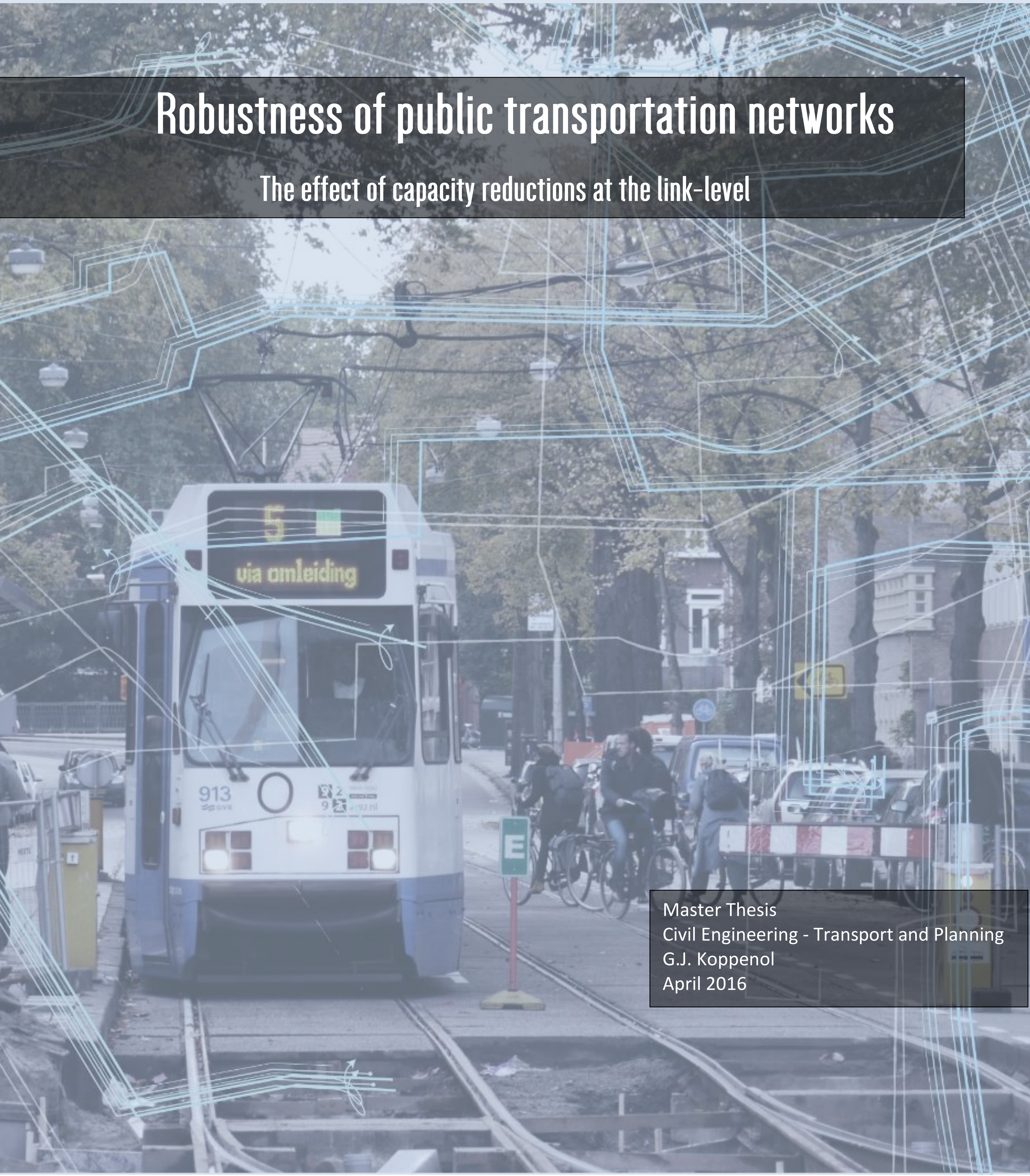
Robustness of public transportation networks

The effect of capacity reductions at the link-level



Master Thesis
Civil Engineering - Transport and Planning
G.J. Koppenol
April 2016

TU Delft

Stadsregio Amsterdam

Robustness of public transportation networks

Master of Science Thesis

Date: April, 2016

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Preface

This thesis report is the final product of my Master of Science program Transport and Planning at Delft University of Technology. My graduation topic is the robustness of public transportation networks. The research is performed at and in cooperation with Stadsregio Amsterdam, the transport authority for the city region of Amsterdam.

First of all I want to thank all the colleagues of Stadsregio Amsterdam for the warm welcome in the organisation and the sincere interest of them in my research and me as a person. Second, I would like to thank the municipality of Amsterdam for delivering the data I needed and the GVB for the valuable information given during the interviews. Further, I would like to thank my daily supervisors – Oded Cats and Marc van Deventer. Oded, for his valuable and inspiring feedback on all the intermediate concept versions I sent him. Marc, for introducing me to the public transport system of Amsterdam, the city of Amsterdam itself and for the nice lunch conversations about all sorts of subjects. I also want to thank the other members of my graduation committee – Serge Hoogendoorn, Martijn Warnier and Paul Wiggenraad– for their valuable feedback and comments during committee meetings.

Besides, I would like to especially thank the following people: René Vennik, in helping me solve some crucial computer running time problems. Naomi Veenhoven, in helping me with some graphic work for my report and invitation flyer. Rolf Bruins, Edwin Lokker and my brother Dirk, for their valuable feedback on my report.

Finally, I would like to say thanks to my parents, family and friends for the support and trust during my study in general and graduation in particular. Last but not least, I would like to say thanks to my girlfriend Elise for her understanding, trust and support the last years of my study.

Gert-Jaap Koppenol

Rotterdam, April 2016

Summary

Most studies on the quality of PT networks merely focus on PT networks in a non-disturbed situation. The quality of a PT network is, however, not only determined by its performance in a non-disturbed situation. Local and smaller disturbances urge for analysis on how well a network is capable to perform in such situations, because they cannot be completely prevented. Results of such an analysis can be used to reduce the consequences of such disturbances. How well is a PT network capable of maintaining its network performance in case of disruptions? Or in other words, how robust are PT networks?

Two main research objectives are formulated in this research. The first objective is to identify robustness indicators for the relationship between the extent of capacity reduction and the network performance. The second objective is to identify and analyse characteristics of the links explaining the score of the different types of links on the identified robustness indicators. This research aims to explain how urban rail bound public transport networks are affected by capacity reductions on a link of the network, in other words the robustness of a network. The formulated main research question in this research is the following:

How do attributes of links in a public transport network influence the relationship between a capacity reduction on that link and the performance of the public transport network as a whole?

Background of urban rail bound public transport network disruptions and robustness

As background knowledge a literature review on public transport network robustness is executed and from literature and interviews with public transport field employees is identified how capacity reductions on public transport networks occur and how they affect the network. First topic of the literature review is the relationship between network design and network robustness. The found relationships indicate that next to the pure link characteristics, the surrounding infrastructural (e.g. clustering) and service network characteristics (e.g. service network density) are of influence as well. Second the explanatory link characteristics on link criticality are explored. The found link characteristics emphasize again the influence of service network characteristics (e.g. length of traversing lines) on link criticality. Third the found relationships between capacity reduction and network performance are discussed. Different relationships, from convex to concave were found for different types of disruptions. Planned line disruptions, in general, show convex relationships. Whereas unplanned link disruptions, in general, show concave relationships.

Through interviews with public transport field employees is identified how capacity reductions on public transport networks occur and how they affect the network performance. Disruptions related to robustness are characterized by the fact that they affect infrastructure availability. These disruptions are referred to as major discrete events. This study focuses on a specific type of the major discrete events, namely the planned major discrete events, in other words planned disruptions which affect infrastructure availability. These major discrete events affect the network performance through its effects on the infrastructure network, as a consequence the service network is affected and this again influences the route choice of

travellers. The effect of major discrete events on the infrastructure availability is, in general, a reduction in speed on the link up to a full out of operation. The service network is primarily affected in longer running times on the link, secondary the frequency of traversing lines may need to be reduced. A public transport operator is able to mitigate the effects by rerouting or cutting the line. The changes in the service network affects the utility of the different route alternatives of travellers, consequently the travellers will experience a change in in-vehicle time, waiting time and number of transfers.

For urban rail bound PT networks four types of planned major discrete events are found: construction or maintenance works near the tracks, track works, road works and planned events. All types of planned major discrete events can cause full link unavailability. The first three types can also cause partial link unavailability and local speed limits and an increase in vehicle stand still time. Public transport operators can often mitigate the consequences of these local infrastructure (un)availability situations. Sometimes they will be forced into a do-nothing scenario which can result in deleting the lines or apply a local speed limit for traversing lines depending on the type of infrastructure unavailability (i.e. link is unavailable or local speed limit is at hand). In some cases the PTO will be able to apply mitigation measures like rerouting or cutting traversing lines. Travelers are affected by the chosen mitigation measures. In worst case they get disconnected, but they can also be affected by longer in-vehicle time, waiting times and more needed transfers. This, among others, depends on whether they are going to reroute or not. Because travellers can be affected in numerous ways it is hard to theorize about the total effect of major discrete events on the network performance. It is therefore necessary to model the PT network and apply these disruption and mitigation scenarios to be able to analyse the effect on the network performance.

The network performance indicator

To analyse this effect an indicator needs to be formulated which can measure the performance of transport network as a whole. In this research is examined how the performance of a transport network as a whole can be expressed. Therefore, at first, the network performance indicators used in literature are explored. Network performance is found to be measured in a whole set of indicators, from pure topological indicators (e.g. connectivity) to indicators which measure the effect of travellers on the network performance (e.g. total generalized travel costs). Based on these findings the performance of the network is measured as the total generalized travel costs. This indicator is able to measure the consequences of disruptions on the travellers in in-vehicle time, waiting time and number of transfers. In other words, to calculate the effect of a capacity reduction on a link on the performance of the public transport network as a whole.

Public transport robustness analysis model

For the analysis of robustness of PT networks a new model is developed in which in an efficient way the robustness of links and the robustness of the entire network can be analysed. The modelling is done in 3 steps: network representation, route choice set generation and assignment. The network is represented in two types of network representations used in complex network theory. One represents the infrastructure network (L-space) and the second one the service network (P-space). Route choice set generation is done using Yen's k-shortest path algorithm which is based on Dijkstra's algorithm which is used to find not only the shortest paths but also the closest best alternatives. Through applying three filtering routes the master path set for every OD-pair is found: transfer filter rule, loop less filter rule and dominancy filter rule. Based on the assumption that all path alternatives are independent the assignment is done using a multinomial logit model. The calculated utility per path as input for the logit model is the weighted travel time based on the in-vehicle time, waiting time and number of transfers. Based on the found mitigation measures of PTO's four types of disruption scenarios implementations are proposed: local speed limit, delete traversing lines, reroute traversing lines and cut traversing lines.

The network robustness indicators

Possible robustness indicators used in literature are explored. Four categories of indicators are found: threshold values, local slopes, ratios and areas under curves. The only found indicator that is able to capture the accumulated effect of the total range of capacity reduction scenarios on the network performance is to measure the area under the capacity reduction-network performance curve. As a result two robustness indicators are proposed: Link Criticality and Degrading Rapidity. The first one indicates the accumulated effect of the whole range of capacity reductions on the network performance, the second one indicates the rapidness of degrading to full breakdown network performance.

Potential determinants on the relationship between capacity reduction and network performance

Potential explanatory variables (i.e. determinants) for the two robustness indicators are derived from the identified conceptual framework and findings in literature. They are grouped into three groups of attributes: line, link and passenger demand attributes. Based on the expected explanatory value of the explored determinants the quantitative analysis is executed for the link, line and passenger demand attributes presented in Table 1.

Table 1 Overview of attributes for the quantitative analysis

Attributes for the quantitative analysis:
Link centrality
Density of link area
Saturation of link area
Operational link characteristics
Service intensity on link
Passenger demand on link

All attributes are operationalized into one or more measurable characteristics. Most of the operational variables are topological indicators for link centrality, density of link area or service intensity on the link. Others are derived from the supply data or from the modelling results of the base case scenario. Multiple linear regression is used to analyse the strength of the relationships and whether it is negative or positive. The analysis is executed on the expected Amsterdam rail bound PT network of 2020.

Link criticality Conclusions

In Figure 1 the results of the Link Criticality calculations are presented in a histogram and a graph. In the left figure the distribution of the links over the different values of the Link Criticality of the Amsterdam urban rail bound PT network is given. In the right figure the criticality of the links in the network are presented.

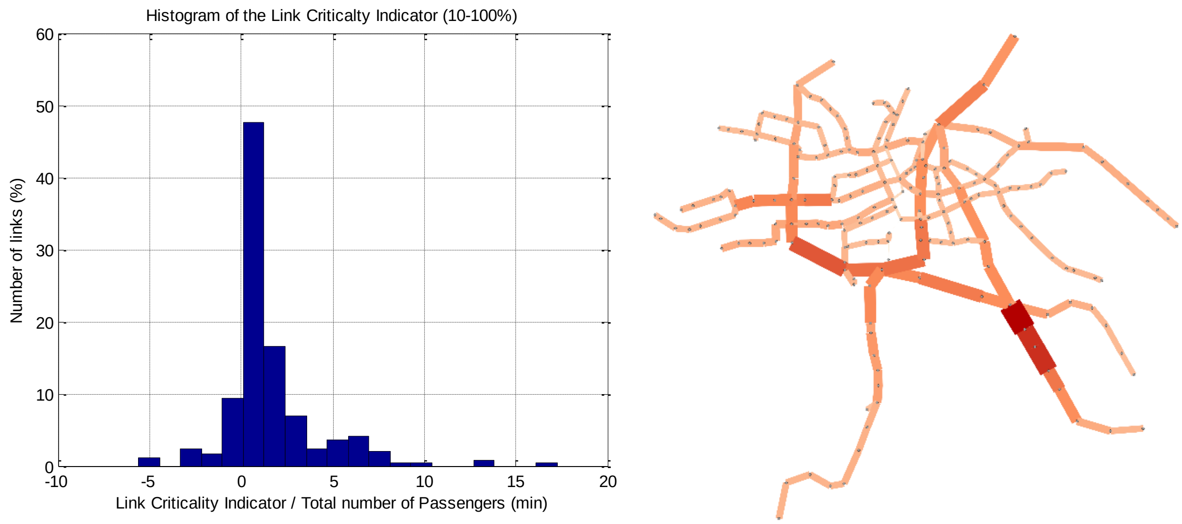

Figure 1 Histogram (left) and a graph (right) of the Link Criticality indicator (10-100%) for the links in the Amsterdam urban rail bound PT network (Thicker and more saturated red links indicate a higher score)

As a result of the multiple linear regression key variables of influence on Link Criticality are found. The total bi-directional flow is the most dominant key variable concerning Link Criticality. The only negative key variable on Link Criticality is the P-space Link Degree Centrality indicator. Another positive determinant on Link Criticality is the Mean speed on the link. The operationalized variables of Link Centrality, Density of Link Area and Saturation of Link Area and Total frequency of traversing lines are not key attributes influencing Link Criticality. This does, however, not imply they are not influencing attributes at all. Especially in the model estimation on the total set of disruption scenarios some minor influencing attributes are found. The attributes of Density of Link Area and Saturation of Link area all are negative determinants for the model estimated on the total set of disruption scenarios. Total frequency of traversing lines and L-space Link closeness centrality indicator where found to be positive determinants in the model estimation on partial capacity reductions only. The L-space link betweenness centrality indicator was not found to be a significant attribute on Link Criticality.

Degrading Rapidity Conclusions

In Figure 2 the results of the Degrading Rapidity calculations are presented in a histogram and a graph. In the left figure the distribution of the links over the different values of the Degrading Rapidity of the Amsterdam urban rail bound PT network is given. In the right figure the Degrading Rapidity of the links in the network are presented.

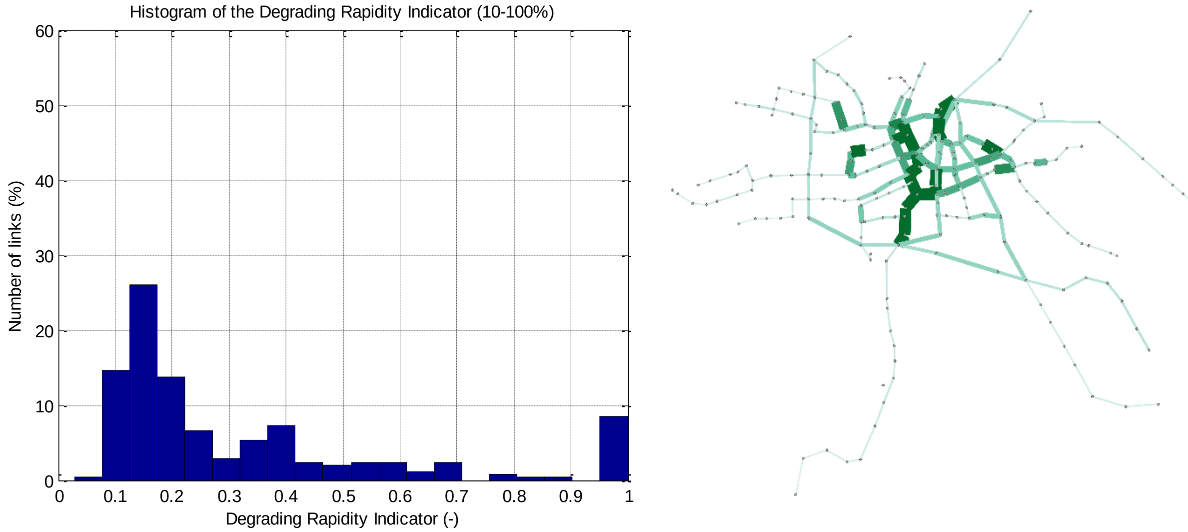

Figure 2 Histogram (left) and a graph (right) of the Degrading Rapidity indicator (10-100%) for the links in the Amsterdam urban rail bound PT network (Thicker and more saturated green links indicate a higher score)

As a results of the multiple linear regression key variables of influence in Degrading Rapidity are found. The L-space Link Closeness Centrality indicator is the most dominant key variable concerning Degrading Rapidity. The L-space link betweenness centrality indicator and the L-space Local clustering coefficient are also a positive key attributes on Degrading Rapidity. The only negative key variable on Degrading Rapidity is the total bi-directional flow on the link. The L-space link degree centrality indicator is a minor attribute only significant for the partial capacity reduction situations. The Degrading Rapidity is not influenced by the attribute of Saturation of Link Area, Operational Link Characteristics and Service intensity on the link.

General conclusions

Density of link area, service intensity and passenger demand are the key variables on PT link robustness. The key is to incorporate these aspects in the design of the network to increase robustness. This can be done by spreading both passengers and PT lines. By designing a network with less PT lines sharing track the local service intensity and passenger demand per link is decreased but also the local infrastructure network density is increased.

Another key variable on PT link robustness is the mean speed on the link. This emphasized the need for the responsible authorities to prevent major discrete events from happening or to reduce the total impact of major discrete events. By, for example, smart planning, in both time and place.

Recommendations

Finally, some recommendations for further research and further improvement of the model are presented. Main items for further research are the increase in detail level of the robustness study (i.e. different disruption types, including other types of mitigation measures or including the surrounding PT network). Another recommendations is to further develop the proposed robustness indicators so that they can be used in, for example, cost benefit analysis and also on the network level.

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List of abbreviations

Abbreviation	Full name/term
CBA	Cost-benefit analysis
DR	Degrading Rapidity
GVB	GVB Exploitatie BV
HTM	HTM Personenvervoer NV
LC	Link Criticality
N/Z-lijn	North/South metro line (In Dutch: Noord/Zuidlijn)
NS	Dutch Railways (In Dutch: Nederlandse Spoorwegen)
OD	Origin-Destination
PT	Public Transport
PTO	Public Transport Operator
SRA	City Region of Amsterdam (In Dutch: Stadsregio Amsterdam)
VOT	Value of time

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Glossary

Track	A structure consisting of a pair of parallel lines of rails.
Public transport link	A representation of a connection between two stops.
Public transport line	A line on which public transit is provided consisting of a sequence of stops connected by public transport links
L-space	A graph representation used for the physical network. Nodes are connected if they are consecutive stops in a given route.
P-space	A graph representation used to account for the presence of public transport lines. Nodes are connected if at least one route between them.

List of symbols

Symbol	Unit	Explanation
β^n	-	Transfer perception factor
β^w	-	Waiting time perception factor
$\sigma_{i,j}$	-	Number of shortest paths from node i to node j
C	List	Edge set representing all direct line connections
$ce_e^{d,P}$	-	P-space link degree centrality indicator for link e
$ce_e^{b,L}$	-	L-space link betweenness centrality indicator for link e
$ce_e^{c,L}$	-	L-space closeness centrality indicator for link e
cl_e^L	-	L-space local clustering coefficient of link e
$ce_e^{d,L}$	-	L-space link degree centrality indicator
d_e	Metre	Traversed distance on link e
d_{ie}	Minute	Travel time over the shortest path between node i and edge e
DR_e	-	Degrading Rapidity of link e
e	-	Indicator for PT edge
E	List	Edge set representing all rail track segments in use
f_r	Vehicles/hour	Service frequency for line r
f_e^{cap}	Vehicles/hour	Service frequency capacity on link e
g	-	Number of directly connected unique nodes
GT	Minute	Generalized travel costs
H	Set	Link set
k	-	Indicator for a path
K	List	Path set
LC_e	Minute	Link criticality of link e
L^a	Matrix	Adjacency L-space
L^d	Matrix	Distance weighted L-space
L^{links}	Table	Infrastructure links for L-space
L^t	Matrix	Travel time weighted L-space
M^L	Cell	Route choice set with sequence of traversed nodes in L-space
M^P	Cell	Route choice set with sequence of traversed nodes in P-space
n_r	Vehicles	Number of needed PT vehicles on line r
n_k	-	Total number of transfers on path k
n_e^r	-	Totten number of traversing lines on link e
O	Matrix	OD-demand matrix
P(k)	-	Number of nodes with k feeding links
P^a	Matrix	Adjacency P-space
P^{lines}	Table	PT Lines for P-space
$P^{line-stop}$	Matrix	Line-stop matrix for P-space

pr_k	-	Probability of choosing path k
P^t	Matrix	Waiting time weighted P-space
q_{ijk}	Number of passengers	Flow of passengers between stop i and stop j over path k
r	-	Indicator for PT line
R	List	Line set with all PT lines in the PT network
sa_e	-	Sum of service frequency saturation of feeding link on link e
s	-	Indicator for PT stop
s_{e^-}	-	Stop downstream of edge e
s_{e^+}	-	Stop upstream of edge e
$s_{r,i}$	-	Stop number i for line r
s^t	-	Indicator for transfer node
S	List	Node set with all stops and stations in the PT network
t_r^{cycle}	Minute	Cycle time on line r
t^{dwell}	Minute	Dwell time
$t_A^{layover}$	Minute	Layover time at terminal A
$t_{x,y}^{run}$	Minute	Running time between stop x and y
$t_{A,B}^{trip}$	Minute	Trip time between terminal A and B
t_e^t	Minute	Planned station-to-station travel time on link e
t_k^t	Minute	Total in-vehicle time for path k
t_c^w	Minute	Waiting time on P-space edge c
t_k^w	Minute	Total waiting time for path k
tt^{con}	Minute	Total generalized travel costs of all remaining connected travellers
$tt^{dis,pen}$	Number of passengers	Total generalized travel costs of all disconnected travellers
μ	-	Logit parameter
u_k	Minute	Utility of path k
u^{delay}	Minute	Extra added travel time for disconnected travellers
$v_e(x)$	Metre/minute	Speed on edge e during disruption scenario x
w	-	Number of edges between unique nodes
$w_{i,j}$	-	Indicator for emptiness of master paths set
x	-	Scenario number
$y_e(x)$	%	Relative reduction in speed on edge e during disruption scenario x
z	Number of passengers	The share of disconnected travellers
z_s^P	-	Node degree in P-space of node s

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1. Introduction

1.1. Problem definition

Public transport (PT) is said to be critical for the futures of nations, regions and cities through its contribution to a better quality of life (APTA, 2007). PT can, for example, contribute to the economy by increasing real estate values and developments. Furthermore, it can ecologically contribute through improvements in air quality and more efficient use of resources and energy. And last but not least it contributes to society by increasing accessibility to all kinds of groups. For example, the poor or the elderly. Research on how to improve the quality of public transport networks is therefore of great importance.

Most studies on the quality of PT networks merely focus on the topological characteristics of a PT network in a non-disturbed situation. The quality of a PT network is, however, not only determined by its topological characteristics and its performance in a non-disturbed situation. On a quite frequent basis PT services do not operate according to plan. In extreme discrete events (e.g. a region wide power failure (NU.nl, 2015; Rijnmond, 2013b) the whole network is out of order and the main focus of research should be on prevention of such breakdowns. In other more recurring events just parts of a PT network are disturbed (Rijnmond, 2013a). These local and smaller disturbances urge for analysis on how well a network is capable to perform in such situations, because they cannot be completely prevented. Results of such an analysis can be used to reduce the consequences of such disturbances. How well is a PT network capable of maintaining its network performance in case of disruptions? Or in other words, how robust are PT networks?

The effect of disruptions on the network performance can be formulated as the robustness of PT networks. Previous PT network robustness studies have mainly focused on static topological characteristics. Angeloudis and Fisk (2006), for example, studied the relationship between connectivity and vertex degree to network robustness in case of random and targeted attacks. Berche et al. (2009) searched for topological network indicators for PT networks indicating the robustness of these networks in case of attacks. Topological network robustness studies, like the previous two, do not include passenger demand distribution in their research. This can give distorted insights in the performance of such networks. Imagine a breakdown of a link, which is not necessarily important in topological terms but intensively used in terms of passenger demand. A study excluding a passenger demand distribution would show that that link is not critical for keeping network performance on a decent level. However, a study including a passenger demand distributions would show that the breakdown of that link would significantly influence the total network performance. Results of studies including passenger demand distributions are therefore much more valuable for stakeholders, like transport authorities and transport operators, than studies excluding passenger demand distributions.

De-Los-Santos et al. (2012) are one of the few that did incorporate a passenger demand distribution in their study. They propose two passenger robustness measures for two cases: without-bridging interruptions and with-bridging interruptions. In the first case a link breakage occurs and passengers have to wait till the link is restored or find an alternative route. In the second case an alternative in the form of a replacing bus service is offered. All previous mentioned studies focused on situations in which a link or node is totally taken out of order. In static studies, in which the time dimension is not taken into account, this means that the affected link is not available for transport purposes at all. In dynamic studies, in which the time dimension ís taken into account, the link is unavailable for transport purposes for a limited amount of time.

Studies in which links or nodes are partly taken out of order -- partial capacity reduction situations -- are rare. Cats and Jenelius (2015b) studied the relation between the extent of temporary reduction in service frequency in the Stockholm PT network on societal costs on two different levels. The first is on the full length of a PT line, the second level is on just a part of a PT line on one link between one stop along the line to another stop along the line. The capacity reduction was applied to all lines as planned disruption and on the most central links as unplanned disruptions. They conclude that for line disruptions service operators should devote disproportional attention to major capacity reductions because such disruptions lead to disproportional consequences and societal costs. For the unplanned link disruptions they conclude that the service operator should devote attention to real-time mitigation measures because a twice as large capacity reduction is likely to result in less than twice as much delay.

In PT networks disruptions take place by which a complete link failure occurs (e.g. bi-directional infrastructure and/or signal breakdown) and disruptions arise through which the service capacity on a public transport line is partially reduced (e.g. cancelled services because of vehicle/personnel shortage). As mentioned in this section we know that both situations have been studied. However, there are also disruptions causing a link not to completely fail but partially reduce the capacity on that specific link locally. Local partial infrastructure breakdown, signal system malfunctions, maintenance and constructions works and traffic incidents can have negative consequences for the local capacity of the link. For instance: lower frequencies, higher running time between stations (e.g. lower speed limits or detours) or smaller trains allowances (e.g. local weigh limits). In case of a local link capacity reduction a transport operator does not necessarily have to let this affect the service capacity of a complete public transport line. Frequencies and size of trains can be kept on normal capacity on the other links by turning parts of the services at a disturbed link or by leaving parts of a train behind at the beginning of a disturbed link. On the other hand, a disruption on the link level can affect multiple lines at once, where a disruption on the line level is kept within the affected line. As far as known the effect of partial capacity reduction on the link level is studied just once (Cats & Jenelius, 2015b) and the researchers looked at unplanned service frequency reduction on just a selection of links only.

The following conclusions can be formulated regarding current research on PT robustness:

- Studies on PT network robustness have mainly focused on the topological characteristics of the network. Hereby neglecting the effect of lines, frequencies and passenger demand distributions;
- Studies on PT network robustness have mainly focused on total link breakdown scenarios and not on partial capacity reduction scenarios despite the occurrence of partial capacity reduction situations in reality;
- Despite the occurrence of local partial capacity reduction situations on the link-level, studies on partial capacity reductions in PT networks have mainly focused on the line-level. The found study incorporating link level capacity reduction only looked at unplanned service frequency reduction on a selection of links.

This study aims to extend research on network robustness as elaborated on in this section, see Table 2. In this table the type of PT disruption modelling is given on the horizontal axis, from left *full link breakage* to right *partial capacity reduction* on the link-level. On the vertical axis the modelling level is given from up only topological characteristics to down dynamic modelling including passenger demand distributions. Both down and right give a higher detail level of analysis, horizontally on the type of disruption and vertically on the modelling level.

Table 2 Categorization of network robustness studies on types of disruptions and detail of modelling

<table>
<tr><td rowspan="3"></td><td colspan="5">Type of PT disruption-modelling</td></tr>
<tr><td colspan="2" rowspan="2"></td><td rowspan="2">Full link breakage</td><td colspan="2">Partial capacity reduction</td></tr>
<tr><td>Line</td><td>Link</td></tr>
<tr><td rowspan="3">Modelling level</td><td></td><td>Topological only</td><td>Robustness to attacks
Considering types of networks
(Angeloudis & Fisk, 2006),
(Berche et al., 2009)</td><td>-</td><td>-</td></tr>
<tr><td rowspan="2">Including passenger demand distribution</td><td>Static</td><td>Passenger robustness measures
(De-Los-Santos et al., 2012)</td><td>-</td><td>The effect of capacity reductions at the link-level
(This study)</td></tr>
<tr><td>Dynamic</td><td>Value of reserve capacity for PT network robustness
(Cats & Jenelius, 2015a)</td><td colspan="2">Impacts of partial capacity reductions on the line and link level
(Cats & Jenelius, 2015b)</td></tr>
</table>

Table 2 shows where the mentioned studies are situated in terms of modelling level and type of disruption modelling and depicts where this study extent the research on PT network robustness. Where most mentioned studies only focus on full link breakage, one mentioned study examined the effect of partial capacity reduction on the line and link level (Cats & Jenelius, 2015b). Where the researchers of the previous mentioned study only focused on the effect of unplanned service frequency reduction on a selected number of links this study will include a full-scan analysis on planned capacity reductions on link level. The focus is on analysing the effect of the extent of capacity reduction on the link level on the performance of a PT network considering the characteristics of these links.

This study is conducted in corporation with the City Region of Amsterdam (SRA). The SRA is responsible for the PT services in 15 municipalities in and around the city of Amsterdam, among which the urban and regional tram and metro network. The SRA emphasize the importance of PT by referring to it as a necessary building block for a healthy and sustainable region (Stadsregio Amsterdam, 2013). To keep the quality of the PT system intact the system has to be changed and maintained to satisfy the passenger demands both in quantity as in quality. The SRA has to cope with two main developments: an increasing PT passenger demand and budget cuts on the BDU[1] (wide purpose investments) of which, amongst other, PT is subsidized (Stadsregio Amsterdam, 2013). Instead of degrading the service level they have chosen to invest a significant amount of money in PT to shorten the running times of vehicles and to improve the reliability of PT in Amsterdam and its region. They formulated this as 'transport more travellers with a better quality against less costs through targeted investments', in other words 'more for less'.

As a consequence, a lot of construction works on top of the already planned maintenance work needs to be carried out in the medium term (i.e. in the years up till 2025). During maintenance and construction work the infrastructure availability and thus the infrastructure network (e.g. track availability and local speed limits) and service network (e.g. running times, routes and frequencies) can and will temporarily change. For the SRA it is of interest to understand what the effect of these construction and maintenance work is on the performance of the network. How will local construction and maintenance work affect the performance of the tram and metro network? Is it better to close an entire link for a short period of time or lower the speed limit for a longer period of time? Is the availability of more alternative routes in the network disproportionally beneficial for the network performance? How robust is the Amsterdam PT network in general and it links in particular? That is the focus of the Amsterdam Case Study as part of this study.

[1] In Dutch: brede doeluitkering: a financing program for regional traffic and transport projects and is financed by the national government.

1.2. Research objective and research questions

1.2.1. Research objectives

There are two main research objectives in this research. The first objective is to identify robustness indicators for the relationship between the extent of capacity reduction and the network performance. The second objective is to identify and analyse characteristics of the links explaining the score of the different types of links on the identified robustness indicators.

Robustness indicators

The relation between a partial reduction in link capacity and network performance does not have to be linear. Situations exist in which this relationship is for example more concave or convex shaped, see Figure 3. The shape of the relationship can indicate the relative sensitivity of a link to major capacity reductions relative to minor capacity reductions. Which can be helpful in analysing robustness.

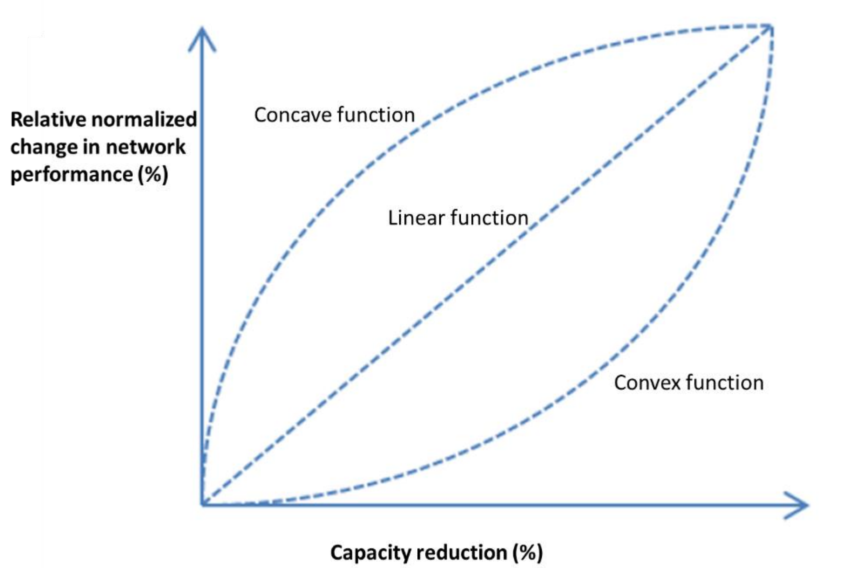


Figure 3 An illustration of possible relations between capacity reduction and change in network performance. Adapted from Cats and Jenelius (2015b).

The first main research objective is to quantify the relationship between capacity reduction and the reduction in network performance into robustness indicators that expresses this relationship. The identified robustness indicators can then be used to compare the scores of the different links for further analysis.

Influencing link characteristics

A non-trivial relationship between the magnitude of a capacity reduction and its consequences on network performance can be expected to be depending on the different link characteristics. For a partial capacity reduction on a link with a high number of passengers traversing it one could expect a large negative effect on the network performance. However, links with a high number of traversing passenger are often situated in a central part of the network where the passenger have, in general, quite some route alternatives. As a consequence the negative effect on the network performance could stay rather limited. By analysing the explanatory power of characteristics of the links (e.g. passenger demand intensity, location of the link in the network) on the identified robustness indicators one is able to calculate the robustness of a link through its characteristics.

1.2.2. Research questions

To achieve the research objectives as formulated in the previous section a main research question and six sub questions are formulated to facilitate an efficient research process. The main research question include all three research objectives. The sub questions are divided into b*ackground knowledge* and *theory and analysis*. For this study, the following main research question is formulated:

How do characteristics of links in a PT network influence the relationship between a partial capacity reduction on that link and the performance of the PT network as a whole?

Background knowledge

The first sub-research questions deals with the background knowledge on transportation network robustness studies. For some time research on transportation network robustness is ongoing. Not only on PT networks, rail bound or not, but also on car networks and aviation networks. Findings and learned lessons from these studies can give a thorough background for this study. It is also possible to apply used concepts in other network robustness studies in this study. The first sub-research question is:

1. *What is the state-of-the-art knowledge on the effect of capacity reductions on the performance of transportation networks in general and PT networks in particular?*

The used terms in the main research question do not speak for themselves. Network performance and partial capacity reductions can be expressed in numerous measures. Network performance can for example be expressed in *network connectivity, total travel time*, *societal costs*, etcetera. Capacity reduction can be expressed in *number of seats per time unit*, *frequency of trains*, *running times between two stations, mean speed* etc. The next two sub-research questions deal with the terms used in the main research question taking into account the scope of the research and the occurrence of partial capacity reductions in reality. The next two sub-research questions are:

2. *What network performance indicators are used in network robustness studies and which one is most appropriate for this study?*
3. *How do infrastructural characteristics (e.g. track availability), in capacity reduction situations, affect the service characteristics of the network (e.g. speed, frequency)?*

Theory and analysis

The last two sub-research questions deal with the two main research objectives as formulated in the previous section. The qualitative relationship between network performance and capacity reduction is going to be quantified in robustness indicators. Next and finally, the characteristics of these links are analysed for their influence on the proposed robustness indicators. The last two sub-research questions are:

4. *Which robustness indicators meaningfully describe the relationship between capacity reduction and network performance?*
5. *What are the key variables of influence on the relationship between capacity reduction and PT network performance in terms of passenger demand, service characteristics and topological characteristics?*

1.3. Definition of PT network robustness

An overview of used definitions of network robustness can be found in section 2.1. For this study network robustness will be defined as a small adapted form of the definition formulated by M. Yap (2014):

Public transport network robustness is the extent to which the network is able to maintain its function under circumstances which strongly deviate from plan.

The definitions incorporates the effect of disturbances on the extent to which the network is able to perform instead of only looking at the ability of the system to stay intact. Next, it incorporates the specific characteristic of the service network (i.e. plan).

The definition states that it only considers circumstances that strongly deviate from plan. This emphasizes the difference with PT reliability which is referred to as the ability of the system to perform under minor quasi continuous events (Lee, 2013; N. Van Oort, 2011) whereas PT robustness is referred to as the ability of the system to perform under major discrete events (i.e. circumstances which strongly deviate from plan) (Tahmasseby, 2009; M. Yap, 2014). A fully conclusive distinction is difficult to make but in general the minor quasi continuous events are smaller recurrent disturbances only influencing parts of the service network (e.g. local delays). Major discrete events are understood to be non-recurrent events that severely affect both the service and infrastructure network (M. Yap, 2015).

1.4. Scientific and practical relevance

Scientifically this study will be relevant in introducing robustness indicators that will meaningfully describe the relationships between capacity reduction and network performance. Subsequently it will show the explanatory power of link characteristics on the relationships between capacity reduction on the link level and network performance. Further it will result in an operational numerical model, not specifically tailored to the rail bound Amsterdam PT network, for the analyses of PT network robustness.

This study will give useful insights for policy and design decisions on different levels. On a strategic level the study can give insights in the effect of infrastructural adaptations like adding missing turns in the infrastructure network of the tram on the robustness of the network. Knowledge of the effect of partial capacity reductions on different links on the performance of the network can be useful on the tactical level for the choice of the execution of planned construction and maintenance work. For example, on some links it might show that a full link closure or major capacity reduction gives the same reduction in network performance as a low capacity reduction. In this case the results suggest that for maintenance or construction works on this link it is more efficient to choose for a full link closure for a short period of time instead of a small capacity reduction for a longer period of time.

1.5. Research scope

The research will only focus on urban rail-bound PT network, in which the operation frequency and transport demand is very high. Partial capacity reductions scenarios are expected to be more distinctive in these cases to full link breakdown than in low frequency scenarios.

Insights in the robustness of these PT networks will only focus on planned capacity reductions on the link level. By choosing to only look at planned capacity reduction the use of static modelling instead of dynamic modelling is justified. In case of planned disruptions en-route route choice decisions occur much less than in case of unplanned disruptions.

1.5.1. Modelling scope

The modelling approach will be on the macro-level; modelling flows per link not including individual choice behaviour of public transport passengers. For insights on the link robustness infrastructural characteristics, passenger flow characteristics and service frequencies are expected to be sufficient and as a consequence micro-level modelling is not necessary.

The focus of PT network robustness in this study will only be on the aggregate level of flows and not individual passengers or vehicles. Within the urban-rail bound PT network robustness the focus will not be on reliability of technical installations or vehicles.

The scope of the modelling is on the relationship between capacity reduction and network performance and not on the relationship between capacity reduction and passenger demand distribution. The passenger demand distribution is as a consequence assumed to be inelastic. People are not expected to change their origin or destination in case of local capacity reductions on the network.

1.5.2. Case study scope

The case study will be the rail-bound Amsterdam PT network, this will include the metro and tram network. Trains and busses are not incorporated. All trams and metro services are executed by the GVB Exploitatie BV (GVB) and in almost all cases of planned capacity reductions on these networks the alternatives they offer will be on these networks (W. van Oort & de Hoog, 2015).

The disruption scenarios will be based on the occurring planned disruptions and corresponding mitigating measures implemented by the GVB in reality.

These scenarios will be run on a single variant of the possible Amsterdam PT networks in 2020. In this variant the newest part of the Amsterdam metro network, the North/South metro line (N/Z-lijn) is included and the tram network is adapted to the presence of the N/Z-lijn.

1.6. Thesis outline

The structure of this thesis is as follows. First in Chapter 2 an literature review of PT network robustness studies, PT network performance indicators and potential PT robustness indicators are given. This corresponds with the purple boxes in the background knowledge phase in Figure 4. In Chapter 3 the theory on urban rail bound disruptions and their consequences are discussed and in Chapter 5 the conceptual framework on link characteristics an link robustness is presented. These two chapters correspond with the two boxes in the theory phase in Figure 4. In Chapter 4 the implementation and design of the model is presented together with the formulated network performance and robustness indicators and the implementation of the disruption scenarios. In Chapter 6 an overview of the case study is given together with the processing of the used data. This thesis ends with the data analysis in Chapter 7 and conclusions and recommendations in Chapter 8.

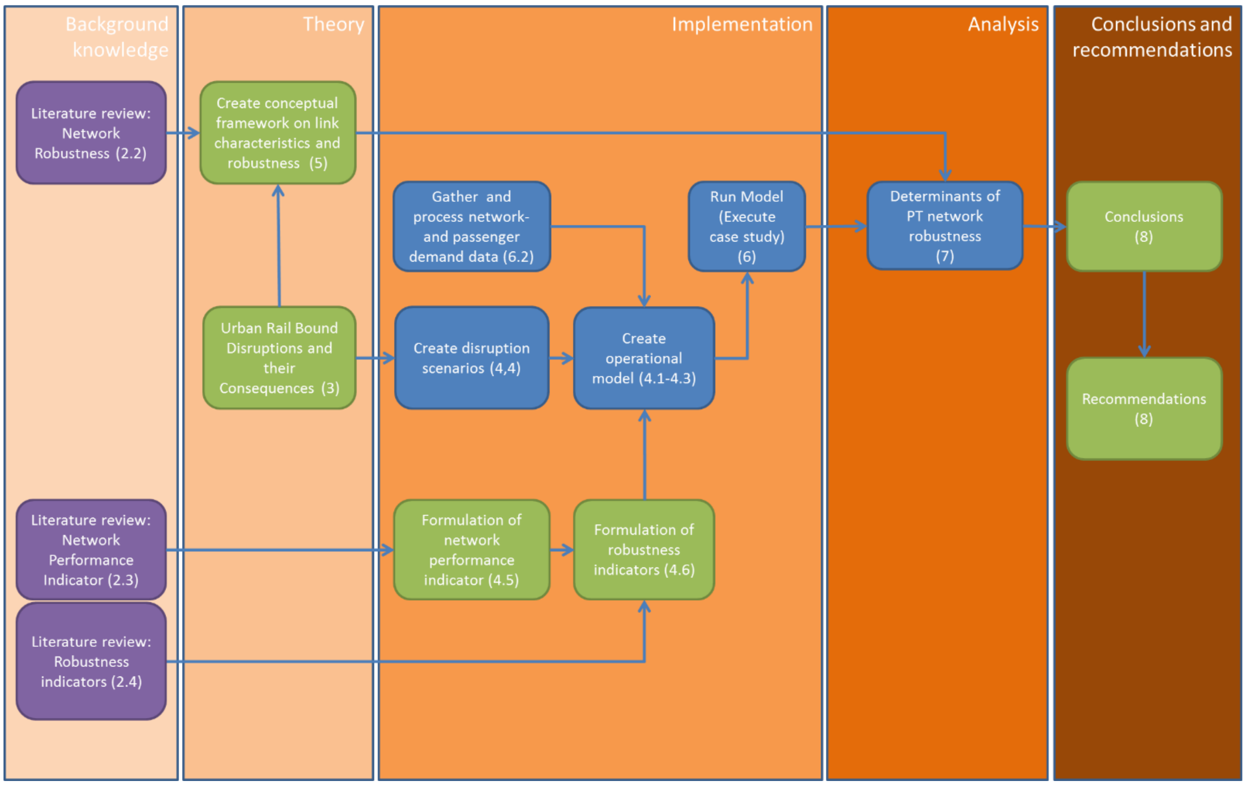


Figure 4 Thesis outline (in purple: literature review; in blue: modelling and analysis; in green: other; the numbers between brackets refer to corresponding chapters and sections)

2. Literature review on PT network robustness

Before commencing studying the effect of capacity reductions on the link level, literature is studied. Analysing PT network robustness starts with defining what network robustness is. This is addressed in Section 2.1. To explore the potential determinants on the relationship between capacity reduction and network performance the relationship between network design and network robustness and the influencing factors on link criticality are reviewed in literature (Section 2.2). The following research question is discussed in this section:

'(1) What is the state-of-the-art knowledge on the effect of capacity reductions on the performance of transportation networks in general and PT networks in particular?'

Furthermore, literature is reviewed to explore what network performance indicators (Section 2.3) and network robustness indicators (Section 2.4) are used in network robustness studies. This supports answering the following research questions:

'(2) What network performance indicators are used in network robustness studies and which one is most appropriate for this study?'

'(4) Which robustness indicators meaningfully describe the relationship between capacity reduction and network performance?'

The chapter ends with a literature synthesis in Section 2.5 and conclusions in 2.6.

2.1. Definitions of network robustness

Robustness relates to something that is generally referred to as being strong or in good condition. The Oxford Dictionaries (2015) defines robustness as "the ability to withstand or overcome adverse conditions or rigorous testing". In this case we are specifically speaking of network robustness. Network studies can be applied to all sorts of disciplines and fields. They vary between ecological studies (Sole & Montoya, 2001) to economic (Haldane & May, 2011) and biological studies (Motter et al., 2008) to engineering (Albert et al., 2004). For this reason the definitions are not as univocal as one would like.

For a number of transport network studies (i.e. power grids, roads and public transport) definitions on network robustness are given in Table 3. For power grid studies, robustness is generally referred to as the ability to keep the system intact in the event of disturbances. Disturbances, especially in the case of power grid networks, can cause cascading failures (Ash & Newth, 2007) which, in turn, cause malfunctioning of the total system. This explains the focus of the power grid network robustness definitions on the ability to keep the system intact.

In other disciplines and fields, robustness not only refers to the extent to which a network remains intact but also to the extent to which it still performs in the case of disturbances. In studies on road transport network robustness (Knoop et al., 2012; Snelder et al., 2012) this is often referred to as 'the extent to which a network is able to maintain its function'.

Table 3 Some, direct and derived, definitions of network robustness

Some definition of Robustness:	Discipline/field:	Literature:
The ability of a system to avoid malfunctioning when a fraction of its elements fail	Power grids	(Koç et al., 2014)
The ability of the system to retain its system structure intact when exposed to perturbations	Power grids	(Holmgren, 2007)
The ability of the network to maintain its functionality under conditions that deviate from the normal conditions	Road transport	(Knoop et al., 2012)
The extent to which, under pre-specified circumstances, a network is able to maintain the function for which it was originally designed	Road transport	(Snelder et al., 2012)
A system's ability to withstand and recover from shocks	Public transport	(Cats, 2016)
The robustness of a public transport system is the ability to withstand or quickly recover from disturbances such as infrastructural and vehicular malfunctions and planned maintenance without significant reduction in the performance of the system (in terms of travel times etc.)	Public transport	(Cats & Jenelius, 2015a)
The capacity of the network to absorb the impact of disturbances in its links and nodes while maintaining operability in conditions similar to those found in a normal situation	Public transport	(Rodríguez-Núñez & García-Palomares, 2014)
The amount of stress that can be absorbed before failure occurs	Public transport	(Wang et al., 2015)
The extent to which the network is able to maintain the function it was originally designed for under circumstance which strongly deviate from plan	Public transport	(M Yap et al., 2014)

The definition of Snelder et al. (2012) considers all kinds of disturbances on routes, nodes and links that lead to a complete loss of function or a partial degradation of its elements. It considers the performance of a complete network and is also applicable to PT networks.

For PT networks it is not only the design of the infrastructural network that is of importance, but also the design of the service network (i.e. frequency, capacity and routes of PT lines). It is only the definition of M. Yap (2014) that refers specifically to the service network as a 'plan'. Other definitions add a resilience component to the PT robustness definition by incorporating the ability to recover from shocks or disturbances (Cats & Jenelius, 2015a; Cats, 2016) or they describe robustness as the ability of the system to remain intact or absorb stress before failure occurs (Rodríguez-Núñez & García-Palomares, 2014; Wang et al., 2015), comparable with the definitions used in power grid network robustness studies.

2.2. The state-of-the-art on PT network robustness

On the topic of PT network robustness there are a large number of combinations of characteristics and topics that can be studied. These characteristics are, for example, the level of detail in the study (e.g. only looking at the topological characteristics or incorporating passenger demand distributions), the choice to incorporate the time dimension (e.g. dynamic or static modelling), the degree of disruptions examined both in terms of time (e.g. link closure without incorporating the time dimension or a link closure for a certain period of time), space (e.g. disruptions on link or line level) and severity (e.g. full link closures or partial capacity reductions).

As mentioned in Section 1.1 most PT network robustness studies have looked at full link closures and the static effects on the network measured in topological terms. Only recently researchers have started to investigate the effect of partial capacity reductions on network performance in public transportation networks. The studies on PT network robustness are discussed in relation to three subjects: network design and network robustness (Section 2.2.1), influencing factors on link criticality (Section 2.2.2) and the relationship between capacity reduction and network performance (Section 2.2.3).

2.2.1. Network design and network robustness

Network design has influence on network robustness, this includes the infrastructural network and the service network design. For example, a link failure in the middle of a single straight line network immediately cuts the network in two whereas a link failure in a circular line network keeps all the nodes connected. In the first case, travellers are not able to travel from every origin to every destination whereas in the second case they are. The circular line is then said to be more robust in the case of link failures than the straight line network.

Infrastructure design

Most PT networks cannot be represented by merely a circular line or a straight line but consist of multiple lines with multiple characteristics. Which makes the research on network robustness more complex. In most PT network robustness studies graph theory is used to describe the form and dimensions of the network and to measure robustness (Angeloudis & Fisk, 2006; Berche et al., 2009; Derrible & Kennedy, 2010; von Ferber et al., 2012). Derrible and Kennedy (2011) made a clear overview of the applications of graph theory and network science to transit networks.

PT networks can be classified into two classes of complex networks based on the node degree distribution of the network. Node degree is the number of edges incident to the node. The two classes of networks are: scale-free networks and high connectivity, low degree networks (Angeloudis & Fisk, 2006). Examples for both network classes can be found in Figure 5, with on the left an example of a high connectivity, low degree network and on the right an example of a scale-free network. The upper figures show the node degree distribution of the represented network below. The high connectivity, low degree networks can be described by a bell-curve or a normal distribution and the scale-free networks with a power law distribution. A power law node degree distribution can be described by the following equation:

Equation 1 Power law node degree distribution

$$P(k) \sim k^{-\gamma}$$

Where k is the number of links and $P(k)$ is the number of nodes with k links. A node degree distribution described by a power law distribution of a larger γ represents a more hub and spoke-like network (i.e. more lower degree nodes and less higher degree nodes).

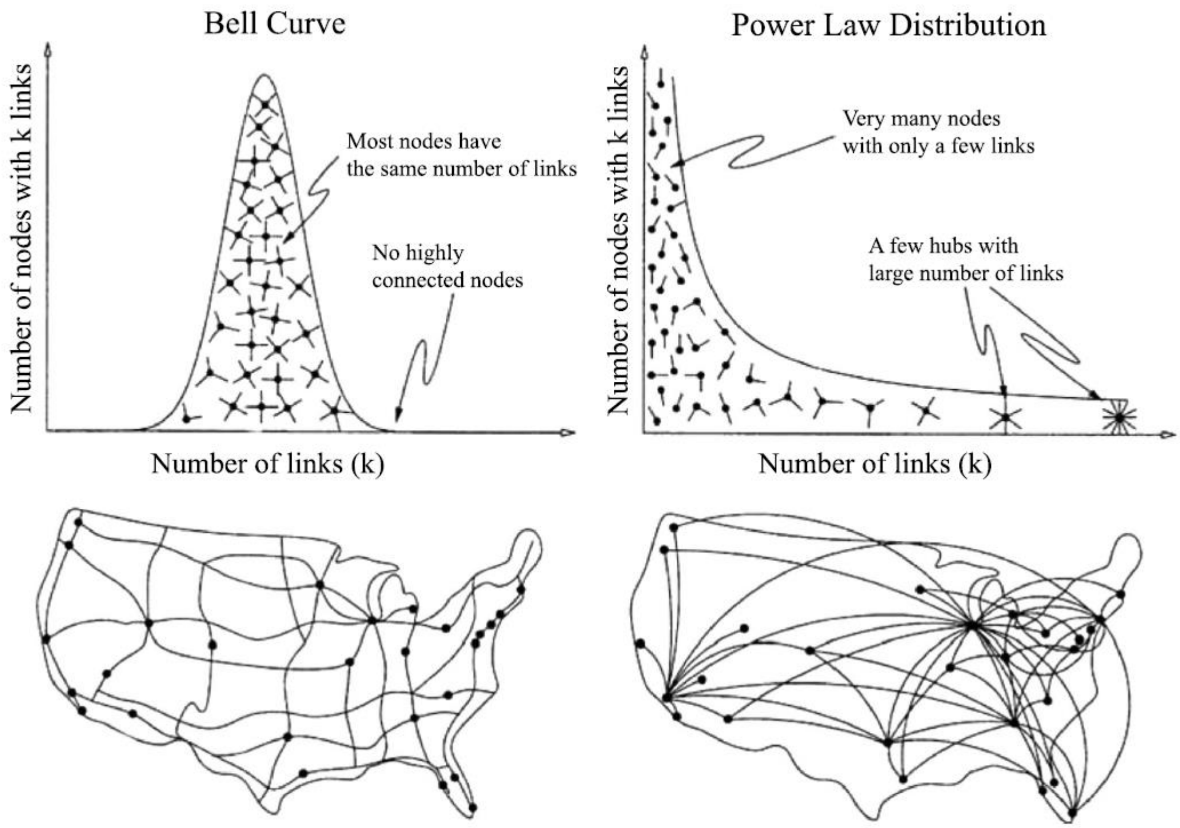


Figure 5 Distribution of nodes according to their degree (number of connecting links) for two types of networks. With examples of both networks. Left: high connectivity, low degree network (bell curve) ; Right: scale-free network (power law distribution). Adapted from Figure 4 in Derrible and Kennedy (2011)

Scale-free PT networks (i.e. networks that can be described by a power-law distribution) are, in general, less robust to targeted link failures than the networks that can be described by a normal distribution (Angeloudis & Fisk, 2006). Scale-free networks described with a smaller value of γ in the power law distribution are, in general, more robust than those described with a larger value of γ (Berche et al., 2009). This means that networks with more nodes and more connecting links and fewer nodes with fewer connecting links are more robust than networks with a large number of less connected links and just one or two super-connected links. In the first case the network offers many more route alternatives than in the second case, which improves the robustness of the network.

So types of classes and types of node degree distributions can help to explain robustness. But also the mean node degree is explanatory. The higher the mean of the node degrees the more robust a network will be (von Ferber et al., 2012; X. Zhang et al., 2015). As a consequence of the higher mean node degree the network will contain more cycles, which improves robustness by adding alternative routes for different origin/destination (OD) pairs (Derrible & Kennedy, 2010; Rodríguez-Núñez & García-Palomares, 2014; X. Zhang et al., 2015). A cycle can be defined as a route with the same start and ending node, which does not traverse the same edge more than once. With the number of cycles as a proxy of robustness, Derrible and Kennedy (2010) conclude that more clustered networks perform better in terms of robustness (i.e. more clustered networks contain more cycles and are thus more robust). The extent to which a network is clustered is expressed into a clustering coefficient which quantifies how close a network is to being a clique (i.e. a complete graph).

From all these insights into the relationship between the design and the robustness of PT networks, recommendations for improving network robustness can be formulated. In general, more route alternatives need to be created to improve robustness. This can be accomplished by creating more loops (i.e. cycles) (X. Zhang et al., 2015) and thus more transfer options in the network. In general, PT networks are centre focused and mainly consist of radial lines. In this case a good option to create more loops is by adding more circular lines to the network (Rodríguez-Núñez & García-Palomares, 2014; X. Zhang et al., 2015). This results at the same time in more transfer options on the crossing of these circular lines with the radial lines. The effect of adding cross-radial lines in the improvement of PT network robustness is also shown by Cats (2016) in his comparative study of the current and future PT network of Stockholm. In general, for smaller networks the robustness can be increased by focusing on creating transfer stations and for larger networks the robustness can be increased by creating additional transfer possibilities throughout the network (Derrible & Kennedy, 2010).

Service network design

In PT networks the transport is done by the public transport vehicles that run according to a certain plan. As a consequence, the robustness of a PT network is also depending on the service network. Firstly, more intertwined timetable design with multiple lines having to share tracks is less robust than systems with dedicated line systems (Angeloudis & Fisk, 2006). However, lines crossing each other should be stimulated to add more transfer options (Berche et al., 2009) and the density of network lines should be increased (von Ferber et al., 2012). For the latter option the infrastructure network design should of course be more dense as well.

2.2.2. Influencing factors on link criticality

The closure of a link because of a disruption affects the network performance. The extent to which the network performance is affected depends on the characteristics of the link. Is the link traversed by a lot of travellers? Is it centrally situated in the network or is it more separately situated in the outskirts of the network? It all influences the effect of a disruption on the network performance and thus affects its criticality.

Both demand level and supply level factors influence the criticality of links. Demand level factors encompass infrastructural characteristics but also service network characteristics. An infrastructural influencing factor on link criticality is network density. Links that are positioned in a low network density part of the network are shown to be the most critical ones (Angeloudis & Fisk, 2006; Berche et al., 2009; Derrible & Kennedy, 2010; Rodríguez-Núñez & García-Palomares, 2014; von Ferber et al., 2012; X. Zhang et al., 2015). A found influencing service network factor is the service intensity on the link. Studies show that links that are traversed by more lines are more critical (Angeloudis & Fisk, 2006) and that links that are part of a very long line (i.e. lines with a large number of connecting stations) are more critical (Rodríguez-Núñez & García-Palomares, 2014).

The found supply level factor influencing the criticality of links is the number of traversing passenger on the link; if a large number of passengers traverse a link, it is more critical (Rodríguez-Núñez & García-Palomares, 2014).

By dynamically modelling a temporary disruption on selected network segment of the Stockholm PT network Cats and Jenelius (2014) were able to show that link centrality (i.e. the position of the link in the network) does not necessarily imply link criticality in terms of robustness and that the criticality of a link also depends on the level of information that is available to passengers.

2.2.3. Relationships between capacity reductions and network performance

The robustness of networks in case of full link closures is significantly more assessed than the robustness of networks in case of partial capacity reductions. The studies that did focus on the network robustness in case of partial capacity reductions pay a specific interest to the relationship between the extent of partial capacity reductions and the network performance. The relationship is able to show the relative sensitivity of a link to major capacity reductions relative to minor capacity reductions. This can be proportional (i.e. a linear relationship), disproportional vulnerable to major capacity reductions (i.e. a convex relationship) and disproportional vulnerable to minor capacity reductions (i.e. a concave relationship).

In theory this relationship can have an infinite number of forms depending on, amongst other, the chosen type of capacity reduction (i.e. reduction in travel time or frequency and applied to lines or links) and the network performance indicator (i.e. an absolute decrease in network performance or a relative decrease in network performance and what type of network performance indicator is being used). In theory this relationship can be concave, convex, linear and everything in between, with the note that usually the relationship is monotonically ascending or descending.

When the network performance is expressed in associated disruption costs and the capacity reduction is expressed in planned line-level frequency reduction the found relationships are found to be concave functions, linear functions or convex functions (Cats & Jenelius, 2015b), see figure 6. Concave functions express that small capacity reductions have disproportional negative effects on the network performance, linear functions express a proportional relationship between the capacity reduction and the network performance and convex functions express that large capacity reductions have disproportional negative effects on the network performance.

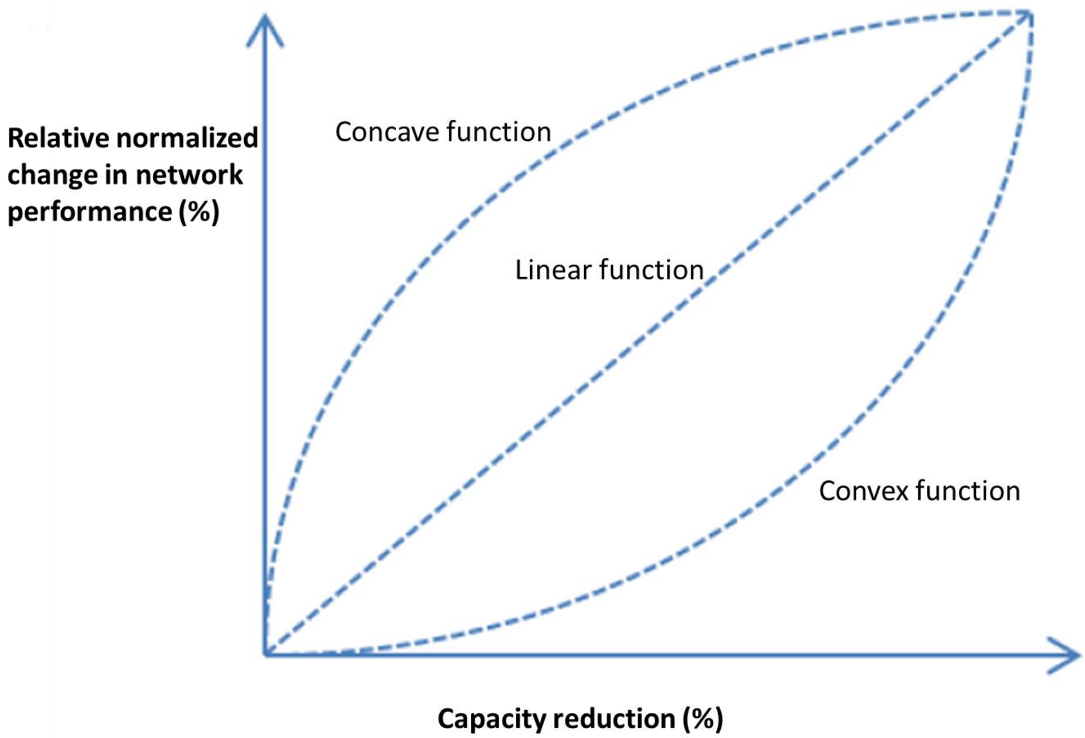


Figure 6 An illustration of possible relations between a capacity reduction and the network performance reduction. Adapted from Cats and Jenelius (2015b).

Planned capacity reductions on the line-level show convex relationships between the capacity reduction and the change in total passenger welfare for nearly all lines in the Stockholm PT network. Planned line disruptions which cause major capacity reductions (expressed in reduced frequency on the whole line) will, as a consequence, have disproportional negative effect on the network performance (Cats & Jenelius, 2015b).

Furthermore, Cats and Jenelius (2015b) modelled unplanned capacity reductions on a selected number of links. For all selected links the relationship between the unplanned capacity reduction and the network performance reduction are concave. Small disruptions on these links already lead to a significant loss in the network performance; a double severe disruption does not double the negative effects on the network performance.

2.3. PT network performance indicators

Before commencing on analysing the PT robustness it is wise to define capacity reduction and network performance. An elaboration on the used network performance indicators is done in this section.

A network performance indicator in a robustness study must be able to grasp the effects of disruptions on the network. The effects of disruptions occur on different levels of detail for which different indicators are available. The use of these indicators will, as a consequence, depend on the level of detail in the study. The studies in which PT network performance indicators are used are discussed on the basis of three groups of studies: infrastructure studies (i.e. studies in which exclusively the infrastructure network is incorporated), service network studies (i.e. studies in which the service network is also included) and passenger demand studies (i.e. studies in which the OD-demand distribution is also used). The indicators that are used in the studies with a lower level of detail are also applicable on the studies with a higher level of detail. Therefore these indicators are discusses as well.

2.3.1. Infrastructure studies

The network performance indicators used in infrastructure studies are, in general, topological indicators based on graph theory. The network performance in these studies is, for example, expressed into connectivity, centrality (Berche et al., 2009; von Ferber et al., 2012) or the largest connected component of the network (Wang et al., 2015). Connectivity expresses the minimum number of links that needs to be removed to disconnect the remaining nodes from each other; network centrality is an indicator of how central a node is situated in the network. These indicators are by Faturechi and Miller-Hooks (2014) referred to as topological measures of effectiveness (MOE). These infrastructure studies and corresponding topological indicators neglect the presence of PT lines and corresponding service network design for which the same disruptions can have different effects on the network performance.

2.3.2. Service network studies

Studies which do include the service network are able to include transfers and travel times in the network performance indicator. By specifically modelling the service network of the PT network, one is able to use the same topological indicators mentioned in the previous section on this representation (this can be a P-space representation and is explained in Section 4.1). In addition, the network performance can be expressed in the mean travel time between all the OD-pairs or the mean number of transfers over all the OD-pairs. Berche et al. (2009) represented the largest metro networks in the world in both L-space and P-space and examined the effect of random attacks on the performance of the network. They expressed the effect into the change of the size of the largest cluster and the average inverse mean shortest path length; this is an example of a topological indicator used in P-space.

2.3.3. Passenger demand studies

In the previous types of study it is demonstrated that travel time and transfers can be incorporated in the network performance. That is, however, without incorporating the effect of the passenger demand distribution; this can give distorted insights in the robustness of the networks. Take, for example, a disruption on a link on which a lot of routes from different OD-pairs travers. When the network performance is expressed intro mean travel time the network performance would be overestimated if the demand on the corresponding routes are very low. In this case it is better to express the network performance into total system travel time which includes the passenger demand.

In the types of study in this section MOE can be expressed functional, topological and economic. This is in contrast to the previous categories of studies in which most indicators had to be topological and just some were functional. This is a consequence of the fact that functional MOE focus on serviceability and are expressed into, for example, travel time, distance or flow for which passenger demand has to be incorporated. Economic MOE are mainly a monetarization of the functional MOE combined with real costs as insurance damage, exploitation costs etc. The monetarization is, in general, done by taking the product of the functional MOE with values of time (VOT). Examples of studies in which the network performance is expressed into an economic MOE are Cats and Jenelius (2015a, 2015b) and Hobbs (2015).

But also available non-monetarized network performance indicators can give more detailed insights in the effects of disruptions in studies in which the passenger demand is included. This can be expressed into only a certain number of passengers or into a combination of a certain number of passengers and their travel times. The effect of a disruption on the network performance can, for example, be expressed into the share of cut-off, affected or delayed passengers (Cats & Jenelius, 2015b). These indicators, however, do not express the extent to which these passengers are delayed or affected. Therefore the indicators which combine the number of affected passenger and their delay need to be used. This can, for example, be expressed into increase in the total travel time (i.e. the sum of the travel time over each passenger in the network) or the average travel time (i.e. the mean travel time of a traveller in the network) (Cats & Jenelius, 2015b). By comparing the loss of the total travel time or the average travel time of the disrupted situation with the normal situation, one is able to quantify the effect of the disruption.

By taking the ratio of the total travel time of the normal situation network and the total travel time of the network in case of disruption, the indicator can be normalized (De-Los-Santos et al., 2012). This indicator can be used as a robustness indicator. In the next section other types of robustness indicators are discussed.

2.4. Types of robustness indicators

To be able to come to a PT robustness indicator which meaningfully describes the relationship between capacity reduction and network performance a review on used types of indicators in comparable relationships is executed. Most of these indicators were found through the overview article on indicators of system resilience of Hosseini et al. (2015) and the overview article on measures on the performance of transportation infrastructure systems in disasters of Faturechi and Miller-Hooks (2014).

The found measures can be generally categorized in *threshold values in curves*, *local slopes in curves*, *ratios on curve values* and *areas under curves*. The four categories of measures will be discussed on the basis of the two figures below (see Figure 7). The left figure shows the network performance for every capacity reduction for four given edges. The right figure shows the same data as the left figure, but then normalized by dividing for every edge the network performance at every capacity reduction by the maximum network performance at 100% capacity reduction.

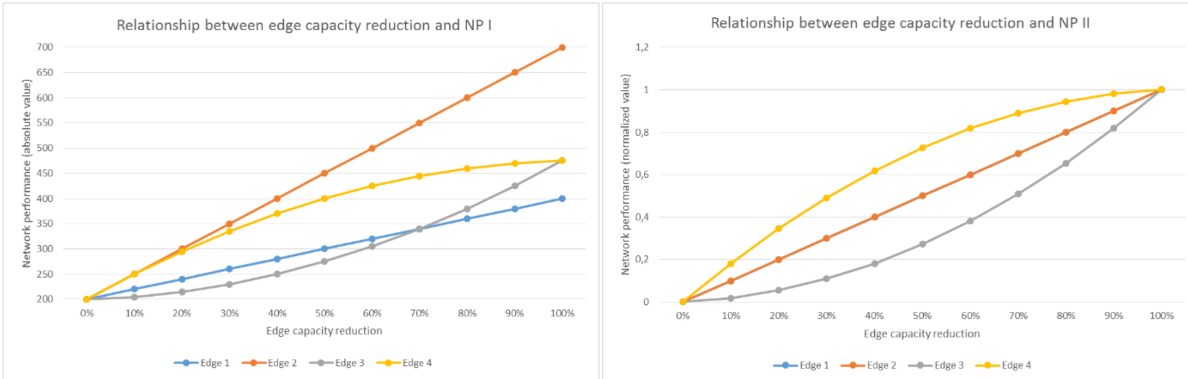


Figure 7 Graphs of relationship between capacity reduction and network performance based on fictive numbers for four different edges. The graph on the left gives the absolute values, the graph on the right is normalized by dividing the network performance by its maximum network performance at a capacity reduction of 100%.

To choose a robustness indicator in a structured way, different criteria are established on which the found types of robustness indicators are tested. The five established criteria are shown in Table 4. The proposed robustness indicator needs to be able to indicate two aspects of the network performance–capacity reduction curve: the accumulated effect (i.e. the criticality of the link) and the form (i.e. the relative sensitivity to minor or major capacity reductions). This is captured in the two first criteria.

Further, it is of added value when the indicator in pure or adapted form can be used in project appraisals to cost-benefit analysis (CBA) and when the indicator can be translated from link level to network level. This study focuses on link level robustness, expansion of robustness studies could however be in network comparison. An indicator which can be translated to network level would therefore be of added value. Studies on changes to public transport networks are in the end expressed in monetary terms used in CBA. It would therefore be of added value when the indicators can be translated to monetary terms. This is captured in the third and fourth criteria

And last but not least the results of robustness studies are in the end input for, among others, policy makers. The indicator should therefore be comprehensible for people who are not that into mathematical indicators and equations.

Table 4 Robustness indicator criteria

Robustness indicator criteria
Able to capture the accumulated effect of capacity reductions
Able to capture the form of the curves
Possible to translate to monetary terms
Possible to translate to network level
General comprehensible

2.4.1. Threshold value in curves

A first category of measures which are, amongst other, applied in robustness (Berche et al., 2009) and resilience studies (Murray-Tuite, 2006) is the measurement of the Network Performance at a certain threshold of the capacity reduction or vice versa. The threshold can be applied to every value of the capacity reduction or the network performance.

An example of the measurement of the network performance at a certain threshold of the capacity reduction is done for a 50% capacity reduction in Figure 8. The application of this measure on the left graph only indicates the effect of a 50% capacity reduction on the network performance. Comparison between the edges is therefore only possible on the 50% capacity reduction scenario. This indicator does not give any information on the effect on the network performance for the other capacity reduction scenarios.

When for all edges the network performance-capacity reduction curves are monotonically increasing and more or less show a convex, concave or linear relationship, the threshold value indicator on a 50% capacity reduction can indicate the form of the relationship. That is to say, when the normalized network performance for a 50% capacity reduction is larger than 0.5 then the curve is concave, when it is smaller than 0.5 the curve is convex and when it is equal to 0.5 the curve is linear. This indication on the function types does not hold when the curves deviate from this standard three functions.

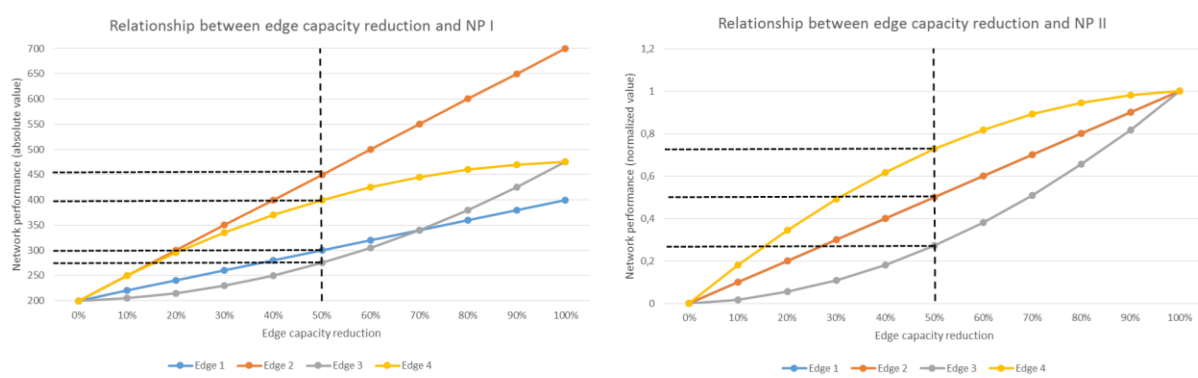


Figure 8 Graphs of relationship between capacity reduction and network performance based on fictive numbers for four different edges. In both graph a 50% capacity reduction threshold is depicted by the vertical line, the horizontal lines indicate the corresponding values of network performance for the different edges.

Another example of the application of the threshold value measure is the corresponding capacity reduction for a network performance threshold value. An example of the application of this type of threshold value measure is given in Figure 9. The applied measure in the first graph indicates at what percentage of capacity reduction the increase in network performance is exceeded. This measure indicates the criticality of the different edges but does not give an indication on the form of the curve.

The application on the second graph does indicate form in the same way as it does in the application in the same graph in Figure 8. When the 0.5 network performance threshold measure indicates a capacity reduction smaller than 50% the curve is concave, when this measure indicates a capacity reduction larger than 50% the curve is convex and when the measure indicates a capacity reduction of exactly 50% the curve is linear. These indications are again only applicable under the assumption of monotonically increasing network performance-capacity reduction curves being more or less a convex, concave or linear relationships.

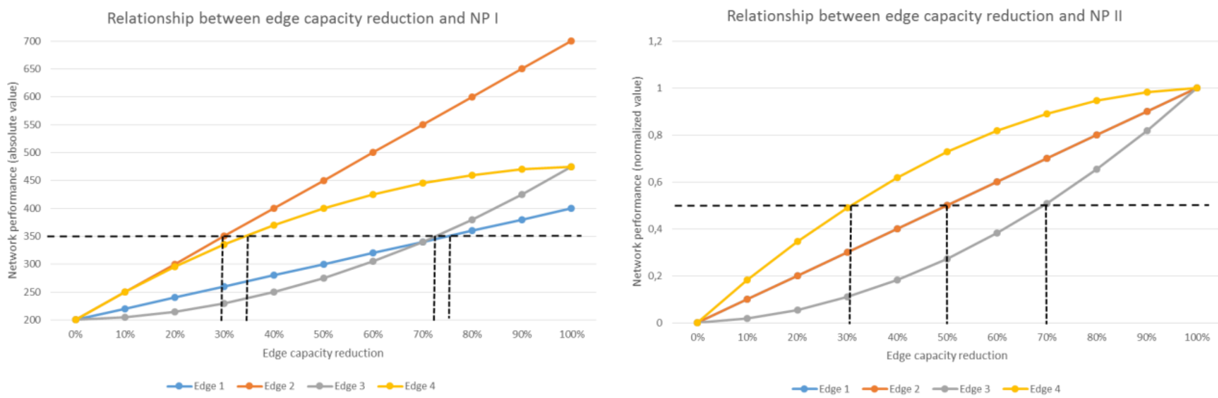


Figure 9 Graphs of relationship between capacity reduction and network performance based on fictive numbers for four different edges. In the left graph the network performance threshold at 350 is indicated by the horizontal line, the vertical lines indicate the corresponding values of the capacity reduction for the different edges. In the right graph the network performance threshold at 0.5 is indicated by the horizontal line, the vertical lines indicate the corresponding value of the capacity reduction for the different edges.

2.4.2. Local slopes in curves

The second category of measures is the measurement of the slopes of the relationships at some specified point. This is for example done in a study on freight resilience measures (Adams et al., 2012). Examples for the application of this category of indicators are given in Figure 10.

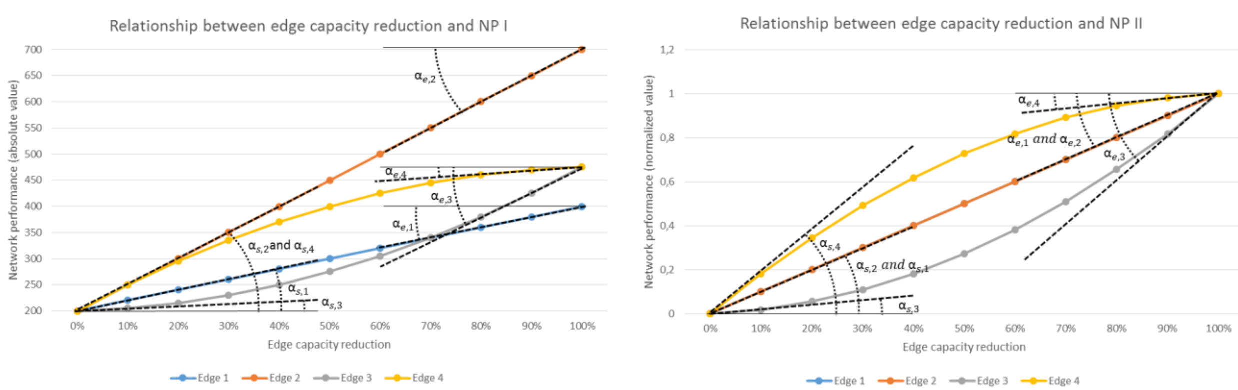


Figure 10 Graphs of relationship between capacity reduction and network performance based on fictive numbers for four different edges. The slopes for both graphs for all edges are indicated with α at 0% and 100% capacity reduction points.

The measure at the 0% capacity reduction point in the first graph indicates the criticality of the edge in case of small capacity reduction; steeper slope indicates a higher edge criticality. The measure in the same graph at the 100% capacity reduction point indicates the criticality of the edge in case of large capacity reduction; in general, a steeper slope indicates a higher edge criticality. However, this does depend on the slope at the end but also on the absolute value of the curve at the 100% capacity reduction point.

The measure applied in the right graph in Figure 10 does, with the same assumptions as in Section 2.4.1, indicate the relative sensitivity of the network performance. The measure at the 0% capacity reduction point in the second graph indicates the network performance sensitivity to smaller capacity reductions and the measure at the 100% capacity reduction point in the second graph indicates the network performance sensitivity to larger capacity reductions. For both measures a steeper slope indicates a higher sensitivity.

The application of this measure in both graphs is not able to indicate the total effect of an edge on the network robustness assessing all the disruption scenarios. The application of this indicator on the right graph does, however, give an indication on the form of the curve and thus on the sensitivity of the edge on different percentage of capacity reduction.

2.4.3. Ratios of curve values

A third category of measures are ratios of performance values for different situations or scenarios. Example of the use of this category of measure can be found in studies on system resilience. Application in these studies is done by measuring the difference between system performance before and after disruption (Faturechi et al., 2014; Faturechi & Miller-Hooks, 2014; Miller-Hooks et al., 2012; Nair et al., 2010; Omer et al., 2014). Another found variant of this measure is the ratio between the system performance after disruption and the worst-case degraded performance level (Cox et al., 2011; Orwin & Wardle, 2004).

In most cases applying a ratio measure on the normalized graph does not make any sense, because dividing by zero is not possible and the value of the network performance at a 100% capacity reduction relative to the maximum network performance at 100% always returns 1. However, applying some ratio measures in the graph with absolute values does give insights. Two variants of ratio measures are discussed here.

The first ratio measure is calculated by dividing the network performance for full link breakdown (a capacity reduction of 100%) by the maximum network performance for full link breakdown over all the edges. This measure returns the effect of a full link breakdown relative to the maximum effect of a full link breakdown. It does make it possible to compare the effects of different links. It is however not possible to compare the robustness for different network or to assess the effects of partial capacity reductions.

The second ratio measure is calculated by dividing the network performance for full link breakdown (a capacity reduction of 100%) by the network performance for the base case scenario (a capacity reduction of 0%). This returns the effect of a full link breakdown relative to the base case scenario. This makes it possible to compare the effects for different edges and by taking the sum or the mean it can be used to assess robustness of the network as a whole. The downside of this ratio is, again, that it does not asses the partial capacity reductions.

Both ratios above can also be applied to network performances for other (partial) capacity reductions. But, it will never be possible to indicate the accumulated effect over the entire range of capacity reductions.

2.4.4. Area under curves

The first measure able to capture the accumulated effect of the total range of capacity reduction on the network performance is measuring the area under the graph. In some reviewed studies this is done in a simple way by calculating the triangle between the network performance for a 100% capacity reduction and the network performance for the base case scenario (see Figure 11) (L. Zhang et al., 2009; Zobel, 2011) and in a more sophisticated way by taking the integral of the relationship. (Bruneau et al., 2003; Cats & Jenelius, 2015b; Enjalbert et al., 2011; Faturechi & Miller-Hooks, 2014; Omer et al., 2011; Rose, 2007; Vugrin et al., 2010; Vugrin et al., 2011; Vugrin & Turnquist, 2012).

By calculating the area in the simple way (see red triangle in Figure 11) it is possible to measure the accumulated effect of the total range of capacity reductions but only with the assumption that the curve is linear. This is only the case for the application of the measure in the left graph, applying this method to the right graph will return the same value for every edge because of the normalization. With the simple area measure it is possible to compare the total effect of the different edges and by taking the sum or the mean it can be used to assess robustness of the network as a whole. But, by assuming all relationships to be linear, the measure is not able to reflect on the form of the curve if they deviate from being linear.

By taking the integral of the curves it is possible to measure the accumulated effect of the total range of capacity reduction without an assumption on the form of the curve. Applying this to the first graph makes it also possible to compare the accumulated effect of the different edges and by taking the sum or the mean over the different edges it can be used to assess robustness of the network as a whole. The application on the first graph, however, does not make it possible to reflect on the form. A concave and a convex function with a different network performance for a full link breakdown can have a same value in this measure. This is not the case when applied to the second graph. When applied to the second graph the measure reflects on the form of the graph but you will not be able to compare the total effects of the different edges on network performance because of the normalization.

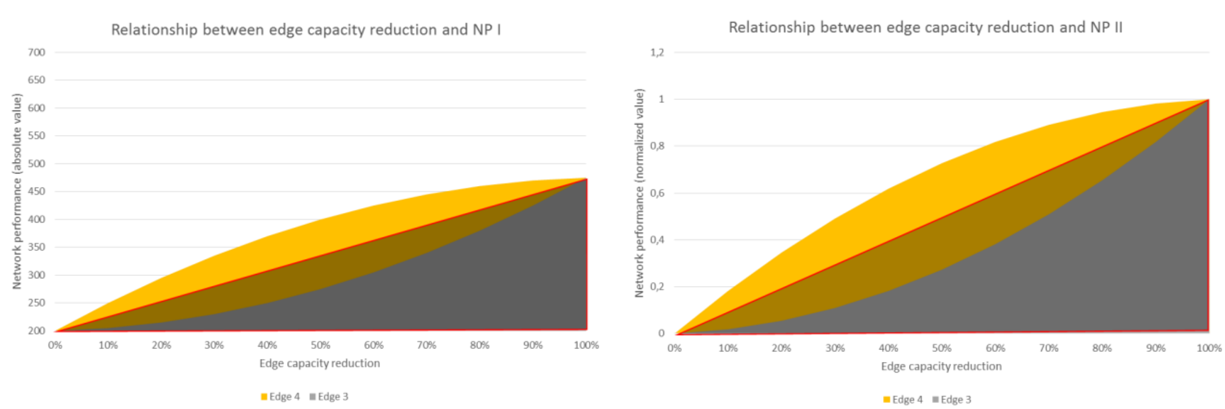


Figure 11 Graphs of relationship between capacity reduction and network performance based on fictive numbers for two different edges. The gray area is the area under the curve of edge 3, the curve under edge 4 is indicated by the sum of the gray and yellow areas. The red triangle indicates the simple area measure.

2.5. Synthesis

Synthesis on PT network robustness is visualized in the synthesis matrix in Table 5 based on the following characteristics of the study: the modelled disruption, whether mitigation measures are included, research detail level and the type of network performance indicator.

Table 5 Synthesis matrix of literature review on PT network robustness

	Modelled disruption			Research detail level			Type of network performance indicator			
				Topological only						
	Full link or node breakdown	Partial capacity reduction	Mitigation measure included	Infrastructure network only	Service network included	Including passenger assignment	Topological	Travel time	Generalized travel costs	Welfare (monetarized)
Angeloudis and Fisk (2006)	✓			✓			✓			
Berche et al. (2009)	✓				✓		✓			
Derrible and Kennedy (2010)	NA	NA			✓		✓			
De-Los-Santos et al. (2012)	✓		✓			Static			✓	
von Ferber et al. (2012)	✓			✓			✓			
Cats and Jenelius (2014)	✓					dynamic				✓
Rodríguez-Núñez and García-Palomares (2014)	✓					static		✓		
Cats and Jenelius (2015a)	✓					dynamic				✓
Cats and Jenelius (2015b)		✓				dynamic				✓
X. Zhang et al. (2015)	✓	✓		✓			✓			
Cats (2016)	✓					static		✓		
This study	✓	✓	✓			static			✓	

In the early years of studies on PT robustness the main focus was on full link breakdown and the detail level stayed at the topological level. Just recently studies on the effect of partial capacity reductions emerged, this included partial increase in travel time on the link level (X. Zhang et al., 2015) and also frequency reductions on both the link and link level (Cats & Jenelius, 2015b). Only one study is found including mitigation measures in the robustness study, De-Los-Santos et al. (2012) examined the effect of a bus alternative in case of full link breakdown. Only the past three years passenger demand assignment is applied in the modelling regarding robustness of PT network. Cats and Jenelius (2014, 2015a, 2015b) used the dynamic transit operation and assignment model BusMezzo to dynamically assign passenger demand. The other found researchers applied static assignment. The increase in detail level of the studies is also reflected in the applied network performance indicator. In the first years of PT Robustness studies only topological indicators are applied, just recently travel time is included as network performance indicator. The latter type of network performance indicator is just in different levels of detail, in other words including waiting and transfers (i.e. generalized travel costs) and expressed in monetary terms (i.e. welfare) or not.

In this study the passenger demand is included in a static assignment and the network performance is expressed in generalized travel costs (i.e. including transfer and waiting time). Full link breakdown and partial capacity reduction disruptions are modelled including mitigation measures (i.e. rerouting and short-turning).

Based on the five criteria the types of robustness indicators are assessed, see Table 6. The *Areas under curves* indicator is the only indicator able to capture the accumulated effect of capacity reductions. All types of indicators are able to capture the form of the curves, however for the first three types assumptions are needed for the in between lying values. The easiest way to translate an indicator to monetary terms is when the indicator can be expressed in minutes. This is only possible for the *threshold values in curves* and *Areas under curves* the other two indicators are dimensionless number or degrees. In some sort of way all types of indicators can be translated to the network level by taking the mean or the sum of the link level scores. Only the *local slopes in curves* indicator is somewhat difficult to explain to other people, in contrast to the other three indicator types.

Table 6 Scores of the different types of robustness indicators on the five robustness indicator criteria

Robustness indicator criteria	Threshold values in curves	Local slopes in curves	Ratios on curve values	Areas under curves
Able to capture the accumulated effect of capacity reductions	No	No	No	Yes
Able to capture the form of the curves	Yes	Yes	Yes	Yes
Able to translate to monetary terms	Yes	No	No	Yes
Able to translate to network level	Yes	Yes	Yes	Yes
General comprehensible	Yes	No	Yes	Yes

The *Areas under curves* indicator seems to be the most meaningfully type of indicator for this robustness study. The score card of Table 6 is used for the proposed robustness indicators discussed in Section 4.6.

2.6. Conclusions

This chapter addressed the literature review on the topics of PT network robustness, PT network performance indicators and types of robustness indicators.

First topic of the literature review on PT network robustness was the relationship between network design and network robustness. The found relationships indicated that next to the pure link characteristics the surrounding infrastructural (e.g. clustering) and service network characteristics (e.g. service network density) are of influence as well. Second the explanatory link characteristics on link criticality were explored. The found link characteristics emphasize again the influence of service network characteristics (e.g. length of traversing lines) on link criticality. Third the found relationships between capacity reduction and network performance were discussed. Different relationships, from convex to concave, were found for different types of disruptions. Planned line disruptions, in general, show convex relationships. Whereas unplanned link disruptions, in general show, concave relationships. This does not give a clear direction on what the relationships looks like for planned link disruptions and therefore emphasizes the importance of this study.

Second, the network performance indicators used in literature were explored. Network performance was found to be measured in a whole set of indicators, from pure topological indicators (e.g. connectivity) to indicators which measure the effect of travellers on the network performance (e.g. total weighted travel time).

Third, possible robustness indicators used in literature were explored. Four categories of indicators were found: threshold values, local slopes, ratios and areas under curves. The only found indicator that was able to capture the accumulated effect of the total range of capacity reduction scenarios on the network performance is to measure the area under the capacity reduction-network performance curve.

The found studies indicate the black spot in the research of network robustness on partial capacity reduction on the link level. The found used network performance indicators and robustness indicators will be used for formulating appropriate indicators for this study (see Chapter 4). The different insights on the influencing characteristics give a first hint to what link characteristics could be of influence on the link robustness. This will be used in the exploration of influencing link characteristics on link robustness in Chapter 5. But first is studied how capacity reductions look like in reality, this is discussed in Chapter 3 on urban rail bound disruptions and its consequences.

3. Urban rail bound disruptions and their consequences

In chapter 2 influencing characteristics on network and link robustness levels were identified from literature. Most of the reviewed studies applied a theoretical full link breakdown neglecting the service network (e.g. PT lines, frequencies etc.). In reality, however, partial capacity reduction take place and the public transport operator (PTO) is not able to neglect the service network. In this chapter the relationships between infrastructure supply, transport service network and travellers is discussed for urban rail bound disruptions. The following question is addressed in this chapter:

'(3) How do infrastructural characteristics (e.g. track availability), in capacity reduction situations, affect the service characteristics of the network (e.g. speed, frequency)?'

First in general an overview is given of the stochastic forces acting on PT systems (Section 3.1) which are presented in Figure 12. Next, a categorization of major discrete events is given (Section 3.2) and the effect of these events on the infrastructure supply (Section 3.3). The chapter ends with how these infrastructure supply reductions affect the transport service network (Section 3.4) and how these change in transport service network affect travellers (Section 3.5). All sections are indicated in the figure below.

- Categorization of major discrete events (Section 3.2)
- Effect on infrastructure supply (Section 3.3)
- Effect on the transport service network (Section 3.4)
- Effect on travellers (Section 3.5)

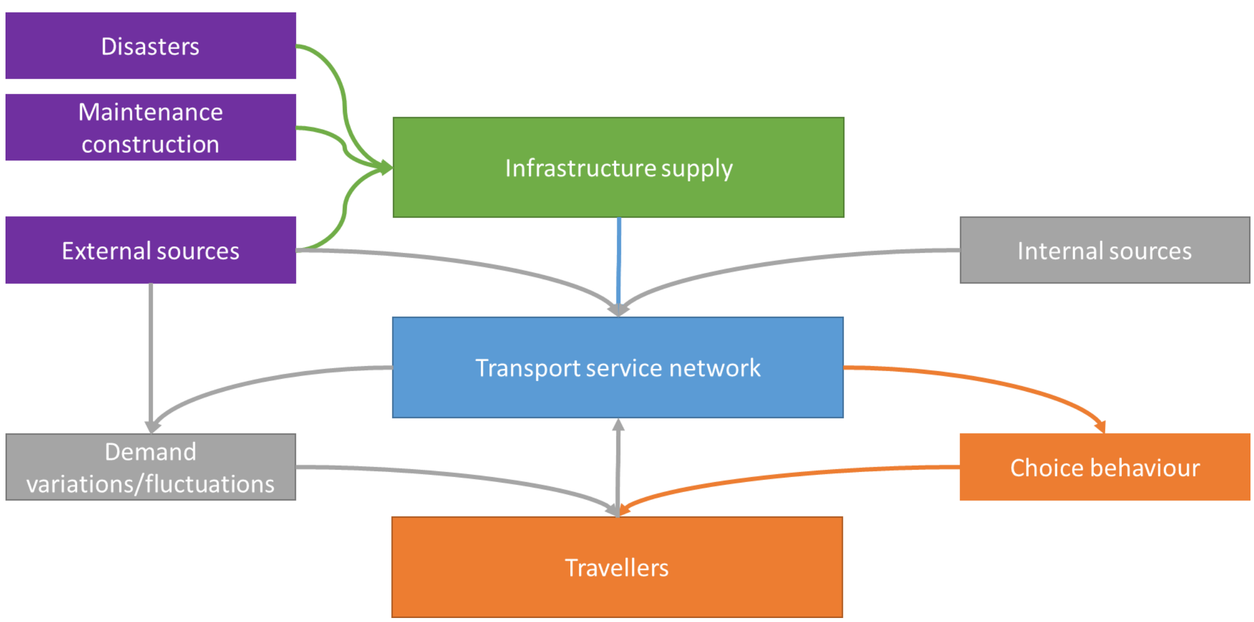


Figure 12 Forces acting on key elements of public transport systems. Modified from Tahmasseby, S. (2009).

3.1. Forces acting on PT systems

Existing models describing transport systems are clear frameworks to assess occurring disruptions with their corresponding consequences. One of these models is the layer model (Schoemaker et al., 1999) used in research and teaching in the transport sector, see Figure 13. The layer model consists of an activity layer, a transport service layer and a traffic service layer. Between the three layers two markets are situated. The first one is the transport market that describes the interaction between the activity demand of persons and the transport service supply. The transport market consists of a supply pattern in space and time for the transportation of people and goods. The second market is the traffic market, which describes the traffic demand and supply. Traffic demand can be seen as a traffic demand pattern in space and time and traffic supply consists of various infrastructure and regulations for using this infrastructure.

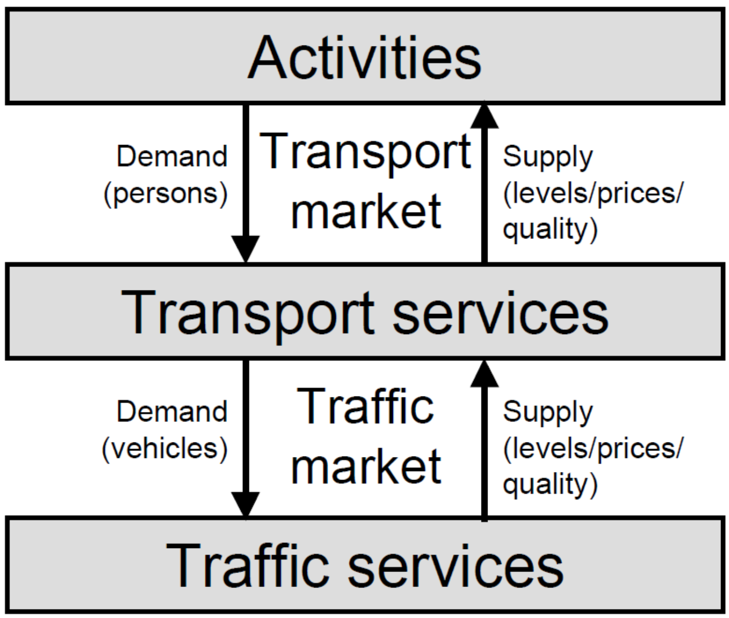


Figure 13 Layer model of the transportation system. Reprinted from Tahmasseby (2009).

Based on above presented layer model, Tahmasseby (2009) identified sources of variations in all three layers for urban PT networks. These are variations in the infrastructure network, public transport service network and travellers' behaviour. The translation of the layer model for a public transport system including forces acting on these layers can be found in Figure 12 in the previous section.

The infrastructure supply (i.e. the traffic service layer in the layer model) can be affected by maintenance and construction, disasters and external sources like incidents, public events and bad weather. Large snow piles or flooding can close an infrastructure segment or blockade a switch. During maintenance and construction some part of the infrastructure can get taken out of service because of safety precautions. If a demonstration is held on a square where tracks are situated these can be blocked by the protesters.

The transport service network (i.e. the transport service layer in the layer model) is subsequently affected by the disruptions in the infrastructure network but can also be directly affected by internal and external sources and by travellers (i.e. the activity layer in the layer model). If infrastructure is not available PT vehicles are not able to run them and lines have to be taken out of service, rerouted or cut. The direct effect of external sources can be exemplified by a collision in which a PT vehicle is not able to run for at least some time or strong winds making it for some vehicles impossible to run according to schedule. Next to these external effects the internal sources can also affect the transport service network. For the public transportation of passengers, vehicles and personnel are needed and the supply of both can vary through for example diseases, labour disability and vehicle failures. As a consequence, the service frequency or the capacity of vehicles may need to be decreased temporarily. Demand variations by, for example, large public events (e.g. soccer game or large concert) may affect the service network design and/or performance. An operator can choose to adapt its service network to these higher demands or let the demand affect the performance of the service network. High demands can cause delays for the vehicles through, for example, higher boarding and alighting times.

Last, but not least, the travellers behaviour (i.e. the activity layer in the layer model) is affected by demand fluctuations and individual choice behaviour. The demand fluctuations are affected by external sources like public events (e.g. soccer games or big events), both demand fluctuations and individual choice behaviour are at their turn affected by the transport service network (e.g. new service network supply makes other route choices more attractive and changes the demand supply).

3.2. Categorization of disruptions on PT networks

Categorization of disruptions on PT networks is helpful to be able to draw general conclusions on the effect of all the different forces acting on key elements of the PT system. Categorization can be done in numerous ways, for example:

- the severity of the disruptions in terms of infrastructure unavailability;
- the frequency of occurrence;
- the duration of the event;
- planned versus unplanned.

On an even higher level, which goes beyond the PT network only, disruptions can be categorized in disruptions where humans are in danger and where they are not. The GVB for example categorizes every disruption report into BOGT[2] (Fire, accident, hazardous substances and terror) or VUS[3] (disturbance, failure and disruption) (W. van Oort & de Hoog, 2015).

In his dissertation on the impacts of distortions and random variations on transit services Tahmasseby (2009) divided the disruptions in two categories: minor ongoing continuous events and major discrete events. Minor ongoing continuous events refer to events that have a higher frequency of occurrence but a low severity; this category is in general linked to PT reliability. Major discrete events refer to events that have a low frequency of occurrence but a high severity; this category is in general linked to PT robustness. M. Yap (2015) states that he distinguishes major discrete events from minor ongoing continuous events in

[2] In Dutch: brand, ongeval, gevaarlijke stoffen en terreur
[3] In Dutch: verstoring, uitval en storing

the effect of an event on infrastructure availability: major discrete event affect infrastructure availability whereas the minor ongoing continuous events do not. In the framework of Tahmasseby (2009) the major discrete events only apply to a part of the stochastic forces, as is shown in Figure 12.

In his thesis on the robustness of multi-modal public transport M. Yap (2014) made an overview of occurring major discrete events in categories for train, metro/light rail, tram and bus network, see Table 36 in Appendix A. Most of the categories found by (M. Yap, 2014) are unplanned disruptions (merely the last category 'Large maintenance work' can be a planned disruption) whereas this study focuses on planned disruptions.

In an interview with two project leaders temporary traffic measures of the GVB (W. van Oort & de Hoog, 2015), the urban PTO in Amsterdam, a more comprehensive categorization of planned disruptions was found (see Table 7).

Table 7 Overview of planned major discrete event types for metro and tram network

Metro network	Tram network
Construction or maintenance works near the tracks (e.g. sewage works, construction works etc.)	
Track works (e.g. maintenance or renovation)	
	Road works (e.g. maintenance or renovation)
	Planned events (e.g. a marathon, public events etc.)

As trams run on tracks along public urban streets all four categories of planned major discrete events are applicable for the tram network. The metro on the other hand, generally operates on grade-separated infrastructure with an exclusive right-of-way with as a consequence that the metro network is only affected by *construction or maintenance works near the tracks* and *track works*.

3.3. Effect on infrastructure supply

The effects of planned major discrete events on infrastructure availability is depicted in green in the framework of Tahmasseby (2009) (see Figure 12) as the lines from external sources, disasters and maintenance construction to infrastructure supply.

From a qualitative assessment of the effect of different planned major discrete event categories with the two project leaders temporary traffic measures of the GVB (W. van Oort & de Hoog, 2015) a categorization of infrastructure (un)availability was found:

- All tracks of a link are unavailable
- One direction of the tracks of a link is unavailable
- A local speed limit and extra waiting time for clearing the tracks on all tracks
- A local speed limit and extra waiting time for clearing the tracks in one direction

For the found planned major discrete event categories (see Section 3.2) a likelihood of infrastructure (un)availability was found. This is presented in Table 8.

Table 8 Qualitative assessment of the effect of different planned major discrete event categories on infrastructure availability on W. van Oort and de Hoog (2015)

<table>
<tr><th>Planned major discrete event category</th><th>Qualitative assessment based on expert judgment</th></tr>
<tr><td>Planned events</td><td>• All tracks of a link are unavailable</td></tr>
<tr><td>Construction or maintenance works near the tracks</td><td rowspan="3">• All tracks of a link are unavailable (more likely)
• One direction of the tracks of a link is unavailable (less likely)
• A local speed limit and extra waiting time for clearing the tracks on all tracks (more likely)
• A local speed limit and extra waiting time for clearing the tracks in one direction (less likely)</td></tr>
<tr><td>Road works</td></tr>
<tr><td>Track works</td></tr>
</table>

The GVB project leaders emphasize that all planned major discrete events are unique and that there is no standard infrastructure availability per category. All combinations of situations happen: restrictions in one or two directions and partial or full track capacity reductions. They, however, state that most of the time the track unavailability or speed limit is mostly bidirectional.

The defined categories of infrastructure (un)availability largely correspond with the categories found by M. Yap (2014) from a qualitative assessment based on an interview with a ProRail[4] dispatcher:

- All tracks of link are unavailable
- 1 track of a link is unavailable
- 50% of the tracks of a link is unavailable

Two differences stand out from the two categorizations of infrastructure (un)availability as a consequence of major discrete events. First, M. Yap (2014) found a distinction between 1 track of link blocked and 50% of the track of link blocked. This is due to the fact that his starting point for the categorization is a train network. In train network links with more than 2 tracks are more common than in tram and metro networks. Second, he did not categories partial infrastructure (un)availability. This is probably due to the fact that his scope was on unplanned major discrete events. In these situations less information is available on risks and safety issues on these disrupted links and operators will be more inclined to take the whole link out of service than trying to operate the link partially or with a local speed limit.

Based on his three found categories M. Yap (2014) was even able to do a quantitative assessment on the expressed probability of occurrence of the different unplanned major discrete event types (see Table 37

[4] ProRail is a governmental organization responsible for the Dutch national railway network infrastructure

in Appendix A). The results emphasize the diversity of infrastructural (un)availability effects by major discrete events. None of the categories stands out from the rest.

3.4. Effect on the transport service network

The effects of major discrete events on the network performance do not stop at the infrastructure availability. Public transport operators can adapt their transport service network to the new situation in infrastructure supply. This relation is depicted in blue in the framework of Tahmasseby (2009) in Figure 12 as the relationship between infrastructure supply and the transport service network.

Public Transport Operators have a number of options in case of planned major discrete events, which depends on the severity of the major discrete event (i.e. the category of infrastructure (un)availability) and the infrastructural possibilities of the operator (i.e. rerouting options, turning options). Based on the interview with the GVB experts (W. van Oort & de Hoog, 2015) a categorization of transport service network effects were derived. In general there are four transport service network options due to infrastructure supply reductions: local speed limits, deleting lines, rerouting lines and cutting lines.

Because the bi-directional infrastructure (un)availability categories (i.e. all tracks of links disrupted) occur most often, the focus will only be on these two categories with the following numbers:

1. All tracks of a link are unavailable
2. A local speed limit and extra waiting time for clearing the tracks on all tracks

The transport service network options can be categorized into do-nothing scenarios and mitigation measures scenarios, see Table 9. The first consists of deleting the traversing lines and the application of a local speed limit. Which one is applied depends on the infrastructure (un)availability category. The second category consists of rerouting traversing lines or cutting traversing lines and can both be applied in both infrastructure (un)availability categories.

Table 9 Overview of transport service network options due to infrastructure (un)availability (1: all tracks of a link are unavailable; 2: a local speed limit and extra waiting time for clearing the tracks on all tracks)

Infrastructure (un)availability category	1	2
Do-nothing	• Delete traversing lines	• Apply local speed limit
Apply mitigation measure	• Reroute traversing lines • Cut traversing lines	• Reroute traversing lines • Cut traversing lines

Each transport service network has its consequences and limitations; this is discussed in the following subsections. First the two do-nothing options are given (i.e. delete traversing lines and apply local speed limit) followed by the two mitigation measure options (i.e. reroute traversing lines and cut traversing lines).

3.4.1. Delete traversing lines (Do-nothing scenario one)

The lines traversing a complete unavailable link will not be able to run their original route. When not chosen to apply a mitigation measure, the only options is then to take all the traversing lines out of order (i.e. delete traversing lines). As a consequence the transport service network becomes less connected and probably some stops will not be served anymore (see Figure 14).

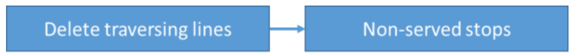


Figure 14 The effects of deleting lines on the transport service network

3.4.2. Apply local speed limit (Do-nothing scenario two)

When a link is not completely unavailable but a local speed limit is in force, the do-nothing scenario does not imply the deletion of the traversing lines. The lines that traverse the link are however affected.

A visualization of a local speed limit is given in figure 15. In this example the speed limit of 10 kilometres per hour is applied to only the most central link between stops 2 and 3. In reality the speed limit and the application length can differ. A local speed limit can, for example, also be a 30 kilometres per hour speed limit on half of the line (e.g. between stops 2 and 4). In theory an unlimited number of combinations is possible. However, because of the resulting effects on trip times and the number of needed vehicles a large number of these combinations is in practice very undesirable.

Figure 15 Visualization of a local speed limit on a link in a single line. The blue circles are served stops, the blue squares are terminal stations and the number in the red circle is the local speed limit.

Effects of local speed limits on the transport service network

The effect of local speed limits on the transport service network is a local higher running time ($t_{2,3}^{run}$ and $t_{3,2}^{run}$). As a consequence the trip times between terminal A and B ($t_{A,B}^{trip}$ and$t_{B,A}^{trip}$) increases. The trip time is the sum of all the station-to-station travel times, in this case only in one direction from terminal A to B:

Equation 2 Trip time between terminal A and B

$$t_{A,B}^{trip} = t_{A,1}^{run} + t_1^{dwell} + t_{1,2}^{run} + t_2^{dwell} + t_{2,3}^{run} + t_3^{dwell} + t_{3,4}^{run} + t_4^{dwell} + t_{4,B}^{run}$$

If the increase in trip time cannot totally be absorbed by an available slack time the cycle time t^{cycle} increases as well. The cycle time t^{cycle} is the sum of the trip times $t_{A,B}^{trip}$ and $t_{B,A}^{trip}$ and the layover times in both terminal A ($t_A^{layover}$) and B ($t_B^{layover}$):

Equation 3 Cycle time between terminal A and B

$$t^{cycle} = t_{A,B}^{trip} + t_{A}^{layover} + t_{B,A}^{trip} + t_{B}^{layover}$$

As a consequence of a larger cycle time t^{cycle} on the line and depending on the length and size of the speed limit a larger number of vehicles n or a lower service frequency f is needed:

Equation 4 Number of needed vehicles

$$n = \left\lceil \frac{t^{cycle} * f}{60} \right\rceil$$

The effect of a local speed limit is represented in a chart in figure 16.

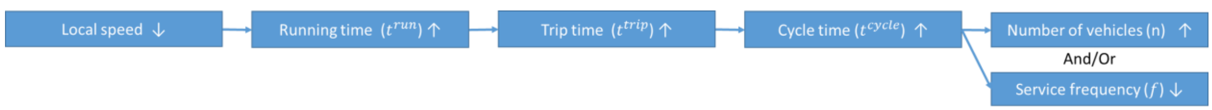


Figure 16 The effects of local speed limits on the transport service network

3.4.3. Reroute traversing lines

In both infrastructure unavailability situations an operator has the option to reroute the traversing lines. This does however depend on the surrounding infrastructure network. A visualization of rerouting on a single line is given in figure 17. In this example the infrastructure between stop 2 and 4 is unavailable and the operator is able to take a detour between stop 2 and 4 traversing the edges from 2 to 5, from 5 to 6 and from 6 to 4 and vice versa. In this case stop 3 is not served anymore and stops 5 and 6 are extra served.

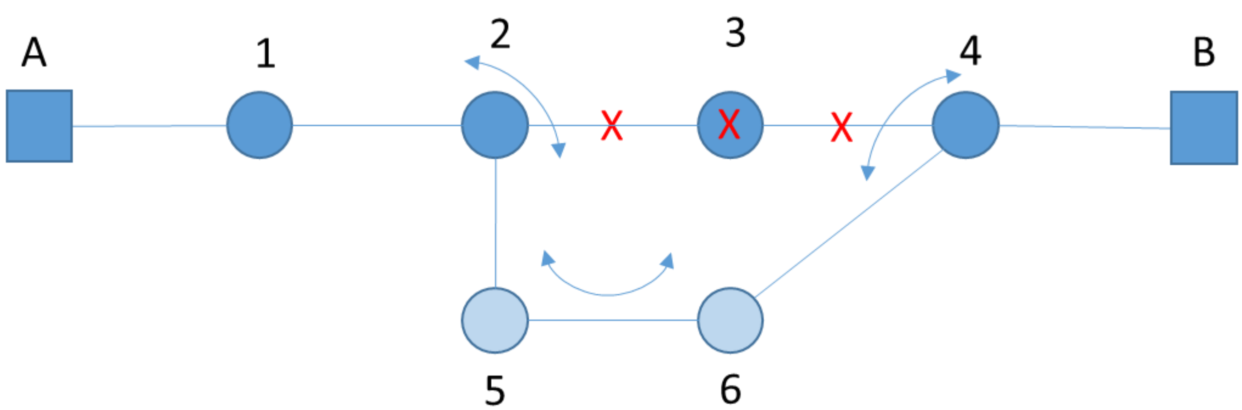


Figure 17 Visualization of rerouting between two nodes on a single line. The dark blue circles are original served stops, the light blue are the extra detour served stops, the blue squares are terminal stations and the red cross indicates the unavailable links and stops.

Effects of rerouting on the transport service network

The effects of rerouting on the transport service network are threefold. First, just like for the local speed limit, the cycle time t^{cycle} increases for which it can be that more vehicles are needed or that the scheduled headway has to be increased (i.e. a lower line service frequency). Second, some stops are not being served anymore and last some stops are served extra or new temporarily stops are opened. The effect of rerouting is represented in a chart in figure 18.

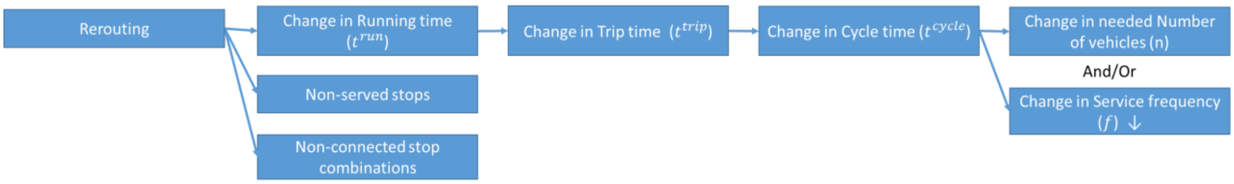


Figure 18 The effects of rerouting on the transport service network

Infrastructural limitations of rerouting

To be able to reroute in urban rail bound PT networks infrastructure has to be available in two respects. First, infrastructure links have to available to run on. So, in case of the example of figure 17 tracks have to be present on links 2-5, 5-6 and 6-3 for vehicles to run on. Second, infrastructure connections are needed to connect the original route to the reroute-route (see figure 19). The situation in figure 19 can, for example, represent the situation just left of stop 4 in the example in figure 17, in which the blue lines indicate the original route from stop 4 to 3 and where the green lines represent the essential connections to the alternative route in direction of stop 6 (purple lines). A similar situation is needed right of stop 2 on the original route.

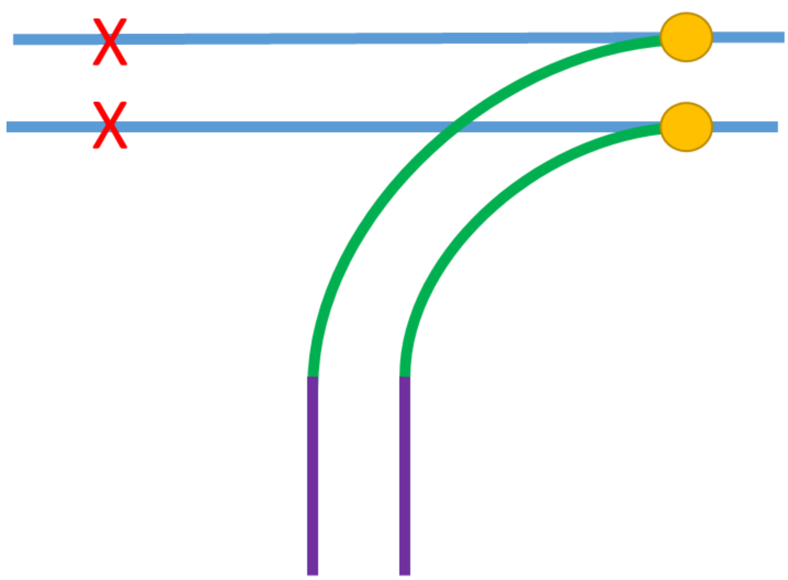


Figure 19 Representation of a connection between an original route and a reroute-route. Lines represent unidirectional tram links. Blue line represents the original line, purple lines the alternative infrastructure links, green the connecting infrastructure between original line and alternative route and yellow the places of switches.

Service network limitations of rerouting

It can be that the route of the detour shares tracks with other lines that are also rerouted or for which that part of the infrastructure is part of their normal route. If the route is intensively used in terms of service frequency (i.e. number of vehicles per hour) it can be that a route alternative is not possible. For example, assume that in the example of figure 17 the link between node 3 and 6 is used near capacity. Adding the line from A to B to this link could make it exceed capacity on this link and as a consequence the rerouting options over this link is not possible.

Other limitations

As a tramline in general crosses multiple other transport networks it is limited by other traffic as well. If a tramline shares space with a very crowded car link it is not desirable to reroute like that, also emergency services can have a voice in this. Further it can be that traffic lights on some intersections are tuned such that rerouting is not desirable.

3.4.4. Cut traversing lines

The second mitigation measure is to cut the traversing lines. A visualization of cutting a single line is given in figure 20. In this example the links between stop 2 and 4 are unavailable. Vehicles that run from terminal A are turned after stop 2 and vehicles from terminal B are turned after stop 4 as indicated by the double sided arrowed lines. The original line from A to B is cut into two lines from A to 2 and from B to 4.

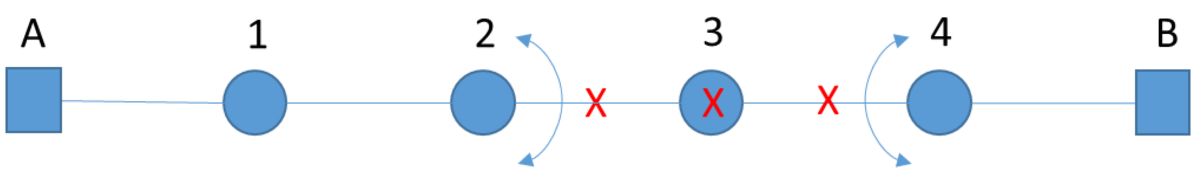


Figure 20 Visualization of cutting lines between two nodes on a single line. The dark blue circles are original served stops, the, the blue squares are terminal stations, the red cross indicates the unavailable links and stop, the double sided arrowed line indicates where vehicles will turn.

Effects of cutting lines on the transport service network

The effects of line cutting on the transport service network are threefold. Because of the unavailability of a number of links some stops are not served anymore. In case of line cutting there is no rerouting past the unavailable links and the line does not connect some stop combinations anymore. If there are no alternative routes in the networks the stop combinations are not served anymore at all. Third effect is that the original line changes into two separate lines with accompanying running times and served stops. As a consequence of these changes a different number of vehicles is needed or the scheduled headway (i.e. service frequency) has to be changed. The effect of cutting lines is represented in figure 21.

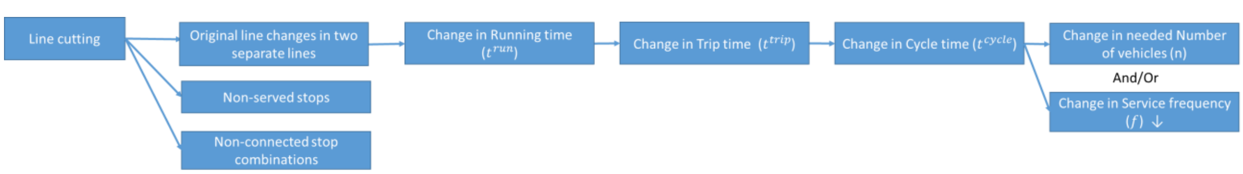


Figure 21 The effects of cutting lines on the transport service network

Infrastructural limitations

For a line to be cut the infrastructure has to be such that the vehicles are able to change running direction by turning. For one direction vehicles, like is the case for most tram models, infrastructure has to be available for trams to turn. This can be turning loops or wyes (i.e. triangular junctions). For two direction vehicles a crossover switch is sufficient. In Appendix B the use of these infrastructure elements for turning are visualized. If these infrastructural turning options are not available cutting a line is limited. Sometimes there are no turning loops or wyes available but because of available other infrastructure it is possible to make an artificial turning loop by using available infrastructure links in the networks, see figure 22.

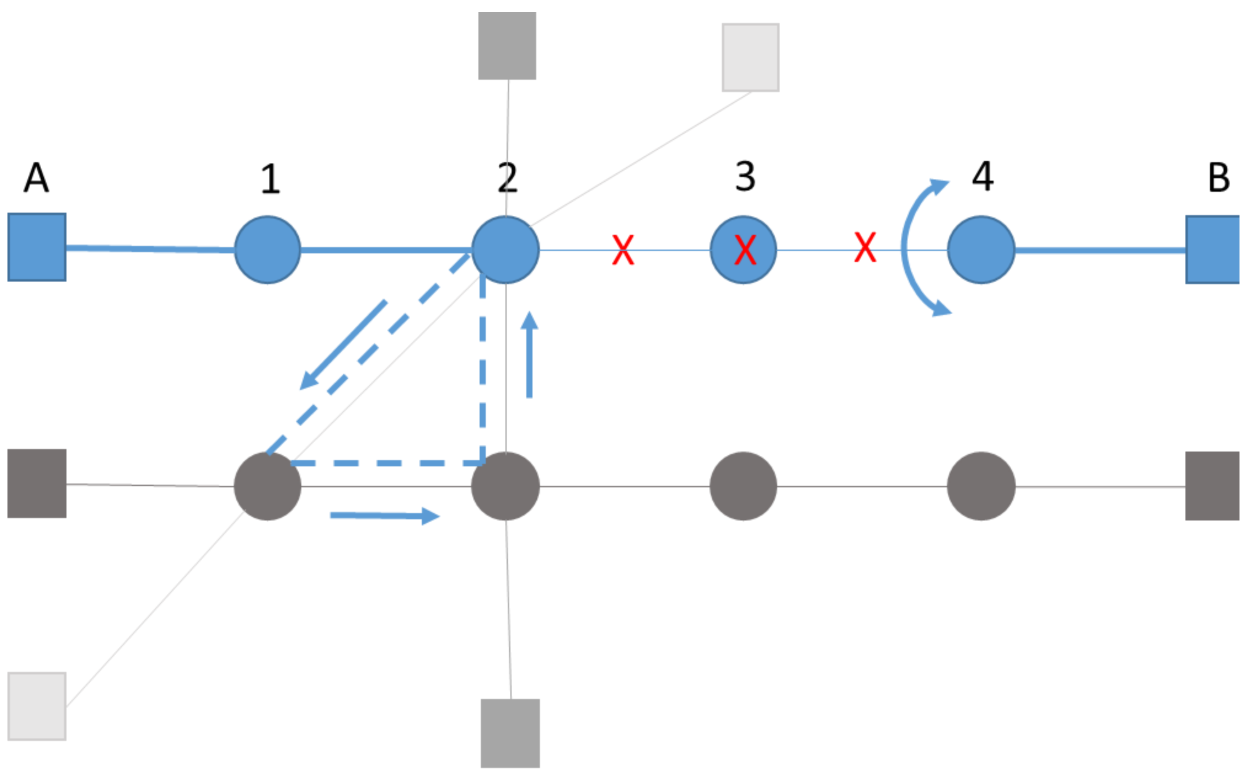


Figure 22 Example of the use of other infrastructure as artificial loop in case of line cutting. Grey scale colours are other lines, the blue is the cutted line. Squares are terminal stations, circles are stops. Red crosses indicate link and stop unavailability. Double sided arrows indicate turning point. Dotted line indicates the artificial loop.

3.4.5. Other effects of infrastructure supply on network service

In theory and in practice multiple other network service adaptations are possible in case of infrastructure unavailability. These adaptations are mostly custom work very much depending on the local situation. Example of these other service adaptations are combining lines or rerouting parts of lines to local hubs. Also the use of other PT vehicles in case of infrastructure unavailability is possible. One could think of temporarily replacing bus services or using the national railway trains.

3.5. Effect on travellers

From all the possible transport service network options in case of partial infrastructure unavailability three consequences affect the travellers route choice:

- Non-served stops
- Change in service frequency
- Extra served stops
- Change in running time

A non-served stop affects the travellers for which this stop was their destination or origin. When we look at the PT network as an isolated network (i.e. no other modalities to arrive at the destination) these travellers are disconnected and will not be able to make the trip they want in this specific situation. In reality, travellers will explore alternative modalities, alternative destinations or alternative moments of travelling when the stop is served again.

A change in service frequency, which in case of disruptions is mostly a decrease in service frequency, affects the waiting time of travellers using the affected lines. This applies to the travellers who start their trip at this line but also the travellers who transfer to this line. Because of the increase in waiting time their total travel time increases on this route. When this increase exceeds a certain threshold and other route alternatives become more attractive, travellers will start taking detours.

When, as a result of rerouting, stops get extra served this can decrease the waiting time of travellers starting their trip at this stop and travellers who transfer at this stop. Because of the decrease in waiting time the total travel time of these travellers decrease on these routes.

The change in running time between different stops because of the local speed limit affects the in-vehicle time of travellers. This can directly when a traveller traverses this part of the network or indirectly when a travellers choses to take detour. This is the case when the change in running time affects the in-vehicle time such that a certain threshold is exceeded. As a consequence, other route alternatives become more attractive and travellers will start taking detour. This can result in an increase in waiting time and number of transfers. In reality, when the PT route alternatives become less attractive, travellers will explore alternative modalities, alternative destinations or alternative moments of traveling.

Summarizing, travellers are affected in numerous ways depending on their origin and destination, original route choice and the changes in the transport service network. Directly, the in-vehicle time and waiting time can change through changes in running time and service frequencies. When these changes in running time and service frequency surpass a certain threshold travellers can choose to reroute, which can change their number of transfers and also their waiting time and in-vehicle time. Non-served stops cause travellers to be disconnected and extra served stops can positively influence the route alternative of travellers.

3.6. Conclusions

This chapter addresses the relationships between urban rail bound disruptions and its effect on travellers, through infrastructure network and service network changes. Disruptions related to robustness are characterized by the fact that they affect infrastructure availability. These disruptions are referred to as major discrete events.

For urban rail bound PT networks four types of planned major discrete events are found: construction or maintenance works near the tracks, track works, road works and planned events. All types of planned major discrete events can cause full link unavailability. The first three types can also cause partial link unavailability and local speed limits and increase in vehicle waiting time.

Public transport operators can often act on these local infrastructure (un)availability. Sometimes they will be forced into a do-nothing scenario which can result in deleting the traversing lines or applying a local speed limit. Which one depends on the type of infrastructure unavailability (i.e. a link is unavailable or a local speed limit is at hand). In some cases the PTO will be able to apply mitigation measures like rerouting or cutting traversing lines.

Travellers are affected by the chosen mitigation measures. In worst case they get disconnected, but they can also be affected by longer in-vehicle time, waiting times and more needed transfers. This, among others, depends on whether they are going to reroute or not.

Because travellers can be affected in numerous ways it is hard to theorise about the total effect of major discrete events on the network performance. It is therefore needed to model the PT network and apply these disruption scenarios to be able to analyse the effect on the network performance. The methodology (i.e. model) used for this analysis is presented in the next chapter.

4. Methodology

In this chapter the developed model for the robustness analysis of a PT network is presented. Based on a given supply (i.e. a given service network) and demand (i.e. a given OD demand distribution) the model is able to calculate the network performance of the PT network for different disruption scenarios. Based on the derived network performances the robustness of the model can be calculated.

A simplified graphical representation of this model can be seen in Figure 23. The numbers between the brackets refer to the corresponding sections in this chapter. The model starts with the base network which is the given undisturbed supply network. Depending on the chosen network for which the robustness study is going to be executed, this can be the current PT network, a future PT network or even a made-up network. This network is then represented in L-space and P-space, these representation forms and how they are generated are discussed in Section 4.1. After the network is represented the route choice set for every OD-pair is generated (Section 4.2) and the passenger demand is assigned to the network (Section 4.3). The previous three steps are repeated for all disruption scenarios (Section 4.4). After all scenarios are run the network performance is calculated (Section 4.5) for all scenarios. When all network performances are calculated the robustness indicators are calculated (Section 4.6).

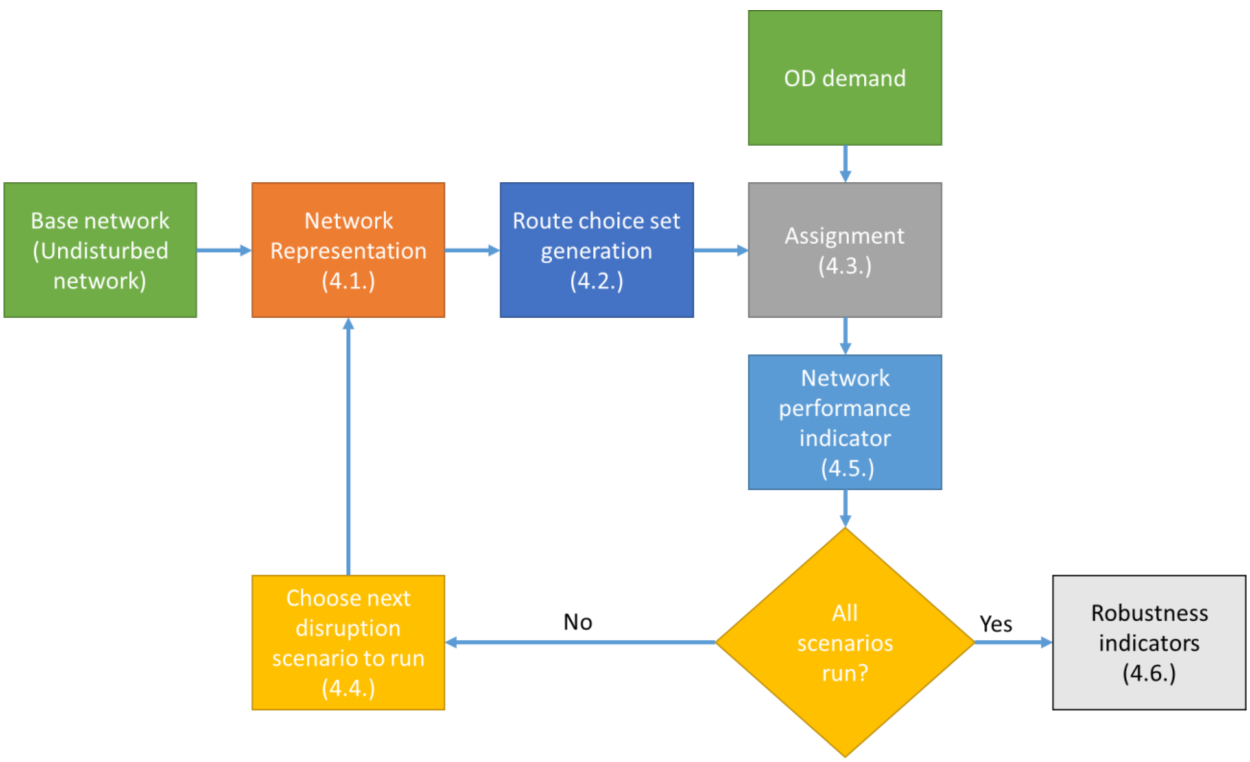


Figure 23 Graphical representation of the model structure (Numbers between brackets refer to corresponding sections)

4.1. Network representation

Before one is able to come to a network representation which can be used to generate route choice sets and assign travellers, the network needs to undergo some intermediate steps. An overview of the network representation process is given in Figure 25. An elaboration on the steps and processes in the overview is given in this section.

The PT network at hand for the robustness study is modelled in two types of network representations used in complex network theory. The first type is a representation used for the physical network (i.e. infrastructure network) and is called L-space. In L-space nodes are connected if they are consecutive stops in a given route (Barthélemy, 2011), the connections are considered regardless of lines (Derrible & Kennedy, 2011). To be able to account for the existence of transfers in the network the PT network is also represented in P-space. In this type of representation the nodes are connected if there is at least one route between them (Barthélemy, 2011), in this way the model accounts for the presence of lines here as well. For an example of a three-line transit system and its corresponding L-space and P-space representation see Figure 24.

Both graphical representations can be represented in matrix form to be able to numerically model and adjust the network in a program like MATLAB[5]. The edges can have properties of direction and weight. A network is called a *directed network* if the edges have a source node and a target node and a network is called a *weighted network* if the links have weights representing the resistance of this links (e.g. distance or travel time). A *directed network* can be a *uni-directional network* or a *bi-directional network.* In the first case edges can only have one direction, in the second case the edges can have two directions.

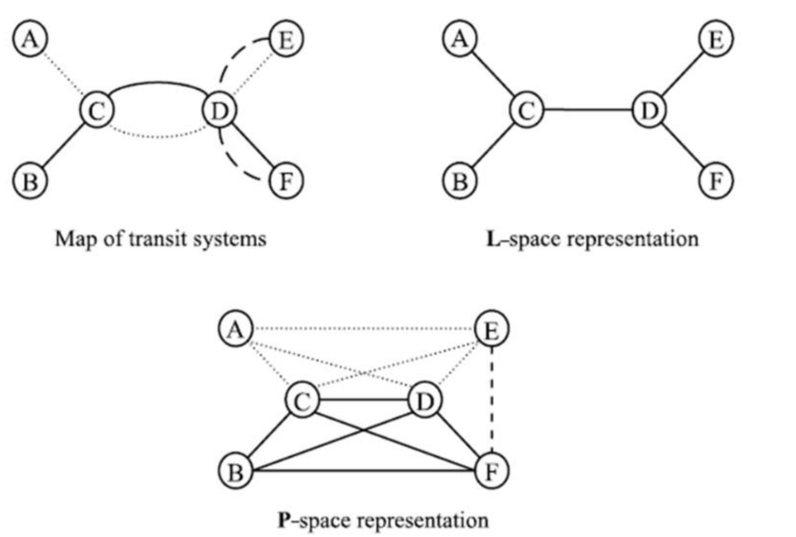


Figure 24 Network representation of a three-line transit system: line 1 (dots), line 2 (solid), line 3 (dash) - Upper left: map of transit system; Upper right: L-space graph (only connections are considered regardless of lines); Down center: P-space graph (all stations part of the same lines are directly connected with one another to account for the presence of lines here as well); Source: Adapted from figure 5 in Derrible and Kennedy (2011)

[5] MATLAB is a numerical computing environment for matrix manipulations, implementation of algorithms, data plotting etc.

This model uses *bi-directional weighted* and *bi-directional unweighted networks*, the first types are referred to as *Travel Time Weighted L-space*, *Waiting Time Weighted P-space* and *Distance Weighted L-space*, the second type is referred to as the *Adjacency L-space* (or respectively *Adjacency P-space)*.

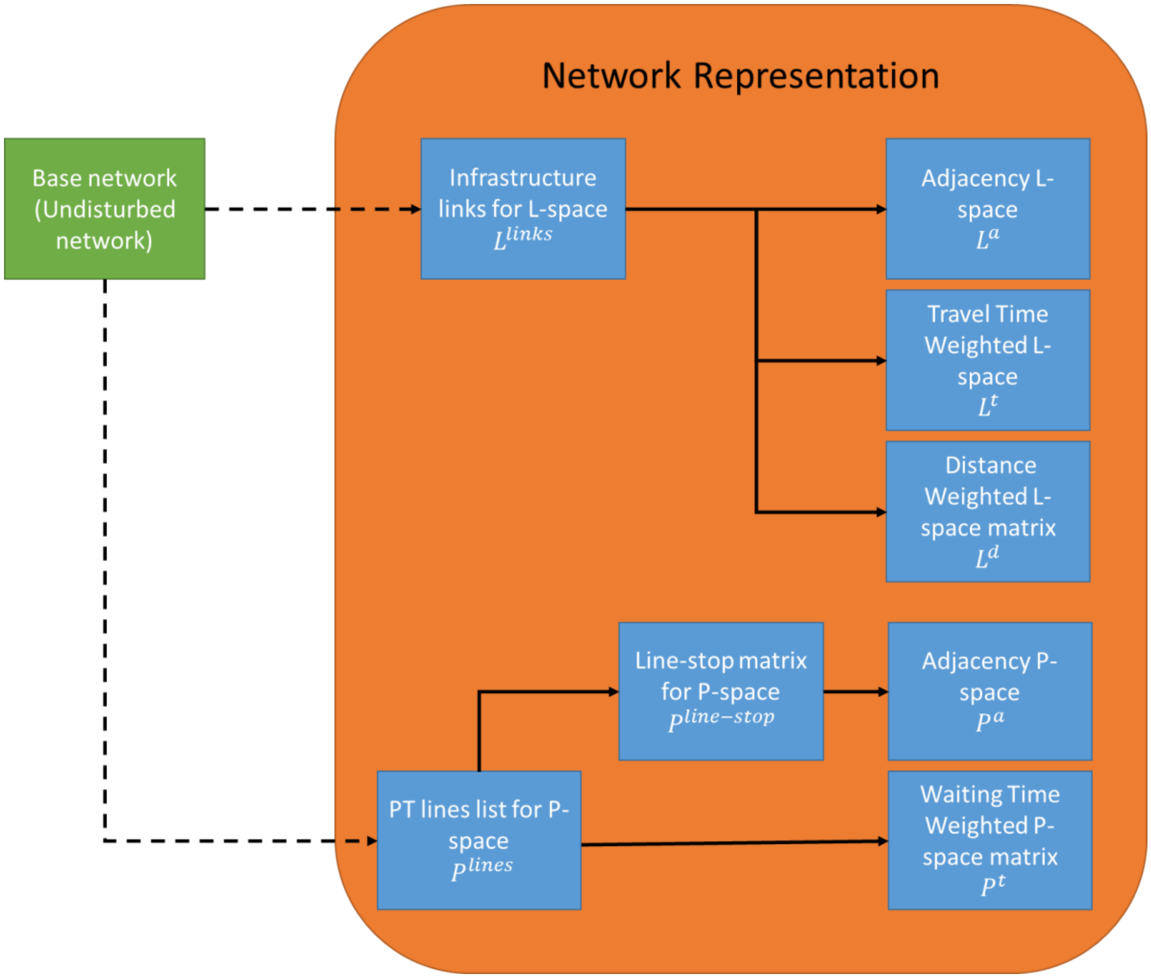


Figure 25 Overview of network representations process from real network to L-space and P-space representations

Before a map of a transit system can be written as a L-space or P-space matrix, the infrastructure network and service network have to be written in intermediate forms, see Figure 25. The intermediate form for the L-space L^{links} consist of all the edges which represent the existence of infrastructure between those nodes with a weight representing resistance of this edge (e.g. distance and travel time) and is referred to as *Infrastructure links for L-space*. For this model the weight is represented as the station-to-station travel time between two stops, expressed in minutes, and as the distance between two stops, expressed in meters. For the three-line transit system example of Figure 24, this would give a table like Table 10. For this intermediate representation of the three-line transit system it is assumed that all edges have a station-to-station travel time of 12 minutes except for the edge between C and D for which the station-to-station travel time is 18 minutes. Note that in this case it is assumed that the routes for all lines between C-D, D-E and E-F have the same station-to-station travel time, in reality these station-to-station travel times can differ and extra edges have to be added to represent this. An edge e is thus defined by an origin node s_{e^-}, an destination node s_{e^+}, a planned station-to-station travel time t_e^t and the traversing distance of the edge d_e.

Table 10 Infrastructure links for L-space L^{links}

Origin Node s_{e^-}	Destination Node s_{e^+}	Station-to-station travel time (min) t_e^t	Distance (m) d_e
A	C	1.2	300
C	D	1.8	450
D	E	1.2	300
E	D	1.2	300
D	C	1.8	450
C	A	1.2	300
B	C	1.2	300
D	F	1.2	300
F	D	1.2	300
C	B	1.2	300

The intermediate form for the P-space graph is expressed in two steps. The first intermediate representation P^{lines} is given in a table in which for every line the stops and the frequency is given and is referred to as *PT lines list for P-space*. An example for the three-line transit system example of figure 24 is given in table 11. The frequency in the last column is given in number of vehicles per hour. A line $r \in P^{lines}$ is thus defined by a type of PT vehicle (e.g. 1 for tram 2 for metro), a corresponding service frequency f_r and a sequence of stops $r = (s_{r,1}, s_{r,2}, \dots, s_{r,|r|})$ where $s_{r,1}$ and $s_{r,|r|}$are the origin and destination terminals, respectively.

Table 11 PT lines list for P-space P^{lines}

Line number	Indicator for type of PT vehicle	f_r (#vehicles/hour)	$s_{r,1}$	$s_{r,2}$	$s_{r,n}$	$s_{r,\vert r\vert}$
1	1	10	A	C	D	E
2	2	15	B	C	D	F
3	2	10	E	D		F

The second intermediate representation of P-space of the PT network $P^{line-stop}$ is given in a matrix with on the x-axis all nodes $s \in S$ where node set S represents all stops and stations in the PT network and on the y-axis all the lines $r \in P^{lines}$. This representation is referred to as *Line-stop matrix for P-space*. Each cell $p_{i,j}^{line-stop}$ equals one if node $i \in S$ is part of a line $j \in P^{lines}$ (i.e. node $s \in r$) and zero if not. An example for the three-line transit system example of Figure 24 is given in Table 12.

Table 12 Line-stop matrix for P-space $P^{line-stop}$

Line number	A	B	C	D	E	F
1	1	0	1	1	1	0
2	0	1	1	1	0	1
3	0	0	0	1	1	1

From these three tables above all L-space and P-space representations are derived. The *Adjacency L-space*, *Travel Time Weighted L-space* and *Distance Weighted L-space* are represented as matrices, respectively, $L^a(S,E)$, $L^t(S,E)$, $L^d(S,E)$ in which link set $E \subseteq S \times S$ represents rail track segments, in use, between stops $s \in S$. In the *Adjacency L-space matrix* (see Table 13) cell $l_{i,j}^a$ equals one if nodes $i,j \in S$ are connected and zero if not; in the *Travel Time Weighted L-space matrix* (see Table 14) cell $l_{i,j}^t$ equals the planned station-to-station travel time t_e^t associated with traversing each link $e \in E$, in the *Distance Weighted L-space matrix* (see Table 15) cell $l_{i,j}^d$ equals the distance d_e associated with traversing each link $e \in E$. The subscript i indicates origin node s_{e^-} and subscript j indicates destination node s_{e^+}.

Table 13 Adjacency L-space matrix L^a

	A	B	C	D	E	F
A	0	0	1	0	0	0
B	0	0	1	0	0	0
C	1	1	0	1	0	0
D	0	0	1	0	1	1
E	0	0	0	1	0	0
F	0	0	0	1	0	0

Table 14 Travel Time Weighted L-space matrix L^t

	A	B	C	D	E	F
A	0	0	1.2	0	0	0
B	0	0	1.2	0	0	0
C	1.20	1.2	0	1.8	0	0
D	0	0	1.8	0	1.2	1.2
E	0	0	0	1.2	0	0
F	0	0	0	1.2	0	0

Table 15 Distance Weighted L-space matrix L^d

	A	B	C	D	E	F
A	0	0	300	0	0	0
B	0	0	300	0	0	0
C	300	300	0	450	0	0
D	0	0	450	0	300	300
E	0	0	0	300	0	0
F	0	0	0	300	0	0

The *Adjacency P-space* and *Waiting Time Weighted P-space* are represented as matrices, respectively, $P^a(S,C)$, $P^t(S,C)$ in which node set S represents all stops and stations; link set $C \subseteq S \times S$ represents direct line connections between stops and stations. In the *Adjacency P-space matrix* (see Table 16) cell $p^a_{i,j}$ equals one if nodes $i,j \in S$ are linked with a direct line and zero if not and in the *Waiting Time Weighted P-space matrix* (see Table 17) cell $p^t_{i,j}$ equals the weights associated with each link $c \in C$, where each element t^w_c denotes the waiting time corresponding with traversing link c in P-space and equals zero if $i,j \in S$ are not linked with a direct line. The waiting time t^w_c, in minutes, for link $c \in C$ is considered to be half the headway time of the traversed link and is calculated as follows:

Equation 5 Waiting time on link c

$$t^w_c = \frac{\frac{60}{\sum_{c \in r} f_r}}{2}$$

In which $c \in r$ denotes that link c belongs to line r, implying that $c = (s_{c-}, s_{c+})$ with $s_{c-}, s_{c+} \in r$. Each link c is thus associated with a set of lines $R_c = \{r \in R | c \in r\}$ that traverse the link.

Table 16 Adjacency P-space matrix P^a

	A	B	C	D	E	F
A	0	0	1	1	1	0
B	0	0	1	1	0	1
C	1	1	0	1	1	1
D	1	1	1	0	1	1
E	1	0	1	1	0	1
F	0	1	1	1	1	0

Table 17 Waiting Time Weighted P-space matrix P^t

	A	B	C	D	E	F
A	0	0	3	3	3	0
B	0	0	2	2	0	2
C	3	2	0	1.2	3	2
D	3	2	1.2	0	1.5	1.2
E	3	0	3	1.5	0	3
F	0	2	2	1.2	3	0

4.2. Route Choice Set Generation

The first step in assigning travellers to the network is to generate a route choice set. The most straightforward route choice set consist of the one shortest path and, as a consequence, all travellers will be assigned to this shortest path. However, the perceived travel time of paths can differ for different travellers. That is to say, for example, some travellers dislike transfers more than others and some travellers dislike waiting time more than others. As a result travellers will in reality not all take the shortest paths, but they will spread over a set of paths. How this set of paths is generated will be discussed in this section.

To make sure the generated route choice set contains only realistic route options the route choice set is filtered with some filtering rules. The Route Choice Set Generation process is built up in 3 steps with corresponding processes and filtering rules, see figure 26 for an overview. In the first step *initial path sets* are created in which all paths are placed with the minimum number of transfers or the minimum number of transfers plus one (filter rule #1). From the *initial path sets* the *logical constrained (LogCon) path sets* are created in which all paths of the *initial path sets* are placed when they do not contain any loops (filter rule #2). In the last step, the *LogCon path sets* are transformed into the *master path sets* in which all the paths in the *LogCon path sets* are placed if it is not dominated by another path (filter rule #3). An elaboration on the different steps, path sets and filtering rules is given in this section.

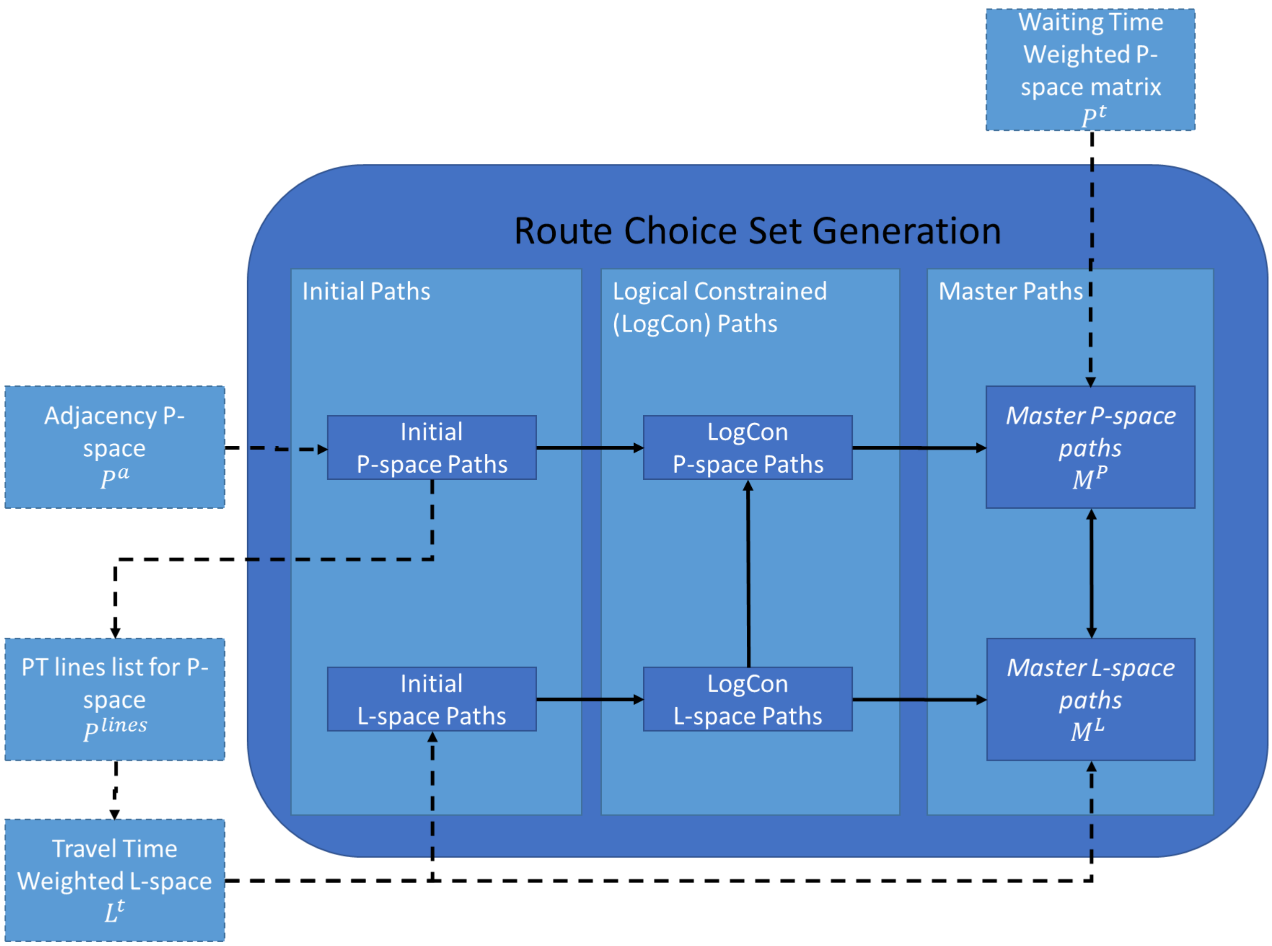


Figure 26 Overview of the Route Choice Set Generation process.

The first applied rule in the Route Choice Set Generation process is that routes with more than one extra transfer than the minimum number of transfers needed are not accepted in the route choice set. In the model this rule is implemented in the pre-programmed Yen's algorithm (Yen, 1971) in JAVA (thinkingscale.com, 2015). Yen's algorithm is based on Dijkstra's algorithm. Dijkstra's algorithm (Dijkstra,

1959) finds the shortest path between a chosen OD-pair, Yen's algorithm is then capable of finding the best alternative after the shortest path found by Dijkstra's. Yen's algorithm is called a k-shortest loopless path algorithm, which indicates that the algorithm is able to find the k shortest loopless paths if they exist. The underlying assumption for this rule is that the disutility for a transfer is that high that the corresponding utility in shorter in-vehicle time never outweighs the disutility associated with two extra transfers, but that the disutility of one extra transfer can be outweighed by the utility of the corresponding shorter in-vehicle time.

Filter rule # 1: Only paths with the minimum number of transfers or with the minimum number of transfers plus one are part of the route choice set.

The adapted Yen's algorithm is run on the *Adjacency P-space matrix* P^a (see Table 16) with as result an initial route choice set called *Initial P-space Paths*. This route choice set consists of all possible paths in P-space satisfying rule #1. The paths are expressed in a sequence of numbers representing the sequence of traversed nodes in P-space.

The sequence of traversed nodes in P-space does not correspond one-to-one to the corresponding traversed nodes in L-space. For example, in the three-line transit network of Figure 24 the shortest path in P-space between B and E would give the sequence of nodes B-D-E. In L-space, however, there is no edge between B and D. Therefore for every edge c in the found paths in the *Initial P-space Paths* the corresponding sequence of traversed nodes in L-space are searched. This is done by establishing a line set R_c^{ini} in which all lines $r \in R_{ini}$ traverse edge c. Hence, $c \in r$ which denotes that link c belongs to line r, implying that $c = (s_{c-}, s_{c+})$ with $s_{c-}, s_{c+} \in r$. Each link c is thus associated with a set of lines $R_c = \{r \in R | c \in r\}$ that traverse the link. For all these lines the total station-to-station travel times between the nodes s_{c-} and s_{c+} are calculated using the *Travel time weighted L-space* L^t. The found sequence of stops between s_{c-} and s_{c+} of the line with the lowest total station-to-station travel time is added between these stops in the *Initial P-space Paths* for every edge c. The new, as a consequence, found path is added to the route choice set with corresponding sequence of nodes in L-space and is called *Initial L-space Paths.* For the three-line transit network example of the P-space path between B and E gives the sequence of nodes B-C-D-E.

Table 18 Initial P-space Paths (left) and Initial L-space Paths (right) for OD-pair A-C of the three-line transit system example of figure 4 where the cells represent the k-shortest paths abiding rule #1

Initial P-space Paths (A-C)	Initial L-space Paths (A-C)
A-C	A-C
A-D-C	A-C-D-C
A-E-C	A-C-D-E-D-C

Because the k shortest paths were first looked for in P-space the *Initial Paths* route choice sets contain routes which have loops which are undesirable in the route choice set. Take for example the OD-pair A-C of the three-transit example of Figure 24. The *Initial Paths* route choice sets for this OD-pair can be found in Table 18. The second and third shortest paths both contain loops: in the second path the C-D edge is traversed twice and in the third path both edge C-D and D-E are traversed twice. It is very unrealistic that travellers run by their destination stop to exit at a stop after their destination stop to transfer to the opposing running train to eventually get to their destination stop. It is assumed that travellers choose to exit the train at the first option for which a train stops at their destination stop and the traveller is thus able to exit. This results in a second rule for the route choice set:

Filter rule #2: Only paths without loops are part of the route choice set

In the *Initial L-space paths* route choice set the paths with loops are identified by looking for edges which are present twice in the paths. These paths are deleted from the route choice set in both *Initial L-space paths* and *Initial P-space paths*, resulting in the route choice sets *LogCon L-space paths* and *LogCon P-space Paths.* For the A-C OD-pair of the three-line transit example this results in a single path route choice set of A-C.

The three-line transit example is a very simple network without any cycles, with the consequence that no real route alternatives exist. Real world urban rail bound PT networks do however have cycles with as consequence the occurrence of real route alternatives. When the networks become larger the number of route alternatives is growing likewise. Lots of these route alternative are however no realistic choice options for the passengers.

An example for an unrealistic route alternative for the Amsterdam rail bound PT network (see Figure 27) is travelling from Waterlooplein to Amsterdam Centraal Station passing Amsterdam Zuid. This second route alternative (yellow line) is not filtered out of the route choice set in the *LogCon paths* route choice set because it does not violate the first two rules (i.e. the one transfer in Amsterdam Zuid is the minimum number of transfers plus one and the alternative does not have any loops). These types of routes are however unrealistic choice options because of relatively long in-vehicle-times, waiting times or a combination of long in-vehicle times, waiting times and numbers of transfers. By applying dominancy rules these unrealistic choice options can be excluded from the route choice set. The application of dominancy rules is based on the assumption that the less attractive characteristics of the dominated alternatives excludes them for the by the decision maker considered route choice set (Androutsopoulos & Zografos, 2009). Cats (2011) formulates this as: “An alternative that is not better than another alternative in the choice set in any aspect and is worse than this alternative in at least one aspect is regarded as dominated”. These dominated paths are also knows as Pareto non-optimal, the non-dominated paths are part of the set of paths described by the Pareto front. It is assumed that travellers only consider paths which are part of the Pareto front, and paths known as Pareto non-optimal (i.e. dominate paths) are not part of the consideration set. This results in the third filtering rule.

Filter rule #3: A path that is not better than another alternative in the choice set in number of transfers, total in-vehicle time and total waiting time and is worse than this alternative in at least one of these aspects is not part of the route choice set.

In the example of the route alternative of Figure 27 the second route alternative is dominated by the first route alternatives on every aspect (e.g. it has more transfers, longer waiting times and longer running times) and is therefore excluded from the route choice set. Application of the third rule on the *LogCon paths* route choice set results in the *Master L-space paths* and *Master P-space paths* route choice sets with assumed realistic route alternatives. *Master Paths* is represented in two cells: *Master L-space paths* $M^L\ (i,j,k)$ represents the sequence of traversed nodes L-space and *Master P-space paths* $M^P(i,j,k)$ represents the sequence of traversed nodes in P-space for every path k between node i and j.

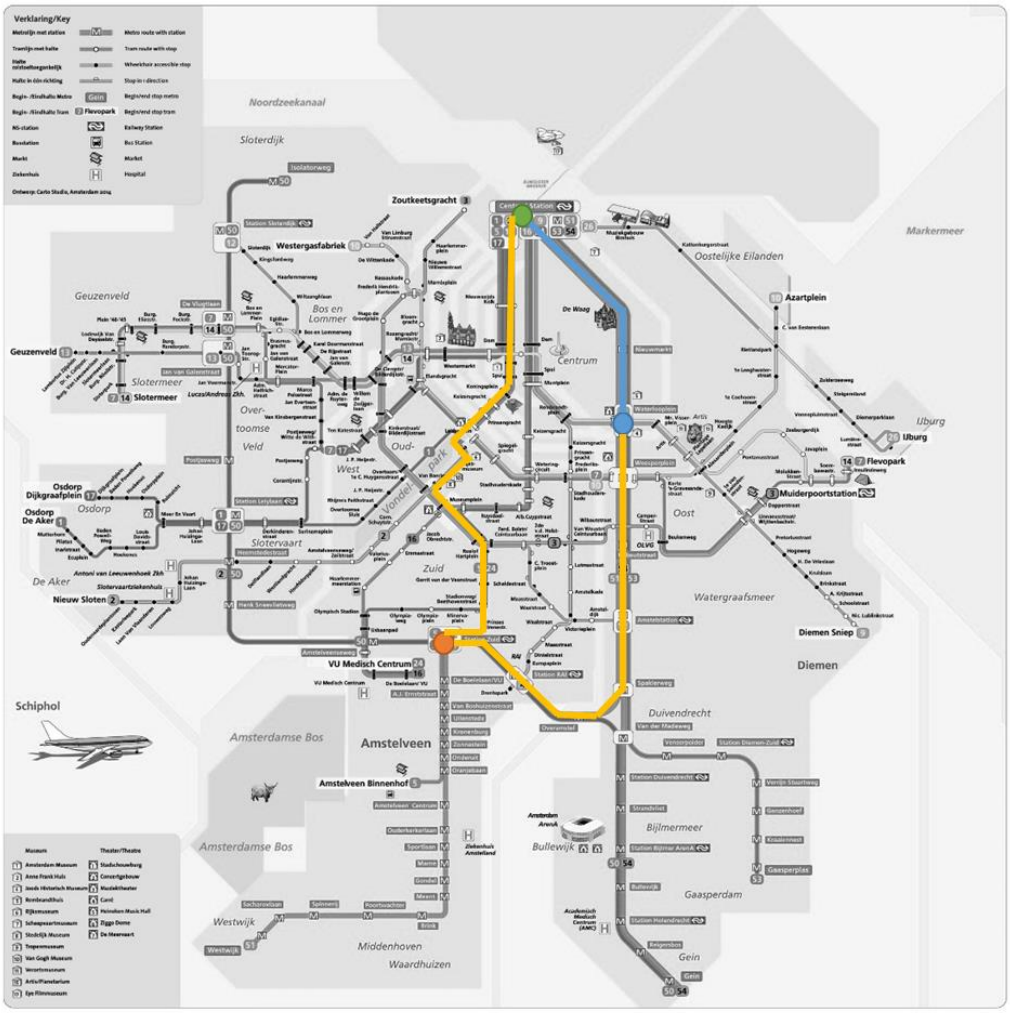


Figure 27 Two route alternatives (first, blue line; second, yellow line) from origin Waterlooplein (blue dot) to destination Amsterdam Central Station (Green dot) and transfer station Amsterdam Zuid (orange dot). Source: adapted from GVB railkaart 2015 (Gemeentelijk Vervoerbedrijf, 2015b)

4.3. Assignment

Next step of the model is the assignment of traveller flows over the network given the OD passenger demand. This is done based on the *Master paths* route choice sets and the characteristics of these paths derived from the weighted P-space and L-space matrices, see figure 28. For every path $k \in M$ the weighted travel time is calculated based on the in-vehicle time t_k^t , number of transfers n_k and waiting time t_k^w. The weighted travel time is the (dis)utility on which the probability of choosing paths is based, from now on the weighted travel time is referred to as the utility u_k. These utility u_k of path k is translated into probabilities of choosing path k (pr_k) and consequently the OD demand is assigned to the network given those probabilities per path.

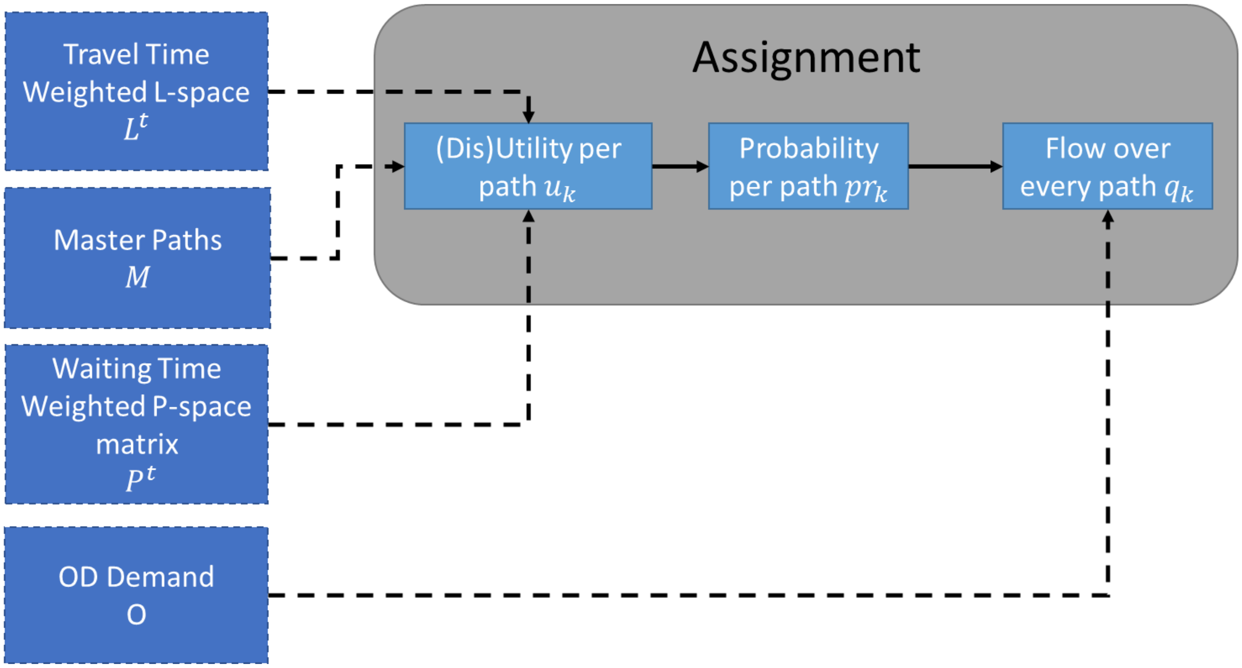


Figure 28 Overview of the assignment process

The used assignment method described by the overview is a stochastic assignment, an assignment method in which the variability in travellers' perception of route costs is taken into account. In this method multiple routes are modelled but congestion effects are not taken into account. For each route in a network the objective and subjective route costs are distinguished. The objective costs are the engineering costs as measured/estimated by the modeller, the subjective costs are perceived by each traveller. It is assumed that there is a distribution of perceived route costs for each route with the objective costs as the mean, as shown in Figure 29.

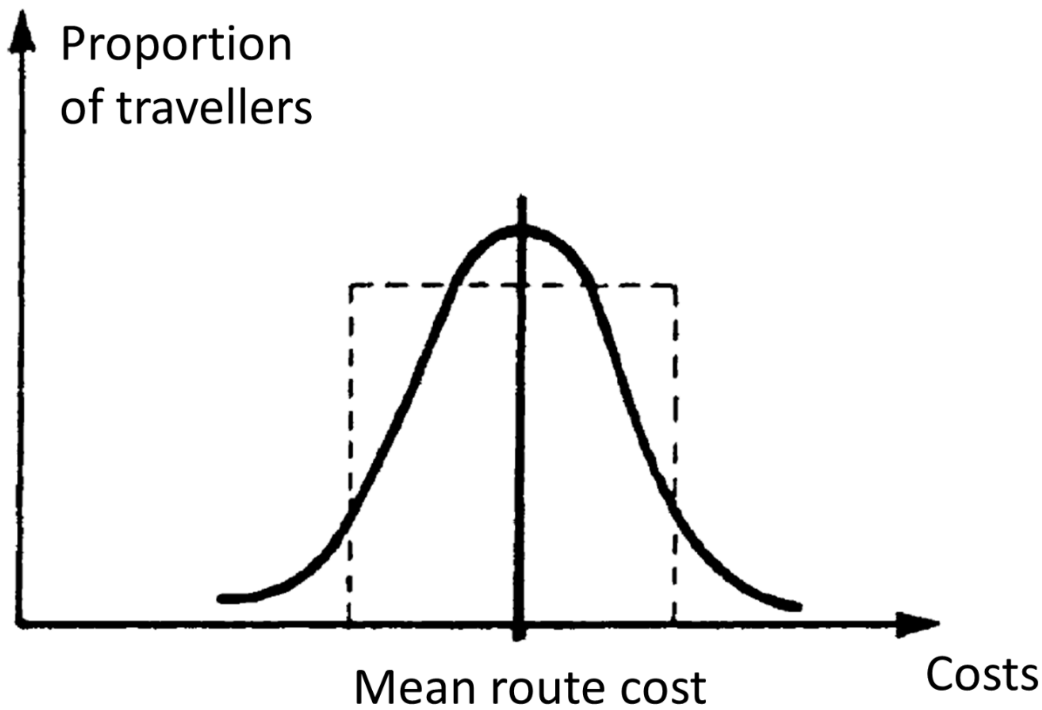


Figure 29 Distribution of perceived costs on a route. Source: Adapted from figure 10.4 in de Dios Ortuzar and Willumsen (2011)

The mean route costs are referred to as the utility u_k of path $k \in M$ and is calculated as follows:

Equation 6 Utility u of path k

$$u_k = t_k^t + \beta^w * t_k^w + \beta^n * n_k$$

The utility u_k represents the weighted travel time for path $k \in M$ and is calculated as the sum of the total station-to-station travel time t_k^t (i.e. the total in-vehicle time or IVT), the weighted waiting time relative to the IVT ($\beta^w * t_k^w$) and the weighted number of transfers relative to the IVT ($\beta^n * n_k$). The βs represent the relative importance of the corresponding variable. Different values for the βs within the same utility function can be used for different perceptions in waiting times or transfers. For example, waiting at a railway station with a lot of shops and leisure activity options is often perceived less heavy than waiting alongside a busy road with only a bus shelter. The utilities u_k are transformed into probabilities pr_k per path by a multinomial logit model. If $K_{i,j}$ is the set of paths in the master set for OD-pair $i - j$ than the probability of choosing path k is then:

Equation 7 Probability p of choosing path k between origin i and destination j

$$pr_{i,j,k} = \frac{\exp(-\mu * u_{i,j,k})}{\sum_{k_{i,j} \in K_{i,j}} \exp(-\mu * u_{i,j,k})}$$

The parameter μ can be used to control the spread of probabilities among the available paths. When the parameter μ has higher value, the probabilities are more spread over the alternative paths. The value of

μ is mostly between 0 and 1. The multinomial logit model as previously presented is based on the assumption that all path alternatives are independent. With overlap in routes and modes (e.g. trams and metro) this might not be completely realistic. To express the correlation between alternatives one can adapt the multinomial logit model with a commonality factor (Cascetta et al., 1996) or use the Path-Size Logit model as proposed by Ben-Akiva and Bierlaire (1999). For now, the paths in the model are assumed independent alternatives.

Last step in the assignment is to assign a given OD-demand over the given routes in the route choice sets. The OD-demand is represented as a matrix $O(S)$ in which node set S represents transfer and terminus stops and stations and where each cell $o_{i,j}$ equals the passenger demand of travellers willing to travel from stop $i \in S$ to stop $j \in S$ within a certain time period. For the three-transit line example of figure 24 this matrix O could look like Table 19.

Table 19 OD-demand matrix O

	A	B	C	D	E	F
A	0	300	600	1000	200	60
B	200	0	800	150	50	150
C	800	450	0	600	100	100
D	650	300	650	0	450	40
E	1200	50	200	1500	0	600
F	750	100	100	600	300	0

The flow over every route $q_{i,j,k}$ is calculated as the product of the total demand from stop i to j ($o_{i,j}$) and the probability of the path $k \in K$ ($pr_{i,j,k}$):

Equation 8 The flow q over path k between origin i and destination j

$$q_{i,j,k} = o_{i,j} * pr_{i,j,k}$$

4.4. Disruption scenario implementation

In the previous section is presented how based on a created network representation (Section 4.1) route choice sets are generated (Section 4.2) and how travellers are assigned to the network (Section 4.3). Unplanned major discrete events (as discussed in Section 3.4) affect the infrastructure availability and, indirectly, the transport service network. Network representation scenarios are created which meaningfully represent these situation of unplanned major discrete events to analyse how this affects the network performance and network robustness.

The disruption scenarios in this model are link-based bi-directional percentage wise capacity reductions on stop-stop relationships based on the two bi-directional infrastructure availability categories from Section 3.4:

1. All tracks of a link are unavailable
2. A local speed limit and extra waiting time for clearing the tracks on all tracks

Based on the second category link capacity is defined as the mean speed on a stop-stop relationship $e \in L$ which is an edge in L-space linking origin node $s_{e^-} \in S$ and destination node $s_{e^+} \in S$. The mean speed v_e on link e is defined by the length d_e of the link divided by the planned station-to-station travel time t_e^t:

Equation 9 Mean speed on link e in the base case scenario

$$v_e(0) = \frac{d_e}{t_e^t}$$

Each link $e \in E$ is assigned with a certain base mean speed $v_e(0)$. The mean speed v_e on link e during scenario x can then be defined as:

Equation 10 Mean speed on link e during scenario x

$$v_e(x) = v_e(0) * (1 - y_e(x))$$

Where $y_e(x)$ is the relative reduction in speed on link e during scenario x. These reductions are applied with steps of 10% from $y_e(0) = 0$ to $y_e(10) = 1$ so that $Y_e = [0,1]$. The lower bound of Y_e corresponds to normal operations, the upper bound value implies a complete breakdown of the link corresponding with the first infrastructure availability category of Section 3.4: all tracks of a link are unavailable.

The effect of a capacity reduction on a link on the service network is not only affected by the extent of capacity reduction but also by the possible mitigation measures taken by the PTO (see Section 3.4). In Table 20 is presented for which capacity reduction scenario which service network options are possible. Which mitigation measure the PTO will choose during which capacity reduction scenario does depend, amongst other, on how the network performs in these situations. The implementation of the disruption scenario swill be discussed according to the possible do nothing scenarios and mitigation measures as shown in Table 20.

Table 20 Application area of possible do nothing scenarios and mitigation measures for the different capacity reduction scenarios, in brackets the corresponding model implementation sections

<table>
<tr><th>Scenario x=</th><th>0</th><th>1-9</th><th>10</th></tr>
<tr><td>Do-nothing</td><td>Base case scenario (4.4.1)</td><td>Apply local speed limit (4.4.2)</td><td>Delete traversing lines (4.4.3)</td></tr>
<tr><td rowspan="2">Mitigation measure</td><td rowspan="2"></td><td colspan="2">Reroute traversing lines (4.4.4)</td></tr>
<tr><td colspan="2">Cut traversing lines (4.4.5)</td></tr>
</table>

4.4.1. Base case scenario

First scenario is the base case scenario in which no capacity reduction is applied. This scenario corresponds to $y_e(0) = 0$ value and is identical for application on every link.

4.4.2. Apply local speed limit

The implementation of the apply local speed limit-scenario can be done for the capacity reduction scenarios $y_e(1,2 \dots, 8, 9) = [0.1, 0.2, \dots ,0.8, 0.9]$). An overview of the implementation of a partial capacity reduction on a link on the network representation is given in Figure 30. The disruption is applied on a certain link $e \in L^{links}$, *infrastructure links for L-space*, through an increase in the corresponding station-to-station travel time. As a consequence of the increase in station-to-station travel time in L^{links} the corresponding cell in the *Travel Time Weighted L-space* L^t also changes. Because the number of available vehicles is limited it is possible that it influences the lines traversing the disrupted link, i.e. a decrease in service frequency on these lines. A service frequency change is in this case applied to the *PT lines list for P-space* P^{lines} and as a consequence of the service frequency also a change in waiting time on some edges in the *Waiting Time Weighted P-space matrix* P^t. A more detailed overview of the effects of a partial capacity reduction on the network representation and corresponding calculations is given below.

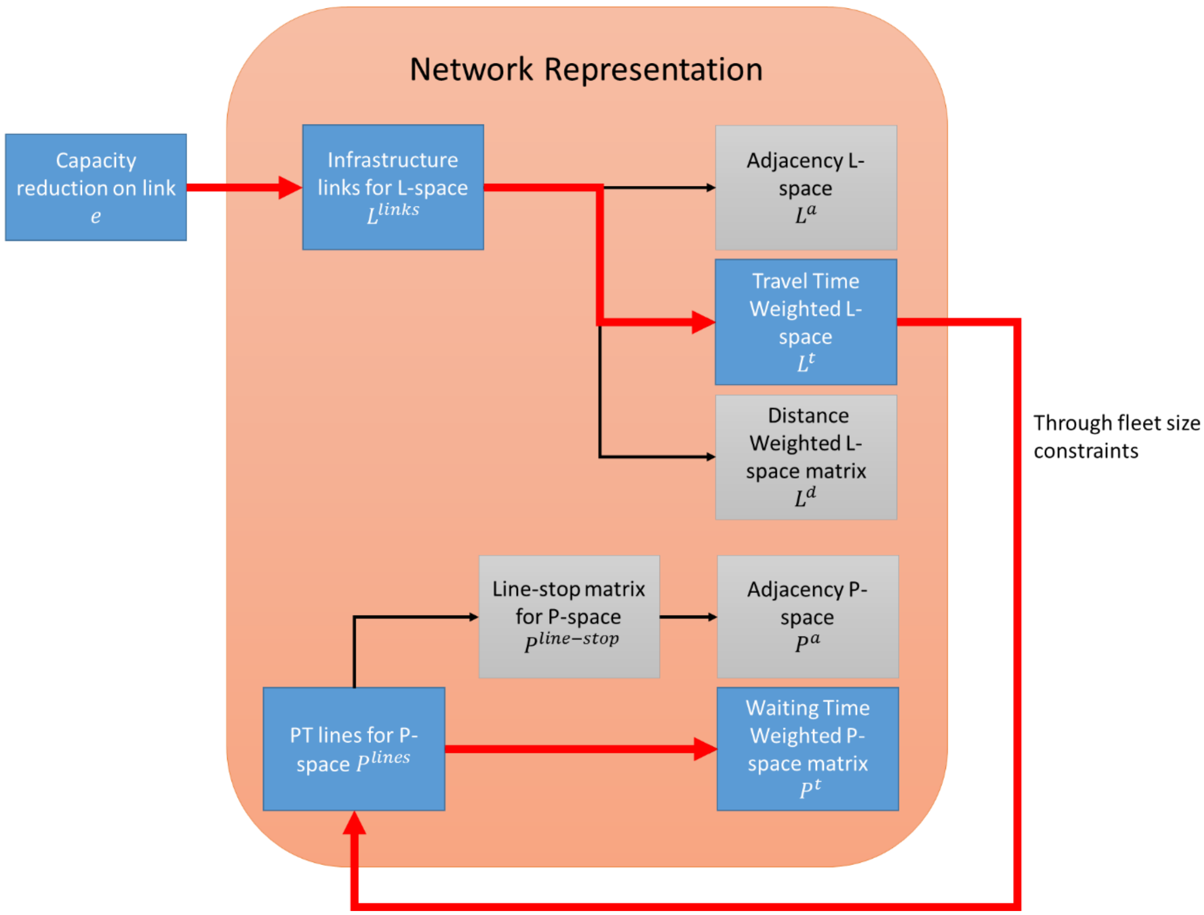


Figure 30 An overview of the implementation of a partial capacity reduction on a link on the corresponding network representations

The partial capacity reduction in speed has as primary effect an increase in station-to-station travel time on the specific link $e \in E$ which gives a new, disrupted, station-to-station travel time on link e of:

Equation 11 Disrupted station-to-station travel time on link e

$$t_e^t(x) = \frac{d_e}{v_e(x)}$$

This new travel time is applied to every line that traverses link e, i.e. all lines in line set $R_e = \{r \in R | e \in r\}$ by changing the *Travel Time Weighted L-space matrix* L^t. The original planned station-to-station travel time $t_e^t(0) \in L^t$ is replaced by the disrupted station-to-station travel time $t_e^t(x)$ on link e.

As a secondary effect the increase in travel time $\Delta t_e^t = t_e^t(x)$ -$t_e^t(0)$ on link $e \in E$ results in an increase in cycle time t_r^{cycle} for every line r in set $R_e = \{r \in R | e \in r\}$. Normally, the cycle time consists of running times, dwell times and layover times, this is discussed in Section 3.4.2. In this model the cycle time is the sum of all station-to-station travel times of all links part of line r and twice a given layover time.

Equation 12 Cycle time of line r

$$t_r^{cycle} = \sum_{e \in r} t_e^t + 2 * t_r^{layover}$$

Corresponding to the original cycle time $t_r^{cycle}(0)$ and the original frequency $f_r(0)$ a number of vehicles $n_r(0)$ is needed. The number of needed vehicles for the base case scenario is calculated as follows:

Equation 13 Number of needed vehicles on line r for the base case scenario

$$n_r(0) = \left\lceil \frac{t_r^{cycle}(0) * f_r(0)}{60} \right\rceil$$

It is assumed that the fleet size is fixed and the vehicles of one line are only available on that line. When the travel time on the disrupted link is increased significantly the base case number of needed vehicles can be insufficient and the service frequency has to be decreased. The new frequency of a certain line is calculated as follows:

Equation 14 Frequency of line r during scenario x

$$f_r(x) = \left\lfloor \frac{n_r(0) * 60}{t_r^{cycle}(x)} \right\rfloor$$

The new frequencies are calculated and applied in the *PT lines for P-space* P^{lines} on all lines $r \in R_e$ and subsequently applied to the *Waiting Time Weighted P-space* P^t on the cells for every cell $c \in P^t$ corresponding to the links affected by the change in frequency. See Section 4.1 for a more thorough explanation.

4.4.3. Delete traversing lines

In the full link breakdown disruption scenarios the disrupted link $e \in E$ is deleted from the L-space (L^a and L^t). In the *Infrastructure links for L-space* L^{links} the row with the disrupted link is deleted. As a consequence the corresponding cell $l_{i,j}^a$ in Adjancency L-space matrix L^a and the corresponding cell $l_{i,j}^t$ in Travel Time Weighted L-space matrix L^t is set to 0.

An overview of a deleting lines disruption scenario on a link on the network representation is given in Figure 31. In this do-nothing scenario for a full link breakdown ($Y_e(10) = 1$) all lines r traversing link e are determined, i.e. all lines in lines set lines $R_e = \{r \in R | e \in r\}$. All this lines $r \in R_e$ are taken away from the *PT lines for P-space* and *Line-stop matrix for P-space*, respectively P^{lines} and $P^{line-stop}$. As a consequence, the cells which indicate a connection only connected by one of the lines $r \in R_e$ will be set to zero in the *Adjacency* and *Waiting Time Weighted P-space matrix* (respectively P^a and P^t). The value of the other cells in the *Waiting Time Weighted P-space matrix* P^t connected by one of the lines $r \in R_e$ will have to be recalculated, depending on the available connection of other lines the value will be zero or only partly decreased. Recalculation is done by Equation 5 in Section 4.1. Also the edges in L-space that are only connected by one or more of the lines $r \in R_e$ will be set to zero in both the *Adjacency* and the *Travel Time Weighted L-space matrix* (Respectively L^a and L^t).

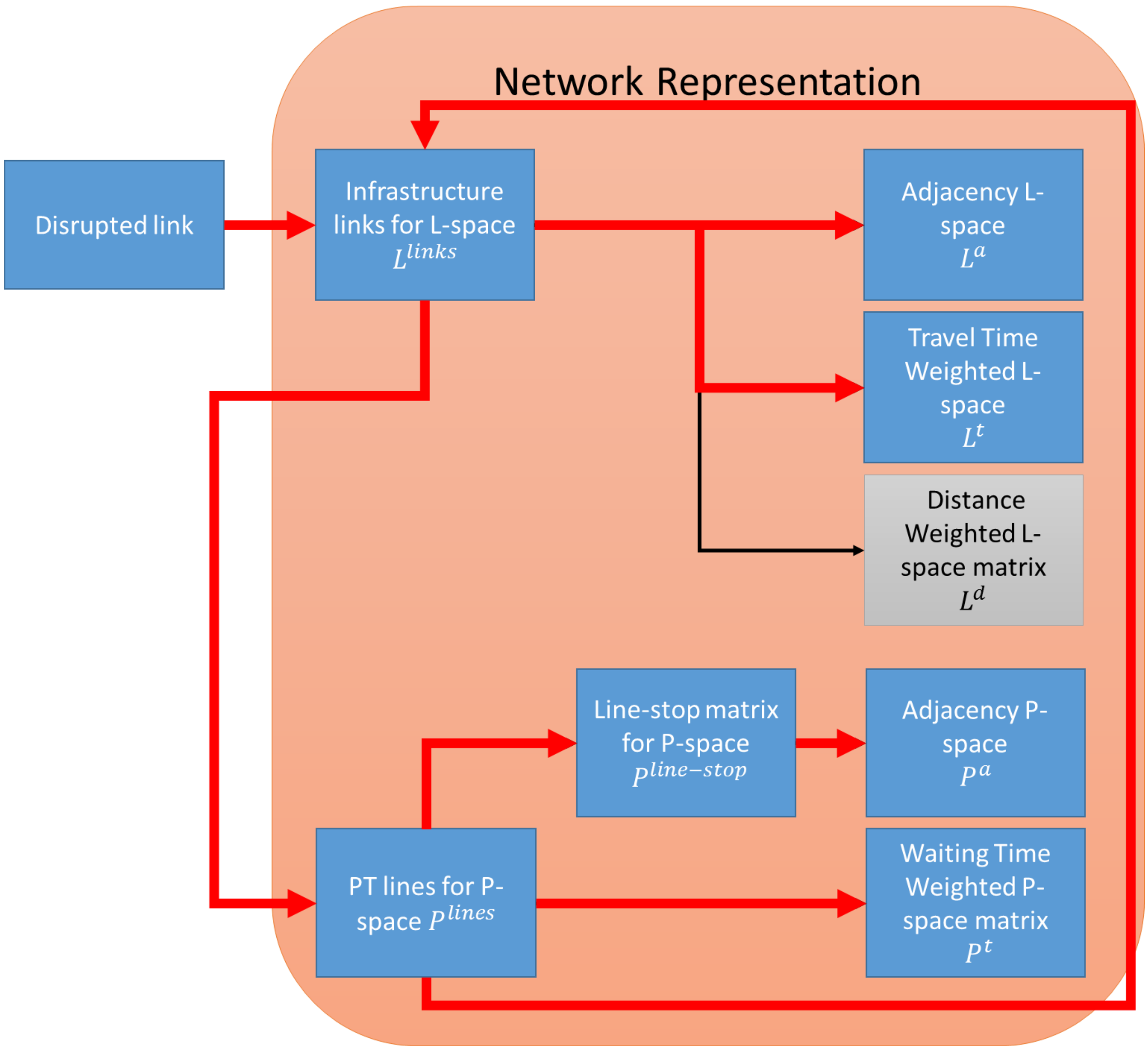


Figure 31 An overview of the implementation of a deleting line disruption scenario on a link on the corresponding network representations

4.4.4. Reroute traversing lines

One of the mitigation measures which can be applied by a PTO is rerouting of the traversing lines. An overview of a rerouting lines disruption scenario on a link on the network representation is given in Figure

32. In the rerouting disruption scenario all lines r traversing link e are determined, i.e. all lines in lines set $R_e = \{r \in R | e \in r\}$. The rerouting of lines $r \in R_e$ are done in sequence in the order of frequency f_r from high to low. When this is not decisive the line with the longest cycle time t_r^{cycle} is done first.

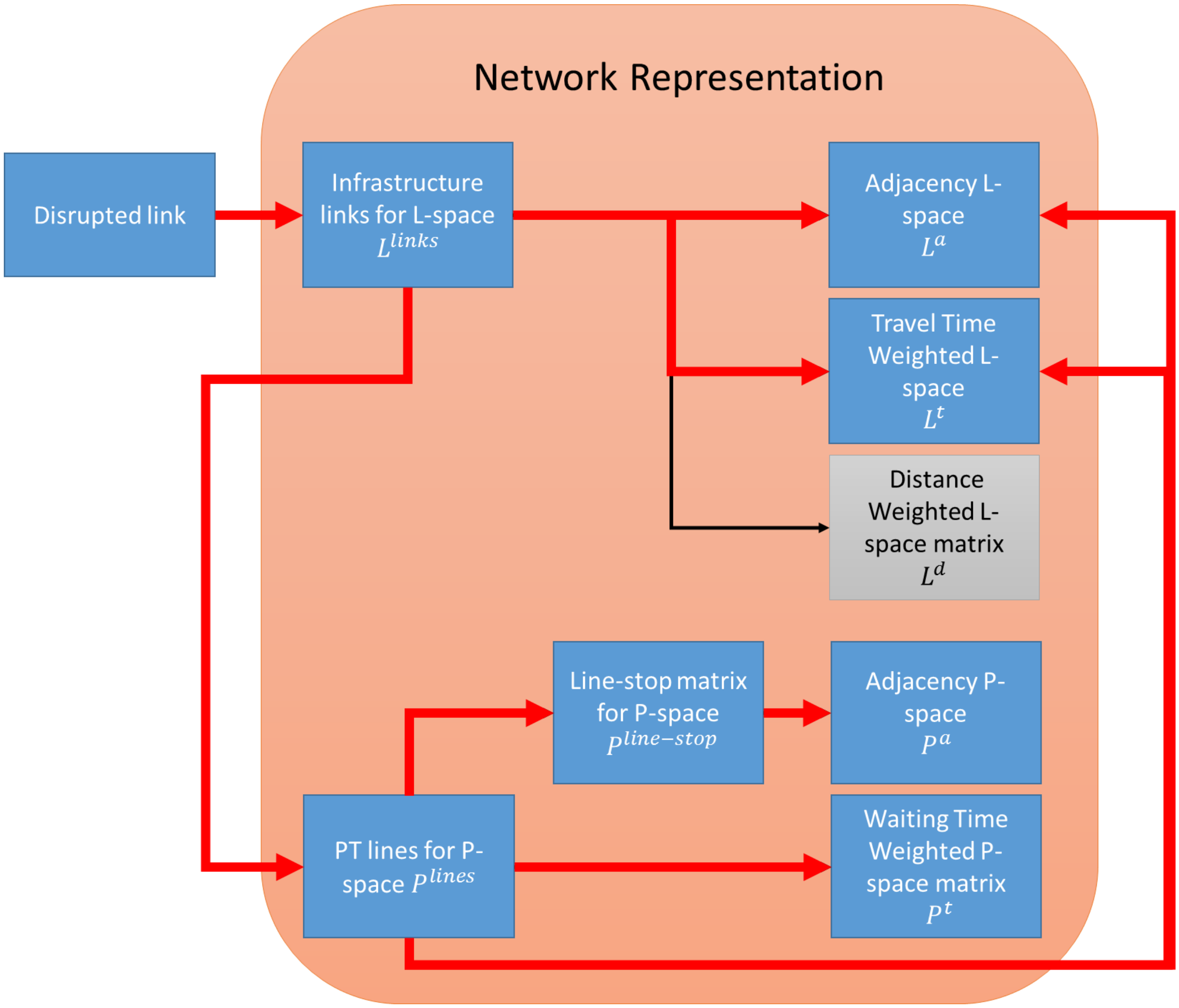


Figure 32 Overview of the implementation of a rerouting disruption scenario on a link on the corresponding network representations

When link e was connecting stop $s_{e-} \in r$ and stop $s_{e+} \in r$ for line r, this connection will be replaced by a detour route. A new shortest path is found between stop s_{e-} and s_{e+} by using the Dijkstra Algorithm (Dijkstra, 1959). This new path is added in r between stop $s_{e-} \in r$ and $s_{e+} \in r$. For every adapted PT line r is checked whether the new PT line is loopless. To check whether and where in the line a loop is present the following check is done: For every stop $s_{r,i} \in r$ the stop before $s_{r,i-1} \in r$ and the stop after $s_{r,i+1} \in r$ are compared. When $s_{r,i+1} = s_{r,i-1}$, the stops $s_{r,i}, s_{r,i-1} \in r$ are deleted and the check is rerun again until only unique stops are part of line r.

For this new line the cycle time is calculated. Same as for the partial capacity reductions the cycle time t_r^{cycle} of some lines r will increase because of the longer travel time t_r^t of the new replacing detour route.

This affects the service frequency f_r on the line. The service frequency changes are similarly applied as in Section 4.4.2.

The infrastructure can have a service frequency constraint so that some reroute options are not available. After recalculating the new frequencies it is checked whether the sum of frequencies on all edges h on the new route between s_{e-} and stop s_{e+} does not exceed the capacity service frequency on this edge, see Equation 15.

Equation 15 Check if the total frequency on link h does not exceed the frequency capacity of link h

$$\sum_{h \in r} f_r < f_h^{cap} \; for \; all \; h \in H$$

where f_r is the service frequency of line r, f_h^{cap} is the service frequency capacity of link h and H is the set of links h on the route between s_{e-} and stop s_{e+}. When the frequency on one of the edges is exceeded, an alternative shortest path is looked for and applied between stop s_{e-} and s_{e+} all steps explained above between this step and checking on exceeding frequency capacity are done again until the frequency does not exceed the frequency capacity or no alternative route options is found. When the last scenario is the case the line will be deleted.

When the rerouting is done for all lines $r \in R_e$ the new formed lines replace the old ones in the *PT lines for P-space* P^{lines}. Based on these changes the *Line-stop matrix for P-space* $P^{line-stop}$ and the *Adjacency* and *Waiting Time Weighted P-space matrices* (respectively P^a and P^t) are updated. Further the *Infrastructure links for L-space* L^{links} is updated according to the serviced edges from the *PT lines for P-space* P^{lines}. Based on these changes the *Adjacency* and *Travel Time Weighted L-space matrices* (respectively L^a and L^t) are updated.

4.4.5. Cut traversing lines

The second mitigation measure that can be applied by a PTO is cutting the traversing lines. An overview of the implementation of a cutting lines disruption scenario on a link on the network representation is given in Figure 33. In the cutting lines disruption scenario all lines r traversing link e are determined, i.e. all lines in lines set $R_e = \{r \in R | e \in r\}$. For all these lines $r \in R_e$ two new lines are added to the *PT Lines for P-space* P^{lines}. When a line $r \in R$ is defined by a sequence of stops $r = (s_{r,1}, s_{r,2}, \dots, s_{r,|r|})$ and the disrupted link e normally connects stop $s_{e-} \in r$ and stop $s_{e+} \in r$ for line r, the first new line $r_{new,1}$ will be defined as a sequence of stops $r_{new,1} = (s_{r,1}, s_{r,2}, \dots, s_{e-})$. The second new line $r_{new,2}$ will be defined as a sequence of stops $r_{new,2} = (s_{e+}, \dots, s_{r,|r|})$. The original lines $r \in R_e$ are deleted and the new lines $r_{new,1}$ and $r_{new,2}$ will have the same frequency as their original line r.

In the case of, for example, a one-direction vehicle it is not realistic for a vehicle to be able to turn at every stop (see Section 3.4.4). Therefore a set of transfer nodes S^t is available which contains nodes $s^t \in S^t$ where PT lines lines cross, join, disjoin or where the serviced network attaches to extra infrastructure which can be used for rerouting. When the new lines $r_{new,1}$ and $r_{new,2}$ contain one or more transfer nodes s^t, i.e. $s^t \in r_{new,1}$ and $s^t \in r_{new,2}$, the new lines (i.e. the sequence of traversed stops) will end at the transfer node s^t closest to respective nodes s_{e-}or s_{e+} instead of at these nodes s_{e-} and s_{e+}. The new lines $r_{new,1}$ and $r_{new,2}$ will change from $r_{new,1} = (s_{r,1}, s_{r,2}, \dots, s_{e-})$ and $r_{new,2} = (s_{e+}, \dots, s_{r,|r|})$ to $r_{new,1} = (s_{r,1}, s_{r,2}, \dots, s^{t})$ and $r_{new,2} = (s^{t}, \dots, s_{r,|r|})$. When no transfer node is part of the new line, i.e. $s^{t} \notin r$ the new lines remain as they were.

As a consequence of the change in the *PT lines for P-space* P^{lines} the *Line-stop matrix for P-space* $P^{line-stop}$ and the Adjacency and Waiting Time Weighted P-space (respectively P^{a} and P^{t}) are updated.

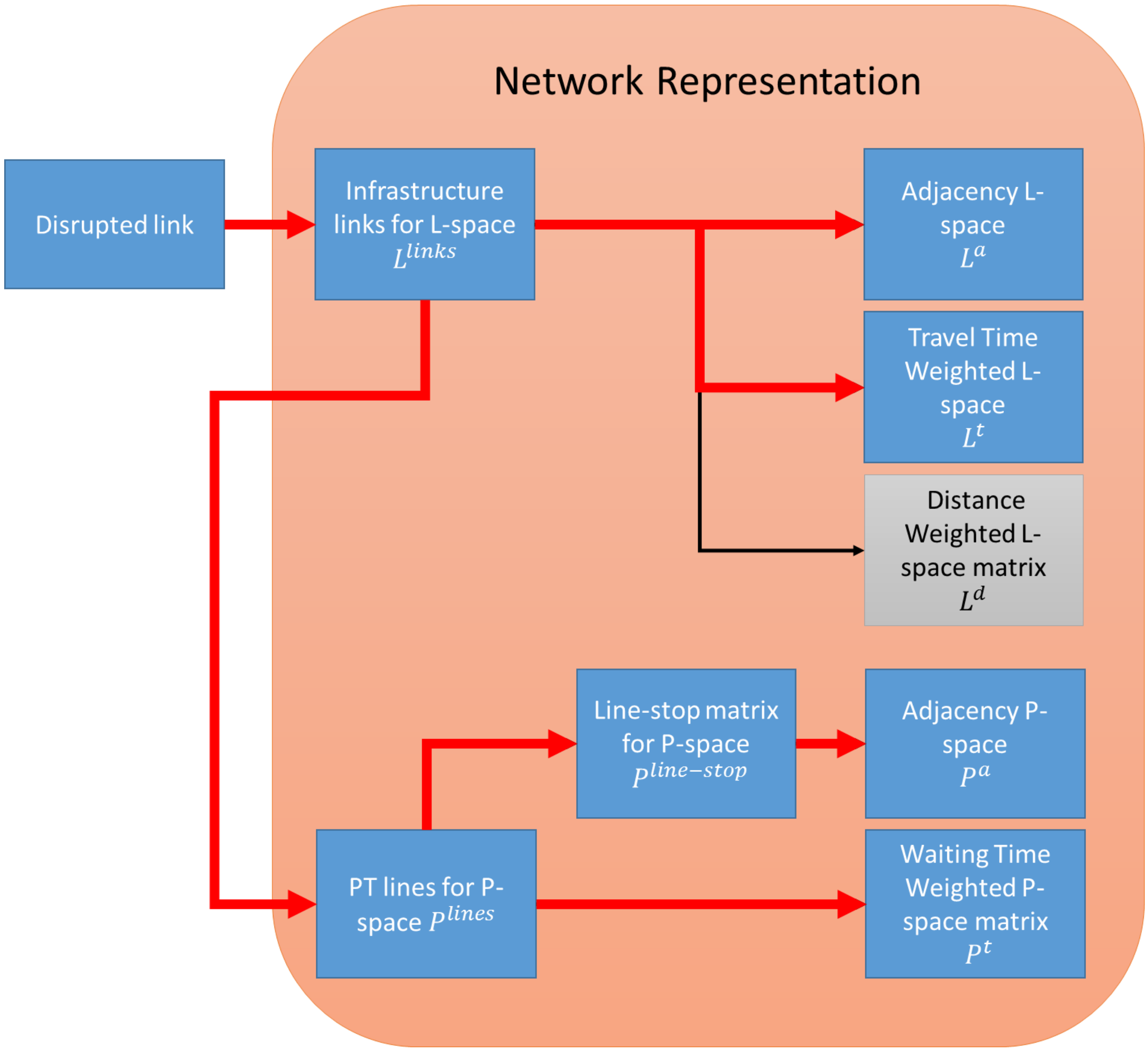


Figure 33 Conceptual overview of a cutting lines disruption scenario on a link on the corresponding network representations

4.4.6. Other mitigation measures

As mentioned in Section 3.4.5 other mitigation measures are possible, like combining lines or rerouting lines to local hubs. Within the scope of this research it is chosen not to develop and apply the implementation of these disruption scenarios. For future research or specific research cases the implementation of other specific mitigation measures can be developed and applied.

4.4.7. Implications on route choice set generation

Through the changes in network representations as a consequence of the disruption scenarios the route choice sets will change. Some routes are not possible anymore because of a full link breakdown or because a line is rerouted. As a consequence, some OD-pairs will have an empty route choice set and the passenger demand on this OD-pair becomes disconnected.

For other routes their absolute or relative attractiveness changes. Some route become so much less attractive that they will not be part of the route choice set anymore, other routes will stay in the route choice set but will become less attractive where their competing routes become relatively more attractive.

To be able to measure the effects of these disruptions through, amongst other, the changes in the route choice sets network performance indicators are formulated. This is discussed in the next section.

4.5. Network performance indicators

Network performance is measured in this study using two indicators: traveller group composition indicator and travel time losses indicator. The first indicator gives an indication on the disintegration of the network (see Section 4.5.1). The second indicator includes both the effect of disconnected and delayed travellers (see Section 4.5.2).

4.5.1. Traveller group composition indicator

In a non-disrupted situation all travellers can reach their destination. In a situation of disruption, however, it can occur that some travellers become disconnected. In this study this is only possible for the following disruption scenarios: delete traversing lines, reroute traversing lines and cut traversing lines. For the apply local speed limit-scenario the presence of disconnected travellers is not necessarily true. Disconnected travellers are defined as:

Travellers that cannot execute their trip because of network disintegration. These travellers have no path available between their origin and destination.

One way of indicating the network performance is by calculating the share of disconnected travellers. The share of disconnected travellers is calculated as follows

Equation 16 Traveller group composition indicator z as the share of disconnected travellers

$$z = \frac{\sum_{i \in S} \sum_{j \in S} o_{i,j} * \omega_{i,j}}{\sum_{i \in S} \sum_{j \in S} o_{i,j}}$$

where $\omega_{i,j}$ equals one if master paths set $M_{i,j}$ is empty.

4.5.2. Travel time loss indicator

The traveller group composition indicator expresses how many travellers become disconnected. It does, however, not express the effect of the disruption on the travellers who remain connected. Therefore a network performance indicator g is chosen which expresses the total experienced travel time of all remaining connected travellers (tt^{con}) and an experience travel time penalty for disconnected travellers ($tt^{dis,pen}$), see Equation 17.

Equation 17 Traveller time loss indicator GT as the total generalized travel costs

$$GT = tt^{con} + tt^{dis,pen}$$

The total experienced travel time of all remaining connected travellers is:

Equation 18 The total experience travel time tt^{con} of all remaining connected travellers

$$tt^{con} = \sum_{i \in S} \sum_{j \in S} \sum_{k \in M_{i,j}} \left(q_{i,j,k} * u_{i,j,k} * \left(1 - \omega_{i,j}\right)\right)$$

Where $q_{i,j,k}$ is the total flow of travelers and $u_{i,j,k}$ is the total experience travel time on path k between origin i and destination j. The parameter $\omega_{i,j}$ equals one if master paths set $M_{i,j}$ is empty.

The experience travel time penalty for disconnected travellers is

Equation 19 The total experience travel time penalty $tt^{dis,pen}$ for disconnected travelers

$$tt^{dis,pen} = \sum_{i \in S} \sum_{j \in S} \sum_{k \in M_{i,j}(e,0)} \left(q_{i,j,k}\left(y_e(0)\right) * \omega_{i,j} \; * \left(u_{i,j,k}\left(y_e(0)\right) + u^{delay}(e)\right)\right)$$

Where $q_{i,j,k}\left(y_e(0)\right)$ is the total flow of travelers and $u_{i,j,k}(y_e(0))$ is the total experience travel time on path k between origin i and destination j during disruption scenario 0 on edge e. The parameter $\omega_{i,j}(y_e(x))$ equals one if master paths set $M_{i,j}\left(y_e(x)\right)$ is empty during disruption scenario x on edge e. The extra added travel time for disconnected travellers is the maximum mean delay over all OD-pairs and is

Equation 20 The extra added travel time u^{delay} for disconnected travellers

$$u^{delay}(e) = \max_{x \in X, i \in S, j \in S, \omega_{i,j}=0} \left(\frac{\sum_{k \in M_{i,j}(e,x)} (q_{i,j,k} * u_{i,j,k}) - \sum_{k \in M_{i,j}(e,0)} \left(q_{i,j,k}\left(y_e(0)\right) * u_{i,j,k}\left(y_e(0)\right)\right)}{o_{i,j}} \right)$$

4.6. Robustness indicators

To analyse the network robustness two indicators are proposed which are based on the travel time loss indicator of previous section. The two indicators are:

- Link Criticality (LC)
- Degrading Rapidity (DR)

Link Criticality indicates the accumulated effect of the whole range of capacity reduction on a link on the network performance. Degrading Rapidity indicates the relative criticality for low capacity reduction relative to high capacity reductions. Both indicators are presented in the next sections, their definitions are derived from the found indicators of Section 2.4.4.

Before the indicators are presented it is discussed how the Network Performance – Capacity Reduction curve is established on which these indicators are applied. For every link e a curve is established in which for each extent of capacity reduction the network performance is presented. As is presented in Table 20 in Section 4.4 a PTO has the option to apply a mitigation measure or to do nothing. As a consequence of the choice of the PTO the network performance for every extent of capacity reduction can differ. Based on the assumption that a PTO will always try to keep the network performance as good as possible it is assumed that a PTO will choose the option that will keep the network performance indicator as low as possible. For the scenarios $x = 1, ..9$ the PTO can choose to do nothing which means that a local speed limit is applied ($GT^{speed\ limit}$) or to apply rerouting ($GT^{rerouting}$) or cutting ($GT^{cutting}$) lines. As a consequence the network performances for the scenarios $x = 1, ..9$ are

Equation 21 The network performance GT for the scenarios x=1,…,9

$$GT_{for\ x=1,2,\ldots,8,9} = min\left(GT^{speed\ limit}, GT^{rerouting}, GT^{cutting}\right)$$

For the $x = 10$ scenarios the PTO can choose to do nothing which means deleting the traversing lines ($GT^{deleting}$) or to apply rerouting ($GT^{rerouting}$) or cutting (GT^{cuting}) lines. As a consequence the network performance for the $x = 10$ scenario is

Equation 22 The network performance GT for the scenario x=10

$$GT\left(y_e(10)\right) = min\left(GT^{deleting}, GT^{rerouting}, GT^{cuting}\right)$$

4.6.1. Link Criticality indicator

Link criticality (LC) is defined as the accumulated effect of the whole range of capacity reductions on the network performance. This is calculated as the sum of the network performances for the 10 capacity reduction scenarios and the network performance at the base case scenario:

Equation 23 Link Criticality indicator LC_e

$$LC_e = \sum_{x \in X} \left(GT\left(y_e(x)\right) - GT\left(y_e(0)\right)\right)$$

Where X is the total set of disruption scenarios from $x = 1$ to $x = 10$. Graphically the link criticality indicator LC_e corresponds with the area under the absolute network performance-capacity reduction curve. An example of two of these curves are presented in Figure 34 below for two examples. The link criticality indicator score for edge 1 corresponds with the sum of the yellow area. The link criticality indicator score for edge 2 corresponds with the grey striped area.

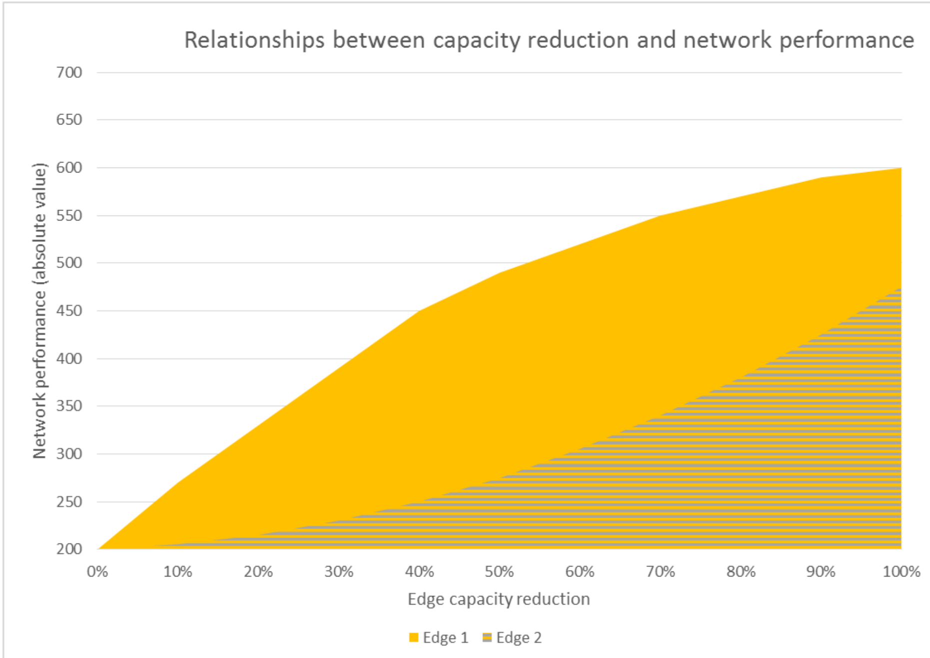


Figure 34 Example of two absolute Network Performance - capacity reduction curves for two example edges (1 and 2)

4.6.2. Degrading Rapidity indicator

Degrading Rapidity (DR) of a link is defined as the accumulated normalized relative effect of the whole range of capacity reductions on the network performance. It indicates the relative criticality for minor capacity reductions relative to major capacity reductions. This is calculated as the sum of the relative increase in network performance for the ten capacity reduction scenarios and the base case scenario:

Equation 24 Degrading rapidity indicator DR_e

$$DR_e = \frac{1}{x} * \sum_{x \in X} \frac{GT\big(y_e(x)\big) - GT\big(y_e(0)\big)}{GT\big(y_e(10)\big)}$$

Where X is the total set of disruption scenarios from $x = 1$ to $x = 10$. Graphically the degrading rapidity indicator DR_e corresponds with the area under the relative normalized Network Performance – capacity reduction curve. An example of two of these curves are presented in Figure 35 below for the two example edges of Figure 34. The degrading rapidity indicator score for edge 1 corresponds with the sum of the yellow area. The degrading rapidity indicator score for edge 2 corresponds with the grey striped area.

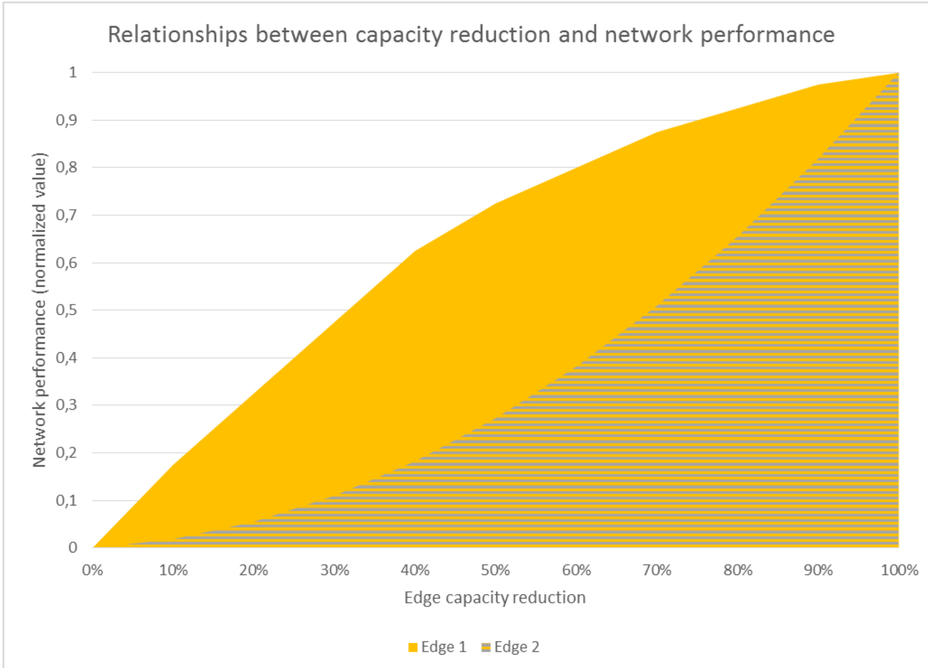


Figure 35 Example of two relative normalized Network Performance - capacity reduction curves for two edges (1 and 2)

4.7. Conclusions

For the analysis of robustness of PT networks a new model is developed in which in an efficient way the robustness of links and indirectly the robustness of the entire network can be analysed. The modelling is done in three steps: network representation, route choice set generation and assignment. The network is represented in two types of network representation used in complex network theory. One represents the infrastructure network (L-space) and the second the service network (P-space).

Route choice set generation is done using Yen's k-shortest path algorithm which is an algorithm based on Dijkstra's algorithm which is used to find not only the shortest paths but also the closest best alternatives. Through applying three filtering routes the master path set for every OD-pair is found: transfer filter rule, loop less filter rule and dominancy filter rule.

Based on the assumption that all path alternatives are independent the assignment is done using a multinomial logit model. The calculated utility per path as input for the logit model is the weighted travel time based on the in-vehicle time, waiting time and number of transfers. Based on the found mitigation measures of PTO's four types of disruption scenarios implementations are proposed: local speed limit, delete traversing lines, reroute traversing lines and cut traversing lines.

The network performance indicator used as input for the robustness indicators is based on the travel time loss of remaining connected travellers and on a travel time penalty for disconnected travellers. Two robustness indicators are proposed: link criticality indicator and degrading rapidity indicator. The first one indicates the accumulated effect of the whole range of capacity reductions on the network performance, the second one indicates the relative criticality for minor capacity reductions relative to major capacity reductions.

In this chapter is shown how robustness can be analysed, before the actual analysis starts a chapter on expected influencing variables is presented on which the analysis is based. The expected influencing variables (or determinants) are presented in Chapter 5 after which the case study is presented (see Chapter 6) on which the analysis is executed.

5. Determinants of link robustness

The robustness of a link depends on different characteristics, from link specific characteristics (e.g. the mean speed on a link) to network-wide characteristics (e.g. local network density) and from topological characteristics (e.g. link centrality) to passenger demand characteristics (e.g. passenger demand on a link). To find out which characteristics are determinant for the robustness of a link a conceptual framework is developed. This framework is presented in Section 5.1.

Next step is to analyse how much these explored characteristics influence the link robustness. Before this analysis can take place the characteristics have to be operationalized (see Section 5.2). The operationalized characteristics are used in a full-scan analysis of a network in the presented model of the previous chapter.

In Section 5.3 of this chapter the expected determinants for analysis on both proposed network robustness indicators are discussed. In the second last section (5.4) the chosen method for the quantitative analysis is discussed. The chapter ends with some conclusions.

5.1. Conceptual framework on link robustness

The conceptual framework on link robustness is presented in Figure 36. Link robustness is affected by the change in network performance due to a disruption on that link. This depends on the number of affected travellers and the extent of the impact of the disruptions. The impact of the disruptions depends on the operational link characteristics and can be mitigated by rerouting options for vehicles and travellers. The expected determinants are presented in three groups of attributes: Passenger demand attributes, line attributes and link attributes.

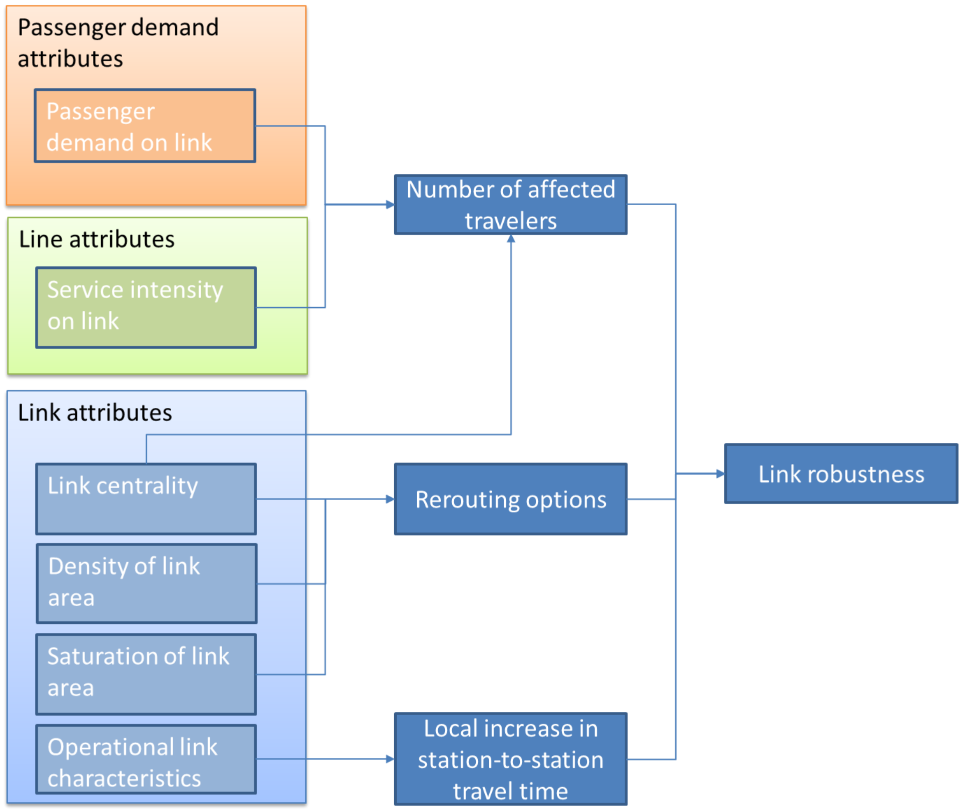


Figure 36 Conceptual framework on link robustness

5.1.1. Passenger demand attributes

The link robustness does, as mentioned before, depend on the number of affected travellers. If a higher number of travellers traverse the disrupted link the effect on the link robustness is greater. Therefore, the *passenger demand on the disrupted link* is expected to be a determinant of link robustness.

5.1.2. Line attributes

Not only the travellers that traverse the disrupted link are affected but also travellers that traverse lines that are affected by the disruption on the link. One can imagine that the *service intensity* (i.e. the number of traversing lines or the total frequency of traversing lines) influences the number of affected travellers and as a consequence influences the link robustness. For example, when a link with a high service intensity is disrupted a high number of travellers are affected and, hence, the link is more critical.

5.1.3. Link attributes

Another expected determinant for link robustness is the *link centrality* (i.e. how centrally is the link situated). In general more travellers will traverse more centrally situated links and therefore more travellers are affected. On the other hand, a more centrally situated link will have more rerouting options and the impact of the disruption on that link is more easily mitigated.

A more specific determinant for rerouting options is expected to be the network density of the link area and the saturation of the link area (i.e. the normal service frequency versus the frequency capacity). If the network density of the link area is higher, more rerouting options are expected to be at hand. However, if these links are highly saturated then rerouting options are limited. Therefore both *density of link area* and *saturation of link area* are expected to be determinants of link robustness.

Primarily the disruption impact depends on the local increase in station-to-station travel time, which depends on the *operational link characteristics* (e.g. a same size speed limit on a longer link will have a greater increase in station-to-station travel time than on a shorter link).

5.2. Operationalization of attributes

The analysis will only focus on attributes that can be quantitatively analysed with the model presented in Chapter 4. The presented attributes are operationalized into different characteristics. The characteristics are presented in three groups corresponding with the groups presented in the previous chapter: *passenger demand attributes*, *line attributes* and *link attributes* (see Figure 37).

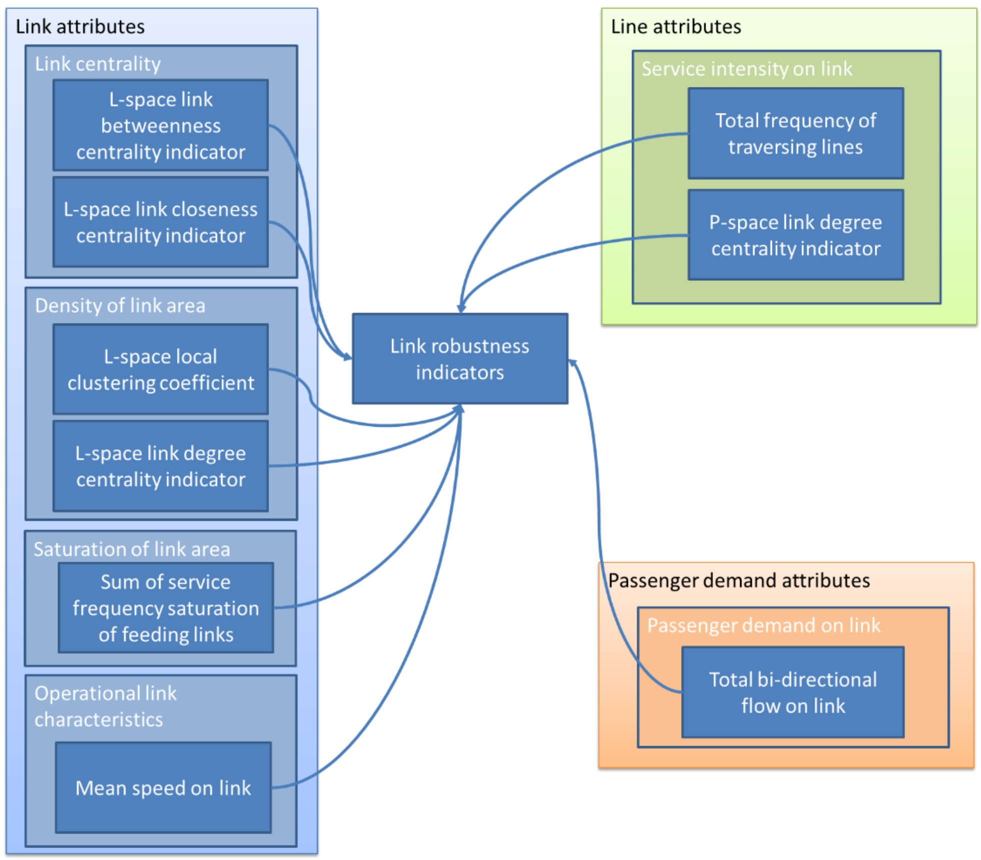


Figure 37 Operationalized characteristics for the analysis of their influence on the robustness indicators

5.2.1. Operationalization of passenger demand attributes

The *passenger demand on the link* is the only passenger demand attribute and can be defined in different measurable characteristics related to the flow that traverses the link in the non-disrupted situation. It is chosen to measure the *passenger demand on the link* as the *total bi-directional flow* which traverses the edge in the base case scenario (see Table 21) as

Equation 25 The passenger demand on link e as the bi-directional flow on link e during base case scenario

$$q_e^{bi-dir} = q_e^+ + q_e^-$$

where q_e^+ and q_e^- are the flows in the base case scenario between, respectively, nodes s_{e^-} and s_{e^+} and s_{e^+} and s_{e^-}.

Table 21 Operationalization of passenger demand attributes

Passenger demand attributes:	Operationalized attributes:
Passenger demand on link	Total bi-directional flow on link

5.2.2. Operationalization of line attributes

The line attributes of a link in general refer to the *service intensity on the link* and can be defined in different measurable characteristics. Most of them are related to the number of traversing lines or the characteristics of these traversing lines. The *service intensity on the link* is operationalized into the *total frequency of traversing lines*, the *total number of traversing lines* and the *P-space link degree centrality indicator* (see Table 22).

Table 22 Operationalization of line attributes

Line attributes:	Operationalized attributes:
Service intensity on the link	Total frequency of traversing lines
	P-space link degree centrality indicator

The *total frequency of traversing lines* is the sum of the service frequency of all traversing lines which is calculated as

Equation 26 The service intensity on link e as the total frequency of traversing lines on link e during base case scenario

$$f_e^{tot} = \sum_{e \in r} f_r$$

where f_r is the frequency of line r in base case scenario.

The *P-space link degree centrality indicator* is an adapted form of the node degree centrality indicator and is an indicator of the number of stops served without transfer from edge e. The *P-space link degree centrality indicator* is calculated as

Equation 27 The service intensity on link e as the P-space link degree centrality indicator of link e during base case scenario

$$ce_e^{d,P} = z_{s_{e-}}^p + z_{s_{e+}}^p - 2$$

where $z_{s_{e-}}^p$ and $z_{s_{e+}}^p$ is the node degree, or the number of edges connected to the node, in P-space of respectively the downstream nodes s_{e-} and s_{e+} of edge e in the base case scenario.

5.2.3. Operationalization of link attributes

The set of link attributes contain *link centrality, density of link area, saturation of link area* and *operational link characteristics*. The *link centrality* and *density of link area* can be operationalized into indicators based on topological indicators used in other studies (Barthélemy, 2011). An overview of the operationalized attributes can be found in Table 23.

Table 23 Operationalization of link attributes

Link attributes:	Operationalized attributes:
Link centrality	L-space link betweenness centrality indicator
	L-space closeness centrality indicator
Density of link area	L-space local clustering coefficient
	L-space link degree centrality indicator
Saturation of link area	Sum of service frequency saturation of feeding links
Operational link characteristics	Mean speed on the link

The *L-space link betweenness centrality indicator* is an indicator for the number of traversing paths and is an adapted form of the node betweenness centrality indicator and is calculated as

Equation 28 The link centrality of link *e* as the L-space link betweenness centrality indicator for link *e* during base case scenario

$$ce_e^{b,L} = \sum_{i \neq j} \frac{\sigma_{ij}(e)}{\sigma_{ij}}$$

Where σ_{ij} is the number of shortest paths going from node i to node j and $\sigma_{ij}(e)$ is the number of shortest paths going from node i to node j traversing edge e. The shortest path from node i to node j is the path with the lowest weighted travel time in the master paths route choice set $M(i,j)$.

The *L-space closeness centrality indicator* is an indicator for the closeness of the link to all other nodes. It is calculated as an adapted form of the node closeness centrality indicator (Lin & Ban, 2013) as

Equation 29 The link centrality of link *e* as the L-space closeness centrality indicator for link *e* base case scenario

$$ce_e^{c,L} = \frac{1}{\sum_{i \neq s_{e-} \neq s_{e+}} d_{ie}}$$

Where d_{ie} is the travel time over the shortest path between node i to edge e, which is the shortest path from node i to node s_{e-} or node s_{e+}. The shortest path from node i to the nodes s_{e-} and s_{e+} is the path with the lowest weighted travel time in the master paths route choice set M.

The *L-space local clustering coefficient* is an indicator for the density of the local surround network. It is calculated as an adapted form of the local clustering coefficient of a node (Lin & Ban, 2013) and is calculated as

Equation 30 Density of link area of link e as the L-space local clustering coefficient of link e during base case scenario

$$cl_e^L = \frac{2 * w}{g(e) * (g(e) - 1)}$$

Where $g(e)$ is the number of unique nodes directly connected to one of the two closest transfer or terminus nodes to edge e and w is the number of edges between the unique nodes directly connected to one of the two closest transfer or terminus nodes.

The *L-space link degree centrality indicator* is an indicator for the number of feeding links. It is calculated as an adapted form of the degree centrality indicator of a node (Lin & Ban, 2013) as

Equation 31 Density of link area of link e as the L-space link degree centrality of link e during base case scenario

$$ce_e^{d,L} = z_{s_{t,e-}}^l + z_{s_{t,e+}}^l - 2$$

Where $z_{s_{t,e-}}^l$ and $z_{s_{t,e+}}^l$ is the node degree, or the number of edges connected to the node, in L-space of respectively the downstream and upstream transfer or terminus node $s_{t,e-}$ and $s_{t,e+}$ of edge e.

The saturation of the link area is operationalized as the sum of service frequency saturation of feeding links. It is calculated as

Equation 32 Saturation of the link area of link e as the sum of service frequency saturation of feeding links of link e during base case scenario

$$sa_e = \frac{1}{|H|} \sum_{h \in H} \frac{f_h}{f_h^{cap}}$$

where f_h is the frequency of link h in link set H during base case scenario. Links $h \in H$ are the links connected to one of the two closest transfer or terminus nodes to edge e, not being the edge e. Frequency f_h^{cap} is the service frequency capacity of edge h.

The operational link characteristic is chosen to be the mean speed on the link and is calculated the same way as in Equation 9 in Section 4.4 as

Equation 33 Operational link characteristics of link e as the mean speed on link e during base case scenario

$$v_e(0) = \frac{d_e}{t_e^t}$$

5.3. Expected determinants of robustness indicators

Robustness is measured using two different indicators: link criticality and degrading rapidity. For both robustness indicators the expected influence of the chosen characteristics on these indicators is presented. These hypotheses are the basis for the analysis presented in Chapter 7. The expected influences are presented for link criticality first and degrading rapidity second.

5.3.1. Link criticality determinants

The expected influences of link characteristics on link criticality are presented in Figure 38. The relationships are discussed on the basis of the three groups of attributes: *link*, *line* and *passenger demand attributes*.

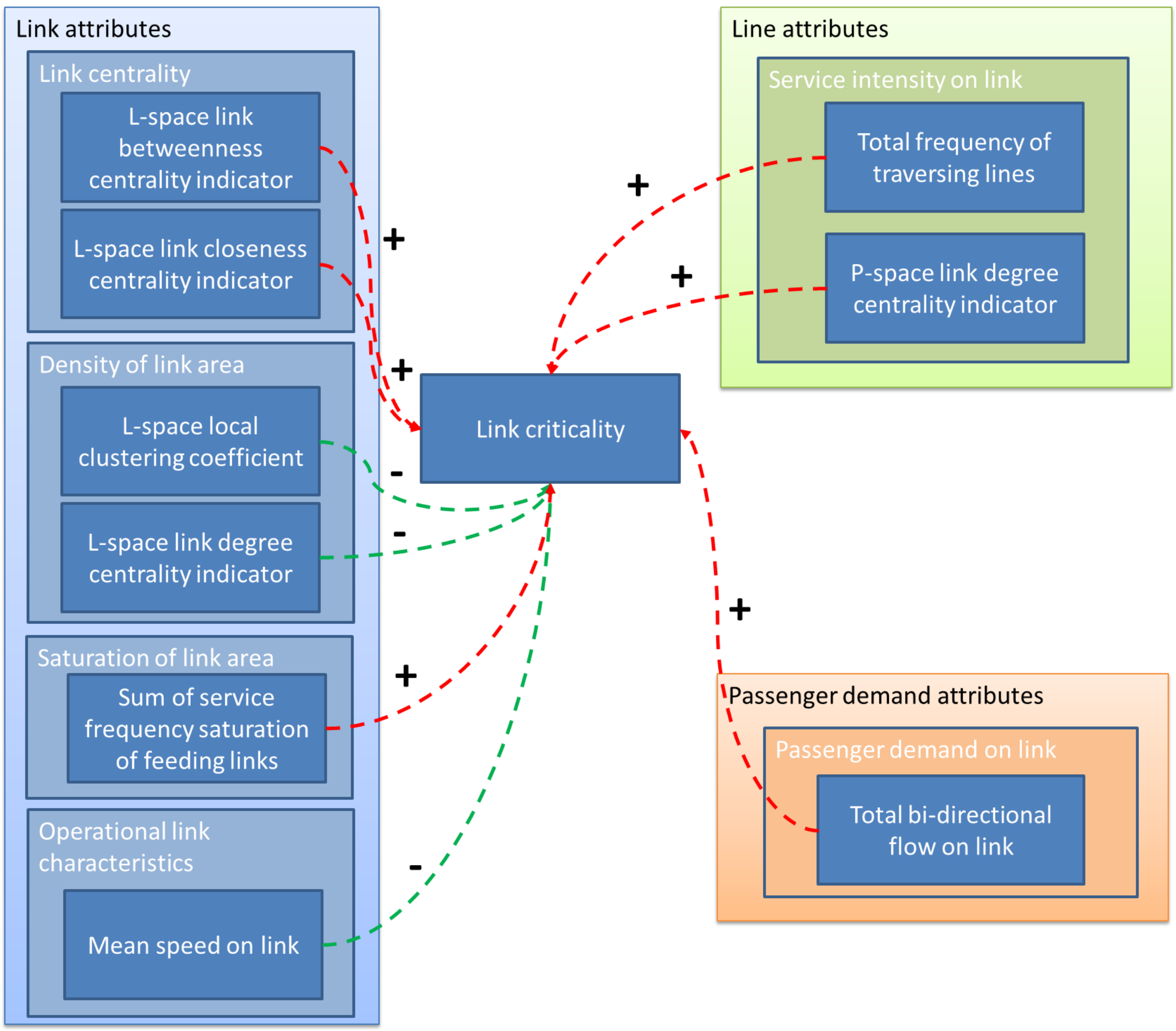


Figure 38 Expected influence of attributes on link criticality (plus: positive expected relationship, minus: negative expected relationship)

Link attribute determinants

When a link is situated more central in the network it is more likely that more lines traverse the link and a larger number of travellers traverse this link. In case of a disruption on this link more travellers are expected to be affected and so a higher link criticality is expected for a link situated more central in the network.

Supported by findings in literature a disrupted link in a high network density area, on the other hand, is expected to have a lower link criticality (Angeloudis & Fisk, 2006; Berche et al., 2009; Derrible & Kennedy, 2010; Rodríguez-Núñez & García-Palomares, 2014; von Ferber et al., 2012; X. Zhang et al., 2015). In the non-disrupted situation travellers have more alternative routes, so it can be that the number of affected travellers is primarily lower. Secondary, travellers have more rerouting options in case of disruptions making the link less critical for disruptions and so a lower link criticality is expected for links situated in a more dense area of the network.

When the surrounding network of a link is highly saturated, rerouting is harder to implement. As a consequence the effects of the disruption are less well mitigated than in situations where rerouting is easier. This is the case when the surrounding network is barely saturated. Therefore the correlation between link area saturation and link criticality is expected to be positive

The operational service quality is also of influence on the link criticality. When in the normal, non-disrupted situations a link is serviced with a higher speed the station-to-station travel time on this link is lower. A percentage wise decrease of the speed will as a consequence result in a higher increase in station-to-station travel time for a link with a lower mean speed than one with a higher mean speed. A link with a higher mean speed is expected to have a lower link criticality than a link with a lower mean speed.

Line attribute determinants

A link in a PT network facilitates the service network, i.e. the PT lines of the network traverse the links of the network. One can imagine that a intensively used link will have a higher criticality than a less intensively used line. For full link breakdown situations this relationship is found to be true by Angeloudis and Fisk (2006), Berche et al. (2009) and Rodríguez-Núñez and García-Palomares (2014). It is explained by the expectation that more travellers are affected if more lines are affected. Secondly, if more lines are affected a larger part of the network is directly or indirectly affected through higher waiting times and number of transfers.

Passenger demand attribute determinants

When over a link a higher number of travellers traverse the edge in the normal non-disrupted situation the link is expected to have a higher score on the link criticality indicator (Rodríguez-Núñez & García-Palomares, 2014). If more travellers traverse the edge more travellers are affected and hence the link is expected to be more critical in robustness terms.

5.3.2. Degrading rapidity determinants

The expected influences of link characteristics on Degrading Rapidity are presented in Figure 39. The relationships are again discussed on the basis of the three groups of attributes: *link*, *line* and *passenger demand attributes*.

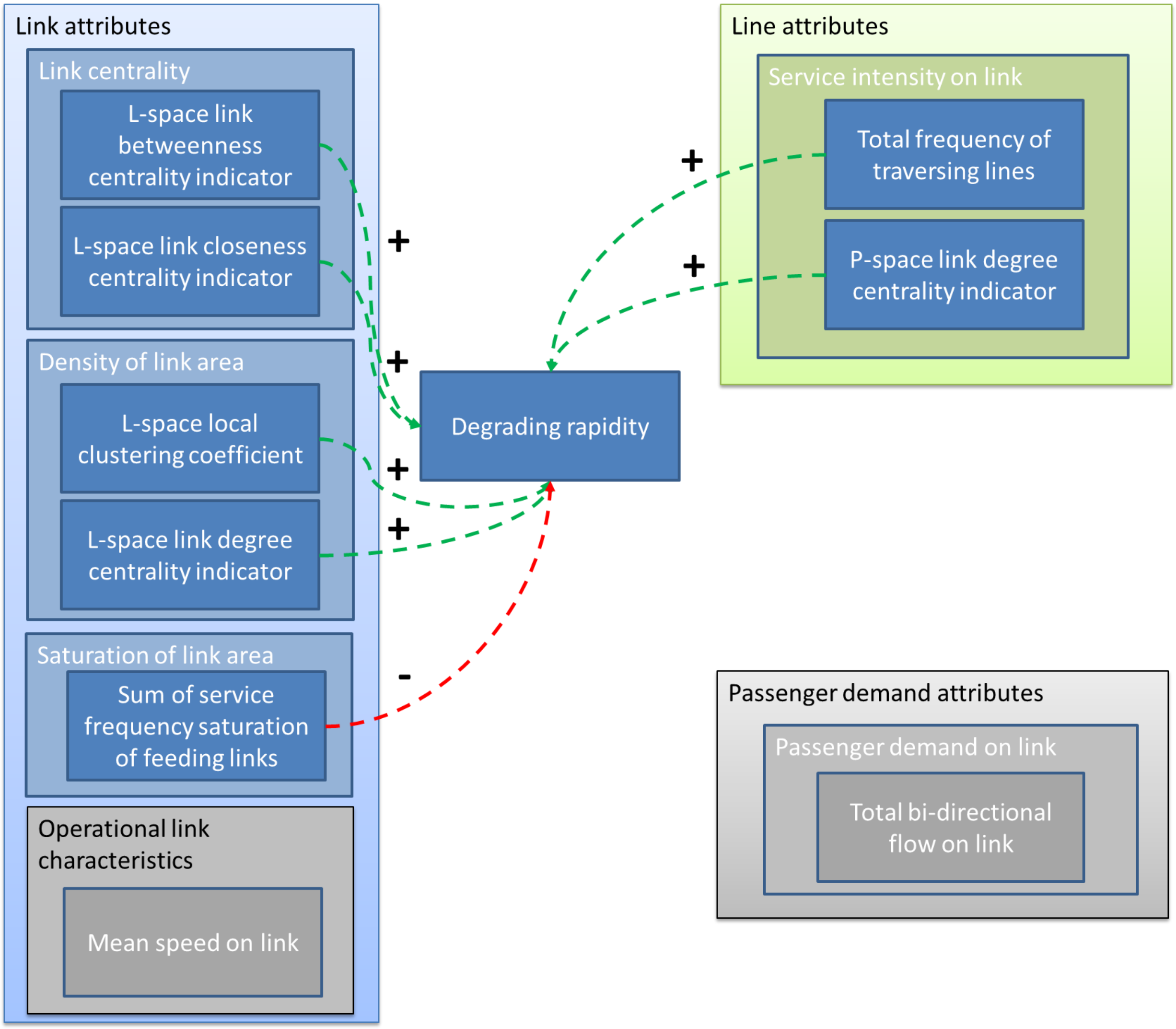


Figure 39 Expected influence of attributes on degrading rapidity (plus: positive expected relationship, minus: negative expected relationship)

Link attributes

Because of existence of more alternative rerouting options in a more central situated link such a link is expected to have a higher degrading rapidity value. A higher degrading rapidity value means that a link is relatively more sensitive to minor capacity reductions than to major capacity reductions.

The same explanation applies to network density. When a link is situated in a high density part of the network, travellers have more rerouting options. Choosing a rerouting options will in general take place in major capacity reductions situations. So when a link is situated in a high density part of the network, these links will be relatively less sensitive for minor capacity reductions and a higher degrading rapidity value is expected.

As mentioned in the previous section rerouting is harder to implement in higher saturated surroundings of a link. Rerouting as mitigation measure is in general only effective for large disruptions (i.e. full link breakdown or major capacity reductions). As rerouting is harder to implement in higher saturated link areas the correlation between saturation of link area and degrading rapidity is expected to be negative.

The effect of the mean speed on a link on the link criticality is found in its absolute value. As the degrading rapidity is an indicator on the relative values, it is not expected to be a correlation between mean speed and degrading rapidity.

Line attributes

The service network characteristics, like the operational service intensity, are also expected to be of influence on the degrading rapidity. If a higher number of lines traverse a link, a higher number of lines is affected throughout the network through lower frequencies as a consequence of higher cycle times. Travelers are able to limit the consequences of the disruption through rerouting during minor capacity reductions. During major capacity reductions, however, the travellers are, despite rerouting, indirectly affected by higher waiting times through lower frequencies.

Passenger demand attributes

The relationship between the passenger demand on the link and link criticality is also found in its absolute value. More demand results in more affected travellers, and as a consequence the link has a higher link criticality. As degrading rapidity is an indicator on normalized relative values, the passenger demand intensity is expected not to have an effect on degrading rapidity.

5.4. Multiple linear regression

Multiple linear regression is used to perform the quantitative analysis on the relationships between the attributes and the indicators. This section gives a brief introduction to multiple linear regression. Through multiple linear regression this research aims to quantify the strength of the relationships between the robustness indicators and the link, line and passenger demand attributes.

Multiple linear regression is used to explain the relationship between one continuous dependent variable (e.g. one of the link robustness indicators) and two or more independent variables (e.g. the identified attributes). The general form of the multiple linear regression is defined as

$$y_i = \beta_0 + \beta_1 * x_{i1} + \beta_2 * x_{i2} + \cdots + \beta_p * x_{ip} \; for \; i = 1,2, \dots n$$

where y_i is the dependent variable. $x_{i1}, x_{i2,\dots,}\, x_{ip}$ are called the independent variables. $\beta_1, \beta_2, \dots, \beta_p$ are the regression coefficients and β_0 is the intercept. The use of multiple linear regression is supported by the assumption that the data is normally distributed, that a linear relationship exists between the dependent variable and the independent variables and that the independent variables are not too highly correlated. The latest assumption is tested by the correlation tool in excel before the linear regression is executed.

For this analysis multiple linear regression will be executed on the dependent variable *link criticality* and on the dependent variable *degrading rapidity*. The independent variables are the explored attributes from Section 5.3. And the estimated regression coefficients will show the strengths of the relationships between the attributes and the indicators and whether it is negative or positive. The linear regression is executed using the Microsoft Office Excel add-in Analysis ToolPack.

5.5. Conclusions

In this chapter the potential explanatory variables (i.e. determinants) for the two robustness indicators are derived from the presented conceptual framework. They are grouped into three groups of attributes: line, link and passenger demand attributes. All attributes are operationalized into one or more measurable characteristics.

Based on the expected explanatory value of the explored determinants the quantitative analysis will be executed for the link, line and passenger demand attributes presented in Table 24.

Table 24 Overview of attributes for the quantitative analysis

Attributes for the quantitative analysis:
Link centrality
Density of link area
Saturation of link area
Operational link characteristics
Service intensity on link
Passenger demand on link

Multiple linear regression will be used to analyse the strength of the relationships and whether it is negative or positive. The analysis will be executed on the expected Amsterdam rail bound PT network of 2020, this case study is presented in the next chapter. The actual results and analysis are presented in Chapter 7. Whether the hypothesized direction (i.e. negative or positive) of the relationships is correct will be discussed in the last chapter of this report.

6. Case Study Description: Amsterdam tram and metro network 2020

The analysis of the determined relationships will be analysed in the case study of the Amsterdam rail bound PT network of 2020. In this chapter the case study network and implementation details are described. In the first section (6.1) the present PT lines in the network with their characteristics (e.g. frequency, form and route) are presented. The preparation data used for this study is described in the subsequent section (6.2). The implementation details, the validation and the modelled disruption scenarios of the case study in the model are presented in respectively Section 6.3, Section 6.4 and Section 6.5. The chapter is summarized in the last section.

6.1. Network description

The scope of this case study is the Amsterdam urban rail bound PT network 2020. The current network is going to change drastically and in the near future the urban rail bound PT network of Amsterdam will be expended with a new metro line: the Noord/Zuidlijn (N/Z-lijn) (N. Z. Gemeente Amsterdam, 2015). The opening of this new metro line is going to have a significant effect on the tram network as well.

It is chosen to execute the case study on the urban rail bound 2020 network of model run3a which was executed in the Amsterdam traffic model GENMOD2013 (D. I. V. e. V. Gemeente Amsterdam, 2013). This decision is taken in consultation with the GVB and the SRA. The urban rail bound PT network in run3a is expected to have the closest resemblance with the future implemented network after commissioning of the N/Z-lijn. Run3a is one of the runs executed for the input for the business case N/Z-lijn and the route network vision for the urban PT network after commissioning of the N/Z-lijn (Amsterdam, 2013). All numbers, figures and indicators are based on the information given in this document.

The Amsterdam urban rail bound PT network 2020 consists of two modalities: tram (see, Figure 40) and metro (see, Figure 41). The serviced rail bound network consists of 270 km of rail, of which 190 km is used by the tram and 80 km is used by the metro. Next to the used rail for the normal operation situation, extra rail is available for rerouting. For tram this is an extra 20 km of rail and for metro this consists of more than 2 km of rail. In 2020 Amsterdam is expected to have a population of more than 820,000 inhabitants (D. I. V. e. V. Gemeente Amsterdam, 2013). Based on the results of the model run of the Amsterdam traffic model Genmod2013 the total expected number of travellers on the urban rail bound PT network in the two evening peak hours is approximately 107,000.

The Amsterdam urban rail bound PT Network 2020 consists of 4 metro lines and 14 tramlines. In Table 25 a summary is given for all lines with line ID, origin, destination, service frequency and form of line. Line 52 in Figure 41 is the new N/Z-lijn, the current metro line 51 from Amstelveen to Amsterdam Centraal Station is replaced in the PT 2020 network by the tram lines G and G+ in Figure 40. The total frequency of trams increases with about 10% in the 2020 network in relation to the current network. The increase in frequency is mostly applied to tangential and diametrical lines, the sum of frequencies on the radial lines even decrease. The tangential and diametrical lines are all linked to the new N/Z-lijn.

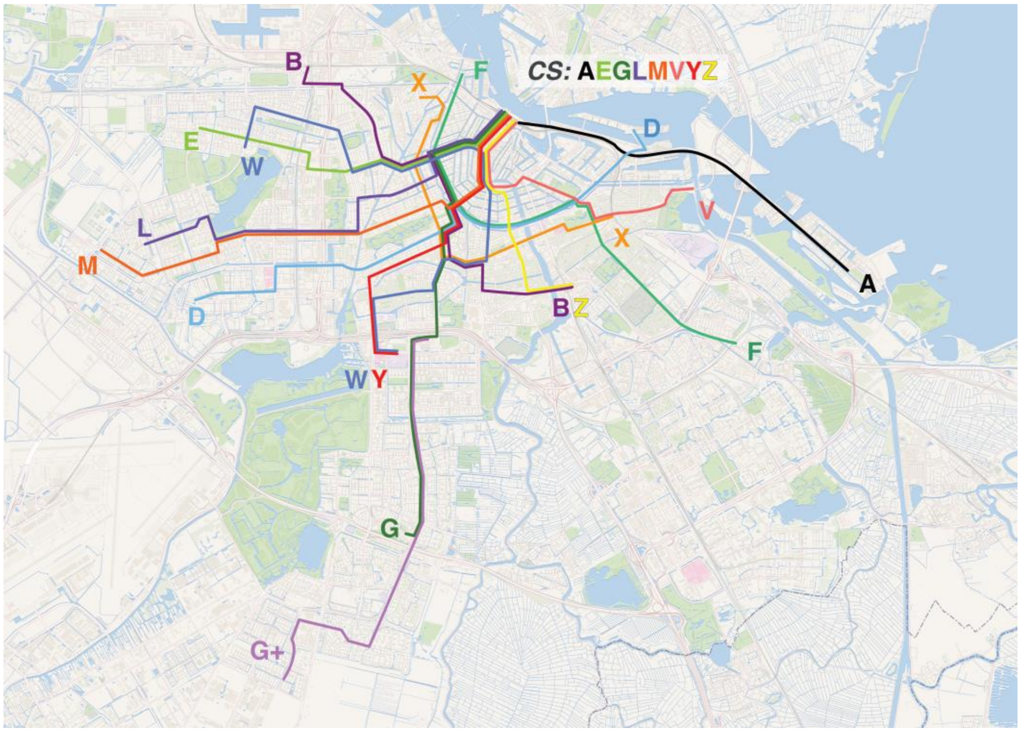


Figure 40 Amsterdam tram network 2020 for case study

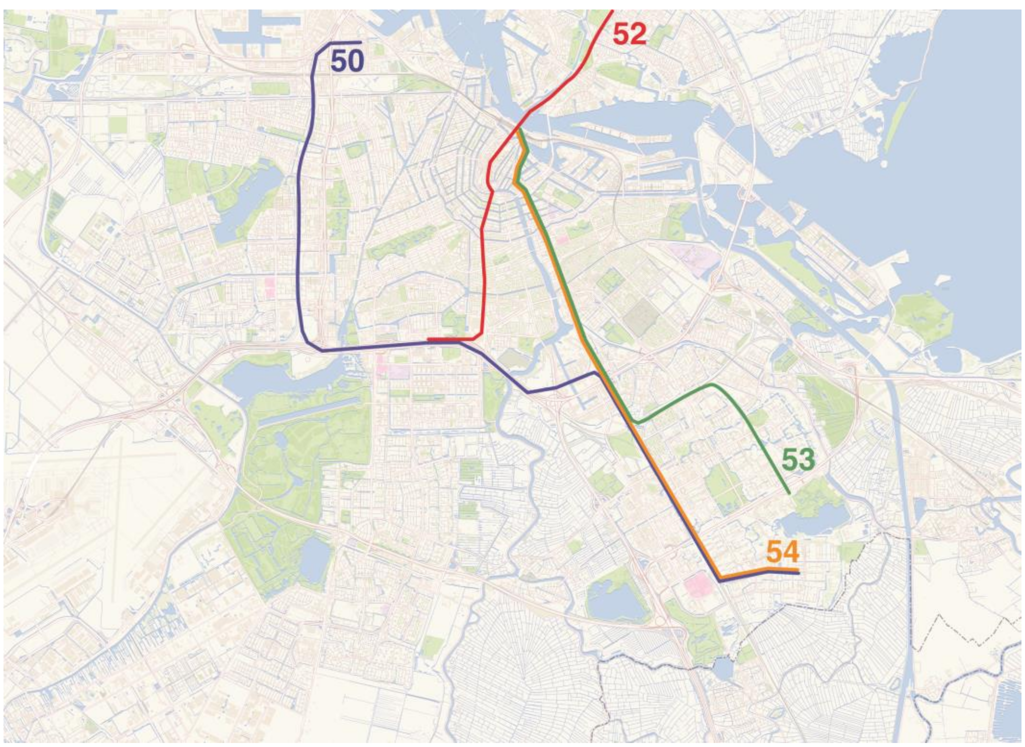


Figure 41 Amsterdam metro network 2020 for case study

Table 25 Line summary of the Amsterdam rail bound PT network 2020

Mode	LineID	From	To	Via	Frequency (departures/hour)	Form
Metro	50	Isolatorweg	Gein	-	11	Tangential
	52	Station Noord	Station Zuid	-	15	Diametrical
	53	CS	Gaasperplas	-	11	Radial
	54	CS	Gein	-	11	Radial
Tram	A	IJburg	CS	-	15	Radial
	B	Station Amstel	Sloterdijk	Leidseplein	8	Diametrical
	D	Nieuw Sloten	Azartplein	Leidseplein	8	Tangential
	E	Geuzenveld	CS	Rozengracht	8	Radial
	F	Diemen Sniep	Zoutkeetsgracht	Leidseplein	8	Tangential
	G	Binnenhof	CS	Leidseplein, Rozengracht	8	Radial
	G+	Westwijk	Station Zuid	-	8	Radial
	L	Osdorp Dijkgraafplein	CS	Rozengracht	10	Radial
	M	Osdorp De Aker	CS	Leidseplein, Leidsestraat	12	Radial
	V	Flevopark	CS	Artis	10	Radial
	W	VU	Slotermeer	Vijzelstraat - Dam - Rozengracht	8	Diametrical
	X	Muiderpoort	Westerpark	Ceintuurbaan - Van Baerlestraat	8	Tangential
	Y	VU	CS	Leidsestraat	8	Radial
	Z	Station Amstel	CS	Utrechtsestraat	8	Radial

6.2. Data preparation

The data for this case study was delivered by the municipality of Amsterdam, the owner of the Amsterdam traffic model GENMOD2013. The supply data was delivered in comma separated files, which are input files for the OmniTRANS model (DAT.Mobility, 2015) used for visualization of the GENMOD2013 network. The demand data was delivered in excel sheets with for every line for every mode the boarding and alighting per stop, this data is the output data of model run3a. All the data represents the situation in the two evening peak hours (16:00-18:00), for both supply and demand. How this data was processed for this case study is discussed in Appendix C for the supply data and Appendix D for the demand data.

The results of the supply data preparation are presented in the next section (Section 6.3). The total OD-demand as a result of the demand data preparation is 107,490 passengers as a total for the evening peak hours (16:00-18:00).

6.3. Implementation details

The network representation for the Amsterdam PT network 2020 consists of 216 unique stops (i.e. nodes) of which 195 are served by tramlines and 39 are served by metro lines. Of these stops 42 stops are indicated as transfer node for the base case scenario, as discussed in Section 4.4.5. These nodes are linked by 492 directed edges in L-space of which 78 are traversed by metro lines and 414 by tram lines. These same nodes are linked by 6746 directed edges in P-space. Except for the first step of the route choice set generation the whole case study modelling is executed in MATLAB. This first step of the route choice set is executed in JAVA. Visualizations are generated in Gephi, examples of visualizations for L-space and P-space are given in Figure 42 and Figure 43 below. As a reminder, L-space represents the physical infrastructure where nodes are connected if they are consecutive stops in a given route; P-space accounts for the presence of PT lines where nodes are connected if there is at least one PT line connecting them.

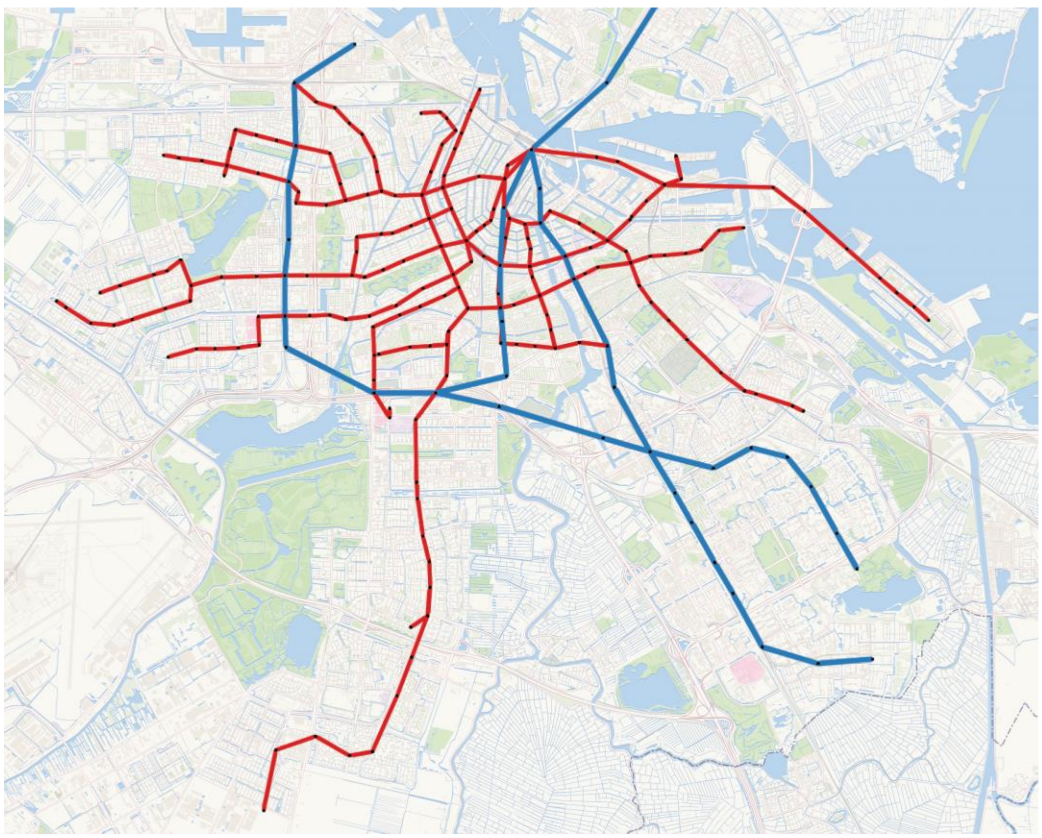


Figure 42 Amsterdam rail bound PT network L-space representation (red is tram lines, blue is metro lines)

Figure 43 Amsterdam rail bound PT network P-space representation

Next to the used infrastructure in the original, non-disrupted, situation some non-traversed infrastructure is available for rerouting measures. In the tram network these are 13 bidirectional links (Figure 44), in the metro network this is 1 bidirectional link (Figure 45).

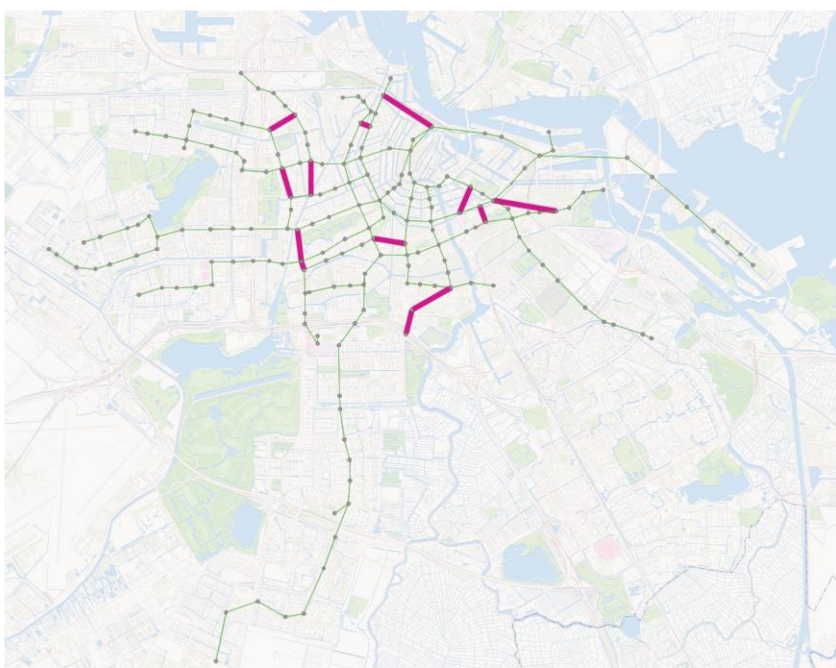


Figure 44 Tram network available for rerouting, thin green is traversed in the base scenario network, thick purple is extra available infrastructure

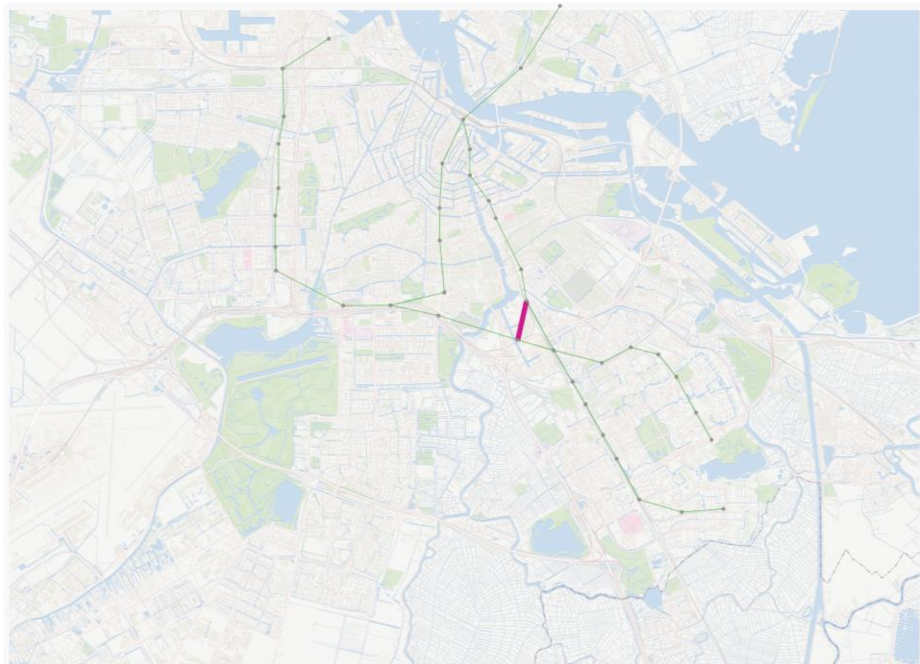


Figure 45 Metro network available for rerouting, green is traversed in the base scenario network, purple is extra available infrastructure

6.4. Modal validation

The model is mainly validated by applying face validity through expert judgement of SRA employees. This is done in three ways:

1. General impression of the assignment on the whole network
2. The passenger distribution on alternative paths between some specific OD-pairs
3. Comparing the passenger flow on some section in the network with the original run3a results

As a result of the model validation the parameters for the utility and logit function were estimated. These parameter values are presented in Table 26.

Table 26 Estimated logit and utility function parameters after model validation

Parameters	Description	Value (minutes)
β^{w}	Waiting time experience factor	1.5
β^{n1}	Transfer penalty type 1	7 (5+2)
β^{n2}	Transfer penalty type 2	10 (5+5)
β^{n3}	Transfer penalty type 3	12 (5+7)
μ	Logit parameter	0.1

The value of the waiting time experience factor is equal to the value used in the traffic model of the metropole region Amsterdam VENOM2015[6] (Kieft, 2015). The transfer penalty is differentiated into three

[6] In Dutch: Verkeerskundig Noordvleugel Model

types based on the vertical distance which has to be travelled during transfer. The transfer penalty is composed of a base transfer penalty and a proxy for the walking time. The base transfer penalty is equal to five minutes for every transfer penalty type, this is equal to the value used in VENOM2015 (Kieft, 2015). The proxy for the walking time is based on expert judgment and is respectively two, five and seven minutes. Each PT line is assigned to a category. All tram lines are in category one, the N/Z-lijn (line 52) is in category three and all other metro lines are in category two. The underlying assumption for this categorization is that the tram network is in general situated on the surface level, the N/Z-lijn in general at a -2 level and the other metro lines at -1 or +1. An overview of the occurring transfers in the Amsterdam rail bound PT network with example transfer nodes and corresponding transfer penalty type can be found in Table 27.

Table 27 Type of transfer with corresponding transfer penalty type

Type of transfer (from/to)	Example 1:	Example 2:	Transfer penalty type
Tram (1) –Tram (1)	Dam	Bilderdijk-Kinkerstraat	1
Tram (1) – Other metro (2)	Waterlooplein	Weesperplein	2
Tram (1) – N/Z-lijn (3)	Rokin	Vijzelgracht	3
Other metro (2) – N/Z-lijn (3)	Amsterdam Zuid	-	2
Other metro (2) – Other Metro (2)	Spaklerweg	-	2

The logit parameter μ is estimated based on the assessed passenger distribution on alternative paths between some specific OD-pairs and is set equal to 0.1.

6.5. Scenario Design

For the Amsterdam PT Network 2020 a total of 2914 scenarios are executed, see Table 28. The base case scenario is executed just once because it is equal for every edge. The nine partial capacity reduction scenarios are applied on al 246 bi-directional edges, so a disruption on an edge is applied in both directions. For the 207 bi-directional tram edges the full link breakdown situation is executed for all three scenarios: deleting lines, rerouting and cutting lines. Because rerouting in the metro network is almost impossible through infrastructural constraints, the full link breakdown situation is only executed for the deleting lines and cutting lines scenario.

Table 28 Overview of applied scenarios for the Amsterdam rail bound PT Network

	Scenarios				
	Base case scenario	Apply local speed limit	Delete traversing lines	Reroute traversing lines	Cutting lines
y_e	0	[0.1,0.2,…,0.9]	1	1	1
Number of runs	1	246*9	246	123	246

The running time of one scenario is approximately 45 minutes on a standard PC, of which twelve minutes in JAVA and 35 minutes in MATLAB. Of this 45 minutes of running time 44 minutes are used for the route choice set generation and one minute of assignment. The sum of the running times of the full-scan for the Amsterdam rail bound PT network is 49 days.

6.6. Summary

The case study is chosen to be executed on the Amsterdam urban rail bound public transport network in 2020 because in this year it is expected that the N/Z-lijn is in operation and the tram network is adapted to this major change. Data for the case study is delivered by the municipality of Amsterdam, the owner of the former Amsterdam traffic model GENMOD2013. The complete network consists of 216 unique stops (i.e. nodes) which are linked by 492 directed edges. The network is extended with 28 directed edges for possible rerouting. The public transport network consists of four metro lines and fourteen tram lines. The total expected number of travellers on the urban rail bound PT network in the two evening peak hours is approximately 107,000.

The model is validated by applying face validity through expert judgment of SRA employees, the resulting estimated logit and utility function parameters can be found in Table 29. The transfer penalty is differentiated for different types of transfers: Tram-Tram (1), Tram-N/Z-lijn (3) and other (2).

Table 29 Estimated logit and utility function parameters after model validation

Parameters	Description	Value (minutes)
β^w	Waiting time experience factor	1.5
β^{n1}	Transfer penalty type 1	7 (5+2)
β^{n2}	Transfer penalty type 2	10 (5+5)
β^{n3}	Transfer penalty type 3	12 (5+7)
μ	Logit parameter	0.1

A total of 2914 scenarios are executed on this case study. All applied as a bi-directional capacity reduction as one of the local speed limits (246*9), deleting lines (246), rerouting lines (123) or cutting lines (246). Rerouting is not possible for every edge. The running time of one scenario is approximately 45 minutes on a standard PC, of which twelve minutes in JAVA and 35 minutes in MATLAB. Of this 45 minutes of running time 44 minutes is used for the route choice set generation and one minute of assignment. The sum of the running times of the full-scan for the Amsterdam rail bound PT network is 49 days.

7. Results and analysis

The analysis is executed on the Amsterdam Urban Rail bound PT network, the results of this analysis are presented in this chapter. First an overview of results of the base case scenario is given in Section 7.1. Subsequently the descriptive analysis is presented for both robustness indicators (see Section 7.2). The chapter ends with the determinants from the different estimated models of the Multiple Linear Regression (see Section 7.3).

7.1. Assignment result base case scenario

As a result of the traveller assignment each link has a certain intensity of use (i.e. a number of traversing travellers), which can be seen in Figure 46. It is clear that the metro links are significantly more intensively used than the tram network. The mean number of travellers over a link is more than four times larger for the metro links than for the tram links. The centre of gravity of the network seems to be at metro line 50 and 52 near Amsterdam Zuid. In the tram network the Rozengracht, Overtoom and 'de Binnenring' (i.e. Marnixstraat-Weteringschans) are the more intensively used parts. On almost all outskirts of the network the intensity drops to zero, except for Amsterdam Noord where a lot of travellers are expected to transfer to the underlying bus network.

Figure 46 Base case scenario assignment results - the bi-directional flow intensity in the evening peak hour

In the table below (see Table 30) an overview is given of the mean trip parameters per traveller. The In-Vehicle time is the largest part of the travel time, thanks to the high-frequency network the waiting time is relatively low. The mean number of transfers per traveller is relatively high, this is probably caused by the more feeding tram lines to the metro line 52 (North/South line). The analysis is however not focussed on the passenger loads per link but on the robustness of these links, therefore the robustness analysis is discussed in the following sections.

Table 30 Mean trip parameters per traveller in the base case scenario

Mean trip parameters in base case scenario	
In-Vehicle time (min)	14.30
Waiting time (min)	3.73
Number of transfers (-)	0.54

7.2. Descriptive analysis

For every link in the network, all the disruption scenarios (see Section 6.5) are applied and the network performance is calculated as presented in Section 4.5 and Section 4.6. For every link the robustness indicators are calculated from the network performance results of the disruption scenarios. The descriptive analysis will be presented with respect to the Link Criticality indicator and the Degrading Rapidity indicator.

7.2.1. Link Criticality indicator

For the analysis of the accumulated effect of the whole range of capacity reduction on a link the Link Criticality Indicator is proposed (see Equation 23 in Section 4.6.1). The application of this indicator to the results of the Amsterdam Urban Rail Bound PT Network is presented in a histogram in Figure 47. Most of the links have a positive Link Criticality Indicator value between zero and five minutes of extra generalized travel costs per traveller. The highest value of the Link Criticality Indicator is approximately 17 minutes of extra generalized travel costs per traveler. It indicates the criticality of the link between Van der Madeweg and Duivendrecht, this link is indicated in Figure 48 as the reddest and thickest link.

A number of links has a negative Link Criticality value which implies that the application of a mitigation measure enhances the network performance relative to the base case scenario (i.e. the non-disrupted situation). This is the case for 21 edges in the Amsterdam urban rail bound PT network. The network is, apparantely, not optimally designed in terms of generalized travel costs.

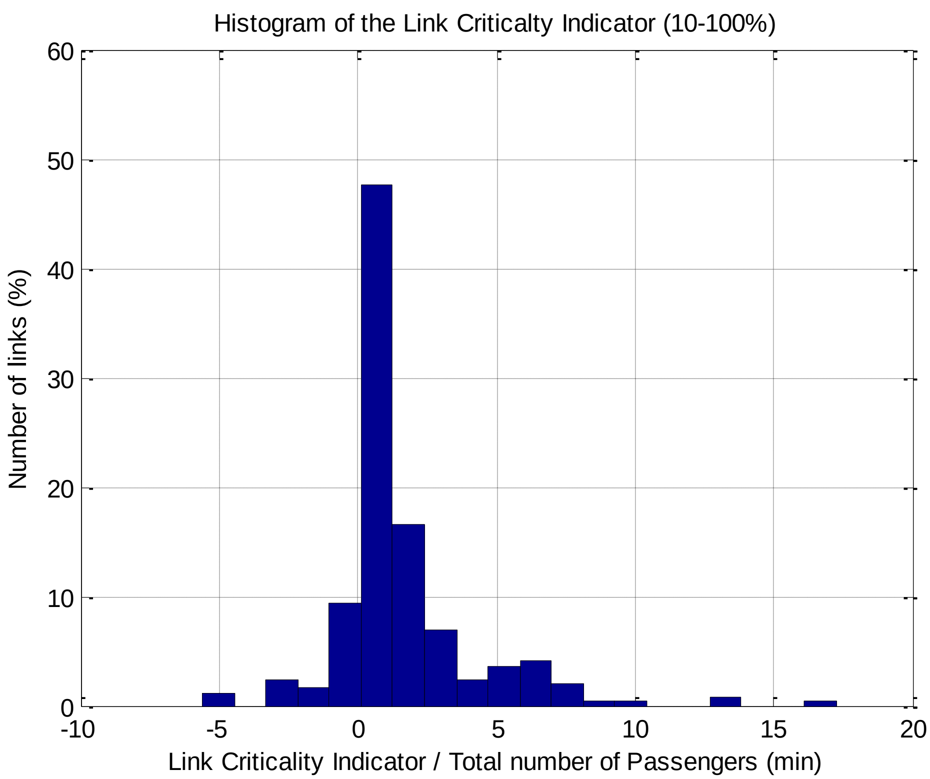


Figure 47 Histogram of the Link Criticality indicator (10-100%) of all links in the Amsterdam urban rail bound PT network

In Figure 48 the Link Criticality indicator values are presented in the Amsterdam Urban Rail Bound PT Network. The most critical links appear on the intensively used metro lines. The less critical links are situated in the centrally situated high density places of the tram network. This seems to be in line with the hypothesized determinants in Chapter 5 of link centrality and passenger demand on the link. The centrally situated links are less critical and the links with a high passenger demand are more critical.

The most critical links in both the tram and the metro network are found on the more isolated outlying branches. In the metro network the most critical links are the links between Amsterdam Centraal Station and Amsterdam Noord, the links in the South East and the connecting link between the Amstelveenseweg and the Henk Sneevlietweg. The latter one is not an outlying branch, but the best alternative route for travellers implies quite a detour (i.e. over 'De Buitenring' – Van Baerlestraat is the closest alternative). In the tram network the increased Link Criticality is also found on outlying branches at, for example, the Cornelis Lelylaan and the line to Amstelveen. This indicates the potential determinant value of network density on Link Criticality.

Figure 48 Graph of the Link Criticality indicator (10-100%) for the links in the Amsterdam urban rail bound PT network (Thicker and more saturated red links indicate a higher score)

As partial capacity reductions are not as much investigated as full link breakdown scenarios in robustness studies a special attention is given to these situations. The histogram of the Link Criticality indicator for the 10 to 90% capacity reductions is presented in Figure 49. Except for the fact that all links have a lower Link Criticality now because only nine scenarios are incorporated, the histogram gives a similar picture as the one for the full set of capacity reduction scenarios. The decrease in Link Criticality values is however larger than 10% (i.e. the value of one scenario less in an original scenario set of 10 scenarios) which indicates the enormous impact of the full link breakdown scenarios on the Link Criticality indicator.

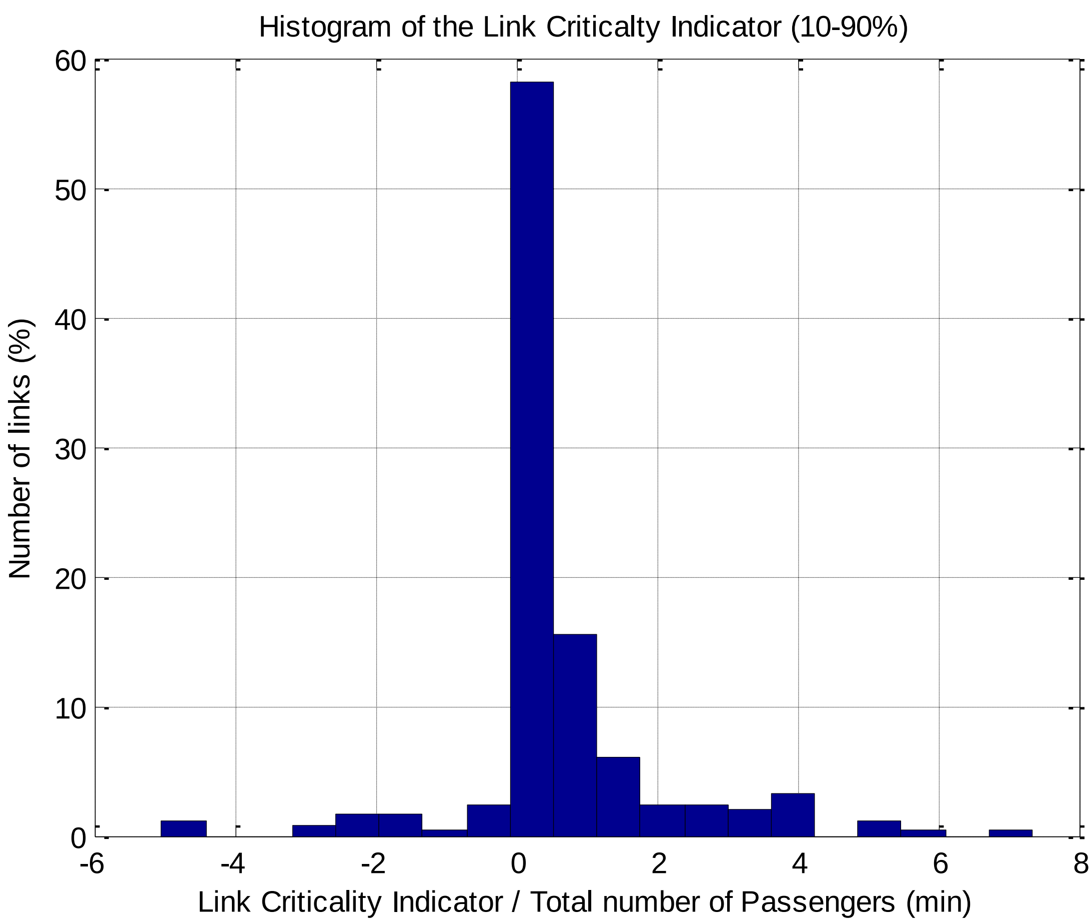


Figure 49 Histogram of the Link Criticality indicator (10-90%) of all links in the Amsterdam urban rail bound PT network

In Figure 50 the Link Criticality values are presented in the Amsterdam urban rail bound PT network graph. The most critical links still seem to be on the intensively used metro lines. The biggest difference with the graph for the Link Criticality values for the total set of scenarios are the links in the South East. These links are a bit toned in comparison with the Link Criticality indicator for the full set of scenarios. A lot of travellers get disconnected at these links during full link breakdown with a large increase in generalized travel costs as result. Finally the longer links now seem to have a higher Link Criticality which indicates that the increase in local station-to-station travel time operationalized in mean speed of the link is indeed a potential determinant.

Figure 50 Graph of the Link Criticality indicator (10-100%) for the links in the Amsterdam urban rail bound PT network (Thicker and more saturated red links indicate a higher score)

7.2.2. Degrading rapidity indicator

Not only the accumulated effect of the different capacity reductions scenarios are of interest, but also the difference in effects between, for example, smaller partial capacity reductions and full link breakdowns. The *normalized change in generalized travel costs – capacity reduction curves* for all links in the Amsterdam urban rail bound PT network are shown in Figure 51. In general the curves are monotonically increasing. The majority of the curves have a more convex-like shape (i.e. the degrading rapidness to full link breakdown network performance is lower), but also linear shaped, concave shaped and other types of curves are present.

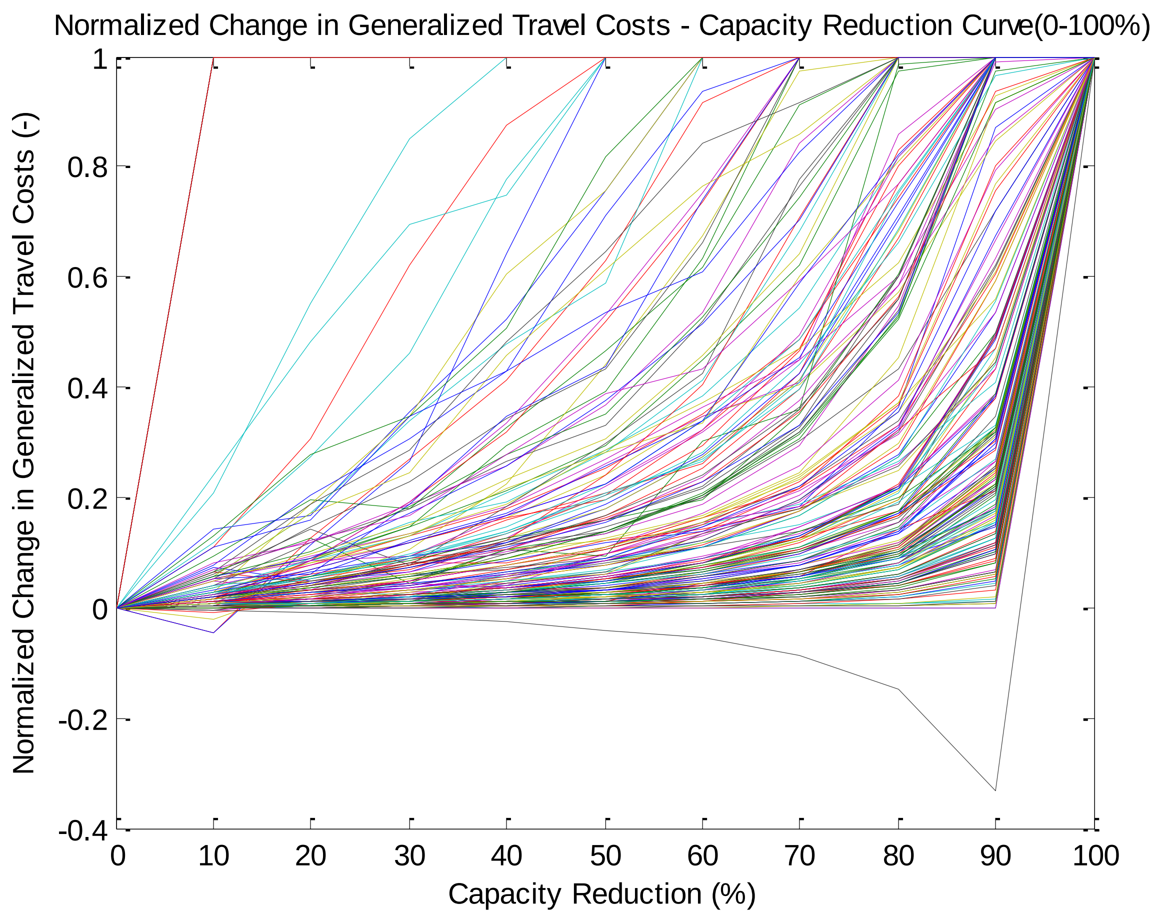


Figure 51 Normalized change in generalized travel costs - capacity reduction curve (0-100%) for all links in the Amsterdam urban rail bound PT network

The red line on the upper left represents the situation for which the network performs better when a mitigation measure is applied than in the base case scenario. The black curve in the lower right differs greatly from the other curves, for in this curve the delete lines scenario (for 100 % capacity reduction) gives the lowest network performance indicator value. As a consequence results in negative relative network performance values for the 10% to 90% capacity reductions.

For a further quantitative analysis the Degrading Rapidity indicator (see Equation 24 in Section 4.6.2) is propoped. The application of this indicator to the results of the Amsterdam urban rail bound PT network is presented in a histogram in Figure 52. The histogram shows an exponential like distribution of the Degrading Rapidity indicator. Most of the links appear to have a low score between 0.1 and 0.25, this corresponds with the higher number of convex-like shaped *Normalized change in generalized travel costs – capacity reduction curves*. The relatively high number of scores between 0.95 and 1 deviates from the general trend in the histogram. These values correspond again with the situation for which the network performs better when a mitigation measure is applied than in the base case scenario.

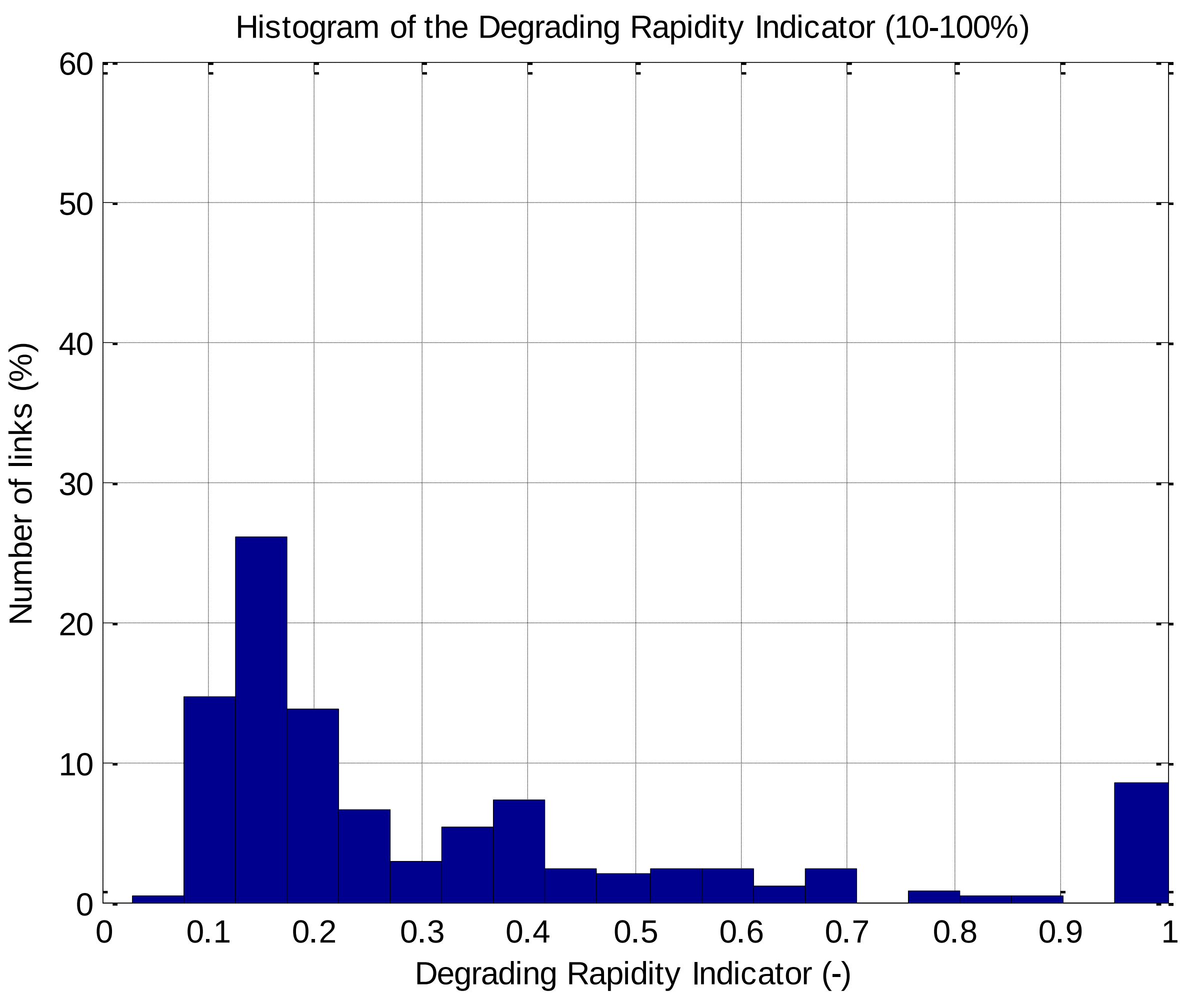


Figure 52 Histogram of the Degrading Rapidity indicator (10-100%) for all links in the Amsterdam urban rail bound PT network

High degrading rapidity means that the point at which the increase in capacity reduction no longer results in an increase in network performance is achieved at a minor capacity reduction rate. This implies that route alternatives with comparable utility are present which are not affected by the disruption on the link.

Figure 53 presents the placed in the network where the different scores on the Degrading Rapidity indicator can be found. The less rapid degrading links seem to occur on the outliers of the network. The higher scores on the Degrading Rapidity indicator are found in the centrally situated high density parts of the network with a lot of parallel alternatives. Especially parallel to line 52 (the N/Z-lijn) a high number of links with a high Degrading Rapidity are found. It seems that the N/Z-lijn offers attractive route alternatives for a lot of links parallel situated to the line.

Figure 53 Graph of the Degrading Rapidity indicator (10-100%) for the links in the Amsterdam urban rail bound PT network (Thicker and more saturated green links indicate a higher score)

Also for the Degrading Rapidity indicator the partial capacity reductions are separately examined. In Figure 54 the *Normalized change in generalized travel costs – capacity reduction curve* for the partial capacity reduction scenarios is given. Again, the vast majority of links show a convex-like shape (i.e. the degrading rapidness to full link breakdown network performance is lower). The curves are less convex than the curves for the total set of scenarios because of the lower generalized travel costs of the 90% capacity reduction scenario in comparison to full link breakdown (i.e. 100% capacity reduction scenario).

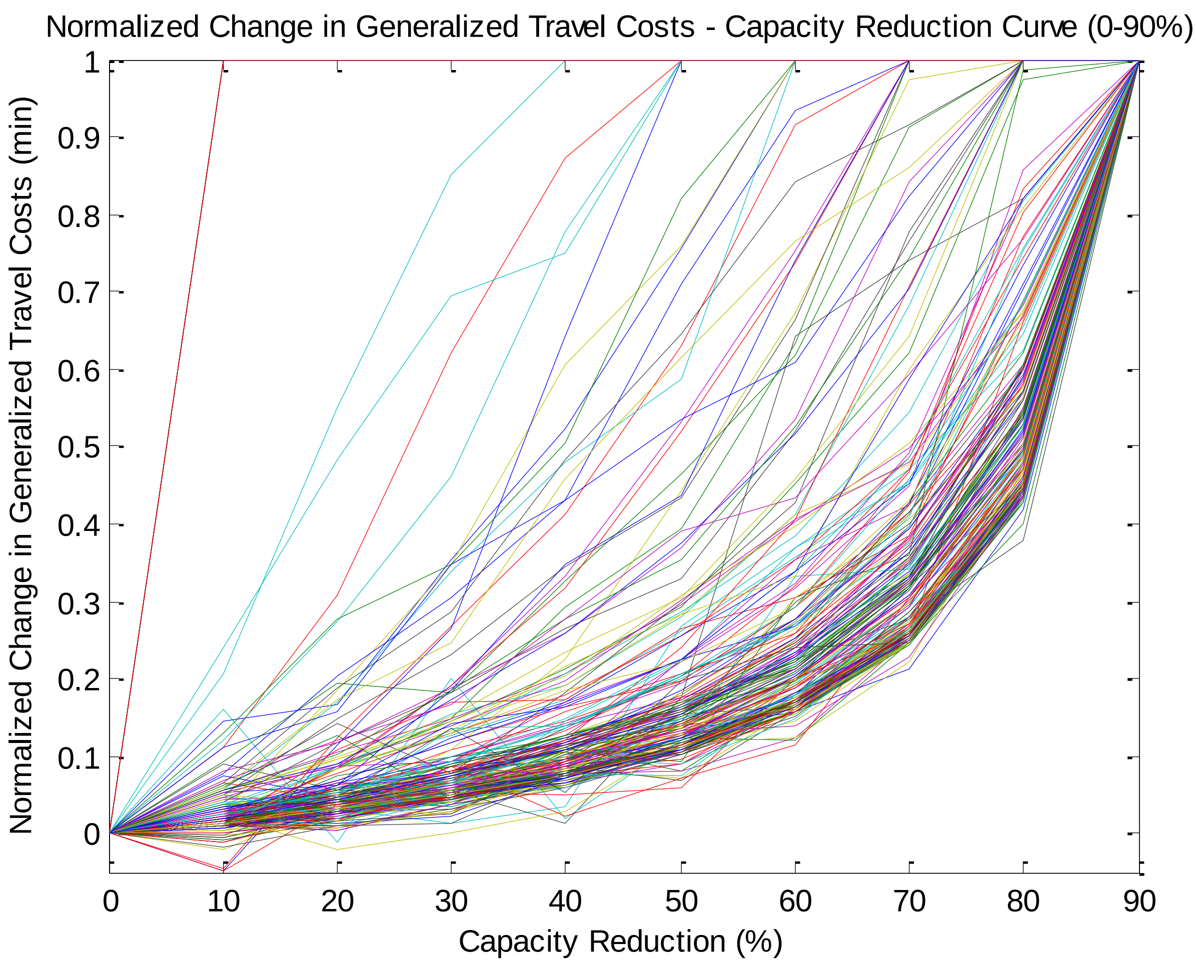


Figure 54 Normalized change in generalized travel costs - capacity reduction curve (0-90%) for all links in the Amsterdam urban rail bound PT network

The histogram of the Degrading Rapidity indicator for the partial capacity reductions show an even clearer exponential distribution, see Figure 55. Again, a deviating peak is visible between 0.95 and 1, clearly the 21 disruption non-sensitive links. Most striking is the clear cut-off of degrading rapidity indicator values lower than 0.2. The minimum degrading rapidity to the network performance corresponding with the 90% capacity reduction is clearly higher than the minimum degrading rapidity to the network performance corresponding with a full link breakdown. The mean degrading rapidity, however, is lower for the partial capacity reductions only than for the full set of scenarios.

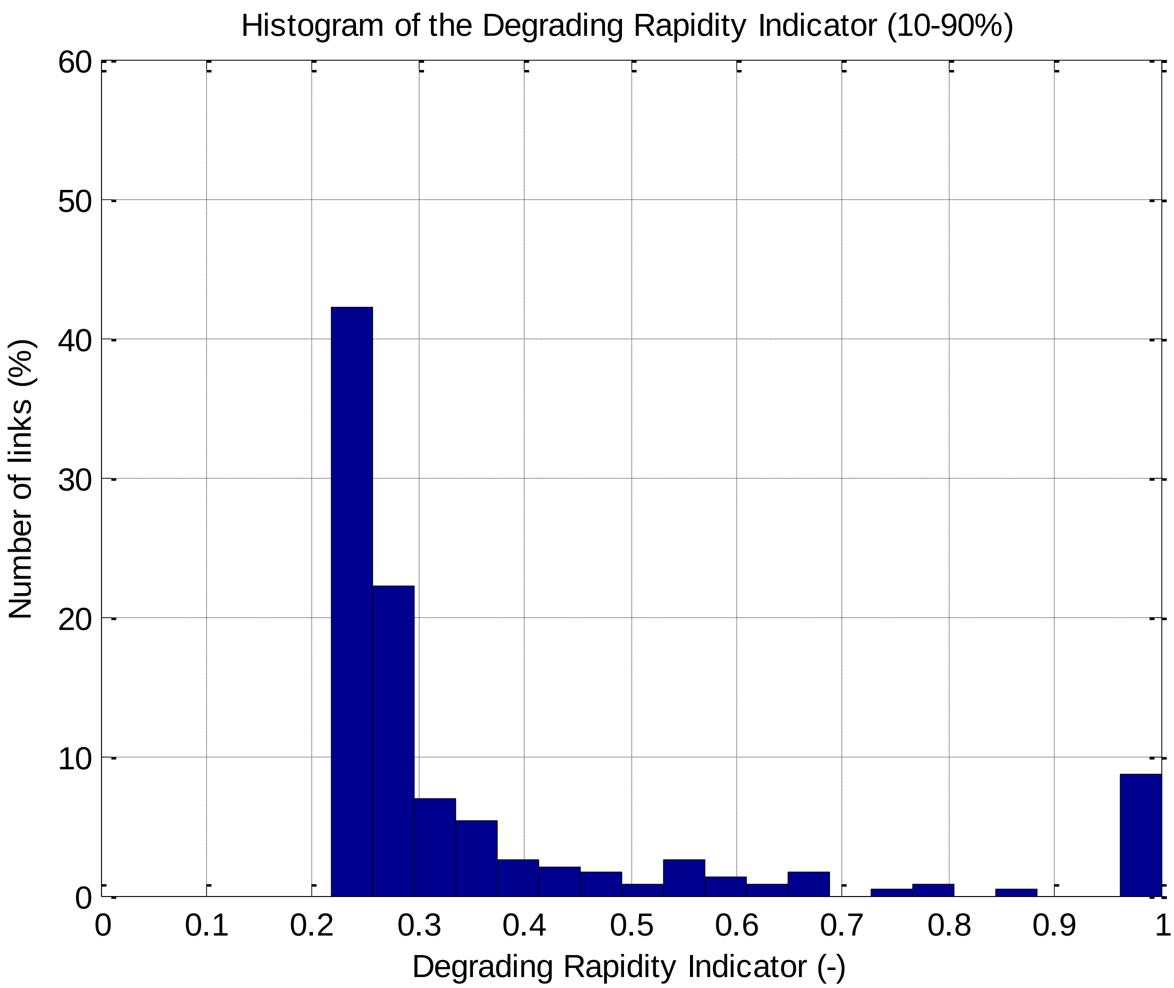


Figure 55 Histogram of the Degrading Rapidity indicator (10-90%) for all links in the Amsterdam urban rail bound PT network

When the degrading rapidity values are presented in the Amsterdam urban rail bound PT network graph, no clear differences in location are found in relation to the graph of the Degrading Rapidity values for the total set of scenarios. A very low degrading rapidity indicator means that the network performance at the highest capacity reduction situation is significantly larger than all other partial capacity reduction situations. A very high degrading rapidity indicator means that the network performance for all (partial) capacity reductions is comparable. One can imagine that both situation are undesirable. In the first case, a disruption on a certain link will not be mitigated, not even for the major capacity reductions and hence the link is, depending on the number of traversing travellers as well, very critical. In the second case the necessity of the presence of the link can be doubted, as this means that travellers are already choosing another route alternative at very minor capacity reductions on the link. This is the case for the 21 links with a high degrading rapidity value in the Amsterdam urban rail bound PT network. For partial capacity reduction expressed in reduction in speed, a degrading rapidity value of 0.23 is the base situation, indicating that the link is not robust at all. Every score higher than that is more robust. By comparing the Degrading Rapidity indicator histograms of 10-90% to 10-100% capacity reductions, it can be seen that the network performance is disproportionally affected by full link breakdown in relation to the partial capacity reductions.

Figure 56 Graph of the Degrading Rapidity indicator (10-90%) for the links in the Amsterdam urban rail bound PT network (Thicker and more saturated green links indicate a higher score)

7.3. Results of multiple linear regression

Multiple linear regression (MLR) models are estimated for both the full set of scenarios of capacity reduction as well as for the scenarios with partial capacity reduction only (10-90%). For both types the regression model is executed on an initial links set and a filtered link set, see Table 31. In general the initial link set consists of all links and the filtered link set consists of all links with the exception of the links containing negative network performance values for all capacity reduction situations (i.e. the links for which the network performs better in disruption situations). In both sets the link which has negative normalized network performance values for 10 to 90% capacity reductions (i.e. the link for which the network performs better than base case when deleting the traversing line) is left out.

Table 31 Overview of MLR estimations and corresponding links in set.

		Link Criticality	Degrading Rapidity
Initial link set	Full set of scenarios 10-100%	N=246	N=245
	Partial capacity reductions 10-90%	N=246	N=246
Filtered link set	Full set of scenarios 10-100%	N=225	N=224
	Partial capacity reductions 10-90%	N=225	N=225

7.3.1. Attribute values of links

The calculated values of the operationalized attributes for all links in the Amsterdam urban rail bound PT network are presented in Table 38 and Table 39 in Appendix E. Visualizations of these values are presented in the figures below to provide insight in the distribution of these values on the links of the network.

The attribute values are presented in the same order which has been used in the operationalization section of Chapter 5: first the passenger demand attributes, subsequently the line attributes and finishing with the link attributes.

Passenger demand attributes

The only passenger demand attribute is the total bi-directional flow on the link, this is presented in Figure 57. The metro links are the more intensively used links in the network, likewise the more centrally situated links are more intensively used. Most outlier links of the network have a low passenger demand, except for the branch to the North.

Figure 57 Total bi-directional flow on the links as the passenger demand on the link (thicker and more colour saturated links have a higher value)

Line attributes

The line attributes are: total frequency of traversing lines and the P-space link degree centrality indicator. Both attributes are expected indicators for service intensity on the link, see Figure 58. The service intensity expressed in total frequency of traversing lines shows that the intensity is highest at the lines towards central station and the East metro line. The P-space link degree centrality indicator is a measure for the direct connectivity to other stops, so if more lines traverse the link, it is connected to more stops and the P-space link degree centrality indicator value is higher. It is clearly higher around hub-like metro stations and the high density area of the tram network in the centre of the city.

Figure 58 Two attributes representing the service intensity on the link: Left the total frequency of traversing lines, right the P-space link degree centrality indicator for all links (thicker and more colour saturated links have a higher value)

Link attributes

The link attributes are link centrality, density of link area, saturation of link area and operational link characteristics. The link attributes values of the links are presented in the same order.

Link centrality is expressed in the two topological indicators: L-space link betweenness centrality and L-space link closeness centrality. The first indicates the relative number of shortest paths that traverse the link, the second indicates the closeness of the link to all other nodes. The betweenness centrality highlights the centrality of 'De Binnenring' of the tram network and the Jan Evertsenstraat in the North West of the network. The closeness centrality indicators emphasizes the centrality of 'De Binnenring' and the N/Z-lijn.

Figure 59 Two attributes representing link centrality: left the L-space link Betweenness centrality, right the L-space link closeness centrality indicator (thicker and more colour saturated links have a higher value)

The density of the link area is also expressed in two topological indicators: L-space local clustering coefficient and the degree centrality indicator. The first indicator quantifies how close the area of the link is to being a complete graph (i.e. a fully connected graph). The second indicators quantifies the number of feeding links to the link under examination. The L-space local clustering coefficient explicitly shows separated local high density areas of the network. In the North-West near Station Amsterdam Sloterdijk, in the centre of the tram network at the West of 'De Binnenring' and in the South near Station Amsterdam Zuid. The degree centrality indicator highlights the hub-like working of the North-South line and the high density of the inner city Tram Network.

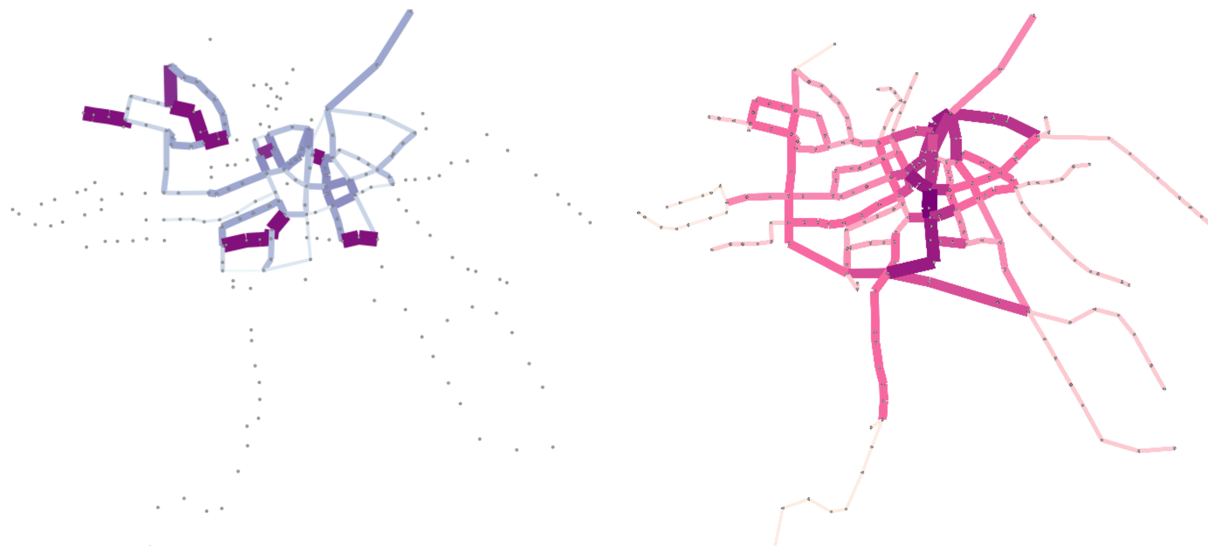


Figure 60 Two attributes representing density of link area: left the L-space local clustering coefficient, right the L-space link degree centrality indicator (thicker and more colour saturated links have a higher value)

The saturation of link area expressed in the service frequency saturation of feeding links clearly indicates the saturation of the area around Amsterdam Centraal Station but also of the isolated branches which are connected to some higher saturated links.

Figure 61 Saturation of link area as the sum of service frequency saturation of feeding links (thicker and more colour saturated links have a higher value)

The mean speed on the link clearly indicates the higher speeds on the metro network and the IJ-tram the branch to the east from Amsterdam Centraal Station. The lower mean speeds on the central tram links are easily distinguishable.

Figure 62 Operational link characteristic as the mean speed on the link (thicker and more colour saturated links have a higher value)

Correlation of attributes

Some of the chosen attributes are likely to correlate with each other, especially the indicators used as an expression for the same attribute (e.g. closeness and betweenness centrality for the link's centrality). In Table 32 an overview of the correlation coefficients is given. Strong to moderate positive relationships are found between the topological centrality indicators (i.e. closeness, betweenness and degree centrality in both L-space and P-space). A weak to moderate positive relationships is found between the total bi-directional flow on the link, the mean speed on the link and the total frequency of traversing lines.

Table 32 Correlation coefficients for the attributes

	L-space Link Betweenness Centrality Indicator	*L-space link Closeness Centrality Indicator*	*L-space Local clustering coefficient*	*L-space link degree centrality indicator*	*Sum of service frequency saturation of feeding links*	*Mean speed on link*	*Total frequency of traversing lines*	*P-space link degree centrality indicator*	*Total bi-directional flow on link*
L-space Link Betweenness Centrality Indicator	1,000								
L-space link Closeness Centrality Indicator	0,497	1,000							
L-space Local clustering coefficient	0,065	0,227	1,000						
L-space link degree centrality indicator	0,355	0,710	0,212	1,000					
Sum of service frequency saturation of feeding links	-0,008	0,213	-0,166	-0,027	1,000				
Mean speed on link	-0,108	-0,036	-0,298	-0,119	0,004	1,000			
Total frequency of traversing lines	0,576	0,409	-0,044	0,288	0,211	0,117	1,000		
P-space link degree centrality indicator	0,535	0,778	0,205	0,545	0,195	-0,138	0,608	1,000	
Total bi-directional flow on link	0,404	0,566	-0,043	0,475	0,088	0,481	0,476	0,398	1,000

7.3.2. Link Criticality parameter estimation results

In Table 33 the results of the multiple linear regression are shown for four estimated models for the Link Criticality parameter, however only significant results are shown. The models are estimated on the full set of disruptions scenarios (i.e. from 10 to 100%) for both the initial link set (all links) and the filtered set (disruption sensitive links) and estimated on the partial capacity reduction scenarios (i.e. from 10 to 90%) for both the initial link set and the filtered set. The results are discussed on the basis of these four estimated models.

Table 33 Link Criticality parameter estimation results, shown is the regression coefficients and between brackets the t-stat for the significant variables

	Total		Partial	
	Initial	Filtered	Initial	Filtered
Adjusted R square:	0.565	0.678	0.654	0.891
Intercept	65685.903 (1.362)	184392.184 (2.475)	39332.930 (1.677)	-94003.797 (-4.123)
L-space Link Betweenness Centrality Indicator	-	-	-465563.700 (-2.449)	-
L-space link Closeness Centrality Indicator	-	-	-	463483753.360 (2.542)
L-space Local clustering coefficient	-321206.622 (-2.506)	-342191.550 (-3.043)	-	-
L-space link degree centrality indicator	-	-22773.962 (-2.946)	-	-
Sum of service frequency saturation of feeding links	-	-376869.856 (-2.811)	-	-
Mean speed on link	6352.227 (3.665)	3964.359 (2.668)	2780.042 (3.249)	3054.993 (7.716)
Total frequency of traversing lines	-	9904.922 (4.064)	-4949.655 (-3.902)	-
P-space link degree centrality indicator	-2222.097 (-6.410)	-1098.181 (-2.803)	-706.499 (-3.597)	-458.754 (-3.584)
Total bi-directional flow on link	38.452 (11.167)	38.927 (11.318)	27.144 (15.191)	21.526 (23.277)

Total Initial

In the estimated model on the total initial set, four attributes are significant on a 95% confidence interval. L-space local clustering coefficient (i.e. density of link area) and P-space link degree centrality indicator (i.e. service intensity on the link) have a negative regression coefficient, implying a negative relationship between these attributes and Link Criticality. In the first case this means that a local higher network density implies a lower Link Criticality and in the second case a higher service intensity implies a higher connectivity to other nodes which results in a lower Link Criticality. The mean speed on the link and total bi-directional flow on the link have a positive regression coefficient, implying a positive relationships between these attributes and Link Criticality. In the first case this implies that a higher mean speed on a link results in a higher Link Criticality and in the second case a higher total bi-directional flow on the link implies a higher Link Criticality.

Total filtered

In the estimated model on the total filtered set, seven attributes are significant on a 95% confidence interval. L-space Local Clustering Coefficient, L-space Link Degree Centrality Indicator, Sum of Service Frequency Saturation of Feeding links and P-space Link Degree Centrality Indicator have a negative regression coefficient, implying a negative relationship between these attributes and Link Criticality. The implications for L-space Local Clustering Coefficient and P-Space Link Degree Centrality indicator is already explained. For the L-space Link Degree Centrality indicator a negative regression coefficient implies that a link with more feeding links will show lower Link Criticality. The negative regression coefficient for the sum of service frequency saturation of feeding links means that the more saturated the link area is the lower the Link Criticality. The mean speed on the link, the total frequency of traversing lines and the total bi-directional flow on the link have positive regression coefficients. The implications for these regression coefficients are already explained for the mean speed on the link and total bi-directional flow on the link. The positive regression coefficient for the total frequency of traversing lines implies that the higher the total frequency of traversing lines the lower the Link Criticality.

Partial Initial

In the estimated model on the partial initial set five attributes are significant on a 95% confidence interval. L-space Link Betweenness Centrality indicator, Total Frequency of traversing lines and P-space Link Degree Centrality indicator have a negative regression coefficient. For the P-space Link Degree Centrality indicators the implications are already explained. For the L-space Link Betweenness centrality a negative regression coefficient implies that if a larger amount of shortest paths traverses the link the Link Criticality is lower. For the total frequency of traversing lines this implies that if the total frequency of traversing lines is higher the Link Criticality is lower. The mean speed on the link and the total bi-directional flow on the link have positive regression coefficients. The implications for the positive regression coefficients for both attributes are already explained.

Partial Filtered

Four parameters are significant in the estimated model on the partial filtered set. P-space Link Degree Centrality Indicator has a negative regression coefficient, the implications of these results are explained before. The other three attributes have a positive regression coefficient. For the mean speed on the Link and Total bi-directional flow on the link the implications are already explained. The positive regression coefficient for the L-space Link Closeness centrality Indicator means that the closer the link is to all other nodes, the higher the Link Criticality.

The above table answers the questions which determinants are of significant influence on the Link Criticality and whether this influence is positive or negative. For the extent of influence one has to incorporate the effect of the value size of the different determinants. An overview of calculated effect sizes can be found in Appendix F and it is visualized in the following figures (see Figure 63 and Figure 64).

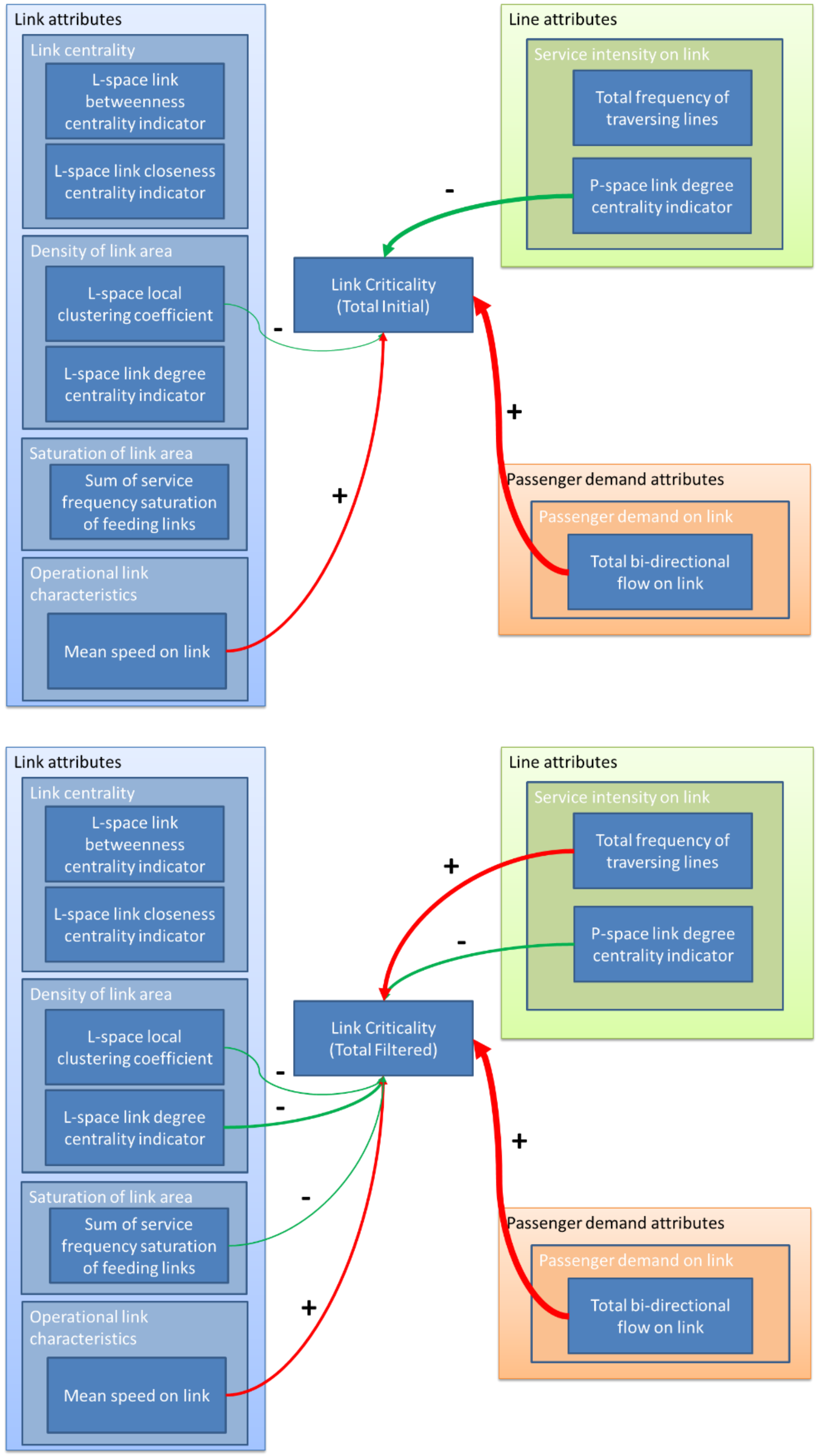


Figure 63 Overview of effect sizes of the significant determinants in the two estimated models for the total set of disruption scenarios for Link Criticality.

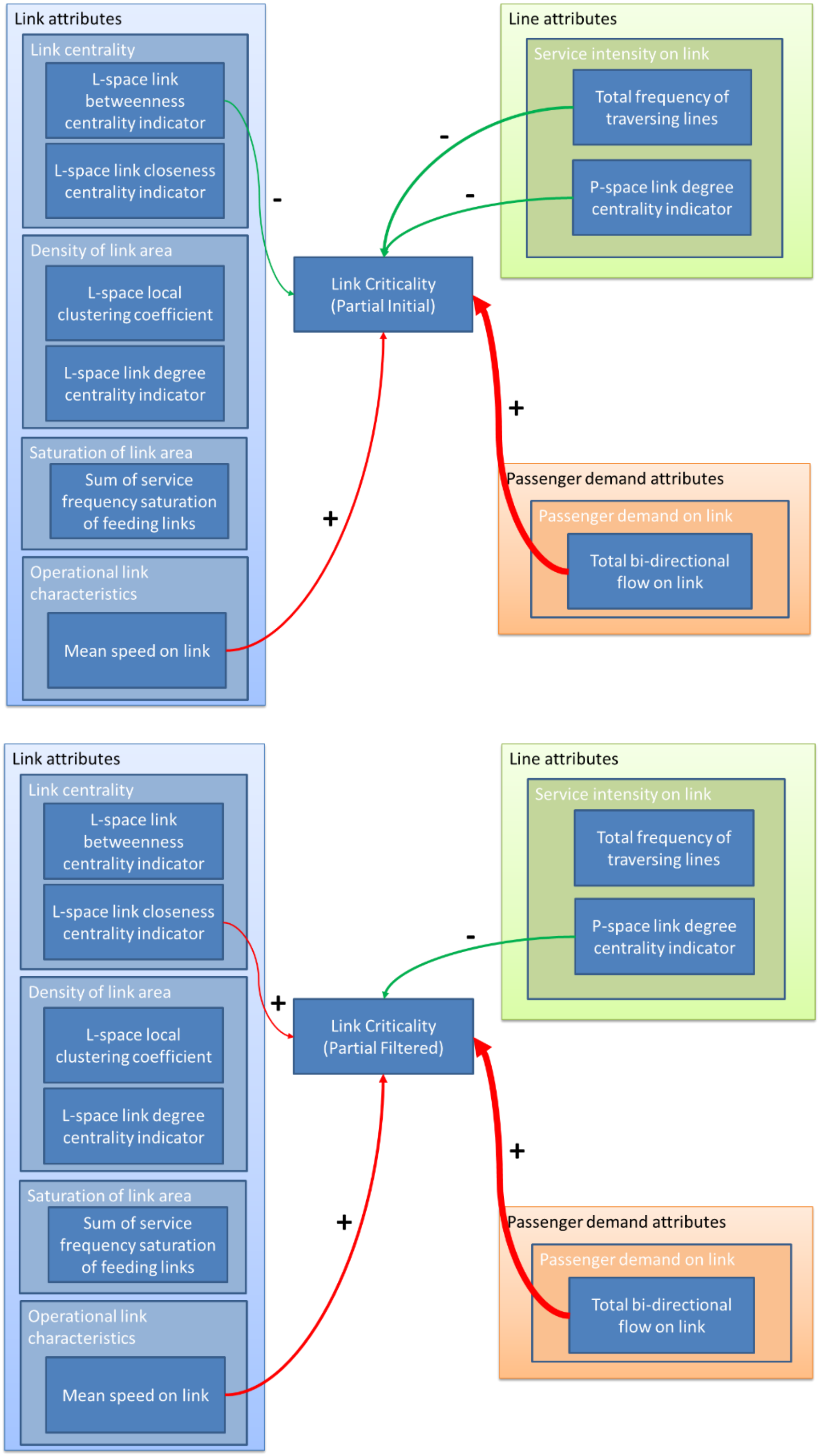


Figure 64 Overview of effect sizes of the significant determinants in the two estimated models for partial capacity reduction situations on Link Criticality

In all estimated models the effect size of the total bi-directional flow on the link is significantly bigger than the effect sizes of the other determinants. Other large effect sizes can be seen for the P-space link degree centrality indicator and the mean speed on the link. In the two models where the total frequency of traversing lines is a significant determinant this has one of the larger effect sizes. The determinants of *link centrality*, *density of link area* and *saturation of link area* have, in general, the smallest effect sizes.

General determinants

In all the four estimated models the total bi-directional flow on the link and the mean speed on the link are positive determinants on Link Criticality. Likewise is the P-space link degree centrality a negative determinant on Link Criticality in all four. These three determinants are also the three determinants with the largest effect size. Links with a lot of traversing passenger and a higher mean speed on the link seem to have a higher Link Criticality than the links with a small number of traversing passengers and a lower mean speed. An explanation for the first relationship can be found in the fact that more passengers are affected when the passenger demand is higher. Because the mean speed on the link is higher the absolute increase in travel time is higher and hence the affected travellers have a higher increase in travel costs.

An explanation for the negative relationship between P-space link degree centrality, as a determinant for service intensity on the link, and Link Criticality can be found in the correlation of this attribute with other attributes. P-space link degree centrality is highly correlated with the attributes for Link Centrality (i.e. L-space link betweenness and closeness centrality) and density of link area (i.e. L-space link degree centrality). This could indicate that the highest service intensity is found in the most dense and central areas of the network and hence travellers have a lot of alternatives in case of disruptions. As a consequence a disruption on a link with a high P-space link degree centrality is less critical than a disruption on a link with a low P-space link degree centrality.

Difference in determinants between the total set of disruption scenarios and partial capacity reductions

The attributes of density of link area are only significant determinants for the total set of scenarios. The negative relationships indicates that links situated in a more dense part of the network have a lower criticality. This can be explained through the fact that in these parts of the network there are more alternative mitigation measures and hence the criticality of these links is lower. The fact that the attributes of Density of Link Area are not significant for the partial capacity reductions only explains the value of the Density of the Link Area, its value is more in the mitigation measure options and less in the travellers rerouting options.

The attributes for Link Centrality are only significant determinants for the partial capacity reduction scenarios. The direction of the relationship (i.e. positive or negative) and which attributes are significant for which model estimation differs. For the total set of links (i.e. the set including the links that are not sensitive for disruptions) the betweenness centrality indicator is a negative determinant. This indicates that links that facilitate relatively more shortest paths have in general a lower Link Criticality. This seems quite contradictory, if more shortest paths traverse a link one can expect that more travellers are affected and hence the Link Criticality is higher. The number of traversing shortest paths is apparently not a one to one indicator on passenger demand. More research is needed to explain this found negative relationship.

For the filtered set (i.e. only the links that are disruption sensitive) the L-space link closeness centrality indicator is a positive determinant. This indicates that links containing a lower total distance to all other nodes (i.e. more centrally situated) have a higher criticality than links with a higher distance to all other nodes. A part of the explanation can be found in the moderately positive linear relationships between this attribute and the Total bi-directional flow on the link. That is to say, more centrally situated links are traversed by more passengers and hence, during disruption, more passengers are affected.

Difference in determinants between initial set and filtered set

The two models on the filtered set (i.e. the set without the disruption insensitive) have more significant determinants than the initial set (i.e. the set with all links). The disruption insensitive links are apparently somewhat disrupting on the analysis of the relationship between the attributes and the Link Criticality. This can also be seen in the model fits of the models, the models on the filtered set have a significantly higher model fit than the models on the initial set.

Special cases

The total frequency of traversing lines is a determinant in two of the four model estimations. Further research is needed to determine why the relationships differs between the Total Frequency of Traversing Lines and Link Criticality for the model on Total Filtered and Partial Initial.

7.3.3. Degrading Rapidity parameter estimation results

In Table 34 the results of the multiple linear regression are shown for four estimated models for the Degrading Rapidity parameters. The models show results for the full set of disruptions scenario (i.e. from 10 to 100%) for the initial link set (all links) and the filtered set (disruption sensitive links) and the partial capacity reduction scenarios (i.e. from 10 to 90%) for the initial link set and the filtered set. The estimated models are comparable for initial and filtered set. Therefore, the results are discussed on the basis of the total set and the partial set, initial and filtered set combined.

Table 34 Degrading Rapidity parameter estimation results, shown is the regression coefficients and between brackets the t-stat

	Total		Partial	
	Initial	Filtered	Initial	Filtered
Adjusted R square:	0,516	0,439	0,446	0,333
Intercept	-0,607 (-9,156)	-0,288 (-5,758)	-0,347 (-5,705)	-0,006 (-0,153)
L-space Link Betweennesss Centrality Indicator	0,891 (2,434)	0,937 (3,552)	0,762 (2,274)	0,664 (3,246)
L-space link Closeness Centrality Indicator	5818,467 (10,874)	3617,968 (8,637)	3974,432 (6,881)	1777,131 (4,802)
L-space Local clustering coefficient	0,276 (2,251)	0,314 (3,533)	-	0,184 (2,633)
L-space link degree centrality indicator	-	-	0,018 (2,222)	0,011 (2,120)
Sum of service frequency saturation of feeding links	-	-	-	-
Mean speed on link	-	-	-	-
Total frequency of traversing lines	0,008 (3,505)	-	0,008 (3,996)	-
P-space link degree centrality indicator	-	-	-	-
Total bi-directional flow on link	-1,948E-05 (-6,090)	-7,650E-06 (-3,385)	-2,301E-05 (-7,905)	-9,466E-06 (-5,354)

Total set

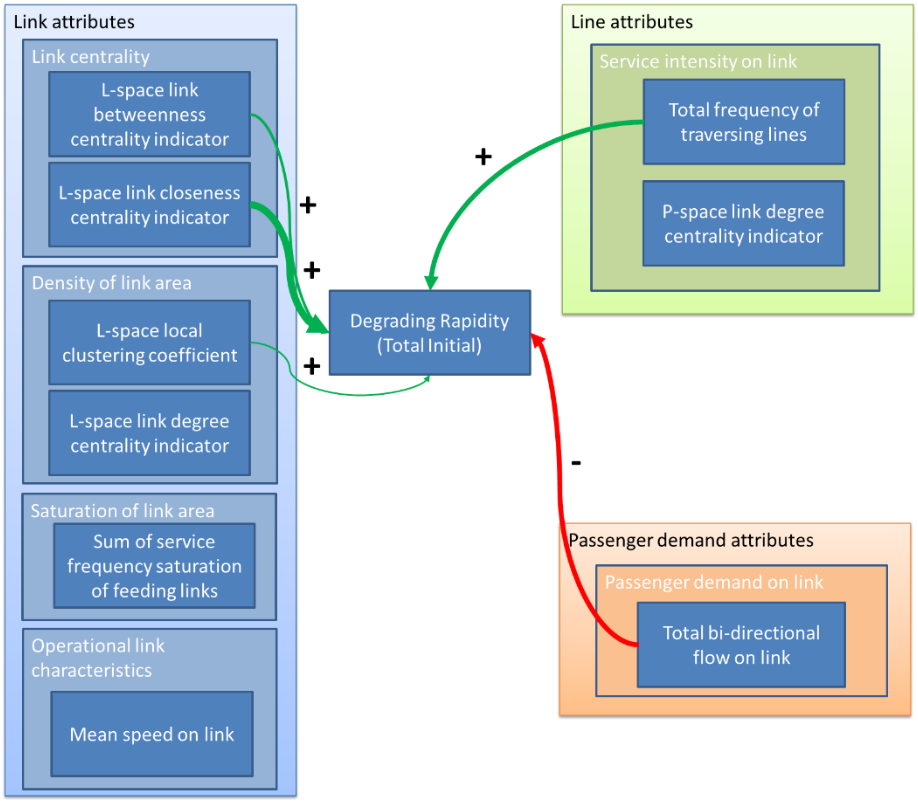
In the model estimation on the total initial set, five attributes are significant on a 95% confidence interval, in the model estimation on the total filtered set, four attributes are. In the significant parameters the total frequency of traversing lines is missing for the total filtered set. The total bi-directional flow on the link is the only parameter with a negative regression coefficient. This implies that if the total bi-direction flow on a link is higher, the Degrading Rapidity is lower. The L-space link betweenness centrality indicator, the L-space link closeness centrality Indicator, the L-space local clustering coefficient and the total frequency of traversing lines have positive regression coefficients. In the first case this implies that if more shortest paths traverse a link the Degrading Rapidity of this link is higher. In the second case this implies that links which are in general closer to all other nodes in the network have a higher Degrading Rapidity. The third case implies that links which are situated in a locally higher density area have a higher Degrading Rapidity. The last case implies that links with a high total frequency of traversing lines show a higher Degrading Rapidity.

Partial set

In the model estimations on both partial sets also five significant attributes are found on a 95% confidence interval. For the initial partial set these are the same as for the total initial set except L-space local clustering coefficient is missing and L-space link degree centrality indicator is added. The positive regression coefficient for L-space link degree centrality indicator implies that links containing more feeding links have a higher degrading rapidity than links with less feeding links. The implications of the other regression coefficients can be found under total set. The five significant attributes on the partial filtered set are the same as for the total filtered set except that L-space link degree centrality indicator is added. Implications of the regression coefficients can be found in the previous explanations.

The above table answers the questions which determinants are significant influence on the Degrading Rapidity and whether this influence is positive or negative. For the extent of influence one has to incorporate the effect of the value size of the different determinants. An overview of calculated effect sizes can be found in Appendix F and is visualized in the following figures (see Figure 65 and Figure 66).

In all estimated models for Degrading Rapidity, the L-space link closeness centrality has the biggest effect size, 3 to 7 times larger than the smallest effect size. This is followed by the effect size of total bi-directional flow on the link, 2 to 4 times as large as the smallest effect size. The smallest effect sizes are seen at the determinants for density of link area and L-space link betweenness centrality.



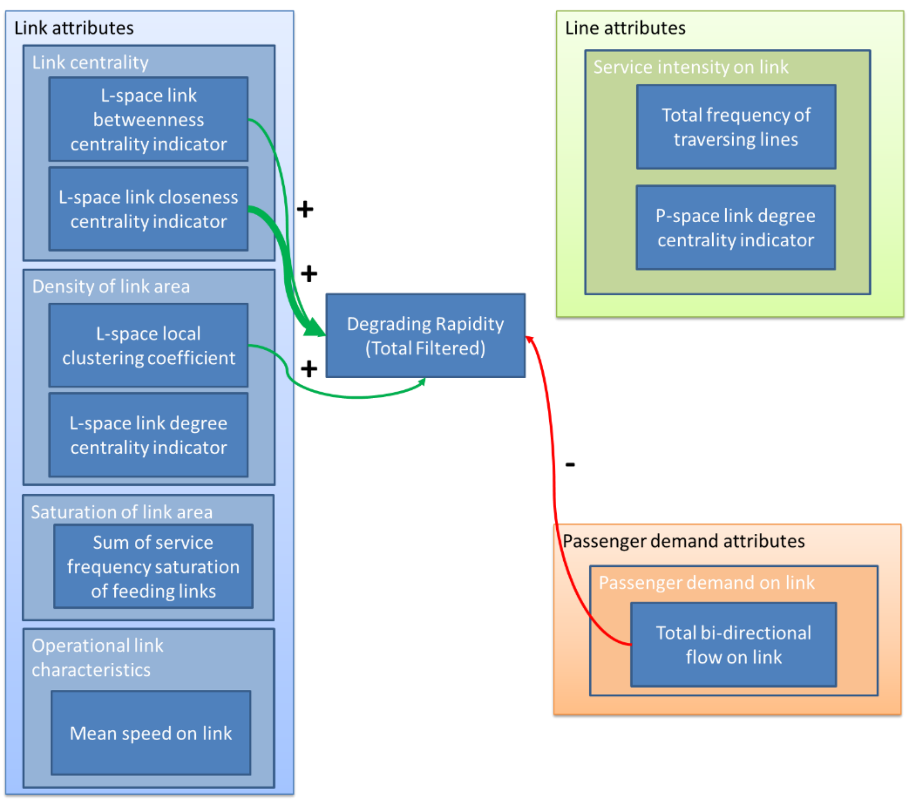
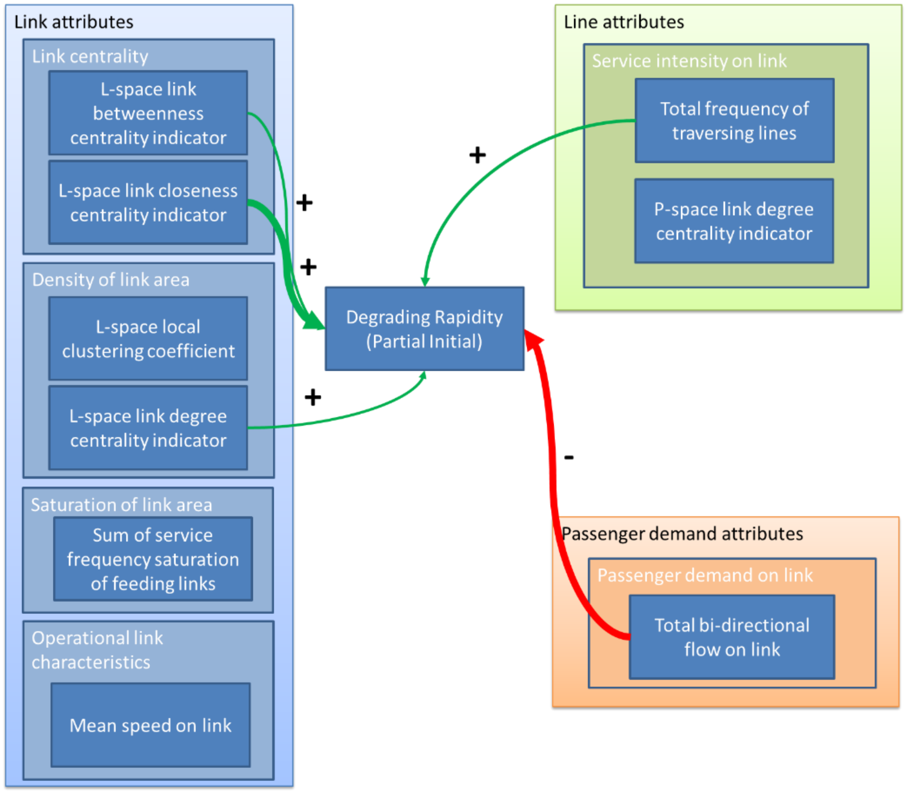


Figure 65 Overview of effect sizes of the significant determinants for the two estimated models of the total set of scenarios on Degrading Rapidity



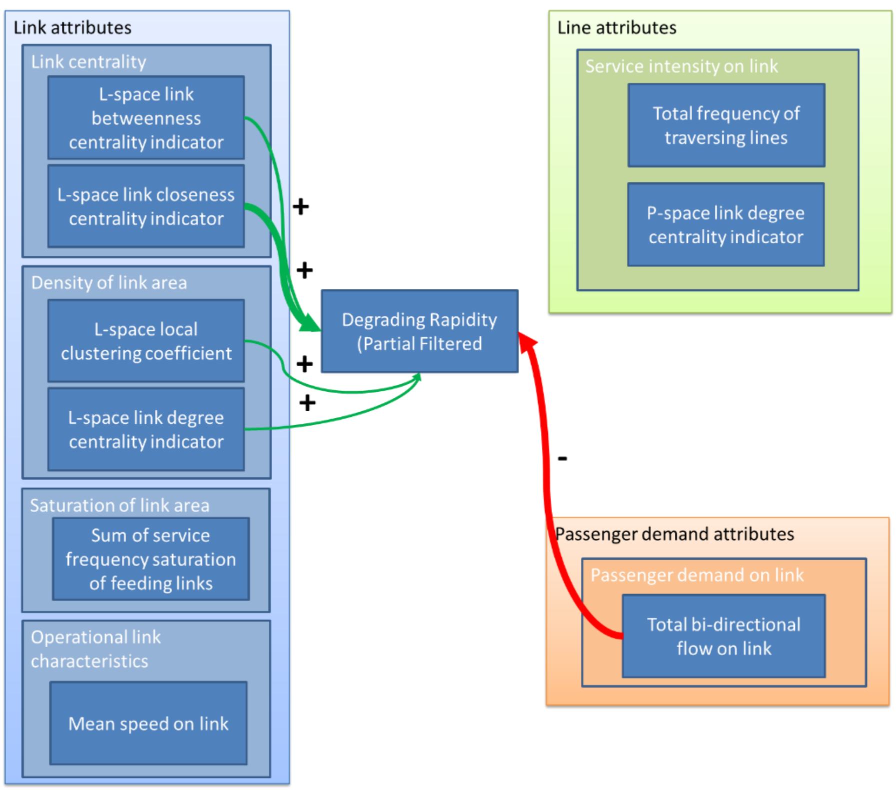


Figure 66 Overview of effect sizes of the significant determinants for the two estimated models of the partial capacity reduction situations on Degrading Rapidity

General determinants:

In all four estimated models the L-space link betweenness and L-space link closeness centrality indicators are positive determinants. Likewise the total bi-directional flow traversing the link is a negative determinant in all estimated models. In contrast with the models estimated on the Link Criticality, the P-space link degree centrality indicator, the mean speed on the link and the sum of service frequency saturation are no significant determinants for Degrading Rapidity.

The positive relationships between the Link Centrality attributes (i.e. the L-space link betweenness centrality indicator and the L-space link closeness centrality indicator) and Degrading Rapidity means that a more centrally situated link has a higher Degrading Rapidity. This can be explained by the fact that in more centrally situated areas, more route alternatives are present for the passenger but also for the PT lines. The threshold that no extra passengers are affected is reached earlier because of the more alternative route options.

The negative relationships between total bi-directional flow and Degrading Rapidity means that links that have more traversing passengers have a lower Degrading Rapidity (i.e. no or less levelling off of the network performance - capacity reduction curve). By taking a closer look at the passenger demand distribution, it can be seen that the highest intensity of the passenger demand is on the metro lines and on tram links where rerouting is also not possible (e.g. the tram links crossing the metro line at Lelylaan). It looks like the passenger demand is highest on links where rerouting is not possible (i.e. all metro links and isolated tram links) and this has a negative influence on Degrading Rapidity. Further research should determine whether this hypothesis holds.

Difference in determinants between the total set of disruption scenarios and partial capacity reductions

In general the Degrading Rapidity determinants are the same for partial capacity reductions and the total set of scenario. The L-space link degree centrality indicator is only a positive determinant for the partial capacity reductions situations. The effect size of the passenger demand seems to be a little larger for partial capacity reductions where the effect size of closeness centrality is larger for the total set of scenarios.

The positive relationship between L-space link degree centrality indicator and Degrading Rapidity for partial capacity reductions situations implies that links with more feeding links have a higher Degrading Rapidity. As this determinant is an attribute for network density this can be explained through the fact that more rerouting options are available and hence the network performance – capacity reduction curve levels off faster. Further research is needed to determine why this attribute is not significant for the total set of scenarios. The same applies to the difference in effect sizes of passenger demand and closeness centrality.

Difference in determinants between initial set and filtered set

The total frequency of traversing lines is only a positive determinant for the two models of the initial set. The positive relationship means that links with a higher total frequency of traversing lines have a higher Degrading Rapidity. The fact that this attribute is only significant for the initial set implies that the links that are not sensitive to disruptions have in general a higher total frequency of traversing lines. The positive relationship can be explained by the fact that the frequency of traversing lines is in general higher in more central and dense areas of the network and hence more rerouting options are available.

Special Cases

L-space local clustering coefficient is a positive determinant in three of the four models. The positive relationship means that links with a higher L-space local clustering coefficient have in general a higher Degrading Rapidity. The same explanation as is presented for the L-space link degree centrality indicator is valid here.

8. Conclusions and recommendations

In this chapter conclusions are derived from the research (Section 8.1). Second the practical implications of the results and conclusions are presented (Section 8.2). The chapter ends with recommendations for further research (Section 8.3.1) and improvement of the developed model (Section 8.3.2).

8.1. Conclusions

This research aims to explain how urban rail bound PT networks are affected by capacity reductions on a link of the network, in other words the robustness of a network. The formulated main research question in this research is the following:

How do attributes of links in a public transport network influence the relationship between a capacity reduction on that link and the performance of the public transport network as a whole?

The topic is researched trough literature review, interviews with employees of the SRA (i.e. the transport authority) and GVB (i.e. the public transport operator) and by modelling the Amsterdam urban rail bound public transport network as case study. In the conclusions the developed model is first discussed and thereafter the general conclusions on robustness of public transportation networks are presented.

8.1.1. Public transport assignment model

For this study an assignment model is developed which is able to calculate the performance of a given network with a given passenger demand. The model is able to change the given network as a consequence of a local capacity reduction. The development of the model creates the opportunity to extend the research on PT networks by, among other, a full-scan analysis on planned capacity reductions on the link level. The model is able to simulate full link breakdowns as well as partial capacity reduction with which it is able to extend the little research done on the effect of partial capacity reductions.

The model also includes the implementation of mitigation measures like rerouting or cutting lines. This means that it is able to model the effect of a change in the infrastructure availability on the service network (i.e. changes in routes, changes in running times and changes in frequency). With rerouting, service frequency constraints on links are taken into account in order to prevent impossible rerouting on high saturated parts of the network. The implementation of modelling mitigation measures, even without service frequency capacity constraints, is not often seen in the studies found in literature.

As a consequence of the changes in the service network, the model is able to change the assignment of the passenger demand on the network. This includes the changes in in-vehicle time, waiting time and the number of transfers. The (dis)utility of a transfer is differentiated for different types of transfers (i.e. different for tram-tram than for metro-tram). The inclusion of passenger demand, transfers and waiting time makes it possible to express the network performance into generalized travel costs. This indicator not only includes in-vehicle time, but also disutility of transfers and waiting time experience by travellers.

8.1.2. Robustness of urban rail bound public transport networks

As a part of the research into robustness of public transport networks this study identified how capacity reductions on public transport networks occur and how they affect the network. Specifically, the focus is on the effects of planned major discrete events, in other words planned disruptions which affect infrastructure availability. These major discrete events affect the network performance through its effects on the infrastructure network, as a consequence the service network is affected and this again influences the route choice of travellers. The effect of major discrete events on the infrastructure availability is, in general, a reduction in speed of the link up to a full out of operation. The service network is primarily affected in longer running times on the link, secondary the frequency of traversing lines may need to be reduced. A public transport operator is able to mitigate the effects by, among other, rerouting or cutting the line. The changes in the service network affects the utility of the different route alternatives of travellers, consequently the travellers will experience a change in in-vehicle time, waiting time and number of transfers.

As, in the end, the travellers are affected by the disruption in the infrastructure network through the changes in the service network, a network performance indicator is searched for which is able to incorporate the changes the travellers experiences. This includes the experienced transfer disutility and the waiting time. As a result the network performance is expressed as the total generalized travel costs. This indicator is able to measure the consequences of the just presented sequence of effects, in other words to calculate the effect of a capacity reduction on a link on the performance of the public transport network as a whole.

To be able to assess network robustness, literature is reviewed to come up with robustness indicators meaningfully describing the relationship between a capacity reduction and the network performance. Meaningfully being able to express the accumulated effect of the whole range of capacity reductions or the relative sensitivity of different extends of capacity reductions. Further it needs to be able to translate to monetary terms, able to translate to network level and general comprehensible. As a result two robustness indicators are proposed: Link Criticality and Degrading Rapidity. Link Criticality indicates the accumulated effect of the whole range of capacity reductions on the network performance, Degrading Rapidity indicates the rapidness of the performance of the network degrading to full link breakdown network performance as a result of reducing the capacity on a link.

Based on literature findings and the formulated conceptual framework on link robustness a number of attributes of links are identified that potentially influence the relationships between a capacity reduction on a link and the performance of the public transport network as a whole. The attributes that are selected for the quantitative analysis are shown in Table 35. All six attributes are operationalized in one or more operational variables. Most of the operational variables are topological indicators for link centrality, density of link area or service intensity on the link. Others are derived from the supply data and the modelling results of the base case scenario.

Table 35 Attributes tested in the quantitative analysis

Attributes for the quantitative analysis:
Link centrality
Density of link area
Saturation of link area
Operational link characteristics
Service intensity on link
Passenger demand on link

The attributes are tested on the modelling results of a case study on the Amsterdam rail bound PT network 2020. Multiple linear regression is used to estimate the influence of an attribute on the robustness indicators (i.e. the relationship between the capacity reduction on a link and the network performance). In total eight models are estimated, four on Link Criticality and four on Degrading Rapidity. It is chosen to apply the model on the total set of scenarios and on the partial capacity reduction scenarios only and on the total set of links and a subset of links. The choice to apply on a subset of links is prompted by the modelling results which showed that for some links the generalized travel costs decreased for all disruption scenarios as a result of the application of a mitigation measure. Therefore in the subset of links these links, as discussed, are excluded. The key attributes of influence on the relationship between capacity reduction and network performance are based on the model estimations on the subset of links only. The estimated attributes on the total set of links incorporate the influence on sub-optimality of the network in terms of generalized travel costs. Therefore these links are excluded in the determination of the key variables. The key attributes are the attributes which are significant for both the model estimated on the total set of scenarios and the model estimated on the partial capacity reduction scenarios only. The estimated key variables of influence on the relationship between capacity reduction and network performance are discussed on the basis of the two formulated robustness indicators: Link Criticality and Degrading Rapidity.

Link Criticality

The significant influence of the operationalized variables and how strong the influence is of each significant variable is visualized in Figure 67. The total bi-directional flow is the most dominant key variable concerning Link Criticality. The higher the total bi-directional flow on the link the more critical the link. The only negative key variable on Link Criticality is the P-space link degree centrality indicator. The higher the P-space link degree centrality indicator the less critical the link. This negative relationship might be explained by the high correlation of the attributes with Link Centrality and density of link area attributes. In other words, high service intensity is found in the most dense and central areas of the network where travellers have more alternative route options. And as a consequence the link is less critical. Another positive determinant on Link Criticality is the mean speed on the link. The higher the mean speed the higher the Link Criticality.

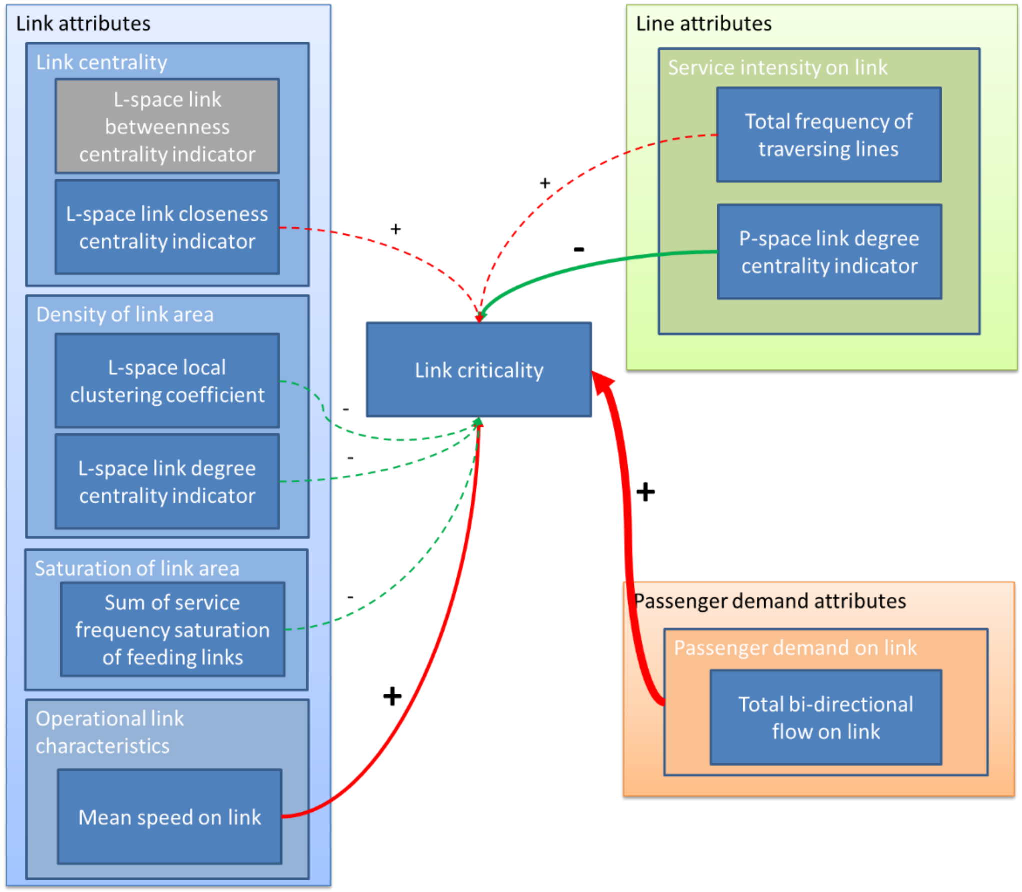


Figure 67 Key attribute (thick solid lines) and minor attributes (thin dashed lines) influencing Link Criticality

The operationalized variables of link centrality, density of link area and saturation of link area and total frequency of traversing lines are not key attributes influencing Link Criticality. This does, however, not imply they are not influencing attributes at all. Especially in the model estimation on the total set of disruption scenarios some minor influencing attributes are found. The attributes of density of link area and saturation of link area all are negative determinants for the model estimated on the total set of disruption scenarios. The higher the density of the link area the less critical the link is. Total frequency of traversing lines and L-space link closeness centrality indicator where found to be positive determinants in the model estimation on partial capacity reductions only. This means that the higher the total frequency of traversing lines and the closer the link is to all other nodes the more critical the link is. The L-space link betweenness centrality indicator was not found to be a significant attribute on Link Criticality.

Degrading Rapidity

The significant influence of the operationalized variables and how strong the influence is of each significant variable is for Degrading Rapidity visualized in Figure 68. The L-space link closeness centrality indicator is the most dominant key variable concerning Degrading Rapidity. The closer a link is to all nodes in the network (i.e. the more central the link is situated) the faster the performance of the network degrades to full link breakdown network performance. The L-space link betweenness centrality indicator and the L-space local clustering coefficient are also positive key attributes on Degrading Rapidity. A higher L-space link betweenness centrality indicator means that a higher share of shortest paths traverses this link and a higher L-space local clustering coefficient means that the surrounding nodes of the link are more connected to each other. The only negative key variable on Degrading Rapidity is the total bi-directional flow on the link. When the passenger demand on the link in the base case scenario is high the network degrades slower to full link breakdown network performance.

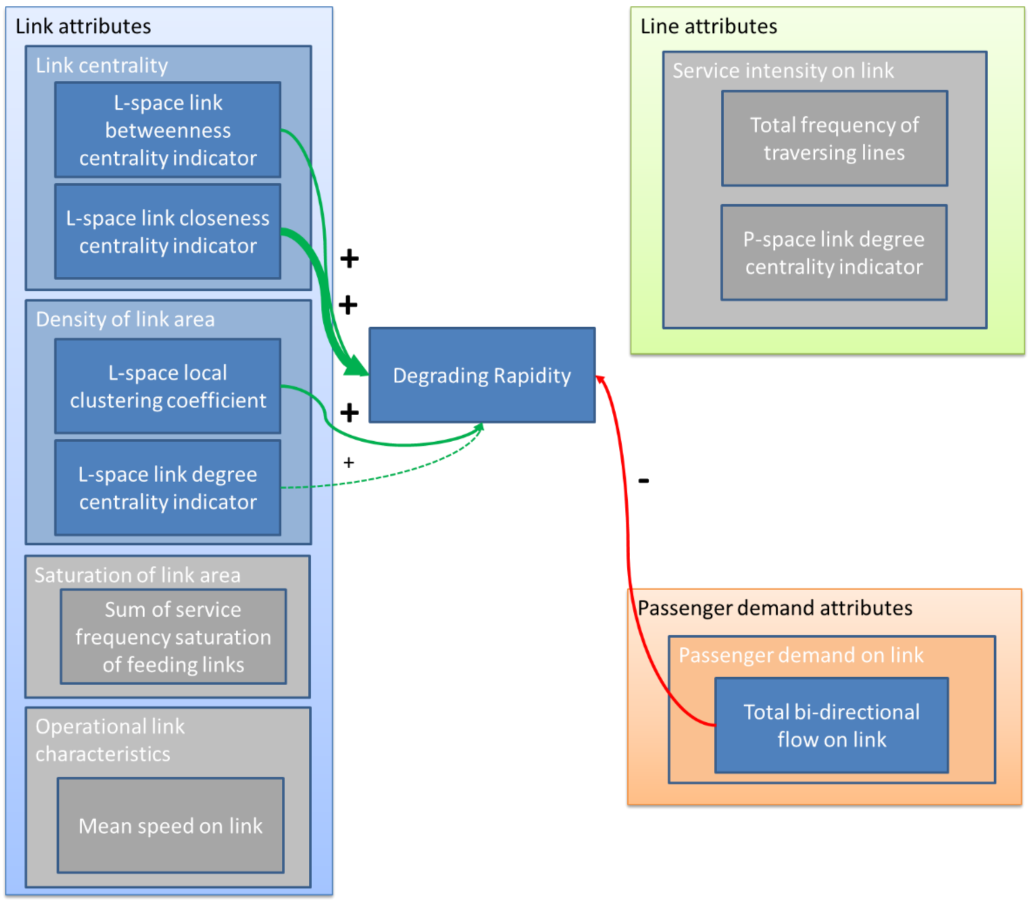


Figure 68 Key attributes (thick solid lines) and minor attribute (thin dashed line) influencing Degrading Rapidity

The L-space link degree centrality indicator is a minor attribute only significant for the partial capacity reduction situations. It implies that a link with more feeding links is degrading more rapid to full link breakdown network performance. The Degrading Rapidity is not influenced by the attribute of saturation of link area, operational link characteristics and service intensity on the link.

General conclusions

For Link Criticality a clear difference exist in significant variables for the total set of scenarios and the partial capacity reductions only. When the full link breakdown is part of the scenario set the density of link area and saturation of link area are significant. Whereas these attributes are not significant for the set with partial capacity reductions only. The mitigation effect on the full link breakdown scenario through possible mitigation measures in the higher density parts of the network appear to be significantly bigger than for the partial capacity reductions alone.

Comparing the found determinants with the findings in literature some similarities and additions are found. From topological studies on network robustness to full link breakdown situations already found the determinants of density and service intensity. Now it is found that these determinants also hold for partial capacity reductions. Studies including passenger demand assignment on network robustness of full link breakdown also showed that the passenger demand is a significant determinant. Now it is found that this variable is also a determinant for partial capacity reductions.

New found determinants on network robustness are link centrality and the operational characteristics of mean speed on the link. Especially the latter one gives a new insight on PT network robustness. That the local size of the disruption is of influence on the criticality of a link gives the responsible authority the possibility to decrease the effect of such a disruption on the network performance. The practical implication of this finding and others is discussed in the next section.

8.2. Practical implications

In this section the practical implications of the conclusions of the previous section are discussed. First the practical implications on robustness of PT networks in general are discussed, followed by the practical implications on the Amsterdam urban rail bound PT network and some future practical applications of the developed model.

8.2.1. Robustness of urban rail bound public transport networks

The found determinants on robustness of PT networks are relevant for the strategic and tactical level of PT networks in general. The practical implications of robustness of urban rail bound PT networks are discussed on the isolated urban rail bound network in general (i.e. from a PTO perspective) and secondly from a passenger perspective by incorporating among other the underlying transport networks.

On the strategic level it is recommended to design networks such that the high criticality of links are mitigated and the robustness of the network is enhanced. The network can be improved by taking into account the found determinants: service intensity on the link, density of link area, passenger demand on the link and the mean speed on the link. A more robust PT network is created by spreading both passengers and PT lines. The first is off course encouraged by the second. By designing a network with less PT lines sharing track the local service intensity is decreased but also the local infrastructure network density is increased. By disentangling public transport lines and by creating comparable attractive route alternatives for passengers the passenger demand is spread which also contributes to a better network robustness. The spread of passengers over the different route alternatives can be enhanced by tuning the departure times of the different public transport lines. To even further improve the robustness the separate infrastructure of the disentangled lines should be connected so rerouting on the infrastructure of the other line is possible.

On the tactical level two implications arise. First is the prevention of occurring major discrete events. Second, is reducing the total impact of the major discrete events. On the prevention of occurring planned major discrete events the planned event (e.g. a marathon or a public event) is the most important type. In the planning of such public events the local government should be informed about the criticality of the different links in the network so that these should be avoided as much as possible. Maintenance, construction, track and road works cannot be prevented completely and hence the total impact of the major discrete events should be kept to a minimum. And there the Degrading Rapidity comes into play. For maintenance, construction, track and road works the local road or rail authority generally have the option to accept a partial capacity reduction (i.e. a lower local speed limit) for a longer period of time or a full link breakdown for a shorter term. In general, on links with high Degrading Rapidity the authority should choose a full link breakdown for a shorter term and on links with a low Degrading Rapidity the authority should choose a minor partial capacity reduction for a longer period of time. Before applying a partial capacity reduction one should calculate the consequential effects. As the significant influence of the operational characteristics showed the local disruptions size needs to be taken into account. The local increase in station-to-station running time does not only have to imply an increase in in-vehicle time for the traversing passengers but can also mean a longer waiting time for other passengers due to the decrease of the service frequency of traversing lines. As the passenger demand is one of the main determinants on link criticality it is also recommended for a public transport operator to incorporate this in its choice to apply a partial capacity reduction. It could be that in a specific season of the year a certain section of the network is less intensively used than in another season of the year. It is therefore recommended to imply the major discrete events in the dips of passenger demand when available. These dips can also be used on day basis (i.e. doing maintenance in the night or between the peak hours).

From a passenger perspective the robustness of a PT networks cannot be seen isolated without alternative transport networks. If a passenger has a decent alternative in, for example, the national railway system when the local urban rail bound PT network is disrupted it would not be too bad in comparison with no alternative at all. For the PTO it is wise to have temporary measures ready for the most critical links. Contacts and plans with other operators (e.g. local or regional bus operator or the national railways) should be ready so that the performance of the (extended) network keeps to a decent level (i.e. passenger are not disconnected and not heavily delayed).

8.2.2. Robustness of Amsterdam urban rail bound PT network

In this section is discussed how, based on the findings of the previous section, robust the Amsterdam urban rail bound PT network is and how the network robustness can be enhanced.

The most robust (i.e. least critical links) are found in the centre part of the network. In general links with a lot of parallelism are less critical, as can be seen in the centre part of the network (e.g. Dam-Nieuwezijds Voorburgwal and Elandsgracht-Leidseplein) where the density is high. From the Link Criticality results of the Amsterdam urban rail bound PT network it is clear that the most critical links are the metro lines. Especially the isolated branches of the metro lines in the the South-East and the North. That links which are on isolated branches are critical can also be seen in the tram network. The tram links to Osdorp traversing the Cornelis Lelylaan are also one of the most critical links in the network.

It has to be noticed that these results are based on an isolated urban rail bound PT network. In other words, alternative public transport networks like the bus network and the national rail network is not taken into account. Both networks could enhance robustness of the public transport network as a whole. On the other hand, the high passenger demands on the tram and metro network are not easily taken over by a bus service. The urban rail bound PTO is expected to prefer enhancing PT robustness within the urban rail bound network rather than enhancing robustness through the multi-modal network of busses, trams, metros and trains. Rerouting travellers through vehicles of other operators means revenue losses. Therefore the network robustness is discussed from both perspectives, first from an operator perspective second from a multi-modal perspective.

If sufficient funds would be available for improving robustness of the Amsterdam urban rail bound PT network some infrastructural extensions are proposed. As the most critical links are the metro links, most can be gained in making these link less critical. For decreasing link criticality within the Amsterdam urban PT Network it would therefore be a good idea to connect the ends of the network at for example the South-East (i.e. making a ring) and the end at the Isolatorweg for the metro (i.e. closing the ring). And also connecting the ends at Nieuw-Sloten towards Osdorp in the west of the tram network. These extensions make it possible to spread the service intensity and the travellers by offering alternative routes. The addition of cycles also offers rerouting options when disruptions occur. Another, cheaper but still expensive, infrastructural investment to create more options in rerouting is to connect the 'Oostlijn' and the N/Z-lijn at Amsterdam Centraal Station and Amsterdam Zuid, making rerouting possible. This means that the infrastructure has to be physically connected and the capacity has to be increased on both lines so that a higher service frequency is possible. As a consequence the criticality of the links on the N/Z-lijn, Oostlijn and the southern Ringlijn will decrease and the robustness will increase.

Fortunately for the local authorities, alternatives are available (i.e. the local bus network and the national railway) which enhance the robustness of the PT network as a whole (i.e. the multi-modal perspective). On most metro links (i.e. the most critical links) alternative routes are possible for the traveller on the nation railway. These are physical parallel routes on the south east of the metro network or an alternative with a little detour for the 'Oostlijn'. On other critical parts of the network alternatives exist in the local high quality bus network: R-net. Examples are the future Westtangent for the travellers normally using the tram line on the Cornelis Lelylaan and the R-net busses crossing the South-East isolated metro branches. So in general can be said that the Amsterdam PT network as a whole is very robust, but when purely looking at the urban rail bound PT network some links are quite vulnerable.

Instead of implying very expensive infrastructural investment or loosing revenue to other operators the operator and the local authority could also look at the cause of disruptions. Specifically for the critical links the transport authority and the PTO should prevent major discrete events from happening or at least decrease the frequency of occurrence to a minimum. On the strategic level this means that construction, maintenance, track and road works need to be kept to a minimum on these links. Specifically for these links, there must be steered on maintenance-free or low-maintenance tracks, platforms etc. Also third parties should be insisted to choose maintenance-free or low-maintenance options. On some links this is already implemented but now the local authorities have to possibility to see whether this is applied to all identified critical links. Other options is to keep conflicts with third parties as low as possible on these links: less trams on roads and no cables and pipes near the tracks.

8.2.3. Possible practical modelling applications

The developed model which is used for this robustness study can also be used for other applications like network design optimization. The modelling results showed some disruptions scenarios which performed better than the base case scenario. When, for example, different network design alternatives are being considered the developed model is able to calculate which alternative performs better in terms of generalized travel costs. More detailed insights in local passenger behaviour from the model can even further improve the design of a considered alternative.

Secondly, the model can be applied in researching the need and necessity of local infrastructural connections which can be used during disruptions through rerouting and short-turning of vehicles. The model is able to quantify the performance of the network during disruptions with and without such an infrastructure connection.

Third, the model can be used to choose the most optimal mitigation measure. The model is able to calculate the network performance as a result of applying a certain mitigation measure. When multiple mitigation measures are possible the model can be used to choose the most optimal one.

And finally, the model can be used to simulate specific situation of, for example, construction or maintenance projects. Situations can be calculated for which is considered to choose a long term partial capacity reduction or a short term full link breakdown. The model is able to quantify the performance of the network for both situations. And consequently comparing is possible.

8.3. Recommendations

In this section recommendations are formulated for further research (8.3.1) and for further improvement of the model developed in this study (8.3.2).

8.3.1. Recommendations for further research

The formulated recommendations in this section are based on the results found in this study.

- First, it is recommended to incorporate more tailor made mitigation measures used by the PTO. This includes mitigation measures used within the network (i.e. combining lines and rerouting lines to local hubs) but also mitigation measure outside the network (i.e. informing people to use the train instead of the metro). In this way an even more realistic outcome of Link Criticality and Degrading Rapidity can be achieved.
- Second, it is recommended to examine not only the bi-directional stop-stop capacity reduction in speed but to explore also the other types of major discrete events. This included one-directional capacity reductions and capacity reductions on complete sections of the network.
- Third, it is recommended to include the length in time of the planned disruption in the assessment of the Link Criticality. Differences will exist between the running time of the disruption for partial capacity reductions and full link breakdowns, but also for both at different locations.
- Fourth, it is recommended to examine the probability of occurrence of the partial capacity reductions. In this study a full set of partial capacity reductions is examined from a 10% speed reduction to a 100% speed reduction. Some of the partial capacity reduction scenarios can, however, be less likely to happen than others.
- Fifth, it is recommended to examine the elasticity of passenger demand (i.e. the effect of planned major discrete events on the passenger demand). In this study the passenger demand is assumed

to be inelastic (i.e. the passenger demand stays the same for every scenario). The trip generation, mode choice and departure time of travellers can, however, be affected by the change in the network as a results of a planned major discrete event.

- Sixth, it is recommended to also study the effects of the unplanned major discrete events. Robustness of a public transport network not only depends on the occurrence of planned major discrete events but even more on the occurrence of the unplanned counterparts.
- Seventh, it is recommended to study the underlying process of the influence of the found key variables on the relationship between capacity reduction and network performance. This study found some key variables and the size of their influence. The underlying process of this influence is, however, not revealed.
- Eight, the key variables are assessed by multiple linear regression. However 30 to 60% of the relationship between capacity reduction and network performance is not explained by the found variables. It is therefore recommended to search for other potential determinants and to consider another approach to reveal the key variables on the examined relationship. The linearity between the independent variables and the dependent variables and that the data is normally distributed is not determined so other approaches could be more successful.
- Ninth, it is recommended to assess the robustness of the PT network of Amsterdam including the other public transport modalities like the bus and the train network. Now the Link Criticality is overestimated on some links because the alternative routes in the bus and train network are not incorporated.
- Tenth, it is recommended to asses not only the robustness of the urban rail bound PT network of Amsterdam but also the robustness of urban rail bound PT networks around the world. Different findings from different networks can give new insights on other determinants or strengthen the findings of this study.
- And finally, it is recommended to further develop the proposed robustness indicators in two ways. First is to develop the robustness indicators such that they can be used not only on link level but also on network level. Meaning, that the robustness of networks can be measured in one number so that different networks can be compared in their robustness scores. Second is to develop the robustness indicators such that they can be meaningfully used in cost-benefit analysis of project appraisals.

8.3.2. Recommendations for further improvement of the model

The formulated recommendations are based on a reflection on assumptions made in this study.

- First, it is recommended to improve the model by incorporating the infrastructural limitations of turning and rerouting PT vehicles. In this study it is assumed that at every node every turning options is possible. In reality not all turning options are available through, for example, the absence of track curves at these options. Also it is assumed that turning of vehicles is possible at every transfer node, in reality this is not everywhere possible through the absence of infrastructure. By incorporating this in the model the applied mitigation measures become even more realistic.

- Second, it is recommended to explore other assignment modelling options. The logit model assumes that all path alternatives are independent. These assumptions can be questioned in the case of two path alternatives that are largely overlapping. By implementing an assignment model what does correct for the property of these paths the assignment may be more valid.
- Third, it is recommended to include crowding in the utility function of the assignment model. Crowding affects the attractiveness of routes for travellers but it can also make routes impossible to use for all travellers who want to travel the route. This can, for example, happen when alternative routes are fully saturated by rerouting travellers because of a disruption elsewhere. The model can be improved by incorporating this effect in the model.
- Fourth, it is recommended to validate the developed assignment model with realization data. Above two recommendations are expected to improve the model's validity, this can be determined by validating the model with realization data like chip card data. By implementing the current network in the model and validate the model on the realized data it can be checked which model to use and how to calibrate the different parameters of the assignment model.
- Fifth, it is recommended to detail the partial capacity reduction scenarios based on further research into the applied size of a lower local speed limit. Now the speed is reduced on the gross mean speed (i.e. the total distance of the link divided by the total station-to-station-travel time). In reality the speed reduction is applied to the net mean speed only (i.e. the distance between leaving a stop to arriving at the next stop divided by the running time between the stops). It changes the increase in travel time, incorporating this in the capacity reduction scenarios makes the results more realistic.
- Sixth, it is recommended to explore the effect of a speed reduction in the frequency limitation on that link. A PT vehicle takes a certain space on the link, when the speed is reduced to a certain extent this can decrease the possible service frequency on that link. Incorporating the effect of a speed reduction on the service frequency in the model would improve the disruption scenario results.
- And finally, it is recommended to improve the implementation of mitigation measures. Both mitigation measures (i.e. rerouting and cutting lines) are rule-based implemented based on a number of assumptions. First turning at every transfer node is not always feasible and desirable through, for example, interference with other traffic. Second, rerouting is done line by line. This does, however, not always result in the system optimal solution. For a better disruptions scenario result it is recommended to implement more tailor made mitigation measures including combining lines, rerouting lines to local hubs, optimization of rerouting lines and short turning at feasible locations.

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Appendix A Categories of major discrete events

In his thesis on the robustness of multi-modal public transport M. Yap (2014) made an overview of occurring major discrete events in categories for train, metro/light rail, tram and bus network, see Table 36. The major discrete events on the train network were found in the public database of the national railway system (Rijdendetreinen, 2015). As no public database is available for local urban PT networks as metro, tram and bus networks, these major discrete events were found at the internal database of the Dutch urban PT operator of the Hague, The Hague Tramway Company (HTM). The functional categorization of the different event types is done on their general predictability, frequency of occurring and severity.

Table 36 Overview of major discrete event categories for metro / light rail and tram network, adapted from M. Yap (2014)

Train network	Metro / light rail network	Tram network	Bus network
Vehicle breakdown	Vehicle breakdown	Vehicle breakdown	Vehicle breakdown
Major incident	Major incident	Major incident	Major incident
Switch failure	Switch failure	Switch failure	
Blockage	Blockage	Blockage	Blockage
Restrictions by emergency services	Restrictions by emergency services	Restrictions by emergency services	Restrictions by emergency services
Defect/ damaged bridge	Defect/ damaged bridge	Defect/ damaged bridge	Defect/ damaged bridge
Power failure	Power failure	Power failure	
Defect track	Defect track	Defect track	
Defect overhead wire	Defect overhead wire	Defect overhead wire	
Signal failure	Signal failure		
Suicide			
Level crossing failure			
Damaged train viaduct			
Copper theft			
Large maintenance work	Large maintenance work	Large maintenance work	Large maintenance work

Severity was expressed in infrastructure availability for which M. Yap (2014) calculated the probability of occurrence of the effect on infrastructure availability (see Table 37) for the different major discrete event types.

Table 37 Effect of different major discrete event types on infrastructure availability, adapted from M. Yap (2014)

1 track of link blocked	50% of the tracks of link blocked: 50 link availability	100% of the tracks of link blocked: 0% link availability
Vehicle breakdown	Signal failure: p=0.9	Signal failure: q = 1 – p = 0.1
Defect track	Switch failure: p = 0.8	Copper theft: q = 1 – p = 0.2
	Copper theft: p = 0.8	Level crossing failure q = 1 – p = 0.2
	Level crossing failure: p=0. 9	Switch failure: q = 1 – p = 0.2
		Power failure
		Defect overhead wire
		Defect / damaged rail bridge
		Damaged train viaduct
		Suicide
		Major incident
		Restrictions by emergency services
		Track blockage

Appendix B Use of infrastructure for turning

The use of a wye or a loop during turning a one-direction vehicle are given in Figure 69 . The blue arrows depict then normal running directions, the red arrows depict the turning moves.

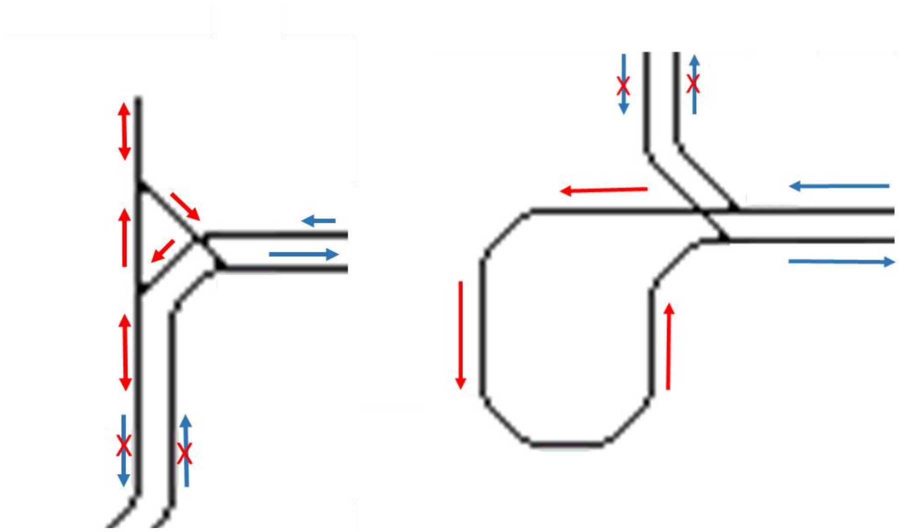

Figure 69 Example of the use of a wye (i.e. a triangular junction) (left) and a loop (right) in case of line cutting

For two direction vehicles a crossover switch is sufficient, see Figure 70. If these infrastructural turning options are not available points of turning and thus cutting a line is limited.

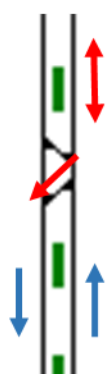

Figure 70 Example of the use of a crossover in case of line cutting

Appendix C Supply data preparation

The supply data is processed in three general steps: extraction, filtering and processing. The supply data processing is discussed in three sections corresponding with these general steps. A flowchart overview of the data processing is given in Figure 71. The cursive names between brackets in the following sections refer to the blocks in this flowchart figure.

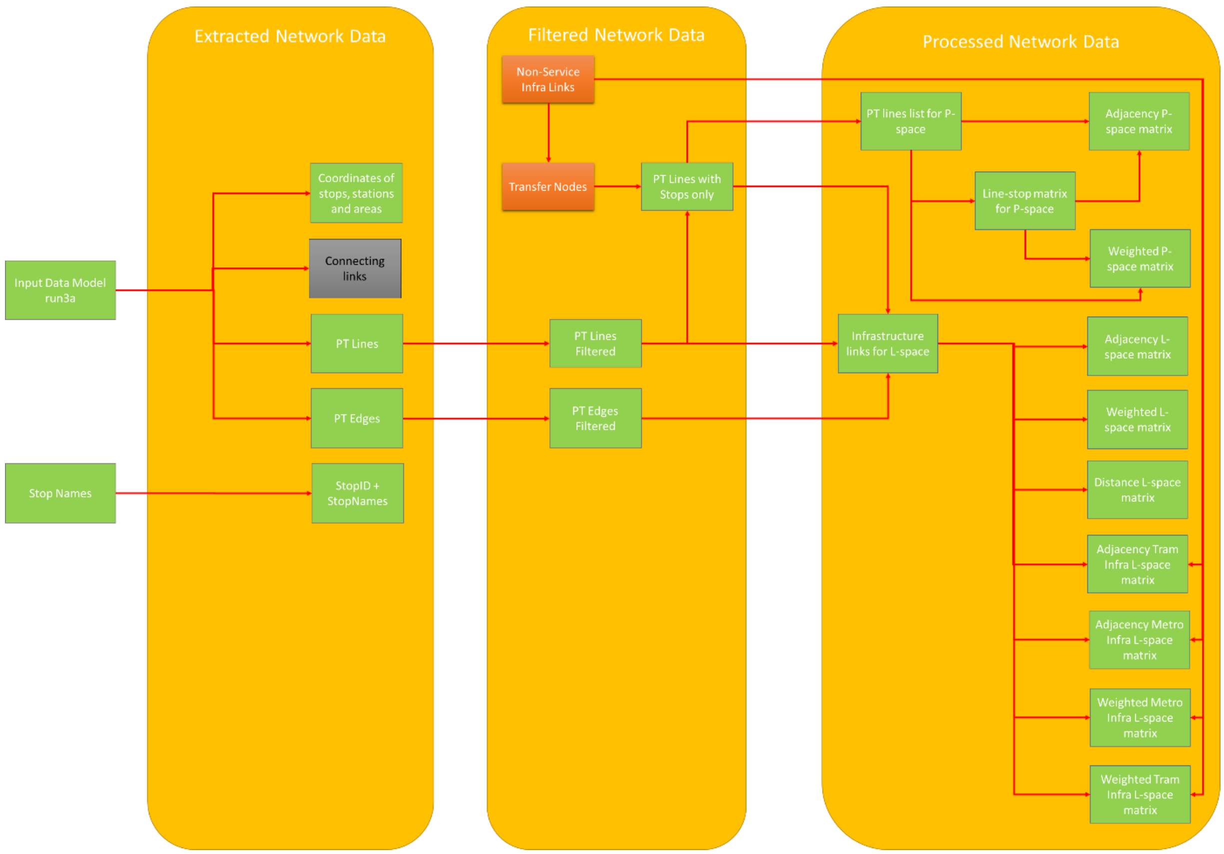

Figure 71 Flowchart overview of data preparation (Green boxes are directly or indirectly provided data from GenMOD2013 run3a, orange boxes is manually added data or data from other source, grey box is excluded data)

Network Data Extracting

The supply data was given in two .csv-files with input data for the OmniTRANS runs (*Input Data Model run3a*) and a separate file with the names of the stop IDs (*Stop Names*). The second file with data (*Stop Names*) is only used for validating the data manually. From the first file four types of data were extracted: coordinates of stops, stations and areas (*Coordinates of stops, stations and areas)*, connecting links for transfers and between areas and stops (*Connecting links*), the PT Lines with corresponding sequence of nodes (*PT Lines*) and links for the PT Lines (*PT Edges*).

Manually a set of infra sections are identified which can be (potentially) used for rerouting (*Non-Service Infra Links*). Generally these are sections where rail is situated in the current situation but which are not used by the service network in this PT Network 2020. An overview of these infrastructure sections is given in Section 6.3. In some cases the distance and travel times on these links were given in the *PT Edges* data, in the rest of the cases the data was found in occupancy rate monitoring of the GVB (Gemeentelijk Vervoerbedrijf, 1998, 2015a).

Manually a set of nodes are added which represent transfer nodes (*Transfer Nodes*). These transfer nodes are nodes where lines cross, join, disjoin or where the serviced network attaches to the extra infrastructure which can be used for rerouting (*Non-Service Infra Links*). In some cases stops which are in Genmod2013 seen as separate stops are in this model seen as one. A complete overview of these nodes can be found in Section 6.3.

The data on the coordinates of stops, stations and areas is used for visualization purposes only. The coordinates were given in Rijksdriehoekscoördinaten which is used in the Netherlands on national scale. These coordinates were translated in Mercator coordinates to be able to use them in Gephi.

The connecting links data is not used in the case study and so this data is excluded in the further process.

Network Data Filtering

The PT lines are represented with a lineID, PT vehicle ID, frequency, vehicle capacity, PTO ID and sequence of traversed nodes of which at the positive numbers the node is a stop and for the negative numbers is not. The PT Lines were given for every PT line, so also for the other PT modalities. In the first step the lines of the other modalities then metro and tram were excluded (*PT Lines Filtered*). And because not all information per PT line was needed the, for example, vehicle capacity and PTO id were excluded. To come to a representation of a line with only the sequence of stops, the intermediate points (the negative Node numbers) were excluded (*PT Lines with stops only*).

The links for the PT Lines were represented with a Start Node, Stop Node, area Code, distance between nodes and travel time between nodes for bus, tram, metro and train. The area code and travel times for bus and train are excluded because this data was not needed for further modelling.

Network Data Processing

The first network representation (as discussed in Section 4.1) is derived from the *PT Lines with Stops only*, *PT Lines Filtered* and *PT Edges Filtered* (*Infrastructure links for L-space*). The *PT Lines with Stops only* gives all the stop-stop edges. *PT Edges Filtered* is, however, given for node-node edges and not necessarily stop-stop edges. *PT Lines filtered* is therefore needed to calculate the travel time and distances for the stop-stop edges. The second network representation (*PT lines for P-space*) is derived from *PT Lines with Stops only*. How these first two network representation are translated into the other representations of L-space and P-space is explained in Section 4.1.

As the tram network and metro network are not connected by rail infrastructure rerouting of trams on the metro network and vice versa is impossible. Therefore extra representation need to be generated to represent the network which can be used for metro rerouting and tram rerouting. The Tram Infra L-space matrices represent the sum of serviced tram network and non-service tram infra links (*Adjacency Tram Infra L-space matrix* and *Weighted Tram Infra L-space Matrix*). The Metro Infra L-space matrices represent the sum of serviced metro network and non-service metro infra links (*Adjacency Metro Infra L-space matrices* and *Weighted Metro Infra L-space Matrix*).

Appendix D Demand data preparation

As mentioned before the demand data is given as boarding and alighting per stop per mode per line (*Model output Data model run 3a*). A flowchart overview of the demand data preparation is given in Figure 72.In the first step the boarding and alighting per stop is extracted for the metro and tram lines only (*Access and Egress per stop per line*). In the next step the boarding per stop is aggregated over all lines and the alighting per stop is aggregated over all lines. As a result the production and attraction per stop is generated. These represents, however, trips and not passengers. Therefore the production and attraction is divided by the mean number of trips per traveller. Based on expert judgments of employees of the SRA the mean number of trips per traveller is assumed to be 1.4. This means that the mean number of transfers per passenger is 0.4. For the current Amsterdam Tram and Metro network this is quite high, but because of the opening of the N/Z-line and the corresponding adaptations of the tram network this is expected to be reasonable.

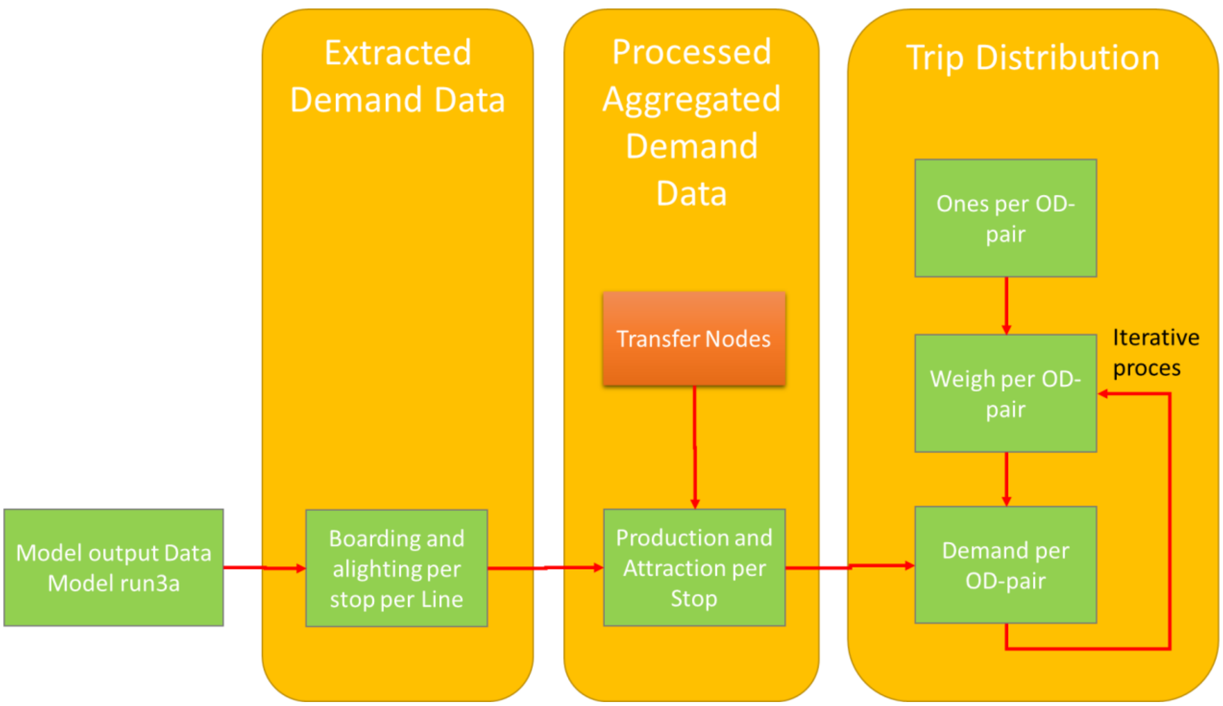

Figure 72 Flowchart overview of demand data preparation (Green boxes are directly or indirectly provided data from GenMOD2013 run3a, orange boxes is manually added data or data from other source)

Through iterative proportional fitting with the RAS-algorithm the demand per OD-pair is generated (Bacharach, 1965). This is done in the pre-programmed iterative proportional fitting algorithm in MATLAB (The Mathworks, 2009). The input matrix for this model is a matrix with in every cell a one (*ones per OD-pair)*, the row sums is equal to the production per stop, the columns sums to the attraction per stop and the precision for convergence is chosen to be equal to 5%. The assumption for the input matrix to be ones, is that on the short distance the network competes with bicycles and walking and on the long distance with the national train network and car network. Both distance of the shortest paths or the inverse of the distance of the shortest paths as input are therefore not really sensible. Data on number of inhabitants and number of jobs on which the original weighs per OD-pair could be derived was not available.

Appendix E Overview of attribute values per link

In the two tables below the calculated attributes (see Section 5.2) for all edges are presented. In Table 38 the nearest main nodes, the line attributes and the passenger demand attributes are shown. In Table 39 the link attributes are presented.

Table 38 Overview of line and passenger demand attributes for all links (1/2)

	Link Nodes		Nearest Main Nodes		Line attributes			Passenger demand attributes
Link Number	Name	Name	Name	Name	Link capacity (veh/hour)	Total frequency of traversing lines (/hour)	P-space link degree centrality indicator (-)	Total bi-directional flow on link
1	Zijlplein	Dr. H. Colijnstraat	Zijlplein	Burg. Röellstraat	40	8	34	185
2	Burg. Van Leeuwenlaan	Dr. H. Colijnstraat	Burg. Röellstraat	Zijlplein	40	8	34	387
3	Burg. Van Leeuwenlaan	Burg. Röellstraat	Zijlplein	Burg. Röellstraat	40	8	59	656
4	Plein 40-45/Burg. De Vlugtlaan	Lod. van Deysselstraat	De Vlugtlaan	Burg. Röellstraat	40	8	64	109

5	Plein 40-45/Burg. De Vlugtlaan	Burg. Eliasstraat	Burg. Röellstraat	De Vlugtlaan	40	8	64	337
6	Burg. Röellstraat	Sloterpark	Burg. Röellstraat	Sloterpark	40	8	74	2
7	Burg. Röellstraat	Lod. van Deysselstraat	Burg. Röellstraat	De Vlugtlaan	40	8	74	70
8	Burg. Röellstraat	Burg. Rendorpstraat	Burg. Röellstraat	Jan van Galenstraat	40	8	59	1122
9	Burg. Fockstraat	Burg. Eliasstraat	De Vlugtlaan	Burg. Röellstraat	40	8	64	337
10	Kingsfordweg	Haarlemmerweg	Sloterdijk	Jan Evertsenstraat	40	8	46	1513
11	Kingsfordweg	Sloterdijk	Jan Evertsenstraat	Sloterdijk	40	16	65	1438
12	Haarlemmerweg	Wiltzanghlaan	Sloterdijk	Jan Evertsenstraat	40	8	46	1513
13	Wiltzanghlaan	Bos en Lommerweg	Sloterdijk	Jan Evertsenstraat	40	8	46	1639
14	Bos en Lommerweg	Rijpstraat	Sloterdijk	Jan Evertsenstraat	40	8	46	1728
15	Bos en Lommerplein	De Vlugtlaan	Mercatorplein	De Vlugtlaan	40	8	82	701
16	Jan van Galenstraat	Rijpstraat	Jan Evertsenstraat	Sloterdijk	40	8	46	1728
17	Jan van Galenstraat	Jan Evertsenstraat	Sloterdijk	Jan Evertsenstraat	40	8	84	1903
18	Van Hallstraat	v. Limburg Sirumstraat	Van Hallstraat	De Clercqstraat	40	8	38	53
19	v. Limburg Sirumstraat	De Wittenkade	Van Hallstraat	De Clercqstraat	40	8	38	453
20	Nassaukade	Frederik Hendrikplantsoen	Van Hallstraat	De Clercqstraat	40	8	38	765
21	De Wittenkade	Nassaukade	Van Hallstraat	De Clercqstraat	40	8	38	453
22	Frederik Hendrikplantsoen	Hugo de Grootplein	Van Hallstraat	De Clercqstraat	40	8	38	888
23	Marnixplein	Bloemgracht	Zoutkeetsgracht	Rozengracht	40	8	46	1364
24	Hugo de Grootplein	De Clercqstraat	Van Hallstraat	De Clercqstraat	40	8	96	992
25	Zoutkeetsgracht	Haarlemmerplein	Zoutkeetsgracht	Rozengracht	40	8	46	154
26	Haarlemmerplein	Nw. Willemsstraat	Zoutkeetsgracht	Rozengracht	40	8	46	719
27	Nw. Willemsstraat	Marnixplein	Zoutkeetsgracht	Rozengracht	40	8	46	1179
28	Muziekgeb. Bimhuis	Jan Schaeferbrug	CS	Rietlandpark	40	15	18	3804
29	Jan Schaeferbrug	Rietlandpark	CS	Rietlandpark	40	15	45	3485
30	Azartplein	C. van Eesterenlaan	Azartplein	Rietlandpark	40	8	52	216
31	Prinsengracht	Frederiksplein	Rembrandtplein	Frederiksplein	20	8	71	254
32	Ecuplein	Baden Powellweg	De Aker	Meer en Vaart	40	12	46	1315

33	Ecuplein	Inarisstraat	Meer en Vaart	De Aker	40	12	46	647
34	Surinameplein	Overtoomsesluis	Surinameplein	1e Con. Huygensstraat	40	12	60	8010
35	Surinameplein	Derkinderenstraat	Surinameplein	Lelylaan	40	22	74	9901
36	Surinameplein	Corantijnstraat	Surinameplein	Bilderdijkstraat	40	10	58	2191
37	Hoekenes	Osdorpplein	Dijkgraafplein	Meer en Vaart	40	10	42	1246
38	Hoekenes	Baden Powellweg	Meer en Vaart	Dijkgraafplein	40	10	42	891
39	Baden Powellweg	Dijkgraafplein	Meer en Vaart	Dijkgraafplein	40	10	42	677
40	Osdorpplein	Ruimzicht	Dijkgraafplein	Meer en Vaart	40	10	42	1255
41	Ruimzicht	Meer en Vaart	Dijkgraafplein	Meer en Vaart	40	10	58	2023
42	Jan Voermanstraat	Jan Tooropstraat	Mercatorplein	Jan van Galenstraat	40	8	34	1913
43	Jan Voermanstraat	Adm. Helfrichstraat	Jan van Galenstraat	Mercatorplein	40	8	34	1943
44	Jan Tooropstraat	Jan van Galenstraat	Mercatorplein	Jan van Galenstraat	40	8	53	1878
45	Jan van Galenstraat	Mercatorplein	De Vlugtlaan	Mercatorplein	40	8	74	814
46	Jan van Galenstraat	Bos en Lommerplein	Mercatorplein	De Vlugtlaan	40	8	64	812
47	Adm. Helfrichstraat	Mercatorplein	Jan van Galenstraat	Mercatorplein	40	8	59	1949
48	Mercatorplein	Marco Polostraat	Mercatorplein	Jan Evertsenstraat	40	16	84	3031
49	Marco Polostraat	Jan Evertsenstraat	Mercatorplein	Jan Evertsenstraat	40	16	103	3109
50	Jan Evertsenstraat	Willem de Zwijgerlaan	Jan Evertsenstraat	De Clercqstraat	40	24	122	5324
51	Willem de Zwijgerlaan	De Clercqstraat	Jan Evertsenstraat	De Clercqstraat	40	24	138	5792
52	De Clercqstraat	Bilderdijkstraat	De Clercqstraat	Bilderdijkstraat	40	8	118	1761
53	De Clercqstraat	Rozengracht	De Clercqstraat	Rozengracht	40	24	186	5491
54	Rozengracht	Bloemgracht	Rozengracht	Zoutkeetsgracht	40	8	132	1376
55	Rozengracht	Elandsgracht	Rozengracht	Elandsgracht	60	34	186	8024
56	Ten Katestraat	Bilderdijkstraat	Surinameplein	Bilderdijkstraat	40	10	62	2776
57	Ten Katestraat	J.P. Heyestraat	Bilderdijkstraat	Surinameplein	40	10	42	2388
58	Postjesweg	Corantijnstraat	Bilderdijkstraat	Surinameplein	40	10	42	2225
59	Postjesweg	Witte de Withstraat	Surinameplein	Bilderdijkstraat	40	10	42	2328
60	Witte de Withstraat	J.P. Heyestraat	Surinameplein	Bilderdijkstraat	40	10	42	2388
61	Bilderdijkstraat	Elandsgracht	Bilderdijkstraat	Elandsgracht	40	10	118	2821

62	Bilderdijkstraat	1e Con. Huygensstraat	Bilderdijkstraat	1e Con. Huygensstraat	40	8	84	2003
63	Elandsgracht	Leidseplein	Elandsgracht	Leidseplein	60	24	186	6825
64	Van Baerlestraat	Museumplein	Van Baerlestraat	Museumplein	60	32	159	8583
65	Van Baerlestraat	Hobbemastraat	Van Baerlestraat	Leidseplein	60	32	168	8457
66	Van Baerlestraat	C. Schuytstraat	Van Baerlestraat	Heemstedestraat	40	8	118	3492
67	Van Baerlestraat	1e Con. Huygensstraat	Van Baerlestraat	1e Con. Huygensstraat	40	8	135	2962
68	Overtoomsesluis	Rhijnvis Feithstraat	Surinameplein	1e Con. Huygensstraat	40	12	46	8035
69	Rhijnvis Feithstraat	J.P. Heyestraat	Surinameplein	1e Con. Huygensstraat	40	12	46	8311
70	J.P. Heyestraat	1e Con. Huygensstraat	Surinameplein	1e Con. Huygensstraat	40	12	66	8307
71	1e Con. Huygensstraat	Leidseplein	1e Con. Huygensstraat	Leidseplein	60	12	152	8071
72	Westermarkt	Rozengracht	Dam	Rozengracht	60	34	183	6985
73	Mr. Visserplein	Waterlooplein	Alexanderplein	Waterlooplein	40	10	51	1177
74	Mr. Visserplein	Plantage Kerklaan	Waterlooplein	Alexanderplein	40	10	32	1045
75	Spui	Koningsplein	Dam	Leidseplein	40	20	70	7028
76	Koningsplein	Keizersgracht	Dam	Leidseplein	40	20	70	7023
77	Muntplein	Rembrandtplein	Muntplein	Rembrandtplein	40	18	83	1387
78	Muntplein	Rokin	Muntplein	Rokin	40	26	118	2105
79	Muntplein	Keizersgracht	Muntplein	Vijzelgracht	40	8	89	616
80	Rembrandtplein	Waterlooplein	Rembrandtplein	Waterlooplein	40	10	61	1074
81	Rembrandtplein	Keizersgracht	Rembrandtplein	Frederiksplein	20	8	39	261
82	Prinsengracht	Keizersgracht	Leidseplein	Dam	40	20	70	7008
83	Prinsengracht	Leidseplein	Dam	Leidseplein	40	20	144	7008
84	Keizersgracht	Vijzelgracht	Muntplein	Vijzelgracht	40	8	113	644
85	Keizersgracht	Prinsengracht	Rembrandtplein	Frederiksplein	20	8	26	254
86	Plantage Kerklaan	Plantage Badlaan	Waterlooplein	Alexanderplein	40	10	32	887
87	Leidseplein	Spiegelgracht	Leidseplein	Vijzelgracht	60	16	153	9899

88	Leidseplein	Hobbemastraat	Leidseplein	Van Baerlestraat	60	32	185	8508
89	Spiegelgracht	Vijzelgracht	Leidseplein	Vijzelgracht	60	16	125	9889
90	Stadhouderskade	Woustraat	Frederiksplein	Woustraat	40	8	46	284
91	Stadhouderskade	Frederiksplein	Woustraat	Frederiksplein	40	8	71	297
92	K. 's Gravesandestraat	Alexanderplein	Weesperplein	Alexanderplein	60	16	103	4737
93	K. 's Gravesandestraat	Weesperplein	Alexanderplein	Weesperplein	60	16	108	4801
94	Beukenweg	Camperstraat	Wijttenbachstraat	Wibautstraat	40	8	38	738
95	Beukenweg	Wijttenbachstraat	Wibautstraat	Wijttenbachstraat	40	8	75	664
96	1e Leeghwaterstraat	1e Coehoornstraat	Rietlandpark	Alexanderplein	30	8	52	1647
97	1e Leeghwaterstraat	Rietlandpark	Alexanderplein	Rietlandpark	30	8	62	1430
98	Rietlandpark	C. van Eesterenlaan	Rietlandpark	Azartplein	40	8	62	613
99	Rietlandpark	Zuiderzeeweg	Rietlandpark	IJburg	40	15	45	3013
100	1e Coehoornstraat	Hoogte Kadijk	Rietlandpark	Alexanderplein	40	8	52	1694
101	Hoogte Kadijk	Alexanderplein	Rietlandpark	Alexanderplein	40	8	85	1934
102	Alexanderplein	Plantage Badlaan	Alexanderplein	Waterlooplein	40	10	75	883
103	Alexanderplein	1e v. Swindenstraat	Alexanderplein	Wijttenbachstraat	40	18	97	2727
104	Dapperstraat	Wijttenbachstraat	Flevopark	Wijttenbachstraat	40	18	90	1156
105	Muiderpoort	Dapperstraat	Flevopark	Wijttenbachstraat	40	18	68	745
106	Muiderpoort	Molukkenstraat	Wijttenbachstraat	Flevopark	40	10	50	428
107	Soembawaweg	Insulindeweg	Wijttenbachstraat	Flevopark	40	10	32	366
108	Molukkenstraat	Soembawaweg	Wijttenbachstraat	Flevopark	40	10	32	411
109	Insulindeweg	Flevopark	Wijttenbachstraat	Flevopark	40	10	32	0
110	1e v. Swindenstraat	Wijttenbachstraat	Alexanderplein	Wijttenbachstraat	40	18	94	2619
111	Wijttenbachstraat	Pretoriusstraat	Wijttenbachstraat	Diemen	40	8	79	1357
112	Zuiderzeeweg	Strandeiland	Rietlandpark	IJburg	40	15	18	2371
113	Steigereiland	Vennepluimstraat	Rietlandpark	IJburg	40	15	18	1904
114	Steigereiland	Strandeiland	IJburg	Rietlandpark	40	15	18	2363
115	Ruisrietstraat	IJburg	Rietlandpark	IJburg	40	15	18	630
116	Ruisrietstraat	Diemerparklaan	IJburg	Rietlandpark	40	15	18	1300

117	Vennepluimstraat	Diemerparklaan	Rietlandpark	IJburg	40	15	18	1324
118	De Aker	Pilatus	De Aker	Meer en Vaart	40	12	46	108
119	Inarisstraat	Pilatus	Meer en Vaart	De Aker	40	12	46	647
120	Baden Powellweg	De La Sallestraat	De Aker	Meer en Vaart	40	12	46	1669
121	Sloterweg	Antwerpenbaan	Heemstedestraat	Osdorperweg	40	8	52	1093
122	Sloterweg	Aletta Jacobslaan	Osdorperweg	Heemstedestraat	40	8	52	1118
123	Louis Davidsstraat	Meer en Vaart	De Aker	Meer en Vaart	40	12	60	1889
124	Louis Davidsstraat	De La Sallestraat	Meer en Vaart	De Aker	40	12	46	1669
125	Kasterleepark	Antwerpenbaan	Osdorperweg	Heemstedestraat	40	8	52	432
126	Kasterleepark	Osdorperweg	Heemstedestraat	Osdorperweg	40	8	52	255
127	Johan Huizingalaan	Aletta Jacobslaan	Heemstedestraat	Osdorperweg	40	8	52	2079
128	Johan Huizingalaan	Heemstedestraat	Osdorperweg	Heemstedestraat	40	8	71	2571
129	Delfland	Westlandgracht	Heemstedestraat	Van Baerlestraat	40	8	52	3011
130	Delfland	Heemstedestraat	Van Baerlestraat	Heemstedestraat	40	8	71	3002
131	Hoofddorpplein	Westlandgracht	Van Baerlestraat	Heemstedestraat	40	8	52	3014
132	Hoofddorpplein	Zeilstraat	Heemstedestraat	Van Baerlestraat	40	8	52	3174
133	Zeilstraat	Valeriusplein	Heemstedestraat	Van Baerlestraat	40	8	52	3217
134	Valeriusplein	C. Schuytstraat	Heemstedestraat	Van Baerlestraat	40	8	52	3390
135	Haarlemmermeerstation	Valeriusplein	Stadionplein	Museumplein	40	8	36	1388
136	Haarlemmermeerstation	Stadionplein	Museumplein	Stadionplein	40	8	64	1324
137	Valeriusplein	Emmastraat	Stadionplein	Museumplein	40	8	36	1495
138	Emmastraat	J. Obrechtstraat	Stadionplein	Museumplein	40	8	36	1503
139	J. Obrechtstraat	Museumplein	Stadionplein	Museumplein	40	8	85	1530
140	Museumplein	Roelof Hartplein	Museumplein	Roelof Hartplein	40	24	146	7322
141	Ijsbaanpad	Amstelveenseweg	Stadionplein	Amstelveenseweg	40	16	110	2083
142	Ijsbaanpad	Stadionplein	Amstelveenseweg	Stadionplein	40	16	92	2089
143	Olympiaweg	Stadionplein	Gerrit v.d. Veenstraat	Stadionplein	40	8	78	797
144	Olympiaweg	Olympiaplein	Stadionplein	Gerrit v.d. Veenstraat	40	8	64	798

145	Olympiaplein	Minervanplein	Stadionplein	Gerrit v.d. Veenstraat	40	8	64	805
146	Stadionweg	Gerrit v.d. Veenstraat	Stadionplein	Gerrit v.d. Veenstraat	40	8	81	824
147	Stadionweg	Minervanplein	Gerrit v.d. Veenstraat	Stadionplein	40	8	64	820
148	Prinses Irenestraat	Station Zuid	Gerrit v.d. Veenstraat	Station Zuid	40	8	72	3484
149	Albert Cuypstraat	Ceintuurbaan	Vijzelgracht	Ceintuurbaan	40	8	104	411
150	Albert Cuypstraat	Vijzelgracht	Ceintuurbaan	Vijzelgracht	40	8	113	406
151	Roelof Hartplein	Gerrit v.d. Veenstraat	Roelof Hartplein	Gerrit v.d. Veenstraat	40	16	128	4347
152	Roelof Hartplein	Ceintuurbaan	Roelof Hartplein	Ceintuurbaan	40	24	151	5724
153	2e v.d. Helstraat	Ceintuurbaan	Woustraat	Ceintuurbaan	40	8	91	2333
154	2e v.d. Helstraat	Woustraat	Ceintuurbaan	Woustraat	40	8	52	2340
155	Woustraat	Lutmastraat	Woustraat	Victorieplein	40	8	46	244
156	Cornelis Troostplein	Ceintuurbaan	Victorieplein	Ceintuurbaan	40	8	95	872
157	Cornelis Troostplein	Scheldestraat	Ceintuurbaan	Victorieplein	40	8	46	788
158	Lutmastraat	Jozef Israelkade	Woustraat	Victorieplein	40	8	26	196
159	Scheldestraat	Maasstraat	Ceintuurbaan	Victorieplein	40	8	46	732
160	Maasstraat	Waalstraat	Ceintuurbaan	Victorieplein	40	8	46	732
161	Waalstraat	Victorieplein	Ceintuurbaan	Victorieplein	40	8	58	676
162	Victorieplein	Jozef Israelkade	Victorieplein	Woustraat	40	8	48	187
163	Victorieplein	Amsteldijk	Victorieplein	Amstel	40	16	70	771
164	Gerrit v.d. Veenstraat	Prinses Irenestraat	Gerrit v.d. Veenstraat	Station Zuid	40	8	69	3472
165	Amsteldijk	Amstel	Victorieplein	Amstel	40	16	89	763
166	Pretoriusstraat	Hogeweg	Wijttenbachstraat	Diemen	40	8	46	1210
167	Hogeweg	Hugo de Vrieslaan	Wijttenbachstraat	Diemen	40	8	46	1183
168	Hugo de Vrieslaan	Kruislaan	Wijttenbachstraat	Diemen	40	8	46	1040
169	Kruislaan	Brinkstraat	Wijttenbachstraat	Diemen	40	8	46	874
170	Brinkstraat	Arent Krijtstraat	Wijttenbachstraat	Diemen	40	8	46	563

171	Arent Krijtstraat	Schoolstraat	Wijttenbachstraat	Diemen	40	8	46	487
172	Schoolstraat	Lublinkstraat	Wijttenbachstraat	Diemen	40	8	46	467
173	Vumc	VU medisch centrum	Vumc	Amstelveenseweg	40	16	92	0
174	VU medisch centrum	Amstelveenseweg	Vumc	Amstelveenseweg	40	16	110	523
175	Lublinkstraat	Diemen	Wijttenbachstraat	Diemen	40	8	46	380
176	Buikslotermeerplein	Van Hasseltweg	Buikslotermeerplei n	CS	40	15	12	5386
177	Van Hasseltweg	CS	Buikslotermeerplei n	CS	40	15	140	7547
178	Isolatorweg	Sloterdijk	Isolatorweg	Sloterdijk	40	11	60	1319
179	Postjesweg	Lelylaan	Jan van Galenstraat	Lelylaan	40	11	74	11981
180	Postjesweg	Jan van Galenstraat	Lelylaan	Jan van Galenstraat	40	11	54	10834
181	Derkinderenstraat	Lelylaan	Surinameplein	Lelylaan	40	22	93	9776
182	Johan Huizingalaan	Lelylaan	Meer en Vaart	Lelylaan	40	22	93	5711
183	Johan Huizingalaan	Piet Wiedijkstraat	Lelylaan	Meer en Vaart	40	22	74	5354
184	Piet Wiedijkstraat	Meer en Vaart	Lelylaan	Meer en Vaart	40	22	74	5282
185	Nieuwmarkt	CS	Waterlooplein	CS	40	22	153	14951
186	Nieuwmarkt	Waterlooplein	CS	Waterlooplein	40	22	54	14630
187	Henk Sneevlietweg	Heemstedestraat	Amstelveenseweg	Heemstedestraat	40	11	63	18635
188	Spaklerweg	Amstel	Van der Madeweg	Amstel	40	22	73	16445
189	Spaklerweg	Van der Madeweg	Amstel	Van der Madeweg	40	22	50	15693
190	VU	Boshuizenstraat	Station Zuid	Oranjebaan	40	16	54	6528
191	Boshuizenstraat	Uilenstede	Station Zuid	Oranjebaan	40	16	54	5817
192	Uilenstede	Kronenburg	Station Zuid	Oranjebaan	40	16	54	5648
193	Kronenburg	Zonnestein	Station Zuid	Oranjebaan	40	16	54	4825
194	Zonnestein	Onderuit	Station Zuid	Oranjebaan	40	16	54	4468
195	Onderuit	Oranjebaan	Station Zuid	Oranjebaan	40	16	54	4042
196	Oranjebaan	Binnenhof	Oranjebaan	Binnenhof	40	8	47	669
197	Oranjebaan	Ouderkerkerlaan	Oranjebaan	Westwijk	40	8	40	2513
198	Ouderkerkerlaan	Sportlaan	Oranjebaan	Westwijk	40	8	26	1975
199	Overamstel	Van der Madeweg	Station Zuid	Van der Madeweg	40	11	49	12391

200	Sportlaan	Meent	Oranjebaan	Westwijk	40	8	26	1445
201	Van der Madeweg	Duivendrecht	Van der Madeweg	Gein	40	22	56	14099
202	Van der Madeweg	Venserpolder	Van der Madeweg	Gaasperplas	40	11	43	5112
203	Venserpolder	Diemenz	Van der Madeweg	Gaasperplas	40	11	24	5050
204	Diemenz	Verrijn Stuartweg	Van der Madeweg	Gaasperplas	40	11	24	3349
205	Verrijn Stuartweg	Ganzenhoefstation	Van der Madeweg	Gaasperplas	40	11	24	1695
206	Ganzenhoefstation	Kraaiennest	Van der Madeweg	Gaasperplas	40	11	24	1037
207	Meent	Brink	Oranjebaan	Westwijk	40	8	26	1142
208	Brink	Poortwachter	Oranjebaan	Westwijk	40	8	26	976
209	Poortwachter	Sacharovlaan	Oranjebaan	Westwijk	40	8	26	468
210	Sacharovlaan	Westwijk	Oranjebaan	Westwijk	40	8	26	298
211	Duivendrecht	Strandvliet	Van der Madeweg	Gein	40	22	50	11219
212	Strandvliet	Bijlmer	Van der Madeweg	Gein	40	22	50	10998
213	Bijlmer	Bullewijk	Van der Madeweg	Gein	40	22	50	6397
214	Bullewijk	Holendrecht	Van der Madeweg	Gein	40	22	50	4483
215	Kraaiennest	Gaasperplas	Van der Madeweg	Gaasperplas	40	11	24	233
216	Holendrecht	Reigersbos	Van der Madeweg	Gein	40	22	50	1835
217	Reigersbos	Gein	Van der Madeweg	Gein	40	22	50	989
218	Frederiksplein	Weesperplein	Frederiksplein	Weesperplein	60	16	122	9199
219	Frederiksplein	Vijzelgracht	Frederiksplein	Vijzelgracht	60	16	139	9393
220	Dam	Rokin	Dam	Rokin	40	26	178	3320
221	Dam	Nieuwezijds Kolk	Dam	CS	60	46	192	8649
222	Dam	Spui	Dam	Leidseplein	40	20	152	7126
223	Dam	Westermarkt	Dam	Rozengracht	60	34	191	7286
224	CS	Nieuwezijds Kolk	CS	Dam	60	46	209	8659
225	CS	Rokin	CS	Rokin	40	15	195	18053
226	CS	Muziekgeb. Bimhuis	CS	Rietlandpark	40	15	143	4098
227	CS	Dam	CS	Dam	60	18	251	4176
228	Waterlooplein	Weesperplein	Waterlooplein	Weesperplein	40	22	99	14815
229	Weesperplein	Wibautstraat	Weesperplein	Wibautstraat	40	22	103	17011
230	Wibautstraat	Camperstraat	Wibautstraat	Wijttenbachstraat	40	8	58	896

231	Wibautstraat	Amstel	Wibautstraat	Amstel	40	22	93	17740
232	Wibautstraat	Woustraat	Wibautstraat	Woustraat	40	8	72	2162
233	Ceintuurbaan	Vijzelgracht	Ceintuurbaan	Vijzelgracht	40	15	153	20705
234	Ceintuurbaan	Europaplein	Ceintuurbaan	Station Zuid	40	15	78	20523
235	Europaplein	Station Zuid	Ceintuurbaan	Station Zuid	40	15	58	19533
236	Station Zuid	Amstelveenseweg	Station Zuid	Amstelveenseweg	40	11	116	20049
237	Station Zuid	RAI	Station Zuid	Van der Madeweg	40	11	70	13285
238	Station Zuid	VU	Station Zuid	Oranjebaan	40	16	79	7296
239	Sloterdijk	De Vlugtlaan	Sloterdijk	De Vlugtlaan	40	11	92	6397
240	De Vlugtlaan	Burg. Fockstraat	De Vlugtlaan	Burg. Röellstraat	40	8	82	412
241	Jan van Galenstraat	De Vlugtlaan	Jan van Galenstraat	De Vlugtlaan	40	11	86	8455
242	Jan van Galenstraat	Burg. Rendorpstraat	Jan van Galenstraat	Burg. Röellstraat	40	8	53	1120
243	Rokin	Vijzelgracht	Rokin	Vijzelgracht	40	15	142	19307
244	Lelylaan	Heemstedestraat	Lelylaan	Heemstedestraat	40	11	101	17162
245	Amstelveenseweg	Henk Sneevlietweg	Amstelveenseweg	Heemstedestraat	40	11	82	19392
246	RAI	Overamstel	Station Zuid	Van der Madeweg	40	11	36	12526

Table 39 Overview of link attributes for all links (2/2)

	Link Nodes		Link attributes					
Link Number	Name	Name	L-space Link Betweenness Centrality Indicator (-)	L-space link Closeness Centrality Indicator (-)	L-space Local clustering coefficient (-)	L-space link degree centrality indicator (-)	Sum of service frequency saturation of feeding links (-)	Mean speed on link (km/hour)
1	Zijlplein	Dr. H. Colijnstraat	0,009259	0,000101	0,333333	3	0,2	24,61765
2	Burg. Van Leeuwenlaan	Dr. H. Colijnstraat	0,018432	0,000104	0,333333	3	0,2	16,01299
3	Burg. Van Leeuwenlaan	Burg. Röellstraat	0,027519	0,000113	0,333333	3	0,2	15,47191
4	Plein 40-45/Burg. De Vlugtlaan	Lod. van Deysselstraat	0,017808	9,99E-05	0,1	6	0,225	19,31429
5	Plein 40-45/Burg. De Vlugtlaan	Burg. Eliasstraat	0,025861	0,000103	0,1	6	0,225	18,48387
6	Burg. Röellstraat	Sloterpark	0,008893	0,00011	0,333333	3	0,2	14,67568
7	Burg. Röellstraat	Lod. van Deysselstraat	0,01096	0,00011	0,1	6	0,225	22,02273
8	Burg. Röellstraat	Burg. Rendorpstraat	0,037382	0,000115	0,1	6	0,225	23,39189
9	Burg. Fockstraat	Burg. Eliasstraat	0,034259	0,000105	0,1	6	0,225	18
10	Kingsfordweg	Haarlemmerweg	0,017614	0,000115	0,166667	4	0,3875	16,60645
11	Kingsfordweg	Sloterdijk	0,010228	0,000135	0,166667	4	0,2875	21,46835
12	Haarlemmerweg	Wiltzanghlaan	0,024763	0,000118	0,166667	4	0,3875	18,53226
13	Wiltzanghlaan	Bos en Lommerweg	0,032127	0,000121	0,166667	4	0,3875	14,57851

14	Bos en Lommerweg	Rijpstraat	0,039406	0,000124	0,166667	4	0,3875	18,27632
15	Bos en Lommerplein	De Vlugtlaan	0,041064	0,000138	0,333333	5	0,27	18,50746
16	Jan van Galenstraat	Rijpstraat	0,046684	0,000127	0,166667	4	0,3875	14,65487
17	Jan van Galenstraat	Jan Evertsenstraat	0,053833	0,000146	0,166667	4	0,3875	17,56098
18	Van Hallstraat	v. Limburg Sirumstraat	0,009259	0,000118	0	3	0,466667	21,1875
19	v. Limburg Sirumstraat	De Wittenkade	0,018432	0,000121	0	3	0,466667	10,51546
20	Nassaukade	Frederik Hendrikplantsoen	0,03652	0,00013	0	3	0,466667	21,27273
21	De Wittenkade	Nassaukade	0,027519	0,000125	0	3	0,466667	14,08475
22	Frederik Hendrikplantsoen	Hugo de Grootplein	0,045435	0,000135	0	3	0,466667	21,68
23	Marnixplein	Bloemgracht	0,03652	0,000144	0	3	0,577778	19,02439
24	Hugo de Grootplein	De Clercqstraat	0,052412	0,000164	0	3	0,466667	17,56291
25	Zoutkeetsgracht	Haarlemmerplein	0,009259	0,000128	0	3	0,577778	10,78924
26	Haarlemmerplein	Nw. Willemsstraat	0,018432	0,000133	0	3	0,577778	19,11429
27	Nw. Willemsstraat	Marnixplein	0,027519	0,000138	0	3	0,577778	15,50847
28	Muziekgeb. Bimhuis	Jan Schaeferbrug	0,065461	0,000133	0,095238	8	0,401042	20,45455
29	Jan Schaeferbrug	Rietlandpark	0,057494	0,000139	0,095238	8	0,401042	29,22772
30	Azartplein	C. van Eesterenlaan	0,009259	0,000119	0	3	0,338889	20,16867
31	Prinsengracht	Frederiksplein	0,026314	0,00017	0,2	5	0,286667	12,2381
32	Ecuplein	Baden Powellweg	0,03652	0,000108	0	2	0,4	17,52212
33	Ecuplein	Inarisstraat	0,027519	0,000105	0	2	0,4	21,216
34	Surinameplein	Overtoomsesluis	0,093906	0,000142	0,166667	5	0,28	10,34783
35	Surinameplein	Derkinderenstraat	0,132063	0,000141	0,1	5	0,33	30
36	Surinameplein	Corantijnstraat	0,046727	0,000143	0	5	0,3	16,21875
37	Hoekenes	Osdorpplein	0,027519	0,000106	0	2	0,425	16,15385
38	Hoekenes	Baden Powellweg	0,018432	0,000104	0	2	0,425	28,14706
39	Baden Powellweg	Dijkgraafplein	0,009259	0,000101	0	2	0,425	26,41379
40	Osdorpplein	Ruimzicht	0,03652	0,000109	0	2	0,425	17,95238
41	Ruimzicht	Meer en Vaart	0,044143	0,000125	0	2	0,425	16,7
42	Jan Voermanstraat	Jan Tooropstraat	0,054694	0,00012	0,166667	5	0,27	11,94175
43	Jan Voermanstraat	Adm. Helfrichstraat	0,059819	0,000123	0,166667	5	0,27	24,35821
44	Jan Tooropstraat	Jan van Galenstraat	0,04832	0,000144	0,166667	5	0,27	19,51402

45	Jan van Galenstraat	Mercatorplein	0,053661	0,000131	0,333333	5	0,27	8,093617
46	Jan van Galenstraat	Bos en Lommerplein	0,046856	0,000118	0,333333	5	0,27	10,82243
47	Adm. Helfrichstraat	Mercatorplein	0,065332	0,000134	0,166667	5	0,27	15,6087
48	Mercatorplein	Marco Polostraat	0,123859	0,000134	0,333333	4	0,3	20,11475
49	Marco Polostraat	Jan Evertsenstraat	0,12963	0,000147	0,333333	4	0,3	14,55446
50	Jan Evertsenstraat	Willem de Zwijgerlaan	0,180405	0,000151	0	5	0,32	14,46429
51	Willem de Zwijgerlaan	De Clercqstraat	0,185724	0,000167	0	5	0,32	17,89595
52	De Clercqstraat	Bilderdijkstraat	0,062145	0,000172	0,133333	6	0,35	15,68675
53	De Clercqstraat	Rozengracht	0,163738	0,000189	0,066667	6	0,388889	13,29282
54	Rozengracht	Bloemgracht	0,044251	0,000183	0	3	0,577778	16,62162
55	Rozengracht	Elandsgracht	0,171361	0,000189	0,2	5	0,403333	16,88276
56	Ten Katestraat	Bilderdijkstraat	0,069703	0,000157	0	5	0,3	15,37226
57	Ten Katestraat	J.P. Heyestraat	0,064621	0,00014	0	5	0,3	14,36735
58	Postjesweg	Corantijnstraat	0,051615	0,000131	0	5	0,3	15
59	Postjesweg	Witte de Withstraat	0,05562	0,000135	0	5	0,3	16,67485
60	Witte de Withstraat	J.P. Heyestraat	0,060099	0,000137	0	5	0,3	10,73684
61	Bilderdijkstraat	Elandsgracht	0,064621	0,000186	0,3	5	0,323333	14,43956
62	Bilderdijkstraat	1e Con. Huygensstraat	0,066279	0,000168	0,2	6	0,233333	17,3871
63	Elandsgracht	Leidseplein	0,14565	0,000207	0,2	6	0,386111	22,09249
64	Van Baerlestraat	Museumplein	0,210896	0,000183	0,1	5	0,346667	22,05882
65	Van Baerlestraat	Hobbemastraat	0,207407	0,000187	0	7	0,328571	22,68293
66	Van Baerlestraat	C. Schuytstraat	0,09059	0,000181	0,066667	6	0,336111	18,83871
67	Van Baerlestraat	1e Con. Huygensstraat	0,067937	0,000189	0,1	6	0,327778	17,34715
68	Overtoomsesluis	Rhijnvis Feithstraat	0,099354	0,000141	0,166667	5	0,28	15,21429
69	Rhijnvis Feithstraat	J.P. Heyestraat	0,104608	0,000144	0,166667	5	0,28	16,39726
70	J.P. Heyestraat	1e Con. Huygensstraat	0,109948	0,000165	0,166667	5	0,28	22,40625
71	1e Con. Huygensstraat	Leidseplein	0,101507	0,000206	0,133333	7	0,342857	16,36364
72	Westermarkt	Rozengracht	0,121232	0,000186	0,133333	7	0,511905	16,2381
73	Mr. Visserplein	Waterlooplein	0,045112	0,000149	0,1	6	0,377778	14,46429
74	Mr. Visserplein	Plantage Kerklaan	0,041193	0,000134	0,1	6	0,377778	18,28571
75	Spui	Koningsplein	0,081051	0,00016	0,190476	8	0,460417	12,94737

76	Koningsplein	Keizersgracht	0,082903	0,000161	0,190476	8	0,460417	7,826087
77	Muntplein	Rembrandtplein	0,064083	0,000162	0,333333	4	0,375	12,36242
78	Muntplein	Rokin	0,080448	0,000175	0,166667	5	0,41	9,14094
79	Muntplein	Keizersgracht	0,031869	0,00016	0,1	7	0,369048	13,84314
80	Rembrandtplein	Waterlooplein	0,041021	0,000156	0,2	5	0,44	13,30233
81	Rembrandtplein	Keizersgracht	0,027519	0,000147	0,2	5	0,286667	11,2766
82	Prinsengracht	Keizersgracht	0,085444	0,000163	0,190476	8	0,460417	8,459016
83	Prinsengracht	Leidseplein	0,087575	0,0002	0,190476	8	0,460417	10,09859
84	Keizersgracht	Vijzelgracht	0,03202	0,000197	0,1	7	0,369048	16,53333
85	Keizersgracht	Prinsengracht	0,027132	0,000132	0,2	5	0,286667	9,966102
86	Plantage Kerklaan	Plantage Badlaan	0,038049	0,000131	0,1	6	0,377778	15,12397
87	Leidseplein	Spiegelgracht	0,159281	0,000202	0,107143	9	0,338889	19,23664
88	Leidseplein	Hobbemastraat	0,210314	0,000204	0	7	0,328571	18,07101
89	Spiegelgracht	Vijzelgracht	0,157709	0,000198	0,107143	9	0,338889	15,11278
90	Stadhouderskade	Woustraat	0,025452	0,00015	0,2	6	0,255556	13,87261
91	Stadhouderskade	Frederiksplein	0,029888	0,000171	0,2	6	0,255556	7,304348
92	K. 's Gravesandestraat	Alexanderplein	0,146469	0,000163	0,1	6	0,377778	15,54331
93	K. 's Gravesandestraat	Weesperplein	0,154866	0,000174	0,1	6	0,377778	22,01538
94	Beukenweg	Camperstraat	0,031934	0,000136	0,066667	6	0,4	12,24
95	Beukenweg	Wijttenbachstraat	0,025797	0,000147	0,066667	6	0,4	19,30435
96	1e Leeghwaterstraat	1e Coehoornstraat	0,041602	0,000133	0,133333	6	0,308333	17,06667
97	1e Leeghwaterstraat	Rietlandpark	0,031611	0,000134	0,133333	6	0,308333	16,21519
98	Rietlandpark	C. van Eesterenlaan	0,017463	0,00013	0	3	0,338889	17,2973
99	Rietlandpark	Zuiderzeeweg	0,061499	0,000132	0	3	0,280556	40,85813
100	1e Coehoornstraat	Hoogte Kadijk	0,049742	0,000136	0,133333	6	0,330556	3,918367
101	Hoogte Kadijk	Alexanderplein	0,055965	0,000159	0,133333	6	0,330556	20,45963
102	Alexanderplein	Plantage Badlaan	0,035314	0,000159	0,1	6	0,377778	14,97345
103	Alexanderplein	1e v. Swindenstraat	0,123277	0,00016	0,133333	6	0,261111	15,63934
104	Dapperstraat	Wijttenbachstraat	0,05422	0,000142	0	3	0,216667	19,48673
105	Muiderpoort	Dapperstraat	0,045435	0,00013	0	3	0,216667	16,63043
106	Muiderpoort	Molukkenstraat	0,03652	0,000127	0	3	0,35	20,75676

107	Soembawaweg	Insulindeweg	0,018432	0,000109	0	3	0,35	21,46479
108	Molukkenstraat	Soembawaweg	0,027519	0,000112	0	3	0,35	14,31933
109	Insulindeweg	Flevopark	0,009259	0,000106	0	3	0,35	30
110	1e v. Swindenstraat	Wijttenbachstraat	0,119294	0,000145	0,133333	6	0,261111	12,32143
111	Wijttenbachstraat	Pretoriusstraat	0,078682	0,000143	0	3	0,366667	21,07317
112	Zuiderzeeweg	Strandeiland	0,054264	0,000117	0	3	0,280556	27,2129
113	Steigereiland	Vennepluimstraat	0,03652	0,000108	0	3	0,280556	27,66279
114	Steigereiland	Strandeiland	0,045435	0,000113	0	3	0,280556	30,39
115	Ruisrietstraat	IJburg	0,009259	9,94E-05	0	3	0,280556	30,77143
116	Ruisrietstraat	Diemerparklaan	0,018432	0,000102	0	3	0,280556	22,68571
117	Vennepluimstraat	Diemerparklaan	0,027519	0,000104	0	3	0,280556	23,07692
118	De Aker	Pilatus	0,009259	9,97E-05	0	2	0,4	13,3125
119	Inarisstraat	Pilatus	0,018432	0,000102	0	2	0,4	27,28696
120	Baden Powellweg	De La Sallestraat	0,045435	0,000112	0	2	0,4	24,11392
121	Sloterweg	Antwerpenbaan	0,027519	0,000111	0	3	0,25	21,53846
122	Sloterweg	Aletta Jacobslaan	0,03652	0,000114	0	3	0,25	16,88889
123	Louis Davidsstraat	Meer en Vaart	0,061951	0,000123	0	2	0,4	17,67568
124	Louis Davidsstraat	De La Sallestraat	0,054264	0,000115	0	2	0,4	23,69748
125	Kasterleepark	Antwerpenbaan	0,018432	0,000108	0	3	0,25	18,67925
126	Kasterleepark	Osdorperweg	0,009259	0,000105	0	3	0,25	26,62185
127	Johan Huizingalaan	Aletta Jacobslaan	0,045435	0,000118	0	3	0,25	23,67188
128	Johan Huizingalaan	Heemstedestraat	0,051163	0,000151	0	3	0,25	19,82432
129	Delfland	Westlandgracht	0,062877	0,000127	0,066667	6	0,336111	19,36364
130	Delfland	Heemstedestraat	0,055922	0,000154	0,066667	6	0,336111	20,928
131	Hoofddorpplein	Westlandgracht	0,067873	0,00013	0,066667	6	0,336111	15,67241
132	Hoofddorpplein	Zeilstraat	0,073127	0,000132	0,066667	6	0,336111	19,54412
133	Zeilstraat	Valeriusplein	0,078768	0,000137	0,066667	6	0,336111	18,0355
134	Valeriusplein	C. Schuytstraat	0,084711	0,000143	0,066667	6	0,336111	16,79426
135	Haarlemmermeerstation	Valeriusplein	0,037274	0,000129	0,166667	4	0,433333	23,08696
136	Haarlemmermeerstation	Stadionplein	0,030986	0,00013	0,166667	4	0,433333	21,26897
137	Valeriusplein	Emmastraat	0,043562	0,000132	0,166667	4	0,433333	20,32759

138	Emmastraat	J. Obrechtstraat	0,049806	0,000136	0,166667	4	0,433333	19,83333
139	J. Obrechtstraat	Museumplein	0,055706	0,000174	0,166667	4	0,433333	17,34653
140	Museumplein	Roelof Hartplein	0,163157	0,000178	0,166667	4	0,433333	17,95489
141	Ijsbaanpad	Amstelveenseweg	0,03174	0,000152	0,1	5	0,27	13,77551
142	Ijsbaanpad	Stadionplein	0,038394	0,000127	0,1	5	0,27	21,66667
143	Olympiaweg	Stadionplein	0,017851	0,000131	0,333333	4	0,3	12,54545
144	Olympiaweg	Olympiaplein	0,02373	0,000122	0,333333	4	0,3	26,976
145	Olympiaplein	Minervanplein	0,029974	0,000125	0,333333	4	0,3	17,03077
146	Stadionweg	Gerrit v.d. Veenstraat	0,043152	0,000149	0,333333	4	0,3	13,38889
147	Stadionweg	Minervanplein	0,03652	0,000128	0,333333	4	0,3	15,96429
148	Prinses Irenestraat	Station Zuid	0,067614	0,000189	0,133333	6	0,320833	17,28497
149	Albert Cuypstraat	Ceintuurbaan	0,029307	0,000189	0,107143	10	0,323333	10,69014
150	Albert Cuypstraat	Vijzelgracht	0,031115	0,000197	0,107143	10	0,323333	16,50794
151	Roelof Hartplein	Gerrit v.d. Veenstraat	0,115159	0,000172	0,333333	4	0,4	16,36098
152	Roelof Hartplein	Ceintuurbaan	0,131029	0,000197	0,133333	7	0,335714	16,09692
153	2e v.d. Helstraat	Ceintuurbaan	0,055426	0,000189	0,066667	8	0,29375	10,09655
154	2e v.d. Helstraat	Woustraat	0,052476	0,000154	0,066667	8	0,29375	18,3375
155	Woustraat	Lutmastraat	0,024354	0,000149	0,166667	5	0,24	14,12658
156	Cornelis Troostplein	Ceintuurbaan	0,045995	0,000187	0	7	0,335714	10,2
157	Cornelis Troostplein	Scheldestraat	0,041322	0,000138	0	7	0,335714	16,22069
158	Lutmastraat	Jozef Israelkade	0,018454	0,00012	0,166667	5	0,24	13,78512
159	Scheldestraat	Maasstraat	0,034733	0,000134	0	7	0,335714	14,94545
160	Maasstraat	Waalstraat	0,028531	0,000132	0	7	0,335714	19,08571
161	Waalstraat	Victorieplein	0,022459	0,000135	0	7	0,335714	20,44444
162	Victorieplein	Jozef Israelkade	0,012295	0,000135	0,166667	5	0,24	15,41538
163	Victorieplein	Amsteldijk	0,021038	0,000132	0,333333	4	0,375	20,04444
164	Gerrit v.d. Veenstraat	Prinses Irenestraat	0,073342	0,000149	0,133333	6	0,320833	19,94191
165	Amsteldijk	Amstel	0,013587	0,000145	0,333333	4	0,375	20,15455
166	Pretoriusstraat	Hogeweg	0,071662	0,000129	0	3	0,366667	16,93846
167	Hogeweg	Hugo de Vrieslaan	0,063006	0,000125	0	3	0,366667	17,72368
168	Hugo de Vrieslaan	Kruislaan	0,054264	0,00012	0	3	0,366667	18,65806

169	Kruislaan	Brinkstraat	0,045435	0,000116	0	3	0,366667	21,58475
170	Brinkstraat	Arent Krijtstraat	0,03652	0,00011	0	3	0,366667	17,83333
171	Arent Krijtstraat	Schoolstraat	0,027519	0,000105	0	3	0,366667	13,29897
172	Schoolstraat	Lublinkstraat	0,018432	0,000103	0	3	0,366667	21,23316
173	Vumc	VU medisch centrum	0,009259	0,000113	0	3	0,316667	15,69231
174	VU medisch centrum	Amstelveenseweg	0,018131	0,00015	0	3	0,316667	13,33945
175	Lublinkstraat	Diemen	0,009259	9,89E-05	0	3	0,366667	13,80645
176	Buikslotermeerplein	Van Hasseltweg	0,009259	0,000183	0,166667	5	0,473333	45,24891
177	Van Hasseltweg	CS	0,018432	0,000199	0,166667	5	0,473333	45,78541
178	Isolatorweg	Sloterdijk	0,009022	0,000134	0	2	0,2375	50,8125
179	Postjesweg	Lelylaan	0,053553	0,000166	0,133333	6	0,341667	28,22222
180	Postjesweg	Jan van Galenstraat	0,046619	0,000152	0,133333	6	0,341667	35,51852
181	Derkinderenstraat	Lelylaan	0,126529	0,000162	0,1	5	0,33	26,59794
182	Johan Huizingalaan	Lelylaan	0,125754	0,00016	0	5	0,33	26,85714
183	Johan Huizingalaan	Piet Wiedijkstraat	0,121792	0,000131	0	5	0,33	31,76471
184	Piet Wiedijkstraat	Meer en Vaart	0,110401	0,000127	0	5	0,33	28,26446
185	Nieuwmarkt	CS	0,046059	0,000187	0,142857	8	0,405208	40,89908
186	Nieuwmarkt	Waterlooplein	0,046555	0,000149	0,142857	8	0,405208	24,48
187	Henk Sneevlietweg	Heemstedestraat	0,074074	0,000164	0	6	0,291667	29,0365
188	Spaklerweg	Amstel	0,09031	0,000143	0	5	0,41	30,04167
189	Spaklerweg	Van der Madeweg	0,083096	0,000155	0	5	0,41	41,00518
190	VU	Boshuizenstraat	0,121792	0,000125	0	6	0,254167	32,30769
191	Boshuizenstraat	Uilenstede	0,113652	0,000119	0	6	0,254167	16,57143
192	Uilenstede	Kronenburg	0,105426	0,000116	0	6	0,254167	26,94118
193	Kronenburg	Zonnestein	0,097115	0,000112	0	6	0,254167	22,04762
194	Zonnestein	Onderuit	0,088717	0,000109	0	6	0,254167	22,75
195	Onderuit	Oranjebaan	0,073062	0,000107	0	6	0,254167	24,37795
196	Oranjebaan	Binnenhof	0,009259	0,000104	0	2	0,3	30,32
197	Oranjebaan	Ouderkerkerlaan	0,051572	0,000117	0	2	0,3	33,10345
198	Ouderkerkerlaan	Sportlaan	0,054264	0,000113	0	2	0,3	33
199	Overamstel	Van der Madeweg	0,044875	0,000155	0	7	0,375	38,25

200	Sportlaan	Meent	0,045435	0,000109	0	2	0,3	38,80473
201	Van der Madeweg	Duivendrecht	0,063006	0,00015	0	3	0,366667	33,38961
202	Van der Madeweg	Venserpolder	0,053359	0,00015	0	3	0,458333	43,11429
203	Venserpolder	Diemenz	0,045435	0,000114	0	3	0,458333	31,52055
204	Diemenz	Verrijn Stuartweg	0,03652	0,00011	0	3	0,458333	31,56164
205	Verrijn Stuartweg	Ganzenhoefstation	0,027519	0,000107	0	3	0,458333	29,34783
206	Ganzenhoefstation	Kraaiennest	0,018432	0,000104	0	3	0,458333	35,44785
207	Meent	Brink	0,03652	0,000105	0	2	0,3	25,4359
208	Brink	Poortwachter	0,027519	0,000102	0	2	0,3	29,87838
209	Poortwachter	Sacharovlaan	0,018432	9,93E-05	0	2	0,3	31,82143
210	Sacharovlaan	Westwijk	0,009259	9,59E-05	0	2	0,3	40,53488
211	Duivendrecht	Strandvliet	0,054264	0,000138	0	3	0,366667	27,74436
212	Strandvliet	Bijlmer	0,045435	0,000133	0	3	0,366667	33,09934
213	Bijlmer	Bullewijk	0,03652	0,000127	0	3	0,366667	28,53333
214	Bullewijk	Holendrecht	0,027519	0,000123	0	3	0,366667	37,62069
215	Kraaiennest	Gaasperplas	0,009259	0,0001	0	3	0,458333	41,15094
216	Holendrecht	Reigersbos	0,018432	0,000118	0	3	0,366667	39,6
217	Reigersbos	Gein	0,009259	0,000112	0	3	0,366667	46,03125
218	Frederiksplein	Weesperplein	0,160745	0,00018	0,2	6	0,372222	27,48252
219	Frederiksplein	Vijzelgracht	0,158872	0,000199	0,142857	8	0,285417	20,52229
220	Dam	Rokin	0,080685	0,000192	0,2	7	0,504762	14,38278
221	Dam	Nieuwezijds Kolk	0,074182	0,000175	0	9	0,443519	8,757225
222	Dam	Spui	0,079931	0,000177	0,190476	8	0,460417	14,125
223	Dam	Westermarkt	0,116774	0,000181	0,133333	7	0,511905	10,75781
224	CS	Nieuwezijds Kolk	0,067959	0,00019	0	9	0,443519	20,46067
225	CS	Rokin	0,023213	0,000204	0,066667	8	0,505208	40,02128
226	CS	Muziekgeb. Bimhuis	0,061003	0,000185	0,095238	8	0,401042	26,55224
227	CS	Dam	0,011542	0,000201	0	9	0,547222	13,48553
228	Waterlooplein	Weesperplein	0,052218	0,000177	0,1	6	0,355556	31,60274
229	Weesperplein	Wibautstraat	0,0764	0,00018	0,2	6	0,338889	32,70968
230	Wibautstraat	Camperstraat	0,038221	0,000155	0,066667	6	0,4	11,89381

231	Wibautstraat	Amstel	0,087489	0,000156	0,1	5	0,38	35,16867
232	Wibautstraat	Woustraat	0,04677	0,000163	0,2	6	0,316667	16,2619
233	Ceintuurbaan	Vijzelgracht	0,03904	0,000213	0,107143	10	0,288333	31,44828
234	Ceintuurbaan	Europaplein	0,052864	0,000205	0,071429	9	0,302778	39,56757
235	Europaplein	Station Zuid	0,030922	0,00021	0,071429	9	0,302778	45,86266
236	Station Zuid	Amstelveenseweg	0,065633	0,000198	0,047619	7	0,332143	37,68966
237	Station Zuid	RAI	0,057709	0,000191	0	7	0,375	38,22472
238	Station Zuid	VU	0,105426	0,000188	0	6	0,254167	22,28125
239	Sloterdijk	De Vlugtlaan	0,01938	0,000139	0,3	5	0,23	38,16949
240	De Vlugtlaan	Burg. Fockstraat	0,041301	0,000135	0,1	6	0,225	18,30508
241	Jan van Galenstraat	De Vlugtlaan	0,031331	0,000143	0	6	0,225	31,2
242	Jan van Galenstraat	Burg. Rendorpstraat	0,044552	0,000142	0,1	6	0,225	21,4125
243	Rokin	Vijzelgracht	0,031804	0,000202	0,133333	8	0,372917	37,3526
244	Lelylaan	Heemstedestraat	0,066645	0,000171	0	6	0,341667	28,25373
245	Amstelveenseweg	Henk Sneevlietweg	0,074828	0,000164	0	6	0,291667	49,56044
246	RAI	Overamstel	0,052821	0,000144	0	7	0,375	49,29368

Appendix F Effect size of Determinants

In the Table 40 below the absolute difference between the lowest addition from the determinant and the highest addition from the determinant to the Link Criticality value is given. The normalized difference is also given, this is calculated as the absolute difference divided by the minimum absolute difference. The same is done for the Degrading Rapidity indicator in Table 41.

Table 40 Effect sizes of determinants on Link Criticality

Link Criticality		L-space Link Betweenness Centrality Indicator	L-space link Closeness Centrality Indicator	L-space Local clustering coefficient	L-space link degree centrality indicator	Sum of service frequency saturation of feeding links	Mean speed on link	Total frequency of traversing lines	P-space link degree centrality indicator	Total bi-directional flow on link
Min		8,89E-03	9,59E-05	0,000	2,000	0,200	3,92	8	12	0
Max		2,11E-01	2,13E-04	0,333	10,000	0,578	50,81	46	251	20705
Total Initial	Absolute difference	-	-	1,07E+05	-	-	2,98E+05	-	5,31E+05	7,96E+05
	Normalized difference	-	-	1	-	-	3	-	5	7
Total Filtered	Absolute difference	-	-	1,14E+05	1,82E+05	1,42E+05	1,86E+05	3,76E+05	2,62E+05	8,06E+05
	Normalized difference	-	-	1	2	1	2	3	2	7
Partial Initial	Absolute difference	7,95E+03	-	-	-	-	1,30E+05	1,88E+05	1,69E+05	5,62E+05
	Normalized difference	1	-	-	-	-	16	24	21	71
Partial filtered	Absolute difference	-	5,41E+04	-	-	-	1,43E+05	-	1,10E+05	4,46E+05
	Normalized difference	-	1	-	-	-	3	-	2	8

Table 41 Effect sizes of determinants on Degrading Rapidity

Degrading Rapidity		L-space Link Betweennesss Centrality Indicator	L-space link Closeness Centrality Indicator	L-space Local clustering coefficient	L-space link degree centrality indicator	Sum of service frequency saturation of feeding links	Mean speed on link	Total frequency of traversing lines	P-space link degree centrality indicator	Total bi-directional flow on link
Min		8,89E-03	9,59E-05	0,000	2,000	0,200	3,92	8	12	0
Max		2,11E-01	2,13E-04	0,333	10,000	0,578	50,81	46	251	20705
Total Initial	Absolute difference	0,180	0,679	0,092	-	-	-	0,304	-	0,403
	Normalized difference	2	7	1	-	-	-	3	-	4
Total Filtered	Absolute difference	0,189	0,422	0,105	-	-	-	-	-	0,158
	Normalized difference	2	4	1	-	-	-	-	-	2
Partial Initial	Absolute difference	0,154	0,464	-	0,144	-	-	0,304	-	0,476
	Normalized difference	1	3	-	1	-	-	2	-	3
Partial filtered	Absolute difference	0,134	0,207	0,061	0,088	-	-	-	-	0,196
	Normalized difference	2	3	1	1	-	-	-	-	3