Noise Considerations for Translinear Filters

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Abstract—Translinear filters exhibit unconventional noise characteristics, due to the internal nonlinear behavior. In this paper, the maximal signal-to-noise-ratio and the dynamic range properties of translinear filters are discussed, and the relation between these two specifications and the inherent instantaneous companding is described.

Index Terms—Comanding, continuous-time filters, noise, translinear.

I. INTRODUCTION

The dynamic translinear (DTL) principle, or “log-domain” principle, constitutes a promising new implementation technique for linear filters [1]–[6]. Using the DTL principle, which states that a linear time derivative of a collector current can be realized through a product of a capacitance current and one or more collector currents [6], a wide variety of linear and nonlinear differential equations (DE’s) can be implemented.

The dynamic range (DR) and the maximal signal-to-noise ratio (SNR), defined in Section II, are important specifications for linear filters. The noise behavior of TL filters is, however, different from conventional filter implementation techniques, due to the explicit dependence on the exponential voltage-to-current transfer function of the bipolar transistor.

This paper discusses the noise properties of TL filters. In Section III, the DR specifications of opamp-MOSFET-C, $g_mC$, and TL filters are compared. The companding nature of TL filters, which is related to both the DR and the maximal SNR is discussed in Section IV. Finally, the maximal SNR of TL filters is treated in Section V.

II. DEFINITIONS OF DR AND (MAXIMAL) SNR

In the literature, several definitions for the DR and the maximal SNR are commonly used. Therefore, to clarify the discussion, this section explicitly defines these quantities.

By definition:

- the (maximal) SNR equals the (maximal) ratio of the signal power to the noise power at the same time;
- the DR equals the ratio of the maximal signal power to the minimum acceptable signal power; the latter is usually taken as equal to the noise power in the absence of any signals, and this convention will be adopted in this paper.

In conventional filters, the DR and the maximal SNR are equal, since the noise floor is constant. Hence, the maximal SNR is obtained for the maximum value of the signal power, which is determined by an application-specific specification of the distortion level, e.g., <1% total harmonic distortion (THD).

As pointed out in [7], a single noise figure cannot adequately describe the noise behavior of TL filters, or companding filters in general. Due to signal noise intermodulation, the maximal SNR can be much smaller than the DR.

III. DYNAMIC RANGE

Static translinear (STL) circuits do not have a good reputation with respect to noise. Since DTL circuits are in a way very similar to STL circuits [6], it is likely that they also inherit the noise characteristics. Therefore, it is interesting to compare the DR properties of $g_mC$ and opamp-MOSFET-C filters, the most popular implementation techniques to date [8]–[10], with the DR specifications of TL filters. Note that a comparison is made with respect to the DR and not to the maximal SNR, thus, due to the definition of the DR, excluding the signal noise intermodulation. This intermodulation is the topic of Section V.

The comparisons are made under the practically relevant restriction of a low supply voltage. Though DTL circuits find wider application than only linear DE’s, in this section, the comparisons are restricted to linear filters.

To obtain an indication of the DR properties of TL, $g_mC$ and opamp-MOSFET-C filters, we compare the $RC$ and the diode-C subcircuits shown in Fig. 1. The DR of a complete filter is strongly related to the DR of these elementary building blocks. For both filter sections, the low-pass transfer function $I_{in} \rightarrow I_{out}$ is considered. Under certain presumptions, to be discussed later, the circuit shown in Fig. 1(a) is represen-
tative for opamp-MOSFET-"C filters. The circuit shown in Fig. 1(b) represents both TL filters and “bipolar $g_m C$” filters, for which the transconductances comprise bipolar transistors only. The DR properties of the two filter sections are first compared based on a simplified approach. The influence of low-voltage implementation issues, tunability, low-power operation, high-frequency performance, and class AB operation is discussed next.

For the $RC$ filter section, the current signal swing is limited, due to the supply voltages, to $V_{DD}/(2R)$. Assuming class A operation, the signal swing in the diode-$C$ filter is limited by the dc bias current $I_{dc}$.

The only noise source in the $RC$ filter is due to the resistor $R$. The double-sided noise current power spectral density is given by $2kT/R$. The noise bandwidth of the filter equals $1/(2RC)$. Hence, the equivalent noise power is found to be $kT/(CR^2)$.

In the diode-$C$ circuit, the power spectral density of the shot noise in the bias point equals $qI_{dc}$. In comparing the two filters, both the capacitance value and the bandwidth of the filter are assumed to be equal. As a result, the relation between $R$ and $I_{dc}$ is given by $RL_{dc} = U_T$, where $U_T$ is the thermal voltage. Hence, the noise bandwidth of the TL filter equals $I_{dc}/(2CU_T)$, and the equivalent noise power is found to be $qP_{dc}/(2CU_T)$.

Dividing the maximal signal swing by the total amount of noise, we find the dynamic ranges $\text{DR}_{RC}$ and $\text{DR}_{g_m C}$ for the $RC$ and the diode-$C$ filter sections, respectively. This yields

$$\text{DR}_{RC} = \frac{CV_{DD}^2}{4kT}, \quad (1)$$

$$\text{DR}_{g_m C} = \frac{2CU_T}{q}. \quad (2)$$

Equations (1) and (2) represent upper limits of the DR. In practice, these values have to be divided by the square of the crest factor of the specific signal being processed.

To compare the dynamic range properties, we divide $\text{DR}_{RC}$ by $\text{DR}_{g_m C}$:

$$\frac{\text{DR}_{RC}}{\text{DR}_{g_m C}} = \frac{V_{DD}^2}{8U_T}. \quad (3)$$

Obviously, in principle, application of filters based on linear resistors yields a much better DR. For example, even for a low supply voltage of 1 V, and $U_T = 26$ mV, $\text{DR}_{RC}$ and $\text{DR}_{g_m C}$ differ by a factor of 185, or equivalently, 22.7 dB. Since the minimal power consumption of a filter is fundamentally related to the desired DR [9], the voltage swings should be preferably rail-to-rail [9], [11]. This is realized in the $RC$ section, but not in the diode-$C$ subcircuit, where the voltage swing is only $U_T$, corresponding to a current swing of $I_{dc}$. This explains the large difference between $\text{DR}_{RC}$ and $\text{DR}_{g_m C}$.

However, the conclusion drawn from (3) is not absolute. Many adventitious factors that affect the DR are not incorporated in (1) and (2). Their influence will be discussed next.

A. Opamp-MOSFET-"C Filters

The opamp-MOSFET-"C technique is the only method to realize filters with rail-to-rail signal swings and low noise levels [9]. In opamp-MOSFET-"C filters, large voltage swings are possible, due to the fact that the quadratic behavior of the MOS transistor in strong inversion is not very nonlinear. Thus, the subcircuit shown in Fig. 1(a) can be used to represent this class of filters. Based on the simple MOS square law equation, it is even possible, in theory, to obtain a perfectly linear transconductance, which extends the voltage swings [12]. Consequently, these filters can be made to approach the fundamental limit regarding the minimal power consumption for a certain specified DR [9].

Unfortunately, at low supply voltages, opamp-MOSFET-"C filters become difficult to implement [8]–[10], resulting in a lower DR than indicated by (1). Due to the requirement for strong inversion operation, very low voltage operation becomes only possible by using an on-chip charge pump to drive the gate voltages high. In addition, the tuning range of these filters is quite limited; it is only just enough to cope with process tolerances [8].

B. MOS $g_m C$ Filters

The class of $g_m C$ filters can be divided into the categories of “MOS $g_m C$” and “bipolar $g_m C$” based on MOS and bipolar transconductors, respectively. With respect to opamp-MOSFET-"C filters, the excess noise of the transconductors in MOS $g_m C$ filters results in a factor 2 to 3 lower DR. Since most of the other characteristics of MOS $g_m C$ filters are very similar to opamp-MOSFET-"C filters, MOS $g_m C$ filters will not be discussed here.

C. Bipolar $g_m C$ Filters

Since the bipolar transistor is an exponential device, the circuit shown in Fig. 1(b) can be used to represent bipolar $g_m C$ filters. Equation (3) shows that the DR of bipolar $g_m C$ filters is generally worse in comparison with opamp-MOSFET-"C filters, since the voltage swings are limited to $U_T$. In practice, however, the voltage swings are even smaller due to the strongly nonlinear nature of the bipolar transistor. Therefore, most often the differential pair is used instead of a single transistor to eliminate even order distortion. Nevertheless, the voltage swings remain limited to only $0.2U_T$ for a THD of 1% [10]. The application of emitter degeneration resistors is often not allowed as this severely reduces the tuning range.

Transconductance linearization techniques using linear combinations of collector currents are not as effective for bipolar as for MOS transconductors and cannot increase the maximal voltage swings above 100 mV [13]. Whereas for MOS transconductors exact linearization is possible, as the square
law is a polynomial, exact linearization of a bipolar transcon-ductance is fundamentally impossible, since the exponential function is transcendental.

Although the small voltage swings in bipolar $g_{mC}$ filters have a negative influence on the DR properties, on the other hand, it makes them very suitable for operation at low supply voltages [8], [10], [13]. The DR is, to first order, independent of the supply voltage. Further, bipolar $g_{mC}$ filters exhibit very wide tuning ranges, and potential for high-frequency and low-power operation.

D. Translinear Filters

Since the diode-$C$ circuit shown in Fig. 1(b) represents both bipolar $g_{mC}$ and TL filters, these two types of filters have many characteristics in common, e.g., excellent tunability and potential for low-voltage and low-power operation.

A major difference is formed by the possible signal swings. Owing to the application of the DTL principle, in theory, TL filters offer a perfectly linear current-mode transfer function. Hence, the maximal signal swings in TL filters are larger than in bipolar $g_{mC}$ filters. A DR comparison between a bipolar $g_{mC}$, a log-domain, and a tanh filter, reported in [3], shows that the latter two outperform the $g_{mC}$ filter by 13 and 10 dB, respectively.

A very important aspect of log-domain filters is the possibility of class AB operation, which can be used to increase the DR [3], [4]. For example, in [4], a DR of 65 dB is reported, in connection with a maximal SNR of 52.5 dB. A generic class AB setup is shown in Fig. 2. A bipolar valued signal is split up into two unipolar signals, which are subsequently processed by different signal paths. Since the two different signal paths only have to process unipolar signals, no dc bias current is needed. Hence, the noise floor is decoupled from the maximal current signal swings, in contrast with the class A setup shown in Fig. 1(b). Class AB operation is possible due to the fact that the linearization mechanism of TL filters is theoretically exact.

E. Summary

To conclude, in principle, opamp-MOSFET-$C$ filters are the best choice for a large DR, in the area of tuneable continuous-time filters. However, in low voltage environments, or for applications where a large tuning range is required, TL filters and bipolar $g_{mC}$ are more suitable.

Fig. 3. Generic output structures of (a) log-domain filters, (b) tanh filters, and (c) sinh filters.

Fig. 4. The second-order derivatives of the $V–I$ transfer functions of the circuits shown in Fig. 3.

Owing to the theoretically exact linearization mechanism, TL filters form an interesting and competitive alternative to $g_{mC}$ filters. Especially when class AB operation is applied, the DR of TL filters exceeds the DR obtainable with bipolar $g_{mC}$ filters.

IV. Companding

In TL filters, the voltages are logarithmically related to the currents. Therefore, these circuits are in some way instantaneously companding. The exact nature of the companding affects the relation between the DR and the maximal SNR of a circuit. In this section, we apply the strict definition of companding, where the input compression and output expansion factors are larger than one. Only for filters that are companding according to this definition can the DR be significantly larger than the maximal SNR.

The companding properties of log-domain, tanh, and sinh filters [3] can be characterized by the $V–I$ transfer functions of the generic output structures of these circuits, depicted in Fig. 3. The $V–I$ “expansion” functions are given by $\exp x$, $\tanh x$, and $\sinh x$, respectively, where $x$ represents the normalized capacitance voltage swing. Fig. 4 depicts the second-order derivatives of the $V–I$ transfer functions. For an output stage complying to the strict definition of (instantaneous) expansion, this second-order derivative should be strictly positive for $x > 0$ and strictly negative for $x < 0$.

Fig. 4 demonstrates that the sinh transconductor is indeed companding. This fact is related to the inherent class AB
properties of \( \tanh \) filters [3]. Large output current swings are thus facilitated without the requirement of a large dc bias current. Hence, the DR can be much larger than the maximal SNR.

The second-order derivative of the \( \tanh \) function shows the opposite behavior with respect to the \( \sinh \) transconductor. In other words, the differential pair output stage, shown in Fig. 3(b), implements a compression operation [5]. The signal swing is limited by the tail current. As a result, the DR and the maximal SNR will not differ much.

A similar conclusion can be reached for log-domain filters operated in class A. The second-order derivative of the function \( \exp \) is positive both for \( x > 0 \) and \( x < 0 \). Thus, these filters are not strictly companding, and hence, the maximal SNR and the DR will be approximately equal.

However, this conclusion is not valid for log-domain filters operated in class AB. Using the setup shown in Fig. 2, only positive signals have to be processed. Since for \( x > 0 \) the second-order derivative of the function \( \exp \) does correspond to the characteristics of an expansion stage, class AB operated log-domain filters are companding. This results in a significantly improved DR over the class A counterpart. It is interesting to note that \( \tanh \) filters cannot be operated in class AB, since \( \tanh x \) constitutes a compression for both positive and negative signals.

V. MAXIMAL SNR

Due to the internal nonlinear behavior, causing signal \( \times \) noise intermodulation, the noise floor in a TL filter is signal-dependent. Since class A operated TL filters, i.e., log-domain and \( \tanh \) filters, are not companding, nonlinear noise effects are not very pronounced. In class AB operated filters, however, the intermodulation noise will dominate for large values of the signal power. The intermodulation noise power is proportional to the signal power, and consequently, the maximal SNR is fundamentally limited. Thus, only the DR, and not the maximal SNR, can be improved through companding [10].

The limited value of the maximal SNR can be explained intuitively from the first-order TL filter depicted in Fig. 5. For this circuit, class AB operation can be established using the setup shown in Fig. 2.

Fig. 6 shows that this filter can be redrawn as a cascade of two ports. It is obvious that the SNR of a chain of two ports is limited by the element(s) having the lowest SNR. A translation of this fact to the circuit shown in Fig. 6 indicates that the maximal SNR of the class AB TL filter is limited by the transistors with the lowest SNR value. In this context, the “SNR of a bipolar transistor” equals the ratio of the mean square value of the collector current \( I_{C} \), to the power of the collector shot noise current \( i_{C} \).

Since the mean square value \( P_{i_{C}} \) of the collector current is proportional to \( I_{C}^2 \), and the noise power \( P_{i_{C}} \) over a certain noise bandwidth \( B_{i_{C}} \), is proportional to \( I_{C} \), the SNR of a bipolar transistor is proportional to \( I_{C} \).

In Fig. 5, for increasing input signal swings and class AB operation, the average collector currents of \( Q_{1} \) and \( Q_{4} \) increase accordingly. Hence, the SNR of \( Q_{1} \) and \( Q_{4} \) increases. The average collector currents of \( Q_{2} \) and \( Q_{3} \), however, remain equal to \( I_{o} \). Thus, for large signals, the SNR of these two transistors is equal to the SNR increases by 20 dB per decade.

The (double-sided) power spectral density of the transconductor. Since the mean square value \( I_{m} \) is the quiescent current level of the (geometric mean) class AB splitter, \( m \) is the modulation index, \( \omega \) is an in-band frequency. The parameter values used in this plot are \( I_{m} = I_{0} = 1 \mu A \), \( C = 10 \) pF, and \( U_{T} = 26 \) mV. For low values of \( m \), the SNR increases by 20 dB per decade. For high values of \( m \), the SNR saturates to a value of 62.1 dB, as predicted by (4).

A. Seevinck’s Class AB Integrator

Although exhibiting an externally linear transfer function, the class AB integrator proposed by Seevinck in [2], and shown in Fig. 8, is in a way a nonlinear DTL circuit as it implements two nonlinear DE’s. The two TL loops of this integrator are described by

\[
CU_{T}I_{out1,2} + I_{out1}I_{out2} = I_{o}I_{int,2}.
\]

A linear integrator is obtained through subtraction of these equations. The overall input and output current equal \( I_{in} = I_{in,1} - I_{in,2} \) and \( I_{out} = I_{out1} - I_{out2} \), respectively.

Fig. 7 shows that the noise performance of both class AB filters is almost identical. The SNR of both filters converges to the same maximal value given by (4). However, for low values of the modulation index, Seevinck’s circuit performs slightly better, about 0.6 dB, due to a lower noise floor in the quiescent point. This noise floor can be decreased further, up to 1 dB, by increasing the term proportional to \( I_{out1}I_{out2} \) in (5).

1 The influence of the base resistance noise is assumed to be negligible.
B. Noise Due to Out-of-Band Signals

An interesting situation is the coexistence of a large signal and a small signal in a companding TL filter. Suppose the small signal is the desired output signal and the large signal is outside the pass band. Now, in conventional filters, the large signal will limit the maximal SNR at the output, as it occupies a large part of the available DR of the filter. Naturally, the same effect applies to companding filters. However, in these filters, the SNR at the output will be further decreased, as the large out-of-band signal will increase the internal noise level [7]. This effect makes companding filters less suitable, e.g., for intermediate-frequency filtering [9], unless some form of linear prefiltering is used [7].

Fig. 9 displays the resulting SNR for an in-band signal as a function of the amplitude of an out-of-band signal, for the class AB filter shown in Fig. 5. The applied input signal equals \( I_{in} (\sin \omega_1 t + n \sin \omega_2 t) \), where \( \omega_1 \) and \( \omega_2 \) are the in-band and out-of-band frequencies, respectively. The modulation index \( m \) of the in-band signal equals \([0, 1, 10]\), and \( n \) denotes the modulation index of the out-of-band signal.

The figure demonstrates the expected behavior. Clearly, the effect is more pronounced for in-band signals with a small amplitude, i.e., a low value of \( m \).

VI. Conclusions

A comparison of the DR properties of different filter implementation techniques shows that class AB operated TL filters constitute an interesting and competitive alternative for low-voltage or low-power applications. The maximal SNR of a TL filter, however, is fundamentally limited. This is due to signal X noise intermodulation, which causes the noise level to increase when large signals (even when out-of-band signals) are being processed.

REFERENCES


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