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MODE INTERACTION WITH STIFFENED PANELS

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Summary

Previous work on mode interaction refers to a column model consisting of two equal flanges. This model is extremely sensitive to imperfection. Actual panel structures are unsymmetrical: the plate side has more cross section than the topside of the stiffeners; and the plate side is more affected by local buckling than the topside. This effect is exaggeratedly represented in a model where the stiffeners do not participate in local buckling. It appears that the sensitivity to imperfections of this model is very little and restricted to geometric parameters R in the near vicinity of $R = 1$. These two models represent extreme cases. The position of actual top-hat stiffened panels between these extremes is being explored. It appears that mode interaction is less severe than with the two flange model though still significant.

Symbols

K	compressive force
L	half of column length
P	compressive force of a plate strip
R	geometric parameter; K_E/K_λ
W	deflection of the axis of the structure
b	width of plate strip
j	radius of gyration of the cross section
λ	half wave length in local buckling
w	deflection of plate strip
α	amplitude of plate imperfection: plate thickness
β	amplitude of column imperfection: j
ϵ	compressive strain at edge of strip
η	stiffness reduction factor
λ	half wave length: width of strip

Subscripts:

E	refers to Euler buckling load
b	refers to overall buckling load
λ	refers to local buckling load.

1. Introduction surveying the problem

Thin-walled struts and panels are subject to local and to overall buckling. The usual conception that the structure where these buckling stresses are equal is optimal was attacked already in 1962 by Koiter and Skaloud [1]. They pointed at the possibility of unfavourable effect of mode interaction. This observation became confirmed by experimental evidence when panels failed explosively at stresses smaller than both the local and the Euler buckling stress. A first attempt to analyse this problem related to a simplified strut consisting of two equal load carrying flanges interconnected by webs without axial stiffness [2, 3, 4]. It demonstrates, that the equilibrium at the local buckling load K_λ

is instable over a range of the geometrical parameter R close to unity, R being the ratio of Euler buckling load K_E and K_L ; that consequently the structure is imperfection sensitive. Further the strength reductions due to the imperfections: initial waviness of the flanges and initial curvature of the strut were established. Using these results Thompson and Lewis [5] have shown that the effect of initial waviness moves the optimum from $R = 1$ to $R < 1$, eroding it more and more with increasing imperfection.

The two-flange model presents an extreme case of mode interaction, because the flanges are strongly and equally affected by local buckling and because the structure contains no components which temper the flange behaviour. The final object of this investigation is to establish the significance of mode interaction for heavily stiffened panels as used in aircraft wing structure. Here the cross section at the topside of the stringers is much smaller than the cross section at the plateside. Also the b/h -ratios of the strips at the two sides are quite different. This lack of symmetry of the two sides can have a major effect on the sensitivity to mode interaction.

A useful first reconnaissance in this area would be the overshoot of exaggerating the dissymmetry. A pronounced case of this type is the thin plate reinforced by stiffeners which do not participate in local buckling of the plate. It represents the lower extreme of the phenomenon of mode interaction in contrast to the two-flange model which is representative for the upper extreme. Tvergaard's work [6,7] falls in this area. He investigated the integrally stiffened plate where the participation of the stiffeners in local buckling is confined to torsion. An important feature is that the compressive load is applied in the centroid of the cross section. However with unsymmetric structures, subject to local buckling the neutral plane in bending moves away from the side most affected by local buckling. The load in the centroid then causes bending. This exaggerates the strength reduction in comparison to the situation with a multi-bay panel.

So as to avoid bending below the buckling load the boundary conditions will be that the panel is rigidly restrained at its ends. Moreover these boundary conditions are realistic. In actual structures panels comprise a number of bays separated by supporting frames. The part of the structure between the centres of the first and the third bay corresponds to the panel clamped at its ends. In two consecutive bays the buckling mode yields increased compression at the plateside of one bay and decreased strain at the plate side of the next one. Due to the nonlinearity of the load-strain relation the bending stiffness of the first bay decreases but it increases in the adjacent one, thereby tempering the destabilisation of the combination of the bays. This concept leads to the conclusion that compression tests should be carried out on clamped structures when non-linear behaviour occurs.

It is to be expected that the model representing the lower extreme of mode interaction, will behave very mildly as to mode interaction.

Having considered these extreme cases the next problem is to locate between these extremes the position of panels where the stiffener is thin-walled and participates in local buckling.

The local buckling mode depends on quite a number of geometric parameters, such as the ratios of the widths of the strips composing the cross section and the ratios of the strip thickness. The coherence of the several strips requires equal longitudinal wave length of all strips, therefore different ratios of wave length and strip width. Equality of buckling stresses means that the buckling coefficients of the strips will differ from each other. Equality of wave length and buckling stress is being achieved by bending moments occurring between the strips at their junction. These moments can have a stabilizing effect (positive elastic restraint) or a destabilizing effect (negative elastic restraint). The solution of the mode interaction problem requires the determination of the local buckling mode and of K_L . Methods are available.

Next comes the post-buckling behaviour of the perfect structure and the behaviour of imperfect structures. This is a very complex problem.

Let be assumed that the stiffnesses of edge restraints remain constant with increasing edge strain. Then the problem is to establish the stiffness in compression as a function of the edge strain of a strip for arbitrary wave length and arbitrary edge restraint, whereas these restraints at the two edges can have any ratio, positive or negative. This is a tremendous problem though its solution does not even solve the problem of panel behaviour under compression without bending, where the strains at all edges are equal. The cause of this unsufficiency is that under the assumption of constant edge restraint the increase of edge rotations with increased compression will be different for adjoining strips. The more so with arbitrary imperfections of the individual strips. So in fact the stiffness of the edge restraints must vary during loading so as to yield compatible edge rotations.

The need to establish the bending stiffness of the panel introduces an additional problem. Plate strips not parallel to the panel plane obtain unequal edge strains due to bending. The position of the neutral plane is unknown and therefore the ratio of the edge strains. Consequently the behaviour of plates with arbitrary ratio of edge strains, arbitrary ratio of wave length to plate width, arbitrary ratio of edge restraints and arbitrary amount of edge restraint would have to be investigated. These ingredients are required to solve the mode interaction problem of this type of stiffened panel. If they were available the remaining interaction problem would not be too cumbersome. The capital problem is the one on strip behaviour just defined.

Since ignorance of the amount of the imperfections of actual structures excludes the possibility of theoretical prediction of the strength of these structures exact solution of the problem seems to have little practical value and would be too ambitious. On the other hand it serves a real need to explore the extent of the zone of R in which interaction presents reduction of strength and the order of magnitude of the reduction in the affected zone.

Therefore the logical step after having considered the two "extreme" models is to consider a model which is not identical to but representative of real panel structures. The problem of the behaviour of its composing plate strips should not be too complex.

2. Local buckling confined to one side of the panel

Panels with large stiffener spacing have local buckling stresses much lower than the overall buckling stress; R is far above unity. Then K_b is not affected by imperfections and can be established with the Euler formula thereby taking into account the effective modulus of the plate. Unfavourable mode interaction occurs only when R is close to unity. This means heavily reinforced panels, where stringer and plate sections do not differ much. As a representative structure has been taken the case where stringer and plate section are equal. The results obtained with ratios between $\frac{1}{3}$ and 3 are only slightly different.

The method of solution is almost a special case of the analysis given in section 6.

Fig. 1 shows the $K_b/K_0 - R$ - curve for the perfect plate and for one with the imperfection $\alpha = 0,05$. With the perfect structure the reduction factor of the bending stiffness for $K_b > K_0$ is 0,8. This reduces the transition zone at $K_b = K_0$ to $1 < R < 1,25$. The equilibrium at $K_b = K_0$ is instable over the narrow range $1,0 < R < 1,1$. These upper limits are very low in comparison to those of the two-flange model, where they are 2,45 and 1,725 resp.

With the imperfection $\alpha = 0,05$ K_b is smaller than K_0 up to $R=1,15$. The maximal reduction at $R = 1$ is 9%, whereas the two-flange model with $\alpha = 0,05$ has 17%.

Fig. 2 depicts the load-strain curve together with the slopes of the load-shortening curves at the bifurcation $K = K_b$. The equilibrium at K_b is unstable only for $R < 1,01$. The steepest tangent at $R = 0,85$ is much milder than with the two-flange model.

Mode interaction appears to be confined to a narrow region in the immediate vicinity

of $R=1$. The strength there is slightly below K_ℓ .

3. Simplified panel model

The selected structure is a panel stiffened by tophat stringers (fig. 3). The ratio of strip width and thickness is arbitrary. The cross sections of plate and stringers have equal area. The local buckling strain has been established by means of a straightforward method [8] as

$$\epsilon_\ell = 1,340 \frac{\pi^2}{12(1-\nu^2)} \frac{h^2}{b^2}.$$

The half wave length of the mode is $1,90 b$. The mode shape and the edge rotations are shown in fig. 3. The joints of plate and stringer have been assumed in the intersection of the plate and the side strips of the stringer, whereas in real structure the rivet joint will be somewhere near the centre of the flanges. Choosing the joint along the rivet row would involve that the edge deflection of the plate strips would be non-zero, which would complicate the problem quite seriously. Another difficulty would be to deal with the flange loaded along the joint. However under these realistic conditions the deflections at the intersection of plate and side strips would be close to zero, as can be conjectured by inspection of fig. 3. Further considering that the flanges contain only 14% of the cross section a rude approximation of their behaviour seems acceptable.

The kind of restraint offered by the edges of the strips appears from the curvature of the mode. Plate strip 2a very clearly obtains support from plate-strip 2b and stringer strip 4; these latter then having negative restraint. The upper edge of strip 4 is being restrained by strip 1. The figure also mentions the coefficients of restraint C defined by:

$$\frac{\text{edge moment}}{\text{edge rotation}} = - C \frac{\text{bending stiffness of plate}}{\text{plate width}}.$$

With strip 4 C_s and C_a refer to the symmetrical and anti-symmetrical part of the mode. The figures indicate that the wide plate strip 2a, being the weakest part of the structure, gets very stiff edge restraint from the adjacent strips.

The model selected for the analysis of mode interaction is a simplified version of the basic structure because of various reasons. Only those strips are represented which have symmetrical buckling mode: strips nrs. 1, 2a and 2b. The flanges nr. 3 can be omitted since their contribution to the bending stiffness of the panel is very small compared to that of the plate. Also the axial load carried by the stringer sides nr. 4 is being neglected so as to avoid serious complications arising from unequal restraint stiffness at upper and lower edge and unequal edge strains with panel bending. The function of the side strips to maintain the integrity of the assembly is being preserved. These simplifications affect the radius of gyration j only slightly, it increases from $0,377b$ to $0,394b$.

In this investigation the emphasis falls on the effect of stiffness reduction of the stringer top nr. 1, which is the essential difference with the structure where the stiffeners do not participate in local buckling. The model, now confined to strips nrs. 1 and 2, is again a two-flange model. The characteristic differences with the previous two-flange model are:

- 1, the areas of the cross sections of the flanges are quite different;
- 2, the ratio of half wave length to plate width $\ell/b \neq 1$;
- 3, the strips are elastically restrained at their edges.

A further simplification is that the coefficients of edge restraint C of the strips are assumed to be constant through the whole range of edge strains and equal to their value with local buckling given in fig. 3. Finally it is being assumed that at the transition of strips 2a and 2b the in plane edge forces vanish. A better approximation would be to assume constant lateral displacement. But this would involve the necessity to solve another post buckling problem, whereas the present assumption permits to use the solution of the problem as occurring with respect to strip nr. 1. To solve this latter problem had appeared to be a very laborious

operation. When evaluating these various simplifications it should be kept in mind that exact quantitative knowledge of an actual structure is not being envisaged. The object being to establish to what extent mode interaction affects the panel strength. For this kind of orientation in the problem area a qualitatively correct representation of the behaviour of the structural elements seems acceptable.

Further assumptions have to be made on the imperfection of the several strips. For obvious reasons they have been taken similar to their local buckling mode shown in fig. 3. The size of the imperfections however has no relation to the wave amplitudes in local buckling. It has been assumed that the ratios of the initial deflections in the centre of the strips to the strip widths is equal for the three strips. Three degrees of imperfection have been assumed (Table I).

Table I

Imperfections α = amplitude: strip thickness

strip	I	II	III
1	0,012	0,024	0,036
2a	0,025	0,050	0,075
2b	0,010	0,020	0,030

4. The elastically restrained strip

The plate strip is supported along its edges and has symmetric elastic restraint. With the perfect strip the post-buckling wave length can have any value. With the imperfect strip the wave length of the imperfection is also arbitrary; the lateral shape of the imperfection is similar to the buckling mode at that wave length; its amplitude is αh .

The governing parameters are: for the strip of width b the wave length parameter $\lambda = l/b$, the coefficient of restraint C , the imperfection α and the compressive strain parameter ϵ/ϵ_ℓ . Instead of C can be taken the local buckling coefficient k defined by

$$\sigma_\ell = k^2 \frac{\pi^2 E}{12(1-\nu^2)} \left(\frac{h}{b}\right)^2. \quad (4.1)$$

The relation between C and k is

$$C = -2\pi \frac{k}{\lambda} \left(\beta \operatorname{Tgh} \frac{\pi}{2} \beta + \gamma \operatorname{tg} \frac{\pi}{2} \gamma \right)^{-1},$$

where

$$\beta = \frac{1}{\lambda} (k\lambda + 1)^{\frac{1}{2}}, \quad \gamma = \frac{1}{\lambda} (k\lambda - 1)^{\frac{1}{2}}.$$

The local buckling mode is

$$w = wh \frac{\operatorname{Cosh} \frac{\pi}{2} \beta \cos \gamma \eta - \cos \frac{\pi}{2} \gamma \operatorname{Cosh} \beta \eta}{\operatorname{Cosh} \frac{\pi}{2} \beta - \cos \frac{\pi}{2} \gamma} \sin \xi, \quad (4.2)$$

where $\xi = \pi x/l$, $\eta = \pi y/b$ and wh is the amplitude of the deflection.

The method by which the $P/P_\ell - \epsilon/\epsilon_\ell$ relation has been established is as usual:

- 1, assume the deflection for any ϵ/ϵ_ℓ to be described by (4.2), where w is unknown;
- 2, solve the Airy stress function from its linear differential equation;
- 3, solve w from the equilibrium condition in the direction normal to the plate by means of the Ritz-Galerkin method.

The $P-\epsilon$ relation so established is in parametric form for Poisson's ratio $\nu=0,3$,

$$\epsilon/\epsilon_k = \frac{\omega}{\omega+\alpha} + \frac{1}{2} A\omega (\omega+2\alpha), \quad (4.3)$$

$$P/P_k = \frac{\omega}{\omega+\alpha} + \frac{1}{2} B\omega (\omega+2\alpha), \quad (4.4)$$

where A and B are highly complicated functions of λ and k, too lengthy to be reproduced here.

The non-linearity of the P- ϵ -relation is expressed by the functions η , η' and μ , occurring in the formulae describing the behaviour of the model.

$$\eta = d(P/P_k)/d(\epsilon/\epsilon_k) = (1 + BX)/(1+AX), \quad (4.5)$$

$$\eta' = d\eta/d(P/P_k) = -3 (A-B)XY [\alpha(1+AX)^2(1+BX)]^{-1}, \quad (4.6)$$

$$\mu = \frac{1}{6} (2\eta'^2 - \eta\eta'') = - (4-5BX)Y\eta' [6\alpha(1+AX)(1+BX)]^{-1}, \quad (4.7)$$

where $X = Y^3/\alpha$ and $Y = \omega+\alpha$.

The values of λ and k pertaining to local buckling of the panel yield A and B given in table II together B/A. The post-buckling stiffness of the perfect plate strips being $\eta = B/A$. For comparison table II gives these figures also for simply supported strips at the same values of λ and at $\lambda = 1$.

Table II

Stiffness parameters A and B

strip nr.	λ	C	k^2	A	B	$\eta=B/A$
1	1,90	-2,42	1,340	2,1183	1,5067	0,7113
2a	0,76	12,9	5,82	1,1722	0,4463	0,3807
2b	1,90	-2,59	0,931	4,370	3,476	0,7954
-	0,76	0	4,31	1,7402	0,6432	0,3696
-	1,00	0	4	1,1536	0,4710	0,4083
-	1,90	0	5,89	0,2726	0,1441	0,5288

It appears that for $C = 0$ the post-buckling stiffness increases with λ . This is plausible because increased waviness (smaller λ) makes the strip more flexible. Comparing η for $C = 0$ and $C \neq 0$ at constant λ it appears that $C > 0$ yields a slight increase of η , whereas $C < 0$ increases the stiffness much.

5. Overall buckling of the perfect model

The cross section of strip i is $A_i = F_i A$, where A is the total cross section of the model. The strips 2a and 2b can be combined to a flange 2 with the cross section $A_2 = A_{2a} + A_{2b}$ and with the effective cross section

$$(\eta F)_2 A = [(\eta F)_{2a} + (\eta F)_{2b}] A. \quad (5.1)$$

Then the bending stiffness of the model is

$$\eta_b EI = \frac{(\eta F)_1 (\eta F)_2}{(\eta F)_1 + (\eta F)_2} EAc^2$$

$$\text{Since } EI = \frac{F_1 F_2}{F_1 + F_2} EAc^2$$

and by definition $F_1 + F_2 = 1$ follows

$$\eta_b = \frac{\eta_1 \eta_2}{(\eta F)_1 + (\eta F)_2} \quad (5.2)$$

The dimensions given in fig. 1 yield $\eta_2 = 0,4992$ and $\eta_b = 0,6576$.

$$\text{Fig. 4 depicts } K_b/K_k = \eta_b R \quad (5.3)$$

At $R > 1,52$ K_b exceeds K_k .

The panel of length $2 L_0$ (parameter R_0) is at $K = K_0$ in neutral equilibrium. Its overall buckling mode consists of 2 parts with different stiffness to infinitesimal deflection. In part I flange nr. 1 is at the concave side of the mode therefore in the post-buckling range and flange nr. 2 in the unbuckled condition. Then its bending stiffness following from (5.2.) is

$$\eta_I = \frac{\eta_1}{(\eta F)_1 + F_2}$$

In part II the condition of the 2 flanges is reversed yielding

$$\eta_{II} = \frac{\eta_2}{F_1 + (\eta F)_2}$$

Having equal buckling loads the lengths L_I and L_{II} are not equal, but their sum is $2 L_0$. It follows

$$R_0 = 4(\eta_I^{\frac{1}{2}} + \eta_{II}^{\frac{1}{2}})^{-2} = 1,258$$

Panels of greater length than $2 L_0$ ($1 < R < R_0$) are at $K = K_0$ instable to infinitesimal deflection. This instable range of R is much smaller than with the two-flange model ($1 < R < 1,725$) but wider than with the panel where the stiffeners do not participate in local buckling ($1 < R < 1,11$).

6. The buckling load of the imperfect model

With respect to infinitesimal deflection the imperfect model is in neutral equilibrium at K_b following from (5.1, 2, 3) after establishing η of the several strips by means of (4.5).

For a given ϵ/ϵ_0 , equal for the 3 strips, follows from (4.3) their ω . Then (4.4) yields $(P/P_0)_i$, whereupon the compressive force can be established

$$K = \sum P_i = K_0 \sum (P/P_0)_i F_i \quad (6.1)$$

The reduction factor of the bending stiffness with respect to infinitesimal deflection derives from (4.5) and (5.1, 2) whereupon the geometric parameter R at which K is the buckling load can be established by means of (5.3).

The K_b/K_0 - R -curves pertaining to the three cases of imperfection (tabel I) are shown in fig. 5. The maximal reductions due to initial waviness occurring at $R = 1$ are resp. 9, 12,5 and 14%. For $R > 1,37$ K_b exceeds K_0 . With R close to unity the strength reduction by strip imperfection appears to be not negligible.

So as to answer the question about the character of the equilibrium at K_b , whether it is stable or instable, the behaviour at small finite deflections has to be established. The possibility exists that $(dK/dW_0^2)_b \neq 0$ due to the non-linearity of the K - ϵ -relation. Then at the deflection W

$$K_b + K_0 t \left(\frac{W_0}{2j}\right)^2, \quad (6.2)$$

where $t = d(K/K_0)/d(W_0/2j)^2$.

The condition of equilibrium in the deflected state is

$$\frac{d^2}{dx^2} \left[M + K_b \left(1 + \frac{1}{\eta_b R} t \left(\frac{W_0}{2j}\right)^2 \right) W \right] = 0 \quad (6.3)$$

The last term of this equation is of third degree in W ; then M has to be established as well up to terms of the third degree. This can be done by expressing the strip loads P_i by the truncated Taylor expansion

$$P = P_b + \frac{dP}{d\epsilon} (\epsilon - \epsilon_b) + \frac{1}{2} \frac{d^2P}{d\epsilon^2} (\epsilon - \epsilon_b)^2 + \frac{1}{6} \frac{d^3P}{d\epsilon^3} (\epsilon - \epsilon_b)^3 \quad (6.4)$$

The flange strains have the relation

$$\epsilon_1 - \epsilon_2 = c \frac{d^2W}{dx^2} \quad (6.5)$$

The compressive force K and the bending moment M are resp. $K = P_1 + P_2$, (6.6)

$$M = (P_1 \frac{\gamma_2}{\gamma_1 + \gamma_2} - P_2 \frac{\gamma_1}{\gamma_1 + \gamma_2}) \cdot c, \quad (6.7)$$

where $\gamma_i = (\eta F)_i$.

Eliminating $\epsilon_1, \epsilon_2, P_1, P_2$ from the equations (6.2, 4, 5, 6, 7), thereby neglecting terms of higher order than the third, the relation between M, W and t is obtained

$$M/K_b c = (1 - A_3 \frac{1}{4\rho} t v_o^2) v'' + A_1 \frac{R}{\rho} v''^2 - (A_2 - 2A_1^2) \frac{R^2}{2} v'''^3 + M_b/K_b c, \quad (6.8)$$

$$\text{where } v'' = \frac{L^2}{\pi^2} \frac{d^2 W/c}{dx^2}$$

$$\rho = j^2/c^2 = F_1 F_2$$

and A_1, A_2, A_3 are functions of η and its derivatives

$$A_1 = (-\gamma_2^2 \phi_1 + \gamma_1^2 \phi_2)/(\gamma_1 + \gamma_2)^2$$

$$A_2 = (\gamma_2^2 \mu_1 + \gamma_1^2 \mu_2)/(\gamma_1 + \gamma_2)^3$$

$$A_3 = 2(\gamma_2 \phi_1 + \gamma_1 \phi_2)/(\gamma_1 + \gamma_2)^2$$

$$\gamma_1 = \eta_1 F_1, \gamma_2 = \gamma_{2a} + \gamma_{2b} = (\eta F)_{2a} + (\eta F)_{2b}$$

$$\phi_1 = -\frac{1}{2} \eta_1', \phi_2 = -\frac{1}{2\gamma_2} [(\eta' \gamma)_{2a} + (\eta' \gamma)_{2b}]$$

$$\mu_2 = \frac{1}{\gamma_2} [(\mu \gamma)_{2a} + (\mu \gamma)_{2b}] - \frac{1}{\gamma_2^2} (\eta'_{2a} - \eta'_{2b})^2.$$

Substitution of M into (6.3) yields the differential equation

$$\{v'' [1 + A_1 \frac{R}{\rho} v'' - (A_2 - 2A_1^2) \frac{R^2}{2} v'''^2 - A_3 \frac{1}{4\rho} t v_o^2] + V(1 + \frac{1}{4\rho \eta_b R} t v_o^2)\}'' = 0. \quad (6.9)$$

The expression between brackets represents the modification of the bending stiffness with small finite deflections.

The solution of this equation is not unique; the amplitude V_0 is undetermined within the range of small finite deflections. Substitution of the solution into (6.9)

yields a left hand side which is the sum of terms of the orders V_0, V_0^2 and V_0^3 .

Since V_0 is undetermined each of these terms must vanish which yields 3 linear

differential equations for the 3 components of V . $V = V_1 + V_2 + V_3$, where

V_1, V_2, V_3 are resp. of the order V_0, V_0^2, V_0^3 .

Their solution for the panel of length $2L$ clamped at its ends is

$$V = V_{10} (\cos \xi + 1) + \frac{1}{6} A_1 \frac{R}{\rho} V_{10}^2 (\cos 2\xi - 1) + \frac{1}{32} (A_2 + \frac{2}{3} A_1^2) \frac{R^2}{\rho^2} V_{10}^3 (\cos 3\xi + 1) \quad (6.10)$$

and

$$t = \frac{d K/K_\ell}{d(W_o/2j)^2} = -\frac{3}{4} \frac{A_2 - \frac{10}{9} A_1^2}{\frac{1}{\eta_b R} + A_3} \cdot \frac{R^2}{\rho}. \quad (6.11)$$

The slope of the load shortening curve at the bifurcation K_b is

$$\left[\frac{d K/K_\ell}{d(-\Delta L/L \epsilon_\ell)} \right]_b = \left\{ (\gamma_1 + \gamma_2) \left[1 - \frac{1}{3} \gamma_1 \gamma_2 \frac{(\frac{1}{\eta_b R} + A_3)^2}{A_2 - \frac{10}{9} A_1^2} \right]^{-1} \right\}_b. \quad (5.11)$$

The load shortening curves in the three cases of imperfection (table I), together with the tangents at the bifurcation are given in fig. 6. The equilibrium at K_b appears to be unstable for $R < 1,1$.

In comparison to the "extreme" cases, [2] and fig. 2, the peaks of the load-shortening curves at the bifurcation are much sharper with the "simplified panel". This implies that a panel without imperfection of the panel axis would fail explosively at K_b . Presumably it does not imply that the sensitivity to axis imperfection is very great. More significant for this effect is the negative slope $d(K/K_b)/d(W/2j)$. Fig 7 compares these slopes in case II ($\alpha_2 = 0,05$) with the slopes in the "extreme" cases, both at $\alpha = 0,05$. It appears that the lower extreme (section 2) is insensitive to axis imperfection and that the maximal slope with the "simplified" panel is half of the maximal slope with the two-flange model. The slope is of the order of the slope with the two-flange model at $\alpha = 0,1$, where initial curvature adds little to the reduction of strength. The effect of axis imperfection is a matter of further investigation.

7. Conclusions

Tophat stiffened panels are subject to mode interaction. With the geometric parameter $R > 1,35$ the imperfections: initial waviness of the plate strips and initial curvature of the panel axis have negligible effect. However the overall buckling load K_b depends on the post-buckling characteristics of the structure. This means that theoretical determination of K_b requires knowledge of the reduction of the bending stiffness of the panel in the post-buckling state. This problem is as yet unsolved.

When $R \gg 1$ the stiffeners are negligibly little affected by local buckling. Then the effective bending stiffness can be established with available methods. With $R < 1,35$ K_b is below both Euler and local buckling load; the reduction depending on the amount of initial waviness and to some, presumably less, extent on initial curvature. The maximal reduction occurring at $R = 1$ is in the order of 10%. Scatter of imperfections yields scatter of strength. Therefore the strength of structures with R close to unity cannot be predicted with great precision. A number of tests on identical specimen has to be carried out.

The strength of pin-ended panels is not representative for the strength of multibay panels. The position of the neutral plane in bending shifts with increasing compressive strain away from the side of the plate, causing bending before buckling. Tests should be carried out on clamped specimen.

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FIG.1

FIG. 2

FIG. 3

FIG. 4

FIG. 5

FIG. 6

FIG. 7

Rapport 180

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