Coalitional games in energy and analytics markets

Raja, A.A.

DOI
10.4233/uuid:28daa621-5070-48e3-8c92-b19b300aec75

Publication date
2023

Document Version
Final published version

Citation (APA)
Raja, A. A. (2023). *Coalitional games in energy and analytics markets*. [Dissertation (TU Delft), Delft University of Technology]. https://doi.org/10.4233/uuid:28daa621-5070-48e3-8c92-b19b300aec75

COALITIONAL GAMES IN ENERGY AND ANALYTICS MARKETS

COALITIONAL GAMES IN ENERGY AND ANALYTICS MARKETS

Proefschrift

ter verkrijging van de graad van doctor
aan de Technische Universiteit Delft,
op gezag van de Rector Magnificus Prof.dr.ir. T.H.J.J. van der Hagen,
voorzitter van het College voor Promoties,
in het openbaar te verdedigen op
maandag 13 maart 2023 om 10:00 uur

door

Aitazaz Ali RAJA
Master of Science i elektroteknik,
Politecnico di Milano, Italy

geboren te Abbottabad, Pakistan.

Dit proefschrift is goedgekeurd door de promotoren.

Samenstelling promotiecommissie bestaat uit:

Rector Magnificus,	voorzitter
Dr. ing. S. Grammatico,	Technische Universiteit Delft, promotor
Prof. dr. ir. B. De Schutter,	Technische Universiteit Delft, promoter
Onafhankelijke leden:	
Prof. dr. ir. L.J. de Vries	Technische Universiteit Delft
Prof. dr. P. Palensky	Technische Universiteit Delft
Prof. dr. P. Pinson	Imperial College London, United Kingdom
Dr. ing. J. Kazempour	Technical University of Denmark, Denmark
Dr. ing. N. Paterakis	Technische Universiteit Eindhoven

This research was supported by the Netherlands Organization for Scientific Research (NWO) under research project P2P-TALES (grant n. 647.003.003) and by the European Research Council under research project COSMOS (grant n. 802348).

Truth is sought for its own sake . . .

Ibn al-Haytham (Alhazen)

Contents

SUMMARY

The main themes of this thesis are the design and analysis of payoff distribution methods for situations where agents collaborate to generate a utility. For modeling such scenarios, we majorly focus on the coalitional game theoretic framework that provides mathematical formalism to study the behavior of rational agents when they cooperate for selfish interests [69]. We utilize the tools from coalitional game theory to develop mechanisms for demand-side energy management, namely, energy coalitions, peer-to-peer energy trading (P2P), and real-time local electricity markets, that can help accelerate the energy transition [106]. For the solution of resulting games, we design distributed algorithms that converge to a payoff distribution characterized by *stability* and *fairness*. The primary approach to convergence analysis of proposed algorithms relies on the operator theory and fixed-point iterations. Finally, we also propose payoff distribution criteria for a wagering-based forecasting market that can help energy generation sources to improve their forecast.

We start with developing the framework of robust coalitional games, a class of dynamic coalitional games where the actual coalitional values are unknown but vary within known bounds. We formalize a solution of robust games, i.e., the “robust core" and design two distributed algorithms, namely, distributed payoff allocation and distributed bargaining, that converge to a consensual payoff distribution in the robust core [13]. We show convergence of both algorithms that are executed on time-varying communication networks. We motivate our setup by energy storage optimization as an application of coalitional resource utilization, which aligns with the idea of sharing economy [65].

The algorithms designed for robust coalitional games have a high computational burden. Thus, for the implementation of large-scale systems, we exploit the geometric structure of the solution set, i.e., the core, to develop faster payoff distribution algorithms. We demonstrate the performance of our algorithms by proposing a bilateral peer-to-peer (P2P) energy trading scheme that empowers prosumers to have control over trading their energy resources [111]. Our market model (assignment game [98]) allows buyers to have heterogeneous preferences (product differentiation) over the energy sellers, which can be economic, social, or environmental. For solving the P2P market, we execute a novel distributed negotiation process over a time-varying communication network that guarantees stable trading prices in a coalitional game theoretic sense and satisfies the desired economic properties. Furthermore, as the different points of the core set treat buyer and seller sides differently, we also designed a bilateral negotiation mechanism that enables participants to reach a trading contract (τ-value) [77], which fairly divides the resulting market welfare among buyers and sellers.

To generalize the setup of robust coalitional games in which we assume that the core remains within certain bounds, we introduce a framework for online coalitional games. In these games, the time variation of coalitional values is not restricted to a set and, consequently, the solution as well. For a dynamic setting of the online games, we then pro-

pose two online distributed algorithms for real-time payoff distribution. The goal of a real-time payoff distribution in an online coalitional game setup is to track a consensus on the payoff distribution solutions, namely, Shapley value [97] and the core. The online setup allows us to model real-time markets operating on fast time scales, for example, a real-time local electricity market [42]. In such markets, we can execute market clearing very close to the time of delivery, which enables accurate forecasting for the integration of small to medium-scale RES.

High quality forecasts can help deal with uncertainty effecting the processes, for example, uncertainty in weather effecting the wind energy generation [75]. Therefore, we design a platform for improving predictions via implicit pooling of private information in return for possible remuneration [124]. Specifically, we design a wagering-based forecast elicitation market platform, where a buyer intending to improve their forecasts posts a prediction task, and sellers respond to it with their forecast reports and wagers. This market delivers an aggregated forecast to the buyer (pre-event) and allocates a payoff to the sellers (post-event) for their contribution. Our mechanism is history-free and general, i.e, it does not consider previous performance of forecasters and can accommodate various forms of predictive distributions, namely, binary, multi-category and continuous. For such a mechanism, we propose a payoff distribution criterion and prove that it satisfies several desirable economic properties.

SAMENVATTING

De hoofdthema's van dit proefschrift zijn het ontwerp en de analyse van uitbetalingsdistributiemethoden voor situaties waarin agenten samenwerken om een hulpprogramma te genereren. Voor het modelleren van dergelijke scenario's richten we ons voornamelijk op het theoretische raamwerk van coalities dat wiskundig formalisme biedt om het gedrag van rationele agenten te bestuderen wanneer ze samenwerken voor egoïstische belangen [69]. We gebruiken de tools uit de coalitiespeltheorie om mechanismen te ontwikkelen voor energiebeheer aan de vraagzijde, namelijk het delen van energiebronnen via P2P, peer-to-peer energiehandel (P2P) en realtime lokale elektriciteitsmarkten, die kunnen helpen de energieproductie te versnellen transitie [106]. Voor de oplossing van resulterende spellen ontwerpen we gedistribueerde algoritmen die convergeren naar een uitbetalingsverdeling die wordt gekenmerkt door *stability* en *fairness*. De primaire benadering van convergentieanalyse van voorgestelde algoritmen is gebaseerd op de operatortheorie en vaste-puntiteraties. Ten slotte stellen we ook uitbetalingsverdelingscriteria voor voor een op weddenschappen gebaseerde voorspellingsmarkt die energieopwekkingsbronnen kan helpen hun voorspelling te verbeteren.

We beginnen met het ontwikkelen van het raamwerk van robuuste coalitiespellen, een klasse van dynamische coalitiespellen waarbij de werkelijke coalitiewaarden onbekend zijn maar binnen bekende grenzen variëren. We formaliseren een oplossing van robuuste games, d.w.z. de 'robuuste kern' en ontwerpen twee gedistribueerde algoritmen, namelijk gedistribueerde uitbetalingstoewijzing en gedistribueerde onderhandelingen, die convergeren naar een consensuele uitbetalingsverdeling in de robuuste kern [13]. convergentie van beide algoritmen die worden uitgevoerd op in de tijd variërende communicatienetwerken.

De algoritmen die zijn ontworpen voor robuuste coalitiespellen hebben een hoge rekenbelasting. Voor de implementatie van grootschalige systemen maken we dus gebruik van de geometrische structuur van de oplossingsset, d.w.z. de kern, om snellere algoritmen voor uitbetalingsdistributie te ontwikkelen. We demonstreren de prestaties van onze algoritmen door een bilateraal peer-to-peer (P2P) energiehandelssysteem voor te stellen dat prosumenten in staat stelt controle te hebben over de handel in hun energiebronnen [111]. Ons marktmodel (toewijzingsspel [98]) stelt kopers in staat om heterogene voorkeuren (productdifferentiatie) te hebben ten opzichte van de energieverkopers, die economisch, sociaal of ecologisch kunnen zijn. Voor het oplossen van de P2P-markt voeren we een nieuw gedistribueerd onderhandelingsproces uit via een in de tijd variërend communicatienetwerk dat stabiele handelsprijzen garandeert in een coalitiespel-theoretische zin en voldoet aan de gewenste economische eigenschappen. Bovendien, aangezien de verschillende punten van de kernset de koper- en verkoperzijde verschillend behandelen, hebben we ook een bilateraal onderhandelingsmechanisme ontworpen dat deelnemers in staat stelt een handelscontract (τ-waarde) [77] te bereiken, dat de resulterende marktwelvaart bij kopers en verkopers.

Om de opzet van robuuste coalitiespellen te veralgemenen waarbij we ervan uitgaan dat de kern binnen bepaalde grenzen blijft, introduceren we een raamwerk voor online coalitiespellen. In deze spellen is de tijdsvariatie van coalitiewaarden niet beperkt tot een set en dus ook de oplossing. Voor een dynamische setting van de online games stellen we vervolgens twee online gedistribueerde algoritmen voor voor realtime uitbetalingsdistributie. Het doel van een realtime uitbetalingsdistributie in een online coalitiegame-opstelling is om een consensus te volgen over de uitbetalingsdistributie-oplossingen, namelijk Shapley-waarde [97] en de kern. De online opzet stelt ons in staat om realtime markten te modelleren die werken op snelle tijdschalen, bijvoorbeeld een realtime lokale elektriciteitsmarkt [42]. In dergelijke markten kunnen we marktclearing zeer dicht bij het moment van levering uitvoeren, wat nauwkeurige voorspellingen mogelijk maakt voor de integratie van kleine tot middelgrote RES.

Hoogwaardige voorspellingen kunnen helpen om te gaan met onzekerheid die de processen beïnvloedt, bijvoorbeeld onzekerheid in het weer die de opwekking van windenergie beïnvloedt [75]. Daarom ontwerpen we een platform voor het verbeteren van voorspellingen via impliciete pooling van privé-informatie in ruil voor een eventuele vergoeding [124]. In het bijzonder ontwerpen we een op weddenschappen gebaseerd marktplatform voor het uitlokken van voorspellingen, waar een koper die van plan is zijn voorspellingen te verbeteren, een voorspellingstaak plaatst en verkopers hierop reageren met hun voorspellingsrapporten en weddenschappen. Deze markt levert een geaggregeerde prognose aan de koper (pre-event) en kent een uitbetaling toe aan de verkopers (post-event) voor hun bijdrage. Ons mechanisme is geschiedenisvrij en algemeen, d.w.z. het houdt geen rekening met eerdere prestaties van voorspellers en is geschikt voor verschillende vormen van voorspellende distributies, namelijk binair, multi-categorie en continu. Voor een dergelijk mechanisme stellen we een uitbetalingsverdelingscriterium voor en bewijzen we dat het aan verschillende wenselijke economische eigenschappen voldoet.

List of Symbols

Basic Operators and Relations

$:=$	left-hand side is defined by right-hand side
$\mid$	such that
$\in$	is element of
$\exists$	there exist
$\forall$	for all
$\Rightarrow$	implies
$\Leftrightarrow$	if and only if

Sets, Spaces and Set Operators

$\mathbb{N}$	set of natural numbers
$\mathbb{R}$	set of real numbers
$\mathbb{R} \geq 0$	set of nonnegative real numbers
$\mathbb{R}_{>0}$	set of positive real numbers
$\mathbb{R}^n$	set of real n-dimensional vectors
$\mathbb{R}^{n \times m}$	set of real n-times- m-dimensional matrices
$A \cup B$	union of the sets A and B
$A \cap B$	intersection of the sets A and B
$A \subset B$	A is subset of B
$A \subseteq B$	A is a subset or equal to B
$A \setminus B$	set of elements of A but not in B
$A \times B$	Cartesian product of the sets A and B
$\prod_{i=1}^{N} A_i$	$:= A_1 \times \ldots \times A_N$, Cartesian product of $A_1, \ldots, A_N$

Operations on Vectors and Matrices

$\mathbf{0}$	matrix/vector with all elements equal to 0
$\mathbf{1}$	matrix/vector with all elements equal to 1
$v^\top$	transpose of the vector v
v_i	i-th component of the vector v
$\|v\|$	2-norm of the vector v
$\mathrm{col}(v_1,\ldots,v_N)$	$:= \left[v_1^\top,\ldots,v_N^\top\right]^\top$ stacked vector of vectors
$\mathrm{diag}(v_1,\ldots,v_N)$	diagonal matrix with $v_1,\ldots,v_N$ on the diagonal
$(v_k)_{k\in\mathbb{N}}$	sequence of vectors v_k
$v \perp w$,	with $v, w \in \mathbb{R}^n$, if $v_i w_i = 0$ for all $i \in \{1,\ldots,n$
$M^\top$	transpose of the matrix M
M^{-1}	inverse of the matrix M
$[M]_{i,j} = M_{i,j}$	element in position (i,j) of the matrix M
$M_1 \otimes M_2$	Kronecker productof the matrices M_1 and M_2

Coalitional Game Theory

x_i	payoff of agent i
$\boldsymbol{x}$	$:= \mathrm{col}(x_1,\ldots,x_N)$, collective payoff
$\boldsymbol{x}_{-i}$	$:= \mathrm{col}(x_1,\ldots,x_{i-1},x_{i+1},\ldots,x_N)$
$\mathcal{I}$	$:=$ set of indices of agents
S	coalition of agents i
$v(S)$	utility of coalition S

Operator Theory

Id	identity operator
proj_Ω	projection onto the set Ω
$\mathrm{overproj}_\Omega$	over projection onto the set Ω
$\mathrm{dom}(F)$	domain of the operator F
$\mathrm{fix}\,(F)$	fixed-point sét of the operator F
$\mathrm{range}(F)$	range of the operator F

1

INTRODUCTION

In this chapter, we provide background and motivation for this thesis by presenting challenges and opportunities in energy management and market design domains for aiding the so-called energy transition. We discuss our research objectives and state the organization of the thesis.

1

1.1. ENERGY TRANSITION

IN recent years, climate change emerged as a critical challenge faced by humanity. As a response to this challenge, global initiatives are being taken in which reform of the energy sector has a central role. The pathway of this reform in the energy sector is termed the energy transition, i.e., the transformation from fossil-based to renewable energy sources (RES) to limit the energy-related carbon emissions [103]. It is estimated that renewable energy and energy efficiency measures have the potential to achieve 90% of the required carbon reductions [29]. Within the energy sector, a considerable contribution to decarbonization can be achieved via electrification of consumption while replacing fossil fuel-generated electricity with low/zero-carbon sources [120]. It is important to note that the energy transition is not limited to the replacement of energy sources, instead it is a paradigm shift that concerns the entire system. This system includes all connected technical, economic, and social sectors. Figure 1.1 shows an ongoing transition in the energy systems.

In the power sector, the energy transition is being led by the rapid deployment of distributed energy resources (DER), such as demand-side generators, storage units, electric vehicles, and flexible loads [10]. These decentralized resources spurring on the demand-side increase the operational complexity of the system, particularly because of the associated uncertainty. We can mitigate this uncertainty linked to DERs by employing demand-side management techniques (e.g., community storage) and by utilizing the relevant data [108]. Demand-side management limits the impact of uncertainty to reach the main system while data and analytics allow making high quality predictions thereby equipping operators to make more informed decisions.

In this thesis, we focus primarily on coalitional solutions for demand-side management and acquiring energy analytics that can help accelerate the energy transition [106]. Such solutions reside in the intersection of technical, economic, and social sectors. Thus, their implementation requires the employment of sophisticated mechanisms on complex infrastructures. Fortunately, with the widespread deployment of sensing, information, and communication technology our systems can support establishment of smart solutions. For a demand-side energy management we design mechanisms for energy communities [65], peer-to-peer energy trading (P2P) [109] and real-time local electricity markets [42]. Furthermore, for market-based analytics we propose a real-time payoff mechanism and a wagering based forecasting market that can help generation sources to improve their forecast.

1.1.1. ENERGY COALITIONS

Motivated by energy transition, consumer behavior across the spectrum of markets is experiencing a disruptive shift. Driven by social conscience to limit the use of resources and to reduce wastage, consumers are moving from a “sole-ownership” to a “shared-ownership” model with guaranteed access [65]. This model is regarded as a shared economy where examples include car/bike-sharing systems, etc. The applications of sharing economy in the power sector (e.g., community energy storage) can improve cost-effectiveness, while grid asset management becomes more efficient [65]. Community-scale collaborative applications have considerable environmental benefits because the self-consumption of locally generated green energy can be maximized to reduce CO_2

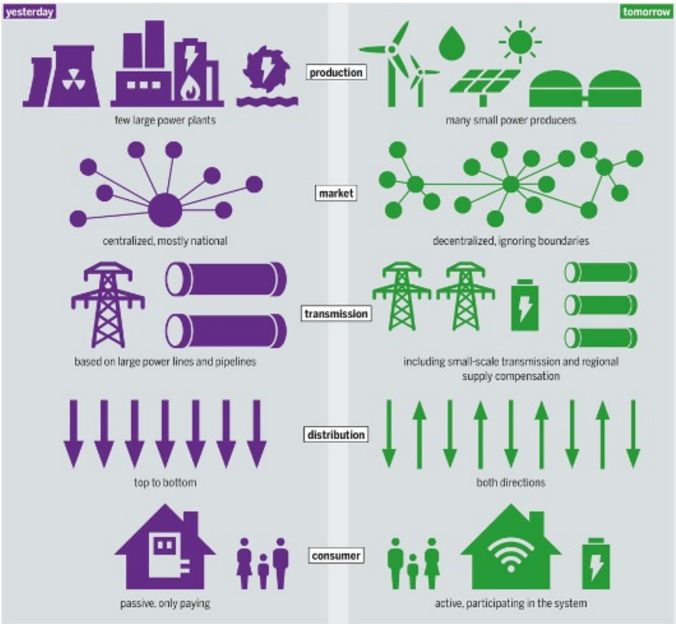


Figure 1.1: Transition of energy systems [28], CC BY 4.0

emissions. While the value of the sharing economy is not in question, remarkably, structured platforms to support the deployment of shared energy services, and algorithmic tools to enable such services, require a significant amount of work [48]. In this thesis, we develop coalitional game-theoretic mechanisms for energy coalitions utilizing the principles of sharing economy.

1.1.2. P2P ELECTRICITY MARKETS

In recent years, power systems have started to evolve towards a more decentralized management driven by the decentralized generation sources. However, electricity markets maintained conventional top-down approach for resource allocation and pricing based [47]. To realize their full potential, prosumers should engage more actively with electricity markets. Currently the direct participation of prosumers in the whole sale energy market is technically and economically non-viable, hence small-scale prosumers interact with aggregating entities such as retailers to deliver their excess energy to the grid [66]. Retailers usually offer a considerably lower price for the energy sold by prosumers, e.g. feed-in-tariff (FiT), compared to the buying price they charge [44]. To ensure an economically appealing role of prosumers, P2P energy trading represents a disruptive demand side energy management strategy [111] that enables prosumers to exchange energy on their own terms of transactions. However, modelling the P2P interaction of self-interested agents with possibly conflicting interests poses a significant mathematical challenge.

In the literature, researchers have utilized various approaches to design P2P energy

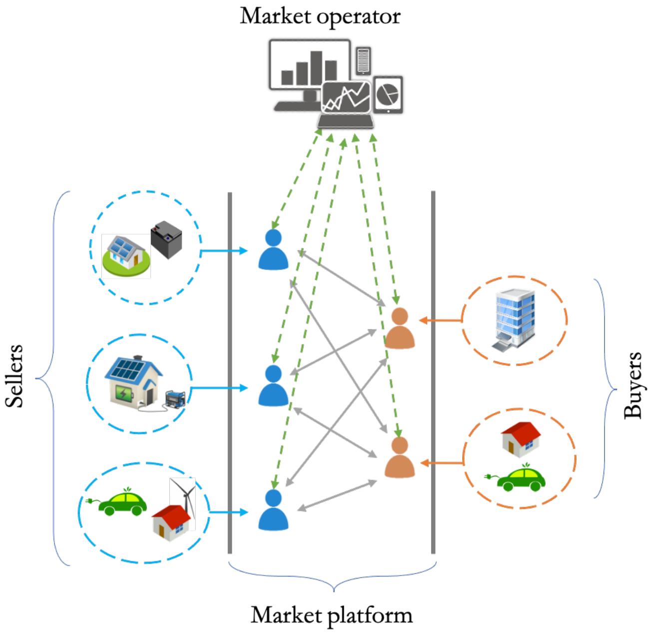


Figure 1.2: Conceptual design of bilateral P2P energy trading platform

trading mechanisms. For example, the authors in [105] and [74] present optimization based methods, [68] formulates P2P as a matching market, in [115] and [25], the authors propose a two-tier market for two-levels of energy exchange, i.e., within a group and with other groups of energy agents. The blockchain based implementations to address privacy and security in P2P platforms are also addressed [84], [5]. Finally, coalitional game theory has also received attention from smart-grid researchers in designing of P2P platforms [109], [112]. Generally, A decision-making process of agents in a P2P paradigm envisions the application of distributed intelligence to controlling and optimizing the behaviour of the consumers that interact through P2P trading networks. Design challenges involve simultaneous consideration of individual and collective objectives to achieve coordination [104], establishment of an interoperable architecture to allow for interactions between the heterogeneous domains of energy actors and system operators and consideration of a pressing issues of data privacy [116]. Furthermore, efficient P2P platforms require development of new rigorous mathematical models to improve distributed computation techniques to tackle the tractability concerns related to modelling of complex multi-actor and multi-interest energy systems. These systems integrate large amounts of data from different sources, time scales and granularity. Figure 1.2 shows the conceptual design of bilateral P2P energy trading platform. The International Energy Agency in [35] presents the basic architecture and components required for the implementation of P2P systems. In this thesis, we attempt to address above mentioned challenges by coalitional game theoretic formulation of P2P systems and design distributed algorithms with convergence guarantees for the solution of resulting games.

1.1.3. Real-time local electricity markets

One of the key challenges in the large-scale integration of RES is the inherent uncertainty [131]. This uncertainty considerably complicates the implementation of demand-side management techniques like local electricity markets, including P2P, that are envisioned for small RES generators. The prosumers participating in these markets utilize forecasts of generation or demand for bidding. The accuracy of these forecasts can be improved by decreasing the lead time [75]. In other words, market clearing closer to the time of delivery, i.e., in real-time can mitigate the possible imbalance caused by the uncertainty associated with RES. The adoption of real-time markets has shown a significant potential in the power system sector [118]. However, very little work has been done towards real-time local electricity markets [42]. The mechanisms for such markets require a large amount of information exchange and execution of negotiation processes on fast time scales. In this thesis, we introduce a framework for online coalitional games to model a real-time market clearing for local electricity markets.

We note that the applications described in Sections 1.1.1, 1.1.2 and 1.1.3 share some important characteristics.

- *Large-scale:* They can involve a large population of decision makers or agents, i.e., prosumers.
- *Self-interested:* The decision makers cooperate or compete to achieve their desired local objectives via local decision making. Each decision maker is interested in maximizing their own utility regardless of the interest of the other decision makers.
- *Shared Resources:* Although the objectives of decision makers are local, they share common resources and infrastructures e.g., community energy storage, the common power grid, etc.
- *Locality of information:* The agents have a limited knowledge of the overall system as the communication among them is local and with *neighbors* only. This limited information transmission is mandated by the privacy concern.

The research objectives in designing demand-side management solutions for energy transition are as follows:

- **(Obj 1)** Present a coalitional game theory based mathematical framework for energy coalitions that enforce principles of sharing economy and design market mechanisms that enable real-time and P2P energy trading;
- **(Obj 2)** Design consensus-based distributed algorithms that are scalable, fast, robust and privacy preserving by possibly exploiting the geometric structure of equilibrium solution.

1.1.4. Market-based analytics

The energy transition is characterised by the rise of new actors in the power sector with complex energy behavior like distributed energy resources, prosumers, electric vehicles,

1

and storage. All these energy actors bring high uncertainty to the energy network's operation and threaten the system's security [10]. However, at the same time, these energy actors also generate vast amount of data that can be utilized to mitigate the associated uncertainty by predicting their behavior. Such predictions can allow system operators to efficiently fulfill balancing and congestion management requirements, exploit flexibility on the demand side and help energy generators to make more profitable biding at the energy markets [95].

Often, the data and information is deemed proprietary which is collected and held by different owners. The data of one owner can be of an additional value for other owners as well as for data consumers. However, the data owned by firms or individuals are perceived to have a cost when exposed. For businesses, this cost can be in terms of competitive disadvantage, and for individuals, in terms of privacy loss. This cost of exposing data can be compensated by offering monetary incentive via market-based platforms, i.e., data markets. Various designs of data markets are proposed in the literature including the platforms that allow bilateral exchange of data, i.e., data in return for data [91], iterative auction mechanisms for the exclusive allocation of data [21] and more recently a regression market framework for the forecasting tasks modeled as regression problems by [82] and [45].

Designing data markets face two main challenges of data valuation and privacy preservation [1]. The value of data of a particular provider for a buyer, in a market setting, is in principle a combinatorial problem, computation requirements for which grow exponentially with increase in number of providers. Furthermore, for personal data providers the market setup causes depressed prices with increased participation or because of the presence of externality thus possibly making it non-attractive for the participants [1]. Similarly, for data owners the competition may come down to collecting more and more data for maximizing monetary rewards that consequently will diminish the value of individual contribution.

The issues of data valuation and privacy can be addressed, to some extent, by the so-called information markets [59]. In these markets, the participants post their predictions about a given uncertain event and are remunerated according to their prediction's quality. The authors in [53] and [56] present forecast elicitation platforms with formal mathematical guarantees on *desirable* economic properties. Inspired from these forecast elicitation platforms, in this thesis, we design a payoff mechanism for a wagering-based forecasting market with single buyer and multiple sellers. Furthermore, we also design an online mechanism to evaluate real-time payoffs for markets operating in highly dynamic environments. The research objectives in designing market-based anlytics platforms for aiding energy transition are as follows:

- **(Obj 3)** Present a framework of a market-based platform for probabilistic forecast elicitation with formal mathematical guarantees on *desirable* economic properties;
- **(Obj 4)** Design an online distributed payoff mechanism for near real-time energy forecasting markets possibly operating with streaming data.

Remark 1. *The mechanisms we design for energy coalitions, P2P paradigm, real-time markets and market-based analytics platforms are general and can be adopted for wide* *area of applications. We focus on energy applications to stay consistent with our motivation of accelerating energy transition.*

1.2. STRUCTURE OF THE THESIS

Figure 1.3 presents the outline of the thesis and the connections of the chapters.
The contents of each chapter and the related publications are summarized next.
In Chapter 2, we review some mathematical tools and theoretical concepts from coalitional game theory, operator theory and fixed point theory, frequently exploited throughout the thesis.
Appendix A collects the basic notation adopted throughout the thesis.

1.2.1. CHAPTER 3: PAYOFF DISTRIBUTION IN ROBUST COALITIONAL GAMES

In this chapter, we develop two distributed algorithms for payoff distribution in dynamic coalitional games where the actual coalitional values are unknown but vary within known bounds. The algorithms are presented as fixed-point iterations of time-varying operators, namely nonexpansive and paracontractions. Furthermore, we motivate our framework of “robust coalitional games" by an energy storage optimization application. This chapter partially answers key research question (Q1) and is based on the following publications:

- A. Raja and S. Grammatico, "Payoff Distribution in Robust Coalitional Games on Time-Varying Networks", IEEE Transactions on Control of Network Systems, vol. 9, no. 1, pp. 511-520, 2022.
- A. Raja and S. Grammatico, "On the approachability principle for distributed payoff allocation in coalitional games", 21st IFAC World Congress, vol. 53, no. 2, pp. 2690-2695, 2020.

1.2.2. CHAPTER 4: PEER-TO-PEER ENERGY TRADING VIA COALITIONAL GAMES

In this chapter, we model a peer-to-peer (P2P) electricity market that enables prosumers to bilaterally trade energy. For a solution of proposed P2P market, we designed a novel distributed negotiation algorithm that utilizes the geometric structure of the equilibrium solution to improve the convergence speed. The convergence analysis of this algorithm is fundamentally based on the theory developed in Chapter 3. The mechanism guarantees *stable* trading prices and satisfies the desired economic properties. This chapter is based on the following publication:

- A. Raja and S. Grammatico, "Bilateral Peer-to-Peer Energy Trading via Coalitional Games", IEEE Transactions on Industrial Informatics, 2022, *(Early Access)*.

1.2.3. CHAPTER 5: PEER-TO-PEER ELECTRICITY MARKET FOR RESIDENTIAL PROSUMERS

This chapter presents a similar market model for P2P energy trading as in Chapter 4 but characterises the equilibrium point with a different notion of *fairness*. Specifically,

1

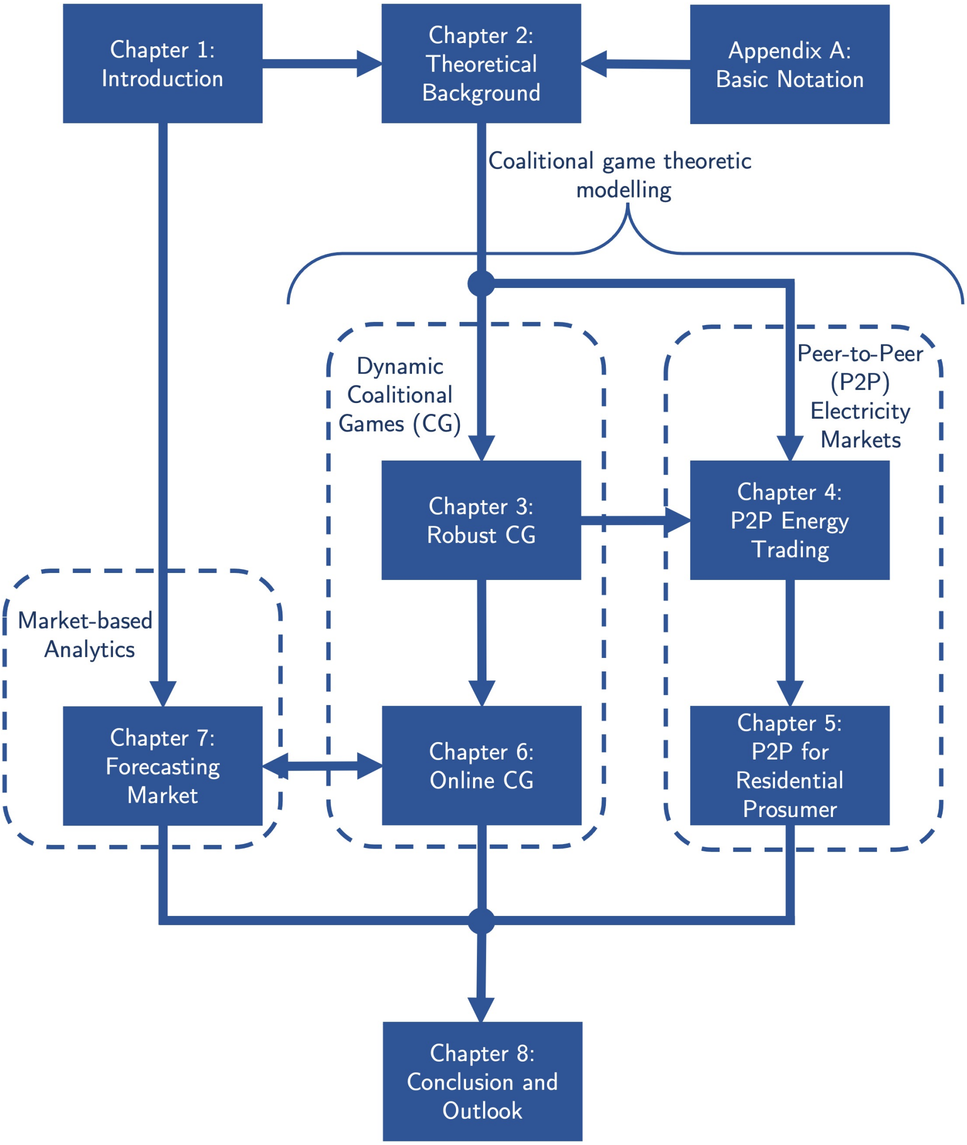


Figure 1.3: Outline of the thesis. Arrows indicate *read-before* relations.

the bilateral negotiation mechanism enables participants to converge to the τ-value, a solution that fairly divides the resulting market welfare among buyers and sellers. This chapter is based on the following publication:

- A. Raja and S. Grammatico, "A fair Peer-to-Peer Electricity Market model for Residential Prosumers", 2021 IEEE PES Innovative Smart Grid Technologies Europe

(ISGT Europe), 2021.

1.2.4. CHAPTER 6: ONLINE COALITIONAL GAMES FOR REAL-TIME PAYOFF DISTRIBUTION

In this chapter, we relax the assumption on dynamic coalitional values made in Chapter 3, i.e., time variation within known bounds. This allows us to introduce a framework for online coalitional games. We then propose two online distributed algorithms that track coalitional game theoretic solutions for a real-time payoff distribution. The convergence is shown to be in the neighbourhood of the time-varying solutions. This chapter is based on the following paper:

- A. Raja and S. Grammatico, "Online coalitional games for real-time payoff distribution with applications to energy markets", (*under review for journal publication*).

1.2.5. CHAPTER 7: A MARKET FOR TRADING FORECASTS

In this chapter, we propose a market-based analytics platform with a single buyer (client) and multiple sellers. The market design is based on a wagering mechanism that enables a client to improve their prediction via implicit pooling of information by the sellers. For a solution, we propose a payoff mechanism and prove that it satisfies several desirable economic properties. This chapter is based on the following paper:

- A. Raja, P. Pinson, J. Kazempour, and S. Grammatico, "A Market for Trading Forecasts: A Wagering Mechanism", arXiv:2205.02668 [econ.TH], (*under review for journal publication*).

1.2.6. CHAPTER 8: CONCLUSION AND OUTLOOK

In this chapter, we draw some concluding remarks regarding the results obtained and presented in the previous chapters. We also discuss the achievement of this thesis on addressing the the key research objectives. Finally, we suggest some open questions for future research.

2

THEORETICAL AND MATHEMATICAL BACKGROUND

In this chapter, we review the mathematical framework of coalitional game theory and results from operator theory that are frequently exploited throughout the thesis. We refer to [11] for exhaustive collection of operator-theoretic results and definitions.

2

2.1. COALITIONAL GAME THEORY

COALITIONAL game theory provides an analytical framework and mathematical formalism, to study the behavior of selfish and rational agents, when they are willing to cooperate. In this thesis, we focus on a particular class of coalitional games, namely transferable utility (TU) coalitional game, which consists of a set of agents indexed by $\mathcal{I} = \{1,\ldots,N\}$ who cooperate to receive a higher individual return compared to that due to non-cooperative actions. The utility generated by this cooperation is defined by a value function v [69]. The TU property implies that the total utility evaluated by a value function v can be divided among the coalition members, according to a defined criteria. Formally,

Definition 1 (Coalitional game). *Let $\mathcal{I} = \{1,\ldots,N\}$ be a set of agents. A coalitional game is a pair $\mathcal{G} = (\mathcal{I}, v)$ where $v : 2^{\mathcal{I}} \to \mathbb{R}$ is a value function that assigns a real value, $v(S)$, to each coalition $S \subseteq \mathcal{I}$. $v(\mathcal{I})$ is the value of so-called grand coalition. By convention, $v(\emptyset) = 0$.* □

For any coalitional game it is desired that the formation of large coalition is never detrimental to any of the involved agents, i.e., no group of agents can do worse by cooperating instead of acting non-cooperatively. This pertains to the mathematical property of superadditivity.

Definition 2 (Superadditivity). *A coalitional game $\mathcal{G} = (\mathcal{I}, v)$ is superadditive if for all $S_1, S_2 \subset \mathcal{I}$ it holds that*

$$v(S_1 \cup S_2) \geq v(S_1) + v(S_2) \qquad s.t. \quad S_1 \cap S_2 = \emptyset.$$

□

Thus, in a superadditive game, cooperation is always beneficial which makes the formation of a grand coalition $\mathcal{I}$ a rational choice. Next, the value generated by a coalition S, i.e., $v(S)$, is distributed among the members of S as a payoff.

Definition 3 (Payoff vector). *Let $(\mathcal{I}, v)$ be a coalitional game and $S \subseteq \mathcal{I}$ be a coalition. For each $i \in S$, the element x_i of a payoff vector $\boldsymbol{x} \in \mathbb{R}^{|S|}$ represents the share of agent i of the value $v(S)$.* □

The two important characteristics of a payoff vector are *rationality* and *efficiency*. For a game with a grand coalition $\mathcal{I}$, a payoff vector $\boldsymbol{x} \in \mathbb{R}^N$ is said to be efficient if $\sum_{i \in \mathcal{I}} x_i = v(\mathcal{I})$. In words, all of the value generated by grand coalition will be distributed among the agents. Second, a payoff vector is rational if for every possible coalition $S \subseteq \mathcal{I}$ we have $\sum_{i \in S} x_i \geq v(S)$. Note that this should also hold for singleton coalitions $S = \{i\}$ i.e. $x_i \geq v(i), \forall i \in \mathcal{I}$. It means that, payoff allocated to each agent should be at least equal to what they can get individually or by forming any coalition S other than $\mathcal{I}$.

We assume that each agent $i \in \mathcal{I}$ acts rationally and efficiently in the game. Mathematically, this means that the payoff vector, given in Definition 3, proposed by each agent must belong to its *bounding set*, as formalized next.

Definition 4 (Bounding set)**.** *For a coalitional game* $\mathcal{G} = (\mathcal{I}, v)$*, the set*

$$\mathcal{X}_i := \Big\{ x \in \mathbb{R}^N \mid \quad \textstyle\sum_{j\in\mathcal{I}} x_j = v(\mathcal{I}), \\ \textstyle\sum_{j\in S} x_j \geq v(S), \forall S \subset \mathcal{I} \text{ s.t. } i \in S \Big\} \tag{2.1}$$

denotes the bounding set of an agent $i \in S$. □

Since a rational agent i agrees on a payoff vector in its bounding set $\mathcal{X}_i$ thus, a mutually agreed payoff shall belong to the intersection of the bounding sets of all the agents. Interestingly, this intersection corresponds to the core, the solution concept that relates to the disinterest of agents, or a sub-coalition of agents, in defecting a grand coalition, i.e., stability of a grand coalition [92].

Definition 5. *(Core): The core* $\mathcal{C}$ *of a coalitional game* $(\mathcal{I}, v)$ *is the following set of payoff vectors:*

$$\mathcal{C} := \{x \in \mathbb{R}^N \mid \sum_{i\in\mathcal{I}} x_i = v(\mathcal{I}), \sum_{i\in S} x_i \geq v(S), \forall S \subseteq \mathcal{I}\}, \tag{2.2}$$

where, the term $\sum_{i\in\mathcal{I}} x_i = v(\mathcal{I})$ *ensures the efficiency and* $\sum_{i\in S} x_i \geq v(S)$ *shows the rationality of a payoff.* □

We note that by using the bounding sets, as in Definition 4, we can write the core as $\mathcal{C} = \bigcap_{i=1}^{N} \mathcal{X}_i$.

The core set ensures stability of the grand coalition however, it does not comply with the notion of *fairness*. In fact, different core allocations can treat agents differently with the possibility of some agents being at advantageous position over the others. Here, the idea of fairness is characterized by axioms presented by Shapley in [97] and the unique payoff allocation that satisfies these *fairness* axioms is known as the Shapley value. Note that the Shapley value does not necessarily belong to the core set.

Definition 6 (Shapley value)**.** *For a coalitional game* $\mathcal{G} = (\mathcal{I}, v)$*, let* Π *be the set of all* $(N!)$ *permutations of the grand coalition* $\mathcal{I}$ *and, for an ordering of agents* $\sigma \in \Pi$*, let* $\mathcal{P}_i^\sigma$ *be the set of predecessors of* i *in* σ *with* $\mathcal{P}_1^\sigma = \emptyset$*. Then, for every agent* $i \in \mathcal{I}$ *the Shapley value* $\boldsymbol{\phi}(v)$ *assigns the payoff* $\phi_i(v)$ *given by:*

$$\phi_i(v) = \frac{1}{N!} \sum_{\sigma\in\Pi} (v(\mathcal{P}_i^\sigma \cup \{i\}) - v(\mathcal{P}_i^\sigma)). \tag{2.3}$$

□

Here, we refer to the term $(v^k(\mathcal{P}_i^\sigma \cup \{i\}) - v^k(\mathcal{P}_i^\sigma))$ in (6.1) as incremental marginal contribution which shows the value added by an agent i when it joins the coalition. In words, the Shapley value assigned to an agent i is its incremental marginal contribution averaged over all permutations.

All the definitions above are provided for the static form of the coalitional games, i.e., the value function does not depend on time. However, in this thesis we also address situation with time-varying value functions and consequently the time-varying cores. In dynamic context, a coalition generates utility at each time $k \in \mathbb{N}$ as defined by a value function v^k.

Definition 7 (Coalitional game [73, Sec. II-A])**.** *Let $\mathcal{I} = \{1,\dots,N\}$ be a set of agents. For each time $k \in \mathbb{N}$, an instantaneous coalitional game is a pair $\mathcal{G}^k = (\mathcal{I}, v^k)$ where $v^k : 2^{\mathcal{I}} \to \mathbb{R}$ is a value function that assigns a real value, $v^k(S)$, to each coalition $S \subseteq \mathcal{I}$. A dynamic coalitional game is a sequence of instantaneous games, i.e., $\mathcal{G} = (\mathcal{I}, (v^k)_{k\in\mathbb{N}})$.* □

For an instantaneous game, an instantaneous value of a coalition S has to be distributed as a payoff $\boldsymbol{x}^k \in \mathbb{R}^{|S|}$ and a rational and efficient payoff of an agent i belongs to its bounding set [73, Sec. II-B].

$$\mathcal{X}_i^k := \Big\{ x \in \mathbb{R}^N \mid \; \textstyle\sum_{j\in\mathcal{I}} x_j = v^k(\mathcal{I}), \quad \textstyle\sum_{j\in S} x_j \geq v^k(S), \forall S \subset \mathcal{I} \text{ s.t. } i \in S \Big\} \tag{2.4}$$

Similarly, the core $\mathcal{C}$ of an instantaneous coalitional game $\mathcal{G}^k = (\mathcal{I}, v^k)$, $k \in \mathbb{N}$, is the following set of payoff vectors:

$$\begin{aligned} \mathcal{C}(v^k) \;&:= \Big\{ x \in \mathbb{R}^N \mid \textstyle\sum_{i\in\mathcal{I}} x_i = v^k(\mathcal{I}), \\ &\qquad \textstyle\sum_{i\in S} x_i \geq v^k(S), \forall S \subseteq \mathcal{I} \Big\}, \\ &= \textstyle\bigcap_{i=1}^N \mathcal{X}_i^k, \end{aligned} \tag{2.5}$$

2.2. OPERATOR THEORY

In this section, we briefly introduce the definitions of the mathematical tools from operator theory and the framework in which we utilize those tools. These definitions are central to the convergence analysis of our proposed algorithms.

2.2.1. OPERATORS

Let the two non-empty sets be $\mathcal{A} \subseteq \mathbb{R}^n$ and $\mathcal{B} \subseteq \mathbb{R}^m$ then the notation $T : \mathcal{A} \rightrightarrows \mathcal{B}$ shows that the operator T maps every $x \in \mathcal{A}$ to a set $T(x) \subset \mathcal{B}$. If $T(x)$ is singleton then we can represent the mapping from $\mathcal{A}$ to $\mathcal{B}$ as $T : \mathcal{A} \to \mathcal{B}$. Few notations that we will use to characterize operator T are,

- domain as $\mathrm{dom}(T) := \{x \in \mathcal{A} \mid T(x) \neq \emptyset\}$;
- range as $\mathrm{range}(T) := T(\mathcal{A})$;
- fixed-points set as $\mathrm{fix}\,(T) := \{x \in \mathcal{A} \mid x \in T(x)\}$.

Examples: Next, we provide some examples of operators that we use frequently in this thesis.

- The identity mapping Id is such that $\mathrm{Id}(x) = x$;
- For a closed convex set $C \subseteq \mathbb{R}^n$, the mapping $\mathrm{proj}_C : \mathbb{R}^n \to C$ denotes the projection onto C, i.e., $\mathrm{proj}_C(x) := \mathrm{arg\,min}_{y\in C} \|y - x\|$;
- An over-projection operator is denoted by $\mathrm{overproj}_C := 2\mathrm{proj}_C - \mathrm{Id}$.

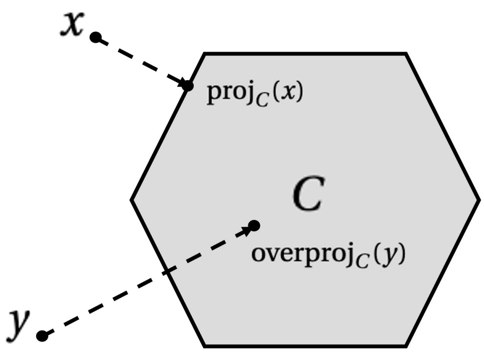


Figure 2.1: Illustration of projection and overprojection of x and y on set C.

2.2.2. OPERATOR PROPERTIES

Let us introduce some properties of the operators that we use in the sequel to prove the convergence of our proposed algorithms. For a detailed mathematical exposition on these properties, we refer the reader to [11].
A mapping $T:\mathbb{R}^n \to \mathbb{R}^n$ is

1. Lipschitz continuous if there exists a scalar $\ell > 0$ such that
$$\|T(x)-T(y)\| \leq \ell\|x-y\|, \quad \forall x,y \in \mathbb{R}^n.$$
2. Nonexpansive if it is 1-Lipschitz continuous, i.e.,
$$\|T(x)-T(y)\| \leq \|x-y\|, \quad \forall x,y \in \mathbb{R}^n.$$
3. Averaged if there exists a constant $\alpha \in (0,1)$ such that
$$\|T(x)-T(y)\|^2 \leq \|x-y\|^2 - \frac{1-\alpha}{\alpha}\|(\mathrm{Id}-T)(x)-(\mathrm{Id}-T)(y)\|^2, \quad \forall x,y \in \mathbb{R}^n.$$
4. Contractive if it is ℓ-Lipschitz continuous with $\ell \in (0,1)$, hence
$$\|T(x)-T(y)\| < \|x-y\|, \quad \forall x,y \in \mathbb{R}^n.$$
5. Paracontraction if
$$\|T(x)-y\| < \|x-y\|, \qquad \forall x,y \in \mathbb{R}^n$$
such that $x \notin \mathrm{fix}(T), y \in \mathrm{fix}(T)$.

2.3. FIXED POINT THEORY

A variety of optimization and consensus tracking problems can be formulated as that of finding fixed points of operators with certain properties. Mathematically, for a mapping $T:\mathbb{R}^n \rightrightarrows \mathbb{R}^n$ the set of fixed points consists of states $x^* \in \mathbb{R}^n$ that satisfy $x^* \in T(x^*)$. We denote this set by $\mathrm{fix}(T)$. Next we present some fundamental results on existence (Browder's fixed-point theorem) and uniqueness of the fixed points of operators.

Theorem 1 (Existence by Browder)**.** *Let $C \subset \mathbb{R}^n$ be a non-empty, bounded, closed and convex set. Let $T : C \to C$ be a nonexpansive operator. Then,* $\mathrm{fix}(T) \neq \emptyset$. □

Theorem 2 (Uniqueness by Banach)**.** *Let $T : \mathbb{R}^n \to \mathbb{R}^n$ be a contraction operator. Then, the fixed point of T is unique, i.e.,* $\mathrm{T} = \{x^*\}$. □

We note that the nonexpansive and paracontraction operators can have several fixed points, e.g. a nonexpansive mapping Id, which is not a contraction, has infinitely many fixed points as $\mathrm{Id}(x) = x$, for all $x \in \mathbb{R}^n$. Similarly, for a paraontraction operator proj_C, i.e., projection on a set C the set of fixed points is C, denoted by $\mathrm{fix}(\mathrm{proj}_C) = C$.

To find the fixed-points of operators with certain properties (e.g contractive) plethora of iterative algorithms have been proposed in the literature. Generally, the basic structure of such algorithms is composed of repeated application of an operator T to generate a sequence $(x^k)_{(k\in\mathbb{N})}$ such that $x^k \to x^* \in \mathrm{fix}(T)$. A fixed-point algorithm in its most basic form is the so-called Banach-Picard iteration which, for all $k \in \mathbb{N}$, is given as

$$x^{k+1} = T(x^k). \tag{2.6}$$

The iteration in (2.6) converges to a fixed-point of an averaged mapping T [11, Prop. 5.16]. However, Banach-Picard iteration may fail to reach a a fixed-point for an operator that is merely nonexpansive. In this case, we can use a relaxation of iteration in (2.6) known as Krasnosel'skii–Mann fixed-point iteration, stated as

$$x^{k+1} = x^k + \lambda^k (T(x^k) - x^k) \tag{2.7}$$

where $(\lambda^k)_{k\in\mathbb{N}}$ is a sequence in $[0,1]$ such that $\sum_{k\in\mathbb{N}} \lambda^k \left(1-\lambda^k\right) = +\infty$.
In this sequel, we formulate our proposed algorithms as fixed-point iterations and use operator-theoretic tools, presented above, to prove their convergence.

2.4. DISTRIBUTED FORMULATION

In this thesis, we mainly design algorithms for learning a solution of a coalitional game, i.e., a payoff for each agent $i \in \mathcal{I}$ as in Definition 3. We approach this multi-agent scenario in a distributed paradigm. Thus, our proposed algorithms are generally formulated as iterative procedures in which, at each step, an agent i estimates a payoff distribution $\boldsymbol{x}_i \in \mathbb{R}^N$ for all agents by applying an operator on the proposals of *neighboring* agents. To cast this problem as a fixed-point iteration we use the so-called stacked vector notation which we elaborate next. An elementary form of fixed-point iteration for an agent i is given as

$$\boldsymbol{x}_i^{k+1} = T_i(\boldsymbol{x}^k)$$

where $\boldsymbol{x} \in \mathbb{R}^{N^2}$ is a stacked vector of the form

$$\boldsymbol{x} = \begin{bmatrix} \boldsymbol{x}_1 \\ \boldsymbol{x}_2 \\ \vdots \\ \boldsymbol{x}_N \end{bmatrix}.$$

Similarly for the mapping $T : \mathbb{R}^{N^2} \to \mathbb{R}^{N^2}$ where

$$T(\boldsymbol{x}) = \begin{bmatrix} T_1(\boldsymbol{x}_1) \\ T_2(\boldsymbol{x}_2) \\ \vdots \\ T_N(\boldsymbol{x}_N) \end{bmatrix}$$

the iteration in collective compact form becomes $\boldsymbol{x}^+ = T(\boldsymbol{x})$. Finally, in the distributed setting payoffs estimated by agents independently, i.e., $(\boldsymbol{x}_i)_{i\in\mathcal{I}}$ must eventually reach consensus.

Definition 8 (Consensus set)**.** *The consensus set* $\mathcal{A} \subset \mathbb{R}^{N^2}$ *is defined as:*

$$\mathcal{A} := \{\mathrm{col}(\boldsymbol{x}_1, \ldots, \boldsymbol{x}_N) \in \mathbb{R}^{N^2} \mid \boldsymbol{x}_i = \boldsymbol{x}_j, \forall i, j \in \mathcal{I}\}. \tag{2.8}$$

□

3

PAYOFF DISTRIBUTION IN ROBUST COALITIONAL GAMES

In this chapter, we consider a sequence of transferable utility (TU) coalitional games where the actual coalitional values are unknown but vary within known bounds. As a solution to the resulting family of games, we formalize the notion of “robust core". Our main contribution is to design two distributed algorithms, namely, distributed payoff allocation and distributed bargaining, that converge to a consensual payoff distribution in the robust core. We adopt an operator-theoretic perspective to show convergence of both algorithms executed on time-varying communication networks. An energy storage optimization application motivates our framework for “robust coalitional games".

3.1. INTRODUCTION

Coalitional game theory provides a framework to study the behavior of selfish and rational agents when they cooperate effectively. This willingness to cooperate arise from the aspiration of gaining a higher return, compared to that for behaving as individuals [69].

Specifically, a transferable utility (TU) coalitional game consists of a set of agents and a value/characteristic function that provides the *value* of each of the possible coalitions [69]. Multi-agent decision problems modelled by TU coalitional games arise in many application areas, such as demand-side energy management [43] and energy storage sharing [22], in various areas of communication networks [92] and as the foundation of coalitional control [30].

One key problem studied by coalitional game theory is the distribution of the value generated by cooperation. Along this research direction, several solution concepts have been proposed with special attention to criteria like stability and fairness. In payoff distribution, stability means that none of the agents has an incentive to defect the coalition. Perhaps the most studied solution concepts in coalitional games that ensures the *stability* of a payoff is the core. The second criterion, i.e., *fairness* means that the payoff for an agent should reflect its contribution to or impact in the game. A seminal work on the axiomatic characterization of fairness is that of Shapley [97], where the unique value satisfying the fairness axioms is in fact known as the Shapley value which depends on the marginal contribution of each agent. The later depicts the impact each agent has on the collective value of the coalition. Other related solution concepts are also proposed in the literature, e.g. the nucleolus and the kernel [61].

We consider the problem of finding a payoff distribution that encourages cooperation, i.e., belongs to the core [85]. Now, to evaluate such a payoff, the value of each possible coalition is required, which seems implausible in many practical applications, mainly because an agent cannot be certain about the values that collaborations may generate. However, one can assume that an agent does hold a belief about the value of some possible collaborations via informed estimation or mere experience. In practice, this brings uncertainty to the coalitional values and, consequently, to the core set. It follows that one should consider solutions that are robust to uncertainty on the coalitional values and we do that via the notion of *robust* core.

The robustness aspect in coalitional games falls into the framework of dynamic TU coalitional games, which has been studied in the literature. Among others, the authors in [31] analyzed the time consistency of the Shapley value and the core under the temporal evolution of the game. Then, the authors in [57] characterized three versions of core allocations for a dynamic game where the worth of the coalitions varies over time according to the previous allocations. In both papers, the coalitional values at the current time are determined endogenously and depend on previous events. In [55], the authors consider a finite sequence of exogenously defined coalitional games, where the agents receive a payoff at each stage of the sequence and consequently, the final utility of an agent depends on the whole stream of payoffs.

Robust coalitional games are the subclass of dynamic TU coalitional games where the coalitional values are unknown and exogenous. In [15], Bauso and Timmer characterized robust allocation rules for the dynamic coalitional game where the average value of each coalition is known with certainty, while at each instant, the coalitional value fluctuates within a bounded polyhedron. The static version of their setup, called cooperative interval games, is presented by the authors in [6], where the coalitional values are considered yet to be uncertain within some bounded intervals. In their setup, they have introduced the interval solutions, which assign a closed real interval as a payoff to each agent instead of a single real value. In [73], Nedich and Bauso have presented a distributed bargaining algorithm for finding a solution in the core under the framework of robust games and dynamic average games. Inspired by the motivation of cooperative interval games and the setup in [73], we consider the value generated by each coalition to vary within certain bounds.

3.1.1. MOTIVATIONAL EXAMPLE

Let us consider the energy optimization application inspired by [43] which justifies a dynamic robust coalitional game model. Consider a group of N prosumers, each of whom owns a renewable energy source (RES) and energy storage (ES). Together they form an energy coalition $\mathcal{I}$ where the participating agents operate their ES systems collectively to minimize their total energy cost. When the energy coalition has an excess of energy, they can store it in an ES for later utilization and any additional energy can be sold to a retailer. The retailer buys energy and remunerates, a few hours ahead of the delivery time. The coalition considers the corresponding remuneration for optimizing their ES operation and consequently minimizing the associated cost function.

Now, the cost saving as a result of the collaborative operation should be distributed in such a way that each prosumer is satisfied by its share, and hence the coalition remains intact. To achieve this, the agents assert their position by presenting the estimated cost saving of possible energy sub-coalitions, $S \subseteq \mathcal{I}$, which they could have been part of and use them to define acceptable payoffs, namely payoffs in the core. Since there is uncertainty in the RES generation, the cost savings, $v(S)$, of each sub-coalition $S \subseteq \mathcal{I}$ is uncertain. How the agents share this saving under such uncertainty is a part of the solution generated by an iterative payoff distribution methods.

Let b_i^t represent the charge or discharge of energy by the ES of prosumer i at time t. Further, denote the net energy demand of prosumer i by q_i^t and let p_{s}^t and p_{b}^t be an electricity sell price and buy price at time t, respectively. Let $\mathrm{proj}_{\geq 0}(x)(\mathrm{proj}_{\leq 0}(x))$ denote the projection onto the non-negative (non-positive) orthant. Then, the energy cost function of any energy sub-coalition $S \subseteq \mathcal{I}$ for a time period of length K is given as:

$$F_S(\boldsymbol{b}) := \sum_{t=1}^K \Big\{ p_{\mathrm{b}}^t \Big(\sum_{i \in S} \mathrm{proj}_{\geq 0}(q_i^t + b_i^t) \Big) + p_{\mathrm{s}}^t \Big(\sum_{i \in S} \mathrm{proj}_{\leq 0}(q_i^t + b_i^t) \Big) \Big\},$$

where $\boldsymbol{b} \in \mathbb{R}^{NK}$ contains the ES charge and discharge profiles of all the N agents over the K time steps.
For a given coalition S, the coalitional energy cost for the time period of length K is defined as:

$$c(S) := \min_{\mathbf{b}} F_S(\mathbf{b}), \tag{3.1}$$

and the cost saving during this period, $v(S)$, as the difference between the sum of the costs of the coalitions of the individual agents in S and the cost of the coalition itself, namely,

$$v(S) := \sum_{i \in S}\{c_i\} - c(S). \tag{3.2}$$

Note that, the cost $c(S)$ is unknown but bounded, from above when each agent $i \in S$ has RES generation equal to the installed capacity, which gives minimum value of net energy consumption $q_i^{\min}$ for the whole period K, and from below when there is no generation, hence $q_i^{\max}$. Due to these bounds, the cost saving $v(S)$ is also bounded. Let $\underline{c}(S)$ be the coalitional cost corresponding to $q_i^{\max}$, and let $\overline{c}_i$ be the individual cost corresponding to $q_i^{\min}$, $i \in S$ then $v(S) \leq \sum_{i \in S}\{\overline{c}_i\} - \underline{c}(S)$. Here, the uniform bound on the coalitional values and the fixed value of grand coalition, for a period of length K, allows us to consider the setup of robust games presented in [73]. We refer to [22], [9] for other engineering problems that can be modeled as robust coalitional games.

3.1.2. CONTRIBUTION

We propose two payoff distribution algorithms within the framework of robust coalitional games:

- We formalize the notion of *robust* core, a set of payoffs that stabilizes a grand coalition under variations in the coalitional values (Section 3.2);
- We develop a distributed payoff allocation algorithm where, given the bounds on the coalitional values, agents communicate and negotiate only locally, i.e., with their neighbors, over a time-varying and repeatedly-connected communication network. We show that the proposed algorithm converges to a common payoff allocation in the *robust* core (Section 6.3);
- We generalize the distributed bargaining protocol in [73] and prove its convergence to a mutually agreed payoff in the *robust* core. We assume similar communication requirements for the bargaining protocol as for the allocation process; but less information on the coalitional bounds is available to the agents (Section 3.4);
- We introduce some tools from operator theory (*paracontraction, nonexpansive* operators and Krasnoselskii-Mann fixed-point iterations) to the domain of coalitional games which allows us to generalize existing results and in turn to propose faster algorithms. This approach represents a new general analysis framework for coalitional games.

We refer the reader to Appendix A for basic notation and Chapter 2, Section 2.2.2 for operator-theoretic definitions.

3.2. BACKGROUND ON ROBUST COALITIONAL GAMES

In this chapter, we consider a similar class of dynamic coalitional games as in [73], where an instantaneous value of each coalition $v^k(S)$ belongs to a finite set bounded by a minimum and a maximum value, i.e., $\underline{v}(S) \leq v^k(S) \leq \overline{v}(S)$. This restriction of values on v^k gives rise to a family of games which we collectively regard as a robust coalitional game.

Definition 9 (Robust coalitional game)**.** *Let $\mathcal{I} = \{1,\ldots,N\}$ index a set of agents. A robust coalitional game $\mathcal{R} = (\mathcal{I},\mathcal{V})$, is a set of instantaneous coalitional games $(\mathcal{I}, v^k)$ with $v^k \in$*

$\mathcal{V} := \{u_1, u_2, \ldots, u_n\}$ with $|\mathcal{V}| < \infty$, for all $k \in \mathbb{N}$, where each u_l is a value function such that $\underline{u}(S) \le u_l(S) \le \overline{u}(S)$ for all $S \subset \mathcal{I}$ and $u_l(\mathcal{I}) = \overline{u}(\mathcal{I})$. □

In words, a robust coalitional game $(\mathcal{I}, \mathcal{V})$ is a family of a finite number of instantaneous coalitional games such that the value of the grand coalition $\mathcal{I}$ is fixed. This setup adequately addresses the practical scenario of negotiations where after the formation of the grand coalition its value becomes certain. However, to compute a core payoff in (2.5) anticipated values of sub-coalitions are also required, which involves uncertainty. We note that our formulation of robust coalitional game is called “robust game" in [73]. Next, we formalize the core of a robust coalitional game as the robust core.

Definition 10 (Robust core)**.** *For a robust coalitional game $\mathcal{R} = (\mathcal{I}, \mathcal{V})$, the robust core is the intersection of all the possible instantaneous core sets, i.e.,*

$$\mathcal{C}_0 := \bigcap\nolimits_{v \in \mathcal{V}} \mathcal{C}(v). \tag{3.3}$$

□

Remark 2. *Let $\mathcal{R} = (\mathcal{I}, \mathcal{V})$ be a robust coalitional game. If there exists $\overline{v} \in \mathcal{V}$ such that for all $k \in \mathbb{N}$, $v^k(\mathcal{I}) = \overline{v}(\mathcal{I})$ and $v^k(S) \le \overline{v}(S)$ for any coalition $S \subset \mathcal{I}$, then, $\mathcal{C}_0 = \mathcal{C}(\overline{v})$ and thus $\mathcal{C}_0 \subseteq \mathcal{C}(v^k)$ for all $v^k \in \mathcal{V}$. Consequently, if $\mathcal{C}_0 \neq \varnothing$ then $\mathcal{C}(v^k) \neq \varnothing$ for all $k \in \mathbb{N}$.* □

In the sequel, we deal with the grand coalition only, therefore, we use the core as the solution concept. We note from (2.5) that the core $\mathcal{C}(v^k)$ is closed and convex. Furthermore, the robust core $\mathcal{C}_0$ in (3.3) is assumed to be nonempty throughout the chapter. Nonemptiness implies that even under the variations in coalitional values, a mutually agreeable payoff exists.

Assumption 1. *The robust core is non-empty, $\mathcal{C}_0 \neq \varnothing$.* □

Next, we discuss a possible strategy for finding a payoff that belongs to core, $\mathcal{C}_0$ in (3.3) of a robust game. Since centralized methods for finding a payoff $\boldsymbol{x} \in \mathcal{C}_0$ do not capture realistic scenarios of interaction among autonomous selfish agents, we propose distributed methods that allow agents to autonomously reach a common agreement on a payoff distribution. We note that our analysis and methods can be adapted for games with empty cores by defining a quasi-core (ε-core) set [93].

Remark 3. *Given $\varepsilon > 0$, the ε-core is the set $\mathcal{C}_\varepsilon(v) := \{x \in \mathbb{R}^N \mid \sum_{i \in \mathcal{I}} x_i = v(\mathcal{I}), \sum_{i \in S} x_i \ge v(S) - \varepsilon, \forall S \subseteq \mathcal{I}\}$. Thus, the core in (2.5) is an $\varepsilon-$core with $\varepsilon = 0$. The ε-core models the cost of coalition formation where ε is a minimum threshold for a gain in coalitional value, below which it is not rational for a coalition $S \subset \mathcal{I}$ to defect the grand coalition.* □

The two payoff distribution methods which we focus on are distributed payoff allocation and distributed bargaining. The former is an iterative procedure in which, at each step, an agent i proposes a payoff distribution $\boldsymbol{x}_i \in \mathbb{R}^N$ by averaging the proposals of neighboring agents and by introducing an innovation factor. This procedure aspires to reach a mutually agreed payoff among agents. In a bargaining process, to propose a payoff distribution $\boldsymbol{x}_i \in \mathbb{R}^N$, an agent i, after averaging the proposals of all agents, makes it compliant to its own interest. Bargaining procedure also aspires to reach a mutually agreed payoff. Thus, in both methods, the proposed payoff distributions $(\boldsymbol{x}_i)_{i\in\mathcal{I}}$ must eventually reach consensus as in (2.8).

In the sequel, we consider the problem of iteratively computing a mutually agreed, payoff vector in the core, i.e., $\boldsymbol{x}^k \to \bar{\boldsymbol{x}} \in \mathcal{A} \cap \mathcal{C}^N$. We address this problem via distributed algorithms under the payoff allocation and bargaining frameworks. Both algorithms, starting from any initial payoff proposal $\boldsymbol{x}^0$, converge to some consensual payoff in the robust core in (3.3).

3

3.3. DISTRIBUTED PAYOFF ALLOCATION

In coalitional games, the agents cooperate because they foresee a higher individual payoff compared to non-cooperative actions. A payoff that can sustain such cooperation, referred as a stable payoff, shall satisfy the criteria in (2.5). Thus, the goal of a payoff allocation process is to let the agents achieve a consensus on a stable payoff in a distributed manner. During the allocation process, each agent proposes a payoff for all the involved agents based on the previous proposals of his neighbors and an innovation term.

In this section, we propose a payoff allocation in the context of robust coalitional games, where the value function v, at each iteration k, takes a value within the given bounds. We model our setup in a distributed paradigm, where each agent estimates the coalitional values independently, hence different agents can assign different values to the same coalition. In context of a robust coalitional game $(\mathcal{I},\mathcal{V})$, this distributed evaluation of the coalitional values implies that at each negotiation step, an agent can choose any value function v from a family $\mathcal{V}$, without central coordination. However, the determination of the family $\mathcal{V}$ is application-specific and generally involves a central evaluation before the distributed negotiation process. We prove that even under the distributed evaluation of the value function and variation of the coalitional values, the proposed algorithm converges to a stable payoff distribution. In particular, our goal is to construct a distributed fixed-point algorithm, using which the agents can reach consensus (2.8) on a payoff distribution in the robust core (3.3).

3.3.1. DISTRIBUTED PAYOFF ALLOCATION ALGORITHM

Consider a set of agents $\mathcal{I} = \{1,\dots,N\}$ who synchronously propose a distribution of utility at each discrete time step $k \in \mathbb{N}$. Specifically, each agent $i \in \mathcal{I}$ proposes a payoff distribution $\boldsymbol{x}_i^k \in \mathbb{R}^N$, where the jth element denotes the share of agent j proposed by agent i at iteration $k \in \mathbb{N}$.

Let the agents communicate over a time-varying network represented by a graph $G^k = (\mathcal{I},\mathcal{E}^k)$, where $(j,i) \in \mathcal{E}^k$ means that there is an active link between the agents i and j at iteration k and they are then referred as neighbours. Therefore, the set of neighbors of agent i at iteration k is defined as $\mathcal{N}_i^k := \{j \in \mathcal{I} | (i,j) \in \mathcal{E}^k\}$. We assume that at each iteration k an agent i observes only the proposals of its neighbouring agents. Furthermore, we assume that the union of the communication graphs over a time period of length Q is connected. The following assumption is typical for many works in multi-agent coordination, e.g. [70, Assumption 3.2].

Assumption 2 (Q–connected graph)**.** *For all $k \in \mathbb{N}$, the union graph $(\mathcal{I}, \cup_{l=1}^{Q} \mathcal{E}^{l+k})$ is strongly connected for some integer $Q \geq 1$.* □

The edges in the communication graph G^k are weighted using an adjacency matrix $W^k = [w^k_{i,j}]$, whose element $w^k_{i,j}$ represents the weight assigned by agent i to the payoff distribution proposed by agent j, $\boldsymbol{x}^k_j$. Note that, for some j, $w^k_{i,j} = 0$ implies that $j \notin \mathcal{N}^k_i$ hence, the state of agent i is independent from that of agent j. We assume the adjacency matrix to be doubly stochastic with positive diagonal, as assumed in [70, Assumption 3.3], [71, Assumption 2, 3].

Assumption 3 (Stochastic adjacency matrix)**.** *For all $k \geq 0$, the adjacency matrix $W^k = [w^k_{i,j}]$ of the communication graph G^k satisfies following conditions:*

1. *It is doubly stochastic, i.e., $\sum_{j=1}^N w_{i,j} = \sum_{i=1}^N w_{i,j} = 1$;*
2. *its diagonal elements are strictly positive, i.e., $w^k_{i,i} > 0, \forall i \in \mathcal{I}$;*
3. *$\exists\, \gamma > 0$ such that $w^k_{i,j} \geq \gamma$ whenever $w^k_{i,j} > 0$.* □

Assumptions 9 and 17 ensure that the agents communicate sufficiently often to each other and have sufficient influence on the resulting allocation. We further assume that the elements of communication matrix W^k take values from a finite set hence, finitely many adjacency matrices are available.

Assumption 4 (Finitely many adjacency matrices)**.** *The adjacency matrices $\{W^k\}_{k\in\mathbb{N}}$, of the communication graphs belong to $\mathcal{W}$, a finite family of matrices that satisfy Assumption 17, i.e., $W^k \in \mathcal{W}$ for all $k \in \mathbb{N}$.* □

This assumption on the adjacency matrices allows us to exploit important results from the literature regarding finite families of mappings for proving convergence of our algorithms.

In our setup, at iteration k, each agent i proposes a payoff allocation $\boldsymbol{x}^{k+1}_i$, for all agents $j \in \mathcal{I}$, as a convex combination of its estimate $\boldsymbol{x}^k_i$ and an innovation term. To generate the latter, agent i first takes an average of the observed estimates of its neighbors $\boldsymbol{x}^k_j$, $j \in \mathcal{N}^k_i$, weighted by an adjacency matrix, and then applies an operator T^k_i on the evaluated average.

Specifically, we propose the following update rule for each agent $i \in \mathcal{I}$:

$$\boldsymbol{x}^{k+1}_i = (1-\alpha_k)\boldsymbol{x}^k_i + \alpha_k T^k_i \left(\textstyle\sum_{j=1}^N w^k_{i,j}\boldsymbol{x}^k_j\right),$$

that is, in collective compact form,

$$\boldsymbol{x}^{k+1} = (1-\alpha_k)\boldsymbol{x}^k + \alpha_k \boldsymbol{T}^k \boldsymbol{W}^k(\boldsymbol{x}^k), \tag{3.4}$$

where $(\alpha_k)_{k\in\mathbb{N}} \in [\epsilon, 1-\epsilon]$ for some $\epsilon \in (0, 1/2]$, $\boldsymbol{T}^k(\boldsymbol{x}) := \mathrm{col}(T^k_1(\boldsymbol{x}_1), \ldots, T^k_N(\boldsymbol{x}_N))$ and $\boldsymbol{W}^k := W^k \otimes I_N$ represents an adjacency matrix.

In (3.4), we require the operator T^k_i to be nonexpansive and its fixed-point set to include the robust core in (3.3). For example, T^k_i can be the projection onto the core, i.e., $T^k_i = \mathrm{proj}_{\mathcal{C}(v^k)}$.

Assumption 5 (Nonexpansiveness)**.** *For all $k \in \mathbb{N}$, the operator $\boldsymbol{T}^k$ in (3.4) is such that $\boldsymbol{T}^k \in \mathcal{T}$, where $\mathcal{T}$ is a finite family of nonexpansive operators such that $\bigcap_{\boldsymbol{T} \in \mathcal{T}} \operatorname{fix}(\boldsymbol{T}) = \mathcal{C}_0^N$, with $\mathcal{C}_0$ being the robust core in (3.3).* □

Let us elaborate on this assumption in context of a robust coalitional game $\mathcal{R} = (\mathcal{I}, \mathcal{V})$, as in Definition 9. Here, for all $k \in \mathbb{N}$, we assume that an instantaneous core $\mathcal{C}(v^k)$ in (2.5) generated by the value function $v^k \in \mathcal{V}$ is the fixed-point set of an operator T_i^k for all $i \in \mathcal{I}$ which implies that $\operatorname{fix}(\boldsymbol{T}^k) = \mathcal{C}^N(v^k)$. Consequently, the intersection of the fixed-point sets of the operators $\boldsymbol{T}^k \in \mathcal{T}$ corresponds to the robust core in (3.3), i.e., $\bigcap_{\boldsymbol{T} \in \mathcal{T}} \operatorname{fix}(\boldsymbol{T}) = \bigcap_{v \in \mathcal{V}} \mathcal{C}^N(v) = \mathcal{C}_0^N$. Furthermore, we note that having a finite family of nonexpansive operators implies that the value function v^k can only take finitely many values within a specified set. This limitation does not pose a significant hindrance in practical scenarios. First, because the number of discrete values inside bounded intervals can be arbitrarily large and secondly, because the most common interpretation of value is in a monetary sense, which is always rounded off to some currency division.

Next, we assume that each $\boldsymbol{T}^k \in \mathcal{T}$ appears at least once in every Q iterations of (3.4), with Q being the integer in Assumption 9, which can be arbitrarily large.

Assumption 6. *Let Q be the integer in Assumption 9. The operators $(\boldsymbol{T}^k)_{k\in\mathbb{N}}$ in (3.4) are such that, for all $n \in \mathbb{N}$, $\bigcup_{k=n}^{n+Q} \{\boldsymbol{T}^k\} = \mathcal{T}$, with $\mathcal{T}$ as in Assumption 16.* □

This assumption ensures that the resulting robust core in (3.3) correspond to all the value functions that belong to the family $\mathcal{V}$. Under Assumptions 1–6, we can guarantee the convergence of the state in iteration (3.4) to some payoff in the set $\mathcal{A} \cap \mathcal{C}_0^N$, as formalized in the following statement.

Theorem 3 (Convergence of payoff allocation)**.** *Let Assumptions 1–6 hold and the step sizes satisfy $\alpha_k \in [\epsilon, 1-\epsilon]$ for all $k \in \mathbb{N}$, for some $\epsilon > 0$. Then, starting from any $\boldsymbol{x}^0 \in \mathbb{R}^{N^2}$, the sequence $(\boldsymbol{x}^k)_{k=0}^{\infty}$ generated by the iteration in (3.4) converges to some $\bar{\boldsymbol{x}} \in \mathcal{A} \cap \mathcal{C}_0^N$, with $\mathcal{A}$ as in (2.8) and $\mathcal{C}_0$ being the robust core (3.3).* □

3.3.2. CONVERGENCE ANALYSIS

To prove the convergence of the payoff allocation process in (3.4), we build upon a well-known result on time-varying nonexpansive mappings, presented by Browder in [18]. To proceed, let us first define the notion of admissible sequence and then recall Browder's result.

Definition 11 (Admissible sequence ([18], Def. 5))**.** *A function $j : \mathbb{N}_{>0} \to \mathcal{D} \subseteq \mathbb{N}_{>0}$ is said to be an admissible sequence of integers in $\mathcal{D}$ if for each integer $r \in \mathcal{D}$, there exists $m(r) \in \mathbb{N}_{>0}$ such that the image under the function j of $m(r)$ successive integers contains r, i.e., $r \in \{j(n), j(n+1), \ldots, j(n+m(r))\}$, for all $n \in \operatorname{dom}(j)$.* □

For example, every p–periodic sequence, i.e., $\{j^k\}_{k\in\mathbb{N}}$ where $j^{k+p} = j^k$, is admissible with $m(r) = p$ for all $r \in \operatorname{ran}(j)$ and a sequence $\{j^k = k\}_{k\in\mathbb{N}}$ is a non-admissible sequence.

Lemma 1 ([18], Thm. 5)**.** *Let $(U_r)_{r\in\mathcal{D}}, \mathcal{D} \subseteq \mathbb{N}_{>0}$, be a (finite or infinite) sequence of nonexpansive mappings such that $C = \bigcap_{r\in\mathcal{D}} \operatorname{fix}(U_r) \neq \varnothing$. Let $(\alpha_k)_{k\in\mathbb{N}}$ be a sequence where*

$\alpha_k \in [\epsilon, 1-\epsilon]$ for some $\epsilon \in (0,1/2]$, and let $(j^k)_{k\in\mathbb{N}}$ be an admissible sequence of integers in $\mathcal{D}$. Then, the sequence $(\boldsymbol{x}^k)_{k\in\mathbb{N}_{>0}}$ generated by

$$\boldsymbol{x}^{k+1} := (1-\alpha_k)\,\boldsymbol{x}^k + \alpha_k U_{j^k}(\boldsymbol{x}^k)$$

converges to some $\bar{\boldsymbol{x}} \in C$. □

Next, we recall some useful properties of nonexpansive and paracontraction operators.

3

Lemma 2 (Doubly stochastic matrix ([32], Prop. 5), ([41], Prop. 3))**.** *If W is a doubly stochastic matrix then, the linear operator defined by the matrix $W \otimes I_n$ is nonexpansive. Moreover, if the operator $(W \otimes I_n)(\cdot)$ satisfies Assumption 17 then, it is also a paracontraction with respect to the mixed vector norm $\|\cdot\|_{2,2}$.* □

The fixed-point sets of nonexpansive and paracontraction operators relate to their compositions as follows.

Lemma 3 (Composition of nonexpansive operators ([12], Prop. 4.49))**.** *Let $T_1, T_2 : \mathbb{R}^n \to \mathbb{R}^n$ be nonexpansive operators with respect to the norm $\|\cdot\|$. Then, the composition $T_1 \circ T_2$ is also nonexpansive with respect to the norm $\|\cdot\|$. Moreover, if either T_1 or T_2 is a paracontraction and* $\mathrm{fix}(T_1) \cap \mathrm{fix}(T_2) \neq \varnothing$ *then,* $\mathrm{fix}(T_1 \circ T_2) = \mathrm{fix}(T_1) \cap \mathrm{fix}(T_2)$. □

Lemma 4 (Composition of paracontracting operators ([32], Prop. 1))**.** *Suppose $M_1, M_2 : \mathbb{R}^n \to \mathbb{R}^n$ are paracontractions with respect to same norm $\|\cdot\|$ and* $\mathrm{fix}(M_1) \cap \mathrm{fix}(M_2) \neq \varnothing$. *Then, the composition $M_1 \circ M_2$ is a paracontraction with respect to the norm $\|\cdot\|$ and* $\mathrm{fix}(M_1 \circ M_2) = \mathrm{fix}(M_1) \cap \mathrm{fix}(M_2)$. □

The Lemmas provided above are convenient operator-theoretic tools that help us in keeping our proofs elegantly brief. Using these tools, let us prove the following Lemma which we exploit later in the proof of Theorem 9.

Lemma 5. *Let $\boldsymbol{T}_1, \ldots, \boldsymbol{T}_q$ be a set of nonexpansive operators with $\bigcap_{r=1}^q \mathrm{fix}(\boldsymbol{T}_r) = C$. Let the composition of the adjacency matrices that satisfy Assumption 17, i.e., $W_q W_{q-1} \cdots W_1$ represent a strongly connected graph. Let $\boldsymbol{W}_r := W_r \otimes I_N$. Then, $\bigcap_{r=1}^q \mathrm{fix}(\boldsymbol{T}_r \boldsymbol{W}_r) = \mathcal{A} \cap C$, where $\mathcal{A}$ is the consensus set in (2.8).* □

Proof. By Lemmas 15 and 3, $\mathrm{fix}(\boldsymbol{T}_r \boldsymbol{W}_r) = \mathrm{fix}(\boldsymbol{T}_r) \cap \mathrm{fix}(\boldsymbol{W}_r)$ hence, $\bigcap_{r=1}^q \mathrm{fix}(\boldsymbol{T}_r \boldsymbol{W}_r) = \mathrm{fix}(\boldsymbol{T}_r) \cap \mathrm{fix}(\boldsymbol{W}_r) \cap \cdots \cap \mathrm{fix}(\boldsymbol{T}_1) \cap \mathrm{fix}(\boldsymbol{W}_1)$. By Lemmas 15 and 10, $\bigcap_{r=1}^q \mathrm{fix}(\boldsymbol{W}_r) = \mathrm{fix}(\boldsymbol{W}_q \cdots \boldsymbol{W}_1)$ where, by the Perron-Frobenius theorem, $\mathrm{fix}(\boldsymbol{W}_q \cdots \boldsymbol{W}_1) = \mathcal{A}$. Since $\bigcap_{r=1}^q \mathrm{fix}(\boldsymbol{T}_r) = C$, we conclude that $\bigcap_{r=1}^q \mathrm{fix}(\boldsymbol{T}_r \boldsymbol{W}_r) = \mathcal{A} \cap C$. □

Given these results, we are now ready to prove Theorem 9.

Proof. (Theorem 9). Let us define the operator $\boldsymbol{U}_f := \boldsymbol{T}_f \boldsymbol{W}_f$ with $\boldsymbol{T}_f \in \mathcal{T}$ and $W_f \in \mathcal{W}$, where $\boldsymbol{W}_f := W_f \otimes I_N$. We note that, by Assumptions 15 and 16 there are only finitely many such operators and therefore, we can define the operator family $\mathcal{U} := \{\boldsymbol{U}_f\}_{f=1}^F$. Let $l : \mathcal{U} \to \mathbb{N}$ be a function such that $l(\boldsymbol{U}_f)$ gives the maximal length of the sequence which

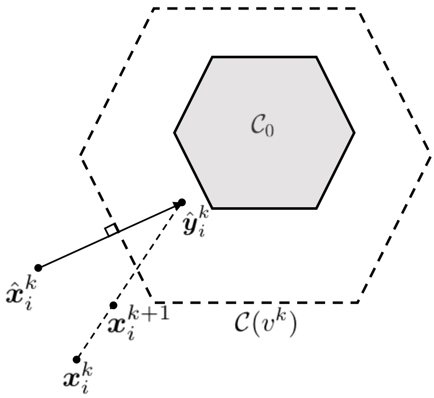


Figure 3.1: Illustration of the payoff allocation proposed by an agent i, as in (3.5) where $\hat{\boldsymbol{y}}_i^k := \text{overproj}_{\mathcal{C}(v^k)} \hat{\boldsymbol{x}}_i^k$.

contains the operator $\boldsymbol{U}_f$. Furthermore, let $\mathcal{D} = \{f \mid l(\boldsymbol{U}_f) < \infty\} \subseteq \{1, \dots F\}$, i.e., the set of indices of the operators that occur at least once in a finite length interval. Since F is finite, there always exist an integer representing the length of sequences in which each index $f \in \mathcal{D}$ appears at least once, thereby fulfilling the admissibility condition in Definition 11. Thus, by Lemmas 1 and 3, the iteration in (3.4) converges to some $\bar{\boldsymbol{x}} \in \bigcap_{f \in \mathcal{D}} \text{fix}(\boldsymbol{U}_f)$.

Let $\mathcal{K}_L$ be the interval of a sequence containing L consecutive operators from the family $\{\boldsymbol{U}_f\}_{f \in \mathcal{D}}$ such that $\bigcap_{k \in \mathcal{K}_L} \text{fix}(\boldsymbol{U}^k) = \bigcap_{f \in \mathcal{D}} \text{fix}(\boldsymbol{U}_f)$. As we can choose an arbitrarily long interval, without loss of generality, let $L \geq Q$, with Q being the integer in Assumptions 9 and 6. Then, it holds that $\bigcap_{k \in \mathcal{K}_L} \text{fix}(\boldsymbol{U}^k) \subseteq \bigcap_{k \in \mathcal{K}_Q} \text{fix}(\boldsymbol{U}^k)$ because, having a longer interval of operators can either reduce the intersection set or leave it unchanged. Finally, by Lemma 5, $\bigcap_{k \in \mathcal{K}_Q} \text{fix}(\boldsymbol{U}^k) = \mathcal{A} \cap \mathcal{C}_0^N$. □

3.3.3. DISCUSSION

Let us now visualize a proposal of an agent i in (3.4) by employing an over-projection operator, i.e., $T_i^k = \text{overproj}_{\mathcal{C}(v^k)}$ which is a nonexpansive operator, see [[12], Prop. 4.2]. For brevity, let $\hat{\boldsymbol{x}}_i^k := \sum_{j=1}^N w_{i,j}^k \boldsymbol{x}_j^k$. Then, the proposal of an agent i reads as:

$$\boldsymbol{x}_i^{k+1} = (1 - \alpha_k)\boldsymbol{x}_i^k + \alpha_k \text{overproj}_{\mathcal{C}(v^k)} \hat{\boldsymbol{x}}_i^k, \tag{3.5}$$

where $\alpha_k \in [\epsilon, 1 - \epsilon]$ for some $\epsilon \in (0, 1/2]$.

In Figure 3.1, we illustrate an arbitrary instance of (3.5), where the proposed payoff allocation $\boldsymbol{x}_i^{k+1}$ does not belong to the instantaneous core $\mathcal{C}(v^k)$ and hence it is not an acceptable payoff, even for agent i. Nevertheless, as stated in Theorem 9, repeated payoff allocations by all agents will eventually reach an agreement on the payoff in the robust core $\mathcal{C}_0$. Note that, in a payoff allocation process, intermediate allocation proposals can be irrational and therefore, the adoption of this process by a rational agent shall be motivated, e.g. via *mechanism design*, where a central authority incentivize a cooperative behavior among agents to derive the process towards equilibrium.

We remark that the number of possible coalitions grows exponentially in N, i.e., 2^N, hence so does the computations required to evaluate the core by an individual agent. This feature is inherent in coalitional games where the aim is to democratize systems by providing agents with decision autonomy [73], [14].

3.4. DISTRIBUTED BARGAINING PROTOCOL

In this section, we propose a bargaining protocol under a typical negotiation framework and a distributed paradigm, similar to the payoff allocation in Section 6.3. Specifically, at iteration k, each agent $i \in \mathcal{I}$ proposes a payoff distribution that belongs to its negotiation set, referred to as the bounding set $\mathcal{X}_i^k$ in (2.1). The intersection of negotiation sets represents the set of all plausible deals, i.e., the core and mutual agreement of agents on one such deal concludes the bargaining process. This struck deal corresponds to the final payoff distribution.

3.4.1. DISTRIBUTED BARGAINING ALGORITHM

For our distributed bargaining protocol, we use a similar setup as the payoff allocation algorithm (3.3.1). Briefly, we consider a set of agents $\mathcal{I} = \{1,\dots,N\}$, each of whom proposes a payoff distribution $\boldsymbol{x}_i^k \in \mathbb{R}^N$ at each iteration $k \in \mathbb{N}$. These agents communicate over a sequence of time-varying network graphs $(G^k)_{k\in\mathbb{N}}$, that satisfies Assumption 9, and the corresponding adjacency matrices $(W^k)_{k\in\mathbb{N}}$ satisfy Assumptions 17 and 15.

During the negotiation, at each iteration k, an agent i first takes an average of the estimates of neighboring agents $\boldsymbol{x}_j^k$, $j \in \mathcal{N}_i^k$, weighted by an adjacency matrix W^k, and then applies an operator M_i^k on the resulting average. Specifically, we propose the following negotiation protocol for each agent $i \in \mathcal{I}$:

$$\boldsymbol{x}_i^{k+1} = M_i^k\left(\textstyle\sum_{j=1}^N w_{i,j}^k \boldsymbol{x}_j^k\right),$$

that is, in collective compact form,

$$\boldsymbol{x}^{k+1} = \boldsymbol{M}^k(\boldsymbol{W}^k\boldsymbol{x}^k), \tag{3.6}$$

where $\boldsymbol{M}^k(\boldsymbol{x}) := \mathrm{col}(M_1^k(\boldsymbol{x}_1),\dots,M_N^k(\boldsymbol{x}_N))$ and $\boldsymbol{W}^k := W^k \otimes I_N$ represents an adjacency matrix.

In (5.5) we require the operator M_i^k to be paracontraction, not necessarily a non-expansive operator as in (3.4). Utilizing a paracontraction operator allows us to prove convergence of our bargaining algorithm without the need of $\alpha-$averaging with the inertial term $\boldsymbol{x}^k$, as required for payoff allocation in (3.4). Furthermore, in (5.5), we also require the fixed-point set of M_i^k to be the bounding set in (2.1), i.e., $\mathrm{fix}(M_i^k) = \mathcal{X}_i^k$. Therefore, $\mathrm{fix}(\boldsymbol{M}^k) = \bigcap_{i=1}^N \mathcal{X}_i^k = \mathcal{C}(v^k)$ and for a robust coalitional game $(\mathcal{I},\mathcal{V})$, it holds that $\bigcap_{v^k\in\mathcal{V}} \mathcal{C}(v^k) = \mathcal{C}_0$.

Assumption 7 (Paracontractions). *For all $k \in \mathbb{N}$, $\boldsymbol{M}^k$ in (5.5) is such that $\boldsymbol{M}^k \in \mathcal{M}$, where $\mathcal{M}$ is a finite family of paracontraction operators such that $\bigcap_{\boldsymbol{M}\in\mathcal{M}} \mathrm{fix}(\boldsymbol{M}) = \mathcal{C}_0^N$ with $\mathcal{C}_0$ being the robust core in (3.3).* □

Similar to the payoff allocation setup, we also assume that each $\boldsymbol{M}^k \in \mathcal{M}$ appears at least once in every Q iterations of (5.5), with Q being the integer in Assumption 9.

Assumption 8. *Let Q be the integer in Assumption 9. The operators $(\boldsymbol{M}^k)_{k\in\mathbb{N}}$ in (5.5) are such that, for all $n \in \mathbb{N}$, $\bigcup_{k=n}^{n+Q}\{\boldsymbol{M}^k\} = \mathcal{M}$, with $\mathcal{M}$ as in Assumption 12.* □

Next, we formalize the main convergence result of the bargaining protocol in (5.5).

Theorem 4 (Convergence of bargaining protocol)**.** *Let Assumptions 1–15, 12–13 hold. Then, starting from any $\boldsymbol{x}^0 \in \mathbb{R}^{N^2}$, the sequence $(\boldsymbol{x}^k)_{k=0}^{\infty}$ generated by the iteration in (5.5) converges to some $\bar{\boldsymbol{x}} \in \mathcal{A} \cap \mathcal{C}_0^N$, with $\mathcal{A}$ as in (2.8) and $\mathcal{C}_0$ being the robust core (3.3).* □

3.4.2. CONVERGENCE ANALYSIS

We prove the convergence of the bargaining protocol in (5.5) by building upon a result related to the time-varying paracontractions, presented in [27].

Lemma 6 ([27], Thm. 1)**.** *Let $\mathcal{M}$ be a finite family of paracontractions such that $\bigcap_{M\in\mathcal{M}} \mathrm{fix}(M) \neq \varnothing$. Then, the sequence $(\boldsymbol{x}^k)_{k\in\mathbb{N}}$ generated by $\boldsymbol{x}^{k+1} := M^k(\boldsymbol{x}^k)$ converges to a common fixed-point of the paracontractions that occur infinitely often in the sequence.*

□ In the following lemma, we provide a technical result about the composition of paracontractions which we exploit later in the proof of Theorem 6.

Lemma 7. *Let Q be the integer in Assumption 9. Let $\boldsymbol{M}_1,\ldots,\boldsymbol{M}_Q$ be paracontraction operators with $\bigcap_{r=1}^{Q} \mathrm{fix}(\boldsymbol{M}_r) =: C$ and let $W_Q W_{Q-1}\cdots W_1$ be the composition of the adjacency matrices where $W_r \in \mathcal{W}$, with $\mathcal{W}$ as in Assumption 15. Let $\boldsymbol{W}_r := W_r \otimes I_N$. Then, the composed mapping $\boldsymbol{x} \mapsto (\boldsymbol{M}_Q\boldsymbol{W}_Q \circ\cdots\circ \boldsymbol{M}_1\boldsymbol{W}_1)(\boldsymbol{x})$*

(i) is a paracontraction with respect to norm $\|\cdot\|_{2,2}$;

(ii) $\mathrm{fix}(\boldsymbol{M}_Q\boldsymbol{W}_Q \circ\cdots\circ \boldsymbol{M}_1\boldsymbol{W}_1) = \mathcal{A}\cap C$,

where $\mathcal{A}$ is the consensus set in (2.8). □

Proof. (i): It follows directly from Lemmas 15 and 10.
(ii): By Lemmas 15 and 10, $\mathrm{fix}(\boldsymbol{M}_Q\boldsymbol{W}_Q \circ\cdots\circ \boldsymbol{M}_1\boldsymbol{W}_1) = \mathrm{fix}(\boldsymbol{M}_Q)\cap\cdots\cap\mathrm{fix}(\boldsymbol{M}_1)\cap\mathrm{fix}(\boldsymbol{W}_Q)\cap\cdots\cap\mathrm{fix}(\boldsymbol{W}_1)$. Again, by Lemmas 15 and 10, $\bigcap_{r=1}^{Q}\mathrm{fix}(\boldsymbol{W}_r) = \mathrm{fix}(\boldsymbol{W}_Q\cdots\boldsymbol{W}_1)$ and since the composition $\boldsymbol{W}_Q\cdots\boldsymbol{W}_1$ is strongly connected, by the Perron-Frobenius theorem, $\mathrm{fix}(\boldsymbol{W}_Q\cdots\boldsymbol{W}_1) = \mathcal{A}$. Finally, as $\bigcap_{r=1}^{Q}\mathrm{fix}(\boldsymbol{M}_r) = C$, $\mathrm{fix}(\boldsymbol{M}_Q\boldsymbol{W}_Q \circ\cdots\circ \boldsymbol{M}_1\boldsymbol{W}_1) = \mathcal{A}\cap C$. □

Given these preliminary results, we are now ready to present the proof of Theorem 6.

Proof. (Theorem 6) Let us define the sub-sequence of $\boldsymbol{x}^k$ for all $k \in \mathbb{N}$ as $\boldsymbol{z}^t = \boldsymbol{x}^{(t-1)Q}$ for each $t \geq 2$ with Q being the integer in Assumptions 9 and 13. Then,

$$\boldsymbol{z}^{t+1} = \boldsymbol{M}^{tQ-1}\boldsymbol{W}^{tQ-1} \circ\cdots\circ \boldsymbol{M}^{(t-1)Q}\boldsymbol{W}^{(t-1)Q}\boldsymbol{z}^t \tag{3.7}$$

for $t \geq 2$. It follows from assertion 1 of Lemma 12 that the maps $\boldsymbol{x} \longmapsto (\boldsymbol{M}^{tQ-1}\boldsymbol{W}^{tQ-1}\circ\cdots\circ \boldsymbol{M}^{(t-1)Q}\boldsymbol{W}^{(t-1)Q})(\boldsymbol{x})$, $t \geq 2$ are all paracontractions. Also, under Assumption 15, there can be only finitely many such maps. Furthermore, by assertion 2 of Lemma 12, the set of fixed-points of each map is $\mathcal{A}\cap\mathcal{C}^N$. Thus, by Lemma 8, the iteration in (4.10) converges to some $\bar{\boldsymbol{z}} \in \mathcal{A}\cap\mathcal{C}^N$. □

3.4.3. DISCUSSION

In our proposed bargaining process in (5.5), let $\boldsymbol{M}^k = \text{proj}_{\mathcal{X}^k}$, for all $k \in \mathbb{N}$, which is a paracontraction [[12], Prop. 4.16]. Then, the resulting iteration, i.e., $\boldsymbol{x}^{k+1} = \text{proj}_{\mathcal{X}^k}(\boldsymbol{W}^k \boldsymbol{x}^k)$ reduces to the bargaining protocol presented in [73]. In that setup, the communication graphs and adjacency matrices also satisfy our Assumptions 9 and 17, respectively. The bargaining algorithm in [73] lies within our bargaining framework, but with the exception that, in our setup, the value function v^k can only take finitely many values in a bounded set. We emphasize that our framework provides an agent with the flexibility to choose a paracontraction operator, not necessarily a projection. This allows an agent to propose a payoff on the boundary or in the interior of its bounding set.

Finally, we note that in the bargaining process, each agent requires the coalitional value bounds of his coalitions only to evaluate the bounding set, hence lower information requirement compared to the payoff allocation in Section 6.3.

3.5. NUMERICAL SIMULATIONS

In this section, we present numerical illustrations of two realistic scenarios modeled as coalitional games with uncertain coalitional values. In the first scenario, we present a collaboration among three firms for providing abstract services; in the second scenario, we simulate the motivational application introduced in Section 3.1. Our goal for presenting the former is to illustrate the robust core and to differentiate between the structure of payoff allocation and bargaining processes during the negotiation stages. Therefore, we use only three agents (firms) to be able to illustrate the outcome graphically in dimension 2. Further, in the second simulation scenario, we demonstrate a more comprehensive application, namely cooperative energy storage optimization in a smart grid framework.

3.5.1. ILLUSTRATIVE EXAMPLE

Consider three firms $\mathcal{I} = \{1,2,3\}$, which individually provide certain services to their customers. These firms can improve their efficiency by collaborating activities and hence generate a higher value. This collective value is a remuneration of services agreed upon by a customer and the coalition of firms in advance. To make this collaboration viable, all three firms have to agree upon their share of the generated value. The resulting scenario is a coalitional game among firms, a solution to which is an agreed payoff distribution in the core.

The core allocation in (2.5) depends on the value of all possible sub-coalitions. In our example, the firms know with certainty about their individual values $v(\{i\})$ and the value of the grand coalition $v(\mathcal{I})$, i.e., the final contract. However, the sub-coalitions are never formed and hence their values are unknown. We assume that the values of the sub-coalitions are random within a bounded interval. Under the above conditions, the coalitional game among the three firms takes the form of a robust coalitional game. Thus, we can apply the robust payoff distribution methods proposed in Sections 6.3 and 3.4.

The coalitional values of this coalitional game among firms are given in Table 3.1. For example, at each iteration k, the value function of the coalition $\{1,2\}$, i.e., $v(\{1,2\})$, takes

Table 3.1: Coalitional values for the illustrative example

$v(\{i\}), i \in \mathcal{I}$	$v(\{1,2\})$	$v(\{1,3\})$	$v(\{2,3\})$	$v(\{1,2,3\})$
1	$\{2,3,4\}$	$\{2,3,4\}$	$\{3,4,5\}$	8

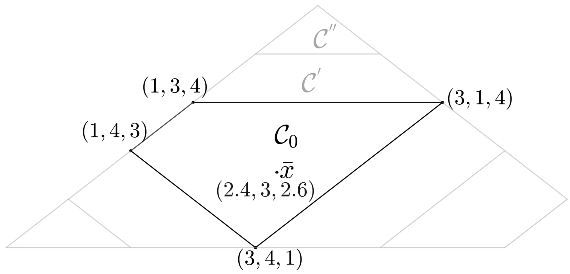


Figure 3.2: Three instances of the core set, $\mathcal{C}_0, \mathcal{C}', \mathcal{C}''$ and final payoff allocation $\bar{\boldsymbol{x}}$.

its value randomly from the set $\{2,3,4\}$ with uniform probability. The possibility of realizing only an integer value, with uniform probability, satisfies the assumption of finite operator families in Theorems 9–6 and also ensures that the resulting sequence satisfies Assumption 6. Furthermore, we consider a fixed, strongly connected communication graph which therefore satisfies Assumptions 9–15. For the initial proposals, we assume that each agent allocates entire value of coalition to itself, e.g. the initial proposal by firm 1 will be $\boldsymbol{x}_1(1) = [8\ 0\ 0]^\top$. Next, we evaluate the payoff distributions generated by payoff allocation algorithm in (3.4) and the bargaining protocol in (5.5).

DISTRIBUTED PAYOFF ALLOCATION

For implementation, we choose an over-projection operator, which is nonexpansive, and the step size $\alpha_k = 0.5$ for all $k \in \mathbb{N}$. The resulting iteration for each agent i is as in (3.5). In Figure 3.2, we depict two arbitrary instances of the core set $\mathcal{C}', \mathcal{C}''$ and the robust core $\mathcal{C}_0$ in (3.3). The allocation process in (3.4) converges to consensus on the payoff allocation, $\bar{\boldsymbol{x}} = [2.4, 3, 2.6]$, which belongs to the robust core, i.e., $\mathcal{A} \cap \mathcal{C}_0^N$. An allocation in the robust core ensures that even under uncertainty on coalitional values, the collaboration will emerge as the only rational choice. We note that, in payoff allocation process each firm does not need to have deterministic information of the core, which is weaker from the usual assumption of coalitional games [14]. In fact, here the firms only know the bounds on coalitional values.

DISTRIBUTED BARGAINING PROTOCOL

We implement the iteration in (5.5), by using the projection operator, which is a paracontraction and therefore, it satisfies the assumptions of Theorem 6. In Figure 3.3, we show an arbitrary negotiation step during the bargaining process. Here, a firm i agrees

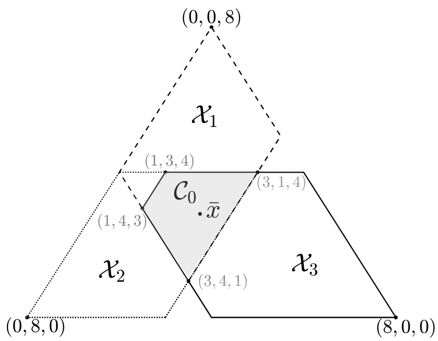


Figure 3.3: An instance of bargaining process showing the bounding sets of the agents, $\mathcal{X}_1$, $\mathcal{X}_2$, $\mathcal{X}_3$, and the robust core $\mathcal{C}_0 = \bigcap_{i\in\mathcal{I}} \mathcal{X}_i$. $\bar{\boldsymbol{x}}$ is the final payoff vector.

with the payoff distribution only if it belongs to its bounding set $\mathcal{X}_i$. Thus, any mutually agreed payoff distribution must belong to the intersection of bounding sets, i.e., $\mathcal{C} = \bigcap_{i\in\mathcal{I}} \mathcal{X}_i$. Because of the uncertainty in the values of sub-coalitions, the bounding sets vary with iterations resulting in an instantaneous core as in (2.5). The bargaining process in (5.5) ensures convergence to the intersection of the instantaneous cores, i.e., the robust core $\mathcal{C}_0$ in (3.3). Thus, in our example, the resulting payoff distribution $\bar{\boldsymbol{x}} = [2.33, 2.833, 2.833]$ belongs to the set $\mathcal{A} \cap \mathcal{C}_0^N$.

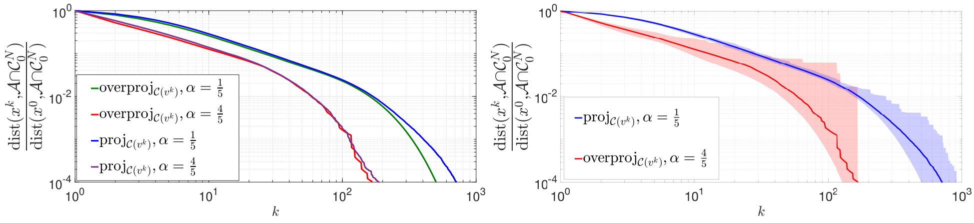
Compared with the payoff allocation process, the knowledge requirement for the firms in the bargaining protocol is even weaker. Here, the firms are required to know the bounds on the values of their own sub-coalitions only, which is a reasonable assumption for a cooperation scenario.

3.5.2. COOPERATIVE ENERGY STORAGE OPTIMIZATION

In this subsection, we simulate the cooperative ES optimization problem described in Section 3.1 as a motivational example. We partially adapt the optimization setup from [43] and, additionally, introduce uncertainty in the RES generation.

PROBLEM SETUP

Consider N prosumers in an energy coalition $\mathcal{I}$, each equipped with RES generation and ES system. Our goal is to cooperatively optimize ES systems, by considering them as a single collective storage, for minimizing the coalitional cost in (3.1) and, distribute the resulting cost savings, i.e., coalitional value in (3.2) among prosumers. Moreover, the share of each prosumer, i.e., the payoff should belong to the robust core in (3.3). We compute the coalitional value of each coalition $S \subseteq \mathcal{I}$ for a time period of length K by solving a linear optimization problem. We assume that the ES system of each prosumer i has an energy capacity of $e_i \geq 0$, a charge and discharge limit, $\overline{b}_i \geq 0$ and $\underline{b}_i \geq 0$ respectively, a charge and discharge efficiency η_i^{ch} and $\eta_i^{\mathrm{dc}} \in (0,1)$, respectively. We also consider an initial state of charge for each ES, $\mathrm{SoC}_i^0 \in [0,1]$ where 1 represents a fully charged battery. We denote the amount of energy stored and released from agent i's ES during time t be b_i^{t+} and b_i^{t-}, respectively.



(a) (b)

Figure 3.4: (a) Sampled average of the trajectories of $\mathrm{dist}(\boldsymbol{x}^k, \mathcal{A}\cap\mathcal{C}_0^N)/\mathrm{dist}(\boldsymbol{x}^0, \mathcal{A}\cap\mathcal{C}_0^N)$ for distributed allocation algorithm with operator $\mathrm{proj}_{\mathcal{C}(v^k)}$ for $\alpha = 1/5, 4/5$ and $\mathrm{overproj}_{\mathcal{C}(v^k)}$ for $\alpha = 1/5, 4/5$. (b) Sampled average of selected trajectories with spread of samples shown by shaded region.

Next, let us denote the vectors representing charge and discharge energies of all prosumers by $\boldsymbol{b}^-$ and $\boldsymbol{b}^+$. Moreover, because of the difference in buying and selling prices of electricity, let us divide the coalitional net load into a positive part $\boldsymbol{L}^+$, which corresponds to the energy bought from the grid, and a non-positive part $\boldsymbol{L}^-$, which represents the energy sold to the grid. These four vectors are the decision variables of our ES optimization problem that computes the coalitional cost $c(S)$ for each coalition $S \subseteq \mathcal{I}$ as follows:

$$\min_{\substack{\boldsymbol{b}^+, \boldsymbol{b}^-, \\ \boldsymbol{L}^+, \boldsymbol{L}^-}} \sum_{t=1}^{K} \left\{ p_b^t \sum_{i\in S} L_i^{t+} + p_S^t \sum_{i\in S} L_i^{t-} \right\} \tag{12a}$$

$$\text{s.t.} \quad L_i^{t-} \leq 0 \leq L_i^{t+} \tag{12b}$$

$$\textstyle\sum_{i\in S}(b_i^{t+} + b_i^{t-} + q_i^t) \leq \sum_{i\in S} L_i^{t+} \tag{12c}$$

$$\textstyle\sum_{i\in S}(b_i^{t+} + b_i^{t-} + q_i^t) = \sum_{i\in S}(L_i^{t+} + L_i^{t-}) \tag{12d}$$

$$\underline{b}_i \leq b_i^{t-} \leq 0 \leq b_i^{t+} \leq \overline{b}_i \tag{12e}$$

$$\textstyle\sum_{t=1}^{K} \left(b_i^{t+}\eta_i^{\mathrm{ch}} + b_i^{t-}/\eta_i^{\mathrm{dc}}\right) = 0, \quad \forall i \in S \tag{12f}$$

$$0 \leq e_i \mathrm{SoC}_i^0 + \textstyle\sum_{t=1}^{m} \left(b_i^{t+}\eta_i^{\mathrm{ch}} + b_i^{t-}/\eta_i^{\mathrm{dc}}\right) \leq e_i \tag{12g}$$

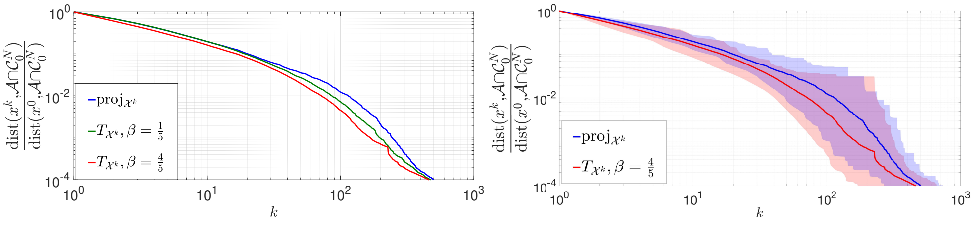
$$\forall i \in S, \forall t \in [1,K], \forall m \in [1,K].$$

The constraints (12e)–(12g) are related to the physical limitations of ES systems. Specifically, (12e) represents the limitation on the rate of charge/discharge, (12g) represents energy storage capacity and (12f) ensures that the state of charge of each ES at the end of the horizon K is same as the initial, i.e., $\mathrm{SoC}_i^K = \mathrm{SoC}_i^0$. For further details, we refer to [43].

To proceed, we introduce uncertainty in the net energy consumption q_i^t, since the generation of RES is uncertain. However, q_i^t can only realize values from the interval $[q_i^{\min}, q_i^{\max}]$ as explained in Section 3.1. Here, these bounds refer to the optimistic and conservative forecasts.

ES OPTIMIZATION AS A ROBUST COALITIONAL GAME

Let us now put the optimisation setup, presented above, in the perspective of the payoff distribution problem. At the first stage, the grand prosumer coalition $\mathcal{I}$ optimizes



(a) (b)

Figure 3.5: (a) Sampled average of the trajectories of $\text{dist}(\boldsymbol{x}^k, \mathcal{A}\cap\mathcal{C}_0^N)/\text{dist}(\boldsymbol{x}^0, \mathcal{A}\cap\mathcal{C}_0^N)$ for distributed bargaining with operator $\text{proj}_{\mathcal{X}^k}$ and $T_{\mathcal{X}^k} := (1-\beta)\text{proj}_{\mathcal{X}^k}(\cdot) + \beta\text{overproj}_{\mathcal{X}^k}(\cdot)$ for $\beta = 1/5, 4/5$. (b) Sampled average of trajectories with spread of samples shown by shaded region.

their energy operation collectively via an aggregator over a time horizon of length K and sells any expected excess of energy (available at each time interval t) to the retailer. The coalition performs this process in advance and gets remunerated by the retailer. The additional value gained by the coalition as a result of the cooperation is given by (3.2). At the second stage, the attained coalitional value, i.e., $v(\mathcal{I})$ is distributed among the agents so that the payoff to each agent belongs to the robust core in (3.3). Thus, for the payoff distribution, an aggregator computes the value $v(S)$ for all $S \subset \mathcal{I}$ by solving the optimization problem presented above. To account for the uncertainty in the RES generation, the aggregator computes the bounds on the coalitional values as $\underline{v}(S) \leq v(S) \leq \overline{v}(S), S \subset \mathcal{I}$ and communicates the vector v containing these bounds to all the agents, who in turn initiate the payoff distribution process. We remark that, a central entity is not necessarily required for the evaluation of coalitional values, except for the grand coalition.

This scenario with uncertainty requires a robust solution and thus demonstrates the practicality of our payoff distribution algorithms. Furthermore, since the core set is not a singleton, it is possible that certain payoff vectors favour some specific agents [93]. Consequently, a central computation of the payoffs might be unacceptable for the prosumers. Instead, distributed mechanisms defer this responsibility on the prosumers as they arrive at a mutual agreement by themselves.

SIMULATIONS STUDIES

For the numerical simulation, we select a time horizon of $K = 6$ hours and an interval $t = 1$ hour. We consider a coalition of 6 prosumers where each prosumer i is equipped with the battery of energy capacity $e_i = 7$ kWh, a maximum charge power $\overline{b}_i = 3.5$ kW, a maximum discharge power $\underline{b}_i = 3.5$ kW, both charge and discharge efficiencies of $\eta_i^{\text{ch}} = \eta_i^{\text{dc}} = 95\%$ and an initial state of charge $\text{SoC}_i^0 = 50\%$. We put the bounds of optimistic and conservative forecast on the RES generation of each agent and randomly generate net consumption scenarios. We then evaluate the coalitional value in (3.2) for each scenario and compute the bounds $\underline{v}(S)$ and $\overline{v}(S), S \subset \mathcal{I}$. We then run 100 different trajectories of payoff distribution processes. We assume, for each trajectory, that agents initially allocate the whole value $v(\mathcal{I})$ to themselves. Also, to make sure that every prosumer's payoff proposal receives adequate importance and sufficient exposure, during negotiation, we assume a strongly connected communication graph among them which satisfies Assumptions 9 and 17. Furthermore, as the coalitional value in cooperative energy optimization is in monetary terms, the prosumers consider reasonably rounded of units (dollars, cents etc.) which results in a finite set of points between the bounds $\underline{v}(S)$ and $\overline{v}(S)$, according to Assumption 6.

Moreover, for the distributed allocation process in (3.4), the whole coalitional value vector v is communicated to the agents whereas, for the bargaining process in (5.5) only the value of agent's own coalitions are communicated. Finally, the agents initiate a robust coalitional game to reach the consensus on a payoff which belongs to the robust core in (3.3). This payoff guarantees the stability of the grand coalition which in turn has considerable operational benefits for the power grid [43].

We first report the numerical results for the distributed allocation process. In Figure 3.4a, we compute the average of the sample trajectories obtained by 100 runs and report the normalized distances $\text{dist}(\boldsymbol{x}(k), \mathcal{C}_0 \cap \mathcal{A})/\text{dist}(\boldsymbol{x}(0), \mathcal{C}_0 \cap \mathcal{A})$, for the projection and over-projection operators, by varying the parameter α. We can observe that an over-projection operator with higher value of α results in faster convergence. In Figure 3.4b, we provide the spread of the sample trajectories to depict the best and worst convergence scenarios in our sample set.

Lastly, we simulate the distributed bargaining process in (5.5) and report the average of the sample trajectories. In Figure 3.5a, we show the comparison of the normalized distances. We conduct the analysis by utilizing the projection operator and the convex combination of projection and over-projection operators, i.e., $T_{\mathcal{X}^k} := (1-\beta)\text{proj}_{\mathcal{X}^k}(\cdot) + \beta \text{overproj}_{\mathcal{X}^k}(\cdot)$ for varying β. Both the operators are paracontraction operators [12]. Figure 3.5b shows the spread of the sample trajectories.

3.6. CONCLUSION

We have addressed the problem of payoff distribution in robust coalitional games over time-varying networks where the goal is to make players reach a consensus on the payoff distribution that belongs to the robust core. We have shown that distributed payoff allocation and bargaining algorithms, with known coalitional value bounds and based on nonexpansive and paracontraction operators, e.g. over-projections, and network averaging converge consensually to the robust core, even with varying coalitional values.

4

Peer-to-Peer Energy Trading via Coalitional Games

Democratization of energy is envisioned to be an essential step towards the energy transition. Thus, to empower prosumers we propose a bilateral peer-to-peer (P2P) energy trading scheme under single-contract and multi-contract market setups, both as an assignment game, a special class of coalitional games. The proposed market formulation allows for efficient computation of a market equilibrium while keeping the desired economic properties offered by the coalitional games. Furthermore, our market model allows buyers to have heterogeneous preferences (product differentiation) over the energy sellers, which can be economic, social, or environmental. To address the problem of scalability in coalitional games, we design a novel distributed negotiation mechanism that utilizes the geometric structure of the equilibrium solution to improve the convergence speed. Our algorithm enables market participants (prosumers) to reach a consensus on a set of "stable" and "fair" bilateral contracts which encourages prosumer participation. The negotiation process is executed with virtually minimal information requirements on a time-varying communication network that in turn preserves privacy. We use operator-theoretic tools to rigorously prove its convergence. Numerical simulations illustrate the benefits of our negotiation protocol and show that the average execution time of a negotiation step is much faster than the benchmark.

4.1. INTRODUCTION

Modernization of power systems is rapidly materializing under the smart grid framework. A major part of this transformation is taking place on the consumer side, due to the increasing penetration of distributed energy resources (DER) along with the deployment of communication and control technologies. These technologies enable consumers to have an active interaction with the grid by an informed control over their energy behavior, thus they are referred as “prosumers".

To realize their full potential, prosumers should engage more actively with energy markets. Currently the direct participation of prosumers in the whole sale energy market is technically and economically non-viable. Hence, small-scale prosumers interact with aggregating entities such as retailers to deliver their excess energy to the grid [66]. Retailers usually offer a considerably lower price for the energy sold by prosumers, e.g. feed-in-tariff (FiT), compared to the buying price that they charge [44]. Moreover, with increasing number of participants, the benefits to prosumers in recent FiT schemes become quite marginal.

To ensure an economically appealing role of prosumers, peer-to-peer (P2P) energy trading represents a disruptive demand side energy management strategy [111]. In fact, P2P markets enable prosumers to locally exchange energy on their own terms of transactions. This direct control over trading allows prosumers to make profitable interactions, thus it encourages wider participation [109]. Furthermore, such a local exchange of energy at the demand side also provides significant benefits to the system operators for example in terms of peak shaving [65], lower investments in grid capacity [62] and improvement in overall system reliability [66].

However, there are strong mathematical challenges in designing a comprehensive P2P energy market mechanism which seeks a market equilibrium while incorporating a self-interested decision-making attitude by the participants [114]. Despite its mathematical sophistication, the mechanisms need to be easily interpretable for the participation of laypersons, e.g. residential prosumers. Along this direction, researchers have recently presented several interesting formulations. In the literature, P2P energy markets are proposed under various architectures that can be broadly categorized as centralized markets (community-based trading), decentralized markets (bilateral trading) and combinations there of. The features of centralized market architecture include an indirect interaction of market participants via assisting platforms, no negotiatory role for market participants and single market wide energy trading price, evaluated centrally. Among others, in [65] Moret and Pinson present a centralized local energy market where groups of prosumers (energy collectives) interact with each other and with the system operator via a community manager to make energy exchanges. In [67], Morstyn and McCulloch treat energy as a heterogeneous product that can be differentiated based on the attributes of its source. Centralization is achieved by a platform agent that is supposed to maximize social welfare by setting prices and that enables energy exchanges among prosumers and with the wholesale electricity market. In [24], Vazquez, Al-Skaif and Rueda present a community based market that models the decision making into three sequential steps, solved using distributed optimization, where the energy is exchanged via a local pool at single clearing price.

Decentralized P2P markets can allow for direct buyer-seller (bilateral) interaction with possibly different energy trade price for each bilateral contract. In [105], Sorin, Bobo and Pinson formulate a decentralized P2P market architecture based on a multi-bilateral economic dispatch with a possibility of product differentiation, where the solution is obtained by solving a distributed optimization problem. Another decentralized P2P market is presented by Morstyn, Teytelboym and McCulloch in [68], which is formulated as a matching market that seeks a stable bilateral contract network. In both works, prosumers are allowed to make bilateral contracts, i.e., each energy transaction can take place at a different price. In [74], Nguyen also presents a decentralized P2P market with a clearing mechanism based on the alternating direction method of multipliers.

Within the industrial informatics community, P2P energy platforms have received strong research attention under both centralized and decentralized architectures. Recently, in [115], the authors propose a two-tier market corresponding to inter and intra-region interactions where a DSO acts as a representative for each region and the price of energy trade between regions is evaluated centrally. In [25], the authors also present a two-level market for trading energy with and within energy communities. Each community is represented by an aggregator that decides inter-community trading price whereas intra-community trading is done at a fixed price. Both the works propose distributed optimization based market solutions and don't allow for bilateral economic interaction on prosumer (peer) level. A decentralized P2P market is analysed in [7] with the possibility of a bilateral trade however, their focus is on the trading preferences of prosumers rather than the mechanism design. The blockchain based implementations to address privacy and security in P2P platforms are also addressed [84]. Most of the works reviewed above lack any discussion on the economic properties of the proposed trading strategies, which are critical for the practicality of any market mechanism. Next, we build a case for our proposal of a P2P market mechanism that allows for a decentralized (fully P2P) interaction and also ensures desirable economic properties.

4

4.1.1. MARKET DESIGN

The key desirable properties of electricity market design are market efficiency, incentive compatibility, cost recovery and revenue adequacy [94]. Unfortunately, by Hurwicz's impossibility theorem [49], no market mechanism can satisfy all four properties simultaneously and a trade-off has to be found. Centralized electricity markets are usually cleared based on the locational marginal pricing, which only satisfies cost recovery and revenue adequacy. Similarly, the VCG mechanism, utilized in several P2P market designs satisfies market efficiency, incentive compatibility, and cost recovery making financial deficit possible for the market operator. In our context of P2P market design where the participants are relatively small prosumers, hence are not generally capable of exercising market power, we can reasonably assume them to be truthful, thus enforcing incentive compatibility. The other market properties can be satisfied by the solutions of canonical coalitional games, thus they provide the required mathematical foundation for the design of P2P markets. We refer the reader to [92] for details about the classes (canonical, coalition formation, and coalitional graph) of coalitional games. In the general setup of coalitional games, the solutions with the required properties, such as the core, suffer the issue of scalability as all possible sub-coalitions, i.e, $2^N - 1$ for N agents, must be considered. Based on these considerations, here we model the P2P electricity market as an assignment game, a special class of coalitional games, for which the solution requires the information about coalition pairs only. This solves the scalability issue while keeping the desired market properties [98]. Furthermore, our model also allows for *bilateral* interactions where buyers can exercise their preferences over the sellers, as well as their energy sources which, in our opinion, captures the true spirit of P2P trading. Specifically, in this chapter, we propose an easily interpretable decentralized (bilateral) P2P energy market that allows for a heterogeneous treatment of energy by utilizing concepts from coalitional game theory for mechanism design and for proving the plausibility of the equilibrium solution.

4.1.2. LITERATURE REVIEW

Coalitional game theory provides rigorous analytical tools for the cooperative interactions among agents with selfish interests, and in fact it has received a strong attention from smart-grid researchers recently. For instance, the authors in [109] propose a P2P energy trading scheme in which prosumers form a coalition to trade energy among themselves at a (centralized) mid-market rate which in turn ensures the stability of the coalition [112]. In [110], the authors formulate a coalition formation game for P2P energy exchange among prosumers, and the resulting coalition structure is shown to be stable. The price of exchange is determined by a central auctioneer based on a double auction mechanism. Another coalition formation game is presented in [113], which allows prosumers to optimize their battery usage for P2P energy trading. The outcome is shown to be stable, optimal and prosumer-centric. The authors in [60] propose decentralized bilateral negotiation among prosumers for energy exchange via coalition formation, but without considering coalitional-game-theoretic stability. Coalition formation games have also been utilized for cooperative charging of electric vehicles (EVs). The authors in [127] consider EVs in different regions with different discharging prices. Based on this difference, EVs form a coalition structure to exchange energy. Then in [119], the authors formulate a coalition formation game among private charging piles to optimally provide charging services to EVs by sharing their resources. In both works the considered games are non-super-additive, thus they employee a merge-and-split protocol to reach a stable coalition structure. The protocol does not determine the price of exchange but only a stable match. In Table 4.1, we provide a comparison between the features of our P2P market model and those of the most relevant literature. We note that in Table 4.1, stability is considered in the context of coalitional game theory and we mark the presence of guarantees on the market properties only if they are explicitly discussed in the paper.

To the best of our knowledge, the literature on coalitional game theoretic formulation of P2P markets lacks a development of bilateral P2P model via most widely studied and easily interpretable class of coalitional games, i.e., canonical coalitional games. Along with the mathematical rigor provided by the game formulation, its straight forward interpretation is also an important feature for a P2P market design, intended to encourage the participation of small prosumers with low technical knowledge. Though canonical games seem the most natural approach to model bilateral market, the hindrance in its adoption comes from high computational complexity and coalition stabilizing contract prices might not exist, i.e., the core set might be empty. To address these, here, we

Table 4.1: Comparison with the state-of-the-art employing coalitional game theory

Features	[109]	[110]	[113]	[60]	This chapter
Bilateral contracts	×	×	×	✓	✓
Guarantees on market properties	✓	×	×	×	✓
Product differentiation	×	×	×	×	✓
Distributed computation	×	×	×	✓	✓
Stability of market	✓	✓	✓	×	✓

model P2P energy trading as a canonical coalitional game that allows for bilateral energy trading contracts and guarantees the existence of *stable* contract prices that represent a competitive equilibrium of the market. Furthermore, the negotiation mechanism enables market participants for an efficient and convenient settlement on the *stable* and *fair* contract prices.

4.1.3. CONTRIBUTION

Our key contributions are summarized next:

- We formulate P2P energy trading as a bilateral assignment game (coalitional game), which is easily interpretable and allows for product differentiation to accommodate the heterogeneous preferences of buyers. This novel formulation ensures the existence of a “stable" set of bilateral contracts that is an equilibrium (Section 5.3). Furthermore, our market formulation ensures the desirable economic properties of the mechanism, which are market efficiency, cost recovery, and revenue adequacy;
- We develop *single-contract* and *multi-contract* setups of bilateral P2P energy market with different computational burdens and features (Section 4.3.2 and 4.3.3);
- We develop a novel distributed negotiation mechanism presented as a fixed-point iteration where buyers-sellers communicate locally over a possibly time-varying communication network. We exploit the geometrical structure of the *core* solution together with operator theory to formulate our algorithm via linear operations, thus considerably reducing the computational complexity of the negotiation, strongly improving over [86], [13]. We show that the mechanism converges to a payoff allocation in the core of the assignment game (Section 4.4);
- We present our algorithm in a generalized form which enables fast convergence and allows participants to negotiate for “fair" contracts in the interior of the core set [98]. The level of information requirement in our mechanism preserves privacy among the market participants. Furthermore, our algorithm is based on consensus protocols, which are easier to analyze and embed on real hardware, instead of

dual variables (e.g. in [74]), to reach a common price vector among the participants.

For basic notation and operator-theoretic definitions see Appendix A and Chapter 2, Section 2.2.2, respectively.

4.2. Background on Assignment Games

4.2.1. Assignment games

An assignment game models a bilateral one-to-one matching market with the primary objective of finding optimal assignments between the two sides, for example, matching buyers to sellers [98]. Thus, let us refer to the sets of agents on the two sides of the market as buyers and sellers and denote them by $\mathcal{I}_{\mathcal{B}}$ and $\mathcal{I}_{\mathcal{S}}$, respectively. Here, each seller $j \in \mathcal{I}_{\mathcal{S}}$ owns a good for which declares the value of at least c_j; whereas, for each buyer $i \in \mathcal{I}_{\mathcal{B}}$, the ceiling worth of the good of seller j is $h_{i,j}$. Then, the value function that gives value to a simplest *meaningful* coalition, i.e., a buyer-seller pair, reads as:

$$v(i,j) = \max\{0, h_{i,j} - c_j\}. \tag{4.1}$$

Here, with a slight abuse of notation, we refer to $v(\{i,j\})$ by $v(i,j)$. We note that any assignment which is favorable to both parties must satisfy $h_{i,j} > c_i$. Furthermore, one-sided coalitions generate no value, i.e., $v(S) = 0$ if $S \subseteq \mathcal{I}_{\mathcal{B}}$ or $S \subseteq \mathcal{I}_{\mathcal{S}}$, thus only mixed coalitions are meaningful.

Interestingly, the buyer-seller pairs alone suffice to determine the market completely. Using this observation, we define an assignment matrix $M = [v(i,j)]$ for all pairs $(i,j) \in \mathcal{I}_{\mathcal{B}} \times \mathcal{I}_{\mathcal{S}}$.

Definition 12 (Value function). *Let $\mathcal{I}_{\mathcal{B}} = \{1,\dots,N_{\mathcal{B}}\}$ and $\mathcal{I}_{\mathcal{S}} = \{1,\dots,N_{\mathcal{S}}\}$ be the sets of buyers and sellers, respectively. Let $M = [v(i,j)]_{(i,j)\in\mathcal{I}_{\mathcal{B}}\times\mathcal{I}_{\mathcal{S}}}$ be an assignment matrix with $v(i,j)$ as in (4.1). Given $\mathcal{B} \subseteq \mathcal{I}_{\mathcal{B}}$ and $\mathcal{S} \subseteq \mathcal{I}_{\mathcal{S}}$, let $\mathcal{P}(\mathcal{B},\mathcal{S})$ be the set of all possible matching configurations between $\mathcal{B}$ and $\mathcal{S}$, where a matching configuration is a set of two-sided matchings such that a seller (buyer) is matched with at most one buyer (seller). Then, the value function $v_M : \mathcal{I}_{\mathcal{B}} \cup \mathcal{I}_{\mathcal{S}} \to \mathbb{R}$ is defined as, $v_M(\mathcal{B}\cup\mathcal{S}) = \max_{P\in\mathcal{P}(\mathcal{B},\mathcal{S})} \sum_{(i,j)\in P} v(i,j)$.* □

Let us now formally define an assignment game.

Definition 13 (Assignment game). *Let $\mathcal{I}_{\mathcal{B}} = \{1,\dots,N_{\mathcal{B}}\}$ and $\mathcal{I}_{\mathcal{S}} = \{1,\dots,N_{\mathcal{S}}\}$ be the sets of buyers and sellers, respectively. An assignment game is a pair $\mathcal{M} = (\mathcal{I}_{\mathcal{B}} \cup \mathcal{I}_{\mathcal{S}}, v_M)$, where the value function v_M is as in Definition 12.* □

For a game with a grand coalition $\mathcal{I}$ we assume that each agent $i \in \mathcal{I}$ is rational and demands an efficient payoff vector. Mathematically, this means that the payoff vector proposed by each agent must belong to its bounding set as in (2.1). We note from (2.1) that Bounding half space is closed and convex, a polytope with special geometry, thus we can represent the bounding set as the intersection of *bounding half-spaces*.

Definition 14 (Bounding half-spaces). *For a coalitional game $(\mathcal{I}, v)$ and a coalition $S \subset \mathcal{I}$ the bounding half-space is a set $H(S) := \{x \in \mathbb{R}^N \mid \sum_{i\in S} x_i \geq v(S)\}$. Moreover, let the set*

$$\mathcal{H}_i := \{H(S) \mid S \subset \mathcal{I}, i \in S\} \tag{4.2}$$

denote the set of all bounding half-spaces corresponding to the set of rational and efficient payoffs for an agent i, i.e., the bounding set $\mathcal{X}_i$ in (2.1). □

Now, using half-spaces as in Definition 14, we can write the bounding set as $\mathcal{X}_i = \bigcap_{S\subseteq\mathcal{I}|i\in S} H(S)$.
Since a rational agent i agrees only on a payoff in its bounding set $\mathcal{X}_i$ thus, a mutually agreed payoff shall belong to the intersection of the bounding sets of all the agents. Interestingly, this intersection corresponds to the core,

Next, we reformulate the core in (2.2) for assignment games.

Definition 15 (Core of assignment game)**.** *The core $\mathcal{C}_M$ of an assignment game $(\mathcal{I}_\mathcal{B}\cup\mathcal{I}_\mathcal{S}, v_M)$ is the following set:*

$$\mathcal{C}_M := \{(x',x'')\in\mathbb{R}^{N_\mathcal{B}}\times\mathbb{R}^{N_\mathcal{S}} \mid \textstyle\sum_{i\in\mathcal{I}_\mathcal{B}} x'_i + \sum_{j\in\mathcal{I}_\mathcal{S}} x''_j = v_M(\mathcal{I}_\mathcal{B}\cup\mathcal{I}_\mathcal{S}), x'_i + x''_j \geq v(i,j) \text{ for all } (i,j)\in\mathcal{I}_\mathcal{B}\times\mathcal{I}_\mathcal{S}\}. \tag{4.3}$$

□

Remark 4 (Non-emptiness of core [98])**.** *An assignment game (as in Definition 13) has a non-empty core.* □

We note that the core of an assignment game is defined by two sided pair coalitions instead of all possible coalitions in (2.2), which considerably reduces the complexity of solving an assignment game. Thus, an assignment game presents a more practical approach, compared to the general coalitional game theory, towards formulating a bilateral P2P market.

For an optimally matched pair $(i,j)\in\mathcal{I}_\mathcal{B}\times\mathcal{I}_\mathcal{S}$, the payoff (x'_i, x''_j) determines the contract price $\lambda_{i,j}$. In a bilateral trade, buyer i pays to seller j the difference of the price they initially offered and his payoff, i.e., $\lambda_{i,j} = h_{i,j} - x'_i$. For brevity, in the sequel, we use the collective payoff vector for buyers and sellers, i.e., $x = \mathrm{col}(x', x'')$, where $x'\in\mathcal{I}_\mathcal{B}$ and $x''\in\mathcal{I}_\mathcal{S}$.

We remark that, for P2P markets modelled as coalitional games it is not plausible to adopt centralized methods for computation of a payoff in the core because the core set is not singleton and different core payoffs can favor different sides (buyers or sellers) of the market. In fact, each core set has a buyer optimal and a seller optimal point as the two extremes [98]. Thus, it raises the possibility of biased behavior of the central operator which may jeopardize the confidence of market participants. Furthermore, in practice, bilateral agreements should be directly negotiated by the self-interested agents. Thus, in Section 4.4 we propose a distributed solution mechanism in which the agents negotiate to autonomously reach a mutual agreement, i.e., consensus on the payoff vector and consequently on the trading prices as in (2.8).

4.3. P2P MARKET AS AN ASSIGNMENT GAME

4.3.1. MODELLING

In this section, we present two setups of a bilateral P2P energy market as assignment games namely, single-contract market and multi-contract market. The participants of

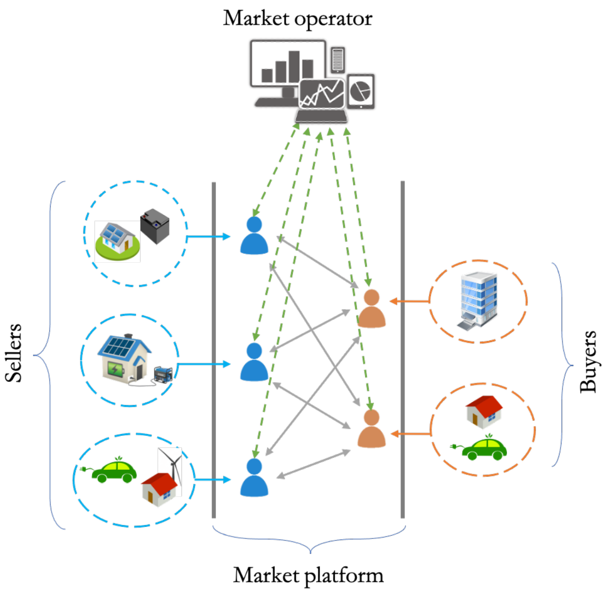


Figure 4.1: Illustrative scheme of a bilateral P2P energy market.

the market are partitioned into buyers and sellers where, a seller is a prosumer who owns an energy source including renewable (RES) and/or energy storage (ES) with an excess energy available, for a trading period, while a buyer can be a mere consumer as well. The market is operated by a central coordinator (market operator) who has complete information of buying and selling bids, and is also responsible for maximizing the overall market welfare. In Figure 4.1, we illustrate the high level concept of the proposed P2P energy market structure.

Let c_j denote the valuation of a seller $j \in \mathcal{I}_{\mathcal{S}}$ for each unit (e.g. 1 KWh) of energy and let s_j represent the total energy offered; then, an offer of a seller j is given by a pair (c_j, s_j). Similarly, we denote the energy demand of a buyer $i \in \mathcal{I}_{\mathcal{B}}$ by d_i and his valuation for the energy offered by seller j by $\alpha_{i,j} p_i$ where, $\alpha_{i,j}$ is the preference factor assigned by buyer i to seller j. The preference factor allows a buyer to differentiate between the offered energy and can depend on several metrics, such as source of energy (green vs. brown), location of the seller, user rating, etc. Furthermore, p_i is a base price that the buyer i is willing to pay for each unit of energy, hence they present their bid as $(\alpha_{i,j} p_i, d_i)$. Next, we impose some practical limitations on the valuations of buyers and sellers to make our P2P market setup economically appealing for the participants.

Prosumers generally sell energy to the grid via a retailer, thus an offer higher than the retailer's remuneration makes it favorable for the sellers to join the P2P market instead. Furthermore, the rationality of the buyer demands that his offers are not higher than the cost of energy from the grid. Let g_b and g_s denote the buying price and the selling price of energy provided by the grid, respectively. Then, the buyer i should offer a seller j a higher energy price than that of the grid, but not more than the grid's selling price, i.e.,

$$\alpha_{i,j} p_i \in (g_b, g_s]. \tag{4.4}$$

Analogously, we require the selling price of seller j to fulfill the similar limitations,

$$c_j \in [g_b, g_s). \tag{4.5}$$

We remark that similar assumptions are also made in [110]. These assumptions are reasonable as the feed-in tariffs have seen a decreasing trend and were also discontinued in some regions [109]. Nevertheless, if the feed-in tariff offered by the grid is very high, then it will impact the prosumer participation in the P2P market.

4.3.2. SINGLE-CONTRACT MARKET

In a single-contract P2P market setup, the buyers and sellers make one-to-one bilateral contracts that generate certain utility (value) for both. Let buyer $i \in \mathcal{I_B}$ and seller $j \in \mathcal{I_S}$ make a bilateral contract; then, the contract generates the value,

$$v(i,j) = \max\{0, \alpha_{i,j} p_i - c_j\} \min\{s_j, d_i\} \tag{4.6}$$

Let us elaborate on the formulation of the bilateral contract value in (5.3). First, the contract is only viable when buyer's valuation of the energy is higher than seller's demand, i.e., $\alpha_{i,j} p_i > c_j$. If $s_j \geq d_i$, then the welfare generated by each traded unit is given by $\alpha_{i,j} p_i - c_j$ where, the total traded units are d_i. Now after the bilateral contract, the excess energy of the seller $(s_j - d_i)$ is sold to the grid. Analogously if $d_i > s_j$. We note that, the value of a non-viable contract will be zero.

Due to the bilateral structure of our P2P market, we can express the worth of possible contracts in a matrix form, which further allows us to model the market welfare maximization as an assignment problem. Let $M = [v(i,j)]_{(i,j)\in\mathcal{I_B}\times\mathcal{I_S}}$ be an assignment matrix where each element $v(i,j)$ represents the value of a bilateral contract between buyer i and seller j. Then, we denote the corresponding assignment game by $\mathcal{M} = (\mathcal{I}_B \cup \mathcal{I}_S, v_M)$. The resulting value function of an assignment game $v_M(S)$ utilized by the market operator is given by the following assignment problem, for each $S \subseteq \mathcal{I}$:

$$\mathbb{P}(S): \begin{cases} \max\limits_{\mu} \sum\limits_{i\in\mathcal{I_B}\cap S} \sum\limits_{j\in\mathcal{I_S}\cap S} v(i,j)\mu_{i,j} \\ \text{s.t.} \sum\limits_{i\in\mathcal{I_B}\cap S} \mu_{i,j} \leq 1 \qquad \forall j \in \mathcal{I_S}\cap S \\ \quad\ \sum\limits_{j\in\mathcal{I_S}\cap S} \mu_{i,j} \leq 1 \qquad \forall i \in \mathcal{I_B}\cap S \end{cases} \tag{4.7}$$

with matching factors $\mu_{i,j} \in \{0,1\}$, where $\mu_{i,j} = 1$ represents the matching between buyer i and seller j. The problem (5.4) determines the optimal assignment of buyers to sellers and the constraints imposed on the matching factors ensure that one buyer is matched to only one seller, i.e., one-to-one matching. By using the results of the assignment problem in (5.4) the market operator can evaluate the core of the game, as in (2.2). We note that, even though the assignment problem in (5.4) is a combinatorial optimization problem, because of its special structure it can be solved in polynomial time using specifically designed algorithms like the Hungarian algorithm. Next, we list the notable features of our bilateral P2P market design:

- Existence: There always exist a set of bilateral contracts which is satisfactory for all of self-interested participants. In other words, the core of a bilateral P2P energy market is always non-empty (Remark 4).
- Product differentiation: Buyers can prioritise sellers or the categories of sellers via preference factors $\alpha_{i,j}$, based on the desired criteria (e.g. green energy).
- Mechanism properties: Market formulation ensures the desirable economic properties of the clearing mechanism.
- Social optimality: The bilateral contracts maximize the overall welfare of the market and the contract price is negotiated internally between buyers and sellers.

4.3.3. Multi-contract market

The formulation of a P2P market presented in Section 4.3.2 is a one step single-contract bilateral market where each buyer can make an energy trade with only one seller and vice versa. Therefore, even though the proposed formulation maximizes the overall welfare, the market participants on both sides can have partially fulfilled energy trades. Hence, in this section we extend the single-contract P2P energy market to accommodate multiple contracts between buyers and sellers which in turn allows for the complete fulfilment of energy trades.

For an assignment market, we model multiple contracts between buyers and sellers by granulation of energy demand or offered into the units (packets) of fixed size (e.g. 1 KWh). Consequently, the matching takes place between these units of energy. Another way of looking at this setup is that each market participant (buyer or seller) is represented in the market by multiple agents, with each agent offering or demanding single unit of energy. Hence, the number of agents representing each participant in the market are equal to the number of energy units offered/demanded. To provide further flexibility, in our multi-contract model, we allow participants to associate different trading characteristics (e.g. valuation, energy source) to each traded unit. For example, a seller can offer energy units from RES (green) and an energy storage (possibly brown); similarly a buyer can bid higher for the energy needed for the critical tasks and lower for the deferrable tasks. In the mathematical formulation, we interpret each agent as an independent seller or buyer, thus the resulting value $v(i,j)$ generated by contracts is similar to the expression in (5.3) for single traded unit ($d_i = s_i = 1$ unit), i.e., $v(i,j) = \max\{0, \alpha_{i,j} p_i - c_j\}$, and the corresponding assignment game $\mathcal{M} = (\mathcal{I}_B \cup \mathcal{I}_S, v_M)$ is solved using the assignment problem in (5.4).

In the multi-contract setup, in addition to maximizing the overall welfare, we can maximize the energy traded inside the bilateral P2P energy market by varying the level of granulation. Specifically, if all the contracts are viable and $\sum_{i\in\mathcal{I}_B} d_i > \sum_{j\in\mathcal{I}_S} s_j$, then by selecting the appropriate size of single energy unit, we can ensure that the total energy offered will be traded inside the P2P market, bilaterally.

The additional features of this multi-contract setup, however come at the cost of higher computational burden due to increased number of agents, representing the trade

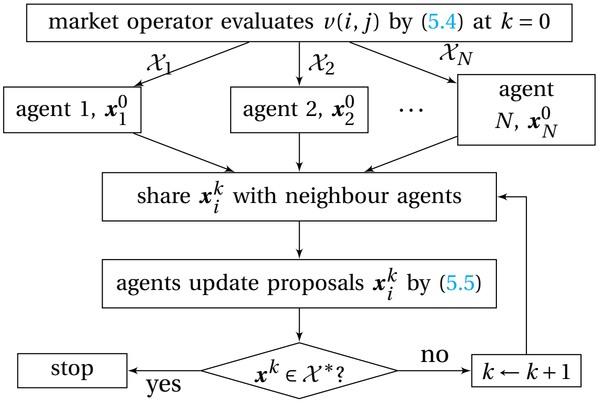


Figure 4.2: Flowchart of our proposed bilateral P2P market mechanism.

of each energy unit. We note that the appropriate size of the energy unit, decided by the market operator, can limit the number of agents and the associated computational burden. Furthermore, single-contract and multi-contract setup can be deployed in different contexts. For example, the former is more suitable for implementation in the larger scales, whereas, the later can bring additional features for localized implementation with lower number of participants such as in energy communities.

4.4. DISTRIBUTED SOLUTION MECHANISM

After the market operator solves (5.4), in the second stage of our design, the participants negotiate among themselves for a bilateral agreement on the trading price. Here, our goal is to enable the participants to autonomously reach a consensus on a set of bilateral contract prices such that no party can raise any objection on the contracts. Therefore, we propose a novel negotiation mechanism that allows for faster convergence rates, thus it is suitable for both single and multi-contract market setups. We model our algorithm in a distributed architecture, where a central market operator with complete information of the game initially transmits information of the bounding sets in (2.1) to the respective agents (market participants). The knowledge of a bounding set implies that each agent knows the values of their own coalitions only. After receiving the required information, each agent distributedly proposes a payoff allocation for all the agents. We prove that even with the partial information available, the proposed solution mechanism converges to a stable payoff distribution. The mechanism for the proposed bilateral P2P electricity market is detailed in Figure 4.2. In particular, we design a distributed fixed-point algorithm, using which the agents can reach consensus (2.8) on a payoff distribution in the core of the P2P market in (5.4).

4.4.1. DISTRIBUTED NEGOTIATION MECHANISM

We consider a bilateral negotiation process in which, at each negotiation step k, a buyer (seller) can communicate with a set of *neighbouring* sellers (buyers) to bargain for their payoff. Therefore, we model their communication over a time-varying network represented by a bipartite graph $G^k = (\mathcal{I}_{\mathcal{B}} \times \mathcal{I}_{\mathcal{S}}, \mathcal{E}^k)$, where for $i \in \mathcal{I}_{\mathcal{B}}$ and $j \in \mathcal{I}_{\mathcal{S}}$, $(i,j) \in \mathcal{E}^k$ means that there is an active link between buyer i and seller j at iteration k and they are then referred as neighbours.

We assume that at each iteration k an agent i observes only the proposals of its neighbouring agents. Furthermore, we assume that each buyer-seller pair communicates at least once during a time period of length Q (arbitrarily large), which ensures that the agents communicate sufficiently often. In other words, we assume that the union of the communication graphs over a time period of length Q is connected. This assumption is fairly common in multi-agent coordination, e.g. [70, Assumption 3.2].

Assumption 9 ($Q-$connected graph)**.** *For all $k \in \mathbb{N}$, the union graph $(\mathcal{I}_{\mathcal{B}} \times \mathcal{I}_{\mathcal{S}}, \cup_{l=1}^{Q} \mathcal{E}^{l+k})$ is strongly connected for some integer $Q \geq 1$.* □

4

The edges (links) in the communication graph G^k are weighted using an adjacency matrix $W^k = [w_{i,j}^k]$, whose element $w_{i,j}^k$ represents the weight assigned by agent i to the payoff proposal of agent j, $\boldsymbol{x}_j^k$ where, for some j, $w_{i,j}^k = 0$ implies that the agent i does not negotiate with agent j at iteration k, i.e., $(i,j) \notin \mathcal{E}^k$. We note that in a P2P market the buyers (sellers) do not negotiate among themselves hence, $w_{i,j}^k = 0$, for all $(i,j) \in \mathcal{I}_{\mathcal{B}}$ $(\mathcal{I}_{\mathcal{S}})$. Furthermore, to ensure that all the agents have sufficient influence on the resulting payoff distribution, we assume the adjacency matrix to be doubly stochastic with positive diagonal, which means that an agent always gives some weight to his previous proposal.

Assumption 10 (Stochastic adjacency matrix)**.** *For all $k \geq 0$, the adjacency matrix $W^k = [w_{i,j}^k]$ of the communication graph G^k is doubly stochastic, i.e., $\sum_{j=1}^{N} w_{i,j} = \sum_{i=1}^{N} w_{i,j} = 1$, its diagonal elements are strictly positive, i.e., $w_{i,i}^k > 0$, for all $i \in \mathcal{I}$ and $\exists\, \gamma > 0$ such that $w_{i,j}^k \geq \gamma$ whenever $w_{i,j}^k > 0$ [70, Assumption 3.3].* □

We further assume that the elements of the communication matrix W^k take values from a finite set hence, finitely many adjacency matrices are available.

Assumption 11 (Finitely many adjacency matrices)**.** *The adjacency matrices $\{W^k\}_{k\in\mathbb{N}}$ of the communication graphs belong to $\mathcal{W}$, a finite family of matrices that satisfy Assumption 17, i.e., $W^k \in \mathcal{W}$ for all $k \in \mathbb{N}$.* □

This assumption on the adjacency matrices is purely technical and allows us to exploit important results from the literature, for proving convergence of our negotiation mechanism. We remark that the set of adjacency matrices can be arbitrarily large hence Assumption 15 poses no practical limitation on our negotiation mechanism, which we propose next.

At each negotiation step k, an agent i bargains by proposing a payoff distribution $\boldsymbol{x}_i^k \in \mathbb{R}^N$, for all the agents. To evaluate a proposal, they first take an average of the estimates of neighboring agents, $\boldsymbol{x}_j^k$ such that $(i,j) \in \mathcal{E}^k$, weighted by an adjacency matrix W^k, $\sum_{j=1}^{N} w_{i,j}^k \boldsymbol{x}_j^k$. Next, agent i utilizes a partial game information in the form of a bounding half-space $H_i^k \in \mathcal{H}_i$ as in (4.2) of a bounding set in (2.1). An agent selects the half-spaces from the set $\mathcal{H}_i$ such that each bounding half-space appears at-least once in every Q negotiation steps with Q as in Assumption 9. In practice, one way of selecting these half-spaces can be a predefined sequence that is arbitrarily chosen by each agent.

Finally, agent i projects the average $\hat{\boldsymbol{x}}_i^k := \sum_{j=1}^{N} w_{i,j}^k \boldsymbol{x}_j^k$ on the bounding half-space. Thus, the algorithm reads as

$$\boldsymbol{x}_i^{k+1} = \mathrm{proj}_{H_i^k}(\hat{\boldsymbol{x}}_i^k). \tag{4.8}$$

The protocol in (4.8) allows agents to propose a payoff at each negotiation step that is acceptable for them. Let us further generalize the iteration in (4.8) by replacing the projection operator, proj(·), with a special class of operators namely, paracontractions. This generalization enables the agents to choose any paracontraction operator T_i^k, for evaluating a payoff proposal $\boldsymbol{x}_i^k$, which in turn allows for a faster convergence to the interior of the core in (4.3). The latter is an important feature because the interior of the core is associated with the fairness of the payoff in assignment games. Specifically, for each $i \in \mathcal{I}$, we propose the negotiation protocol $\boldsymbol{x}_i^{k+1} = T_i^k(\hat{\boldsymbol{x}}_i^k)$, that in collective form, reads as the fixed-point iteration

$$\boldsymbol{x}^{k+1} = \boldsymbol{T}^k(\boldsymbol{W}^k \boldsymbol{x}^k), \tag{4.9}$$

where $\boldsymbol{T}^k(\boldsymbol{x}) := \mathrm{col}(T_1^k(\boldsymbol{x}_1), \dots, T_N^k(\boldsymbol{x}_N))$ and $\boldsymbol{W}^k := W^k \otimes I_N$ represents an adjacency matrix. In (5.5), we require the paracontraction operator T_i^k to have $H_i^k \in \mathcal{H}_i$ in (4.2) as fixed-point set, i.e., $\mathrm{fix}(T_i^k) = H_i^k$.

Assumption 12 (Paracontractions)**.** *For $k \in \mathbb{N}$, $\boldsymbol{T}^k$ in (5.5) is such that $T_i^k \in \mathcal{T}$, where $\mathcal{T}$ is a finite family of paracontraction operators such that* $\mathrm{fix}(T_i) = H_i$ *with $H_i \in \mathcal{H}_i$ in (4.2).* □

Here, for utilizing the negotiation mechanism in iteration (5.5), an agent can choose any operator T_i that satisfies Assumption 12. This choice can affect the speed of convergence, as demonstrated in Section 4.5, and also the specific limit point inside the core. Examples of paracontractions include the projection on a closed convex set C, $\mathrm{proj}_C(\cdot)$, and the convex combination of projection and over-projection operators, i.e., $T = (1-\beta)\mathrm{proj}_C(\cdot) + \beta \mathrm{overproj}_C(\cdot)$ with $\beta \in [0,1)$.
We also assume that each $\boldsymbol{T}^k \in \mathcal{T}^N$ appears at least once in every Q iterations of (5.5), with Q as in Assumption 9.

Assumption 13. *Let Q be the integer in Assumption 9. The operators $(\boldsymbol{T}^k)_{k\in\mathbb{N}}$ in (5.5) are such that, for all $n \in \mathbb{N}$, $\bigcup_{k=n}^{n+Q}\{\boldsymbol{T}^k\} = \mathcal{T}^N$, with $\mathcal{T}$ as in Assumption 12.* □

Next, we formalize our main convergence result for the negotiation mechanism in (5.5).

Theorem 5 (Convergence of negotiation mechanism)**.** *Let Assumptions 9–13 hold. Let $\mathcal{X}^* := \mathcal{A} \cap \mathcal{C}_M^N$ with $\mathcal{A}$ as in (2.8) and $\mathcal{C}_M$ being the core in (4.3). Then, starting from any $\boldsymbol{x}^0 \in \mathbb{R}^{N^2}$, the sequence $(\boldsymbol{x}^k)_{k=0}^{\infty}$ generated by the iteration in (5.5) converges to some $\bar{\boldsymbol{x}} \in \mathcal{X}^*$.*

□ We provide the proof of Theorem 6 in Appendix. We remark that, presenting the mechanism as a fixed-point iteration and in terms of operators allows us to utilize results from operator theory to keep our convergence analysis general and brief.

4.4.2. Technical discussion

Theorem 6 shows that the repeated proposals by all agents, generated by our negotiation mechanism, eventually reach an agreement on a payoff that belongs to the intersection of the bounding sets, i.e., the core. This core payoff allows us to compute the *stable* contract prices. Let the payoff of buyer i and seller j be x_i and x_j respectively then, the contract price is $\lambda_{i,j} = \alpha_{i,j} p_i - x_i$. We note that the core of an assignment game has a special structure with two extreme points, i.e. buyer optimal and seller optimal at its boundary. A buyer optimal payoff is the worst core payoff for the seller side and vice versa. Thus, the payoff in the interior of the core corresponds to a fairer allocation and a consensus on such allocation can be achieved via the proposed algorithm in (5.5).

Let us now mention the features of our algorithm that enhance its practicality.First, for the negotiation, the market participants do not require full information of the game but only the values of their own contracts represented by the bounding sets in (2.1), which is privacy preserving. Such a lower information requirement of our mechanism is a considerable benefit over the algorithm presented in [13], which requires each participant to have complete information of the corresponding core set in (2.2). Secondly, utilizing the half-spaces $H_i \in \mathcal{H}_i$ as the fixed-point sets of the operators $T_i \in \mathcal{T}$ in (5.5) allows us to design $\mathcal{T}$ as a set of linear operators. For example, let $\mathbf{e}_k$ be the vector of coefficients of the inequality that defines the bounding half-space, i.e., $H_i^k = \{y \in \mathbb{R}^N \mid \mathbf{e}_k^\top y \geq \eta\}$. Then, we can write the iteration in (4.8) as

$$\boldsymbol{x}_i^{k+1} = \hat{\boldsymbol{x}}_i^k + \frac{\eta - \mathbf{e}_k^\top \hat{\boldsymbol{x}}_i^k}{\|\mathbf{e}_k\|^2} \mathbf{e}_k,$$

if $\hat{\boldsymbol{x}}_i^k \notin H_i^k$ [12, Example 28.16]. This closed-form expression reduces the computational burden of our algorithm greatly since no optimization problem should be solved at each iteration. Clearly, both privacy preservation and low computational burden are highly desirable features of a market mechanism.

4.5. Numerical simulations

In this section, we simulate the proposed bilateral P2P energy market with prosumers employing the negotiation mechanism designed in Section 4.4. We conduct the analysis for the time slots that incur peak prices and have considerable PV generation and compare it with the conventional approach of trading with the grid via aggregators and retailers, to demonstrate the effectiveness of our algorithm and show the economic benefits for the prosumers. Next, the size of the time slots should be decided by the market operator considering the variation in the demand and production of energy at prosumer level. For our economic analysis, we use hourly time slots of four peak hours for each day over a week and for convergence analysis we consider 100 scenarios of prosumer demand and generation to report their average and spread of samples. Furthermore, as our focus is on the economic and algorithmic design of the market mechanism, we do not consider network constraints (as also done in [109], [112]) and remark that their incorporation would not effect the resulting market properties.

Table 4.2: Profiles of buyers and sellers

Buyers	Concern for environment	Concern for rating	Sellers	Energy source
B1	✓	×	S1	PV (green)
B2	×	×	S2	Storage (brown)
B3	×	✓	S3	PV (green)
B4	✓	✓	S4	fossil (brown)

4.5.1. SIMULATION SETUP

We consider 4 residential prosumers with energy deficiency and 4 with surplus to act as buyers and sellers, respectively for each time slot. During different sessions of the market, prosumers can vary between the roles of sellers and buyers, depending on their energy profiles. However, during each session (time slot) a prosumer acts as either a buyer or a seller. The energy deficiency and surplus of each prosumer lies within the range of [2, 8]. Next, we purposefully build the profiles of prosumers to show diverse participation and to emphasize various features of our P2P market designs. To buy energy for a given time-slot, a buyer i enters a P2P market with its bid and demand $(\alpha_{i,j} p_i, d_i)$ as in (5.3) where, the factor $\alpha_{i,j}$ represents his preference valuation for the energy offered by seller j. For the design of preference factor, we let buyer specify his level of environmental concern (preference to the green energy) α_i^{g} on the scale of $\{0,\cdots,5\}$ and his concern to seller's user rating $\alpha_j^{\mathrm{r}} \in \{0,\cdots,5\}$ by $\gamma_i^{\mathrm{r}} \in \{0,1\}$, with 0 being indifference to the associated factor. Let us indicate the energy type of seller j by $\gamma_j^{\mathrm{g}} \in \{0,1\}$ with 1 specifying green energy then, the preference factor is evaluated as $\alpha_{i,j} = 1 + 0.1(\alpha_i^{\mathrm{g}}\gamma_j^{\mathrm{g}} + \alpha_j^{\mathrm{r}}\gamma_i^{\mathrm{r}})$. The value of α_i^{g} is randomly chosen for the buyers who include the environmental concern in their profiles, given in Table 4.2, and the consumer rating of each seller is chosen randomly from the range of $[3,5]$. We note that our design of preference factor is arbitrary and the market operator can design it differently to include other considerations.

The buyers choose base valuation of the energy p_i such that their bid is higher than the grid's buying price $g_{\mathrm{b}} = 0.05$ £/kWh and not more than the grid's selling price $g_{\mathrm{s}} = 0.17$ £/kWh as in condition (4.4) [44]. Furthermore, the sellers choose their valuation c_j less than the grid's selling price g_{s} as in (4.5).

4.5.2. BILATERAL P2P ENERGY MARKET

In our P2P market setup, at the first stage, the market operator performs the optimal matching as formulated in (5.4), which results in the optimal buyer-seller pairs. Next, the participants adopt the negotiation mechanism in (5.5) to mutually decide the bilateral contract prices. In Figure 5.1, we show, for a particular time slot, the convergence of our negotiation algorithm using the operators $\mathrm{proj}_{H^k}(\cdot)$ and $T_{H^k} := (1-\beta)\mathrm{proj}_{H^k}(\cdot) + \beta\mathrm{overproj}_{H^k}(\cdot)$, for single-contract and multi-contract market setups. We report the average of 100 samples of convergence trajectories obtained by varying energy offer and demand conditions and the sequence of half-spaces, i.e., negotiation strategy of each

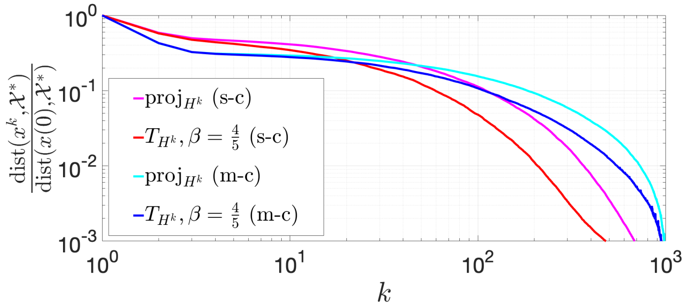


Figure 4.3: Trajectories of $\text{dist}(\boldsymbol{x}^k, \mathcal{X}^*)/\text{dist}(\boldsymbol{x}^0, \mathcal{X}^*)$ for distributed bilateral negotiation with operators proj_{H^k} and $T_{H^k} := (1-\beta)\text{proj}_{H^k}(\cdot) + \beta\text{overproj}_{H^k}(\cdot)$ for single-contract (s-c) and multi-contract (m-c) market setups.

4

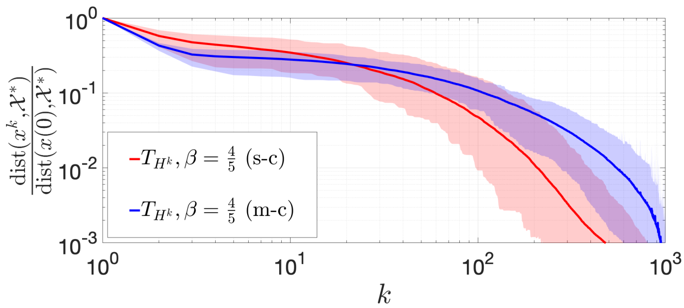


Figure 4.4: Sampled average of the trajectories in Figure 5.1, with spread of samples shown by shaded region.

agent (see Section 5.4). We can observe that the operator T_{H^k} results in the faster convergence, as we claimed in Section 4.4. We remark that the convergence speed of negotiation in a multi-contract market decreases with an increase in the level of energy granulation. In Figure 4.4 we plot the spread of the sample trajectories to illustrate the best and worst convergence scenario for both single-contract and multi-contract setups using the operator T_{H^k}. Next, we benchmark the computational performance of our algorithm. We choose a static case of a distributed bargaining algorithm proposed in [72] for payoff allocation in coalitional games, as a benchmark. In Figure 4.5, we present the trajectories of our algorithm and the benchmark. Since, the benchmark algorithm utilizes the whole bounding set instead of just the bounding half space in (4.2) at each iteration, it proceeds faster initially. However, in the long run, our proposed algorithm performs better. Lower information requirement in our approach makes the execution of a negotiation step considerably faster. In this simulation scenario, the average execution time of a negotiation step is about 40× times faster than the benchmark.

Now, to evaluate the economic benefit for prosumers we setup P2P markets for peak hours and show the average change in revenues and costs of sellers and buyers, respectively, in Figure 5.3. The revenues of sellers are higher and the costs of buyers are lower in both the market setups compared to the trade with the grid/retailer. We also observe that for the market with similar category of participants (e.g. residential) and adequate participation from both sides (buyers and sellers), the economic performance

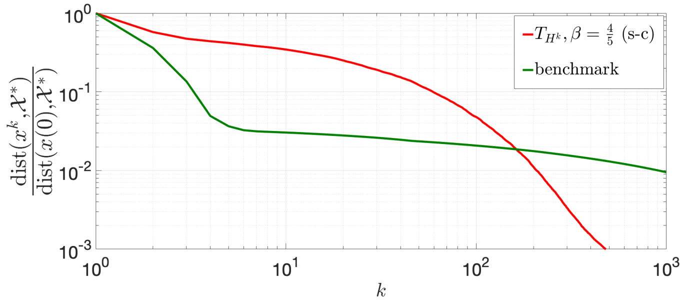


Figure 4.5: Comparison of the trajectories of dist$(\boldsymbol{x}^k, \mathcal{X}^*)$/dist$(\boldsymbol{x}^0, \mathcal{X}^*)$ for our distributed bilateral negotiation (red) and the benchmark (green).

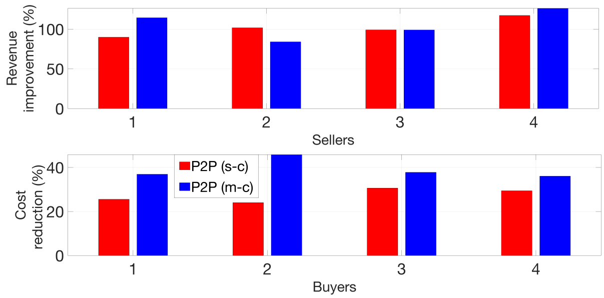


Figure 4.6: Average revenue improvement (sellers) and average cost reduction (buyers) in single-contract (s-c) and multi-contract (m-c) P2P setups compared to trading with the grid.

of single-contract and multi-contract markets is comparable. This is due the fact that similar excesses and demands of energy reduces the trade outside the P2P market. We also remark that moving from single-contract to a multi contract market can maximize the total amount of energy traded inside P2P market hence increasing the overall market welfare but it does not guarantee individual improvements for all parties. This is because a random payoff inside the core set in (4.3) can assign a higher share of the value generated by a buyer-seller pair to the either side of the market. For instance, this can be observed in Figure 5.3 for seller 2. In Figure 4.7, we observe that the proposed P2P market designs strongly encourage prosumer participation in bilateral energy trading and, in particular, the multi-contract setup increases the internal energy trade.

Now, observe from (5.4) that allowing for product differentiation (e.g. on environmental or social basis) can increase the overall social welfare of the market compared to mere economic considerations. This increase comes from the higher user satisfaction which is achieved by catering to their personal preferences. Therefore, we define a metric of user satisfaction as the number of times the buyers with green energy preferences are matched to the green sellers in 100 scenarios of our study. In Figure 4.8, we compare this user satisfaction with a traditional market that only considers economic factors. The figure shows that product differentiation offers higher user satisfaction and thus encourages prosumer participation.

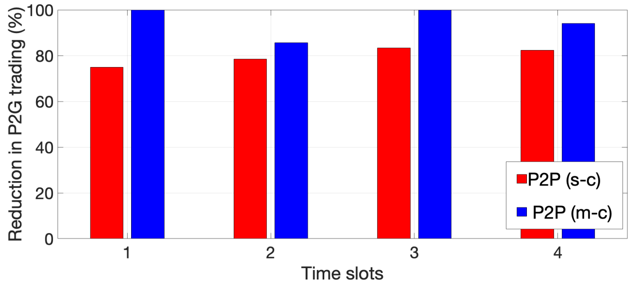


Figure 4.7: Reduction in energy traded with the grid via single-contract (s-c) and multi-contract (m-c) P2P market setups.

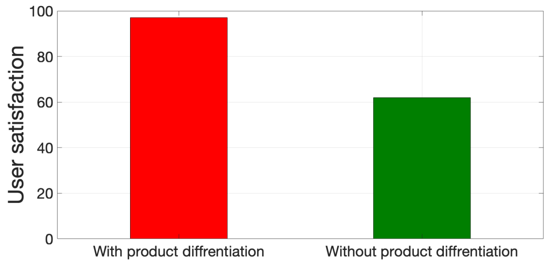


Figure 4.8: User satisfaction for the buyers with a preference for green energy in P2P market setups with and without product differentiation.

Finally, in Table 4.3, we present the computational times of an agent's negotiation process in proposed P2P market mechanism to numerically show the scalability with respect to the market size. We note that because of the distributed implementation the negotiation protocol of agents run in parallel on their personal computational resource. Here, the simulations are executed in MATLAB 2020b installed on a laptop computer with 2.3 GHz Intel Core i5 and 8GB RAM. These numerical results can help in deciding how far ahead in time from the actual energy delivery should such markets operate, depending on the expected level of prosumer participation (e.g. the size of energy community). Furthermore, we can conclude that the adoption of the assignment game formulation provides opportunity for the practical implementation of reasonably large P2P markets while guaranteeing contract prices that represent a competitive market equilibrium. We note that the regulator can also decide the market size systematically, e.g. on geographical basis and also put the eligibility criteria on the power capacity of installed generation source. Such regulatory restrictions are often imposed on trading mechanisms for example, in Queensland, Australia, a prosumer cannot participate in a feed-in-tariff program if they have solar panels beyond 5kW capacity [113].

4.6. CONCLUSION

We have formulated P2P energy trading as an assignment game (coalitional game) over time-varying communication networks and proposed a novel distributed negotiation al-

Table 4.3: Avg. negotiation time per agent with market size

Agents	40	60	80	100
Negotiation time (seconds)	4.1437	10.5930	19.9979	43.9713

gorithm as a clearing mechanism that guarantees stable trading prices in a coalitional game theoretic sense and satisfies the desired economic properties.

The proposed bilateral P2P energy market designs namely, single-contract and multi-contract, encourage prosumers to participate by making P2P trading a favourable choice, considering their economic and social priorities. Furthermore, enabling product differentiation increases user satisfaction and allows for a higher overall market welfare. Finally, the negotiation mechanism via paracontraction operators enables faster convergence to a consensus on a set of bilateral contract prices that represent a competitive equilibrium and belong to the core.

An interesting extension of our work would be the design of online mechanisms for real-time markets where the core set varies over time, thus accommodating for the short-term uncertainty in RES generation and demand.

4.7. APPENDIX

To prove the convergence of (5.5), as stated in Theorem 6, we first provide useful results regarding paracontractions.

Lemma 8 ([27], Thm. 1)**.** *Let $\mathcal{T}$ be a finite family of paracontractions such that $\bigcap_{T\in\mathcal{T}} \mathrm{fix}(T) \neq \varnothing$. Then, the sequence $(\boldsymbol{x}^k)_{k\in\mathbb{N}}$ generated by $\boldsymbol{x}^{k+1} := T^k(\boldsymbol{x}^k)$ converges to a common fixed-point of the paracontractions that occur infinitely often in the sequence.*

□

Lemma 9 (Doubly stochastic matrix ([32], Prop. 5))**.** *If W is a doubly stochastic matrix then, the linear operator defined by the matrix $W \otimes I_n$ under Assumption 17 is a paracontraction with respect to the mixed vector norm $\|\cdot\|_{2,2}$.* □

Lemma 10 (Composition of paracontracting operators ([32], Prop. 1))**.** *Suppose $T_1, T_2 : \mathbb{R}^n \to \mathbb{R}^n$ are paracontractions with respect to same norm $\|\cdot\|$ and $\mathrm{fix}(T_1) \cap \mathrm{fix}(T_2) \neq \varnothing$. Then, the composition $T_1 \circ T_2$ is a paracontraction with respect to the norm $\|\cdot\|$ and $\mathrm{fix}(T_1 \circ T_2) = \mathrm{fix}(T_1) \cap \mathrm{fix}(T_2)$.* □

Lemma 11 (Stacked vector of paracontractions ([32], Prop. 4))**.** *Suppose each map $T_1, \ldots, T_m$ is a paracontraction with respect to $\|\cdot\|_2$. Then, the map $\boldsymbol{T} := \mathrm{col}(T_1, \ldots, T_N)$ is a paracontraction with respect to $\|\cdot\|_{2,2}$.* □

Using these properties, we now show that the sequence of operators generated by the iteration in (5.5) is a paracontraction and the set of its fixed-points is a consensus in the intersection of the fixed-point sets of the operators.

Lemma 12. *Let Q be the integer in Assumption 9. Let $\boldsymbol{T}_1, \ldots, \boldsymbol{T}_Q$ be paracontraction operators with $\bigcap_{r=1}^{Q} \mathrm{fix}(\boldsymbol{T}_r) =: C$ and let $W_Q W_{Q-1} \cdots W_1$ be the composition of the adjacency* *matrices where $W_r \in \mathcal{W}$, with $\mathcal{W}$ as in Assumption 15. Let $\boldsymbol{W}_r := W_r \otimes I_N$. Then, the composed mapping $\boldsymbol{x} \mapsto (\boldsymbol{T}_Q \boldsymbol{W}_Q \circ \cdots \circ \boldsymbol{T}_1 \boldsymbol{W}_1)(\boldsymbol{x})$*

(i) is a paracontraction with respect to norm $\|\cdot\|_{2,2}$;

(ii) $\text{fix}(\boldsymbol{T}_Q \boldsymbol{W}_Q \circ \cdots \circ \boldsymbol{T}_1 \boldsymbol{W}_1) = \mathcal{A} \cap C$,

where $\mathcal{A}$ is the consensus set in (2.8). □

Proof. (i): It follows directly from Lemmas 15 and 10.
(ii): By Lemmas 15 and 10, $\text{fix}(\boldsymbol{T}_Q \boldsymbol{W}_Q \circ \cdots \circ \boldsymbol{T}_1 \boldsymbol{W}_1) = \text{fix}(\boldsymbol{T}_Q) \cap \cdots \cap \text{fix}(\boldsymbol{T}_1) \cap \text{fix}(\boldsymbol{W}_Q) \cap \cdots \cap \text{fix}(\boldsymbol{W}_1)$. Again, by Lemmas 15, 10 $\bigcap_{r=1}^{Q} \text{fix}(\boldsymbol{W}_r) = \text{fix}(\boldsymbol{W}_Q \cdots \boldsymbol{W}_1)$ and since the composition $\boldsymbol{W}_Q \cdots \boldsymbol{W}_1$ is strongly connected, by the Perron-Frobenius theorem, $\text{fix}(\boldsymbol{W}_Q \cdots \boldsymbol{W}_1) = \mathcal{A}$. Furthermore, as $\bigcap_{r=1}^{Q} \text{fix}(\boldsymbol{T}_r) = C$, $\text{fix}(\boldsymbol{T}_Q \boldsymbol{W}_Q \circ \cdots \circ \boldsymbol{T}_1 \boldsymbol{W}_1) = \mathcal{A} \cap C$. □

4

With these results, we are now ready to prove Theorem 6.

Proof. (Theorem 6) Let us define the sub-sequence of $\boldsymbol{x}^k$ for all $k \in \mathbb{N}$ as $\boldsymbol{z}^t = \boldsymbol{x}^{(t-1)Q}$ for each $t \geq 2$ with Q being the integer in Assumptions 9 and 13. Then,

$$\boldsymbol{z}^{t+1} = \boldsymbol{T}^{tQ-1} \boldsymbol{W}^{tQ-1} \circ \cdots \circ \boldsymbol{T}^{(t-1)Q} \boldsymbol{W}^{(t-1)Q} \boldsymbol{z}^t \tag{4.10}$$

for $t \geq 2$. It follows from Lemma 11 and assertion 1 of Lemma 12 that the maps $\boldsymbol{x} \longmapsto (\boldsymbol{T}^{tQ-1} \boldsymbol{W}^{tQ-1} \circ \cdots \circ \boldsymbol{T}^{(t-1)Q} \boldsymbol{W}^{(t-1)Q})(\boldsymbol{x})$, $t \geq 2$ are all paracontractions. Also, under Assumption 15, there can be only finitely many such maps. Furthermore, by assertion 2 of Lemma 12, the set of fixed-points of each map is $\mathcal{X}^*$. Thus, by Lemma 8, the iteration in (4.10) converges to some $\bar{\boldsymbol{z}} \in \mathcal{X}^*$. □

5

Peer-to-Peer Electricity Market for Residential Prosumers

Coalitional game theory offers rigorous tools for modelling cooperative interaction of agents. In this chapter, we utilize those tools to model a bilateral peer-to-peer (P2P) energy trading scheme for residential prosumers with a simplified entry to the market. We formulate the market as an assignment game, a special class of coalitional games. For solving the resulting decision problem, we design a bilateral negotiation mechanism that enables matched buyer-seller pairs to reach a consensus on a set of "stable" and "fair" trading contracts. The proposed negotiation process can be executed on possibly time-varying communication networks with virtually minimal information requirements that in turn preserves privacy among prosumers. Numerical simulations illustrate the beneficial features of our P2P market model and negotiation protocol.

5.1. INTRODUCTION

Decarbonization of energy systems is one of the key agenda of the climate action plan and to achieve this goal, power systems are envisioning large scale integration of renewable energy sources (RES). Wide scale decentralized deployment of RES, especially photo-voltaic (PV), is being undertaken by the small prosumers (e.g. residential) which brings them at the center of this transformation [20]. Thus, many demand-side tools are being developed for the technical and economic integration of the residential prosumers.

Local or community based electricity markets can effectively facilitate the distributed deployment of RES by managing the associate uncertainty locally and by providing financial benefits. Therefore, such market based solutions have received considerable attention from smart-grid researchers, especially towards the peer-to-peer (P2P) market paradigm [111]. P2P markets provide prosumers with the direct control over the trade of their energy sources on their own terms of transactions to make profitable interactions. Thus, it encourages wider prosumer participation and also provides significant benefits to the system operators for example in terms of peak shaving [65], and lower investments in grid capacity [62].

However, the design of the local P2P electricity markets presents mathematical and structural challenges. The whole sale market in EU requires sellers to enter with the complex offers, which requires high level of technical abilities. Such structure cannot be replicated in the markets where the participants are laypersons (residential prosumers). Thus, the key tasks are to design: a mechanism which seeks a market equilibrium while incorporating a self-interested decision-making attitude by the participants and a structure simple enough to encourage the entry by residential prosumers. Therefore, in this chapter, we first design a bilateral P2P market that simplifies the entry of a typical residential prosumer and then we present an algorithm that enables prosumers to converge to a *fair* and *stable* market solution, in context of coalitional game theory.

Coalitional game theory provides analytical tools to study the cooperative interaction of selfish and rational agents and thus, holds adequate prospects for the design of P2P markets. The authors in [109] propose a P2P energy trading scheme in which prosumers form a coalition to trade energy among themselves, at a mid-market rate which ensures the stability of the coalition. In [44] the authors use coalitional game theory to formulate a community based architecture for local energy exchange to minimize overall energy cost. A coalition formation game is formulated by the authors in [110] for P2P energy exchange among prosumers, and the resulting coalition structure is shown to be stable. The price of exchange is determined by the double auction mechanism. Another coalition formation game is presented by the authors in [113], that allows prosumers to optimize their battery usage for P2P energy trading. In [52] the authors present an iterative procedure for peer matching among prosumers, who then undertake a bilateral negotiation to come to an agreement on the price and quantity of energy trade, but without coalitional game theoretic guarantees.

In this chapter, we model P2P energy trading as an assignment game, a special class of coalitional games, that allows for bilateral contracts and for a mutual settlement on the *fair* and *stable* contract prices via distributed negotiation. Broadly, we propose a bilateral P2P electricity market and a solution mechanism within the framework of coalitional game theory. Our key contributions are summarized next:

- We formulate bilateral P2P energy trading as an assignment game which simplifies the prosumer participation, amidst of RES uncertainty, and allows for product differentiation. Our formulation ensures the existence of a “stable" set of bilateral contracts (Section 5.3);
- We develop a distributed negotiation mechanism where the buyer (seller) communicates only with the matched seller (buyer) and the market operator, and we show that the mechanism converges to the τ-value in the core of the assignment game while preserving the privacy among the market participants (Section 4.4);

For basic notation see Appendix A and refer to Chapter 2, Section 2.2.2 for operator-theoretic definitions.

5.2. PAYOFF IN ASSIGNMENT GAMES

5

Assignment games (see detailed mathematical background in Chapter 4) are a class of coalitional games that model a two sided matching market with the objective of finding optimal assignments between the opposite sides for a bilateral trade [98]. The payoff (x'_i, x''_j) as a result of a bilateral trade between an optimally matched pair $(i, j) \in \mathcal{I}_\mathcal{B} \times \mathcal{I}_\mathcal{S}$ determines the bilateral contract price $\lambda_{i,j}$. The contract price is defined as the difference of the bid of buyer i and his payoff, i.e., $\lambda_{i,j} = h_{i,j} - x'_i$. A set of *efficient* and *rational* payoff vectors is called the core as in (4.3). However, the core set is not singleton and different core payoffs can favor different sides of the market [98]. The two extreme points of the core are referred as the buyer optimal payoff $(\overline{x}', \underline{x}'')$ and the seller optimal payoff $(\underline{x}', \overline{x}'')$. Therefore, it is important to identify a *fair* payoff that belongs to the core. Next, we provide one such *fair* payoff, namely the τ-value in context of assignment games [77].

The τ-value is generally defined as an average of the utopic payoff and the minimal rights payoff where, the utopia payoff is regarded as the maximal payoff an agent can receive in the core and the minimal rights payoff is what an agent can guarantee himself. In context of an assignment game $(\mathcal{I}_\mathcal{B} \cup \mathcal{I}_\mathcal{S}, v_M)$, the utopia payoff for a buyer i is given by his marginal contribution to the grand coalition, which is also the buyer optimal, i.e., $\overline{x}'_i = v_M(\mathcal{I}_\mathcal{B} \cup \mathcal{I}_\mathcal{S}) - v_M(\mathcal{I}_\mathcal{B} \cup \mathcal{I}_{\mathcal{S}\setminus\{i\}})$. Furthermore, the minimal rights payoff of buyer i that is optimally matched with seller j is given as $\underline{x}'_i = v_M(\mathcal{I}_\mathcal{B} \cup \mathcal{I}_{\mathcal{S}\setminus\{j\}}) - v_M(\mathcal{I}_{\mathcal{B}\setminus\{i\}} \cup \mathcal{I}_{\mathcal{S}\setminus\{j\}})$. Finally, the τ-value of an assignment game is as follows:

$$\tau(v_M) = \frac{(\overline{x}', \underline{x}'') + (\underline{x}', \overline{x}'')}{2} \tag{5.1}$$

Remark 5 (τ-value in core [77])**.** *The τ-value of an assignment game (as in Definition 13) belongs to the core.* □

Using the fact that the τ-value is a midpoint between the buyer-optimal and the seller-optimal core payoff, we can define a set of favorable payoffs for each side.

Definition 16 (Favorable payoff)**.** *For buyer i in an optimally matched pair (i, j), the set of favorable payoffs is*

$$\mathcal{X}_i := \{x'_i \in \mathbb{R} \mid x'_i \geq \frac{(\overline{x}'_i + \underline{x}'_i)}{2}, x'_i + x''_j = v(i,j)\}. \tag{5.2}$$

□

In the sequel, we associate the idea of fairness to the τ-value and use it as a solution concept for our bilateral P2P market design. In practice, bilateral agreements should be directly negotiated by self-interested agents. Thus, we propose a distributed solution mechanism in which the agents negotiate bilaterally to autonomously reach a consensus on the payoff.

5.3. P2P MARKET AS AN ASSIGNMENT GAME

In this section, we formulate a bilateral P2P energy market as assignment game where the participants are partitioned into buyers and sellers. A prosumer who owns an energy source is regarded as a seller while a buyer can be a mere consumer as well. For modelling a seller, we consider a typical residential prosumer who is not present at home during high PV generation hours on the weekdays. Therefore, it makes high economic sense for such prosumer to sell the energy produced in the market. Let us elaborate on the models of the market participants namely, sellers, buyers and the market operator. First, a sellers' offer is composed of a rated power of the generation source and the price per KWh of energy for the period of availability. Such offer structure, with easily known parameters, greatly simplifies the process of prosumer's entry into the market which in fact is an important practical requirement for enabling the participation of residential prosumers (layman) in P2P markets. Another way of looking at our model is that a seller offers to rent out his generation source for the desired time period. Our market design also creates an opportunity for the data markets in energy systems that allows participants to share their generation and demand data for additional financial or operational benefits. This data is then utilized by the market operator to optimize amidst of uncertainty. We note that our model can accommodate other energy sources as well (e.g. ES) but we maintain our focus on PV as it is most widely adopted RES at residential level.

5

A buyer enters the market with the energy demand, bid per KWh and the preference factor that allows a buyer to prioritise sellers based on the desired criteria (e.g. green energy, seller rating). Finally, the market is operated by a central operator who has complete information of bids and offers, and is also responsible for maximizing the market welfare.

Let c_j denote the price demanded by a seller $j \in \mathcal{I}_\mathcal{S}$ per KWh of energy and let s_j represent the rated power of the offered energy source; then, an offer of a seller j is given by a pair (c_j, s_j). Similarly, let us denote the energy demand of a buyer $i \in \mathcal{I}_\mathcal{B}$ by d_i and his preference factor for seller j by $\alpha_{i,j}$. Furthermore, let p_i be a base price that the buyer i is willing to pay for each unit (1 KWh) of energy, hence he presents its bid as $(\alpha_{i,j} p_i, d_i)$. Next, we impose some practical limitations on the buyer bids and seller offers to make our market setup economically rational for the participants. Let g_b and g_s denote the buying price and the selling price of energy provided by the grid, respectively. Then, the rational buyer i should offer a seller j a higher energy price than that of the grid, but not more than the grid's selling price, i.e., $\alpha_{i,j} p_i \in (g_\text{b}, g_\text{s}]$ and analogously, the seller j satisfies $c_j \in [g_\text{b}, g_\text{s})$.

The energy generation by RES (PV) is inherently uncertain thus to encourage prosumer participation we transfer the responsibility of accounting for this uncertainty from a seller to the market operator. We achieve this by allowing seller to only include the rated power of his energy asset instead of the energy. Therefore, a stochastic market mechanism is required. For this purpose we use scenario modeling of uncertainty in RES generation. Let the set of generation scenarios of future be $\mathcal{F}$ and denote the probability of occurrence of the scenario $f \in \mathcal{F}$ by ρ_f. Also, let us denote the generation forecast of seller j's energy source in scenario f by $\hat{s}_j(f)$ then the corresponding expected value is given by $\mathbb{E}[\hat{s}_j] = \sum_{f\in\mathcal{F}} \rho_f \hat{s}_j(f)$. We note that, without the loss of generality, the uncertainty can also be considered in demand. However, in this chapter we assume the demand forecast to be deterministic. Next, we formulate a P2P energy market as an assignment game.

In our P2P market setup, the sellers and buyers make bilateral contracts that generate certain utility (value) for both. Let buyer $i \in \mathcal{I_B}$ and seller $j \in \mathcal{I_S}$ make a bilateral contract then, the value $\hat{v}(i,j)$ generated by this contract reads as

$$\hat{v}(i,j) = \begin{cases} (\max\{0, \alpha_{i,j} p_i - c_j\}) d_i & \text{if } \mathbb{E}[\hat{s}_j] \geq d_i \\ (\max\{0, \alpha_{i,j} p_i - c_j\}) \mathbb{E}[\hat{s}_j] & \text{otherwise.} \end{cases} \tag{5.3}$$

5

Let us elaborate on the bilateral contract value in (5.3). First, the contract is only viable when buyer's bid of the energy is higher than seller's offer, i.e., $\alpha_{i,j} p_i > c_j$. Then, in the first case, i.e., $\mathbb{E}[\hat{s}_j] \geq d_i$, trading each unit generates the welfare equal to $\alpha_{i,j} p_i - c_j$ where, the total traded units are d_i. Furthermore, the excess energy of the seller $(s_j - d_i)$ is sold to the grid. The second case has a similar explanation. We note that, the value of a non-viable contract will be zero and that if $s_j = d_i$ then the two cases are equivalent.

Now, let us define an assignment matrix $M = [\hat{v}(i,j)]_{(i,j)\in\mathcal{I_B}\times\mathcal{I_S}}$ where each element $\hat{v}(i,j)$ represents the value of a bilateral contract between buyer $i \in \mathcal{I_B}$ and seller $j \in \mathcal{I_S}$. Then, the corresponding assignment game is given by $\mathcal{M} = (\mathcal{I}_B \cup \mathcal{I_S}, v_M)$. To solve the resulting game, the market operator first evaluates the value $v_M(S)$, for each $S \subseteq \mathcal{I}$, by solving the following assignment problem:

$$\mathbb{P}(S): \begin{cases} \max\limits_{\mu} \sum\limits_{i\in\mathcal{I_B}\cap S} \sum\limits_{j\in\mathcal{I_S}\cap S} \hat{v}(i,j)\mu_{i,j} \\ \text{s.t.} \sum\limits_{i\in\mathcal{I_B}\cap S} \mu_{i,j} \leq 1 \qquad \forall j \in \mathcal{I_S} \cap S \\ \quad\; \sum\limits_{j\in\mathcal{I_S}\cap S} \mu_{i,j} \leq 1 \qquad \forall i \in \mathcal{I_B} \cap S \end{cases} \tag{5.4}$$

with matching factors $\mu_{i,j} \in \{0,1\}$, where $\mu_{i,j} = 1$ represents the matching between buyer i and seller j. The constraints imposed on the matching factors ensure one-to-one matching. We note that the sellers and buyers can also enter as multiple agents to ensure adequate energy trading in the case of participation discrepancy on two sides of the market. The results obtained by solving the assignment problem in (5.4) enable the market operator to evaluate the optimal buyer-seller assignment and the marginal contribution of the agents. Following are the notable features of our P2P market design:

- Existence: There always exist a set of unobjectionable bilateral contracts for all participants, i.e., the core of a bilateral P2P market is always non-empty (Remark 4).

- Product differentiation: Buyers can assign priority to seller characteristics (e.g. green energy, location of seller) via preference factors $\alpha_{i,j}$.
- Convenience: Residential prosumers do not require any technical tools or methods to offer suitable amount of energy for given time interval (offer includes power rating) thereby simplifying the market entry.
- Bilateral negotiation: Optimal bilateral contracts are assigned centrally by the market operator but the contract price is negotiated internally between matched buyer and seller, thus preserving the inter-prosumer privacy.

5.4. DISTRIBUTED BILATERAL NEGOTIATION

5

After the market operator's evaluation of the optimal assignment, members of each matched pair negotiate between themselves for a bilateral agreement on the trading price. The goal of our mechanism design is to enable the matched pairs to independently reach a consensus on a fair bilateral contract price such that the corresponding collective vector of payoffs belong to the core in (4.3). Thus, to achieve our goal, we propose a distributed bilateral negotiation mechanism, where a central market operator with complete information of the game transmits to the market participants only their marginal contribution to the grand coalition (P2P market). After receiving the required information, each agent distributedly proposes a payoff allocation to his matched agent. We prove that utilizing such a limited information, the proposed solution converges to a fair and stable payoff distribution, i.e, to the τ-value.

We consider a bilateral negotiation process in which, at each negotiation step k, a buyer (seller) communicates with the matched seller (buyer) to bargain for his payoff. We present the process of negotiation for a matched pair $\mathcal{E}_{i,j} \in \mathcal{E} := \{(i,j) \mid \mu_{i,j} = 1, \text{ for all } (i,j) \in \mathcal{I_B} \times \mathcal{I_S}\}$ where, $\mathcal{E}$ is a set of matchings in an optimal assignment attained by solving (5.4) for the grand coalition $(\mathcal{I}_B \cup \mathcal{I}_S)$. The matched pair communicates over a directed network link

weighted using an adjacency matrix $W^k = [w^k_{i,j}]$, whose element $w^k_{i,j}$ represents the weight assigned by agent i to the payoff proposal of his matched agent j, $\boldsymbol{x}^k_j$. Here, the time-variation k refers to the variation of the weights assigned by each agent to the proposal of the corresponding matched agent.

Furthermore, we assume the adjacency matrix to be stochastic with the positive entries, which means that an agent always gives some weight to his previous proposal and the proposal of the matched agent.

Assumption 14 (Stochastic adjacency matrix). *For all $k \geq 0$, the adjacency matrix $W^k = [w^k_{i,j}]$ of the communication links is row-stochastic and $\exists\, \gamma > 0$ such that $w^k_{i,j} \geq \gamma$.* □

We further make a technical assumption on the elements of the adjacency matrix W^k, i.e., they belong to a finite set hence, finitely many adjacency matrices are available.

Assumption 15 (Finitely many adjacency matrices). *The adjacency matrices $\{W^k\}_{k\in\mathbb{N}}$ of the communication graphs belong to $\mathcal{W}$, a finite family of matrices that satisfy Assumption 17, i.e., $W^k \in \mathcal{W}$ for all $k \in \mathbb{N}$.* □

At each negotiation step k, an agent i bargains by proposing a payoff distribution $\boldsymbol{x}_i^k \in \mathbb{R}^2$, for both of the agents in a matched pair (i,j). To evaluate a proposal, he first takes an average of the estimate of the matched agent $\boldsymbol{x}_j^k$ and his own proposal weighted by an adjacency matrix W^k, $\sum_{j\in\mathcal{E}_{i,j}} w_{i,j}^k \boldsymbol{x}_j^k$. Next, agent i receives the information of his marginal contribution in the market by the market operator, which allows agent i to evaluate the set of his favorable payoffs $\mathcal{X}_i$, as in (5.2). We note that our algorithm does not require evaluation of the complete core as for the algorithms presented in [13]. After receiving the required information, agent i projects the average $\hat{\boldsymbol{x}}_i^k := \sum_{j\in\mathcal{E}_{i,j}} w_{i,j}^k \boldsymbol{x}_j^k$ on the set of favorable payoffs $\mathcal{X}_i$. Thus, the iteration reads as $\boldsymbol{x}_i^{k+1} = \mathrm{proj}_{\mathcal{X}_i}(\hat{\boldsymbol{x}}_i^k)$. We can generalize this iteration by replacing the projection operator, proj(·), with a special class of operators namely, paracontractions. This generalization enables us to utilize operator theory for showing the convergence of our algorithm later. The protocol we propose for an agent $i \in \mathcal{E}_{i,j}$ is $\boldsymbol{x}_i^{k+1} = T_i(\hat{\boldsymbol{x}}_i^k)$, that in collective form, for negotiation between pair (i,j), reads as the fixed-point iteration

$$\boldsymbol{x}_{(i,j)}^{k+1} = \boldsymbol{T}(\boldsymbol{W}^k \boldsymbol{x}_{(i,j)}^k), \tag{5.5}$$

5

where $\boldsymbol{T}(\boldsymbol{x}) := \mathrm{col}(T_1(\boldsymbol{x}_1), T_2(\boldsymbol{x}_2))$ and $\boldsymbol{W}^k := W^k \otimes I_4$ represents an adjacency matrix. In (5.5), we require the operator T_i to have $\mathcal{X}_i$ in (5.2) as fixed-point set, i.e., $\mathrm{fix}(T_i) = \mathcal{X}_i$.

Assumption 16 (Paracontractions)**.** *The operator* $\boldsymbol{T}$ *in (5.5) is such that* $T_i \in \mathcal{T}$*, where* $\mathcal{T}$ *is a finite family of paracontraction operators such that* $\mathrm{fix}(T_i) = \mathcal{X}_i$ *with* $\mathcal{X}_i$ *in (5.2).* □

The iteration in (5.5) provides the bilateral negotiation process that will be executed by each matched pair $(i,j) \in \mathcal{I}_B \times \mathcal{I}_S$ independently. Next, we provide our main convergence result.

Theorem 6 (Convergence of bilateral negotiation)**.** *Let Assumptions 17 – 16 hold. Let* $\mathcal{X}_{(i,j)} := \mathcal{X}_i \cap \mathcal{X}_j$ *with* $\mathcal{X}_i$ *as in (5.2). Then, starting from any* $\boldsymbol{x}_{(i,j)}^0$*, the sequence* $(\boldsymbol{x}_{(i,j)}^k)_{k=0}^{\infty}$ *generated by the iteration in (5.5) converges to* $\boldsymbol{x}_{(i,j)}^* \in \mathcal{X}_{(i,j)}^2 \cap \mathcal{A}$ *with* $\mathcal{A}$ *as in (2.8) and the collective payoff vector* $\boldsymbol{x}^* \in \prod_{(i,j)\in\mathcal{E}} \mathcal{X}_{(i,j)}$ *is the* τ*-value in (5.1) thus belongs to the core,* $\mathcal{C}_M$ *in (4.3).* □

5.5. NUMERICAL SIMULATIONS

In this section, we demonstrate the effectiveness of proposed algorithm by conducting numerical simulations of our bilateral P2P market design for time slots with high PV generation. We consider 3 residential prosumers with energy deficiency and 3 with surplus to act as buyers and sellers, respectively. Sellers are equipped with either a PV source of capacity 2 - 5 kWp or an ES of 4 kWh. The buyers assign preference level to each seller using the priority factor $\alpha_{i,j} \in [1, 1.5]$ with 1 being indifference to any criteria, e.g. green source (PV) vs brown source (ES), etc. Furthermore, buyers choose base valuation of the energy p_i such that their bid is higher than the grid's buying price $g_\mathrm{b} = 0.05$ £/kWh and not more than the grid's selling price $g_\mathrm{s} = 0.17$ £/kWh and the sellers choose their valuation c_j less than the grid's selling price g_s [44]. To account for the uncertainty, we use three PV generation scenarios.

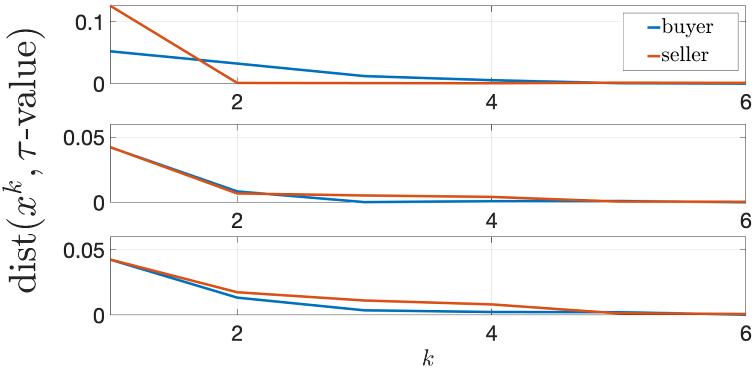


Figure 5.1: Trajectories of dist($\boldsymbol{x}^k$, τ-value) via bilateral negotiation algorithm with operator $T_i := \text{proj}_{\mathcal{X}_i}$ for optimally matched buyer-seller pairs.

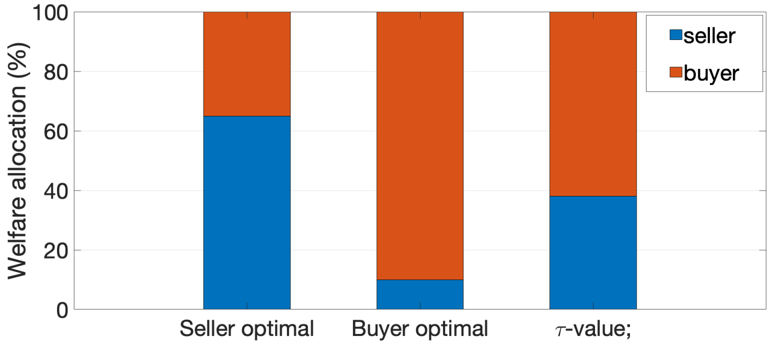


Figure 5.2: Percentage distribution of total welfare between buyers and sellers via seller optimal, buyer optimal and τ-value payoffs.

In our P2P market setup, first the market operator evaluates the optimal trading pairs of buyers and sellers using the formulation in (5.4). Then, each matched pair internally negotiates for the contract prices via mechanism in (5.5). In Figure 5.1, we show the convergence of bilateral negotiation algorithm to the respective τ-value payoffs. In Figure 5.2 we show the welfare allocation by respective points in the core. As the payoff is negotiated bilaterally the gain of buyer corresponds to the loss of seller and vice versa thus, we observe that, the τ-value payoff provides more fair treatment to both sides. Next, we illustrate the economic benefit of trading inside the P2P market, compared to trading with the grid, in Figure 5.3.

5.6. CONCLUSION

We have modelled P2P energy trading as an assignment game (coalitional game) and proposed a bilateral negotiation process as a clearing mechanism. The proposed P2P electricity market model encourages prosumers to participate by providing ease of accessibility, flexibility of choice and economic benefits, i.e., higher revenue (sellers) and

Figure 5.3: Average revenue improvement (sellers) and average cost reduction (buyers) via seller optimal, buyer optimal and τ-value payoffs compared to energy trading with the grid.

lower energy costs (buyers) compared to trading with the grid. Furthermore, the bilateral negotiation mechanism enables participants to reach a trading contract (τ-value) which fairly divides the resulting market welfare among buyers and sellers.

5.7. APPENDIX

To prove the convergence of (5.5), as stated in Theorem 6, we first provide useful property of a paracontraction operator.

Lemma 13 ([32], Thm. 2)**.** *Let* $\mathcal{M} = \{M_1, \dots, M_m\}$ *be a set of paracontractions such that* $\bigcap_{M \in \mathcal{M}} \mathrm{fix}(M) \neq \varnothing$. *Let the communication graph be connected and consider the iteration* $\boldsymbol{x}^{k+1} = \boldsymbol{M}(\boldsymbol{W}^k(\boldsymbol{x}^k))$, *where* $\boldsymbol{M}(\boldsymbol{x}) := \mathrm{col}(M_1(\boldsymbol{x}_1), \dots, M_m(\boldsymbol{x}_m))$. *Then, the state* $\boldsymbol{x}^k$ *converges to a state in the set* $\mathcal{A} \cap \mathrm{fix}(\boldsymbol{M})$ *as* $k \to \infty$. □

Proof. By (5.2), $\mathcal{X}_{(i,j)} = \frac{(\overline{x}'_i, \underline{x}''_j) + (\underline{x}'_i, \overline{x}''_j)}{2}$. Let Assumptions 17 and 16 hold then, by Lemma 13 the iteration in (5.5) converges to $\mathcal{A} \cap \mathcal{X}^2_{(i,j)}$. Next, by Definition in (5.1) the collective payoff $\boldsymbol{x}^*$ is the τ-value thus, by Remark 5, $\boldsymbol{x}^* \in \mathcal{C}_M$. □

6

ONLINE COALITIONAL GAMES FOR REAL-TIME PAYOFF DISTRIBUTION

Motivated by the markets operating on fast time scales, we present a framework for online coalitional games with time-varying coalitional values and propose real-time payoff distribution mechanisms. Specifically, we design two online distributed algorithms to track the Shapley value and the core, the two most widely studied payoff distribution criteria in coalitional game theory. We show that the payoff distribution trajectory resulting from our proposed algorithms converges to a neighborhood of the time-varying solutions. We adopt an operator-theoretic perspective to show the convergence of our algorithms. Numerical simulations of a real-time local electricity market and cooperative energy forecasting market illustrate the performance of our algorithms: the difference between online payoffs and static payoffs (Shapley and the core) to the participants is little; online algorithms considerably improve the scalability of the mechanism with respect to the number of market participants.

6.1. Introduction

A technological transformation is currently underway converting key infrastructures, such as power grids, commerce, and trading platforms, into highly dynamic complex systems. In these domains, predictive decision-making and operational planning under uncertainty traditionally rely on forecasts. The reliability of a forecast generally decreases as the lead time increases, especially for systems operating in highly dynamic environments. Thus, acting closer to the time of occurrence of an event decreases the chance of inaccurate or erroneous decision-making. Alongside, the sweeping technological advances across sectors like communication, sensing, data acquisition, and computation are making time-ahead decision-making a dormant approach. Therefore, we need methodologies and mechanisms that make use of real-time data streams and respond to the fast dynamics of the underlying system via online decision-making [26].

Among the systems operating in highly dynamic environments, here we focus on real-time markets. The adoption of real-time markets has shown significant potential in the power system sector [118]. In particular, the increased presence of distributed energy resources (DERs) and demand response (DR) programs on the consumer side allow system operators to utilize them for providing demand-supply balancing services in real time [80]. Unlike conventional generators, the response time of DERs and DR fulfills the operational requirements of participation in real-time balancing markets. In [42], the authors build a model for the real-time operation of a recent market paradigm, i.e., peer-to-peer (P2P) markets. P2P markets envision a bilateral trade of renewable energy among small prosumers. As the accuracy of forecasts can be improved by decreasing the lead time [75], market clearing closer to the time of delivery can mitigate the possible imbalance caused by the uncertainty associated with RES. In both balancing and P2P markets, the key enabling feature is the computational speed of the clearing mechanism. The mechanisms for such markets require a large amount of information exchange and execution of negotiation processes. Consequently, in the context of real-time markets, the computational time for market clearing can be higher than the gap between two market instances.

Another marketplace operating in a dynamic environment that has recently gained a lot of interest from both academia and industry, is the data market [17]. With the emergence of machine learning across all business and social sectors, the need for quality training data has grown enormously. One way to ensure data availability is by creating a market that compensates data providers. Various structures and mechanisms are proposed in the literature for data markets including bilateral exchange of data [91] and a regression market framework for wind power forecasting [45]. In general, assigning a value to a particular data set among many is inherently a combinatorial problem. The authors in [33] address the problem of data valuation for a specified machine learning algorithm under a static market structure. Here, we are interested in the mechanisms that can handle continuous data streams, hence real-time data markets. In this direction, the authors in [2] present a real-time data market for buying and selling training data and propose a mechanism to fairly compensate the data providers. However, the compensation in their mechanism is computed offline. In the presence of continuous data streams and combinatorial complexity of data valuation, offline solutions cannot be executed in the time scales that match the dynamics of the underlying process. Therefore, in this paper, we adopt a game-theoretic approach to design online market mechanisms for real-time markets operating at fast time scales. We present these mechanisms in a general form that is applicable to several domains. For the data markets in [2], the online formulation enables us to better remunerate the market players under continuous data streams. Similarly, for the real-time P2P market of [42], we can employ online market mechanisms grounded in coalitional game theory, which offers mathematical tools for analysing the interaction of self-interested agents and provides guarantees of *fairness* or stability on the remuneration criteria. From an economic perspective, these properties are highly desirable for a payoff distribution mechanism.

In this paper, we focus on a particular class of coalitional games, namely transferable utility (TU) coalitional game, which consists of a set of agents $\mathcal{I}$ and a value function v that assigns a value $v(S)$ to each possible coalition of agents $S \subset \mathcal{I}$. Collectively, a TU coalitional game is represented by a pair $(\mathcal{I}, v)$ [69]. Multi-agent decision-making problems modeled by coalitional games arise in many application areas, such as energy systems [43], [22] and communication networks [92]. In particular, we study markets modeled as coalitional games. Coalitional game theory studies the mechanism of the distribution of the value generated by cooperation to respective agents. Two key solution concepts that undertake the task of value distribution (payoff) are the Shapley value and the core. The Shapley value addresses the fairness aspect, which implies that the payoff for an agent should reflect its impact on the game. This property is ensured by the axiomatic characterization of fairness [97]. The core payoff ensures that no agent has any incentive to defect the coalition and thus addresses stability.

We consider the problems of evaluating both fair and stable payoff allocations, i.e., the Shapley value and the core payoff respectively, under a dynamic coalitional game setting. Essentially, our work lies at the intersection of time-varying optimization and dynamic coalitional games. In the direction of the former, algorithms proposed in the literature [100], [102] track trajectories of the optimizers of the time-varying optimization problems up to asymptotic error bounds, under the assumption of strong convexity. The problem of payoff allocation in dynamic coalitional games has also been studied in the literature. Among others, the authors in [57] characterize the core allocations when the coalitional values vary over time and are dependent on previous events. In [15], Bauso and Timmer propose payoff allocation rules for a dynamic game where the coalitional value fluctuates within a bounded polyhedron while the average value of each coalition over time is known. As we are seeking to design iterative algorithms, a closer work is [73] by Nedich and Bauso. The paper considers a core payoff allocation in a sequence of games where the intersection of all the corresponding cores is non-empty. Further generalization of their work is presented by the authors in [88] under the same assumption on the core sets. However, in the context of real-time markets it is not reasonable to assume that the coalitional values evolve only within a particular set or that we have knowledge about the average coalitional value over time. Thus, the assumptions made on the non-empty intersection of the solution sets in the works mentioned above make their algorithms inapplicable to real-time markets. Considering these short-comings, in this paper, we drop the assumptions on the knowledge of average coalitional values as well as of non-empty intersection of solution sets to formulate coalitional games in an online paradigm and in turn propose solutions for real-time market setups.

A typical problem of real-time markets modelled as a coalitional game is the exponential computational complexity of an equilibrium solution which usually makes exactly evaluating the core and Shapley value impractical. Therefore, we introduce online distributed payoff allocation algorithms that instead of evaluating at each time instant the exact solution, track the solutions of the continuously-varying coalitional games up to an asymptotic error bound. Among all energy-related markets, here we focus on the advanced real-time markets that are operating at a high frequency, where the time interval between the opening and the clearing of the market is not enough to compute a coalitional solution in an offline manner. We note that online mechanisms are instead not necessary or suitable for traditional centralised wholesale markets.

Before listing our contributions, let us further motivate our setting through an example. Note that, here, we present our example in a general setting to show the extent of our contribution. Later, we simulate the energy-related market as a specific case of this motivational setting.

6.1.1. Motivational example

Let us consider an online forecast valuation scheme, inspired by [53] and [89], for pooling the information and expertise held by different owners and generating a combined forecast. First, let us introduce the forecasting markets that are designed to predict an event e.g. renewable energy generation [95]. Generally, in such markets, the market participants (forecasters) sell predictions in the form of a probability distribution; then the true outcome of the event is observed and the market pays each expert based on the *quality* of their predictions. Let there be a central platform $\mathcal{L}$ designed for a prediction task, e.g. to predict the wind energy generation. Consider a set of N forecasters, $\mathcal{I}$, that have expertise in making such predictions. To generate accurate predictions, the forecasters take into account various factors, e.g. wind speed and overall weather conditions effecting the wind energy generation. Each provider $i \in \mathcal{I}$ posts a bid to the forecasting market. To achieve the forecast valuation, the following steps are performed:

- A client posts a prediction task Y to the central platform;
- Each forecaster $i \in \mathcal{I}$ posts their prediction f_i of the announced task;
- The platform combines these forecasts using a pooling method [117] and the resulting aggregate forecast $\hat{f}$ is delivered to the client;
- After the event occurs, the client announces a reward γ corresponding to the improvement that they achieved in decision making. Then, the quality of posted predictions is evaluated and the reward is distributed fairly among the forecasters as a payoff $\boldsymbol{x}$.

In our setting, we consider high frequency events with fast dynamics which thus requires an online forecast valuation scheme. This process of eliciting a combined forecast, i.e., cooperative forecasting results in an online coalitional game among the forecasters, represented by a triplet $(\mathcal{I}, \mathcal{L}, v^k)$. The setup of real-time valuation results in a time varying value function v^k, where $v^k(S)$ represents the utility of a client attained by a cooperative forecast of coalition $S \subset \mathcal{I}$. After the occurrence of the event, forecasters negotiate

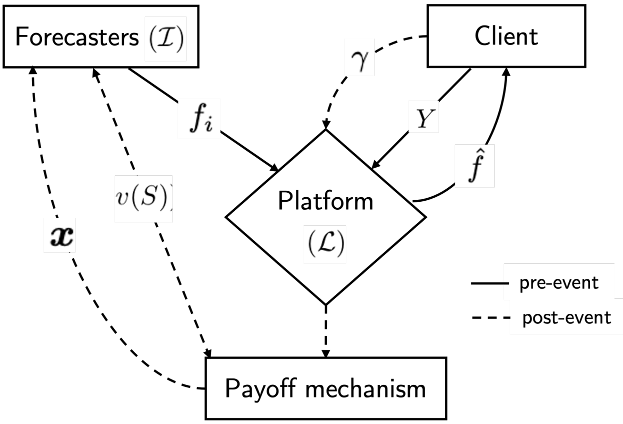


Figure 6.1: Overview of an online data valuation scheme in the context of a cooperative forecasting market.

their share of resulting value according to a criteria that acknowledges their individual contributions in predicting that event. In Figure 6.1, we present a cooperative forecasting scheme with an online payoff distribution mechanism. In the literature, the Shapley value is utilized for payoff allocation in an offline setting for similar markets [50], [3] as it fulfills the key criterion of a fair forecast valuation scheme. For further details on the criterion, we refer to [33]. In this chapter, we design online algorithms for the most widely used payoff distribution methods in the coalitional games, namely the core and the Shapley value.

6.1.2. CONTRIBUTION

- We introduce the concept of online tracking of solutions (Shapley value and the core) in context of coalitional game theory;
- We develop a novel distributed online payoff allocation algorithm to track the Shapley value up to an asymptotic error bound. We also present the static version of the algorithm which converges to the Shapley value exactly (Section 6.3);
- We relax the assumption on the core sets of the sequence of coalitional games in [88] and present an online algorithm to track the payoff allocation in a neighborhood of the core. We show that the proposed algorithm is asymptotically consistent, i.e., converges to the core payoff exactly in the absence of dynamics (Section 6.3.2);
- We introduce an operator-theoretic analysis for the design of online algorithms in the domain of dynamic coalitional games, which allows us to generalize existing results.

We note that, even though we focus on mechanism design of energy-related real-time markets, our solutions can be applied to other applications of cooperative game theory as well. For instance, a community based energy storage optimisation presented in [43] can be addressed in an online fashion to mitigate the effects of uncertainty in load and RES generation. Similarly, the coalition between EV owners and their work places for optimal charging proposed in [130] can implement online payoff distribution to make use of accurate data on generation and dynamic pricing. A real-time fair pricing can be achieved for a ride-hailing service proposed in [96]. Next, we provide preliminaries on online coalitional games. For basic notation see Appendix A and refer to Chapter 2, Section 2.2.2 for operator-theoretic definitions.

6.2. PRELIMINARIES

For the necessary mathematical background on coalitional game theory in dynamic context we refer the reader to Chapter 2. Next, we provide preliminaries for proposed online payoff distribution processes to track Shapley value and the core.

The Shapley value, as in definition 6, is given by

$$\phi_i(v^k) = \frac{1}{N!} \sum_{\sigma \in \Pi} (v^k(\mathcal{P}_i^\sigma \cup \{i\}) - v^k(\mathcal{P}_i^\sigma)). \tag{6.1}$$

where the term $(v^k(\mathcal{P}_i^\sigma \cup \{i\}) - v^k(\mathcal{P}_i^\sigma))$ in (6.1) is referred as incremental marginal contribution. this term shows the value added by an agent i when it joins the coalition. Thus, the Shapley value assigned to an agent i is its incremental marginal contribution averaged over all permutations. We note from (6.1) that to evaluate its Shapley payoff an agent needs to know the value of all possible coalitions, which is impractical for many real-world applications and renders distributed computation useless. For the purpose of designing a distributed algorithm, we identify the orderings $\sigma \in \Pi$ for which an agent i can evaluate the incremental marginal contributions of all the agents with only the knowledge of the coalitional values of its own coalitions. These orderings are the ones in which i joins the coalition at first position. To clarify further, we present the following example.

Example 1. *Let us consider a three player coalitional game* $(\{a,b,c\}, v)$. *Here, agent a can compute the incremental marginal contributions for ordering* (a,b,c) *and* (a,c,b) *as* $(v(\{a\}, v(\{a,b\} - v(\{a\}, v(\{a,b,c\} - v(\{a,b\})$ *and* $(v(\{a\}, v(\{a,c\} - v(\{a\}, v(\{a,b,c\} - v(\{a,c\}))$, *respectively by knowing the values of its own coalitions only. However, for the ordering* (b,c,a) *the incremental marginal contributions are* $(v(\{b\}, v(\{b,c\} - v(\{b\}, v(\{a,b,c\} - v(\{b,c\})$ *and to evaluate them, agent a requires the knowledge of* $v(\{b,c\})$ *which is unreasonable as the coalition* (b,c) *is not its coalition.* □

To exploit the observation from Example 1, in the sequel, we define the marginal contribution vector $\hat{\boldsymbol{m}}_i$ that is the average of incremental marginal contribution vectors corresponding to those orderings for which an agent i can evaluate with minimal information.

Definition 17 (Marginal contribution vector). *Let* $\pi_i \subset \Pi$ *be the set of permutations of the grand coalition* $\mathcal{I}$ *in which an agent i occupies the first position. For each ordering* $\sigma \in \Pi$, *let* $\boldsymbol{m}_\sigma \in \mathbb{R}^N$ *be a vector of incremental marginal contributions with* j*th element* $m_j^\sigma = v^k(\mathcal{P}_j^\sigma \cup \{j\}) - v^k(\mathcal{P}_j^\sigma)$. *Then, for every agent* $i \in \mathcal{I}$, *the marginal contribution vector* *is*

$$\hat{\boldsymbol{m}}_i = \frac{1}{(N-1)!}\sum_{\sigma\in\pi_i}\boldsymbol{m}_\sigma. \tag{6.2}$$

□

Now, the Shapley value, in terms of marginal contribution vectors, becomes $\boldsymbol{\phi}(v) = \frac{1}{N}\sum_{i\in\mathcal{I}}\hat{\boldsymbol{m}}_i$.

Next, we note that for a dynamic coalitional game the solution also varies with time and that the complexity of both the solutions, i.e., the core in (2.2) and the Shapley value in (6.1) grows exponentially with the number of agents. Therefore, guaranteeing convergence to a solution payoff vector for each instantaneous game is not necessarily possible, especially in highly dynamic settings, e.g. real-time applications, where computational and communication bottlenecks can hinder the exact tracking of a solution trajectory. Therefore, in the sequel, we propose a distributed online algorithm to track the Shapley value and provide bounds on the asymptotic error, defined as the “distance" between the evaluated payoff vector and the Shapley value. Furthermore, we also design a distributed online algorithm that provides bound on the asymptotic error for tracking the core set.

In general, a distributed online payoff allocation method is an iterative procedure in which, at each step, an agent i proposes a payoff distribution $\boldsymbol{x}_i \in \mathbb{R}^N$ by averaging the proposals of neighboring agents and by then making it compliant to its own interest. The allocation procedure aspires to reach a mutually agreed payoff in the core. Thus, the proposed payoff distributions $(\boldsymbol{x}_i)_{i\in\mathcal{I}}$ must also pursue consensus, as in (2.8).

In the sequel, first we consider the problem of computing a trajectory of payoff vectors that converges to the Shapley value up to a bounded error, i.e., $\limsup_{k\to\infty}\|\boldsymbol{x}^k - \boldsymbol{\Phi}^k\|$ is small. Then, we address the problem of tracking the core set such that $\limsup_{k\to\infty}\operatorname{dist}(\boldsymbol{x}^k, \mathcal{A}\cap\mathcal{C}(v^k))$ is small. In both problems, an online payoff distribution is achieved via discrete-time distributed algorithms.

6.3. DISTRIBUTED ONLINE PAYOFF ALLOCATION

In this section, we propose a payoff distribution in the context of online coalitional games, where the value function v varies with time k on a fast scale hence, the exact computation of the solution for each instantaneous game is not computationally achievable. Therefore, our goal is to design a distributed algorithm to compute a payoff trajectory that tracks a solution reasonably well. We remark that we analyse the most conservative case where agents evaluate one iteration per sample v^k. The tracking performance can be improved with multiple iterations per sample, depending on the lead time of a market. Let a set of agents $\mathcal{I} = \{1,\dots,N\}$ synchronously propose a distribution of utility at each discrete time step $k \in \mathbb{N}$, i.e., each agent $i \in \mathcal{I}$ proposes a payoff distribution $\boldsymbol{x}_i^k \in \mathbb{R}^N$, where the jth element denotes the share of agent j proposed by agent i at step $k \in \mathbb{N}$.

Let the agents communicate over a time-varying network represented by a graph $G^k = (\mathcal{I}, \mathcal{E}^k)$, where $(j,i) \in \mathcal{E}^k$ means that there is an active link between the agents i and j at iteration k and they are then referred as neighbours. Therefore, the set of neighbors of agent i at iteration k is defined as $\mathcal{N}_i^k := \{j \in \mathcal{I} | (i,j) \in \mathcal{E}^k\}$. We assume that at each iteration k the communication graph is connected. The edges in the communication graph G^k are weighted using an adjacency matrix $W^k = [w_{i,j}^k]$, whose element $w_{i,j}^k$ represents the weight assigned by agent i to the payoff distribution proposed by agent j, $\boldsymbol{x}_j^k$. Note that, for some j, $w_{i,j}^k = 0$ implies that $j \notin \mathcal{N}_i^k$ hence, the state of agent i is independent from that of agent j. We assume the adjacency matrix to be doubly stochastic with positive diagonal elements, as assumed in [70, Assumption 3.3], [71, Assumptions 2, 3].

Assumption 17 (Stochastic adjacency matrix)**.** *For all $k \geq 0$, the adjacency matrix $W^k = [w_{i,j}^k]$ of the communication graph G^k satisfies following conditions:*

1. *It is symmetric and doubly stochastic, i.e., $\sum_{j=1}^N w_{i,j} = \sum_{i=1}^N w_{i,j} = 1$;*
2. *its diagonal elements are strictly positive, i.e., $w_{i,i}^k > 0, \forall i \in \mathcal{I}$;*
3. *$\exists\, \gamma > 0$ such that $w_{i,j}^k \geq \gamma$ whenever $w_{i,j}^k > 0$.* □

Assumption 17 ensure that the agents communicate sufficiently often to each other and have sufficient influence on the resulting allocation. Finally, we propose distributed discrete-time algorithms of the form:

$$\boldsymbol{x}_i^{k+1} = M_i^k(\boldsymbol{x}_i^k),$$

where $\boldsymbol{x}_i^k \in \mathbb{R}^N$ is agent i's estimate of the payoff allocation of all the agents and and M_i^k is a time-varying update operator. We can write the above iteration for all agents in collective compact form:

$$\boldsymbol{x}^{k+1} = \boldsymbol{M}^k(\boldsymbol{x}^k), \tag{6.3}$$

where $\boldsymbol{M}^k(\boldsymbol{x}) := \mathrm{col}(M_1^k(\boldsymbol{x}_1), \dots, M_N^k(\boldsymbol{x}_N))$. Next, we assume a bound on the time variation of the fixed-point of the time-varying operators $\boldsymbol{M}^k$ in (6.3).

Assumption 18 (Bounded time variations)**.** *Let $(\boldsymbol{M}^k)_{k\in\mathbb{N}}$ be the sequence of operators in (6.3). The distance between the fixed-points of two consecutive operators is bounded, i.e., $\sup_{k\in\mathbb{N}} \sup_{(\bar{\boldsymbol{x}}^k, \bar{\boldsymbol{x}}^{k+1}) \in \mathrm{fix}(\boldsymbol{M}^k) \times \mathrm{fix}(\boldsymbol{M}^{k+1})} \|\bar{\boldsymbol{x}}^{k-1} - \bar{\boldsymbol{x}}^k\| \leq \delta$, for some $\delta > 0$.* □

We note that Assumption 18 bounds the time variations of the fixed point sets of the time-varying operators, rather than the Euclidean distance between the optimal points at consecutive times, i.e., $\|\bar{\boldsymbol{x}}^{k-1} - \bar{\boldsymbol{x}}^k\| \leq \delta$, which is standard in the time-varying optimization [100, Assumption 1], [101, Theorem 1]. Next, we present an online payoff allocation algorithm where we design the operator M_i^k with the Shapley payoff as its fixed-point set, i.e., $\mathrm{fix}(M_i^k) = \boldsymbol{\phi}(v^k)$. We note that in the context of Shapley payoff distribution, Assumption 18 relates to the dynamics of the coalitional game and implies a bounded variation of the Shapley value from one time step to the next.

6.3.1. ONLINE TRACKING OF THE SHAPLEY ALLOCATION

Let us formulate the distributed tracking of the Shapley value via time-varying operators and provide convergence results. The problem of computing the Shapley value for a static coalitional game can be formulated as an unconstrained convex optimization problem with the objective of achieving a consensus on the Shapley value, i.e.,

$$\min_{\boldsymbol{x}} \frac{1}{2} \sum_{i \in \mathcal{I}} \|\boldsymbol{x} - \hat{m}_i\|^2, \tag{6.4}$$

where $\hat{m}_i$ is a marginal contribution vector as in (6.2). Here, we consider dynamic coalitional games executed on time-varying networks and design an algorithm in a distributed paradigm, thus the marginal contribution vector is also time-varying. Each agent i minimizes a local objective function $f_i^k = \frac{1}{2}\left\|\boldsymbol{x}_i - \hat{m}_i^k\right\|^2$. For solving the resulting optimization problem, an agent i can adopt a gradient based algorithm. Let $\boldsymbol{y}_i^k := \sum_{j=1}^N w_{i,j}\boldsymbol{x}_j$, then the state update is given as

$$\boldsymbol{x}_i^{k+1} = \boldsymbol{y}_i^k - \alpha\nabla f_i^k(\boldsymbol{y}_i^k).$$

In operator-theoretic terms, we can define an operator $\boldsymbol{M}^k$ in (6.3) as a composition of a gradient step operator and a consensus operator, i.e., $\boldsymbol{M}^k = (\mathrm{Id} - \alpha\nabla f^k)\circ\boldsymbol{W}^k$ where $\boldsymbol{W}^k := W^k \otimes I_N$ represents an adjacency matrix. We note that for a strongly convex function f the operator $\boldsymbol{M}$ is a contraction mapping, a fact we use later on to prove convergence of proposed algorithm.

Assumption 19 (Contractions)**.** *For all $k \in \mathbb{N}$, the operator $\boldsymbol{M}^k$ in (6.3) is such that $\boldsymbol{M}^k \in \mathcal{M}$, where $\mathcal{M}$ is a family of contraction operators with contraction factor $L_k \in (0,1)$.*

□

Finally, the compact and simplified iteration takes the following form:

$$\boldsymbol{x}^{k+1} = (1-\alpha)\boldsymbol{W}^k\boldsymbol{x}^k + \alpha\hat{\boldsymbol{m}}^k. \tag{6.5}$$

The authors in [128] present an iteration based on the static version of the operator $\boldsymbol{M}$, i.e., $\boldsymbol{z}^+ = \boldsymbol{M}\boldsymbol{z}$ and show an inexact convergence which achieves an asymptotic error bound $O(\alpha)$ with respect to the consensus optimizer $\bar{\boldsymbol{z}}$. In our setting, $\bar{\boldsymbol{z}}$ refers to the Shapley value $\boldsymbol{\phi}$. Here we derive a bound for an online setting in terms of $O(\alpha)$-neighborhood as defined in [128, Lemma 1] under the same conditions on the step size α. We note that the solution in the context of online coalitional games means convergence of the payoff allocation trajectory to a neighbourhood of the time-varying Shapley value, as shown by the following convergence result for (6.5).

Theorem 7 (Convergence of online Shapley allocation)**.** *Let Assumptions 17– 19 hold. Starting from any $\boldsymbol{x}^0 \in \mathbb{R}^{N^2}$, the error norm $||\boldsymbol{x}^k - \boldsymbol{\Phi}^k||$ generated by the iteration in (6.5) satisfies the following bound:*

$$\|\boldsymbol{x}^k - \boldsymbol{\Phi}^k\| \leqslant \hat{L}_k\|\boldsymbol{x}^0 - \boldsymbol{\Phi}^0\| + \frac{1-(\bar{L}_k)^{k-1}}{1-\bar{L}_k}\delta + O(\alpha),$$

where $\hat{L}_k := \prod_{i=1}^{k-1} L_i, \bar{L}_k = \max_k L_k$ and $\boldsymbol{\Phi}^k := \boldsymbol{\phi}^k \otimes \boldsymbol{1}_N$ is the Shapley value in (6.1) where $O(\alpha)$ is as in [128, Lemma 1]. Therefore, we have that $\lim_{k\to\infty} \|\boldsymbol{x}^k - \boldsymbol{\Phi}^k\| \leqslant \frac{1}{1-\bar{L}_k}\delta + O(\alpha)$.

6

□

The result of Theorem 7 asserts that the sequence $(\boldsymbol{x}^k)_{k\in\mathbb{N}}$ tracks the trajectory of the Shapley value up to a bound that linearly depends on the parameter δ, which comes from Assumption 18 and relates to the time variability of the Shapley allocation of a dynamic coalitional game in Definition 7. We provide the proof of Theorem 7 in Appendix.

We note that if the coalitional game is static then by using the setting in [126, Theorem 1] we can design a distributed algorithm that converges to the Shapley allocation. Let us present a corollary for the static case.

Corollary 1 (Convergence to Shapley allocation)**.** *Let Assumptions 17 and 19 hold. Let Assumption 18 hold with* $\delta = 0$. *Then, starting from any* $\boldsymbol{x}^0 \in \mathbb{R}^{N^2}$, *the sequence* $(\boldsymbol{x}^k)_{k=0}^{\infty}$ *generated by the iteration*

$$\boldsymbol{x}^{k+1} = (1-\alpha_k)\boldsymbol{W}^k\boldsymbol{x}^k + \alpha_k\hat{\boldsymbol{m}}^k,$$

converges to the Shapley value in (6.1), i.e., $\boldsymbol{x}^k \to \boldsymbol{\Phi}$, *where* $(\alpha_k)_{k\in\mathbb{N}} \in (0,1)$ *such that* $\alpha_k \to 0, \sum_{k\in\mathbb{N}}\alpha_k = +\infty, \sum_{k\in\mathbb{N}}|\alpha_{k+1}-\alpha_k| < +\infty$. □

Discussion: The solutions offered by coalitional game theory have interesting mathematical properties, but their computational complexity poses a challenge to their utilization in real-world applications. As the evaluation of the Shapley value requires the computation of the value of all possible permutations of the set of agents, the computational time increases exponentially with the number of agents. This challenge makes it impractical to utilize the Shapley payoff allocation in almost real-time. In this direction, the distributed structure of proposed algorithm in (6.1) mitigates the problem of high computational times by logically distributing the computational burden among the agents. Furthermore, it democratizes the negotiation process by autonomizing the decision making of agents, which is an important feature of liberal markets.

For coalitional games, the payoff allocated via the Shapley value guarantees fairness. However, it does not ensure the stability of a grand coalition $\mathcal{I}$, i.e., the Shapley value does not necessarily belong to the core in (2.2). As a consequence, if a coalition structure is not encouraged externally, then the Shapley payoff might not provide an adequate incentive for agents to join a coalitional game. Therefore, it is highly desirable to design a distributed algorithm for an online tracking of the core in dynamic coalitional games.

6.3.2. ONLINE TRACKING OF A CORE ALLOCATION

Let us now turn our attention towards the problem of tracking the core solution in (2.2) for online coalitional games. As the core is a set which in dynamic game setting varies with time, the problem takes the form of distributively tracking a time-varying set. Let us make an assumption on the non-emptiness of the core.

Assumption 20. *The core of each instantaneous game* $(\mathcal{I}, v^k)$, *is non-empty, i.e.,* $\mathcal{C}(v^k) \neq \varnothing$ *for all* $k \in \mathbb{N}$. □

For an agent i, the problem of tracking the core set $\mathcal{C}(v^k)$ can be formulated as an unconstrained time-varying convex optimization problem with objective of minimizing the distance of agent's payoff allocation estimate from its bounding set in (2.1). Mathematically, each agent i has an objective function $f_i^k := \frac{1}{2}\|\hat{\boldsymbol{x}}_i^k - \mathrm{proj}_{\mathcal{X}_i(v^k)}(\hat{\boldsymbol{x}}_i^k)\|^2 + \frac{\gamma}{2}\|\hat{\boldsymbol{x}}_i^k - \boldsymbol{x}_i^{k-1}\|^2$ with $\gamma > 0$. Thus, the optimization problem takes the following form:

$$\begin{cases} \min\limits_{\boldsymbol{x}_i} \frac{1}{2}\|\hat{\boldsymbol{x}}_i^k - \mathrm{proj}_{\mathcal{X}_i(v^k)}(\hat{\boldsymbol{x}}_i^k)\|^2 + \frac{\gamma}{2}\|\hat{\boldsymbol{x}}_i^k - \boldsymbol{x}_i^{k-1}\|^2 \\ \text{s.t. } \hat{\boldsymbol{x}}_i^k = \sum_{j=1}^N w_{i,j}^k \boldsymbol{x}_j^k \end{cases} \tag{6.6}$$

The optimization problem in (6.6) can be solved by using an iteration based on the forward operator $\mathrm{Id} - \alpha\nabla f_i$ which is a contraction mapping for a strongly convex and a strongly smooth function f_i. In our setup, for each time step $k \in \mathbb{N}$, an agent i updates its state as

$$\boldsymbol{x}_i^{k+1} = (1 - \alpha - \alpha\gamma)\hat{\boldsymbol{x}}_i^k + \alpha\mathrm{proj}_{\mathcal{X}_i(v^k)}(\hat{\boldsymbol{x}}_i^k) + \alpha\gamma\boldsymbol{x}_i^{k-1}, \tag{6.7}$$

where $\mathcal{X}_i$ is a bounding set in (2.1). In a stacked vector notation the forward operator applied on (6.6) gets composed with the consensus operator, i.e., $(\mathrm{Id} - \nabla f) \circ \boldsymbol{W}(\cdot)$. Let us further generalize the iteration in (6.7) by replacing the projection operator, $\mathrm{proj}(\cdot)$, with contractions in Assumption 19. This generalization enables the agents to choose any contraction operator T_i^k for evaluating a payoff $\boldsymbol{x}_i^k$. For consistency we require the fixed-point set of T_i^k to be the bounding set in (2.1), i.e., $\mathrm{fix}(T_i^k) = \mathcal{X}_i(v^k)$. Consequently, $\mathrm{fix}(\boldsymbol{T}^k) = \bigcap_{i=1}^N \mathcal{X}_i(v^k) = \mathcal{C}(v^k)$, the instantaneous core set. The contraction property allows us to prove the convergence of the state $\boldsymbol{x}^k$ to the set $\mathcal{A} \cap \mathcal{C}^N$ up to a specified error bound. Specifically, we propose the following online allocation protocol:

$$\boldsymbol{x}^{k+1} = \boldsymbol{T}^k(\boldsymbol{W}^k\boldsymbol{x}^k), \tag{6.8}$$

where the operator $\boldsymbol{M}^k := \boldsymbol{T}^k(\boldsymbol{W}^k(\cdot))$ as in iteration (6.3) is a sequence of time-varying contraction operators corresponding to the time-varying core set being tracked via time-varying communication network. This formulation of online tracking in terms of operators allows us to use the existing results from operator theory and to generalize the algorithms in [73] and [88] by dropping their assumption that the intersection of time-varying cores is non-empty. Furthermore, the operator theoretic analysis allows us to keep our proofs brief and elegant. Next, we formalize the convergence result of online tracking of the core allocation.

Theorem 8 (Online core payoff allocation)**.** *Let Assumptions 20–19 hold. Then, starting from any* $\boldsymbol{x}^0 \in \mathbb{R}^{N^2}$*, the error norm* $||\boldsymbol{x}^k - \bar{\boldsymbol{x}}^k||$ *generated by the iteration in (6.3) satisfies the following bound:*

$$\|\boldsymbol{x}^k - \bar{\boldsymbol{x}}^k\| \leq \hat{L}_k\|\boldsymbol{x}_0 - \bar{\boldsymbol{x}}_0\| + \frac{1-(\bar{L}_k)^{k-1}}{1-\bar{L}_k}\delta,$$

where $\hat{L}_k = \prod_{i=1}^{k-1} L_i, \bar{L}_k = \max_k L_k$ *and* $\bar{\boldsymbol{x}}^k \in \mathcal{A} \cap \mathcal{C}^N(v^K)$*, with* $\mathcal{A}$ *as in (2.8) and* $\mathcal{C}$ *being the core (2.2). Therefore, it holds that* $\lim_{k\to\infty}\|\boldsymbol{x}^k - \bar{\boldsymbol{x}}^k\| \leq \frac{\delta}{1-\bar{L}_k}$. □

We provide the proof of Theorem 8 in Appendix. Note that we are addressing the problem of tracking the core of a dynamic coalitional game, thus the result of Theorem 8 shows the convergence of the sequence $(\boldsymbol{x}^k)_{k\in\mathbb{N}}$ to a neighborhood of the core set that depends on the parameter δ as in Assumption 18, which bounds the variability over time of the coalitional game. For the problem of tracking the core, the variability of the game can be bounded by assuming non-empty intersection of the two consecutive cores, i.e., $\mathcal{C}(v^{k-1}) \cap \mathcal{C}(v^k) \neq \varnothing$. Note that, if the game is static, then the iteration in (6.8) converges to the common point in the core set, i.e., the agents employing the algorithm will reach consensus on the core payoff distribution. Thus, the online payoff distribution protocol in (6.8) is asymptotically consistent [100], which is an important feature of online algorithms.

6.3.3. Discussion

To use the payoff distribution algorithm in (6.8), each agent requires information on its own bounding set in (2.1) only that can be evaluated using the values of its own coalitions. Thus, this negotiation via bounding sets maintains inter-agent privacy. It is reasonable to assume that the agents have knowledge of their own coalitions.

We note that the centralized version of online tracking in the context of time-varying convex optimization is presented by Simonetto in [99]. However, centralized methods for tracking a payoff in the core do not capture scenarios of interaction among autonomous self-interested agents. Furthermore, as the core is a set in which different payoffs treat agents differently, a centralized evaluation will demand the trust of agents on the central entity, which is undesirable in many real-world applications, e.g. peer-to-peer energy exchange [88]. Thus, we propose a distributed method in (6.8) that allows agents to autonomously track a core payoff distribution.

6

Interestingly, for the class of games (e.g. convex games) where the Shapley value belongs to the core, the online tracking of Shapley value via the iteration in (6.5) implicitly tracks the core and vice versa via the algorithm in (6.8).

6.4. Real-time market applications

In this section, we illustrate numerically the scenarios of two real-time markets, i.e., a forecasting market and a local electricity market, modeled as the dynamic coalitional games. In the first scenario, we present a distributed tracking of the Shapley value for an online data valuation scheme; in the second scenario, we simulate a real-time local electricity market and track the time-varying core payoff as an online market solution.

6.4.1. Collaborative forecasting market

We simulate the near real-time collaborative forecasting market described in Section 3.1 for an application of wind power generation. Here, we model a market for trading point forecasts instead of probabilistic forecasts to remain consistent with the most widely adopted practice for wind power prediction [83]. Normally, wind energy is forecasted for horizons of hours ahead. However, if the wind power penetration in a system reaches a certain high level, it becomes crucial for the system's security to also have forecasts with a lead time ranging from 1 to 30 minutes. These short-term to near real-time predictions are required for various operations in the power systems, e.g. by the transmission system operator (TSO) for the continuous balance of the power system, as an input to the (offshore) wind farm controllers, and for the operation of wind-storage systems providing system regulation [81]. Therefore, we design a market-based prediction system for near

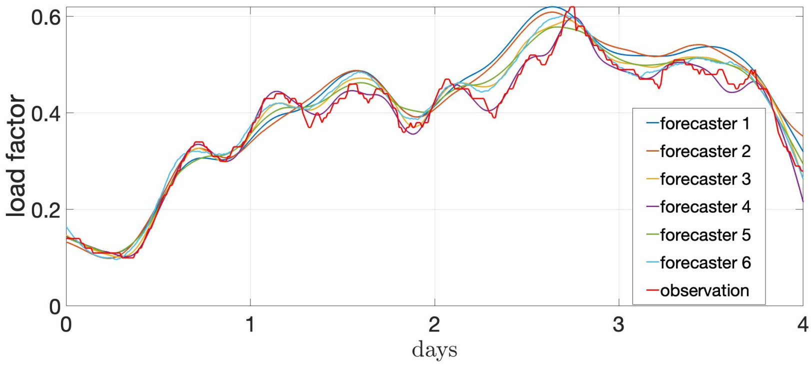


Figure 6.2: Agents' forecasts of wind energy generation with a lead time of 5 minutes and corresponding observation.

real-time wind energy forecasting based on online coalitional games.

PROBLEM SETUP

Consider a client's platform $\mathcal{L}$ (e.g. TSO, wind farm owner, energy trader, etc.) that uses a wind energy forecast to optimise decision-making in highly dynamic environments. The client organises a collaborative forecasting market with the task of predicting wind energy generation Y^k at time instant k for time $k+m$, where m is on the scale of a few minutes. We consider N forecasters (agents) that register on the client's platform to participate in the near real-time collaborative forecasting market. Each forecaster $i \in \mathcal{I}$ posts a point forecast f_i^{k+m} at time k, which is a conditional expectation of Y^{k+m}. Then, the client uses linear pooling to evaluate an aggregated forecast $\hat{f}_{\mathcal{I}} = \sum_{i\in\mathcal{I}} \frac{1}{|\mathcal{I}|} f_i$. After the event occurs and the actual wind energy generation ω is observed, the client's platform evaluates the quality of the aggregated forecast. Then, the client announces the reward ϕ to be distributed among the forecasters according to the quality of their predictions.
In the literature, the most widely used criteria to evaluate the quality of forecasts are the so-called scoring rules [36]. For our work, we use absolute error (AE) as a scoring rule which is used for the evaluation of point forecasts. Let the reported prediction by a forecaster i, be f_i and let ω be the actual outcome, then their AE is given as $\text{AE}_i = |f_i - \omega|$.
We can now formulate this collaborative forecasting market as a coalitional game by letting the value of a coalition $S \subset \mathcal{I}$ to be $(1-\text{AE})$ of its combined forecast, i.e., $v(S) = 1 - |\hat{f}_S - \omega|$. Each forecaster evaluates the values of its own coalitions and utilizes the online protocol in (6.5) to distributedly track the Shapley payoff. The payoff represents the share of each forecaster in the reward evaluated by Shapley value. Note that, we compute the Shapley value as a payoff factor which corresponds to the monetary payoff that an agent will receive. The correspondence of payoff factor to monetary payoff is application specific and depends on the gain in the monetary utility of the client because of the collaborative forecast. For instance, in our example of wind energy forecasting, the payoff can correspond to the improvement in utility by optimal operation of combined wind-hydro power plants or by avoiding an imbalance charge in the market. To keep our focus on the market mechanism, we do not consider monetary payoffs and remark that

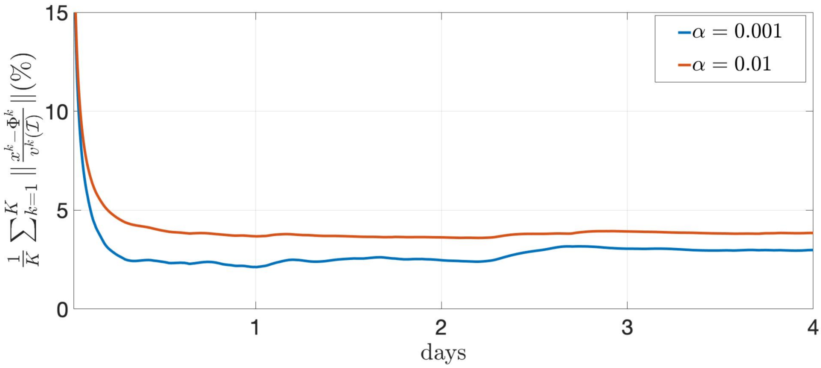


Figure 6.3: Trajectory of mean cumulative tracking error $\frac{1}{K}\sum_{k=1}^{K} \| \frac{\boldsymbol{x}^k - \boldsymbol{\Phi}^k}{v(\mathcal{I})} \|$, where $\boldsymbol{\Phi}$ is the Shapley allocation.

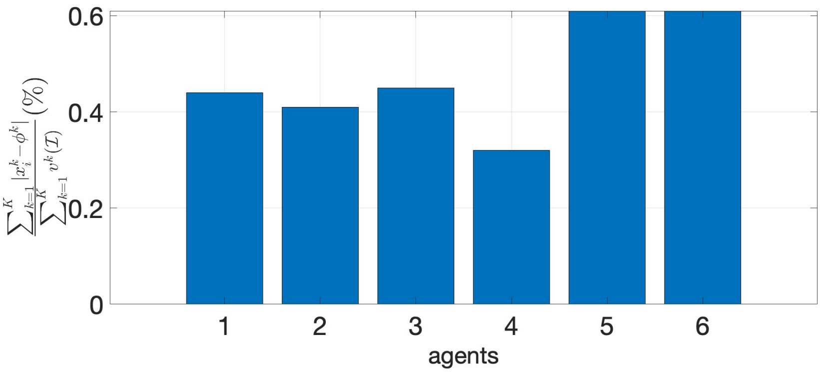


Figure 6.4: Difference in payoff received by online PD and the Shapley payoff $|\boldsymbol{x}_i^k - \boldsymbol{\phi}^k|$ proportional to the total value generated in the market $\sum_{k=1}^{K} v^k(\mathcal{I})$.

their incorporation would not affect the resulting solution properties.

SIMULATION STUDY

To illustrate the collaborative forecasting market, we consider that a client sets up a micro market with the task of forecasting wind energy generation in Germany with a lead time of 5 minutes. Let 6 forecasters (agents) register at the client's platform for providing the forecast reports. Each forecaster posts their prediction of wind power in the form of a point forecast at time k for lead time $k+5$ minutes. The client then aggregates the reported forecasts to generate a collaborative prediction and utilizes the mechanism in (6.5) for real-time payoff distribution. Let this market run continuously for 4 days to create a time series. Here, we use synthetic data to simulate forecasters' predictions generated using the forecast and actual measurements provided by the Spotrenewables and interpolate it to get the required resolution for the period of 25-28/05/2022. Fig. 6.2 shows the agents' forecasts and corresponding observations in terms of the capacity factor, i.e., normalized to the theoretical maximum of wind power plant for a 4-day period.

The high accuracy of generated forecasts simulates the near real-time forecasting effect. Next, in Fig. 6.3, we present the tracking performance of our algorithm in (6.5) for different values of α. We compute the tracking error by evaluating the online payoff and the Shapley payoff (static case) for the market game at each instant k as $\frac{1}{K}\sum_{k=1}^{K}\|\frac{\boldsymbol{x}^k-\boldsymbol{\Phi}^k}{v(\mathcal{I})}\|$. In words, we report the norm of the normalized difference between the online payoff and the Shapley payoff accumulated over time K. This cumulative tracking error is less than 4% for both values of α. Generally, for short-term to near real-time energy-related markets, the forecast accuracy is high and there is a low variation from one time-step to the next. Thus under such setups, our algorithm shows promising performance. We stress that the tracking performance of our algorithm depends on the dynamics of an underlying problem. Abrupt changes in the value function $v^k(S)$ can increase the tracking error significantly. Fig. 6.4 shows the difference in forecasters' payoff over four days with the Shapley payoff.

6.4.2. REAL-TIME LOCAL ELECTRICITY MARKET

In this subsection, we simulate a real-time local energy trading with an electricity market setup inspired by [42]. In our proposed setup, the prosumers and consumers participate in a local electricity market, established within the community, to trade energy internally rather than with a grid. The economic viability of such a market setup is based on the assumption that the buyers value energy higher than the grid's buying price and not more than the grid's selling price. Similarly, the sellers choose their valuation less than the grid's selling price. We note that these assumptions are common in the literature [44]. Traditionally, electricity markets are organized in a day-ahead setting with some intra-day arrangements for balancing purposes. However, due to uncertainty in RES and consumer load, at the level of a community, the market-clearing so far ahead of delivery can be considerably problematic for the system operator, responsible for system security. One way to mitigate the effect of uncertainty is organizing a market close to the time of delivery. In this direction, we design an online market mechanism based on dynamic coalitional game theory for a real-time market model. The dynamic formulation incorporates an evolving energy demand and RES generation that change with time. In this market setup, ideally, the goal is to maximize the social welfare of the local electricity market and distribute the resulting amount among participants such that the payoff should belong to the core in (2.2). However, in a real-time clearing setup, it is not possible to compute a payoff in the core exactly, thus we track it via the online mechanism in (6.8).

PROBLEM SETUP

We consider a simplified setup with N agents in an energy community $\mathcal{I}$, some equipped with RES generation (prosumers). We compute the coalitional value of each coalition $S \subseteq \mathcal{I}$ for a time instant k by solving a linear optimization problem. At each k, an agent either belongs to a set of buyers $\mathcal{S}_b$ or sellers $\mathcal{S}_s$ where, $\mathcal{S}_b \cup \mathcal{S}_s = S$. Let us denote the energy demand or generation of an agent i at a time instant k by E_i^k and the corresponding utility function coefficient by p_i^k. Here, we take the utility function coefficient of a seller as negative, i.e., $p_i < 0$ if $i \in \mathcal{I}_s$. We compute the coalitional value $v^k(S)$ for each coalition

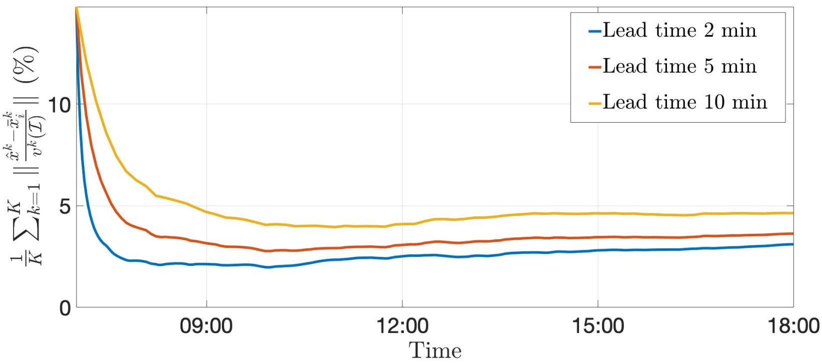


Figure 6.5: Trajectory of mean cumulative tracking error $\frac{1}{K}\sum_{k=1}^{K} \| \frac{\hat{\boldsymbol{x}}^k - \bar{\boldsymbol{x}}_i{}^k}{v(\mathcal{I})} \|$, where $\hat{\boldsymbol{x}} = \frac{1}{N}\sum_{i\in\mathcal{I}} \boldsymbol{x}_i$ and $\bar{\boldsymbol{x}}_i$ is the core allocation.

$S \subseteq \mathcal{I}$ as follows:

$$v^k(S) = \begin{cases} \max\limits_{(E_i)_{i\in S}} \sum\limits_{i\in S} p_i^k E_i^k \\ \text{s.t.} \quad 0 \le (E_i^k)_{i\in S} \le (\bar{E}_i{}^k)_{i\in S} \\ \qquad \sum\limits_{i\in S_s} E_i^k - \sum\limits_{i\in S_b} E_i^k = 0. \end{cases} \tag{6.9}$$

The constraints in (11) show instantaneous generation and consumption limits of sellers and buyers, respectively, and a power balance. We note that only mixed coalitions, i.e., with buyers and sellers, will produce a value, a fact that reduces the computational burden. At every market instant, each agent computes its bounding set in (2.1) and then proposes a payoff via the online protocol in (6.8). To compute their bounding sets, agents need to compute the values of only their own coalitions, which requires information on individual coalitional values. We assume our local market to be established in an advanced paradigm, in combination with futuristic data markets for energy systems like [40]. This allows the agents to acquire the information required for computing the bounding sets. At the first instant of the market, agents allocate the whole value $v(\mathcal{I})$ to themselves, which is in accordance with their rational and self-interested nature. The goal here is to maximize the social welfare of the local electricity market and then distribute the resulting amount among participants such that the payoffs track the core. Note that the unserved demand and unutilized generation will be traded with the grid.

Simulation study

For the numerical simulation, we consider a small local electricity market of 10 participants where the seller agents are equipped with PV systems and buyers are consumers. We use real data of PV generation and consumer load, recorded at 10-minute intervals, provided by a smart-grid demonstration project in the UK named Customer-Led Network Revolution (CLNR) [23]. We analyse market-clearing with the lead time of $k+2$, $k+5$, and $k+10$ minutes. For the 2 and 5 minute lead time, we interpolate CLNR's data to achieve the required resolution. Furthermore, we only consider the time slots

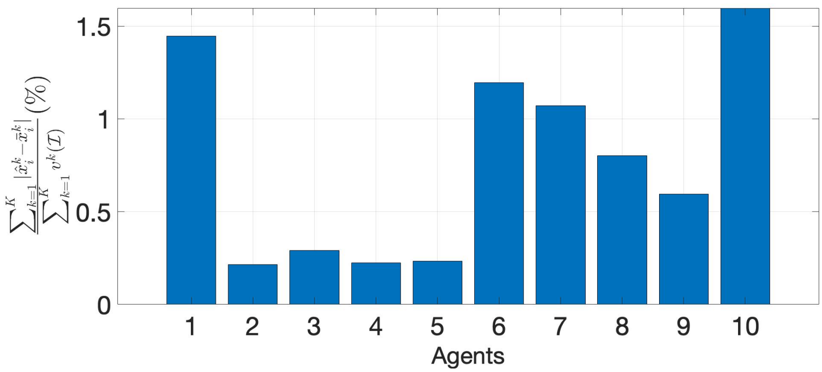


Figure 6.6: Difference in payoff received by online PD and the core payoff $|\hat{\boldsymbol{x}}_i^k - \bar{\boldsymbol{x}}_i{}^k|$ proportional to the total value generated in the market $\sum_{k=1}^{K} v^k(\mathcal{I})$, with lead time of five minutes.

6

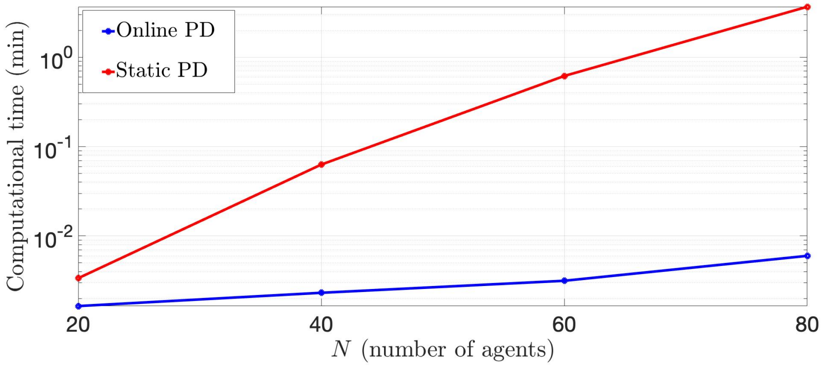


Figure 6.7: Computational time of an agent i for online payoff distribution (PD) and static PD case.

that have considerable PV generation during the day to demonstrate the effectiveness of our algorithm. At each market instance, the seller agents post the energy available to trade and its asking price. While buyer agents post their energy demand and willingness to pay for it. After receiving offers and demands, the participants negotiate to divide the optimal welfare of the market $v^k(N)$ by (6.9). We note that in an online setting, we track a consensus among agents on a core payoff instead of exact convergence to it. Therefore, because of the distributed formulation of our algorithm, the payoff proposals of agents at each market instant can differ and a criterion is required to allocate a mutually agreed payoff. In this simulation study, we select an average of all proposals to allocate a payoff $\hat{\boldsymbol{x}}$, where $\hat{\boldsymbol{x}} = \frac{1}{N}\sum_{i\in\mathcal{I}} \boldsymbol{x}_i$. For evaluating the tracking error, we let the algorithm converge to a consensus on a core payoff allocation, i.e., $\hat{\boldsymbol{x}}_i = \hat{\boldsymbol{x}}_j$ for all $i, j \in \mathcal{I}$. Finally, in Fig. 6.5, we report mean cumulative tracking error $\frac{1}{K}\sum_{k=1}^{K} \| \frac{\hat{\boldsymbol{x}}^k - \bar{\boldsymbol{x}}_i{}^k}{v(\mathcal{I})} \|$ that shows the core tracking capability of the algorithm. Since with a lead time of 10 minutes the market conditions (generation and load) change more from one market-clearing instance to another than with a lead time of 2 minutes, the tracking error is higher in the former case. This observation is consistent with the result in Theorem 8. Interestingly, for our market setup, the cumulative tracking error is below 5% even in the 10-minute case. Next, in Fig. 6.6, we present the difference in the payoff of each agent from the core payoff for a lead time of 5 minutes. The difference is at most 1.6% only, thus supporting the financial viability of an online payoff distribution in real-time markets. Finally, to report a comparison of computational times of online payoff distribution with the static case across the market size, we simulate a time-varying version of the bilateral P2P market presented in [87]. In Fig. 6.7, we show that exactly computing the core payoff is not feasible for fast-paced markets.

6.5. Conclusion

In this paper, we propose a real-time payoff distribution in online coalitional games where the goal is to track a consensus on the payoff distribution solutions, namely, Shapley value and the core. We have shown that an online paradigm of coalitional games provides promising tools for modeling collaborative systems working in environments with fast dynamics, e.g., such as real-time markets. The proposed distributed algorithms based on contraction operators adequately track the payoff distribution solutions. Our examples of local electricity market and collaborative forecasting market show the extent of energy-related applications that can be formulated with our proposed online framework. Numerical simulations illustrate the benefits of our online protocol and show that under the bounded variation in coalitional values a reasonable aggregate difference in online payoff and corresponding exact solutions can be achieved. Thus, online algorithms address the problem of scalability in real-time markets well modeled as coalitional games.

Next, we envision a competition platform to test the performance of the proposed online market mechanism and the behavior of participants in practical scenarios. Such a setup should provide useful insights for real-world implementation of the mechanism. An interesting extension of our work would be to incorporate long-term forecasts in the online formulation to provide better performance for events with high volatility.

6.6. APPENDIX

To prove the convergence of iteration in (6.5) and (6.7), as stated in Theorem 7 and Theorem 8, respectively, we first provide useful results regarding contraction operators.

Lemma 14 ([99], Thm. 3.1)**.** *Let* $\{M^k\}_{k\in\mathbb{N}}$ *be a sequence of contraction operators with* $\{L^k\}_{k\in\mathbb{N}}$ *as corresponding contraction factors such that* $\text{fix}(M^K)_{k\in\mathbb{N}} \neq \emptyset$*. Let Assumption 18 hold. Then, the error norm* $||x^k - \bar{x}^k||$ *generated by* $x^{k+1} := M^k(x^k)$ *converges as:*

$$\|x^k - \bar{x}^k\| \leq \hat{L}_k \|x_0 - \bar{x}_0\| + \frac{1-(\bar{L}_k)^{k-1}}{1-\bar{L}_k}\delta,$$

where $\bar{x}^k \in \text{fix}(M^k)$, $\hat{L}_k = \prod_{i=1}^{k-1} L_i$ *and* $\bar{L}_k = \max_k L_k$. □

Lemma 15 (Doubly stochastic matrix ([32], Prop. 5))**.** *If* W *is a doubly stochastic matrix then, the linear operator defined by the matrix* $W \otimes I_n$ *under Assumption 17 is a paracontraction with respect to the mixed vector norm* $\|\cdot\|_{2,2}$.

□

Lemma 16 (Composition of a contraction and paracontraction operator ([11], Prop. 4.49))**.** *Suppose* $T_1 : \mathbb{R}^n \to \mathbb{R}^n$ *is a contraction operator and* $T_2 : \mathbb{R}^n \to \mathbb{R}^n$ *is a paracontraction with respect to same norm* $\|\cdot\|$ *and* $\text{fix}(T_1) \cap \text{fix}(T_2) \neq \emptyset$*. Then, the composition* $T_1 \circ T_2$ *is a contraction and* $\text{fix}(T_1 \circ T_2) = \text{fix}(T_1) \cap \text{fix}(T_2)$. □

6

With these results, we are now ready to present the proofs of Theorems 7 and 8.

Proof. (Theorem 7) Let us formulate the iteration in (6.5) as $\boldsymbol{x}^{k+1} = \boldsymbol{M}^k(\boldsymbol{x}^k)$ where $\boldsymbol{M}^k := (\text{Id} - \alpha\nabla f^k) \circ \boldsymbol{W}^k$. Then, by Lemmas 15 and 16 $(\boldsymbol{x}^k)_{k\in\mathbb{N}}$ generates a sequence of contraction operators. For a time-invariant case, i.e., $\boldsymbol{z}^{k+1} = \boldsymbol{M}\boldsymbol{z}^k$ by [128, Lemma 1] $\boldsymbol{z}^k \to \bar{\boldsymbol{z}}$ as $\|\boldsymbol{z}^k - \bar{\boldsymbol{z}}\| = O(\alpha)$ where $\bar{\boldsymbol{z}}$ is optimizer of the problem in (6.4), i.e., $\bar{\boldsymbol{z}} = \boldsymbol{\phi}(v) = \frac{1}{N}\sum_{i\in\mathcal{I}} \hat{\boldsymbol{m}}_i$. Now, in time-varying case, under Assumption 18 the time variation of $\boldsymbol{M}^k$ is bounded, thus the application of Lemma 14 completes the proof. □

Proof. (Theorem 8) For the iteration in (6.8), it follows from Lemma 15 that $\text{fix}(\boldsymbol{T}^k \circ \boldsymbol{W}^k) = \text{fix}(\boldsymbol{T}^k) \cap \text{fix}(\boldsymbol{W}^k) = C^N(v^k) \cap \mathcal{A}$. By Lemmas 15 and 16, the iteration in (6.8) generates a sequence of time-varying contraction operators. Under Assumption 18 the time variation of $\boldsymbol{T}^k$ is bounded, thus the application of Lemma 14 completes the proof. □

7

A Market for Trading Forecasts

Several tech enterprises are collecting vast amounts of data that they deem proprietary, for example, social media platforms. These data owners extract predictive information of varying quality and relevance from data depending on quantity, inherent information content and their own technical expertise. Aggregating these data and heterogeneous predictive skills, which are distributed in terms of ownership, can result in a higher collective value for a prediction task. In this chapter, we envision a platform for improving predictions via implicit pooling of private information in return for possible remuneration. Specifically, we design a wagering-based forecast elicitation market platform, where a buyer intending to improve their forecasts posts a prediction task, and sellers respond to it with their forecast reports and wagers. This market delivers an aggregated forecast to the buyer (pre-event) and allocates a payoff to the sellers (post-event) for their contribution. We propose a payoff mechanism and prove that it satisfies several desirable economic properties, including those specific to electronic platforms. Furthermore, we discuss the properties of the forecast aggregation operator and scoring rules to emphasise their effect on the sellers' payoff. Finally, we provide numerical examples to illustrate the structure and properties of the proposed market platform.

7.1. Introduction

Forecasting plays a central role in planning and decision-making. Thus, it has always received substantial attention from researchers and practitioners. For a comprehensive review of forecasting and methodological advances, we refer the reader to an encyclopedic article by [79]. To produce good quality predictions, forecasters rely on high-quality data and sophisticated mathematical models. Often, the data are collected and held by different owners at different locations, i.e., distributed in terms of geography and ownership. The pooling of this distributed data can generate additional value. For example, logistics companies can exchange their data on consumer behavior to improve their forecast of future inventory demand. Such a forecast improvement by combining or accessing more data from distributed sources is demonstrated in several studies, see [8] and [63], for the example of energy applications. The general results such that forecasts can be improved through combination is already well-known within the forecasting community. However, in practice, the data owned by firms or individuals are perceived to have a cost when exposed. For businesses, this cost can be in terms of competitive disadvantage, and for individuals, in terms of privacy loss. Therefore, to utilize the distributed data, we aim at designing platforms for the pooling of predictive information. Such platforms allow for a monetary transfer from the buyer to the sellers, who are then compensated for the costs incurred for data collection, processing, modelling, etc., without explicit exposure of their private data. Because of the market context, in this work, we do not consider the infrastructural cost associated with the data.

We position our work in the area of market-based analytics, which can be broadly categorised into *data markets* and *information markets* depending on whether the traded product is the raw data or extracted information. Both these platforms have received increasing attention in the last few decades. In data markets, the key task is data valuation based on the contribution of each data seller to a learning task posted by a data buyer (the *client*), typically at a central platform [2], [33]. The market platform determines the monetary compensation that corresponds to the data value. Another significant factor in designing data markets is the cost of seller's privacy loss [34], which plays an important role in determining the value of data, see [107] and [1]. For details on data markets, we refer the reader to a comprehensive review by [17].

Data markets empower data owners (sellers) to have control over the exposure of their private resources and allow buyers to obtain high-quality training data for their learning algorithms and prediction tasks. Despite their huge potential, data markets are not free from limitations and challenges. First, determining the contribution of a particular data set for a buyer is, in principle, a combinatorial problem because of the possible overlap of information among the data sets [2]. Thus, the computational requirements for data valuation grow exponentially with the increase in the number of sellers and consequently for the evaluation of remuneration. Second, each seller can have different sensitivity to their data privacy, which makes it challenging to design a privacy-preserving mechanism. Both these issues can be addressed, to some extent, by so-called information markets.

Information markets [59], encompass the trade of a much broader category of information goods like news, translations, legal information, etc. However, here, we focus on the frameworks of forecasting platforms that can be categorised under the information markets. In this direction, *prediction markets* gained popularity beyond the academic circles [16], [125]. Prediction markets generate aggregate forecasts of uncertain future events, from dispersed information, by utilizing the notion of "wisdom of crowds". For example, in a prediction market designed for forecasting the election result, the share price of political candidates indicates the aggregate opinion on the probability of a candidate's win. Different from the structure of prediction markets, we design an information market for the improvement of buyer's forecast. This improvement offered by the forecasters is remunerated via a mechanism with formal mathematical guarantees on *desirable* economic properties like budget balanced, truthfulness, etc. [53]. Thus, in terms of design, our work is closer to the markets proposed for forecast elicitation with formal guarantees. In these works, typically, the sellers report their beliefs about a future event. Then, after the event occurs, the sellers are ranked according to the quality of their forecasts, evaluated by a *scoring rule* [53], [39].

An approach different from contribution-based reward, i.e., "winner takes it all", is proposed in [123]. Interesting to note that rewarding the best encompasses many real-world forecasting settings. For example, Netflix offered 1M USD to the team with the best prediction on how users would rate movies [123]. Even though popular in forecasting competitions, the "winner takes it all" approach ignores the fact that the forecasts other than the best one can still provide additional information. Therefore, in line with the idea of pooling the distributed information, we pursue the mechanisms that aggregate information provided by all the sellers and reward according to the quality.

In our work, we particularly take inspiration from the self-financed *wagering* market setup of [56] that features a weighted-score mechanism. In their setup, each player posts a prediction report of an event and wagers a positive amount of money into a common pool. After the occurrence of the event, the wager pool is redistributed among the players according to their relative individual performance. The payoff function is a weighted mixture of strictly proper scoring functions that satisfies several desirable economic properties. Such self-financed mechanisms create a competition of forecasting skills but do not include criteria for utilizing these forecasts, thus ignoring their value for a particular application or for an observer. In other words, there is no external agent who is aggregating, utilizing, and rewarding the resulting forecast based on the utility it generates. Differently from the setting in [56], in this chapter, we design a mechanism that considers both the forecasting skill of the players and the utility of the forecasts.

7

We consider a situation where a client (see [53]) posts a forecasting task on the market platform, along with the monetary reward they are willing to pay for an improvement in their own belief. In response, the sellers report their forecasts along with their wagers. A central operator then aggregates these forecasts, considering the wagers as corresponding weights, and passes to the client for planning or decision making. We note that, unlike prediction markets, where their mechanism inherently elicits an aggregated information in terms of stock prices, the aggregation of forecasts here has to be performed methodically [122]. Thus, our first goal is to select a suitable aggregation method that reflects players' wagers into the aggregated forecast. Next, a central operator evaluates the quality and contribution of each reported forecast and their corresponding payoffs. Our framework requires a payoff function with a utility component that rewards a contribution to the forecast improvement and a competitive component that evaluates the relative performance of sellers to reward or penalize accordingly. Thus, our second goal is to design a collective payoff function, with utility and competitive components that enjoys desirable economic properties.

7.1.1. CONTRIBUTION

We propose a marketplace for aggregate forecast elicitation using a wagering mechanism focused on improving the client's utility in terms of an improvement in their forecast. The proposed market model (Section 7.3.1) is general and history-free. It is general in the sense that tasks from any application area can be posted in the form of binary, discrete, or continuous random variables. History-free implies that we do not utilize past data on sellers' performance or market outcome, i.e., each instance of the market is set up independently. Then, we provide requirements for the aggregation of forecast reports by utilizing corresponding wagers and compare the quantile averaging to the linear pooling method as examples. (Section 7.3.2). Finally, we design a payoff function that rewards the skill of forecasters relative to each other as well as their contribution to the improvement of the utility of the client. We show that the proposed payoff function satisfies the desirable economic properties (Section 7.3.2).

7.2. PRELIMINARIES

7.2.1. FORECASTING TASK

Forecasting is a key requisite for decision making and planning employed in diverse situations for example, to predict a candidate's probability of winning the election, to project an economic condition of a country, businesses forecast their sales growth for production planning, renewable energy producers make an energy generation forecast for bidding in the market, etc. The diversity in the purpose of forecasting also translates into the types of forecasting tasks faced by a decision-maker. Broadly speaking, we can categorize forecasts into point forecasts, probabilistic forecasts, and scenarios [37], [64].

Point forecasts do not communicate the uncertainty associated with the possible outcomes of an event, hence an incomplete picture is delivered to a decision-maker. This shortcoming of point forecasts is resolved by probabilistic forecasts that provide decision-makers with the comprehensive information about potential future outcomes. Thus, in this paper we focus on probabilistic forecasts. A probabilistic forecast consists in a prediction of the probability distribution function (PDF) or of some summary measures of a random variable Y. These summary measures can be quantile forecasts or prediction intervals [38]. The market framework proposed in this chapter covers all types of probabilistic forecasts, given that the forecast evaluation method satisfies the property of being *strictly proper*. However, in the sequel, we focus on forecasting tasks in terms of PDFs for better exposition. We consider the single category, multi-category, and continuous forecasting tasks. Mathematically, these types of forecasts relate to forecasting of binary, discrete, and continuous random variables, respectively. Therefore, these cases suffice to cover most forecasting tasks we find in practice. Let us describe these forecasting tasks for uncertain events and provide relevant examples.

A single category task covers the binary events where the probability of an event happening is forecasted. For example a hedge fund predicting a return from a prospective investment has a single category forecasting task, i.e., will the quarterly growth of prospective investment be greater than x%? In the probabilistic forecasting framework the task will translate into; "the probability of the quarterly growth being greater than x%". For a multi-category forecast, take an example of a farming company that wants to predict seasonal rainfall in categories of light, moderate and heavy. Here, the forecast is in the form of a discrete probability distribution, e.g., the rainfall in the upcoming season being {light, moderate, heavy} has probability distribution $\{0.2, 0.5, 0.3\}$. An even more comprehensive probabilistic information can be obtained by forecasting an event in terms of a continuous probability distribution. For example, a wind energy producer bidding in an electricity market can obtain the whole uncertainty associated with the day-ahead energy generation event by obtaining a forecast in terms of a probability density function [81], [129].

In all three forms of forecasting presented above, the decision-makers, i.e., the hedge fund, farming company, and energy producer, can also have the in-house capability of forecasting. However, they expect that additional data and expertise can help them improve the quality of their forecasts for better planning and decision making, which in turn can lead to a higher utility. One way to achieve such a quality improvement is by designing a forecasting market platform where the data and expertise of the expert forecasters can be pooled in return for a competitive reward, depending on the contribution of each expert. When a decision-maker utilizes such a platform for forecast improvement, they expect experts to report their beliefs truthfully instead of gaming the market for higher rewards. Furthermore, the decision-maker requires the improvement offered by the experts to be measurable by formalized criteria. Both, the guaranteed truthful reporting and numeric evaluation of the quality of probabilistic forecast can be achieved by so-called *scoring rules*.

7.2.2. QUALITY, SKILL AND SCORING

At a forecast pooling platform, a scoring rule is required for quantifying the improvement in the forecast to be used by the decision-maker. Furthermore, it allows us to rank the forecasters to assign rewards according to their contributions. We note that this assessment is performed in an ex-post sense, i.e., after the event has occurred.

Definition 18 (Scoring rule)**.** *Let r be a reported probabilistic forecast and ω represent the event observed eventually. Then, a scoring rule $s : (r, \omega) \to \mathbb{R}$ provides a summary measure that assigns a real value for the evaluation of a probabilistic forecast r in view of the realization ω.*

In context of a marketplace for forecast elicitation, the role of the scoring rule $s(r, \omega)$ is to encourage the players to do their best in generating valuable predictive information, as well as in incentivizing their honest reporting. These tasks can be achieved by selecting scoring rules that satisfy certain properties. Next, we discuss the properties of scoring rules that we need in this work.

PROPERTIES OF SCORING RULES

First, we can incentivize that the forecasters report their beliefs truthfully, by rewarding them according to a strictly proper scoring rule [39].

Definition 19 (Strictly proper scoring rule). *Let a player report a probabilistic forecast r of an uncertain event Y. Let an outcome ω of an event be distributed according to the probability distribution p. Then, a real-valued function $s(\cdot,\omega)$ is called strictly proper when*

$$\mathbb{E}_p[s(r,\omega)] < \mathbb{E}_p[s(p,\omega)], \text{ for all } r \neq p.$$

Here, let ϱ be the support of p and f_{PDF} be the probability density function. Then, $\mathbb{E}_p[s(p,\omega)] = \int_\varrho s(p,\omega) f_{PDF}(p)dp$.

Later, we utilize a strictly proper scoring rule for our payoff criteria to measure the quality of the probabilistic forecasts and reward the players accordingly. There are many such rules reported in the literature, e.g., Brier score, logarithmic score, quadratic score, etc. [121]. In principle, a scoring rule is chosen based on the properties suitable for the application. Here, for a strictly proper score rule we consider two more properties of non-local and sensitivity to distance [39]. These properties consider a complete PDF, while ranking, and allocate a higher reward to a forecaster that concentrates the probability more around the realized event. This corresponds to rewarding a higher forecasting skill on forecaster's behalf. Next, we describe two other properties of scoring rules which we relate later on to show the effect of the choice of scoring rules on the payoff mechanism. This choice is important for implementing our proposed market design in practical scenarios.

Definition 20 (Non-local scoring [121]). *Let the forecasters report a PDF of an event Y and we observe the corresponding outcome ω. Then, a scoring rule is called local if the score depends only on the probability (for a categorical event), or likelihood (for a continuous variable), assigned to ω. Conversely, the rule is not local if it depends on the entire reported PDF.*

Definition 21 (Sensitivity to distance [51]). *Let r be a predictive PDF and R the corresponding cumulative distribution function (CDF). Then, a CDF R' is more distant from the value x than R if $R' \neq R, R'(y) \geq R(y)$ for $y \leq x$, and $R'(y) \leq R(y)$ for $y \geq x$. Consequently, a scoring rule s is said to be sensitive to distance if $s(r,\omega) > s\left(r',\omega\right)$, whenever R' is more distant from R.*

In other words, a scoring rule that allocates a higher score to the player whose report has assigned higher probability to the values closer to the observed value as compared with probabilities assigned to the values farther from the true value is said to be sensitive to the distance [121]. Later in Section 7.4.4, we numerically illustrate the properties of locality and sensitivity to distance for building a better intuition and providing a comparison between scoring rules.

7.3. PROPOSED FORECAST ELICITATION MARKET DESIGN

We consider a setting of a market with a single buyer and multiple sellers for eliciting a probabilistic forecast in the form of a probability distribution of an uncertain future event. In our setting, we refer to a buyer as a client and sellers as players or forecasters. A client posts a forecasting task on the market platform and announces a rate of monetary compensation for improvement in their own belief. Players with resources and expertise

7

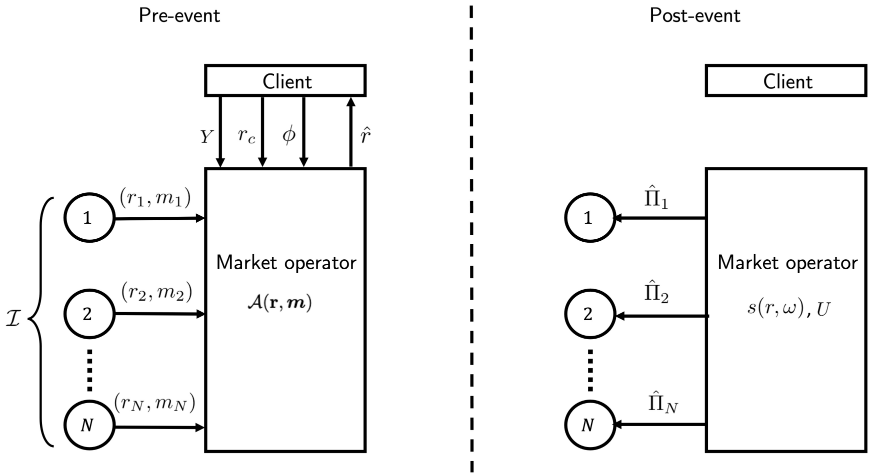


Figure 7.1: Market structure showing information flow and pre and post event evaluations. The delivery of $\hat{r}$ occurs after all the inputs are received.

in forecasting the posted task respond by reporting their forecasts along with the wagers. The market then aggregates the received information and delivers it to the client. This aggregated forecast, in turn, is expected to generate a utility for a client in terms of operational improvement. The resulting utility, considering the announced reward rate, is then distributed among the players such that it corresponds to their contribution. We note that the proposed mechanism can generally be used for the forecast eliciting of any event that can generate utility, such as the movement of a stock. Next, we formally describe our market model, and later we show the properties of the corresponding payoff distribution function.

7.3.1. MARKET MODEL AND PARTICIPANTS

CLIENT

Let there be a client i_c who is interested in improving their forecast (e.g., a generation forecast for their renewable energy asset). We parameterize a client through the following quantities:

- *Forecasting task* Y, an uncertain event that the client wants to predict better;
- *Forecast report* r_c, client's own forecast which is used as a reference for improvement;
- *Reward rate* $\phi > 0$, a monetary value that the client offers for per unit improvement in prediction.

A client can post a task Y in the form of a single category forecast (e.g., probability of energy generation being $[0.4, 0.6]$ per unit), a multi-category forecast (e.g., discrete probability distribution of energy generation in the intervals $\{[0.4, 0.6], [0.6, 0.8]\}$ per unit) and a continuous forecast (e.g., probability density function of energy generation). We note that the market design can also accommodate reports in the form of cumulative distribution functions. In the sequel, we represent the forecast reports of all three forms by r to keep the focus primarily on proposed mechanism, which holds for all forms of predictive distributions.

PLAYERS

Let $\mathcal{I} = \{1,\dots,N\}$ be the set of players that are forecasting experts in the area of a prediction task. We parameterize a player through the following quantities:

- *Forecast report* r_i, a prediction of forecasting task Y generated using player i's data resources and expertise; players try to improve r_c in return for a monetary reward;
- *Wager* $m_i > 0$ which accompanies the report r_i and expresses player i's confidence on their forecast.

A wager is associated with the player's confidence because it decides the level of impact their prediction has on the resulting forecast. Furthermore, in proposed payoff function, wagers also influence the reward (penalty) of the players.

MARKET OPERATOR

A central market operator manages the platform where a client and the players arrive with respective parameters. This operator is also responsible for maintaining transparency in the market process and is assumed to be honest. The functions of a market operator are:

- evaluation of an aggregated forecast $\hat{r}(\boldsymbol{m},\boldsymbol{r})$, where $\boldsymbol{r}$ represents a set of predictive distributions $\{r_i\}_{i=1}^N$ posted by the players and $\boldsymbol{m}$ is the vector of corresponding wagers;
- evaluation of the score $s(r_i,\omega)$ of each player $i \in \mathcal{I}$, after observing the outcome ω;
- evaluation of the utility U that corresponds to the improvement in client's forecast; thus, in case of improvement the utility $U \propto \phi(s(\hat{r},\omega) - s(r_c,\omega))$ and is zero otherwise.
- evaluation of the payoff $\hat{\Pi}_i$ of each player $i \in \mathcal{I}$.

Here, after the occurrence of the event, the market operator observes the true outcome ω and evaluates the score $s(r_i,\omega)$ of each player $i \in \mathcal{I}$, which shows how "good" was the forecast reported by player i. Then, the operator evaluates the utility $U(s(\hat{r},\omega), s(r_c,\omega), \phi)$ allocated by the client and distributes it among the players that have contributed to the improvement. For transparency, the market operator publicly posts the reward rate, forecast aggregation method, scoring rule and utility evaluation method, as agreed with the client. The individual predictions posted by the players can be kept private and only an aggregated forecast is delivered to the client. In Figure 7.1, we show the schematic structure of the proposed market with all participants and stages. Note that the allocated utility U depends on the improvement a client has made and, for the purposes of this work, we treat it as an exogenously specified value. Further details of the forecast aggregation methods, the payoff function and their properties are discussed in the sequel.

Remark 6. *An important benefit of the proposed market architecture is that the client cannot access the underlying features; instead they only receive an aggregated forecast. This mitigates a key challenge faced by data markets where sellers are hesitant to release their proprietary data streams as they are freely replicable.*

The mechanism design of this market model requires three main components: (i) an aggregation operator (to combine forecasts), (ii) a scoring rule, and (iii) a payoff allocation mechanism. Our goal is to design a history-free mechanism, i.e., a mechanism that does not require the past data or reputation of the players to compute a solution. This allows us to keep our market general, where clients can post diverse tasks in various forms without an assumption of a repetitive market with a pre-specified task. We note that, in the sequel, we use and drop the arguments from the notations depending on the necessity. Next, we present the components of our market mechanism and discuss their properties.

7.3.2. MECHANISM DESIGN

AGGREGATION OPERATOR

After the players have submitted their reports and wagers, in response to the client's forecasting task, the market operator creates a collective forecast $\hat{r}$, using an aggregation operator. Then, the client utilizes the resulting aggregated forecast for the decision making which in turn generates some utility. An improvement in the client's forecast r_c is rewarded at a pre-announced rate ϕ by the client. Therefore, the selection of the forecast aggregation operator constitutes an important part of the mechanism design.

Combining of probabilistic forecasts can be achieved via weighted averaging of predictive distributions. In this method, a weight assigned to a prediction reflects its relative accuracy determined by the historical data [54]. In other words, the predictions of players are weighted by their historical *performance* and have a corresponding impact on the evaluation of an aggregated forecast. Although logical, such methods are not useful for history-free mechanisms. Thus, in our proposed mechanism, the performance of a player is associated with their confidence in the reported prediction. Here, the players quantify this confidence via a wagering amount. This allows assigning an appropriate weightage to the individual forecasts while combining, which can improve the quality of an aggregated forecast. It also allows our mechanism to penalize (reward) forecasters for low (high) quality predictions, proportional to their influence on the aggregated forecast via wagers. We present this penalizing property of the payoff function named stimulant in the sequel.

Definition 22 (Aggregation operator)**.** *An aggregation operator $\mathcal{A} : (\mathbf{r}, \boldsymbol{m}) \rightarrow \hat{r}$ takes a set of predictive reports $\{r_i\}_{i=1}^N$ and a vector of corresponding wagers $\boldsymbol{m} \in \mathbb{R}^N$ as inputs, to evaluate a combined prediction $\hat{r}$.*

Two candidate methods that fulfil the criteria of aggregation operator are the so-called *linear opinion pool* (LOP) and the *quantile averaging* (QA). In terms of distributional forecasts, linear averaging of the probability forecasts can be viewed as vertical combining and averaging the quantiles can be seen as horizontal combining [58]. Therefore, these two methods can be regarded as two extreme cases in averaging. The

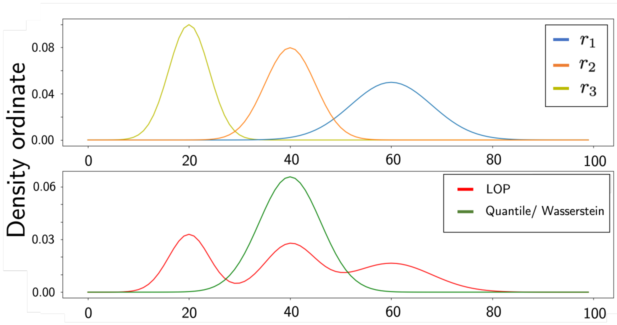


Figure 7.2: Comparison of LOP and quantile averaging/Wasserstein barycenter as aggregation operator

first method LOP is the most widely used method in literature [54], as well as in practice and has several extensions such as weighted linear opinion pool and optimally weighted linear opinion pool.

Definition 23 (Linear opinion pool)**.** *Let $\mathcal{I} = \{1,\ldots,N\}$ be a set of players. Let r_i be the forecast report of player $i \in \mathcal{I}$ and m_i be the corresponding wager. Then LOP is merely an average of all the reports weighted by wagers as $\sum_i \hat{m}_i r_i$ where $\hat{m}_i = \frac{m_i}{\sum_{j\in\mathcal{I}} m_j}$.*

For the optimally weighted extension, the weights m_i for all $i \in \mathcal{I}$, are evaluated by setting up an optimization problem considering the past data of the same market. However, even with optimized weights, the LOP suffers the problem of over-dispersed (under-confident) forecasting, meaning that the aggregate forecast evaluated via LOP has higher dispersion than the individual reports [90]. The authors in [90] propose a re-calibration method to improve the combined forecast resulting from the LOP, where the re-calibration parameters are evaluated by utilizing past data. Thus, this re-calibration method is not suitable for our history-free market mechanism. Next, we explore the quantile averaging which, interestingly, also corresponds to the Wasserstein barycenter [4] of the reported forecasts.

Definition 24 (Quantile averaging)**.** *Let $\mathcal{I} = \{1,\ldots,N\}$ be a set of players. For each player $i \in \mathcal{I}$, let r_i be the forecast report in terms of probability distribution function and R_i be the corresponding cumulative distribution function. Then, the average quantile forecast is given by $\hat{r}_{QA} = \sum_i \hat{m}_i R_i^{-1}$.*

In Figure 7.2, we present an illustration for the comparison of the aggregate forecasts evaluated via LOP and QA with equal weights (wagers). It provides an intuition for how the QA keeps the shape of individual forecasts reported as widely used parametric families of distributions, e.g., normal distribution. Consequently, it also maintains the properties of those parametric families that can comparatively provide more meaningful aggregation for decision-makers. [58] show some useful properties of the aggregated forecast evaluated via QA. For instance, an aggregated forecast attained by QA is sharper than that by LOP and each of its even central moments is less than or equal to those of the LOP [58, Prop. 8]. In a memory less market, like the proposed one, a prediction which is sharper around the observation can provide better information to the decision makers and thus is regarded as of higher quality.

We note that the QA can also be interpreted as the report that minimizes the Wasserstein distance $W(\cdot,\cdot)$ from all the forecast reports, i.e., $\hat{r} = \min_r \sum_{i=1}^{N} W(r, r_i)$, which corresponds to the Wasserstein barycenter. We refer the reader to [4] for further details on the Wasserstein distance and barycenter.

Remark 7. *The preference of one forecasts aggregation method over the other is primarily an empirical design choice that is largely application dependent.*

SCORING RULES

In this subsection, we specify a scoring function $s(r,\omega)$ to evaluate the quality of the forecast in an ex-post sense. We present a continuous ranked probability score (CRPS), as a strictly proper score function for elicitation of a forecast in terms of a probability density function. CRPS is non-local and sensitive to distance (see Section 7.2.2). For single category and multi-category prediction tasks, we present scores with same properties as of CRPS in 7.7.1. Note that, to stay consistent with the literature, we define scoring rules as negatively oriented, i.e., the lower, the better. However, for our design of the payoff function, presented later, we need a positively oriented scoring. Thus, in the sequel, we re-orient scoring rules for illustrative examples.

Definition 25 (Continuous ranked probability score)**.** *For an event of interest x, let the probability density function reported by a player be r, and let ω be the event that actually occurred. Let R denote the cumulative distribution. Then, the continuous ranked probability score is defined as*

$$CRPS(R,\omega) = \int_{-\infty}^{\infty} [R_r(x) - R_\omega(x)]^2 \, dx \tag{7.1}$$

where

$$R_\omega(x) = \begin{cases} 0 & \text{if } x < \omega \\ 1 & \text{if } x \geq \omega \end{cases}$$

In words, the CRPS presents a distance between the probabilistic forecast r and the truth ω.

Note that we can conveniently re-orient the CRPS depending on the application. For example, renewable energy production can be normalized to obtain a continuous random variable $P_g \in [0,1]$. Then, we can re-orient the scoring function by defining $s(r,\omega) = 1 - \text{CRPS}$ and consequently $s(r,\omega) \in [0,1]$. With all the components defined, we are now read to propose a wagering-based payoff mechanism and its desired economic properties.

PAYOFF ALLOCATION MECHANISM

A payoff function is central to the design of a market mechanism as it distributes the pool of wagers $\sum_j m_j$ and the generated utility U among the market players according to their *performance*. Therefore, it is critical for the design of a payoff function that it encourages market participation, on one hand by clearly reflecting the player's relative contribution, and on the other hand by enabling the delivery of valuable information to the client. The payoff functions are characterized by several desirable properties that can be proven mathematically, e.g., budget balanced, individual rationality, etc.

For the design of a payoff function, we take inspiration from [56], where the authors present a self-financed wagering mechanism for competitive forecast elicitation. The payoff function in [56] rewards the skill of the player relative to the other players by re-distributing the wagers and is shown to satisfy several interesting properties. Such self-financed markets work in the absence of a particular client with a task hence, the payoff is only based on the skill component of the players and does not involve any utility component. In other words, a player is rewarded for being better than other players regardless of the value or utility of their forecast. However, our market model in Section 7.3.1 involves a client with a specified task and, therefore, our model involves an external payment associated with the utility of the client. Consequently, we need a payoff function that distributes the utility generated by the forecast, i.e., a monetary gain corresponding to an improvement in client's operational decisions, apart from rewarding the forecasting skill of the players. In practice, the incentive by the client can implicitly help in improving the forecast quality and in growing the size of the market. For instance, a player who believes their competitors are better informed than them will not enter a market with only a skill payoff, as in [56]. On the other hand, if the same player believes that their data can provide valuable information and insights to the client in terms of probabilistic forecast, they will be encouraged to enter our market considering the reward from a utility component. Let us first propose a payoff function, and then we present its desirable economic properties.

7

We divide the payoff function in two parts, one representing the allocation from the wager pool and another from the client's allocated utility. The former evaluates the relative forecasting skill of a player, and the latter compensates for their contribution to an improvement of the client's utility U. Let the wager payoff of a player i be

$$\Pi_i(\boldsymbol{r}, \boldsymbol{m}, \omega) := m_i \left(1 + s(r_i, \omega) - \frac{\sum_j s(r_j, \omega)\, m_j}{\sum_j m_j}\right). \tag{7.2}$$

This term evaluates the relative performance of the players, considering the relative quality of the forecasts and the amounts wagered. It shows that the reward of player i, i.e., $\Pi_i(\boldsymbol{r}, \boldsymbol{m}, \omega) - m_i$ equals the difference between its performance (confidence and quality) and the average performance of the players. We note that wager payoff can also generate a loss for the players such that they can lose the amount wagered. We refer to it as a penalty to players for posting low-quality forecasts which plays an important role in showing that our payoff criterion incentivizes truthful reporting by the participants. Now, let us define an indicator $\mathbb{1}_{\{a>b\}}$ that takes value 1 if $a > b$ and 0 otherwise. Then, an overall payoff is given as

$$\hat{\Pi}_i = \underbrace{\Pi_i}_{\text{skill component}} + \underbrace{\mathbb{1}_{\{U>0\}} \left(\frac{\tilde{s}(r_i, \omega)\, m_i}{\sum_j \tilde{s}(r_j, \omega)\, m_j} U\right)}_{\text{utility component}}, \tag{7.3}$$

where $\tilde{s}(r_i,\omega) = \mathbb{1}_{\{s(r_i,\omega)>\bar{s}\}} s(r_i,\omega)$ and $\bar{s} := s(r_c,\omega)$. Here, the utility component depends on an improvement offered by the player beyond the client's own resources r_c. Thus, to be eligible for a share of an allocated utility U, first, there should be an improvement in the client's resulting forecast, i.e., $U > 0$, and second, the score of player i, $s(r_i,\omega)$ should be greater than the score of the client. Here, the utility payoff of a player is always non-negative but a wager payoff can also create a loss, i.e., $\Pi_i - m_i < 0$ is possible. The possibility of a loss encourages players to compete in improving the forecast by employing better models and acquiring more meaningful data. We note that the client can achieve negative utility as well, i.e., the forecast becomes worst than their own prediction. However, again with a penalty imposed by the wagering part of the payoff function, it is expected from risk-averse players to report high-quality forecasts. Next, we provide a brief explanation of some desirable properties of a payoff function.
Desirable properties: The properties are adapted from [56] and here we include their explanations in context of the payoff function in (7.3).

i) *Budget-balance*: A mechanism is budget-balanced if the market generates no profit and creates no loss, i.e., $\sum_{i\in\mathcal{I}} \hat{\Pi}_i = \sum_{i\in\mathcal{I}} m_i + U$. In other words, the generated utility and the wager pool must be completely distributed, as a payoff, among the players.

ii) *Anonymity*: A mechanism satisfies anonymity if the payoff received by a player does not depend on their identity; rather it depends only on the forecast reports and the realization of an uncertain event. Formally, for any permutation σ of $\mathcal{I}$, the payoff $\hat{\Pi}_i\left((r_i),(m_i),\omega,U\right) = \hat{\Pi}_{\sigma(i)}\left(\left(r_{\sigma^{-1}(i)}\right),\left(m_{\sigma^{-1}(i)}\right),\omega,U\right)$ for all $i \in \mathcal{I}$.

iii) *Individually rational*: Let the belief of a player $i \in \mathcal{I}$ about an event be p. Then, a mechanism is individually rational if for any wager $m_i > 0$ there exists r_i^* such that an expected profit of a player is non-negative, i.e., $\mathbb{E}_p[\hat{\Pi}_i((\boldsymbol{r}_{-i}, r_i^*), \boldsymbol{m}, \omega, U) - m_i] \geq 0$, for any vector of wagers $\boldsymbol{m}_{-i}$ and reports $\boldsymbol{r}_{-i}$.
Individual rationality encourages the participation of players by ensuring a non-negative expected profit according to their beliefs.

iv) *Sybilproofness*: A truthful mechanism is sybilproof if the players cannot improve their payoff by creating fake identities and copies of their identities. Formally, let the reports $\boldsymbol{r}$ and vectors of wagers $\boldsymbol{m}$ and $\boldsymbol{m}'$ be such that for a subset of players $S \subset \mathcal{I}$ the reports $r_i = r_j$ for $i,j \in S$, the wagers $m_i = m_i'$ for $i \notin S$ and that $\sum_{i\in S} m_i = \sum_{i\in S} m_i'$. Then, the sybilproofness implies that, for all $i \notin S$, $\hat{\Pi}_i(\boldsymbol{r},\boldsymbol{m},\omega,U) = \hat{\Pi}_i\left(\boldsymbol{r},\boldsymbol{m}',\omega,U\right)$ and that $\sum_{i\in S} \hat{\Pi}_i(\boldsymbol{r},\boldsymbol{m},\omega,U) = \sum_{i\in S} \hat{\Pi}_i\left(\boldsymbol{r},\boldsymbol{m}',\omega,U\right)$.
We note that the Shapley value, a solution used to evaluate data in market setting, suffers the drawback of being prone to replication, i.e., players can increase their payoff by creating fake copies of themselves [2]. This consideration takes special importance in markets dealing with forecasts as the data are a freely-replicating good.

v) *Conditionally truthful for players*: A mechanism is conditionally truthful if the player does not have enough information or influence over the payoff function to manipulate it for their benefit. Thus, reporting their true belief becomes the best strategy for a risk averse player.

This definition of conditional truthfulness considers practical situations for the players and the market operation. Truthfulness of a mechanism encourages the players to post their true belief at the market platform thus, fulfilling the client's expectation of having an access to the honest assessments of the experts about an event.

vi) *Truthful for the client*: A mechanism is truthful for a client, in terms of reported prediction, if the client's expected allocated utility is minimized by reporting their true belief p as their own forecast, i.e., $\mathbb{E}_p\left[U(s(\hat{r},\omega), s(r_c,\omega), \phi)\right] > \mathbb{E}_p\left[U(s(\hat{r},\omega), s(p,\omega), \phi)\right]$ is satisfied for all $r_c \neq p$. We note that the truthfulness of the client concerns the prediction report r_c and not the reward rate ϕ. Since, with our single-buyer design, it is not possible to elicit their true willingness to pay.

vii) *Stimulant*: Let a player i's payoff be the sum of skill and utility components, i.e., $\pi_i(\boldsymbol{r}, (\boldsymbol{m}_{-i}, m_i), \omega, U) = \pi_i^s(\boldsymbol{r}, (\boldsymbol{m}_{-i}, m_i), \omega) + \pi_i^u(\boldsymbol{r}, (\boldsymbol{m}_{-i}, m_i), \omega, U)$. Let the wager be $m_i' > m_i$. Then, this payoff is monotonic if it holds that for the skill component, either

$$0 < \mathbb{E}_p\left[\pi_i^s(\boldsymbol{r}, (\boldsymbol{m}_{-i}, m_i), \omega) - m_i\right] < \mathbb{E}_p\left[\pi_i^s\left(\boldsymbol{r}, \left(\boldsymbol{m}_{-i}, m_i'\right), \omega\right) - m_i'\right]$$

or

$$0 > \mathbb{E}_p\left[\pi_i^s(\boldsymbol{r}, (\boldsymbol{m}_{-i}, m_i), \omega) - m_i\right] > \mathbb{E}_p\left[\pi_i^s\left(\boldsymbol{r}, \left(\boldsymbol{m}_{-i}, m_i'\right), \omega\right) - m_i'\right].$$

In words, a mechanism is monotonic if a player's expected profit, as well as loss from the skill component, increases by increasing the wager. Now, for a utility factor, let $U > 0$ and $s(r_i, \omega) > \bar{s}$. Then,

$$\pi_i^u(\boldsymbol{r}, (\boldsymbol{m}_{-i}, m_i), \omega) < \pi_i^u\left(\boldsymbol{r}, \left(\boldsymbol{m}_{-i}, m_i'\right), \omega\right).$$

These properties encourage the players to post higher wagers considering their confidence in their forecasts thus we refer to them as stimulant. Importantly, it also justifies weighting the forecasts by the corresponding wagers while creating an aggregate forecast. We note that, for real-world applications, the market operator can place lower and upper bounds on the amounts of wagers considering the viability of the market.

Now, we show that the proposed payoff criterion in (7.3) satisfies all the desirable properties described above.

Theorem 9 (Characteristics of payoff allocation). *Let $s(r,\omega) \in [0,1]$ be a strictly proper score function. Then, the payoff function given in (7.3) is (i) budget-balanced, (ii) anonymous, (iii) individually rational, (iv) sybilproof, (v) conditionally truthful for players, (vi) truthful for client, and (vii) stimulant.*

We provide the proof of the theorem in Appendix to improve the reading flow.

7

Table 7.1: Profit (payoff - wager) evaluation for forecast reports in Figure 7.3 and its sensitivity to wagers

(a)

Players	1	2	3
Wager	100	100	100
Scores	0.9430	0.8450	0.4830
Profit	546	481.39	−27.40

(b)

Players	1	2	3
Wager	100	100	500
Scores	0.9430	0.8450	0.4830
Profit	552.85	488.24	−41.10

7.4. ILLUSTRATIVE EXAMPLES

In this section, we illustrate several numerical examples to provide some intuition on the proposed market model and to numerically demonstrate the properties of the proposed payoff function in (7.3). For all the illustrations, we use a beta distribution, with parameters (α, β), as a base predictive density. We then vary its parameters to simulate potential forecast reports of different players. We acknowledge that these reports might not represent a real-world scenario. However, these examples are sufficient to illustrate and discuss interesting properties of the payoff function.

7.4.1. EFFECT OF WAGER AMOUNT

Let a client post a prediction task Y on a market platform along with their own forecast that has a score of 0.5, i.e., $s(r_c, \omega) = 0.5$. In response, let the players $\mathcal{I} = \{1, 2, 3\}$ post the predictive densities of a random variable $Y \in [0, 1]$, as shown in Figure 7.3. Though, in reality we expect the reports by expert forecasters to be concentrated around nearby values but here we consider an extreme case to emphasize our observations. First, we evaluate the players' payoff for equal wagers and then increase the wager of player 3 to demonstrate the stimulant property of the payoff function, defined in Section 7.3.2. Suppose the market operator announces the cap on the wager amount, i.e., maximum value a player can wager, $\bar{m} = 500$. The case of equal wagers in Table 7.1a shows a loss for player 3, taken from their wager, for posting a sharp predictive density concentrated far from the realized event, $\omega = 0.8$. The corresponding aggregate prediction $\hat{r}_a$, shown in Figure 7.3, has a score of 0.867. Here, the score of player 3 is lower then clients score and thus it doesn't receive any share from utility payoff. We note that the score of each player is given by a positively oriented scoring rule (1-CRPS) and the utility of a client is assumed to be specified exogenously. Next, for the case in Table 7.1b, the wager of player 3 is increased to maximum, which results in the increase of loss. This implies that showing more confidence via higher wager on a "bad" forecast will result in a higher loss which is an important consequence as a higher wager by player 3 resulted in the reduced quality of the aggregated prediction $\hat{r}_b$, as shown in Figure 7.3, with $s(\hat{r}_b, \omega) = 0.822$. This example illustrates the justification for using wagers as weights in the aggregation method. It also demonstrates how using a wager as a player's confidence results in a fair penalty or reward for them.

7

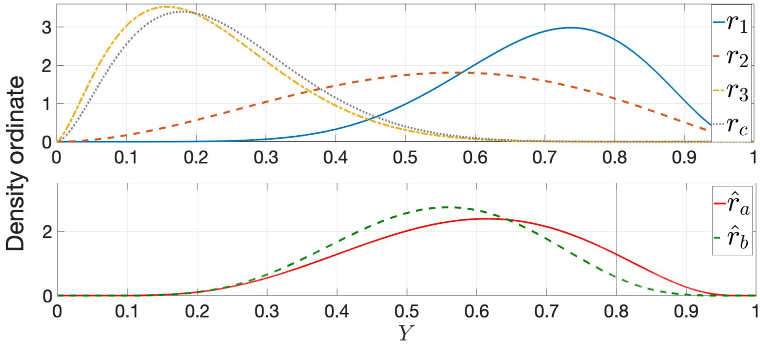


Figure 7.3: Plot on the top shows the reports on density forecast of random variable $Y \in [0,1]$ by market participants and the bottom plot shows aggregate density forecasts for wagering case (a) and (b) as in Table 7.1a and 7.1b, respectively. The vertical line is at the realization, $\omega = 0.8$.

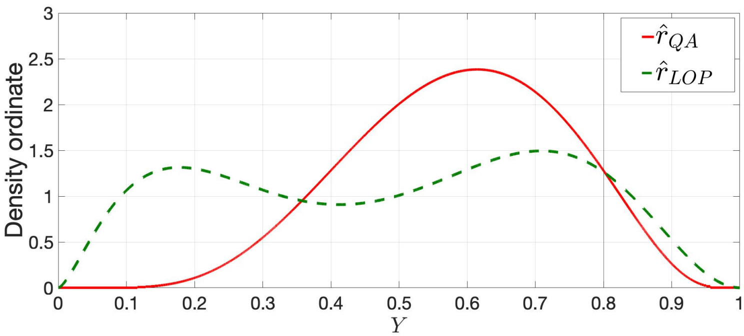


Figure 7.4: Plots of aggregate predictive densities obtained by quantile averaging $\hat{r}_{QA}$ and linear pooling $\hat{r}_{LOP}$ in an equal wagering case.

Table 7.2: Sybilproofness of profit (payoff - wager) in proposed mechanism

(a)

Players	1	2
Wager	100	100
Scores	0.9430	0.8450
Profit	532.30	467.69

(b)

Players	1	$2(a)$	$2(b)$
Wager	100	40	60
Scores	0.9430	0.8450	0.8450
Profit	532.30	187.07	280.61

7

7.4.2. COMPARISON OF QA AND LOP

In Figure 7.4, we present the comparison of aggregate predictive distributions obtained via quantile averaging $\hat{r}_{QA}$ and linear pooling $\hat{r}_{LOP}$. It is evident how $\hat{r}_{LOP}$ can be problematic for a decision-maker. The loss of sharpness translates into lower scores for linear opinion pool as well where, $s(\hat{r}_{LOP}, \omega) = 0.817$ compared with $s(\hat{r}_{QA}, \omega) = 0.867$. Furthermore, for commonly used parametric distributions quantile averaging maintains the shape of the distribution, while linear pooling does not.

7.4.3. DEMONSTRATION OF SYBILPROOFNESS

Now, we illustrate the property of sybilproofness (see Section 7.3.2), which in truthful mechanisms prevents players from manipulating identities. Sybilproofness of payoff function is specially important for electronic platforms. Table 7.2a shows profit and scores of two players with reported predictive densities r_1 and r_2, as in Figure 7.3. Now, let the player 2 create a fake identity and appear in the market as $2(a)$ and $2(b)$ with different wagers, as reported in Table 7.2b. We note that, even after identity manipulation, the collective profit of both identities of player 2 remained the same as with the true identity. Consequently, it does not affect the player 1 as well.

7.4.4. SENSITIVITY OF SCORING RULES

In this section, we demonstrate various properties of scoring rules to emphasize their effect on the design of a payoff function. Generally, the choice of a scoring rule depends on the application area of the prediction task. Thus, these illustrations are important to provide useful insights to the practitioners for adopting the proposed mechanism to a particular application. The choice of scoring rules can also affect the willingness of players to participate and constitute an important part of the design.

7

LOCAL VS. NON-LOCAL SCORING

Different scoring rules differ in their sensitivity to the variation in prediction quality. For applications where sharp predictions are required because of the high stakes, the scoring rules with higher sensitivity can perform better. Let us now compare the sensitivity of CRPS and log score by varying parameters (α, β) of predictive densities. To illustrate these effects across the variation in single parameter α, we fix the mean of densities and then evaluate β as $\beta = \frac{\alpha(1-\text{mean})}{\text{mean}}$. We note that in parametric case the variation in parameters simulates the varying quality or features utilized to construct the predictive densities. In Figure 7.5b, we show the predictive beta distributions for different values of α and the corresponding CRPS and log scores. As the log score depends only on the realization ω, it has considerable variation for given predictive densities. Whereas, CRPS takes complete information into account thus varies slightly with the slight change in densities. The scoring rules are selected essentially by considering the nature of the prediction task at hand. We note that our results hold for all strictly proper scoring rules, including the normalized log score.

SENSITIVITY TO DISTANCE

In this example, we illustrate the impact of the scoring rules' sensitivity to distance (see Definition 21). Let the three forecasters E_1, E_2 and E_3 provide a normal-

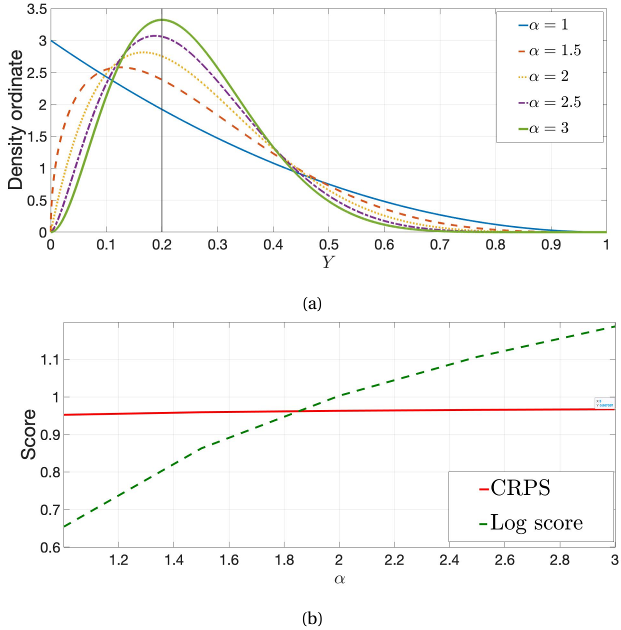


Figure 7.5: (a) Predictive beta distributions with same mean = 0.25 where, for each given α, $\beta = \frac{\alpha(1-\text{mean})}{\text{mean}}$. (b) Comparison of scores assigned to predictive distributions via CRPS and log-score.

7

ized multi-category probabilistic forecasts for the energy generation y of a wind producer for intervals $\{[0-0.2],(0.2-0.4],(0.4-0.6],(0.6-0.8],(0.8-1]\}$ per-unit represented by $\{1,2,3,4,5\}$. Let the reported probabilistic forecasts of E_1, E_2 and E_3 be $\{0.1,0.1,0.6,0.1,0.1\}$, $\{0,0.2,0.6,0.2,0\}$ and $\{0.2,0,0.6,0,0.2\}$, respectively. Suppose we observe the actual wind production in the third interval, i.e., $y = 3$. Let us now assess the quality of the forecasts using quadratic and ranked probability scoring (RPS) rules (see [121] and Section 7.7.1 for mathematical expressions). Here, E_1 receives a quadratic score of 0.8 while E_2 and E_3 receive 0.76. We first observe that all three forecasters have assigned a probability of 0.6 to the realized value of y. Next, we note that E_2 assigns the remaining probability of 0.4 to the intervals 2 and 4, that are adjacent to the realized interval, i.e., 3, while E_3 assigns it to the farthest (more distant) intervals. This probability assignment shows comparatively a better forecasting skill on behalf of E_2. However, their scores are same, which shows that the quadratic scoring is not sensitive to distance. In comparison, RPS assigns 0.975, 0.98 and 0.96 to the predictions of E_1, E_2 and E_3, respectively. We note that RPS acknowledges the concentration of probability around the observation and assigns highest score to E_2. Thus, RPS is sensitive to distance which can be important for the practitioners while designing a payoff function.

7.5. WIND ENERGY FORECASTING: A CASE STUDY

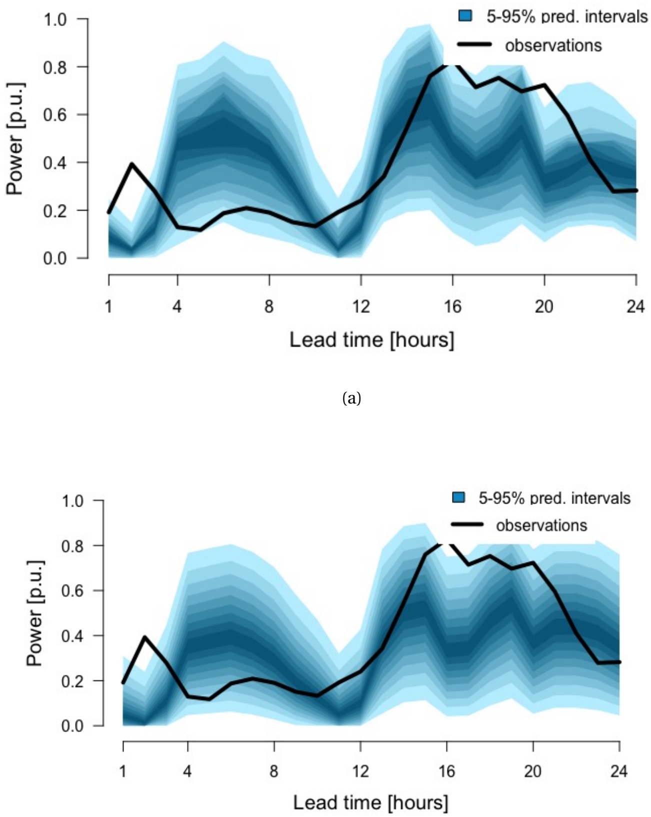
In this section, we present an energy forecasting application of the proposed market mechanism. Here, we differentiate forecasters based on their forecasting skill and resourcefulness. In former, the players utilize same data but different models (forecasting skill) to construct predictive densities and vice versa in the latter. This differentiation criteria covers an important feature of forecasting market that it creates a competition of both resourcefulness (data) and forecasting skill among the players. The aim of this case study is to demonstrate the compensation allocated by our market mechanism for eliciting forecasts evaluated by experts based on their private information and skills. Elicited forecasts are aggregated and delivered to the client.

7.5.1. SIMULATION SETUP

Consider a wind energy producer who wants to improve its generation forecast for more informed bidding in an electricity market, thereby avoiding a penalty for causing an imbalance. For this purpose, the energy producer arrives at the wagering based forecasting market, described in Section 7.3, as a client. We assume that the client submits the task of forecasting the next 24-hours of wind energy generation. In response, let the forecasters $\mathcal{I}$ submit the probabilistic forecasts along with their wagers. The market operator evaluates the scores of submitted forecasts on hourly basis and compensates accordingly. For our case study, we use an open data set from the Global Energy Forecasting Competition 2014, GEFcom2014 [46] and an open-source toolkit ProbCast by [19]. The wind power measurements are normalized and thus take values in $[0,1]$. For the market setup, we assume fixed utility U, offered by the client, to analyse scores and the share of each player's payoff $\hat{\Pi}_i$ in $\sum_i m_i + U$. We note that, in reality, the compensation provided by the client depends on the operational benefits that they receive through an improvement in their forecast. Next, we first present a simpler case of wind energy forecasting with 2 players evaluate the resulting payoff allocation, as in (7.3), and later we move to more extensive cases.

7.5.2. FORECASTING MARKET WITH 2 PLAYERS

Let the players $\mathcal{I} = \{1,2\}$ provide wind energy generation forecast for the next 24 hours. Here, we assume that both forecasters have the same data but they utilize different models to generate predictive densities for wind energy forecasting. Selection of a particular forecasting model can be seen as a forecasting skill of a player thus, the players have different forecasting skills. In this case, player 1 provides their wager m_1 and the forecast report r_1 as a parametric distribution, i.e., an inflated beta distribution as proposed by [78] generated by using a generalised additive model GAMLSS. Whereas, player 2 utilizes gradient boosted regression trees to generate non-parametric predictive densities and submits the forecast report r_2 along with the wager m_2. Let the market operator announce wager bounds such that $m_1, m_2 \in [10,100]$. We assume that the score of the client's own forecast is constant at 0.5 for all 24 hours. Such a low score shows that the client has a low-quality forecast and consequently, for our data, the players will be eligible for utility payoff at each hour. After receiving the reports, the market operator evaluates an aggregate forecast $\hat{r}$ and delivers it to the wind energy producer (client), who in turn uses it for operational planning. Figures 7.6a and 7.6b show the reports of player 1 and player 2, i.e., r_1 and r_2, respectively. The hourly observations represent the realization ω, i.e., the actual wind energy generation during the corresponding hour. After the forecasting period has passed, the market operator evaluates the score of each



(a)

(b)

Figure 7.6: (a) Wind energy generation forecast reported by player 1 via inflated beta distribution, i.e., r_1. (b) Wind energy generation forecast reported by player 2 via non-parametric predictive density, i.e., r_2. Observations represent realization ω.

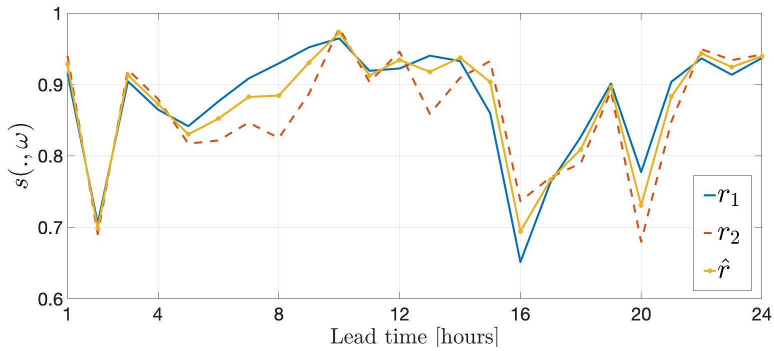


Figure 7.7: CRPS of forecasts reported by player 1 (r_1), player 2 (r_2) and an aggregate forecast $\hat{r}$.

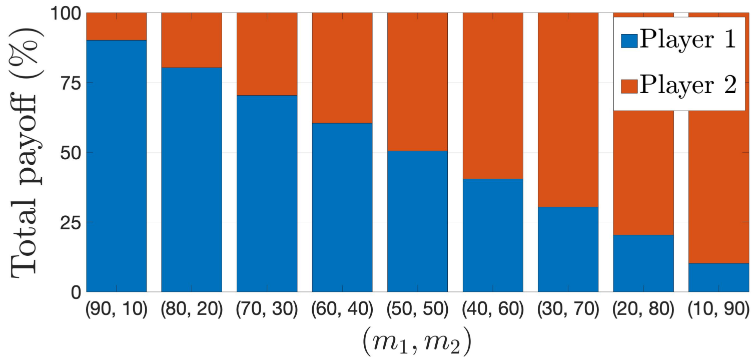


Figure 7.8: Players' total payoff of 24 hours as a share of money pool $\sum_i m_i + U$ for different wagers.

player and that of an aggregate forecast. Figure 7.7 shows the scores (CRPS) of r_1, r_2 and $\hat{r}$. We note that the aggregate forecast $\hat{r}$ evaluated via quantile averaging, as in Definition 24, depends on the wagers of the players, and Figure 7.7 is the case of equal wagers. The difference in the scores of both players is not much as their reported predictive densities follow a similar trend. Though the score rank of players varies at different hours the parametric forecaster performs slightly better in a cumulative sense, for this particular instance of market. If this variation in score rank is considerable the aggregate forecast can score better than both players. We illustrate this fact later in our case study. Next, we show players' total payoff for 24 hours as a share of money pool $\sum_i m_i + U$. The payoff, as in (7.3), also depends on wagers m_i and in the case of equal wagers it corresponds directly to the scores. To observe the effect of wager, in Figure 7.8 we plot payoff across different wager pairs. As both players offer improvement and the scores of both players do not differ much, the stimulant property of our payoff function, explained in Section 7.3.2, allocates higher payoff to high wagering player.

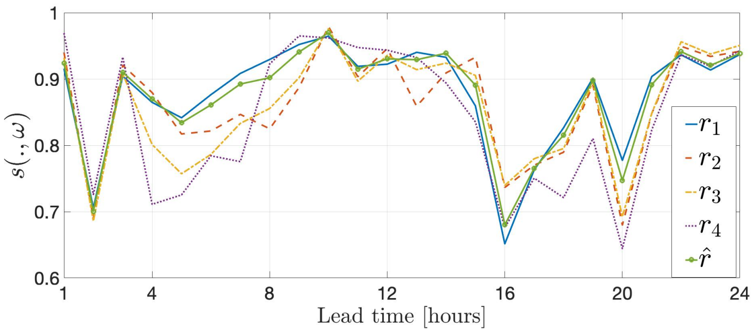


Figure 7.9: CRPS of forecasts reported by players (r_1, r_2, r_3 and r_4) and an aggregate forecast $\hat{r}$.

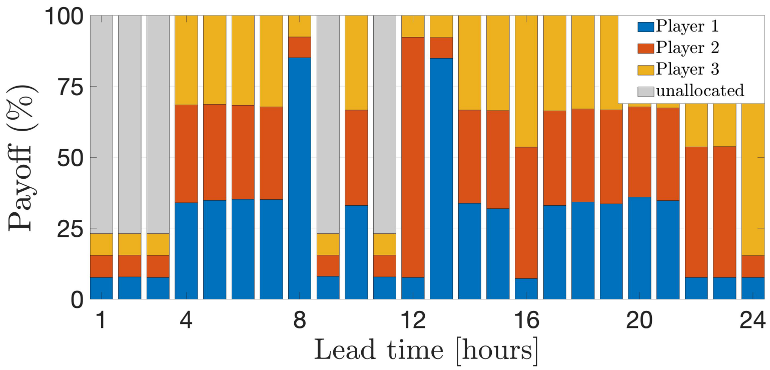


Figure 7.10: Players' hourly payoff as a share of money pool $\sum_i m_i + U$, assuming equal wagers.

Table 7.3: Total score (CRPS) of reported forecasts over 24 hour period.

Report	r_1	r_2	r_3	r_4	$\hat{r}$
Total score	21.0480	20.6978	20.6090	20.2514	21.0074

7.5.3. FORECASTING MARKET WITH 4 PLAYERS

Now, let two more players join the market referred as player 3 and player 4. We assume that these new players have the same forecasting skill, i.e., both players utilize same forecasting method. However, the data held/ collected by the players is different. Player 3 holds the data of wind forecasts, as predictor, at the height of 10 m above the ground level whereas player 4 has the data of wind forecasts at 100 m above ground level. Wind forecast being a key predictor effects the quality of energy generation forecasts. The quality of all 4 reports is evaluated by CRPS and is presented in Figure 7.9 along with the score of an aggregate forecast. In Table 7.3, we report total scores of all forecast reports over the period of 24 hours. Interestingly, for this market instance, the score of aggregate forecast $s(\hat{r}, \omega)$ is higher than that of individual forecast reports of all the players.

To analyse the hourly payoff allocation when the client has a forecast report of a reasonable quality, we assume player 4 to be the client, i.e., $r_c = r_4$, as in (7.3). Consequently, according to proposed payoff function in (7.3), a player becomes eligible for a utility payoff only when it offers an improvement to the client, i.e., scores higher than the client. Assuming a fixed utility payoff U, we present players' payoff allocation in Figure 7.10. We can observe that for the first 3 hours the score of client's forecast report (r_4) in Figure 7.9 is higher than the players thus, the payoff distribution occurs only from the wager pool $\sum_i m_i$. As we consider a fixed utility component U, there remains an unallocated utility payoff component which is returned back to the client. In contrast, if utility component depends on the forecast improvement of the client then $U = 0$ in case of first 3 hours. Next, observe that at the 12th hour only player 2 offers slight improvement, i.e., scores higher than the client (see Figure 7.9) thus, they receive the whole offered utility payoff.

7.6. CONCLUSION

We have designed a marketplace for revealing an aggregate forecast by eliciting truthful individual forecasts from a group of forecasters. In the proposed model, a client with a prediction task calls for forecasts on a market platform and announces a monetary reward for it. The forecasters respond with predictive reports and wagers showing their confidence. The platform aggregates the forecasts and delivers them to the client. Here, the utilized aggregation criteria allows us to make our mechanism a one-shot history-free method that does not account for the forecaster's performance in the past. Next, upon the realization of the event, it allocates payoffs to the forecasters depending on the quality of their forecasts. We have proposed a payoff function with skill and utility components that depend on the relative forecast quality of a forecaster and their contribution to improving the forecast of the client, respectively. We show that the proposed payoff allocation satisfies several desirable economic properties, including budget balance, anonymity, conditional truthfulness, sybilproofness, individual rationality, and stimulant. The simplicity of scoring-based market design, with a wagering mechanism, allows it to cater diverse forecasting tasks with forecasting reports taking forms of discrete to continuous probability distributions.

From the success story of platforms like NUMERAI [76], we see a high potential for real-world aggregative forecasting marketplaces. Differently from current implementations, the mechanism proposed in this chapter is designed for the improvement of predictions and provides theoretical guarantees on the monetary compensation that can encourage and retain the participation of experts. Next, we envision a competition platform to test the performance of the proposed market model and the behavior of players in practical scenarios. Such an experimental setup would help us gain further insights for real-world implementation. Furthermore, our market setup opens several paths for applied modelling of information eliciting platforms and their analysis. An important step is to design a mechanism for online predictions based on streaming data and in turn analyse if it maintains the economic properties discussed in this chapter. Another interesting research avenue is to design models that value the reputation of forecasters (historic credits) as well.

7.7. APPENDIX

7.7.1. SCORING RULES

Let us present strictly proper scoring rules for single-category and multi-category reporting that are non-local and sensitive to distance (see Section 7.2.2). A strictly proper scoring rule which is non-local and can be used for eliciting a single-category forecast for binary events, is the Brier score.

Definition 26 (Brier score)**.** *Let the probability of occurrence of an event x, reported by a player, be r and let ω be the actual outcome. Then, the Brier score is given as*

$$\mathrm{BS} = (r - \omega)^2 . \tag{7.4}$$

Interestingly, a generalization of the Brier score known as ranked probability score (RPS), which is also non-local and sensitive to distance, can be used for multi-category forecasting tasks where the reports are in the form of discrete probability distributions.

Definition 27 (Ranked probability score)**.** *Let the multi-category forecasting task have J categories. Let $r(i)$ be the forecasted probability of outcome i and $\omega(j)$ represents if the category j has occurred. Then, the ranked probability score is defined as*

$$\mathrm{RPS} = \sum_{i=1}^{J} (R(i) - O(i))^2 \tag{7.5}$$

with $R(i) = \sum_{j=1}^{i} r(j)$ and $O(i) = \sum_{j=1}^{i} \omega(j)$.

7.7.2. PROOF OF THEOREM 9

Let us now provide the proof of the properties mentioned in Theorem 9.

1. *Budget balance:* For any vector of reports $\boldsymbol{r}$, wagers $\boldsymbol{m}$ and an outcome ω,

$$\begin{aligned}\sum_i \hat{\Pi}_i &= \sum_i \Pi_i(\boldsymbol{r},\boldsymbol{m},\omega) + \sum_i \frac{\tilde{s}(r_i,\omega)\, m_i}{\sum_j \tilde{s}\left(r_j,\omega\right) m_j} U \\ &= \sum_i m_i + \sum_i s(r_i,\omega)\, m_i - \left(\sum_i m_i\right)\left(\frac{\sum_j s\left(r_j,\omega\right) m_j}{\sum_j m_j}\right) \\ &+ \sum_i \frac{\tilde{s}(r_i,\omega)\, m_i}{\sum_j \tilde{s}\left(r_j,\omega\right) m_j} U \\ &= \sum_i m_i + U.\end{aligned}$$

2. *Anonymous:* Let σ be any permutation of $\mathcal{I}$. For any $\boldsymbol{r},\boldsymbol{m},\omega$, and i,

$$\begin{aligned}\hat{\Pi}_{\sigma(i)}\left(\left(r_{\sigma^{-1}(j)}\right)_{j\in\mathcal{I}},\left(m_{\sigma^{-1}(j)}\right)_{j\in\mathcal{I}},\omega,U\right) &= m_{\sigma^{-1}(\sigma(i))}\Bigg(1+s\left(r_{\sigma^{-1}(\sigma(i))},\omega\right) \\ &- \frac{\sum_j s\left(r_{\sigma^{-1}(j)},\omega\right) m_{\sigma^{-1}(j)}}{\sum_j m_{\sigma^{-1}(j)}} \\ &+ \frac{s\left(r_{\sigma^{-1}(\sigma(i))},\omega\right)}{\sum_j s\left(r_{\sigma^{-1}(j)},\omega\right) m_{\sigma^{-1}(j)}} U\Bigg) \\ &= m_i\Bigg(1+s(r_i,\omega) - \frac{\sum_j s\left(r_j,\omega\right) m_j}{\sum_j m_j} \\ &+ \frac{s(r_i,\omega)}{\sum_j s\left(r_j,\omega\right) m_j} U\Bigg) \\ &= \hat{\Pi}_i\left(\left(r_j\right)_{j\in\mathcal{I}},\left(m_j\right)_{j\in\mathcal{I}},\omega,U\right).\end{aligned}$$

3. *Individually rational:* The skill factor Π_i of the payoff function in (7.3) is individually rational by Theorem 1 in [56] and the utility factor is always non-negative. Thus, the payoff $\hat{\Pi}_i$ is individually rational, i.e., $\mathbb{E}[\hat{\Pi}_i - m_i] \geq 0$.

4. *Sybilproofness:* Let a vector of reports $\boldsymbol{r}$ and vectors of wagers $\boldsymbol{m}$ and $\boldsymbol{m}'$ such that for a subset of players $S \subset \mathcal{I}$ the reports $r_i = r_j$ for $i, j \in S$, the wagers $m_i = m'_i$ for $i \notin S$ and that $\sum_{i\in S} m_i = \sum_{i\in S} m'_i$. Let players $i \in S$ post a common forecast report r then, for any $i \notin S$,

7

$$
\begin{aligned}
\hat{\Pi}_i(\boldsymbol{r},\boldsymbol{m},\omega,U) =& m_i\Bigg(1+s(r_i,\omega)-\frac{\sum_{j\notin S} s\left(r_j,\omega\right)m_j+s(r,\omega)\sum_{j\in S} m_j}{\sum_{j\notin S} m_j+\sum_{j\in S} m_j}\\
&+\frac{\tilde{s}(r_i,\omega)}{\sum_{j\notin S} s\left(r_j,\omega\right)m_j+s(r,\omega)\sum_{j\in S} m_j}U\Bigg)\\
=& m_i'\Bigg(1+s(r_i,\omega)-\frac{\sum_{j\notin S} s\left(r_j,\omega\right)m_j'+s(r,\omega)\sum_{j\in S} m_j'}{\sum_{j\notin S} m_j'+\sum_{j\in S} m_j'}\\
&+\frac{\tilde{s}(r_i,\omega)}{\sum_{j\notin S} s\left(r_j,\omega\right)m_j'+s(r,\omega)\sum_{j\in S} m_j'}U\Bigg)\\
=& \hat{\Pi}_i(\boldsymbol{r},\boldsymbol{m}',\omega,U).
\end{aligned}
$$

Additionally, for all $i \in S$

$$
\begin{aligned}
\sum_{i\in S}\hat{\Pi}_i(\boldsymbol{r},\boldsymbol{m},\omega,U) =& \sum_{i\in S} m_i\Bigg(1+s(r,\omega)-\frac{\sum_{j\notin S} s\left(r_j,\omega\right)m_j+s(r,\omega)\sum_{j\in S} m_j}{\sum_{j\notin S} m_j+\sum_{j\in S} m_j}\Bigg)\\
&+\sum_{i\in S}\frac{\tilde{s}(r,\omega)\,m_i}{\sum_{j\notin S} s\left(r_j,\omega\right)m_j+s(r,\omega)\sum_{j\in S} m_j}U\\
=&\left(\sum_{i\in S} m_i\right)\Bigg(1+s(r,\omega)-\frac{\sum_{j\notin S} s\left(r_j,\omega\right)m_j+s(r,\omega)\sum_{j\in S} m_j}{\sum_{j\notin S} m_j+\sum_{j\in S} m_j}\Bigg)\\
&+\frac{\tilde{s}(r,\omega)\sum_{i\in S} m_i}{\sum_{j\notin S} s\left(r_j,\omega\right)m_j+s(r,\omega)\sum_{j\in S} m_j}U\\
=&\left(\sum_{i\in S} m_i'\right)\Bigg(1+s(r,\omega)-\frac{\sum_{j\notin S} s\left(r_j,\omega\right)m_j'+s(r,\omega)\sum_{j\in S} m_j'}{\sum_{j\notin S} m_j'+\sum_{j\in S} m_j'}\Bigg)\\
&+\frac{\tilde{s}(r,\omega)\sum_{i\in S} m_i'}{\sum_{j\notin S} s\left(r_j,\omega\right)m_j'+s(r,\omega)\sum_{j\in S} m_j'}U\\
=&\sum_{i\in S}\hat{\Pi}_i\left(\boldsymbol{r},\boldsymbol{m}',\omega,U\right).
\end{aligned}
$$

5. *Conditionally truthful for players:* The skill factor Π_i of the payoff function in (7.3) is truthful by Theorem 1 in [56]. Furthermore, for $U > 0$ utility becomes proportional to the strictly proper score function given that $U \propto \phi(s(\hat{r},\omega) - s(r_c,\omega))$. Hence, players can maximize utility by reporting their true belief $\mathbf{p}$, i.e., $\mathbb{E}_p\left[U(s(\mathcal{A}(\mathbf{p},\boldsymbol{m}),\omega),s(r_c,\omega),\phi)\right] > \mathbb{E}_p\left[U(s(\mathcal{A}(\mathbf{r},\boldsymbol{m}),\omega),s(p,\omega),\phi)\right]$ is satisfied for all $\mathbf{r} \neq \mathbf{p}$. Finally, a player does not have enough information and influence on term $\left(\frac{\tilde{s}(r_i,\omega)m_i}{\sum_j \tilde{s}(r_j,\omega)m_j}\right)$ in (7.3) to create a beneficial arbitrage between skill and utility factors. Thus, we conclude that the payoff $\hat{\Pi}_i$ is conditionally truthful in practical situations.

6. *Truthful for client:* From the design of utility, i.e, $U \propto \phi(s(\hat{r},\omega) - s(r_c,\omega))$, it is proportional to the strictly proper score function. Furthermore, the predictions of forecasters r_i, for all $i \in \mathcal{I}$ are independent of client's forecast r_c. Thus,

the expected utility is minimized when the client posts their true belief p, i.e., $\mathbb{E}_p\left[U(s(\hat{r},\omega), s(r_c,\omega), \phi)\right] > \mathbb{E}_p\left[U(s(\hat{r},\omega), s(p,\omega), \phi)\right]$.

7. *Stimulant:* For a player $i \in \mathcal{I}$, the skill factor Π_i of the payoff function in (7.3) is monotone by Theorem 1 in [56] and the utility factor is proportional to wager m_i thus, the payoff $\hat{\Pi}_i$ is stimulant.

■

8

CONCLUSION AND OUTLOOK

Motivated by fast evolving energy scenario, in this thesis, we developed mechanisms for cooperative and market-based solutions for the energy transition. In this final chapter, we summarize our main contributions to the literature on cooperative game theory and the design of forecasting markets. We discuss the extent to which the research objectives have been achieved. Finally, we conclude the thesis by discussing some open challenges and future research directions.

8.1. CONCLUSION

The main objectives of this thesis are to develop payoff distribution mechanisms for cooperative and market-based energy management strategies that can help accelerate the energy transition. We proposed coalitional game theoretic solutions for payoff distribution in the context of energy resource sharing, P2P energy trading, and real-time energy markets. Furthermore, we also designed a payoff mechanism for a wagering-based forecasting market for the elicitation of probabilistic energy forecasts with desirable economic properties. Next, we summarize the contributions of this thesis and conclude the chapters by relating to our research objectives that are as follows:

- **(Obj 1)** Present a coalitional game theory based mathematical framework for energy coalitions that enforce principles of sharing economy and design market mechanisms that enable real-time and P2P energy trading;
- **(Obj 2)** Design consensus-based distributed algorithms that are scalable, fast, robust and privacy preserving by possibly exploiting the geometric structure of equilibrium solution.
- **(Obj 3)** Present a framework of a market-based platform for probabilistic forecast elicitation with formal mathematical guarantees on *desirable* economic properties;
- **(Obj 4)** Design an online distributed payoff mechanism for near real-time energy forecasting markets possibly operating with streaming data.

8

The first contribution (chapter 3) is the development of two distributed algorithms for payoff distribution in coalitional resource utilization, which aligns with the idea of sharing economy. We have formulated this problem of payoff distribution in the context of robust coalitional games over time-varying networks where the goal is to make players reach a consensus on the payoff distribution that belongs to the robust core. We motivated our setup by coalitional energy storage optimization where prosumers form a coalition to share their storage for jointly minimizing the cost of energy bought from the grid. For a solution, we have designed distributed payoff allocation and bargaining algorithms. Furthermore, we have shown that proposed algorithms, with known coalitional value bounds and based on nonexpansive and paracontraction operators, e.g. over-projections, converge consensually to the robust core, even with varying coalitional values. Therefore, this chapter partially achieves objectives 1 and 2.

The algorithms designed in chapter 3 solve robust coalitional games, but their computational burden grows exponentially with the number of agents. Thus, for the implementation of large-scale systems like P2P markets, we exploit the geometric structure of the solution set, i.e., the core, to develop faster payoff distribution algorithms. We presented this contribution in chapter 4, where to further liberalize the electricity market and empower prosumers to have control over trading their generation, we have formulated P2P energy trading as an assignment game (coalitional game) over time-varying communication networks. For solving the resulting game, we have proposed a novel distributed negotiation algorithm as a clearing mechanism that guarantees stable trading prices in a coalitional game theoretic sense and satisfies the desired economic properties.

The proposed bilateral P2P energy market designs namely, single-contract and multi-contract, encourage prosumers to participate by making P2P trading a favourable choice, considering their economic and social priorities. Furthermore, enabling product differentiation increases user satisfaction and allows for a higher overall market welfare. Finally, the negotiation mechanism via paracontraction operators enables faster convergence to a consensus on a set of bilateral contract prices that represent a competitive equilibrium and belong to the core. This chapter achieves the objectives of designing a scalable and distributed mechanism for P2P energy trading (objective 1 and 2) with coalitional game theoretic solution.

In chapter 5, we utilized a similar model of the P2P electricity market as in Chapter 4, i.e., an assignment game (coalitional game). However, as the core in Chapter 4 is a set, its different points treat buyer and seller sides differently. Thus, we designed a bilateral negotiation mechanism that enables participants to reach a trading contract (τ-value) which fairly divides the resulting market welfare among buyers and sellers. The proposed P2P electricity market model encourages prosumers to participate by providing ease of accessibility, flexibility of choice, and economic benefits, i.e., higher revenue (sellers) and lower energy costs (buyers) compared to trading with the grid.

The algorithms presented above can model uncertainty under the framework of robust coalitional games in which we assume that the core varies with time albeit within certain bounds resulting in finite core sets. However, this setup of robust games cannot model the real-time payoff distribution in which the variation of a game is not restricted within bounds and consequently the solution. Therefore, we introduce a framework of online coalitional games for real-time payoff distribution mechanism in chapter 6.

A central problem in designing a local electricity market for the integration of small to medium scale RES is managing the associated uncertainty. We have addressed this problem by designing a real-time market mechanism, i.e., clearing the market very close to the delivery time, which enables accurate forecasting. The goal of a real-time payoff distribution in an online coalitional game setup is to track a consensus on the payoff distribution solutions, namely, Shapley value and the core. We have shown that online coalitional games provide promising tools for modeling cooperative systems working in environments with fast dynamics, e.g., the real-time markets. Furthermore, in such settings, our proposed distributed algorithms based on contraction operators adequately track the payoff distribution solutions. In the context of local electricity markets, our results imply that we can guarantee a prosumer that, asymptotically, their payoff will be in the neighborhood of equilibrium. Such guarantees allowed us to address our research objective of enabling real-time energy trading (obj 1).

Finally, to address research objectives 3 and 4, in chapter 7, we design a platform for improving predictions via implicit pooling of private information in return for possible remuneration. Motivated by the fact that forecasts play a critical role in operational efficiency and viability of energy systems, we design a wagering-based forecast elicitation market platform, where a buyer intending to improve their forecasts posts a prediction task, and sellers respond to it with their forecast reports and wagers. This market delivers an aggregated forecast to the buyer and allocates a payoff to the sellers for their contribution. We have proposed a payoff function with skill and utility components that depend on the relative forecast quality of a forecaster and their contribution to improving the forecast of the client, respectively. We showed that the proposed payoff allocation satisfies several desirable economic properties. These theoretical guarantees on monetary compensation can encourage and retain the participation of forecasting experts. The simplicity of scoring-based market design, with a wagering mechanism, allows it to cater to diverse forecasting tasks, with forecasting reports taking forms of discrete to continuous probability distributions.

8.2. OUTLOOK

This thesis has investigated some payoff distribution algorithms for energy resources sharing, P2P energy trading and forecast improvement schemes. The mechanisms that have been developed can be further improved, extended, and evaluated for comprehensive scenarios. Hence, an outlook for future research is outlined as follows:

- The distributed algorithms presented in chapter 3 and 4 for payoff distribution converge to random points in the core set. As different points in the core treat agents differently, an interesting addition can be to design the operators of algorithms such that the convergence point inside the core can be characterised with additional properties like fairness.

- The setups of P2P markets proposed in chapter 4 and 5 require central operator for optimal matching. This requires an assumption of trust in the central operator. To autonomise the process of bilateral matching, a coalition formation game can be designed at the first stage that optimally pairs complementary prosumers to maximize operational performance.

- In the real-time payoff distribution process presented in chapter 6, we track the time-varying coalitional game theoretic solutions without considering any information on the future. It can be interesting to explore if the asymptotic bounds on tracking error can be improved by combining a prediction method with the tracking. For example, we can evaluate the payoff of prosumers in a local electricity market considering long-term predictions of RES and demand. Then, we adjust those payments online in a real-time setup. In case of low forecast errors, the performance of a real-time payoff distribution algorithm might be improved.

- Our focus has been on providing theoretical guarantees for proposed algorithms, and we illustrated their features via numerical examples and small case studies. We can gain useful insights into the performance of algorithms in practical scenarios by conducting large-scale case studies on real data.

- For the forecast elicitation mechanism in chapter 7, we envision a competition platform to test the performance of the proposed market model and the behavior of players in practical scenarios. Such an experimental setup would help us gain further insights for real-world implementation. Furthermore, our market setup opens several paths for applied modelling of information eliciting platforms and their analysis. An important step is to design a mechanism for online predictions based on streaming data and in turn analyse if it maintains the economic properties discussed in this paper. Another interesting research avenue is to design models that value the reputation of forecasters (historic credits) as well.

A

Basic Notation

- Given a mapping $M:\mathbb{R}^n \to \mathbb{R}^n, \mathrm{fix}(M) := \{x \in \mathbb{R}^n \mid x = M(x)\}$ denotes the set of its fixed points.
- For a closed set $C \subseteq \mathbb{R}^n$, the mapping $\mathrm{proj}_C\colon \mathbb{R}^n \to C$ denotes the projection onto C, i.e., $\mathrm{proj}_C(x) = \arg\min_{y\in C} \|y - x\|$.
- An over-projection operator is denoted by $\mathrm{overproj}_C := 2\mathrm{proj}_C - \mathrm{Id}$.
- For a set S the power set is denoted by 2^S.
- $A \otimes B$ denotes the Kronecker product between the matrices A and B.
- For $x_1, \ldots, x_N \in \mathbb{R}^n$, $\mathrm{col}((x_i)_{i\in(1,\ldots,N)}) := \left[x_1^\top, \ldots, x_N^\top\right]^\top$.
- For a norm $\|\cdot\|_p$ on $\mathbb{R}^n$ and a norm $\|\cdot\|_q$ on $\mathbb{R}^m$, the mixed vector norm $\|\cdot\|_{p,q}$ on $\mathbb{R}^{mn}$ is defined as $\|x\|_{p,q} = \|\mathrm{col}(\|x_1\|_p, \cdots, \|x_m\|_p)\|_q$.
- $\mathrm{dist}(x, C)$ denotes the distance of x from a closed set $C \subseteq \mathbb{R}^n$, i.e., $\mathrm{dist}(x, C) := \inf_{y\in C} \|y - x\|$.
- For a closed set $C \subseteq \mathbb{R}^n$ and $N \in \mathbb{N}$, $C^N := \prod_{i=1}^N C_i$.

Bibliography

[1] D. Acemoglu, A. Makhdoumi, A. Malekian, and A. Ozdaglar, "Too much data: Prices and inefficiencies in data markets", *American Economic Journal: Microeconomics*, vol. 14, no. 4, pp. 218–56, 2022.

[2] A. Agarwal, M. Dahleh, and T. Sarkar, "A marketplace for data: An algorithmic solution", in *Proceedings of the 2019 ACM Conference on Economics and Computation*, 2019, pp. 701–726.

[3] A. Agarwal, M. Dahleh, and T. Sarkar, "A marketplace for data: An algorithmic solution", in *ACM EC 2019 - Proceedings of the 2019 ACM Conference on Economics and Computation*, 2019, pp. 701–726, ISBN: 9781450367929. DOI: 10.1145/3328526.3329589. arXiv: 1805.08125. [Online]. Available: https://doi.org/10.1145/3328526.3329589.

[4] M. Agueh and G. Carlier, "Barycenters in the Wasserstein space", *SIAM Journal on Mathematical Analysis*, vol. 43, no. 2, pp. 904–924, 2011.

[5] M. K. AlAshery, Z. Yi, D. Shi, *et al.*, "A blockchain-enabled multi-settlement quasi-ideal peer-to-peer trading framework", *IEEE Trans. Smart Grid*, vol. 12, no. 1, pp. 885–896, 2020.

[6] S. Z. Alparslan-Goek, S. Miquel, and S. H. Tijs, "Cooperation under interval uncertainty", *Mathematical Methods of Operations Research*, vol. 69, no. 1, pp. 99–109, 2009.

[7] T. AlSkaif, J. L. Crespo-Vazquez, M. Sekuloski, G. van Leeuwen, and J. P. Catalão, "Blockchain-based fully peer-to-peer energy trading strategies for residential energy systems", *IEEE Trans. Ind. Informat.*, vol. 18, no. 1, pp. 231–241, 2021.

[8] J. R. Andrade and R. J. Bessa, "Improving renewable energy forecasting with a grid of numerical weather predictions", *IEEE Transactions on Sustainable Energy*, vol. 8, no. 4, pp. 1571–1580, 2017.

[9] E. Baeyens, E. Y. Bitar, P. P. Khargonekar, and K. Poolla, "Coalitional aggregation of wind power", *IEEE Transactions on Power Systems*, vol. 28, no. 4, pp. 3774–3784, 2013.

[10] P. Basak, S. Chowdhury, S. H. nee Dey, and S. Chowdhury, "A literature review on integration of distributed energy resources in the perspective of control, protection and stability of microgrid", *Renewable and Sustainable Energy Reviews*, vol. 16, no. 8, pp. 5545–5556, 2012.

[11] H. H. Bauschke and P. L. Combettes, *Convex analysis and monotone operator theory in Hilbert spaces*, 2nd ed. Springer, 2017.

[12] H. H. Bauschke and P. L. Combettes, *Convex analysis and monotone operator theory in Hilbert spaces*, 2nd ed. Springer, 2017.

[13] D. Bauso and G. Notarstefano, "Distributed n-player approachability and consensus in coalitional games", *IEEE Transactions on Automatic Control*, vol. 60, no. 11, pp. 3107–3112, 2015.

[14] D. Bauso and G. Notarstefano, "Distributed n-player approachability and consensus in coalitional games", *IEEE Transactions on Automatic Control*, vol. 60, no. 11, pp. 3107–3112, 2015, ISSN: 00189286. DOI: 10.1109/TAC.2015.2411873. arXiv: 1310.1874.

[15] D. Bauso and J. Timmer, "Robust dynamic cooperative games", *International Journal of Game Theory*, vol. 38, no. 1, pp. 23–36, 2009.

[16] J. E. Berg, F. D. Nelson, and T. A. Rietz, "Prediction market accuracy in the long run", *International Journal of Forecasting*, vol. 24, no. 2, pp. 285–300, 2008.

[17] D. Bergemann and A. Bonatti, "Markets for information: An introduction", *Annual Review of Economics*, vol. 11, pp. 85–107, 2019.

[18] F. E. Browder, "Convergence theorems for sequences of nonlinear operators in banach spaces", *Mathematische Zeitschrift*, vol. 100, no. 3, pp. 201–225, 1967.

[19] J. Browell and C. Gilbert, "Probcast: Open-source production, evaluation and visualisation of probabilistic forecasts", in *2020 International Conference on Probabilistic Methods Applied to Power Systems (PMAPS)*, IEEE, 2020, pp. 1–6.

[20] D. W. Cai, S. Adlakha, S. H. Low, P. De Martini, and K. M. Chandy, "Impact of residential pv adoption on retail electricity rates", *Energy Policy*, vol. 62, pp. 830–843, 2013.

[21] X. Cao, Y. Chen, and K. R. Liu, "Data trading with multiple owners, collectors, and users: An iterative auction mechanism", *IEEE Transactions on Signal and Information Processing over Networks*, vol. 3, no. 2, pp. 268–281, 2017.

[22] P. Chakraborty, E. Baeyens, K. Poolla, P. P. Khargonekar, and P. Varaiya, "Sharing storage in a smart grid: A coalitional game approach", *IEEE Transactions on Smart Grid*, vol. 10, no. 4, pp. 4379–4390, 2018.

[23] CLNR, *Customer-led network revolution*, http://www.networkrevolution.co.uk/resources/project-data/, accessed May 30, 2022, 2012.

[24] J. L. Crespo-Vazquez, T. A. Skaif, A. M. Gonzalez-Rueda, and M. Gibescu, "A Community-Based Energy Market Design Using Decentralized Decision-Making under Uncertainty", *IEEE Trans. Smart Grid*, vol. 3053, no. c, pp. 1–11, 2020, ISSN: 19493061. DOI: 10.1109/TSG.2020.3036915.

[25] S. Cui, Y.-W. Wang, Y. Shi, and J.-W. Xiao, "An efficient peer-to-peer energy-sharing framework for numerous community prosumers", *IEEE Trans. Ind. Informat.*, vol. 16, no. 12, pp. 7402–7412, 2019.

[26] E. Dall'Anese, A. Simonetto, S. Becker, and L. Madden, "Optimization and learning with information streams: Time-varying algorithms and applications", *IEEE Signal Processing Magazine*, vol. 37, no. 3, pp. 71–83, 2020.

[27] L. Elsner, I. Koltracht, and M. Neumann, "Convergence of sequential and asynchronous nonlinear paracontractions", *Numerische Mathematik*, vol. 62, no. 1, pp. 305–319, 1992, ISSN: 0029599X. DOI: 10.1007/BF01396232.

[28] *Energy atlas 2018: Facts and figures about renewables in europe*, Accessed: 19-Sep- 2022, 2018. [Online]. Available: https://eu.boell.org/en/energy-atlas-2018.

[29] *Energy transition*, Accessed: 2022-01-09. [Online]. Available: https://www.irena.org/energytransition.

[30] F. Fele, J. Maestre, and E. Camacho, "Coalitional control: Cooperative game theory and control", *IEEE Control Systems Magazine*, vol. 37, no. 1, pp. 53–69, 2017.

[31] J. A. Filar and L. A. Petrosjan, "Dynamic cooperative games", *International Game Theory Review*, vol. 2, no. 01, pp. 47–65, 2000.

[32] D. Fullmer and A. S. Morse, "A Distributed Algorithm for Computing a Common Fixed Point of a Finite Family of Paracontractions", *IEEE Transactions on Automatic Control*, vol. 63, no. 9, pp. 2833–2843, 2018, ISSN: 15582523. DOI: 10.1109/TAC.2018.2800644.

[33] A. Ghorbani and J. Zou, "Data shapley: Equitable valuation of data for machine learning", in *International Conference on Machine Learning*, PMLR, 2019, pp. 2242–2251.

[34] A. Ghosh and A. Roth, "Selling privacy at auction", in *Proceedings of the 12th ACM Conference on Electronic Commerce*, 2011, pp. 199–208.

[35] *Global observatory on peer-to-peer energy trading – analysis - iea*, Accessed: 19-Sep- 2022, 2021. [Online]. Available: https://www.iea.org/articles/global-observatory-on-peer-to-peer-energy-trading.

[36] T. Gneiting, "Making and evaluating point forecasts", *Journal of the American Statistical Association*, vol. 106, no. 494, pp. 746–762, 2011.

[37] T. Gneiting, "Quantiles as optimal point forecasts", *International Journal of forecasting*, vol. 27, no. 2, pp. 197–207, 2011.

[38] T. Gneiting and M. Katzfuss, "Probabilistic forecasting", *Annual Review of Statistics and Its Application*, vol. 1, pp. 125–151, 2014.

[39] T. Gneiting and A. E. Raftery, "Strictly proper scoring rules, prediction, and estimation", *Journal of the American Statistical Association*, vol. 102, no. 477, pp. 359–378, Mar. 2007, ISSN: 01621459.

[40] C. Goncalves, P. Pinson, and R. J. Bessa, "Towards data markets in renewable energy forecasting", *IEEE Transactions on Sustainable Energy*, vol. 12, no. 1, pp. 533–542, 2020.

[41] S. Grammatico, "Proximal dynamics in multiagent network games", *IEEE Transactions on Control of Network Systems*, vol. 5, no. 4, pp. 1707–1716, 2017.

[42] Z. Guo, P. Pinson, S. Chen, Q. Yang, and Z. Yang, "Online optimization for real-time peer-to-peer electricity market mechanisms", *IEEE Transactions on Smart Grid*, vol. 12, no. 5, pp. 4151–4163, 2021.

[43] L. Han, T. Morstyn, and M. McCulloch, "Constructing prosumer coalitions for energy cost savings using cooperative game theory", in *2018 Power Systems Computation Conference (PSCC)*, IEEE, 2018, pp. 1–7.

[44] L. Han, T. Morstyn, and M. McCulloch, “Incentivizing prosumer coalitions with energy management using cooperative game theory”, *IEEE Transactions on Power Systems*, vol. 34, no. 1, pp. 303–313, 2018.

[45] L. Han, P. Pinson, and J. Kazempour, “Trading data for wind power forecasting: A regression market with lasso regularization”, *Electric Power Systems Research*, vol. 212, p. 108 442, 2022.

[46] T. Hong, P. Pinson, S. Fan, H. Zareipour, A. Troccoli, and R. J. Hyndman, “Probabilistic energy forecasting: Global energy forecasting competition 2014 and beyond”, *International Journal of Forecasting*, vol. 32, no. 3, pp. 896–913, 2016.

[47] J. Hu, R. Harmsen, W. Crijns-Graus, E. Worrell, and M. van den Broek, “Identifying barriers to large-scale integration of variable renewable electricity into the electricity market: A literature review of market design”, *Renewable and Sustainable Energy Reviews*, vol. 81, pp. 2181–2195, 2018.

[48] Z. Huang, H. Yu, Z. Peng, and M. Zhao, “Methods and tools for community energy planning: A review”, *Renewable and sustainable energy reviews*, vol. 42, pp. 1335–1348, 2015.

[49] L. Hurwicz, “On informationally decentralized systems”, *Decision and organization*, 1972.

[50] R. Jia, D. Dao, B. Wang, *et al.*, “Efficient task-specific data valuation for nearest neighbor algorithms”, in *45th International Conference on Very Large Data Bases (VLDB)*, ACM, 2019, pp. 1610–1623.

[51] V. R. R. Jose, R. F. Nau, and R. L. Winkler, “Sensitivity to distance and baseline distributions in forecast evaluation”, *Management Science*, vol. 55, no. 4, pp. 582–590, 2009.

[52] M. Khorasany, A. Paudel, R. Razzaghi, and P. Siano, “A new method for peer matching and negotiation of prosumers in peer-to-peer energy markets”, *IEEE Transactions on Smart Grid*, vol. 12, no. 3, pp. 2472–2483, 2020.

[53] D. M. Kilgour and Y. Gerchak, “Elicitation of probabilities using competitive scoring rules”, *Decision Analysis*, vol. 1, no. 2, pp. 108–113, 2004.

[54] M. Knüppel and F. Krüger, “Forecast uncertainty, disagreement, and the linear pool”, *Journal of Applied Econometrics*, vol. 37, no. 1, pp. 23–41, 2022.

[55] L. Kranich, A. Perea, and H. Peters, “Core concepts for dynamic tu games”, *International Game Theory Review*, vol. 7, no. 01, pp. 43–61, 2005.

[56] N. S. Lambert, J. Langford, J. Wortman, *et al.*, “Self-financed wagering mechanisms for forecasting”, in *Proceedings of the 9th ACM Conference on Electronic Commerce*, 2008, pp. 170–179.

[57] E. Lehrer and M. Scarsini, “On the core of dynamic cooperative games”, *Dynamic Games and Applications*, vol. 3, no. 3, pp. 359–373, 2013.

[58] K. C. Lichtendahl Jr., Y. Grushka-Cockayne, and R. L. Winkler, “Is it better to average probabilities or quantiles?”, *Management Science*, vol. 59, no. 7, pp. 1594–1611, 2013.

[59] F. Linde and W. G. Stock, *Information Markets: A Strategic Guideline for the I-Commerce*. Association for Information Science & Technology, 2011.

[60] F. Luo, Z. Y. Dong, G. Liang, J. Murata, and Z. Xu, "A distributed electricity trading system in active distribution networks based on multi-agent coalition and blockchain", *IEEE Trans. Power Syst.*, vol. 34, no. 5, pp. 4097–4108, 2018.

[61] M. Maschler, "The bargaining set, kernel, and nucleolus", *Handbook of game theory with economic applications*, vol. 1, pp. 591–667, 1992.

[62] E. Mengelkamp, J. Gärttner, K. Rock, S. Kessler, L. Orsini, and C. Weinhardt, "Designing microgrid energy markets: A case study: The brooklyn microgrid", *App. Ener.*, vol. 210, pp. 870–880, 2018.

[63] J. W. Messner and P. Pinson, "Online adaptive lasso estimation in vector autoregressive models for high dimensional wind power forecasting", *International Journal of Forecasting*, vol. 35, no. 4, pp. 1485–1498, 2019.

[64] J. M. Morales, A. J. Conejo, H. Madsen, P. Pinson, and M. Zugno, "Renewable energy sources—modeling and forecasting", in *Integrating Renewables in Electricity Markets*, Springer, 2014, pp. 15–56.

[65] F. Moret and P. Pinson, "Energy collectives: A community and fairness based approach to future electricity markets", *IEEE Trans. Power Syst.*, vol. 34, no. 5, pp. 3994–4004, 2018.

[66] T. Morstyn, N. Farrell, S. J. Darby, and M. D. McCulloch, "Using peer-to-peer energy-trading platforms to incentivize prosumers to form federated power plants", *Nature Energy*, vol. 3, no. 2, pp. 94–101, 2018.

[67] T. Morstyn and M. D. McCulloch, "Multiclass energy management for peer-to-peer energy trading driven by prosumer preferences", *IEEE Trans. Power Syst.*, vol. 34, no. 5, pp. 4005–4014, 2018.

[68] T. Morstyn, A. Teytelboym, and M. D. McCulloch, "Bilateral contract networks for peer-to-peer energy trading", *IEEE Trans. Smart Grid*, vol. 10, no. 2, pp. 2026–2035, 2019, ISSN: 19493053. DOI: 10.1109/TSG.2017.2786668.

[69] R. B. Myerson, *Game theory*. Harvard University Press, 2013.

[70] A. Nedic, A. Olshevsky, and W. Shi, "Achieving geometric convergence for distributed optimization over time-varying graphs", *SIAM Journal on Optimization*, vol. 27, no. 4, pp. 2597–2633, 2017.

[71] A. Nedic, A. Ozdaglar, and P. A. Parrilo, "Constrained consensus and optimization in multi-agent networks", *IEEE Transactions on Automatic Control*, vol. 55, no. 4, pp. 922–938, 2010.

[72] A. Nedić and D. Bauso, "Dynamic coalitional tu games: Distributed bargaining among players' neighbors", *IEEE Transactions on Automatic Control*, vol. 58, no. 6, pp. 1363–1376, 2013. DOI: 10.1109/TAC.2012.2236716.

[73] A. Nedich and D. Bauso, "Dynamic coalitional TU games: Distributed bargaining among players' neighbors", *IEEE Transactions on Automatic Control*, vol. 58, no. 6, pp. 1363–1376, 2013, ISSN: 00189286. DOI: 10.1109/TAC.2012.2236716.

[74] D. H. Nguyen, "Optimal solution analysis and decentralized mechanisms for peer-to-peer energy markets", *IEEE Trans. Power Syst.*, 2020.

[75] T. Nielsen, H. Madsen, H. A. Nielsen, *et al.*, "Short-term wind power forecasting using advanced statistical methods", in *The European Wind Energy Conference, EWEC 2006*, 2006, 9–pages.

[76] NUMERAI, *NUMERAI, the hardest data science tournament on the planet*, https://numer.ai/, Accessed: 2022-01-07, 2022.

[77] M. Núñez and C. Rafels, "The assignment game: The τ-value", *International Journal of Game Theory*, vol. 31, no. 3, pp. 411–422, 2002, ISSN: 00207276. DOI: 10.1007/s001820300127.

[78] R. Ospina and S. L. Ferrari, "Inflated beta distributions", *Statistical Papers*, vol. 51, no. 1, pp. 111–126, 2010.

[79] F. Petropoulos, D. Apiletti, V. Assimakopoulos, *et al.*, "Forecasting: Theory and practice", *International Journal of Forecasting*, vol. 38, no. 3, pp. 705–871, 2022.

[80] S. Pineda and A. J. Conejo, "Using electricity options to hedge against financial risks of power producers", *Journal of Modern Power Systems and Clean Energy*, vol. 1, no. 2, pp. 101–109, 2013.

[81] P. Pinson, "Very-short-term probabilistic forecasting of wind power with generalized logit–normal distributions", *Journal of the Royal Statistical Society: Series C (Applied Statistics)*, vol. 61, no. 4, pp. 555–576, 2012.

[82] P. Pinson, L. Han, and J. Kazempour, "Regression markets and application to energy forecasting", *TOP*, pp. 1–41, 2022.

[83] P. Pinson, H. Madsen, H. A. Nielsen, G. Papaefthymiou, and B. Klöckl, "From probabilistic forecasts to statistical scenarios of short-term wind power production", *Wind Energy: An International Journal for Progress and Applications in Wind Power Conversion Technology*, vol. 12, no. 1, pp. 51–62, 2009.

[84] N. R. Pradhan, A. P. Singh, N. Kumar, M. Hassan, and D. Roy, "A flexible permission ascription (fpa) based blockchain framework for peer-to-peer energy trading with performance evaluation", *IEEE Trans. Ind. Informat.*, 2021.

[85] A. A. Raja and S. Grammatico, "On the approachability principle for distributed payoff allocation in coalitional games", in *21st IFAC World Congress*, 2020.

[86] A. A. Raja and S. Grammatico, "Payoff distribution in robust coalitional games on time-varying networks", *IEEE Trans. Control Netw. Syst.*, early access. DOI: 10.1109/TCNS.2021.3113269.

[87] A. A. Raja and S. Grammatico, "A fair peer-to-peer electricity market model for residential prosumers", in *2021 IEEE PES Innovative Smart Grid Technologies Europe (ISGT Europe)*, IEEE, 2021, pp. 1–5.

[88] A. A. Raja and S. Grammatico, "Payoff distribution in robust coalitional games on time-varying networks", *IEEE Transactions on Control of Network Systems*, 2021.

[89] A. A. Raja, P. Pinson, J. Kazempour, and S. Grammatico, "A market for trading forecasts: A wagering mechanism", *arXiv preprint arXiv:2205.02668*, 2022.

[90] R. Ranjan and T. Gneiting, "Combining probability forecasts", *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, vol. 72, no. 1, pp. 71–91, 2010.

[91] M. Rasouli and M. I. Jordan, *Data sharing markets*, 2021. [Online]. Available: https://arxiv.org/abs/2107.08630.

[92] W. Saad, Z. Han, M. Debbah, A. Hjørungnes, and T. Başar, "Coalitional game theory for communication networks", *IEEE Signal Processing Magazine*, vol. 26, no. 5, pp. 77–97, 2009, ISSN: 10535888. DOI: 10.1109/MSP.2009.000000.

[93] D. Schmeidler, "The nucleolus of a characteristic function game", *SIAM Journal on Applied Mathematics*, vol. 17, no. 6, pp. 1163–1170, 1969.

[94] F. C. Schweppe, M. C. Caramanis, R. D. Tabors, and R. E. Bohn, *Spot pricing of electricity*. Springer Science & Business Media, 2013.

[95] M. Shamsi and P. Cuffe, "Prediction markets for probabilistic forecasting of renewable energy sources", *IEEE Transactions on Sustainable Energy*, vol. 13, no. 2, pp. 1244–1253, 2021.

[96] S. Shao, A. Mittal, R. Twumasi-Boakye, and A. Gupta, "Fair pricing of ridehailing services with asymmetric demand and travel time", *IEEE Transactions on Control of Network Systems*, 2021.

[97] L. S. Shapley, "A value for n-person games", *Contributions to the Theory of Games*, vol. 2, no. 28, pp. 307–317, 1953.

[98] L. S. Shapley and M. Shubik, "The assignment game i: The core", *International Journal of Game Theory*, vol. 1, no. 1, pp. 111–130, 1971.

[99] A. Simonetto, *Time-varying convex optimization via time-varying averaged operators*, 2017. arXiv: 1704.07338. [Online]. Available: https://arxiv.org/pdf/1704.07338.pdf.

[100] A. Simonetto, E. Dall'Anese, S. Paternain, G. Leus, and G. B. Giannakis, "Time-varying convex optimization: Time-structured algorithms and applications", *Proceedings of the IEEE*, vol. 108, no. 11, pp. 2032–2048, 2020.

[101] A. Simonetto, A. Koppel, A. Mokhtari, G. Leus, and A. Ribeiro, "Prediction-correction methods for time-varying convex optimization", in *2015 49th Asilomar Conference on Signals, Systems and Computers*, IEEE, 2015, pp. 666–670.

[102] A. Simonetto and G. Leus, "Distributed asynchronous time-varying constrained optimization", *Conference Record - Asilomar Conference on Signals, Systems and Computers*, vol. 2015-April, no. 1, pp. 2142–2146, 2015, ISSN: 10586393. DOI: 10.1109/ACSSC.2014.7094854.

[103] B. D. Solomon and K. Krishna, "The coming sustainable energy transition: History, strategies, and outlook", *Energy Policy*, vol. 39, no. 11, pp. 7422–7431, 2011.

[104] E. Sorin, L. Bobo, and P. Pinson, "Consensus-based approach to peer-to-peer electricity markets with product differentiation", *IEEE Transactions on Power Systems*, vol. 34, no. 2, pp. 994–1004, 2018.

[105] E. Sorin, L. Bobo, and P. Pinson, "Consensus-Based Approach to Peer-to-Peer Electricity Markets with Product Differentiation", *IEEE Trans. Power Syst.*, vol. 34, no. 2, pp. 994–1004, 2019, ISSN: 08858950. DOI: 10.1109/TPWRS.2018.2872880. arXiv: 1804.03521.

[106] T. Sousa, T. Soares, P. Pinson, F. Moret, T. Baroche, and E. Sorin, "Peer-to-peer and community-based markets: A comprehensive review", *Renewable and Sustainable Energy Reviews*, vol. 104, pp. 367–378, 2019.

[107] S. Spiekermann, A. Acquisti, R. Böhme, and K.-L. Hui, "The challenges of personal data markets and privacy", *Electronic Markets*, vol. 25, no. 2, pp. 161–167, 2015.

[108] G. Strbac, "Demand side management: Benefits and challenges", *Energy policy*, vol. 36, no. 12, pp. 4419–4426, 2008.

[109] W. Tushar, T. K. Saha, C. Yuen, P. Liddell, R. Bean, and H. V. Poor, "Peer-to-peer energy trading with sustainable user participation: A game theoretic approach", *IEEE Access*, vol. 6, pp. 62 932–62 943, 2018.

[110] W. Tushar, T. K. Saha, C. Yuen, T. Morstyn, H. V. Poor, R. Bean, *et al.*, "Grid influenced peer-to-peer energy trading", *IEEE Trans. Smart Grid*, vol. 11, no. 2, pp. 1407–1418, 2019.

[111] W. Tushar, T. K. Saha, C. Yuen, D. Smith, and H. V. Poor, "Peer-to-Peer Trading in Electricity Networks: An Overview", *IEEE Trans. Smart Grid*, vol. 11, no. 4, pp. 3185–3200, 2020, ISSN: 19493061. DOI: 10.1109/TSG.2020.2969657. arXiv: 2001.06882.

[112] W. Tushar, T. K. Saha, C. Yuen, *et al.*, "A motivational game-theoretic approach for peer-to-peer energy trading in the smart grid", *App. Ener.*, vol. 243, pp. 10–20, 2019.

[113] W. Tushar, T. K. Saha, C. Yuen, *et al.*, "A coalition formation game framework for peer-to-peer energy trading", *App. Ener.*, vol. 261, p. 114 436, 2020.

[114] W. Tushar, C. Yuen, H. Mohsenian-Rad, T. Saha, H. V. Poor, and K. L. Wood, "Transforming energy networks via peer-to-peer energy trading: The potential of game-theoretic approaches", *IEEE Signal Processing Magazine*, vol. 35, no. 4, pp. 90–111, 2018.

[115] M. H. Ullah and J.-D. Park, "A two-tier distributed market clearing scheme for peer-to-peer energy sharing in smart grid", *IEEE Trans. Ind. Informat.*, vol. 18, no. 1, pp. 66–76, 2021.

[116] C. Véliz and P. Grunewald, "Protecting data privacy is key to a smart energy future", *Nature Energy*, vol. 3, no. 9, pp. 702–704, 2018.

[117] K. F. Wallis, "Combining density and interval forecasts: A modest proposal", *Oxford Bulletin of Economics and Statistics*, vol. 67, pp. 983–994, 2005.

[118] Q. Wang, C. Zhang, Y. Ding, G. Xydis, J. Wang, and J. Østergaard, "Review of real-time electricity markets for integrating distributed energy resources and demand response", *Applied Energy*, vol. 138, pp. 695–706, 2015.

[119] Y. Wang, Z. Su, J. Li, *et al.*, "Blockchain-based secure and cooperative private charging pile sharing services for vehicular networks", *IEEE Transactions on Vehicular Technology*, 2021.

[120] *What do we mean by energy transition?*, Accessed: 2022-01-09. [Online]. Available: https://www.enelgreenpower.com/learning-hub/energy-transition.

[121] R. L. Winkler, J. Muñoz, J. L. Cervera, *et al.*, "Scoring rules and the evaluation of probabilities", *Test*, vol. 5, no. 1, pp. 1–60, 1996, ISSN: 18638260.

[122] R. L. Winkler, Y. Grushka-Cockayne, K. C. Lichtendahl Jr., and V. R. R. Jose, "Probability forecasts and their combination: A research perspective", *Decision Analysis*, vol. 16, no. 4, pp. 239–260, 2019.

[123] J. Witkowski, R. Freeman, J. Vaughan, D. Pennock, and A. Krause, "Incentive-compatible forecasting competitions", in *Proceedings of the AAAI Conference on Artificial Intelligence*, vol. 32, 2018.

[124] J. Wolfers and E. Zitzewitz, "Prediction markets", *Journal of economic perspectives*, vol. 18, no. 2, pp. 107–126, 2004.

[125] J. Wolfers and E. Zitzewitz, *Prediction markets in theory and practice*, 2006. [Online]. Available: https://www.nber.org/papers/w12083.

[126] I. Yamada and N. Ogura, "Hybrid steepest descent method for variational inequality problem over the fixed point set of certain quasi-nonexpansive mappings", 2005.

[127] R. Yu, J. Ding, W. Zhong, Y. Liu, and S. Xie, "Phev charging and discharging cooperation in v2g networks: A coalition game approach", *IEEE Internet of Things Journal*, vol. 1, no. 6, pp. 578–589, 2014.

[128] K. Yuan, Q. Ling, and W. Yin, "On the convergence of decentralized gradient descent", *SIAM Journal on Optimization*, vol. 26, no. 3, pp. 1835–1854, 2016.

[129] Z. Zhou, A. Botterud, J. Wang, *et al.*, "Application of probabilistic wind power forecasting in electricity markets", *Wind Energy*, vol. 16, no. 3, pp. 321–338, 2013.

[130] M. Zima-Bockarjova, A. Sauhats, L. Petrichenko, and R. Petrichenko, "Charging and discharging scheduling for electrical vehicles using a shapley-value approach", *Energies*, vol. 13, no. 5, p. 1160, 2020.

[131] R. H. Zubo, G. Mokryani, H.-S. Rajamani, J. Aghaei, T. Niknam, and P. Pillai, "Operation and planning of distribution networks with integration of renewable distributed generators considering uncertainties: A review", *Renewable and Sustainable Energy Reviews*, vol. 72, pp. 1177–1198, 2017.

List of Publications

Journals and preprints

A. Raja and S. Grammatico, "Payoff Distribution in Robust Coalitional Games on Time-Varying Networks", IEEE Transactions on Control of Network Systems, vol. 9, no. 1, pp. 511-520, 2022.

A. Raja and S. Grammatico, "Bilateral Peer-to-Peer Energy Trading via Coalitional Games", IEEE Transactions on Industrial Informatics, pp. 1-10, 2022.

A. Raja and S. Grammatico, "Online coalitional games for real-time payoff distribution with applications to energy markets", arXiv:2301.12280 [eess.SY] (*under review for journal publication*).

A. Raja, P. Pinson, J. Kazempour, and S. Grammatico, "A Market for Trading Forecasts: A Wagering Mechanism", arXiv:2205.02668 [econ.TH], (*accepted for publication in International Journal of Forecasting*).

Conference proceedings

A. Raja and S. Grammatico, "On the approachability principle for distributed payoff allocation in coalitional games", 21st IFAC World Congress, vol. 53, no. 2, pp. 2690-2695, 2020.

A. Raja and S. Grammatico, "A fair Peer-to-Peer Electricity Market model for Residential Prosumers", 2021 IEEE PES Innovative Smart Grid Technologies Europe (ISGT Europe), 2021.

Acknowledgements

Saying this 4-year ride was a bit bumpy would be an understatement however, all's well that ends well. At the finish line of this intense but rewarding journey, I want to express my sincerest gratitude to God, foremost, and then to some amazing people that were part of it in various capacities. I start with dedicating this thesis to my parents whom I owe every success in my life. Specially to my (late) father, who was waiting for me to return home with a doctorate but left too early.

Next, I cannot thank enough my supervisor dr. Sergio Grammatico for his endless support, patience, and constant availability. Any scientific prowess I demonstrate is undoubtedly the result of his excellent mentorship. His concern for my progress was so high that he provided me feedback even during weekends or holidays (my apologies for putting you in that position). I look towards him as an inspiration for scientific rigor, productivity, and articulation.

My gratitude also goes to my co-promotor, Prof. Bart De Schutter, for his calming presence and insightful swift responses. I particularly thank him for adjusting his holiday plan because of my defense date.

A special recognition to Prof. Jalal Kazempour and Prof. Pierre Pinson for hosting me at DTU during my research visit. It was an incredibly rewarding stay where I thoroughly enjoyed our technical discussions and brainstorming sessions. I have learned a lot from Jalal's pragmatic insights and Pierre's width of knowledge, whose productivity never ceases to amaze me.

I would like to thank the kindest staff at TU Delft: Erica, Marieke, Heleen, Francy, for taking away all our bureaucratic pains away allowing us to focus on research. They are integral to this process.

I was also incredibly fortunate to meet a number of wonderful friends in my research group. I thank them for being there. To Filippo, Alfiya, Giuseppe, Carlo, Wicak, Barbara for the best cookies and help in writing maths, Mattia for planting jokes (funny if you understand) in conversations, Suad for increasing Muslim population in the group and making halal BBQs possible, and Emilio the witty one.

I am really grateful to the whole well-connected Pakistani community in Delft for making my stay wonderful with amazing food, lighter conversations, and always being there for help. These gatherings kept me going. Not to forget the person who introduced an introvert like me to this community; Mohsan, it's not the same without you. I also thank my other roommates: Asad bhai and Hamid for being very easygoing people. To my dearest neighbor Hassan for weekly dinners and Mubariz for being a brother with motivating suggestions and a personification of stress relief therapy, we are lucky to have you.

I am profoundly grateful to my family; the loving sisters who always think the best for me and are always trying to help even from far away, I owe a lot to you all. My extended maternal family whom I consider a part of my own home for your ever-available selfless support. I also extend gratitude to my aunt, cousins Maj. Waqar and Rafi who were there supporting my mother during my father's illness when I couldn't reach.

I am thankful to all my teachers who encouraged me and played a role in making me reach where I am today. To my all long term friends who were a constant support and help whenever I needed. Faisal, Yawar and Akmal for taking care of some necessary arrangements back home for me.

Finally, I reserve this special place to thank my love, my wife, who is a source of warmth, festivity, and happiness in my life. For your efforts to bring organisation to the chaos I usually create. I thank you for completing me.