Numerical Simulation of Free Edge Delamination in Graphite–Epoxy Laminates under Uniaxial Tension

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ABSTRACT

The free edge delamination of a graphite–epoxy laminate is simulated using non-linear finite element analysis. Generalised plane strain elements and interface elements have been developed for this purpose. The material behaviour is assumed to be governed by a mode I crack model including strain softening. In this paper a formulation is proposed for strain loading in non-linear analysis under arc length control. In addition, the constitutive model and the finite element formulation for the interface elements are discussed. Mesh objectivity and the influence of fracture energy on delamination growth and laminate strength are examined in the analyses of $[\pm 25, -25, 90]_s$, laminates. Results show that the presented approach does not suffer from mesh dependency, which implies that size effects can be described properly.

1 INTRODUCTION

A failure mode often encountered in structural composite laminates is delamination, a phenomenon which has great influence on the structural integrity. In the last two decades many research activities have been concentrated on the complex mechanisms of delamination. Analytical models were developed for the treatment of free edge stress distributions,\textsuperscript{1−3} which were held primarily responsible for the initiation of delamination. A historical review of these models is given by Pagano and Soni.\textsuperscript{4} Furthermore, procedures to predict delamination onset and growth
have been developed. In general these techniques are based on the principle of virtual crack extension. They have been applied rather successfully by Wang\textsuperscript{4,6} and O'Brien,\textsuperscript{7} amongst others. Because of the mesh size dependency and stress singularities it is commonly believed that the use of stress-based failure criteria does not result in any relevant results as far as free edge delamination is concerned. However, Kim and Soni\textsuperscript{8} indicated that an average stress approach combined with an anisotropic failure criterion results in an accurate prediction of the onset of delamination.

In this paper an alternative procedure is developed for the prediction of delamination onset and growth. The performance of the method is demonstrated by means of the analyses of free edge delamination in [\(\pm 2\theta_{/0}\)], graphite–epoxy specimens under uniaxial tension. In the examples 12-noded cubic generalised plane strain elements with three translational degrees of freedom in each node have been used. These elements, which are assumed to remain elastic during the loading process, give an accurate representation of the stress concentrations near the free edges without the need for extreme mesh refinement. The individual plies are connected by cubic interface elements\textsuperscript{9,10} which are well suited for modelling the geometric discontinuity that arises during delamination, which can either be gradual (softening-type behaviour) or perfectly brittle. Emphasis is put on the mesh sensitivity of delamination growth and the effects of fracture energy and laminate thickness on the limit load.

2 GENERALISED PLANE STRAIN ELEMENTS

In free edge delamination testing specimens are subjected to a uniaxial load. It is assumed that at a certain distance from the ends of the specimen the in-plane displacements are independent of the \(x\) coordinate (Fig. 1). This

![Fig. 1. Geometry of the \([\pm 2\theta_{/0}\])\_n laminate.](image)
allows us to introduce for the displacement field of a cross-section \(^4,3,7\)

\[
\begin{align*}
  u_x(x, y, z) &= e_x x + u_x(y, z) \\
  u_y(x, y, z) &= u_y(y, z) \\
  u_z(x, y, z) &= u_z(y, z)
\end{align*}
\]  

(1)

with \(e_x\) the strain that is prescribed in the \(x\) direction of the specimen. If we define a differential operator \(L\)

\[
L^T = \begin{bmatrix}
0 & 0 & 0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial y} \\
0 & \frac{\partial}{\partial y} & 0 & \frac{\partial}{\partial z} & 0 \\
0 & 0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial y} & 0
\end{bmatrix}
\]  

(2)

and a matrix \(H\) which contains the interpolation polynomials, \(u = Ha\), the strains \(\varepsilon\) can be related to the nodal displacements \(a\) through

\[
\varepsilon = LH\alpha + e_i
\]  

(3)

where \(e_i = (e_x, 0, 0, 0, 0)\) denotes a vector that contains the externally applied strain \(e_x\). With the strain displacement matrix \(B = LH\) the element stiffness matrix can be obtained following standard finite element procedures.\(^{11}\)

For the incremental strains \(\delta \varepsilon\) in a non-linear analysis we can write

\[
\delta \varepsilon_j = B\delta a_j + \delta \lambda_j \varepsilon_i
\]  

(4)

where \(j\) denotes the iteration number and \(\delta \lambda_j = \Delta \lambda_j - \Delta \lambda_{j-1}\) is the change in value of the load parameter \(\lambda\) from iteration \(j - 1\) to iteration \(j\). The incremental nodal displacement vector is given by \(\delta a_j\). From eqn (4) it follows with the tangential stress–strain matrix \(D\) that the incremental stresses are given by

\[
\delta \sigma_j = DB\delta a_j + \delta \lambda_j D\varepsilon_i
\]  

(5)

If \(\sigma_{i-1}\) represents the total stress vector at the end of iteration \(j - 1\) the total stress at the end of iteration \(j\) reads

\[
\sigma_j = \sigma_{j-1} + DB\delta a_j + \delta \lambda_j D\varepsilon_i
\]  

(6)
The expressions for the internal and external force vectors for strain
loading then read
\[
\begin{align*}
\mathbf{f}_j &= \int_V B^T (\mathbf{\sigma}_j - \lambda_j \mathbf{D} \mathbf{e}_j) \, dV \\
\mathbf{f}_o &= -\lambda_0 \int_V B^T \mathbf{D} \mathbf{e}_1 \, dV
\end{align*}
\]  
(7)
where \( \lambda_0 \) is the total load factor at the beginning of the current step. Denoting the structural stiffness matrix by \( \mathbf{K} \) the incremental displacements can be solved iteratively from
\[
\mathbf{K} \mathbf{\delta a}_j = \mathbf{f}_o - \mathbf{f}_{j-1}
\]  
(8)

3 INDIRECT DISPLACEMENT CONTROL FOR STRAIN LOADING

A major drawback with strain- or load-controlled calculations is the fact that no limit points can be passed. Riks\textsuperscript{12} developed an 'arc length' method to overcome these limitations. Herein the incremental load factor is constrained by the norm of the incremental displacement vector. Although the arc length control method has proved to be rather successful, it has been reported to fail in situations of highly localised failure. It was suggested by de Borst\textsuperscript{13} that in these cases the displacement norm should be determined considering only the dominant degrees of freedom.

In conventional strain loading the incremental nodal displacements are determined from eqn (8) in an iterative manner. In an arc length modification of strain loading this process can be represented by the following set of equations\textsuperscript{13} (for iteration \( j \)):
\[
\begin{align*}
\mathbf{\delta a}_j^i &= -K_j^{-1} \left[ \lambda_0 \int_V B^T \mathbf{D} \mathbf{e}_j \, dV + \int_V B^T (\mathbf{\sigma}_{j-1} - \lambda_{j-1} \mathbf{D} \mathbf{e}_j) \, dV \right] \\
\mathbf{\delta a}_j^e &= -K_j^{-1} \int_V B^T \mathbf{D} \mathbf{e}_1 \, dV \\
\mathbf{\delta a}_j &= \mathbf{\delta a}_j^i + \delta \lambda_j \mathbf{\delta a}_j^e \\
\Delta \mathbf{a}_j &= \Delta \mathbf{a}_{j-1} + \mathbf{\delta a}_j
\end{align*}
\]  
(9)  
(10)  
(11)  
(12)
where \( \Delta \mathbf{a}_j \) denotes the total incremental displacement vector. It is determined from the requirement that the crack opening displacement (COD) of the interface between two plies where delamination occurs has a constant value for each iteration:
\[
\delta (\text{COD}) = 0 \Rightarrow \mathbf{\delta a}_j^o - \mathbf{\delta a}_j^o = 0
\]  
(13)
where $\delta u^t_n$ is the change in displacement in the thickness direction of the laminate of node $n$ from iteration $j-1$ to $j$.

4 INTERFACE ELEMENTS AND CONSTITUTIVE RELATIONS

The individual plies of the laminate are connected by interface elements (Fig. 2). These elements have the ability to model the geometrical discontinuity which is introduced by delamination. In the elastic stage of the calculation no additional deformations are allowed in the finite element model because of these interface elements. Therefore a sufficiently high dummy stiffness has to be supplied. In contrast to continuum elements, interface elements do not keep track of stresses and strains at the integration points but consider tractions and relative displacements. With differential operator matrix $L$ and the interpolation matrix $H$ defined as

$$
L = \begin{bmatrix}
-1 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 1 & 0 \\
0 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 \\
\end{bmatrix}
H = \begin{bmatrix}
n & 0 & 0 & 0 & 0 \\
n & 0 & 0 & 0 & 0 \\
n & 0 & n & 0 & 0 \\
n & 0 & 0 & n & 0 \\
n & 0 & 0 & 0 & n \\
n & 0 & 0 & 0 & 0 \\
\end{bmatrix}
$$

(14)

and the nodal displacement vector

$$
a = (u^1_1, u^1_2, \ldots, u^i_n, u^i_2, \ldots, u^i_k, u^i_2, \ldots, u^i_l)
$$

(15)

the relative displacements are related to the nodal displacements through

$$
\Delta u = LHa
$$

(16)

In eqn (14) $n$ contains the interpolation functions. If we denote $D$ as the tangent stiffness relation the tractions $t = (t_n, t_s, t_t)$ are obtained from

$$
t = D \Delta u
$$

(17)

![Fig. 2. Cubic 3D line interface element with nodal degrees of freedom.](image)
For a line interface element in a generalised plane strain situation the element stiffness matrix is derived as

\[ K = \int_{\zeta = -1}^{\zeta = +1} B^T D B \frac{\partial x}{\partial \zeta} d\zeta \]  \hspace{1cm} (18)

Once the elastic limit in an integration point of the interface element is exceeded, the traction-relative displacement relation \( D \) becomes non-linear and is determined by a discrete crack model. In the present model crack initiation is supposed to be purely in mode I. A crack arises when the normal traction \( t_n \) exceeds the tensile strength \( f_t \). It is recognised that a stress criterion cannot predict the onset of the delamination correctly. Depending on the type of material the traction and stiffness in a mode I crack may abruptly or gradually reduce to zero. The first phenomenon is called brittle failure, whereas the gradual decrease of the crack traction and stiffness with increasing relative displacement is called softening behaviour. The use of a softening type of response results in a rate-controlled delamination. Figure 3 shows four different types of non-linear response.

Once mode I failure has occurred a resulting mode II/mode III stiffness of zero is supplied. For the derivation of the non-linear stiffness relation we will use a decomposed approach\(^{9,10}\) in which the total relative displacement consists of an elastic part \( \Delta u^e \) and an inelastic part which is equal to the crack relative displacement \( \Delta u^c \). In the crack model the incremental tractions \( \Delta t \) for the intact material are given by

\[ \Delta t = D \Delta u^e \]  \hspace{1cm} (19)

The incremental crack relative displacements \( \Delta \Delta u^c \) are related to the incremental tractions according to

\[ \Delta t = D^c \Delta \Delta u^c \]  \hspace{1cm} (20)

where \( D^c \) is the crack stiffness matrix which is dependent on the mode I post-crack relation. The incremental relative displacement is written as

\[ \Delta \Delta u = \Delta \Delta u^e + \Delta \Delta u^c \]  \hspace{1cm} (21)
Substituting eqns (19) and (20) into eqn (21) results in
\[ \Delta t = \left[ D^{-1} + (D^*)^{-1} \right]^{-1} \Delta \Delta u \] (22)
or using the Sherman–Morrison–Woodbury formula
\[ \Delta t = \left( D - D(D^* + D)^{-1} D \right) \Delta \Delta u \] (23)
This relation reduces to zero when brittle failure is concerned.

5 DELAMINATION IN [+25/−25/90]s GRAPHITE–EPoxy LAMINATES

For the investigation of the mesh dependency of delamination growth a 6-ply \([±25/90]s\), graphite–epoxy laminate (see Table I for material properties) was subjected to a uniaxial strain load. Since delamination is initiated at the 90/90-ply interface a pure mode I crack extension results. The dimensions of the cross-section of the laminate are 25.0 \(\times\) 0.792 mm with a ply thickness equal to 0.132 mm.

<table>
<thead>
<tr>
<th>Table I</th>
<th>Material properties for AS-3501-06 Graphite Epoxy [N/mm²]¹⁴</th>
</tr>
</thead>
<tbody>
<tr>
<td>(E_{11})</td>
<td>140 (\times) 10⁶</td>
</tr>
<tr>
<td>(E_{22})</td>
<td>11 (\times) 10⁷</td>
</tr>
<tr>
<td>(E_{33})</td>
<td>11 (\times) 10⁷</td>
</tr>
</tbody>
</table>

Due to the symmetry of the laminate only a quarter of the cross-section was modelled (Fig. 4) using cubic generalised plane strain elements. Translations in the z direction along the y axis and translations in the x and z directions along the y axis were prevented. Cubic line interface elements were supplied between the 90° plies. A 4 \(\times\) 4 Gauss integration scheme was used for generalised plane strain elements. For the interface elements a nodal lumping scheme was applied since Gaussian integration leads to oscillatory traction profiles.¹⁰ For the dummy stiffness of the interface elements a value of 10⁺⁸ N/mm² was substituted; the tensile strength was

![Fig. 4. Dimensions of the finite element mesh.](image-url)
chosen equal to \( f_t = 51.6 \, \text{N/mm}^2 \). A linear softening relation determines the post-crack response in the interface elements.

Mesh objectivity was examined using three different meshes with a varying number of elements over the width of the specimen. In all cases the element height was chosen equal to the ply thickness. The part of the specimen within 5 mm of the free edge was modelled using 50, 100, 200 and 400 elements respectively for each ply (element lengths 0.1, 0.05, 0.025 and 0.0125 mm). The remaining 7.5 mm was modelled using three elements per ply. To achieve a rate-controlled delamination a fracture energy \( G_c = 0.175 \, \text{N/mm} \) was supplied. Figure 5(a) and (b) show the results for the three different meshes. Upon mesh refinement the different analyses converge to the same solution. Hence the results are not influenced by the different element sizes. A deformed geometry of the mesh with 100 elements per ply for a delamination of 4.37 mm is presented in Fig. 6. The scale of deformation is 1:10.

To investigate the effect of fracture energy on crack growth and the ultimate load capacity three different values for the fracture energy were chosen: \( G_c = 0.05 \, \text{N/mm} \), \( G_c = 0.125 \, \text{N/mm} \) and \( G_c = 0.175 \, \text{N/mm} \). The mesh with element lengths of 0.05 mm was used in the analyses. Figure 7(a) and (b) demonstrate that a quadratic dependency exists between fracture energy and the ultimate strain \( \varepsilon_u \), which agrees with conclusions reached in

![Deformed model of one half of the laminate.](image)
Fig. 7. Effect of fracture energy $G_c$. (a) Axial load versus applied uniaxial strain. (b) Applied uniaxial strain versus delamination length.

Fig. 8. Effect of laminate thickness on ultimate uniaxial strain. (a) Axial load versus applied uniaxial strain. (b) Applied uniaxial strain versus delamination length.

Fig. 9. Ultimate strain for $[+25_1/-25_2]_3$ (n = 1, 2, 3) laminates.
Ref 5–7. This implies that the present approach is rather sensitive to the value of the fracture energy. Three analyses on \([\pm 25\pi/25\theta_0\pi, ]_n\) \((n = 1, 2, 3)\) laminates were performed to assess the capability of our approach to deal with size effects. In the analyses a fracture energy \(G_c\) equal to 0.175 N/mm and an element length of 0.05 mm were used. Figure 8(a) and (b) illustrate the effect of the laminate thickness on the laminate response. From Figs 8(b) and 9 it is observed that the ultimate strain depends inversely on the square root of the laminate thickness, as indicated by the dashed line. Wang and O'Brien reported the same dependency.\(^*\)\(^7\)

6 CONCLUDING REMARKS

A non-linear finite element approach is proposed for the analysis of free edge delamination problems in composite laminates. Although a stress criterion is used for the initiation of delamination no influence of mesh refinement on the ultimate strength of the laminates is encountered. This is a result of the softening type of response after cracking. Furthermore, the method results in a proper treatment of size effects. It is mentioned that the values as obtained for the ultimate strengths of the laminates do not correspond with the results presented in Ref. 6. This is because the initial thermal stresses that are present in the laminates due to the forming process were not taken into account.

REFERENCES


