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How do steel fibers improve the shear capacity of reinforced concrete beams without stirrups?

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Abstract

Even though the structural behavior steel fiber reinforced concrete (SFRC) has been extensively researched, structural applications are still limited. One barrier to its implementation is the lack of mechanical models that describe the behaviour of SFRC members failing in shear. This paper reviews the effect of steel fibers on the different mechanisms of shear transfer and combines the observations from the literature regarding the parameters that affect the shear capacity of SFRC. Additionally, a selection of currently available expressions for the shear capacity of SFRC is presented. This paper reviews the current state-of-the-art on the shear capacity of SFRC elements without shear reinforcement, shows the lacks in our current understanding on the shear behaviour of SFRC elements without shear reinforcement, and outlines the steps necessary to address these lacks. The presented work aims to be a framework for (experimental) efforts addressing the shear capacity of SFRC members without shear reinforcement.

Keywords

Aggregate interlock, Cracking, Dowel action, Fiber properties, Fiber type, Mechanical properties, Review, Steel fibers, Shear.
1 Introduction

Since concrete is strong in compression but weak in tension, researchers and practitioners have been searching for ways in which the tension in concrete structures can be carried since the beginning of the use of structural concrete. One option to improve the tensile strength of concrete is the use of fibers dispersed in the mixture, with steel fibers the most commonly used fiber material. This option was suggested as early as 1874 in a patent of A. Bernard for concrete strengthened with steel splinters [1]. Even though steel fiber reinforced concrete (SFRC) and the behavior of structural members with SFRC have been extensively researched in the past, the structural applications are not often extended to the main load-carrying elements of a structure. Some building and construction codes that govern in a number of countries, such as ACI 318-14 [2], do not include provisions for the design of structural elements in SFRC. At the moment, the application of SFRC is generally limited to industrial buildings, small housing projects, joints with a closing function, and tunnels [3]. Potential applications include high-strength prestressed concrete bridge girders [4], more durable high-performance bridge deck slabs, as well as girders and joists in building frames.

One barrier to the structural use of SFRC is the lack of mechanical models that describe the behaviour of SFRC members failing in shear [4-6], especially members without shear reinforcement. While most beams are provided with stirrups because of the high shear demands, slabs are typically designed without stirrups. The use of steel fibers as shear reinforcement is interesting for the design of one-way and two-way slabs, as well as to reduce reinforcement congestion in heavily reinforced beams. As such, construction time and costs can be reduced [7].

The classic set of experiments by Narayanan and Darwish [8] showed that the increase in shear strength attributable to the steel fibers varied from 13% to 170%. The question remains,
however, how steel fibers contribute to the shear-carrying capacity of structural concrete members. To answer this question, this review paper first summarizes the effect of adding fibers on the mechanical properties of concrete, and then revises what is known regarding the effect of adding fibers on the different shear-carrying mechanisms. Then, different parameters that influence the shear capacity of SFRC are discussed, and ultimately an overview of the current equations (and the assumptions that lie at the basis of these equations) for predicting the shear capacity of SFRC elements without stirrups is given. Finally, the way forward in addressing the needs for research and in practice are outlined and discussed.

2 Mechanical properties of SFRC

2.1 Introduction

To understand how the addition of steel fibers influences the different shear-carrying mechanisms, we first need to revise the influence of the fibers on the mechanical properties of concrete. The largest influence on the mechanical properties occurs for long fibers with a large aspect ratio [9].

2.2 Tensile strength

Since the fibers can carry tension after the tensile strength of concrete is reached and cracking occurs, the improved tensile strength of SFRC and post-cracking behavior is its most significant mechanical property [7, 10]. Although codified approaches exist for uniaxial tension tests, the flexural strength is often determined instead of the tensile strength [11]. The drawback of using flexural strength tests (or other types of indirect tests), is that additional sources of uncertainty are introduced in the material characterization. For this purpose, a beam test (with or without notch, depending on the governing code) is carried out. What is reported (deflection or
crack mouth opening displacement CMOD) and used [12-14] to determine the tensile strength also depends on the code [15-20] under consideration, and results in discrepancies between these approaches [21]. For these tests, multiple points on the measured diagrams must be calibrated to determine the uniaxial tensile strength properties. For ACI 544.3R-08 [16] the reported properties are the first crack flexural strength, the peak post-cracking strength, and the toughness. When the tensile strength of an SFRC mix cannot be measured, and only information about the compressive strength is available, the tensile strength can be estimated as [22]:

\[
f_{\text{fF}} = 0.97\left(f_{\text{cu}}\right)^{0.5} + 0.295\left(f_{\text{cu}}\right)^{0.5} F + 1.117F
\]

(1)

and \( F \) is the fiber factor, which is a measure of the fiber-matrix interfacial bond, calculated as a function of the fiber length \( L_f \), the fiber diameter \( d_f \), the fiber volume fraction \( V_f \) and the fiber bond factor \( \rho_f \), which depends on the type of fiber (i.e., straight, hooked, crimped,…):

\[
F = \frac{L_f}{d_f} V_f \rho_f
\]

(2)

The cracking moment of reinforced concrete members increases as a result of the increased tensile strength [23]. A practical result of the improved tensile resistance and post-cracking behavior is that in SFRC members, temperature and shrinkage reinforcement is not required [24].

2.3 Compressive strength

Adding steel fibers does not significantly increase the compressive strength of concrete [22, 25], but the post-peak rate of strength loss is smaller, since the fibers mitigate post-peak splitting cracks [4, 26].
2.4 Cracking in reinforced concrete

In reinforced concrete (RC) members, the addition of steel fibers reduces crack width and spacing. The reduction in spacing [8] can be estimated with the crack control coefficient $\kappa$:

$$\kappa = \frac{50}{L_f / d_f} \leq 1.0$$  \hspace{1cm} (3)

Another method to calculate the reduced crack spacing, as recommended by the fib Model Code 2010 [18], is based on reducing the tensile strength in the equation with a linear relation between spacing and tensile strength. However, experimental evidence [23] indicates that this method does not lead to realistic results for the crack spacing and width. Therefore, the tension tie model that lies at the basis of the fib Model Code recommendations is reevaluated for the influence of fibers [4], resulting in a tension chord model [27], which takes into account the nonlinear bond between concrete and fibers [28] and the improved tension stiffening resulting from fibers bridging the cracks and carrying tension [27]. The tension chord model was also extended for ultra-high performance concrete (UHPC) and to include the effects of creep and shrinkage [29].

The tension stiffening effect can be modeled by considering three components (concrete bond, steel fibers, and mild steel reinforcement) separately [30]. Experimental results showed that the increasing the fiber dosage and the reinforcement ratio resulted in a lower contribution of bond between the concrete and reinforcing bar.

Given the complex mechanics of the topic of cracking in SFRC, the best results are obtained for the combination of finite element modeling and probabilistic studies [31, 32]. When crack width and spacing are reduced, the crack kinematics change. Since crack kinematics are a driving factor for determining the contribution of the different shear-carrying mechanisms, we cannot separate the cracking behavior in SFRC beams failing in shear from the analysis of the...
shear capacity [33, 34]. Additionally, the influence of the reduce crack width and crack spacing on the size effect in shear has a direct effect on the shear capacity [35].

2.5 *Flexural strength and ductility in reinforced concrete*

To estimate the capacity of SFRC beams subjected to bending, sectional analysis can be used. Figure 1 shows the sectional analysis proposed by RILEM TC 162-TDF [36], with the detailed stress-strain relationship in Figure 2. The effect of the steel fibers is considered for the part of the cross-section that is under tension, represented by the tensile stress block in Figure 1.

The stresses identified in Figure 2 and used in the stress blocks in Figure 1 are:

\[ \sigma_1 = 0.7 f_{\text{fcm,fl}} (1.6m - d) \text{ with } d \text{ in m} \]  \( \text{(4)} \)

\[ \sigma_2 = 0.45 f_{R,k} k_h \]  \( \text{(5)} \)

\[ \sigma_3 = 0.37 f_{R,k} k_h \]  \( \text{(6)} \)

The size effect factor is:

\[ k_h = 1.0 - 0.6 \frac{h - 12.5 \text{cm}}{47.5 \text{cm}} \text{ for } 12.5 \text{cm} \leq h \leq 60 \text{cm} \text{ with } h \text{ in cm} \]  \( \text{(7)} \)

Using the following expression for the Young’s modulus of the fiber reinforced concrete:

\[ E_c = 9500 \sqrt{f_{\text{fcm}}} \text{ with } f_{\text{fcm}} \text{ in MPa} \]  \( \text{(8)} \)

we can then determine the strains:

\[ \varepsilon_1 = \frac{\sigma_1}{E_c} \]  \( \text{(9)} \)

\[ \varepsilon_2 = \varepsilon_1 + 0.1\% \]  \( \text{(10)} \)

\[ \varepsilon_3 = 25\% \]  \( \text{(11)} \)
Figure 1. Sectional analysis for SFRC: (a) cross-section; (b) strains; (c) stresses.

Figure 2. Stress-strain relationship for fiber reinforced concrete.

Using simplified assumptions and a rectangular stress block results in the sectional analysis shown in Figure 3, which also shows the force resultants. The effect of the fibers is represented by an additional force resultant $T_c$, see Figure 3. With these simplified assumptions, the flexural capacity can then be estimated as [37, 38]:

\[
\sigma_c = \frac{M}{I} + T_c
\]
\[
M = \frac{1}{2} \rho f_{y}bd^2 (2 - \eta) + 0.83Fbd^2 (0.75 - \eta)(2.15 + \eta)
\]

with

\[
\eta = \frac{\rho f_{y} + 2.32F}{0.85f'_{c} + 3.08F}
\]

Figure 3. Sectional analysis of SFRC: (a) cross-section; (b) strains; (c) stresses and resultant forces.

Similar expressions \[28\] can be derived when a parabolic stress-strain relationship is used for the concrete instead of the rectangular stress block diagram \[39\], and when the stress-strain diagram of the SFRC under tension from Figure 2 is considered instead of an equivalent stress block \[36\].

While the presented concepts for flexure are useful for design, it should be kept in mind that experiments \[40\] showed that sectional analysis overestimates the flexural capacity of SFRC beams, and that the overestimation increases linearly as the cross-sectional area increases. Adding fibers increases the ductility of SFRC beams in flexure \[41\] for fiber volume fractions between 0.5% and 0.75% \[42\], but not for fractions of 0.375%. 

-9-
2.6 Fatigue behavior

An additional mechanical property to consider is the fatigue strength of SFRC elements. One should distinguish between fatigue with stress reversals (seismic conditions) and without stress reversals (repeated loads, such as vehicles passing). For fatigue loading with stress reversals, the crack bridging ability of the fibers is negatively affected [43], so that for seismic conditions a minimum of transverse steel reinforcement is still required in beams. Other experiments [44] however showed an improved resistance of shear-critical SFRC beams as compared to RC beams under cyclic loading. For fatigue tests without stress reversals, the improvement thanks to the fibers is clear [45-47]. The failure mode is fiber pull-out (when crack coalescence is significant), or breaking of the fibers otherwise [48]. The fatigue life of SFRC under compression can be described by the relation between strain and number of cycles, and a three-parameter Weibull distribution[49]. When the life-cycle cost of concrete members under repeated loads is considered, SFRC outperforms RC.

2.7 Creep and shrinkage

A final mechanical property to consider is the effect of creep and shrinkage on the long-term strength of SFRC elements. For steel fiber concrete mixes of UHPC [50], the magnitude of the tensile creep and drying shrinkage depends on the curing conditions. To reduce creep, thermal treatment should be used. Creep was also reduced in the UHPC mixes with fibers as compared to the mixes without fibers [50, 51]. Similarly, for normal strength concrete, experiments showed that creep is smaller for specimens with steel fiber than for plain concrete specimens[52]. For precracked steel fiber beams without longitudinal reinforcement under sustained loading [53], experiments showed that for small crack widths, stable responses up to 18 months were measured. Experiments on similar specimens subjected to a sustained loading for
90 days showed additionally that fiber slenderness and fiber content significantly influence the effect of load ratio on the flexural creep response of SFRC [54], and for experiments that included unloading and different load levels, the influence of the irreversible part of the crack opening displacement and the loading level were found to significantly affect the creep deformation [55]. Additional, the fiber type affects the creep performance: the interfacial transition zone between the hooked-end of the fiber and the concrete matrix influences the pull-out creep behaviour of a single fiber, which then influences the uni-axial tensile creep performance of the mixture [56]. Drying was found to result in increased crack widths, but had not influence on the global damage state [57]. Similarly, the total shrinkage of steel fiber mixes was found to be 15% less than the total shrinkage in plain concrete [58].

For members with longitudinal steel reinforcement, experiments on normal strength SFRC beams with stirrups under sustained loading resulted in the conclusion that adding steel fibers reduces the long-term deflections at load levels above 50% of the design value of the flexural capacity [59], and expressions for the long-term deflections have been derived [60, 61]. The efficiency of the fibers was found to decrease as crack widths increase, after which the longitudinal reinforcement acts to reduce the secondary creep deflection rate [62].

3 Mechanisms of shear transfer

3.1 Overview of shear transfer mechanisms in SFRC

When it is known how steel fibers affect the mechanical properties of concrete, it becomes possible to understand the effect of steel fibers on the different mechanisms of shear transfer in reinforced concrete beams without stirrups. For slender beams, the shear resistance can be analysed based on the different shear-carrying mechanisms by studying the free-body
diagram on a crack [63], see Figure 4. Just as for reinforced concrete beams, the concrete compression zone provides shear resistance $V_{cz}$, the flexural reinforcement provides shear resistance through dowel action $V_d$, and aggregates protruding from the crack face provide shear resistance through aggregate interlock $V_a$. The contribution of residual tension in the concrete across the crack is now replaced by the tensile strength of the SFRC across the crack, $\sigma_{f,cr}$.

![Figure 4. Overview of mechanisms of shear transfer for SFRC.](image)

3.2 **Capacity in compression zone**

The first mechanism that contributes to the shear resistance is the load-carrying capacity of the uncracked concrete in the compression zone [64-68]. In RC, the contribution of the compression zone to the shear resistance can be found by integrating the shear stresses over the depth of the compression zone [68], and is estimated to contribute between 20% [66] and 40% [65] to the total shear capacity in RC.

As discussed earlier, the effect of fibers on the compressive strength of concrete is limited. From this perspective, the addition of fibers does not change the capacity in the compression zone. However, on observing the simplified sectional analysis from Figure 3, it becomes clear that the influence of the fibers on the capacity in the compression zone is related to the height of the compression zone. Adding steel fibers changes the horizontal equilibrium...
(see Figure 3), as a result of adding the resultant $T_c$ in SFRC. The height of the compression zone will thus be larger in SFRC than in RC. As such, the capacity in the compression zone will be larger in SFRC than in RC. The fibers also provide post sliding shear capacity when a compressive normal stress is applied across the crack. Finally, when the critical shear crack propagates, failure can occur when this crack extends into the compression zone and the shear-carrying mechanisms of the capacity of the uncracked concrete in the compression zone is lost.

3.3 Tension across crack

Since cracks in concrete are not clean breaks, tension can be transferred across the crack when the crack width is small enough (i.e. microcracking). In RC, the contribution of residual tension across a crack [69, 70] is only significant for small members, where the crack widths are small enough for tension-softening to take place.

In SFRC, on the other hand, the shear-resisting mechanism of tension across the crack is provided by the fibers bridging the crack. The effectiveness of this action depends on the fiber-matrix interfacial bond, the fiber fracture and pullout properties, the maximum aggregate size, the shear yielding of the fibers, the fiber geometry, the fiber content, and the fiber aspect ratio [71]. Additionally, the fiber orientation is important for the shear strength, and fibers that are perpendicular to the crack plane are the most effective [72]. Stroeven [73] pointed out that the fibers reinforcing the crack should be distributed according to a function of $\sin^2 \theta$ with respect to crack plane or loading axis, with $\theta$ the angle enclosed by loading direction and fiber, and that the fibers are not distributed randomly as often assumed. Stereological principles lie at the basis of this theoretical conclusion.

Several authors have developed expressions to quantify the contribution of steel fibers to the shear capacity, see Table 1. The inclination of the critical diagonal crack $\alpha$ was measured to
be between 25 to 36°, and can be taken for design as \( \alpha \approx 30° \). The factor \( k_f \) considers the
contribution of flanges in T-sections (=1 for rectangular sections). \( \alpha \) is a factor that takes the
random distribution of fibers into account (\( \approx 3/8 \)), and \( \eta_l \) is a length factor used to account for the
variability of the fiber embedment length across the cracking plane (\( \approx 0.5 \)). Note that the
expression of Khuntia et al. [74] is almost the same as the expression of Mansur et al. [75]. For
hooked fibers, a more elaborate expression that considers the bond strength of the fibers as well
as the bond strength of the hook has been derived [4]. The contribution of the fibers has also
been studied based on microcrack initiation and growth [76].

Analyzing the background of the expressions in Table 1 reveals that main assumptions
are that the contribution of the fibers is directly linked to the average pullout force per fiber on
the crack plane. This pullout force is again related to the average fiber-matrix interfacial bond
strength \( \tau \). This procedure can be illustrated with the expression of Singh and Jain [7]. The
contribution of the fibers is the vertical projection of the tensile force along the length of the
diagonal crack \( T_{f,d} \), which can be linked to the tensile stress \( \sigma_{fu} \) resisted by fibers bridging a unit
area of the inclined crack:

\[
V_F = T_{f,d} \cos \alpha = \sigma_{fu} b (d - c) \cot \alpha
\]  \hspace{1cm} (14)

where

\[
\sigma_{fu} = N \times f
\]  \hspace{1cm} (15)

with \( N \) the number of fibers crossing a unit area of the inclined crack:

\[
N = 0.5 \frac{V_F}{\pi r_f^2}
\]  \hspace{1cm} (16)

and \( f \) the average pullout force per fiber:

\[
f = \tau \pi p_f d_f \frac{L_f}{4}
\]  \hspace{1cm} (17)
Substituting Eq. (17) and Eq. (16) into Eq. (15) and then into Eq. (14) results in the expression form Eq. (18) in Table 1. Different assumptions and estimations for the magnitude of the fiber-matrix interfacial bond strength are given in Table 1. Expressions that are based on different assumptions are the expressions by Amin and Foster [77], which are embedded in the analysis of the Modified Compression Field Theory. The expressions of Aoude et al. [78] do not only consider the effect of the fiber pullout force, but also force required to pull out the hooked end of hooked fibers.

Table 1. Overview of expressions for the contribution of steel fibers to the shear capacity.

<table>
<thead>
<tr>
<th>Author(s)</th>
<th>Ref</th>
<th>Expression</th>
<th>Eq.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Singh and Jain</td>
<td>[7]</td>
<td>$V_F = 0.5\tau F b (d - c) \cot \alpha$</td>
<td>(18)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>with $\tau = 0.85\sqrt{f'_c}$ for hooked-end fibers</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\tau = 0.75\sqrt{f'_c}$ for crimped fibers</td>
<td></td>
</tr>
<tr>
<td>Mansur et al.</td>
<td>[75]</td>
<td>$V_F = 0.41\tau F b d$</td>
<td>(19)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>with $\tau = 0.68\sqrt{f_c}$</td>
<td></td>
</tr>
<tr>
<td>Narayanan and Darwish</td>
<td>[8]</td>
<td>$V_F = 0.41\tau F b d$</td>
<td>(20)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>with $\tau = 4.15$ MPa</td>
<td></td>
</tr>
<tr>
<td>Khuntia et al.</td>
<td>[74]</td>
<td>$V_F = 0.25F \sqrt{f_c} b d$</td>
<td>(21)</td>
</tr>
<tr>
<td>Kwak et al.</td>
<td>[79]</td>
<td>$V_F = 0.8 \times 0.41\tau F b d$</td>
<td>(22)</td>
</tr>
<tr>
<td>RILEM</td>
<td>[36]</td>
<td>$V_F = 0.7k_j k \tau b d$</td>
<td>(23)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(24)</td>
</tr>
</tbody>
</table>
\[
k = 1 + \sqrt{\frac{200mm}{d}} \leq 2
\]
\[
\tau = 0.12 f_{Rk,4}
\]

**Amin and Foster** [77]

\[V_p = 0.7k_f bdf_{f,5}\]

\[k_f = \cot \theta \leq 1.28\]

\[\theta = 29^\circ + 7000 \varepsilon_x\]

\[
\varepsilon_x = \frac{M}{2E_x A_y} + 0.5V \cot \theta
\]

**Tahenni et al.** [80]

\[V_p = \tau \left(1 - 20V_f\right) \frac{V_f L_f}{d_f} bd\]

\[\tau = 0.28 \sqrt{f_c} \quad \text{for plain fibers}\]

\[\tau = 0.5 \sqrt{f_c} \quad \text{for fibers with improved bond}\]

**Aoude et al.** [78]

\[V_p = \frac{V_f}{A_f} \alpha \eta \theta 0.83 \left(\tau \pi d_f \frac{L_{f,straight}}{2} + \Delta P'\right) b_w d_v \cot \theta\]

with \[\Delta P' = 0.565 f_{f,s} d_f^2\] for hooked fibers

\[
\tau = \begin{cases} 
2 - 3MPa & \text{for } f_c' \leq 50MPa \\
3.4 - 4.5 MPa & \text{for } 50MPa < f_c' \leq 70MPa \\
5 - 6MPa & \text{for } f_c' > 70MPa
\end{cases}
\]

---

### 3.4 Dowel action

Dowel action is the contribution of the flexural reinforcement to resist the opening and slipping of the shear crack [81-83]. The maximum resistance that can be developed through
dowel action is related to the tensile strength of the concrete cover, which fails through splitting when the dowel action force becomes too large. Since the tensile strength of SFRC is improved as compared to RC, dowel action is improved in SFRC [8]. Bond between the concrete and reinforcement also influences the dowel action. In SFRC, the bond properties (especially the resistance against splitting bond failure) are improved as compared to RC [26], resulting in a stronger dowel action in SFRC. Additionally, crack propagation in SFRC is slower, so that the propagation of dowel cracks is slower in SFRC and the dowel action resistance is larger [84, 85].

Preliminary measurements with DIC [86] showed that dowel action contributes to 10-35% of the shear resistance in SFRC.

3.5 Aggregate interlock

Aggregate interlock [82, 87-93] is the resistance provided by the contact between aggregates protruding from both crack surfaces. Traditionally, to evaluate the slipping and sliding of the shear crack, direct shear tests on plain concrete specimens are used. For SFRC, direct shear tests measure both the aggregate interlock capacity as well as the tensile capacity of the fibers crossing the crack. Moreover, experiments indicate that the aggregate interlock capacity is lower in reinforced concrete specimens than in the plain concrete specimens tested in direct shear tests [94, 95].

From shear tests on steel fiber concrete specimens without flexural reinforcement, a number of hypotheses for the aggregate interlock mechanism in SFRC have been formulated:

- Increasing the volume fraction of fibers from 0% to 0.5% to 1% and 1.5% increases the aggregate interlock capacity almost linearly [96]. Different expressions where derived for different concrete classes and fiber aspect ratios.
For example, the expression for concrete class C30 and a fiber aspect ratio $L/f_d = 65$ is [96]:

\[ \tau_{c30,65} = 26.74V_f + 34.52 \]  \hspace{1cm} (33)

- The fiber aspect ratio does not influence the aggregate interlock capacity [96] for hooked-end fibers with an aspect ratio of 65 and 80.

- Fibers with flattened ends result in better aggregate interlock capacity than crimped fibers [97] for fiber volume fractions varying between 0% and 2%. For the flattened-end fibers the increase in shear strength was almost linear with the increase in fiber volume fraction, whereas a plateau was observed for the crimped fibers.

- The enhancement of the aggregate interlock capacity is larger for high strength concrete (65 MPa) than for normal strength concrete (28 MPa) as a result of the improved bond between the fibers and the concrete matrix [98]. For this series of experiments, the fiber volume fraction was constant at 1%.

Adding fibers also makes the failure mode more ductile [99-101]. The sliding and slipping across the crack can be described with $w - \delta$ relationships based on the MCFT for use with finite element models [4].

From direct shear tests (push-off tests) of SFRC with flexural steel reinforcement, it was found that the shear effectiveness of the fibers depends on the amount of reinforcement crossing the crack plane [102].

The aggregate interlock capacity in SFRC has not been studied much in experiments. Based on theoretical concepts, one can expect the aggregate interlock capacity to be larger in SFRC than in RC. The addition of fibers results in a smaller crack spacing and width, so that for
larger loads, the crack width in SFRC is smaller than in RC. As such, for the same load, more
load can be carried through aggregate interlock for the SFRC beam than in RC [103]. However,
preliminary measurements [104] demonstrated that the crack width to slip ratio is smaller in
SFRC, which results in larger aggregate interlock stresses. Many shear tests in SFRC use a
maximum aggregate size of 10 mm. For RC, using a smaller maximum aggregate size is a
conservative approach, as larger aggregate sizes improve the aggregate interlock capacity.
However, for SFRC, this assumption may not hold true [26], because for SFRC a smaller
aggregate size results in less disturbance of the bond between the steel fibers and the concrete
matrix, which increases the shear strength.

3.6 Arching action

Arching action is the shear-resisting mechanism that results from the compressive strut
between the load and the support [105-107]. This mechanism significantly increases the shear
capacity for beams with short shear spans. Not all expressions for the shear capacity take this
effect into account, which may result in overly conservative predictions for the shear capacity of
members with short shear spans [75].

According to [8] and [79], the addition of steel fibers improves arching action in SFRC,
so that the increase in shear capacity for beams with small values of $a/d$ was measured to be
significantly larger than for beams with larger $a/d$ ratios. This effect is attributed to the additional
compression across the crack provided by the fibers, which helps to sustain the shear transfer
across the critical crack [108]. The softening effect of the diagonal concrete strut becomes less
[109], and the deformation characteristics improve [110] as compared to reinforced concrete.
3.7 Discussion

Just as for reinforced concrete [63] there is a disagreement in the literature on how much shear is resisted by each of these mechanisms. Extensive measurements in reinforced concrete beams failing in shear showed that the distribution among the shear-resisting mechanisms depends on a number of parameters, such as the amount of longitudinal reinforcement, the concrete compressive strength, the type and size of the aggregates, etc. To quantify the distribution among the shear-resisting mechanisms in SFRC, experiments with extensive measurements (direct strain readings or non-contact readings through digital image correlation [108, 111-114]) are necessary. DIC analysis of a small number of experiments [86] indicated that the steel fiber bridging effect is constant at approximately 20% of the total shear resisting strength.

While most authors contribute the larger shear resistance of SFRC as compared to RC to the tensile strength of the steel fibers crossing the crack, Zarrinpour and Chao [86] contribute the increased capacity to the delay that takes place in the development and propagation of the critical shear crack. As a result, the depth of the compression zone remains larger, even at higher loads, and the authors speculated that most of the shear resistance of SFRC is provided by this larger and highly compressed compression zone, which remains stable even when diagonal cracks propagate into it. They postulated that the aggregate interlock contribution to the ultimate shear strength of SFRC is negligible. Others [84] have argued that the importance of the aggregate interlock effect is a function of the shear-span-to-depth ratio, and that aggregate interlock loses its importance for beams with \( a/d < 2.5 \), where arching action becomes the governing shear-resisting mechanism.
4 Parameters that affect the shear capacity of SFRC beams

4.1 Concrete compressive strength

In RC, the concrete compressive strength is often considered one of the most important parameters affecting the shear capacity [115]. Since the influence of fibers on the concrete compressive strength is negligible, one could argue that the influence of the concrete compressive strength on the shear capacity of SFRC should be the same as for RC. However, this logic separates the influence of the fibers from the other shear-carrying mechanisms.

Contradictory conclusions result from different series of experiments. Some authors [116] have found a larger increase in the shear capacity when fibers are added to the mix for high strength concrete than for normal strength concrete. This increase was attributed to the larger tension zone resulting from horizontal equilibrium for higher strength concrete, resulting in a larger contribution of the steel fibers crossing the tension crack. Similarly, a larger increase in shear capacity thanks to the addition of steel fibers was observed for lightweight concrete as compared to normal weight concrete [117]. Others [118], however, report no difference in the increase in shear capacity resulting from the inclusion of steel fibers for high strength concrete as compared to normal strength concrete.

Analyzing the influence of the different shear-carrying mechanisms, one could expect the following influence from the concrete compressive strength on the shear capacity of SFRC beams:

- Comparing concrete with and without fibers, it is expected that the contribution of the concrete compression zone will be larger for the concrete with fibers, since the height of the compression zone will be larger as a result of the horizontal equilibrium. On the other hand, comparing normal and high strength concrete with fibers, the height
of the compression zone and thus the contribution of the concrete under compression will be smaller for the high strength concrete than for the normal strength concrete.

- Comparing normal and high strength concrete with fibers, a larger tension zone and thus larger contribution of the fibers in tension can be expected for the high strength concrete.
- The addition of fibers to high strength concrete is expected to result in an increase in dowel action as compared to high strength concrete without fibers. This effect should be related to the tensile strength of the fiber reinforced concrete.
- Increasing the concrete compressive strength can reduce the aggregate interlock capacity, as the crack becomes more smooth when aggregates rupture. This effect is expected to influence the shear capacity both for high strength concrete elements with and without fibers.
- The influence of increasing the concrete compressive strength for concrete with and without fibers on the arching action capacity can be related to the increased compressive resistance of the resulting compressive arch.

4.2 Amount of flexural steel

For reinforced concrete, increasing the amount of flexural steel will increase the shear capacity, as the dowel action is improved. For SFRC, experiments showed that the increase depends on the type of fiber [119], showing an increase in shear capacity with an increasing amount of flexural steel for only some fiber types [85]. Other experiments [84] showed that the increase in shear capacity for increasing amounts of flexural steel was larger for SFRC than for RC, as a result of the larger tensile strength of SFRC. As such, larger dowel forces can develop before spalling of the cover occurs.
Certainly, dowel action is the most important shear-carrying mechanism influenced by the amount of flexural steel. As for the remaining shear-carrying mechanisms, one can expect the following influence of the amount of flexural reinforcement:

- As the amount of flexural steel increases, a larger compression zone is necessary to achieve horizontal equilibrium in the cross-section. As such, the contribution of the of the uncracked concrete in compression increases as the amount of flexural steel increases. For the SFRC as compared to RC, this effect is expected to be larger, as the resultant of the concrete under tension requires an even larger compression zone.

- It is expected that an increase in flexural steel results in an increase in the contribution of the steel fibers across the crack, as larger amounts of flexural reinforcement keep crack widths smaller at similar load levels, which increases the contribution of the steel fibers.

- Flexural steel can provide a clamping force on the crack, which enhances aggregate interlock. However, fibers are not expected to influence this effect.

- A larger amount of flexural steel can increase arching action, as a larger compression force can balance the tensile resultant in the reinforcement. However, fibers are not expected to play a role in the effect of increasing the amount of flexural steel on the shear capacity.

### 4.3 Shear span to depth ratio

For RC members, the shear span to depth ratio \((a/d)\) influences the failure mode, as known from Kani’s valley [115]. Often, the most critical position for shear is considered at \(a/d = 2.5\) [120], although Kani pointed out that the critical position is not a preset value, but depends,
among other on the longitudinal reinforcement ratio. Experiments on building slabs with lateral restraint found a critical position at $a/d = 7.3$ [121].

The addition of steel fibers alters the critical position for shear that divides the flexure- and shear-critical zone [122], since the fibers start to act as shear reinforcement. Indeed, various series of experiments have shown that, even for beams with $a/d$ smaller than 2.5 with fibers, the failure mode becomes flexure instead of shear because the fibers act as shear reinforcement. Adding [37] steel fibers reduces the shear domain in Kani’s valley [123]. The critical $a/d$ ratio to induce shear failure also seems to decrease as the fiber content increases [75]. For large fiber volume fractions, the shear valley disappears as the mode of failure changes. Additionally, the shear domain in Kani’s valley is influenced by the effective depth $d$ of the member. For larger members, the size effect in shear makes the beam more shear-critical, which means that Kani’s valley is extended. Further experiments on the size effect in shear-critical SFRC beams, and its link to other parameters such as the influence of $a/d$ and the reinforcement ratio, are necessary.

An ANN-based analysis of a database of 209 experiments on SFRC beams failing in shear [124] showed that $a/d$ is the most influential parameter affecting the shear strength of the SFRC beams from the database under consideration. However, one should bear in mind that ANN-based analysis only gives insight in the available data from the database under consideration. It does not explain nor give insight in the underlying mechanics of the problem, and it is limited to the available experimental data. As such, it can be highlighted again that further experiments on the link between $a/d$ and the overall member depth are necessary.

From a theoretical perspective, the effect of changing the $a/d$ ratio on SFRC can be expected to have an influence on the arching action only. Arching action becomes more important as $a/d$ decreases, and can be larger in SFRC than in RC. The effect of the other
mechanisms can be related to changes in the internal forces and moments, but not on a direct
influence on the mechanics of the problem.

4.4 Member height

The size effect in shear on RC members is well-known [125-131], although researchers
disagree on the mechanics behind this effect, and the correct mathematical expression for the
size effect [132-135].

As the mechanics of SFRC members without stirrups failing in shear is even more
complex, similar disagreements are found in the literature. In fact, there is no consensus on
whether or not the size effect in shear in SFRC members exists. An analysis of a database of 139
tests [79] showed no apparent size effect. The authors gave two possible explanations for this
observation: i) the increased ductility of SFRC as compared to RC; and ii) the limited number of
experiments on large beams in the database. On the other hand, a parametric analysis of a
database of 85 experiments using artificial neural networks (ANNs) showed that the accuracy of
the matrix-based expression of the ANN improved significantly when the effective depth is
considered as an input [136].

Experiments on beams with a total height of 455 mm and 685 mm [119] showed only a
7% decrease in shear strength for the higher beams as compared to the smaller beams, which led
the authors to conclude that for the tested range of beam heights, the size effect is negligible. For
beams with heights ranging from 181 mm to 887 mm [137], a size effect in SFRC beams was
observed, which was larger than the size effect in shear in RC beams with stirrups. However, a
careful analysis of these results shows that the reported shear stress in these experiments does not
consider the contribution of the self-weight of the beam to the shear stress at the ultimate. When
the shear stress at failure is recalculated including the self-weight, virtually no size effect is observed.

For lightweight SFRC beams with heights varying between 308 mm and 1000 mm, experiments [138] showed a size effect in shear and no size effect in flexure. Similarly, [139] found a size effect when testing T-girders with a height of 800 mm and rectangular beams with a height of 250 mm. The authors reported that for the deeper beams, the crack opening at failure became larger, which reduces the contribution of the fibers to the shear capacity. This observation may explain the seemingly contradictory conclusions reported in the literature: as long as the crack width and spacing remains small enough, and the fibers can bridge the crack and carry tension across the crack, no size effect occurs. However, when the crack width and spacing becomes larger, the fibers cannot carry tension across the crack anymore, and a size effect as for RC members can be observed.

From a theoretical perspective, we need to make assumptions on the size effect itself before we can analyze the influence of increasing member height on the different shear-carrying mechanisms. Assuming that the size effect is driven by the fact that deeper members have wider cracks, we can expect the following influence of increasing member height on the shear-carrying mechanisms in SFRC:

- As the member height increases, the tension capacity of the steel fibers across the crack will be less, as the crack width will be larger. Fibers can have a positive effect here in keeping the crack width smaller.

- As the member height increases, the aggregate interlock capacity will be less, as the crack width will be larger. Fibers can have a positive effect here in keeping the crack width smaller.
4.5 Fiber properties

The following characteristics define the properties of fibers and, as such, their influence on the shear capacity: the fiber geometry, the fiber type (including the anchorage characteristics of the fiber), and the fiber material properties. Since this work focuses on steel fibers only, we will focus here on the geometry of the fiber and the fiber type.

The different types of steel fibers that are available are (among others): straight fiber, crimped fiber, hooked fiber, and corrugated fiber, see Figure 5. The contribution of the steel fibers to the shear resistance of the SFRC can be divided in two parts: the tensile strength and pullout strength of the main part of the fiber, and the anchorage of the end part of the fiber in the concrete (if such anchorage is provided, as for hooked fibers). While smooth and stocky fibers are preferable from the point of view of workability, such fibers have a smaller shear-carrying capacity through tension because of the lack of an end deformation (as for hooked or crimped fibers). Typically, the effect of the type of fiber is considered through the fiber bond factor $\rho_f$ in the fiber factor $F$, see Eq. (2). Some theories [140], however, calculate the contribution to the shear capacity by expressing the pullout strength and the anchorage strength of a particular fiber, and thus studying the behavior of the fibers at the meso-scale.

![Fiber types](image)

**Figure 5: Most commonly used fiber types.**

The fiber geometry can be expressed based on the aspect ratio $L_f/d_f$. Experiments [141] (on SFRC beams with stirrups) showed an increase in the load at which the first shear crack
appeared for an increase in aspect ratio. Longer fibers have been observed to allow a larger opening of the critical crack before failure [119]. Another set of experiments [26] showed an increase in shear strength by 32\% when increasing the aspect ratio from 55 to 80 for $V_f = 0.75\%$ and $\rho = 2.06\%$, and a decrease in shear strength by 15\% when increasing the aspect ratio from 55 to 80 for $V_f = 0.75\%$ and $\rho = 1.56\%$. For other combinations of values of $V_f$ and $\rho$, either an increase, decrease, or virtually no change was found when increasing the aspect ratio from 55 to 80. These results show that no clear conclusions can be drawn on the influence of the aspect ratio on the shear capacity.

### 4.6 Amount of fibers

When evaluating the influence of the amount of fibers on the shear capacity of SFRC elements, typically two questions are evaluated:

i) “Which quantity of fibers should be added to avoid a shear failure?”, and

ii) “How does the amount of fibers influence the amount of tension that can be carried across the crack?”.

To answer the first question, many series of experiments have been carried out [79, 142]. In general, an analysis [43] showed that, depending on the fiber aspect ratio, the mechanical anchorage, and fiber tensile strength, fiber volume contents between 0.5\% and 1.0\% effectively control cracking and result in elements with a similar behavior as those with shear reinforcement. Based on the analysis of a database of 147 SFRC beams [143], the author showed that the addition of 0.75\% of steel fibers increases the shear strength of concrete above $0.3\sqrt{f'_c}$ ($f'_c$ in MPa). Dinh et al. [119] echoed this conclusion and stated that a volume fraction of at least 0.75\% of hooked steel fibers can be used as minimum shear reinforcement in beams with normal strength concrete. Other researchers have confirmed this finding based on nonlinear finite
element models [144]. Imam et al. [37] argue that not the fiber volume fraction but the fiber factor should be used to express the transition from the domain of shear failures to the domain of flexural failures.

The second question is addressed in some experimental series as well. Sahoo and Sharma [71] found that the increase in shear strength provided by steel fibers is reduced when the fiber content exceeds 1%. Similarly, when testing beams with 1%, 2%, and 3% of fibers [5], an increase in shear capacity was found for increasing the fiber content from 1% to 2%, whereas increasing the fiber content to 3% either changed the failure mode to flexure (for \(a/d = 4.5\)), decreased the shear capacity (for \(a/d = 2.5\)), or increased the shear capacity (for \(a/d = 3.5\)). In the experiments of Dinh [26], smaller increases in the shear capacity were observed when the fiber volume fraction exceeded 0.75%. Similarly, Ashour et al. [145] found that the increase in shear capacity for an increase in fiber volume fraction depends on the shear span to depth ratio: for \(a/d = 1\) the shear strength increased by 96.6% when the fiber volume fraction was increased from 0% to 1.5%, whereas for \(a/d = 6\) the shear strength increased by 32.2%.

An ANN-based analysis [146] using a database of 129 experiments showed that the influence of the aspect ratio, the concrete compressive strength, the effective depth, and the amount of flexural steel reduces as the fiber volume fraction increases. This observation shows that the shear-carrying mechanisms are interrelated, and heavily influenced by a variety of parameters.
5 Available expressions

5.1 Code equations

Not all concrete codes prescribe provisions that include the shear-enhancing effect of the addition of fibers. Table 2 gives an overview of formulas that are available in current codes and guidelines. Older references that are not included here are the previous versions of the French guidelines [147, 148]. The safety factor in Eq. (35) \( \gamma_{cf} \gamma_E \) includes the safety coefficient \( \gamma_E \) so that \( \gamma_{cf} \gamma_E = 1.5 \), to cover the uncertainty on extrapolating formulae developed for concrete with \( f_{ck} \leq 90 \) MPa to higher strengths. Eq. (40) is valid for rectangular cross-sections and T-sections. Expressions for other geometries are available in the French guidelines. A minimum value of \( \theta = 30^\circ \) is recommended in Eq. (36). In Eq. (37), the value of \( K \) can be estimated as \( K = 1.25 \) unless the element is small. When the width \( b_w \) and height \( h \) are both less than 5\( l_f \), the value of \( K = 1.75 \).

When tension tests are carried out, the value of \( K \) for the mix under consideration can be determined as the ratio between the average peak of the load-strain or moment-strain diagram obtained with the specimens cast in the suitability test and the average peak of the load-strain or moment-strain diagram obtained on sawn specimens used in the suitability tests. For small members, one should use instead the ratio of the average peak of the load-strain or moment-strain diagram obtained with the specimens cast in the suitability test and the lowest peak of the load-strain or moment-strain diagram obtained on sawn specimens used in the suitability tests. Since Eq. (42) is based on the German National Annex to the Eurocode, the value of \( C_{Rd,fc} = 0.15 \) (not 0.18, the recommended value of EN 1992-1-1:2004 [149]). The reinforcement ratio is limited to \( \rho \leq 2\% \). The material factor for concrete is \( \gamma_c = 1.5 \), whereas \( \gamma_{ci}^f = 1.25 \). The factor \( \alpha_f \) = 0.85 accounts for long term effects. The value of \( A_f^l \) in Eq. (45) is \( b_w \times d \) with \( d \) limited to 1.5 m, which corresponds to the maximum beam size that has been tested. The value of \( k_f^l \) in Eq.
(44) is 0.5 for shear, assuming that tensile stresses of the steel fibers in the inclined crack are half of those determined in beam tests. For cross-sections subjected to axial stresses, the value of \( V_{Rd,cf} = 0 \), since insufficient experimental results are available for beams under a combination of shear and axial loading [10].

The expression from the fib Model Code 2010 [18] is based on EN 1992-1-1:2004 [149], and thus uses \( C_{Rd,c} = 0.18 \), \( \gamma_c = 0.15 \), \( \rho \leq 2\% \), and \( k \) as given in Eq. (46).

ACI 318-14 [2] allows the use of deformed steel fibers in volume fractions greater than or equal to 0.75% as minimum shear reinforcement in normal-strength concrete beams as long as the shear strength \( V_u \) lies in the range \( 0.50\phi V_c \leq V_u \leq \phi V_c \), where \( \phi \) is the strength-reduction factor. In addition to the specified minimum fiber content, the ACI Building Code also prescribes a flexural performance criterion based on the ASTM C1609 [15] four-point bending test for the acceptance of steel fibers as minimum shear reinforcement.

The recommendations from RILEM [36] are also based on the expression from EN 1992-1-1:2004 [149] (including \( \rho \leq 2\% \)) for the concrete contribution to the shear capacity.

In analyzing the expression in Table 2 it is clear that the French and German guidelines and the RILEM equations separate the contribution of the concrete from the fiber contribution. These approaches use an empirically derived inclined cracking load for the concrete contribution, whereas the effect of the fibers is studied based on the mechanics of fibers bridging the crack – as such, they combine two different philosophies. As discussed earlier, the addition of fibers influences all shear-carrying mechanisms – separating terms may simplify computations, but is mechanically incorrect. The fib expressions combine the effect of concrete and fibers, but the resulting expression is empirical and does not consider the contributions of the different shear-carrying mechanisms.
Comparing the available code equations to a database of 488 experiments [150] showed that the average ratio of tested to predicted shear capacity with the Eurocode was slightly conservative, and that the Eurocode expressions resulted in the lowest coefficient of variation. As such, designers can use the Eurocode expressions to estimate the shear capacity of SFRC members without stirrups until models based on the mechanics of the problem are available.

Table 2: Overview of existing codes and guidelines that include the effect of fibers on the shear capacity. The expressions are for SFRC elements without stirrups and without prestressing or axial load.

<table>
<thead>
<tr>
<th>Country</th>
<th>Ref</th>
<th>Expression</th>
<th>Eq.</th>
</tr>
</thead>
<tbody>
<tr>
<td>France</td>
<td>[151]</td>
<td>(V_{Rd} = V_{Rd,c} + V_{Rd,f})</td>
<td>(34)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(V_{Rd,c} = \frac{0.21}{\gamma_f \gamma_E} f_{ck}^{1/2} b_n d)</td>
<td>(35)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(V_{Rd,f} = \frac{A_f \sigma_{Rd,f}}{\tan \theta})</td>
<td>(36)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(\sigma_{Rd,f} = \begin{cases} \frac{1}{K_{\gamma_c}} \frac{1}{w_{lim}} \int_{0}^{w_{lim}} \sigma_f(w) dw &amp; \text{for strain softening or low strain hardening} \ \frac{1}{K_{\gamma_c}} \int_{0}^{w_{lim} - \varepsilon_{lim}} \sigma_f(\varepsilon) d\varepsilon &amp; \text{for high strain hardening} \end{cases} )</td>
<td>(37)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(w_{lim} = \max(w_n, w_{max}))</td>
<td>(38)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(\varepsilon_{lim} = \max(\varepsilon_n, \varepsilon_{max}))</td>
<td>(39)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(A_f = b_n z)</td>
<td>(40)</td>
</tr>
<tr>
<td>Germany</td>
<td>[19]</td>
<td>(V_{f/Rd,c} = V_{Rd,c} + V_{Rd,cf})</td>
<td>(41)</td>
</tr>
</tbody>
</table>
\[ V_{Rd,c} = \frac{C_{Rd,c}}{\gamma_c} k \left( 100 \rho f_{ck} \right)^{1/3} b_u d > V_{Rd,c,\text{min}} \quad (42) \]

\[ V_{Rd,cf} = \frac{\alpha_f f_{cR,u} b_u h}{\gamma_{ct}} \quad (43) \]

\[ f_{cR,u} = k_G \cdot 0.37 f_{cfk,1.2} \quad (44) \]

\[ k_G = 1.0 + 0.5 A_t' \leq 1.7 \quad (45) \]

\[ k = 1 + \sqrt{\frac{200 \text{mm}}{d}} \quad (46) \]

fib [18]

\[ V_{Rd} = V_{Rd,F} = \frac{C_{Rd,c}}{\gamma_c} k \left( 100 \rho \left( 1 + 7.5 \frac{f_{\text{Fuk}}}{f_{ck}} \right) f_{ck} \right)^{1/3} b_u d \quad (47) \]

\[ f_{ck} = \begin{cases} 0.3(f_{ck})^{2/3} & \text{for concrete grades } \leq \text{C50} \\ 2.12 \ln \left( 1 + 0.1(f_{ck} + 8 \text{MPa}) \right) & \text{for concrete grades } > \text{C50} \end{cases} \quad (48) \]

RILEM [36]

\[ V_{Rd} = V_{cd} + V_{fd} \quad (49) \]

\[ V_{cd} = 0.12k \left( 100 \rho f_{ck} \right)^{1/3} b_u d \quad (50) \]

\[ V_{fd} = 0.7k_f k_{\tau_{ld}} b_u d \quad (51) \]

\[ k_f = 1 + n \left( \frac{h_f}{b_u} \right) \left( \frac{h_f}{d} \right) \leq 1.5 \quad (52) \]

\[ n = \frac{b_f - b_u}{h_f} \leq 3 \text{ and } n \leq \frac{3b_u}{h_f} \quad (53) \]

\[ \tau_{fd} = 0.12 f_{Rk,4} \quad (54) \]
5.2 Expressions proposed in the literature

In the literature, a plethora of expressions for the shear capacity of SFRC elements failing in shear is present. Table 3 gives a selection of empirical formulas from the literature. The expression of Sarveghadi et al. [152] is developed as a simplification of an ANN-based expression in the form of a matrix. Other matrix-based expressions can be found in the literature [136, 146, 153-156]. Greenough and Nehdi [157] expression is a simplified version of an expression developed using a genetic algorithm. In this expression, \( \eta_o \) is the orientation factor [158], which can be taken as 0.41, but which can be larger in thin web members, and \( \tau \) is the fiber matrix interfacial bond strength, which can be estimated as 4.15 MPa [159, 160].

Dinh et al. [161] use an expression that combines the ultimate limit state for flexure with shear failure, and that ignores the contributions of dowel action and aggregate interlock. Shear failure is supposed to be triggered by crushing of the concrete above the neutral axis. As such, the equilibrium on the cross-section that results in the height of the compression zone \( c \) is at the ultimate limit state for flexure. An angle of 45° is assumed for the shear crack. The average tensile stress in the tension zone \( \sigma_{tu} \) should be based on ASTM 1609 tests [15].

The expressions by Kwak et al. [79], Li et al. [84], Eq. (78) and (79) by Ashour et al. [145], the expression by Narayanan and Darwish [8], and the expression by Arslan [5, 162] are written in the form of Zsutty’s empirical equation [163] for predicting the shear capacity of RC elements, including the dowel action through the coupling of \( \rho \) and \( a/d \). Narayanan and Darwish [8] neglected the contribution of the aggregate interlock capacity when they derived an expression for the shear capacity of SFRC. To find the height of the compression zone, Arslan uses the quadratic equation proposed by Zararis and Papadakis [164], which does not take the contribution of the steel fibers into account.
Kwak et al. [79]’s equation, Shin et al. [165]’s equation, Ashour et al. [145]’s equation, and Narayanan and Darwish [8, 166]’s equation use $v_b$ from Eq. (57).

The Imam et al. [37, 167] equation and the first set of expressions from Yakoub [168] are developed based on the expression by Bažant and Kim [125], which includes the size effect factor based on fracture mechanics considerations. These approaches do not follow the idea that the shear capacity can be determined by studying the shear-carrying mechanisms, but instead are based on the study of fracture mechanics of quasi-brittle materials. Yakoub introduces the geometry factor $R_g$ to replace the bond factor $\rho_f$. The geometry factor $R_g$ depends on the fiber type, and equals 0.83 for crimped fibers, 1.00 for hooked fibers, and 0.91 for round fibers. For other fiber types, the values of $R_g$ can be found in [168].

The expressions of Khuntia et al. [74], Mansur et al. [75], and Ashour et al. [145] were developed to be used with the ACI 318 building code [2], which is based on an empirically determined inclined cracking shear, and not on the contributions of the shear-carrying mechanism. Mansur et al. [75]’s formula uses the procedure of Swamy and Al-Ta’an [169] to determine $\sigma_{tu}$, which (see Eq. (73)) combines the fiber orientation factor $\eta_o$ from Romualdi and Mandel [158] ($\eta_o = 0.41$ for fibers with a 3D random orientation), the fiber length correction factor $\eta_l$ from Cox [170], the fiber spacing from Swamy et al. [160], and the bond stress $\tau$ proposed by Swamy and Mangat [171]. Sharma’s equation [172] is given in a similar format as the shear expression from ACI 318, and uses the relation of Wright [173] to link the concrete compressive strength to the tensile strength of concrete. The second set of equations by Yakoub [168] (Eqs. (97) - (101)) are a modification of the Canadian standard CSA A23.3-04 [174] that includes the contribution of fibers to the shear capacity.
Table 3. Selection of empirical formulas from literature to determine shear capacity of SFRC without stirrups and without prestressing or axial load.

<table>
<thead>
<tr>
<th>Authors</th>
<th>Ref</th>
<th>Expression</th>
<th>Eq.</th>
</tr>
</thead>
</table>
| Sarvegha di et al.     | [152]     | \[
V_u = \rho + \frac{\rho}{v_b} + \frac{1}{a} \left( \frac{a}{d} \left( f'_t \rho (\rho + 2) \left( f'_t - \frac{3}{\nu_b} \right) + f'_t \right) \right) + v_b \] b_u d  
\]
                                                                                                                                                                                                 | (55) |
|                        |           | \[
\nu_b = 0.79 f'_c  
\]
                                                                                                                                                                                                 | (56) |
|                        |           | \[
\nu_b = 0.41\tau F \text{ with } \tau = 4.15 \text{ MPa}  
\]
                                                                                                                                                                                                 | (57) |
| Kwak et al.            | [79]      | \[
V_u = \left[ 3.7f'_{spfc} \left( \rho \frac{d}{a} \right)^{\frac{2}{3}} + 0.8v_b \right] b_u d  
\]
                                                                                                                                                                                                 | (58) |
|                        |           | \[
f'_{spfc} = \frac{f_{cuf}}{(20 - \sqrt{F})} + 0.7 + 1.0\sqrt{F} \text{ in MPa}  
\]
                                                                                                                                                                                                 | (59) |
|                        |           | \[
e = \left\{ \begin{array}{ll}
1 & \text{for } \frac{a}{d} > 3.4 \\
3.4 & \text{for } \frac{a}{d} \leq 3.4
\end{array} \right.  
\]
                                                                                                                                                                                                 | (60) |
| Greenough and Nehdi    | [157]     | \[
V_u = \left[ 0.35 \left( 1 + \sqrt{\frac{400}{d}} \right) (f'_c) ^{0.18} \left( 1 + F \right) \rho \frac{d}{a} ^{0.4} + 0.9\eta_s\tau F \right] b_u d  
\]
                                                                                                                                                                                                 | (61) |
| Khuntia et al.         | [74]      | \[
V_u = \left[ (0.167 + 0.25F) f'_c \right] b_u d  
\]
                                                                                                                                                                                                 | (62) |
| Li et al.              | [84]      | \[
V_u = \left[ 1.25 + 4.68 \left( f'_{RF} f'_{spfc} \right) ^{\frac{1}{4}} \left( \rho \frac{d}{a} ^{\frac{1}{3}} d^{-\frac{1}{3}} \right) \right] b_u d \text{ for } \frac{a}{d} \geq 2.5  
\]
<pre><code>                                                                                                                                                                                             | (63) |
</code></pre>
<table>
<thead>
<tr>
<th>Authors</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shin et al.</td>
<td>$V_u = \left[ 0.19 f_{ypc} + 93 \rho \frac{d}{a} + 0.834 v \right] b_u d$ for $\frac{a}{d} \geq 3$ (65)</td>
</tr>
<tr>
<td></td>
<td>$V_u = \left[ 0.22 f_{ypc} + 217 \rho \frac{d}{a} + 0.834 v \right] b_u d$ for $\frac{a}{d} &lt; 3$ (66)</td>
</tr>
<tr>
<td>Imam et al.</td>
<td>$V_u = 0.6\psi \sqrt{\omega \left( f_{yc} \right)^{0.44} + 275 \sqrt{\omega \left( \frac{a}{d} \right)^3}} b_u d$ (67)</td>
</tr>
<tr>
<td></td>
<td>$\psi = 1 + \frac{\sqrt{5.08}}{d_u} \sqrt{1 + \frac{d}{25d_u}}$ (68)</td>
</tr>
<tr>
<td></td>
<td>$\omega = \rho(1+4F)$ (69)</td>
</tr>
<tr>
<td>Sharma</td>
<td>$V_u = \left( \frac{2}{3} \times 0.8 \sqrt{f_{yc} \left( \frac{d}{a} \right)^{0.25}} \right) b_u d$ (70)</td>
</tr>
<tr>
<td>Mansur et al.</td>
<td>$V_u = V_c + \sigma_u b_u d$ (71)</td>
</tr>
<tr>
<td></td>
<td>$V_c = \left( 0.16 \sqrt{f_{yc} + 17.2 \frac{\rho Vd}{M}} \right) b_u d \leq 0.29 \sqrt{f_{yc}} b_u d$ (72)</td>
</tr>
<tr>
<td></td>
<td>$\sigma_u = 3.2\eta_1 \eta_2 F \tau$ with $\tau = 2.58$MPa (73)</td>
</tr>
<tr>
<td></td>
<td>$\eta_1 = 1 - \frac{\tanh \left( \beta \frac{L}{2} \right)}{\beta \frac{L}{2}}$ (74)</td>
</tr>
</tbody>
</table>
\[
\beta = \frac{2\pi G_m}{\sqrt{E_f A_f \ln \left(\frac{S}{r_f}\right)}}
\]

\[S = 25 \frac{d_f}{V_f L_f}\]

<table>
<thead>
<tr>
<th>Reference</th>
<th>Equation</th>
</tr>
</thead>
</table>
| Ashour et al. [145] | \[
V_u = \left[\left(0.7\sqrt{f_{f_c}} + 7F\right)\frac{d}{a} + 17.2\rho\frac{d}{a}\right]b_u d
\]
| | \[
V_s = \left[2.11\sqrt{f_{f_c}} + 7F\right]\left(\rho\frac{d}{a}\right)^{0.333} b_s d \text{ for } \frac{a}{d} \geq 2.5
\]
| | \[
V_s = \left[2.11\sqrt{f_{f_c}} + 7F\right]\left(\rho\frac{d}{a}\right)^{0.333} \left(\frac{25}{a} + v_h \left(2.5-a\right)\frac{a}{d}\right) b_s d \text{ for } \frac{a}{d} < 2.5
\]
| Narayan and Darwish [8] | \[
V_u = \left[e \left(0.24 f_{spfc} + 80\rho\frac{d}{a}\right) + v_h\right] b_u d
\]
| | \[
e = \begin{cases} 1 & \text{for } \frac{a}{d} > 2.8 \\ 2.8 \frac{d}{a} & \text{for } \frac{a}{d} \leq 2.8 \end{cases}
\]
| Lee et al. [28] | \[
V_{ci,req} \leq V_{ci,cap} \text{ and } V_{ce,req} \leq V_{ce,cap}
\]
| | \[
V_{ci,req} = \frac{\Delta f_{ss} \rho}{2} b_w \left(d - c_{simp}\right)
\]
| | \[
\frac{M_i}{d - c_{simp}} - f_{sf} b_w \left(d - c_{simp}\right)
\]
| | \[0 \leq \Delta f_{ss} = \frac{3}{A_y} \leq f_y
\]
| | \[f_{sf} = 0.772 F \text{ (MPa)}
\]
\[ V_{ci,\text{cap}} = \frac{0.18A\sqrt{f_c'}}{0.31 + 0.686 \eta_c w_f} b_w (d-c_{\text{simp}}) \] (87)

\[ \eta_{ci} = \frac{0.47d}{h-d} \] (88)

\[ w_f = \varepsilon_u S_{m_x} = \frac{\Delta f_{sx}}{E_s} \times 3(h-d) \] (89)

\[ V_{cc,\text{req}} = V_{\text{tot}} - V_{ci,\text{req}} \] (90)

\[ V_{cc,\text{cap}} = 0.52 \sqrt{f_c'} \times b_w c_{\text{simp}} \]

<table>
<thead>
<tr>
<th>Author</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dinh et al [161]</td>
<td>[ V_u = 0.13A \sigma_f + \sigma_u b(d-c) ] (91)</td>
</tr>
<tr>
<td>Arslan et al [5, 162]</td>
<td>[ V_u = \left[ \left( 0.2 \left( f_c' \right)^{2/3} \frac{c}{d} + \sqrt{\rho (1+4F) f_c'} \right) \frac{3}{a} \right] b_w d ] (92)</td>
</tr>
<tr>
<td>Yakoub [168]</td>
<td>[ V_u = \left[ 0.83\delta \sqrt{f_c'} + 249.28 \left( \frac{\rho}{a} \right) + 0.405 \frac{L_d}{V_f R_a} \frac{d}{a} \sqrt{f_c'} \right] b_w d \text{ for } \frac{a}{d} \leq 2.5 ] (94)</td>
</tr>
</tbody>
</table>

\[ \left( \frac{c}{d} \right)^2 + \left( \frac{600 \rho}{f_c'} \right) \left( \frac{c}{d} \right) - 600 \rho = 0 \] (93)

\[ V_u = \left[ 0.83\delta \sqrt{f_c'} + 249.28 \left( \frac{\rho}{a} \right) + 0.162 \frac{L_d}{V_f R_a} \frac{d}{a} \sqrt{f_c'} \right] b_w d \text{ for } \frac{a}{d} \geq 2.5 \] (95)
\[
\xi = \frac{1}{\sqrt{1 + \frac{d}{25d_n}}}
\]

\[
V_s = 2.5 \left( \frac{0.40}{1+1500\varepsilon_e} \times \frac{1300}{1000+s_w} \right) \sqrt{f'_c} \left( 1 + 0.7 \frac{L_f}{d_f} V_f R_s \right) \frac{d}{a} b_a d_v \text{ for } \frac{a}{d} \leq 2.5
\]

\[
V_s = \left( \frac{0.40}{1+1500\varepsilon_e} \times \frac{1300}{1000+s_w} \right) \sqrt{f'_c} \left( 1 + 0.7 \frac{L_f}{d_f} V_f R_s \right) b_a d_v \text{ for } \frac{a}{d} \geq 2.5
\]

\[
d_v = \max \left( 0.9d, 0.72h \right)
\]

\[
\frac{M}{d_v} + V
\]

\[
\varepsilon_s = \frac{2E_s A_s}{d_v}
\]

\[
s_{x,t} = \frac{35s}{16 + d_n} \geq 0.85s_x \text{ and } s_x \approx d_v
\]

### 5.3 Mechanical models

Models that are based on mechanics are also available. Lee et al. [28] developed the dual potential capacity model, which studies the capacity and demand on the tension side and the compression side of the cross-section, neglecting aggregate interlock and dowel action. In Table 3, the simplified, closed-form solution of the model is presented. The simplified model is based on the assumption of a shear crack under 45°, on the simplification of the calculation of the height of the compression zone as \(c_{simp}\) (with \(c_{simp} = 0.3d\) for rectangular sections, \(c_{simp} = 0.2d\) for flanged sections, and \(c_{simp} = 0.5d\) near end supports), and the relation between \(f_{sf}\) and \(F\) was taken from [175] (with \(V_f\) as a value between 0 and 1, not a percentage).
Additionally, a number of proposals for extending the Modified Compression Field Theory (MCFT) to include the effect of steel fibers are available in the literature. The MCFT uses smeared cracking, which is also used in the proposals to extend the MCFT for SFRC. One proposal [103] included the effect of fibers on the constitutive models for cracked concrete, resulting in cracked SFRC as a new material. The outcome of this analysis is general expressions for the fiber contribution to the stress transfer across the leading crack for the ultimate and post-ultimate response. Another proposal [4] relates the tensile stress in a steel fiber at a crack with the average tensile stress, which can then be used to determine the strain profiles, including net strain and slip, in the SFRC member. The stresses can then be found using the constitutive equations. This proposal has been programmed into the VecTor2 software and has been compared to the more general approach of the Disturbed Stress Field Model (DSFM) [176, 177].

Other researchers used panel tests [178] to derive the constitutive equations [179] of SFRC [180] for using with the MCFT. The substantial tension-softening that was observed in the panel tests is reflected by the proposed constitutive equations. An approach at the meso-level that studies the rotation of each fiber separately has also been proposed in the literature [181], and a closed-form solution based on this method has been proposed as well [182]. For practicing engineers, the engineering model [183] based on the MCFT and proposed for inclusion in the fib Model Code [18] is the most practical way to use the MCFT for SFRC concrete. In a more general approach, Hwang et al. [184] included the contribution of steel fibers to different softened truss models, and then proposed a model STM-SF (softened truss model with steel fibers).

Finally, a number of plasticity-based approaches are available. A simple modification of the lower-bound plasticity-based model for an RC beam in shear has resulted in a practical method to predict the shear capacity of a SFRC beam [185], which resulted in good and
conservative predictions when compared to a limited number of experiments. A more extensive
approach for using the concepts of the theory of plasticity to the shear capacity of SFRC is
available as well [186]. In recent years, not many approaches based on the theory of plasticity
have been developed. The exception is Spinella’s method [187], in which the crack sliding model
is extended to take into account the arching effect for deep beams, the post-cracking tensile
strength of steel fiber reinforced concrete and its ability to control sliding along shear cracks, and
the influence of steel fibers on the size effect in shear.

6 Discussion

6.1 Can we separate the influence of the steel fibers?

Most theories contribute the influence of the steel fibers only to the shear-resisting
mechanism of the tension across the crack. For computational purposes, this approach may be
attractive, but it does not represent the complex mechanics of how SFRC elements resist shear.
Foster et al. [182] showed that the concrete and fiber are coupled and they advise against the
separation of terms. The components are tied through the critical crack width.

Similarly, [188] showed that the effect of adding fibers is different for beams with and
without stirrups. This observation indicates that it is not correct to simply add the contributions
of the concrete, stirrups, and fibers to find the total shear capacity, but that these mechanisms are
interrelated. These authors also comment on the finer crack distribution as the fiber volume
fraction increases. Indeed, it seems necessary to study the effect of crack spacing and width to be
able to evaluate the contribution of the different shear-carrying mechanisms.
6.2 Towards mechanical models

At the moment, a number of mechanical models that were developed for shear in RC beams without stirrups have been extended to include the effect of fibers. As discussed in section 5.3, a number of researchers proposed how to include the effect of fibers into the MCFT [189], either by replacing the uncracked and cracked concrete materials in the MCFT with uncracked and cracked SFRC [103], based on an assumption of smeared cracking [4, 176, 177], or by deriving new constitutional equations based on panel tests of SFRC [178-180]. The MCFT lies at the basis of the shear expressions in the fib Model Code 2010 [18]. Proposals to improve the fib Model Code expressions for SFRC have been developed [181, 183]. A more general softened truss model that includes fibers has also been proposed [184]. Other researchers have developed proposals for predicting the shear capacity of SFRC beams without stirrups based on the theory of plasticity, see [185-187].

At the moment, none of the existing (mechanical) models that are proposed for SFRC has managed to capture the full behavior and mechanics of the problem under study.

6.3 Research needs

A mechanical model of a SFRC beam without stirrups failing in shear should combine all shear-carrying mechanisms: the capacity of the concrete in the compression zone, the aggregate interlock, the dowel action, the tension across the crack carried by the fibers, and the arching action (for members with a small shear span to depth ratio). The influence of fibers on each of these mechanism is as follows. For the contribution of the compression zone, the addition of fibers to the cross-section changes the horizontal equilibrium in the cross-section, and thus the height of the compression zone. The tension across the crack carried by the fibers has been studied extensively in the past, as indicated in section 3.3. The effect of fibers on the aggregate
interlock should be studied through the influence of the fibers on the crack width and spacing. The influence of fibers on dowel action should be studied as a function of the increased tensile strength of SFRC, resulting in the potential for the development of a larger dowel action force. Finally, it should be investigated how fibers improve arching action (as indicated by preliminary experiments).

Then, the relation between these different mechanisms should be studied, to develop a full description of the behavior of SFRC beams without stirrups failing in shear. The first need for research is to understand the behavior of SFRC elements without shear reinforcement, the influence of fibers on the different shear-carrying mechanisms, and the relation between the shear-carrying mechanisms in SFRC. Experimental studies in which these mechanisms can be quantified, for example by analyzing crack kinematics with photogrammetry [113, 190], are necessary to address this research need.

The second research need is more practical in nature: how much fibers should be added to a concrete mix for a certain structural element so that the fibers act as shear reinforcement instead of providing steel stirrups? Experimental results [116] showed that providing 0.75% of fibers fulfills the ACI 318-14 [2] code requirements for minimum shear reinforcement, since the shear capacity of the SFRC beams was more than 1.5 time the shear capacity of the RC beam without stirrups. However, further testing is necessary to address this issue. Moreover, to answer this question, the first research need should be tackled as well, so that one can understand what happens in the cross-section to apply steel fibers on large scale in structural elements. For this purpose, construction codes and standards should include the contribution of fibers to the shear capacity in their provisions.
7 Summary and conclusions

Adding steel fibers to concrete alters some of its mechanical properties: the tensile strength is improved, crack width and crack spacing are reduced, fatigue life is generally improved, but the concrete compressive strength not influenced. As such, in a sectional analysis, the influence of the fibers can be studied by adding a stress block (or equivalent) to the tension side of the concrete, and the flexural capacity can be determined.

When it comes to the shear capacity of steel fiber reinforced concrete (SFRC) members without shear reinforcement, the influence of the fibers should be studied based on the different shear-carrying mechanisms. The larger compression zone results in a larger shear capacity of the concrete compression zone in SFRC as compared to RC. The fibers carry tension across the crack, which provides shear resistance. Dowel action is improved through the improved concrete-reinforcement bond and through the larger resistance of the cover against splitting. The effect of fibers on aggregate interlock should be studied through the crack width and spacing in SFRC. Arching action seems to be improved by the inclusion of fibers, but requires further study. Then, the relation between these mechanisms should be studied to predict the ultimate shear strength.

For the parameters that influence the shear strength of SFRC, the results in the literature on the influence of fibers on the effect of the concrete compressive strength, amount of flexural steel, member height, and fiber properties are inconclusive. A mechanical model based on the different shear-carrying mechanisms could be a tool to understand these seemingly contradictory results reported in the literature. The effect of the shear span to depth ratio on the shear capacity is influenced by the amount of fibers, and the resulting valley of shear failures versus flexural failures becomes smaller as the fiber volume fraction increases.
A number of existing equations are based on the combination of a concrete contribution and a contribution of the steel fibers to the shear capacity. When the concrete contribution is based on an empirically determined inclined cracking load and the contribution of the steel fibers on the capacity of fibers under tension crossing the crack, the resulting equations are mix concepts that mechanically do not work together. Again, this observation shows the need for a mechanics-based model that includes the influence of fibers on all shear-carrying mechanisms. In parallel to this required effort, code provisions that include the beneficial effect of fibers and that address how fibers can be used as shear reinforcement should be developed and included in the building codes, so that SFRC can be implemented on a larger scale in practice for cases where the use of SFRC reduces the life-cycle cost of a structure.

Notation List

The following symbols are used in this paper:

- $a$: shear span
- $b$: width of section
- $b_f$: width of flange
- $b_w$: web width when the web is in tension, otherwise the smallest width of the cross-section in the tension zone
- $c$: height of compression zone
- $c_{simp}$: simplified value of the height of the compression zone
- $d$: effective depth
- $d_a$: maximum size of the aggregate
- $d_v$: shear depth
- $d_f$: fiber diameter
1. $e$  
   effect of shear span to depth ratio

2. $f$  
   average pullout force per fiber

3. $f_{1.5}$  
   average residual tensile strength at 1.5 mm

4. $f_c$  
   concrete compressive strength

5. $f_c'$  
   specified concrete compressive strength

6. $f_{cfrk, t.2}$  
   characteristic value of post-cracking flexural strength for a deflection of 3.5 mm

7. $f_{ck}$  
   characteristic concrete compressive strength

8. $f_{ck'}$  
   characteristic value of the axial tensile strength of concrete matrix

9. $f_{fcm}$  
   mean flexural tensile strength of fiber reinforced concrete

10. $f_{ctR u}$  
    uniaxial tensile strength of SFRC

11. $f_{cu'}$  
    design cube compressive strength of concrete

12. $f_{cu'}$  
    cube compressive strength of fiber reinforced concrete

13. $f_{tcm}$  
    average fiber concrete compressive strength

14. $f_{flF}$  
    flexural strength of fiber reinforced concrete

15. $f_{Ftuk}$  
    characteristic value of post-cracking strength for ultimate crack opening

16. $f_y$  
    yield strength of steel of fiber

17. $f_{R,1}$  
    stress at CMOD of 0.5 mm

18. $f_{R,4}$  
    stress at CMOD of 3.5 mm

19. $f_{Rk,4}$  
    characteristic residual flexural strength for the ultimate limit state at a CMOD of 3.5 mm

20. $f_{sf}$  
    local stress increase in steel fibers in crack

21. $f_{spfc}$  
    splitting tensile strength of fiber reinforced concrete

22. $f_{t'}$  
    specified splitting tensile strength of concrete

23. $f_y$  
    yield strength of steel
1 $h$  height  
2 $h_f$  height of flange  
3 $k$  size effect factor  
4 $k_f$  factor that considers the contribution of flanges in T-sections (=1 for rectangular sections)  
5 $k_F^f$  factor that considers the orientation of the fibers  
6 $k_G^f$  size factor, which accounts for the fact that fibers are better distributed in larger elements  
7 $k_h$  size effect factor  
8 $k_{\theta}$  factor that takes into account the inclination of the compression struts according to the Modified Compression field theory [189]  
9 $n$  parameter for effect of geometry of flanged sections  
10 $r_f$  fiber radius  
11 $s_x$  crack spacing parameter  
12 $s_{xe}$  equivalent crack spacing factor  
13 $v_b$  fiber contribution to shear strength  
14 $v_u$  ultimate shear strength  
15 $w$  crack width  
16 $w_f$  crack width at the level of the tension reinforcement  
17 $w_{lim}$  limiting crack width  
18 $w_{max}$  maximum crack width permitted by the code  
19 $w_u$  ultimate crack width, i.e. the value attained at the ULS for resistance to combined stresses on the outer fiber under the moment exerted in this section  
20 $z$  internal lever arm  
21 $A_{ct}$  effective area $b_w \times d$, with $d$ limited to 1.5 m
1. $A_f$ cross-sectional area of the steel fiber
2. $A_s$ amount of longitudinal tension reinforcement
3. $A_{ef}$ area of fiber effect
4. $C_c$ resultant of concrete under compression
5. $C_{Rd,c}$ calibration factor used in the shear formula of Eurocode 2
6. $E_c$ Young’s modulus of the concrete
7. $E_f$ fiber modulus of elasticity
8. $E_s$ Young’s modulus of reinforcement steel
9. $F$ fiber factor
10. $G_m$ matrix shear modulus
11. $K$ orientation coefficient
12. $L_f$ fiber length
13. $L_{f,straight}$ length of the straight portion of the fiber
14. $M$ acting bending moment
15. $M_{fl}$ bending moment capacity
16. $M_i$ external moment
17. $N$ number of fibers crossing a unit area of the inclined crack
18. $R_g$ geometry factor
19. $R_{sup}$ support reaction
20. $S$ fiber spacing
21. $S_{mx}$ crack spacing
22. $T_c$ resultant of concrete under tension
23. $T_{f,d}$ resultant of fiber tension along length of diagonal crack
1. $T_s$ resultant of steel under tension
2. $V$ acting shear force in cross-section
3. $V_a$ shear resistance from aggregate interlock
4. $V_{ax}$ projection on $x$-direction of aggregate interlock resultant
5. $V_{ay}$ projection on $y$-direction of aggregate interlock resultant
6. $V_c$ concrete contribution to shear capacity
7. $V_{cc,cap}$ capacity in the compression zone of the section
8. $V_{cc,req}$ demand on the compression zone of the section
9. $V_{cd}$ design value of concrete contribution to shear capacity
10. $V_{ci,cap}$ capacity in the tension zone of the section
11. $V_{ci,req}$ demand on the tension zone of the section
12. $V_{cz}$ shear resistance provided by the concrete compression zone
13. $V_d$ shear resistance from dowel action
14. $V_f$ fiber volume fraction
15. $V_{fd}$ design value of contribution of fibers to shear capacity
16. $V_F$ shear capacity from fibers
17. $V_{Rd}$ design shear capacity
18. $V_{Rd,c}$ concrete contribution to design shear capacity
19. $V_{Rd,c,min}$ lower bound of the shear capacity
20. $V_{Rd,f}$ fiber contribution to design shear capacity, denomination in French code and fib Model Code
21. $V_{Rd,cf}$ fiber contribution to design shear capacity, denomination in German code
22. $V_{Rd,fc}$ design shear capacity of fiber reinforced concrete

-50-
1 \( V_{tot} \)  total shear on cross-section
2 \( V_u \)  ultimate shear strength
3 \( \alpha \)  inclination of the critical diagonal crack
4 \( \alpha_A \)  factor that takes the random orientation of the fibers into account
5 \( \alpha_c' \)  factor that accounts for the long term effects
6 \( \beta \)  fiber and matrix property factor developed by Cox [170]
7 \( \gamma_{cf} \)  concrete material factor, notation used in the French guideline
8 \( \gamma_c \)  concrete material factor, notation used in the German guideline
9 \( \gamma_{\alpha f} \)  partial factor for tensile strength of fiber reinforced concrete
10 \( \gamma_E \)  additional safety factor
11 \( \delta \)  crack slip
12 \( \varepsilon_1 \)  strain at maximum tensile stress of SFRC
13 \( \varepsilon_2 \)  strain at \( \sigma_2 \)
14 \( \varepsilon_3 \)  25\%, last point on stress-strain diagram of SFRC
15 \( \varepsilon_c \)  strain in concrete
16 \( \varepsilon_{cu} \)  ultimate strain in concrete
17 \( \varepsilon_{el} \)  elastic strain
18 \( \varepsilon_{lim} \)  limiting strain
19 \( \varepsilon_{\max} \)  maximum strain
20 \( \varepsilon_s \)  strain in steel
21 \( \varepsilon_{si} \)  strain in steel due to moment \( M_i \)
22 \( \varepsilon_x \)  strain at mid-height in the cross-section
ε_u  ultimate strain at the ULS for bending combined with axial forces on the outer fiber

η  factor that takes into account effect of fibers on bending moment capacity

η_o  fiber orientation factor

η_cr  crack concentration factor

η_l  a length factor used to account for the variability in the fiber embedment length across the cracking plane

κ  crack control coefficient

λ  lightweight factor

ξ  size effect factor from Bažant and Kim [125]

ρ  reinforcement ratio

ρ_f  bond factor: 0.5 for straight fibers, 0.75 for crimped fibers, 1 for hooked fibers

σ_1  maximum tensile stress in SFRC

σ_2  post-peak tensile stress in SFRC

σ_3  tensile stress in SFRC for a strain of 25‰

σ_c  stress in fiber reinforced concrete

σ_f  stress in fibers

σ_{fu}  tensile stress resisted by fiber bridging a unit area of an inclined crack

σ_{f(ε)}  experimentally determined relation between stress in fiber concrete and strain

σ_{f(w)}  experimentally determined relation between post-cracking stress and crack width w

σ_{f,cr}  shear resistance provided by fibers in tension across the crack

σ_{Rd,f}  residual tensile strength of fiber reinforced cross-section
average stress at the ultimate limit state in the equivalent tensile stress block used for bending moment analysis of SFRC
bond capacity of fiber-matrix interface
expression for shear capacity of fiber concrete for concrete class C30 and aspect ratio 65
design bond capacity of fiber-matrix interface
size effect factor from Imam et al. [37]
reinforcement ratio that includes the effect of fibers
angle of compression strut
local stress increase in the tension reinforcement
mechanical contribution of hook of fiber to the pull-out strength of the fiber

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