A new vertical approximation for the numerical simulation of non-hydrostatic free surface flows
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In civil engineering applications short surface waves, short internal waves, bores, flow and wakes behind sills or steps, buoyant plumes, (buoyant) jets occur for which the pressure in water deviates notably from the hydrostatic pressure as currently assumed in hydrostatic flow solvers such as Delft3D-Flow of WL/Delft Hydraulics or TRIWAQ of Rijkswaterstaat.

Casulli & Stelling (1998) present therefore a method of solving non-hydrostatic free-surface flows. This report presents a new vertical approximation for the non-hydrostatic pressure when solving all momentum equations for incompressible free-surface flows and demonstrates its accuracy and efficiency through test cases for surface waves. The new method is comparable to a Boussinesq type of model but it is not limited to just wave-dominated flows.

Using a single layer or three layers, these tests show the correct frequency dispersion of a short standing surface wave, the stationary propagation of a solitary wave and the spatial dispersion of wave energy after passing a sill (Beji & Batjes experiment). These results show improvements compared to the original method (Casulli & Stelling, 1998) and with computational effort comparable to hydrostatic solvers. For short free surface waves.

Among other subjects, we recommend the extension of the research code to three dimensions as well as research towards an optimal solution procedure for solving the pressure if many vertical layers are required.

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Summary

Non-hydrostatic flows with a free surface occur whenever the vertical temporal or spatial acceleration or a vertical driving force yields significant deviations from the hydrostatic pressure. Some examples in civil engineering applications are short surface waves, short internal waves, bores, flow and wakes behind sills or steps, buoyant plumes, (buoyant) jets etc..

This report presents a new vertical approximation of the non-hydrostatic pressure when solving all momentum equations for incompressible free-surface flows. The method applies Hermite interpolation with hydrodynamic pressure and its vertical gradient as unknowns. The computational grid is Cartesian, staggered and fixed such that the free surface displaces through the vertical grid using concepts of flooding and drying. Advection is treated explicitly and, for simplicity of testing, stratification and turbulence model(s) are not yet implemented and the flow is confined to a single vertical plane (2DV say). The explicit treatment of the hydrostatic pressure in conjunction with the simplicity of the single-step approach is expected to yield computational efforts of the order of magnitude of the hydrostatic version of Delft3D-Flow or TRIWAQ. For simplicity of testing only a direct pressure solver is implemented with computational effort proportional to the number of vertical layers cubed.

This report presents test cases for surface waves only. Using a single layer or three layers, these tests show the correct frequency dispersion of a standing surface wave (short seiche), the stationary propagation of a solitary wave and the reasonably correct spatial dispersion of waves after passing a sill (Beji & Batjes experiment). These results show improvements compared to the previous method (Casulli & Stelling, 1998). For short free surface waves, the new method is comparable to a Boussinesq type of model but it is not limited to just wave-dominated flows.

We recommend:
- The extension of the new method from two to three dimensions;
- For wave studies, the inclusion of weakly reflective boundary conditions;
- For wave studies, the treatment of breaking and rolling waves in terms of momentum exchange and turbulence production;
- The implementation of (an) adequate turbulence closure model(s), including turbulence production by bores, breaking waves and orbital motions;
- The extension to density-stratified flows and to internal waves;
- Research towards the optimal way of solving the elliptic system of equations for the hydrodynamic pressure;
- Test cases dedicated to stationary flows over a backward-facing step, steep sill, wavy bed and buoyant plume and (buoyant) jet;
- Study towards a rigid-lid approach for removal of time step limitations imposed by free-surface (barotropic) modes for climatological, oceanographic and lake studies.
1 Introduction

1.1 Non-hydrostatic models

Numerical flow models that solve the 3D shallow water equations are sufficiently accurate for large-scale flow phenomena: such as tidal and wind-driven flows in coastal seas, lakes, estuaries and rivers. The shallow water approximation assumes that the horizontal length scale is much larger than the vertical scale. The vertical momentum equation is simplified to the hydrostatic pressure relation, neglecting vertical accelerations. The vertical velocity component is integrated from the continuity equation and coupled to the mobile free water surface.

For flow phenomena at smaller scales such as the flow over a bed with rapidly varying slopes or in the vicinity of hydraulic structures or subjected to short waves, the hydrostatic approximation is no longer valid. The governing equations are then the full 3D Reynolds-averaged Navier-Stokes equations. A number of papers, Casulli and Stelling (1998), Casulli, (1999) have been written on non-hydrostatic free surface numerical models. Recently, Zijlstra (2000) reported the extension of the 3D free-surface model TRIAQ of Rijkswaterstaat with the effect of the non-hydrostatic pressure, using a pressure correction method. The pressure correction method for the hydrodynamic pressure was also tested by Gaartniers (1995). He showed that this method has still good convergence if the horizontal and vertical length scale are of the same magnitude, but that this method is less efficient if the aspect ratio between the vertical and horizontal grid size decreases. Weilbeer and Jankowski (2000) extended the finite element model TELEMAC3D with the effect of the non-hydrostatic pressure. Chen (2000) combined an Eulerian-Lagrangian method with a pressure-correction method.

In this report we present a new vertical approximation of the non-hydrostatic pressure, based on a Hermitian method. It can deal with non-hydrostatic pressure effects using a few layers only (1-3 say). Even for a single-layer depth-averaged system, the results improve and, for short free surface waves, are comparable to a Boussinesq type of model. For stratified flows the number of layers depends on the vertical density gradients and then the method presented here is less efficient. The stratified case requires further research.

1.2 The Cassulli and Stelling method

In the models of Casulli and Stelling (1998) the discretisation of the momentum equations is based on a staggered Cartesian grid where the velocity points are at the sides of the grid boxes with length $\Delta x$, width $\Delta y$ and height $\Delta z$. The layer interfaces are strictly horizontal so that an inclined bed is represented as a staircase. A drying-and-flooding algorithm at the free surface allows it to displace freely along the fixed vertical grid. The pressure is defined in grid-box centres, see Figure 1.1. In the cell below the free surface, the
hydrodynamic pressure is assumed to be zero. This implies that for a single layer, the system reduces to a depth-averaged as well as strictly hydrostatic model.

\[ z = \zeta(x, t) \]

\[ [\zeta, w]_{m,k} \rightarrow [q]_{m,k} \rightarrow [w]_{m,k-1} \]

\[ u_{n-2,n} \rightarrow u_{n,n} \rightarrow u_{n,n-1} \]

\[ z = d(x) \]

Figure 1.1: Staggered grid used by Casulli and Stelling

In most 3D hydrostatic flow models (TRIWAQ, Delft3D-FLOW, TELEMAC-3D, ECOM3), the so-called sigma transformation is applied for obtaining a numerical grid that is boundary-fitted to the (mobile) free surface as well as the bed. For non-hydrostatic models, however, this sigma transformation yields a non-symmetric difference operator for the hydrodynamic pressure. Zijlema (2000) implemented a non-hydrostatic model using boundary-fitted vertical co-ordinates. Consequently, the free surface as well as the bed are grid lines and the boundary conditions at the free surface and bottom for the hydrodynamic pressure are easily implemented. Zijlema reports that, due to the sigma transformation, the discrete Laplacian has 25 non-zero diagonals but nevertheless the resulting system of equations is solved efficiently using a BICGSTAB algorithm.

Instead, the time integration of the non-hydrostatic model of Casulli and Stelling (1998), Casulli (1999) is based on a fractional-step method where the hydrostatic and hydrodynamic component of the pressure are considered separately. In the hydrostatic step, the free-surface gradient is integrated implicitly for avoiding time step limitations due to the CFL stability condition based on the phase speed of long free surface waves. In the hydrodynamic step, a non-hydrostatic correction is computed such that the 3D velocity field is divergence free (incompressibility condition). At the free surface position of the hydrostatic step and at the open boundaries, the hydrodynamic pressure is set to zero. The local mass conservation is assured by the finite-volume discretisation of the incompressibility condition.

For describing the essence of the original Casulli & Stelling approach, we present a two dimensional system using a single horizontal and vertical dimension. For simplicity, the viscosity terms, the Coriolis acceleration and the baroclinic pressure have been omitted. The momentum equations are then given by:
\[
\frac{Du}{Dt} + \frac{\partial p}{\partial x} = 0
\]
\[
\frac{Dw}{Dt} + \frac{\partial p}{\partial z} = -g \tag{1.1}
\]

where \(u(x,z,t)\) is the velocity component in the horizontal direction, \(w(x,z,t)\) is the velocity component in the vertical direction and \(p(x,z,t)\) is the total kinematic pressure i.e. pressure divided by density \(\rho_0\), assumed constant (Boussinesq' assumption). Likewise, the 2D conservation of volume is expressed by the incompressibility condition:

\[
\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \tag{1.2}
\]

The position of the free surface is assumed to be a single-value function \(z = \zeta(x,t)\) thereby excluding overtopping waves. The bed is defined at \(z = -d(x)\). Integrating the continuity equation over the water depth and using the kinematic condition at the free surface yields the following relation:

\[
\frac{\partial \zeta}{\partial t} + \frac{\partial}{\partial x} \int_{z=-d(x)}^{z=\zeta(x,t)} u \, d'z' = 0 \tag{1.3}
\]

Further, the normalized total pressure is split into a hydrostatic \(p_h\) and a hydrodynamic part \(q\):

\[
p = p_h + q \quad \text{where} \quad \frac{\partial p_h}{\partial z} = -g \quad \text{and} \quad p_h \{x, \zeta(x,t), t\} = 0 \tag{1.4}
\]

In view of (1.4), the momentum equations are now expressed as:

\[
\frac{Du}{Dt} + g \frac{\partial \zeta}{\partial x} + \frac{\partial q}{\partial x} = 0
\]
\[
\frac{Dw}{Dt} + \frac{\partial q}{\partial z} = 0 \tag{1.5}
\]

The Casulli and Stelling (1999) time integration of (1.5) is based upon a fractional step approach and this is described below.

**First Step : hydrostatic pressure**

In the first step, the pressure is hydrostatic and the free surface gradient is integrated implicitly. Advection is approximated an explicit Eulerian-Lagrangian finite difference operators defined as \(L_x\) and \(L_z\) in (1.6) below. Likewise, in (1.6) \(\delta \cdot / \delta x\) denotes the second-order central difference approximation to the first-order spatial derivative.
\[ \frac{u_{m,k}^* - \bar{u}_{m,k}^n}{\Delta t} + \bar{u}_{m,k}^n \bar{L}_x \bar{u}_{m,k}^n + \bar{w}_{m,k}^n \bar{L}_z \bar{u}_{m,k}^n + g \frac{\delta \zeta}{\delta x} = 0 \]

\[ \frac{w_{m,k}^* - \bar{w}_{m,k}^n}{\Delta t} + \bar{w}_{m,k}^n \bar{L}_x \bar{w}_{m,k}^n + \bar{u}_{m,k}^n \bar{L}_z \bar{w}_{m,k}^n = 0 \]

where \( u^* \) and \( w^* \) denote the approximations to the horizontal and vertical velocity component after the hydrostatic step, \( \bar{u} \) and \( \bar{w} \) are the approximation to the velocity components after the complete time step. Note that through (1.6), the horizontal velocity field satisfies the global continuity equation.

**Second Step: hydrodynamic pressure**

In the second step, the velocity vector of the first step is corrected by the hydrodynamic pressure gradient. The hydrodynamic pressure gradient is determined by requiring that the velocity field after every complete time step is locally divergence free (local continuity equation). At the free surface and at the open boundaries, the hydrodynamic pressure is set equal to zero. The physical meaning of this boundary condition is that the hydrostatic approximation is assumed to be valid at these boundaries. At the solid walls and at the bottom, the impermeability condition applied in both sub-steps yields the Neumann boundary condition with zero hydrodynamic-pressure gradient.

\[ \frac{u_{m,k}^{n+1} - u_{m,k}^*}{\Delta t} + \frac{\delta q}{\delta x} = 0 \]

\[ \frac{w_{m,k}^{n+1} - w_{m,k}^*}{\Delta t} + \frac{\delta q}{\delta z} = 0 \]

\[ \frac{\delta u_{m,k}^{n+1}}{\delta x} + \frac{\delta w_{m,k}^{n+1}}{\delta z} = 0 \]

\[ q|_{z=\zeta} = \]

\[ \bar{n} \cdot \nabla q|_{z=d} = 0 \]

Substitution of the discretized momentum equations into the incompressibility equation yields to a Poisson equation for the hydrodynamic pressure. The difference operator is symmetric and positive definite.

The time step restrictions because of this splitting into a hydrostatic and a non-hydrostatic step are quite severe for truly non-hydrostatic flows. Therefore, Casulli proposed a correction for the position of the free surface in the hydrodynamic step such that in essence the splitting error is reduced. Casulli proposed the following relation:
\[ g \tilde{\zeta} = g \zeta^* + q|_{z=z^*} \quad (1.8) \]

with \( \zeta^* \) the hydrostatic approximation of the free surface of the first step.

Condition (1.8) replaces the zero hydrodynamic-pressure condition at the free surface. Substitution of the momentum equations into the incompressibility condition leads to a symmetric, positive definite system of equations. It can be solved by a Conjugate Gradient Iterative method.

Despite of this improvement accurate approximations of the so-called Beji-Battjes experiment still require many vertical layers. This leads to a computational effort that would hamper 3D applications, although Casulli reports efficient 3D applications.

1.3 Improvements

Stelling and Busnelli (2000) extended the approach of Casulli and Stelling (1998) with a momentum conservative scheme for the horizontal advection terms and using the k-\( \varepsilon \) turbulence model. The numerical advection scheme was developed for simulating shallow water flows with large gradients in water depth, see Stelling et al. (1998). Conservation of momentum is the most important aspect near discontinuities in the solution, such as steep bottom gradients, hydraulic jumps or breaking waves.

Zijlema (2000) discretised the 3D Reynolds-averaged Navier-Stokes equations in boundary-fitted vertical co-ordinates. His vertical momentum equation is still based on the Cartesian velocity \( w \) and not on the transformed vertical velocity as Zijlema (2000) replaced the fractional-step approach by a pressure-correction method. The latter removes the error introduced by the splitting method for the non-linear terms that are integrated implicitly (bottom friction, vertical viscosity and advection). Zijlema presents results both for stratified and non-stratified test cases. Note that in his test cases, the bottom slopes are relatively small (\( < 1/10 \)). Further, note that for discontinuities, e.g. the backward facing step, the sigma transformation becomes singular.

This concludes our brief summary of previous and related methods of solving the full Navier-Stokes equations with a (mobile) free surface. This section ends with an overview of the remaining contents of this report.

In Chapter 2, we present a new vertical approximation to the hydrodynamic pressure. This approximation is based on a Hermitian method allowing for a very small number of vertical layers (1-3 say) for simulating short waves. It is a discrete analogue of the Boussinesq-type of wave models: the vertical dependency of the hydrodynamic pressure is resolved by a finite series of splines. The splines are described by the pressure itself and its vertical derivative. Even in the case of a single vertical layer, this new approximation yields the correct frequency dispersion relation of short waves, as demonstrated in Chapter 3 for three test cases. For stratified flows the number of layers depends on the vertical density gradients and then the new method is less efficient. Finally, Chapter 4 presents conclusions and recommendations.
2 The new numerical method

2.1 Introduction

This section describes our new numerical method for solving the 3D Reynolds-averaged Navier Stokes equations as tested in this study. In the new method the operator splitting of Casulli and Stelling (1998) is replaced by the explicit integration of the free surface gradient. The latter imposes the time step limit

$$\Delta t \leq \frac{\Delta x}{\sqrt{g H}} \quad (2.1)$$

through the CFL condition for long free-surface waves. The condition (2.1) however, was not restrictive for the test cases presented in this report.

In the horizontal direction a so-called staggered grid is used. In this grid the water levels as well as the hydrodynamic pressure are defined in the horizontal centre of control volumes (grid boxes), the horizontal velocity components at cell interfaces and the vertical velocity component at the layer interfaces, as shown below.

$$\begin{bmatrix}
\zeta, w, \partial q \over \partial z, q \\
\end{bmatrix}_{m,k}$$

$$\begin{bmatrix}
w, \partial q \over \partial z, q \\
\end{bmatrix}_{m,k-1}$$

$$\begin{bmatrix}
w, \partial q \over \partial z, q \\
\end{bmatrix}_{m,k-2}$$

Figure 2.1 Flow variables defined on a staggered grid.

In the vertical discretisation of the vertical momentum equation an Hermitian method is applied in which both the hydrodynamic pressure as well as its vertical derivative are unknown. The hydrodynamic pressure and its first-order vertical derivative are defined at the vertical layer interfaces in the centre of the horizontal computational cell.

Following the Hermitian method, the first-order vertical derivative of the hydrodynamic pressure is now related to the pressure through:
\[
\frac{q_{m,k} - q_{m,k-1}}{\Delta z_k} = \frac{1}{2} \left[ \frac{\partial q}{\partial z} \right]_{m,k} + \frac{1}{2} \left[ \frac{\partial q}{\partial z} \right]_{m,k-1}
\]

(2.2)

In the following Section 2.2 we describe the discretisation for the multiple-layer case and Section 2.3 deals with the depth-averaged or single-layer case.

### 2.2 The multiple-layer case

In this section we describe the numerical method for the multiple-layer case. For simplicity of notation, the governing equations are the Navier-Stokes equations for an incompressible fluid in just two dimensions with a single horizontal co-ordinate. In the present implementation, horizontal and vertical viscosity terms have been omitted because these terms are of minor importance for the test cases considered here. Note that Stelling and Busnelli (2000) implemented a two-equation turbulence model in a non-hydrostatic model and our numerical method can also be extended with these dissipative terms.

For discretising the equations, we introduce a set of strictly horizontal levels

\[
\{ z_k \mid k = 0, 1, \ldots, k_{\text{max}} \} \quad \text{with} \quad \forall x \quad z_0 \leq -d(x) \leq \zeta(x, t) \leq z_{k_{\text{max}}}
\]

(2.3)

and a horizontal grid with gridlines \( x_m \). Using the definition of the horizontal levels \( z_k \) and the horizontal gridlines \( x_m \), the control volumes are defined by the intersection of the water column with the bottom or with the free surface. Consequently, the layer thickness defined at the centre of the control volume (pressure point) reads:

\[
\Delta z^n_{m,k} = \min \left[ z^n_{m+1} - z^n_k, -d_m - z_{k-1} \right]
\]

(2.4)

A control volume is wet, i.e. occupied by fluid, as long as \( \Delta z^n_{m,k} > 0 \) holds. For each \( x_m \) point, the number of wet control volumes depends on the levels of the free surface and of the bed. The vertical index of the first control volume below the free surface is denoted as \( k_{\text{upp}}(m) \) and the first control volume above the bed is denoted as \( k_{\text{bottom}}(m) \) where \( m \) refers to the \( x_m \) point.

Note that at a \( u \)-velocity point, the thickness of the wet layer (local height of control volume) is not uniquely defined. This thickness depends on the procedure of determining the layer thickness for the bottom layer and free-surface layer in the \( u \)-velocity point. In the present implementation we applied a so-called upwind approach by which the layer thickness depends on the direction of the depth-averaged flow \( U \) through

\[
\Delta z^n_{m,\text{upp}} = \begin{cases} 
    z^n_{m+1} - z_{k_{\text{upp}}-1} & \text{if } U^n_m > 0 \\
    z^n_{m+1} - z_{k_{\text{upp}}-1} & \text{if } U^n_m < 0 \\
    \max(z^n_{m+1}, z^n_{m+1}) - z_{k_{\text{upp}}-1} & \text{if } U^n_m = 0
\end{cases}
\]

(2.5)
and

\[ \Delta z_{\text{bottom}}^n = \begin{cases} z_{\text{bottom}} + d_m & \text{if } U_m^n > 0 \\ z_{\text{bottom}} + d_{m+1} & \text{if } U_m^n < 0 \\ z_{\text{bottom}} + \min(d_{m+1}, d_m) & \text{if } U_m^n = 0 \end{cases} \]  

(2.6)

This particular upwind approach guarantees a positive total water depth at water-level points (horizontal centres of control volumes).

Given the staggered Cartesian grid, the Navier-Stokes equations are discretized as follows:

\[ \frac{u_{m,k}^{n+1} - u_{m,k}^n}{\Delta t} + u_{m,k}^n \frac{L_x u_{m,k}^n + \overline{w_{m,k}^{n \times 2}}}{\Delta t} + L_xu_{m,k}^n + g \frac{\delta \zeta^n}{\delta x} + \frac{1}{2} \frac{\delta}{\delta x} \left[ q_{m,k}^{n+1} + q_{m,k-1}^{n+1} \right] = 0 \]

\[ \frac{w_{m,k}^{n+1} - w_{m,k}^n}{\Delta t} + u_{m,k}^n \frac{L_x w_{m,k}^n + \overline{w_{m,k}^{n \times 2}}}{\Delta t} + \frac{\partial}{\partial z} \left[ q_{m,k}^{n+1} \right] = 0 \]  

(2.7)

\[ \frac{\delta u_{m,k}^{n+1}}{\delta x} + \frac{\delta w_{m,k}^{n+1}}{\delta z} = 0 \]

\[ \frac{\zeta_{m,k}^{n+1} - \zeta_{m,k}^n}{\Delta t} + \frac{\delta}{\delta x} \sum_{\text{top}} \Delta z_{\text{bottom}}^n + U_{m,k}^{n+1} = 0 \]

In (2.7), for the approximations \( L_x \) and \( L_z \) of the advection terms a momentum-conserving scheme such as of (Stelling and Busnelli, 2000) may be used. Such a momentum-conserving scheme is of importance near local flow discontinuities e.g. steps in the bed or bores. In (2.7), the explicit discretisation of the advection terms impose the following time step limitation by the so-called CFL-condition for advection:

\[ \Delta t \leq \min \left( \frac{\Delta x}{|U|}, \frac{\Delta z}{|W|} \right) \]  

(2.8)

For a strictly momentum-conserving balance, in the global sense, special attention should be given to the discretisation of the non-hydrostatic pressure gradient in the bed layer. In Section 2.3, this discretisation is presented for the single-layer or depth-averaged case.

At the free surface the kinematic condition leads to:

\[ \frac{w_{m,\text{top}}^{n+1} - \zeta_{m,k}^n}{\Delta t} + u_{m(\text{top})}^{n+1} \frac{\delta \zeta^n}{\delta x} \]  

(2.9a)

and at the bottom:
\[ W_{m, \text{bottom}}^{n+1} = -u_{m(U), \text{bottom}}^{n+1} \frac{\delta d}{\delta x} \]  \hspace{1cm} (2.9b)

The index \( m(U) \) is dependent on the flow direction. Following Eq. (2.5) and (2.6). We use an upwind approach: \( m \) for \( U > 0 \) and \( m+1 \) for \( U < 0 \).

Substituting the momentum equations in the incompressibility condition yields:

\[
-\Delta t \frac{\delta}{\delta x} \left[ \frac{\delta}{\delta x} \left[ q_{m,k}^{n+1} - \frac{1}{4} \Delta z_{m,k}^{n+1} \left( \frac{\partial q}{\partial z} m,k + \frac{\partial q}{\partial z} m,k-1 \right) \right] \right] - \Delta t \frac{\delta}{\delta z} \frac{\partial q}{\partial z} m,k^{n+1} = -\Delta t \left( \frac{\partial q}{\partial z} m,k^{n+1} \right) \]

\[
-\frac{\delta}{\delta x} \left[ F_u[u_{m,k}^{n+1}] - \Delta t g \frac{\delta z}{\delta x} \right] - \frac{\delta F_u[w_{m,k}^{n+1}]}{\delta z}
\]

with

\[ q_{m,k}^{n+1} = -\frac{1}{2} \sum_{l=1}^{10} \Delta z_{m,l} \left[ \left( \frac{\partial q}{\partial z} m,l \right) + \left( \frac{\partial q}{\partial z} m,l+1 \right) \right]. \]

This is an implicit equation for the vertical hydrodynamic pressure gradient. In (2.10), the finite-difference operators \( F_u \) and \( F_w \) contain the temporal explicit terms of the momentum equations:

\[
F_u[u_{m,k}^{n}] = u_{m,k}^{n} - \Delta t \, u_{m,k}^{n} L_x u_{m,k}^{n} - \Delta t \, \frac{w_{m,k}^{n}}{L_z} \, L_z u_{m,k}^{n},
\]

\[
F_w[w_{m,k}^{n}] = w_{m,k}^{n} - \Delta t \, \frac{w_{m,k}^{n}}{L_z} \, L_z w_{m,k}^{n} - \Delta t \, w_{m,k}^{n} L_z w_{m,k}^{n},
\]

(2.11)

The discretisation of the incompressibility equation leads to \( k_{\text{max}} \) linear equations in the unknown vertical hydrodynamic pressure gradients which are defined at the layer interfaces. The number of unknowns is \( k_{\text{max}} + 1 \) i.e. the set (2.11) is undetermined. For yielding a well-posed system, the discretisation of the kinematic boundary condition at the bottom is added. This addition involves the substitution of the discretized momentum equations in the kinematic boundary condition at the bed and yields for \( U > 0 \):

\[
-\Delta t \left( \frac{\partial q}{\partial z} m, \text{bottom} \right)^{n+1} - \frac{\delta}{\delta x} \left[ q_{m, \text{bottom}+1}^{n+1} - \frac{1}{4} \Delta z_{m, \text{bottom}+1}^{n+1} \left( \frac{\partial q}{\partial z} m, \text{bottom}+1 \right) + \left( \frac{\partial q}{\partial z} m, \text{bottom} \right) \right] \frac{\delta d}{\delta x} = \]

\[
- \left( F_u[u_{m, \text{bottom}}^{n}] - \Delta t g \frac{\delta z}{\delta x} \right) \frac{\delta d}{\delta x} - F_w[w_{m, \text{bottom}}^{n}]
\]

(2.12)

with

\[ q_{m, \text{bottom}+1}^{n+1} = -\frac{1}{2} \sum_{l=\text{bottom}+1}^{10} \Delta z_{m,k} \left[ \left( \frac{\partial q}{\partial z} m,l \right) + \left( \frac{\partial q}{\partial z} m,l+1 \right) \right]. \]
The kinematic boundary condition (2.9a) at the free surface is automatically fulfilled and this can be shown as follows. Add the discrete local continuity equations for each control volume from the bottom until the free surface:

\[ w_{m,\text{top}}^{n+1} - w_{m,\text{bottom}}^{n+1} + \sum_{\text{top}}^{\text{bottom}} \Delta z_{m,l}^{n} \frac{\delta u_{m,l}^{n+1}}{\delta x} = 0 \]  (2.13a)

Using the definition of the vertical grid:

\[ \frac{\delta \Delta z_{m,l}^{n}}{\delta x} = \begin{cases} 
0 & l \neq \text{bottom} \land l \neq \text{top} \\
\frac{\delta d_{m}}{\delta x} & l = \text{top} \\
\frac{\delta z_{m}^{n}}{\delta x} & l = \text{bottom}
\end{cases} \]  (2.14)

gives

\[ w_{m,\text{top}}^{n+1} - w_{m,\text{bottom}}^{n+1} - u_{m(l),\text{top}}^{n+1} \frac{\delta z_{m}^{n}}{\delta x} - u_{m(l),\text{bottom}}^{n+1} \frac{\delta d_{m}}{\delta x} + \sum_{\text{bottom}}^{\text{top}} \frac{\delta \left( \Delta z_{m,l}^{n} u_{m,l}^{n+1} \right)}{\delta x} = 0 \]  (2.13b)

Substitution of the discrete kinematic boundary condition at the bottom and the global continuity equation results into the discretized kinematic boundary condition at the free surface. That this relation is exactly fulfilled on the staggered grid is not straightforward.

### 2.3 Solution Method

The resulting linear system of equations has a special block structure. Let \( a_{m,k,n,l} \) be the matrix element describing the coupling between the pressure gradient \( (\partial q / \partial z)_{m,k} \) in the grid point with index \( m \) and at layer interface \( k \). Likewise, define the pressure gradient \( (\partial q / \partial z)_{n,l} \) in the grid point with index \( n \) and at layer interface \( l \). Further, let \( m_{max} \) be the number of horizontal gridpoints and \( k_{max} \) be the number of vertical layers. In the horizontal direction then there is just a coupling between the unknowns at grid point \( m \) and the unknowns at its neighbouring grid points \( m+1 \) and \( m-1 \).

In the vertical direction, the unknown at layer interface \( k \) is coupled with the unknown at the layer interface from \( k-1 \) until the index of the top layer. The matrix elements \( a_{m,k,n,l} \) are nonzero for \( n=m-1,m,m+1 \) and \( l = k-1(l)k_{top} \). For the third index only three nonzero elements have to be stored and it is possible to save computer memory with a factor which is proportional with the number of horizontal grid points.

The matrix \( A \) of \( m_{max} \times (k_{max}+1) \) \( m_{max} \times (k_{max}+1) \) elements with a block structure can be reduced to a matrix \( A' \) of \( m_{max} \times (k_{max}+1) \times 3 \times (k_{max}+1) \) elements. The matrix elements are defined by:
\[ a_{m,k,-1,l}^* = a_{m,k,m-1,l} \]
\[ a_{m,k,0,l}^* = a_{m,k,m,l} \]
\[ a_{m,k,1,l}^* = a_{m,k,m+1,l} \]  \hfill (2.15)

The matrix elements of \( A^* \) are nonzero. The number of nonzero elements grows linearly with the number of horizontal grid points but the number of nonzero elements increased squared with the number of vertical layers. Contrary to the Casulli/Stelling method, the resulting system of equations is not symmetric and not positive definite. The new system of equations is solved via Gaussian elimination. The elimination algorithm is based on the special band structure of the matrix. The computational effort is proportional to the number of scalar operations. The number of operations (adding and multiplication) is \( O(m m_{\text{max}} \times k \text{max}^2) \). If the number of layers in the vertical is small (<5) then the use of a direct solution method is not restrictive, however, for many layers this approach is less efficient.

### 2.4 The depth-averaged case

For a single layer, the approximation presented in Section 2.2 corresponds to a Boussinesq type of model. Figure 2.2 shows the arrangement of the variables on the vertical grid for a single-layer system. The hydrodynamic pressure and the first-order vertical derivative are defined at the free surface and at the bed and in the centre of the horizontal computational cell:

\[
\begin{bmatrix}
\zeta, w, \frac{\partial q}{\partial z}, q
\end{bmatrix}_{m,1}
\]

\[
\begin{bmatrix}
w, \frac{\partial q}{\partial z}, q
\end{bmatrix}_{m,0}
\]

Figure 2.2 Variables arranged for the single-layer or depth-averaged case.

In Figure 2.2, \( U(x,t) \) is the depth-averaged horizontal velocity component. The first vertical derivative of the hydrodynamic pressure is related to the pressure itself through:

\[
\frac{q_{m,1} - q_{m,0}}{H_m} = 2 \left[ \frac{\partial q}{\partial z} \right]_{m,1} + 2 \left[ \frac{\partial q}{\partial z} \right]_{m,0}
\]  \hfill (2.16)

The depth-averaged momentum equation is derived for a control volume of length \( \Delta x \) and height \( H \). This equation is extended with a quadratic friction term, with \( C \) the Chézy resistance coefficient. Special attention is paid to the vertical integration of the non-hydrostatic pressure gradient:
\[ \int_{z=-d(x)}^{z=\zeta(x,t)} \frac{\partial q(x,z')}{\partial x} \, dz' = \frac{\partial}{\partial x} \int_{z=-d(x)}^{z=\zeta(x,t)} q(x,z') \, dz' - q_0(x) \frac{\partial d(x)}{\partial x} - q_1(x) \frac{\partial \zeta(x,t)}{\partial x} \] (2.17)

For the present applications we assume that the non-hydrostatic pressure at the free surface is zero. Consequently, the third term in (2.17) is zero so that just the second stock term appears in the depth-averaged momentum equation. In the RHS of (2.17), the integral may be approximated with different numerical integration rules using the pressure variables that are defined at the bottom and free surface in the cell centres: \(q_{m,0}, q_{m,1}, (\partial q / \partial z)_{m,0}\) and \((\partial q / \partial z)_{m,1}\). The best results where obtained with the most simple approximation for the integral in the RHS of (2.17) i.e.:

\[ \frac{\partial}{\partial x} \int_{z=-d(x)}^{z=\zeta(x,t)} q(x,z') \, dz' - q_0(x) \frac{\partial d(x)}{\partial x} \approx \frac{H_{m+1,0}^n q_{m+1,0} - H_{m,0}^n q_{m,0}}{2 \Delta x} + \frac{q_{m,0} + q_{m+1,0}}{2} \frac{\partial d_m}{\partial x} \] (2.18)

Using the relation between the pressure and the vertical pressure gradient, the following expression is obtained for the depth-averaged momentum equation:

\[ \frac{U_{m}^{n+1} - U_{m}^{n}}{\Delta t} + U_{m}^{n} L_{x} U_{m}^{n} + g \frac{\delta z_{m}^{n}}{\delta x} + g \frac{\frac{U_{m}^{n} U_{m}^{n+1}}{H_{m}^{n} U}}{C^2} \]

\[ - \frac{1}{4 H_{m}^{n+1} \Delta x} \left( \left( H_{m+1,1}^{n} \right)^{2} \left( \frac{\partial q}{\partial z} \right)_{m+1,1}^{n+1} + \left( \frac{\partial q}{\partial z} \right)_{m+1,0}^{n+1} \right) - \left( H_{m,0}^{n} \right)^{2} \left( \frac{\partial q}{\partial z} \right)_{m,0}^{n+1} + \left( \frac{\partial q}{\partial z} \right)_{m,1}^{n+1} \right) \] (2.19)

\[ + \frac{1}{4} \left( \frac{\partial q}{\partial z} \right)_{m+1,1}^{n+1} + \frac{\partial q}{\partial z} \right)_{m+1,0}^{n+1} + \left( \frac{\partial q}{\partial z} \right)_{m,1}^{n+1} + \frac{\partial q}{\partial z} \right)_{m,0}^{n+1} \frac{\delta d_m}{\delta x} = 0 \]

The discretised vertical momentum equations at the free surface and bottom are defined through

\[ \frac{W_{m,1}^{n+1} - W_{m,1}^{n}}{\Delta t} + U_{m}^{n} L_{x} W_{m,1}^{n} + \left[ \frac{\partial q}{\partial z} \right]_{m,1}^{n+1} = 0 \] (2.20)

\[ \frac{W_{m,0}^{n+1} - W_{m,0}^{n}}{\Delta t} + U_{m}^{n} L_{x} W_{m,0}^{n} + \left[ \frac{\partial q}{\partial z} \right]_{m,0}^{n+1} = 0 \]

The incompressibility relation is given by:
\[ H_m \frac{SU_m^{n+1}}{\delta x} + W_m^{n+1} - W_m^{n+1} = 0 \]  
\[ \frac{z_m^{n+1} - z_m^n}{\Delta t} + \frac{\delta}{\delta x} \left( H_m^n U_m^n \right) = 0 \]  
(2.21)

At the free surface the kinematic condition leads to:

\[ W_{m,0}^{n+1} = \frac{z_m^{n+1} - z_m^n}{\Delta t} + U_m^n \frac{\delta z_m}{\delta x} \]  
(2.22a)

and at the bottom:

\[ W_{m,0}^{n+1} = -U_m^n \frac{\delta d}{\delta x} \]  
(2.22b)

Substitution of the kinematic boundary conditions in the incompressibility condition gives the depth-averaged continuity equation. Via substitution of the momentum equations in the incompressibility condition we obtain:

\[ -\Delta t H_m^n \frac{\delta}{\delta x} \left\{ \frac{1}{4H_m^n U_m^n b_u[U_m^n]\Delta x} \left[ \left( H_m^n \frac{\delta q}{\delta z} \right)_{m+1,1}^{n+1} + \left( \frac{\partial q}{\partial z} \right)_{m+1,0}^{n+1} \right] - (H_m^n q) \left( \frac{\partial q}{\partial z} \right)_{m+1,1}^{n+1} + \left( \frac{\partial q}{\partial z} \right)_{m+1,0}^{n+1} \right\} \right. \]

\[ -\Delta t H_m^n \frac{\delta}{\delta x} \left[ \frac{1}{4b_u[U_m^n]} \left( \frac{\partial q}{\partial z} \right)_{m+1,1}^{n+1} + \left( \frac{\partial q}{\partial z} \right)_{m+1,0}^{n+1} + \left( \frac{\partial q}{\partial z} \right)_{m+1}^{n+1} \right] \frac{\delta d_m}{\delta x} \right] \]

\[ -\Delta t \left( \frac{\partial q}{\partial z} \right)_{m+1,1}^{n+1} - \left( \frac{\partial q}{\partial z} \right)_{m,0}^{n+1} = -H_m^n \frac{\delta}{\delta x} \left[ F_u[U_m^n] - \frac{\Delta t g}{b_u[U_m^n]} \frac{\delta z}{\delta x} \right] - F_w[W_{m,1}^n] - F_w[W_{m,0}^n] \]

(2.23)

This is an implicit equation for the vertical hydrodynamic pressure gradients at the free surface and the bottom. The finite difference operators \( F_u \) and \( F_w \) include the explicit terms of the momentum equations.

\[ F_u[U_m^n] = \frac{U_m^n - \Delta t U_m^n L_x U_m^n}{b_u[U_m^n]} \]  
(2.24)

\[ F_w[w_{m,k}^n] = w_{m,k}^n - \Delta t U_m^n L_x w_{m,k}^n + \Delta t w_{m,k}^n L_z w_{m,k}^n \]

with
\[ b_n \left[ U_m^n \right] = 1 + \Delta t \frac{g[U_m^n]}{H_m^{n+1} C^2} \]

The substitution of the momentum equation into the kinematic boundary condition at the bed yields:

\[
\frac{\Delta t}{4b_n[U_m^n] \Delta x} \left[ \left( H_m^{n+1} \right)^2 \left( \left( \frac{\partial q}{\partial z} \right)_{m+1,1}^{n+1} + \left( \frac{\partial q}{\partial z} \right)_{m+1,0}^{n+1} \right) - \left( H_m^n \right)^2 \left( \left( \frac{\partial q}{\partial z} \right)_{m,1}^{n+1} + \left( \frac{\partial q}{\partial z} \right)_{m,0}^{n+1} \right) \right] \frac{\delta d_m}{\delta x} = \]

\[
- \frac{\Delta t}{4b_n[U_m^n]} \left( \left( \frac{\partial q}{\partial z} \right)_{m+1,1}^{n+1} + \left( \frac{\partial q}{\partial z} \right)_{m+1,0}^{n+1} + \left( \frac{\partial q}{\partial z} \right)_{m,1}^{n+1} + \left( \frac{\partial q}{\partial z} \right)_{m,0}^{n+1} \right) \left( \frac{\delta d_m}{\delta x} \right)^2 - \Delta t \left( \frac{\partial q}{\partial z} \right)_{m,0}^{n+1} = - F_n [w_{m,0}^n] \]

\[
(2.25)
\]

The resulting linear system of equations has the following special block structure. Let \( a_{m,k,n,l} \) be the matrix element describing the coupling between the pressure gradient \( \left( \frac{\partial q}{\partial z} \right)_{m,k} \) in the grid point with index \( m \) and at layer interface \( k \) and \( \left( \frac{\partial q}{\partial z} \right)_{n,l} \) with index \( n \) and at layer interface \( l \); the indices \( k \) and \( l \) are 0 or 1.

Let \( m_{max} \) be the number of horizontal grid points. In the horizontal direction then there is just a coupling between the unknown at grid point \( m \) and the unknown at its neighbouring points \( m+1 \) and \( m-1 \).

In the vertical direction the unknown at the bottom (index 0) is coupled to the unknown at the free surface (index 1). The matrix elements \( a_{m,k,n,l} \) are nonzero for \( n=m-1,m,m+1 \) and \( l = 1,2 \). For the third index just three nonzero elements have to be stored and it is possible to save computer memory with a factor proportional to the number of horizontal grid points.

The matrix \( A \) of \( m_{max} \times 2 \times m_{max} \times 2 \) elements with a block structure can be reduced to a matrix of \( m_{max} \times 2 \times 3 \times 2 \) elements. The resulting system of equations is not-symmetric and not-positive definite. The system of equations is solved via Gaussian elimination. The elimination algorithm is based on the special band structure of the matrix.
3 Verification

The numerical method has been verified by the three following test cases. The first test case is a standing wave in a closed basin where the depth is of the same order as the horizontal length. The oscillating basin shows the difference between a hydrostatic and a non-hydrostatic simulation. This test case was applied by different authors Gaarthuis (1995), Casulli and Stelling (1998), Zijlema (2000), Chen (2000). The second test case is the so-called Beiji & Battjes experiment as used by Gaarthuis (1995), Casulli and Stelling (1998) and Zijlema (2000). The third test case is a solitary wave as demonstrated by Weilbeer and Jankowski (2000) and for the non-hydrostatic case this wave should preserve its shape.

3.1 Standing wave

For demonstrating the influence of the non-hydrostatic pressure we consider the free oscillations in a closed basin where the depth (10 m) is of the same order as the length (20 m), see also Zijlema (2000). For these dimensions, vertical accelerations are not longer negligible. The wave speed $c = \sqrt{gH}$ of the hydrostatic long wave solution is independent of the frequency but the wave speed for shorter waves is given by the dispersion relation:

$$c = \frac{\omega}{k} = \sqrt{\frac{g}{k} \tanh(kH)} \quad (3.1)$$

with $\omega$ the wave's angular frequency and $k$ its wave number and this wave speed depends on its frequency. Hence a non-hydrostatic model is necessary for calculating the solution correctly.

As initial condition, the free surface elevation is defined as

$$\zeta(x) = \zeta_0 \cos\left(\frac{\pi x}{10}\right) \quad (3.2)$$

with $\zeta_0$ the amplitude of the standing wave. For this test case, we choose $\zeta_0 = 0.1$ m. According to the dispersion relation (3.1), the wave period should be 3.59 s.

For simulating the seiching in the closed basin, we applied a horizontal grid with 1 m grid size and three vertical layers using 0.1 s time step without bed-friction. For a hydrostatic and a non-hydrostatic simulation, Figure 3.1 compares the time series of the water level at $x=17.5$ m, being near the closed boundary. As anticipated, the wave period of the hydrostatic simulation is 2.0 s and it does not correspond to the expected wave period.

This simulation is repeated with the depth-averaged non-hydrostatic model and Figure 3.2 presents the time series again at $x=17.5$ m. Using just a single layer, the hyperbolic vertical
profile for pressure is not resolved and the wave period for a single layer is computed less accurately than with 3 layers.

**Standing wave in closed basin**

$x = 17.5 \text{ m}$

Figure 3.1: Time series water level at $x = 17.5 \text{ m}$, for non-hydrostatic and hydrostatic model with 3 vertical layers.

**Standing wave in closed basin**

$x = 17.5 \text{ m}$

Figure 3.2: Time series for water level at $x = 17.5 \text{ m}$ using the non-hydrostatic model with three vertical layers and a single-layer or depth-averaged non-hydrostatic model.

The present method using an explicit time integration of the hydrostatic pressure gradient has no splitting error and Figure 3.1 shows that the wave is not damped due to numerical dissipation.
3.2 The Beji and Battjes experiment

At Delft Hydraulics in a 30 m long laboratory flume, the propagation of regular waves over a longshore bar was observed, for details see Beji and Battjes (1994). Figure 3.2 presents the bottom profile. The still water depth is 0.4 m but it is reduced to 0.1 m over the submerged trapezoidal bar. At the end of the flume, a plane beach, with a slope of 1/25, serves as a wave absorber. A sinusoidal wave with 1 cm amplitude 2.02 s period is imposed at the left open boundary. Above the upward slope, the wave front becomes steeper and then generates higher harmonics each propagating with its particular wave speed thereby generating a complex wave pattern. This experiment was simulated by Casulli and Stelling (1998) and Zijlema (2000) using a non-hydrostatic model and also by Borsboom et al. (2000) using a Boussinesq type of model.

We simulate this experiment with a 0.015 m horizontal grid size and 0.01 s time step with one as well as with three layers.

![Figure 3.3: Geometry of Beji & Battjes experiment](image)

The water levels of the simulations are compared with the observed water levels (Beji & Battjes, 1994). The results are presented at the same locations in (Casulli & Stelling, 1998) and (Zijlema, 2000). Zijlema shows results with a hydrostatic approximation.

For the depth-averaged approach, Figure 3.4 and 3.5 demonstrate a good correspondence between the simulation and the measurements. In the last station, behind the bar (Figure 3.6) the results differ and the single-layer approximation is not accurate. Instead, Figure 3.9 shows improved results behind the bar, using three vertical layers. Zijlema (2000) used ten equidistant sigma layers but in our approach, based on Hermitian approximations for the pressure, just three layers yield accurate results.
Figure 3.4: Water levels at $x=13.5$ m; the new single-layer or depth-averaged non-hydrostatic model compared to observations.

Figure 3.5: Water levels at $x=15.7$ m; the new single-layer or depth-averaged non-hydrostatic model compared to observations.
Figure 3.6: Water level at x = 19.0 m; the new single-layer or depth-averaged non-hydrostatic model compared to observations.

Figure 3.7: Water level at x = 13.5 m; the new single-layer or depth-averaged non-hydrostatic model compared to observations.
Figure 3.8: Water level at \( x = 15.7 \) m; the new single-layer or depth-averaged non-hydrostatic model compared to observations.

Figure 3.9: Water level at \( x = 19.0 \) m; the new single-layer or depth-averaged non-hydrostatic model compared to observations.
3.3 Solitary wave

The third test case considers the propagation of a solitary wave in a long channel. This test case was also used by Weilbeer and Jankowski (2000) and Mooiman and Verboom (1992). The solitary wave is a non-linear wave of finite amplitude which is not a solution of the shallow water equations and can be computed only with a non-hydrostatic model. A feature of the solitary wave is that it is neither preceded nor followed by any free surface disturbance. Neglecting bottom friction and viscosity and far away from the boundaries, a solitary wave travels over a horizontal bottom without changing its shape and velocity. The accuracy of the numerical model can be evaluated by comparing the solution with the theoretical one for an infinitely long channel. Let $A$ be the wave height, $c$ the celerity. The velocity components and the waterlevel are given by:

$$u(x,z,t) = \sqrt{gd} \frac{A}{d} \sec^{2} \left[ \sqrt{\frac{3A}{4d^{3}}} (x-ct) \right]$$
$$w(x,z,t) = \sqrt{3gd} \left( \frac{A}{d} \right)^{2} \frac{z}{d} \sec^{2} \left[ \sqrt{\frac{3A}{4d^{3}}} (x-ct) \right] \tanh \left[ \sqrt{\frac{3A}{4d^{3}}} (x-ct) \right]$$
$$\zeta(x,t) = d + A \sec^{2} \left[ \sqrt{\frac{3A}{4d^{3}}} (x-ct) \right]$$

(3.3)

$$p(x,z,t) = \rho_0 g (\zeta - z)$$
$$c(x,t) = \sqrt{g (d + \zeta)}$$

The solitary wave is prescribed as initial condition and the evolution in the time is computed for a 600 m long and 10 m deep channel using 1 m grid size and 0.1 s time step. The solitary wave is prescribed as initial condition and the evolution in time is computed. The initial crest position is 80 m.
Figure 3.10: Time series for water level at $x=80$ m and $x=200$ m simulated by the new single-layer or depth-averaged non-hydrostatic model.

Figure 3.10 and 3.11 show that the non-hydrostatic model conserves the shape and the amplitude of the solitary wave, see also Weilbeer and Jankowski (2000).

Figure 3.11: Spatial variation of water level, initial and after 20 s and 40 s as simulated by the new single-layer or depth-averaged non-hydrostatic model.
4 Conclusions and recommendations

4.1 Conclusions

- For non-hydrostatic free-surface flows, this report presents a new numerical method that appears to be more accurate than the original Stelling-Casulli method (1998);
- Three test cases, all dedicated to surface waves, demonstrate that even with a few layers already significant non-hydrostatic effects are computed with reasonable accuracy;
- Because of the limited number of layers, the computational effort per time step is moderate;
- The explicit treatment of the hydrostatic pressure in conjunction with the simplicity of the single-step approach is expected to yield computational efforts of the order of magnitude of the hydrostatic version of Delft3D-Flow or TRIWAQ.

4.2 Recommendations

- The extension of the new method from two to three dimensions;
- For wave studies, the inclusion of weakly reflective boundary conditions;
- For wave studies, the treatment of breaking and rolling waves in terms of momentum exchange and turbulence production;
- The implementation of (an) adequate turbulence closure model(s), including turbulence production by bores, breaking waves and orbital motions;
- The extension to density-stratified flows and to internal waves;
- Research towards the optimal way of solving the elliptic system of equations for the hydrodynamic pressure;
- Test cases dedicated to stationary flows over a backward-facing step, steep sill, wavy bed and buoyant plume and (buoyant) jet;
- Study towards a rigid-lid approach for removal of time step limitations imposed by free-surface (barotropic) modes for climatological, oceanographic and lake studies.
References


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