraft Cooling Units .



(c)  $d_0 = 130$ .

Fig. 2. Illustration of the rolling horizon approach and the update of the prognostic information,  $\tau = 5$  days, PH = 15 days.

### 3.1. Aircraft Cooling Units (CUs)

All considered aircraft are equipped with 4 identical Cooling Units (CUs). The CUs are part of the Cooling System, which cools the air of the aircraft's galleys. Fig. 4 shows a schematic overview of one CU, consisting of a condenser, a flash tank, an evaporator and a compressor. Fig. 5 shows a schematic overview of the Cooling System in an aircraft, where there are 4 CUs that are integrated with a Pump, Galley Cooling Units and Air Heat Exchangers.

#### 3.2. Health indicator for CUs

As the CU (the aircraft) is increasingly used over time, the filter gets clogged, accelerating the compressor wear, which ultimately leads to a failure. We consider nine sensors monitoring the CUs. Fig. 3 shows the

mean and maximum sensor measurement per day until failure for one CU and for each of the nine available sensors. For the purpose of our analysis, the data sets are anonymized.

Let  $\delta_d$  denote the flight time during the *d*th day when this CU is in use, i.e.,  $\delta_d$  is the number of valid sensor measurements larger than a threshold  $\varphi = 0$ . Let  $y_{d,b}^s$  denote the *b*th valid sensor measurement during day *d* for this CU, generated by a sensor *s*. We normalize the measurements during day *d* as follows:

$$\tilde{y}_{d,b}^{s} = \frac{y_{d,b}^{s}}{\max_{s} - \max_{b \in 1, \dots, \delta_{d}}(y_{d,b}^{s})},$$
(1)

with  $\max_{\boldsymbol{s}}$  the overall maximum measurement generated by sensor  $\boldsymbol{s}.$ 

We then define the health indicator  $m_d^i$  of CU *i* at day *d* as follows:

$$n_d = \frac{1}{n} \sum_{j=d-n}^d \frac{1}{\delta_j} \sum_{b=1}^{\delta_j} \tilde{y}_{j,b}^s, \quad n > 1.$$
(2)

Our health indicator combines the increasing maximum sensor measurement towards failure (see Fig. 3) and the increasing mean sensor measurement towards failure (see again Fig. 3), while it is at the same time independent of the length of the flights during a day d. For our analysis, we select for the health indicator the sensor with the largest correlation coefficient with the time to failure [32,33], which in our case is sensor 8 with a correlation coefficient of 0.77. Fig. 6 shows the health indicator obtained 30 days before failure for 5 CUs. For all CUs, the increase in the health indicator accelerates towards failure.

# 3.3. RUL prognostics for CUs

Based on the health indicator  $m_d$ , we now determine the RUL prognostics for each of these components. There are two phases for the health indicator. In the first phase, this component is only monitored and the health indicator  $m_d$  is recorded every day d.

As soon as the health indicator reaches a prognostics threshold  $T^P$ , i.e., as soon as  $m_d > T^P$ , a second phase begins where a prognostic for the RUL of this component is determined. In this second phase, we consider the true degradation level of this component, denoted by  $x_d$ , and the health indicator  $m_d$  at day d as follows:

$$x_d = x_{d-1} + \alpha_d \lambda_d e^{\lambda_d d},\tag{3}$$

$$m_d = x_d + v_d, \tag{4}$$

where  $\alpha_d \sim N(\mu_{\alpha}, \sigma_{\alpha}^2), \lambda_d \sim N(\mu_{\lambda}, \sigma_{\lambda}^2)$ , and  $\nu_d \sim N(0, \sigma_{\nu}^2)$  are model parameters.

The exponential functional form in Eq. (3) is assumed since the cumulative damage in a component has an effect on the degradation rate of the component [34]. An exponential degradation model is a good approximation for non-linear degradation processes such as corrosion, bearing degradation and the deterioration of LED lighting [35–39]. The CU can also be seen as subject to accelerated wear due to increasing filter clogging.

Next, we estimate the RUL of this component using a particle filtering algorithm (see, for instance, [40]). We consider recorded health indices  $m_d$  for this component up to the current day d. Based on these indices, we estimate the RUL of this component as follows. We initialize  $x_0$  with the measured health levels prior to the second phase. We consider n initial particles  $(x_0^{(i)}, \alpha_0^{(i)}, \lambda_0^{(i)})$ ,  $i \in \{1, 2, ..., n\}$ , each with initial weight 1/n. Then, new particles  $(x_d^{(i)}, \alpha_d^{(i)}, \alpha_d^{(i)})$  are obtained as follows:

$$x_{d}^{(i)} = x_{d-1}^{(i)} + \alpha_{d}^{(i)} \lambda_{d}^{(i)} \exp(\lambda_{d}^{(i)}d),$$
(5)

where  $\alpha_d^{(i)}$  and  $\lambda_d^{(i)}$  are realizations of the random variables  $\alpha_d$  and  $\lambda_d$ , respectively.

As *d* increases, and new measurements are available, the weights of the particles are updated and normalized with

$$p(m_d | x_d^{(i)}) = \frac{1}{\sqrt{2\pi}\sigma_v} exp\left(-\frac{1}{2}\left(\frac{m_d - x_d^{(i)}}{\sigma_v}\right)^2\right).$$



Fig. 3. Mean and maximum sensor measurement per day for one CU for all nine available sensors. This CU fails at day 48.

Now, given the weights of the particles, these particles are resampled [14] and, again, their weights are updated as 1/n. Lastly, the RUL  $z_d$  of this component is predicted at the current day *d* based on the re-sampled particles and the measurements up to and including day *d*, where the RUL  $z_d$  is defined as:

$$RUL = \inf\{z_d : x(d+z_d) \ge D | x_0, x_1, \dots, x_d\},$$
(6)

where *D* is a pre-defined failure threshold,  $x_0, x_1, \ldots, x_d$  are the estimated degradation levels of this component at days  $0, 1, \ldots, d$ , respectively, and  $x(d + z_d)$  is the predicted degradation level at time  $d + z_d$ . We use Eq. (6) to predict the RUL  $z_d^i$  of each individual particle *i* in the particle filtering algorithm as follows:

$$z_d^i = \inf\{z_d^i : x_{d+z_d^i}^{(i)} \ge D|x_0^{(i)}, x_1^{(i)}, \dots, x_d^{(i)}\}.$$
(7)

Here,  $x_0^{(i)}, x_1^{(i)}, \dots, x_d^{(i)}$  are the estimated degradation levels of particle *i* at days  $0, 1, \dots, d$ , respectively, and  $x_{d+z_d}^{(i)}$  is the predicted degradation level of particle *i* at time  $d + z_d$ .

Lastly, the probability that the RUL equals  $z_d$  at current day d is approximated by:

$$p(RUL = z_d | m_0, m_1, \dots, m_d) = \sum_{i=1}^n w_d^{(i)} \mathcal{D}(z_d - z_d^i),$$
(8)

where  $w_d^i$  is the weight of the *i*th particle, and  $\mathcal{D}(.)$  is a Dirac function:

$$\mathcal{D}(y) = \begin{cases} 1 & y = 0, \\ 0 & y \neq 0. \end{cases}$$
(9)

From Eq. (8), which provides the pdf of the RUL obtained at current day *d* for a component  $c \in C_a$  of aircraft  $a \in A$ , we obtain the probability  $P_{acd^*}^{fail}$  that this component *c* of aircraft *a* fails before some future day  $d^* > d$  as follows:

$$P_{acd^*}^{\text{fail}} = P(RUL \le (d^* - d)).$$
(10)



Fig. 4. Schematic overview of a Cooling Unit.

Thus, given a current day d, Eq. (10) determines the probability of failure before a day  $d^* > d$  for a specific CU. If, however, the CU is in the first, monitoring-only phase, than we assume that  $P_{acd^*}^{\text{fail}} = 0.001$ .

### 3.4. Results — prognostics for CU

Following the methodology in Section 3.3, we determine the RUL prognostics for CUs using 1000 particles,  $\sigma_v = 0.01$ , n = 10 days, D = 22 and  $T^P = 11$ . Furthermore, we determine  $\mu_{\alpha}$ ,  $\mu_{\lambda}$ ,  $\sigma_{\alpha}^2$  and  $\sigma_{\lambda}^2$  using Maximum Likelihood Estimation of  $\alpha$  and  $\lambda$  on the log transformation of Eq. (3) on the available data [41]. Fig. 7 shows the pdf of the RUL and the distribution of  $P_{acd}^{fail}$  of a CU c of an aircraft a estimated at day 339 (15 days before failure), day 344 (10 days before failure) and at day 349 (5 days before failure) since the start of the monitoring phase. The RUL estimation is precise, i.e., the actual RUL always falls within the probability distribution of the predicted RUL, while the uncertainty in the prediction is low. For all prediction horizons, the actual RUL



Fig. 5. Schematic overview of the Cooling System.



Fig. 6. The health indicator  $m_d^i$  for 5 CUs *i* 30 days before failure.

is slightly underestimated. For this CU, it takes on average 14.4 s to estimate the RUL distribution using a computer with an Intel Core i7 processor at 2.11 GHz and 8Gb RAM.

#### 4. Predictive maintenance planning model for a fleet of aircraft

Using the prognostics obtained in Section 3, as well as information about the availability of maintenance slots and spare components, we now introduce a linear integer program to plan the maintenance of multiple aircraft systems of repairable components. This model is applied, using a rolling horizon approach, for a planning time window of *PH* days  $[d_0, ..., d_0 + PH)$  (see Section 2.6 and Fig. 2), and for a fleet of aircraft.

We first introduce some additional notation and definitions.

**Definition 1.** An aircraft is said to be *critical* when the probability that this aircraft is in an *AOG-condition* at the end of the planning time window  $[d_0, ..., d_0 + PH)$  exceeds a reliability threshold *r*, i.e.,  $P_{a(d_0+PH)}^{AOG} \ge r$ .

Let  $A_r \subseteq A$  denote the set of critical aircraft at the beginning of the planning time window  $[d_0, \dots, d_0 + PH)$ .

Let  $G_a$  denote the set of all possible subsets of the components of aircraft  $a \in A_r$  that can be replaced in the planning time window

 $[d_0, \ldots, d_0 + PH)$ , such that  $P_{a(d_0+PH)}^{AOG} < r$ . We assume that once a component is replaced in a planning time window, then this component cannot fail anymore in the same time window. The set  $G_a$  depends on the configuration of the multi-component system. To illustrate  $G_a$ , we discuss an example of a system where the components are linked in series, i.e., if one component fails, the whole system fails. Let critical aircraft *a* have a system consisting of 4 components in series,  $C_a = \{1, 2, 3, 4\}$ . Let the probability of failure for component  $k \in \{1, 2\}$  by day  $d_0 + PH$  be  $P_{ak(d_0+PH)}^{fail} > r$ . Let the probability of failure for component  $k \in \{3, 4\}$  by day  $d_0 + PH$  be  $P_{ak(d_0+PH)}^{fail} \ll r$ . Then, at least component 1 and 2 must be replaced to ensure that  $P_{AOG}^{AOG} < r$ . The set  $G_a$  of component subsets that can be replaced to avoid the aircraft being in an AOG-condition is thus:

$$G_a = \{\{1,2\},\{1,2,3\},\{1,2,4\},\{1,2,3,4\}\}$$

We now introduce the decision variables, objective function and constraints of the predictive maintenance planning model with RUL prognostics and limited spare components.

#### Decision variables

We consider the following decision variable.

$$X_{acs} = \begin{cases} 1, & \text{component } c \in C_a \text{ of aircraft } a \in A \\ & \text{is replaced in maintenance slot } s \in S_a, \\ 0, & \text{otherwise.} \end{cases}$$

We also consider the following three auxiliary variables which (i) keep track of the maintenance planning for an entire aircraft, (ii) keep track of the number of leased components at the end of a day, and (iii) keep track of the number of newly leased components during a day. First,

$$Y_{as} = \begin{cases} 1, & \text{aircraft } a \in A \text{ is assigned to slot } s \in S_a, \\ 0, & \text{otherwise.} \end{cases}$$

Here, the auxiliary variable  $Y_{as}$  is defined by the decision variables  $X_{acs}$  as follows:

$$Y_{as} \ge X_{acs}, \qquad \qquad \forall a \in A, \, \forall c \in C_a, \, \forall s \in S_a \tag{11}$$

$$Y_{as} \le \sum_{c \in C_a} X_{acs}, \qquad \qquad \forall a \in A, \ \forall s \in S_a, \qquad (12)$$

where Eq. (11) ensures that when a component  $c \in C_a$  of aircraft  $a \in A$  is replaced in maintenance slot  $s \in S_a$ , the entire aircraft is assigned to



(a) The estimated pdf of the RUL for one CU c of aircraft a estimated at day 339 since the start of the monitoring phase. The actual RUL is 15 days.



(c) The estimated pdf of the RUL for one CU c of aircraft a estimated at day 344 since the start of the monitoring phase. The actual RUL is 10 days.



(e) The estimated pdf of the RUL for one CU c of aircraft a estimated at day 349 since the start of the monitoring phase. The actual RUL is 5 days.



(b)  $P_{acd}^{fail}$  of CU c of aircraft a, estimated at day 339 since the start of the monitoring phase. The actual failure time is at day 354.



(d)  $P_{acd}^{\text{fail}}$  of CU c of aircraft a, estimated at day 344 since the start of the monitoring phase. The actual failure time is at day 354.



(f)  $P_{acd}^{fail}$  of CU c of aircraft a, estimated at day 349 since the start of the monitoring phase. The actual failure time is at day 354.

Fig. 7. The RUL prognostic results for three consecutive time windows.

maintenance slot *s*. Eq. (12) ensures that when an aircraft is assigned to a maintenance slot, at least one component of this aircraft is replaced.

Second, we define the number of leased spare parts at the end of day  $d\in [d_0,\ldots,d_0+PH+\Delta)$  as:

$$L_{d} = \max\{0, \sum_{a \in A} \sum_{c \in C_{a}} \sum_{\substack{s \in S_{a}:\\d_{s} \leq d < d_{s} + \Delta}} X_{acs} - S_{d}^{\text{begin}}\},$$
  
$$\forall d \in [d_{0}, \dots, d_{0} + PH + \Delta),$$
(13)

where  $S_d^{\text{begin}}$  is the number of spare components initially available at the beginning of day *d* (see Section 2.6). Eq. (13) defines the number

of leased spare components to be the number of components in repair at the beginning of day *d*, minus the number of initially available spare components. If a component is replaced on day  $d \in [d_0, ..., d_0 + PH)$ , then this component is in repair until day  $d + \Delta$ .

Third, we define  $L_d^{\text{new}}$  to be the number of newly leased spare parts during day  $d \in [d_0, \ldots, d_0 + PH + \Delta)$ . The following two constraints apply for  $L_d^{\text{new}}$ :

$$L_d^{\text{new}} = \max\{0, L_d - L_{d-1}\} \quad \forall d \in [d_0 + 1, \dots, d_0 + PH + \Delta) \quad (14)$$

$$L_{d_0}^{\text{new}} = \max\{0, L_{d_0} - \max\{0, S_{d_0-1}^{\text{begin}}\}\}. \quad (15)$$

Eqs. (13), (14) and (15) are linearized exactly with the use of binary dummy variables, following [42, Chapter 4.5].

# Objective function

We consider the following objective function that minimizes the total costs with the maintenance of multiple aircraft systems.

$$\min \sum_{a \in A} \sum_{c \in C_a} \left[ \left[ \sum_{s \in S_a} X_{acs} \frac{c^{\text{fix}} + P_{acd_s}^{\text{fail}} \cdot c^{\text{ex}}}{d_s - d_{ac}^{\text{install}}} \right] + \left[ (1 - \sum_{s \in S_a} X_{acs}) \frac{c^{\text{fix}} + P_{ac(d_0 + PH)}^{\text{fail}} \cdot c^{\text{ex}}}{d_0 + PH - d_{ac}^{\text{install}}} \right] \right] + \sum_{a \in A} \sum_{s \in S_a} Y_{as} \cdot c_s + \frac{d_0 + PH + \Delta - 1}{d_0 + PH + \Delta - 1} (L_d \cdot c^{\text{Ld}} + L_d^{\text{new}} \cdot c^{\text{Lf}}).$$
(16)

The first term of Eq. (16) represents the expected cost of replacing a component. This cost is incurred either when the replacement is planned within the planning time window  $[d_0, \ldots, d_0 + PH)$ , or later when the decision to replace is postponed to the beginning of the next planning time window. In the first case, a fixed repair cost  $c^{\text{fix}}$ is incurred, plus a cost  $c^{ex}$  when the component is actually failed at the moment of replacement. This cost is normalized with the number of days the component is in use  $d_s - d_{ac}^{\text{install}}$ , i.e., it is preferred to use the component as long as possible. In the second case, we consider the cost of postponing the replacement, which contains the same costs  $c^{\text{fix}}$  and  $c^{\text{ex}}$ , relative to the earliest possible replacement time when the decision is postponed. Overall, the first term of Eq. (16) trades-off between replacing a component in the current time window (a lower exploitation time of the component, but also a lower probability of failure) or postponing the replacement to a later time window (a higher exploitation time of the component, but also a higher probability of failure).

The second term of Eq. (16) represents the costs of assigning an entire aircraft to a maintenance slot.

The last term of Eq. (16) represents the cost of leasing spare components for an entire fleet of aircraft.

#### Constraints

We consider the following constraints:

$$\sum_{s \in S_a} Y_{as} \le 1, \qquad \qquad \forall a \in A$$
 (17)

$$\sum_{a \in A} Y_{as} \le m_s, \qquad \qquad \forall s \in S$$
 (18)

$$\exists g \in G_a : \sum_{\substack{c \in g \\ a_s < d'_n}} \sum_{\substack{s \in S_a : \\ a_s < d'_n}} X_{acs} \ge |g|, \qquad \qquad \forall a \in A_r$$

where 
$$d_a^r = \underset{d \in \{d_0+1,\ldots,d_0+PH\}}{\operatorname{arg\,min}} \{P_{ad}^{\operatorname{AOG}} | P_{ad}^{\operatorname{AOG}} \ge r\},$$
 (19)

$$X_{acs} \in \{0,1\}, \qquad \qquad \forall a \in A, \ \forall s \in S_a, \ \forall c \in C_a$$
(20)

 $Y_{ac} \in \{0,1\}, \qquad \qquad \forall a \in A, \ \forall s \in S_a \ (21)$ 

$$L_d, L_d^{\text{new}} \in \mathbb{N}^+. \qquad \forall d \in \{d_0, \dots, d_0 + PH + \Delta\}$$
(22)

Constraint (17) ensures that each aircraft is assigned to at most one maintenance slot within the planning time window. Constraint (18) ensures that the number of aircraft assigned to a maintenance slot *s* does not exceed the slot's capacity  $m_s$ . Constraint (19) ensures that the probability that an aircraft is in an AOG-condition does not exceed a reliability threshold *r* within the planning time window. To prevent that an aircraft  $a \in A_r$  is in an AOG-condition, a subset of the components must be replaced before  $d_a^r$ , where  $d_a^r$  is the first day *d* within the

time window  $[d_0 + 1, ..., d_0 + PH)$  when  $P_{ad_a^{AOG}}^{AOG} \ge r$ . To ensure that  $P_{a(d_0+PH)}^{AOG} < r$ , i.e., that the probability of an AOG-condition for aircraft  $a \in A_r$  does not exceed the reliability threshold, all the components in at least one subset  $g \in G_a$  have to be replaced, i.e., all |g| components of the subset g are replaced. This constraint is linearized exactly with the use of binary dummy variables, following [42, Chapter 3.6]. Finally, Constraints (20), (21) and (22) define the domains of the decision variables.

# 5. Results — Predictive maintenance planning of Cooling Units for a fleet of aircraft

In this section, we illustrate the maintenance planning model (see Section 4) for a fleet of |A| = 13 homogeneous, wide-body aircraft. Each aircraft is equipped with N = 4 identical Cooling Units (CUs) in the Cooling System, as introduced in Section 3. First, we discuss the Cooling Units system and its *k*-out-of-*N* system's configuration in Section 5.1. In Section 5.2 we illustrate the maintenance planning model for this multicomponent, *k*-out-of-*N* system. Lastly, in Section 5.3 the computational time of the model is discussed for different sizes of aircraft fleet.

# 5.1. k-Out-of-N system of Cooling Units

Each aircraft is equipped with N = 4 Cooling Units (CUs), which are linked in a *k*-out-of-*N* system. Here, the Minimum Equipment List (MEL) requires that k = 2 [28]. An aircraft is thus allowed to fly (i.e., not in an AOG-condition) if at least k + 1 = 3 or more CUs are operational. However, if exactly k = 2 CUs are operational, then the aircraft is still allowed to fly for a maximum of V = 10 days [28]. Otherwise, the aircraft is in an *Aircraft-On-Ground* condition, which is defined as follows:

**Definition 2.** An aircraft is in an *Aircraft-On-Ground* (AOG) condition as soon as i) (N - k) + 1 or more components fail, or ii) (N - k) components have been failed for more than V days.

The probability  $P_{ad}^{AOG}$  that an aircraft  $a \in A$  with a *k*-out-of-*N* system is in an AOG-condition at the beginning of day *d*, is as follows:

 $P_{ad}^{AOG} = P(i \in \{(N - k) + 1, ..., N\} \text{ components fail before}$ the beginning of day *d*, or exactly (N - k)components fail before the beginning of day d - V)

For the case of the Cooling System with N = 4, k = 2 and V = 10, the probability of an aircraft being in an AOG-condition at day *d* is:

$$P_{ad}^{AOG} = \prod_{i=1}^{4} P_{aid}^{fail} + \sum_{i=1}^{4} (1 - P_{aid}^{fail}) \prod_{\substack{l=1\\l\neq i}}^{4} P_{ald}^{fail} + \sum_{i=1}^{3} \sum_{j=i+1}^{4} P_{ai(d-10)}^{fail} P_{aj(d-10)}^{fail} \prod_{\substack{l=1\\l\neq \{i,j\}}}^{4} (1 - P_{ald}^{fail}).$$
(23)

In Section 4, we define that the set  $G_a$  contains all subsets of  $C_a$  that could be replaced to ensure that  $P_{a(d_0+PH)}^{\rm AOG} < r$ , i.e., the set of components that could be replaced to avoid having the aircraft in an AOG-condition. To illustrate  $G_a$  for the Cooling System, we discuss the following example. Let a critical aircraft  $a \in A_r$  (see Definition 1) have N=4 CUs, i.e.,  $C_a=\{1,2,3,4\}$ . Furthermore, let r=0.01 and PH=15 days. Then  $P_{a(d_0+15)}^{\rm AOG}$  is the sum of i) the probability that three of four components fail by day  $d_0+15$ , and (ii) the probability that two components fail by day  $d_0+15-10$  and no components fail between day  $d_0+15-10$  and day  $d_0+15$  (see Eq. (23)). Moreover, let the probabilities that components 1, 2 3 and 4 fail by day  $d_0+15$  be  $P_{a1(d_0+15)}^{\rm fail}=1$ ,  $P_{a2(d_0+15)}^{\rm fail}=0.05$ ,  $P_{a3(d_0+15)}^{\rm fail}=0.05$  and  $P_{a4(d_0+15)}^{\rm fail}=0.001$ . Lastly, let the probabilities that components 1, 2, 3 and 4 fail by day  $d_0+15-10$  be  $P_{a1(d_0+5)}^{\rm fail}=1$ ,  $P_{a2(d_0+5)}^{\rm fail}=0.02$ ,  $P_{a3(d_0+5)}^{\rm fail}=0.02$  and  $P_{a3(d_0+5)}^{\rm fail}=0.001$ .

Table 1





Fig. 8. Maintenance planning for 50 days, from day 1465 to 1515 for a fleet of 13 wide-body aircraft..

Parameter values for the maintenance planning model in Section 4.

	1 0
Costs	
$C^{\mathrm{fix}}$	$10^{4}$
C <sup>ex</sup>	$5 \cdot 10^{3}$
$C^{\mathrm{Lf}}$	$4 \cdot 10^{4}$
$C^{\mathrm{Ld}}$	10 <sup>3</sup>
Rolling horizon parameters	
PH	15 days
τ	5 days
CU-related parameters	
Ν	4 CUs
k	2 CUs
Δ	4 weeks
V	10 days
S <sub>0</sub> <sup>begin</sup>	3 CUs
Reliability-related parameters	
r	0.01

In this example, the set of replaced components must include at least component {1}, or components {2,3} to ensure that  $P^{AOG}_{a(d_0+15)} < 0.01$ . Thus, the set of component subsets that can be replaced to solve the aircraft criticality (see Definition 1) is:

$$\begin{split} G_a = & \{\{1\}, \{2,3\}, \{1,2\}, \{1,3\}, \{1,4\}, \{1,2,3\}, \{1,2,4\}, \\ & \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\}. \end{split}$$

#### 5.2. Maintenance planning

In this section, we illustrate the maintenance planning model (see Section 4) for a fleet of |A| = 13 homogeneous, wide-body aircraft. The initial stock of spare CUs for this fleet of 13 aircraft at day 0 is  $S_0^{\text{begin}} = 3$ . Moreover, the first  $\tau = 5$  days of each maintenance planning in the rolling horizon approach are fixed. In general, various planning horizons  $\tau$  can be considered. The other parameter values for our proposed maintenance planning model are given in Table 1.

In practice, it is assumed that there are two types of maintenance slots for the aircraft: (i) aircraft-specific slots, which are dedicated to one specific aircraft, and (ii) generic slots, which can be used by all aircraft. We assume that at most two aircraft can be maintained at the same time in a generic slot, i.e.,  $m_s^{\text{generic}} = 2$ . In extreme cases, when there are very few aircraft-specific slots or a large number of aircraft, this capacity could be increased. One generic slot is available every day. Lastly, we assume that the cost  $c_s$  of a maintenance slot s is  $c_s^{\text{generic}} = 10^4$  for a generic slot and  $c_s^{\text{specific}} = 1$  for an aircraft-specific slots that have

Table 2

The compo	nents that	are rep	laced in	the	maintenance	planning	in	Fig.	8
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The components that are replaced in the maintenance planning in right					
$a \in A$	$c \in C_a$	Day of slot s <sub>d</sub>	Failed at replacement?	Actual RUL	
10	3	1465	No	9 days	
8	2	1480	Yes	-	
8	3	1480	No	11 days	
4	4	1484	No	6 days	
2	3	1508	No	9 days	
3	3	1508	Yes	-	

been used in practice by the fleet of 13 aircraft. On average, an aircraft has 35 of these aircraft-specific maintenance slots per year.

Fig. 8 shows the final maintenance planning of the fleet of 13 aircraft for a period of 50 days, using a rolling horizon approach with planning time windows of PH = 15 days, of which each time the first  $\tau = 5$  days are fixed. In this period, 6 CUs are replaced, 1 CU is leased and the total maintenance costs of the CUs is 137.203. These results are obtained in 3.3 s with the Gurobi solver version 9.0.2 with standard settings (branch-and-cut algorithm), implemented in Python, using an Intel Core i7 processor at 2.11 GHz and 8Gb RAM. The model is initialized with a random installation time for each CU,  $d_{ac}^{install} \sim U(80, 200)$  days before the start of the maintenance planning.

In Fig. 8, the aircraft-specific maintenance slots available for each aircraft during the 50 days period are depicted. There is also a generic slot available every day. The planning results show that aircraft  $a \in \{3, 4, 10\}$  are assigned to an aircraft-specific maintenance slot, while aircraft  $a \in \{2, 8\}$  are assigned to generic slots. Regarding the aircraft-specific slots, aircraft 3 is planned to be maintained during day 1508, aircraft 4 during day 1484 and aircraft 10 during day 1465. Regarding the generic slots, aircraft 2 is assigned to a generic slot at day 1508 and aircraft 8 during day 1480.

The components that are replaced in the maintenance planning of 50 days are given in Table 2. Aircraft 8 is assigned to a maintenance slot at day 1480, during which two components, CU 2 and 3, are replaced. For the other aircraft, only one component per maintenance slot is replaced. Out of the 6 replacements, 4 components are replaced before they fail (66%). On average 8.75 days of the RUL are wasted when a component is replaced before its failure time. During the 50 days considered, there is one new component leased at day 1484 (i.e.  $L_{1484}^{\text{new}} = 1$ ). This component is leased until day 1492 (i.e.  $L_d = 1\forall d \in [1484, ..., 1492]$  while  $L_d = 0 \forall d \in [1465, ..., 1483] \cup [1493, ..., 1515]$ ).

To illustrate the dynamic character of our rolling horizon approach, Figs. 9 and 10 show three rolling time windows, which correspond to the last several days in Fig. 8. Fig. 9 shows the prognostics at the beginning of each time window. Only the CUs that have not failed yet,



Fig. 9. The prognostics at the beginning of planning time windows [1495, 1510), [1500, 1515), [1505, 1520).

but that are in the second phase of the prognostics at the beginning of the time window, are shown. These prognostics are used as input in the maintenance planning model in Fig. 10.

At the beginning of time window [1495, 1510), two CUs are in the second phase of the prognostics: CU 2 of aircraft 3 and CU 2 of aircraft 2 (see Fig. 9). CU 3 of aircraft 3 is already failed. This aircraft is therefore critical, and some components have to be replaced before day 1508. In contrast, all CUs of aircraft 2 are still functional, and this aircraft is therefore not critical. For this time window [1495, 1510), there are no spare CUs available until day 1508. Aircraft 2 has no generic slots after or on day 1508, and the replacement of CU 2 of aircraft 2 is therefore not scheduled. However, a replacement of a CU of aircraft 3 has to be scheduled before day 1508, i.e., before a spare CU becomes available, due to the required reliability of each aircraft. The replacement of CU 2, with a predicted near-future failure, is therefore scheduled in the aircraft-specific slot at day 1500 (see Fig. 10), and it is planned to lease a CU. The maintenance planning of the first five days, [1495, 1499], is fixed. Since there is no maintenance planned in the first five days, no maintenance is thus executed and no CUs are leased.

In the next time window, [1500, 1515), CU 2 of aircraft 3 and CU 2 of aircraft 2 are not failed yet (see Fig. 9). With the updated prognostics for CU 2 of aircraft 3, some components have to be replaced before day 1511 in this time window, instead of before day 1508. Aircraft 3 is therefore scheduled to be repaired in the generic slot during day 1508, when a spare CU becomes available. Both CU 2 and CU 3 of aircraft 3 are failed by this day, and one of them (CU 3) is selected for replacement in a specific slot. As before, the first five days of this maintenance planning, [1500, 1504], are now fixed.

In the third time window, [1505, 1520), both CU 2 of aircraft 2 and CU 2 of aircraft 3 have failed. However, CU 3 of aircraft 2 is now in the second phase of the prognostics (i.e., predicted to fail in the near-future) as well. The aircraft is therefore critical; some components have

Table 3

	Size of fleet of aircraft				
	13	30	60	90	120
Total computation time [sec] (60 months planning)	71	179	482	752	1239
Average computation time [sec] (one time window — 15 days)	0.04	0.14	0.44	0.73	1.22

to be replaced before day 1517. An aircraft-specific slot for aircraft 2 is available on day 1507. However, no spare CU is available then. Since using a generic slot is much cheaper than leasing a spare CU, the replacement of CU 3 of aircraft 2 is scheduled in a generic slot at day 1508. The maintenance actions planned from day 1505 to day 1509 are fixed, which means that the maintenance planned on day 1508 (see Fig. 8) is now fixed.

# 5.3. Computation time vs size of aircraft fleet

Table 3 shows the total computational time required to obtain a maintenance planning for 60 months for different aircraft fleet sizes. Here, the number of spare CUs and the capacity of the generic slots is proportional to the fleet size. We also include the average computation time required to solve the maintenance planning problem for one time window (15 days). These computation times are obtained using a computer with an Intel Core i7 processor at 2.11 GHz and 8Gb RAM. For an aircraft fleet as large as 140 aircraft, a total of 1239 s are needed to obtain a maintenance planning for 60 months, with an average computation time of 1.22 s to solve the problem for one time window of 15 days.

# 6. Prognostic-based maintenance vs. corrective and preventive maintenance

In this section, we compare our proposed prognostic-based maintenance model with limited spare components (see Section 4) with a corrective and a preventive maintenance strategy (see [43,44]), for the *k*-out-of-*N* systems. For these two maintenance strategies, we also consider a limited amount of spare components and fixed maintenance slots. Corrective and preventive maintenance strategies are often used in the practice of aircraft maintenance [14,45,46].

# Corrective maintenance (CM) for k-out-of-N systems of repairables with limited spares

We consider a corrective maintenance (*CM*) strategy where the system is maintained only when k = 2 or more components of the system are failed (see also Definition 2). We plan the aircraft maintenance in the following order of priority: First, the maintenance for all aircraft already in an AOG-condition (see Definition 2) is planned. An aircraft in an AOG-condition is assigned to the earliest available maintenance slot. When there are  $f \ge k$  failed components in the aircraft, at least f - 1 failed components are replaced in this maintenance slot. If there are not enough spare components, then extra components are leased so that all f - 1 failed components can be replaced.

Second, all aircraft with k = 2 failed components that are not yet in an AOG-condition (see Definition 2), are assigned to maintenance slots. Such an aircraft is maintained in the earliest available aircraft-specific slot, as long as this does not lead to an AOG-condition. Otherwise, the aircraft is maintained in the earliest available maintenance slot, irrespective of the type of slot. At least 1 failed component is replaced. If there are not enough spare components, then extra components are leased.

Last, all remaining failed components in the two types of aircraft above are replaced as well, as long as there are enough spare components.



Fig. 10. The maintenance planning of three iterations of the rolling horizon approach for time windows [1495, 1510), [1500, 1515) and [1505, 1520).

Table 4

95% CI - Long-term performance of PM,	CM and Prog.M, with T — the total number
of replacements, and NF - the total numb	per of replacements of non-failed components.

or replace.	menney and m	the total number	of replacements of ne	in nanea componento
	95% CI	95% CI	95% CI	95% CI
	AOG events	Leases	Replacements	Total costs (mil)
CM	[0.71, 0.82]	[21.6, 22.3]	[112.2, 113.1] (T)	[3.05, 3.10]
PM	[0.08, 0.11]	[4.43, 4.78]	[134.7, 135.6] (T)	[2.26, 2.29]
Prog.M	0.0	[3.90, 4.19]	[105.1, 106.0] (T)	[1.57, 1.60]
			[0,12, 00.0] (11)	

Preventive maintenance (PM) for k-out-of-N systems of repairables with limited spares

We consider a preventive maintenance (PM) strategy where the system is maintained to prevent a system failure. To prevent that the entire system fails, i.e., at least k + 1 components are failed, or k components are failed for more than V days, we replace components as soon as they fail, provided spare components are available. First, the aircraft for which the system has k = 2 or more failed components in the remaining aircraft are replaced as well. These aircraft can only be assigned to aircraft-specific slots. Furthermore, no spare components can be leased to replace these failed components.

We analyze CM, PM and the prognostics-based maintenance planning model for a fleet of 13 aircraft for a period of 60 months using Monte Carlo simulation with a 1000 simulation runs. All parameters and costs are the same as in Table 1. Fig. 11 shows the performance of CM, PM and our proposed prognostics-based maintenance planning model. Table 4 gives 95% confidence intervals. Fig. 11(a) shows the expected number of times an aircraft is in an AOG-condition (see Definition 2) for the three strategies. This is called an AOG-event. The results show that the CM strategy leads to the highest number of expected AOG-events.

Fig. 11(b) shows the expected number of leased spare components per strategy. The most spare components are leased for the CM strategy. Both the PM strategy and the prognostic maintenance planning model need relatively few spare components.

The total number of replacements T, and the number of replacements of non-failed components NF, is given in Fig. 11(c). For the CM and PM strategies, by definition, only failed components are replaced. The number of total replacements is highest for the PM strategy, because components are replaced as soon as they fail (provided that there are enough spare components). In contrast, for the CM strategy, failed components are replaced only when there are at least k = 2 failed components in a system. For the prognostic-based maintenance planning, the total number of replacements is the lowest because components that fail are not necessarily immediately replaced. When the probability of an AOG-condition for an aircraft exceeds the reliability threshold r, it is often more beneficial to replace the component(s) that have a failure predicted in the near-future, thus saving repair costs. Here, for on average 88 out of the 106 replacements, the components are not failed at the time of replacement.

Lastly, the total expected maintenance costs are given in Fig. 11(d). For all strategies, the repair costs constitute the largest fraction of the total costs, while the slot costs constitute the smallest fraction of the total costs. The total costs are the highest for the *CM* strategy, while the prognostic maintenance planning has the lowest total costs.

Overall, the results of our case study show that the prognosticsbased maintenance planning model is most beneficial, with the lowest expected maintenance costs and the lowest expected number of AOG-events.



(c) The total number of replacements (denoted by T), and the number of replacements of non-failed components (denoted by NF)

(d) The total costs with the maintenance

Fig. 11. The expected long-term performance of *PM*, *CM* and prognostics-based maintenance model (*Prog.M*) for a period of 60 months and a fleet of 13 wide-body aircraft, including 95% confidence intervals (CI).

# 7. Conclusion

An integrated approach from sensor data to RUL prognostic algorithms, to maintenance planning is proposed for a fleet of aircraft, each equipped with a multi-component system of repairable components. RUL prognostics are updated over time with new sensor measurements. In turn, the maintenance planning takes the RUL prognostics into account to schedule component replacements in a rolling horizon fashion. As a case study, a fleet of wide-body aircraft, each equipped with a system of Cooling Units, is considered. First, a model-based RUL prognostic is developed for these aircraft Cooling Units. Second, these prognostic models are integrated into a rolling horizon maintenance planning model. Here, the planning also takes into account a limited stock of spare components, as well as available maintenance slots. Moreover, a reliability constraint is imposed on each considered system. The results show that by integrating prognostics into the maintenance planning, components are replaced in anticipation of failure without wasting their useful life. Specifically, in our numerical example, 66% of the replaced components were not failed at the moment of replacement. Also, the wasted useful life of the replaced components is limited to an average of 8.75 days. When compared with other maintenance strategies, the results show that our proposed prognosticsbased maintenance planning model reduces the costs by 48% relative to a corrective maintenance strategy and by 30% relative to a preventive maintenance strategy. Overall, our approach shows how RUL prognostics could be integrated into a dynamic, rolling horizon maintenance planning model and what the performance is to be expected.

As future work, we plan to further develop and test prognostic models for the Cooling Units. In particular, we will focus on the quantification of the uncertainty of the prognostics, using (extended) Kalman filters and other Bayesian inference sampling methods. We also plan to further extend our maintenance planning model taking into account dynamically changing repair costs, to illustrate the long-term condition of the repairable components. Moreover, we plan to analyze several other types of corrective and preventive maintenance strategies, using a larger range of performance indicators. Here, we aim to also contrast additional maintenance strategies such as predictive maintenance combined with opportunistic maintenance. Also, we plan to consider gradually decreasing planning time windows, based on the RUL prognostics. Lastly, we plan to relax the assumption that a repaired CU is "as-good-as-new", and instead consider imperfect repairs. With such extensions, we aim to obtain an increasingly closer-to-implementation prognostics-driven maintenance planning model.

# CRediT authorship contribution statement

**Ingeborg de Pater:** Conceptualization, Methodology, Software, Validation, Formal analysis, Data curation, Writing - original draft, Writing - review & editing, Visualization. **Mihaela Mitici:** Conceptualization, Methodology, Validation, Formal analysis, Writing - original draft, Writing - review & editing, Supervision, Funding acquisition.

## Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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