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## **ORIGINAL ARTICLE**



# Joint scheduling of vessels and vessel service providers for enhancing the efficiency of the port call process

 $Shahrzad \, Nikghadam^1 \cdot Ratnaji \, Vanga^2 \cdot Jafar \, Rezaei^2 \cdot Lori \, Tavasszy^2$ 

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# Abstract

As ports are experiencing heavier traffic, the pressure to improve port call processes is increasing. Port call optimization (PCO) is one of these improvement initiatives, enabling the arrival of vessels to the port just-in-time when the vessel services, like pilotage, towage, and mooring, are all readily available. Otherwise, vessels that sailed at full speed to arrive at the port may have to wait, idling at anchorage, occupying space, burning fuel, and leading to increased congestion. One of the main challenges in the implementation of PCO is determining the time at which availability of these services can be guaranteed. The paper addresses this challenge by presenting a model that jointly schedules vessels and service providers. It extends the current approaches to allow application to larger and busier ports, where repositioning times for pilots and tugboats is highly variable and vessels experience waiting times between services. The problem is formulated as a mixed-integer linear programming one and is modelled in continuous time. We test alternative scheduling strategies using three different objective functions, based on the current 'first-come-first-serve' approach, a minimal level of service, and the best capacity utilization. The model is applied on data made available by the Port of Rotterdam, and it provides a full-service schedule for vessels and service providers.

Keywords Scheduling  $\cdot$  Port call optimization  $\cdot$  Port management  $\cdot$  Port services  $\cdot$  Technical-nautical services  $\cdot$  Mathematical modelling

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# 1 Introduction

As global trade continues to grow, maritime traffic is on the rise, leading to increasingly busy ports (UNCTAD 2022). Larger vessels are now calling at ports more frequently, occupying more resources for longer periods (UNCTAD 2022). As a result, the conventional practice of accommodating vessels on a first-comefirst-serve basis is becoming increasingly challenging to maintain. Generally, when confirming requests from vessels for arrival or departure, ports assume that vessel services, such as pilotage and towage, will be available at the requested time. However, simultaneous arrival of several vessels at port may create a peak demand for pilots and tugboats. As these resources are finite and cannot be dimensioned only for peak demands, long waiting times become inevitable under a 'first-come-first-served' practice (Lind et al. 2023). Previous studies show that peak demand for pilots and tugboats can reach up to 200% and 600% of normal demand, respectively (Nikghadam et al. 2023). In these peak periods, the absence of appropriate scheduling has negative implications for the efficient use of resources. Service providers that need to rush to deliver their services in peak demands will have idle resources in low demand periods. In addition, when resources are not available, vessels that sailed full speed to arrive at the port may now have to wait, idling at anchorage or terminals, occupying space, burning fuel, and leading to increased congestion. Therefore, the conventional way of working (first-come-first-served) has several shortcomings from a safety, environmental, and economic perspective.

As ports are experiencing heavier traffic, the need to optimize port call procedures is increasing. Port call optimization (PCO) is one of the initiatives undertaken by maritime stakeholders to address this need (IMO 2020). PCO concerns the efficient planning of port call processes, with the goal of reducing vessel waiting times at ports. It ensures that all relevant parties facilitate an efficient port call, from when vessels depart from their previous port of call until they arrive at their destination port, complete their cargo operations, and leave the port again. The benefits of PCO are numerous. Besides the cost-saving benefits and environmental sustainability of the slow steaming experienced by the vessels and shipping companies, PCO also benefits the other actors within the port including terminals, port authorities, and service providers (IMO 2020). It leads to timely port services, more efficient utilization of service providers' resources, improved terminal planning, reductions in greenhouse gas emissions, and shortening the waiting times of vessels at ports (IMO 2020). Analysis undertaken by Maritime Strategies International (ABS 2020) shows that annual CO<sub>2</sub> emission savings amount to approximately 11%. As an example, for a container ship calling at the Port of Rotterdam, PCO can lead to significant fuel savings and a 15% reduction in call time (IMO 2020).

A crucial aspect of PCO is the proactive involvement of port authorities in guiding the timely arrival and departure of vessels. If port authorities can assess the vessels' *requested times*, prior to their actual arrival, for providing them with a feedback based on port's resource availabilities, vessels can adjust their speed

to arrive just-in-time (JIT) when their resource availabilities are guaranteed. This proactive involvement of port authorities allows vessels to arrive and depart when all the required resources, such as pilot, tugboats, and boatmen, are readily available, based on the eventual *scheduled time*, when service providers have confirmed their availability. One of the key challenges ports face in the implementation of PCO is determining the time based on which they can guarantee resource availabilities. It is challenging because a full-service schedule must be created for multiple service providers. Also, the servicing sequence of vessels must be decided based on expected arrivals and available resources. Therefore, ports need tools to help them simultaneously schedule vessels and service providers.

Until recently, the scheduling literature in the port context has focussed on individual port resources, like tugboat and pilot scheduling under fixed vessel arrival times. For example, Wei et al. (2020) determine tugboat schedules such that the time tugboats travel back and forth between their assignments are minimized. Pilot scheduling studies minimize pilotage service, repositioning, and delay costs for pilot organizations see e.g. Wu et al. (2020). The only study that has recently considered pilotage and towage services together to schedule vessels is by Abou Kasm et al. (2021). Their results showed that significant improvements can be obtained by scheduling of vessels given service providers' resource constraints. Interestingly, however, the study assumed that vessels are already waiting to be serviced. Thus, they will always experience a waiting time. This will put more pressure on the resource scheduling process than with proactive scheduling, where incoming vessels can slow down to arrive at their scheduled times. In addition, their study involved some non-trivial assumptions, which made it difficult to apply the model to larger and busy ports. Here, there is an increased chance of waiting times for vessels during the services, when the successive services are needed. Also, because of the stronger spatial dispersion of port, the time taken by the service provider's resources to move between assignments may show large variations. Finally, the objective of scheduling was limited to minimizing the longest waiting times, while alternative scheduling strategies could also be interesting. Especially a strategy to minimize the total waiting times of vessels for services would, by definition, provide more relief to the overall port, and deserves to be studied. Particularly in busy ports, the effect of this strategy on overall throughput is expected to be relevant.

In short, especially for large and busy ports, appropriate tools are lacking to design joint and proactive port call schedules for vessels and service providers. Our study addresses this gap. We propose an extended optimization model and study alternative objectives, testing these for the illustrative case of the large and busy port of Rotterdam.

The remainder of this paper is organized as follows; Section 2 reviews the relevant literature. Section 3 briefly specifies the system on which the model is built. Section 4 presents the solution of the model for a practical example and discusses the results. Section 5 presents the main findings and recommendations.

# 2 Literature review

The question of efficient scheduling of resources for port vessel services has been addressed by several scholars. Decisions considered by these studies include tactical capacity decisions as well as operational deployment decisions for the service providers (Lee and Song 2017). Their scheduling objectives are diverse, including minimizing the sum or maximum of port stay time, waiting time, handling time, service completion time, or delayed departures of vessels. One of the pioneering scheduling studies was on tugboat fleet management by Jaikumar and Solomon (1987). This study minimized the number of tugboats required to serve a given number of vessels in the port. Later tugboat scheduling studies aimed to minimize a variety of objectives such as the latest completion time of all services, total waiting time of vessels, or total towage operation costs including repositioning and penalty costs (Ilati et al. 2014; Wang et al. 2014; Wei et al. 2021). A variety of modelling techniques, such as integer programming, mixed-integer programming (Kang et al. 2020), and mixed-integer non-linear programming has been proposed. Various heuristic or metaheuristic approaches have been developed (Wang et al. 2014), some focussed on towage services for container terminals, and others on inland barge operations (Zhen et al. 2018). While earlier tugboat scheduling studies were deterministic, the latest one incorporates various uncertainties (Kang et al. 2020). In another study, Wei et al. (2021) argue that it is unrealistic to assume that all towage requests are known beforehand and propose a model that dynamically updates tugboat schedules.

Unlike the extensive research on tugboat scheduling, pilot scheduling studies are limited. This limited attention may be because pilot organizations are less efficiency-driven than tugboat companies (Nikghadam et al. 2022). Pilot organizations in many ports are associations of self-employed pilots that have a monopolistic position and a public mandate to provide pilotage services. Two out of three pilot scheduling studies focus on minimizing costs for the pilot organization. Wu et al. (2020) proposed a model in which they minimize the total pilotage costs which consist of delay, service, and repositioning costs. Jia et al. (2020) integrate pilot scheduling into vessel traffic management. Their model schedules pilots, considering the utilization of fairways, anchorage areas, and terminal basins as constraints of the model. Similar to Wu et al. (2020), their model minimizes the total pilotage delay, service, and repositioning costs. Additionally, they consider the costs of unsatisfied vessel service requests (Jia et al. 2020). Besides these cost minimization models, another study by Lorenzo-Espejo et al. (2021) proposes a model that configures extended breaks for pilots given their off-day preferences and labour regulations.

In the above-mentioned resource scheduling studies, the attention was on the supply side of the port services assuming fixed vessel arrival and departure times. In those studies, the decision to be made was to determine which resource to be deployed to which vessel, assuming that vessels should be serviced according to a prespecified order. Previous studies have considered different objectives in their modelling, which were mainly related to the service providers' interests

such as maximizing tugboat utilization, minimizing (total or maximum) towage service times, or minimizing total pilotage costs. However, there has been limited attention to the demand side of the services i.e. scheduling vessels. Only a recent study by Abou Kasm et al. (2021) considers pilot and tugboat resource capacity constraints to schedule the vessels. Their model shows that vessel scheduling is crucial for improved resource utilization and customer satisfaction. However, the model has some limitations which are particularly relevant for larger, busier ports. First, the waiting times between subsequent services are ignored. In large ports, these waiting times can be significant. In the Port of Rotterdam, for instance, 63% of the incoming vessels wait for towage, after the pilot has boarded with waiting times of up to 40 min (Molkenboer 2020). A second limitation is that the repositioning times of pilots and tugboats between assignments are assumed to be constant. However, in larger ports, this time is highly variable as the spatial dispersion of assignments also varies. A third issue closely related to the above is the discretized treatment of time in their model. An advantage of a discrete representation of time is that it reduces the complexity of the solution approach. However, as inter-service waiting times and variations in repositioning times are introduced, time intervals in the optimization need to be very short, which again increases complexity. A continuous time approach is much more refined, increases the solution space, and gives more accurate results. Such a model has not yet been studied in this context, however.

Given the above-mentioned gaps in the literature, our study contributes with an optimization model which schedules vessels together with vessel service providers. The proposed model considers the proactive involvement of port authorities in the port call process by assessing the requested times, thus providing vessels with feedback on when vessel services would be available. The model is continuous in time, and it considers inter-service waiting times and sequence-dependent repositioning time of resources. In the next section, the system is described which is the basis for the modelling problem.

# **3** Problem definition

In this section, we further explain the characteristics of the problem and present the details of operations, services, service providers, and resources.

Consider that V vessels are calling the port at any time period. They can be either incoming or outgoing vessels. Incoming vessels enter the port from the sea and sail towards their allocated berth. Outgoing vessels are located at berth, sail out of the port, and travel towards their next destinations. Both these sets of vessels may require assistance while travelling through congested, often narrow channels, thus requiring the assistance of services such as pilotage, towage, and mooring for safe passage. Different service providers offer these services, namely pilot organization, tugboat company, and boatmen organization. Each service provider has several resources to provide their service. We assume that each service provider has only one primary type of resource. Pilots, tugboat fleets, and boatmen teams are considered the main type of resources for the pilot organization, tugboat company, and boatmen organization, respectively. We also assume that pilots, tugboat fleet, and boatmen crew, are identical. For example, we assumed that every pilot could serve any vessel. As such, we simplified the use of pilotage certificates and tugboats bollard pull, since we do not expect them to significantly alter the conclusions drawn from our findings. Each vessel, either incoming or outgoing, goes through j operations. The sequence of operations for incoming and outgoing vessels are illustrated in Fig. 1a and b, respectively.

For an incoming vessel, the sequence of operations is as follows: When an incoming vessel is still at sea, the vessel (agent) sends an estimated time of arrival at the pilot boarding place, also called ETA-pilot boarding place. This time is the requested starting time ( $R_i$ ) for incoming vessels. When the vessel arrives at the port, it asks the PA for clearance and the vessel's captain starts communicating with

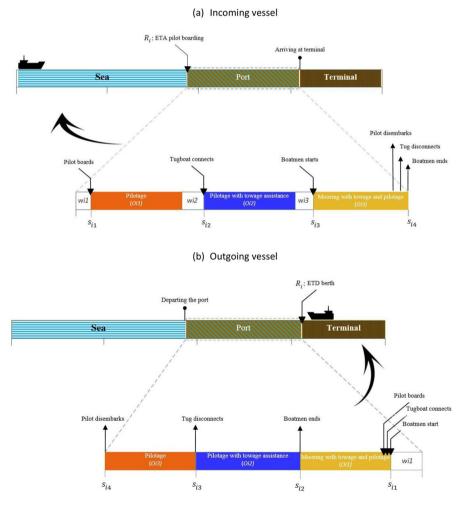


Fig. 1 Sequence of operations for a incoming and b outgoing vessels

the pilot organization to take a pilot on board for pilotage. A pilot sails from its station to the boarding place to board the vessel. After the pilot has boarded the vessel, the vessel enters the harbour. This operation between the pilot on boarding and tugboat engagement is considered operation 1 of incoming vessels. Next, the pilot and the vessel's captain command the tugboats to provide towage services. The number of tugboats required typically depends on the vessel size. Tugboats sail from either their station or previous assignments to connect to the vessel for towage. The tugboats tow/push the vessel arrives at its berth, boatmen start their service to help moor the vessel, which begins operation 3. Only after the completion of this operation, the services of an incoming vessel are considered complete, and the assigned pilot, tugboats, and boatmen are released for their subsequent assignments or can move back to their respective stations. Note that, during the successive operations, the resources of earlier operations remain occupied. Figure 1a illustrates the sequence of services for incoming vessels.

Outgoing vessels request their servicing start time by sending an Estimated Time of Departure time from their berth, called ETD-berth. This time is the requested starting time  $(R_i)$  for outgoing vessels. If the service providers confirm the requested time, clearance is given by the PA, the servicing can begin at the requested time. The unmooring operation is carried out when the pilot, the required number of tugboats, and the boatmen team are ready. Unmooring is operation 1 of an outgoing vessel. This operation is done by boatmen, with towage assistance, and under the pilot's command. With the completion of the unmooring operation, the boatmen will be released, and operation 2 will start. During operation 2, the vessel manoeuvres through harbour with the tugboat's assistance, still under the pilot's command. When tugboat assistance is no longer needed, the tugboats disconnect, and operation 3 starts, where only the pilot is on board to help the vessel sail out of the port. Operation 3 continues until the vessel is out of the port, and the pilot disembarks the vessel to go to its next assignment (or station). With the pilot disembarkation and completion of operation 3, the service for the outgoing vessel will end. Note that, unlike the services for an incoming vessel where resources are seized successively at the start of their respective operation, all resources are occupied at the start of the operations for the outgoing vessels, and they are released after the completion of their respective operations. Figure 1b presents the sequence of operations for outgoing vessels.

The operating times of a vessel refer to the duration of the sailing time in reaching its berth (incoming vessel) or in reaching sea from its berth (outgoing vessel) and are typically estimated from past data and considered as known. In the Port of Rotterdam, for instance, the maximum allowed sailing speed for each vessel class and historical data are used to estimate the operation times (Verduijn 2017). Operation times of different vessels differ depending on several factors, such as vessel size and berth location. The time that operation *j* of vessel *i* takes is denoted mathematically as  $O_{ij}$ . This time enables the service providers to approximate their service start time. For example, the boatmen team assigned to an incoming vessel would expect the vessel arrival time as  $s_{i3} = R_i + O_{i1} + O_{i2}$ , where  $s_{i3}$  is the estimated time

of starting mooring operations for the incoming vessel,  $R_i$  is the vessels' requested time, and  $O_{i1}$  and  $O_{i2}$  are the durations of operations 1 and 2 for vessel *i*, respectively. Each operation is assumed to be non-pre-emptive, which means that each operation  $O_{ij}$  cannot be interrupted once it starts. For example, the boarded pilot can only leave the occupied vessel to serve another once the service ends. This is almost always the case in reality.

The sequence of operations for both types of vessels is fixed and known (see Fig. 1a and b). Vessels with pilotage exceptions are excluded in this study since the vessels without a pilot are not allowed to take tugboats either. Further, we assumed that each vessel required one pilot and one boatmen team. However, its towage requirements vary in number. Bigger vessels may require up to 4 tugboats, while smaller vessels do not require towage assistance. In the case of smaller vessels, the number of tugboats needed is assigned zero (An example is presented in the next section).

Service providers are individual companies that perform vessel services with their resources. We assume that the pilot organization, tugboat company, and boatmen organization have *P* identical pilots, *T* identical tugboat fleet, and *B* identical boatmen crew, respectively. Each resource may serve multiple vessels successively during the scheduling period but not simultaneously. For example, one pilot cannot serve two vessels simultaneously. Initially, each resource is assumed to start from its station (denoted as *c*) and has to return to its station or travel to its next assignment after completing the current one. Each resource can only make one complete tour by starting from its own stations and returning there finally. Subtours are not allowed (see Fig. 2). The resources that are not assigned to any assignment can stay at their stations. The transportation time to and from its station is considered explicitly. Each resource may finish its assignments is significant, and they are indicated by  $D_{ik}^p$ ,  $D_{ik}^t$ ,  $D_{ik}^b$  for pilot, tugboat, and boatmen, respectively. The repositioning time between the assignments is shorter for the pilot and tugboats if the

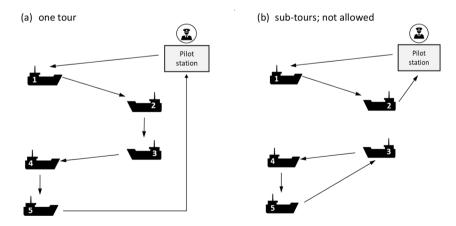


Fig. 2 Sequence of pilot repositioning for serving five vessels; **a** a correct tour starting and ending at the station, **b** includes subtours which is not allowed

resource serves a different type of vessel type (incoming, outgoing) in succession. Alternatively, when a resource successively serves two incoming (or two outgoing) vessels, the repositioning time becomes much longer. For example, when a pilot is assigned to two incoming vessels successively, the pilot has to travel back to the port entry after the disembarking the first vessel, to start its next incoming assignment. Thus, based on the current and next assignment combinations, each resource might travel in five ways (incoming–incoming, incoming–outgoing, outgoing to outgoing, incoming to station, outgoing–station). However, for the boatmen, as their assignments are always at the berth, such sequence dependency is typically not significant.

The challenge is that in peak times the port can only grant some of all the requested times  $(R_i)$ . During peak demand, when the service providers are not readily available, the port may suggest a new scheduled starting time  $(s_{i1})$ . This implies that the scheduled starting time of vessel *i* has a deviation of  $w_{i1} = s_{i1} - R_i$  from the requested time. We assume that the scheduled starting times must follow the vessels' requested times  $(s_{i1} \ge R_i)$ , which is a natural constraint. After operation 1 starts with the deviation  $w_{i1}$ , there is still a possibility of facing deviations in the next consecutive operations (referred to as  $w_{i2}$  and  $w_{i3}$ ). Since the vessels' starting and operation times define the service providers' assignments and their reposition-ing durations between them, the model needs to schedule the vessels and service providers simultaneously. A port may pursue different optimization strategies in this scheduling. We formulate and explore three separate strategies as objective functions as the following:

- Strategy (I): Minimizes the total sum of deviations of scheduled from requested times [Min Sum] Minimize ∑<sub>i∈V</sub> ∑<sub>j=1</sub><sup>J</sup> w<sub>ij</sub>
- Strategy (II): Minimizes the maximum of deviations of scheduled from requested times [Min Max] Minimize  $\max_{i \in V} \sum_{j=1}^{J} w_{ij}$
- Strategy (III): Minimizes the deviation of the scheduled from the requested starting time [FCFS] Minimize  $\sum_{i=1}^{n} w_{1j}$ .

Note that these objective functions represent alternative strategies for ports. Hence, they are not multiple objectives of a single model. Multi-objective optimization is suitable for modelling problems, where several conflicting objective functions need to be considered simultaneously. In this study, we explore three scheduling strategies and understand their trade-offs among them to determine the best strategy. Strategy (I) focusses on minimizing the total sum of  $w_{i1} + w_{i2} + w_{i3}$  for all vessels. Strategy (II) minimizes the maximum of  $w_{i1} + w_{i2} + w_{i3}$  for all vessels. Strategy (III) minimizes the total sum of  $w_{i1}$  for all vessels focussing only on starting the services as soon as possible, disregarding the inter-service waiting time of vessels i.e.  $w_{i2} + w_{i3}$ . Each of these strategies has its own strengths, enabling ports to adapt to different scheduling objectives. The above-mentioned problem definition can be formulated as the *mixed-integer linear programming model* (MILP) that appears in Appendix. The model aims to find optimal solutions for the alternative scheduling strategies. The main decision variables are the scheduled starting time of operation j of vessel  $i(s_{ij})$ , and the deviation between the scheduled and requested starting time of operation j of vessel  $i(w_{ij})$ . The model's main input parameters are the duration of operation j of vessel  $i(O_{ij})$ , the requested starting time of the first operation of vessel  $i(R_i)$ , the required number of tugboats for vessel  $i(N_i)$ , and the repositioning times of resources.

# 4 Application

In this section, we apply the model to an illustrative case and use it to study the effect of the three scheduling strategies discussed earlier (Sect. 3). Our analysis is based on historical data from the Port of Rotterdam, the biggest port in Europe, with about 30,000 seagoing vessels visiting annually. In the first subsection, we present input parameters extracted from our dataset. Further subsections show the results from the application of the model to the case, and in the final subsection, we discuss the key results.

# 4.1 Input parameters

Table 1 shows the vessel-related input parameters of the case. We used the data obtained from the port call data of the port of Rotterdam on a random date and time (Verduijn 2017). We considered an instance of 12 vessels, equivalent to a work-load of approximately 2 h in the Port of Rotterdam. The port serves 150 incoming and outgoing vessels per day (75 incoming + 75 outgoing vessels per 24 h  $\simeq$  on average 12 vessels per 2 h). Specific information, such as the date and vessel's name, is removed from the table for confidentiality reasons. The vessels requested times (ETA-Pilot boarding place for incoming vessels and ETD-berth for outgoing

Vessel	Movement type	The required	Requested	Operation times			
i		number of tugboats $N_i$	starting time R <sub>i</sub>	$\overline{O_{i1}}$	<i>O</i> <sub><i>i</i>2</sub>	<i>O</i> <sub><i>i</i>3</sub>	
<i>v</i> <sub>1</sub>	In	3	12:45	01:00	01:15	00:30	
<i>v</i> <sub>2</sub>	In	2	12:45	01:00	00:50	00:30	
<i>v</i> <sub>3</sub>	In	2	12:45	00:50	00:50	00:20	
$v_4$	In	2	13:15	00:50	00:55	00:30	
<i>v</i> <sub>5</sub>	In	0	13:15	00:30	00:30	00:20	
$v_6$	In	2	13:15	01:00	01:15	00:30	
<i>v</i> <sub>7</sub>	Out	0	13:30	00:20	00:30	00:40	
v <sub>8</sub>	Out	0	12:30	00:20	00:30	00:55	
<i>v</i> <sub>9</sub>	Out	2	12:50	00:30	00:40	00:50	
<i>v</i> <sub>10</sub>	Out	2	13:00	00:30	00:40	01:00	
<i>v</i> <sub>11</sub>	Out	0	13:10	00:20	00:30	01:00	
v <sub>12</sub>	Out	0	13:15	00:20	00:40	01:00	

Table 1 Input parameters of the case



vessels) and their operations times are obtained from the same dataset. The durations $O_{i1}$ ,  $O_{i2}$ , and  $O_{i3}$  of each vessel are shown in Table 1. Column  $R_i$  represents the requested starting time of operation 1. The requested starting times of operations 2 and 3 for incoming vessels are calculated by equations  $R_i + O_{i1}$  and  $R_i + O_{i1} + O_{i2}$ , respectively. For the incoming vessel 1, the requested ETA-pilot boarding time is 12:45, the requested towage starting time is 13:45, and the mooring starting time is 13.45 + 01:15 = 15:00. Hence, the requested completion time of services is 15:00 + 0:30 = 15:30.

In the selected time interval (12:00–14:00), the service providers' overall resource availability for servicing these 12 vessels is approximated as the following: 8 pilots and 10 tugboats, and 3 boatmen teams. To approximate the available capacity, we excluded 25% of their total capacity for shifting vessels and 15% for the scheduled breaks of the crew. The distribution of vessel movements in a day is approximately 37%, 37%, and 25% for incoming, outgoing, and shifting voyages, respectively. We assumed that at 12:00, all the resources were based at their stations. The repositioning time (in min) for a pilot and tugboat,  $D_{ik}^{p}$ , and  $D_{ik}^{t}$ , between two con-

incoming outgoing

secutive assignments are assumed to be as the following  $incoming \begin{pmatrix} 00:50 & 00:30 \\ 00:30 & 00:50 \end{pmatrix}$ , outgoing  $\begin{pmatrix} 00:50 & 00:30 \\ 00:30 & 00:50 \end{pmatrix}$ ,

# incoming outgoing

*incoming*  $\begin{pmatrix} 00:40 & 00:20\\ 00:20 & 00:40 \end{pmatrix}$ , matrices, respectively. For example, if a pilot outgoing

serves an incoming vessel consecutively, after completion of serving incoming vessels at berth, it has to reposition to port entry which takes about 50 min. This repositioning time will be shorter (about 30 min) if the pilot assigns to an outgoing vessel. The repositioning time between two consecutive assignments for a boatmen team is assumed to be always 20 min.

# 4.2 Results

This section presents the findings of our experiments with input data of Sect. 4.1 to compare the results of each scheduling strategy. The proposed model (presented in Appendix) is coded in Python and solved by the Gurobi Optimization solver.

# 4.2.1 Strategy (I): [Min sum]

The [Min sum] objective function minimizes the sum of total deviations of scheduled times of vessels from their requested times. Table 2 presents the optimal solution obtained. The results show that with the current combination of available resources (8 pilots, 10 tugboats, and 3 boatmen teams), only *some* vessels are served at their requested times, and the starting times of the others have been postponed. For example, incoming vessel  $v_1$  's requested ETA-pilot boarding place was 12:45, but it can only get the service at 15:20. Given its operation times, (as in Table 1) the scheduled starting time of towage and boatmen becomes 16:20 and 17:35, respectively, and the scheduled completion of services is 18:05. The assigned resources are

 $b_2$ 

Vessel i	R <sub>i</sub>	s <sub>i1</sub>	s <sub>i2</sub>	<i>s</i> <sub><i>i</i>3</sub>	<i>s</i> <sub>i4</sub>	Assigned pilot p	Assigned tugboats t	Assigned boatmen team b
<i>v</i> <sub>1</sub>	12:45	15:20	16:20	17:35	18:05	$p_1$	$t_1, t_2, t_3$	$b_3$
<i>v</i> <sub>2</sub>	12:45	12:50	13:50	14:40	15:10	$p_5$	$t_5, t_8$	$b_1$
<i>v</i> <sub>3</sub>	12:45	12:50	13:40	14:30	14:50	$p_4$	$t_3, t_4$	$b_2$
$v_4$	13:15	14:45	15:35	16:30	17:00	$p_3$	$t_9, t_{10}$	$b_1$
<i>v</i> <sub>5</sub>	13:15	13:15	13:45	14:15	14:35	$p_2$	-	$b_3$
$v_6$	13:15	13:15	14:15	15:30	16:00	$p_7$	$t_6, t_7$	$b_1$
<i>v</i> <sub>7</sub>	13:30	15:05	15:25	15:55	16:35	$p_2$	-	$b_3$
v <sub>8</sub>	12:30	12:30	12:50	13:20	14:15	$p_3$	-	$b_2$
<i>v</i> <sub>9</sub>	12:50	12:50	13:20	14:00	14:50	$p_1$	$t_2, t_9$	$b_1$
v <sub>10</sub>	13:00	13:00	13:30	14:10	15:10	$p_8$	$t_3, t_{10}$	$b_3$
<i>v</i> <sub>11</sub>	13:10	13:00	13:30	14:00	15:00	$p_6$	-	$b_2$

Table 2 Vessels' schedule based on strategy (I) [Min sum]

pilot  $p_1$ , tugboats  $t_1, t_2, t_3$ , and boatmen team  $b_3$ . This schedule suggests a deviation of 155 min from vessel  $v_1$  's requested starting time, with no inter-service waiting time. As another example, outgoing vessel  $v_8$  is scheduled at its requested time. Pilot  $p_3$  and boatmen team  $b_2$  are assigned to this vessel, with no tugboats, given that this vessel does not require towage assistance.

17:20

 $p_4$ 

Table 3 shows the optimal schedule for each resource. For example, it shows that vessel  $v_1$  is the second assignment of pilot  $p_1$ . As shown in Table 3, the pilot  $p_1$ , starts operations from its station, and serves the outgoing vessel  $v_0$ , and incoming vessel  $v_1$  before returning to its station. Tugboat  $t_1$  serves vessels  $v_{10}$  and  $v_1$  consecutively. Each boatmen team serves four vessels. For example, boatmen team  $b_1$  serves vessels  $v_0, v_2, v_6$ , and  $v_4$  before they return to their station. The order of assignments for the pilots and tugboat shows that they are assigned successively to incoming and outgoing vessels to minimize the repositioning time. However, such a pattern is not observed for the boatmen team, as their repositioning time is fixed. The last row of Table 3 shows the total repositioning time of pilots, tugboats, and boatmen. Below, we compare them with the results of other strategies.

# 4.2.2 Strategy (II): [Min max]

The Min max optimization problems have multiple solutions. Tables 4 and 5 present one of the optimal solutions obtained. Table 4 shows that, similar to strategy (I), only some of the vessels can be served at their requested times, resulting in the postponement of other services. Comparison of Tables 4 and 5 with Table 2 and Table 3 shows that the optimal schedule of the vessels and the services provided are different for these two strategies. A comparison of the repositioning times of Table 3 with Table 5 shows that the total repositioning time of tugboats and boatmen is equal in these two strategies. However, strategy (I) yielded a schedule, where the repositioning time of pilots is shorter.

v<sub>12</sub>

13:15

13:00

15:40

16:20

Pilots' schedule		Tugboats' schedule		Boatmen team's schedule	edule
pilot <i>p</i>	Order of assignments	Tugboat t	Order of assignments	Boatmen b	Order of assignments
$p_1$	$c^{\mathrm{P}}, v_{9}, v_{1}, c^{\mathrm{P}}$	<i>t</i> <sub>1</sub>	$c^{\mathrm{t}}, v_{\mathrm{10}}, v_{\mathrm{1}}, c^{\mathrm{t}}$	$b_1$	$c^{\rm b}, v_9, v_2, v_6, v_4, c^{\rm b}$
$p_2$	$c^{\mathrm{p}}, v_{5}, v_{7}, c^{\mathrm{p}}$	$t_2$	$c^{\mathfrak{t}}, v_{9}, v_{1}, c^{\mathfrak{t}}$	$b_2$	$c^{\mathrm{b}}, \nu_{\mathrm{8}}, \nu_{\mathrm{11}}, \nu_{\mathrm{3}}, \nu_{\mathrm{12}}, c^{\mathrm{b}}$
$p_3$	$c^{\mathrm{p}}, v_{\mathrm{8}}, v_{\mathrm{4}}, c^{\mathrm{p}}$	$t_3$	$c^{\mathrm{t}}, v_3, v_1, c^{\mathrm{t}}$	$b_3$	$c^{\mathrm{b}}, \nu_{10}, \nu_{5}, \nu_{7}, \nu_{1}, c^{\mathrm{b}}$
$p_4$	$c^{\mathrm{p}}, v_{3}, v_{12}, c^{\mathrm{p}}$	$t_4$	$c^{\mathrm{t}}, \nu_{3}, c^{\mathrm{t}}$		
$P_5$	$c^{\mathrm{p}}, v_{2}, c^{\mathrm{p}}$	$t_5$	$c^{\mathfrak{t}}, v_2, c^{\mathfrak{t}}$		
$P_6$	$c^{\mathrm{p}}, v_{\mathrm{11}}, c^{\mathrm{p}}$	$t_6$	$c^{\mathrm{t}}, v_{6}, c^{\mathrm{t}}$		
$P_7$	$c^{\mathrm{p}}, v_6, c^{\mathrm{p}}$	$t_7$	$c^{\mathrm{t}}, v_{6}, c^{\mathrm{t}}$		
$P_8$	$c^{\mathrm{p}}, \nu_{\mathrm{10}}, c^{\mathrm{p}}$	$t_8$	$c^{\mathrm{t}}, v_2, c^{\mathrm{t}}$		
		$t_9$	$c^{\mathfrak{t}}, v_9, v_4, c^{\mathfrak{t}}$		
		$t_{10}$	$c^{\mathfrak{t}}, \nu_{10}, \nu_{4}, c^{\mathfrak{t}}$		
Total repositioning time 680 min	680 min	Total repositioning time of 640 min	time of 640 min	Total repositioning time of 300 min	time of 300 min
of pilots		tugboats		boatmen	

i	$R_i$	<i>s</i> <sub><i>i</i>1</sub>	s <sub>i2</sub>	s <sub>i3</sub>	<i>s</i> <sub>i4</sub>	Assigned pilot p	Assigned tugboats t	Assigned boatmen b
<i>v</i> <sub>1</sub>	12:45	13:45	14:45	16:00	16:30	$p_1$	$t_1, t_2, t_3$	<i>b</i> <sub>3</sub>
$v_2$	12:45	13:40	14:40	15:30	16:00	$p_4$	$t_6, t_{10}$	$b_2$
<i>v</i> <sub>3</sub>	12:45	12:50	13:40	14:30	14:50	$p_3$	$t_7, t_9$	$b_3$
$v_4$	13:15	14:45	15:35	16:30	17:00	$p_6$	$t_5, t_6$	$b_1$
$v_5$	13:15	13:15	13:45	14:15	14:35	$p_8$	-	$b_2$
$v_6$	13:15	15:20	16:20	17:35	18:05	$p_5$	$t_4, t_7$	$b_1$
$v_7$	13:30	15:20	15:40	16:10	16:50	$p_3$	-	$b_1$
$v_8$	12:30	12:30	12:50	13:20	14:15	$p_6$	-	$b_1$
<i>v</i> <sub>9</sub>	12:50	12:50	13:20	14:00	14:50	$p_5$	$t_5, t_6$	$b_2$
$v_{10}$	13:00	13:10	13:40	15:30	17:00	$p_7$	$t_4, t_{10}$	$b_1$
<i>v</i> <sub>11</sub>	13:10	13:35	13:55	16:00	17:20	$p_2$	-	$b_3$
<i>v</i> <sub>12</sub>	13:15	15:20	15:40	16:20	17:20	$p_8$	-	$b_3$

Table 4 Vessels schedule based on strategy (II) [Min max]

#### 4.2.3 Strategy (III) [FCFS]

This section presents the results for strategy [FCFS], where the objective function minimizes the deviations of vessels' scheduled starting times from their requested starting times. Since the requested starting time of some vessels are the same, this strategy also may have more than one solution. Tables 6 and 7 show one of the schedules obtained by this strategy for vessels and service providers, respectively. A comparison of the repositioning times of Tables 3, 5, and 7 shows that strategy (III) resulted in a schedule where the repositioning time for both pilots and tugboats is longer than strategy (I) and (II). Figure 3 compares and illustrates the repositioning times for the three strategies. The repositioning time of the boatman was equal to strategy (I) and (II) because the repositioning time of boatmen is not sequence-dependent.

To compare the performance of the three strategies, in Table 8 we summarize the results in terms of deviations from the requested starting times and inter-service waiting times. The first column under each strategy (referred  $asw_{i1}$ ) is the deviation of vessel *i*'s scheduled starting time from its requested time. The terms  $\sum_{j=2}^{3} w_{ij}$  represent the inter-service waiting time of vessel*i*, whereas  $\sum_{j=1}^{3} w_{ij}$  is the sum of the previous two terms and refers to the total waiting time for resources. Table 8 shows that strategies (I) and (II) can determine schedules with the minimum total deviation and the minimum of maximum deviation from the requested times, respectively. Strategy (III) yielded a schedule, where deviations from the vessels' requested starting times are 0, while in strategy (III), it is significant. Figure 4 visually compares these three strategies.

In Fig. 4, the chequered bar appears only in strategy (III), which shows that, unlike strategy (III), strategy (I) and (II) found schedules, where the inter-service waiting times of vessels are 0. Take, for example, vessel 6. When scheduled by

Pilot $p$ Order of assignments $p_1$ $c^p, v_1, c^p$ $p_2$ $c^p, v_1, c^p$ $p_3$ $c^p, v_3, v_7, c^p$ $p_4$ $c^p, v_2, c^p$ $p_5$ $c^p, v_9, v_6, c^p$ $p_6$ $c^p, v_{91}, c^p$ $p_7$ $c^p, v_{91}, c^p$	Tugboat t			
		Order of assignments	Boatmen b	Order of assignments
	$t_1$	$c^{\mathrm{t}}, v_{\mathrm{l}}, c^{\mathrm{t}}$	$b_1$	$c^{\mathrm{b}}, v_{\mathrm{8}}, v_{\mathrm{10}}, v_{7}, v_{4}, v_{6}, c^{\mathrm{b}}$
	$t_2$	$c^{\mathrm{t}}, \nu_{\mathrm{l}}, c^{\mathrm{t}}$	$b_2$	$c^{\mathrm{b}}, v_{\mathrm{9}}, v_{\mathrm{5}}, v_{\mathrm{2}}, c^{\mathrm{b}}$
	$t_3$	$c^{\mathrm{t}}, \nu_{\mathrm{l}}, c^{\mathrm{t}}$	$b_3$	$c^{\mathrm{b}}, \nu_{11}, \nu_{3}, \nu_{12}, \nu_{1}, c^{\mathrm{b}}$
	$t_4$	$c^{\mathrm{t}}, \nu_{\mathrm{10}}, \nu_{\mathrm{6}}, c^{\mathrm{t}}$		
	$t_5$	$c^{\mathrm{t}}, v_9, v_4, c^{\mathrm{t}}$		
	$t_6$	$c^{\mathrm{t}}, v_9, v_2, c^{\mathrm{t}}$		
	$t_{7}$	$c^{\mathrm{t}}, v_3, v_6, c^{\mathrm{t}}$		
$P_8$ $c^{\rm p}, v_5, v_{12}, c^{\rm p}$	$t_8$	$c^{\mathrm{t}}, \nu_4, c^{\mathrm{t}}$		
	$t_9$	$c^{\mathrm{t}}, v_3, c^{\mathrm{t}}$		
	$t_{10}$	$c^{\mathrm{t}}, \nu_{\mathrm{10}}, \nu_{\mathrm{2}}, c^{\mathrm{t}}$		
Total repositioning time 760 min of pilots	Total repositioning time of tugboats	640 min	Total repositioning time of boatmen teams	300 min

 Table 5
 Service providers' schedule based on strategy (II) [Min max]

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 Biters' and the

i	R <sub>i</sub>	<i>s</i> <sub><i>i</i>1</sub>	<i>s</i> <sub><i>i</i>2</sub>	<i>s</i> <sub><i>i</i>3</sub>	<i>s</i> <sub>i4</sub>	Assigned pilot p	Assigned tugboats t	Assigned boatmen b
<i>v</i> <sub>1</sub>	12:45	12:50	15:35	16:50	17:20	$p_1$	$t_1, t_2, t_3$	$b_1$
$v_2$	12:45	12:50	15:10	16:00	16:30	$p_4$	$t_8, t_9$	$b_1$
<i>v</i> <sub>3</sub>	12:45	12:50	13:40	14:30	14:50	$p_7$	$t_7, t_{10}$	$b_1$
$v_4$	13:15	15:20	16:10	17:05	17:35	$p_2$	$t_6, t_{10}$	$b_3$
$v_5$	13:15	13:15	13:45	14:15	14:35	$p_6$	-	$b_2$
$v_6$	13:15	14:45	18:00	19:15	19:45	$p_3$	<i>t</i> <sub>1</sub> , <i>t</i> <sub>5</sub>	$b_1$
$v_7$	13:30	15:20	15:40	16:10	16:50	$p_7$	-	$b_1$
$v_8$	12:30	12:30	12:50	13:20	14:15	$p_3$	-	$b_1$
<i>v</i> <sub>9</sub>	12:50	12:50	13:20	14:00	14:50	$p_2$	$t_2, t_9$	$b_3$
<i>v</i> <sub>10</sub>	13:00	13:00	13:30	14:10	15:10	$p_5$	$t_3, t_4$	$b_2$
<i>v</i> <sub>11</sub>	13:10	13:10	13:30	14:00	15:00	$p_8$	_	$b_1$
<i>v</i> <sub>12</sub>	13:15	15:05	15:25	16:05	17:05	$p_6$	-	$b_2$

Table 6 Vessels schedule based on strategy (III) [FCFS]

strategy (I) and (II), the deviations of scheduled starting times from the requested times are 0 and 125 min, respectively. The inter-service waiting times are 0 in both. With strategy (III), this vessel's starting time has a deviation of 90 min from its requested time. In addition, its inter-service waiting time is 135 min. The other observation is that the maximum deviation of the scheduled starting time from the requested time for strategy (I) is larger than that of strategy (II). The former equals 155 min for vessel 1, while the latter equals 125 min for vessel 6. We discuss the results further in the next subsection.

## 4.3 Discussion

This subsection discusses the results of Sect. 4.2.

The application of the model for an illustrative case of the large and busy port of Rotterdam confirms that the model generates the full-service schedule for the vessels and service providers, considering the repositioning and inter-service waiting times. Compared to the current 'first-come-first-served' approach, the results show that significant time savings can be obtained by joint scheduling of vessels and service providers. Both the vessels and the service providers experience these time savings. Therefore, in peak times, when not all the requested times of vessels can be accommodated, ports can use the proposed modelling tool to provide vessels with feedback on their *scheduled times*. Accordingly, vessels can slow down to arrive JIT and be served immediately without waiting between the services.

The order of the resource assignments (as in Tables 3, 5, and 7) shows that the model succeeded to assign the resources successively to incoming and outgoing vessels. This ordering enabled minimizing the repositioning time of resources so that the resources are used more efficiently. The efficient use of resources, in return, helped schedule vessels closer to their requested times.

Pilots' schedule		Tugboats' schedule		Boatmen team's schedule	0
Pilot p	Order of assignments	Tugboat t	Order of assignments	Boatmen b	Order of assignments
$p_1$	$c^{\mathrm{p}}, v_{\mathrm{l}}, c^{\mathrm{p}}$	$t_1$	$c^{\mathrm{t}}, \nu_{\mathrm{l}}, \nu_{\mathrm{6}}, c^{\mathrm{t}}$	$b_1$	$c^{\mathrm{b}}, \nu_{\mathrm{8}}, \nu_{\mathrm{11}}, \nu_{\mathrm{3}}, \nu_{7}, \nu_{2}, \nu_{\mathrm{1}}, \nu_{\mathrm{6}}, c^{\mathrm{b}}$
$p_2$	$c^{\mathrm{p}}, v_{\mathrm{g}}, v_{\mathrm{d}}, c^{\mathrm{p}}$	$t_2$	$c^{\mathfrak{t}}, v_9, v_1, c^{\mathfrak{t}}$	$b_2$	$c^{\mathrm{b}}, v_{10}, v_{5}, v_{12}, c^{\mathrm{b}}$
$p_3$	$c^{\mathrm{p}}, \nu_{\mathrm{8}}, \nu_{\mathrm{6}}, c^{\mathrm{p}}$	$t_3$	$c^{\mathrm{t}}, v_{\mathrm{10}}, v_\mathrm{1}, c^{\mathrm{t}}$	$b_3$	$c^{\mathrm{b}}, v_{\mathrm{9}}, v_{\mathrm{4}}, c^{\mathrm{b}}$
$P_4$	$c^{\mathrm{p}}, v_2, c^{\mathrm{p}}$	$t_4$	$c^{\mathrm{t}},  u_{10}, c^{\mathrm{t}}$		
$P_5$	$c^{\mathrm{p}}, v_{\mathrm{10}}, c^{\mathrm{p}}$	$t_5$	$c^{\mathrm{t}}, \nu_{6}, c^{\mathrm{t}}$		
$P_6$	$c^{\mathrm{p}}, v_5, v_{12}, c^{\mathrm{p}}$	$t_6$	$c^{\mathrm{t}}, \nu_4, c^{\mathrm{t}}$		
$p_7$	$c^{\mathrm{p}}, v_3, v_7, c^{\mathrm{p}}$	$t_7$	$c^{\mathrm{t}}, \nu_3, c^{\mathrm{t}}$		
$P_8$	$c^{\mathrm{p}}, v_{\mathrm{l1}}, c^{\mathrm{p}}$	$t_8$	$c^{\mathrm{t}}, \nu_2, c^{\mathrm{t}}$		
		$t_9$	$c^{\mathfrak{t}}, v_9, v_2, c^{\mathfrak{t}}$		
		$t_{10}$	$c^{\mathrm{t}}, \nu_3, \nu_4, c^{\mathrm{t}}$		
Total repositioning time 770 min of pilots	770 min	Total repositioning Time of tugboats	660 min	Total repositioning time of boatmen	300 min

Joint scheduling of vessels and vessel service providers for...

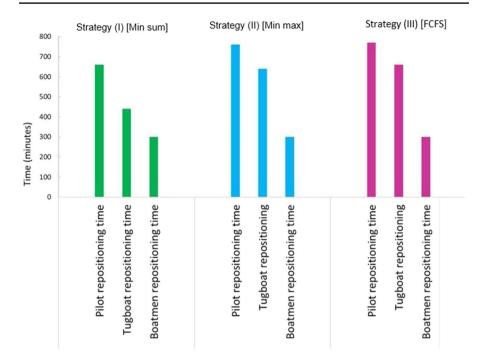


Fig. 3 The visual comparison of repositioning times for three different strategies

Vessel i	Strate	gy (I) [Min	sum]	Strate	Strategy (II) [Min max]			Strategy (III) [FCFS]		
	w <sub>i1</sub>	$\sum_{j=2}^{3} w_{ij}$	$\sum_{j=1}^{3} w_{ij}$	w <sub>i1</sub>	$\sum_{j=2}^3 w_{ij}$	$\sum_{j=1}^3 w_{ij}$	w <sub>i1</sub>	$\sum_{j=2}^3 w_{ij}$	$\sum_{j=1}^{3} w_{ij}$	
<i>v</i> <sub>1</sub>	155	0	155	60	0	60	5	105	110	
$v_2$	5	0	5	55	0	55	5	80	85	
<i>v</i> <sub>3</sub>	5	0	5	5	0	5	5	0	5	
$v_4$	90	0	90	90	0	90	125	0	125	
<i>v</i> <sub>5</sub>	0	0	0	0	0	0	0	0	0	
$v_6$	0	0	0	125	0	125	90	135	225	
v <sub>7</sub>	95	0	95	110	0	110	110	0	110	
$v_8$	0	0	0	0	0	0	0	0	0	
V9	0	0	0	0	0	0	0	0	0	
v <sub>10</sub>	0	0	0	10	0	10	0	0	0	
v <sub>11</sub>	0	0	0	25	0	25	0	0	0	
v <sub>12</sub>	125	0	125	125	0	125	110	0	110	
Sum	475	0	475	605	0	605	450	320	770	
Max			155			125			225	

 Table 8
 The comparison of schedules for three different strategies (values are in min)

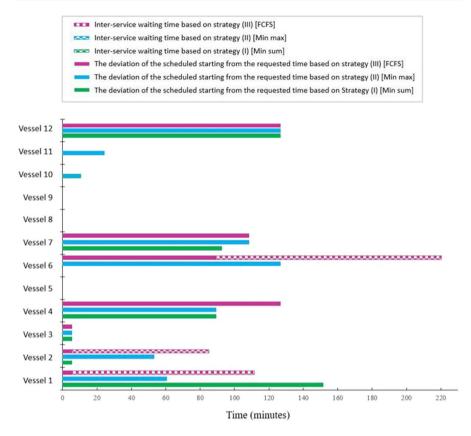


Fig. 4 The visual comparison of schedules for three different strategies (values are in min)

Ports may use the model by adopting different scheduling strategies; each of these has its strengths and weaknesses. For example, strategy (I) minimized the total sum of deviations from the requested times. By definition, the average deviation from the requested starting time and inter-service waiting time experienced by vessels is the smallest of the three scenarios. However, it may be considered unfair as the deviation from the requested time is larger for some vessels (vessels  $v_1$  and  $v_{12}$ ) compared to others. In order to split the deviations among the vessels more evenly, strategy (II) can be employed.

Both strategies (I) and (II) took the inter-service waiting times into account and found schedules with zero inter-service waiting times. This indicates that the model assigns all the resources such that the waiting time for successive resources after servicing has started (by pilot boarding) is avoided for efficient use of resources. However, this important factor is ignored in strategy (III). In this strategy, the model aims to schedule vessel servicing start times as close to their requested times as possible. This results in longer inter-service waiting times. Existing models in the literature (Abou Kasm et al. 2021) have only considered the starting time of services. Ignoring the

inter-service waiting times has a risk of shifting the waiting times to later stages where resources have been occupied. In practice, too, this important aspect of FCFS needs attention. In many ports today, serving vessels based on FCFS is still the most common servicing principle (Yıldırım et al. 2020), putting extra pressure on service providers' resources by occupying them unnecessarily exacerbating the vessel's waiting times.

The comparison of repositioning times for different strategies showed that strategy (I) outperformed all the other strategies in minimizing repositioning times. This indicates that strategy (I) is the best strategy for the efficient use of resources, particularly in busier and larger ports where repositioning time is a factor for efficient use of service providers' resources. In summary, both strategies (I) and (II) are advantageous from the service provider's point of view. Strategy (I) should be considered to increase their utilization, whereas strategy (II) can be applied to be equally fair to all vessel operators.

In this study, we assumed that all vessels are equally important for the port. However, certain vessels may have higher priorities, in need to be served at their requested times (Imai et al. 2001). To investigate such strategies, strategy (I) can be further extended by assigning different weight factors to different vessels. This comparison would provide insights into which prioritization strategies are most beneficial. This weighted sum strategy requires an investigation with port managers and shipping companies to assign priorities.

We note that the joint scheduling of vessels and service providers requires close cooperation of all the parties, to adapt their resource deployments. Our earlier work (Nikghadam et al. 2022) confirms that the service providers of the Port of Rotterdam are willing to engage in the proposed form of cooperation. Highlighting that all service providers would benefit from joint scheduling is essential to incentivize their participation (Nikghadam et al. 2023). Finally, the proactive involvement of the PAs in the port call process may require ports to suggest and schedule vessels for times earlier than their requested times. For example, a certain level of earliness can be easily achieved for some vessels in certain circumstances resulting in more efficient PCO. Therefore, exploring decision support systems to schedule the vessels earlier than the requested times can be advantageous. However, our model is designed to schedule the vessels for times later than their requested times. We made this assumption because speeding up to arrive earlier at the port can be costly for vessels due to increased fuel consumption (IMO 2020). Therefore, the compensation schemes for this request need to be further investigated as the costs would be experienced by the vessels that have to rush, and the benefits are shared among all the parties, both the service providers as well as the vessels. In connection to the above, benefits of proactive scheduling (while vessels are still underway), as modelled here, include fuel savings due to slow steaming and reduced anchorage. Future work could take these benefits into account as well.

# 5 Conclusion

This study addressed one of the main challenges ports have been facing in the implementation of PCO: determining the time based on which they can guarantee their resource availabilities. We proposed a novel mathematical model which

provides a full-service schedule created for vessels and vessel service providers. The schedule is decided according to vessels' requested arrival and departure times and the service providers' resource availability. The model is relevant for larger and busier ports: it is continuous in time and considers inter-service waiting times and sequence-dependent repositioning time of resources, i.e. pilots, tugboats, and boatmen. It can be solved to optimality using exact solution approaches. Three alternative scheduling strategies were formulated via different objective functions. The model runs using the data that are generally available at port authorities.

Our illustrative case shows that time savings can be obtained by joint scheduling of vessels and service providers, compared to the usual FCFS servicing principle. Especially in periods of peak demand, when not all requested times of vessels can be granted, ports can use the model to create a full-service schedule. Strategy (I) minimizes the total sum of deviations of the scheduled from the requested times. This strategy is the best strategy for the efficient utilization of resources. However, the disadvantage of this strategy is that it may lead to higher deviations for some vessels over others. Alternatively, ports that prefer to split the deviations more evenly are advised to use Strategy (II), which minimizes the maximum of deviations of the scheduled from the requested times. The present 'first-come-first-served' approach, studied as strategy (III), resulted in schedules, where the scheduled starting times are closest to their requested starting times. However, the vessels had to wait significantly longer after their servicing started. This strategy has a risk of shifting the waiting times to later stages, where resources have been occupied.

Future research could consider the following. In practice, vessel service requirements may vary with external conditions such as the weather. Future research can extend our model by including stochastic aspects of the port call process such as uncertain vessel arrival and departure times or servicing durations. Another extension of our model could be dynamic rescheduling according to updated ETA and ETDs, where the model runs and reiterates as the updated requested times are sent by the vessels. Third, one could relax the assumption of identical service providers by considering various pilotage certificates, and tugboat types. Our model can also be extended by including additional port call services, such as bunkering, or multiple pilotage services in the case of river navigation. Finally, as the problem studied in this study is NP-hard, the computational time increases strongly when the problem size gets larger. Future work can focus on developing efficient algorithms to solve large-sized problems in shorter times.

# Appendix

This section presents the mathematical formulation of the problem. The sets, decision variables, and parameters used in the formulation are presented below.

Sets

*i*, *k* : Vessels, *i*, *k*  $\in$  *V*, *V* = {*V*<sup>in</sup>  $\cup$  *V*<sup>out</sup>}, where *IV* indicates the number of vessels *J* : set of operations, *j* = {1, ..., *J*}, *J* = 3

Sets

P: Pilots,  $p = \{1, ..., P\}$ T: Tugboat fleet,  $t = \{1, ..., T\}$ B: Boatmen crew,  $b = \{1, ..., B\}$ C: Stations,  $c = \{c^{p}, c^{t}, c^{b}\}$ , where  $c^{p}, c^{t}, c^{b}$  indicate pilot, tugboat and boatmen stations, respectivelyDecision variables

 $s_{ij}$ : The scheduled starting time of operation j of vessel i

 $s_{i4}$ : The scheduled completion time of the last operation of vessel *i* 

 $w_{ij}$ : The deviation between the scheduled and requested time of *j*th operation of vessel *i* 

if pilot p is assigned to operation *j* of vessel *i*  $x_{iir}^{p} \ge \begin{cases} 0 \end{cases}$ otherwise  $y_{ikn}^{\mathrm{p}} \colon \left\{ \begin{array}{l} 1 \\ 0 \end{array} \right.$ if pilot p transports from *i*to k;  $i, k \in V \cup \{c^p\}$ otherwise  $x_{ijr}^{t}$ :  $\begin{cases} 1\\ 0 \end{cases}$ if tugboat t is assigned to operation *j* of vessel *i* otherwise  $y_{ikt}^{t}$ :  $\begin{cases} 1 \\ 0 \end{cases}$ if tugboat t transports from *i* to *k*;  $i, k \in V \cup \{c^t\}$ otherwise  $x^{\mathrm{b}}_{iik}$ :  $\begin{cases} 1\\ 0 \end{cases}$ if boatman b is assigned to operation *j* of vessel *i* otherwise  $y^{\mathrm{b}}_{ikb}$ :  $\left\{ \begin{array}{l} 1 \\ 0 \end{array} \right.$ if boatman b transports from *i* tok;  $i, k \in V \cup \{c^{b}\}$ otherwise

 $z_i^p$ : The order which vessel *i* is served by its pilot (auxiliary variable for subtour elimination)

 $z_i^t$ : The order which vessel *i* is served by its tugboat(s) (auxiliary variable for subtour elimination)

 $z_i^{b}$ : The order which vessel *i* is served by its Boatman (auxiliary variable for subtour elimination)

(Input) parameters

 $O_{ij}$ : The duration of operation j of vessel i

 $N_i$ 

The required number of tugboats for vessel *i* 

 $D_{ik}^{p}$ : The duration of repositioning time of pilot p from i to k;  $i, k \in V \cup \{c^{p}\}$ 

 $D_{ik}^{t}$ : The duration of repositioning time of tugboat *t* from *i* to *k*;  $i, k \in V \cup \{c^{t}\}$ 

 $D_{ik}^{b}$ : The duration of repositioning time of boatman b from i to k;  $i, k \in V \cup \{c^{t}\}$ 

 $D_{c^{p}k}^{p}$ : The duration of repositioning time of pilot p from the pilot station (c<sup>p</sup>) to vessel k

 $D_{c^{t}k}^{t}$ : The duration of repositioning time of tugboat t from the tugboat station ( $c^{t}$ ) to vessel k

 $D_{c^{b_{k}}}^{b}$ : The duration of repositioning time of boatman b from the tugboat station  $(c^{b})$  to vessel k

 $R_i$ : The requested starting time of first operation for vessel *i* (for incoming vessels that is ETA at pilot boarding place, for outgoing vessels that is ETD at berth)

Model

 $\begin{array}{ll} \text{Minimize } \sum_{i \in V} \sum_{j=1}^{J} w_{ij} & (1) \\ \text{Subject to} & (2) \\ s_{i1} - R_i - w_{i1} = 0 \quad \forall i \in V \\ s_{ij} + O_{ij} + w_{ij+1} - s_{ij+1} = 0 \quad \forall i \in V, j = \{1, \dots, J-1\} \end{array}$ 

$\begin{split} \sum_{i=1}^{P} \sum_{\lambda_{1D}}^{h} \sum_{\lambda_{2D}}^{h} = 1  \forall i \in V  (4) \\ \begin{cases} \lambda_{1D}^{h} = \lambda_{DD}^{h} \\ \lambda_{2D}^{h} = \lambda_{DD}^{h} \\ \forall i \in V, p = \{1, \dots, P\} \end{cases}  (5) \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\$	hour	
$\begin{cases} \lambda_{1D}^{\mu} = \lambda_{DD}^{\mu} & \forall i \in V, p = \{1, \dots, P\} \end{cases} $ (5) $\sum_{I=1}^{r} \lambda_{ID}^{I} = N_{I}  \forall i \in V  (6) \\ \{\lambda_{1D}^{I} = \lambda_{DD}^{I}  \forall i \in \{V^{in}\},  t = \{1, \dots, T\} \end{cases} $ (7) $\{\lambda_{ID}^{I} = \lambda_{DD}^{I}  \forall i \in \{V^{out}\}, t = \{1, \dots, T\} \end{cases} $ (8) $\sum_{I=1}^{D} \lambda_{ID}^{I} = \lambda_{DD}^{I}  \forall i \in \{V^{out}\}, t = \{1, \dots, T\} \end{cases} $ (8) $\sum_{I=1}^{D} \sum_{i=1}^{J} \lambda_{iD}^{b} = 1  \forall i \in V \qquad (9) \\ \lambda_{ID}^{b} + \lambda_{DD}^{c} = 0  \forall i \in \{V^{out}\}, t = \{1, \dots, B\} \end{cases} $ (10) $\lambda_{DD}^{b} + \lambda_{DD}^{c} = 0  \forall i \in \{V^{out}\}, t = \{1, \dots, B\} \end{cases} $ (11) $2\lambda_{ID}^{b} + \lambda_{DD}^{c} = 0  \forall i \in \{V^{out}\}, t = \{1, \dots, B\} \end{cases} $ (11) $2\lambda_{ID}^{b} + \lambda_{DD}^{c} = 0  \forall i \in \{V^{out}\}, t = \{1, \dots, B\} \end{cases} $ (11) $2\lambda_{ID}^{b} + \lambda_{DD}^{c} = 0  \forall i \in \{V^{out}\}, t = \{1, \dots, B\} \end{cases} $ (11) $2\lambda_{ID}^{b} + \lambda_{DD}^{c} = 0  \forall i \in \{V^{out}\}, t = \{1, \dots, P\} \end{cases} $ (13) $2kev(e^{i})_{i\neq k} \lambda_{BD}^{c} = \sum_{k\in V(e^{i})} j_{kp}^{k} + \sum_{k\in V(e^{i})} j_{kp}^{k} \geq 2\lambda_{ID}^{p} \qquad \forall i \in V = \{e^{i}\}, p = \{1, \dots, P\} $ (14) $y_{ID}^{c} = 0  \forall i \in V \cup \{e^{i}\}, p = \{1, \dots, P\} \end{cases} $ (15) $4 \leq c_{1}^{2} \leq  V   \forall i \in V \qquad (16) \\ j_{1D}^{2} = 0  \forall i \in V \cup \{e^{i}\}, p = \{1, \dots, T\} \end{cases} $ (17) $2\lambda_{ID}^{i} - \lambda_{I}^{k} = 1  \forall i \in V \cup \{e^{i}\}, j \in \{1, \dots, T\} \end{cases} $ (19) $\sum_{k\in V(e^{i}), j\neq k} \lambda_{Rk}^{i} \leq 1  \forall i \in V \cup \{e^{i}\}, j \in \{1, \dots, T\} \end{cases} $ (20) $y_{ID}^{i} = 0  \forall i \in V \cup \{e^{i}\}, i \in \{1, \dots, T\} \end{cases} $ (21) $\sum_{k\in V(e^{i}), j\neq k} \lambda_{Rk}^{i} \leq 1  \forall i \in V \cup \{e^{i}\}, j \in \{1, \dots, T\} \end{cases} $ (21) $\sum_{k\in V(e^{i}), j\neq k} \lambda_{Rk}^{i} \leq 1  \forall i \in V \cup \{e^{i}\}, k \in V, i \notin k $ (23) $2 \times \sum_{j=1}^{j} \lambda_{DD}^{i} = 1 \leq k \in V(e^{i}), j \notin k \end{pmatrix} \lambda_{I, k}^{i} \in V, i \notin k $ (23) $2 \times \sum_{j=1}^{j} \lambda_{DD}^{i} = 1 \leq k \in V(e^{i}), j \notin k \end{pmatrix} \lambda_{I, k}^{i} \in V \cup \{e^{i}\}, k \in \{1, \dots, B\} $ (24) $\sum_{k\in V(e^{i}), j \notin k \end{pmatrix} \lambda_{RD}^{i} \in V \cup \{e^{i}\}, b = \{1, \dots, B\} $ (25) $\sum_{k\in V(e^{i}), j \notin k \end{pmatrix} \lambda_{L}^{i} \in V \cup \{e^{i}\}, b = \{1, \dots, B\} $ (26) $\lambda_{D}^{i} = 0  \forall i \in V \cup \{e^{i}\}, b = \{1, \dots, B\} $ (27) $\sum_{k\in V(e^{i}), j \notin k \end{pmatrix} \lambda_{RD}^{i} \in V \cup \{e^{i}\}, b \in \{1, \dots, B\} $ (26) $\lambda_{D}^{i} = 0  \forall i \in V \cup \{e^{i}\}, b = \{1, \dots, B\} $ (27) $\sum_{k\in V(e^{i}), j \notin k \end{pmatrix} \lambda_{L}^{i} \in V \mid k \in V $ (28) $\lambda_{D}^$	$\sum_{p=1}^{P} x_{i1p}^{p} = 1  \forall i \in V$	(4)
$ \begin{split} & \sum_{i=1}^{T} x_{i,i}^{i} = N_{i}  \forall i \in V  (6) \\ \begin{cases} x_{1i}^{i} = 0 \\ x_{12}^{i} = x_{i,i}^{i}  \forall i \in \{V^{\mathrm{in}}\},  t = \{1, \dots, T\} \\ \end{cases} & (7) \\ \begin{cases} x_{1i}^{i} = x_{i,i}^{i}  \forall i \in \{V^{\mathrm{oul}}\},  t = \{1, \dots, T\} \\ \end{cases} & (8) \\ \end{cases} \\ \begin{cases} x_{1i}^{i} = x_{i,i}^{i} = 0  \forall i \in \{V^{\mathrm{oul}}\},  t = \{1, \dots, T\} \\ \end{cases} & (8) \\ \end{cases} \\ \end{cases} \\ \begin{cases} x_{1i}^{i} = x_{i,i}^{i} = 0  \forall i \in \{V^{\mathrm{oul}}\},  t = \{1, \dots, T\} \\ \end{cases} & (9) \\ x_{i,j}^{i} = 0  \forall i \in \{V^{\mathrm{oul}}\},  b = \{1, \dots, B\} \\ \end{cases} & (10) \\ x_{2i,j}^{i} + x_{2i,j}^{i} = 0  \forall i \in \{V^{\mathrm{oul}}\},  b = \{1, \dots, B\} \\ \end{cases} & (11) \\ 2x_{i,j}^{i} - 1 \leq \sum_{k \in V \cup \{e^{i}\},  p^{k} + \sum_{k \in V \cup \{e^{i}\},  p = \{1, \dots, P\} \\ \end{cases} & (11) \\ 2x_{i,j}^{i} - 1 \leq \sum_{k \in V \cup \{e^{i}\},  p^{k} + \sum_{k \in V \cup \{e^{i}\},  p = \{1, \dots, P\} \\ \end{cases} & (13) \\ \sum_{k \in V \cup \{e^{i}\}, \neq k} y_{kj}^{i} = \sum_{k \in V \cup \{e^{i}\}, \neq k} y_{kjj}^{i}  \forall i \in V \cup \{e^{i}\},  p = \{1, \dots, P\} \\ \end{cases} & (14) \\ y_{i,j}^{i} = 0  \forall i \in V \cup \{e^{i}\},  p = \{1, \dots, P\} \\ \end{cases} & (15) \\ 1 \leq z_{i}^{i} \leq  V   \forall i \in V \\ \end{pmatrix} & (16) \\ 2x_{i}^{i} - z_{k}^{i} + 1 \leq  V (1 - \sum_{p=1} y_{kp}^{i})  \forall i,  k \in V, i \neq k \\ \end{cases} & (17) \\ 2x_{i}^{i} - 1 \leq \sum_{k \in V \cup \{e^{i}\}, \neq k} y_{kli}^{i} = \sum_{k \in V \cup \{e^{i}\}, \neq k} y_{kli}^{i} \leq 2x_{i,j}^{i}  \forall i \in V, i = \{1, \dots, T\} \\ \end{cases} & (18) \\ \sum_{k \in V \cup \{e^{i}\}, \neq k} y_{kli}^{i} \leq 1  \forall i \in V \cup \{e^{i}\}, i \in \{V \cup V \cup e^{i}\}, i \in \{V \cup \{e^{i}\}, i \in \{V \cup \{e^{i}\}, j \in \{V$	$\begin{cases} x_{i1p}^p = x_{i2p}^p & \forall i \in V, p = \{1, \dots, P\} \end{cases}$	(5)
$\begin{cases} x_{11}^{k} = x_{12}^{k}, \forall i \in \{V^{in}\},  t = \{1, \dots, T\} \end{cases} $ $(7)$ $\begin{cases} x_{12}^{k} = x_{23}^{k}, \forall i \in \{V^{in}\},  t = \{1, \dots, T\} \end{cases} $ $(7)$ $\begin{cases} x_{12}^{k} = x_{23}^{k}, \forall i \in \{V^{in}\},  t = \{1, \dots, T\} \end{cases} $ $(8)$ $\begin{cases} \sum_{k=1}^{p} \sum_{j=1}^{k} x_{j0}^{k} = 0  \forall i \in \{V^{out}\},  b = \{1, \dots, B\} \end{cases} $ $(10)$ $x_{12}^{k} + x_{23}^{k} = 0  \forall i \in \{V^{out}\},  b = \{1, \dots, B\} \end{cases} $ $(11)$ $2x_{1p}^{p} - 1 \leq \sum_{k \in V \cup (q)} y_{kp}^{k} + \sum_{k \in V \cup (q)} y_{kp}^{k} \leq 2x_{12}^{k}  \forall i \in V, p = \{1, \dots, P\} \end{cases} $ $(12)$ $\sum_{k \in V \cup (q), j \neq k} y_{kp}^{k} = 1  \forall i \in V \cup \{c^{k}\},  p = \{1, \dots, P\} \end{cases} $ $(13)$ $\sum_{k \in V \cup (q), j \neq k} y_{kp}^{k} = 1  \forall i \in V \cup \{c^{k}\}, p = \{1, \dots, P\} \end{cases} $ $(14)$ $y_{ip}^{k} = 0  \forall i \in V \cup \{c^{k}\}, p = \{1, \dots, P\} \end{cases} $ $(15)$ $2x_{1q}^{k} - 1 \leq \sum_{k \in V \cup (q), j \neq k} y_{kp}^{k} + \sum_{k \in V \cup (q), j \neq k} y_{kp}^{k} = V \cup \{c^{k}\}, p = \{1, \dots, P\} \end{cases} $ $(15)$ $2x_{1q}^{k} - 1 \leq V \cup \{c^{k}\}, p = \{1, \dots, P\} $ $(15)$ $2x_{1q}^{k} - 1 \leq V \cup \{c^{k}\}, p = \{1, \dots, P\} $ $(16)$ $2x_{1q}^{k} - 1 \leq \sum_{k \in V \cup (q), j \neq k} y_{kp}^{k} + \sum_{k \in V \cup (q), j \neq k} y_{ki}^{k} \leq 2x_{12}^{k}  \forall i \in V, t = \{1, \dots, T\} $ $(17)$ $2x_{1q}^{k} - 1 \leq \sum_{k \in V \cup (q), j \neq k} y_{ki}^{k} + \sum_{k \in V \cup (q), j \neq k} y_{ki}^{k}  \forall i \in V \cup \{c^{k}\}, t = \{1, \dots, T\} $ $(16)$ $2x_{1q}^{k} - 1 \leq \sum_{k \in V \cup (q), j \neq k} y_{ki}^{k} = 1  \forall i \in V \cup \{c^{k}\}, t = \{1, \dots, T\} $ $(17)$ $2x_{1q}^{k} - 1 \leq \sum_{k \in V \cup (q), j \neq k} y_{ki}^{k}  \forall i \in V \cup \{c^{k}\}, t = \{1, \dots, T\} $ $(19)$ $\sum_{k \in V \cup (q), j \neq k} y_{ki}^{k} = 1  \forall i \in V \cup \{c^{k}\}, t \in V \cup \{c^{k}\}, t = \{1, \dots, T\} $ $(20)$ $y_{ii}^{k} = 0  \forall i \in V \cup \{c^{k}\}, t = \{1, \dots, T\} $ $(21)$ $2x_{ij}^{k} - x_{ki}^{k} + 1  \forall  V  \times (1 - \sum_{i=1}^{m} y_{ki}^{k})  \forall i \in V \cup \{c^{k}\}, t \in k \end{cases}$ $(22)$ $2x_{ij}^{k} - x_{ki}^{k} + y_{ki}^{k} = \sum_{k \in V \cup \{c^{k}\}, i \neq k}  \forall i \in V \cup \{c^{k}\}, b \in \{1, \dots, B\} $ $(24)$ $\sum_{k \in V \cup \{c^{k}\}, k \neq y_{ki}^{k}} = \sum_{k \in V \cup \{c^{k}\}, k \notin k}  \forall i \in V \cup \{c^{k}\}, b \in \{1, \dots, B\} $ $(25)$ $\sum_{k \in V \cup \{c^{k}\}, k \neq y_{ki}^{k}} = \sum_{k \in V \cup \{c^{k}\}, k \in V, i \neq k} $ $(26)$ $y_{ib}^{k} = 0  \forall i \in V \cup \{c^{k}\},$		
$\begin{cases} x_{11t}^{1} = x_{2t}^{1} & \forall i \in \{V^{\text{out}}\}, t = \{1, \dots, T\} \end{cases} $ $(8)$ $\begin{aligned} \sum_{ijb}^{B} = \sum_{ij}^{J} x_{ijb}^{b} = 1  \forall i \in V \qquad (9) \\ x_{ijb}^{b} + x_{ibb}^{b} = 0  \forall i \in \{V^{\text{in}}\},  b = \{1, \dots, B\} \qquad (10) \\ x_{ijb}^{b} + x_{ibb}^{b} = 0  \forall i \in \{V^{\text{out}}\},  b = \{1, \dots, B\} \qquad (11) \\ 2x_{ijp}^{p} - 1 \leq \sum_{k \in V \cup \{r^{0}\}} y_{kp}^{k} + \sum_{k \in V \cup \{r^{0}\}} y_{kp}^{k} \leq 2x_{ijp}^{p}  \forall i \in V, p = \{1, \dots, P\} \qquad (13) \\ \sum_{k \in V \cup \{r^{0}\}, i \neq k} y_{kp}^{k} \leq 1  \forall i \in V \cup \{r^{0}\}, p = \{1, \dots, P\} \qquad (14) \\ y_{ijp}^{p} = 0  \forall i \in V \cup \{r^{0}\}, p = \{1, \dots, P\} \qquad (15) \\ 1 \leq z_{i}^{p} \leq  V   \forall i \in V \qquad (16) \\ x_{ijp}^{p} - z_{k}^{k} + 1 \leq  V (1 - \sum_{p=1} y_{ikp}^{p})  \forall i, k \in V, i \neq k \qquad (16) \\ z_{ij}^{p} - z_{k}^{k} + 1 \leq  V (1 - \sum_{p=1} y_{ikp}^{p})  \forall i, k \in V, i \neq k \qquad (17) \\ 2x_{i2r}^{1} - 1 \leq \sum_{k \in V \cup \{r^{1}\}, i \neq k} y_{kit}^{k} + \sum_{k \in V \cup \{r^{1}\}, i \neq k} y_{kit} \leq 2x_{i2r}^{1}  \forall i \in V, t = \{1, \dots, T\} \qquad (13) \\ \sum_{k \in V \cup \{r^{1}\}, i \neq k} y_{kit}^{k} \leq 1  \forall i \in V \cup \{r^{1}\}, t = \{1, \dots, T\} \qquad (15) \\ 2x_{i2r}^{k} - 1 \leq \sum_{k \in V \cup \{r^{1}\}, i \neq k} y_{kit}^{k} + \sum_{k \in V \cup \{r^{1}\}, i \neq k} y_{kit} \leq 2x_{i2r}^{1}  \forall i \in V, t = \{1, \dots, T\} \qquad (15) \\ 2x_{i2r}^{1} - 1 \leq \sum_{k \in V \cup \{r^{1}\}, i \neq k} y_{kit}^{k}  \forall i \in V \cup \{r^{1}\}, t = \{1, \dots, T\} \qquad (20) \\ y_{it}^{k} = 0  \forall i \in V \cup \{r^{1}\}, t = \{1, \dots, T\} \qquad (21) \\ 2x_{i}^{k} =  V   \forall i \in V \qquad (22) \\ z_{i}^{1} - z_{k}^{k} + 1 \leq  V  \times (1 - \sum_{r=1}^{T} y_{kit}^{k})  \forall i, k \in V, i \neq k \qquad (22) \\ z_{i}^{1} - z_{k}^{k} + 1 \leq  V  \times (1 - \sum_{r=1}^{T} y_{kit}^{k})  \forall i, k \in V, i \neq k \qquad (23) \\ 2x \sum_{j=1}^{J} x_{jib}^{b} - 1 \leq \sum_{k \in V \cup \{r^{k}\}, k \neq k}  k \in V \cup \{r^{k}\}, k \neq V \in V \cup \{r^{k}\}, k \neq k \in V \cup \{r^{k}\}, k \neq K \in V \mid r \in V  (22) \\ z_{k}^{k} - z_{k}^{k} + 1 \leq  V  \times (1 - \sum_{k=1}^{B} y_{kk}^{k})  \forall i \in V \cup \{r^{k}\}, k \in V \mid r \in V \mid r \in V  (26) \\ z_{k}^{k} b \in V \mid r \in V  (26) \\ z_{k}^{k} b \in V \mid r \in V  (26) \\ z_{k}^{k} b \in V \mid r \in V  (26) \\ z_{k}^{k} b \in V \mid r \in V  (26) \\ z_{k}^{k} b \in V \mid r \in V  (26) \\ z_{k}^{k} b \in V \mid r \in V  (26) \\ z_{k}^{k} b \in V \mid r \in V  (26) \\ z_{k}^{k} b $		(6)
$\begin{cases} \sum_{k=V}^{I_{11}} - \sum_{i=1}^{O_{2}} \forall i \in \{V^{\text{out}}\}, t = \{1, \dots, T\} \end{cases}$ $\sum_{b=1}^{B} \sum_{j=1}^{J} x_{ijb}^{b} = 1  \forall i \in V \qquad (9)$ $x_{i1b}^{i_{1}} + x_{i2b}^{b} = 0  \forall i \in \{V^{\text{in}}\},  b = \{1, \dots, B\} \qquad (10)$ $x_{i2b}^{i_{1}} + x_{i3b}^{i_{2}} = 0  \forall i \in \{V^{\text{out}}\},  b = \{1, \dots, B\} \qquad (11)$ $2x_{i1p}^{i_{1}} - 1 \leq \sum_{k \in V \cup \{e^{k}\}} y_{kp}^{i_{k}} + \sum_{k \in V \cup \{e^{k}\}} y_{kp}^{i_{k}} \leq 2x_{i1p}^{i_{1}}  \forall i \in V, p = \{1, \dots, P\} \qquad (12)$ $\sum_{k \in V \cup \{e^{k}\}} y_{kp}^{i_{k}} = 1  \forall i \in V \cup \{e^{k}\}, p = \{1, \dots, P\} \qquad (13)$ $\sum_{k \in V \cup \{e^{k}\}} y_{kp}^{i_{k}} = \sum_{k \in V \cup \{e^{k}\}} y_{kp}^{i_{k}}  \forall i \in V \cup \{e^{k}\}, p = \{1, \dots, P\} \qquad (14)$ $y_{i1p}^{i_{p}} = 0  \forall i \in V \cup \{e^{k}\}, p = \{1, \dots, P\} \qquad (15)$ $z_{i}^{i_{p}} - z_{i}^{k} + 1 \leq  V  (1 - \sum_{p=1}^{p} y_{ikp}^{i_{k}})  \forall i, k \in V, i \neq k \qquad (17)$ $2x_{i2r}^{i_{2}} - 1 \leq \sum_{k \in V \cup \{e^{k}\}, i \neq k} \sum_{k \in V \cup \{e^{k}\}, k \in V$	$\begin{cases} x_{i1t}^{t} = 0 \\ x_{i2t}^{t} = x_{i3t}^{t} \end{cases} \forall i \in \{V^{\text{in}}\},  t = \{1, \dots, T\} \end{cases}$	(7)
$ \begin{aligned} x_{lib}^{h} + x_{lab}^{h} &= 0  \forall i \in \{V^{in}\},  b = \{1, \dots, B\} \end{aligned} \tag{10} \\ x_{lib}^{h} + x_{lab}^{h} &= 0  \forall i \in \{V^{out}\},  b = \{1, \dots, B\} \end{aligned} \tag{11} \\ 2x_{lip}^{p} - 1 \leq \sum_{k \in V \cup \{e^{k}\}} y_{lip}^{p} + \sum_{k \in V \cup \{e^{k}\}} y_{lip}^{k} \geq 2x_{lip}^{p}  \forall i \in V, p = \{1, \dots, P\} \end{aligned} \tag{12} \\ \sum_{k \in V \cup \{e^{k}\}, j \neq k} y_{lip}^{p} = \sum_{k \in V \cup \{e^{k}\}, j \neq k} y_{kip}^{p}  \forall i \in V \cup \{c^{k}\}, p = \{1, \dots, P\} \end{aligned} \tag{13} \\ \sum_{k \in V \cup \{e^{k}\}, j \neq k} y_{lip}^{p} = \sum_{k \in V \cup \{e^{k}\}, j \neq k} y_{kip}^{p}  \forall i \in V \cup \{c^{k}\}, p = \{1, \dots, P\} \end{aligned} \tag{14} \\ y_{lip}^{p} = 0  \forall i \in V \cup \{c^{k}\}, p = \{1, \dots, P\} \end{aligned} \tag{16} \\ z_{lip}^{p} - z_{k}^{k} + 1 \leq  V (1 - \sum_{p=1} y_{lip}^{p})  \forall i, k \in V, i \neq k \end{aligned} \tag{17} \\ 2x_{lip}^{i} - 1 \leq \sum_{k \in V \cup \{e^{k}\}, i \neq k} \sum_{k \in V \cup \{e$	$\begin{cases} x_{i1t}^{t} = x_{i2t}^{t} & \forall i \in \{V^{\text{out}}\}, t = \{1, \dots, T\} \\ x_{i3t}^{t} = 0 & \forall i \in \{V^{\text{out}}\}, t = \{1, \dots, T\} \end{cases}$	(8)
$ \begin{aligned} x_{lib}^{h} + x_{lab}^{h} &= 0  \forall i \in \{V^{in}\},  b = \{1, \dots, B\} \end{aligned} \tag{10} \\ x_{lib}^{h} + x_{lab}^{h} &= 0  \forall i \in \{V^{out}\},  b = \{1, \dots, B\} \end{aligned} \tag{11} \\ 2x_{lip}^{p} - 1 \leq \sum_{k \in V \cup \{e^{k}\}} y_{lip}^{p} + \sum_{k \in V \cup \{e^{k}\}} y_{lip}^{k} \geq 2x_{lip}^{p}  \forall i \in V, p = \{1, \dots, P\} \end{aligned} \tag{12} \\ \sum_{k \in V \cup \{e^{k}\}, j \neq k} y_{lip}^{p} = \sum_{k \in V \cup \{e^{k}\}, j \neq k} y_{kip}^{p}  \forall i \in V \cup \{c^{k}\}, p = \{1, \dots, P\} \end{aligned} \tag{13} \\ \sum_{k \in V \cup \{e^{k}\}, j \neq k} y_{lip}^{p} = \sum_{k \in V \cup \{e^{k}\}, j \neq k} y_{kip}^{p}  \forall i \in V \cup \{c^{k}\}, p = \{1, \dots, P\} \end{aligned} \tag{14} \\ y_{lip}^{p} = 0  \forall i \in V \cup \{c^{k}\}, p = \{1, \dots, P\} \end{aligned} \tag{16} \\ z_{lip}^{p} - z_{k}^{k} + 1 \leq  V (1 - \sum_{p=1} y_{lip}^{p})  \forall i, k \in V, i \neq k \end{aligned} \tag{17} \\ 2x_{lip}^{i} - 1 \leq \sum_{k \in V \cup \{e^{k}\}, i \neq k} \sum_{k \in V \cup \{e$	$\sum_{k=1}^{B} \sum_{i=1}^{J} x_{i:k}^{b} = 1  \forall i \in V$	(9)
$ \begin{split} & \sum_{k_{12b}^{b}} + x_{13b}^{b} = 0  \forall i \in \{V^{\text{out}}\},  b = \{1, \dots, B\} \\ & (11) \\ & 2x_{11p}^{i} - 1 \leq \sum_{k \in V \cup \{c^{b}\}} y_{ikp}^{k} + \sum_{k \in V \cup \{c^{b}\}} y_{kip}^{p} \leq 2x_{11p}^{b}  \forall i \in V, p = \{1, \dots, P\} \\ & (12) \\ & \sum_{k \in V \cup \{c^{b}\}, i \neq k} y_{ikp}^{b} \leq 1  \forall i \in V \cup \{c^{b}\}, p = \{1, \dots, P\} \\ & (13) \\ & \sum_{k \in V \cup \{c^{b}\}, i \neq k} y_{ikp}^{b} = \sum_{k \in V \cup \{c^{b}\}, i \neq k} y_{kip}^{b}  \forall i \in V \cup \{c^{b}\}, p = \{1, \dots, P\} \\ & (14) \\ & y_{iip}^{ip} = 0  \forall i \in V \cup \{c^{b}\}, p = \{1, \dots, P\} \\ & (15) \\ & 1 \leq z_{i}^{p} \leq  V   \forall i \in V \\ & (16) \\ & z_{i}^{1} - z_{k}^{2} + 1 \leq  V (1 - \sum_{p=1} y_{ikp}^{b})  \forall i, k \in V, i \neq k \\ & (17) \\ & 2x_{i2r}^{i} - 1 \leq \sum_{k \in V \cup \{c^{i}\}, i \neq k} y_{kir}^{i} + \sum_{k \in V \cup \{c^{i}\}, i \neq k} y_{kir}^{i} \leq 2x_{i2r}^{i}  \forall i \in V, t = \{1, \dots, T\} \\ & \sum_{k \in V \cup \{c^{i}\}, i \neq k} y_{ik}^{i} \leq 1  \forall i \in V \cup \{c^{i}\}, t = \{1, \dots, T\} \\ & (14) \\ & y_{iir}^{i} = 0  \forall i \in V \cup \{c^{i}\}, t = \{1, \dots, T\} \\ & \sum_{k \in V \cup \{c^{i}\}, i \neq k} y_{kir}^{i} \leq 1  \forall i \in V \cup \{c^{i}\}, t = \{1, \dots, T\} \\ & (15) \\ & \sum_{k \in V \cup \{c^{i}\}, i \neq k} y_{kir}^{i} \leq 1  \forall i \in V \cup \{c^{i}\}, t \in V, i \neq k \\ & (23) \\ & 2 \times \sum_{j=1}^{j} x_{jib}^{i} - 1 \leq \sum_{k \in V \cup \{c^{i}\}, i \neq k} y_{kib}^{i} + \sum_{k \in V \cup \{c^{i}\}, i \neq k} y_{kib}^{i} \leq 2 \sum_{j=1}^{j} x_{jb}^{i}  \forall i \in V,  b = \{1, \dots, B\} \\ & \sum_{k \in V \cup \{c^{k}\}, i \neq k} y_{kib}^{k} = 1  \forall i \in V \cup \{c^{k}\}, b = \{1, \dots, B\} \\ & \sum_{k \in V \cup \{c^{k}\}, i \neq k} y_{kib}^{k} = \sum_{k \in V \mid \{c^{k}\}, i \neq k} y_{kib}^{k}  \forall i \in V \cup \{c^{k}\}, b = \{1, \dots, B\} \\ & \sum_{k \in V \cup \{c^{k}\}, i \neq k} y_{kib}^{k} = \sum_{k \in V \mid \{c^{k}\}, i \neq k} y_{kib}^{k}  \forall i \in V \cup \{c^{k}\}, b = \{1, \dots, B\} \\ & y_{iib}^{k} = 0  \forall i \in V \cup \{c^{k}\}, b = \{1, \dots, B\} \\ & y_{iib}^{k} = 0  \forall i \in V \cup \{c^{k}\}, b = \{1, \dots, B\} \\ & z_{iib}^{k} =  V   \forall i \in V \\ & z_{iib}^{k} = y_{iib}^{k}  \forall i \in V \cup \{c^{k}\}, b = \{1, \dots, B\} \\ & z_{iib}^{k} = y_{iib}^{k} = y_{iib}^{k}  \forall i, k \in V, i \neq k, p = \{1, \dots, P\} \\ & z_{iib}^{k} = z_{iib}^{k} =  V  \mid \forall i \in V \\ & z_{iib}^{k} = z_{iib}^$	$\begin{aligned} z_{ilb}^{b} + z_{i2b}^{b} &= 0  \forall i \in \{V^{in}\},  b = \{1, \dots, B\} \end{aligned}$	(10)
$\begin{split} & 2x_{i1p}^{p} - 1 \leq \sum_{k \in V \cup \{c^{k}\}} y_{ikp}^{p} + \sum_{k \in V \cup \{c^{k}\}} y_{kip}^{p} \leq 2x_{i1p}^{p}  \forall i \in V, p = \{1, \dots, P\} \end{split} \tag{12} \\ & \sum_{k \in V \cup \{c^{k}\}, i \neq k} y_{ikp}^{p} \leq \sum_{k \in V \cup \{c^{k}\}, i \neq k} y_{kip}^{p}  \forall i \in V \cup \{c^{k}\}, p = \{1, \dots, P\} \end{aligned} \tag{13} \\ & \sum_{k \in V \cup \{c^{k}\}, i \neq k} y_{ikp}^{p} = \sum_{k \in V \cup \{c^{k}\}, i \neq k} y_{kip}^{p}  \forall i \in V \cup \{c^{k}\}, p = \{1, \dots, P\} \end{aligned} \tag{14} \\ & y_{iip}^{p} = 0  \forall i \in V \cup \{c^{k}\}, p = \{1, \dots, P\} \end{aligned} \tag{16} \\ & \sum_{i}^{p} - c_{k}^{p} + 1 \leq  V (1 - \sum_{p=1})y_{ikp}^{p})  \forall i, k \in V, i \neq k \end{aligned} \tag{16} \\ & \sum_{i}^{p} - c_{k}^{p} + 1 \leq  V (1 - \sum_{p=1})y_{ikp}^{p})  \forall i, k \in V, i \neq k \end{aligned} \tag{17} \\ & 2x_{i2r}^{1} - 1 \leq \sum_{k \in V \cup \{c^{k}\}, i \neq k} y_{kit}^{k} + \sum_{k \in V \cup \{c^{k}\}, i \neq k} y_{kit}^{k} \leq 2x_{i2r}^{1}  \forall i \in V, t = \{1, \dots, T\} \end{aligned} \tag{18} \\ & \sum_{k \in V \cup \{c^{k}\}, i \neq k} y_{iki}^{k} \leq 1  \forall i \in V \cup \{c^{k}\}, t = \{1, \dots, T\} \end{aligned} \tag{19} \\ & \sum_{k \in V \cup \{c^{k}\}, i \neq k} y_{iki}^{k} = \sum_{k \in V \cup \{c^{k}\}, i \neq k} y_{kit}^{k} = \sum_{k \in V \cup \{c^{k}\}, i \neq k} y_{kit}^{k}} \forall i \in V \cup \{c^{k}\}, t = \{1, \dots, T\} \end{aligned} \tag{20} \\ & y_{i1}^{i} = 0  \forall i \in V \cup \{c^{k}\}, i = \{1, \dots, T\} \end{aligned} \tag{21} \\ & \leq z_{i}^{i} - z_{k}^{i} + 1 \leq  V  \times (1 - \sum_{i=1}^{T} y_{ik}^{k}))  \forall i, k \in V, i \neq k \end{aligned} \tag{22} \\ & z_{i}^{i} - z_{k}^{i} + 1 \leq  V  \times (1 - \sum_{i=1}^{T} y_{ik}^{k}))  \forall i, k \in V, i \neq k \end{aligned} \tag{22} \\ & \sum_{i}^{i} - z_{k}^{i} + 1 \leq  V  \times (1 - \sum_{i=1}^{T} y_{ik}^{k})  \forall i \in V \cup \{c^{k}\}, b = \{1, \dots, B\} \end{aligned} \tag{24} \\ & \sum_{k \in V \cup \{c^{k}\}, i \neq k} y_{kib}^{k} = \sum_{k \in V \cup \{c^{k}\}, i \neq k} y_{kib}^{k}  \forall i \in V \cup \{c^{k}\}, b = \{1, \dots, B\} \end{aligned} \tag{26} \\ & y_{ib}^{k} = 0  \forall i \in V \cup \{c^{k}\}, b \in \{1, \dots, B\} \end{aligned} \tag{27} \\ & \leq z_{i}^{k} - c_{k}^{k} + 1 \leq  V  \times (1 - \sum_{k=1}^{B} y_{kib}^{k})  \forall i, k \in V, i \neq k \end{aligned} \tag{28} \\ & z_{i}^{k} - c_{k}^{k} + 1 \leq  V  \times (1 - \sum_{k=1}^{B} y_{kib}^{k})  \forall i, k \in V, i \neq k, p = \{1, \dots, P\} \end{aligned} \tag{28} \\ & z_{i}^{k} - z_{k}^{k} + 1 \leq  V  \times (1 - \sum_{k=1}^{B} y_{kib}^{k})  \forall i, k \in V, i \neq k, p = \{1, \dots, P\} \end{aligned} \tag{28} \\ & z_{i}^{k} - b_{i}^{k} \leq z_{ki} + b_{i}^{k} = z_{ki} + b_{i}^{k} = z_{ki} + z_{ki} \in z_{ki} + b_{i}^{k} = z_{ki} + z_{ki} \in z_$		(11)
$ \begin{split} & \sum_{k \in V \cup \{c^{k}\}, i \neq k} y_{kp}^{k} = \sum_{k \in V \cup \{c^{k}\}, i \neq k} y_{kp}^{k}  \forall i \in V \cup \{c^{k}\}, p = \{1, \dots, P\}  (14) \\ & y_{lip}^{p} = 0  \forall i \in V \cup \{c^{p}\}, p = \{1, \dots, P\}  (15) \\ & 1 \leq z_{i}^{p} \leq  V   \forall i \in V  (16) \\ & z_{i}^{p} - z_{k}^{p} + 1 \leq  V (1 - \sum_{p=1} y_{ikp}^{p})  \forall i, k \in V, i \neq k  (17) \\ & 2x_{i2t}^{i} - 1 \leq \sum_{k \in V \cup \{c^{i}\}, i \neq k} y_{ikt}^{i} + \sum_{k \in V \cup \{c^{i}\}, i \neq k} y_{kit}^{i} \leq 2x_{i2t}^{i}  \forall i \in V, t = \{1, \dots, T\}  (18) \\ & \sum_{k \in V \cup \{c^{i}\}, i \neq k} y_{ikt}^{i} = \sum_{k \in V \cup \{c^{i}\}, i \neq k} y_{kit}^{i}  \forall i \in V \cup \{c^{i}\}, t = \{1, \dots, T\}  (20) \\ & y_{i1t}^{i} = 0  \forall i \in V \cup \{c^{i}\}, i \neq y_{kit}^{i}  \forall i \in V \cup \{c^{i}\}, t = \{1, \dots, T\}  (21) \\ & 1 \leq z_{i}^{i} \leq  V   \forall i \in V  (2^{i}), i \neq y_{kit}^{i}  \forall i, k \in V, i \neq k  (22) \\ & z_{i}^{i} - z_{k}^{i} + 1 \leq  V  \times (1 - \sum_{t=1}^{T} y_{ikt}^{t})  \forall i, k \in V, i \neq k  (23) \\ & 2 \times \sum_{j=1}^{J} x_{jb}^{ib} - 1 \leq \sum_{k \in V \cup \{c^{k}\}, i \neq k} y_{kib}^{ib} + \sum_{k \in V \cup \{c^{k}\}, i \neq k} y_{kib}^{ib} \leq 2 \sum_{j=1}^{J} x_{jjb}^{ib}  \forall i \in V,  b = \{1, \dots, B\}  (24) \\ & \sum_{k \in V \cup \{c^{k}\}, i \neq k} y_{kib}^{ib} \leq 1  \forall i \in V \cup \{c^{b}\}, b = \{1, \dots, B\}  (25) \\ & \sum_{k \in V \cup \{c^{k}\}, i \neq k} y_{kib}^{ib} = \sum_{k \in V + \{c\}, i \neq k} y_{kib}^{ib}  \forall i \in V \cup \{c^{b}\}, b = \{1, \dots, B\}  (27) \\ & 1 \leq z_{b}^{i} \leq  V   \forall i \in V  (2^{k}), k \in V, i \neq k,  (29) \\ & s_{ib}^{i} - z_{k}^{i} + 1 \leq  V  \times (1 - \sum_{b=1}^{B} y_{ib}^{ib})  \forall i, k \in V, i \neq k  (29) \\ & s_{ib}^{i} - z_{k}^{i} + 1 \leq  V  \times (1 - \sum_{b=1}^{B} y_{ib}^{ib})  \forall i, k \in V, i \neq k,  (29) \\ & s_{ik}^{i} + D_{ik}^{i} \leq s_{k1} + M(1 - y_{ik}^{ib})  \forall i, k \in V, i \neq k,  (29) \\ & s_{ik}^{i} + D_{ik}^{i} \leq s_{k1} + M(1 - y_{ik}^{ib})  \forall i, k \in V, i \neq k, p = \{1, \dots, P\}  (31) \\ & s_{ik}^{i} v_{i}^{i} + s_{i3i \in (VW)} + D_{ik}^{i} \mid v_{i} \in v_{i} \leq s_{k2k \in \{VW} + s_{k1k \in (VW)} + M(1 - y_{ik}^{ib})  i, k \in V, i \neq k, p = \{1, \dots, T\}  (31) \\ & s_{ik}^{i} v_{i}^{i} + s_{ij}^{i} \in (0, 1)  \forall i, k \in V, j = \{1, \dots, J\}, p \in \{1, \dots, T\}, b \in \{1, \dots, B\}  (34) \end{aligned}$	$2x_{i1p}^{p} - 1 \le \sum_{k \in V \cup \{c^{p}\}} y_{ikp}^{p} + \sum_{k \in V \cup \{c^{p}\}} y_{kip}^{p} \le 2x_{i1p}^{p}  \forall i \in V, p = \{1, \dots, P\}$	(12)
$ \begin{split} & \sum_{k \in V \cup \{c^{k}\}, i \neq k} y_{kp}^{k} = \sum_{k \in V \cup \{c^{k}\}, i \neq k} y_{kp}^{k}  \forall i \in V \cup \{c^{k}\}, p = \{1, \dots, P\}  (14) \\ & y_{lip}^{p} = 0  \forall i \in V \cup \{c^{p}\}, p = \{1, \dots, P\}  (15) \\ & 1 \leq z_{i}^{p} \leq  V   \forall i \in V  (16) \\ & z_{i}^{p} - z_{k}^{p} + 1 \leq  V (1 - \sum_{p=1} y_{ikp}^{p})  \forall i, k \in V, i \neq k  (17) \\ & 2x_{i2t}^{i} - 1 \leq \sum_{k \in V \cup \{c^{i}\}, i \neq k} y_{ikt}^{i} + \sum_{k \in V \cup \{c^{i}\}, i \neq k} y_{kit}^{i} \leq 2x_{i2t}^{i}  \forall i \in V, t = \{1, \dots, T\}  (18) \\ & \sum_{k \in V \cup \{c^{i}\}, i \neq k} y_{ikt}^{i} = \sum_{k \in V \cup \{c^{i}\}, i \neq k} y_{kit}^{i}  \forall i \in V \cup \{c^{i}\}, t = \{1, \dots, T\}  (20) \\ & y_{i1t}^{i} = 0  \forall i \in V \cup \{c^{i}\}, i \neq y_{kit}^{i}  \forall i \in V \cup \{c^{i}\}, t = \{1, \dots, T\}  (21) \\ & 1 \leq z_{i}^{i} \leq  V   \forall i \in V  (2^{i}), i \neq y_{kit}^{i}  \forall i, k \in V, i \neq k  (22) \\ & z_{i}^{i} - z_{k}^{i} + 1 \leq  V  \times (1 - \sum_{t=1}^{T} y_{ikt}^{t})  \forall i, k \in V, i \neq k  (23) \\ & 2 \times \sum_{j=1}^{J} x_{jb}^{ib} - 1 \leq \sum_{k \in V \cup \{c^{k}\}, i \neq k} y_{kib}^{ib} + \sum_{k \in V \cup \{c^{k}\}, i \neq k} y_{kib}^{ib} \leq 2 \sum_{j=1}^{J} x_{jjb}^{ib}  \forall i \in V,  b = \{1, \dots, B\}  (24) \\ & \sum_{k \in V \cup \{c^{k}\}, i \neq k} y_{kib}^{ib} \leq 1  \forall i \in V \cup \{c^{b}\}, b = \{1, \dots, B\}  (25) \\ & \sum_{k \in V \cup \{c^{k}\}, i \neq k} y_{kib}^{ib} = \sum_{k \in V + \{c\}, i \neq k} y_{kib}^{ib}  \forall i \in V \cup \{c^{b}\}, b = \{1, \dots, B\}  (27) \\ & 1 \leq z_{b}^{i} \leq  V   \forall i \in V  (2^{k}), k \in V, i \neq k,  (29) \\ & s_{ib}^{i} - z_{k}^{i} + 1 \leq  V  \times (1 - \sum_{b=1}^{B} y_{ib}^{ib})  \forall i, k \in V, i \neq k  (29) \\ & s_{ib}^{i} - z_{k}^{i} + 1 \leq  V  \times (1 - \sum_{b=1}^{B} y_{ib}^{ib})  \forall i, k \in V, i \neq k,  (29) \\ & s_{ik}^{i} + D_{ik}^{i} \leq s_{k1} + M(1 - y_{ik}^{ib})  \forall i, k \in V, i \neq k,  (29) \\ & s_{ik}^{i} + D_{ik}^{i} \leq s_{k1} + M(1 - y_{ik}^{ib})  \forall i, k \in V, i \neq k, p = \{1, \dots, P\}  (31) \\ & s_{ik}^{i} v_{i}^{i} + s_{i3i \in (VW)} + D_{ik}^{i} \mid v_{i} \in v_{i} \leq s_{k2k \in \{VW} + s_{k1k \in (VW)} + M(1 - y_{ik}^{ib})  i, k \in V, i \neq k, p = \{1, \dots, T\}  (31) \\ & s_{ik}^{i} v_{i}^{i} + s_{ij}^{i} \in (0, 1)  \forall i, k \in V, j = \{1, \dots, J\}, p \in \{1, \dots, T\}, b \in \{1, \dots, B\}  (34) \end{aligned}$	$\sum_{k \in V \cup \{c^{p}\}, i \neq k} y_{ikp}^{p} \le 1  \forall i \in V \cup \{c^{p}\}, p = \{1, \dots, P\}$	(13)
$ \begin{split} &1 \leq z_{i}^{p} \leq  V   \forall i \in V \qquad (16) \\ &z_{i}^{p} - z_{k}^{p} + 1 \leq  V (1 - \sum_{p=1} y_{ikp}^{p})  \forall i, k \in V, i \neq k \qquad (17) \\ &2x_{i2t}^{1} - 1 \leq \sum_{k \in V \cup \{c^{1}\}, i \neq k} y_{ikt}^{1} + \sum_{k \in V \cup \{c^{1}\}, i \neq k} y_{kit}^{1} \leq 2x_{i2t}^{1}  \forall i \in V, t = \{1, \dots, T\} \qquad (18) \\ &\sum_{k \in V \cup \{c^{1}\}, i \neq k} y_{ikt}^{1} \leq 1  \forall i \in V \cup \{c^{1}\}, t = \{1, \dots, T\} \qquad (19) \\ &\sum_{k \in V \cup \{c^{1}\}, i \neq k} y_{ikt}^{1} = \sum_{k \in V \cup \{c^{1}\}, i \neq k} y_{kit}^{1}  \forall i \in V \cup \{c^{1}\}, t = \{1, \dots, T\} \qquad (20) \\ &y_{iit}^{1} = 0  \forall i \in V \cup \{c^{1}\}, t = \{1, \dots, T\} \qquad (21) \\ &1 \leq z_{i}^{1} \leq  V   \forall i \in V \qquad (22) \\ &z_{i}^{1} - z_{k}^{1} + 1 \leq  V  \times (1 - \sum_{i=1}^{T} y_{iki}^{1})  \forall i, k \in V, i \neq k \qquad (22) \\ &z_{i}^{1} - z_{k}^{1} + 1 \leq  V  \times (1 - \sum_{i=1}^{T} y_{iki}^{1})  \forall i, k \in V, i \neq k \qquad (23) \\ &2 \times \sum_{j=1}^{J} x_{jb}^{b} - 1 \leq \sum_{k \in V \cup \{c^{b}\}, i \neq k} y_{bb}^{b} + \sum_{k \in V \cup \{c^{b}\}, i \neq k} y_{kib}^{b} \leq 2 \sum_{j=1}^{J} x_{jb}^{b}  \forall i \in V,  b = \{1, \dots, B\} \qquad (24) \\ &\sum_{k \in V \cup \{c^{k}\}, i \neq k} y_{ikb}^{b} \leq 1  \forall i \in V \cup \{c^{b}\}, b = \{1, \dots, B\} \qquad (25) \\ &\sum_{k \in V \cup \{c^{k}\}, i \neq k} y_{bib}^{b} = \sum_{k \in V + \{c\}, i \neq k} y_{kib}^{b}  \forall i \in V \cup \{c^{b}\}, b = \{1, \dots, B\} \qquad (26) \\ &y_{jib}^{b} = 0  \forall i \in V \cup \{c^{b}\}, b = \{1, \dots, B\} \qquad (27) \\ &1 \leq z_{i}^{b} \leq  V   \forall i \in V \qquad (28) \\ &z_{i}^{b} - z_{k}^{b} + 1 \leq  V  \times (1 - \sum_{b=1}^{B} y_{ikb}^{b})  \forall i, k \in V, i \neq k \qquad (28) \\ &z_{i}^{b} - z_{k}^{b} + 1 \leq  V  \times (1 - \sum_{b=1}^{B} y_{ikb}^{b})  \forall i, k \in V, i \neq k \qquad (29) \\ &s_{i4} + D_{ik}^{p} \leq s_{k1} + M(1 - y_{ik}^{b})  \forall i, k \in V, i \neq k, p = \{1, \dots, P\} \qquad (30) \\ &s_{i4} V_{ie}^{p} + s_{i3i \in (V \cong 1} + D_{ik, i \in V, i \neq k}^{b} \leq s_{k3k \in \{V^{m}\}} + s_{k1k \in (V \otimes 1} + M(1 - y_{ik}^{b})_{i,k \in V, i \neq k}  t = \{1, \dots, T\} \qquad (31) \\ &s_{i4} v_{ie}^{p} (w_{i}) + s_{i3i \in (V \otimes 1} + D_{ik}^{b} \leq s_{k3k \in \{V^{m}\}} + s_{k1k \in (V \otimes 1} + M(1 - y_{ik}^{b})_{i,k \in V, i \neq k}  b = \{1, \dots, B\} \qquad (32) \\ &w_{ij}, s_{ij}^{c}, z_{i}^{c}, z_{i}^{c}, z_{i}^{c} \in 0  \forall i \in \{1, \dots, J\}, p = \{1, \dots, J\}, p \in \{1, \dots, T\}, b \in \{1, \dots, B\} \qquad (34) \end{aligned}$	$\sum_{k \in V \cup \{c^{p}\}, i \neq k} y_{ikp}^{p} = \sum_{k \in V \cup \{c^{p}\}, i \neq k} y_{kip}^{p}  \forall i \in V \cup \{c^{p}\}, p = \{1, \dots, P\}$	(14)
$ \begin{split} z_{i2t}^{p} - z_{k}^{p} + 1 &\leq  V (1 - \sum_{p=1} y_{ikp}^{p})  \forall i, k \in V, i \neq k \\ (17) \\ 2x_{i2t}^{t} - 1 &\leq \sum_{k \in V \cup \{c^{i}\}, i \neq k} y_{ikt}^{t} + \sum_{k \in V \cup \{c^{i}\}, i \neq k} y_{kit}^{t} \leq 2x_{i2t}^{t}  \forall i \in V, t = \{1, \dots, T\} \\ (18) \\ \sum_{k \in V \cup \{c^{i}\}, i \neq k} y_{ikt}^{t} &\leq 1  \forall i \in V \cup \{c^{i}\}, t = \{1, \dots, T\} \\ (19) \\ \sum_{k \in V \cup \{c^{i}\}, i \neq k} y_{ikt}^{t} &\equiv \sum_{k \in V \cup \{c^{i}\}, i \neq k} y_{kit}^{t}  \forall i \in V \cup \{c^{i}\}, t = \{1, \dots, T\} \\ (20) \\ y_{itt}^{t} = 0  \forall i \in V \cup \{c^{i}\}, t = \{1, \dots, T\} \\ (21) \\ 1 \leq z_{i}^{t} \leq  V   \forall i \in V \\ (22) \\ z_{i}^{t} - z_{k}^{t} + 1 \leq  V  \times (1 - \sum_{t=1}^{T} y_{ikt}^{t})  \forall i, k \in V, i \neq k \\ (23) \\ 2 \times \sum_{j=1}^{J} x_{ijb}^{ib} - 1 \leq \sum_{k \in V \cup \{c^{k}\}, i \neq k} y_{ikb}^{b} + \sum_{k \in V \cup \{c^{k}\}, i \neq k} y_{kib}^{b} \leq 2 \sum_{j=1}^{J} x_{ijb}^{b}  \forall i \in V, b = \{1, \dots, B\} \\ \sum_{k \in V \cup \{c^{k}\}, i \neq k} y_{ikb}^{b} \leq 1  \forall i \in V \cup \{c^{k}\}, b = \{1, \dots, B\} \\ \sum_{k \in V \cup \{c^{k}\}, i \neq k} y_{ikb}^{b} \leq 1  \forall i \in V \cup \{c^{k}\}, b = \{1, \dots, B\} \\ \sum_{k \in V \cup \{c^{k}\}, i \neq k} y_{ikb}^{b} = \sum_{k \in V + \{c\}, i \neq k} y_{kib}^{b}  \forall i \in V \cup \{c^{k}\}, b = \{1, \dots, B\} \\ (25) \\ \sum_{k \in V \cup \{c^{k}\}, i \neq k} y_{ikb}^{b} \leq 1  \forall i \in V \cup \{c^{k}\}, b = \{1, \dots, B\} \\ (26) \\ y_{iib}^{b} = 0  \forall i \in V \cup \{c^{b}\}, b = \{1, \dots, B\} \\ (27) \\ 1 \leq z_{i}^{b} \leq  V   \forall i \in V \\ (28) \\ z_{i}^{b} - z_{k}^{b} + 1 \leq  V  \times (1 - \sum_{b=1}^{B} y_{kb}^{b})  \forall i, k \in V, i \neq k \\ (29) \\ s_{i4} + D_{ik}^{p} \leq s_{k1} + M(1 - y_{ikp}^{p})  \forall i, k \in V, i \neq k, p = \{1, \dots, P\} \\ (30) \\ s_{i4} \forall i \in \{V^{m}\} + s_{i3} i \in \{V^{m}\} + D_{ik}^{i} i \in V , i \neq k, p = \{1, \dots, P\} \\ s_{i4} \forall i \in \{V^{m}\} + s_{i3} i \in \{V^{m}\} + D_{ik}^{b} i \in V , i \neq k \\ s_{i3} i \forall i \in \{V^{m}\} + s_{i3} i \in \{V^{m}\} + D_{ik}^{b} i \in V , i \neq k \\ s_{i3} k \in \{V^{m}\} + s_{i3} i \in \{V^{m}\} + D_{ik}^{b} i \in V , i \neq k \\ s_{i3} k \forall i \in \{V^{m}\} + s_{i3} i \in \{V^{m}\} + D_{ik}^{b} i \in V, i \neq k \\ s_{i3} k \forall i \in \{V^{m}\} + s_{i3} i \in \{V^{m}\} + D_{ik}^{b} i \in V, i \neq k \\ s_{i3} k \forall i \in \{V^{m}\} + s_{i3} i \in \{V^{m}\} + D_{ik}^{b} i \in V, i \neq k \\ s_{i3} k \forall i \in \{V^{m}\} + s_{i3} i \in \{V^{m}\} + D_{ik}^{b} i \in $	$y_{iip}^{p} = 0  \forall i \in V \cup \{c^{p}\}, p = \{1, \dots, P\}$	(15)
$\begin{aligned} & 2x_{12T}^{1} - 1 \leq \sum_{k \in V \cup \{c^{1}\}, i \neq k} y_{ikt}^{1} + \sum_{k \in V \cup \{c^{1}\}, i \neq k} y_{kit}^{1} \leq 2x_{12T}^{1}  \forall i \in V, t = \{1, \dots, T\} \end{aligned} \tag{18} \\ & \sum_{k \in V \cup \{c^{1}\}, i \neq k} y_{ikt}^{1} \leq 1  \forall i \in V \cup \{c^{1}\}, t = \{1, \dots, T\} \end{aligned} \tag{19} \\ & \sum_{k \in V \cup \{c^{1}\}, i \neq k} y_{ikt}^{1} = \sum_{k \in V \cup \{c^{1}\}, i \neq k} y_{kit}^{1}  \forall i \in V \cup \{c^{1}\}, t = \{1, \dots, T\} \end{aligned} \tag{20} \\ & y_{iit}^{1} = 0  \forall i \in V \cup \{c^{1}\}, t = \{1, \dots, T\} \end{aligned} \tag{21} \\ & 1 \leq z_{i}^{1} \leq  V   \forall i \in V \end{aligned} \tag{22} \\ & z_{i}^{1} - z_{k}^{1} + 1 \leq  V  \times (1 - \sum_{t=1}^{T} y_{ikt}^{1})  \forall i, k \in V, i \neq k \end{aligned} \tag{23} \\ & 2 \times \sum_{j=1}^{J} x_{ijb}^{ib} - 1 \leq \sum_{k \in V \cup \{c^{1}\}, i \neq k} y_{ikb}^{b} + \sum_{k \in V \cup \{c^{1}\}, i \neq k} y_{kib}^{b} \leq 2 \sum_{j=1}^{J} x_{ijb}^{ib}  \forall i \in V,  b = \{1, \dots, B\} \end{aligned} \tag{24} \\ & \sum_{k \in V \cup \{c^{1}\}, i \neq k} y_{ikb}^{b} \leq 1  \forall i \in V \cup \{c^{1}\}, b = \{1, \dots, B\} \end{aligned} \tag{25} \\ & \sum_{k \in V \cup \{c^{1}\}, i \neq k} y_{ikb}^{b} = \sum_{k \in V + \{c\}, i \neq k} y_{kib}^{b}  \forall i \in V \cup \{c^{1}\}, b = \{1, \dots, B\} \end{aligned} \tag{26} \\ & y_{ib}^{1b} = 0  \forall i \in V \cup \{c^{1}\}, b = \{1, \dots, B\} \end{aligned} \tag{26} \\ & y_{ib}^{1b} \leq 0  \forall i \in V \cup \{c^{1}\}, b = \{1, \dots, B\} \end{aligned} \tag{27} \\ & 1 \leq z_{i}^{1} \leq  V   \forall i \in V \end{aligned} \end{aligned} \tag{28} \\ & z_{i}^{1} - z_{k}^{1} + 1 \leq  V  \times (1 - \sum_{b=1}^{B} y_{ib}^{b})  \forall i, k \in V, i \neq k \end{aligned} \tag{28} \\ & z_{i}^{1} - z_{k}^{1} + 1 \leq  V  \times (1 - \sum_{b=1}^{B} y_{ib}^{b})  \forall i, k \in V, i \neq k \end{aligned} \tag{28} \\ & z_{i}^{1} - z_{k}^{1} + 1 \leq  V  \times (1 - \sum_{b=1}^{B} y_{ib}^{1})  \forall i, k \in V, i \neq k \end{aligned} \tag{29} \\ & s_{i4} + D_{ik}^{1} \leq s_{k1} + M(1 - y_{ikp}^{1})  \forall i, k \in V, i \neq k, p = \{1, \dots, P\} \end{aligned} \tag{29} \\ & s_{i4} + U_{ik}^{1} + s_{i2i\in \{V^{ca}\}} + D_{ik}^{1} + U_{ikki, k \in V, i \neq k} \leq s_{k2k\in \{V^{ca}\}} + s_{k1k\in \{V^{ca}\}} + M(1 - y_{ikt}^{1})_{i, k \in V, i \neq k} \end{cases} \end{aligned} \end{aligned} \tag{29} \\ & s_{i4} \forall i \in V^{1} + s_{i2i\in \{V^{ca}\}} + D_{ik}^{1} + U_{ikki, k \in V, i \neq k} \leq s_{k3k\in \{V^{ca}\}} + s_{k1k\in \{V^{ca}\}} + M(1 - y_{ikt}^{1})_{i, k \in V, i \neq k} \end{cases} \end{aligned} \Biggr$		(16)
$ \begin{split} & \sum_{k \in V \cup \{c^{i}\}, i \neq k} y_{ikt}^{i} \leq 1  \forall i \in V \cup \{c^{l}\}, t = \{1, \dots, T\} \\ & (19) \\ & \sum_{k \in V \cup \{c^{i}\}, i \neq k} y_{ikt}^{i} = \sum_{k \in V \cup \{c^{i}\}, i \neq k} y_{kit}^{i}  \forall i \in V \cup \{c^{l}\}, t = \{1, \dots, T\} \\ & (20) \\ & y_{it}^{i} = 0  \forall i \in V \cup \{c^{l}\}, t = \{1, \dots, T\} \\ & (21) \\ & 1 \leq z_{i}^{i} \leq  V   \forall i \in V \\ & (22) \\ & z_{i}^{i} - z_{k}^{i} + 1 \leq  V  \times (1 - \sum_{i=1}^{T} y_{ikt}^{i})  \forall i, k \in V, i \neq k \\ & (23) \\ & 2 \times \sum_{j=1}^{J} x_{jb}^{ib} - 1 \leq \sum_{k \in V \cup \{c^{b}\}, i \neq k} y_{jkb}^{b} + \sum_{k \in V \cup \{c^{b}\}, i \neq k} y_{kib}^{b} \leq 2 \sum_{j=1}^{J} x_{ijb}^{ib}  \forall i \in V,  b = \{1, \dots, B\} \\ & 2 \times \sum_{j=1}^{J} x_{ijb}^{ib} - 1 \leq \sum_{k \in V \cup \{c^{b}\}, i \neq k} y_{ikb}^{b} + \sum_{k \in V \cup \{c^{b}\}, i \neq k} y_{kib}^{b} \leq 2 \sum_{j=1}^{J} x_{ijb}^{ib}  \forall i \in V,  b = \{1, \dots, B\} \\ & \sum_{k \in V \cup \{c^{b}\}, i \neq k} y_{ikb}^{b} \leq 1  \forall i \in V \cup \{c^{b}\}, b = \{1, \dots, B\} \\ & \sum_{k \in V \cup \{c^{b}\}, i \neq k} y_{ikb}^{b} = \sum_{k \in V + \{c\}, i \neq k} y_{kib}^{b}  \forall i \in V \cup \{c^{b}\}, b = \{1, \dots, B\} \\ & y_{ib}^{b} = 0  \forall i \in V \cup \{c^{b}\}, b = \{1, \dots, B\} \\ & y_{ib}^{b} = 0  \forall i \in V \cup \{c^{b}\}, b = \{1, \dots, B\} \\ & z_{i}^{b} - z_{k}^{b} + 1 \leq  V  \times (1 - \sum_{b=1}^{B} y_{ib}^{b})  \forall i, k \in V, i \neq k \\ & (28) \\ & z_{i}^{b} - z_{k}^{b} + 1 \leq  V  \times (1 - \sum_{b=1}^{B} y_{ib}^{b})  \forall i, k \in V, i \neq k, p = \{1, \dots, P\} \\ & s_{i4} + D_{ik}^{i} \leq s_{k1} + M(1 - y_{ib}^{i})  \forall i, k \in V, i \neq k, p = \{1, \dots, P\} \\ & s_{i4} + U_{ik}^{b} \leq s_{k1} + M(1 - y_{ik}^{b}) = \sum_{i, k \in V, i \neq k} \leq s_{k2k} \in \{V^{in}\} + s_{k1k} \in V^{iost}\} + M(1 - y_{ikt}^{b})_{i, k \in V, i \neq k} \\ & s_{i4} \forall i \in \{V^{in}\} + s_{i3i \in \{V^{oin}\}} + D_{ik}^{b} y_{i, k \in V, i \neq k} \leq s_{k3k} \in \{V^{in}\} + s_{k1k \in \{V^{oos}\}} + M(1 - y_{ikt}^{b})_{i, k \in V, i \neq k} \\ & w_{ij}, s_{ij}^{c}, z_{i}^{c}, z_{i}^{b} \geq 0  \forall i \in \{1, \dots, I\}, j = \{1, \dots, J\}, p = \{1, \dots, T\}, b = \{1, \dots, B\} \\ & \forall i \in \{V^{in}\} \\ & y_{ij}^{b} y_{ij}^{b} y_{ij}^{b} y_{ij}^{b} \in \{0, 1\}  \forall i, k \in V, j = \{1, \dots, J\}, p = \{1, \dots, T\}, b \in \{1, \dots, T\}, b \in \{1, \dots, B\} \\ & \forall i \in \{1, \dots, I\}, p \in \{1, \dots, I\}, p \in \{1, \dots, T\}, b \in \{1, \dots, I\}, p \in \{1, \dots, I\}, p \in \{1, \dots, I\}, p \in $		(17)
$ \begin{split} & \sum_{k \in V \cup \{c^k\}, i \neq k} y_{ikt}^{t} = \sum_{k \in V \cup \{c^k\}, i \neq k} y_{kit}^{t}  \forall i \in V \cup \{c^l\}, t = \{1, \dots, T\} \end{aligned} \tag{20} \\ & y_{itt}^{t} = 0  \forall i \in V \cup \{c^t\}, t = \{1, \dots, T\} \end{aligned} \tag{21} \\ & 1 \leq z_i^{t} \leq  V   \forall i \in V \end{aligned} \tag{22} \\ & z_i^{t} - z_k^{t} + 1 \leq  V  \times (1 - \sum_{t=1}^{T} y_{ikt}^{t})  \forall i, k \in V, i \neq k \end{aligned} \tag{23} \\ & 2 \times \sum_{j=1}^{J} x_{ijb}^{b} - 1 \leq \sum_{k \in V \cup \{c^b\}, i \neq k} y_{ikb}^{b} + \sum_{k \in V \cup \{c^b\}, i \neq k} y_{kib}^{b} \leq 2 \sum_{j=1}^{J} x_{ijb}^{b}  \forall i \in V,  b = \{1, \dots, B\} \end{aligned} \tag{24} \\ & \sum_{k \in V \cup \{c^b\}, i \neq k} y_{ikb}^{b} \leq 1  \forall i \in V \cup \{c^b\}, b = \{1, \dots, B\} \end{aligned} \tag{25} \\ & \sum_{k \in V \cup \{c^b\}, i \neq k} y_{ikb}^{b} = \sum_{k \in V + \{c\}, i \neq k} y_{kib}^{b}  \forall i \in V \cup \{c^b\}, b = \{1, \dots, B\} \end{aligned} \tag{26} \\ & y_{iib}^{b} = 0  \forall i \in V \cup \{c^b\}, b = \{1, \dots, B\} \end{aligned} \tag{27} \\ & 1 \leq z_i^{b} \leq  V   \forall i \in V \end{aligned} \tag{28} \\ & z_i^{b} - z_k^{b} + 1 \leq  V  \times (1 - \sum_{b=1}^{B} y_{ibb}^{b})  \forall i, k \in V, i \neq k \end{aligned} \tag{29} \\ & s_{i4} + D_{ik}^{b} \leq s_{k1} + M(1 - y_{ikp}^{b})  \forall i, k \in V, i \neq k, p = \{1, \dots, P\} \end{aligned} \tag{29} \\ & s_{i4} + D_{ik}^{b} \leq s_{k1} + M(1 - y_{ikp}^{b})  \forall i, k \in V, i \neq k, p = \{1, \dots, P\} \end{aligned} \tag{29} \\ & s_{i4} + Q_{ik}^{b} \leq V \cup \{w^{a}\} + s_{i3i \in \{V \cup a^{a}\}} + D_{ik}^{b} (s_{ik \in V, i \neq k} \leq s_{k2k \in \{V^{in}\}} + s_{k1k \in \{V \cup a^{a}\}} + M(1 - y_{ikt}^{b})_{i,k \in V, i \neq k} & b = \{1, \dots, B\} \end{aligned} \tag{22} \\ & w_{ij}, s_{ij}^{c}, z_{i}^{c}, z_{i}^{c} \geq 0  \forall i \in \{1, \dots, I\}, j = \{1, \dots, J\}, p = \{1, \dots, T\}, b = \{1, \dots, B\} \end{aligned}$	$2x_{i2t}^{t} - 1 \le \sum_{k \in V \cup \{c^{t}\}, i \ne k} y_{ikt}^{t} + \sum_{k \in V \cup \{c^{t}\}, i \ne k} y_{kit}^{t} \le 2x_{i2t}^{t}  \forall i \in V, t = \{1, \dots, T\}$	
$ \begin{aligned} y_{iit}^{1} = 0  \forall i \in V \cup \{c^{t}\}, t = \{1, \dots, T\} \\ (21) \\ 1 \leq z_{i}^{t} \leq  V   \forall i \in V \\ (22) \\ z_{i}^{t} - z_{k}^{t} + 1 \leq  V  \times (1 - \sum_{t=1}^{T} y_{ikt}^{t})  \forall i, k \in V, i \neq k \\ (23) \\ 2 \times \sum_{j=1}^{J} x_{jib}^{b} - 1 \leq \sum_{k \in V \cup \{c^{b}\}, i \neq k} y_{ikb}^{b} + \sum_{k \in V \cup \{c^{b}\}, i \neq k} y_{kib}^{b} \leq 2 \sum_{j=1}^{J} x_{ijb}^{b}  \forall i \in V,  b = \{1, \dots, B\} \\ (24) \\ \sum_{k \in V \cup \{c^{b}\}, i \neq k} y_{ikb}^{b} \leq 1  \forall i \in V \cup \{c^{b}\}, b = \{1, \dots, B\} \\ (25) \\ \sum_{k \in V \cup \{c^{b}\}, i \neq k} y_{ikb}^{b} = \sum_{k \in V + \{c\}, i \neq k} y_{kib}^{b}  \forall i \in V \cup \{c^{b}\}, b = \{1, \dots, B\} \\ (26) \\ y_{ib}^{b} = 0  \forall i \in V \cup \{c^{b}\}, b = \{1, \dots, B\} \\ (27) \\ 1 \leq z_{i}^{b} \leq  V   \forall i \in V \\ (28) \\ z_{i}^{b} - z_{k}^{b} + 1 \leq  V  \times (1 - \sum_{b=1}^{B} y_{ikb}^{b})  \forall i, k \in V, i \neq k \\ (29) \\ s_{i4} + D_{ik}^{b} \leq s_{k1} + M(1 - y_{ikp}^{b})  \forall i, k \in V, i \neq k, p = \{1, \dots, P\} \\ s_{i4} \forall_{ie}\{V^{m}\} + s_{i3i \in \{V^{out}\}} + D_{iki,k \in V, i \neq k}^{b} \leq s_{k2k \in \{V^{in}\}} + s_{k1k \in \{V^{out}\}} + M(1 - y_{ikb}^{i})_{i,k \in V, i \neq k}  b = \{1, \dots, B\} \\ s_{i4} \forall_{ie}\{V^{m}\} + s_{i2i \in \{V^{out}\}} + D_{iki,k \in V, i \neq k}^{b} \leq s_{k3k \in \{V^{in}\}} + s_{k1k \in \{V^{out}\}} + M(1 - y_{ikb}^{i})_{i,k \in V, i \neq k}  b = \{1, \dots, B\} \\ (32) \\ w_{ij}, s_{ij}, z_{i}^{p}, z_{i}^{j}, z_{i}^{b} \geq 0  \forall i = \{1, \dots, I\}, j = \{1, \dots, J\}, p = \{1, \dots, P\}, t = \{1, \dots, T\}, b = \{1, \dots, B\} \\ (34) \end{aligned}$		
$\begin{split} & \sum_{i} \leq  V   \forall i \in V \tag{22} \\ & z_{i}^{1} - z_{k}^{1} + 1 \leq  V  \times (1 - \sum_{i=1}^{T} y_{iki}^{1})  \forall i, k \in V, i \neq k \end{aligned}$ $\begin{aligned} & (23) \\ & 2 \times \sum_{j=1}^{J} x_{ijb}^{b} - 1 \leq \sum_{k \in V \cup \{c^{b}\}, i \neq k} y_{ikb}^{b} + \sum_{k \in V \cup \{c^{b}\}, i \neq k} y_{kib}^{b} \leq 2 \sum_{j=1}^{J} x_{ijb}^{b}  \forall i \in V,  b = \{1, \dots, B\} \end{aligned}$ $\begin{aligned} & \sum_{k \in V \cup \{c^{b}\}, i \neq k} y_{ikb}^{b} \leq 1  \forall i \in V \cup \{c^{b}\}, b = \{1, \dots, B\} \end{aligned}$ $\begin{aligned} & (24) \\ & \sum_{k \in V \cup \{c^{b}\}, i \neq k} y_{ikb}^{b} \leq 1  \forall i \in V \cup \{c^{b}\}, b = \{1, \dots, B\} \end{aligned}$ $\begin{aligned} & (25) \\ & \sum_{k \in V \cup \{c^{b}\}, i \neq k} y_{ikb}^{b} = \sum_{k \in V + \{c\}, i \neq k} y_{kib}^{b}  \forall i \in V \cup \{c^{b}\}, b = \{1, \dots, B\} \end{aligned}$ $\begin{aligned} & (26) \\ & y_{ib}^{b} = 0  \forall i \in V \cup \{c^{b}\}, b = \{1, \dots, B\} \end{aligned}$ $\begin{aligned} & (27) \\ & 1 \leq z_{i}^{b} \leq  V   \forall i \in V \end{aligned}$ $\begin{aligned} & (28) \\ & z_{i}^{b} - z_{k}^{b} + 1 \leq  V  \times (1 - \sum_{b=1}^{B} y_{ikb}^{b})  \forall i, k \in V, i \neq k \end{aligned}$ $\begin{aligned} & (29) \\ & s_{i4} + D_{ik}^{b} \leq s_{k1} + M(1 - y_{ikp}^{b})  \forall i, k \in V, i \neq k, p = \{1, \dots, P\} \end{aligned}$ $\begin{aligned} & s_{i4} \forall i \in \{V^{m}\} + s_{i3} i \in \{V^{om}\} + D_{ik}^{b} i_{k \in V, i \neq k} \leq s_{k2k \in \{V^{im}\}} + s_{k1k \in \{V^{om}\}} + M(1 - y_{ikp}^{b})_{i,k \in V, i \neq k} & b = \{1, \dots, R\} \end{aligned}$ $\begin{aligned} & s_{i4} \forall i \in \{V^{im}\} + s_{i2} i \in \{V^{om}\} + D_{ik}^{b} i_{k \in V, i \neq k} \leq s_{k2k \in \{V^{im}\}} + s_{k1k \in \{V^{om}\}} + M(1 - y_{ikp}^{b})_{i,k \in V, i \neq k} & b = \{1, \dots, R\} \end{aligned}$ $\begin{aligned} & s_{i4} \forall i \in \{V^{im}\} + s_{i2} i \in \{V^{om}\} + D_{ik}^{b} i_{k \in V, i \neq k} \leq s_{k3k \in \{V^{im}\}} + s_{k1k \in \{V^{om}\}} + M(1 - y_{ikb}^{b})_{i,k \in V, i \neq k} & b = \{1, \dots, R\} \end{aligned}$ $\begin{aligned} & s_{i4} \forall i \in \{V^{im}\} + s_{i2} i \in \{V^{om}\} + D_{ik}^{b} i_{k \in V, i \neq k} \leq s_{k3k \in \{V^{im}\}} + s_{k1k \in \{V^{om}\}} + M(1 - y_{ikb}^{b})_{i,k \in V, i \neq k} & b = \{1, \dots, R\} \end{aligned}$		
$\begin{aligned} z_{i}^{t} - z_{k}^{t} + 1 &\leq  V  \times (1 - \sum_{i=1}^{T} y_{iki}^{t})  \forall i, k \in V, i \neq k \end{aligned} \tag{23} \\ 2 \times \sum_{j=1}^{J} x_{ijb}^{b} - 1 &\leq \sum_{k \in V \cup \{c^{b}\}, i \neq k} y_{ikb}^{b} + \sum_{k \in V \cup \{c^{b}\}, i \neq k} y_{kib}^{b} \leq 2 \sum_{j=1}^{J} x_{ijb}^{b}  \forall i \in V,  b = \{1, \dots, B\} \end{aligned} \tag{24} \\ \sum_{k \in V \cup \{c^{b}\}, i \neq k} y_{ikb}^{b} \leq 1  \forall i \in V \cup \{c^{b}\}, b = \{1, \dots, B\} \end{aligned} \tag{25} \\ \sum_{k \in V \cup \{c^{b}\}, i \neq k} y_{ikb}^{b} = \sum_{k \in V + \{c\}, i \neq k} y_{kib}^{b}  \forall i \in V \cup \{c^{b}\}, b = \{1, \dots, B\} \end{aligned} \tag{26} \\ y_{iib}^{b} = 0  \forall i \in V \cup \{c^{b}\}, b = \{1, \dots, B\} \end{aligned} \tag{26} \\ y_{iib}^{b} = 0  \forall i \in V \cup \{c^{b}\}, b = \{1, \dots, B\} \end{aligned} \tag{27} \\ 1 \leq z_{i}^{b} \leq  V   \forall i \in V \end{aligned} \tag{28} \\ z_{i}^{b} - z_{k}^{b} + 1 \leq  V  \times (1 - \sum_{b=1}^{B} y_{ikb}^{b})  \forall i, k \in V, i \neq k \end{aligned} \tag{29} \\ s_{i4} + D_{ik}^{p} \leq s_{k1} + M(1 - y_{ikp}^{p})  \forall i, k \in V, i \neq k, p = \{1, \dots, P\} \end{aligned} \tag{30} \\ s_{i4} v_{i6} \{v^{in}\} + s_{i3i \in \{V^{out}\}} + D_{ik i, k \in V, i \neq k}^{b} \leq s_{k2k \in \{V^{in}\}} + s_{k1k \in \{V^{out}\}} + M(1 - y_{ikp}^{b})_{i, k \in V, i \neq k} \end{aligned} \tag{32} \\ w_{ij}, s_{ij}^{p}, z_{ij}^{p}, z$		
$\begin{aligned} 2 \times \sum_{j=1}^{J} x_{ijb}^{b} - 1 &\leq \sum_{k \in V \cup \{c^{b}\}, i \neq k} y_{ikb}^{b} + \sum_{k \in V \cup \{c^{b}\}, i \neq k} y_{kib}^{b} &\leq 2 \sum_{j=1}^{J} x_{ijb}^{b}  \forall i \in V,  b = \{1, \dots, B\} \end{aligned} \tag{24} \\ \sum_{k \in V \cup \{c^{b}\}, i \neq k} y_{ikb}^{b} &\leq 1  \forall i \in V \cup \{c^{b}\}, b = \{1, \dots, B\} \end{aligned} \tag{25} \\ \sum_{k \in V \cup \{c^{b}\}, i \neq k} y_{ikb}^{b} &\leq \sum_{k \in V + \{c\}, i \neq k} y_{kib}^{b}  \forall i \in V \cup \{c^{b}\}, b = \{1, \dots, B\} \end{aligned} \tag{26} \\ y_{ib}^{b} &= 0  \forall i \in V \cup \{c^{b}\}, b = \{1, \dots, B\} \end{aligned} \tag{26} \\ y_{ib}^{b} &= 0  \forall i \in V \cup \{c^{b}\}, b = \{1, \dots, B\} \end{aligned} \tag{27} \\ 1 \leq z_{i}^{b} \leq  V   \forall i \in V \end{aligned} \tag{28} \\ z_{i}^{b} - z_{k}^{b} + 1 \leq  V  \times (1 - \sum_{b=1}^{B} y_{ikb}^{b})  \forall i, k \in V, i \neq k \end{aligned} \tag{29} \\ s_{i4} + D_{ik}^{b} \leq s_{k1} + M(1 - y_{ikp}^{b})  \forall i, k \in V, i \neq k, p = \{1, \dots, P\} \end{aligned} \tag{30} \\ s_{i4} \forall i \in \{V^{m}\} + s_{i3i \in \{V^{out}\}} + D_{iki,k \in V, i \neq k}^{b} \leq s_{k2k \in \{V^{in}\}} + s_{k1k \in \{V^{out}\}} + M(1 - y_{ikt}^{t})_{i,k \in V, i \neq k} t = \{1, \dots, T\} \end{aligned} \tag{31} \\ s_{i4} \forall i \in \{V^{m}\} + s_{i2i \in \{V^{out}\}} + D_{iki,k \in V, i \neq k}^{b} \leq s_{k3k \in \{V^{in}\}} + s_{k1k \in \{V^{out}\}} + M(1 - y_{ikt}^{b})_{i,k \in V, i \neq k} t = \{1, \dots, T\} \end{aligned} \tag{32} \\ w_{ij}, s_{ij}^{i}, z_{i}^{j}, z_{i}^{j}, z_{i}^{b} \geq 0  \forall i \in \{1, \dots, I\}, j = \{1, \dots, J\}, p = \{1, \dots, P\}, t = \{1, \dots, T\}, b = \{1, \dots, B\} \end{aligned}$		
$\begin{split} \sum_{k \in V \cup \{c^b\}, i \neq k} y_{ikb}^b &\leq 1  \forall i \in V \cup \{c^b\}, b = \{1, \dots, B\} \end{split} \tag{25}$ $\sum_{k \in V \cup \{c^b\}, i \neq k} y_{ikb}^b &= \sum_{k \in V + \{c\}, i \neq k} y_{kib}^b  \forall i \in V \cup \{c^b\}, b = \{1, \dots, B\} \end{aligned} \tag{26}$ $y_{iib}^b &= 0  \forall i \in V \cup \{c^b\}, b = \{1, \dots, B\} \end{aligned} \tag{27}$ $1 \leq z_i^b \leq  V   \forall i \in V \end{aligned} \tag{28}$ $z_i^b - z_k^b + 1 \leq  V  \times (1 - \sum_{b=1}^B y_{ikb}^b)  \forall i, k \in V, i \neq k \end{aligned} \tag{29}$ $s_{i4} + D_{ik}^p \leq s_{k1} + M(1 - y_{ikp}^p)  \forall i, k \in V, i \neq k, p = \{1, \dots, P\} \end{aligned} \tag{30}$ $s_{i4\forall i \in \{V^{in}\}} + s_{i3i \in \{V^{out}\}} + D_{ik i, k \in V, i \neq k}^b \leq s_{k2k \in \{V^{in}\}} + s_{k1k \in \{V^{out}\}} + M(1 - y_{ikp}^t)_{i, k \in V, i \neq k} $ $s_{i4\forall i \in \{V^{in}\}} + s_{i2i \in \{V^{out}\}} + D_{ik i, k \in V, i \neq k}^b \leq s_{k2k \in \{V^{in}\}} + s_{k1k \in \{V^{out}\}} + M(1 - y_{ikp}^t)_{i, k \in V, i \neq k} $ $s_{i4\forall i \in \{V^{in}\}} + s_{i2i \in \{V^{out}\}} + D_{ik i, k \in V, i \neq k}^b \leq s_{k3k \in \{V^{in}\}} + s_{k1k \in \{V^{out}\}} + M(1 - y_{ikp}^t)_{i, k \in V, i \neq k} $ $s_{i4\forall i \in \{V^{in}\}} + s_{i2i \in \{V^{out}\}} + D_{ik i, k \in V, i \neq k}^b \leq s_{k3k \in \{V^{in}\}} + s_{k1k \in \{V^{out}\}} + M(1 - y_{ikb}^b)_{i, k \in V, i \neq k} $ $s_{i4\forall i \in \{V^{in}\}} + s_{i2i \in \{V^{out}\}} + D_{ik i, k \in V, i \neq k}^b \leq s_{k3k \in \{V^{in}\}} + s_{k1k \in \{V^{out}\}} + M(1 - y_{ikb}^b)_{i, k \in V, i \neq k} $ $s_{ij}, z_i^p, z_i^p, z_i^p, z_b^i \geq 0  \forall i = \{1, \dots, I\}, j = \{1, \dots, J\} $ $s_{ij}, x_{ij}^p, x_{ij}^n, x_{ij}^n, x_{ij}^n, x_{ij}^b \in \{0, 1\}  \forall i, k \in V, j = \{1, \dots, J\}, p = \{1, \dots, P\}, t = \{1, \dots, T\}, b = \{1, \dots, B\} $		
$\begin{split} \sum_{k \in V \cup \{c^b\}, i \neq k} y_{ikb}^b &= \sum_{k \in V + \{c\}, i \neq k} y_{kib}^b  \forall i \in V \cup \{c^b\}, b = \{1, \dots, B\} \end{split} \tag{26}$ $y_{iib}^b &= 0  \forall i \in V \cup \{c^b\}, b = \{1, \dots, B\} \tag{27}$ $1 \leq z_i^b \leq  V   \forall i \in V \tag{28}$ $z_i^b - z_k^b + 1 \leq  V  \times (1 - \sum_{b=1}^B y_{ikb}^b)  \forall i, k \in V, i \neq k \tag{29}$ $s_{i4} + D_{ik}^p \leq s_{k1} + M(1 - y_{ikp}^p)  \forall i, k \in V, i \neq k, p = \{1, \dots, P\} \tag{30}$ $s_{i4\forall i \in \{V^m\}} + s_{i3i\in\{V^{out}\}} + D_{iki,k\in V, i \neq k}^t \leq s_{k2k\in\{V^m\}} + s_{k1k\in\{V^{out}\}} + M(1 - y_{ikp}^t)_{i,k\in V, i \neq k} t = \{1, \dots, T\} \tag{31}$ $s_{i4\forall i \in \{V^m\}} + s_{i2\forall i \in \{V^{out}\}} + D_{ik\forall i, k\in V, i \neq k}^t \leq s_{k3k\in\{V^m\}} + s_{k1k\in\{V^{out}\}} + M(1 - y_{ikp}^t)_{i,k\in V, i \neq k} t = \{1, \dots, T\} \tag{32}$ $w_{ij}, s_{ij}^t, z_i^t, z_i^b \geq 0  \forall i = \{1, \dots, I\}, j = \{1, \dots, J\} \tag{33}$ $x_{ijp}^p, x_{ij}^t, x_{ijb}^t \in \{0, 1\}  \forall i, k \in V, j = \{1, \dots, J\}, p = \{1, \dots, P\}, t = \{1, \dots, T\}, b = \{1, \dots, B\} \tag{34}$		
$\begin{aligned} y_{iib}^{b} &= 0  \forall i \in V \cup \{c^{b}\}, b = \{1, \dots, B\} \end{aligned} \tag{27} \\ 1 \leq z_{i}^{b} \leq  V   \forall i \in V \qquad (28) \\ z_{i}^{b} - z_{k}^{b} + 1 \leq  V  \times (1 - \sum_{b=1}^{B} y_{ikb}^{b})  \forall i, k \in V, i \neq k \qquad (29) \\ s_{i4} + D_{ik}^{p} \leq s_{k1} + M(1 - y_{ikp}^{p})  \forall i, k \in V, i \neq k, p = \{1, \dots, P\} \qquad (30) \\ s_{i4\forall i \in \{V^{in}\}} + s_{i3i\in\{V^{out}\}} + D_{ikki,k\in V, i\neq k}^{t} \leq s_{k2k\in\{V^{in}\}} + s_{k1k\in\{V^{out}\}} + M(1 - y_{ikt}^{t})_{i,k\in V, i\neq k}  t = \{1, \dots, T\} \qquad (31) \\ s_{i4\forall i \in \{V^{in}\}} + s_{i2\forall i \in (V^{out}\}} + D_{ik\forall i, k\in V, i\neq k}^{t} \leq s_{k3k\in\{V^{in}\}} + s_{k1k\in\{V^{out}\}} + M(1 - y_{ikt}^{b})_{i,k\in V, i\neq k}  b = \{1, \dots, B\} \qquad (32) \\ w_{ij}, s_{ij}^{p}, z_{i}^{t}, z_{i}^{b} \geq 0  \forall i = \{1, \dots, I\}, j = \{1, \dots, J\}, p = \{1, \dots, P\}, t = \{1, \dots, T\}, b = \{1, \dots, B\} \qquad (34) \end{aligned}$		
$\begin{aligned} 1 \leq z_{i}^{b} \leq  V   \forall i \in V \end{aligned} \tag{28} \\ z_{i}^{b} - z_{k}^{b} + 1 \leq  V  \times (1 - \sum_{b=1}^{B} y_{ikb}^{b})  \forall i, k \in V, i \neq k \end{aligned} \tag{29} \\ s_{i4} + D_{ik}^{p} \leq s_{k1} + M(1 - y_{ikp}^{p})  \forall i, k \in V, i \neq k, p = \{1, \dots, P\} \end{aligned} \tag{30} \\ s_{i4\forall i \in \{V^{in}\}} + s_{i3i \in \{V^{out}\}} + D_{ik,i,k \in V, i \neq k}^{t} \leq s_{k2k \in \{V^{in}\}} + s_{k1k \in \{V^{out}\}} + M(1 - y_{ikt}^{t})_{i,k \in V, i \neq k}  t = \{1, \dots, T\} \end{aligned} \tag{31} \\ s_{i4\forall i \in \{V^{in}\}} + s_{i2i \in \{V^{out}\}} + D_{ik\forall i,k \in V, i \neq k}^{t} \leq s_{k2k \in \{V^{in}\}} + s_{k1k \in \{V^{out}\}} + M(1 - y_{ikt}^{t})_{i,k \in V, i \neq k}  b = \{1, \dots, T\} \end{aligned} \tag{32} \\ w_{ij}, s_{ij}^{t}, z_{i}^{t}, z_{i}^{b} \geq 0  \forall i = \{1, \dots, I\}, j = \{1, \dots, J\} \end{aligned} \tag{33} \\ x_{ijp}^{t}, x_{ijt}^{t}, x_{ijb}^{t} \in \{0, 1\}  \forall i, k \in V, j = \{1, \dots, J\}, p = \{1, \dots, P\}, t = \{1, \dots, T\}, b = \{1, \dots, B\} \end{aligned}$		
$\begin{aligned} z_{i}^{b} - z_{k}^{b} + 1 &\leq  V  \times (1 - \sum_{b=1}^{B} y_{ikb}^{b})  \forall i, k \in V, i \neq k \end{aligned} \tag{29} \\ s_{i4} + D_{ik}^{p} &\leq s_{k1} + M(1 - y_{ikp}^{p})  \forall i, k \in V, i \neq k, p = \{1, \dots, P\} \end{aligned} \tag{30} \\ s_{i4\forall i \in \{V^{in}\}} + s_{i3i \in \{V^{out}\}} + D_{iki,k \in V, i \neq k}^{t} \leq s_{k2k \in \{V^{in}\}} + s_{k1k \in \{V^{out}\}} + M(1 - y_{ikj}^{t})_{i,k \in V, i \neq k}  t = \{1, \dots, T\} \end{aligned} \tag{31} \\ s_{i4\forall i \in \{V^{in}\}} + s_{i2i \in \{V^{out}\}} + D_{ik\forall i, k \in V, i \neq k}^{t} \leq s_{k3k \in \{V^{in}\}} + s_{k1k \in \{V^{out}\}} + M(1 - y_{ikb}^{t})_{i,k \in V, i \neq k}  b = \{1, \dots, B\} \end{aligned} \tag{32} \\ w_{ij}, s_{ij}^{p}, z_{ij}^{t}, z_{ij}^{b} \geq 0  \forall i = \{1, \dots, I\}, j = \{1, \dots, J\} \end{aligned} \tag{33} \\ x_{ijp}^{p}, x_{ijr}^{t}, x_{ijb}^{t} \in \{0, 1\}  \forall i, k \in V, j = \{1, \dots, J\}, p = \{1, \dots, P\}, t = \{1, \dots, T\}, b = \{1, \dots, B\} \end{aligned}$		
$\begin{aligned} s_{i4} + D_{ik}^{p} &\leq s_{k1} + M(1 - y_{ikp}^{p})  \forall i, k \in V, i \neq k, p = \{1, \dots, P\} \end{aligned} \tag{30} \\ s_{i4\forall i\in\{V^{in}\}} + s_{i3i\in\{V^{out}\}} + D_{iki,k\in V, i\neq k}^{t} &\leq s_{k2k\in\{V^{in}\}} + s_{k1k\in\{V^{out}\}} + M(1 - y_{ikl}^{t})_{i,k\in V, i\neq k}  t = \{1, \dots, T\} \end{aligned} \tag{31} \\ s_{i4\forall i\in\{V^{in}\}} + s_{i2\forall i\in\{V^{out}\}} + D_{ik\forall i,k\in V, i\neq k}^{b} &\leq s_{k3k\in\{V^{in}\}} + s_{k1k\in\{V^{out}\}} + M(1 - y_{ikl}^{b})_{i,k\in V, i\neq k}  b = \{1, \dots, B\} \end{aligned} \tag{32} \\ w_{ij}, s_{ij}^{p}, z_{i}^{t}, z_{i}^{b} \geq 0  \forall i = \{1, \dots, I\}, j = \{1, \dots, J\} \end{aligned} \tag{33} \\ x_{ijp}^{p}, x_{ijt}^{i}, x_{ijb}^{ib} \in \{0, 1\}  \forall i, k \in V, j = \{1, \dots, J\}, p = \{1, \dots, P\}, t = \{1, \dots, T\}, b = \{1, \dots, B\} \end{aligned}$	1	
$\begin{aligned} s_{i4\forall i\in\{V^{in}\}} + s_{i3i\in\{V^{out}\}} + D_{ik,i,k\in V, i\neq k}^{in} \leq s_{k2k\in\{V^{in}\}} + s_{k1k\in\{V^{out}\}} + M(1 - y_{ikl}^{t})_{i,k\in V, i\neq k}  t = \{1, \dots, T\}  (31) \\ s_{i4\forall i\in\{V^{in}\}} + s_{i2\forall i\in\{V^{out}\}} + D_{ik\forall i,k\in V, i\neq k}^{b} \leq s_{k3k\in\{V^{in}\}} + s_{k1k\in\{V^{out}\}} + M(1 - y_{ikb}^{b})_{i,k\in V, i\neq k}  b = \{1, \dots, B\}  (32) \\ w_{ij}, s_{ij}, z_{i}^{p}, z_{i}^{t}, z_{i}^{b} \geq 0  \forall i = \{1, \dots, I\}, j = \{1, \dots, J\}  (33) \\ x_{ijp}^{p}, x_{ijt}^{i}, x_{ijb}^{b} \in \{0, 1\}  \forall i, k \in V, j = \{1, \dots, J\}, p = \{1, \dots, P\}, t = \{1, \dots, T\}, b = \{1, \dots, B\}  (34) \end{aligned}$		
$\begin{split} s_{i4\forall i\in\{V^{in}\}} + s_{i2\forall i\in\{V^{out}\}} + D_{ik\forall i,k\in V, i\neq k}^{b} \leq s_{k3k\in\{V^{in}\}} + s_{k1k\in\{V^{out}\}} + M(1-y_{ikb}^{b})_{i,k\in V, i\neq k}  b = \{1, \dots, B\}  (32) \\ w_{ij}, s_{ij}^{p}, z_{i}^{p}, z_{i}^{j}, z_{i}^{b} \geq 0  \forall i = \{1, \dots, I\}, j = \{1, \dots, J\}  (33) \\ x_{ijp}^{p}, x_{ij}^{i}, x_{ijb}^{b} \in \{0, 1\}  \forall i, k \in V, j = \{1, \dots, J\}, p = \{1, \dots, P\}, t = \{1, \dots, T\}, b = \{1, \dots, B\}  (34) \end{split}$	in inp	
$ \begin{aligned} & w_{ij}, s_{ij}, z_i^{p}, z_i^{t}, z_i^{b} \ge 0  \forall i = \{1, \dots, I\}, j = \{1, \dots, J\} \\ & x_{ijp}^{p}, x_{ijt}^{t}, x_{jb}^{b} \in \{0, 1\}  \forall i, k \in V, j = \{1, \dots, J\}, p = \{1, \dots, P\}, t = \{1, \dots, T\}, b = \{1, \dots, B\} \end{aligned} $ (33)		
$x_{ijp}^{p}, x_{ijt}^{t}, x_{ijb}^{b} \in \{0, 1\}  \forall i, k \in V, j = \{1, \dots, J\}, p = \{1, \dots, P\}, t = \{1, \dots, T\}, b = \{1, \dots, B\} $ (34)		
	$y_{ikpi,k\in V\cup\{c^{P}\}}^{p}, y_{iki,k\in V\cup\{c^{1}\}}^{t}, y_{ikbi,k\in V\cup\{c^{b}\}}^{b} \in \{0,1\}  p = \{1,\dots,P\}, t = \{1,\dots,T\}, b = \{1,\dots,B\}$	(35)

The objective function (1) formulates the scheduling strategy (I) and minimizes the total sum of deviation of the vessel's scheduled times from their requested times. The decision variable  $w_{i1}$  indicates the deviation from their requested starting time. The other decision variables,  $w_{i2}$ , and  $w_{i3}$  are inter-service waiting times, indicating the waiting time of vessels for operations 2 and 3, respectively. Alternatively, strategy (II) could be formulated as Minimizemax  $\sum_{i \in V} \sum_{j=1}^{J} w_{ij}$ . This objective function minimizes the maximum deviation of vessels scheduled times from their requested times. Strategy (III) can be formulated as Minimize  $\sum_{i \in V} w_{1j}$ , which minimizes the sum of deviation of vessels scheduled starting times from their requested starting times.

Constraint (2) defines  $w_{i1}$ , which is the difference between the vessel's requested starting time and scheduled starting time. Constraint (3) assures that the sequence of operations for each vessel is respected. It also defines the inter-service waiting time between the operations. For example, the starting time of operation 2 of an incoming vessel is equal to the sum of starting time of operation 1, the duration of operation 1, and the waiting time of the vessel to start operation 2. Thus,  $s_{i1} + o_{i1} + w_{i2} - s_{i2} = 0$ . Constraint (4) assigns one (and only one) pilot to the first operation of each vessel. Constraint (5) ensures that the assigned pilot remains assigned to the vessel until the completion of the last operation.

Constraint (6) assigns the required number of tugboats  $(N_i)$  to each vessel for starting the towage operations. Constraint (7) assigns tugboats to the second operation of an incoming vessel and ensures that the assigned tugboats remain assigned until the completion of the third operation. The term  $x_{i1t}^t = 0$  ensures that no tugboat is assigned to the first operation of an incoming vessel. Constraint (8) assigns tugboats to the first operation of an outgoing vessel and ensures that the assigned tugboat remains engaged until the completion of the second operation. The term  $x_{i3t}^t = 0$  means that no tugboat is assigned to the third operation of an outgoing vessel and ensures that the assigned tugboat remains engaged until the completion of the second operation. The term  $x_{i3t}^t = 0$  means that no tugboat is assigned to the third operation of an outgoing vessel. Constraint (9) assigns one (and only one) boatman team to each vessel. Constraints (10) and (11) ensure that no boatmen are assigned to the first two operations of incoming vessels and the last two operations of outgoing vessels, respectively.

Constraint (12) assures sequential pilot assignments and specifies that if pilot *p* is assigned to vessel *i*, i.e.  $x_{i1p}^p = 1$ , then in its assignments, either *i* proceeds *k* or vice versa. It indicates that the assigned pilot must come from either the station or a vessel and move to either the station or another vessel after completing the current assignment. If pilot *p* is not assigned to vessel *i* ( $x_{i1p}^p = 0$ ), then the constraint ensures that the pilot does not go from *i* to *k*, which implies that  $\sum_{k \in V \cup \{c^p\}} y_{ikp}^p + \sum_{k \in V \cup \{c^p\}} y_{kip}^p = 0$ . Constraint (13) ensures that each pilot repositions from *i* to *k* only once. Constraint (14) ensures that if pilot *p* repositions from *i* to *k* only once. Constraint (15) ensures that pilot repositioning from *i* to *i* is not defined. Hereby, the diagonal elements of the matrix  $y_{iip}^p$  set to zero. Constraints (16) and (17) are subtour elimination constraints of the well-known Miller–Tucker–Zemlin formulation (Pferschy and Staněk 2017). Together they ensure that each pilot makes only one complete tour. The pilot can only start from its station and return to its station after completing its assignments.

Constraint (18) ensures that if tugboat *t* is assigned to vessel *i* (i.e.  $x_{i2t}^t = 1$ ), then the tugboat should be assigned to *k*, which either immediately succeeds or proceeds *i*. Constraint (19) ensures that each tugboat repositions at most one time from *i* to *k*. Constraint (20) ensures that if tugboat *t* repositions from *i* to *k*, the successive move will start from *k*. Constraint (21) ensures that tugboat repositioning from *i* to *i* is not

defined and set to zero. Constraints (22) and (23) together define subtour elimination constraints of Miller–Tucker–Zemlin formulation for the tugboat (Pferschy and Staněk 2017). They ensure that each tugboat makes only one tour starting from its stations and returning there. Constraint (24) ensures that if boatman team *b* is assigned to the vessel *i* (i.e.  $\sum_{j=1}^{J} x_{ijb}^{b} = 1$ ), then, the boatmen team is either repositioned from *k* or will proceed to it. Constraint (25) ensures that each boatman team repositioning from *i* to *k* is performed at most once. Constraint (26) ensures that if boatmen team has repositioned from *i* to *k*, the successive move will start from *k*. Constraint (27) ensures that the boatmen repositioning from *i* to *i* is set to zero. Constraints (28) and (29) together define subtour elimination constraints of Miller–Tucker–Zemlin formulation for the boatmen (Pferschy and Staněk 2017). They ensure that each boatman makes only one complete tour starting and returning from its stations.

Constraint (30) determines the starting time of operations given the completion time of the earlier assignment of a pilot and the corresponding repositioning time to the next assignment. Constraint (31) determines the starting time of operations given the completion time of the towage assignment and its repositioning time to its next assignment. Constraint (32) determines the starting time of operations given the completion time of the boatmen's assignment and its repositioning time to its next assignment. Constraints (33)–(35) specify the nature of decision variables.

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