Monolithic scintillators and SiPMs in time-of-flight PET detectors

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Chapter 1

Introduction

A general overview is presented of the most important features of positron emission tomography (PET) imaging, the basic physics involved, and the various parameters that contribute to the image quality (sections 1.1 and 1.2). Since this work is focusing on detector technology, a full section (1.3) is dedicated to the different components and operating principles of PET detectors. This provides a background for the main chapters of this thesis, of which the goal and the outline are given in sections 1.4 and 1.5, respectively.

1.1 Medical imaging

Medical imaging is the process of producing visual representations of the anatomy and/or function of (parts of) the human body for use in clinical diagnosis and/or monitoring of diseases, often in a non-invasive way. Many different imaging modalities are available, all revealing different features. Therefore, for a specific diagnostic or therapeutic task, a careful choice of the modality to be used should be made.

The different imaging modalities available today can roughly be subdivided into two categories. One group of methods mainly provides structural and/or anatomical information of organs and tissues, but is less suitable for imaging of biological processes. Examples of such modalities are X-ray imaging, magnetic resonance imaging (MRI), and ultrasound imaging [1],[2]. A second group of medical imaging modalities primarily provides functional and/or molecular information of tissues, e.g. the density of a certain type of cells, proteins, etc. This group includes e.g. functional MRI (fMRI), single photon emission computed tomography (SPECT), and positron emission tomography (PET) [1]–[5].

The principal difference between these two categories is that the first group is mainly based on measuring differences in the interaction of acoustic, electromagnetic, or ionizing radiation applied from an external device with the different structures within the human body, while the second group relies mainly on the mapping of tracer concentrations, i.e. chemically labeled molecules that specifically target a biological or physiological process of interest, such as blood flow, metabolic processes, or receptor density, without significantly disturbing
this process. If the detectable property of these tracers is radioactivity, the molecules are referred to as radiotracers and the imaging technique is referred to as radiomolecular or nuclear imaging.

Of most medical imaging modalities dedicated versions for small animal imaging exist for the *in vivo* imaging of rodents (e.g. mice and rats) [6]. These techniques, also referred to as preclinical imaging, are used for research purposes such as drug development and the investigation of disease mechanisms and interventions. Although the principles of preclinical devices are similar to medical imaging systems, the performance requirements on the instrumentation are usually quite different. For example, much higher image resolution is usually needed due to the smaller dimensions of the rodents. On the other hand, the restrictions on radiation dose and required field-of-view may be less strong. However, this thesis focuses on PET instrumentation technology aimed at medical imaging.

1.1.1 Anatomical and structural imaging modalities

X-ray imaging is a term used to indicate a collection of methods based on the mapping of the attenuation of X-rays in the human body and includes e.g. planar radiography (X-ray photo) and X-ray computed tomography (CT) [1],[2]. In radiography, two-dimensional (2D) images of the attenuation (correlated to the electron density) of X-rays transmitted through the body are acquired. Due to the significantly higher attenuation of skeletal tissues, resulting from the relatively large effective atomic number $Z_{eff}$ and mass density of these calcium-containing structures, one of the main applications of radiography is the imaging of bone fractures (see figure 1.1a).

CT is based on the production of images of cross-sections of the body, commonly referred to as slices. This is accomplished by rotating an X-ray source and a detector around the patient and acquiring transmission images (projections) at many different angles [1],[2]. From these projections, the slices are obtained by tomographic reconstruction algorithms, for example by filtered back-projection, or, if longer computation times can be accepted (in order to obtain better image quality), by means of iterative reconstruction algorithms. Figure 1.2b shows an example of a cross-sectional image obtained by CT. In case that a series (stack) of 2D slices are recorded around a common axis of rotation, three-dimensional (3D) images of the anatomy of a patient can be directly reconstructed. Nowadays, such high resolution 3D CT images can be acquired within seconds. Nevertheless, imaging modalities based on X-rays always require that a tradeoff has to be made between an acceptable radiation dose and the achievable image quality. Additionally, compared to MRI, X-ray
1.1. Medical imaging

MRI is based on the measurement of proton spin density and relaxation times, which are determined by the distribution of hydrogen atoms and the magnetic susceptibility of tissues in the body [1],[2]. High resolution as well as high contrast, especially for soft tissues such as those of the brain, can be achieved without using ionizing irradiation (see figure 1.1b). However, drawbacks are the low sensitivity to molecular targets including tracers (needed for most functional imaging tasks), and the problems associated with the presence of high magnetic fields, e.g. in case of pacemakers or prostheses.

In ultrasound imaging, high-frequency acoustic pulses are sent into the body, which reflect at discontinuities such as the boundaries between different tissues [1],[2]. By measuring the time delay and the intensity of the reflected pulses, an image showing the tissue interfaces can be reconstructed. Advantages of this method are that there is no radioactive dose involved for the patient, the imaging is fast, often a good resolution can be obtained, and the instruments are relatively cost effective. Yet, physical contact is required between the transducer (signal emitter and receiver) and the patient, interference from the skeleton has to be dealt with, and making a whole-body scan is difficult. A well-known application is the monitoring of the unborn fetus (see figure 1.1c).

1.1.2 Molecular and functional imaging modalities

MRI can be used to obtain some functional information in addition to the structural information [7]. In this approach, commonly referred to as functional MRI (fMRI), hemoglobin can be used as a paramagnetic tracer, which is e.g. utilized to measure brain activation.

The nuclear imaging technique SPECT is based on the detection of a
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A radiotracer that emits gamma photons [1],[2],[4]. Such tracers may e.g. be based on the radionuclide \( {^{99m}}\text{Tc} \), which emits gamma photons with an energy of about 140 keV. One or more gamma cameras fitted with collimators, designed to attenuate all but the near-perpendicularly incident gamma photons, are used to record 2D projection images of the radioactivity. In analogy to CT, 2D or 3D tomographic images of the radiopharmaceutical distribution in the body can be obtained by rotating the gamma camera(s) around the patient. In this way, functional information can be obtained using a potentially very large range of radiotracers. This clinical imaging method, however, yields a relatively low image resolution (typically ~ 10 mm in clinical systems) and the gamma photon detection efficiency is limited due to the use of collimators.

PET is based on the coincident detection of the pairs of gamma photons originating from the annihilation of positrons emitted upon the decay of neutron-deficient radionuclides such as \( ^{11}\text{C} \), \( ^{15}\text{O} \), and \( ^{18}\text{F} \) [1]–[5],[8]. The annihilation photons are emitted in almost opposite directions defining a line through the patient along which the annihilation event must have occurred. These so-called lines-of-response (LOR) can be used to reconstruct 2D or 3D tomographic images of the body indicating e.g. the existence and position of cancerous tissues. Figure 1.2a shows an example of a cross-sectional PET image (slice).

Compared to SPECT the sensitivity of clinical PET systems is \( \sim 10^2 – 10^3 \) higher due to the absence of collimators and higher resolution images can be obtained (about 2.5 mm – 4 mm in current clinical systems) [9]. The sensitivity of PET to molecular targets is in the order of \( 10^6 \) times higher than that of fMRI. The high sensitivity often allows for more accurate detection as well as quantification of radiotracer uptake and it facilitates the possibility to perform very fast dynamic (i.e. time-resolved) studies. Nevertheless, a drawback of PET is that it requires a nearby cyclotron for the production of the short-lived positron emitters. Furthermore, the positron range and the acollinearity of the two annihilation photons constitute physical limitations on the image quality of PET.

1.1.3 Multi-modality imaging

The combination of multiple imaging modalities can substantially increase the benefit for physician and patient. For example, in the case of PET, only the radiotracer is visualized, but essentially no anatomical information is provided (see figure 1.2a). CT, on the other side, does show anatomical information, but is insensitive to most physiological functions of interest (see figure 1.2b). Therefore, only by combining both modalities, e.g. using software registration techniques, a functional image can be provided with an anatomical background.
1.1. Medical imaging

This allows e.g. for better localization of tumors (see figure 1.2c). The CT data can furthermore be used to apply a correction for effects of attenuation and scattering of the annihilation photons [10],[11]. Moreover, by using the CT data other signs of a disease might be diagnosed and/or (parts of) tumors might be spotted that do not take up the tracer.

Nowadays, the majority of PET systems sold have an integrated CT scanner. These systems are set up coaxially (i.e. around a common scanner axis), which greatly reduces incorrect image alignment compared to the case of two individual imaging systems. Another advantage is the shorter overall patient setup and imaging time and, therefore, the patient throughput. It should be noted that a scan typically involves a very fast CT acquisition of a few seconds, followed by a PET acquisition of several minutes.

In spite of the good performance of PET/CT systems, they will always be limited by the low soft-tissue contrast of CT and by the impossibility (due to dose restrictions) of concurrent PET/CT image acquisition, prohibiting accurate

Figure 1.2: Example of (a) a cross-sectional PET image showing a liver metastasis, (b) a CT image of the same slice showing three abnormalities (a hemangioma, a tumor, and a blood vessel), and (c) a fused PET/CT image used to find the anatomical location of the PET hot spot, or on the other hand to find if one of the abnormalities in the CT scan is cancerous tissue. Images courtesy of W. V. Vogel, NKI – Antoni van Leeuwenhoek hospital, Amsterdam, The Netherlands. Printed with permission.
correction of patient and organ motion artifacts. Therefore, a novel approach is to integrate PET with an MRI system [12]. This has the advantage of superior and flexible contrast for soft tissue, facilitating oncology applications such as prostate imaging. Since a 4-dimensional (4D) MRI image (time being the fourth dimension) can be made concurrently with the PET image, accurate correction of motion artifacts as well as 4D attenuation correction become possible. In addition, this technology could be used to directly compare functional MRI studies with PET blood flow studies and to temporally correlate MR spectroscopy and PET information in the study of complex metabolic processes. All of this can be done without administering additional radiation dose to the patient. However, since MR intensities correlate with proton densities and tissue-relaxation properties, rather than with electron density and mass density, attenuation correction for PET is more complex [13].

Only recently, integrated PET-MRI are commercially available on a large scale, due to the much greater technical challenges compared to PET-CT caused by the interference of the PET system with the magnetic field and the electromagnetic signals of the MRI system and vice versa. Prototype MRI-compatible PET inserts have been developed for small-animal and brain imaging, and promising results have been obtained with these systems [14].

1.2 PET imaging

Positron emission tomography (PET) is a key medical imaging modality for the diagnosis of cancer and cardiovascular diseases. In oncology, PET is almost always combined with CT. Both modalities provide the physician with 3D information about the patient, but while CT gives information about the morphology of the body, i.e. the position and size of tumors and organs, PET maps a specific physiological or pathological function, such as the metabolism of cells. For example, the most important PET marker used today is $^{18}$F based fluorodesoxyglucose (FDG), which accumulates in cells with high metabolic activity. Then, an accelerated metabolism appears as a hot spot/region in the image and can indicate the existence and position of cancerous tissue. Conversely, in the heart a reduced tracer uptake (cold spot/region) can show aspects of cardiovascular diseases such as a myocardial infarction.

Besides by visual inspection, the physician often evaluates the PET data quantitatively. For example, in oncology the amount of glucose uptake is strongly correlated with the malignancy of a tumor. The high quantitative accuracy of PET may furthermore enable treatment response monitoring: repeatedly measuring FDG uptake during e.g. a chemotherapy course can allow
1.2. PET imaging

the physician to predict the success of the treatment within days or weeks [15]. This makes early adaptation of the treatment strategy possible when necessary.

PET is also rapidly gaining importance in radiation therapy planning. Furthermore, with improved scanners, viz. reduced radiation dose, cancer screening with PET might become an option. Other clinical fields in which PET is applied are neurology, psychiatry, and pharmacology. For example, PET is used for diagnosis and therapy follow-up in Alzheimer and Parkinson’s disease and for the development of new pharmaceuticals.

The diagnostic value of a PET image depends crucially on the image quality, determined by the spatial resolution, signal-to-noise ratio, and quantitative accuracy. In oncology, these factors determine if the physician is able to detect small tumors and metastases, to accurately identify their locations, to distinguish them from simple inflammations, and to quantify their response to treatment.

1.2.1 PET imaging principles

To visualize a specific function with PET, the patient must be injected with a substance, which distributes in the body such that it correlates with the function of interest. Prior to administration, the substance is marked with a positron (β⁺) emitting radionuclide. Once the radiopharmaceutical has been distributed in the body, the patient is positioned in the PET scanner, which essentially consists of a cylindrical configuration of gamma radiation detectors.

The image formation consists of localizing the positron emitters. After its emission, a positron travels a short distance (~1 mm) in the body following a tortuous path through the surrounding tissue while losing energy. When it has lost sufficient energy, the positron annihilates with an electron. The annihilation yields two gamma photons of equal and well-determined energy (511 keV), which leave the body in (nearly) opposite directions (see figure 1.3a).

The detectors of the PET scanner must detect both of these annihilation photons. If two gamma quanta with the correct energy are detected in coincidence within a certain time window, it is assumed that they both originate from the same annihilation event, and that the positron annihilation must have occurred on the line connecting the two detector elements, the so-called line-of-response (LOR) (see figure 1.3b). From a large number (typically several hundreds of millions) of such LORs, a tomographic image can be reconstructed, which resembles the distribution of the positron-labeled substance in the body.

Most PET scanners consist of multiple rings of radiation detectors. These rings may be separated with lead sheets (septa) to prevent scattered and random events (see section 1.2.3.2) in case a scanner operates in 2D mode, i.e. coincidences are only recorded between detector elements within the same ring.
or closely neighboring rings. In 3D mode, coincidences between detectors in any combination of rings are recorded having the advantage of much higher sensitivity, resulting in an improved image quality and shorter acquisition times. However, the image reconstruction is more complex. Nowadays, most scans are made in 3D mode and septa are used less and less.

Many different methods have been developed to reconstruct a visual representation of the radiotracer distribution inside the patient’s body from the acquired LORs. Due to limited computational power, direct inversion methods such as filtered back-projection (FBP) have long been the most widely used reconstruction algorithm. In FBP the data of each LOR is back-projected and filtered, where the type of filter and its cut-off frequency determine the resolution and noise level of the reconstructed image. Although this method is fast and often relatively simple to implement, it ignores a number of physical effects in the data acquisition that degrade the image quality.

In order to compensate for these image degrading effects, iterative reconstruction algorithms have been developed, which can employ models of, for example, geometrical and detector-related effects, limited spatial sampling, Poisson statistics in photon counting, radiotracer decay, and attenuation and scattering of annihilation photons. Such iterative methods yield improved image quality but are computationally much more expensive. The reconstruction procedure starts with an initial estimate of the tracer distribution upon which all projections of this distribution are calculated (forward projection) using a model of the image formation process. These estimated projections and the measured

**Figure 1.3:** Imaging principle of PET. (a) After annihilation of a positron and an electron, two 511 keV photons are emitted in (almost) opposite directions. (b) When two interactions are simultaneously detected within a ring of detectors surrounding the patient, it is assumed that an annihilation occurred on the so-called line-of-response (LOR) connecting the interactions. By recording many LORs the activity distribution can be reconstructed.
projections are compared and the image is updated according to the discrepancies. The procedure is repeated until (to some extent) convergence of the estimate and the measured data is reached. Commonly used iterative reconstruction methods are maximum likelihood expectation maximization (MLEM) [16] and ordered subsets expectation maximization (OSEM), an accelerated algorithm [17].

1.2.2 Spatial resolution

The image spatial resolution represents the minimum distance between two points such that they in principle can be resolved in a PET image as two separate spatial features. It depends on a number of factors such as the physical processes of the positron decay and annihilation, the design of the scanner, the detectors, and on the image reconstruction algorithm.

1.2.2.1 Positron range

An emitted positron travels a certain distance through the surrounding material while losing kinetic energy by interacting with atomic electrons and nuclei. When it has lost sufficient energy, it annihilates with an electron producing two 511 keV photons travelling in (nearly) opposite directions (see figure 1.3a). The positron range, i.e. the maximum distance between the original emission point and the position of annihilation, depends on the atomic number of the material and on the initial kinetic energy of the positrons, which in turn depends on the radionuclide. The radial distribution of annihilation points around the position of positron emission in a homogeneous medium is sharply peaked and isotropic. The contribution $R_{\text{positron}}$ of this distribution to the image spatial resolution expressed in terms of the full width at half maximum (FWHM) is relatively small in clinical PET systems. For the isotopes $^{18}\text{F}$, $^{11}\text{C}$, $^{13}\text{N}$, and $^{15}\text{O}$, $R_{\text{positron}}$ equals 0.10 mm, 0.19 mm, 0.28 mm, and 0.50 mm, respectively [18]. It should be noted that, although often seen as a physical lower boundary for the image spatial resolution, the positron range can in principle be reduced by means of a strong magnetic field, e.g. in case of an integrated PET scanner in an MRI device [19].

1.2.2.2 Acollinearity

When a positron and an electron annihilate, their center of mass may not be in a state of complete rest, if only because the electron is normally part of an atom. As the total momentum is preserved in the annihilation process, the 511 keV gamma photons may not be emitted in exactly opposite directions. The width of the angular distribution of this deviation, referred to as acollinearity, is usually
assumed to be in the order of 0.5 degrees FWHM. As a consequence, the detected LOR may not intersect the real annihilation point. The effect of acollinearity on the image resolution $R_{\text{acollinearity}}$ is largest in the center of the scanner, where its effect in terms of FWHM can be estimated as $R_{\text{acollinearity}} \approx 0.0022 \cdot D_{\text{scanner}}$, and thus increases with increasing scanner diameter $D_{\text{scanner}}$ in units of millimeters [20]. Therefore, this effect is considerable in clinical scanners where $R_{\text{acollinearity}}$ is in the order of ~1.7 mm to ~2.0 mm for ring diameters of 80 cm to 90 cm, respectively.

1.2.2.3 Detector spatial resolution and parallax error

Currently, the largest degrading factor of the image spatial resolution in clinical PET systems is the accuracy by which the positions, where each of the annihilation photons entered or interacted with the detectors, can be determined. This detector spatial resolution $R_{\text{detector}}$ includes the so-called parallax error, i.e. an uncertainty as well as a systematic error of the positioning of the LOR. This error arises for gamma photons that have a large angle of incidence on the detectors and, thus, may first penetrate one or more adjacent detector elements before being detected. In case an event would be assigned to the center of the firing detector element, the line-of-response (LOR) may not be connected to the center that it passes most closely. Parallax errors can be reduced if the detectors provide a way of estimating the depth of interaction (DOI) inside the crystals or when the point is determined where the gamma photon enters the detector front surface.

For all PET detectors based on segmented crystals (see section 1.3.2), the lateral detector spatial resolution $R_{\text{detector}}$ is strongly related to the crystal size. When segmented crystals are grouped into so-called block detectors (see section 1.3.2) involving light-sharing or charge-sharing, $R_{\text{detector}}$ includes a further loss of spatial resolution due to errors in event localization caused by statistical fluctuations in the photosensor signals, scatter within the detector, and imperfections in the block-decoding scheme [20].

1.2.2.4 Image spatial resolution

Combining the contributions mentioned above, the spatial resolution $R_{\text{image}}$ of a PET image for events in the center of the scanner can be estimated as $R_{\text{image}} \approx k_{\text{reconstruction}} \cdot \sqrt{0.25R_{\text{detector}}^2 + R_{\text{acollinearity}}^2 + R_{\text{positron}}^2}$. The factor $k_{\text{reconstruction}}$ takes into account imperfections in the image reconstruction process but also the resolution gain that can be obtained by intelligent iterative reconstruction techniques, e.g. by including models for the positron range and the acollinearity. While $R_{\text{image}}$ of a PET scanner in a clinical imaging situation is, at best, ~5 mm,
the physical effects, viz. the positron range and the acollinearity, contribute only ~2 mm or less [21],[22]. This indicates that in principle a significantly better image resolution can be obtained by improving the detector and system design, provided that the number of recorded LORs is sufficiently large.

1.2.3 Signal-to-noise ratio

The contrast in the reconstructed image is limited by the accuracy of the estimate of the activity concentration in the image voxels, which in turn depends on the image reconstruction method used and on the image signal-to-noise ratio (SNR). The SNR, which for a considerable part is determined by counting statistics, could be improved by measuring longer or to some extent by increasing the activity, but in practice this is usually not desirable. Consequently, the number of measured LORs should be maximized by optimizing the PET scanner’s sensitivity, being defined as the probability of detecting both photons from an annihilation event (see section 1.2.3.1). On the other hand, the influences from noise sources should be minimized (see section 1.2.3.2). At present, in practice, it is usually the limited SNR and, consequently, the need to reduce noise by data smoothing that limit the spatial resolution as well as the contrast in clinical PET images [3],[5],[8].

1.2.3.1 Sensitivity

To obtain high scanner sensitivity, a high gamma photon detection efficiency of the detectors is essential. Since both annihilation photons have to be detected to form a LOR, the scanner sensitivity correlates to the square of the detector’s detection efficiency. In turn, detector detection efficiency is strongly dependent on the material and the thickness of the scintillation crystal.

Furthermore, the scanner sensitivity improves with increasing geometric efficiency, i.e., with increasing solid angle coverage of the field of view (FOV). This can be achieved by reducing the scanner diameter and/or by increasing its axial extent.

As a third option, the sensitivity of a PET system can be increased by decreasing the dead time of the detectors, e.g. by the use of fast scintillators, fast electronic readout, and fast data processing.

1.2.3.2 Noise sources: randoms, scattering, and attenuation

If both 511 keV photons originating from a single emitted positron are detected in two opposite detectors, this is referred to as a true coincidence or simply a ‘true’ (see figure 1.4a). However, due to several error sources, the determined LOR does not always pass through the voxel in which the annihilation took
The first noise source originates from random coincidences, commonly referred to as ‘randoms’. Randoms occur when within the coincidence time window two photons are detected originating from different annihilations (see figure 1.4b). These random coincidences usually result in a more or less uniform enhanced background in the reconstructed image, thus reducing the image SNR. The number of random coincidences has a quadratic dependence on the amount of radioactivity as well as on the detector detection efficiency and a linear dependence on the width of the coincidence time window. The width of this window should thus be kept as narrow as possible while maintaining an appropriate FOV diameter. On the other hand, the coincidence time window should be kept larger than the detector timing resolution, i.e. the accuracy of the determination of the moment of interaction inside the detector, so as not to result in discarding true coincidence events. Therefore, the detector timing resolution should be as accurate as possible to obtain a good SNR.

A second type of image SNR deterioration occurs, when one or both of the annihilation photons undergo single or multiple Compton scattering events inside the patient (see figure 1.4c). As a result, one or both annihilation photons may be deflected from their initial trajectory and will either not be detected or they will be detected by a different detector pair than expected. In the latter case, the event is classified as a scattered coincidence or simply as a ‘scatter’ and the measured LOR is no longer correct. Moreover, both cases reduce the number of trues and, thus, the image SNR. This effect is especially significant in clinical PET, where ~15% of events are scattered in case of 2D mode, and > 40% of events are scattered in 3D whole body PET [3]. The scattering probability depends both on the energy of the emitted photons and on the amount and composition of the material through which the photons are travelling. Since the

Figure 1.4: Three possible types of coincidence events in a PET scanner: (a) trues, (b) randoms, and (c) scatters.
annihilation photons lose energy in a Compton event, scattered photons may be identified and rejected by applying an energy threshold, if the energy of a detected photon can be determined with a high enough accuracy, i.e. with good energy resolution.

A third source of error, somewhat related to the second one, is the photoelectric absorption of one or both annihilation photons within the patient. As a result, gamma photons from annihilations deep inside the body have a lower probability to reach the detector, which without correction results in the apparent ‘darkening’ of the inner part of the body, hampering quantification, and reducing the SNR in the reconstructed image.

To correct for attenuation and the removed scatters, both should be taken into account during reconstruction to prevent distortions of the reconstructed image. One method to correct for attenuation and scattering is to calculate patient specific attenuation and scattering coefficient maps, which in practice is often based on measured data from a CT device [10]. It should be noted that the scattering and attenuation effects do not depend on the annihilation position along a specific LOR, but only on the length of the intersection of the LOR with the patient. This makes attenuation correction, and therefore quantitative imaging, relatively straightforward in PET compared to other imaging modalities.

**Figure 1.5:** (a) Better localization of the annihilation event is obtained by measuring the difference in time-of-flight (TOF) of the gamma photons. (b) During reconstruction of the image only activity is attributed to those voxels along the LOR that are close to the voxel estimated by using the TOF information resulting in a better SNR.
A major research topic for PET to improve the SNR is the use of the difference in the arrival times of the gamma quanta to roughly localize the position of annihilation along the LOR (see figure 1.5a). The positioning accuracy \( \Delta x \) depends directly on the detector coincidence resolving time (CRT) \( \Delta t \): 
\[
\Delta x = \frac{c \cdot \Delta t}{2},
\]
where \( c \) is the speed of light in vacuum. By including this time-of-flight (TOF) information in the image reconstruction, only activity is attributed to those voxels along the LOR that are within \( \Delta x \) from the estimated voxel (see figure 1.5b). As a result, the signal-to-noise ratio of the reconstructed image improves by approximately a factor of \( \sqrt{\frac{D_{\text{patient}}}{\Delta x}} = \sqrt{\frac{2D_{\text{patient}}}{c \Delta t}} \), where \( D_{\text{patient}} \approx 40 \text{ cm} \) is the patient diameter [23],[24]. Thus, significantly better image quality can be obtained (see figure 1.6) and TOF is especially beneficial for obese patients. It is noted that in practice the SNR improvement may be somewhat smaller due to reconstruction effects and the fact that the activity is not distributed homogeneously through the body [8],[24],[25].

For example, a CRT of 600 ps, achieved with commercial state-of-the-art TOF PET scanners, can increase the effective SNR by a factor of \(~2.1\) [21],[26]. For the same effect in conventional PET, one would have to scan 3 times as long, apply 3 times the dose, or use a much larger (and much more expensive) scanner. A LaBr\(_3\):Ce based prototype scanner even achieved a CRT of 375 ps corresponding to a SNR improvement with a factor of \(~2.7\) [27],[28].

Furthermore, if \textit{a priori} knowledge about the geometry of the patient based on CT data is available, TOF information can be used to discard the possibility of events outside of the patient reducing the number of randoms [11]. Moreover,
the use of TOF information leads to faster and more uniform convergence of the reconstruction, improved lesion detectability (see figure 1.6), more homogeneous image quality, and more accurate quantification [29].

1.2.4 Commercial systems

Some of the characteristics of two currently commercially available state-of-the-art TOF PET scanners, viz. the Philips Gemini TruFlight and the Siemens Biograph TruePoint, are listed in table 1.1. Both scanners have detectors based on L(Y)SO crystals coupled to photomultiplier tubes.

1.3 PET detectors

The quality of the detectors of a PET scanner has a large impact on the image quality. However, the design of the detectors is complicated by the wide variety of detector requirements needed for optimal performance. Good spatial as well as DOI resolution of the detector are mandatory for a high image resolution and to prevent parallax errors. A good energy resolution is needed to correct for Compton scattering. For a good contrast in the image, the detectors should have a high sensitivity, minimum dead space, and maximum angular coverage. A good timing resolution of the detectors is needed to correct for randoms and to obtain TOF information. The detector should preferably be insensitive to magnetic fields, to allow for integration in MRI systems. The design should be cost effective, viz. the cost of fabrication, operation and maintenance of the PET

Table 1.1: Some characteristics of current state-of-the-art TOF PET scanners.

<table>
<thead>
<tr>
<th>System</th>
<th>Ring diameter</th>
<th>Nr. and size of crystals</th>
<th>Energy resolution</th>
<th>Spatial resolution</th>
<th>Coincidence resolving time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Philips Gemini</td>
<td>90 cm</td>
<td>28336</td>
<td>11.5%</td>
<td>4.8 mm at 1 cm</td>
<td>585 ps [21]</td>
</tr>
<tr>
<td>TruFlight</td>
<td>4×4×22 mm³</td>
<td></td>
<td></td>
<td>5.0 mm at 10 cm</td>
<td></td>
</tr>
<tr>
<td>Siemens Biograph</td>
<td>84.2 cm</td>
<td>32448</td>
<td>11%</td>
<td>4.4 mm at 1 cm</td>
<td>550 ps [26]</td>
</tr>
<tr>
<td>TruePoint</td>
<td>4×4×20 mm³</td>
<td></td>
<td></td>
<td>5.0 mm at 10 cm</td>
<td></td>
</tr>
</tbody>
</table>
detector modules should be as low as possible. Finally, the detectors should be operable around room temperature, easily exchangeable in case of malfunction, and their performance should be stable in time.

Certain requirements may be in conflict which each other and, thus, it may be necessary to make trade-offs. For example, a crystal geometry providing the best possible spatial resolution does not necessarily provide good energy or timing resolution. It is therefore important to simultaneously study the influence of all components in the detector chain, including the influence of each component on the performance of the other components, to obtain the best possible PET image.

1.3.1 Detector components

Typical PET detectors consist of scintillation crystals read out by photosensors. In such detectors, an incident gamma photon interacts with the scintillator and the deposited energy is converted into optical photons in the visible and/or ultraviolet spectrum. By using optical glue, a light guide, and/or optical fibers, the crystals are optically coupled to the photosensors, which convert the optical signals into electronic signals. Dedicated, high-speed, low-noise amplifiers are usually required to transmit the electronic signals to the readout system, viz. the analog-to-digital convertor (ADC).

1.3.1.1 Scintillator

For the conversion of gamma quanta into optical photons many different scintillation materials are available. A selection of the most suitable ones for PET together with some of their main characteristics, which may be temperature dependent, is listed in table 1.2. Other optical crystal properties, such as refractive index, absorption and scattering, which influence the number of photons detected by the photosensor, will be discussed in chapter 2.

In order to obtain a high sensitivity of the PET system, the gamma photon detection efficiency of the scintillator should be as large as possible. This efficiency depends on the density, the effective atomic number $Z_{\text{eff}}$, and the thickness of the crystal. Additionally, for materials with a higher $Z_{\text{eff}}$, the fraction of photoelectric interactions as well as absorbed secondary Compton photons increases. This is important for PET detectors, since Compton interactions in the detector cannot be distinguished from Compton interactions in the patient and events might have to be discarded. Furthermore, the secondary Compton photon may be detected in another detector, giving rise to an uncertainty in the positioning of the event and decreasing the SNR of the measured signal.
A good energy resolution of the detector can be obtained by choosing a scintillator with a high light yield (the number of created optical photons per unit of absorbed energy). This results in a smaller influence of counting statistics, which scales with the inverse of the square root of the number of detected photons. It is noted that the spread on the number of emitted photons is usually larger than expected from Poisson statistics due to the non-proportionality of the scintillator. Thus, light losses should also be kept as low as possible taking into account highly reflective covering materials, an optimized optical coupling to the photosensor, the geometry of the scintillator, the refractive index, absorption, scattering, surface treatment, and matching of the wavelength range of the emitted photons with the spectral response of the photosensor.

For good timing properties and a small dead time, the scintillator should additionally have short rise and decay times of the emitted optical photon pulse, and a small transient time spread of the optical photons within the crystal.

### Table 1.2: Some characteristics of scintillators for PET. The values are obtained from [32] and references therein. The values for the attenuation lengths, the photofractions, and the light yields correspond to 511 keV gamma photons.

<table>
<thead>
<tr>
<th>Material</th>
<th>Density (g/cm³)</th>
<th>Z_{eff}</th>
<th>Att. length (mm)</th>
<th>Photo-fraction (%)</th>
<th>Light yield (ph/keV)</th>
<th>Decay time (ns)</th>
<th>Emission wav. max (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gd₂SiO₅:Ce (GSO)</td>
<td>6.7</td>
<td>59</td>
<td>14.1</td>
<td>25</td>
<td>8</td>
<td>60</td>
<td>440</td>
</tr>
<tr>
<td>Bi₄Ge₃O₁₂ (BGO)</td>
<td>7.1</td>
<td>75</td>
<td>10.4</td>
<td>40</td>
<td>9</td>
<td>300</td>
<td>480</td>
</tr>
<tr>
<td>LuAlO₃:Ce (LuAP)</td>
<td>8.3</td>
<td>65</td>
<td>10.5</td>
<td>30</td>
<td>11</td>
<td>18</td>
<td>365</td>
</tr>
<tr>
<td>Lu₃Si₂O₇:Ce (LPS)</td>
<td>6.2</td>
<td>64</td>
<td>14.1</td>
<td>29</td>
<td>20</td>
<td>30</td>
<td>380</td>
</tr>
<tr>
<td>Lu₂SiO₅:Ce (LSO)</td>
<td>7.4</td>
<td>66</td>
<td>11.4</td>
<td>32</td>
<td>26</td>
<td>40</td>
<td>420</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>36*, 32*</td>
<td></td>
</tr>
<tr>
<td>LaCl₃:Ce</td>
<td>3.9</td>
<td>60</td>
<td>28.0</td>
<td>15</td>
<td>46</td>
<td>25</td>
<td>350</td>
</tr>
<tr>
<td>LaBr₅:Ce</td>
<td>5.1</td>
<td>47</td>
<td>22.3</td>
<td>13</td>
<td>70</td>
<td>16</td>
<td>380</td>
</tr>
<tr>
<td>LuI₃:Ce</td>
<td>5.6</td>
<td>60</td>
<td>18.2</td>
<td>28</td>
<td>90</td>
<td>6**, 31, 140, 1000</td>
<td>470, 535</td>
</tr>
</tbody>
</table>

* Ca²⁺ co-doped LSO [30]

** 6 ns (20%), 31 ns (30%), 140 ns (22%), 1000 ns (28%)
Essentially, the number of detected photons per unit of time should be as large as possible.

In addition to the above requirements, to minimize the costs and to maximize the design options, the scintillation crystals should be relatively easy to grow and machine, and should preferably not be hygroscopic.

Current state-of-the-art PET systems commonly use LSO:Ce or LYSO:Ce scintillators, which with the use of co-dopants can still be improved [21], [22], [30]. An alternative scintillator is LaBr$_3$:Ce, discovered at Delft University of Technology in 2001 [31]. It is 2.7 times brighter and 2.5 times faster than LSO/LYSO. However, due to the larger attenuation length and the smaller photofraction, thicker crystals might be required resulting in a potential smaller benefit from its favorable properties. Furthermore, its hygroscopicity might hamper the detector design. Finally, the emerging material LuI$_3$:Ce has even higher light yield than LaBr$_3$:Ce, as well as better stopping power and, thus, may be of interest for future detector designs.

### 1.3.1.2 Photosensor

Optimum detector performance requires the photosensor to detect scintillation photons as efficiently and accurately as possible. Therefore, it is mandatory for the photosensor to have high photon detection efficiency (PDE), i.e. the probability that an incident photon is converted to a measurable signal, at the emission wavelengths of the scintillator. The sensor PDE is determined by 1) the probability that a scintillation photon is not reflected on the sensor surface, and 2) the fill factor of the sensor surface, i.e. the ratio of active area to total sensor area, 3) the internal quantum efficiency (QE), i.e. the probability that a photon absorbed in the active area of the sensor gives rise to a signal. Furthermore, to minimize the influence of electronic noise and excess noise (variance of the gain), the photosensor should preferably have a high internal gain. It should additionally have a fast response for a good timing resolution. Finally, the photosensor should preferably not interfere with magnetic fields so as to be MRI compatible, and it should be cost effective.

Nowadays, the most widely used photosensor in PET is the photomultiplier tube (PMT). Its fast rise time of ~1 ns and its high gain of ~$10^6$ allow for fast timing and for good energy resolution. The QE of a PMT is somewhat poor, typically ~25% at 420 nm, although PMTs with a QE of up to ~43% have been developed recently [33]. Despite their good performance, their sensitivity to magnetic fields complicates their use in simultaneous PET/MR imaging. In addition, the devices are not transparent to annihilation photons, prohibiting double-sided readout of the scintillator, and, since the devices are rather bulky, a
close packing of detector modules based on PMTs is difficult.

Another type of photosensor is the Avalanche Photodiode (APD), which compared to a PMT has a significantly higher QE, viz. ~75% at 420 nm and up to ~90% at higher wavelengths. It is furthermore insensitive to magnetic fields and it is very compact, since it is a semiconductor device. However, APDs have a much lower gain of $10^2 – 10^3$ making them more susceptible to electronic noise, which in combination with their relatively large leakage current and excess noise factor results in worse timing and energy resolution. This, together with their slower rise time of ~5 ns prohibits TOF PET.

The silicon photomultiplier (SiPM) is a relatively new type of photosensor combining the advantages of the compactness, the cost effective semiconductor technology, the low operating voltage, the low power consumption, and the magnetic field insensitivity of the APD with the favorable properties of high gain and fast response of the PMT. An SiPM consists of an array of $10^3 – 10^5$ individual APDs (microcells) of ~20 µm – 100 µm working in Geiger mode: each detected photon generates a fast (rise time ~1 ns), well defined single-photoelectron pulse with a very high gain of $10^6$. The individual cells being connected in parallel give rise to a current pulse comprised of the analog sum of the illuminated cells. Furthermore, the SiPM bias voltage is much lower than for a PMT and, thus, the power consumption is much less, and since SiPMs have a smaller timing jitter than PMTs, they enable superior TOF PET. However, the limited number of cells gives rise to a reduced dynamic range, which for fast and bright scintillation pulses may result in a non-proportional response of SiPMs. This is discussed in more detail in chapter 3, where also a model of the response as a function of several SiPM parameters is presented. Finally, SiPMs are more temperature sensitive than PMTs, and compared to APDs the PDE of SiPMs is lower, viz. ~25% – 75% depending on the microcell size, due to the dead space between the cells.

1.3.2 Monolithic scintillator detectors

Many PET detectors consist either of individual crystal elements optically separated by reflective material or of a scintillator block in which e.g. $8 \times 8$ pixels are cut and which is read out by a limited number (e.g. 4) of PMTs in order to decrease both complexity and cost [34]. In the latter design the depth of the cuts varies so as to minimize the probability that adjacent pixels produce similar intensity ratios on the PMTs. The pixel in which the incident gamma photon interacted is localized using the light sharing between the PMTs and Anger logic.

In clinical PET scanners the width of each crystal segment typically is
between 4 mm and 6 mm [21], [22], [35]. PET scanners for small animal imaging or for dedicated high resolution organ imaging usually have pixels of about 1 mm – 2 mm width. Recently, even detectors based on ~0.25 mm and ~0.5 mm wide crystals were developed [36]. However, detectors of these dimensions will probably not enter into clinical systems because of the significant costs for readout and manufacturing [37], and because the image resolution is anyway limited by the acollinearity of the gamma photons (see section 1.2.2.2). Furthermore, smaller crystals cause the number of lines-of-response (LORs) in a scanner to increase, potentially leading to a statistical problem for the reconstruction, if the scan time is short and the injected radioactivity remains low [37].

In the above designs the reflective material separating the crystals may take up a considerable fraction (~10%) of the detector volume, reducing the system sensitivity. Additionally, in this approach always a tradeoff has to be made between system resolution (many pixels) and sensitivity (large dead area). Furthermore, although these designs aim to improve resolution by preventing light spreading, their performance is still limited by inter-crystal scattering of the gamma photons.

Detectors based on a monolithic (i.e. a single, continuous) scintillation crystal read out by one or more pixelated photosensors are considered as alternatives to detectors based on segmented crystals [38], [39]. These monolithic scintillator detectors have proven to exhibit good spatial resolution as well as excellent depth-of-interaction (DOI) correction, and their reduced dead space allows for high system sensitivity. Furthermore, they have the practical advantages that less crystal polishing and surface processing is required, and assembly of the detector is easier. For example, a single monolithic scintillator detector with an area of 4 cm \( \times \) 4 cm would replace 100 pixels of 4 mm \( \times \) 4 mm, which would result for the state-of-the-art TOF PET scanners (section 1.2.4) in a reduction of the number of crystals from ~30000 to ~300.

The determination of the interaction position of a gamma photon in a monolithic scintillator is more complex than for segmented crystals, in which case the position can simply be determined by the position of the crystal where the interaction occurred. In monolithic scintillation detectors the position needs to be estimated from the light intensity distribution on the photosensor pixels. Here, it should be noted that, because of Compton scatter, X-ray fluorescence, etc., the annihilation photon does not always interact in one point inside the crystal, but may interact in multiple points complicating the estimation of the position of interaction(s).
As an illustration, in figure 1.7 a schematic representation of a scintillation event in a monolithic scintillator detector is shown. The upper row of figure 1.7 illustrates how the light distribution might change if the interaction takes place closer to the edge of the detector. The bottom row of the same figure indicates how the DOI might relate to the light distribution. It should be emphasized that this figure is an oversimplified representation and that in practice the light distribution is strongly influenced by reflections on the side and the top of the crystal. Therefore the peak of the light distribution does not always correspond to the lateral interaction coordinates. Furthermore, the light distribution is not as smooth as sketched due to photon statistics, noise, and the limited number of sensor pixels. Moreover, there will always be differences between detectors, e.g. due to variations of the light yield from crystal to crystal, variations of the optical coupling properties, variations of the reflective properties of the enclosing material, and variations of the amplifier gains.

Instead of the interaction position of the annihilation photon, the entry point on the front surface of the crystal might be derived from the measured light distribution on the sensor pixels. By deriving the entry point, the DOI is
intrinsically corrected for.

In practice, it appears very difficult in monolithic scintillators to estimate the interaction/entry position(s) from the measured light distribution based on Anger logics or based on a model describing the light distribution as a function of the interaction position. A better solution may be to use a statistical method, such as a nearest neighbor (NN) method, a maximum likelihood (ML) method, or neural networks. These methods are based on a calibration measurement in which many light distributions are recorded at known interaction positions. A detailed discussion about the calibration and the NN entry point estimation method can be found in chapters 5 and 6. Here, it should be noted that the calibration of the detector may have to be performed at many angles of incidence. Alternatively, the detector could be calibrated only for perpendicularly incident annihilation photons and the DOI may be obtained using some other method (see chapter 6), potentially allowing for much shorter calibration times.

Eventual implementation of these monolithic scintillator detectors would require overcoming a number of technical challenges. Since the scintillation photons might be spread over many pixels, both the influence of the statistical fluctuations on the number of detected photons per pixel and the influence of the electronic noise increase, requiring advanced readout methods and electronics. Also, the positioning of the individual events on the detectors requires extra data processing steps, possibly leading to a very large data stream and increased processing time, necessitating high-bandwidth, high-speed (pre)processing units.

1.4 Thesis research objectives

The diagnostic value of a PET image depends crucially on the image quality, i.e. the spatial resolution, signal-to-noise ratio, and quantitative accuracy. In oncology, these factors determine whether the physician is able to detect the tumors and/or metastases, accurately identify their locations, and distinguish them from ordinary inflammations. The image quality in turn is largely determined by the performance of the PET detector. Thus, for a better quality PET image, detectors require an improved spatial, energy, and timing resolution, detection efficiency, DOI correction, and TOF capability, compared to the current state-of-the-art PET detectors (see table 1.1). Furthermore, the detectors must be economically attractive when implemented in future products and it is desirable that they are compatible with a MRI device.

It should be taken into account that a detector system that outperforms the current state-of-the-art in one or two performance parameters, but performs badly in a third parameter, may be essentially useless. Complicating things
furthermore, certain performance requirements may be in conflict with each other and trade-offs may therefore need to be made. For example, a crystal geometry providing the best possible spatial resolution does not necessarily provide good energy resolution, while read-out electronics optimized for energy resolution may not provide good timing resolution. Therefore, it is important to simultaneously study the influence of each component on the performance of all other components to obtain the best possible PET image.

The primary objectives of this thesis are to understand and characterize the performance of an innovative PET detector concept, based on the recently discovered LaBr$_3$:Ce material in the form of a monolithic scintillator in combination with novel, high-speed, position-sensitive SiPM light sensors to read out the scintillation light distribution from which the position is derived by means of dedicated software algorithms. This concept may open up the possibilities for shorter PET scans, detectability of smaller lesions, reduced patient dose, and multimodality imaging capability, which would be a great advance compared to the current state-of-the-art systems.

1.5 Thesis outline

The research presented in this thesis is organized as follows. Chapter 2 presents measurements on the optical properties of LaBr$_3$:Ce, viz. the refractive index, the optical absorption length, and the optical scattering length. A theoretical model of the absorption has been developed to which the measured values are compared. The results are essential input parameters for accurate Monte Carlo simulations of LaBr$_3$:Ce based detectors. In chapter 3, a model is discussed of the non-proportional response of an SiPM to scintillation light pulses. It accounts for the total number and the temporal distribution of the incident photons as well as for the relevant SiPM parameters, i.e. the recovery time, afterpulsing, crosstalk, and their cross-correlations. This model can be utilized, e.g. to properly design a detector for a given application, to perform corrections on measurements or on energy spectra, to calibrate an SiPM detector for low-level-light measurements, to predict detector performance, and/or to determine difficult-to-measure SiPM parameters. Chapter 4 is dedicated to the performance assessment of a PET detector based on a monolithic LYSO crystal read out by an SiPM array. The experimental setup and the methods to characterize the detector are presented and the spatial resolution, the ability to correct for the DOI, and the energy and timing resolutions obtained with the detector are discussed. Chapter 5 describes improvements of the standard $k$-nearest neighbor method for the estimation of the position of interaction allowing for faster
computation and calibration. Furthermore, the possibility of calibrating with a line source is explored, which would allow for even shorter calibration times. In chapter 6, several new methods for the determination of the DOI in monolithic scintillator detectors with single-sided readout are presented and validated. These methods require only perpendicularly incident gamma photons to obtain calibration data, speeding up the detector calibration significantly, and they need neither detector modifications nor models for the signal variance or light transport, which makes them practical. Finally, in chapter 7, the findings of the preceding chapters are discussed, and an outlook is provided to new technological developments and further research steps that should be taken with regard to PET detectors.

References


Chapter 1. Introduction


Chapter 2

Optical absorption length, scattering length, and refractive index of LaBr$_3$:Ce$^{3+}$


**Abstract**

Despite the large interest in the application of LaBr$_3$:Ce$^{3+}$, little is known yet about its optical properties, as measurements are hampered by the hygroscopicity of LaBr$_3$:Ce$^{3+}$ and because it is not trivial to produce crystals with optically polished surfaces. Here, the absorption and scattering lengths as well as the refractive index of LaBr$_3$:5%Ce$^{3+}$ are determined experimentally for the first time. The refractive index is found to vary from 2.25 to 2.40 depending on wavelength and crystal orientation, which is considerably higher than the value of ~1.9 that has often been assumed to date. Furthermore, a model of the Ce$^{3+}$ absorption and emission probability as a function of wavelength, Ce$^{3+}$ concentration, and scintillation photon traveling distance is developed. This model is used in combination with the measured absorption and scattering lengths to obtain the intrinsic emission spectrum of LaBr$_3$:Ce$^{3+}$ from measured emission spectra. Additionally, the model is used to illustrate the importance of the investigated crystal properties for scintillation detector design. It is demonstrated that for crystals with dimensions in the order of a few centimeters, the fraction of scintillation photons undergoing scattering and/or absorption before reaching the photosensor can be several tens of percents depending on the Ce$^{3+}$ concentration. Finally, it is shown that self-absorption and re-emission of the scintillation photons can have a non-negligible effect on the timing resolution of LaBr$_3$:Ce$^{3+}$ scintillator detectors.
Chapter 2. Optical properties of LaBr$_3$:Ce$^{3+}$

2.1 Introduction

The relatively new scintillator LaBr$_3$:Ce$^{3+}$ has already proved its worth for use in spectroscopy due to its excellent energy resolution [1], [2]. Furthermore, because of its short decay time (~16 ns) [3] and high light yield (~70000 ph/MeV) [4], LaBr$_3$:Ce$^{3+}$ is under investigation by several groups for application in next generation time-of-flight PET systems [5]–[7].

The performance of scintillation detectors may be highly influenced by optical processes in the crystal. For example, absorption and/or scattering in the bulk of the crystal may result in a decreased photon detection efficiency (PDE), and/or in a loss of the information carried by the scintillation photons about the interaction time and position. Furthermore, the reflectance of the crystal surfaces is governed by the refractive index, which in the case of LaBr$_3$:Ce$^{3+}$ is expected to be dependent on both the orientation of the crystal and the polarization of the light due to the birefringence resulting from its hexagonal crystal structure [8], [9].

Therefore, accurate knowledge of all of these optical processes is needed to quantitatively analyze or predict the performance of a scintillation detector. For example, in comprehensive calculations, such as Monte Carlo simulations using e.g. GATE, GEANT4, DETECT2000, LITRANI or similar codes, in which the interactions of each optical photon are modeled, it is mandatory to know the required optical input parameters very accurately [10].

Despite the large interest in LaBr$_3$:Ce$^{3+}$, little was known to date about its refractive index or its optical scattering and absorption properties. These properties are typically derived from transmission and reflection measurements. However, complicating factors, such as the hygroscopicity of LaBr$_3$:Ce$^{3+}$ and the fact that it is not trivial to produce crystals with optically polished surfaces, have prohibited such measurements to date.

The primary goal of the present work is to experimentally determine the optical attenuation length as well as the refractive index of LaBr$_3$:Ce$^{3+}$ as a function of wavelength at room temperature. In addition, a model is developed of the Ce$^{3+}$ absorption and emission probability as a function of wavelength, Ce$^{3+}$ concentration, and photon traveling distance. This model is used in combination with the measured absorption and scattering lengths to derive the intrinsic emission spectrum of LaBr$_3$:Ce$^{3+}$ from emission spectra measured with crystals of different dimensions. Finally, the model is utilized to predict the influence of Ce$^{3+}$ concentration and scintillation photon traveling distance on several detector performance parameters.
2.2 Ce\textsuperscript{3+} absorption and emission

In this section, a model is developed that predicts the probability of scintillation photons of a given energy to propagate over a certain distance in a LaBr\textsubscript{3}:Ce\textsuperscript{3+} crystal with a known Ce\textsuperscript{3+} concentration without undergoing any optical interaction. First a description of the absorption and emission of individual Ce\textsuperscript{3+} ions in the crystal lattice is given, which is then extended to Ce\textsuperscript{3+} doped crystals of finite dimensions.

2.2.1 Ce\textsuperscript{3+} absorption and emission mechanisms

When a photon is absorbed by a Ce\textsuperscript{3+} ion, an electron from the incompletely filled 4f shell (ground state) is excited into the 5d shell (excited state). The probability distribution \( p_{\text{abs,Ce}^{3+}}(E) \) for the absorption of a photon that reaches the Ce\textsuperscript{3+} ion and has the energy \( E \) can be described by the Huang-Rhys model [11], [12]. However, for many purposes the semi-classical approximation describes this distribution adequately. This approach, which utilizes the wave function of the lowest vibrational energy level, results in a Gaussian shaped absorption band [13], [14]:

\[
p_{\text{abs,Ce}^{3+}}(E) = \frac{1}{\sqrt{2\pi}\sigma_{\text{abs}}} \exp\left(-\frac{(E-E_{\text{abs}})^2}{2\sigma_{\text{abs}}^2}\right),
\]

where \( E_{\text{abs}} \) is the peak position of the absorption band and \( \sigma_{\text{abs}} \) its standard deviation. The degenerate 5d levels of the free Ce\textsuperscript{3+} ion are split by the crystal field and spin orbit interaction in minimal two to maximal five components [15]. Nevertheless, in this work only the absorption band corresponding to transitions to the lowest energy 5d\textsubscript{1} level is considered, because transitions to the higher energy levels have negligible influence on the absorption of the scintillation light.

Upon absorption of a photon, the lattice with the excited Ce\textsuperscript{3+} ion first relaxes to its lowest vibrational level by phonon emission. During this relaxation usually no photon emission occurs, because the radiative decay rate is \( \sim 10^{8} \text{ s}^{-1} \) while the phonon decay rate is \( \sim 10^{13} \text{ s}^{-1} \) [15]. The efficiency of the 5d–4f emission in LaBr\textsubscript{3}:Ce\textsuperscript{3+} is considered to be close to 1, since no thermal quenching is observed at room temperature [16]. After photon emission, the lattice with Ce\textsuperscript{3+} in the 4f ground state configuration is in a high vibrational state and again relaxation occurs. The 4f configuration of Ce\textsuperscript{3+} is split into two states due to spin orbit coupling giving rise to the two characteristic Ce\textsuperscript{3+} emission bands. Following similar arguments as those that led to equation 2.1, the probability of emitting a photon with energy \( E \) is given by:
Chapter 2. Optical properties of LaBr$_3$:Ce$^{3+}$

\[
p_{\text{em,} \text{Ce}^{3+}}(E) = \frac{A}{\sqrt{2\pi}\sigma_{\text{em,1}}} e^{-\frac{(E-E_{\text{em,1}})^2}{2\sigma_{\text{em,1}}^2}} + \frac{1-A}{\sqrt{2\pi}\sigma_{\text{em,2}}} e^{-\frac{(E-E_{\text{em,2}})^2}{2\sigma_{\text{em,2}}^2}},
\]  

(2.2)

where $E_{\text{em,1}} < E_{\text{em,2}}$ are the mean energies of the two emission bands with their corresponding widths $\sigma_{\text{em,1}}$ and $\sigma_{\text{em,2}}$, respectively, while $A$ describes the probability of emission at the energy band corresponding to $E_{\text{em,1}}$.

2.2.2 Self-absorption and re-emission

As a result of the relaxation processes in the excited state and in the ground state described in the previous section, the average energy of emission is lower than that of absorption. The energy difference between the mean value of the lowest energy absorption band and that of the highest energy emission band is called the Stokes shift. Depending on the size of the Stokes shift, the emission and absorption bands may overlap and part of the emitted light may be absorbed by other Ce$^{3+}$ ions. This effect is referred to as self-absorption. The Ce$^{3+}$ ions excited by self-absorbed photons may again emit photons following the same mechanism as described in the previous section. This is referred to as re-emission.

Since the direction of emission is randomly distributed, the original direction of the absorbed photon is not preserved during re-emission. Furthermore, the probability that the re-emitted photon has a certain energy $E$ is again described by equation 2.2 and, therefore, its energy may be different from the energy of the absorbed photon. Consequently, when a gamma or X-ray photon is absorbed in a LaBr$_3$:Ce$^{3+}$ crystal and (part of) the energy is transferred to the Ce$^{3+}$ luminescence centers, the spectrum of the scintillation light leaving the crystal is not necessarily equal to the intrinsic emission spectrum of Ce$^{3+}$ ions in LaBr$_3$:Ce$^{3+}$.

2.2.3 Ce$^{3+}$ absorption and emission in LaBr$_3$:Ce$^{3+}$ samples with finite dimensions

To describe the amount of Ce$^{3+}$ absorption as a function of energy and photon path length, we consider a LaBr$_3$:Ce$^{3+}$ crystal with density 5.1 g/cm$^3$ [17] and Ce$^{3+}$ concentration $a$. Thus, in the crystal a fraction $a$ of the La$^{3+}$ ions is assumed to be replaced by Ce$^{3+}$ ions and the Ce$^{3+}$ density $\rho$ can be approximated using molar weights by:

\[
\rho = a \cdot 8.11 \cdot 10^{21} \text{ ions} / \text{cm}^3.
\]  

(2.3)

The optical absorption cross-section of each Ce$^{3+}$ ion $\sigma_{\text{cs,abs}}$ can be considered to consist of two components, a geometrical component $\sigma_{\text{cs,geom}}$, which is
2.3 Experimental methods

assumed to be energy-independent, and an energy-dependent component
described by equation 2.1, resulting in:

\[
\sigma_{cs,abs} = \sigma_{cs,geom} \cdot p_{abs,Ce^{3+}} = \frac{\sigma_{cs,geom}}{2\pi\sigma_{abs}} e^{\frac{(E-E_{abs})^2}{2\sigma_{abs}^2}}. \tag{2.4}
\]

In the absence of other absorption or scattering centers, the mean free path \(\lambda_{abs}\), also referred to as the absorption length, for a photon with energy \(E\) in a LaBr\(_3\):Ce\(^{3+}\) crystal equals:

\[
\lambda_{abs} = \left(\rho \cdot \sigma_{cs,abs}\right)^{-1}. \tag{2.5}
\]

In the presence of other absorption and/or scattering centers with an (energy-dependent) scattering length \(\lambda_{sca}\), it follows that the total attenuation length \(\lambda_{att}\) is determined by:

\[
\lambda_{att} = \frac{1}{\lambda_{abs}} + \frac{1}{\lambda_{sca}}. \tag{2.6}
\]

Thus, the probability of a photon with energy \(E\) to be transmitted over a distance \(d\) within the crystal without undergoing any interaction can be written as:

\[
P_{trans,d} = e^{-d/\lambda_{mi}} = e^{-d/\lambda_{mi}(E)} \left(1 - \frac{\rho\sigma_{cs,geom} e^{\frac{(E-E_{abs})^2}{2\sigma_{abs}^2}}}{\sqrt{2\pi\sigma_{abs}}} \right). \tag{2.7}
\]

Then, using equations 2.2 and 2.7, the probability distribution \(p_{em,d}\) for a scintillation photon that is observed after a distance \(d\) to have the energy \(E\) and to have been neither re-emitted nor scattered is given by:

\[
p_{em,d} = c_{em,d} \cdot p_{trans,d} \cdot p_{em,Ce^{3+}} = c_{em,d} e^{-d/\lambda_{mi}(E)} \left(1 - \frac{\rho\sigma_{cs,geom} e^{\frac{(E-E_{abs})^2}{2\sigma_{abs}^2}}}{\sqrt{2\pi\sigma_{abs}}} \right) \times \left\{ \frac{A}{\sqrt{2\pi\sigma_{em,1}}} e^{\frac{(E-E_{em,1})^2}{2\sigma_{em,1}^2}} + \frac{1-A}{\sqrt{2\pi\sigma_{em,2}}} e^{\frac{(E-E_{em,2})^2}{2\sigma_{em,2}^2}} \right\}, \tag{2.8}
\]

where \(c_{em,d}\) is a normalization constant.

2.3 Experimental methods

2.3.1 Refraction measurements and refractive index

The refractive index \(n\) of LaBr\(_3\):5%Ce\(^{3+}\) was measured as a function of wavelength using a method based on refraction. This method is relatively insensitive to the surface quality of the sample, since it does not utilize measured intensity or polarization, and it can be applied to samples contained within a
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holder with windows on opposite sites, circumventing problems associated with the hygroscopicity of LaBr$_3$:Ce$^{3+}$.

The measurement was performed on a polished LaBr$_3$:5\%Ce$^{3+}$ sample with outer dimensions of 2.2 cm × 2.2 cm × 2.2 cm and with one side beveled at an angle $\alpha$ of ~10 degrees. The sample was contained in an air- and moisture-tight holder equipped with vacuum seals and two quartz windows of 3 mm thickness. All parts were carefully baked out before assembly, which was done in a dry environment (glove-box).

A wavelength-tunable laser was positioned perpendicularly to a flat white screen at a distance $s = 502.5$ cm. The holder with the LaBr$_3$:5\%Ce$^{3+}$ sample was placed on a rotation stage, which could be read out with an accuracy of ~0.1°. This was done such that the beam entered the crystal in the center. The side opposite to the beveled side was perpendicular to the beam at a rotation of $\varphi = 0^\circ$ (see figure 2.1a). At a rotation of the sample of $\varphi = 180^\circ$, where the beveled side is facing the laser, the rotation is expressed as $\chi = \varphi - 180^\circ = 0^\circ$ (see figure 2.1b). In the experiment described here, four crystal orientations were used, viz. $\varphi = -10^\circ$, $\varphi = 0^\circ$, $\chi = 0^\circ$, and $\chi = 10^\circ$.

The distance from the crystal to the screen $s$ and the displacement of the laser spot $m$ compared to its position without sample were measured. This was performed for the set of wavelengths of 353.5, 356.5, 359, 365, 370, 375, 380, 390.5, 400, 405, 425, 441.25, 445, and 707 nm. It should be noted that because of the small lateral displacement of the beam within the crystal (see e.g. figure...
2.3. Experimental methods

For each set of four measurements at a single wavelength, the following relations can be obtained by applying Snell’s law for angles around $\phi$:

$$n \cdot \sin \left( \alpha - \arcsin \left( \frac{\sin(\phi)}{n} \right) \right) - \sin \left( \alpha - \phi + \arctan \left( \frac{m}{s} \right) \right) = 0,$$  

(2.9)

and for angles around $\chi$:

$$n \cdot \sin \left( \alpha - \arcsin \left( \frac{\sin(\alpha - \chi)}{n} \right) \right) - \sin \left( \chi + \arctan \left( \frac{m}{s} \right) \right) = 0.$$  

(2.10)

For each combination of two crystal orientations the appropriate set of equations, i.e. either equations 2.9 and 2.9, equations 2.9 and 2.10, or equations 2.10 and 2.10, were combined into a single equation by solving one of the equations for $\alpha$ and substituting the result in the other equation. This approach was taken because the angle of the beveled side $\alpha$ was not known exactly. Subsequently, the squared error between the left-hand side and the right-hand side of each of the six resulting equations was calculated as a function of $n$.

In order to determine the refractive index $n$ as a function of wavelength $\lambda$, the sum of all squared errors for all wavelengths was then minimized utilizing the Sellmeier equation [18]:

$$n = \sqrt{1 + \frac{B}{1 - C / \lambda^2}},$$  

(2.11)

where $B$ and $C$ are the so-called Sellmeier coefficients. In this work only one effective resonance wavelength is taken into account, i.e. a single set of Sellmeier coefficients is used. This is in correspondence with the assumption that only the absorption band corresponding to the lowest energy component 5d1 influences the absorption of scintillation light (see section 2.2.1). Furthermore, the relatively large uncertainties in the measurements (see section 2.4.1) and the limited range of wavelengths prohibited to resolve the potential (small) influence of other absorption bands on the refractive index.

According to specifications of the manufacturer the crystallographic $c$-axis (i.e. the optical axis) of our crystal was oriented perpendicularly to the laser beam. Therefore, both the refractive indices $n_{||}$ and $n_{\perp}$ for light polarized along and perpendicular to the $c$-axis, respectively, as well as the mean refractive index $n_m$ could be determined.

The uncertainties in $n$ and in the coefficients are determined by first estimating an effective uncertainty on the measured distances $m$ including the projection of all other experimental uncertainties such as the crystal surface levelness. To this end, the measurements at two crystal orientations at each wavelength were considered. From these, one of the measured distances $m_{\text{meas}}$ is
Chapter 2. Optical properties of LaBr$_3$:Ce$^{3+}$

inserted into either equation 2.9 or 2.10, as appropriate. The equation(s) were solved for the second distance $m_{\text{calc}}$ making use of the value of $n$ that was determined earlier, as described above. The error $e_m$ between $m_{\text{calc}}$ and the corresponding $m_{\text{meas}}$ was subsequently determined. This was done for all possible combinations of two crystal orientations at each wavelength, resulting in three values of $e_m$ per crystal orientation per wavelength. Using these values the mean absolute error $\bar{e}_m$ on $m$ was calculated for each wavelength.

Then, for each measured distance $m_{\text{meas}}$, 1000 new values were generated by adding a random value sampled from a uniform distribution with the same mean error $\bar{e}_m$. For each of the resulting 1000 sets of distances $m$, the Sellmeier coefficients were determined by minimizing the mean squared errors in equations 2.9 and/or 2.10. Using these 1000 sets of coefficients the standard deviation (SD) on $n$ could be calculated as a function of wavelength. Since a (linear) correlation between $B$ and $C$ was observed, the SD on these coefficients was determined with respect to a linear fit of this correlation. In other words, the SD of one coefficient was calculated given a fixed value of the other coefficient.

2.3.2 Transmission measurements and attenuation length

Two polished, bare, rectangular LaBr$_3$:5%Ce$^{3+}$ crystals (19.30 mm × 24.30 mm × 99.00 mm and 26.62 mm × 37.92 mm × 70.35 mm) provided by Saint-Gobain Crystals, Nemours, France, were used for transmission measurements. The crystallographic $c$-axis was along the longest dimension of both crystals. Each crystal was placed in an airtight sample holder. Extensive care was taken to not expose the samples to normal air or any other sources of humidity during all steps of sample manufacturing and assembly. On all sides, these holders were equipped with quartz optical windows with well-defined values for the optical attenuation and the refractive index. All planes of the windows and the crystals were specified to be parallel within $< 0.5^\circ$.

The samples in their holders were placed in a PerkinElmer LAMBDA 1050 spectrophotometer in which the beam was split into two light paths with equal optical path lengths to provide a reference spectrum including all light source fluctuations during a measurement. The sample was placed in the second light path as shown schematically in figure 2.2. The divergence of the beam was small, so that the path length of the photons through the crystal was known accurately. A diaphragm was placed in the beam path to prevent photons originating from scattering and luminescence within the crystal to reach the detector. It appeared that, when increasing the diameter of the diaphragm opening, no increase of the light intensity was measurable, indicating that the fraction of measured scattering and luminescence was in fact negligible.
2.3. Experimental methods

The transmission $T_{\text{meas}}$ was measured along the two shortest crystal axes of both crystals, i.e. along 19.30 mm, 24.30 mm, 26.62 mm, and 37.92 mm, in the wavelength range of 340 nm – 700 nm. The internal transmission (i.e. the bulk transmission) of the crystal $\tau_c$ was calculated for each of the measurements as described in the following.

The total transmission $T_c$ of the crystal can be calculated as:

$$T_c = \frac{(1 - R_c)^2 \tau_c}{1 - \tau_c^2 R_c^2},$$

(2.12)

where the crystal surface reflectance $R_c$ is obtained from the refractive index determined in section 2.3.1 using:

$$R_c = \left(\frac{n - 1}{n + 1}\right)^2.$$

(2.13)

Here, $n$ was set to $n_m$, since the beam was unpolarized and perpendicular to the crystallographic $c$-axis. The amount of scattering at the interface layers due to surface imperfections is assumed to be negligible. A definition for the total transmission of a single window $T_w$ similar to equation 2.12 can be obtained, making use of the surface reflectance $R_w$ and the internal transmission $\tau_w$ of the windows as specified by the manufacturer.

The combined transmission of the crystal and a single window is given by:

$$T_1 = \frac{T_w T_c}{1 - R_{w,\text{tot}} R_{c,\text{tot}}},$$

(2.14)

where $R_{c,\text{tot}} = R_c (1 + T_c \tau_c)$ and $R_{w,\text{tot}} = R_w (1 + T_w \tau_w)$ denote the total reflectance of the crystal and the window, respectively, taking into account all internal

Figure 2.2: Schematic overview of the transmission measurement setup. The beam is split in order to obtain a reference spectrum. Although not shown for simplicity of the figure, the reference and measurement beams have equal optical path lengths. A diaphragm is used to prevent detecting reflections as well as luminescence and scattering of the crystal (exemplified by the thin red lines).
Chapter 2. Optical properties of LaBr\(_3\):Ce\(^{3+}\)

reflections. Then, the total measured transmission of the crystal in between the two windows can be calculated in a similar way resulting in:

\[
T_{\text{meas}} = \frac{T_w^2 T_c}{\left(1 - R_{w,\text{tot}} R_{c,\text{tot}} \right)^2 - \left(R_{w,\text{tot}} T_c \right)^2},
\]

which can be solved numerically for the bulk transmission \(\tau_c\). The result, which in section 2.2.3 was referred to as \(P_{\text{trans},d}\), can be expressed in terms of the attenuation length \(\lambda_{\text{att}}\) using equation 2.7. Since a parallel beam is used, \(d\) equals the thickness of the crystal in the transmission measurements.

The above calculation of \(\lambda_{\text{att}}\) was performed for each of the four measurements using \(n = n_m\) as well as \(n_m\) plus and minus its standard deviation. The resulting twelve values per wavelength were used to calculate the mean attenuation length and its standard deviation.

It is assumed that in LaBr\(_3\):5\%Ce\(^{3+}\) no Ce\(^{3+}\) absorption occurs at wavelengths \(\lambda > 370\ \text{nm}\) and that there are no other absorption centers. This implies that \(\lambda_{\text{att}}\) equals the scattering length \(\lambda_{\text{sca}}\) at those wavelengths. This region is linearly fitted and by extrapolating this fit, \(\lambda_{\text{sca}}\) is obtained for shorter wavelengths. Then, using equations 2.6 and 2.7, the absorption length \(\lambda_{\text{abs}}\) is determined from \(P_{\text{trans},d} = \tau_c\).

2.3.3 Emission measurements

Emission spectra were recorded in a moisture free environment using the setup schematically depicted in figure 2.3. A set of four bare, cylindrical LaBr\(_3\):5\%Ce\(^{3+}\) crystals were used, having the same diameter of 25 mm but heights \(h\) of 5 mm, 10 mm, 20 mm, and 40 mm. The cylindrical surfaces of each crystal were kept rough, while the other surfaces were polished. The crystals were excited on one side by 8 keV X-rays generated with an X-ray tube equipped with a Be window, most of which are absorbed within the first \(\sim 10 \ \mu\text{m}\).
of the crystal. On the other side of the crystal a lens and a monochromator with a limited acceptance angle were used to read out the crystal. This setup assured that the scintillation photons traveled at least a distance equal to the crystal height $h$ through the crystal. At the same time, it restricted the fraction of reflected and scattered photons.

### 2.4 Results and discussion

In the following all measurement results are presented as a function of wavelength $\lambda$, although the equations and calculations in section 2.2 were given as a function of energy $E$.

#### 2.4.1 Refractive index

The results of the refractive index measurements are shown in figure 2.4. The error bars correspond to the standard deviations obtained as explained in section 2.3.1. The component $n_\perp$ is depicted in red (dash-dotted), $n||$ in blue (solid), and $n_m$ in green (dashed). The corresponding Sellmeier coefficients defined in equation 2.11 are displayed in table 2.1.

The mean error $\sigma_m$ on the distance $m$ (which itself was typically in the order of 1 m) was calculated to be $\sim$0.4 cm depending on the wavelength (see section 2.3.1). The main contribution to the uncertainty of the refractive index is expected to be the levelness of the crystal surfaces including a small potential tilt of the beveled side. Other contributions may originate from experimental inaccuracies in e.g. the crystal orientation and/or the alignment of the different components in the setup.

The measured values can be compared to the findings of other authors. Chivian et al. measured $n_m = 1.95$ for pure LaBr$_3$ and $n_m = 1.99$ for LaBr$_3$:10%Pr at 633 nm [19]. Our measurements with LaBr$_3$:5%Ce$^{3+}$ yield a

<table>
<thead>
<tr>
<th>Table 2.1: Sellmeier coefficients for the refractive indices $n</th>
<th></th>
<th>$, $n_\perp$, and $n_m$ of LaBr$_3$:5%Ce$^{3+}$ for light polarized along and perpendicular to the c-axis as well as their mean, respectively. The uncertainties denote the standard deviations.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n</td>
<td></td>
<td>$</td>
</tr>
<tr>
<td>$n_\perp$</td>
<td>$2.61 \pm 0.09$</td>
<td>$0.051 \pm 0.003$</td>
</tr>
<tr>
<td>$n_m$</td>
<td>$2.80 \pm 0.06$</td>
<td>$0.048 \pm 0.002$</td>
</tr>
</tbody>
</table>
somewhat larger value of \( n_m = 2.04 \) at this wavelength. Iltis et al. presented the refractive index of pure LaBr\(_3\) as estimated from density functional theory (DFT) simulations, providing values of \( n_\perp = 2.03 \) and \( n_\parallel = 2.09 \) at 380 nm [20], which are somewhat smaller than our measured values of \( n_\perp = 2.2 \) and \( n_\parallel = 2.3 \) at this wavelength. However, these DFT simulations, similar to the work presented in [21], were reported to require a correction to fit experimental data and detailed simulation details are not provided. Finally, Liu et al. calculated electronic dielectric constants \( \varepsilon_\infty \) of 4.77 and 5.18 corresponding to \( n_\perp = 2.18 \) and \( n_\parallel = 2.28 \), respectively, for optical wavelengths [22]. These values are in the range of our measured refractive indices and the difference between \( n_\perp \) and \( n_\parallel \) is in good agreement with our measurements.

### 2.4.2 Absorption and scattering lengths

The attenuation length of LaBr\(_3\):5\%Ce\(^{3+}\) determined from the transmission measurements is shown in figure 2.5. From this result, the scattering length and the absorption lengths were derived as described in section 2.3.2. The linear fit of \( \lambda_{\text{att}} \) in the long wavelength region resulted in: \( \lambda_{\text{sca}} \) (mm) = -104 mm + 0.598 \( \times \) \( \lambda \) (nm). This linear trend continues in the remainder of the wavelength range.
used in the experiment, i.e. up to \( \lambda = 700 \) nm.

In principle, the attenuation length might have been slightly underestimated due to the assumption that the amount of scattering at the crystal surfaces is negligible. However, surface scattering is expected to be much less wavelength dependent than the Ce\(^{3+}\) absorption. Therefore, given the fitting and extrapolation procedures used, it seems plausible that any potential surface scattering may have affected the values obtained for the scattering length only.

The absorption length is fitted with equation 2.5, where \( E_{\text{abs}} \) was kept constant at 4.03 keV corresponding to \( \lambda = 308 \) nm, i.e. the 5d\(_1\) excitation peak position obtained from excitation measurements on LaBr\(_3\):0.5\%Ce\(^{3+}\) at 10 K Van Loef et al. [23]. The fitted values for \( \sigma_{\text{cs,geom}} \) and \( \sigma_{\text{abs}} \) are shown in table 2.2. These values might be slightly influenced by the fact that the 5d\(_1\) excitation peak position might shift by a few nanometer at higher temperatures and at higher concentrations of Ce\(^{3+}\). Using the fit, one may obtain values of the absorption length at \( \lambda < 340 \) nm, while the large amount of absorption prohibits accurate measurement of \( \lambda_{\text{abs}} \) at those wavelengths.

Here, it should be noted that the re-emission probability for absorbed photons in LaBr\(_3\):Ce\(^{3+}\) is \( \sim 1 \) (see sections 2.2.1 and 2.2.2). Therefore, the limited absorption length in the lower wavelength range does not necessarily lead to a loss of scintillation photons in LaBr\(_3\):Ce\(^{3+}\) based detectors.

**Figure 2.5:** Attenuation-, absorption-, and scattering lengths as a function of wavelength. The error bars correspond to the standard deviations and are shown for an arbitrary set of wavelengths chosen for reasons of clarity.
2.4.3 Emission measurements

Emission spectra obtained from the measurements with crystals of different heights are represented by the symbols in figure 2.6. Each spectrum is normalized such that its area equals unity. The effect of self-absorption is clearly visible at the shorter wavelengths and results in a relative increase of the long-wavelength emission band observed for thicker samples.

The four spectra were fitted with equation 2.8, i.e. the probability distribution \( p_{em,d} \) for a scintillation photon that is observed after traversing a distance \( d \) through the crystal to have the energy \( E \) and to have been neither re-emitted nor scattered. As explained in section 2.3.3, an attempt was made to restrict the detected fraction of reflected, re-emitted, and scattered photons. A shared set of fitting parameters was used for \( A, \bar{E}_{em,1}, \sigma_{em,1}, \bar{E}_{em,2} \), and \( \sigma_{em,2} \), which determine the intrinsic emission spectrum of \( \text{LaBr}_3:5\%\text{Ce}^{3+} \) at room temperature, while \( d \) was a free parameter describing the mean traveling distance of the scintillation photons through the crystal before being detected. The fitted spectra are presented as solid lines in figure 2.6 and the values obtained for \( A, \bar{E}_{em,1}, \sigma_{em,1}, \bar{E}_{em,2} \), and \( \sigma_{em,2} \) are given in table 2.2. Figure 2.6 also shows the corresponding intrinsic emission spectrum (black dashed line). It appears that even in the thinnest (5 mm) sample, part of the scintillation light is being absorbed.

The fitted values of the mean photon traveling distances \( d \) in the four crystals were 5.1 mm, 14 mm, 37 mm, and 67 mm. These values are progressively larger than the corresponding crystal heights \( h \) of 5 mm, 10 mm, 20 mm, and 40 mm, respectively, which may be explained by the geometry of the measurement setup. Firstly, due to the larger aspect ratio of the longer crystals, these may act as a light guide towards the back of the crystal facing the monochromator. This could lead to the detection of a larger fraction of the photons that do not travel straight towards the monochromator, despite its limited acceptance angle. These photons may have undergone multiple reflections on the front and back surface.

<table>
<thead>
<tr>
<th>( \sigma_{cs,geom} ) (cm(^2))</th>
<th>( \sigma_{abs} ) (eV)</th>
<th>( \bar{E}_{abs} ) (eV)</th>
<th>( A )</th>
<th>( \sigma_{em,1} ) (eV)</th>
<th>( \bar{E}_{em,1} ) (eV)</th>
<th>( \sigma_{em,2} ) (eV)</th>
<th>( \bar{E}_{em,2} ) (eV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 1.3 \cdot 10^{-20} )</td>
<td>0.13</td>
<td>4.03(^*)</td>
<td>0.60</td>
<td>0.10</td>
<td>3.23(^*)</td>
<td>0.09</td>
<td>3.48(^*)</td>
</tr>
</tbody>
</table>

* Obtained from [23].

** 3.23 eV and 3.48 eV are equivalent to 384 nm and 356 nm, respectively.
2.5. Influence on detector performance

In the following, the significance of the obtained results is discussed. To this end the influence of the measured values of the refractive index and the attenuation as well as on the diffuse side surface. Secondly, the distance between the monochromator and the back of the crystal facing the monochromator is smaller for the longer crystals. Therefore, the solid angle of photons that may reach the monochromator is larger and more reflected photons may be recorded. Additionally, both effects may increase the amount of re-emitted photons detected. This may have artificially enhanced the long-wavelength band resulting in a somewhat larger fitted value for $d$.

It should be noted that the dependency of the emission spectrum on the Ce$^{3+}$ concentration observed by Glodo et al. might also be explained by means of equation 2.8, similar to the dependency on the photon traveling distance in this work.

2.5 Influence on detector performance

Figure 2.6: Measured emission spectra (symbols) of LaBr$_3$:5%Ce$^{3+}$ crystals of different heights excited by 8 keV X-rays. Each spectrum is normalized such that its area equals unity. The dashed black line indicates the intrinsic emission spectrum, obtained by fitting (solid lines) the measured spectra with the emission model, see equation 2.8, for samples with thicknesses of 5 mm, 10 mm, 20 mm, and 40 mm, respectively.
length of \( \text{LaBr}_3: \text{Ce}^{3+} \) on several scintillation detector performance parameters is assessed.

### 2.5.1 Refractive index

The reflective properties of the crystal surfaces are to a large extent determined by the refractive index. It was shown in section 2.4.1 that the refractive index of \( \text{LaBr}_3: \text{Ce}^{3+} \) in the \( \text{Ce}^{3+} \) emission wavelength band measures about 2.25 to 2.40 depending on wavelength and crystal orientation. This is considerably larger than the value of ~1.9 that has often been assumed to date [24]. Therefore, the reflectance of the crystal surfaces may be larger than expected (for example, ~50% larger at perpendicular incidence on a crystal-air interface), while the critical angle may be smaller than expected.

Both effects result in less light escaping the crystal at surfaces that are not coupled to a photosensor, which may lead to reduced light losses at these surfaces. Furthermore, in case the crystal has polished surfaces covered with a diffuse reflector, the ratio of specular and diffuse reflection and, consequently, light output, may be influenced. On the other hand, light may be less easily coupled into the photosensor. Altogether, the light transport and the photon collection efficiency can be quite different than expected.

The fact that the refractive index can vary by ~0.1 depending on crystallographic orientation and light polarization indicates that it may be favorable to adjust the crystal orientation such that the photosensor is coupled to a crystal surface with the lowest refractive index, i.e. a surface perpendicular to the \( c \)-axis. Then, scintillation photons may be coupled more efficiently into the photosensor, while at the same time less light losses may occur at the side surfaces.

### 2.5.2 Fraction of direct photons

The energy, timing, and spatial resolution of a scintillation detector can be strongly influenced by the optical photon transport in the scintillator. As an illustration, a monolithic scintillator detector is considered, which is under investigation for application in PET [25]. Such a detector utilizes a multi-channel photosensor array to decode the position of interaction in a scintillator of several cubic centimeters.

An important parameter determining the performance of a monolithic detector is the fraction of so-called direct photons, i.e. the fraction of all emitted scintillation photons that reach the photosensor without undergoing any optical interaction before being detected. The fraction of direct photons is primarily determined by geometrical considerations, notably the combination of the
acceptance angle of the photosensor array and the distance between the array and the origin of the scintillation photons. However, even if a photon is emitted towards the photosensor at an angle at which it can in principle be detected directly, it may still be scattered, or absorbed (and subsequently re-emitted) before reaching the photosensor. Although such interactions do not make a photon disappear, they generally result in loss of information about its origin and/or time of emission. In the following, we will investigate the influence of these effects on the fraction of direct photons. It should be emphasized that our considerations are restricted to bulk effects only. In practice, internal reflection at the crystal-photosensor interface(s) may be a significant additional factor reducing the fraction of direct photons.

The influence of absorption and scattering on the fraction of direct photons is estimated making use of equation 2.8 in combination with the absorption, scattering, and emission parameters obtained in sections 2.4.2 and 2.4.3. By fixing the normalization constant $c_{\text{em},d}$ at a value of 1, the integral of $p_{\text{em},d}$ over all $E$ predicts the fraction of scintillation photons that do not undergo any interaction in LaBr$_3$:Ce$^{3+}$ as a function of both the Ce$^{3+}$ concentration $a$ and the distance $d$ that the photons need to propagate through the crystal before being detected. The result is shown in figure 2.7.

It appears that for LaBr$_3$:Ce$^{3+}$ crystals with a Ce$^{3+}$ concentration of 5% and a height of $\sim$2 cm to $\sim$4 cm, which may be seen as a typical height for PET detectors, a fraction of $\sim$0.85 of the photons within the acceptance angle of the photosensor is not affected by bulk interactions for scintillation events occurring farthest away from the photosensor. For crystals with higher Ce$^{3+}$ concentrations [26], this factor may even decrease below $\sim$0.6.

### 2.5.3 Scintillation pulse shape and timing resolution

Scintillation photons that are absorbed in the bulk of a LaBr$_3$:Ce$^{3+}$ crystal and subsequently re-emitted carry essentially no spatial information anymore, as the re-emission can be assumed to occur isotropically. However, absorption with subsequent re-emission may also deteriorate the timing resolution of a detector, since re-emission again involves a decay time $\tau_{d,0}$ equal to the lifetime of the 5d state.

Assuming that the photon transit time spread is negligible and that the long-wavelength emission band is not affected by self-absorption, the fraction of absorbed photons $f_{\text{abs}}$ can be approximated as:

$$f_{\text{abs}} = \frac{A_{\text{obs}}}{A} - 1,$$

where $A$ and $A_{\text{obs}}$ are the intrinsic and observed probability of emission at this
Chapter 2. Optical properties of LaBr$_3$:Ce$^{3+}$

band, respectively. Since for LaBr$_3$:Ce$^{3+}$ the intrinsic decay time $\tau_{d,0}$ (~15 ns) is much larger than the intrinsic rise time $\tau_{r,0}$ (~0.4 ns) [26], most self-absorbed photons will not be re-emitted in the first part of the scintillation pulse. Therefore, the slope $S_{\text{obs}}$ of the rising edge of the observed scintillation pulse is expected to be approximately a factor of $1 + f_{\text{abs}}$ smaller than the slope $S$ of the intrinsic scintillation pulse:

$$S_{\text{obs}} \approx \frac{S}{1 + f_{\text{abs}}} \approx \frac{A}{A_{\text{obs}}} S.$$  \hspace{1cm} (2.17)

Consequently, the detector timing resolution may deteriorate by a factor $\sqrt{A_{\text{obs}} / A}$ [27]. At the same time, it is expected that an effective decay time $\tau_{d,\text{eff}}$ will be observed that is longer than $\tau_{d,0}$. Assuming that essentially all photons are re-emitted, this decay time lengthening can be approximated by [28]:

$$\tau_{d,\text{eff}} = \frac{A_{\text{obs}}}{A} \tau_{d,0}.$$  \hspace{1cm} (2.18)

Here, it should be noted that these estimations are for illustrative purposes only. In practice, other effects such as the photon transport may also have a significant effect on the measured pulse shape and, thus, on the timing resolution.

Figure 2.7: Predicted fraction of photons that do not undergo any scattering or absorption upon traveling a given distance (vertical axis) through a LaBr$_3$:Ce$^{3+}$ crystal with a given Ce$^{3+}$ concentration (horizontal axis). The fraction of unaffected photons varies from a value of ~1 at the bottom left of the figure to a value of ~0.45 at the top right of the figure.
2.5. Influence on detector performance

Figure 2.8 shows the relative decay time lengthening, which according to equation 2.17 is equal to the relative decrease of the slope of the rising edge of the scintillation pulse, as would be observed after a given (average) traveling distance of the photons (vertical axis) in the bulk of a LaBr$_3$:Ce$^{3+}$ crystal with a given Ce$^{3+}$ concentration (horizontal axis). The relative increase varies from a value of ~1 at the bottom left of the figure to a value of ~1.45 at the top right of the figure.

Figure 2.8 shows the relative decay time lengthening, which according to equation 2.17 is equal to the relative decrease of the slope of the rising edge of the scintillation pulse, as a function of both the Ce$^{3+}$ concentration $a$ and the (average) distance $d$ traveled by scintillation photons before being detected. The corresponding deterioration of the detector timing resolution is expected to be equal to the square root of this value.

In a LaBr$_3$:Ce$^{3+}$ scintillation detector having a thickness of e.g. ~2 cm, many photons will travel more than a few centimeters. If such detectors would use the commonly available 5% Ce$^{3+}$ concentration, it appears from figure 2.8 that the observed decay time would be ~10% to ~20% slower than expected.

Moreover, for crystals with higher Ce$^{3+}$ concentrations, which in principle have faster intrinsic rise times [26], the relative smaller slope of the observed rising edge can considerably suppress the positive effect of the faster intrinsic rise time. For example, crystals with 30% Ce$^{3+}$ concentration have been reported to have a ~2 times faster rise time than crystals with 5% Ce$^{3+}$ concentration, in principle resulting in a ~1.4 times better timing resolution for small samples.
with negligible self-absorption. However, for larger crystals, self-absorption and re-emission may deteriorate the timing resolution with a factor of ~1.15 to 1.2 depending on the (average) traveling distance of the scintillation photons before being detected. Therefore, the benefit of the higher Ce$^{3+}$ concentration may be reduced to only a factor of ~1.2 in such crystals.

### 2.6 Conclusions

In the present work the refractive index as well as the optical absorption and scattering lengths of LaBr$_3$:5%Ce$^{3+}$ have been determined experimentally for the first time.

The refractive index was measured to be about 2.25 to 2.40 depending on wavelength and crystal orientation. These values are considerably higher than the commonly assumed value of 1.9, for example resulting in a ~50% larger reflectance at perpendicular incidence on a crystal-air interface as well as in a smaller critical angle.

Furthermore, a model of the Ce$^{3+}$ absorption and emission probability as a function of wavelength, Ce$^{3+}$ concentration, and scintillation photon traveling distance has been developed. This model is used in combination with the measured absorption and scattering lengths to obtain the intrinsic emission spectrum of LaBr$_3$:Ce$^{3+}$ from measured emission spectra.

It was demonstrated that in crystals of a few cubic centimeters the fraction of direct photons may be influenced considerably by absorption and scattering, and that the improvement in intrinsic timing resolution of LaBr$_3$:Ce$^{3+}$ with larger Ce$^{3+}$ concentrations may at least partly be canceled by self-absorption and re-emission. These results illustrate the importance of the investigated crystal properties for scintillation detector design.

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### References

References


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A comprehensive model of the response of silicon photomultipliers

Chapter 3

Abstract

The response of a silicon photomultiplier (SiPM) to optical signals is inherently non-proportional due to saturation, afterpulsing, and crosstalk. Existing models of the SiPM response do not account for all of these effects and, therefore, these models are not sufficiently accurate for many applications. In this work a comprehensive model of the SiPM response is developed that is generally applicable to exponentially decaying light pulses and that can be simplified in the case of very short (e.g. laser) light pulses. The model accounts for the total number and the temporal distribution of the incident photons as well as for the relevant SiPM parameters, viz. the recovery time, afterpulsing, crosstalk, and their cross-correlations. The model is shown to correspond well with measurements on an SiPM-based scintillation detector. Furthermore, it is shown to be in agreement with several cases for which the SiPM response is known a priori. Having thus validated the model, its use is demonstrated by predicting the response of the Hamamatsu MPPC S10362-33-050C SiPM to several different scintillators.

3.1 Introduction

The silicon photomultiplier (SiPM), also referred to as, e.g. multi-pixel Geiger-mode avalanche photodiode (GM-APD or G-APD), multi-pixel photon counter (MPPC), solid state photomultiplier (SSPM), single-photon avalanche diode (SPAD) array, or pixelated Geiger-mode avalanche photon detector (PPD), is a
relatively new type of low-level-light sensor [1]–[7]. Since it is fast, has a high
gain, is compact, and is potentially cost-effective, it is increasingly being applied
in many areas, such as high-energy physics, medical imaging, astronomy,
quantum optics, photon counting, fluorescence spectroscopy, and bio-molecular
imaging [8]–[24].

In contrast to many other photosensors, the performance of any SiPM-based
detector may be affected significantly by the inherent non-proportionality of the
SiPM response. Saturation, afterpulsing, and crosstalk are the main contributors
in this, as they exhibit a non-linear relationship to the number of incident
photons detected in a given time interval. Because of these effects it is not trivial
to relate the measured, electronic signal to the corresponding optical signal and
vice versa. Nevertheless, for most applications it is desirable to understand and
quantify the SiPM response, e.g. in order to properly design a detector for a
given application, to perform corrections on measurements or on energy spectra,
to calibrate an SiPM detector for low-level-light measurements, to predict
detector performance, and/or to determine difficult-to-measure SiPM
parameters.

A commonly used simple exponential model of the SiPM response is only
valid for instantaneous light pulses and assumes zero afterpulsing and crosstalk
[25]. Models that account for the recovery of SiPM microcells have also been
proposed, describing the SiPM output amplitude either for a sequence of
instantaneous light pulses [26] or for light pulses of finite duration with a
constant photon flux [27]. However, in many applications these simple models
are not sufficiently accurate, for example because the influences of afterpulsing
and crosstalk are significant, the duration of the light pulse is not negligible
compared to the SiPM recovery time, and/or the photon flux is not constant.
Also, the latter model is based on functions for which no physical justification is
given [27].

The goal of the present work is to develop a comprehensive model of the
SiPM response that is generally applicable to exponentially decaying light
pulses. It accounts for the total number as well as the temporal distribution of
the incident photons, and for the relevant SiPM parameters, viz. the SiPM
recovery time, afterpulsing, crosstalk, and the cross-correlations between these
parameters. It will be shown that the model can be simplified in the case of short
(e.g. laser) light pulses. The model is validated by comparison to measurements
on an SiPM-based scintillation detector. Additionally, it is shown that the model
agrees with several cases for which the response is known a priori. Having thus
validated the model, its use is demonstrated by predicting the response of a
Hamamatsu MPPC S10362-33-050C SiPM coupled to various scintillators.
3.2 Silicon photomultipliers

SiPMs typically comprise between $\sim 10^2$ and $\sim 10^5$ parallel-connected GM-APDs, each reverse-biased at a voltage $V_{\text{bias}}$ and quenched by a resistor $R_q$ [28], [29]. The SiPM output current can be converted into a voltage using a shunt resistor $R_S$ and amplified with a (high input impedance) voltage preamplifier as indicated in figure 3.1.

When a photon is absorbed in one of the GM-APDs, henceforth referred to as (micro)cells, an electron-hole pair may be created and, subsequently, a discharge may be triggered (i.e., the cell fires). These two processes of creating an electron-hole pair and triggering a discharge have a joint probability $\varepsilon(\lambda) \leq 1$, which we define as the internal quantum efficiency (QE) of a single microcell for incident photons with wavelength $\lambda$. For a light pulse consisting of photons with different wavelengths, the effective internal quantum efficiency $\varepsilon$ can be defined as the weighted average of $\varepsilon(\lambda)$. It should be noted that our definition of $\varepsilon$ is consistent with the one generally used for photosensors, i.e. the fraction of absorbed photons that contribute to the measured photocurrent. The internal microcell QE is to be distinguished from the external one, which includes the wavelength- and angle-dependent absorption probability for incident photons. Additionally, the SiPM photon detection efficiency (PDE), i.e. the product of the external microcell QE and the fill factor of the active area, is to be discerned from the so-called detector PDE of e.g. an SiPM-based scintillation detector, which we define as the product of the internal microcell QE $\varepsilon$ and the photon

Figure 3.1: Schematic overview of an SiPM and its readout circuit. The SiPM comprises $\sim 10^2 – 10^5$ GM-APDs, each quenched by a quenching resistor $R_q$ and reverse biased at $V_{\text{bias}}$. The output current of the SiPM is converted into a voltage by the shunt resistor $R_S$ and amplified by a voltage preamplifier.
collection efficiency $f$ (i.e. the fraction of emitted photons that are absorbed in one or more microcells). The detector PDE thus includes effects related to the detector optical geometry, the use of absorbers and/or reflectors, etc. This is illustrated schematically in the left part of figure 3.2.

The direct triggering of a discharge by a photon originating from the light source (e.g. a scintillator or laser) will be called a primary trigger in the remainder of this document. Other trigger mechanisms, such as afterpulsing, crosstalk, and dark counts, will be discussed in section 3.3.2. It is pointed out that for two simultaneous triggers in the same cell, the resulting output charge still corresponds to one fired cell. After a discharge the cell recovers and new photons can be detected. If a discharge is triggered while the cell is still recharging, the output charge corresponding to the second photon will be reduced. These effects are commonly referred to as saturation of the SiPM.

### 3.2.1 SiPM gain and gain non-proportionality

A discharge in a microcell results in a current signal at the output of the SiPM. The gain $M$, defined as the expectation value of the number of output charge carriers due to a single fired cell, can be estimated as:

$$ M = \left( C_D + C_q \right) V_{\text{ob}} / e, $$

where $e$ is the elementary charge, $C_D$ is the microcell capacitance, $C_q$ is the parasitic capacitance of the quench resistor, and $V_{\text{ob}}$ is the voltage-overbreakdown, defined as $V_{\text{ob}} = V_{\text{bias}} - V_{\text{br}}$, where $V_{\text{br}}$ is the breakdown voltage of the GM-APD [30].
For the remainder of this work it is important to notice that the SiPM gain may not be constant during a light pulse with a duration of the same order of magnitude as the duration of the SiPM output pulse. This effect occurs if there is any resistance in series with the SiPM. In the readout scheme used in this work, see figure 3.1, the dominating resistance is $R_S$. In other cases this might be, for example, the input resistance of a charge sensitive pre-amplifier (CSP) and/or the resistance of filter circuitry at the power supply lines.

If one or more photons have created a discharge, the signal current will cause a voltage drop over $R_S$. This, in return, causes $V_{ob}$ to be reduced by the same absolute value. Depending on the number of fired cells and the value of $R_S$, this may lead to a significant reduction of $V_{ob}$. The gain, being proportional to $V_{ob}$, will be reduced by the same fraction. This effect is especially significant around the time when the voltage over $R_S$ peaks. An example of such gain reduction is schematically shown in figure 3.3 as a function of time.

Experiments in which very short light (e.g. laser) pulses are used are not affected by this gain non-linearity, since in those cases the gain has not yet changed significantly at the time of arrival of any of the photons. As illustrated in figure 3.3, however, this effect may lead to a significant reduction of the

![Figure 3.3: Schematic representation of the SiPM gain non-proportionality as a function of time, together with a scintillation pulse and a fast light pulse, illustrating that part of the primary triggers may have a lower gain if the duration of the light pulse is of the same order of magnitude as the duration of the SiPM output pulse.](image-url)
average gain for (e.g. scintillation) light pulses with a duration of the same order of magnitude as the duration of the SiPM output pulse, depending on $R_s$, on $V_{ob}$, and on how many cells were fired at which times prior to the firing of a given cell.

It should be noted that, for fast pulses, the SiPM gain may show another type of electronic non-proportionality, which is typically in the order of a few percent and finds its origin in the variation of the time needed to quench the discharge as a function of the number of coincidently fired cells [29].

### 3.3 SiPM response

Suppose an SiPM is illuminated by a light source generating a pulse of $N_{ph}$ photons. As defined in section 3.2, the photon collection efficiency $f$ equals the fraction of these photons that are absorbed in one or more SiPM microcells, giving rise to a total expected number of $N_{abs} = f N_{ph}$ absorbed photons. The corresponding expectation value of the number of primary triggers then equals $\varepsilon N_{abs} = \varepsilon f N_{ph}$, where $\varepsilon f$ is the detector PDE discussed in section 3.2. We now define the response $\chi$ of an SiPM as the ratio between the expectation value of the total output charge $\bar{N}_{f,eq}$ and the expectation value of the number of primary triggers:

$$\chi \equiv \frac{\bar{N}_{f,eq}}{\varepsilon N_{abs}} = \frac{\bar{N}_{f,eq}}{\varepsilon f N_{ph}}, \quad (3.2)$$

where $\bar{N}_{f,eq}$ is expressed in terms of the equivalent number of fired cells, defined as the ratio of the total signal charge and the charge due to a single-fired microcell. (In this work, expectation values are denoted with a bar on top of the variable.) Figure 3.2 schematically illustrates how $\chi$, together with $\varepsilon$ and $f$, determines the measured SiPM output charge $\bar{N}_{f,eq}$ as a function of $N_{ph}$. It should be noted that the total SiPM output charge in terms of the number of electrons can be obtained by multiplying $\bar{N}_{f,eq}$ with the SiPM gain $M$ defined in equation 3.1.

Since $\bar{N}_{f,eq}$ depends non-linearly on $\varepsilon$, $f$, $N_{ph}$, afterpulsing, crosstalk, and saturation, the SiPM response $\chi$ is not constant. Specifically, if $\varepsilon f N_{ph}$ is not small compared to the number of microcells $N_{cells}$, the response $\chi$ may be smaller than 1 due to saturation. Conversely, for $\varepsilon f N_{ph} \ll N_{cells}$ the response $\chi$ might be larger than 1 due to afterpulsing and crosstalk. It should be noted that $\chi$ is only defined for $\varepsilon f N_{ph} \neq 0$.

#### 3.3.1 Lower limit of SiPM response

A simple model of the effect of saturation on the response of an SiPM has been
3.3. SiPM response

presented by e.g. Stoykov et al. [25]. This model is applicable only to instantaneous light pulses and furthermore assumes that crosstalk, afterpulsing, and dark counts are absent. Under these conditions the lower limit of the SiPM response \( \chi_{\text{lower}} \) is reached, which can be written as:

\[
\chi_{\text{lower}} = \frac{N_{\text{cells}}}{\epsilon f N_{\text{ph}}} \left( 1 - e^{-\frac{\epsilon f N_{\text{ph}}}{N_{\text{dis}}}} \right). \tag{3.3}
\]

3.3.2 Factors affecting the SiPM response

In case the light pulse is not instantaneous but has a finite duration, such as a scintillation light pulse, it is possible that cells are fired again after they have (partially) recovered from a previous discharge. In this case, still neglecting afterpulsing, crosstalk, and dark counts, \( \chi_{\text{lower}} < \chi < 1 \).

A model for \( \chi \) that accounts for the time profile of a scintillation light pulse, the SiPM recovery time, afterpulsing, crosstalk, dark counts, and their cross-correlation, requires an adequate description of these processes, taking into account their temporal characteristics. Here it should be noted that many SiPM parameters are dependent on the voltage-over-breakdown \( V_{\text{ob}} \) as well as on the temperature. Therefore, the response should be calculated for each value of \( V_{\text{ob}} \) and for each temperature separately.

3.3.2.1 Time profile of a scintillation light pulse

In a scintillator, optical photons are created upon absorption of e.g. \( \gamma \)-ray or X-ray radiation. In this work it is assumed that the temporal distribution of this scintillation light pulse can be described adequately by a single exponential decay with time constant \( \tau_d \). It should be noted that the SiPM response model can relatively easily be adapted with a weighted sum of exponentials when necessary. The single-photon emission probability distribution \( p_{\text{sc}}(t) \) of a scintillation light pulse starting at \( t_0 = 0 \) is thus modeled as:

\[
p_{\text{sc}}(t) = \begin{cases} 
 1 - e^{-\frac{t}{\tau_d}}, & t \geq 0 \\
 0, & t < 0
\end{cases}. \tag{3.4}
\]

It is assumed that the travelling time spread of the optical photons in the scintillator is negligible, so that the probability distribution of the photons incident on the light sensor and, therefore, of the primary triggers, can be described with equation 3.4 as well.
Silicon Photomultiplier (SiPM) response model

Chapter 3. SiPM response model

3.3.2.2 SiPM recovery time

Suppose a microcell is triggered at $t = t_1$ by an absorbed photon. The discharge of the microcell then causes a current $I_q$ to flow through the quench resistor. An instantaneous discharge process is assumed, since the discharging of $C_D$ is at least an order of magnitude faster than the recharging. Then, the normalized current pulse $I_{q,\text{norm}}$ through the quench resistor in response to a single discharge starting at $t = t_1$ can be described as:

$$I_{q,\text{norm}}(t) = \begin{cases} 
\frac{1}{\tau_{\text{rec}}} e^{-\frac{t-t_1}{\tau_{\text{rec}}}} , & t \geq t_1 \\
0 , & t < t_1 
\end{cases} \quad (3.5)$$

where $\tau_{\text{rec}}$ is the SiPM recovery time given by [31]:

$$\tau_{\text{rec}} = \left( C_D + C_q \right) R_q . \quad (3.6)$$

Suppose the microcell is fired again by a second absorbed photon at some time $t = t_2$ while it is still recharging. The current pulse then continues as of $t = t_2$ as a new exponential decay according to equation 3.5, but with $t_1$ replaced by $t_2$. This is illustrated in figure 3.4.

More generally, when a cell is fired $k_{\text{max}}$ times and the time differences between triggers $k$ and $k-1$ are $\Delta t_{k,k-1}$, then the total output charge from a single

Figure 3.4: Example of the normalized current pulse through the quench resistor of a microcell induced by two discharges triggered at times $t_1$ and $t_2$. 

\[ \text{Figure 3.4: Example of the normalized current pulse through the quench resistor of a microcell induced by two discharges triggered at times } t_1 \text{ and } t_2. \]

\[ \text{Figure 3.4: Example of the normalized current pulse through the quench resistor of a microcell induced by two discharges triggered at times } t_1 \text{ and } t_2. \]
3.3. SiPM response

microcell \( N_{\text{f,eq,1}} \), expressed in the equivalent number of fired cells, is:

\[
N_{\text{f,eq,1}} = 1 + \sum_{k=2}^{\infty} \left( 1 - e^{-\frac{\Delta t_{k}}{\tau_{\text{eq}}}} \right).
\]  
(3.7)

It is noted that the normalized single microcell signal shape at the output of the SiPM might in fact differ somewhat from \( I_{q,\text{norm}} \), as it is shaped due to the various capacitances and inductances, which might not be lossless, in the SiPM [29], [30]. However, assuming that microcells do not influence each other, the total integrated signal of all cells at the output of the SiPM, \( N_{\text{f,eq}} \), expressed in terms of the equivalent number of fired cells, simply equals the sum of all \( N_{\text{f,eq,1}} \).

3.3.2.3 Afterpulsing

During each discharge there is a possibility that one or more traps in the microcell are occupied by charge carriers from the discharge. These may subsequently be released and trigger an additional discharge. This phenomenon is referred to as afterpulsing. The probability distribution \( p_{\text{nap}} \) of the number of afterpulses originating from a single discharge can be described as a Poisson distribution with expectation value \( N_{\text{ap}} \) [32].

For each trapped charge carrier that initiates an additional discharge, the probability distribution \( p_{\text{ap}}(\Delta t) \) of the elapsed time \( \Delta t \) between the triggering of the discharge causing the afterpulse and the triggering of the afterpulse itself can be described as a sum of multiple exponentials with time constants ranging from \(~10^{-8} \) s to \(~10^{-5} \) s [33]. However, the filled trap density corresponding to the smallest release time constant is at least \(~2 \) times larger than each of the other filled trap densities. Furthermore, when measuring the charge content of SiPM pulses, a time window is often applied with the result that afterpulses occurring a relatively long time after the primary trigger are not recorded. For these reasons only a single trapped-carrier release time constant \( \tau_{\text{ap}} \) is taken into account here:

\[
p_{\text{ap}}(\Delta t) = \begin{cases} 
\frac{1}{\tau_{\text{ap}}} e^{-\frac{\Delta t}{\tau_{\text{ap}}}}, & \Delta t \geq 0 \\
0, & \Delta t < 0
\end{cases}.
\]  
(3.8)

It should be noted that a weighted sum of exponentials could relatively easily be implemented in the SiPM response model when necessary. The effect of saturation on afterpulsing and how to correct for it are discussed in section 3.3.3.

3.3.2.4 Optical crosstalk

Another process, which may occur within a cell during a discharge, is the
emission of optical photons that may reach other cells and trigger additional discharges [34], [35]. This process is referred to as optical crosstalk and is considered to be instantaneous. The probability distribution $p_{\text{oct}}$ of the number of crosstalk-induced triggers due to a single primary trigger, including the possible cascade of crosstalk inducing other crosstalk, is assumed to be Poisson distributed with expectation value $\bar{N}_{\text{oct}}$. Then, if we neglect saturation and crosstalk following afterpulsing, the probability distribution $p_{\text{oct,1}}$ of the number of crosstalk-induced triggers in any given cell due to $\bar{N}_{\text{pt,1}}$ primary triggers in each of the other $N_{\text{cells}} - 1$ cells is Poisson distributed with expectation value:

$$\bar{N}_{\text{oct,1}} = \bar{N}_{\text{oct}} (1 + \bar{N}_{\text{oct}}) \bar{N}_{\text{pt,1}}.$$  

(3.9)

The sum of the number of primary triggers and the number of crosstalk-induced triggers is referred to as the number of photon-induced triggers in the remainder of this document. The effects of saturation and afterpulsing on crosstalk are discussed in section 3.3.3.

### 3.3.2.5 Dark counts

Thermally released charge carriers in the breakdown region of a microcell may give rise to discharges, which are referred to as dark counts. The expectation value of the output charge due to dark counts $\bar{N}_{\text{dc}}$, expressed in terms of the equivalent number of fired cells, can be calculated as the product of the expectation value of the dark count rate $r_{\text{dc}}$ and the integration time window $\Delta t_{\text{meas}}$ used in a measurement:

$$\bar{N}_{\text{dc}} = r_{\text{dc}} \Delta t_{\text{meas}}.$$  

(3.10)

Usually, $\Delta t_{\text{meas}}$ will be of the order of the order of the output signal duration.

When the average time between two dark counts, $r_{\text{dc}}^{-1}$, is much larger than $\tau_{d}$, $\tau_{\text{rec}}$, and $\tau_{\text{ap}}$, the influence of dark counts on the response is negligible, as is the case for the measurements in this work, see section 3.5.1. In case of a very slow scintillation signal, i.e. when $r_{\text{dc}}^{-1}$ is much smaller than $\tau_{d}$, $\tau_{\text{rec}}$, and $\tau_{\text{ap}}$, the response is not expected to suffer from saturation. In that case the influence of $\bar{N}_{\text{dc}}$ can simply be modeled as an offset to the response, where it should be noted that dark counts may also induce afterpulses and crosstalk. Finally, in cases where $r_{\text{dc}}^{-1}$ is of the order of $\tau_{d}$, $\tau_{\text{rec}}$, or $\tau_{\text{ap}}$, the model would require an extension to include dark counts and their cross-correlation with the other effects.

### 3.3.3 SiPM response to scintillation pulses

During a scintillation pulse, many different sequences of the various events described in the previous section are possible in each microcell. In order to calculate the SiPM response, we first calculate the expected output charge of a
3.3. SiPM response

single cell, for a fixed number of photon-induced triggers \( i \) and for fixed numbers of afterpulses \( j, k, \ldots \) for each photon-induced trigger. Then, using the probability of each possible sequence, the expected output charge of a single cell is calculated, from which the SiPM response is finally obtained. The output charge is henceforth expressed in terms of the equivalent number of fired cells, unless mentioned otherwise.

3.3.3.1 Single-microcell output charge

We define \( \psi_{i,j} \) as the expectation value of the output charge of a single cell, in the case of a single photon-induced trigger, giving rise to \( j \) afterpulses. Similarly, \( \psi_{2,j,k} \) denotes the expectation value of the output charge of a single cell, in the case of two photon-induced triggers in that cell, giving rise to \( j \) and \( k \) afterpulses, respectively. For each number of photon-induced triggers \( i > 2 \), the expectation value of the output charge of a single cell, giving rise to fixed numbers of afterpulses for each photon-induced trigger, can be denoted in an equivalent way.

\[ \psi_{i,j}, \psi_{i,j,k}, \ldots \] are functions of \( i, \tau_d, \tau_{\text{rec}}, \tau_{\text{ap}} \), and the numbers of afterpulses \( j, k, \ldots \). They take into account the temporal dependencies of the scintillator decay, the SiPM recovery, and the afterpulsing, each of which were discussed in section 3.3.2. The derivations of \( \psi_{i,j}, \psi_{i,j,k}, \ldots \) are given in Appendix A. For reasons of simplicity it is assumed that the probability of afterpulsing is small enough that afterpulses induced by afterpulses can be neglected. If the number of photon-induced triggers \( i = 0 \), there is no output charge and therefore \( \psi_0 = 0 \).

The probability of the number of afterpulses for each photon-induced trigger is Poisson distributed with probability distribution \( p_{\text{nap}} \) (see 3.3.2.3). We assume that the number of charge carriers trapped during a discharge is proportional to the total number of charge carriers in the discharge. Then, due to the finite recovery time of each cell, the average number of afterpulses for each photon-induced trigger occurring in a scintillation event is smaller than the average number of afterpulses \( \bar{N}_{\text{ap}} \) occurring upon a single discharge. Therefore, the probability distributions \( p_{\text{nap},i} \) of the number of afterpulses of each photon-induced trigger, for \( i \) photon-induced triggers occurring in a single cell, can be described as a Poisson distribution with expectation values \( \bar{N}_{\text{ap},i} \) calculated by multiplying \( \bar{N}_{\text{ap}} \) with the average relative amount of charge per photon-induced trigger:

\[ \bar{N}_{\text{ap},1} = \bar{N}_{\text{ap}} \frac{\psi_{i,0}}{1}, \quad \bar{N}_{\text{ap},2} = \bar{N}_{\text{ap}} \frac{\psi_{2,0,0}}{2}, \quad \bar{N}_{\text{ap},3} = \ldots \ldots \] (3.11)

Then, \( \xi_{i} \), which we define as the expectation value of the output charge of a single cell in the case of one photon-induced trigger, is calculated as the
expectation value of $\psi_{1,j}$:

$$\xi_i = \sum_{j=0}^{\infty} p_{\text{nap},1}(j) \psi_{1,j} . \quad (3.12)$$

Similarly, the output charge of a single cell in the case of two photon-induced triggers, $\xi_2$, is calculated as the expectation value of $\psi_{2,j,k}$:

$$\xi_2 = \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} p_{\text{nap},2}(j) p_{\text{nap},2}(k) \psi_{2,j,k} . \quad (3.13)$$

For $i > 2$, $\xi_i$ can be calculated in an equivalent way.

### 3.3.3.2 Numerical computation of the single-microcell output charge

The calculation of $\psi_{1,j}$, $\psi_{2,j,k}$, ... described in Appendix A result in series that grow rapidly in size with the number of photon-induced triggers $i$ and the corresponding numbers of afterpulses, leading to long computation times. In the remainder of this work it is therefore assumed that the average number $\bar{N}_{\text{ap}}$ of afterpulses due to a single fired microcell is less than $\sim 0.5$. Then, in order for the cumulative distribution function (CDF) of the number of afterpulses to reach $> 99\%$, $\xi_i$ only has to be calculated for up to three afterpulses for each absorbed photon.

If it is furthermore assumed that there are on average less than $\sim 4$ photon-induced triggers per microcell per light pulse, $\chi$ needs to be calculated for up to nine photon-induced triggers per microcell only in order for the CDF of the number of photon-induced triggers to reach $> 99\%$. When working with scintillation detectors this assumption is often satisfied as excessive saturation is usually avoided. It should nevertheless be noted that for $i > 9$, the numerical computation of $\xi_i$ requires so much time, that it may be more practical to obtain additional terms by fitting and extrapolating $\xi_i$ as a function of $i$ as demonstrated in appendix B.

### 3.3.3.3 SiPM output charge and response

Assuming, for the moment, that the scintillation photons are homogenously distributed over the SiPM, the expectation value of the total output charge of a single cell $N_{\text{eq},1}$ in response to a scintillation event can be obtained as follows.

A scintillator with absolute light yield $Y$ is considered, in which a gamma photon with energy $E_\gamma$ is absorbed, so that the number of created scintillation photons at the photopeak position equals $N_{\text{ph}} = E_\gamma Y$. A fraction $f$ of these photons is absorbed in one or more cells of the SiPM and may trigger discharges. Thus, $f$ depends on the crystal aspect ratio, reflective materials, losses within the crystal, etc. Provided that $\varepsilon/N_{\text{cells}} << 1$ and $f N_{\text{ph}} >> 1$, the probability distribution $p_{\text{pt},1}$ of
the number of such primary triggers in any cell is Poisson distributed with expectation value:

\[
\bar{N}_{pt,1} = \frac{\varepsilon}{N_{cells}} f N_{ph} .
\] (3.14)

Taking into account crosstalk as discussed in section 3.3.2.4, where it was assumed that the influence of saturation and afterpulsing on crosstalk is negligible, the probability distribution \(p_{pt,1}(i)\) of the number of photon-induced triggers \(i\) in any cell can be written as a Poisson distribution with expectation value:

\[
\bar{N}_{pt,1} \approx \bar{N}_{pt,1} + \bar{N}_{oct} \left(1 + \bar{N}_{oct}\right) \bar{N}_{pt,1} .
\] (3.15)

Here, use has been made of equation 3.9.

The total output charge of a single cell, \(\bar{N}_{f,eq,1}\), can now be calculated as the expectation value of \(\xi_i\):

\[
\bar{N}_{f,eq,1} = \sum_{i=1}^{\infty} \left[p_{pt,1}(i) \cdot \xi_i\right] .
\] (3.16)

However, the number of crosstalk photons being released during a discharge is proportional to the number of electrons in the discharge [34]. Hence, the average amount of crosstalk per photon-induced trigger due to a scintillation event differs from that due to a single fired cell, because of saturation and afterpulsing. A simple, first order correction on \(\bar{N}_{pt,1}\) can be obtained by multiplying \(\bar{N}_{oct}\) with an approximation of the average charge per photon-induced trigger, viz. \(\bar{N}_{pit,linint} / \bar{N}_{pt,1}\), where \(\bar{N}_{pit,linint}\) is linearly interpolated between the \(\bar{N}_{pt,1}\) corresponding to the two values of \(i\) closest to \(\bar{N}_{pit,1}\).

In this work, a more accurate corrected expectation value of the number of photon-induced triggers in any cell is used, as follows:

\[
\bar{N}_{pit,1,cor} \approx \bar{N}_{pt,1} + \bar{N}_{oct} \frac{\bar{N}_{f,eq,1}}{\bar{N}_{pit,1,cor}} \left(1 + \bar{N}_{oct} \frac{\bar{N}_{f,eq,1}}{\bar{N}_{pit,1,cor}}\right) \bar{N}_{pt,1} ,
\] (3.17)

which in combination with equation 3.16 is solved iteratively. It should be noted that crosstalk following afterpulses is included in the crosstalk due to primary triggers, resulting in a small overestimation of the number of photon-induced triggers and also in a small underestimation of the contribution of afterpulses to the total output charge.

Finally, since the expectation value \(\bar{N}_{f,eq}\) of the total output charge of the SiPM simply equals the product of \(N_{cells}\) and \(\bar{N}_{f,eq,1}\), the SiPM response \(\chi\) can be expressed as:

\[
\chi = \frac{N_{cells} \bar{N}_{f,eq,1}}{\varepsilon f N_{ph}} .
\] (3.18)
Chapter 3. SiPM response model

In cases where it cannot be assumed that the scintillation photons are homogenously distributed over the SiPM, a number of partial responses $\chi_k$ may be defined for $n$ groups of $N_{\text{cells},k}$ microcells, over each of which a fraction $f_k$ of the scintillation photons can be assumed to be absorbed homogenously. For each $k \leq n$, $\chi_k$ can then be calculated using equation 3.18, where $f$ is replaced by $f_k$ and $N_{\text{cells}}$ by $N_{\text{cells},k}$. The total response of the SiPM can then be calculated as the weighted average of the partial responses:

$$\chi = \sum_{k=1}^{n} \frac{f_k}{f} \chi_k,$$

where the number of groups $n$ is an integer between 1 and $N_{\text{cells}}$. For simplicity, it is assumed that the scintillation photons are homogenously distributed over the cells in the remainder of this work.

3.3.4 SiPM response to fast light pulses

If the light pulse is much shorter than the SiPM recovery time, it can be considered instantaneous. Then, if during a discharge a second photon triggers a discharge in the same cell, the output charge of that cell corresponds to only one discharge. Therefore, in case of a fast light pulse, the expectation value of the output charge of a single cell $\bar{N}_{\text{f,eq},1,\text{fast}}$ simplifies from equation 3.16 to:

$$\bar{N}_{\text{f,eq},1,\text{fast}} = \sum_{i=1}^{\infty} \left[ p_{\text{pit},1}(i) \right] \cdot \xi_1 = \left(1 - p_{\text{pit},1}(0)\right) \cdot \xi_1,$$

where it is assumed that crosstalk following afterpulses can be neglected. The response $\chi_{\text{fast}}$ for fast light pulses then simplifies from equations 3.14, 3.15, 3.18, and 3.20 to:

$$\chi_{\text{fast}} = \frac{N_{\text{cells}} \cdot \epsilon f N_{\text{ph}}}{\epsilon f N_{\text{ph}}} \times \left(1 - e^{-\left[1+R_{\text{rec}}(1+R_{\text{cell}})\right] \frac{\epsilon f N_{\text{ph}}}{N_{\text{cell}}} } \right) \frac{\xi_1}{\xi_1},$$

where $\xi_1$ can be simplified from equation 3.12 and from equation 3.29 in appendix A to:

$$\xi_1 = \sum_{j=0}^{\infty} \frac{\bar{N}_{\text{ap}}^j}{j!} e^{-\bar{N}_{\text{ap}}} \left(1 + j - \sum_{k=1}^{j} \frac{1}{k + \frac{\tau_{\text{rec}}}{\tau_{\text{rec}}}} \right),$$

where $k$ is an integer used for summing. It should be noted that $\chi_{\text{fast}}$ calculated in equation 3.21 equals $\chi_{\text{lower}}$ described by the simple exponential model in equation 3.3 if there is no afterpulsing and crosstalk, i.e. if $\bar{N}_{\text{cell}} = 0$ and $\xi_1 = 1$. 

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3.4 Materials, experiments, and methods

The SiPM response model is validated with measurements using two SiPMs type Hamamatsu MPPC S10362-33-050C (3 mm × 3 mm active area with 50 µm × 50 µm microcells [36]), henceforth referred to as detector 1 and detector 2. All measurements were performed at a temperature of 22 ± 1 ºC and at the nominal bias voltages specified by the manufacturer. The response of the SiPMs was measured as function of $N_{ph}$ and compared to the modeled response, using measured parameters of our specific devices as input for the model.

3.4.1 SiPM parameter measurements

The voltage-over-breakdown $V_{ob}$ was determined as described by Seifert et al. [29]. The value $R_q$ of the quench resistors was determined by forward-biasing the SiPM and measuring the IV-curve.

The SiPM gain $M$, i.e. the average output charge due to a single fired cell, was determined as follows. The SiPM output current signal was converted by a shunt resistor $R_S = 15 \, \Omega \pm 0.15 \, \Omega$ into a voltage, which was amplified by a non-inverting voltage amplifier based on a TI OPA847 operational amplifier, providing a gain of 13.27. The amplified signals were sampled with an Acqiris DC282 fast 10-bit digitizer at a sampling rate of 8 GS/s. In each measurement the input voltage range of the digitizer was set for an optimal use of the dynamic range of the digitizer so as to minimize the ADC noise. The sampling time window of 500 ns was chosen to match the length of the SiPM pulses.

The SiPM was illuminated with a Hamamatsu PLP-04 laser (wavelength 633 nm, average pulse duration 50 ps, repetition rate 10 kHz). The intensity of the light was regulated with neutral density filters such that, on average, ~30 photons reached the SiPM. The laser and the digitizer were triggered by the same clock generator.

The average pulse shape was determined by recording 50000 traces, correcting them for offsets, normalizing them to the same peak height and averaging them. The resulting averaged pulse was again corrected for offset so as to remove the homogeneous contribution of dark counts. It is emphasized that the averaged pulse is influenced by afterpulsing, resulting in a larger pulse area. This was corrected for by calculating and subtracting the contribution of afterpulsing by means of the response model.

Subsequently, an additional neutral density filter was added, such that on average only a few photons reached the SiPM. Again, 50000 traces were recorded and corrected for offset. All of these traces were fitted with the averaged pulse using the peak height as the fitting parameter. The obtained peak
heights were multiplied with the corrected area of the averaged pulse and a histogram was created. The resulting comb structure was fitted with a sum of Gaussians, where the distance between the peak positions was set to be constant. This distance is a measure of the gain observed at the output of the pre-amplifier, expressed in V·s. The SiPM gain $M$ for both devices was subsequently calculated by correcting for the pre-amplifier gain and dividing by the shunt-resistor value $R_S$ and the elementary charge $e$.

The recovery time $\tau_{\text{rec}}$ could then be calculated by inserting equation 3.1 into equation 3.6:

$$\tau_{\text{rec}} = \frac{eM}{V_{\text{ob}}} R_S.$$  \hspace{1cm} (3.23)

The SiPM parameters $\bar{N}_{\text{oct}}$, $\bar{N}_{\text{ap}}$, $\tau_{\text{ap}}$, and $r_{\text{dc}}$ were obtained in a similar way as described by Du and Retière [32]. For convenience, the method is briefly summarized and differences and additions are described where necessary. Both SiPMs were connected to a transistor-based, high-bandwidth, low-noise amplifier with a gain of ~50. The amplified signals were sampled with an Acqiris DC282 fast 10-bit digitizer at a sampling rate of 4 GS/s.

The average pulse shape was determined in the same way as described for the determination of $M$. Then, the laser was removed and 50000 random traces were recorded per detector using a time window of 2.5 μs. All traces were analyzed using a pulse finding method that worked in four steps. It located the first pulse by comparing the trace values to a certain threshold. Then, the pulse was fitted with either one or the sum of two average pulses, depending on the rise time and on the number of maxima of the pulse. Here, the starting time(s) of the pulse(s) and amplitude(s) were the fitting parameters. Finally, the starting time(s) and the corresponding amplitude(s) were stored and the fit was subtracted from the trace. This was repeated, until no more pulses were found in the trace.

For all pulses the time difference with the preceding pulse was calculated and stored together with its amplitude. To exclude afterpulses from earlier discharges, only those pulses were used for which the 300 ns before the preceding pulse contained no other pulses. Furthermore, only those pulses were used for which the amplitude of the preceding pulse corresponded to a single discharge.

From the resulting amplitude vs. time-difference diagram (showing a similar horizontal band structure of amplitudes corresponding to 1, 2, 3, … fired cells as presented by Du and Retière [32]), the probability $P_{1,\text{oct}}$ that one fired cell triggers exactly one other cell was determined from the fraction of pulses $f_{1,\text{oct}}$ with exactly one crosstalk-induced trigger (i.e. the band of amplitudes...
corresponding to two fired cells). Given this definition of $P_{1,\text{oct}}$, which is different from the way crosstalk is characterized by Du and Retière [32], one needs to take into account that the same fraction $f_{1,\text{oct}}$ of the single crosstalk-induced triggers also induces crosstalk:

$$P_{1,\text{oct}} = f_{1,\text{oct}} \sum_{k=0}^{\infty} f_{1,\text{oct}}^k = \frac{f_{1,\text{oct}}}{1 - f_{1,\text{oct}}}.$$  \hspace{1cm} (3.24)

For the determination of $f_{1,\text{oct}}$, only time differences larger than 200 ns were taken into account, since crosstalk originating from afterpulses cannot be separated from dark counts in the amplitude vs. time-difference diagram.

The expectation value $\tilde{N}_{\text{oct}}$ of the number of crosstalk-induced triggers was subsequently calculated as:

$$\tilde{N}_{\text{oct}} = \sum_{k=1}^{\infty} 2^{k-1} \left( P_{1,\text{oct}} \right)^k = \frac{(1 - f_{1,\text{oct}}) f_{1,\text{oct}}}{2(1 - 3 f_{1,\text{oct}})},$$  \hspace{1cm} (3.25)

where the factor $2^{k-1}$ accounts for the possibilities to induce $k$ discharges including crosstalk-induced crosstalk.

In addition, a histogram of the time differences of the selected set of recorded pulses was created and normalized to the area of the histogram. The parameters $\tilde{N}_{\text{ap}}, \tau_{\text{ap}}$ and $r_{\text{dc}}$ could then be determined by fitting the histogram with a function $P(t)$, which, because of a misprint in [32], is repeated in its correct form here:

$$P(t) = \left[ 1 - \int_{0}^{t} P_{\text{AP}}(x) \, dx \right] P_{\text{DN}}(t) + \left[ 1 - \int_{0}^{t} P_{\text{DN}}(x) \, dx \right] P_{\text{AP}}(t),$$  \hspace{1cm} (3.26)

where $P_{\text{AP}}(t)$ and $P_{\text{DN}}(t)$ are the probabilities that the first afterpulse or dark count, respectively, occurs in the time interval $t$ and $t + dt$ after the starting pulse.

### 3.4.2 SiPM response measurements

Measurements of the SiPM response were performed using two 3 mm × 3 mm × 5 mm LaBr$_3$:5%Ce scintillators optically coupled to the SiPMs, using a Silicone encapsulation gel (Lightspan LS-3252). All crystal faces not coupled to the SiPM were covered with a highly reflective PTFE-based material (Spectralon). Since LaBr$_3$:5%Ce has a high light yield (70 ph/keV at 662 keV [37]) and a fast decay time ($\tau_d = 16$ ns [38]), significant saturation of the SiPM is expected, resulting in a clear dependency of the response on the number of emitted photons. The SiPM signals were amplified and digitized in the same way as described for the determination of $M$ in the previous section, except that a sampling rate of 4 GS/s was used.

Pulse height spectra were recorded for several isotopes, i.e. $^{125}$I, $^{241}$Am,
Chapter 3. SiPM response model

\[ ^{203}\text{Hg}, \; ^{22}\text{Na}, \; ^{137}\text{Cs}, \; ^{88}\text{Y}, \; \text{and} \; ^{60}\text{Co}, \; \text{emitting gamma photons at 10 different energies between 27.3 keV and 1836 keV. The numbers of emitted photons} \; N_{\text{ph}} \; \text{corresponding to 27.3 keV and 59.5 keV were calculated taking into account the light yield non-proportionality of LaBr}_3:\text{Ce with values of 0.964 and 0.986, respectively [39]. Between 10000 and 50000 pulses were recorded for each isotope. The pulses were corrected for offset by subtracting the mean value of the 25 data points preceding the pulse, and subsequently integrated. In order to express the resulting values in terms of the equivalent number of fired cells, they were divided by the gain} \; M \; \text{determined in the previous section. A histogram was created and the position of the photopeak was determined by fitting it with a Gaussian. Due to the good energy resolution} \; R_{\text{sc}} \; \text{of LaBr}_3:\text{Ce (~3% FWHM at 662 keV) [40] it can be assumed that the SiPM response only affects the photopeak position while negligibly affecting its shape.}

\]

The output charge corresponding to the photopeak position was subsequently corrected for the gain non-proportionality described in section 3.2.1. To this end, the voltage-over-breakdown \( V_{\text{ob}}(t) \) as a function of time was estimated by subtracting the average output pulse corresponding to the photopeak position from the undisturbed \( V_{\text{ob}} \). By multiplying the calculated \( V_{\text{ob}}(t) \) with the probability distribution of the scintillation pulse \( p_{\text{sc}}(t) \) and integrating this product, a weighted average of \( V_{\text{ob}}(t) \) over the duration of the scintillation pulse was calculated. This weighted average was divided by the value of the undisturbed \( V_{\text{ob}} \), yielding the relative gain non-proportionality. It should be noted that in case of a slower or lower light yield scintillator, it may be required to perform this correction on an event-per-event basis.

3.4.3 Detector PDE determination

To calculate the SiPM responses from the measurements and from the model, and to compare them as a function of \( N_{\text{ph}} \), the value of the detector PDE, \( \varepsilon \cdot f \), needs to be determined. Since the detector PDE depends on the detector geometry, the scintillator, the use of reflecting materials, etc., it is difficult to measure it directly. Therefore, the value of \( \varepsilon \cdot f \) was chosen so as to match the modeled and the measured response at the measurement that is least influenced by saturation, i.e. the one at 27.3 keV, corresponding to 1842 emitted photons. Here, literature values of the absolute light yield of LaBr\(_3\):Ce have been used [37], [39]. It should be noted that the model requires the product of \( N_{\text{ph}} \) and \( \varepsilon \cdot f \) as an input. Therefore, in case one uses the model to derive an accurate value of the detector PDE, also the absolute light yield needs to be determined accurately.
3.5 Results and discussion

3.5.1 SiPM parameters measurements

The measured voltage-over-breakdown $V_{ob}$, quench resistor value $R_q$, SiPM gain $M$, and SiPM recovery time $\tau_{rec}$ are given in table 3.1. The corrections on $M$ due to afterpulsing were equal to factors of 1.076 and 1.087 for detector 1 and 2, respectively. The values of $M$ are in good correspondence with those in the data sheet of the manufacturer for the 1 mm $\times$ 1 mm version of the same type of SiPM (Hamamatsu MPPC S10362-11-050C) at the same temperature [41].

The measurements for the determination of $\tilde{N}_{oct}$, $\tilde{N}_{ap}$, $\tau_{ap}$, and $r_{dc}$, see section 3.4.1, yielded an amplitude vs. time-difference diagram and a time-difference histogram with similar features as those presented by Du and Retière [32]. The derived parameter values are given in table 3.1.

The values of $\tilde{N}_{ap}$ and $\tau_{ap}$ are very similar to the values obtained by Du and Retière for the 1 mm $\times$ 1 mm version of the same type of SiPM (Hamamatsu MPPC S10362-11-050C), viz. $\sim$0.13 for $\tilde{N}_{ap}$ and $\sim$15 ns for $\tau_{ap}$ [32]. However, it is emphasized that the determination of $\tilde{N}_{ap}$ and $\tau_{ap}$ may be influenced by the pulse finding method and by the range used for fitting the time-difference histogram.

The dark count rates are close to the value of 6 MHz at 25 °C specified by the manufacturer [36]. This means that a dark count occurs on average every $\sim$180 ns, which is much longer than $\tau_d$. Hence, dark counts can indeed be neglected.

Values for $f_{1,oct}$ of 0.085 and 0.082 were obtained for detector 1 and 2, respectively. The respective probabilities that one fired cell triggers at least one other cell, i.e. 0.093 and 0.089, are very similar to the values obtained by Du and

Table 3.1: Measured SiPM parameter values of the two Hamamatsu MPPC S10362-33-050C devices. All uncertainties quoted are 95% confidence intervals, based either on fits or on estimated measurement uncertainties.

<table>
<thead>
<tr>
<th>Detector</th>
<th>$V_{ob}$ (V)</th>
<th>$R_q$ (kΩ)</th>
<th>$M$ ($10^5$)</th>
<th>$\tau_{rec}$ (ns)</th>
<th>$\tilde{N}_{ap}$</th>
<th>$\tau_{ap}$ (ns)</th>
<th>$r_{dc}$ (MHz)</th>
<th>$\tilde{N}_{oct}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Detector 1</td>
<td>1.26 ± 0.05</td>
<td>145.3 ± 0.5</td>
<td>8.3 ± 0.5</td>
<td>15.3 ± 1</td>
<td>0.124 ± 0.005</td>
<td>25 ± 8</td>
<td>5.6 ± 0.1</td>
<td>0.140 ± 0.005</td>
</tr>
<tr>
<td>Detector 2</td>
<td>1.39 ± 0.05</td>
<td>140.4 ± 0.5</td>
<td>7.9 ± 0.5</td>
<td>12.8 ± 1</td>
<td>0.132 ± 0.005</td>
<td>26 ± 8</td>
<td>5.1 ± 0.1</td>
<td>0.132 ± 0.005</td>
</tr>
</tbody>
</table>
Retière for the 1 mm \( \times \) 1 mm version of the same type of SiPM (Hamamatsu MPPC S10362-11-050C), viz. \(~0.09\) [32].

### 3.5.2 SiPM response measurements

The relative SiPM gain non-proportionality was estimated for the different gamma energies used in the pulse height measurements. The result is shown in figure 3.5. It can be seen that larger energies, which induce larger SiPM signals, result in a larger temporary reduction of the voltage-over-breakdown. It should be noted that due to saturation the weight of the first part of the photon pulse might be relatively larger, resulting in an uncertainty of the relative gain non-proportionality.

All pulse height spectra were corrected for the estimated gain non-proportionality. Figure 3.6 shows the resulting pulse heights as a function of the number of scintillation photons \( N_{ph} \) per event, for both SiPMs.

### 3.5.3 Model validation

The values of the detector PDE of the SiPM-based detectors used in this work were determined to be 0.16 and 0.17 for detector 1 and 2, respectively. Using these values and the parameters measured in section 3.5.1, the response of the
two SiPMs as a function of $N_{ph}$ was predicted with the model. To this end, $\xi_i$ was calculated for $1 \leq i \leq 9$ using the method described in section 3.3.3 and appendix A. However, the expectation value $N_{pid,1}$ of the number of photon-induced triggers in any cell is $\sim 7$ for the largest gamma energy used in the measurements, viz. 1836 keV. Therefore, in order for the cumulative distribution function of the number of photon-induced triggers per cell to reach > 99%, the values of $\xi_i$ were fitted and extrapolated to $i = 14$ as described in appendix B. Figure 3.7 shows both the modeled and the measured response. Good agreement is observed.

There are a number of potential reasons for the remaining discrepancies. They can at least partly be attributed to the uncertainties in the many different measurements performed to determine the various interdependent parameters, including the correction of the gain non-proportionality. The largest contribution to the overall uncertainty is in fact expected from the measurement of $M$, which involves measuring the small signals resulting from only a few photons. Because of this and the relatively large device capacitance, noise causes the determination of $M$ to be difficult. Furthermore, small temperature differences between the gain determination measurement and the SiPM response.
measurement could have contributed to an uncertainty of $M$. These uncertainties of $M$ could indirectly affect the uncertainty in $\varepsilon \cdot f$ and $\tau_{\text{rec}}$ as well. It should be noted that the measurements at the lowest gamma energy, i.e. 27.3 keV, are of large significance to both the modeled and the measured response, since these are used for the determination of $\varepsilon \cdot f$.

A second potential cause of uncertainty concerns the assumption of an instantaneous discharge process. In this simplification the probability is neglected that during the quenching of the discharge, i.e. during the rise time of the current signal through the quench resistor, additional triggers might occur, which do not contribute to the output charge. The rise time of the current signal through the quench resistor can thus be seen as a dead time. The effect of this dead time will be more pronounced when triggers occur more closely together in time. This might result in a small overestimation of the response for the measurements at the higher gamma energies.

### 3.5.4 Model comparison to known cases

The SiPM response was predicted as a function of $N_{\text{ph}}$ for three different cases for which the behavior of the response is known 	extit{a priori}, viz. for a scintillator decay time much longer than $\tau_{\text{rec}}$ and $\tau_{\text{ap}}$, i.e. $\tau_d = 10$ µs; for an instantaneous light pulse without afterpulsing and crosstalk, i.e. $\tau_d = 0$, $\overline{N}_{\text{oct}} = 0$, and $\overline{N}_{\text{ap}} = 0$;
3.5. Results and discussion

and for an instantaneous light pulse including afterpulsing and crosstalk. All other SiPM parameters as well as \( \epsilon \cdot f \) were kept equal to the values determined for detector 1. The result is presented in figure 3.8 in comparison to the simple exponential model given by equation 3.3.

The response for \( \tau_d = 10 \, \mu s \) (blue dashed curve) is nearly constant, which is to be expected since \( \tau_d \) is much larger than \( \tau_{rec} \) and \( \tau_{ap} \). Therefore, on average each cell is fully recovered when it is triggered again by another photon.

For \( \tau_d = 0 \), \( N_{oct} = 0 \), and \( N_{ap} = 0 \) (green solid curve), the model predicts the same response as the simple exponential model (black dash-dotted curve), as expected.

For \( \tau_d = 0 \) including afterpulsing and crosstalk (red dotted curve), the response converges towards the case that \( \tau_d = 10 \, \mu s \) for small \( N_{ph} \) and towards the case that \( \tau_d = 0 \), \( N_{oct} = 0 \), and \( N_{ap} = 0 \) for large \( N_{ph} \). In the first case saturation is absent, while in the second case the recovery of the cells is negligible as the light pulse is instantaneous. Therefore, for large \( N_{ph} \) the response converges towards \( N_{cells} / \epsilon f N_{ph} \) when afterpulsing and crosstalk are neglected, and towards \( \xi_1 N_{cells} / \epsilon f N_{ph} \) when afterpulsing and crosstalk are included.

Figure 3.8: The modeled SiPM response as a function of the number of emitted photons \( N_{ph} \), for \( \tau_d = 10 \, \mu s \) (dashed blue curve), for \( \tau_d = 0 \) with (red dotted curve), and without (green solid curve) after-pulsing and crosstalk, using the SiPM parameters and the detector PDE determined for detector 1. The response \( \chi_{lower} \) calculated with the simple exponential model (equation 3.3) is also shown (dash-dotted black curve).
Consequently, both curves converge towards each other and towards 0. This is indeed observed in figure 3.8. For completeness, it should be noted that the simplified model for fast light pulses (equation 3.21) gives the same result as the full model (equation 3.18) if $\tau_d = 0$.

3.5.5 Response predictions for several scintillators

Finally, having compared the model with experiments and several a priori known cases, its application is demonstrated by predicting the response of the Hamamatsu MPPC S10362-33-050C SiPM to five common scintillators, viz.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig3_9.png}
\caption{Predicted response of a Hamamatsu MPPC S10362-33-050C SiPM as a function of the detector PDE, $\varepsilon f$, for various scintillators irradiated with 511 keV gamma quanta. The light yield and decay time of each scintillator is given in table 3.2.}
\end{figure}

Table 3.2: Scintillator light yields and decay times.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Unit</th>
<th>BGO</th>
<th>CsI:Tl</th>
<th>NaI:Tl</th>
<th>LYSO</th>
<th>LaBr$_3$:Ce</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light yield, $Y$</td>
<td>(ph/keV)</td>
<td>6.3</td>
<td>56</td>
<td>53</td>
<td>27</td>
<td>70</td>
<td>[37]</td>
</tr>
<tr>
<td>Decay time, $\tau_d$</td>
<td>(ns)</td>
<td>300</td>
<td>1000</td>
<td>250</td>
<td>41</td>
<td>16</td>
<td>[38]</td>
</tr>
</tbody>
</table>
3.6 Conclusions

BGO, CsI:Tl, NaI:Tl, LSO:Ce, and LaBr$_3$:Ce, irradiated with 511 keV gamma quanta as used in positron emission tomography (PET). Since the detector PDE, $\varepsilon \cdot f$, depends on the detector geometry, the use of reflecting materials, etc., the response is calculated as a function of $\varepsilon \cdot f$. The values of the light yield $Y$ and the decay time $\tau_d$ of each scintillator, obtained from literature, are listed in table 3.2. In order for the cumulative distribution function of the number of photon-induced triggers per cell to reach > 99% for each scintillator, the values of $\xi_i$ were fitted and extrapolated up to $i = 18$ where necessary. It should be noted that in case of scintillators with longer decay time constants, dark counts and afterpulses with longer time constants may have to be taken into account for more accurate results. The resulting responses are presented in figure 3.9.

It appears that the response for BGO is almost independent of $\varepsilon \cdot f$. Despite the higher light yield of CsI:Tl, its response is also nearly constant due its long decay time. In contrast, a significant reduction of the response may be expected for fast and bright scintillators such as LYSO and LaBr$_3$:Ce, depending on the detector PDE.

3.6 Conclusions

A model of the response of SiPMs that is generally applicable to exponentially decaying light pulses has been developed and validated. In contrast to existing models based on very simple assumptions, it accounts for the total number and the temporal distribution of the incident photons as well as for the SiPM recovery time, afterpulsing, crosstalk, and their cross-correlations. This allows for much more accurate calculations on the performance of SiPMs.

The model was shown to agree well with response measurements on two Hamamatsu MPPC S10362-33-050C SiPMs coupled to LaBr$_3$:Ce crystals. It is also in agreement with several cases for which the response is known a priori. It was shown how the model can be simplified in the case of very short (e.g. laser) light pulses.

To demonstrate the use of the model, the response of the Hamamatsu MPPC S10362-33-050C SiPM to various scintillators was predicted. Another application of the response model was demonstrated during its validation, where it was used to determine the detector PDE, which otherwise would have been difficult to measure (see sections 3.4.3 and 3.5.3).

Further development of the response model, e.g. by including dark counts and the rise time of the current signal through the quench resistor, might make the model even more accurate. However, this would also make the equations in appendix A more complex.
Chapter 3. SiPM response model

As a final remark, it is noted that the C source files, each including a MATLAB gateway routine, for the calculation of $\xi_i$ for $1 \leq i \leq 9$ according to section 3.3.3 and appendix A, are available via the corresponding author.

Appendix A

This appendix shows the derivation of $\psi_{i,j}$, $\psi_{i,j,k}$, ... for $1 \leq i \leq 4$. Higher order terms can be obtained by logically extending the results of these first four terms.

$i = 1$

The expectation value $\psi_{1,j}$ of the output charge of a single cell due to a single photon-induced trigger giving rise to $j$ afterpulses can be obtained using equations 3.4, 3.7 and 3.8. For example, $\psi_{1,1}$ equals the integral over the product of the probability $p_{sc}(t_1)$ that a photon-induced trigger is created after a time interval $t_1$ following the start of a light pulse at $t_0 = 0$, and the probability $p_{ap}(t_{11})$ that an afterpulse is created after a time interval $t_{11}$ following $t_1$, multiplied by the output charge $N_{f,eq,1}(t_{11})$ corresponding to this sequence of events, which is schematically represented in figure 3.10:

$$
\psi_{1,1} = 1! \int_0^\infty \int_0^\infty p_{sc}(t_1)p_{ap}(t_{11})N_{f,eq,1}(t_{11}) \, dt_1 dt_{11} .
$$

(3.27)

The possible chronological permutations of the photon-induced triggers and the afterpulses are described by the factorial terms in front of the integrals.

Similarly, $\psi_{1,2}$ is given by:

$$
\psi_{1,2} = 1!2! \int_0^\infty \int_0^\infty \int_0^\infty p_{sc}(t_1)p_{ap}(t_{11})p_{ap}(t_{11} + t_{12})N_{f,eq,1}(t_{11},t_{12}) \, dt_1 dt_{11} dt_{12} ,
$$

(3.28)
where $t_{12}$ is the time interval between the first and the second afterpulse.

By examining higher order terms and evaluating the integrals, it appears that $\psi_{1,j}$ can be written as:

$$\psi_{1,j} = 1! \cdot j! \left( \frac{1}{\tau_{\text{ap}}} \right) \left[ \prod_{a_1=1}^{j} h(a_1, 0, 0) \left\{ 1 + j - \sum_{a_1=1}^{j} h(a_1, 0, 0) \right\} \right],$$  

(3.29)

where $a_1$ is an integer used for summing as well as multiplication, and $h(a,b,c)$ is defined as:

$$h(a,b,c) = \frac{\tau_d \tau_{\text{ap}} \tau_{\text{rec}}}{a \tau_d \tau_{\text{rec}} + b \tau_{\text{ap}} \tau_{\text{rec}} + c \tau_d \tau_{\text{ap}}}.$$  

(3.30)

$i = 2$

The expectation value $\psi_{2,j,k}$ of the output charge of a single cell due to two photon-induced triggers giving rise to $j$ and $k$ afterpulses, respectively, is again calculated by integrating over the probabilities of all possible sequences of events. For example:

$$\psi_{2,1,1} = 2! \cdot 1! \int_0^\infty \int_0^\infty \int_0^\infty q(t_1, t_{11}, t_2, t_{21}) N_{\text{eq},1}(t_1, t_{11}, t_2, t_{21}) dt_1 dt_{11} dt_2 dt_{21},$$  

(3.31)

where $q(t_1, t_{11}, t_2, t_{21})$ denotes the three possible sequences of events:

$$q(t_1, t_{11}, t_2, t_{21}) =
\begin{align*}
p_{\text{sc}}(t_1) p_{\text{ap}}(t_1) p_{\text{sc}}(t_1 + t_{11} + t_2) p_{\text{ap}}(t_{21}) \\
+ p_{\text{sc}}(t_1) p_{\text{sc}}(t_1 + t_2) p_{\text{ap}}(t_2 + t_{11}) p_{\text{ap}}(t_{11} + t_{21}) \\
+ p_{\text{sc}}(t_1) p_{\text{sc}}(t_1 + t_2) p_{\text{ap}}(t_{21}) p_{\text{ap}}(t_2 + t_{21} + t_{11}).
\end{align*}$$  

(3.32)

The first of these three sequences is schematically represented in figure 3.11.

By examining higher order terms and evaluating the integrals, it appears that $\psi_{2,j,k}$ can be written as:

$$\psi_{2,j,k} = 2! \cdot j! \cdot k! \left( \frac{1}{\tau_d} \right)^{j+k} \left( \frac{1}{\tau_{\text{ap}}} \right) \frac{1}{2} \times
\left[ \sum_{j_1=0}^{k} \left\{ \begin{array}{c}
\frac{j + j_1}{k} \prod_{a_1=1}^{j} h(a_1, 0, 0) \prod_{a_{j_1}=1}^{j_1} h(a_{j_1}, 0, 0) \\
\prod_{a_{j_1}=j_1}^{k} h(a_2, 1, 0) \times
\end{array} \right\} \right],$$  

(3.33)

where $a_1$, $a_2$, and $j_1$ are integers used for summing as well as multiplication, and $h(a,b,c)$ has been defined in equation 3.30.
Chapter 3. SiPM response model

Figure 3.11: Time line of one of the three possible sequences of events corresponding to $\psi_{2,1,1}$. A scintillation pulse starts at $t_0 = 0$. One of the scintillation photons (sc) causes a trigger after some time interval $t_1$, with probability $p_{sc}(t_1)$. This photon-induced trigger is followed by an afterpulse (ap) after some time interval $t_{11}$, with probability $p_{ap}(t_{11})$. The afterpulse is followed by a second absorbed photon causing a trigger after some time interval $t_2$, with probability $p_{sc}(t_1+t_{11}+t_2)$. This second photon-induced trigger is followed by an afterpulse after some time interval $t_{21}$, with probability $p_{ap}(t_{21})$.

$i = 3$

The expectation value $\psi_{3,j,k,l}$ of the output charge of a single cell due to three photon-induced triggers giving rise to $j$, $k$, and $l$ afterpulses, respectively, is obtained similarly. Since the formulas analogous to equations 3.31 and 3.32 for $\psi_{3,1,1,1}$ contain many terms, only the end result is given here:

$$
\psi_{3,j,k,l} = 3! \frac{j! k! l!}{\tau_d^2 \tau_{ap}} \frac{1}{3} \times \left[ \frac{1}{k+j} \left( \begin{array}{c} k+k_l \\ j+j_l \end{array} \right) \times \left\{ \begin{array}{c} j+k_l \\ a_1=1 \\ a_2=j_l \\ \prod_{a_1=1}^{j+k_l} h(a_1,0,0) \prod_{a_2=j_l}^{k+k_l} h(a_2,1,0) \prod_{a_3=k_l}^{l} h(a_3,2,0) \times \end{array} \right\} \text{\begin{array}{c} a_1=1 \\ a_2=j_l \\ \sum_{a_3=k_l}^{l} h(a_3,1,1) - \sum_{a_3=k_l}^{l} h(a_3,2,1) \end{array}} \right\},
$$

(3.34)
where $a_1, a_2, a_3, j_1,$ and $k_1$ are integers used for summing and multiplication.

$i = 4$

The expectation value $\psi_{4,j,k,l,m}$ of the output charge of a single cell due to four photon-induced triggers giving rise to $j$, $k$, $l$, and $m$ afterpulses, respectively, is obtained as:

$$\psi_{4,j,k,l,m} = 4! j! k! l! m! \left( \frac{1}{\tau_d} \right)^3 \left( \frac{1}{\tau_{ap}} \right)^{j+k+l+m} \frac{1}{4} \times \left[ \begin{array}{c} (l+j_1)(k+k_1)(j+j_1) \prod_{a_1=1}^{j+l+j_1} h(a_1,0,0) \prod_{a_2=j_1}^{l+k_1} h(a_2,1,0) \prod_{a_3=k_1}^{l+k_l} h(a_3,2,0) \prod_{a_4=l_1}^{m} h(a_4,3,0) \end{array} \right]$$

where $a_1$, $a_2$, $a_3$, $a_4$, $j_1$, $k_1$, and $l_1$ are integers used for summing and multiplication.

**Appendix B**

The computation of $\xi_i$ may take a very long time if the number of photon-induced triggers in a single cell $i > 9$. As an alternative, additional terms may be obtained by fitting and extrapolating $\xi_i$ as a function of $i$. This is illustrated for the case of detector 1, for which the parameters have been determined in section 3.5. The following function fits the modeled values of $\xi_i$ well for this detector:

$$\xi_{\text{fit}}(i, h_1, h_2, h_3) = \frac{1}{(1 - e^{-h_1})} + h_2(1 - e^{-h_3}),$$

where $h_1$, $h_2$, and $h_3$ are the fit parameters. It should be noted that this function has no physical meaning.

Figure 3.12 shows the fit (solid green curve) and the modeled values of $\xi_i$ (black squares). Since the curve becomes less steep for larger values of $i$, it is expected that possible deviations of the extrapolated values of $\xi_i$ from the modeled ones become smaller with increasing $i$. 
Chapter 3. SiPM response model

Figure 3.12: The modeled values of $\xi_i$ as a function of the number of photon-induced triggers in a single cell $i$ (black squares) fitted using $0 \leq i \leq 9$ and extrapolated to $i = 14$ (solid green curve).

References


References


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References


[38] Saint-Gobain material product sheets for NaI:Tl (10-05), BGO (03-07), CsI:Tl (05-07), LYSO (07-07), and LaBr$_3$:Ce (12-07), http://www.detectors.saint-gobain.com.


Chapter 3. SiPM response model
Chapter 4

A novel, SiPM-array-based, monolithic scintillator detector for PET


Abstract

Silicon photomultipliers (SiPMs) are of great interest to positron emission tomography (PET), as they enable new detector geometries for e.g. depth-of-interaction (DOI) determination, are MR-compatible, and offer faster response and higher gain than other solid-state photosensors such as avalanche photodiodes (APDs). Here we present a novel detector design with DOI correction, in which a position-sensitive SiPM array is used to read out a monolithic scintillator. Initial characterization of a prototype detector consisting of a $4 \times 4$ SiPM array coupled to either the front or back surface of a $13.2 \text{ mm} \times 13.2 \text{ mm} \times 10 \text{ mm}$ LYSO:Ce$^{3+}$ crystal shows that front-side readout (FSR) results in significantly better performance than conventional back-side readout (BSR). Spatial resolutions < 1.6 mm FWHM were measured at the detector center in response to a ~0.54 mm FWHM diameter test beam. Hardly any resolution losses were observed at angles of incidence of up to 45°, demonstrating excellent DOI correction. About ~14% FWHM energy resolution was obtained. The timing resolution, measured in coincidence with a BaF$_2$ detector, equals 960 ps FWHM.

4.1 Introduction

Positron emission tomography (PET) enables imaging of biological processes and is increasingly being used in the clinic as well as in biomedical research [1]–[4]. PET has proven its value in the diagnosing, staging, and restaging of cancer.
It furthermore plays a growing role in e.g. radiotherapy treatment planning, in radionuclide- and chemo-therapy monitoring, and in other fields such as neurology and cardiology.

PET instrumentation is continuously being improved, driven by the need for better image quality and shorter scanning times. At the detector level, this requires higher spatial resolution including correction for depth-of-interaction (DOI) errors, higher sensitivity, improved count-rate performance, and better energy resolution [5]. In addition, clinical PET will benefit from further improvement of time-of-flight (TOF) performance [6]–[7], as TOF-PET with ~600 ps coincidence resolving time (CRT) has already demonstrated significant improvement in image quality, especially in heavier patients [8].

It is furthermore desirable to integrate complementary imaging modalities [9],[10]. For example, the combination of the functional and anatomical imaging capabilities of PET and X-ray CT, respectively, into hybrid PET/CT systems has had tremendous impact within the field of oncology [11]–[13]. At present, various groups are working on the more difficult challenge of combining PET and MRI [10],[14]–[16]. Potential advantages of PET/MRI include the far better soft-tissue contrast of MRI compared to CT and the elimination of the CT dose, which tends to be responsible for most of the overall dose received by the patient during a PET/CT scan. True PET/MRI integration, however, requires PET detectors that are very compact, do not distort the operation of the MRI system, and are insensitive to magnetic fields.

The ideal PET detector would perform optimally with respect to each of the above criteria and be affordable at the same time. Scintillation detectors based on solid-state photosensors are very promising in this respect. In contrast with conventional photomultiplier tubes (PMTs), such sensors can be made MR-compatible. Furthermore, their small size enables novel detector geometries that allow DOI determination as well as a high detector packing fraction to maximize PET system sensitivity. The importance of sensitivity should not be underestimated as the reconstructed resolution of clinical PET images is often limited by the number of acquired counts rather than by the system resolution [7]. Furthermore, combating the inherent physical limits on PET spatial resolution by compensating for positron range and acollinearity in the image reconstruction process requires sufficient statistical quality of the acquired data [9].

Whereas solid-state photosensors such as PIN diodes and avalanche photodiodes (APDs) have been explored by many authors, a particularly interesting new class of devices are silicon photomultipliers (SiPMs) [17]–[24]. These can be fabricated using CMOS technology, offering the possibility of low
cost when made in large quantities. They have gains in the order of $\sim 10^6$ and are very fast, which is crucial for TOF-PET. In fact, CRTs of 237 ps FWHM and 240 ps FWHM have recently been demonstrated using 3 mm $\times$ 3 mm SiPMs coupled to small crystals of, respectively, LaBr$_3$:Ce$^{3+}$ [25] and LYSO:Ce$^{3+}$ [26]. As of very recently, SiPMs can be manufactured into compact arrays that can be used as position-sensitive light sensors in PET detectors [27]–[30].

The aim of this paper is to present an initial characterization of the first SiPM-array based PET detector following the monolithic scintillator concept explored using APD arrays by Maas et al. [31],[32]. This monolithic approach has previously been shown to allow not only high resolution and excellent DOI correction, but also very high system sensitivity [33]. Two readout geometries are compared in this work: front-side readout (FSR) and conventional back-side readout (BSR).

4.2. Materials and methods

4.2.1 Detector

The detector prototype is based on a 13.2 mm $\times$ 13.2 mm $\times$ 10 mm monolithic LYSO:Ce$^{3+}$ scintillator with optically polished surfaces (Crystal Photonics). The 13.2 mm $\times$ 13.2 mm crystal surfaces match the sensitive area of the SiPM array, which is optically coupled to the crystal using Sylgard 527 dielectric gel. All other faces of the crystal are covered with a highly reflective PTFE-based material (Spectralon). The SiPM array (SensL SPMArray 3035G16) is a 4 $\times$ 4 array of SiPM pixels mounted onto a 550 µm thick white float glass substrate using flip chip technology (see inset figure 4.1). The 16 silicon dies are mounted at a pitch of 3.3 mm. Each pixel has an active area of 2.85 mm $\times$ 2.85 mm, made up of 3640 Geiger-mode avalanche photodiodes (microcells). The SiPM array was operated at the manufacturer-specified bias voltage of 29.3 V, exceeding the breakdown voltage by 2.0 V and corresponding to a gain of $\sim 10^6$.

The blue squares in figure 4.1 show the photodetection efficiency (PDE) in air of the 2.85 mm $\times$ 2.85 mm active area of a single SiPM pixel at 2 V above breakdown [SensL, private communication]. It is emphasized that these values are free of any contributions from after-pulsing or crosstalk [34]. The solid black line in figure 4.1 shows the emission spectrum of a 3 mm $\times$ 3 mm $\times$ 3 mm LYSO:Ce$^{3+}$ crystal (Crystal Photonics), measured as described by De Haas and Dorenbos [35]. The effective PDE of the SiPM active area, weighted by the normalized LYSO:Ce$^{3+}$ emission spectrum, equals $\sim 5.9\%$. As the fractional active area of the array equals $\sim 75\%$, the effective PDE of the entire array in air is estimated to be $\sim 4.4\%$. 

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Position estimation

Rather than the interaction point of the annihilation photon, its entry point on the crystal front surface is estimated, using the statistical algorithm described by Maas et al. [32]. For convenience we briefly summarize the method here. As indicated in figure 4.2, reference data are first collected by irradiating the detector with 511 keV photons at a series of known positions \((x_i, y_j)\) and angles of incidence \(\theta_k\) on the crystal front surface. At each position and angle the light distributions of \(n_{\text{ref}}\) reference events are recorded. The entry point of an unknown annihilation photon is subsequently estimated by calculating the sum-of-squared-differences of its light distribution with those of all events in the reference set recorded at the \(\theta_k\) closest to the angle of incidence \(\theta\) of the unknown event. In a PET scanner, \(\theta\) can be estimated from the positions of the two detectors triggering in coincidence [32]. A subset of the reference data consisting of the \(L\) closest matches (‘nearest neighbors’) is selected, and the
4.2. Materials and methods

The most frequently occurring entry point within this subset is assigned to the unknown event.

### 4.2.3 Measurements

Measurements were performed using the setup described by Maas et al. [32]. Briefly, the detector is contained in a temperature-controlled box and can be irradiated at different positions and angles of incidence with a < 1 mm diameter test beam of annihilation photons, defined by placing the detector close to a 0.5 mm diameter $^{22}$Na source and operating it in coincidence with a collimated BGO detector placed on the opposite side of the source. The SiPM signals were preamplified using a 16-channel readout board designed to minimize nonlinearity due to SiPM impedance variations. The design and characteristics of these preamplifiers have been described by Seifert et al. [36]. The preamplified SiPM pulses were shaped and their pulse heights digitized using the multichannel data acquisition system described by Maas et al. [32].

---

**Figure 4.2:** Schematic representation of the nearest-neighbor algorithm used to estimate the entry point $(x,y)$ of the annihilation photon on the front surface of the crystal from the scintillation light distribution measured by the position-sensitive SiPM array. See text for details.
parallel, a trigger signal was generated by adding the 16 SiPM signals by means of a fast summing amplifier on the preamplifier board.

Spatial resolution measurements were performed by recording reference events at a rectangular, equidistant grid of reference beam positions \((x_i, y_j)\), having a pitch of 0.25 mm and covering the entire front surface of the crystal. At non-perpendicular incidence, the same reference grid was used and the lateral crystal surface turned towards the beam was included in the measurement. The reference events were also used as test events, using the leave-one-out method described by Maas et al. [32]. All measurements were conducted at ~24 °C.

Pulse height spectra were derived by correcting the digitized pulse heights of all detector channels for offsets, adding the 16 corrected pulse heights of each event, and normalizing the result such that the center of the full-energy peak corresponded to 511 keV.

The detector timing resolution was determined by placing the detector in coincidence with a BaF\(_2\) crystal on an XP2020Q PMT connected to an Ortec 579 fast filter amplifier (FFA) and an Ortec 935 constant fraction discriminator (CFD). The SiPM sum signal was fed into a second, identical FFA and a LeCroy WavePro 7300 oscilloscope was used to measure the time difference between the CFD logic pulse and the moment at which the second FFA output signal crossed a fixed threshold corresponding to ~10 keV. Only full-energy events were accepted.

**Figure 4.3:** Schematic representation of the readout geometries investigated. Left: front-side readout (FSR) geometry. Right: back-side readout (BSR).
4.3 Results

Measurements were performed in two different readout geometries: front-side readout (FSR), in which the SiPM array is placed on the crystal surface facing the radiation source, and conventional back-side readout (BSR), see figure 4.3. It is emphasized that FSR is possible without significantly disturbing the annihilation photon beam since the SiPM array is very thin and consists of low-Z materials only. Specifically, a Monte Carlo simulation of the detector using GATE [37], showed that the probability of a 511 keV photon undergoing at least one Compton or Raleigh interaction in the SiPM array before being detected (i.e., before undergoing at least one Compton and/or photoelectric interaction in the crystal), equals ~3%.

4.3.1 Spatial resolution

The detector spatial response (i.e., the two-dimensional histogram of the differences between the true and estimated annihilation photon entry points) can in principle be derived at each point of the measurement grid. However, the number of events $n_{\text{ref}}$ recorded at each point is limited, resulting in considerable statistical fluctuations if the histogram is determined for one such point only. As the spatial response appears to be approximately constant over the central area of the detector, the results obtained within the central 3.25 mm × 3.25 mm were combined into a single error histogram.

Figure 4.4 shows the corresponding result obtained in FSR geometry. The full-width-at-half-maximum (FWHM) and the full-width-at-tenth-maximum (FWTM) are shown in table 4.1. It is noted that these result still contain the influence of the ~0.54 mm FWHM diameter test beam. Furthermore, they were obtained at a low energy threshold of ~50 keV applied to the sum of the 16 SiPM signals.

Interestingly, increasing the energy threshold to ~400 keV hardly appears to improve these results (< 1%). On first sight one might expect that low-energy

<table>
<thead>
<tr>
<th>Direction</th>
<th>FWHM (mm)</th>
<th>FWTM (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>1.58</td>
<td>4.06</td>
</tr>
<tr>
<td>y</td>
<td>1.56</td>
<td>4.23</td>
</tr>
</tbody>
</table>

*Table 4.1: FWHM and FWTM of the spatial response at the detector center at normal incidence. Results were obtained in FSR geometry and are not corrected for the ~0.54 mm FWHM diameter test beam.*
events would be positioned less accurately due to the lower amount of scintillation light emitted. However, in most of these events the scintillation light will be emitted from a single (Compton) interaction location. In contrast, many of the events in the full-energy peak appear to involve multiple interactions within the crystal (see section 4.3.4). Events in which the scintillation light is emitted from multiple interaction locations may be more difficult to position than single-interaction events. Apparently, the resulting average positioning accuracy is similar for events in the full-energy peak, most of which involve multiple interactions, and events in the Compton ridge, most of which involve a single interaction, so that the detector spatial resolution becomes almost independent of the energy threshold.

Similar to what was found in APD-based monolithic detectors by Maas et al. [32], the spatial resolution shows some degradation near the detector edges. For example, the solid black line in figure 4.5 shows the FWHM of the detector spatial response in the x-direction as a function of x. At each x, all results

---

Figure 4.4: Detector spatial response to a ~0.54 mm FWHM diameter test beam, measured in FSR geometry at the detector center and at normal incidence using $n_{\text{ref}} = 1000$ and $L = 750$. 
obtained between $x - 0.5$ mm and $x + 0.5$ mm (i.e., over the entire length of the crystal in the $y$-direction) were combined into a 2D error histogram in order to minimize statistical fluctuations. At about $\sim 3$ mm from the crystal edge, the FWHM starts to increase, until it reaches a maximum at $\sim 2$ mm from the edge. At smaller distances the FWHM decreases again, as the error histograms are being truncated on one side by the crystal edge.

The dashed blue line in figure 4.5 shows the FWHM of the detector spatial response in the $y$-direction as a function of $x$, derived from the same error histograms used to obtain the solid black curve. Interestingly, no significant dependence of the FWHM in the $y$-direction on $x$ is observed. Conversely, the FWHM in the $y$-direction was found to depend on $y$, whereas the FWHM in the $x$-direction did not. Hence, the FWHM in a given direction ($x$ or $y$) is only affected by a crystal edge perpendicular to that direction, in agreement with what was found by Maas et al. [32].

**4.3.2 FSR versus BSR**

In a further series of measurements the resolutions obtained in different readout
geometries and at different angles of incidence were compared. In these measurements, the $^{22}\text{Na}$ point source had to be placed at a larger distance from the detector box to allow it to rotate. This resulted in a larger, but constant, test beam diameter of $\sim0.64$ mm FWHM.

The results were analyzed by combining the data obtained over the entire crystal surface into a single error histogram, which we will denote as the ‘average detector spatial response.’ It is noted that these results cannot be

*Figure 4.6:* FWHM and FWTM of the average detector spatial response in the x-direction, measured as a function of the angle of incidence $\theta$ using a $\sim0.64$ mm FWHM diameter test beam. Data were obtained in FSR geometry with $n_{\text{eff}} = 250$ and $L = 1000$.

Table 4.2: Comparison of front- and back-side readout. Values represent the FWHM and FWTM in the x-direction of the detector spatial response at normal incidence, averaged over the entire detector surface, not corrected for the $\sim0.64$ mm FWHM diameter test beam.

<table>
<thead>
<tr>
<th>Readout geometry</th>
<th>FWHM (mm)</th>
<th>FWTM (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Front surface readout</td>
<td>1.86</td>
<td>4.68</td>
</tr>
<tr>
<td>Back surface readout</td>
<td>2.21</td>
<td>5.33</td>
</tr>
</tbody>
</table>
compared directly to those obtained at the detector center (i.e., table 4.1 and figure 4.4), both because of the larger beam diameter and because of the larger FWHM values observed near the detector edges (see figure 4.5).

The FSR and BSR results are compared in table 4.2. FSR appears to perform considerably better than BSR. We therefore focus on FSR in the remainder of this work.

4.3.3 DOI correction

Estimating the annihilation photon entry point has the advantage that DOI errors are, in principle, eliminated [32]. This intrinsic DOI correction was tested by irradiating the detector at different angles of incidence $\theta$ and deriving the average spatial response projected onto a plane perpendicular to the test beam. In this way, the results correlate directly to the uncertainty in the position of the line-of-response (LOR) in a PET scanner.

Figure 4.6 shows the FWHM and FWTM of the average detector response as a function of $\theta$, determined in FSR geometry using the same test beam diameter as in table 4.2. Hardly any spatial resolution losses are observed for angles of incidence of up to 45°. Only the FWTM increases slightly with increasing angle of incidence.

4.3.4 Energy and timing resolution

The black squares in figure 4.7a show a typical pulse height spectrum measured with 511 keV photons. As explained in section 4.2.3, the entire detector surface was uniformly irradiated. The full-energy peak contains ~60% of the total number of counts recorded, while the probability of photoelectric interaction of 511 keV photons in LYSO:Ce$^{3+}$ equals about ~30% of the total probability of interaction. This is attributed to the relatively large size of the crystal: a significant fraction of the detected annihilation photons undergo Compton scattering before being absorbed completely. A Gaussian fit to the full-energy peak is indicated by the solid blue line. The corresponding energy resolution equals 14.2% FWHM at 511 keV.

Figure 4.7b shows the timing spectrum of the detector, measured in coincidence with a BaF$_2$ crystal. The timing resolution equals 960 ps FWHM. As the contribution of the BaF$_2$ crystal is considered negligible, the CRT of two of the SiPM-based detectors in coincidence is expected to be $0.96\sqrt{2} \approx 1.4$ ns FWHM.
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4.4 Discussion and conclusion

A novel, SiPM-array based, high-resolution, monolithic scintillator PET detector with DOI correction has been developed. As the SiPM array is very thin and composed of low-Z materials only, it can be placed on the front surface of the

Figure 4.7: Pulse height spectrum (a) measured by irradiating the entire detector with 511 keV photons. Timing spectrum (b) measured in coincidence with a BaF₂ detector. Both spectra were measured in FSR geometry. Black squares represent measured data, while solid blue lines indicate Gaussian fits.
4.4. Discussion and conclusion

crystal without significantly disturbing the annihilation photon beam. Such FSR geometry appears to provide better performance than conventional BSR, confirming earlier findings obtained with APD-based monolithic PET detectors [31]. The superior performance of FSR is attributed to the fact that ~60% of the annihilation photons are absorbed in the front half of the crystal. Events occurring closer to the SiPM array result in more sharply peaked light distributions that vary more strongly with the position of interaction. Consequently, these events can be positioned more accurately, as has been discussed by Van der Laan et al. [38]. These results illustrate the potential of SiPMs for the development of novel detector designs aiming at, for example, compactness, DOI determination, and MR-compatibility.

In FSR geometry, spatial resolutions < 1.6 mm FWHM were measured at the detector center in response to a ~0.54 mm FWHM diameter test beam. Slightly larger FWHM values were found near the detector edges, very similar to what was found in APD-based monolithic detectors by Maas et al. [32]. Fully characterizing the dependence of the detector spatial response as a function of the position and angle of incidence would allow these effects to be compensated for during iterative image reconstruction, a topic that warrants further research. Hardly any resolution losses were observed at angles of incidence of up to 45°, demonstrating excellent DOI correction. About ~14% FWHM energy resolution was obtained. The single detector timing resolution is estimated to be 960 ps FWHM, translating into a CRT of ~1.4 ns for two detectors in coincidence.

The present results may be compared to those obtained recently by other authors using SiPM arrays. Kolb et al. [28] could resolve a 12 × 12 array of 1.5 mm × 1.5 mm × 10 mm LYSO:Ce³⁺ pixels using a 3 × 3 array, made in-house using 3 mm × 3 mm SiPMs and dedicated light guides. They achieved a single detector timing resolution of 950 ps FWHM and ~14% FWHM energy resolution. España et al. [27] nicely resolved a 4 × 4 array of 1.5 mm × 1.5 mm × 12 mm LYSO:Ce³⁺ pixels using a 6 mm × 6 mm active area, monolithic 2 × 2 SiPM array. They obtained energy resolutions between 11% and 22% FWHM for single crystals at different locations on the array, but provided no information on timing resolution yet. Llosa et al. [29] reported ~15% FWHM energy resolution and a CRT of ~3.3 ns FWHM using 4 mm × 4 mm × 5 mm monolithic LYSO:Ce³⁺ crystals coupled to an SiPM array consisting of 4 × 4 pixels of 1 mm × 1 mm size in a common substrate, in preparation of spatial resolution measurements.

While very similar timing resolutions were obtained with all approaches, the energy resolution tends to be better for monolithic crystals. All detectors appear to achieve good spatial resolution, although it is to be noted that the position
information obtained with pixelated crystals is discrete (crystal identification), whereas our approach yields (pseudo-) continuous coordinates. An advantage of monolithic scintillators is that the reduction of inter-crystal dead space results in higher system sensitivity [33]. Another important advantage of the present approach is the excellent DOI correction.

The spatial resolution obtained in this work is slightly worse than that achieved with APD-based monolithic detectors by Maas et al. [32]. This is mainly attributed to the relatively low PDE of the active area of the present SiPM arrays, viz. ~6%, compared to 25% – 30% for PMTs and up to ~75% for APDs. However, the spatial resolution obtained with APDs is inherently limited by their relatively large excess noise factor and dark current, as analyzed quantitatively by Maas et al. [39]. Due to the relatively low gain of APDs, preamplifier noise is another limiting factor. Thus, further improvement of the SiPM photodetection efficiency might eventually result in better spatial resolution than can be obtained with APDs.

Although the timing resolution obtained with SiPM arrays is considerably better than that of APD-based monolithic PET detectors [32], it is still insufficient for TOF-PET [7]. In fact, obtaining the best possible timing resolution has not been emphasized upon in this work, as the present quality of the SiPM arrays used was not expected to allow CRTs significantly smaller than 1 ns. Nevertheless, CRTs ≤ 240 ps have recently been demonstrated with small LYSO:Ce³⁺ and LaBr₃:Ce³⁺ crystals coupled to 3 mm × 3 mm SiPMs having higher PDE and lower dark current [25],[26], demonstrating that TOF determination with SiPM-based scintillation detectors is in principle feasible.

We conclude that SiPMs are a very promising new class of light sensors for use in PET scintillation detectors and that further improvement of these devices may lead to detectors with unsurpassed overall performance.

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**References**


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References


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Chapter 5

Improved nearest neighbor methods for gamma photon interaction position determination in monolithic scintillator PET detectors


Abstract

Monolithic scintillator detectors have been shown to provide good performance and to have various practical advantages for use in PET systems. Excellent results for the gamma photon interaction position determination in these detectors have been obtained by means of the \(k\)-nearest neighbor (\(k\)-NN) method. However, the practical use of monolithic scintillator detectors and the \(k\)-NN method is hampered by the extensive calibration measurements and the long computation times. Therefore, several modified \(k\)-NN methods are investigated that facilitate as well as accelerate the calibration procedure, make the estimation algorithm more efficient, and reduce the number of reference events needed to obtain a given lateral (x,y)-resolution. These improved methods utilize the information contained in the calibration data more effectively. The alternative approaches were tested on a dataset measured with an SiPM-array-based monolithic LYSO detector. It appears that, depending on the number of reference events, \(~10\%\) to \(~25\%\) better spatial resolution can be obtained compared to the standard approach. Moreover, the methods amongst these that are equivalent to calibrating with a line source may allow for much faster and easier collection of the reference data. Finally, some of the improved methods yield essentially the same spatial resolution as the standard method
using ~200 times less reference data, greatly reducing the time needed for both calibration and interaction position computation. Thus, using the improvements proposed in this work, the high spatial resolution obtainable with the $k$-NN method may come within practical reach and, furthermore, the calibration may no longer be a limiting factor for the application of monolithic scintillator detectors in PET scanners.

5.1 Introduction

Monolithic scintillation crystals read out by position sensitive photosensors are investigated as alternatives to detectors based on segmented crystals in positron emission tomography (PET) [1]–[7]. These monolithic scintillator detectors have several favorable characteristics, especially for whole-body clinical PET systems. They exhibit excellent depth-of-interaction (DOI) correction as well as good spatial resolution, and, since they allow for reduced dead space, high system sensitivity can be obtained [6],[8]. Furthermore, they have several practical advantages, such as easier detector assembly.

The determination of the interaction position of a gamma photon in a monolithic scintillator is more complex than in a detector based on segmented crystals in which the lateral interaction coordinates ($x,y$-plane, see figure 5.1a) are usually determined by crystal segment identification. For example, in a monolithic scintillator detector the position of the sensor pixel with the largest signal does not necessarily correspond to the $(x,y)$-coordinates of the interaction position, since a large fraction of the scintillation photons may be reflected one or more times within the crystal before being detected. Furthermore, the scintillation photons typically spread over many sensor pixels, increasing the relative statistical variance on the number of detected photons per pixel and increasing the influence of electronic noise.

In previous works we determined the entry points of annihilation photons on the crystal front surface using the $k$-nearest neighbor ($k$-NN) method [6],[9]. It was shown that this approach offers excellent spatial resolution while the influence of parallax errors on the calculated line-of-response (LOR) is kept very small. However, the calibration of the detector is cumbersome as it requires the acquisition of a large set of reference data by irradiating the crystal with a thin beam of annihilation photons at a large number of known entry points and at many different angles of incidence.

To illustrate this issue, one could do the following crude extrapolation. Suppose one would use the measurement setup described in section 5.5 of this work, without any further optimization. In that case, due the relatively low source activity and the high degree of collimation of the calibration beam, the
count rate during calibration would be less than 10 counts per second. The corresponding time to calibrate a single detector of several square centimeters at a few thousand entry points and at a few hundred angles of incidence would be in the order of years.

Furthermore, entry point estimation with the standard $k$-NN method is computationally expensive, since the measured light distribution of an event under test has to be compared to those of a large number of reference events.

Again for illustration, an exemplary computation time in the order of ~0.1 s per event might be assumed for a basic, non-optimized implementation of the $k$-NN algorithm on a single CPU. At this rate, the computation time for calculating the positions of interaction for a typical clinical PET scan of several hundreds of millions of coincidences would be in the order of years.

Clearly, the practical advantages of monolithic scintillator detectors and the high spatial resolution achievable with the $k$-NN method may only come within reach for practical use in PET scanners if the calibration procedure can be simplified, the estimation algorithms can be made more efficient, and the number of required reference events can be reduced.

In this work we investigate and compare several possible modifications of the standard $k$-NN method that can be used to fulfill these objectives. Various enhancements of the $k$-NN method reduce the number of reference events needed to obtain a given $(x,y)$-resolution, accelerating both calibration and computation. These modified algorithms utilize the information contained in the calibration data more effectively. Additionally, several of the modified $k$-NN methods enable the use of calibration data obtained with a line source, allowing for much easier and faster calibration.

Each of the improved approaches, as well as combinations thereof, are tested on a dataset measured with an SiPM-based monolithic LYSO detector, by evaluating the spatial resolution as well as the bias sensitivity of the modified estimation methods as a function of irradiation position and amount of reference data. The results are compared to those obtained with the standard $k$-NN method.

## 5.2 Improved detector calibration procedure

The detector calibration can be made much simpler and shorter by using only a perpendicularly incident beam of annihilation photons to collect reference data (see figure 5.1a) instead of calibrating at many angles of incidence as required for entry point determination. In the procedure that is followed in this work, $(x,y)$ interaction coordinates rather than entry points are obtained by means of
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The $k$-NN method. Since this approach requires explicit DOI information, the DOI is to be determined separately by means of another method. It has been shown that this can also be performed using calibration data obtained with perpendicularly incident gamma photons only, while neither detector modifications nor a priori knowledge of the light transport and/or signal variances are needed [4],[10]. It is emphasized that this approach might decrease the calibration time of a single detector by at least two orders of magnitude, since calibration at several hundred different angles of incidence is no longer necessary.

Further acceleration and facilitation of the calibration procedure could be achieved by irradiating the detector front surface along two series of lines, i.e. one series parallel to the $x$-axis and one series parallel to the $y$-axis (figure 5.1b), instead of at a grid of points (figure 5.1a). A line source calibration would require only $n_{\text{pos},x} + n_{\text{pos},y}$ calibration positions instead of $n_{\text{pos},x} \times n_{\text{pos},y}$ positions, where $n_{\text{pos},x}$ and $n_{\text{pos},y}$ denote the numbers of calibration positions in the $x$- and $y$-directions, respectively. Such a procedure could be implemented by either mechanical or electronic collimation. In the first case, one could e.g. use a line source in combination with a slit collimator in between the source and detector. In the second case, one might e.g. place a line source in between the detector under investigation and a coincidence detector equipped with a slit collimator. A

Figure 5.1: (a) Illustration of a monolithic scintillator detector irradiated at a given position $(x,y)$ by a perpendicularly incident beam of gamma photons. (b) The same detector irradiated along a line parallel to the $y$-axis at a given $x$-coordinate.
line source has the advantages that it can contain a much higher activity, it is easier to produce, and the geometric efficiency for the detection of gamma photons can be much higher. This concept of a line source together with the abovementioned approach of calibrating at a single angle of incidence could potentially reduce the calibration time of a single detector from the order of years to less than an hour. However, this requires a position estimation method that can make use of reference data corresponding to a line of irradiation points instead of singular points.

5.3  $k$-nearest neighbor method (Standard Method)

The $k$-nearest neighbor ($k$-NN) method has been introduced by Fix and Hodges [11] and its application for the determination of the entry points of gamma photons in a monolithic scintillator detector has been studied by Maas et al. [2]. In the latter work, a calibration measurement is performed in which a reference dataset is collected by perpendicularly irradiating the monolithic scintillator detector with a thin beam of 511 keV photons at a grid of $n_{\text{pos}}$ known positions (classes). For each irradiation position, the resulting light intensity distributions $I = (I_1, I_2, \ldots, I_N)$ of $n_{\text{ref}}$ reference events are recorded, where $N$ denotes the number of sensor pixels. All of the light patterns are normalized such that the sum of all detector signals equals unity. It should be noted that, due to differences in optical coupling, reflector, scintillator response, etc., each detector may in principle require its own reference dataset.

The unknown ($x, y$)-coordinates of the annihilation photon interaction position of an event under test (an unclassified event) is subsequently estimated as follows. The Euclidean distance

$$D = \sqrt{\sum_{i=1}^{N} (I_{\text{test},i} - I_{\text{ref},i})^2}$$

of the measured light distribution $I_{\text{test}}$ to those of all events in the reference set $I_{\text{ref}}$ is calculated. A subset of the reference data consisting of the $k$ events with the smallest values of $D$ (nearest neighbors) is selected and a histogram of their ($x, y$) irradiation coordinates is created (see figure 5.2). The coordinate corresponding to the maximum value of the histogram, i.e. the most frequently occurring coordinate, is assigned to the unclassified event. In case of multiple maxima (a tie), one of the maxima is selected randomly. It should be noted that a tie could also be solved by increasing or decreasing $k$ until a single maximum is obtained [12].
5.4 Improved nearest neighbor algorithms

The probability of misclassification approaches the theoretical minimum (i.e. the Bayes error probability) for the standard $k$-NN method in the infinite sample case, i.e. if $k$, $n_{\text{pos}}$, and $n_{\text{ref}}$ tend to infinity such that $k / n_{\text{tot}} \rightarrow 0$ [13]. In practice, however, approaching the conditions for optimal performance of the $k$-NN method may be very time consuming. It requires the detector to be irradiated with a large number of annihilation photons at many different known positions. Additionally, the resulting large reference dataset causes the interaction position estimation to be computationally intensive. Moreover, the standard $k$-NN method is not guaranteed to be the optimal way of using the information contained in the neighborhood of unclassified patterns in the finite sample case [14].

One reason for this is that the $k$-NN method implicitly assumes that the $k$ nearest neighbors are contained in a relatively small volume. In practice, however, the Euclidean distance $D$ between the unclassified pattern and one of its closest neighbors is not always negligible. Therefore, it was proposed to give weights to the nearest neighbors based on the distance to the unclassified

![Figure 5.2: Example of an NN-histogram. In the standard k-NN method the most frequently occurring coordinate is assigned to the unclassified event. The improved approaches include e.g. fitting and smoothing the NN-histogram.](image)
5.4. Improved nearest neighbor algorithms

sample, yielding better results in several cases [15],[16]. Nevertheless, it was shown that the standard $k$-NN method still outperforms the weighted method in the infinite sample case [12].

A second reason is that in many cases it cannot be assumed that all classes are represented in the reference dataset, e.g. when the detector irradiation has been performed using a coarse grid. In those cases, an unclassified sample could be far away from any reference sample and most probably belongs to a class for which no reference data has been collected [17].

Yet, another limitation of the $k$-NN method is that it offers no obvious way to cope with uncertainty or imprecision in the labeling of the reference data, e.g. due to a finite irradiation beam width or due to Compton scattering in the crystal. Furthermore, even if patterns are correctly labeled, they may be rather atypical for the particular class. Several adapted $k$-NN methods based on fuzzy sets theory have been proposed for handling imprecision and uncertainty in a classification process, e.g. by giving less weight to atypical patterns than those that are truly representative of the classes [18].

In literature various alternative NN-approaches have been shown to yield better results than the standard method in the finite sample case in a variety of applications. These approaches include the method proposed by Dudani and Macleod, in which weights are assigned to the nearest neighbors based on their distances to the unclassified sample [15],[16], the different untie methods proposed by Bailey and Jain [12], and the kernel based CAP method proposed by Hotta et al. [19]. However, these methods appeared to perform similarly to the standard method for the detector investigated in this work and, therefore, they are not discussed further here.

The alternatives to the standard $k$-NN method that are presented in this work are based on the fact that all nearest neighbors carry some information. This information can be used more effectively, e.g. by smoothing or fitting the NN-histogram, by means of the so-called categorical average patterns (CAP) method, and/or by combining reference data for each dimension analogous to calibrating with a line source. These methods may be less sensitive to misclassification due to statistical fluctuations in case of small reference datasets and, therefore, they may yield similar results as for large reference datasets, allowing for faster calibration. From all the methods that were tested by means of the cross-validation method described in section 5.5, only those that appeared to perform best are discussed in the remainder of this work.

5.4.1 Fit of the $k$-NN-histogram (Fit Lorentzian)

In this method the $k$ nearest neighbors of an unclassified light distribution are
selected and a 2D-histogram of their irradiation positions is created (see figure 5.2). So far this is similar to the standard $k$-NN method. Now, for sufficiently large reference datasets, we can assume that the irradiation coordinate closest to the true position of interaction has the highest probability of being present in the set of nearest neighbors and that this probability decreases for irradiation coordinates further away from this point. Therefore, all points in the histogram carry some information about the true position of interaction. Using more of this information, instead of only the most frequently occurring coordinate, can reduce statistical fluctuations on the interaction position estimation. Here, this is performed by fitting the histogram with a 2D Lorentzian shaped function using a log-likelihood method. Since the actual shape of the histogram is unknown, several peak shaped functions were tested, of which the Lorentzian shaped function yielded the best results. The coordinate that corresponds to the peak position value of the fit is assigned to the unclassified event. It should be noted that this method allows results in continuous coordinates.

5.4.2 Smoothed $k$-NN-histogram (Smoothed)

The 2D irradiation position histogram of the set of $k$ nearest neighbors is smoothed with a moving average filter of $n \times n$ bins. Here, $n = 5$. Thus, each new bin value is based on the average of 25 bin values of the original histogram. Near the edges of the histogram the number of averaging bins is decreased. The coordinate corresponding to the maximum value of the smoothed histogram is assigned to the unclassified event.

5.4.3 Split 1D $k$-NN-histograms (Split 1D Max)

As discussed in section 5.2, collecting reference data with a line source (see figure 5.1b) allows for much easier and faster calibration. If such a line source would be oriented parallel to the $y$-direction and scanned along the $x$-direction, one would obtain reference line classes, in which each event is assigned to an $x$-coordinate only. By following an approach equivalent to the standard $k$-NN method, the $x$-coordinate of an unclassified event can be determined by comparing its light distribution to all events in these reference line classes. Analogous to the standard $k$-NN method, we can assume that the reference line class closest to the true $x$-position of interaction has the highest probability of being present in the set of nearest neighbors and that this probability decreases for line classes further away from this point. The $y$-coordinate can be obtained similarly by comparing the unclassified event to reference line classes obtained by a scan of a line source along the $y$-direction. It should be noted that this approach is enabled by the intrinsic characteristic of the $k$-NN method that one
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One has access to all individual reference events.

The procedure is implemented and assessed in the following way. For the determination of the \(x\)-coordinate, a line source oriented parallel to the \(y\)-direction is emulated, where use has been made of the reference data obtained at a grid of points. This is done by combining all reference events corresponding to a given \(x\)-coordinate into a single class. For all events in such a reference line class only the \(x\)-coordinate of each reference event is used. The Euclidean distances \(D\) of the unclassified light distribution to those of all reference events are calculated (see equation 5.2). Then, a 1D-histogram is created of the \(x\)-coordinates of the irradiation positions of the \(k\) nearest neighbors. The coordinate that corresponds to the maximum bin value is the estimated \(x\)-coordinate. The \(y\)-coordinate is then determined in an equivalent way.

5.4.4 Fit of the split 1D \(k\)-NN-histograms (Split 1D Fit)

This method is equivalent to the previous one, except that the coordinate assigned to the unknown event corresponds to the peak position of a Lorentzian fit of the 1D-histogram.

5.4.5 Categorical average patterns (CAP)

The classification scheme of the so-called categorical average patterns (CAP) method makes somewhat different use of the information contained in the reference data than the previous methods [19]. In this method the \((x,y)\)-position of interaction is estimated by first calculating the Euclidean distances \(D\) of the unclassified light distribution to those of all reference events at a single irradiation position. A subset of these reference events consisting of the \(k_{\text{CAP}}\) nearest neighbors is then selected and an average light distribution is calculated for this subset. This is repeated for each of the \(n_{\text{pos}}\) irradiation positions in the reference dataset. Then, the Euclidean distances of the unclassified light distribution to all of the average light distributions are calculated (see figure 5.3). The coordinate corresponding to the minimum distance is assigned to the unclassified light distribution.

5.4.6 Categorical average patterns smoothed (CAP Smoothed)

This method is equal to the previous one, except that the obtained set of distances of the unclassified light distribution to all of the average light distributions is smoothed with a moving average filter, similar to the filter described in section 5.4.2.
5.4.7 Categorical average patterns 1D (CAP 1D Min)

In this method the $x$- and $y$-coordinate are obtained separately. First, only the $x$-coordinate of each reference event is used. The Euclidean distances $D$ of the unclassified light distribution to those of all irradiation positions with one specific $x$-coordinate are calculated. A subset of these reference events consisting of the $k_{\text{CAP}}$ nearest neighbors is then selected and an average light distribution is calculated for this subset. This is repeated for all $x$-coordinates of the irradiation positions. Then, the distances of the unclassified light distribution to all average light distributions are calculated. The $x$-coordinate corresponding to the minimum distance is assigned to the unclassified light distribution. A similar procedure is used to obtain the $y$-coordinate. This method is analogous to collecting reference data with a line source, similarly to the ‘split 1D’ methods described in sections 5.4.3 and 5.4.4.

**Figure 5.3:** Illustration of the CAP method showing the squared Euclidean distances of an unclassified light distribution to the average light distributions of the $k$ nearest neighbors at all irradiation coordinates $(x,y)$. In the standard CAP method the coordinate corresponding to the minimum distance is assigned to the unclassified light distribution.
5.5 Experimental methods

Measurements were performed with a monolithic scintillator detector based on a 13.2 mm × 13.2 mm × 10 mm monolithic LYSO:Ce³⁺ scintillator with optically polished surfaces (Crystal Photonics). An SiPM array (SensL SPMArray 3035G16) is optically coupled to the crystal using Sylgard 527 dielectric gel. All other faces of the crystal are covered with a highly reflective PTFE-based material (Spectralon).

The SiPM array is a 4 × 4 array of SiPM pixels measuring about 3 mm × 3 mm each and having a pitch of 2.5 mm. Each pixel has an active area of 2.85 mm × 2.85 mm, made up of 3640 microcells. The SiPM array was operated at the manufacturer-specified bias voltage of 29.3 V, exceeding the nominal breakdown voltage by 2.0 V and corresponding to a gain of ~10⁶.

The SiPM signals were preamplified using a 16-channel readout board described by Seifert et al. [20]. The preamplified SiPM pulses were shaped and their pulse heights digitized using the multichannel data acquisition system described by Maas et al. [6].

Reference data were collected by irradiating the detector with perpendicularly incident 511 keV gamma photons at a rectangular, equidistant grid of (x, y)-positions with a pitch of 0.25 mm and covering the entire front surface of the crystal. To this end, a ²²Na source was electronically collimated resulting in a beam with a diameter of ~0.64 mm. A detailed description of the experiments can be found in [9].

5.5.1 Spatial resolution

The spatial resolution was determined using a cross-validation method, i.e. the leave-one-out method described by Maas et al. [6]. The estimated coordinates are subtracted from the corresponding irradiation coordinates and a 2D-histogram is created, representing the detector spatial response. In this work the ~0.64 mm width of the test beam is much smaller than the detector spatial response and, therefore, the histogram approximates the point spread function (PSF) [21]. Since for small sample cases the PSF may suffer from low statistics, the histogram was fitted with an inverse polynomial shaped function

\[
f_{\text{fit}} = \frac{p_1}{1 + p_4 (p_2 x^2 + p_3 y^2) + p_5 (p_2 x^2 + p_3 y^2)^2},
\]

where \( p_i \) are fitting parameters. This function has no physical meaning, but it was chosen from a set of tested fitting functions, as it appeared to fit the PSFs best. An average value of the full width at half maximum (FWHM) and the full
width at tenth maximum (FWTM) was calculated of the cross sections of the fit in the $x$- and $y$-directions as well as the two directions with a 45 degrees angle to the $x$- and $y$-direction. This was repeated for different numbers of nearest neighbors and the minimum FWHM and corresponding FWTM were taken as measures of the spatial resolution.

Each interaction position estimation method described in section 5.4 was tested for different numbers of reference events per irradiation position $n_{\text{ref}}$ and for different total amounts of reference data $n_{\text{tot}}$. This was achieved by subsampling the full reference dataset by randomly removing a given number of events per irradiation position $n_{\text{ref,rem}}$. When $n_{\text{ref,rem}}$ was larger than the remaining $n_{\text{ref}}$, the discarded events were used to create an additional equivalent reference dataset with an equal value of $n_{\text{ref}}$. In the case that $n_{\text{tot}}$ was varied both $n_{\text{ref}}$ and $n_{\text{pos}}$ were decreased. In this way, the mean value and standard deviation of the spatial resolution were obtained for part of the cases.

The spatial resolution was determined using irradiation coordinates from the entire irradiation grid. Since the spatial resolution appeared to be rather different in the center compared to the edges of the detector, it was also determined for the central $6.3 \text{ mm} \times 6.3 \text{ mm}$ as well as for the remaining edges of the detector.

5.5.2 Bias sensitivity

For any algorithm that assigns interaction positions inside a given set of boundaries (e.g. the crystal dimensions), the PSF is truncated and/or asymmetrical near the edges. Systematic measurement errors, such as variations in gain or in offsets, may have a similar effect on the PSF. Such effects are commonly referred to as bias and should be taken into account when modeling the detector spatial response, e.g. for image reconstruction purposes.

Additional bias may arise from the interaction position estimation method itself. In order to compare the relative sensitivity to bias $S$ of the different methods, the following characterization was performed based on the principle that, if each position is irradiated with $n_{\text{ref}}$ events, an unbiased method would assign all irradiation positions $n_{\text{ref}}$ times in the absence of counting statistics.

For each method a 2D-histogram was created of the entire set of estimated interaction positions that were determined using each of the methods described in section 5.4. Subsequently, the number of reference data per irradiation position $n_{\text{ref}}$ was subtracted from each bin value $n_{x,y}$. The result was divided by $n_{\text{ref}}$. Then, it was first summed along the $y$-direction, the absolute value of each element of the resulting vector was calculated, this vector was summed and, finally, this value was divided by the total number of irradiation positions $n_{\text{pos}}$. This results in a relative bias sensitivity
This procedure was repeated with a reversed order of the two summations. By averaging the resulting two values, a measure for the bias sensitivity was obtained. The bias sensitivity was determined using irradiation coordinates of the entire irradiation grid, the central 6.3 mm × 6.3 mm, and the remaining edges of the detector.

\[
S = \frac{\sum_x \sum_y n_{x,y} - n_{\text{ref}}}{n_{\text{pos}}}. \tag{5.4}
\]

5.6 Results and discussion

In the remainder of this section the error bars indicate the standard deviations arising from the results of equivalent reference datasets as discussed in section 5.5.1. It should be noted that for the so-called 1D methods the number of \( n_{\text{ref}} \) corresponds to the number of reference events used for the determination of a single coordinate (\( x \) or \( y \)).

5.6.1 Spatial resolution

The spatial resolution calculated as an average over the central part of the detector surface in terms of FWHM and FWTM is shown in figures 5.4a and 5.4b, respectively, as a function of \( n_{\text{ref}} \), where the irradiation grid spacing was kept constant at 0.25 mm. Figures 5.5a and 5.5b show the same for the detector edges. In all cases all alternative methods outperform the standard method.

The five best performing methods yield approximately equal results, i.e. \(~1.5\) mm in terms of FWHM, at high \( n_{\text{ref}} \) in the central part of the detector. For all methods, the spatial resolution is better in the center of the detector than at the edges, both in terms of FWHM and FWTM. This is in agreement with previous findings [2],[6]. It suggests that the spatial resolution may be substantially improved by enlarging the detector area, resulting in a relatively large central area. Here, it should be noted that this improvement might be limited by a possible increase of statistical fluctuations on the number of detected photons per pixel and/or by electronic noise. Furthermore, a larger detector area might result in increased dead time of the detector, which might influence the count rate performance of the system.

Figures 5.6a and 5.6b show the spatial resolution in terms of FWHM and FWTM, respectively, calculated as an average over the entire detector surface as a function of the number of reference events per irradiation position \( n_{\text{ref}} \). Due to the somewhat small dimensions of the detector, the influence of the edges is
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large and, thus, the relative differences between the methods are comparable to the case of the detector edges. It is shown that all alternative methods yield a better spatial resolution, FWHM as well as FWTM, than the standard method. This improvement varies from ~10% to ~25% depending on $n_{\text{ref}}$.

More specifically, when comparing the CAP methods to their equivalent ‘normal’ methods, i.e. standard method vs. CAP, smoothed vs. CAP smoothed, and split 1D max vs. CAP 1D min, the CAP versions perform better. Apparently, the CAP methods make use of the information contained in the reference events more effectively in the interaction position estimation.

Figure 5.4: Spatial resolution in terms of (a) FWHM and (b) FWTM calculated as an average over the central part of the detector surface as a function of $n_{\text{ref}}$ for all methods investigated using a constant irradiation grid spacing of 0.25 mm.
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Furthermore, the CAP smooth method outperforms all other methods in terms of FWHM, whereas the two fitting methods give better results than the others in terms of FWTM. These two fitting methods yield an almost constant value for the spatial resolution as a function of $n_{\text{ref}}$, i.e. $\sim 1.85$ mm for FWHM and $\sim 4.25$ mm for FWTM.

A noteworthy result is the fact that the 1D methods perform at least as well and in most cases even better than the corresponding 2D method. This could be explained by the fact that, given a total number of reference events, there is more reference data per 1D coordinate.

**Figure 5.5:** Spatial resolution in terms of (a) FWHM and (b) FWTM calculated as an average over the edges of the detector surface as a function of $n_{\text{ref}}$ for all methods investigated using a constant irradiation grid spacing of 0.25 mm.
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Figures 5.7a and 5.7b show the spatial resolution in terms of FWHM and FWTM, respectively, calculated as an average over the entire detector surface as a function of $n_{\text{tot}}$ for all methods investigated using a constant irradiation grid spacing of 0.25 mm.

**Figure 5.6:** Spatial resolution in terms of (a) FWHM and (b) FWTM calculated as an average over the entire detector surface as a function of $n_{\text{ref}}$ for all methods investigated using a constant irradiation grid spacing of 0.25 mm.

Figures 5.7a and 5.7b show the spatial resolution in terms of FWHM and FWTM, respectively, calculated as an average over the entire detector surface, as a function of the total number of reference events $n_{\text{tot}}$. Each value of $n_{\text{tot}}$ in general allows different combinations of $n_{\text{ref}}$ and $n_{\text{pos}}$ (see equation 5.1), each yielding different values for the spatial resolution. This should be taken into account when reducing the amount of reference data to speed up calibration measurements. However, to provide a clear view of the spatial resolution in figures 5.7a and 5.7b, $n_{\text{tot}}$ was binned with logarithmically increasing bin size and for each bin center only the best spatial resolution is displayed.
The figures show that all alternative methods give a better spatial resolution than the standard method at practically all $n_{\text{tot}}$, in terms of FWHM as well as FWTM. Moreover, the CAP smooth method, the CAP 1D Min method, as well as the fitting methods, require ~10 to ~20 times less reference events than the standard method to obtain the same spatial resolution. Furthermore, the two smoothing methods yield a spatial resolution in terms of FWHM that is only slightly deteriorated at ~200 times less reference events, i.e. ~2.0 mm compared to ~1.9 mm.

If the latter result is multiplied by the calibration time reduction that can be

**Figure 5.7:** Spatial resolution in terms of (a) FWHM and (b) FW TM calculated for the entire detector surface a function of $n_{\text{tot}}$ for all methods investigated. As explained in the text, both the irradiation grid spacing and the number of reference events per irradiation point were varied.
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obtained by calibrating at a single angle of incidence and/or by using a line source (see section 5.2), a single detector might be calibrated within minutes. This would enable the calibration of all detectors in a PET system within a reasonable amount of time. Thus, using the improvements proposed in this work, the calibration may no longer be a limiting factor for the application of monolithic scintillator detectors in PET scanners.

This result furthermore implies that the exemplary computation time, given in section 5.1 for the calculation of the positions of interaction for a PET scan of several hundreds of millions of coincidences, might be reduced from to order of years to days. Moreover, it is expected that optimization of the implementation of the position estimation algorithm, parallelization of processes on multiple CPUs, and data preprocessing, may further reduce the computation time by several orders of magnitude. This then would decrease the computation time to the order of minutes, bringing the $k$-NN method within reach for practical use in PET scanners.

Finally, it is noted that when a calibration measurement with a reduced total number of reference data $n_{tot}$ is performed, it should be considered that there may be a combination of the number of reference events per irradiation position $n_{ref}$ and the number of irradiation positions $n_{pos}$ that provides optimum spatial resolution. The most favorable combination of $n_{ref}$ and $n_{pos}$ differs per $k$-NN method and, therefore, should be obtained empirically for each detector and for each implementation of the $k$-NN method separately.

5.6.2 Bias sensitivity

Figures 5.8a, 5.8b, and 5.8c show the bias sensitivity $S$ as defined in section 5.5.2 calculated as an average over the entire detector surface, the central part, and the edges, respectively, as a function of $n_{ref}$ where the irradiation grid spacing was kept constant at 0.25 mm. Considerable differences between the methods are observed when considering the total detector surface. However, it appears that in the detector center the bias sensitivity is small and roughly equal for all methods, in contrast to the edges of the detector. In the latter case the substantial bias sensitivity for the smoothing methods could be explained by the fact that the averaging kernel becomes smaller as well as asymmetrical close to the edges. For the fitting methods the larger bias sensitivity could be attributed to the fact that for interaction positions closer to the edge the shape of the 2D $k$-NN-histogram might be different from the rest of the detector (see sections 5.4.1 and 5.4.2). Furthermore, close to the edges the PSF is cut off and due to this loss of symmetry the fitting may introduce a bias, since the actual shape of the histogram is not necessarily a Lorentzian.
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Figure 5.8: Bias sensitivity $S$ as defined in section 5.5.2 calculated as an average over (a) the entire detector surface, (b) the central part of the detector surface, and (c) the edges of the detector surface, as a function of $n_{\text{ref}}$ for all methods investigated using a constant irradiation grid spacing of 0.25 mm.
5.7 Conclusions

Various methods for improving the standard \(k\)-nearest neighbor method used to determine the gamma photon \((x,y)\)-position of interaction in monolithic scintillator detectors for PET have been investigated.

For a given number of reference events, \(\sim 10\%\) to \(\sim 25\%\) better spatial resolution can be obtained by utilizing the information contained in the full set of nearest neighbors instead of only the one occurring most frequently.

Additionally, several of the alternative methods investigated yield a spatial resolution that is only slightly deteriorated, i.e. \(\sim 2.0\) mm FWHM compared to \(\sim 1.9\) mm FWHM, when using \(\sim 200\) times less reference events. Using these methods, the detector could be calibrated \(\sim 200\) times faster, but also the position estimation using the \(k\)-NN method may be accelerated with up to the same factor.

Moreover, it was shown that the \(k\)-NN method can make use of reference data corresponding to a line of irradiation points instead of singular points. The methods equivalent to calibrating with a line source yielded at least as good results as the standard method, while allowing for much faster and easier collection of the reference data.

It appeared that, compared to the standard method, the relative bias sensitivity of the improved methods was larger at the detector edges. This should be taken into account when modeling the detector spatial response, e.g. for image reconstruction purposes.

This work focused on several improvements of methodology, which may enable use of monolithic scintillators and/or the \(k\)-NN method in PET systems. However, the practical realization of a calibration method as well as a position estimation algorithm for monolithic scintillator detectors in a PET system will require further research as the details of the actual implementation have a strong influence on the calibration and computation times. All aspects of the implementation should be optimized taking into account e.g. the calibration source(s), collimator(s), and data preprocessing methods. In the implementation of the position estimation algorithm, aspects such as code efficiency, parallelization of processes, and data handling, will need to be studied.

It is, however, to be emphasized that the proposed improvements are multiplicative with the gains that can be obtained through implementation efficiency. Thus, the present results form an essential contribution to the development of methods that allow accelerated and facilitated calibration as well as position estimation in monolithic scintillator detectors, while maintaining their good performance.

Finally, we conclude that, using the improvements proposed in this work, the
high spatial resolution obtainable with the $k$-NN method may come within practical reach and, furthermore, the calibration may no longer be a limiting factor for the application of monolithic scintillator detectors in PET scanners.

References


Chapter 5. Improved nearest neighbor methods


Chapter 6

A practical method for depth of interaction determination in monolithic scintillator PET detectors


Abstract

Several new methods for determining the depth-of-interaction (DOI) of annihilation photons in monolithic scintillator detectors with single-sided, multi-pixel readout are investigated. The aim is to develop a DOI decoding method that allows for practical implementation in a positron emission tomography (PET) system. Specifically, calibration data, obtained with perpendicularly incident gamma photons only, are being used. Furthermore, neither detector modifications nor a priori knowledge of the light transport and/or signal variances are required. For this purpose, a clustering approach is utilized in combination with different parameters correlated to the DOI, such as the degree of similarity to a set of reference light distributions, the measured intensity on the sensor pixel(s) closest to the interaction position, and the peak intensity of the measured light distribution. The proposed methods were tested experimentally on a detector comprised of a 20 mm × 20 mm × 12 mm polished LYSO:Ce crystal coupled to a 4 × 4 multi-anode photomultiplier. The method based on the linearly interpolated measured intensities on the sensor pixels closest to the estimated (x,y)-coordinate outperformed the other methods, yielding DOI resolutions between ~1 mm FWHM and ~4.5 mm FWHM depending on the DOI, the (x,y)-resolution, and the amount of reference data used.
6.1 Introduction

Positron emission tomography (PET) detectors consisting of fast and bright monolithic scintillators, such as LYSO:Ce or LaBr₃:Ce, coupled to fast, high gain, multi-pixel photosensors, such as a multi-anode photomultiplier (MAPMT) or a silicon photomultiplier (SiPM) array, provide accurate time-of-flight (TOF) information, have good spatial and energy resolution, and allow for high system sensitivity [1]–[6]. An additional advantage of SiPM-array based detectors is that they are not affected by magnetic fields, which allows for the integration of PET with magnetic resonance imaging (MRI) [7],[8].

An important prerequisite for a PET detector is the ability to detect 511 keV annihilation gamma photons with high efficiency. This imposes the use of thick, high-density scintillation crystals with a high effective atomic number $Z$. However, when thicker crystals are used, the uncertainty about the depth-of-interaction (DOI) of the annihilation photons within the crystals gives rise to parallax errors, i.e. additional random as well as systematic distortions in the acquired lines-of-response (LOR).

It is well established that the use of PET detectors providing DOI information significantly reduces the resulting degradation of the image quality [9],[10]. Furthermore, in a 3D PET system with accurate DOI decoding it is possible to use a larger number of detector rings, i.e. a longer axial field-of-view (FOV), in order to increase the system sensitivity without deteriorating spatial resolution.

More recently, it was shown that DOI information can also be used to correct for time walk due to the varying gamma photon interaction position within the scintillator. This is required for an optimal timing resolution of the detectors in time-of-flight (TOF) PET systems [6],[11].

In literature numerous designs and methods have been proposed for PET detectors with DOI information. These include, for example, stacked scintillator layers with independent read-out, multiple scintillators with different decay time constants (phoswich), detectors using the relation between the measured number of scintillation photons and DOI, double-sided read-out concepts, and modifications of the crystal geometry involving e.g. saw cuts, reflective gaps, and/or the inter-crystal optical interfaces, aimed to achieve DOI-dependent light sharing between detector elements [12]–[23]. However, such designs may give rise to reduced detection efficiency, increased light losses, performance instabilities, and/or calibration difficulties. The manufacturing of such complex detector structures, which may e.g. involve the elaborate mechanical processing of components, the assembly of many crystals, reflectors, and other elements, the coupling of multiple light sensors, and the connection of extensive readout
electronics, may also be relatively expensive.

In contrast, monolithic scintillator detectors read out by multi-channel, position-sensitive light sensors offer simplicity of design. In previous works we employed a readout approach for such detectors in which, instead of the 3D interaction position of the annihilation photon, the entry point on the front surface of the crystal is determined using e.g. neural networks or the \( k \)-nearest neighbor (\( k \)-NN) method [1],[24]. It was shown that excellent spatial resolution and DOI correction can be obtained in this way [3],[4],[25]. However, the calibration of the detector may have to be performed at many angles of incidence, which can be impractical in a full detector ring and which can result in long calibration times.

Whereas the approach of entry point determination offers intrinsic DOI correction, one may also explicitly estimate the DOI from the measured scintillation light distribution. Several DOI decoding methods for monolithic scintillator detectors have been described in literature [6],[26]–[31]. Many of these rely on the correlation between the DOI and the width of the light distribution on the photosensor pixels and/or make use of a model of the light transport in the crystal. However, since a large fraction of the scintillation photons may undergo many interactions (e.g. reflections or scattering) before being detected by the photosensor, it may be difficult to obtain an accurate optical transport model and/or the relationship between the width of the light distribution and the DOI may be relatively weak. Furthermore, some of the approaches give rise to light losses, sometimes applied intentionally to suppress edge effects, but worsening energy resolution and overall detector performance. Finally, some methods require sideways irradiation for calibration, which is practically impossible in a fully assembled PET system.

The goal of the present work is to investigate DOI decoding methods that neither require any detector modifications nor \textit{a priori} knowledge of the light transport and/or signal variances. Moreover, we aim to calibrate the detector with perpendicularly incident annihilation photons only and to use these calibration data to separately estimate both the \((x,y)\)-coordinates and the DOI, allowing for a relatively easy and fast calibration procedure. These objectives follow from the requirement that it should be possible to implement the DOI decoding method in a PET system in a practical way.

The DOI can be estimated using a clustering method, which is based on some parameter correlated to the DOI, such as the peak value of the measured light distribution, and which requires an additional lookup table only [32]. In this paper we study several new measures correlated to the DOI in monolithic scintillator detectors with single-sided, multi-pixel readout. These DOI measures
include the degree of similarity to a set of reference light distributions, the measured intensity on the sensor pixel(s) closest to the interaction position, and the largest measured intensity (peak value) of a light distribution. The proposed methods are tested on a 4 × 4 MAPMT-based monolithic LYSO:Ce detector. The (x,y)-coordinates are estimated by means of the so-called categorical average patterns (CAP) version of the nearest neighbor method [25]. The DOI resolution is investigated as a function of depth, (x,y)-resolution, and the amount of reference data used.

6.2 Methods

The DOI is estimated by means of a clustering method per (x,y)-coordinate. For each event, the (x,y)-position is determined first by means of the CAP method that will be discussed in section 6.2.1. Using the calibration data, obtained with perpendicularly incident gamma photons only, for this (x,y)-coordinate, DOI estimation is subsequently performed based on several different parameters correlated to the DOI, which are described in more detail in section 6.2.2. In section 6.2.3 it is explained how these measures are used in practice to decode the DOI and how they are validated.

6.2.1 CAP nearest neighbor (x,y)-position determination

The CAP method for the (x,y)-position estimation is an alternative to the standard k-nearest neighbor (k-NN) method having the advantage of better spatial resolution with less calibration data [25]. It is based on a reference dataset consisting of normalized light distributions recorded by irradiating a monolithic scintillator detector with 511 keV gamma photons at an equidistant grid of known (x,y)-positions (see figure 6.1) [25]. For each irradiation position, the (normalized) light intensity distributions \( I = (I_1, I_2, \ldots, I_N) \) of \( n_{\text{ref}} \) reference events are recorded, where \( N \) denotes the number of sensor pixels. Each light intensity distribution is normalized such that the sum of the measured intensities \( \sum_{i=1}^{N} I_i \) equals 1. The similarity of an unclassified light distribution \( I_{\text{test}} \) to those of all \( n_{\text{ref}} \) reference events at a single irradiation position \( I_{\text{ref,x,y}} \) is calculated in terms of the Euclidean distance

\[
D = \sqrt{\sum_{i=1}^{N} (I_{\text{test},i} - I_{\text{ref,x,y},i})^2}.
\] (6.1)

A subset of these reference events consisting of the \( k \) closest matches (i.e. with the smallest Euclidean distances \( D \)) is subsequently selected and an average light distribution is calculated for this subset. This is repeated for each
irradiation position. Then, the distances $d_{av}(x,y)$ of the unclassified light distribution to the average light distributions at all of the irradiation $(x,y)$-positions are calculated. The irradiation position corresponding to the minimum distance is estimated as the $(x,y)$-position of the event under investigation.

### 6.2.2 DOI measures

#### 6.2.2.1 Degree of similarity

If a gamma photon interacts close to the multi-pixel photosensor, a small variation of the interaction position gives rise to a large variation of the measured light distribution. This is in contrast with interactions far away from the sensor, where the light distribution may hardly change with the interaction position. In the latter case, assuming that a set of light distributions originating from interactions distributed over the entire crystal is available, there will be many reference light distributions very similar to the light distribution under investigation.

This principle is exploited to decode the DOI by utilizing the CAP nearest neighbor algorithm described in the previous section. This is done by evaluating the distances $d_{av}(x,y)$ of the unclassified light distribution to the average light distributions at all of the irradiated $(x,y)$-positions. In case $d_{av}(x,y)$ is a sharply
peaked distribution around the minimum corresponding to the estimated \((x,y)\)-coordinate, the event under investigation is very dissimilar to all others and is assigned to a depth close to the sensor. In case \(d_{av}(x,y)\) is a flat, wide distribution, the event under investigation is very similar to all others and is assigned to a depth far away from the sensor.

Various DOI measures based on \(d_{av}(x,y)\) were tested and the best performing ones were selected. The first DOI measure is defined as

\[
\text{CAP Max} = \frac{\max\{d_{av}(x, y)\}}{\text{mean}\{d_{av}(x, y)\} \cdot \text{median}\{d_{av}(x, y)\}}.
\] (6.2)

The second measure for the DOI is given by

\[
\text{CAP MD} = \frac{\text{md}\{d_{av}(x, y)\}}{\text{mean}\{d_{av}(x, y)\} \cdot \text{median}\{d_{av}(x, y)\}},
\] (6.3)

where \(\text{md}\{d_{av}(x,y)\}\) denotes the mean difference between all \(d_{av}(x,y)\).

6.2.2.2 Closest and maximum sensor pixel intensity

Gamma photon interactions close to the multi-pixel photosensor give rise to scintillation events having a high optical photon flux on a localized region of the sensor. On the other hand, the optical photons originating from scintillation events far away from the sensor spread over a large area of the sensor and, thus, the measured light distribution is more flat.

This principle is utilized as a measure for the DOI as follows. First, the \((x,y)\)-position is estimated, for which in this work the CAP nearest neighbor method described in section 6.2.1 is used. As a DOI measure, the measured intensity (normalized such that the sum of all sensor pixel values equals unity) on the sensor pixel that is closest to the estimated \((x,y)\)-position can be taken. This method based on the pixel intensity of the closest pixel is henceforth referred to as CL PI. Alternatively, the measured normalized intensities on the (in general four) sensor pixels closest to the estimated \((x,y)\)-coordinate can be (bi)linearly interpolated (LI PI). Further DOI measures are the maximum of the normalized light intensity distribution (similar to the method presented by Ling et al. [32]) or the measured intensity on the pixel having on average the largest value for the calibration events corresponding to the particular \((x,y)\)-position (Max PI, Av Max PI).

6.2.3 Experimental methods

Measurements were performed on a monolithic scintillator detector consisting of a \(4 \times 4\) MAPMT (Hamamatsu H8711-03) optically coupled to a \(20 \text{ mm} \times 20 \text{ mm} \times 12 \text{ mm}\) polished LYSO:Ce crystal using Sylgard 527 dielectric gel. All
other faces of the crystal are covered with a highly reflective PTFE-based material (Spectralon). A 511 keV gamma photon beam with a diameter of ~1 mm was provided by electronically collimating a ~1 mm spherical $^{22}$Na source using a collimated photomultiplier in coincidence with the monolithic scintillator detector. A detailed description of the experiments can be found in [6].

6.2.3.1 Calibration measurements

A calibration measurement was performed in which a reference dataset was collected by irradiating the monolithic scintillator detector with perpendicularly incident 511 keV gamma photons at an equidistant grid of $n_{\text{pos}}$ known (x,y)-positions having a pitch of 2.0 mm and covering the entire front surface of the crystal. For each irradiation position, the resulting light intensity distributions of $n_{\text{ref}}$ reference events within the photopeak were recorded, where $n_{\text{ref}} = 2500$. All of these $n_{\text{pos}} \times n_{\text{ref}}$ light patterns were subsequently normalized such that the sum of all sensor pixel values equals unity.

6.2.3.2 (x,y)-resolution

The (x,y)-resolution was determined using irradiation coordinates from the entire irradiation grid with a cross-validation method, i.e. the leave-one-out method described in more detail in [3],[25]. The full width at half maximum (FWHM) and the full width at tenth maximum (FWTM) of the detector spatial response were calculated for different numbers $k$ of nearest neighbors and the minimum FWHM and corresponding FWTM were taken as measures of the (x,y)-resolution.

To investigate the influence of the amount of reference data used per irradiation position $n_{\text{ref}}$ on the (x,y)-resolution as well as on the DOI resolution (see section 6.2.3.4), the full reference dataset was subsampled by randomly removing a given number of events per irradiation position $n_{\text{ref,remove}}$. When $n_{\text{ref,remove}}$ was larger than the remaining $n_{\text{ref}}$, the discarded events were used to create an additional equivalent reference dataset with an equal value of $n_{\text{ref}}$. In this way, the mean value and standard deviation of the spatial resolution were obtained for part of the cases.

6.2.3.3 DOI determination

First, the probability distribution of the DOI of perpendicularly incident 511 keV gamma photons within an LYSO:Ce scintillator was determined by means of a Monte Carlo simulation, using GATE [33],[34]. In case of multiple interactions due to Compton scattering, an energy-weighted DOI was calculated.
Subsequently, the fractions $f_z$ of the number of average interaction positions were determined within each of the $n_z$ depth ranges obtained by subdividing the crystal height (12 mm) into bins of $12/n_z$ mm. In this work $n_z = 12$ is used.

For each $(x,y)$-irradiation-position a lookup table was created for each of the DOI measures discussed in section 6.2.2, henceforth referred to as the DOI reference dataset. This was done by calculating the value of a given DOI measure for all $n_{ref}$ reference light distributions corresponding to a given $(x,y)$-position. The resulting values were sorted in ascending or descending order depending on the DOI measure. Then, the values that demarcate the fractions $f_z$ of the series of sorted values were selected. These $n_z - 1$ reference values, which estimate the boundaries of the $n_z$ depth ranges, are stored per $(x,y)$-irradiation-position in a lookup table constituting the DOI reference dataset for a certain DOI measure.

The DOI corresponding to any light distribution can now be estimated as follows. First, the $(x,y)$-position is estimated using the CAP nearest neighbor method. Then, the DOI measure is calculated and compared to the reference values in the lookup table corresponding to the DOI measure and the estimated $(x,y)$-position. The DOI is subsequently determined as the $z$-value corresponding to the depth range that is demarcated by the two DOI reference values above and below the DOI measure of the light distribution. It should be noted that if the value of the DOI measure is smaller than all reference values, the first depth range is taken, while if the value of the DOI measure is larger than all reference values, the last depth range is taken.

### 6.2.3.4 DOI resolution

The performance of the different DOI determination methods was tested by recording a validation dataset, which was obtained by irradiating one of the lateral sides of the crystal with perpendicularly incident 511 keV gamma photons at an equidistant grid of $(y,z)$-positions having a pitch of 2.0 mm. For each of these light distributions the DOI was determined based on the DOI reference dataset for front irradiation as described in sections 6.2.3.1, 6.2.3.2, and 6.2.3.3. Then, for each $z$-position used in the side irradiation, an error histogram was created containing the difference between the estimated DOI and the side irradiation $z$-coordinate. The full width at half maximum (FWHM) was subsequently determined representing the DOI resolution as a function of DOI. For the methods based on the degree of similarity, this was repeated for different numbers of nearest neighbors used for the calculation of the DOI measures. The number of nearest neighbors that resulted in the best DOI resolution was selected.
An average value of the DOI resolution over all z-positions used in the side irradiation measurement was calculated for each method. The resulting values were compared to the \((x,y)\) spatial resolution and they were investigated as a function of the amount of reference data \(n_{\text{ref}}\). In the latter case, \(n_{\text{ref}}\) was varied as described in section 6.2.3.2 and a mean value and a standard deviation of the DOI resolution were obtained for part of the cases in a similar way as for the \((x,y)\)-resolution.

### 6.3 Results and discussion

Figure 6.2 shows the \((x,y)\) spatial resolution FWHM and FWTM for different numbers of reference events per irradiation position.

![Figure 6.2: (x,y) spatial resolution FWHM and FWTM for different numbers of reference events per irradiation position.](image)

An average value of the DOI resolution over all z-positions used in the side irradiation measurement was calculated for each method. The resulting values were compared to the \((x,y)\) spatial resolution and they were investigated as a function of the amount of reference data \(n_{\text{ref}}\). In the latter case, \(n_{\text{ref}}\) was varied as described in section 6.2.3.2 and a mean value and a standard deviation of the DOI resolution were obtained for part of the cases in a similar way as for the \((x,y)\)-resolution.

**6.3 Results and discussion**

Figure 6.2 shows the \((x,y)\)-resolution as a function of \(n_{\text{ref}}\). The error bars indicate the standard deviation arising from the results of equivalent reference datasets as discussed in section 6.2.3.2. The spatial resolution deteriorates quickly for smaller amounts of reference data, in agreement with earlier findings [24],[25]. For completion, it is noted that in addition to \(n_{\text{ref}}\) also the irradiation beam width, scintillation photon statistics, electronic noise, and the spacing of the irradiation grid determine the measured \((x,y)\) resolution [35].

The DOI resolution averaged over all irradiation depths as a function of \(n_{\text{ref}}\) is shown in figure 6.3. The error bars indicate the standard deviation arising from the results of equivalent reference datasets as discussed in section 6.2.3.4. The abbreviations in the legend refer to the different DOI measures discussed in
sections 6.2.2.1 and 6.2.2.1. The method based on the linearly interpolated measured intensities on the sensor pixels surrounding the estimated \((x,y)\)-coordinate (LI PI) outperforms the others, yielding a value of \(~3.4\) mm FWHM. The methods based on the degree of similarity both exhibit a comparable DOI resolution. Similar to the \((x,y)\)-resolution, the average DOI resolution deteriorates for \(n_{\text{ref}} < 500\). This can be attributed to the limited \((x,y)\)-resolution and the larger statistical fluctuations of the fraction of events in each depth range from the DOI probability distribution.

Figure 6.4 shows the average DOI resolution as a function of the number of nearest neighbors \(k\) used for the \((x,y)\)-coordinate estimation in the case of \(n_{\text{ref}} = 2500\). For the methods based on the degree of similarity, the number of nearest neighbors used for the calculation of the DOI measures was chosen such that the best DOI resolution was obtained given the value of \(k\). A clear correlation between the DOI resolution and the \((x,y)\)-resolution is observed. At \(k = 100\), where the \((x,y)\)-resolution reaches its optimum, also the DOI resolutions approach their optima. This implies that the DOI resolution may be further improved by optimizing the \((x,y)\)-resolution. This may e.g. be achieved by refining the \((x,y)\) irradiation grid, optimizing the nearest neighbor method, or by filtering outliers from the reference dataset [25],[32].

Here, it is noted that the information obtained about the DOI itself might be

**Figure 6.3:** Average DOI resolution as a function of \(n_{\text{ref}}\) for different DOI methods. The abbreviations in the legend refer to the different DOI measures discussed in sections 6.2.2.1 and 6.2.2.2.
6.3. Results and discussion

used to improve the \((x,y)\)-position determination, e.g. by using different numbers
of nearest neighbors \(k\) per DOI range or by using only reference data
corresponding to the estimated DOI. In such a study the DOI- and \((x,y)\)-
resolution might, in principle, be optimized by iteratively estimating the \((x,y)\)-
positions and the DOI.

The DOI resolution as a function of DOI is shown in figure 6.5. It appears
that the method based on the linearly interpolated measured intensities on the
sensor pixels surrounding the estimated \((x,y)\)-coordinate (LI PI) outperforms the
others at almost all depths. Close to the front surface, i.e. at small DOI, all
methods perform about the same. Furthermore, some methods seem to provide
better DOI resolution at small DOI compared to DOI positions closer to the
center of the crystal. This is attributed to the fact that the DOI error histogram is
truncated at the position corresponding to the top of the crystal, resulting in a
smaller FWHM of the histogram.

The observed improvement of the DOI resolution with increasing DOI can
be explained using arguments that are similar to those given by Salvador et al.
[19] for the DOI resolution in segmented crystals and that also bear similarity to
the arguments given by Van der Laan et al. [36] to explain the improvement of

![Figure 6.5: Average DOI resolution as a function of the number of nearest
neighbors used for the \((x,y)\)-coordinate estimation for different DOI methods, where
\(n_{ref} = 2500\). The curve of the \((x,y)\)-resolution corresponds to the FWHM. The
abbreviations in the legend refer to the different DOI measures discussed in sections
6.2.2.1 and 6.2.2.2.](image)
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Salvador et al. used a DOI measure based on the measured intensity on either one or two photosensors for single-sided and double-sided readout, respectively. It was shown that in the case of single-sided readout, the change of measured intensity as a function of DOI was much larger for events close to the sensor than for events farther away from the sensor. Thus, measured light intensities originating from events far away from the sensor are much more alike than those that are close by. Since this DOI measure bears similarity to the methods based on the intensity of the closest and maximum sensor pixel discussed in this work, this would also explain why the DOI resolution deteriorates for smaller DOI in our monolithic scintillator detector.

6.4 Conclusions

Several new methods for the determination of the DOI in a monolithic scintillator detector were investigated. These methods fulfill essential requirements for practical implementation in a PET system, viz. they utilize the same calibration data, obtained with perpendicularly incident gamma photons only, as used for the \((x,y)\)-position estimation, while neither detector modifications nor models of the light transport and/or signal variances are required.
The different methods were tested experimentally on a 20 mm × 20 mm × 12 mm polished LYSO:Ce crystal coupled to a 4 × 4 pixel MAPMT. DOI resolutions between ~1 mm and ~4.5 mm were obtained depending on the DOI and the amount of reference data used. The method based on the linearly interpolated measured intensities on the sensor pixels surrounding the estimated (x,y)-coordinate (LI PI) outperformed the others providing an average DOI resolution of ~3.4 mm. Another observation was that a significantly better DOI resolution could be achieved by improving the (x,y)-resolution, which may for example be done by refining the irradiation grid used for calibration or by Compton rejection and/or by filtering of the calibration data.

Acknowledgements

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References


Chapter 6. Depth of interaction


Chapter 6. Depth of interaction


Chapter 7

Discussion and outlook

This thesis focuses on the understanding and characterization of PET detectors based on monolithic scintillators. For this purpose, several important optical parameters of the recently discovered LaBr$_3$:Ce material were measured, an analytic model of the output signals of novel silicon photomultiplier (SiPM) photosensors was created, and various improved algorithms for the estimation of the position of interaction were investigated. Furthermore, during the course of this research, arrays of SiPMs useful for implementation in PET detectors became available for the first time. A prototype detector consisting of a monolithic LYSO:Ce crystal coupled to one of these SiPM arrays was constructed and tested. Below, the results and implications of several of these investigations are discussed and an outlook on future research and developments is presented.

7.1 Monolithic scintillator detectors

The early SiPM-based monolithic scintillator detector prototype investigated in chapter 4 showed a spatial resolution of ~1.6 mm FWHM, a coincidence resolving time (CRT) of ~1.4 ns FWHM, and an energy resolution of ~14% FWHM. Thus, it outperformed the current state-of-the-art detectors implemented in commercially available clinical PET scanners essentially only in terms of spatial resolution (see table 1.1 in chapter 1). However, as argued in chapter 4, considerable further performance improvement is expected from the ongoing research on better SiPM-based detectors.

Compared to earlier prototype monolithic detectors based on avalanche photodiode (APD)-arrays, the measured spatial and energy resolutions are slightly worse [1]. This is mainly attributed to the rather low photon detection efficiency (PDE) of the active area of the SiPM arrays that were used, namely ~6%, compared to up to ~75% for APDs. Nevertheless, the spatial and energy resolutions obtained with APDs are inherently limited by their relatively large excess noise factor and dark current. Due to their relatively low gain of ~$10^2$ – $10^3$, preamplifier noise is another limiting factor. Thus, SiPM arrays with improved PDE and even lower dark count rates may in principle provide better spatial and energy resolution than can be obtained with APD arrays. Recently,
Chapter 7. Discussion and outlook

this has been investigated with measurements on a Hamamatsu S1106_050P SiPM array coupled to a monolithic 16 mm × 18 mm × 10 mm LaBr₃:Ce crystal, resulting in an energy resolution of ~6.4% FWHM and a spatial resolution of ~1.6 mm FWHM [2]. Furthermore, measurements using a so-called digital SiPM (see section 7.3) coupled to a monolithic 24 mm × 24 mm × 10 mm LSO:Ce,Ca crystal yielded an energy resolution of ~11% FWHM and a spatial resolution of ~1.0 mm FWHM [3].

The CRT of ~1.4 ns FWHM of our first prototype detector was not yet as good as one might expect based on the fast response of the SiPMs. The measured timing resolution was limited by the PDE, the dark count rate, the presence of two dead pixels of the SiPM array, and, importantly, the fact that the pre-amplifiers were not optimized for timing. With fast, low noise, optimized pre-amplifiers and single Hamamatsu MPPC S10362-11-050C SiPMs, which have much better performance [4], we measured a CRT of 100 ps FWHM for small (3 mm × 3 mm × 5 mm) LaBr₃:Ce crystals [5] and 125 ps FWHM for LSO:Ce,Ca crystals of the same dimensions [6]. When using digital SiPMs coupled to the same small LSO:Ce,Ca crystals, a value of 120 ps FWHM was achieved [6]. For larger scintillation crystals somewhat higher CRTs are expected, since the transient time spread increases as a result of the larger photon path lengths, the larger variation in the interaction positions of the gamma photons, the variation of the pulse shape with the interaction position, and increased scintillation light losses resulting from the relatively larger reflector area as well as the larger dead area. First test measurements performed by Vinke et al. with a 16 mm × 18 mm × 10 mm LaBr₃:Ce crystal to a 4 × 4 Hamamatsu S1106_050P SiPM array yielded a CRT of ~320 ps FWHM [7]. More optimized measurements by Seifert et al. resulted in a CRT of ~200 ps FWHM [2]. First tests with a prototype digital SiPM array coupled to a 24 mm × 24 mm × 10 mm LSO:Ce,Ca crystal provided a CRT of < 350 ps FWHM [8]. In that study it was also shown that making use of the time stamps obtained from multiple photosensor pixels can further improve the timing resolution of monolithic scintillator detectors. In order to approach the excellent timing resolution obtained with small crystals, the optimization of monolithic scintillator detectors, the development of new sensors, and the investigation of optimal time pickoff methods remain of large interest in current research.

The early SiPM-based prototype monolithic scintillator detector that was investigated in chapter 4 provides very similar performance compared to many of the detectors based on pixelated crystal matrices assessed by other authors. In table 7.1 an overview is presented of several detector geometries that have been investigated recently. In addition to this table it is noted that Delfino et al. and
Yamaya *et al.* also provided depth-of-interaction resolutions of ~1.5 mm and 1 mm, respectively. It can be seen that our latest prototype detectors, i.e. the ones described by Seifert *et al.* and Van der Lei *et al.*, have outstanding overall performance.

Although the performance of our prototype detectors can still be improved considerably in several ways, the concept of a monolithic scintillator read out by an SiPM array has large potential for implementation in PET systems, since it has many practical advantages. For example, compared to detectors based on segmented crystals, the number of crystals needed, and therefore the labor involved in crystal surface processing, can be significantly reduced. Moreover,

**Table 7.1:** Overview of prototype PET detectors based on SiPMs. In most detectors LYSO:Ce crystals were used, except for the one in Seifert *et al.* (LaBr₃:Ce) and the ones in Song *et al.*, Yamamoto *et al.*, and Yamaya *et al.* (LGSO). $R_E$ denotes the FWHM energy resolution and $R_S$ denotes the FWHM spatial resolution.

<table>
<thead>
<tr>
<th>Reference</th>
<th>No. of crystals</th>
<th>Size of crystals (mm³)</th>
<th>No. of SiPM pixels</th>
<th>Pixel size (mm²)</th>
<th>$R_E$ (%)</th>
<th>CRT (ps)</th>
<th>$R_S$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Degenhardt [9]</td>
<td>8×8</td>
<td>4×4×22</td>
<td>8×8</td>
<td>4×4</td>
<td>10</td>
<td>290</td>
<td>–</td>
</tr>
<tr>
<td>Delfino [10]</td>
<td>12×12</td>
<td>1×1×10</td>
<td>2×(4×4)</td>
<td>3×3</td>
<td>20</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>España [11],[12]</td>
<td>4×4</td>
<td>1.5×1.5×12</td>
<td>2×2</td>
<td>3×3</td>
<td>11–22</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Hong [13]</td>
<td>4×4</td>
<td>3×3×20</td>
<td>4×4</td>
<td>3×3</td>
<td>19</td>
<td>1600</td>
<td>–</td>
</tr>
<tr>
<td>Kato [14]</td>
<td>4×4</td>
<td>3×3×10</td>
<td>4×4</td>
<td>3×3</td>
<td>14</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Kolb [16]</td>
<td>12×12</td>
<td>1.5×1.5×10</td>
<td>3×3</td>
<td>3×3</td>
<td>14</td>
<td>1400</td>
<td>–</td>
</tr>
<tr>
<td>Llosa [17]</td>
<td>1</td>
<td>12×12×5</td>
<td>8×8</td>
<td>1.5×1.4</td>
<td>16</td>
<td>8500</td>
<td>0.9</td>
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<tr>
<td>Schaart [18]</td>
<td>1</td>
<td>13×13×10</td>
<td>4×4</td>
<td>3×3</td>
<td>14</td>
<td>1400</td>
<td>1.9</td>
</tr>
<tr>
<td>Seifert [2]</td>
<td>1</td>
<td>16×18×10</td>
<td>4×4</td>
<td>3×3</td>
<td>6.4</td>
<td>200</td>
<td>1.6</td>
</tr>
<tr>
<td>Solf [19]</td>
<td>4×4</td>
<td>4×4×22</td>
<td>8×8</td>
<td>4×4</td>
<td>15</td>
<td>680</td>
<td>–</td>
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<tr>
<td>Song [20]</td>
<td>10×10</td>
<td>0.8×0.8×3</td>
<td>4×4</td>
<td>3×3</td>
<td>20</td>
<td>1700</td>
<td>–</td>
</tr>
<tr>
<td>Van der Lei [3]</td>
<td>1</td>
<td>24×24×10</td>
<td>6×6</td>
<td>4×4</td>
<td>11</td>
<td>&lt;350</td>
<td>1.0</td>
</tr>
<tr>
<td>Yamamoto [21]</td>
<td>15×15</td>
<td>0.7×0.7×6</td>
<td>4×4</td>
<td>3×3</td>
<td>15</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Yamaya [22]</td>
<td>12×12×12</td>
<td>1×1×1</td>
<td>6×(4×4)</td>
<td>3×3</td>
<td>11</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>
Chapter 7. Discussion and outlook

The assembly of detectors can be easier and more cost effective. The reduced dead space between crystals allows for highest possible system sensitivity. Furthermore, monolithic scintillator detectors appear to be less sensitive to photosensor deterioration. For example, if one or a few of the photosensor pixels are dead, the detector may still provide acceptable performance. In contrast, in a system with segmented crystals dead pixels lead to missing LORs and/or, in case of light sharing between crystals, to significant distortion of the detector flood map.

The use of SiPMs instead of photomultiplier tubes (PMTs) has the benefit of allowing for integration of PET devices in magnetic resonance imaging (MRI) systems because of their insensitivity to magnetic fields. Moreover, as the SiPM array is very thin and composed of low-Z (atomic number) materials only, it can be placed on the front surface of the crystal without significantly disturbing the incident annihilation photons, enabling novel detector geometries that may allow e.g. more accurate depth-of-interaction (DOI) determination and/or the use of thicker crystals to maximize PET system sensitivity.

Additionally, monolithic scintillator detectors have several intrinsic properties that may be beneficial for performance. Due to the more favorable aspect ratio of monolithic scintillator detectors compared to segmented detectors, the average number of reflections per scintillation photon is smaller. Therefore, the average time to reach the photosensor as well as the spread in the photon transit times are smaller and, furthermore, there are fewer losses within the crystal and the reflective crystal enclosure. These factors are expected to result in better timing and energy resolutions. It would also explain why the energy resolution of monolithic scintillator detectors tends to be better than that of segmented crystals, in spite of the fact that the scintillation light is spread over a relatively large number of sensor pixels, which usually results in a somewhat larger dead area and poorer signal-to-noise ratio. An additional advantage may be that the energy and timing resolution may be less dependent on the depth of interaction. Investigating how this potentially better performance can be exploited in practice will be an interesting research topic for the next years.

7.2 Calibration and interaction position determination

Despite the many advantages of monolithic scintillator detectors mentioned above, at this moment such detectors have not yet been implemented in whole-body clinical TOF PET systems. Reasons can for example be found in more difficult data processing, as the gamma photon interaction position needs to be
determined from the measured light distribution, in contrast to the easier method of crystal segment identification, and in the more complex as well as elaborate calibration of the detectors. Therefore, part of the work presented in this thesis is attributed to the investigation of methods for faster and easier detector calibration that maintain the good performance.

The first step to reach this goal is to calibrate the detector with perpendicularly incident annihilation photons only, rather than at many \((\sim 10^3)\) different angles of incidence, accelerating the calibration procedure with essentially the same factor. Calibration at many angles would be required for the determination of the entry point of gamma photons at the front surface of the detector, which implicitly provides DOI correction. However, if the DOI is determined explicitly, the lateral coordinates can accurately be obtained using perpendicularly incident reference data only. Therefore, in chapter 6, several DOI decoding methods were investigated, which require no additional calibration measurements and for which no detector modifications or models of the light transport and/or signal variances are needed. Determining the 3D position of interaction is also required for optimal timing performance, since the measured pulse shape and arrival time are expected to depend on this position.

Additional acceleration as well as facilitation of the calibration procedure could be achieved by irradiating the detector front surface along two series of lines, viz. one series parallel to the \(x\)-axis and one series parallel to the \(y\)-axis, instead of at a grid of points (see figure 5.1). A line source calibration would require only \(n_{\text{pos,x}} + n_{\text{pos,y}}\) calibration positions instead of \(n_{\text{pos,x}} \times n_{\text{pos,y}}\) positions, where \(n_{\text{pos,x}}\) and \(n_{\text{pos,y}}\) denote the number of calibration positions in the \(x\)- and \(y\)-directions, respectively. Such a procedure could be implemented by either mechanical or electronic collimation. In the first case, one could e.g. use a line source in combination with a slit collimator in between the source and the detector. In the second case, one might e.g. place a line source in between the detector under investigation and a coincidence detector equipped with a slit collimator. A line source has the advantages that it can contain a much higher activity, it is easier to produce, and the geometric efficiency for the detection of gamma photons can be much higher. Therefore, it allows for much faster (several orders of magnitude) and easier collection of the reference data. However, the use of a line source requires a position estimation method that can make use of reference data corresponding to a line of irradiation points instead of singular points. It was demonstrated in chapter 5 that the so-called \(k\)-nearest neighbor (\(k\)-NN) method could handle such calibration data. Moreover, the methods equivalent to calibrating with a line source were shown to yield considerably better results than the standard method for a given total number of
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reference events.

For the \( k \)-NN method it can furthermore be shown that the probability of misclassification, i.e. mispositioning, approaches the theoretical minimum for sufficiently large reference data sets [23]. In [24] it was indeed demonstrated that the spatial resolution obtained with the \( k \)-NN method can approach the Cramér-Rao lower bound. Moreover, in practice the \( k \)-NN method appears to yield unsurpassed spatial resolution, e.g. a 10 mm × 20 mm × 20 mm LYSO:Ce crystal read out from two sides with 8 × 4 APD arrays provided an average resolution of \( \sim 1.8 \text{ mm FWHM} \) [1], a 16 mm × 18 mm × 10 mm LaBr\(_3\):Ce crystal coupled to a 4 × 4 SiPM array resulted in an average resolution of \( \sim 1.6 \text{ mm FWHM} \) [2], and with a 24 mm × 24 mm ×10 mm LSO:Ce,Ca crystal coupled to a (digital) SiPM array an average resolution of \( \sim 1.0 \text{ mm FWHM} \) was obtained [3]. It is noted that these values are not corrected for the \( \sim 0.5 \text{ mm – 0.7 mm test beam diameter} \) and, thus, the intrinsic detector resolution is even better.

A drawback of the \( k \)-NN method is that the estimation of the position of interaction is computationally intensive as is the case for many statistical methods. This may render such methods impractical for use in PET scanners. Yet, considering developments in improved algorithms, faster and better FPGAs, high-bandwidth data transfer technologies, larger memories, and multi-core processing units, the advantages of the \( k \)-NN method may still be within reach. Various methods to considerably speed up the handling and processing of the acquired data may be used, which would allow for the implementation of (better performing) statistical methods, for the use of more cost effective FPGAs, and for higher maximum count rates, all bringing monolithic scintillator detectors closer to implementation in clinical PET systems.

For example, pre-selection of relevant reference data can be performed based on a fast deterministic entry point estimation method, reducing the computation time with a factor up to the fraction of discarded reference events. Further computational speed increases can be obtained either by parameterizing the light distributions on the sensor pixels, or by summing the rows as well as the columns of the light distributions. Then, instead of all pixel values either the parameters or the summed values are used in the estimation algorithm, which could result in a significant reduction of the computation time, especially for detectors with many pixels.

Specifically, computation times of the \( k \)-NN method can be further decreased considerably in several other ways, e.g. by reducing the number of calculations and/or preprocessing the data [25]. In chapter 5, several improvements of the \( k \)-NN method itself were investigated that make better use of the information contained in the full set of nearest neighbors. It was shown that improved
versions of the standard method need two orders of magnitude less reference data to yield essentially the same spatial resolution as the standard method. This greatly reduces the time needed for both calibration and interaction position computation and may bring the high spatial resolution achievable with the \(k\)-NN method within reach for practical use in PET scanners.

Nevertheless, the \(k\)-NN method may still be more computation intensive than other position estimation methods. For example, neural networks can be extremely fast, once they are trained, and can even be implemented directly in hardware. Maximum likelihood estimation (MLE) methods may also be somewhat less computationally intensive depending on their implementation. However, the \(k\)-NN method, although maybe slower, might still be a candidate for practical use, if both FPGA capacity and speed allow for it. Then, it might, for example, have the advantage of requiring less reference data or, on the other hand, yield a better spatial resolution for a given amount of reference data.

An interesting study would be to compare several interaction position estimation methods, such as neural networks, MLE, and the \(k\)-NN method, each having its particular advantages and disadvantages. The performance of the different methods would need to be compared on various aspects relevant for implementation in a clinical PET scanner by testing them on datasets obtained with the same detector(s). This should be done under the conditions of relatively fast and easy calibration, e.g. requiring calibration data from perpendicularly incident gamma photons only and being capable to handle line source calibration data. Then, for example, the spatial resolution could be investigated as a function of the amount of reference data, number of mathematical operations, or computation time on state-of-the-art FPGAs. Related to this, the different methods should be tested with respect to robustness, count rate performance, implementation costs, etc.

### 7.3 New technological developments

In PET instrumentation technology in general, a number of advancements not yet discussed in this thesis seem very promising for high resolution time-of-flight (TOF) PET detectors. For example, the emerging scintillation material \(\text{LuI}_3\text{Ce}\) might be relevant for future detector designs, since it has a \(~30\%\) higher light yield than \(\text{LaBr}_3\text{Ce}\), a \(~10\%\) higher density, as well as a \(~30\%\) higher effective atomic number. However, similar to \(\text{LaBr}_3\text{Ce}\), \(\text{LuI}_3\text{Ce}\) is hygroscopic, which in general may hamper the implementation in PET scanners, since it requires additional air-tight packaging and complicates surface processing of the crystals, production, and maintenance of PET devices. Additionally, the
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The packaging of the crystals introduces additional dead space in the detector ring, deteriorating the performance of the PET scanner.

Therefore, the improvements of the non-hygroscopic material LSO:Ce, for example by co-doping it with Ca resulting in higher light yield and a faster light pulse, is also of great interest for PET detectors [26]. Compared to LaBr₃:Ce and LuI₃:Ce, this material is much denser and has a higher effective atomic number. Therefore, it has the advantages of requiring less material, thinner detectors, and having a larger fraction of photo-events, which usually gives better energy, spatial, and timing resolution. It also emits at higher wavelengths, which usually results in better reflectivity of the surrounding material, less absorption in optical interfaces, and better spectral matching with the sensor quantum efficiency. Thus, although the intrinsic performance of LaBr₃:Ce and LuI₃:Ce is still much better than the improved LSO:Ce,Ca, their overall detector performances need to be compared carefully. For practical use in PET scanners also factors such as cost effectiveness, maintainability, and production should be taken into account.

Finally, the development of SiPMs with integrated digital electronics, the so-called digital silicon photomultipliers (dSiPMs), may offer many benefits for TOF PET detectors [27],[28]. As opposed to analog SiPMs, in which all Geiger-mode cells (microcells) are connected in parallel and have a common current as output, dSiPMs are equipped with digital logic for each microcell that triggers when the cell detects a photon. Using e.g. a 180 nm CMOS process, the necessary digital logic can be placed in between the dSiPM cells, occupying only about 10% of the total area and allowing fill factors above 70%, providing a PDE comparable to that of analog SiPMs. Pulse integration in dSiPMs is realized by digitally counting the number of fired cells. This approach renders the device relatively insensitive to temperature fluctuations and practically immune to electronic noise and interference. It furthermore eliminates the need for (pre)amplification and transmission of analog signals. This makes these devices very interesting for application in future PET/MRI systems where harsh requirements have to be met to avoid unwanted interference between the PET and MRI subsystems. With analog light sensors, this requires short analog paths with differential signals to reduce MRI crosstalk, fast and low-noise preamplifiers, low-threshold discriminators, challenging ASIC implementations, etc. Moreover, in dSiPMs the relatively small amount of faulty cells that are responsible for the majority of dark counts can be simply switched off, which reduces the noise dramatically and increases the production yield – a tremendous advantage for a large area PET detector. Accurate time stamps are enabled by the programmable trigger logic and the on-chip time-to-digital
converters (TDCs).

Nevertheless, the current prototype dSiPMs have still some limitations. For example, in the implementation of the trigger logic the trigger levels corresponding to the second, third, and fourth detected scintillation photon are in fact statistical thresholds. For these trigger levels $\geq 2$, $\geq 3$, and $\geq 4$ photons are required, respectively. Since in these cases triggering occurs at a number of photons that varies per scintillation event, timing performance is deteriorated. Therefore, accurate time stamps can be obtained for the first detected scintillation photon only. This, however, does not necessarily provide the best possible timing performance [29] and it may not be optimal for count rate performance.

In our first investigations of prototype dSiPMs, we assessed the performance of a prototype PET detector based on a digital SiPM array coupled to a 24 mm × 24 mm × 10 mm LSO:Ce,Ca crystal. These measurements provided a spatial resolution of ~1.0 mm FWHM, an energy resolution of ~11%, and a CRT of < 350 ps FWHM [3],[8]. We furthermore developed a model of the probability distribution of the number of fired microcells, i.e. the number of counted scintillation photons, in response to a given amount of energy deposited in a scintillator optically coupled to a dSiPM [30]. Based on this model we additionally proposed a method to correct for the non-linearity of energy spectra measured with dSiPMs. Given the beneficial characteristics of dSiPMs, they are expected to be the subject of many more investigations related to the development of better PET detectors.

References


Chapter 7. Discussion and outlook


References


Chapter 7. Discussion and outlook
Summary

Monolithic scintillators and SiPMs in time-of-flight PET detectors

Positron emission tomography (PET) is one of the key medical imaging modalities in the diagnosis and staging of e.g. cancer, neurodegenerative diseases, and cardiovascular diseases. PET maps a specific physiological function such as receptor density or the metabolism of cells. To create a PET image, the patient must be administered a radiolabeled (positron-emitting) agent (tracer) that distributes within the body such that it correlates with the function of interest. Glucose molecules, for example, accumulate in cells with high metabolic activity. Prior to administration, these molecules are labeled with the positron emitting radionuclide $^{18}$F. The image formation consists of localizing the positron emitters. After emission, the positrons travel a short distance in the surrounding tissue and annihilate with an electron. This process yields two gamma photons of equal and well-determined energy of 511 keV, which leave the body in almost opposite direction. The detectors in the ring of the PET scanner must detect these gamma photons. If two gamma photons with the correct energy are detected in coincidence within a certain time window, then it is assumed that they originate from the same annihilation event, and that the positron annihilation must have occurred on the line connecting the two detector elements. From a large number of such lines-of-response (LORs), a tomographic image can be reconstructed, which resembles the initial distribution of the positron-labeled substance in the body.

The diagnostic value of a PET image depends crucially on the image quality, which to a large extent is determined by the performance of the PET detectors. Thus, for better quality PET images, detectors are required with improved spatial, energy, and timing resolution, detection efficiency, depth-of-interaction (DOI) correction, time-of-flight (TOF) capability, and magnetic resonance imaging (MRI) compatibility, compared to the current state-of-the-art PET detectors. The research presented in this thesis aims to understand, characterize, and improve the performance of an innovative PET detector concept, based on the recently discovered LaBr$_3$:Ce scintillator in the form of a monolithic crystal in combination with novel, high-speed, position-sensitive SiPM light sensors to read out the scintillation light distribution, from which the position of interaction is derived by means of a software position estimation algorithm.
Current state-of-the-art TOF PET systems often use LSO or LYSO scintillators. LaBr\(_3\):Ce is considered as an alternative, because of its higher light yield and faster decay time. However, detector designs based on LaBr\(_3\):Ce are complicated by the hygroscopicity of LaBr\(_3\):Ce, requiring measurements in glove boxes and/or air-tight encapsulation of the crystals. This has as a consequence that currently little is known about the optical properties of this material. Therefore, in chapter 2 a method is presented circumventing these problems to accurately measure the refractive index as well as the optical absorption and scattering lengths. The measured values were compared to a theoretical model of the absorption to parameterize and validate them. These results are essential input parameters for accurate Monte Carlo simulations, which are an important tool in the investigation of detector and system performance as well as their correlation with a single detector parameter.

Silicon photomultipliers (SiPMs) are of great interest to PET, as they enable new detector geometries, e.g. for DOI determination, are MRI compatible, and offer faster response and higher gain than other solid-state photosensors such as avalanche photodiodes (APDs). However, the limited number of microcells gives rise to a reduced dynamic range, which for fast and bright scintillation pulses may result in a non-proportional response of the SiPM. In chapter 3, a model is discussed of this non-proportional response to scintillation light pulses. It accounts for the total number and the temporal distribution of the incident photons as well as for the relevant SiPM parameters, i.e. the recovery time, afterpulsing, crosstalk, and their cross-correlations. This model can be utilized, e.g. to properly design a detector for a given application, to perform corrections on measurements or on energy spectra, to calibrate an SiPM detector for low-level-light measurements, to predict detector performance, and/or to determine difficult-to-measure SiPM parameters.

The concept of a monolithic scintillator read out by an SiPM array was assessed in chapter 4. Such a concept has the advantages of reduced dead space allowing for high system sensitivity, easier crystal surface processing and cost effective detector assembly. The prototype detector consists of a \(4 \times 4\) SiPM array coupled to either the front or back surface of a 13.2 mm \(\times\) 13.2 mm \(\times\) 10 mm LYSO:Ce crystal. As the SiPM array is very thin and composed of low-Z (atomic number) materials only, it can be placed on the front surface of the crystal without significantly disturbing the annihilation photon beam. It was shown that front-side readout geometry results in significantly better performance than conventional back-side readout. Spatial resolutions < 1.6 mm full-width-at-half-maximum (FWHM) were measured at the detector center in response to a \(~0.54\) mm FWHM diameter test beam. Hardly any resolution
losses were observed at angles of incidence of up to 45°, demonstrating excellent DOI correction. About 14% FWHM energy resolution was obtained. The single detector timing resolution, measured in coincidence with a BaF$_2$ detector, equals 960 ps FWHM. These results illustrate the potential of SiPMs for the development of novel detector designs aiming at, for example, compactness, DOI determination and MRI compatibility.

At present, the derivation of the gamma photon entry position from the scintillation light distribution is complicated by the long calculation times and the extensive calibration measurements that are required. Chapter 5 describes several improvements of the standard $k$-nearest neighbor method that was used in chapter 4, reducing these problems considerably. These improvements are based on the fact that all nearest neighbors carry some information about the gamma photon interaction point, which is more effectively used by smoothing or fitting, by combining reference data for each dimension analogous to calibrating with a line source, or by means of the categorical average patterns (CAP) method. The methods were tested on a dataset measured with the SiPM-array-based monolithic LYSO scintillator detector described in chapter 4. It was shown that ~10% to ~25% better spatial resolution can be obtained than with the standard method, depending on the number of reference events. Furthermore, several methods require up to two orders of magnitude less reference data to obtain a spatial resolution that is only slightly deteriorated compared to the standard method, reducing both the calibration and entry point estimation times. Finally, the improved methods equivalent to calibrating the detector with a line source yielded considerably better results than the standard method, which uses a grid of irradiation points to obtain reference data. These last methods would thus allow for a much faster collection of the reference data.

The concept of entry point estimation itself has several drawbacks. It requires calibration data at many angles of incidence and it does not provide explicit DOI information. Therefore, in chapter 6, several new methods for the determination of the DOI in monolithic scintillator detectors with single-sided readout are presented and validated. These have the advantage of requiring only perpendicularly incident calibration data, speeding up the detector calibration significantly. Furthermore, they need neither detector modifications nor models for the signal variance or light transport, making them practical. The methods are based either on the degree of similarity to a set of reference light distributions, on the measured intensity of the sensor pixel(s) closest to the interaction position, or on the largest measured intensity of a light distribution. Tests were performed on a detector comprised of a $20 \text{ mm} \times 20 \text{ mm} \times 12 \text{ mm}$ polished LYSO crystal coupled to a $4 \times 4$ pixel multi-anode photomultiplier.
Summary

The DOI resolution was evaluated as a function of depth, lateral resolution, and number of reference data. The method based on the linearly interpolated measured intensities on the sensor pixels closest to the estimated lateral coordinate yielded DOI resolutions between ~1 mm and ~4.5 mm depending on the DOI and the number of reference data. It was furthermore shown that considerably better DOI resolution could be achieved with an improved lateral spatial resolution.

Based on the results discussed in this thesis it can be concluded that SiPM-based monolithic-scintillator detectors are promising for application in future clinical TOF PET and PET/MRI systems. The implementation of monolithic scintillator detectors in clinical PET scanners still requires further research and engineering, e.g. on (digital) front-end electronics, data handling, data processing, calibration of the system, and optimal system design. In this thesis progress has been made on e.g. accelerated and facilitated detector calibration, which may serve as a basis for the development of full system calibration methods. More fundamental studies such as the measurements of optical scintillator properties of \( \text{LaBr}_3: \text{Ce} \) and the modeling of the response of detectors based on SiPMs are essential for PET system modeling in order to obtain optimal performance. Advances in PET instrumentation technology such as better scintillation materials, e.g. \( \text{LuI}_3: \text{Ce} \), and the development of SiPMs with integrated digital electronics may offer many benefits for TOF PET detectors and, therefore, warrant further research on PET detector technology.
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Monolithische scintillatoren en SiPM’s in time-of-flight PET detectoren

Positronemisstietomografie (PET) is een van de belangrijkste medische beeldvormingstechnieken voor de diagnose en het volgen van bijvoorbeeld kanker, neurodegeneratieve ziektes en hart- en vaatziekten. Deze techniek resulteert in afbeeldingen van specifieke fysiologische functies, zoals receptordichtheid of celmetabolisme. Om een PET-afbeelding te maken, moet een substantie aan een patiënt worden toegediend, die zich zo door het lichaam verspreidt dat de verdeling van de substantie correleert met de functie die onderzocht wordt. Suikermoleculen hopen zich bijvoorbeeld op in cellen met een hoge metabolische activiteit. Deze moleculen worden vooraf gemarkeerd met de positronenemitter $^{18}$F. De beeldvorming bestaat uit het lokaliseren van de positronemitters. De positronen leggen na emissie een korte afstand af in het omringende weefsel en annihileren met een elektron. Dit proces resulteert in twee gammafotonen, die het lichaam in vrijwel tegenovergestelde richting verlaten. De detectoren in de ring van de PET-scanner waarin de patiënt ligt moeten deze gammafotonen detecteren. Als twee gammafotonen tegelijkertijd worden gedetecteerd, dan wordt aangenomen dat ze uit hetzelfde annihilatieproces voortkomen en dat de positronannihilatie heeft plaatsgevonden op de lijn die de twee detectorelementen verbindt. Uit een groot aantal van zulke ‘lines-of-response’ (LOR) kan een tomografisch plaatje worden gereconstrueerd dat overeenkomt met de oorspronkelijke verdeling van de toegediende substantie in het lichaam.

De diagnostische waarde van een PET-afbeelding hangt sterk af van de beeldkwaliteit, die voor een groot gedeelte wordt bepaald door de prestaties van de PET-detectoren. Voor PET-afbeeldingen van hogere kwaliteit zijn in vergelijking met de huidige PET-detectoren onder andere verbeteringen nodig voor de plaats- energie- en tijdsresolutie; detectiewaarschijnlijkheid; correcties voor ‘depth-of-interaction’ (DOI); ‘time-of-flight’ (TOF) bepaling; en ‘magnetic resonance imaging’ (MRI) compatibiliteit. Het onderzoek dat in dit proefschrift is gepresenteerd heeft als doel het begrijpen, karakteriseren en verbeteren van de prestaties van een innovatief PET-detectorconcept. Dit concept maakt gebruik van recentelijk ontdekte scintillatoren die in de vorm van een monolithisch kristallen zijn gekoppeld aan nieuwe, snelle, plaatsgevoelige lichtsensoren voor
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het meten van de intensiteitsverdeling van het licht dat wordt uitgezonden door de scintillator als er een gammafoton wordt geabsorbeerd. Uit deze lichtverdeling wordt de interactiepositie van het gammafoton verkregen met behulp van softwarealgoritmen.

Huidige, moderne TOF-PET-systemen maken vaak gebruik van de scintillatoren LSO of LYSO. Als een potentiële opvolger wordt het materiaal LaBr₃:Ce overwogen vanwege de hogere lichtopbrengst en de snellere afvaltijd. Detectorontwerpen gebaseerd op LaBr₃:Ce worden echter gecompliceerd door de vochtgevoeligheid van dit materiaal, waardoor metingen in een omgeving met droge lucht moeten worden gedaan of de kristallen luchtdicht moeten worden ingepakt. Door deze belemmeringen is er op dit moment weinig bekend over de optische eigenschappen van LaBr₃:Ce. Daarom is in hoofdstuk 2 een methode gepresenteerd die deze problemen omzeilt waarmee nauwkeurige metingen kunnen worden verricht van de brekingsindex en de optische absorptie- en verstrooiingslengte. De gemeten waarden zijn vergeleken met een theoretisch model van de absorptie om ze te parametrizeren en te valideren. De resultaten zijn essentiële inputparameters voor nauwkeurige Monte-Carlosimulaties, die een belangrijk hulpmiddel zijn in het onderzoeken van detector- en systeemprestaties en hun correlaties met bepaalde detectorparameters.

In PET-detectoronderzoek is veel belangstelling voor ‘silicon photomultipliers’ (SiPM) aangezien deze lichtsensoren nieuwe detectorgeometriën mogelijk maken voor bijvoorbeeld DOI bepaling; ze MRI-compatible zijn; en ze een snellere respons en een grote interne versterking hebben dan andere ‘solid state’-lichtsensoren zoals ‘avalanche photodiodes’ (APD). Het beperkt aantal microcellen van een SiPM veroorzaakt echter een verminderd dynamisch bereik, wat voor snelle en heldere scintillatiepulsen kan resulteren in een niet-proportionele respons van de SiPM. In hoofdstuk 3 wordt een model besproken van deze niet-proportionele respons op scintillatielichtpulsen. Dit model is een functie van het totaal aantal en de tijdsverdeling van de binnenkomende scintillatiefotonen en een functie van de relevante SiPM-parameters, d.w.z. de ‘recovery time’, ‘afterpulsing’, ‘crosstalk’ en hun kruiscorrelaties. Het model kan bijvoorbeeld worden gebruikt voor het correct ontwerpen van een detector voor een specifieke toepassing, voor het doen van correcties op metingen of energiespectra, voor het kalibreren van een SiPM-detector voor metingen van lage lichtintensiteiten, voor het voorspellen van detectorprestaties en/of voor het bepalen van SiPM-parameters die moeilijk te meten zijn.

Het concept van een monolithische scintillator uitgelezen door een SiPM-
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matrix is onderzocht in hoofdstuk 4. Dit concept heeft onder andere de voordelen van minder dode ruimte wat een hogere detectiewaarschijnlijkheid mogelijk maakt, eenvoudigere kristaloppervlakbewerking en goedkopere detectormontage. De prototypedetector bestaat uit een $4 \times 4$ SiPM-matrix gekoppeld aan ofwel het achtervlak ofwel het voorvlak van een $13.2 \, \text{mm} \times 13.2 \, \text{mm} \times 10 \, \text{mm}$ LYSO:Ce-kristal. Aangezien de SiPM-matrix erg dun is en louter bestaat uit materialen met lage $Z$ (atomair getal), kan deze aan het voorvlak van het kristal gekoppeld worden zonder de gammafotonbundel significant te verstoren. Er is aangetoond dat het uitlezen van het voorvlak resulteert in significant betere prestaties dan het conventionele uitlezen van het achtervlak. Plaatsresoluties van $< 1.6 \, \text{mm}$ ‘full-width-at-half-maximum’ (FWHM) zijn gemeten in het midden van de detector waarbij een testbundel met een diameter van $\sim 0.54 \, \text{mm}$ FWHM is gebruikt. Vrijwel geen enkel resolutieverlies is zichtbaar bij invalhoeken tot $45^\circ$, wat de uitmuntende DOI-correctie laat zien. Een energieresolutie van ongeveer 14% FWHM is behaald en de tijdsresolutie van een enkele detector gemeten in coïncidentie met een BaF$_2$ detector is 960 ps FWHM. Deze resultaten illustreren wat men zou kunnen verwachten van SiPMs voor nieuwe detectorontwerpen die mikken op bijvoorbeeld compactheid, DOI-correctie en MRI-compatibiliteit.

Hoofdstuk 5 beschrijft een aantal verbeteringen van de standaard $k$-naaste-buur-methode die is gebruikt in hoofdstuk 4 voor de bepaling van het punt van binnenkomst van de gammafotonen in het scintillatiekristal, gebruik makende van de scintillatielichtverdeling. Voor deze standaardmethode zijn lange rekentijden en uitgebreide kalibratiemetingen nodig. Deze problemen kunnen aanzienlijk verminderd worden door gebruik te maken van het feit dat alle naaste buren enige informatie dragen over het punt van binnenkomst van het gammafoton. Dit kan effectiever worden gebruikt door smooth- of fitmethodes, door het combineren van referentiedata per dimensie hetgeen vergelijkbaar is met het kalibreren met een lijnbron of door de zogenaamde ‘categorical average patterns’ (CAP) methode. De methodes zijn getest op de dataset die was gemeten met de detector bestaande uit de SiPM-matrix en het monolithische LYSO-kristal zoals beschreven in hoofdstuk 4. Het bleek dat afhankelijk van het aantal referentie-events $\sim 10\%$ tot $\sim 25\%$ betere plaatsresolutie kan worden behaald dan met de standaardmethode. Verder hebben een aantal methoden tot twee ordes van grootte minder referentiedata nodig om een plaatsresolutie te verkrijgen die slechts minimaal slechter is ten opzichte van de standaardmethode, wat een versnelling oplevert voor zowel de kalibratie als de aanschatting van het punt van binnenkomst. De verbeterde methodes die equivalent zijn aan het kalibreren van de detector met een lijnbron leveren ook
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aanzienlijk betere resultaten dan de standaardmethode, waar de referentiedata is verkregen met een raster van instraalposities. Die laatste methodes zouden dus zorgen voor een veel snellere kalibratiemeting.

Het concept van het bepalen van het punt van binnenkomst van een gammafoton heeft zelf enkele nadelen. Er is kalibratiedata van verschillende hoeken van inval nodig en expliciete DOI informatie wordt niet geleverd. Daarom worden in hoofdstuk 6 een aantal nieuwe methoden voor de bepaling van de diepte van interactie in monolithische-scintillator-detectoren met enkelzijdige uitlezing besproken en getest. Deze methodes hebben als voordeel dat ze alleen kalibratiedata van loodrechte inval nodig hebben, wat de detectorkalibratie significant sneller maakt. Verder zijn ze praktisch, aangezien er geen detectoraanpassingen of modellen voor de signaalvariantie of het lichttransport nodig zijn. De methoden zijn gebaseerd op de mate van gelijkheid met een set referentielichtverdelingen, op de gemeten intensiteit van de sensorpixel(s) die het dichtstbij de interactiepositie zijn, of op de hoogst gemeten intensiteit van een lichtverdeling. De methodes zijn getest met metingen op een detector bestaande uit een $20 \times 20 \times 12$ gepolijst LYSO-kristal gekoppeld aan een $4 \times 4$ pixel ‘multi-anode photomultiplier’. De DOI-resolutie is onderzocht als een functie van DOI, laterale resolutie en de hoeveelheid referentiedata. De methode die is gebaseerd op de lineair geïnterpoleerde, gemeten intensiteiten op de sensorpixels, die zich het dichtstbij de geschatte laterale coördinaat bevinden, leverde DOI-resoluties tussen ~1 mm en ~4.5 mm afhankelijk van de DOI en de hoeveelheid referentiedata. Verder is er aangetoond dat aanzienlijk betere DOI-resolutie kon worden behaald met een betere laterale plaatsresolutie.

Op basis van de in dit proefschrift besproken resultaten kan worden geconcludeerd dat op SiPMs gebaseerde monolithische-scintillator-detectoren veelbelovend zijn voor toepassing in toekomstige klinische TOF PET en PET/MRI systemen. De implementatie van monolithische-scintillator-detectoren in klinische PET-scanners vereist wel verder onderzoek en ontwerp aan bijvoorbeeld (digitale) front-end elektronica, het omgaan met data, het verwerken van data, systeemkalibratie en optimale systeemprestaties. In dit proefschrift is er o.a. vooruitgang geboekt op het gebied van versnelde en vereenvoudigde kalibratie van detectoren, op basis waarvan kalibratiemethoden voor volledige systemen ontwikkeld kunnen worden. Meer fundamentele studies zoals de metingen van de optische eigenschappen van $LaBr_3:Ce$ en het modeleren van de respons van op SiPMs gebaseerde detectoren zijn van elementair belang voor PET-systeemontwerp en voor het verkrijgen van optimale prestaties. Verdere vooruitgang in PET-instrumentatie, zoals de
ontwikkeling van de scintillator LuI$_3$:Ce en de ontwikkeling van SiPMs met geïntegreerde digitale elektronica, kunnen vele voordelen bieden voor TOF-PET-detectoren en zorgen er daarom voor dat verder onderzoek in PET-detectortechnologie nodig blijft.
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Curriculum vitae

Herman van Dam was born in Gorinchem in 1983. He obtained his secondary school degree from the Gymnasium C.S.G. Oude Hoven in Gorinchem in 2001. From 2001 to 2006 he studied Applied Physics at Eindhoven University of Technology. His characterization of plasma density properties for laser wakefield accelerators at the FOM Institute for Plasma Physics “Rijnhuizen” in Nieuwegein resulted in an MSc degree in 2006. At the end of the same year he started his PhD research at Delft University of Technology, in the faculty of Applied Sciences, in the department of Radiation Radionuclides & Reactors, in the section Radiation Detection & Medical imaging. The results of this research are presented in this thesis.
Curriculum vitae
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