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A REVIEW OF ANALYTICAL MODELS FOR DETERMINING THE BEHAVIOR OF METALLIC TUBULAR STRUCTURES SUBMITTED TO AXIAL CRUSHING

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ABSTRACT

This paper reviews analytical models proposed by Abramowicz et al.[1, 2] and Stefan et al.[3] for the axial crushing of metallic tubular structures with square and circular cross-sections. First, a database of experiments for square and circular tubes was created based on the literature. Subsequently, the predictions obtained using these analytical models were compared against the database of experiments to determine the accuracy of these analytical models. The database of experiments was also compared against some results generated using Finite-Element Method (FEM). Furthermore, the sensitivity of the analytical models to various material and geometrical parameters was studied to determine the influence of these parameters on the mean crushing force.

Both models were found to be highly sensitive to the thickness of the tubular structures. It was observed that the models proposed by Abramowicz et al.[1, 2] showed better agreement with experimental results, although an over-prediction was observed for square tubular structures made of materials with a significant difference between the values of yield and ultimate stress. The model proposed by Stefan et al.[3] slightly over-predicted for square tubes and showed reasonable accuracy for circular tubes.

Keywords: Axial crushing, Metallic tubes, Mean crushing force

NOMENCLATURE

c	Side length for square tubes
r	Radius of circular tube
t, h	Wall thickness of a tube
P_m	Mean crushing force
δ_{eff}	Effective crushing length
σ_y	Yield stress
σ_u	Ultimate stress
ε_u	Ultimate strain
σ_0	Characteristic stress
R	Mean radius of circular tube
$2H$	Initial distance between plastic hinges at top and bottom of a basic folding element (Fig. 12, Reference [1])
κ	Crushing efficiency
N_c	Number of corners
L_e	Flange length
θ_i	Internal angle (Fig. 2, Reference [3])
θ_e	External angle (Fig. 2, Reference [3])

1. INTRODUCTION

By 2050, the aviation sector aims to achieve net-zero carbon emissions [4]; which would require new aircraft designs with increased aerodynamic efficiency and sustainable propulsion technologies (SAF, hydrogen, and electric). Such aerodynamically efficient designs tend to have an ovalized fuselage structure, a departure from the jetliners of the present day. Ovalization decreases the available crushing distance between the passenger floor and the ground [5], making the crashworthiness design for these aircraft significantly challenging. Additionally, historical crash data for such aircraft is not yet available. Such a shift to new disruptive aircraft configurations makes it essential to develop new methods for determining crashworthiness in the preliminary design phase, to avoid drastic design changes at later stages of the design process. Disruptive aircraft designs such as Flying-V also pose other significant challenges in terms of crashworthiness. For example, in the case of bird strike crashworthiness for Flying-V, the passengers seated next to the windows are at an increased risk of impact as compared to conventional aircraft due to Flying-V's configuration [6].

Axial crushing is one of the significant energy-absorbing mechanisms in aircraft structures. A typical force versus displacement graph for the axial crushing of a metallic tubular structure consists of 3 phases. In the first phase, the reaction force increases sharply with respect to displacement till a peak force value is reached, followed by a transition phase in which the reaction force undergoes a progressive decline. After the transition phase, the tube enters a region of progressive folding. For the region of progressive folding (Figure 1), the energy absorbed by the structure can be approximated by multiplying the crushed distance with the mean value of the collapse load.

Analytical models that can predict the collapse load are essential to account for the energy absorbed by vertical struts subjected to axial crushing in the preliminary design phase. Several analytical models that deal with the axial crushing of thin-walled tubular structures exist in the literature. Alexander[7] proposed a solution for collapse load based on the final deformed shape of the structure, Abramowicz et al.[1, 2] developed a kinematical method to analyze thin-walled structures, Wierzbicki et al. [8] developed a method for the progressive crushing of circular tubes using super-folding elements, Singace et al. [9] worked on the eccentricity factor used by Wierzbicki et al. [8] and proposed a new equation for the mean force, and recently, Stefan et al. [3] proposed a model which gives a global expression of mean collapse load for thin-walled tubes with a varying number of sides and also for star-shaped cross sections. Since these models propose different expressions for mean force, it is necessary to revisit them in order to identify which of these are better suited for the purpose of design for crashworthiness in the preliminary design phase.

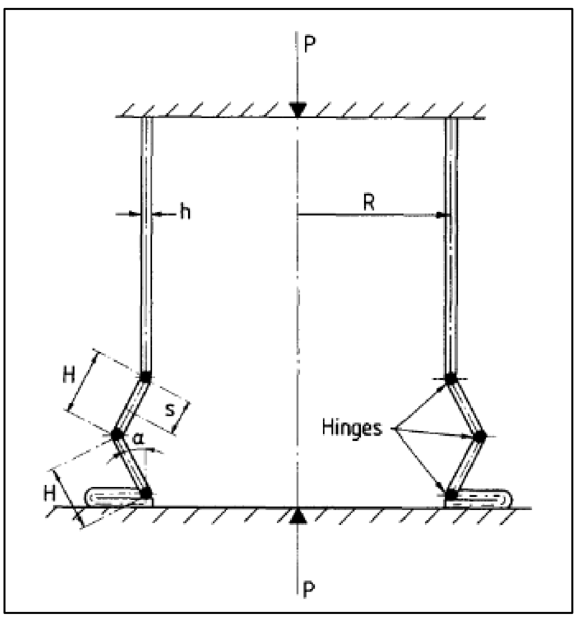

FIGURE 1: PROGRESSIVE FOLDING OF A METALLIC TUBE SUBJECTED TO AN AXIAL CRUSHING LOAD [2]

Comparisons of these models have been made with experimental results in the past, but these have been primarily done as a part of the study conducted to propose these models and sometimes the comparisons deal with a single material. Therefore, it is quite hard to use the available results for analyzing the robustness of analytical models. Since we require to use these models for preliminary structural design, we decided to revisit the comparison of these models with experimental data.

Because such a study for all the available models will consume a significant amount of time, we narrowed down our scope to two models for this work. The first set of models that we considered was proposed by Abramowicz et al. [1, 2] in 1984, these models were chosen because they take a kinematical approach toward the calculation of collapse load, considering the continuity conditions at the propagating hinge lines. The meticulous examination of the folding element undertaken in such an approach should result in a high degree of accuracy in predicting the mean collapse load. The second model that we considered was proposed by Stefan et al. [3] in 2019. The model proposed by Stefan et al. [3] was chosen because it provides a global expression for various cross-sections. This capability to model geometries of varied sizes and shapes is of significant interest to us since we want to use analytical models in the preliminary design phase, at which point the geometry and materials are generally not well defined.

The expressions for collapse load calculations for the models from Abramowicz et. al [1,2] and Stefan et al. [3] are presented below:

Expression of mean collapse load by Abramowicz et al. [1] for square tubes:

$$\frac{P_m}{M_0}.\kappa = 17.76\left(\frac{b}{h}\right) + 4\pi\left(\frac{c}{h}\right)\left(\frac{h}{H}\right) + 9.184\left(\frac{H}{h}\right)\left(\frac{h}{b}\right) \tag{1}$$

Where: $\frac{b}{h} = 0.72\left(\frac{c}{h}\right)^{\frac{1}{3}}$, $\frac{H}{h} = 0.99\left(\frac{c}{h}\right)^{\frac{2}{3}}$ and $M_0 = \frac{\sigma_0 h^2}{4}$

Expression of mean collapse load by Abramowicz et al. [2] for circular tubes:

$$\frac{P_m}{M_0}.\kappa = 2\sqrt{3}\pi\left(\frac{R}{h}\right)\left(\frac{H}{R}\right) + \frac{2}{\sqrt{3}}\pi\left(\frac{R}{h}\right)\left(\frac{H}{R}\right)^2 + 2\pi^2\left(\frac{R}{H}\right) + 2\pi \tag{2}$$

Where: $\frac{H}{R} = \left\{\frac{\pi h}{\sqrt{3}R}\left[1 + \frac{2}{3}\left(\frac{H}{R}\right)\right]^{-1}\right\}^{\frac{1}{2}}$ and $M_0 = \frac{2\sigma_0 h^2}{4\sqrt{3}}$

Generalized expression for calculation of collapse load by Stefan et al. [3]:

$$P_m.\kappa = \sigma_0.t.N_c.\sqrt{[2.\pi - 2(\theta_i + \theta_e)].\pi.L_e.t} \tag{3}$$

The notations used in the above equations (equation 1-3) are described under the section titled “Nomenclature” at the start of this paper.

2. MATERIALS AND METHODS

The datasets for comparison with analytical results were compiled based on the literature (Section 2.1). Following the compilation of the datasets, the material parameters used for the analytical calculations were obtained from the respective literature sources. In cases where the material parameters were not explicitly stated, the provided graphs were digitized and used to extract the required parameters (Section 2.2 and Table 3). A sensitivity analysis of the models was then carried out to better understand the influence of various parameters on the mean force. (Section 3.1). Finally, a comparison was done between the experimental dataset and the obtained analytical results for both square and circular tubular structures (Section 3.2).

Furthermore, a comparison between some experimental and numerical results was also carried out for square tubular structures (Section 3.3). For the Finite Element Method, the simulations were performed in Abaqus using S4R shell elements. The model (Figure 2) was composed of a tube sandwiched between two rigid planes. Displacement along the tube's axis was applied to the top rigid plane, while the bottom plane was encastred. An imperfection was applied to the tube geometry for triggering the progressive failure mechanism. The reaction forces were measured using a reference node placed on the top plate.

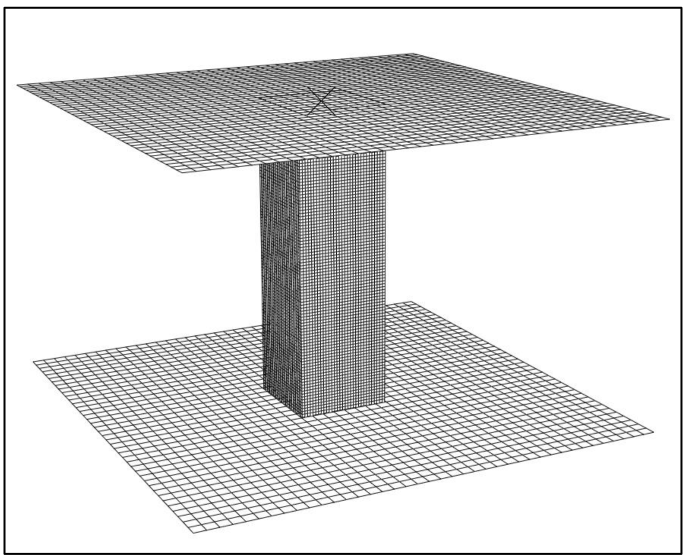

FIGURE 2: FINITE ELEMENT MODEL SETUP AND MESH

2.1 Experimental dataset

To determine the accuracy of these analytical models, the obtained results must be compared with data generated from physical experiments. Since, experimentation is both time taking and costly, for the purpose of this work, data from literature has instead been used. The experimental datasets for square and circular tubular structures have been presented in Table 1 and Table 2.

TABLE 1: EXPERIMENTAL DATASET FOR SQUARE TUBULAR STRUCTURES

S.No.	Reference	Material	c (mm)	t (mm)	P_m (kN)
1	Zhao et al. [10]	Brass (Cu 64 – Zn 36)	35	1.5	18
2			35	1.5	18.09
3	Zhang et al. [11]	Mild steel Q235	40	1.2	16.01
4			40	1.2	16.72
5			40	1.2	15.6
6	DiPaolo et al. [12]	A36	50	1.4	25.5
7			50	1.4	25.7
8		A513	50	1.51	36.5
9			50	1.51	37
10		AISI 316	50	1.47	46.2
11			50	1.47	46.2
12			50	1.49	46.6
13		AISI 304 (S2)	50	1.49	44.6
14			50	1.49	44
15		AISI 304 (S3)	50	1.49	50.9
16			50	1.49	51
17		AISI 304 (S4)	50	1.49	50.6
18			50	1.49	49.3
19			50	1.49	49.2
20	Abramowicz et al. [1]	Mild Steel	49.3	1.625	35.28
21			49.34	1.635	36.71
22			37.11	1.145	20.75
23			37.11	1.145	17.9
24			37.1	1.155	19.75
25			37.06	1.175	18.5

2.2 Parameters for analytical models

In this section, the parameters used in the analytical models for the calculation of mean force are listed along with a short description. These parameters can essentially be divided into 2 categories, geometrical parameters, and material parameters:

A. Geometrical parameters:
   a. Side length of square tube / Radius of circular tube
   b. Wall thickness of the tube

B. Material parameters:
   a. <u>Effective crushing length:</u>
   The effective crushing length for a tube can be given by equation-4:

TABLE 2: EXPERIMENTAL DATASET FOR CIRCULAR TUBULAR STRUCTURES

S.No.	Reference	Material	r (mm)	t (mm)	P_m (kN)
1	Bardi et al. [13]	AL6061-T6	15.28	1.26	21.97
2			15.08	1.66	33.99
3		AL6260-T4	28.94	2.04	42.61
4	Stefan et al. [3]	AA6063-T6	20	1.23	15.8
5			19.5	0.9	8.74
6	Guillow et al. [14]	AA6060-T5	48.95	1.9	43
7			48.5	1	15.4
8			48.5	1	15.8
9			48.5	1	16.25
10			48.95	1.9	44
11			48.75	1.5	30.18
12			48.3	0.56	5.75
13			48.5	1	15.72
14			48.25	0.54	5.06
15			48.15	0.26	1.66
16			48.1	0.22	1.1
17			48.15	0.31	1.84
18			48.15	0.29	1.825
19			48.15	0.31	2
20			48.25	0.52	5.06
21			24.2	1.35	18.7
22			24.2	1.35	17.5
23			24.2	1.35	19
24			24.2	1.35	18.5
25			24.15	1.35	18.75
26			15.2	1.59	24
27			14.5	2.95	63.1
28			28.95	1.92	36.9
29			28.25	0.57	4.5
30			28.55	1.15	15.33
31			28.55	1.15	15.1
32			28.05	0.31	1.64
33			48.1	0.22	1.07

$$\delta_{\text{eff}} = \frac{\text{crushed length}}{\text{original length}} \times 100 \quad (4)$$

b. Characteristic stress:

Characteristic stress or flow stress with respect to the axial crushing of metallic tubular structures is the value of stress required to keep plastically deforming the material. Determining the characteristic stress is very necessary for the crushing calculation for metallic structures. In literature, various expressions for characteristic stress (denoted by σ_0) have been provided:

i. Wierzbicki et al. [8] :

$$\sigma_0 = 0.92\sigma_u \quad (5)$$

ii. Stefan [3]:

$$\sigma_0 = \frac{1}{\varepsilon_u} . \int_0^{\varepsilon_u} \sigma . d\varepsilon \quad (6)$$

iii. Wierzbicki [15]:

$$\sigma_0 = 0.70\sigma_u \quad (7)$$

iv. Yang and Caldwell[16], Zhang and Zhang [17], Langseth and Hopperstad[18], and Hanssen et al. [19]:

$$\sigma_0 = \frac{\sigma_y + \sigma_u}{2} \quad (8)$$

v. Alexander [7]:

$$\sigma_0 = \frac{2}{\sqrt{3}} \sigma_y \quad (9)$$

For the purpose of this work, the characteristic stress has been calculated using Equation-5 for the models proposed by Abramowicz et al. [1, 2] and using Equation-6 for the model proposed by Stefan et al. [3]. The material properties used have been presented in Table 3.

3. RESULTS AND DISCUSSIONS

In the following sub-sections, the sensitivity analysis for the analytical models has been presented (section-3.1), followed by the comparison of analytical calculations with the experimental dataset (section-3.2). Subsequently, a comparison between numerical and experimental results has also been presented (section-3.3).

3.1 Sensitivity analysis

Although these models provide a detailed mathematical treatment of the problem, they are still prone to errors from multiple sources. An inaccuracy in the input parameters as compared to reality can lead to an error in the mean force estimation. Such inaccuracies can be rooted in measurement errors, manufacturing defects, as well as approximation errors.

Since we have used experimental data collected from the literature, it was prudent to analyze the sensitivity of the analytical models to the input parameters.

TABLE 3: YIELD, ULTIMATE AND CHARACTERISTIC STRESS VALUES USED IN THE CALCULATIONS FOR THE MATERIALS PRESENTED IN TABLE-1 AND TABLE-2.

			σ_0	
	σ_y	σ_u	(Eq. 5)	(Eq. 6)
Material	**Unit: MPa**			
Brass (Cu64-Zn36)	173.0	430.0	395.6	291.5
Mild steel Q235	218.5	323.3	297.4	305.5
A36	340.0	364.0	334.9	358.0
A513	400.0	448.0	412.2	431.9
AISI 316	468.0	679.0	624.7	637.2
AISI 304 (S2)	380.0	683.0	628.4	598.1
AISI 304 (S3)	420.0	740.0	680.8	659.9
AISI 304 (S4)	439.0	761.0	700.1	677.1
Mild Steel	264.5	328.5	302.2	311.4
AL6061-T6	300.8	348.3	320.5	328.4
AL6260-T4	128.0	272.8	251.0	200.4*
AA6063-T6	170.0	222.0	204.2	190.7
AA6060-T5	180.0	212.0	195.0	199.1
*Since, stress versus strain curve for this material has not been provided in the reference, the equivalent σ_0 has instead been calculated using Equation-8.				

To perform the sensitivity analysis, each input parameter was varied by +5% and -5% (keeping other parameters constant) to quantify the effect on mean force prediction. The observed results are presented in the following figures (Figure 3 - Figure 6).

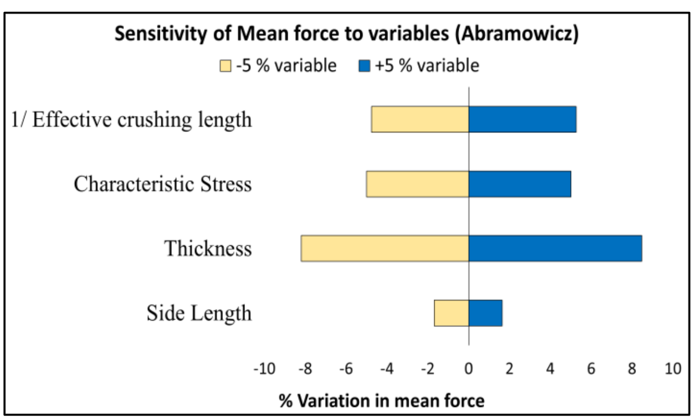

FIGURE 3: SENSITIVITY OF ANALYTICAL MODEL FOR SQUARE TUBES PROPOSED BY ABRAMOWICZ ET AL. [1] TO GEOMETRICAL AND MATERIAL PARAMETERS

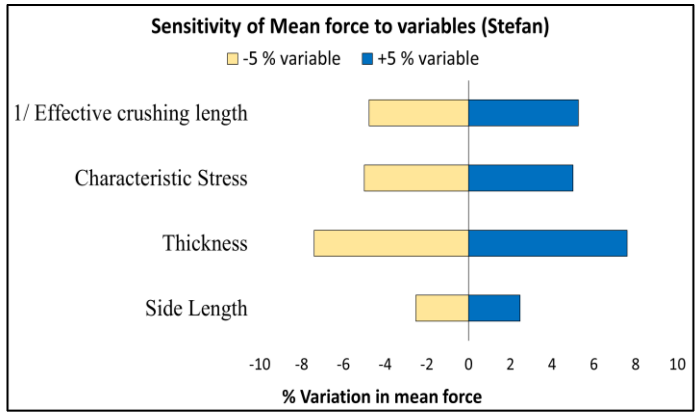

FIGURE 4: SENSITIVITY OF ANALYTICAL MODEL FOR SQUARE TUBES PROPOSED BY STEFAN ET AL. [3] TO GEOMETRICAL AND MATERIAL PARAMETERS

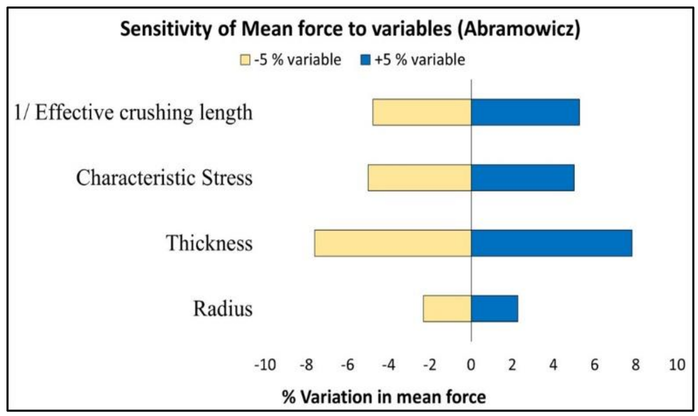

FIGURE 5: SENSITIVITY OF ANALYTICAL MODEL FOR CIRCULAR TUBES PROPOSED BY ABRAMOWICZ ET AL.[2] TO GEOMETRICAL AND MATERIAL PARAMETERS

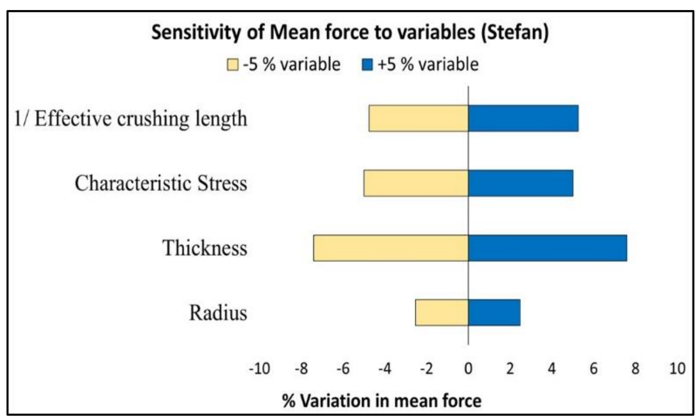

FIGURE 6: SENSITIVITY OF ANALYTICAL MODEL FOR CIRCULAR TUBES PROPOSED BY STEFAN ET AL.[3] TO GEOMETRICAL AND MATERIAL PARAMETERS

We observed that the mean force for all the models was very sensitive to the *"thickness"* of metallic tubes. A ±5% change in thickness results in about ±8% change in the value of mean force. For *"1/Effective crush length"* and *"Characteristic stress"*, we observed an almost proportional change in mean force as compared to the input. Lastly, we observed that the sensitivity of mean force prediction to *"side length/radius"* was relatively low. A ±5% change in *"side length/radius"* results in a change of about ±2% in the value of mean force.

The high sensitivity of mean force value to thickness indicates that manufacturing defects and measurement errors can have a huge impact on the comparison between experiments and results obtained from the analytical models. Since the dimensions used in this study are taken from the literature, we are unable to comment on the manufacturing and measurement accuracy of tubes used in these experimental studies. It is also important to emphasize that characteristic stress is a function of the stress-strain curve. And therefore, the accuracy of mean force calculations using analytical models has a strong dependence on the accurate measurement of stresses and strains during the experiments.

Based on the above observations, due care must be taken while using these analytical models for structural design. The user of these analytical models must always remember that small deviations in input data can lead to significant differences between experimental and analytical results.

3.2 Comparison with experimental results

In this section, the results from the analytical models are compared with the experimental dataset collected from the literature. Figure 7 and Figure 8 present the comparison for the square tubes, while Figure 9 and Figure 10 present the comparison for the circular tubes. For Figure 7 to Figure 11, the straight line indicates the ideal situation where the experimental value is equal to the value obtained using analytical or numerical methods.

From the comparison with experimental data, we observe that the models proposed by Abramowicz et al. [1, 2] do a good job of predicting the mean force for both square and circular tubular structures. Although for the case of square tubes, the model should be used with caution as an over-prediction is observed at loads greater than 40kN (Materials AISI-304 and AISI-316), for the dataset obtained from DiPaolo et al. [12]. Since, the characteristic stress used for the model of Abramowicz et al. [1] is given by equation-5 ($0.92 \times \sigma_{ultimate}$), a possible explanation for this over-prediction could be the big difference between yield and ultimate stress for AISI-304 and AISI-316. Therefore, using a different value of characteristic stress for such materials may yield better results.

The model proposed by Stefan et al. [3] slightly over-predicts in the case of square tubular structures when compared against the experimental results. We observed that better predictions can be obtained when the estimated mean crushing force values are scaled by a factor. The value of this factor has not been reported as the dataset used in this work is a relatively small one and further investigation in this direction is required. The model from Stefan et al.[3] gave reasonably accurate predictions for the case of circular tubes, which combined with its ability to predict the mean crushing force for other polygonal and star-shaped cross sections, makes the model really useful when a lot of different geometries need to be considered during the design process.

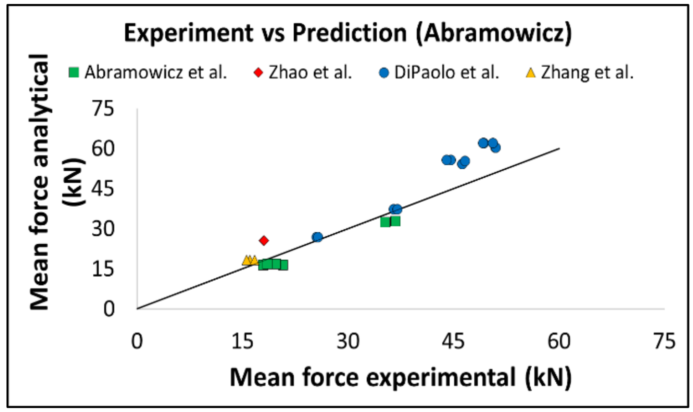

FIGURE 7: ANALYTICAL (ABRAMOWICZ ET AL.[1]) VS EXPERIMENTAL MEAN FORCE PREDICTION FOR SQUARE TUBES

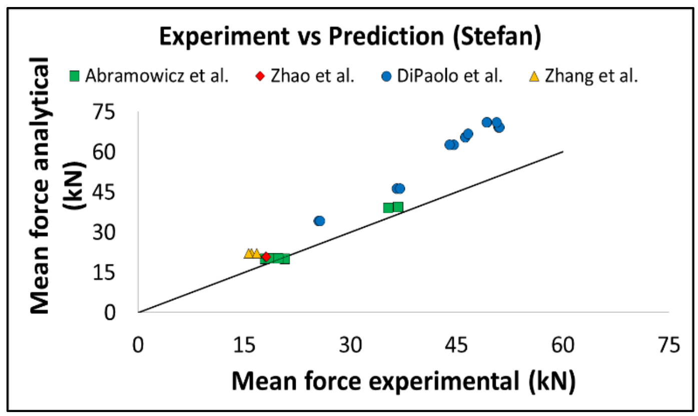

FIGURE 8: ANALYTICAL (STEFAN ET AL.[3]) VS EXPERIMENTAL MEAN FORCE PREDICTION FOR SQUARE TUBES

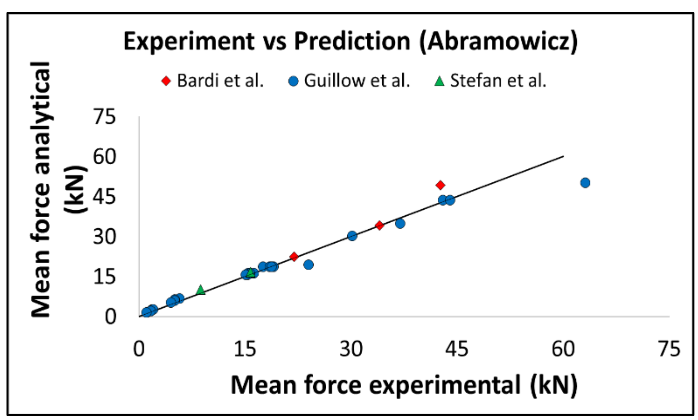

FIGURE 9: ANALYTICAL (ABRAMOWICZ ET AL.[2]) VS EXPERIMENTAL MEAN FORCE PREDICTION FOR CIRCULAR TUBES

Further work on this topic is envisaged including but not limited to expanding the dataset, comparison of the dataset with other proposed models and comparison of results with more data generated through numerical analysis of axial crushing for various cross-sectional shapes.

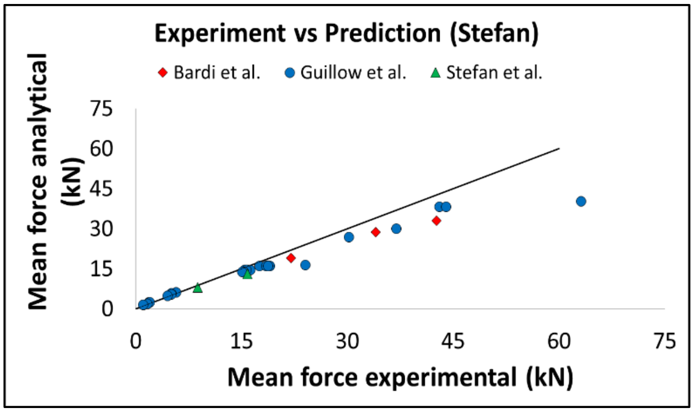

FIGURE 10: ANALYTICAL (STEFAN ET AL.[3]) VS EXPERIMENTAL MEAN FORCE PREDICTION FOR CIRCULAR TUBES

3.3 Comparison of experimental data with finite element

A comparison of experimental results with some Finite Element Method (FEM) results for square tubes was also done (Figure 11). Since crushing simulations using shell elements are usually employed to determine the value of mean force. The idea was to examine the trend between Analytical and Experimental results in comparison to the trend observed between FEM and experimental results. We observed these trends to be similar which increases the desirability of using analytical models while making design decisions in the early design phases.

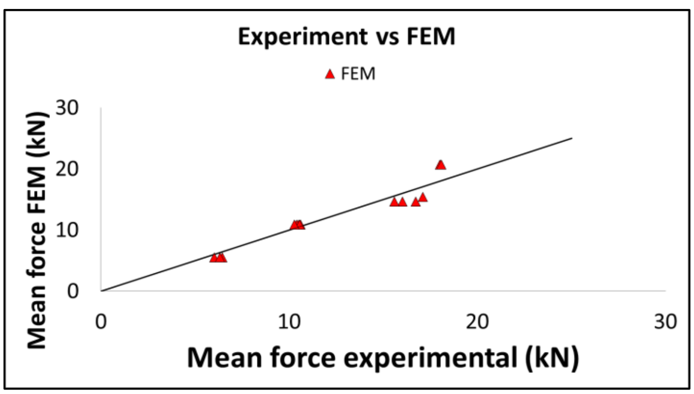

FIGURE 11: FEM VERSUS EXPERIMENTAL MEAN FORCE PREDICTION FOR SQUARE TUBES

4. CONCLUSION

The models proposed by Abramowicz et al.[1, 2] show better agreement with the experiments for the dataset used in the present work. For our experimental dataset, the model from Stefan et al. [3] slightly over-predicts the mean force values for square tubes but shows promising results when the estimated mean force values are scaled by a factor. For the case of circular tubes, the model from Stefan et al. [3] gives reasonably accurate predictions. Both these models should be employed with care as the results tend to be very sensitive to the input parameters.

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REFERENCES

[1] Abramowicz, W. and N. Jones, *Dynamic axial crushing of square tubes.* International Journal of Impact Engineering, 1984. **2**(2): p. 179-208.

[2] Abramowicz, W. and N. Jones, *Dynamic axial crushing of circular tubes.* International Journal of Impact Engineering, 1984. **2**(3): p. 263-281.

[3] Tabacu, S. and C. Ducu, *An analytical solution for the estimate of the mean crushing force of structures with polygonal and star-shaped cross-sections subjected to axial load.* International Journal of Mechanical Sciences, 2019. **161-162**: p. 105010-105010.

[4] *Our Commitment to Fly Net Zero by 2050, International Air Transport Association, https://www.iata.org/en/programs/environment/flynetzero/.*

[5] Oosterom, W.J., *Flying-V Family Design*, 2021, Delft University of Technology, Netherlands

[6] Chen, S., W. van de Waerdt, and S.G.P. Castro, *Design for bird strike crashworthiness using a building block approach applied to the Flying-V aircraft*. 2022, Engineering Archive (www.EngrXiv.org).

[7] Alexander, J.M., *An approximate analysis of the collapse of thin cylindrical shells under axial loading.* The Quarterly Journal of Mechanics and Applied Mathematics, 1960. **13**(1): p. 10-15.

[8] Wierzbicki, T., et al., *Alexander revisited—A two folding elements model of progressive crushing of tubes.* International Journal of Solids and Structures, 1992. **29**(24): p. 3269-3288.

[9] Singace, A.A., H. Elsobky, and T.Y. Reddy, *On the eccentricity factor in the progressive crushing of tubes.* International Journal of Solids and Structures, 1995. **32**(24): p. 3589-3602.

[10] Zhao, H., S. Abdennadher, and R. Othman, *Experimental study of square tubes crushing under impact loading using a modified SHPB.* International Journal of Impact Engineering - INT J IMPACT ENG, 2006. **32**: p. 1174-1189.

[11] Zhang, X. and H. Zhang, *Experimental and numerical investigation on crush resistance of polygonal columns and angle elements.* Thin-Walled Structures, 2012. **57**: p. 25-36.

[12] DiPaolo, B.P. and J.G. Tom, *A study on an axial crush configuration response of thin-wall, steel box components: The quasi-static experiments.* International Journal of Solids and Structures, 2006. **43**(25-26): p. 7752-7775.

[13] Bardi, F.C., H.D. Yun, and S. Kyriakides, *On the axisymmetric progressive crushing of circular tubes under axial compression.* International Journal of Solids and Structures, 2003. **40**(12): p. 3137-3155.
[14] Guillow, S.R., G. Lu, and R.H. Grzebieta, *Quasi-static axial compression of thin-walled circular aluminium tubes.* International Journal of Mechanical Sciences, 2001. **43**(9): p. 2103-2123.
[15] Wierzbicki, T., *Crushing analysis of metal honeycombs.* International Journal of Impact Engineering, 1983. **1**(2): p. 157-174.
[16] Yang, P.D.C. and J.B. Caldwell, *Collision energy absorption of ships' bow structures.* International Journal of Impact Engineering, 1988. **7**(2): p. 181-196.
[17] Zhang, X. and H. Zhang, *Axial crushing of circular multi-cell columns.* International Journal of Impact Engineering, 2014. **65**: p. 110-125.
[18] Langseth, M. and O.S. Hopperstad, *Static and dynamic axial crushing of square thin-walled aluminium extrusions.* International Journal of Impact Engineering, 1996. **18**(7-8): p. 949-968.
[19] Hanssen, A.G., M. Langseth, and O.S. Hopperstad, *Static crushing of square aluminium extrusions with aluminium foam filler.* International Journal of Mechanical Sciences, 1999. **41**(8): p. 967-993.