A Framework for Simultaneous Sensor and Actuator Fault-Tolerant Flight Control

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I. Introduction

In civil aviation, many developments focus on improving the safety levels and reducing the risks that critical faults occur [1]. According to [2], loss of control in-flight is one of the three high-risk accident categories. Some recent accidents caused by actuator faults include [3] and [4]. To avoid these accidents, the flight control systems should be reconfigurable in the presence of actuator faults. This motivates the development of actuator Fault-Tolerant Control (FTC) systems.

Apart from actuator faults, recent airliner accidents indicate that sensor faults can result in critical failures [5]. Some recent accidents caused by sensor failures include [6] and [7]. These examples highlight the importance of sensor Fault Detection and Diagnosis (FDD) systems. In the past few decades, many promising sensor FDD approaches have been proposed [8-13].

Most sensor FDD approaches use the aircraft aerodynamic model, which calculates the aerodynamic forces and moments. Furthermore, these FDD approaches are usually designed based on linear time invariant systems [14]. The aerodynamic forces and moments acting on the aircraft can change significantly under different flight conditions. To reduce the influence of nonlinearities during the whole flight, a Linear Parameter Varying (LPV) system, which may explicitly contain information about the aerodynamic coefficient variations over the flight envelope [15], can be designed. However, the modeling of this LPV system can be time-consuming. Alternatively, the aircraft kinematic model [12] [13] [16-18], which does not require the modeling of an LPV system,

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The fault information provided by FDD systems can be used to compensate the influence of sensor faults. To deal with actuator faults, a reconfigurable controller is required [19]. Various reconfigurable controllers are proposed for aerospace applications [8, 19-22]. 23 uses model predictive control which can handle system input and output constraints. Sliding mode control [24] is also applied to FTC due to its fast convergence property [21]. $H_{\infty}$ control approach has also been proposed to design FTC controllers [25]. Additionally, Backstepping (BS) is one of the popular Lyapunov-based control approaches which can be used for designing FTC systems. However, standard BS is sensitive to model uncertainties. To cope with model uncertainties, Adaptive Backstepping [26] and Command Filtering Backstepping [27] were proposed. More recently, Incremental Backstepping (IBS) [28, 29], which can cope with model uncertainties, is proposed.

Although several sensor FDD systems and actuator FTC systems are proposed [19, 30, 31], research on simultaneous sensor and actuator FTC system is limited. One reason is that state-of-the-art sensor FDD systems make use of actuator information such as commanded control surface deflections. It can lead to incorrect sensor FDD when the actuator fails resulting in wrong information from the actuator. Recently, Zhang and Li [32], Marzat et. al [33] and Alwi et. al [14] consider simultaneous sensor and actuator FDD. However, simultaneous sensor and actuator FTC is not dealt with. Furthermore, performance of existing sensor FDD systems is only shown in offline simulation. In this paper, simultaneous sensor and actuator FTC is considered and the performance is shown in online simulation. To the best of the authors’ knowledge, the present paper is one of the first papers which consider simultaneous aircraft sensor and actuator FTC.

The present paper aims at proposing a framework of FTC systems which is able to deal with simultaneous sensor and actuator faults. The FTC system is composed of a sensor FDD system and a reconfigurable controller. The sensor FDD system is designed by making use of an aircraft kinematic model which does not require actuator deflections. The detection and estimation of the sensor faults are resolved using an Adaptive Three-Step Unscented Kalman Filter (ATS-UKF) [13] which can estimate the state and fault in an unbiased sense. The unbiased state estimation is
provided to the reconfigurable controller such that FTC in the presence of sensor faults is achieved. Regarding actuator and process faults, the IBS approach is applied to reconfigure the controller. By doing this, simultaneous sensor and actuator FTC is achieved. It should be noted that [13] only validates sensor FDD in off-line simulation while the validation in this paper is online. In addition, [13] does not consider Aircraft Heading and Reference System (AHRS) faults and actuator faults.

The performance of the proposed FTC system is validated. The control objective is to control the aircraft attitude. An attitude controller and an angular rate controller are designed based on the IBS approach. Command filters are implemented to take the actuator physical limits into consideration. The aircraft sensor faults contain Air Data Sensors (ADS) and AHRS faults. Aileron and rudder faults are considered. The FTC system is applied to address the sensor and actuator FTC simultaneously and maintain controlled flight.

II. Sensor Fault Reconstruction System Design

In this section, first the model used for sensor fault reconstruction is introduced in section II A. Then the detailed sensor fault reconstruction system design is presented in section II B.

A. Nonlinear aircraft kinematic model including ADS and AHRS faults

The model used for sensor fault reconstruction is the aircraft kinematic model. It should be noted that this model does not require any information of the actuator. The process model including
ADS and AHRS faults is presented as follows [13]:

\[ \dot{V} = (A_{xm} - w_{Ax} - g \sin \theta) \cos \alpha \cos \beta + (A_{ym} - w_{Ay} + g \sin \phi \cos \theta) \sin \beta \]

\[ + (A_{zm} - w_{Az} + g \cos \phi \cos \theta) \sin \alpha \cos \beta \]  

(1)

\[ \dot{\alpha} = \frac{1}{V \cos \beta} \left[ -(A_{xm} - w_{Ax}) \sin \alpha + (A_{zm} - w_{Az}) \cos \alpha + g \cos \phi \cos \theta \cos \alpha \right. \]

\[ + g \sin \theta \sin \alpha \right] + q_m - w_q - \left[ (p_m - w_p) \cos \alpha + (r_m - w_r) \sin \alpha \right] \tan \beta \]

(2)

\[ \dot{\beta} = \frac{1}{V} \left[ -(A_{xm} - w_{Ax} - g \sin \theta) \cos \alpha \sin \beta + (A_{ym} - w_{Ay} + g \sin \phi \cos \theta) \cos \beta \right. \]

\[ - (A_{zm} - w_{Az} + g \cos \phi \cos \theta) \sin \alpha \sin \beta \right] + (p_m - w_p) \sin \alpha - (r_m - w_r) \cos \alpha \]

(3)

\[ \dot{\phi} = (p_m - w_p) + (q_m - w_q) \sin \phi \tan \theta + (r_m - w_r) \cos \phi \tan \theta \]

(4)

\[ \dot{\theta} = (q_m - w_q) \cos \phi - (r_m - w_r) \sin \phi \]

(5)

\[ \dot{\psi} = (q_m - w_q) \frac{\sin \phi}{\cos \theta} + (r_m - w_r) \frac{\cos \phi}{\cos \theta} \]

(6)

Define:

\[ x = [V \alpha \beta \phi \theta \psi]^T, \quad w = [w_{Ax} w_{Ay} w_{Ax} w_{Ay} w_{Az} w_{Az} w_{Ap} w_{Ap} w_{Ap}]^T \]  

(7)

\[ u_m = [A_{xm} A_{ym} A_{zm} p_m q_m r_m]^T = [A_x A_y A_z p q r]^T + w \]  

(8)

where \( x \in \mathbb{R}^n \) is the state vector, \( u_m \in \mathbb{R}^l \) is the measured input vector. \( V \) is the true airspeed, \( \alpha \) is the angle of attack, \( \beta \) is the angle of sideslip. \( \phi, \theta \) and \( \psi \) are the roll angle, pitch angle and yaw angle, respectively. The input vector, denoted as \( u_m \), is the measurement from Inertial Measurement Unit (IMU) sensors. The subscript “m” indicates a measured quantity. \( A_x, A_y, A_z \) are the specific forces, \( p, q, r \) are the roll rate, pitch rate and yaw rate, respectively. \( w \) is the noise in the IMU sensor with covariance matrix defined as:

\[ E[w(t)w^T(t_\tau)] = Q\delta(t - \tau). \]  

(9)

The above process model can be rewritten into the following vector form:

\[ \dot{x}(t) = f(x(t), u_m(t), t) + G(x(t))w(t) \]  

(10)

Further define the following:

\[ y_m = [V_m \alpha_m \beta_m \phi_m \theta_m \psi_m]^T, \quad v = [v_V v_\alpha v_\beta v_\phi v_\theta v_\psi]^T, \quad f = [f_V f_\alpha f_\beta f_\phi f_\theta f_\psi]^T \]  

(11)
where $y_m \in \mathbb{R}^m$ is the measurement vector, $f \in \mathbb{R}^p$ is the fault vector. $[f_V f_0 f_\phi]^T$ denotes the faults in the ADSs and $[f_\theta f_\phi f_\psi]^T$ denotes the faults in the AHRS. $v$ is the output noise vector with

$$E[v(t_i)v^T(t_j)] = R \delta(t_i - t_j). \tag{12}$$

The measurement model including the ADS and AHRS faults is:

$$y_m(t) = h(x(t)) + F(t)f(t) + v(t) \tag{13}$$

$$= H(t)x(t) + F(t)f(t) + v(t), \quad t = t_i, \ i = 1, 2, ... \tag{14}$$

where $H = F = I_{6 \times 6}$.

### B. Design of the sensor fault reconstruction system

To design the ATS-UKF [13], first the condition for fault reconstruction is checked. In this study, $m = 6$, $p = 6$ and rank $F_k = 6$. According to [13, 34], the condition is satisfied and an ATS-UKF can be designed. The ATS-UKF, proposed in [13], is generalized into the following three steps to estimate the states and faults. In [13], only ADS faults are considered while in this paper, both ADS and AHRS faults are considered.

**Step1** Time update

$$X_{i,k|k-1} = X_{i,k-1} + \int_{k-1}^{k} \tilde{f}(X_{i,k-1}, u(t), t)dt \tag{15}$$

$$\hat{x}_{k|k-1} = \sum_{i=0}^{2n} w_i^{(m)} X_{i,k|k-1} \tag{16}$$

$$P_{k|k-1} = \sum_{i=0}^{2n} w_i^{(c)} [X_{i,k|k-1} - \hat{x}_{k|k-1}][X_{i,k|k-1} - \hat{x}_{k|k-1}]^T + Q_d \tag{17}$$

$$Y_{i,k|k-1} = h(X_{i,k|k-1}) \tag{18}$$

$$\hat{y}_k = \sum_{i=0}^{2n} w_i^{(m)} Y_{i,k|k-1} \tag{19}$$

$$P_{xy,k} = \sum_{i=0}^{2n} w_i^{(c)} [X_{i,k|k-1} - \hat{x}_{k|k-1}][Y_{i,k|k-1} - \hat{y}_k]^T \tag{20}$$

$$P_{yy,k} = \sum_{i=0}^{2n} w_i^{(c)} [Y_{i,k|k-1} - \hat{y}_k][Y_{i,k|k-1} - \hat{y}_k]^T + R \tag{21}$$
where $X_{i,k-1}$ is the sigma point vector which is generated using the state estimation and covariance of estimation error at time step $k-1$ denoted as $\hat{x}_{k-1|k-1}$ and $P_{k-1|k-1}$ respectively [33]. $w_i^{(m)}$ and $w_i^{(e)}$ are the weights which can be found in [13, 36]. $\hat{y}_k$ is the estimate of the measurements. $P_{xy,k}$ and $P_{yy,k}$ are covariance matrices. $Q_d$ is approximated by $G(\hat{x}_{k|k-1})Q_G(\hat{x}_{k|k-1})^T \Delta t$ and $\Delta t$ is the time step.

**Step2** Detection, isolation and estimation of the faults

Define $C_k$ as follows:

$$C_k := \frac{1}{N} \sum_{j=k-N+1}^{k} \gamma_j \gamma_j^T$$  \hspace{1cm} (22)

where $\gamma_j = (y_j - \hat{y}_j)$, denotes the innovation at time step $j$. In this paper, $N = 10$.

Let $C_{ii,k}$, $i = 1, 2, ..., m$ denote the $i$th diagonal elements of $C_k$ at time step $k$. The fault detection and isolation is performed through the following:

If $C_{ii,k} > T_i$, $F_{Ai} = 1$. otherwise $F_{Ai} = 0$, $i = 1, 2, ..., m$.

where $F_A = [F_{Av} F_{Aa} F_{Ab} F_{Aa} F_{Ae}]^T$ is the vector of binary alarm indicators. $T_i$ is the vector of thresholds which are designed to detect the faults in the ADS and AHRS respectively. These thresholds are designed based on fault-free cases.

To cope with initial condition errors, we need to calculate the change of the innovation covariance $\Delta C_{ii,k}$ as follow [13]:

$$\Delta C_{ii,k} := C_{ii,k} - C_{ii,k-1}, \ i = 1, 2, ..., m.$$  \hspace{1cm} (23)

When the following inequality holds, the initial measurement update can be regarded as sufficient. The inequality is

$$\Delta C_{ii,k} < \eta_i, \ i = 1, 2, ..., m.$$  \hspace{1cm} (24)

where $\eta_i$, $i = 1, 2, ..., m$ are pre-defined constants which can be tuned to stop the initial measurement update. It should be noted that this initial measurement update is only performed in the first few time steps denoted by $k^*$. In this paper, $k^* = 20$.

If either of the following two conditions is satisfied:
(a) There are no faults detected (i.e., $\|F_A\| = 0$)

(b) $k < k^*$ and $\Delta C_{i,k} > \eta_i$ for all $i = 1, 2, ..., m$

the estimates of the fault and its error covariance matrix are:

$$\hat{f}_k = 0, \quad P^f_k = 0$$  \hspace{1cm} (25)

If neither of conditions (a) and (b) is satisfied, the estimates of the fault and its error covariance matrix are calculated as follows:

$$N_k = (F_k^T P_{yy,k}^{-1} F_k)^{-1} F_k^T P_{yy,k}^{-1}$$  \hspace{1cm} (26)

$$\hat{f}_k = N_k \gamma_k, \quad P^f_k = (F_k^T P_{yy,k}^{-1} F_k)^{-1}$$  \hspace{1cm} (27)

where $\gamma_k$ is the innovation at time step $k$, $\hat{f}_k$ is the estimation of $f_k$ and $P^f_k$ is its error covariance matrix. $N_k$ is the gain matrix which can achieve an unbiased estimation of $f_k$.

**Step3** Measurement update

$$K_k = P_{xy,k} P_{yy,k}^{-1}$$  \hspace{1cm} (28)

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k (y_k - \hat{y}_k - F_k \hat{f}_k)$$  \hspace{1cm} (29)

$$P_{k|k} = P_{k|k-1} - K_k (P_{yy,k} - F_k P^f_k F_k^T) K_k^T$$  \hspace{1cm} (30)

By substituting the functions $\hat{f}$ and $h$ given in Eqs. (10) and (13) into Eqs. (15) and (18) respectively, the fault estimation and state estimation can be achieved which are given in Eqs. (27) (or (25)) and (29) respectively. The state and fault estimation results are used by the controller which is designed in the following section.

**III. Reconfigurable Control: Incremental Backstepping**

This section will introduce the reconfigurable control approach for the actuator FTC. First, in section III A the dynamics of the aircraft attitude and angular rates are presented. Then, the design of the IBS controller for actuator FTC is presented in section III B.
A. Aircraft attitude and angular rate dynamics

In this section, the attitude dynamics and angular rate dynamics of the aircraft are presented, which is the model used for designing the IBS controller. The kinematics of the Euler angles are:

\[
\begin{bmatrix}
\dot{\phi} \\
\dot{\theta}
\end{bmatrix} = \begin{bmatrix}
1 & \sin \phi \tan \theta & \cos \phi \tan \theta \\
0 & \cos \phi & -\sin \phi
\end{bmatrix} \omega
\]  
(31)

with \( \omega = [p \ q \ r]^T \) the rotational rates of the aircraft in the body reference frame. Furthermore, the sideslip angle \( \beta \) has to be kept at zero. The dynamics of the sideslip angle are given in Eq. (3).

Eqs. (31) and (3) can be rewritten into:

\[
\dot{x}_1 = f_1(x_1) + g_1(x_1)x_2
\]  
(32)

where

\[
x_1 = [\phi, \theta, \beta]^T, x_2 = \omega,
\]

\[
f_1 = \begin{bmatrix}
0 \\
0 \\
\frac{1}{V} \left[ -(A_x - g \sin \theta) \cos \alpha \sin \beta + (A_y + g \sin \phi \cos \theta) \cos \beta - (A_z + g \cos \phi \cos \theta) \sin \alpha \sin \beta \right]
\end{bmatrix}
\]

\[
g_1 = \begin{bmatrix}
1 & \sin \phi \tan \theta & \cos \phi \tan \theta \\
0 & \cos \phi & -\sin \phi \\
\sin \alpha & 0 & -\cos \alpha
\end{bmatrix}
\]

(33)

The angular rate dynamics of the aircraft are:

\[
\dot{\omega} = J^{-1}(M - \omega \times J\omega)
\]  
(34)

where \( J \) is the inertia tensor. \( M \) are the moments acting on the aircraft which can be described as:

\[
M = M_a + M_u = M_a + C_{Ma}u
\]  
(35)

where \( M_a \) are the moments except for the moments generated by the control surface deflections. \( M_u \) are the moments generated by the control surface deflections. \( C_{Ma} \) are the coefficients related
to $M_a$, which are denoted as

$$M_a = qSC_1 \begin{bmatrix} C_l(\beta, p, r, M_a) \\ C_m(\alpha, \dot{\alpha}, q, M_a) \\ C_n(\beta, p, r, M_a) \end{bmatrix}, \quad C_M = qSC_1 \begin{bmatrix} C_{l_{sa}} & 0 & C_{l_{sr}} \\ 0 & C_{m_{sa}} & 0 \\ C_{n_{sa}} & 0 & C_{n_{sr}} \end{bmatrix}, \quad C_1 = \text{diag}(b, \bar{c}, b)$$ (36)

where $\bar{q}$ is the dynamic pressure, $M_a$ is the Mach number, $S$ is the wing area, $b$ the wing span and $\bar{c}$ is the mean aerodynamic chord. The control surface deflections are $u = [\delta_a, \delta_e, \delta_r]^T$, which are also the input to the system.

Eq. (34) can be rewritten into the following affine-in-control form:

$$\dot{x}_2 = f_2(x_1, x_2) + g_2 u$$ (37)

where $f_2 = J^{-1}(M_a - \omega \times J\omega)$, $g_2 = J^{-1}C_{M_a}$.

### B. Incremental Backstepping Controller Design

In order to design the IBS controller, we need to rewrite Eq. (37) into an incremental form. Denote the actuator deflections in the previous time step as $u_0$ and the incremental deflections in the current step as $\Delta u$, then the actuator deflections in the current step are:

$$u = u_0 + \Delta u$$ (38)

where $u_0 = [\delta_{a0}, \delta_{e0}, \delta_{r0}]^T$ and $\Delta u = [\Delta \delta_a, \Delta \delta_e, \Delta \delta_r]^T$. $u_0$ can be obtained using the actuator model [28]. According to [37], Eq. (37) can be rewritten into the following:

$$\dot{x}_2 = \dot{x}_{2,0} + g_2 \Delta u$$ (39)

where $\dot{x}_{2,0}$, the derivative of the angular rates in the previous time step, is defined as:

$$\dot{x}_{2,0} := f_2(x_{10}, x_{20}) + g_2 u_0$$ (40)

$\dot{x}_{2,0}$ can be computed by passing $\omega_m ([p_m, q_m, r_m]^T)$ through the following filter:

$$\frac{s^2 \omega_m^2}{s^2 + 2\zeta_n \omega_n s + \omega_n^2}$$ (41)

with $\zeta_n = 0.8$, $\omega_n = 25$ rad/s chosen by trial and error to reduce the influence of noise [37].
Now the complete model used for the controller design can be given as follows:

\[
\begin{align*}
\dot{x}_1 &= f_1(x_1) + g_1(x_1)x_2 \\
\dot{x}_2 &= \dot{x}_{2,0} + g_2\Delta u 
\end{align*}
\]  
(42)  
(43)

Now the design of the controller based on the model Eqs. (42) and (43) can be presented. The control task is to steer \(x_1\) towards a given reference \(y_r = [\phi^{ref}, \theta^{ref}, \beta^{ref}]^T\) with bounded and known derivatives (\(\dot{y}_r, \ddot{y}_r, \ldots\)). Define the following tracking errors:

\[
\begin{align*}
z_1 &= x_1 - y_r \\
z_2 &= x_2 - \alpha_1
\end{align*}
\]  
(44)  
(45)

where \(\alpha_1\) is the virtual control for the state \(x_2\). The derivatives of Eqs. (44) and (45) are:

\[
\begin{align*}
\dot{z}_1 &= \dot{x}_1 - \dot{y}_r \\
\dot{z}_2 &= \dot{x}_2 - \dot{\alpha}_1
\end{align*}
\]  
(46)  
(47)

Substituting Eqs. (42) and (43) into Eqs. (46) and (47), it follows that:

\[
\begin{align*}
\dot{z}_1 &= f_1(x_1) + g_1(x_1)x_2 - \dot{y}_r \\
\dot{z}_2 &= \dot{x}_{2,0} + g_2\Delta u - \dot{\alpha}_1
\end{align*}
\]  
(48)  
(49)

Define \(\bar{z}_1 = z_1 - \chi_1\) and \(\bar{z}_2 = z_2 - \chi_2\). By using the control Lyapunov function \(V = \frac{1}{2}\bar{z}_1^T \bar{z}_1 + \frac{1}{2}\bar{z}_2^T \bar{z}_2\), the desired control laws are defined as:

\[
\begin{align*}
\alpha_1 &= g_1^{-1}(-c_1z_1 - f_1(x_1) + y_r) \\
\Delta u^{des,0} &= g_2^{-1}(-g_1(x_1)\bar{z}_1 - c_2z_2 + \dot{x}^{des}_2 - \dot{x}_{2,0})
\end{align*}
\]  
(50)  
(51)

where \(c_1\) and \(c_2\) are the controller gains. \(\dot{x}^{des}_2\) is the filtered derivative of \(x^{des,0}_2\) which is defined as:

\[
x^{des,0}_2 = [\dot{\phi}^{des,0}, \dot{\theta}^{des,0}, \dot{\beta}^{des,0}]^T = \alpha_1 - \chi_2
\]  
(52)

\(\chi_1\) and \(\chi_2\) are computed as:

\[
\begin{align*}
\dot{\chi}_1 &= -c_1\chi_1 + g_1(x^{des}_2 - x^{des,0}_2) \\
\dot{\chi}_2 &= -c_2\chi_2 + g_2(\Delta u^{des} - \Delta u^{des,0})
\end{align*}
\]  
(53)  
(54)
Define the following variables:

\[
x_{2}^{\text{des},0} = [p_{\text{des},0}^{\text{des}}, q_{\text{des},0}^{\text{des}}, r_{\text{des},0}^{\text{des}}]^T, \quad \dot{x}_{2}^{\text{des}} = [p_{\text{des}}^{\text{des}}, q_{\text{des}}^{\text{des}}, r_{\text{des}}^{\text{des}}]^T
\]

\[
\Delta u_{\text{des},0}^{\text{des}} = [\Delta \delta_{a}^{\text{des},0}, \Delta \delta_{e}^{\text{des},0}, \Delta \delta_{r}^{\text{des},0}]^T, \quad \Delta u_{\text{des}}^{\text{des}} = [\Delta \delta_{a}^{\text{des}}, \Delta \delta_{e}^{\text{des}}, \Delta \delta_{r}^{\text{des}}]^T
\]

\(x_{2}^{\text{des},0}\) is filtered through second-order command filters \([27, 38]\) to compute \(x_{2}^{\text{des}}\) and \(\dot{x}_{2}^{\text{des}}\). \(\Delta u_{\text{des},0}^{\text{des}}\) is filtered through second-order command filters to compute \(\Delta u_{\text{des}}^{\text{des}}\). Take \(x_{2}^{\text{des},0}\) for example, the command filter is as follows \([27, 38]\):

\[
\begin{bmatrix}
\dot{q}_1 \\
\dot{q}_2
\end{bmatrix} =
\begin{bmatrix}
q_2 \\
2\zeta_c\omega_c [S_R(\frac{\omega^2}{2\omega_c}) [S_P(x_{2}^{\text{des},0}) - q_1]) - q_2]
\end{bmatrix}
\]

where \(S_P\) and \(S_R\) are position and rate limit functions \([27, 38]\): Then \(x_{2}^{\text{des}} = q_1\) and \(\dot{x}_{2}^{\text{des}} = q_2\).

The final desired control input \(u_{\text{des}}^{\text{des}}\) is given by:

\[
u_{\text{des}}^{\text{des}} = [\delta_{a}^{\text{des}}, \delta_{e}^{\text{des}}, \delta_{r}^{\text{des}}]^T = u_0 + \Delta u_{\text{des}}^{\text{des}}
\]

This concludes the design for the IBS for the feedback system. The IBS approach is robust to the uncertainties in the plant dynamics term since it does not require the information of the plant dynamics. The robustness with respect to the uncertainties in the control effectiveness as well as actuator faults is analyzed in \([29]\).

IV. Fault-Tolerant Control system for dealing with simultaneous sensor and actuator faults

The proposed FTC system for dealing with simultaneous sensor and actuator faults is presented in Fig. 1. As can be seen from the figure, there are two control loops: attitude control loop and rate control loop. The attitude controller follows the commands \(\phi^{\text{ref}}, \theta^{\text{ref}}, \text{ and } \beta^{\text{ref}}\) and is designed based on the BS control law. The attitude control loop generates the reference commands \(p_{\text{des}}^{\text{des}}, q_{\text{des}}^{\text{des}}, \text{ and } r_{\text{des}}^{\text{des}}\) for the angular rate control loop. The rate control loop is designed based on the IBS approach and it generates the command for the control surface deflections denoted as \(\delta_{a}^{\text{des}}, \delta_{e}^{\text{des}}\) and \(\delta_{r}^{\text{des}}\). The actual control surface deflections are denoted as \(\delta_{a}, \delta_{e}\) and \(\delta_{r}\).

The key point of this simultaneous sensor and actuator FTC system is that the sensor FDD makes use of the kinematic model rather than the dynamic model which uses the actuator deflections as the input. In Fig. 1 it is seen that the sensor FDD system does not require any information.
Fig. 1: Block diagram of the FTC system in the presence of sensor and actuator faults

about the actuators such as commanded actuator deflections. In most aircraft sensor FDD system
designs, sensor FDD is achieved using the commanded or measured control surface deflections. In
some cases, measurements of control surfaces are not available and are derived from the actuator rod
positions [39]. Wrong actuator deflections can be obtained when the control surface is disconnected
from the actuator rod [39]. Consequently, it can lead to performance degradation of the sensor FDD
system.

Reconfigurable controllers can deal with actuator faults. However, it is difficult for these con-
trollers to deal with sensor faults. Therefore, the sensor FDD plays a critical role in this FTC
system. The sensor FDD system not only provides unbiased fault estimation (\(\hat{f}_V, \hat{f}_\alpha, \hat{f}_\beta,\) and \(\hat{f}_\phi,\)
\(\hat{f}_\theta, \hat{f}_\psi\)) but also unbiased state estimation (\(\hat{\dot{V}}, \hat{\dot{\alpha}}, \hat{\dot{\beta}},\) and \(\hat{\dot{\phi}}, \hat{\dot{\theta}}, \hat{\dot{\psi}}\)) such that the performance of the
actuator FTC system is not influenced by sensor faults.

V. Simulation examples

In this section, the proposed FTC system against sensor and actuator faults will be validated. In
Section V A the aircraft model and the fault scenarios are presented. The design parameters used in
the simulation are presented in Section V B. The performance of the sensor FDD system compared
The performance of the FTC system in the presence of simultaneous sensor and actuator faults is shown in Section V.D. Some discussions are presented in Section V.E.

A. Aircraft model and fault scenario

The aircraft model used in this paper is the citation model [22]. Due to the page limit, the interested readers can refer to [22] for a more detailed introduction of the actuator. The actuators are modeled as first-order low-pass filters. The control surfaces are left aileron, right aileron, upper rudder, lower rudder and elevator and the deflections are denoted as $\delta_{al}$, $\delta_{ar}$, $\delta_{ru}$, $\delta_{rl}$ and $\delta_e$ respectively. The desired commands of the ailerons and rudders are as follows:

$$\delta_{des}^{al} = \delta_{des}^{ar} = \delta_{des}^a$$
$$\delta_{des}^{ru} = \delta_{des}^{rl} = \delta_{des}^r$$

This paper considers both sensor faults and actuator faults. The sensor measurements contain noise. The noise covariances of the sensors in the aircraft can be found in [13]. Sensor faults include ADS sensor faults and AHRS faults. All the sensor faults occur simultaneously. The fault scenario of the ADS and AHRS faults is given by dashed lines in Fig. 3. The actuator faults considered in this paper are jamming faults. The specific fault scenario is given in Table 1.

Table 1: Actuator faults

<table>
<thead>
<tr>
<th>Time interval</th>
<th>Actuator</th>
<th>Fault type</th>
<th>Stuck position</th>
<th>Fault unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t &gt; 25 s$</td>
<td>left aileron</td>
<td>Jamming</td>
<td>0.56</td>
<td>[rad]</td>
</tr>
<tr>
<td>$t &gt; 75 s$</td>
<td>upper rudder</td>
<td>Jamming</td>
<td>0.2</td>
<td>[rad]</td>
</tr>
</tbody>
</table>

It can be seen that during $25 s < t < 40 s$, the left aileron and the ADS and the AHRS fail. During $75 s < t < 80 s$, the left aileron, the upper rudder and the ADS and AHRS all fail. Therefore, simultaneous sensor and actuator faults are considered in this paper.
B. FTC system design parameters

In this section, the parameters used for the design of the FTC system is presented. For the sensor FDD, \( \eta \), which is used in Eq. (24) to terminate the initial measurement update, is chosen as:

\[
\eta = [2 \times 10^{-2}, 1 \times 10^{-4}, 1 \times 10^{-5}, 1 \times 10^{-5}, 1 \times 10^{-5}]^T \tag{61}
\]

The threshold \( T \) used for fault detection is chosen as:

\[
T = [8 \times 10^{-1}, 1 \times 10^{-4}, 1 \times 10^{-4}, 5 \times 10^{-4}, 1 \times 10^{-4}, 2 \times 10^{-4}]^T \tag{62}
\]

For the IBS controller, the control gains are chosen as:

\[
c_1 = \text{diag}(5, 5, 5), \quad c_2 = \text{diag}(2, 2, 2) \tag{63}
\]

C. Validation of the FTC system in the presence of sensor faults

In this section, the performance of the FTC system in the presence of sensor faults is validated. It is seen from the dashed lines in Fig. 3 that during \( 20 \text{ s} < t < 40 \text{ s} \) and \( 60 \text{ s} < t < 80 \text{ s} \), all the ADS and AHRS fail simultaneously. The sensor FDD system of the FTC system will be used to reconstruct the sensor faults in order to provide unbiased state estimation for the feedback.

For comparison, the IBS controller without the sensor FDD system is also applied. For this controller, the measurements are directly used by the controller. In this case, the state feedback used by the controller is biased due to the sensor faults.

The results of the IBS controller with and without using the sensor FDD system are given in Figs. 2(a) and 2(b), respectively. As can be seen from Fig. 2(a), even in the presence of sensor faults, \( \phi, \theta \) and \( \beta \) of the aircraft can still follow the reference commands well. In contrast, when the sensor FDD is not used, \( \phi, \theta \) and \( \beta \) of the aircraft can not follow the reference commands when there are sensor faults, as shown in Fig. 2(b). The response of the aircraft is obviously influenced by the wrong sensor measurements. For example, during \( 20 \text{ s} < t < 30 \text{ s} \), the reference command for \( \theta \) is zero. But the aircraft pitches down to -10 deg. This is caused by the \( \theta \) sensor fault, which is a bias fault. Furthermore, during \( 60 \text{ s} < t < 80 \text{ s} \), \( \beta \) reaches 5 deg even when there are no lateral maneuvers. This is not desirable for civil aircraft. The results demonstrate the importance of a sensor FDD system.
The state estimation performance of the sensor FDD system is shown in Fig. 2(a). It is seen that the solid lines can still follow the dashed lines closely. This demonstrates that the sensor FDD system can provide unbiased state estimation even in the presence of sensor faults.

The actuator control surface deflections $\delta_{ar}$, $\delta_e$ and $\delta_{rl}$ of the FTC system with the sensor FDD
Fig. 3: Fault reconstruction of the sensor FDD system in the presence of sensor faults

system are given in Fig. 2(c). It is seen that all the control surface deflections are within the actuator position limits. The control surface deflections $\delta_{ar}$, $\delta_e$ and $\delta_{rl}$ of the FTC system without using the sensor FDD system is shown in Fig. 2(d). As can be seen from the figure, the control surfaces oscillate in the presence of sensor faults. During $70 \, s < t < 80 \, s$, the elevator oscillates frequently, which can even damage the elevator itself.

For the FTC system with sensor FDD, sensor FTC is achieved. The fault reconstruction results using the sensor FDD system are given in Figs. 3(a) and 3(b). It is seen from the figure, although the fault types are different, all faults are reconstructed in an unbiased sense. The oscillatory frequency of the oscillatory faults are also different. This demonstrates the performance of the sensor FDD system which makes use of the ATS-UKF.

D. Validation of the FTC system in the presence of simultaneous sensor and actuator faults

In this section, the performance of the FTC system in the presence of simultaneous sensor and actuator faults is validated. The actuator faults are given in Table 1. The sensor faults are coped with by the sensor FDD system. The actuator faults are dealt with by the IBS approach. The results of the FTC system are given in Fig. 4.
Fig. 4: Results of the FTC system in the presence of simultaneous sensor and actuator faults

(a) Estimated, true and commanded $\phi$, $\theta$ and $\beta$ in the presence of simultaneous sensor and actuator faults. The arrows denote the occurrence of the faults.

(b) Estimated and true $V$, $\alpha$ and $\psi$ in the presence of simultaneous sensor and actuator faults

(c) Incremental actuator deflections in the presence of simultaneous sensor and actuator faults

(d) Actuator deflections in the presence of simultaneous sensor and actuator faults. Dashed lines denote position limits. Arrows denote the occurrence of the faults.

The response of $\phi$, $\theta$ and $\beta$ of the aircraft using the FTC system is shown in Fig. 4(a). As can be seen from the figure, when the actuators are stuck, $\phi$ and $\beta$ will be influenced and deviate from zero. However, $\phi$ and $\beta$ are controlled back to zero thanks to the IBS controller.

The state estimation performance remains satisfactory. Estimated and true $\phi$, $\theta$ and $\beta$ are shown in Fig. 4(a) while those of $V$, $\alpha$ and $\psi$ are shown in Fig. 4(b). It can be seen that the state
estimation is unbiased despite the sensor and actuator faults.

The incremental control surface deflections generated by the IBS controller are shown in Fig. 4(c). The actual control surface deflections $\delta_{ar}, \delta_{e}$ and $\delta_{rl}$ are shown in Fig. 4(d). It is seen from the figure that the control surface deflections change after the occurrence of the faults. Take the ailerons for example, after the left aileron is stuck at a positive position at $t = 25$ s, the IBS controller takes effect. The desired command $\delta_{a}^{des}$ generated by the IBS decreases immediately. Then the right aileron deflects to a negative position. By doing this, the moments generated by the stuck left aileron is counteracted by the right aileron which deflects to a negative position.

It is also obvious that the remaining control authority will decrease due to the stuck aileron. Since the left aileron is stuck at a positive position, it generates a negative rolling moment all the time. Therefore, the control authority of rolling to the right is decreased. Therefore, extreme maneuvers should be performed with caution after the actuator faults.

The fault reconstruction using the sensor FDD system is still satisfactory even in the presence of simultaneous sensor and actuator faults. They are the same as those in Figs. 3(a) and 3(b). Therefore, they are not shown again.

E. Discussion

Through the validations in the previous sections, the following conclusions can be made:

1. The sensor FDD system is critical to maintain the performance in the presence of sensor faults. In Section VC the reconfigurable controller (IBS) has no ability to recover the control in the presence of sensor faults if the sensor FDD is not included. This is expected since the controller relies on the measurements to calculate the error between the reference command and the controlled states. The control objective is to minimize the tracking errors such as those in Eq. (44). Let $z_{\theta}$ denote the tracking error of $\theta$ and assume the wrong measurement is directly used by the controller, then

$$z_{\theta} = \theta_{m} - \theta^{ref} = \theta + f_{\theta} + v_{\theta} - \theta^{ref}$$  \hspace{2cm} (64)

If $z_{\theta}$ is minimized to zero by the controller, it can be readily inferred that the true state $\theta$ deviates from the command $\theta^{ref}$ by $f_{\theta} + v_{\theta}$.
2. Simultaneous sensor and actuator FTC can be achieved by decoupling the sensor FTC from the actuator information. Through the validation in Section VD simultaneous sensor and actuator FTC is achieved by the sensor FDD system and the reconfigurable controller. Since the sensor FDD system does not require the actuator information, it can perform state and fault estimation without being influenced by the actuator faults. The sensor FDD system provides unbiased state and fault estimation such that the reconfigurable controller can perform the reconfiguration without being influenced by sensor faults.

3. Control authority could be reduced after the actuator faults depending on the actuator redundancy. A control allocation technique can be useful for aircraft with actuator redundancy to perform FTC in the presence of actuator faults.

VI. Conclusions

This paper proposes an aircraft Fault-Tolerant Control (FTC) system, which can maintain controlled flight in the presence of simultaneous sensor and actuator faults. The proposed system consists of a sensor Fault Detection and Diagnosis (FDD) system and a reconfigurable controller. The sensor FDD system makes use of the aircraft kinematic model and does not require any information about the actuator deflections. The Adaptive Three-Step Unscented Kalman Filter (ATS-UKF) is used to reconstruct the state and fault in an unbiased sense. The state estimation is used by the controller to achieve sensor FTC. The actuator faults are dealt with by the reconfigurable controller which is the Incremental Backstepping (IBS) in this paper.

The proposed FTC system is validated by simulation examples. The simulated aircraft contains Air Data Sensors (ADS) and Aircraft Heading and Reference System (AHRS) faults as well as stuck actuator faults. The simulation results demonstrate that the proposed FTC system is able to deal with sensor faults as well as actuator faults and maintain the controlled flight.

The simulation results show that sensor and actuator FTC can be achieved simultaneously using model-based analytical redundancy. This FTC system has a potential to be applied to the aircraft to enhance the safety of the aircraft. For future work, the stability of the whole FTC system is worth investigating.
References


