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Some Notes on Static Anchor Chain Curve

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Offshore Technology Conference on behalf of American Institute of Mining, Metallurgical, and Petroleum Engineers, Inc., The American Association of Petroleum Geologists, American Institute of Chemical Engineers, American Society of Civil Engineers, The American Society of Mechanical Engineers, The Institute of Electrical and Electronics Engineers, Inc., Marine Technology Society, Society of Exploration Geophysicists, and Society of Naval Architects & Marine Engineers.

This paper was prepared for presentation at the Second Annual Offshore Technology Conference to be held in Houston, Tex., April 22-24, 1970. Permission to copy is restricted to an abstract of not more than 300 words. Illustrations may not be copied. Such use of an abstract should contain conspicuous acknowledgment of where and by whom the paper is presented.

ABSTRACT

This paper offers fundamentals necessary to solve many of the problems posed in the design and evaluation of mooring systems for offshore operations. In addition to considering the mooring system requirements in place, the actual operational requirements of "running" anchors, dropping, etc., are also considered. Other aspects of mooring systems such as pile anchors and catenary "sinkers" are also covered. Special attention was given to anchor line stretch.

INTRODUCTION

To those involved in the offshore operations of mobile drilling units, derrick barges and pipe laying barges, the evaluation and design of mooring systems is an ever-present problem. The questions to be answered about any offshore mooring situation are many. Guidance in finding these answers is presented in this paper.

Included are fundamentals necessary for selection of proper number of anchor lines, proper fluke area, anchor type and anchor location. In addition, problems related to setting out anchors are considered--dropping distance of anchors, type of dropping, required workboat pull requirements. The subject matter of this paper is relatively new and, of course, References and illustrations at end of paper.

subject to discussion at the conference.

EQUATIONS OF CATENARY

Differential Equation and Solution

The catenary is the curve in which a flexible chain or chord of uniform weight will hang when supported by its ends [Fig. 1].

Differential equation of the catenary can be obtained as follows: let an element of catenary ds , have Forces T and $T + \Delta T$ acting at ends, as shown in Fig. 2.

Because any element with a force $w \cdot ds$ acting downward is in equilibrium, we can construct the force diagram, as shown in Fig. 3. [H_0 is constant at any point on the curve.] From Fig. 3, it is easily seen that $d \tan \phi$ is equal to $[w \cdot ds] / H_0$ or $d[dy/dx]$, since:

$$ds = dx \cdot \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

and

$$d \frac{dy}{dx} = \frac{d^2 y}{dx^2}$$

Setting $dy/dx = u$, we get

$$\frac{du}{dx} = \frac{w}{H_0} \sqrt{1+u^2}$$

$$\frac{wx}{H_0} = \text{Ln} (u + \sqrt{1+u^2}) + C_1$$

This can be written as

$$u = \sinh \left(\frac{wx}{H_0} - C_1 \right) = \frac{dy}{dx}$$

Finally, we have

$$y = \frac{H_0}{w} \cosh \left(\frac{wx}{H_0} - C_1 \right) + C_2$$

If we call $H_0/w = c$, the parameter of the curve and choose the minimum point as the origin

$$C_1 = -c \quad C_2 = 0$$

we have:

$$y = c \left(\cosh \frac{x}{c} - 1 \right) \dots [1]$$

This can also be represented as $y = c \cdot \cosh [x/c]$ by moving the origin to O_1 , as shown in Fig. 1.

The Tight Catenary

In the case of very flat catenary [c large] expansion of y into series gives:

$$y = \frac{x^2}{2c} \dots [2]$$

Eq. 2 is used to derive the δ deflection of the chord at any point on the curve [Fig. 4].

$$\delta = \frac{w \cdot x \cdot (l-x)}{2 H_0} \dots [3]$$

$$\delta = \frac{w \cdot x (l-x)}{2 T \cdot \cos \phi} \dots [3A]$$

The last two formulas were used long years in the shipyards for line shaft alignment.

Basic Equations

The basic equations of the catenary are listed below and apply to Fig. 5.

At any point [x,y].

$$V = w \cdot s \dots [4]$$

$$H = w \cdot c \dots [5]$$

$$T = w \cdot y \dots [6]$$

$$y^2 = s^2 + c^2 \dots [7]$$

$$y = c \cdot \cosh \frac{x}{c} \dots [8]$$

$$s = c \cdot \sinh \frac{x}{c} \dots [9]$$

$$\sqrt{L^2 - B^2} = 2c \cdot \sinh \frac{K}{2c} \dots [10]$$

$$\frac{B}{L} = \tanh \frac{x_m}{c} \dots [11]$$

$$x_m = x_a + \frac{K}{2} \dots [12]$$

$$x_b = x_m + \frac{K}{2} \dots [13]$$

Determination of The Tension in Field

Often it is required to know the tension in an anchor chain or cable of mooring systems or offshore drilling barges when it is impossible to measure the length of the chain. From experience, it has been found that existing tension measuring devices cannot provide accurate readings because of the friction developed in the hawse pipes. Eq. 14 gives an accurate value for tension when the water depth and the angle at the top of the chain are known.

w = weight of chain in water lb/ft

$$T = \frac{z \cdot w}{1 - \cos \phi} \dots [14]$$

$$x = \frac{z}{\frac{1}{\cos \phi} - 1} \cdot \sinh^{-1}(\tan \phi) \dots [15]$$

if ϕ and x are known,

$$T = \frac{w \cdot x}{\cos \phi \cdot \sinh^{-1}(\tan \phi)} \dots [16]$$

H, V, T, x, s, z, ϕ RELATIONS

x, z - H Are Known

Most important relations are as follows.

$$x = \frac{H}{W} \cosh^{-1} \left(\frac{H + z \cdot W}{H} \right) \dots [17]$$

$$s = \sqrt{z \cdot \left(z + \frac{2H}{W} \right)} \dots [18]$$

$$V = s \cdot W \dots [19]$$

$$T = H + z \cdot W \dots [20]$$

$$\sin \phi = \frac{s \cdot W}{H + z \cdot W} \dots [21]$$

$$\cos \phi = \frac{H}{H + zW} \dots [22]$$

$$\text{scope} = \sqrt{1 + \frac{2H}{z \cdot W}} \dots [23]$$

$$C = \frac{H}{W} \dots [24]$$

w, z - T Are Known

$$x = \frac{T - z \cdot W}{W} \cdot \cosh^{-1} \left(\frac{T}{T - z \cdot W} \right) \dots [25]$$

$$s = \sqrt{z \cdot \left(\frac{2T - zW}{W} \right)} \dots [26]$$

$$V = s \cdot W \dots [27]$$

$$H = T - z \cdot W \dots [28]$$

$$\sin \phi = \frac{s \cdot W}{T} \dots [29]$$

$$\cos \phi = 1 - \frac{z \cdot W}{T} \dots [30]$$

$$\text{scope} = \sqrt{\frac{2T}{z \cdot W} - 1} \dots [31]$$

$$C = \frac{T - z \cdot W}{W} \dots [32]$$

Relations with k Parameter

k parameter is the ratio of the T tension to the weight of the chain of which length is equal to the water depth.

$$k = \frac{T}{z \cdot W}$$

Many of catenary formulas may be simplified when they are expressed in terms of k.

Catenary parameter $C = z \cdot (k - 1)$

Catenary length $s = z \cdot \sqrt{2k - 1}$

Catenary distance $x =$

$$z \cdot (k - 1) \cdot \cosh^{-1} \left(\frac{k}{k - 1} \right)$$

For small size computer application:

$$\cosh^{-1} \left(\frac{k}{k - 1} \right) \equiv \ln \left(\frac{\zeta - 1}{\zeta} \right)$$

for $|k| > 1$

where: $\zeta = \frac{k + \sqrt{2k - 1}}{k - 1}$

Horizontal tension $H = \frac{k - 1}{k} \cdot T$

Catenary angle $\phi = \cosh^{-1} \left(\frac{k - 1}{k} \right)$

Sometimes to know the scope is very useful and practical. By definition we have:

$$\text{scope} = \frac{s}{z} = \sqrt{2k - 1}$$

FORCE - DEPARTURE RELATIONS

Another important use of the catenary curve is the calculation of tension - departure relations. That is the change in the tension of a chain due to the movement of floating systems.

Horizontal Departure

The relationship is shown in the sketch in Fig. 8. At the point O_1 the chain tension is that induced by the weight of the chain hanging vertically downward. At all other points, the

chain tension increases with movement of the system to the right of point O_1 . Accordingly, it is possible to find the new tension in a chain after any movement or departure.

The solution of this problem is facilitated by the use of a graph similar to the one in Fig. 9 showing the correlation between departure λ and tension T :

$$\lambda = \frac{H}{W} \cdot \cosh^{-1} \left(\frac{z \cdot W + H}{H} \right) - \sqrt{z \cdot \left(z + \frac{2H}{W} \right)} + z. \quad [33]$$

or

$$\lambda = \frac{T - zW}{W} \cdot \cosh^{-1} \left(\frac{T}{T - zW} \right) - \sqrt{z \cdot \left(\frac{2T - z \cdot W}{W} \right)} + z. \quad [34]$$

As an example, consider a system at Point A inducing a tension of 70 kips into a 2-in. Di-Lok anchor chain in 175 ft of water moving a distance $\Delta\lambda$ equal to 5 ft [$w = 34.15$ lb/ft].

The new tension in the chain, when the system moves to Point B, can be found by entering the correlation curve [Fig. 8] with the initial tension, moving along the abscissa a distance of $\Delta\lambda = 5$ ft and reading the new tension on the curve.

It should be noted that the same results would be obtained if, instead of considering a departure of 5 ft, the length of chain was "taken up" 5 ft.

$$T = .234 \frac{z^3 \cdot W}{(z - \lambda)^2} \dots [35]$$

In terms of k parameter, λ becomes:

$$\lambda = z \cdot \left[1 + (k - 1) \cdot \left(\cosh^{-1} \frac{k}{k - 1} - \frac{\sqrt{2k - 1}}{k - 1} \right) \right] \dots [36]$$

Vertical Departure

Fig. 10 shows a catenary at Point A. It can be considered as departed from O_1 , in horizontal direction and departure equals to λ . It can also be considered as departed from Point O_2 in vertical direction and departure equals to ν . Since $\lambda = \nu$ and from the conditions at Point A, $\mu = z - \lambda = z - \nu$ can be easily calculated, i.e.,

$$H = z \cdot W \cdot (k - 1) \quad \text{if } H \text{ is known}$$

$$T = z \cdot W \cdot k \quad \text{if } T \text{ is known,}$$

and ν values can be obtained in terms of k parameter.

$$\frac{T}{(\mu + \nu) \cdot W} = k_A = \frac{H}{(\mu + \nu) \cdot W} + 1 = \frac{T}{z \cdot W} = \frac{H}{z \cdot W} + 1$$

$$\lambda = \nu = (\mu + \nu) \left[1 + (k - 1) \cdot \left(\cosh^{-1} \frac{k}{k - 1} - \frac{\sqrt{2k - 1}}{k - 1} \right) \right]$$

$$\nu = \mu.$$

$$\left[\frac{1}{(1 - k) \cdot \left(\cosh^{-1} \frac{k}{k - 1} - \frac{\sqrt{2k - 1}}{k - 1} \right) - 1} \right] \dots [37]$$

STRETCH FORMULAS

From geometry we have:

$$\frac{dy}{dx} = \tan \phi$$

$$\frac{dx}{ds} = \cos \phi$$

$$\frac{dy}{ds} = \sin \phi$$

From the elongation of an element of length ds we have:

$$d(\Delta L) \frac{dx}{ds} = \frac{T}{E'A} \cos \phi = d(\Delta x).$$

We obtain:

$$\Delta x = \frac{H}{EA} \int_0^s ds = \frac{H \cdot s}{EA} \quad [38]$$

or in terms of k parameter,

$$\Delta x = z^2 \cdot w \cdot \frac{\sqrt{2k-1}}{EA} \quad [39]$$

Similarly, we obtain:

$$\Delta y = \frac{V \cdot s}{2EA} \quad [40]$$

$$\Delta y = z^2 \cdot w \cdot \frac{2k-1}{2EA} \quad [41]$$

Total elongation may be obtained from

$$d(\Delta L) = \frac{T}{EA} ds = \frac{1}{EA} \sqrt{H^2 + (s \cdot w)^2} \cdot ds \quad [42]$$

$$\Delta L = \frac{w}{2EA} \left\{ \left[c^2 \cdot \ln \left(\frac{s}{c} + \sqrt{1 + \frac{s^2}{c^2}} \right) + s c^2 \sqrt{1 + \frac{s^2}{c^2}} \right] \right\}, \dots [43]$$

or, in terms of k parameter:

$$\Delta L = \frac{z^2 \cdot w}{EA} \cdot \frac{(k-1)^2}{2} \cdot \left[\ln \frac{\sqrt{2k-1} + k}{k-1} + \frac{k \cdot \sqrt{2k-1}}{(k-1)^2} \right] \quad [43A]$$

Another way to obtain ΔL is:

$$\Delta L = \sqrt{\Delta x^2 + \Delta y^2}$$

$$\Delta L = \frac{s}{EA} \cdot \sqrt{H^2 + \left(\frac{sw}{2}\right)^2}, \quad [44]$$

or:

$$\Delta L = \frac{z^2 w}{EA} \cdot \sqrt{2k-1} \cdot \sqrt{k^2 - \frac{3}{2}k + \frac{3}{4}} \quad [45]$$

Comparison between Eq. 43A and Eq. 45 is shown in Fig. 25, where:

A = metallic area for cable

$$E' = 12.0 \times 10^6 \text{ Lb/in}^2$$

For chain:

$$EA = \frac{PL}{\delta}$$

where L = link length

δ = elongation of link

P = acting force for δ elongation

since:

$$\delta \approx .0149 \frac{PR^3}{EI}$$

we have:

$$\frac{P}{\delta} = \frac{EI}{.0149R^3}$$

By assuming

$$L \approx 2R, \quad d \approx \frac{R}{3}$$

we can have:

$$EA = \frac{EIL}{.0149R^3}$$

since

$$A = .7854 d^2,$$

$$I = .0491 d^4$$

$$E' = .755 \times 30 \times 10^6 = 22.7 \times 10^6 \text{ Lb/in}^2$$

d is the diameter of the link bar.

LOCATION OF A DROPPED ANCHOR

Fig. 14 indicates an anchor at Point A departed from Point O, by applying a horizontal force, H, by a tugboat carrying that anchor on her deck. Point A is assumed to be very near the waterline. A vertical force, V, is also applied by the tugboat which causes a change in trim and stability on the tugboat. Cable which

can be wire rope or chain is assumed to be long enough and has a tangent point on the mud line.

Controlled Lowering

If the anchor is lowered to the mudline by a cable from a winch on the tugboat, drop distance is equal to S and can be easily determined if H, w and Z are known.

Free Drop

If the anchor is dropped free, it most probably will fall onto a point, C, located between Points D and E. In this case the drop distance d_z may be expressed in terms of x.

$$d_z = q \cdot x$$

q may be a function of scope, [s/z].

Office-type experience with a long dog chain gave the following results.

[1] q is nearly independent of scope.

[2] In air and water, q varies between 0.93 to 0.97 and 0.91 to 0.95, respectively.

ANCHOR DROPPING DISTANCE

Fig. 15A shows a tugboat pulling a chain of length 2s from a barge. Tugboat speed is assumed to be approximately equal to the payoff speed of the chain. At the moment Link 2 touches to the mudline, Link 1 is the first free link at the barge end. If this was a static condition, anchor dropping distances could be as follows.

1. Controlled lowering:

$$d_z = x + s \dots \dots \dots [46]$$

2. Free drop:

$$d_z = (1 + q) \cdot x \dots \dots \dots [47]$$

Fig. 15B shows the tugboat keeps pulling the chain with a speed equal to payoff speed until a kinetic equilibrium is reached.

We have:

$$d \cdot H = \frac{1}{2} f w d^2, \dots \dots \dots [48]$$

or

$$d = \frac{2H}{f w} \dots \dots \dots [49]$$

where f is the friction coefficient of the chain on mudline and w is the linear weight of chain in water [lb/ft].

Tugboat speed equals to payoff speed, but they decrease together until the tugboat stalls. Kinetic energy of tugboat and chain, water forces, etc., are neglected.

If for some reason the winch operator starts to payoff the chain much faster than tugboat speed, tangent point on the chain would tend to move toward the barge. We could have the following values as shown in Fig. 15C.

1. Controlled lowering,

$$d_z = \frac{2H}{f \cdot w} + s \dots \dots \dots [50]$$

2. Free drop,

$$d_z = \frac{2H}{f \cdot w} + q \cdot x \dots \dots \dots [51]$$

Fig. 15D indicates the anchor dropping distances obtained by an experienced winchman and a tugboat skipper combination as follows.

1. Controlled lowering,

$$d_z = x + d + s \dots \dots \dots [52]$$

2. Free drop,

$$d_z = d + (1 + q) \cdot x \dots \dots \dots [53]$$

For practical reasons, d values should contain efficiency factors including the human factor. We obtain final recommended form of the drop distances:

1. Controlled lowering,

$$d_z = x + n \cdot d + s \dots \dots [54]$$

2. Free drop,

$$d_z = n \cdot d + (1 + q) \cdot x \dots \dots \dots [55]$$

According to Fig. 16, x_{max} is the optimum necessary horizontal distance [barge to anchor] to obtain T_{max} tension in the chain in working condition. Here the shank angle is assumed to be zero; therefore,

$$x_{max} =$$

$$z \cdot (k - 1) \cdot \cosh^{-1} \left(\frac{k}{k - 1} \right) \dots [56]$$

$$\text{where } k = \frac{T_{max}}{z \cdot w}$$

If T_{max} , z and w are known, x_{max} can easily

be obtained.

Necessary tugboat pull can be obtained by trial and error from:

- 1. Controlled lowering,

$$x_{max} = \frac{H}{W} \cdot \cosh^{-1} \left(\frac{z \cdot W + H}{H} \right) + \frac{2nH}{f \cdot W} + \sqrt{z \cdot \left(z + \frac{2H}{W} \right)} \quad [57]$$

- 2. Free drop,

$$x_{max} = (1+q) \cdot \frac{H}{W} \cosh^{-1} \left(\frac{z \cdot W + H}{H} \right) + \frac{2nH}{f \cdot W} \dots \dots \dots [58]$$

The iterative forms of these equations are more useful, especially if a small desk-type computer is available; we can have:

- 1. Controlled lowering,

$$H = \frac{x_{max} - \sqrt{z \cdot \left(z + \frac{2H}{W} \right)}}{\cosh^{-1} \left(\frac{z \cdot W + H}{H} \right) + \frac{2n}{f}} \cdot W \dots \dots \dots [59]$$

- 2. Free drop,

$$H = \frac{x_{max} \cdot W}{(1+q) \cdot \cosh^{-1} \left(\frac{z \cdot W + H}{H} \right) + \frac{2n}{f}} \dots \dots \dots [60]$$

Friction coefficient f of the chain on dry ground is taken as 0.40 < f < 0.90 for the ship launching calculations.

In our case, a minimum value 0.25 for 2n/f is recommended to obtain a conservative value if efficiency and bottom conditions are not known. For example, with a 2-1/2-in. Di-Lok chain:

- W = 53.6 lb/ft in water
- Z = 300 ft water depth
- T_{max} = half of the breaking strength 744,000 lb = 372,000 lb.

The minimum required tugboat pull would be:

- 1. Controlled lowering,

Assume 2n/f = 0.25

$$k = \frac{T_{max}}{z \cdot W} = \frac{372,000}{300 \times 53.6} = 23.1343$$

From Eq. 56, x_{max} = 1,988.1 ft. From Eq. 59, we obtain H = 49,430.6 lb. From Eq. 57, we check x_{max} = 1,988.1 ft.

- 2. Free drop,

Assume 2n/f = 0.25; q = 0.95. From Eq. 60, we have H = 54,163.1 lb. From Eq. 58, we check x_{max} = 1,988.1 ft.

EFFECT OF CURRENT

The effect of current on an anchor chain is represented in Fig. 17. Consider the forces acting on an element of length, ds, in Fig. 18.

$$a = C_D \frac{\gamma}{2g} u^2 = \text{drag force, lb/ft}$$

- C_D = drag coefficient
- g = gravitational acceleration [sq ft/sec]
- γ = specific weight of water [lb/cu ft]
- u = velocity of current [ft/sec]

The force diagram is shown in Fig. 19. From Figs. 18 and 19, we obtain:

$$d \tan \phi = \frac{w + a \cdot \tan \phi}{H} ds \quad [61]$$

$$H = H_0 - \alpha \int_0^s \sin^2 \phi \cdot ds \dots \dots [62]$$

$$V = V_0 - w \cdot s - \alpha \int_0^s \sin \phi \cdot \cos \phi \cdot ds \quad [63]$$

$$x = \int_0^s \cos \phi \cdot ds \dots \dots \dots [64]$$

$$y = \int_0^s \sin \phi \cdot ds \dots \dots \dots [65]$$

Since:

$$\frac{d}{ds} \tan \phi = \frac{1}{\cos^2 \phi} \frac{d\phi}{ds} \quad [66]$$

we have

$$d\phi =$$

$$\frac{w \cdot \cos^2 \phi + \alpha \cdot \sin \phi \cdot \cos \phi}{H_0 - \alpha \int_0^s \sin^2 \phi \cdot ds} \cdot ds \dots \dots \dots [67]$$

For a graphical solution, Eq. 67 can be replaced by

$$\Delta \phi_{n+1} = \frac{w \cdot \cos^2 \phi_n + \alpha \cdot \sin \phi_n \cdot \cos \phi_n}{H_0 - \alpha \cdot \sum_1^n \sin^2 \phi_n \cdot \Delta s} \cdot \Delta s \dots \dots \dots [68]$$

If the lower boundary conditions, H_0 , V_0 , ϕ_0 , are known, we can obtain S , H , V , T , x , y values from the following relations.

$$S_{n+1} = S_n + \Delta s \dots \dots \dots [69]$$

$$H_{n+1} = H_n - \alpha \cdot \sin^2 \phi_n \cdot \Delta s [70]$$

$$V_{n+1} = V_n - w \cdot \Delta s - \alpha \cdot \sin \phi_n \cdot \cos \phi_n \cdot \Delta s \dots \dots [71]$$

$$T_{n+1} = \sqrt{(H_{n+1})^2 + (V_{n+1})^2} [72]$$

$$\phi_{n+1} = \phi_n + \frac{w \cdot \cos \phi_n + \alpha \cdot \sin \phi_n \cdot \cos \phi_n}{H_{n+1}} \cdot \Delta s \dots \dots \dots [73]$$

$$x_{n+1} = x_n + \cos \phi_n \cdot \Delta s [74]$$

$$y_{n+1} = y_n + \sin \phi_n \cdot \Delta s [75]$$

The results can be obtained by a digital computer or a digital plotter. For a variable current, similar results may be obtained by a minor change in the α term. These formulas may be used in the force analysis of a cable controlled submarine. [Application of these formulas will indicate that to neglect the resistance of anchor cables or chains may cause unsafe selection of mooring system for drilling barges operating against currents in deep waters.]

PILE ANCHOR CATENARY

Fig. 20 shows a pile anchor at Point A and a cable hanging down Point O. When Point O moves toward Point O_0 , the tangent point on the mudline moves toward A. When the tangent point reaches Point A, catenary will begin to be a partial catenary. By inspection it can be seen that Point O_0 cannot pass Point O_1 , if it does not stretch. To find any relation between

departure and other variables, Eqs. 4 through 13 may be used. For example, the relation between the horizontal tension, H , and departure parameter, λp , becomes:

From Eq. 5 and Eq. 10, we have:

$$K = 2c \cdot \sinh^{-1} \frac{1}{2c} \sqrt{L^2 - z^2} \dots [76]$$

or

$$K = 2 \frac{H}{w} \sinh^{-1} \frac{w}{2H} \sqrt{L^2 - z^2} \dots [77]$$

and $\lambda p = K - (L - z) \dots \dots \dots [78]$

In terms of k parameter, we can have:

$$\lambda p = 2k \cdot z \cdot \left[\sinh^{-1} \frac{\sqrt{r^2 - 1}}{2k} - (r - 1) \right] \dots \dots \dots [79]$$

where $r = \frac{L}{z}$.

For each r ratio and water depth we can have a similar graph as shown in Fig. 21. In this figure, $z\sqrt{r^2 - 1}$ indicates the asymptotic condition.

CATENARY WITH SINKER

$L < z$ Condition

From the basic equations we can have:

$$h = \sqrt{s_2^2 + c^2} - \sqrt{s_1^2 + c^2} [80]$$

$$(\zeta + c)^2 = s_1^2 + c^2 \dots \dots \dots [81]$$

$$L = \sqrt{(z + h + c)^2 - c^2} - \sqrt{(\zeta + h + c)^2 - c^2} [82]$$

From the equilibrium of the sinker of Weight P , we have

$$s_2 = s_1 + \frac{P}{w} \dots \dots \dots [83]$$

We obtain:

$$\zeta = \sqrt{[L - \sqrt{(z + h + c)^2 - c^2}]^2 + c^2} - (h + c) \dots \dots \dots [84]$$

$$h = \sqrt{[\sqrt{(\zeta + c)^2 - c^2} + \frac{P}{W}]^2 + c^2} - (\zeta + c) \dots \dots \dots [85]$$

ζ can be evaluated by iteration from Eqs. 84 and 85 for each L, z, w, H combination. Departure λ_s can be calculated from:

$$\lambda_s = \lambda + \lambda_p \dots \dots \dots [86]$$

where

$$\lambda = (z - \zeta) \cdot \left[(k-1) \cdot \left(\cosh^{-1} \frac{k}{k-1} - \frac{\sqrt{2k-1}}{k-1} \right) + 1 \right]$$

$$\lambda_p = 2k \cdot (z - \zeta) \cdot \left[\sinh^{-1} \frac{\sqrt{r^2 - 1}}{2k} - (r-1) \right]$$

where

$$r = \frac{L}{z - \zeta}$$

and $H = (z - \zeta) \cdot w \cdot (k-1)$.

L > z Condition

In this case, the cable will begin to form a natural catenary until the tangent point moves to the sinker location. It will form a pile anchor catenary until the sinker starts to rise. $V \geq P + L \cdot w$ after it becomes a catenary with sinker. By using the previous formulations we can obtain a characteristic curve as shown in Fig. 24.

COMPARISON OF ANCHORS

It has been shown that the holding power of anchors varies as follows: [1] holding power = shape factor x [fluke area]^{3/2}, or, for the same family of anchors, [2] holding power will vary linearly with the weight of the anchor.

The following anchor families were examined: [1] Baldt snug - stowing anchor, [2] Baldt stockless anchor, [3] Hall anchor, [4] Admiralty anchor, [5] Pool anchor, [6] Statoc anchor and [7] LWT anchor. The first six were geometrically similar. Weights varied with L³.

Holding powers could be expressed as HP = c · weight.

In LWT anchors, weight varied as

$$(W_L) \text{ Weight} = L^{3.66} = W^{1.22}$$

They were not geometrically similar. Their holding powers were expressed as HP = c · [w_L]^{0.82}. If they were geometrically similar, holding power would be:

$$HP = c \cdot (W_L)^{.82} = c \cdot (W^{1.22})^{.82} = C \cdot W$$

Therefore, true comparison between LWT and other anchors should be done on the fluke area basis for the same soil conditions.

NOMENCLATURE

- A = metallic area
- C = constant
- E = Young modulus
- L = sinker location on catenary
- P = weight of sinker
- T = tension
- W = weight of anchor
- c = parameter of catenary
- d = distance, diameter of link bar
- f = friction coefficient
- g = gravitational acceleration
- k = tension parameter
- q = distance coefficient
- r = ratio
- s = length of catenary
- w = weight of unit length of cable in water
- x, y = coordinates
- z = water depth
- δ = deflection
- ζ = sinker location above mudline
- λ = horizontal departure parameter
- ν = vertical departure parameter
- μ = reduced depth parameter
- ρ = radius of curvature of catenary
- φ = catenary angle
- Δ = increment

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2. Design Manual - Harbor and Coastal Facilities, NAVFAC DM-26, U. S. Naval Publications and Forms Center, Philadelphia [1968] Chapter 6, Sec. 3.

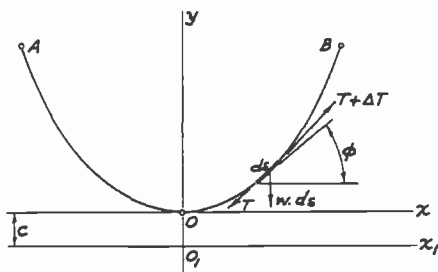


Fig. 1

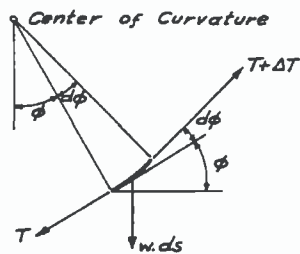


Fig. 2

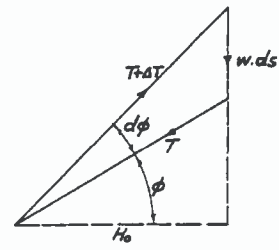


Fig. 3

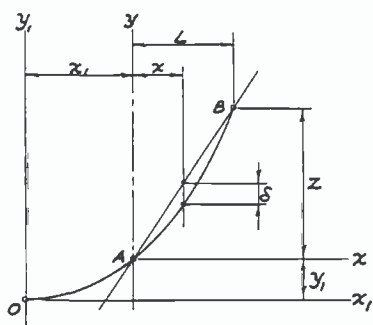


Fig. 4

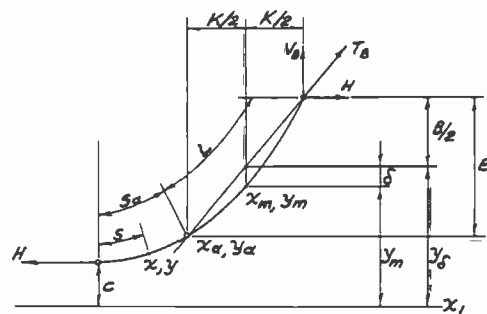


Fig. 5

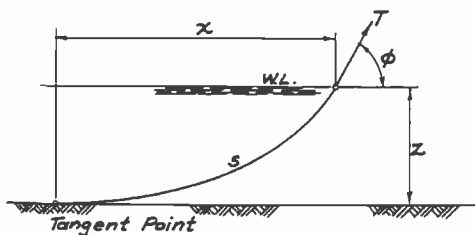


Fig. 6

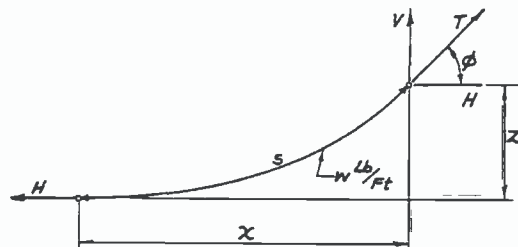


Fig. 7

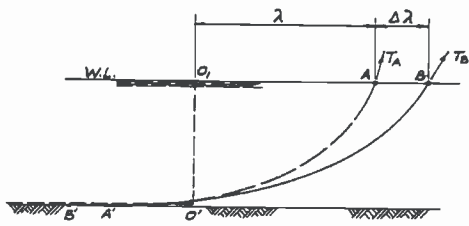


Fig. 8

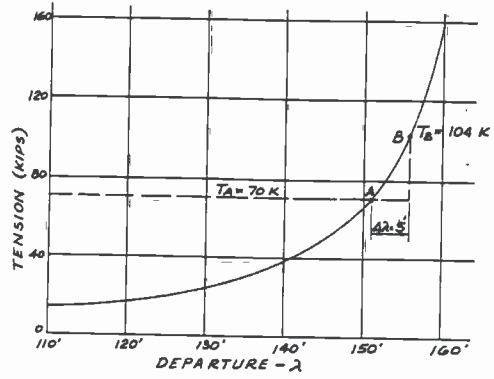


Fig. 9

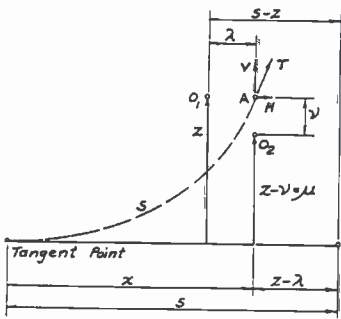


Fig. 10

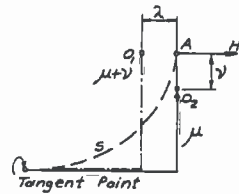


Fig. 11

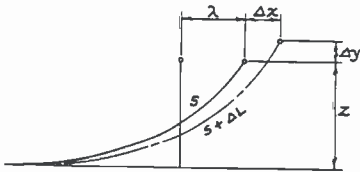


Fig. 12

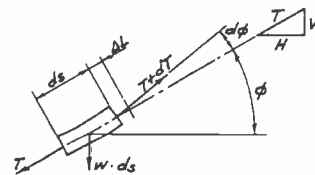


Fig. 13

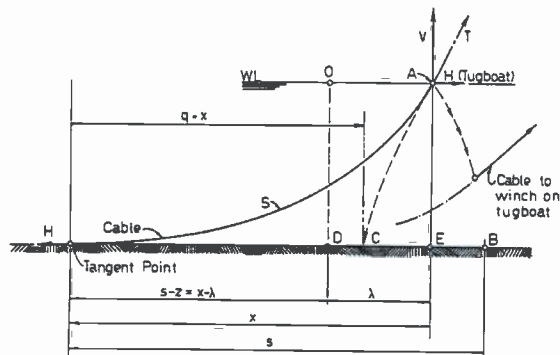


Fig. 14

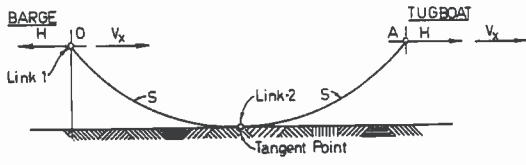


Fig. 15A

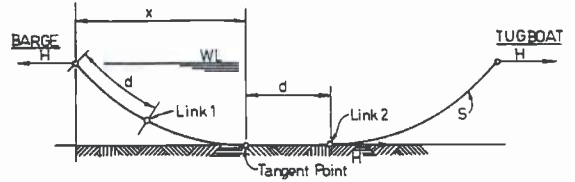


Fig. 15B

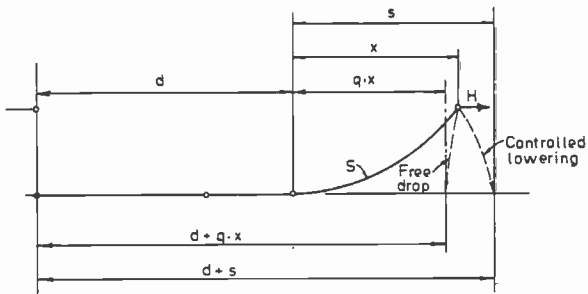


Fig. 15C

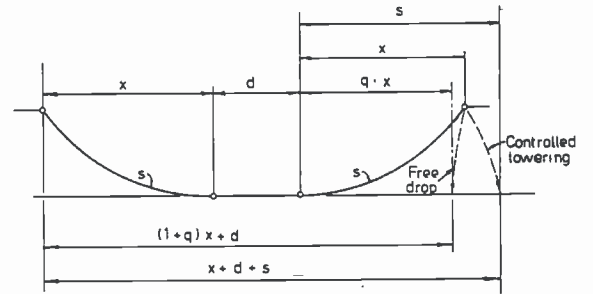


Fig. 15D

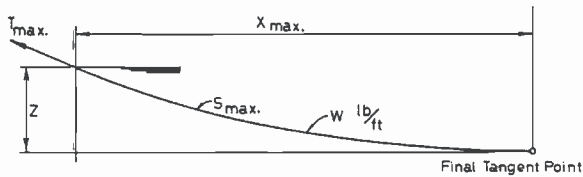


Fig. 16

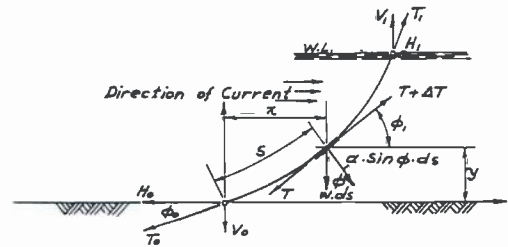


Fig. 17

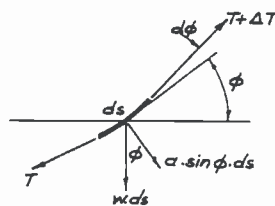


Fig. 18

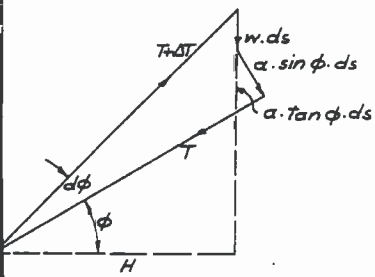


Fig. 19

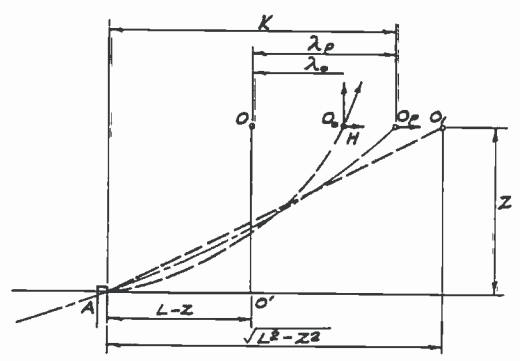


Fig. 20

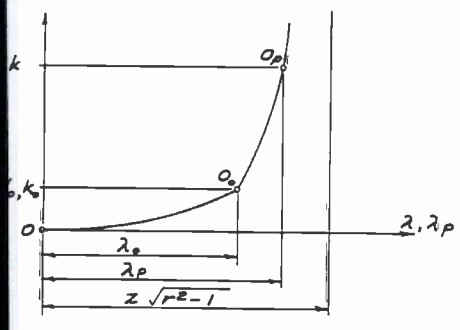


Fig. 21

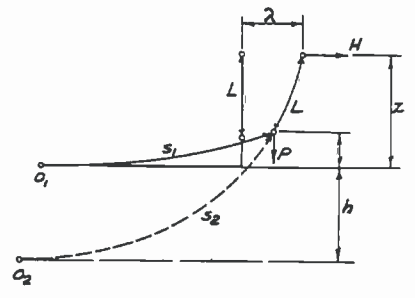


Fig. 22

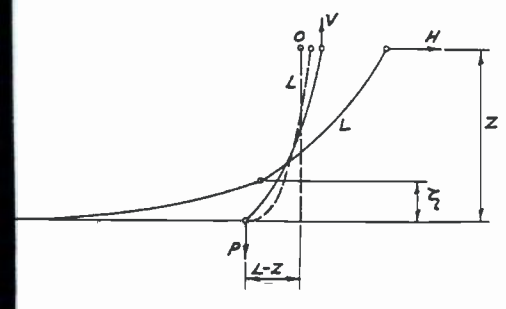


Fig. 23

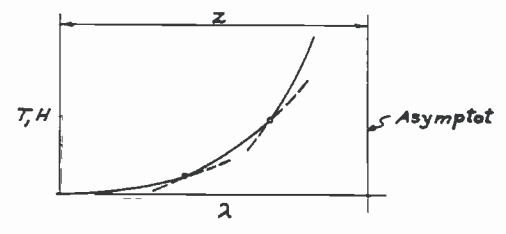


Fig. 24

Fig. 25 - Chain, comparison between stretch formulas.

K+1	CØEFFICIENT ØF FIRST EQUATION	CØEFFICIENT ØF SECONÐ EQUATION
3.0	2.39052976	2.29128785
4.0	5.27894927	5.12347538
5.0	8.87064720	8.67467579
6.0	13.04517744	12.81600562
7.0	17.72940567	17.47140521
8.0	22.87255526	22.58871400
9.0	28.43663978	28.12916636
10.0	34.39182885	34.06244266
11.0	40.71387970	40.36396908
12.0	47.38257904	47.01329599
13.0	54.38073442	53.99305511
14.0	61.69348778	61.28825336
15.0	69.30782978	68.88577502
16.0	77.21224500	76.77401904
17.0	85.39644583	84.94262770
18.0	93.85116842	93.38227883
19.0	102.56801323	102.08452380
20.0	111.53931832	111.04165885
21.0	120.75805727	120.24662157
22.0	130.21775581	129.69290651
23.0	139.91242307	139.37449551
24.0	149.83649422	149.28579972
25.0	159.98478221	159.42161083
26.0	170.35243686	169.77705970
27.0	180.93490990	180.34758107
28.0	191.72792483	191.12888321
29.0	202.72745096	202.11692160
30.0	213.92968061	213.30787608
31.0	225.33100939	224.69813083
32.0	236.92801861	236.28425677
33.0	248.71745995	248.06299603
34.0	260.69624169	260.03124812
35.0	272.86141655	272.18605768
36.0	285.21017080	284.52460350
37.0	297.73981439	297.04418862
38.0	310.44777224	309.74223154
39.0	323.33157620	322.61625811
40.0	336.38885789	335.66389439
41.0	349.61734215	348.88285999
42.0	363.01484104	362.27096212
43.0	376.57924845	375.82609009
44.0	390.30853511	389.54621035
45.0	404.20074402	403.42936184
46.0	418.25398625	417.47365186
47.0	432.46643712	431.67725211
48.0	446.83633259	446.03839521
49.0	461.36196599	460.55537126