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NUMERICAL MODELLING OF COMPRESSIBLE TWO-PHASE FLOWS

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Abstract. *This paper presents a hierarchy of compressible two-phase flow models. It is shown that using Chapman-Enskog expansions in the limit of zero relaxation time toward the various equilibria that exist in these flows, a full set of models of decreasing complexity can be constructed. At the top of this hierarchy, we have a complete seven equation model with two pressures, two velocities and two temperatures while the simplest model is given in the inviscid case by the Euler equations of compressible flows. We also show how some non-equilibrium effects can be retained in these models under the form of second-order dissipative terms. This paper concludes by some representative test-cases.*

1 A general description of an immiscible binary fluid

1.1 Description as a continuous medium

We consider a binary fluid which consists of two components labelled 1 and 2. At the microscopic level, these two fluids are considered as immiscible and unable to mix. Thus for any subdomain Ω of the fluid domain $\mathcal{D}$, the fluid i occupies a volume Ω_i such that $\Omega_1 \cup \Omega_2 = \Omega$ and $\Omega_1 \cap \Omega_2 = \phi$. In each subdomain Ω_k, we assume that the fluid can be described by the Navier-Stokes equations for compressible fluid that writes

$$\begin{aligned}
&\frac{\partial \rho_k}{\partial t} + div \rho_k \underline{u}_k = && 0 \quad \text{for} \quad k = 1,2 \\
&\frac{\partial \rho_k \underline{u}_k}{\partial t} + div \rho_k \underline{u}_k \otimes \underline{u}_k + \nabla(p_k) && = div(\underline{\underline{\tau}}_k) \quad \text{for} \quad k = 1,2 \\
&\frac{\partial \rho_k e_k}{\partial t} + div(\rho_k e_k + p_k)\underline{u}_k && = div(\underline{\underline{\tau}}_k \underline{u}_k) - div(\underline{Q}_k) \quad \text{for} \quad k = 1,2
\end{aligned} \tag{1}$$

with the usual notations, ρ_k denotes here the density of the fluid in the subdomain Ω_k, $\underline{u}_k$ the vector velocities, p_k the pressures and $e_k = \varepsilon_k + \underline{u}_k^2/2$ the specific total energies, with ε_k the specific internal energies. These equations are supplemented by two complete state laws that can be given for instance by two relations of the form $\varepsilon_k = \varepsilon_k(s_k, \rho_k)$ where s_k are the entropies and by the definition of the stress tensors $\underline{\underline{\tau}}_k$ and heat fluxes $\underline{Q}_k$. However, from a macroscopic point of view, the previous description of this binary fluid is inadequate and an homogeneized description have to be preferred. This description will use two distributions called the volume fractions $\alpha_1(\underline{x}, t)$ and $\alpha_2(\underline{x}, t)$ with value in $[0, 1]$ such that for an arbitrary subvolume $\Omega \subset \mathcal{D}$, the volume occupied by fluid i will be :

$$Vol(\Omega_i) = \int_\Omega \alpha_i d\underline{x}$$

Obviously, we have $\alpha_1 + \alpha_2 = 1$. In a region composed of a pure fluid, $\alpha_i \in \{0, 1\}$. However, in Eulerian numerical schemes, the space steps can include many microscopic pure fluid entities and we have to define a thermodynamically consistent homogeneized model for the macroscopic zones where $\alpha \in]0, 1[$. For this purpose we introduce the following definitions. If $Mass(\Omega_i)$ is the mass of component i contained in the volume Ω, the apparent and "true" densities $\tilde{\rho}_i$ and ρ_i will be defined by

$$Mass(\Omega_i) = \int_\Omega \tilde{\rho}_i d\underline{x} \quad \text{and} \quad \rho_i = \tilde{\rho}_i / \alpha_i \tag{2}$$

and we proceed similarly to define the apparent entropies $\tilde{s}_i$ and the "true" entropies s_i of the two components. The thermodynamical states of the two fluids are defined by two complete state equations giving the internal energy ε_i as a function of entropy and density in the form $\varepsilon_i = \varepsilon_i(s_i, \rho_i)$ such that a Gibbs relation holds for the two components :

$$d\varepsilon_i = T_i ds_i - p_i d\tau_i \quad \text{with} \quad \tau_i = 1/\rho_i \ , \ T_i = (\frac{\partial \varepsilon_i}{\partial s_i})_{\tau_i} \quad \text{and} \quad p_i = (\frac{\partial \varepsilon_i}{\partial \tau_i})_{s_i} \tag{3}$$

With these defintions, the thermodynamical state of the mixture appears as a function of the thermodynamical variables $s_1, s_2, \tau_1, \tau_2, \alpha$. To complete the description of the system, we add to these variables, two mechanical variables $\underline{u}_1$ and $\underline{u}_2$ representing the velocities of the two fluids. The binary fluid is then completely described by the 7-vector (in 1-D) $q = (s_1, s_2, \tau_1, \tau_2, \alpha, \underline{u}_1, \underline{u}_2)^t$ and a model for this binary fluid will require to find 7 closure relations for these variables. To formulate, these closure relations, it may be useful to define mixture variables for this binary fluid. The local density and mass fraction Y_i at any $\underline{x}$ and t of the fluid domain is given as :

$$\rho = \tilde{\rho}_1 + \tilde{\rho}_2 = \alpha_1 \rho_1 + \alpha_2 \rho_2 \text{ and } Y_i = \tilde{\rho}_i / \rho = \alpha_i \rho_i / \rho \tag{4}$$

Since entropy and energy are extensive variables, the mixture specific entropy s and specific internal energy are defined by

$$\begin{aligned} \rho s &= \tilde{\rho}_1 s_1 + \tilde{\rho}_2 s_2 = \alpha_1 \rho_1 s_1 + \alpha_2 \rho_2 s_2 \\ \rho \varepsilon &= \tilde{\rho}_1 \varepsilon_1 + \tilde{\rho}_2 \varepsilon_2 = \alpha_1 \rho_1 \varepsilon_1 + \alpha_2 \rho_2 \varepsilon_2 \end{aligned} \tag{5}$$

Similarly, for the mechanical variables $\underline{u}_1$ and $\underline{u}_2$, we introduce the velocity of the centre of mass $\underline{u}$ and the diffusion velocities $\underline{w}_i$ defined by :

$$\underline{u} = Y_1\underline{u}_1 + Y_2\underline{u}_2 \text{ and } \underline{w}_i = \underline{u}_i - \underline{u} \tag{6}$$

1.2 A thermodynamically consistent two-phase model

The usual way to obtain a two-phase model results from taking some average of the single phase equations (1). Standard references on these procedures are the books [10], [7]. The two phase models that results from these techniques are a system composed of two mass conservation equations

$$\frac{\partial \alpha_k \rho_k}{\partial t} + div \alpha_k \rho_k \underline{u}_k = \Gamma_k \quad \text{for} \quad k = 1, 2 \tag{7}$$

two momentum equations

$$\frac{\partial \alpha_k \rho_k \underline{u}_k}{\partial t} + div(\alpha_k \rho_k \underline{u}_k \otimes \underline{u}_k) + \nabla(\alpha_k p_k) = div(\alpha_k \underline{\underline{\tau}}_k) + p_I \nabla \alpha_k + \underline{u}^\Gamma \Gamma_k + \underline{M}_k^d \quad \text{for} \quad k = 1, 2 \tag{8}$$

and two equations expressing the conservation of energy

$$\begin{aligned} \frac{\partial \alpha_k \rho_k e_k}{\partial t} + div \alpha_k (\rho_k e_k + p_k) \underline{u}_k \quad &= div(\alpha_k \underline{\underline{\tau}}_k \underline{u}_k) - div(\alpha_k \underline{Q}_k) + p_I \frac{\partial \alpha_k}{\partial t} \\ &+ h_k^\Gamma \Gamma_k + \underline{M}_k^d . \underline{u}_I + Q_I \quad \text{for} \quad k = 1, 2 \end{aligned} \tag{9}$$

In these equations, Γ_k represents the averaged mass transfer between the phases, $\underline{u}_\Gamma$ and h_k^Γ a velocity and enthalpy whose product with Γ_k model the momentum and energy transfer between the phase associated with mass transfer, p_I is the averaged interface pressure, M_k^d is the averaged momentum transfer between phases, $\underline{u}_I$ is a velocity whose product with M_k^d models the kinetic energy transfer between the phases and finally Q_I is the interface heat transfer.

These equations form an open system and modelling assumptions are needed for the terms $p_I, \Gamma_k, \underline{u}^\Gamma, h_k^\Gamma, \underline{M}_k^d, \underline{u}_I, Q_I, \underline{\underline{\tau}}_k, \underline{Q}_k$ in order to obtain a closed system. Following the phenomenological approach of classical irreversible thermodynamics, these closure relations will be choosen in order to ensure a positive entropy production. Thus, we will write the entropy balance equation in the form

$$\frac{\partial \rho s}{\partial t} + div \underline{F_s} = \Delta_s \tag{10}$$

where $\underline{F_s}$ is the entropy flux and Δ_s, the entropy production term has to be positive.

Using Gibbs relations (3) we obtain after some algebraic manipulations of (7)-(8)-(9)

$$\frac{\partial \rho s}{\partial t} + div(\alpha_1 \rho_1 s_1 \underline{u}_1 + \alpha_2 \rho_2 s_2 \underline{u}_2) = \frac{\alpha_1}{T_1} \underline{\underline{\tau}}_1 : \nabla \underline{\underline{u_1}} + \frac{\alpha_2}{T_2} \underline{\underline{\tau}}_2 : \nabla \underline{\underline{u_2}} \tag{11.1}$$

$$-(\frac{div(\alpha_1 Q_1)}{T_1} + \frac{div(\alpha_2 Q_2)}{T_2}) \tag{11.2}$$

$$+\frac{p_1 - p_I}{T_1} \frac{D_1 \alpha_1}{Dt} + \frac{p_2 - p_I}{T_2} \frac{D_2 \alpha_2}{Dt} \tag{11.3}$$

$$+[(\frac{1}{T_1} - \frac{1}{T_2}) h_1^\Gamma - (\frac{\underline{u}_1}{T_1} - \frac{\underline{u}_2}{T_2}) \underline{u}_\Gamma + (\frac{|\underline{u}_1|^2}{2T_1} - \frac{|\underline{u}_2|^2}{2T_2})] \Gamma_1 \tag{11.4}$$

$$+[\frac{\underline{u}_I - \underline{u}_1}{T_1} - \frac{\underline{u}_I - \underline{u}_2}{T_2}] M_1^d \tag{11.5}$$

$$+[\frac{1}{T_1} - \frac{1}{T_2}] Q_1 \tag{11.6}$$

$$+[\frac{g_1}{T_1} - \frac{g_2}{T_2}] \Gamma_1 \tag{11.7}$$

where $g_k = h_k - s_k T_k$ is the chemical potential. The first two lines are "classical" and are present in the entropy evolution equation for single phase flows. In the sequel, we will assume that the heat flux and viscous tensor models are given as in single phase model by second-order dissipative terms that ensure a positive entropy production. Assume now that the volume fraction evolution equation can be written in the form :

$$\frac{\partial \alpha_k}{\partial t} + \underline{u}_\alpha \nabla \alpha_k = \dot{\alpha_k} \tag{12}$$

then the line (11.3) can be re-written

$$(\frac{p_1 - p_I}{T_1} - \frac{p_2 - p_I}{T_2}) \dot{\alpha_1} + (\frac{p_1 - p_I}{T_1} (\underline{u}_1 - \underline{u}_\alpha) - \frac{p_2 - p_I}{T_2} (\underline{u}_2 - \underline{u}_\alpha)) \nabla \alpha_1 \tag{13}$$

Let us examine the second term of this expression. In [4] is made the following interesting remark. If system (7)-(8)-(9) and (12) is reduced to its first-order term, this system is unconditionally hyperbolic and the field associated to the eigenvalue $\underline{u}_\alpha$ is linearly degenerate if and only if $\underline{u}_\alpha \in \{\underline{u}_1, \underline{u}_2, \underline{u}\}$ where $\underline{u}$ is the centre of mass velocity defined by (6). The values $\underline{u}_1$ or $\underline{u}_2$ correspond to the original model of Baer and Nunziato [1]. Since this property is an important one, we will adopt this recipe and set $\underline{u}_\alpha = \beta \underline{u}_1 + (1 - \beta) \underline{u}_2$ with $\beta \in \{0, 1, Y_1\}$. With this choice, it is easy to define p_I such that the second term in (13) is indentically equal to zero. This gives $p_I = \frac{a_1(1-\beta)}{a_1(1-\beta)+a_2\beta} p_1 + \frac{a_2\beta}{a_1(1-\beta)+a_2\beta} p_1$ with

$a_k = (Y_k T_k)^{-1}$. Then, similarly, the line (11.4) can be set to zero, by an adequate choice of $\underline{u}_\Gamma$ and h_Γ. Finally the entropy evolution equation reduces to

$$\frac{\partial \rho s}{\partial t} + div(\alpha_1 \rho_1 s_1 \underline{u}_1 + \alpha_2 \rho_2 s_2 \underline{u}_2) = \frac{\alpha_1}{T_1} \underline{\underline{\tau}}_1 : \nabla \underline{\underline{u_1}} + \frac{\alpha_2}{T_2} \underline{\underline{\tau}}_2 : \nabla \underline{\underline{u_2}} \quad (14.1)$$

$$-(\frac{div(\alpha_1 Q_1)}{T_1} + \frac{div(\alpha_2 Q_2)}{T_2}) \quad (14.2)$$

$$+(\frac{p_1 - p_I}{T_1} - \frac{p_2 - p_I}{T_2})\dot{\alpha_1} \quad (14.3)$$

$$+[\frac{\underline{u}_I - \underline{u}_1}{T_1} - \frac{\underline{u}_I - \underline{u}_2}{T_2}] M_1^d \quad (14.4)$$

$$+[\frac{1}{T_1} - \frac{1}{T_2}] Q_1 \quad (14.5)$$

$$+[\frac{g_1}{T_1} - \frac{g_2}{T_2}] \Gamma_1 \quad (14.6)$$

and a positive entropy production can therefore be insured if we set

$$\dot{\alpha_1} = \lambda_p \frac{p_1 - p_2)}{\varepsilon_p} \quad (15.1)$$

$$M_1^d = \lambda_u \frac{(\underline{u}_2 - \underline{u}_1)}{\varepsilon_u} \quad (15.2)$$

$$Q_1 = \lambda_T \frac{T_2 - T_1}{\varepsilon_T} \quad (15.3)$$

$$\Gamma_1 = \frac{\lambda_g}{\varepsilon_g} [\frac{g_2}{T_2} - \frac{g_1}{T_1}] \quad (15.4)$$

where the coefficients $\lambda_{\{p,u,T,g\}}$ are of order one while $\varepsilon_{\{p,u,T,g\}}$ are some relaxation time scales.

2 A hierarchy of two-phase models

2.1 Relaxation times

Equation (14) shows that entropy is produced by disequilibriums between the pressures, velocities, temperatures and chemical potentials of the two fluids. Therefore, the natural evolution of the binary fluid drives the system toward an homogeneous state characterized by a single common pressure, velocity, temperature and chemical potential. The time

scales $\varepsilon_{\{p,u,T,g\}}$ measure the velocity of this evolution toward equilibrium.
While these time scales depend on many different conditions and may vary from one experiment to another one, physical evidences show that in a large number of common situations we have

$$0 \sim \varepsilon_p < \varepsilon_u < \varepsilon_T < \varepsilon_g$$

see for instance [12] for some estimate of these time scales in granular materials. This gives the opportunity to derive from the models (7)-(8)-(9) a hierarchy of two-phase models that assume that some equilibrium have been reached.

2.2 Equilibrium models

2.2.1 General procedure

Here, we begin by describing the general procedure allowing to derive reduced models. For a general discussion on relaxation hyperbolic problems, see [17]. In the sequel, we concentrate on first-order terms and neglect dissipative viscous stress or heat flux terms. Consider a first-order system with stiff source term :

$$\frac{\partial U}{\partial t} + A(U)\frac{\partial U}{\partial x} = \frac{R(U)}{\varepsilon} \tag{16}$$

In this equation $U = U(x,t)$ the state vector belongs to Ω, some open subset of $\mathbb{R}^N$.
We are interested in the behavior of the solutions of (16) when the relaxation time ε goes to zero. Therefore, we expect these solutions to be close to $\mathcal{E}$ the subset of $\mathbb{R}^N$ defined by :

$$\mathcal{E} = \{U \in \mathbb{R}^N; R(U) = 0\} \tag{17}$$

Now, to obtain a reduced model, we look for a solution in the form :

$$U = M(u) + \varepsilon V \tag{18}$$

where $M(u)$ (the Maxwellian) stands for a parametrisation of an element of $\mathcal{E}$. Introducing this expression in (16) gives :

$$\begin{aligned} &\frac{\partial M(u)}{\partial t} + A(M(u))\frac{\partial M(u)}{\partial x} - R'(M(u)).V \\ &+\varepsilon[\frac{\partial V}{\partial t} + A(M(u))\frac{\partial V}{\partial x} + [\frac{\partial A}{\partial U_i}V_i]\frac{\partial M(u)}{\partial x} - \frac{1}{2}R''(M(u))(V,V)] = \mathcal{O}(\varepsilon^2) \end{aligned} \tag{19}$$

Let P the projection on $\ker(R'(M(u)))$ in the direction of $\mathrm{Rng}(R'(M(u)))$, multiplying (16) by P gives :

$$\frac{\partial u}{\partial t} + P.A(M(u)).dM_u\frac{\partial u}{\partial x} = \mathcal{O}(\varepsilon) \tag{20}$$

The reduced model of (16) is thus obtained by neglecting the terms of order ε.

2.2.2 The classical one-pressure two fluid model

We now apply this procedure to the basic two-phase model (7)-(8)-(9) and (12) and let the relaxation time $\varepsilon_p \to 0$. For the details of the computation see [14]. The following system is obtained

$$\begin{cases} \dfrac{\partial \alpha_1\rho_1}{\partial t} + \text{div}\,(\alpha_1\rho_1 u_1) & = 0 \quad (21.1) \\ \dfrac{\partial \alpha_2\rho_2}{\partial t} + \text{div}\,(\alpha_2\rho_2 u_2) & = 0 \quad (21.2) \\ \dfrac{\partial \alpha_1\rho_1 u_1}{\partial t} + \text{div}\,(\alpha_1\rho_1 u_1 \otimes u_1) + \alpha_1\nabla p & = 0 \quad (21.3) \\ \dfrac{\partial \alpha_2\rho_2 u_2}{\partial t} + \text{div}\,(\alpha_2\rho_2 u_2 \otimes u_2) + \alpha_2\nabla p & = 0 \quad (21.4) \\ \dfrac{\partial \alpha_1\rho_1 e_1}{\partial t} + \text{div}\,\alpha_1\,(\rho_1 e_1 + p)\,u_1 & = 0 \quad (21.5) \\ \dfrac{\partial \alpha_2\rho_2 e_2}{\partial t} + \text{div}\,\alpha_2\,(\rho_2 e_2 + p)\,u_2 & = 0 \quad (21.6) \end{cases}$$

This is the classical two-fluid system used in many two-phase flow studies [6],[19],[20], [24],[2]. It is known for a long time that this model is not hyperbolic. As a result, the linearized equations are ill-posed as an initial value problem and exponential growth of unstable modes can be expected. This fact causes considerable numerical problems for the approximation of the system (2.2.2). To solve this intrinsic problem in the formulation, several remedies have been suggested. We just mention here the works [5],[23], [22], [13] where pressure corrections are used in order to recover hyperbolicity.

2.2.3 One pressure-One velocity model

A step further toward equilibrium is to consider flows where both the pressure and the velocity are equal. This situation can be investigated in the framework of section 2.2.1 by letting both ε_p and $\varepsilon_u \to 0$. Analysis of this case can be found in [11] and [16]. This

model can be written in term of conservative variables ${}^t(\alpha_1\rho_1, \alpha_2\rho_2, \rho\underline{u}, \rho e, \alpha_2)$ as

$$\left\{\begin{array}{lcll} \dfrac{\partial \alpha_1\rho_1}{\partial t} + \mathrm{div}(\alpha_1\rho_1\underline{u}) & = & 0 & (22.1)\\ \dfrac{\partial \alpha_2\rho_2}{\partial t} + \mathrm{div}(\alpha_2\rho_2\underline{u}) & = & 0 & (22.2)\\ \dfrac{\partial \rho\underline{u}}{\partial t} + \mathrm{div}(\rho\underline{u}\otimes\underline{u}) + \nabla p & = & 0 & (22.3)\\ \dfrac{\partial \rho e}{\partial t} + \mathrm{div}(\rho e + p)\underline{u} & = & 0 & (22.4)\\ \dfrac{\partial \alpha_2}{\partial t} + \underline{u}.\nabla\alpha_2 & = & \alpha_1\alpha_2\dfrac{\rho_1 a_1^2 - \rho_2 a_2^2}{\sum_{k=1}^2 \alpha_{k'}\rho_k a_k^2}\mathrm{div}\underline{u} & (22.5) \end{array}\right.$$

This model is hyperbolic with velocity waves in the direction $\underline{n}$ given by $\underline{u}.\underline{n}-a$, a, $\underline{u}.\underline{n}+a$ where a is the Wood sound speed defined by

$$\frac{1}{\rho a^2} = \frac{\alpha_1}{\rho_1 a_1^2} + \frac{\alpha_2}{\rho_2 a_2^2} \tag{23}$$

2.2.4 Isobaric-Isothermal-One velocity model

If now the relaxation times for pressure, velocity and temperature are small ($\varepsilon_{\{p,u,T\}} \to 0$), the first-order system that we obtain is composed of the well-known multi-component Euler equations :

$$\left\{\begin{array}{lcll} \dfrac{\partial \alpha_1\rho_1}{\partial t} + \mathrm{div}(\alpha_1\rho_1\underline{u}) & = & 0 & (24.1)\\ \dfrac{\partial \alpha_2\rho_2}{\partial t} + \mathrm{div}(\alpha_2\rho_2\underline{u}) & = & 0 & (24.2)\\ \dfrac{\partial \rho\underline{u}}{\partial t} + \mathrm{div}(\rho\underline{u}\otimes\underline{u}) + \nabla p & = & 0 & (24.3)\\ \dfrac{\partial \rho e}{\partial t} + \mathrm{div}(\rho e + p)\underline{u} & = & 0 & (24.4) \end{array}\right.$$

This system is also appropriate to represent multi-component mixture where $\alpha_1 = \alpha_2 = 1$. However, here the requirement of immiscibility of the fluids is translated at the macroscopic level by $\alpha_1 + \alpha_2 = 1$ while the state law is given by the solution of the system $p_2 = p_1$ and $T_2 = T_1$.

2.2.5 Homogeneous Equilibrium model

The last stage in this procedure consists in assuming that pressures, velocities and temperatures as well as the chemical potentials are in equilibrium, $\varepsilon_{\{p,u,T,g\}} \to 0$. The model that results from these assumptions are simply the Euler equations of gas dynamiacs expressing the conservation of mass, momentum and energy

$$\left\{\begin{array}{lcll} \dfrac{\partial \rho}{\partial t} + \mathrm{div}(\rho \underline{u}) & = & 0 & (25.1) \\ \dfrac{\partial \rho \underline{u}}{\partial t} + \mathrm{div}(\rho \underline{u} \otimes \underline{u}) + \nabla p & = & 0 & (25.2) \\ \dfrac{\partial \rho e}{\partial t} + \mathrm{div}(\rho e + p)\underline{u} & = & 0 & (25.3) \end{array}\right.$$

The state law here results from solving the equations $p_2 = p_1, T_2 = T_1$ as well as $g_2 = g_1$. In practice however, for this model, the state law is often given by experimental correlations, see [3] for an exemple.

2.3 First-order Chapman-Enskog expansion and near-equilibrium models

The previous class of models have been obtained from the basic two phase model by assuming that the relaxation times $\varepsilon_{\{p,u,T,g\}}$ vanishes to zero and neglecting terms of order ε in an asymptotic expansion of the solution. It is possible by using a first order Chapman-Enskog expansion of the solution to obtain more accurate models that will keep some influences of non-equilibrium through dissipative terms. We explain here briefly, the principle of this procedure on the example of the pressure-velocity relaxation. More details can be found in [18], [14] or [9]. The basic idea of the Chapman-Enskog expansion is to look for a solution of (16) of the form :

$$U = M(\underline{v}) + \varepsilon V + \mathcal{O}(\varepsilon^2) \quad \text{with} \quad V \in \ \mathrm{Rng}(\mathrm{R}'(\mathrm{M}(\underline{v}))) \tag{26}$$

Introducing Q the projection on $\mathrm{Rng}(R'(M(u)))$ in the direction of $\ker(R'(M(u)))$, we see that (16) is totally equivalent to the following two sub-systems obtained by multiplying it by P and Q :

$$\begin{array}{l} \dfrac{\partial \underline{v}}{\partial t} + PA(M(\underline{v}))\dfrac{\partial M(\underline{v})}{\partial x} \\ +\varepsilon P[\dfrac{\partial V}{\partial t} + A(M(\underline{v}))\dfrac{\partial V}{\partial x} + [\dfrac{\partial A}{\partial U_i}V_i]\dfrac{\partial M(\underline{v})}{\partial x} - \dfrac{1}{2}R''(M(\underline{v}))(V,V)] = \mathcal{O}(\varepsilon^2) \end{array} \tag{27}$$

and

$$\begin{aligned} &QA(M(\underline{v}))\frac{\partial M(\underline{v})}{\partial x} - QR'(M(\underline{v})).V \\ &+Q\varepsilon[\frac{\partial V}{\partial t} + A(M(\underline{v}))\frac{\partial V}{\partial x} + [\frac{\partial A}{\partial U_i}V_i]\frac{\partial M(\underline{v})}{\partial x} - \frac{1}{2}R''(M(\underline{v}))(V,V)] = \mathcal{O}(\varepsilon^2) \end{aligned} \tag{28}$$

Solving (28) for V gives :

$$QR'(M(\underline{v})).V = QA(M(\underline{v})).dM_{\underline{v}}\frac{\partial \underline{v}}{\partial x} + \mathcal{O}(\varepsilon) \tag{29}$$

Since Q is the projection on Rng $(R'(M(\underline{v})))$, equation (29) has a unique solution. Actually, it can be shown (see ([9]) that (29) can be written :

$$V = \mathcal{D}(v)\frac{\partial \underline{v}}{\partial x} + \mathcal{O}(\varepsilon) \tag{30}$$

that establish that the correction terms expressing non-equilibrium have the form of second-order terms. Now, introducing this expression into (27) and neglecting terms of order $\mathcal{O}(\varepsilon^2)$ gives the first-order system :

$$\begin{aligned} &\frac{\partial \underline{v}}{\partial t} + PA(M(\underline{v}))\frac{\partial M(\underline{v})}{\partial x} \\ &= -\varepsilon P[A(M(\underline{v}))\frac{\partial}{\partial x}(\mathcal{D}(v)\frac{\partial \underline{v}}{\partial x}) + [\frac{\partial A}{\partial U_i}(\mathcal{D}(v)\frac{\partial \underline{v}}{\partial x})_i]\frac{\partial M(\underline{v})}{\partial x} - \frac{1}{2}R''(M(\underline{v}))(\mathcal{D}(v)\frac{\partial \underline{v}}{\partial x}, \mathcal{D}(v)\frac{\partial \underline{v}}{\partial x}))] \end{aligned} \tag{31}$$

As an example, we will apply this technique to the one-pressure, one-velocity model described in section 2.2.3. Details of the computation can be found in [9]. The non-equilibrium corrections satisfies :

$$\lambda(u_2 - u_1) = \rho Y_1 Y_2(\frac{1}{\rho_1} - \frac{1}{\rho_2})\frac{\partial p}{\partial x} \tag{32.1}$$

$$\mu(p_2 - p_1) = \alpha_1\alpha_2\frac{C_1 - C_2}{\alpha_1 C_2 + \alpha_2 C_1}\frac{\partial u}{\partial x} \tag{32.2}$$

where $C_k = \rho_k a_k^2$ with a_k the speed of sound in the phase k. One then obtain, the system (compare with the non-disspative system (22))

$$\frac{\partial}{\partial t}(\alpha_1\rho_1) + \frac{\partial}{\partial x}(\alpha_1\rho_1 u) - \varepsilon\frac{\partial}{\partial x}J_1 = 0 \quad (33.1)$$

$$\frac{\partial}{\partial t}(\alpha_2\rho_2) + \frac{\partial}{\partial x}(\alpha_2\rho_2 u) - \varepsilon\frac{\partial}{\partial x}J_2 = 0 \quad (33.2)$$

$$\frac{\partial}{\partial t}(\rho u) + \frac{\partial}{\partial x}(\rho(u)^2 + p) - \varepsilon\frac{\partial}{\partial x}\left(\mu\frac{\partial u}{\partial x}\right) = 0 \quad (33.3)$$

$$\frac{\partial}{\partial t}(\rho e) + \frac{\partial}{\partial x}(\rho e + p)u - \varepsilon\frac{\partial}{\partial x}(h_1 J_1 + h_2 J_2 + u\mu\frac{\partial u}{\partial x}) = 0 \quad (33.4)$$

$$\frac{D\alpha_2}{Dt} - \alpha_1\alpha_2\frac{C_1 - C_2}{\alpha_1 C_2 + \alpha_2 C_1}\frac{\partial u}{\partial x} - \varepsilon\frac{D}{Dt}\left(\frac{1}{\mu}\frac{(\alpha_1\alpha_2)^2(C_1 - C_2)}{(\alpha_1 C_2 + \alpha_2 C_1)^2}\frac{\partial u}{\partial x}\right) = 0 \quad (33.5)$$

where $D/Dt = \partial/\partial t + u.\nabla$ denote the Lagrangian derivative and the expression for J_k and μ are given by :

$$J_k = \frac{(\rho Y_1 Y_2)^2}{\lambda}(\frac{1}{\rho_k} - \frac{1}{\rho'_k})\frac{\partial p}{\partial x} \quad (34)$$

and

$$\mu = \frac{(C_2 - C_1)^2}{\mu(\frac{C_2}{\alpha_2} + \frac{C_1}{\alpha_1})^2} =$$

while the h_k are the specific phase enthalpies.

3 Some applications

3.1 Comparison between the complete two-phase and the isobaric-One velocity model

This numerical experiment consider problems where two phases are simultaneously present at the same location and compare the results obtained with the isobaric-one velocity model (22) with those obtained by the seven equation model composed of ((7)-(8)-(9) and the topological equation (12). The closure assumptions used for this model are the ones of [21]. This computation considers a two-phase shock tube problem where the initial volume fraction is constant and equal to $\alpha_1 = 0.5$ everywhere in the domain. On the left side ($x < 0.5$) the pressure is 10^9 Pa while it is equal to 10^5 Pa on the right side. The velocity is zero at time 0. The discretization is done on a 1000 cells grid and the CFL number is fixed and equal to 0.6. The results are shown at time 200μs. We compare in Figure 1 the results obtained with the model (22) with those obtained by the complete

seven equation model. The numerical method used to solve the seven equation model is the one described in [21], except that the relaxation procedures have been improved as described in [15]. The numerical method used to solve the model (22) is described in [16]. The results are in perfect agreement and this confirms that the five equation model (22) is a correct asymptotic limit of the seven equation model in the limit of zero pressure and velocity relaxation times. In particular, we observe that even if the initial composition of the mixture is constant, it evolves in space and time and that this evolution is the same in the results obtained with the two models.

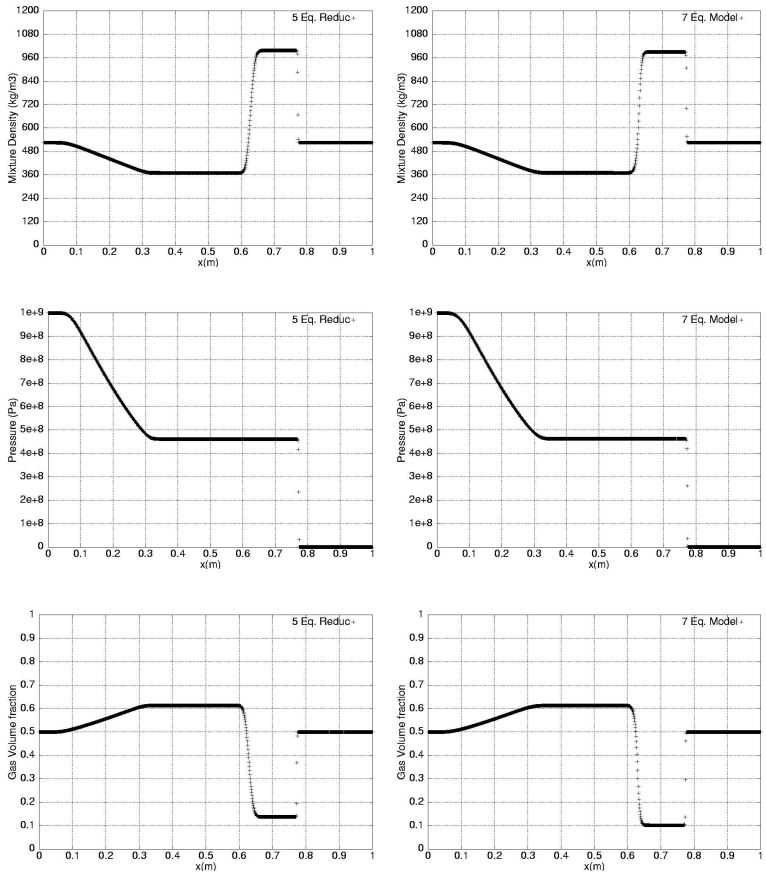

Figure 1: **5 equation reduced model (left) and 7 equation model (right) for a two-phase shock tube problem. Computed solutions with 1000 cells (symbols).**

3.2 A 3-D interface computation

In this second example, we use the isobaric-one velocity model (22) to compute a three-dimensional multifluid interface problem. The 3-D model and numerical method are described in [25] and this test case is from [26]. It computes the interaction of a

shockwave moving through a low density fluid and interacting with a bubble of high density fluid. For this three dimensional test-case, the number of mesh vertices was 1.03M and an explicit three-stage RK was used to advance the solution in time. Figure 2 shows the three-dimensional pressure contours after 720 time steps at which time the shockwave has passed through the bubble reflected off the top boundary and passed through the bubble a second time. These computations shows that the model (22) is not only useful to compute two-phase flows but that it can also be used to compute interface problems between two immiscible compressible fluids.

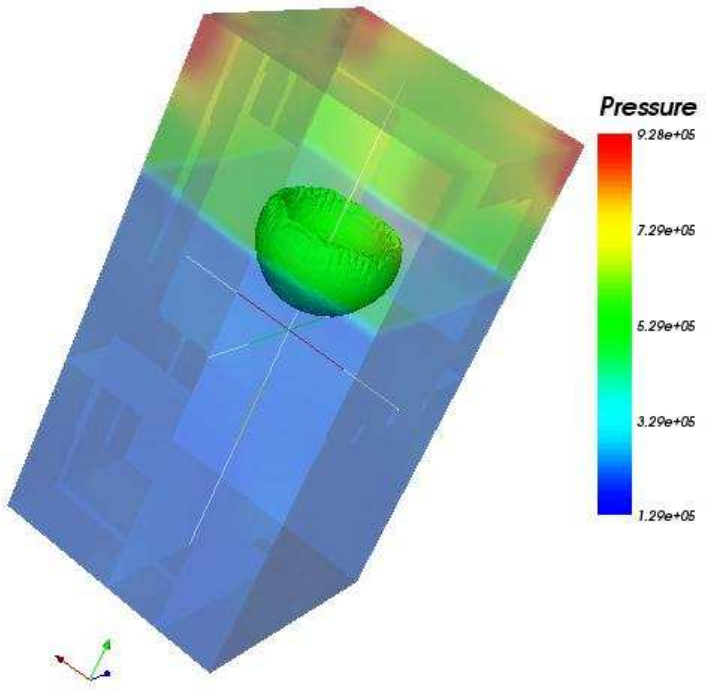

Figure 2: **Pressure contours in a 3-D shock-bubble interaction**

3.3 Phase separation in a pipe

We consider here in a classical two-phase test case designed to investigate the effect of gravity induced phase separation. Initial conditions represent a 7.5 m vertical tube filled with an homogeneous mixture of volume fraction $\alpha_k = 0.5$. Under the effect of

gravity, the two phases will separate and a final situation where the high density fluid lies under the light one will be reached. This situation is interesting because obviously the velocities of the two phase are different. Therefore, it is expected that only a two-velocity model (either the complete seven equation model or the classical isobaric (2.2.2) one) can compute this situation. We show here, that near equilibrium models as described in section 2.3 that retains some non-equilibrium phenomena through dissipative second-order terms are also able to compute this situation. This computation done in [8] uses an isothermal model developped in [18] where a non-equilibrium between the two phase velocities exists throught a Darcy-like law that writes

$$\begin{cases} \alpha_1\rho_1 u_1 &= \alpha_1\rho_1 u \quad -\dfrac{(\rho Y_1 Y_2)^2}{\lambda}\left(\dfrac{1}{\rho_1}-\dfrac{1}{\rho_2}\right)\dfrac{\partial p}{\partial x}+\mathcal{O}(\varepsilon^2) \quad (35.1) \\ \\ \alpha_2\rho_2 u_2 &= \alpha_2\rho_2 u \quad +\dfrac{(\rho Y_1 Y_2)^2}{\lambda}\left(\dfrac{1}{\rho_1}-\dfrac{1}{\rho_2}\right)\dfrac{\partial p}{\partial x}+\mathcal{O}(\varepsilon^2) \quad (35.2) \end{cases}$$

The figure 3 shows the propagation of two waves travelling from the two ends of the pipe that meet in the middle section of the pipe to form a sharp separation between liquid and gas.

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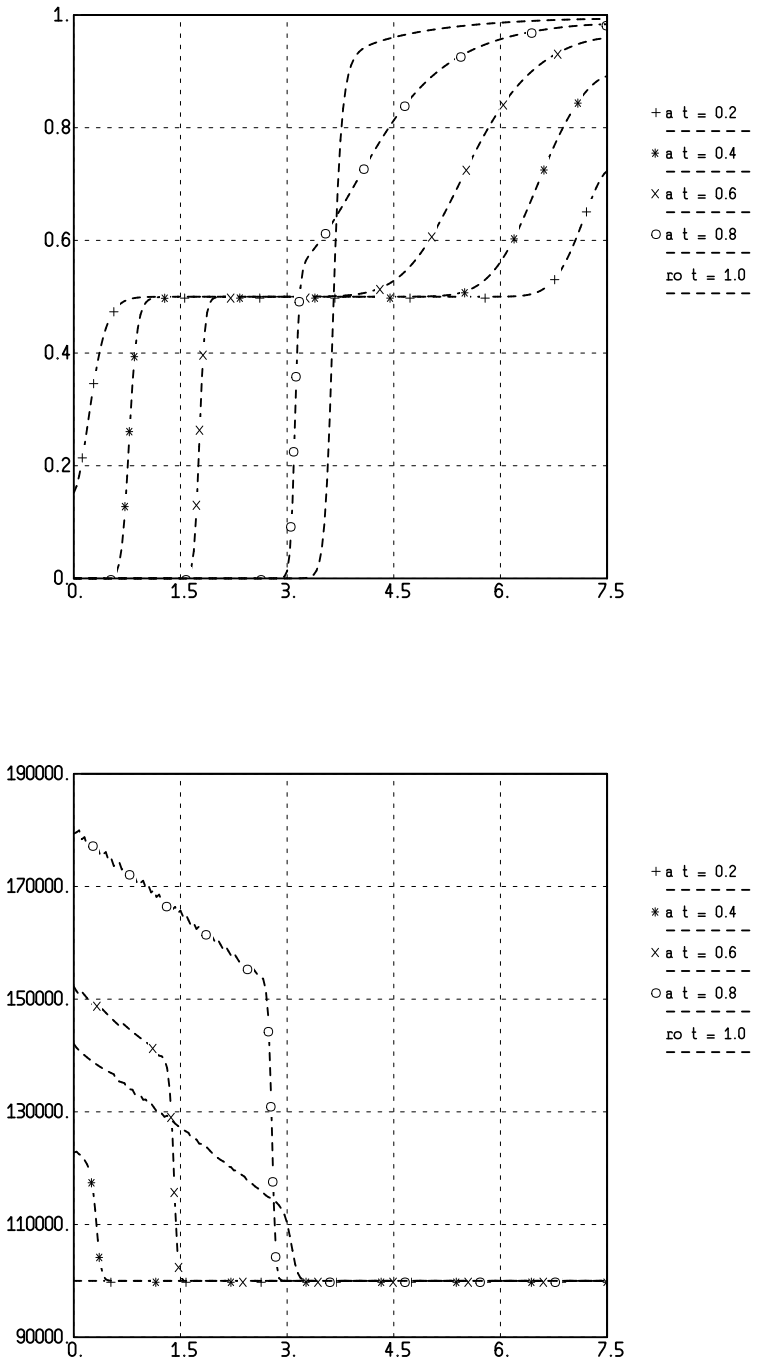

Figure 3: **Volume fraction and pressure evolution in a sedimentation test case(gravity is here in the x-direction)**

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