Tracking of vortices in a turbulent boundary layer

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The motion of spanwise vortical elements and large-scale bulges has been tracked in the outer region between wall-normal distance $z/\delta = 0.11$ and 0.30 of a turbulent boundary layer at $Re_\theta = 2460$. The experimental dataset of time-resolved three-dimensional velocity fields used has been obtained by tomographic particle image velocimetry. The tracking of these structures yields their respective average trajectories as well as the variations thereof, quantified by the root mean square of the trajectory coordinates as a function of time. It is demonstrated that the variation in convection can be described by a dispersion model for infinitesimal particles in homogeneous turbulence, which suggests that these vortical structures and bulges are transported passively by the external velocity field without significant changes in their topology, at least over the present observation time of $1.2\delta/U_e$. However, this does not mean that the structure's convection velocity is equal to the local flow velocity at each instant. Differences of the order of the Kolmogorov or wall friction velocity have been observed for the spanwise vortical elements. In addition, the simultaneous detection and tracking of multiple structures allows an evaluation of the relative velocity between two spanwise vortex elements, which are approximately aligned along the streamwise direction. The typical streamwise distance between such neighbouring structures is found to be around 0.2δ . Their relative velocities are small, especially the streamwise component, which shows less variation as may be expected based on the relative flow velocity statistics for the same separation distance. This appears consistent with the hairpin packet model, which comprises a set of streamwise aligned hairpins travelling coherently. In exceptional cases, however, the structures approach each other rapidly, forcing an interaction on a time scale of the order of $1\delta/U_e$. It is shown that the measured variation in convection velocity can further be used successfully to predict the temporal development of space–time correlation functions starting from the instantaneous correlation map. In this prediction the structures are assumed to convect without change, following our observations.

Key words: boundary layers, turbulent boundary layers, turbulent convection

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1. Introduction

The convection velocity of coherent flow structures in wall-bounded turbulence has received considerable attention, first of all because of a fundamental interest in the dynamic behaviour of these structures. The turbulent flow contains a variety of structures (for a recent survey see Adrian 2007), which, in principle, may all advect at different velocities. We may then further imagine that such a difference in convection velocity ultimately causes the structures to change their relative position and interact, which results in topological changes, such as for instance the generation of a new structure. In that sense convection can be seen as an appropriate starting point for the investigation of coherent structure dynamics.

Unfortunately very little quantitative information is available on the changes in convection velocity from one structure to the next, as most attention has been paid to their average convection velocities. This makes sense, because a statistical treatment is unavoidable. However, as will be shown in this paper, the variation around the mean is of significant importance too, particularly when interpreting space–time correlations and assessing the validity of Taylor’s hypothesis. Therefore, the aim of the present paper is to provide such quantitative information by tracking individual structures in the outer layer of a turbulent boundary layer.

Before proceeding, the main conclusions from earlier work will be summarized, which mainly concerns the average convection. First of all, the average has been shown to be scale-dependent. Kim & Hussain (1993) consider the propagation velocity of turbulent fluctuations in turbulent channel flow at different spatial wavelengths and find that it increases with spanwise scale in the near wall region. Compared with earlier experimental work (e.g. Willmarth & Wooldridge 1962; Wills 1964; Bradshaw 1967), Kim & Hussain (1993) have benefited from the introduction of direct numerical simulations (DNSs), which resolves both the time and length scales in contrast to the experimental methods available at that time, such as hot-wire anemometry. The latter provide time series in a limited number of points (usually two). Therefore, the resulting space–time correlations from those experiments cannot distinguish well between a fast large eddy and a slow moving small eddy.

Similar conclusions are reached by Krogstad, Kaspersen & Rimestad (1998) for the case of a turbulent boundary layer. They find that coherent motions of the order of the boundary layer thickness convect at the local mean velocity and that the velocities drop significantly when the scales are reduced. Interestingly, the recent work by Del Alamo & Jimenez (2009) states that the small scales travel at approximately the local average velocity whereas the larger scales travel at a ‘more uniform speed proportional to the bulk velocity’. The difference is most noticeable, again, close to the wall. This observation is explained by the outer large-scale (global) modes penetrating down to the wall. Their higher wall-normal extent means they convect at rates associated with the flow velocity further from the wall, which is higher on average. The behaviour of the large scales seems to be reversed in the outer layer, where they travel at speeds just below the local average (Del Alamo & Jimenez 2009). Unfortunately, this aspect has not been given much attention.

There may appear to be a contradiction in the observed average convection velocity of the small scales, but, as pointed out by Moin (2009), there is great difficulty associated with the unambiguous definition of a convection velocity from space–time correlation or frequency maps. Hence, the reported differences may be due to the nature of the investigation and the methods employed: experimental methods using two-point correlation functions of ejection, sweep and shear layer events (Krogstad *et al.* 1998) and numerical methods with a new approach based on spectral content

(Del Alamo & Jimenez 2009). In some sense, the present approach of directly tracking features through space and time may be more straightforward.

The average convection velocity has also been found to depend on the flow variable or coherent structure under consideration. Ejections, for instance, travel at distinctly lower speeds than sweeps (Guezennec, Piomelli & Kim 1989; Krogstad *et al.* 1998) and pressure fluctuations near the wall propagate at a much higher velocity than velocity and vorticity perturbations (Kim & Hussain 1993).

Furthermore, convection connects to some very important practical applications such as establishing the most appropriate velocity scale when converting temporal into spatial scales using Taylor's hypothesis of frozen turbulence (Taylor 1938). The validity and the interpretation of the results obtained in this way are both long-standing topics, which have been revived recently (Dennis & Nickels 2008; Del Alamo & Jimenez 2009; Moin 2009) in light of the energetic very-large-scale motions observed in Taylor reconstructions of hot-wire time signals (e.g. Kim & Adrian 1999; Balakumar & Adrian 2007; Hutchins & Marusic 2007).

As recognized before in the papers cited, there exists some variation in the instantaneous convection velocity even between coherent flow structures of the same type. This scatter has not been quantified extensively yet, but its presence is visible, for instance, in the sample probability density function (p.d.f.) for the time delay between detected events at two streamwise locations, as presented by Krogstad *et al.* (1998) in their figure 6. Furthermore, the width of the rim in the turbulent spatio-temporal power spectrum (figure 2 in Moin 2009) illustrates that a single spatial wavelength is associated with a range of temporal frequencies, that is a range of convection velocities (assuming the spatial wave remains undistorted). For channel and boundary layer flow, Wu & Christensen (2006) show p.d.f.s for the streamwise flow velocity at the spanwise vortex locations identified in their two-dimensional measurement plane, which they have assumed to be equal to the vortex convection velocity distribution. The width of these p.d.f.s decreases with increasing wall-normal distance similar to the turbulence intensity. Pirozzoli, Bernardini & Grasso (2008) performed a similar analysis in a supersonic boundary layer, but reported only the average convection velocity in the streamwise and wall-normal direction.

The purpose of the present paper is to provide a further quantification of the variation in the instantaneous convection velocities of coherent structures. This will be based on the detection and tracking of large numbers of individual spanwise vortex elements and large-scale bulges in the logarithmic and outer region of a turbulent boundary layer, as described in § 3. These structures represent different scales, possibly with different associated convection velocities, and are believed to be representative features for wall-bounded turbulence, especially in the context of the hairpin-type models (Adrian 2007). This model features hairpin, cane and arch vortices as the main elements, all of which contain a spanwise vortex element, also known as the (hairpin) head. Moreover, the vorticity associated with such spanwise vortices adds to give the mean shear profile in a turbulent boundary layer, hence they are important. To check for scale dependence, we consider also the convection of the largest vortical structures, the size of which is considered to be of the order of the boundary layer thickness, or bulge scale. As argued before (Elsinga *et al.* 2010) a vortical motion involves at least two independent fluctuating velocity components (for example, the streamwise and wall-normal velocity in the case of a spanwise vortex) and requires them to be of approximately equal amplitude (or energy) and scale. Since the energy spectra for two of the three velocity components start to drop near the bulge scale, significant vortical flow topologies are unlikely to exist beyond that point.

The resulting vortex trajectories yield the average, but more importantly, also the root-mean-square (r.m.s.) convection over time (§ 4). Moreover, the tracking approach allows further analysis and the investigation of the relative velocities between neighbouring structures (§ 4.2). It provides a time scale for the interaction of these eddies and may assist in expanding the instantaneous kinematic description of the spatial organization of coherent structures into a dynamical one in the future. Furthermore, it will be demonstrated that the present results can be used to predict the space–time correlation function of different flow variables, such as the individual velocity components and the invariants of the velocity gradient tensor, given their respective instantaneous spatial auto-correlation functions (§ 5). This provides insight on how these correlations should be interpreted and puts the present results in relation to some of the earlier work, which have relied on such space–time correlations to establish average convection velocities and explore the validity, or range of applicability, of Taylor’s hypothesis.

In this investigation, existing three-dimensional and time-resolved experimental velocity data were used, the details of which are presented in § 2. Simultaneous time and spatial scale information is thus available, which solves some of the mentioned ambiguities related to probe measurements in much the same way as a DNS. The mentioned probe ambiguities refer to the inability to separate fast moving large-scale structure from the slow moving small-scale structures. An additional advantage of the present volumetric approach over, for example, planar measurements, is that no structures are lost during the tracking due to convection out of the observation plane. This advantage applies in particular to the smaller scales of motion, which can move out of the plane completely over shorter times.

2. Experimental dataset

The turbulent boundary layer data was obtained from a time-resolved tomographic particle image velocimetry (PIV) measurement (Elsinga *et al.* 2006; Schröder *et al.* 2008) in the water tunnel of the Laboratory for Aero & Hydrodynamics at TU Delft. The setup and first results have been described in detail by Schröder *et al.* (2011). Furthermore, the data were used before to study the dynamic evolution and lifetimes of the invariants of the velocity gradient tensor, which characterize the local flow topology around a fluid particle (Elsinga & Marusic 2010). For completeness, however, we will briefly recall some of the boundary layer properties here. The zero-pressure gradient boundary layer develops over a 2.5 m long flat plate with an elliptical leading edge at a free stream flow velocity U_e of 0.53 m s^{-1} and with a free stream turbulence level below 0.5 %. Transition is forced 15 cm downstream of the elliptical leading edge by a zig-zag strip. At the measurement location, 2.0 m downstream, the boundary layer thickness δ is 37 mm and the Reynolds numbers Re_θ and Re_τ are 2460 and 800, respectively. An overview of the boundary layer properties is provided in table 1.

The three-dimensional velocity distribution $\mathbf{V}(x, y, z, t)$ is evaluated in an effective volume spanning $1.8\delta \times 1.8\delta$ in streamwise (x) and spanwise (y) direction and covering $0.11 < z/\delta < 0.30$ in the wall-normal (z) direction in five time series of 2 s each. The sampling frequency is 1 kHz corresponding to $70U_e/\delta$ and $\Delta t^+ = 0.47$, which indicates that the flow is well resolved in time. Between subsequent velocity volumes, a fluid element moving with the free stream velocity would advect by approximately 10 wall units. The spatial resolution, taken as the cross-correlation volume linear dimension, is 0.07δ , corresponding to approximately 58 wall units in each direction. Furthermore, 75 % cross-correlation volume overlap is used in both

U_e	0.53 m s^{-1}
δ_{99}	38.1 mm
θ	4.65 mm
u_τ	0.0219 m s^{-1}
u_τ/ν	21.6 mm^{-1}
$Re_\tau = u_\tau \delta/\nu = \delta^+$	800
$Re_\theta = U_e \theta/\nu$	2460

TABLE 1. Overview of boundary layer properties (after Schröder *et al.* 2011).

space and time. The latter is the result of a time stepping method (correlating the volumes corresponding to image 1 with that from image 5 recorded 4 m s later, then image 2 with image 6 and so on), which increases the dynamic range of the measurement.

The vortical structures within the measurement domain were detected by a local evaluation of the velocity gradients (see §3), which were obtained by a second-order regression filter in space and time (Elsinga *et al.* 2010). The frequency response of this filter is similar to the inherent window averaging effect in PIV when taking the filter kernel size equal to the cross-correlation window.

The uncertainties in the present measurement can be assessed based on the r.m.s. of the divergence, which was equal to 3.5 s^{-1} . This value represents the uncertainty in the velocity gradients, since water is incompressible and consequently the measured flow should be divergence free to within experimental uncertainty. The uncertainty in the velocity is then estimated by multiplying the uncertainty in the gradients with the vector spacing, which gives 2.4 mm s^{-1} corresponding to $0.0045U_e$. Note that the velocity fields filtered by means of the mentioned regression are used throughout the paper.

The effect of spatial resolution has been assessed before in Elsinga & Marusic (2010) and Schröder *et al.* (2011). There it was found that the fluctuating velocity and Reynolds shear stress spectra for the present data compare well with other hot-wire measurement capturing 98 % of the energy, which indicates that the energy-containing scales of motion have been adequately resolved.

3. Vortex detection and tracking method

3.1. Spanwise vortex elements

The individual snapshots contain a variety of three-dimensional vortical structures and before following them in time, first a detection method is required to locate individual events. In this paper, the focus will be on spanwise vortex elements, which are defined as regions of positive spanwise swirling strength, $\lambda_{ci,y} \cdot \text{sign}(\omega_y)$. The detected features may not be an isolated structure, but can be part of a larger vortex. For example, a hairpin vortex contains a spanwise vortex element, i.e. the head, with quasi-streamwise vortex elements on either side, i.e. the so-called legs. The spanwise swirling strength, $\lambda_{ci,y}$, used here is the absolute value of the imaginary part of the eigenvalue of the reduced velocity gradient tensor J_{uw} , given by

$$J_{uw} = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial z} \end{bmatrix} \quad (3.1)$$

which considers only the components in the streamwise, wall-normal plane. The eigenvalue $\lambda_{ci,y}$ is non-zero only when the velocity vectors in that plane describe a local swirling motion (Christensen & Adrian 2001). To differentiate between clockwise and counterclockwise swirling motion, the swirling strength is signed according to the local spanwise vorticity component ω_y .

The next step is then to create a template of the feature to be tracked, which is required to automate the subsequent detection of the structures in the instantaneous flow volumes. For this purpose, the average flow structure associated with the spanwise swirling event, $E = \lambda_{ci,y} \cdot \text{sign}(\omega_y)$, is used, which is estimated using a linear stochastic estimation (LSE) approach (Christensen & Adrian 2001). The present event E represents a scalar quantity obtained from the measured velocity distribution. The conditional average of the fluctuating velocity distribution $\langle \hat{V}'(x, y, z) | E(x', y', z') \rangle$ is then estimated as a linear function of E , where the coefficient is obtained by minimizing the mean-square error between the estimate and the true conditional average. This results in

$$\langle \hat{V}'(x' + r_x, y' + r_y, z) | E(x', y', z') \rangle \approx \frac{\langle V'(x' + r_x, y' + r_y, z) \cdot E(x', y', z') \rangle}{\langle E(x', y', z')^2 \rangle} E(x', y', z'). \quad (3.2)$$

As can be seen, the conditional average is estimated by unconditional two-point correlations, which are much easier to compute than the conditional average itself. Furthermore, if it is assumed that in a boundary layer the two-point correlations are (nearly) independent of the streamwise and spanwise coordinate, then the conditional average is only a function of r_x , r_y and z and $\langle \dots \rangle$ in (3.2) denotes averaging over all velocity field snapshots as well as spatial averaging in x and y . Previous investigations using LSE have indicated that $\lambda_{ci,y}$ is associated with the head of a hairpin type vortex (e.g. Christensen & Adrian 2001; Elsinga *et al.* 2010). Hairpins are a very prominent feature in the models for wall-bounded flow (see, for example, Adrian 2007), which has been a main motivation for choosing spanwise swirling strength. An additional reason is that spanwise swirl is more spatially compact than swirling in the other directions, which are associated with the legs of hairpins and can extend well beyond the height of the present measurement volume (examples are shown later in figure 2b,c). Hence, the spanwise element is easier to detect and track accurately. The present conditionally eddy resulting from the LSE is shown in figure 1(a). It reveals a hairpin/arch type structure, as expected. There is a slight remaining asymmetry in the legs of the hairpin, which must be attributed to the underlying correlation maps not being fully converged there. Note that these correlations are rather weak at the location of the legs and it would require a very large dataset to suppress even a small amount of noise. The head, however, is nearly symmetric. In addition, a horn is visible on the top-left side of the hairpin, which is believed to be real and due to contributions from the spanwise vortex elements connected to taller vortical structures extending in the wall-normal direction (see the examples presented in figures 2b and 2c and discussed below). These features are found on either side, but again due to their weak correlation with the spanwise swirling event they appear first on one side in the present iso-surface display. The horns have also been observed by Dennis & Nickels (2011).

From the conditional eddy we take again the distribution of the spanwise swirling strength magnitude and use that for the template (figure 1b). The template has a shape

FIGURE 1. (Colour online available at journals.cambridge.org/flm) (a) Conditional eddy of the average flow associated with a spanwise swirling event at $z/\delta = 0.2$ wall-normal distance. The iso-surface indicates vortical motion revealed by the Q -criterion. (b) Spanwise vortex template derived from the conditional eddy, which is used for detection of spanwise vortex elements. The grey scale indicates the spanwise swirling strength normalized by the peak value.

FIGURE 2. (Colour online) Three examples of spanwise vortex element trajectories (black lines) with corresponding extended vortex structure (iso-surface of constant $Q = 0.13U_e^2\delta^{-2} = 1.1 \times 10^{-4}u_\tau^4\nu^{-2}$) at the start and end of their trajectory, which are marked by the crosses (blue online).

that is slightly elongated in the spanwise direction, and corresponds to the head of the conditional eddy.

Structures are subsequently detected by finding the local maxima in the map resulting from a cross-correlation of the template with the measured instantaneous signed spanwise swirling strength distribution. The location of each maximum can be obtained to a higher, sub-grid-scale, precision by performing a local Gaussian peak fit in all three directions. The peak of the fitted function is taken as the location of the identified vortex structure. The outlined method of feature detection via cross-correlation with a template has similarities with the approaches presented in turbulent wakes by Ferre & Giralt (1989) who applied it on the basis of hot-wire rake measurements or Scarano, Benocci & Riethmuller (1999) who used planar PIV.

Once the spanwise vortices have been identified in the instantaneous volumes, they can be tracked in time using methods similar to those for tracking tracer particles in particle tracking velocimetry (e.g. Lüthi, Tsinober & Kinzelbach 2005). When

tracking a structure, its location at the next time step is first estimated based on the local average flow velocity. If a structure can be detected at this next time instant within a specified search radius from the estimated position (by the method described above), a link is established and the new location is added to the structure's trajectory. The procedure is repeated until the structure leaves the measurement domain or no structure is detected within the search radius (0.04δ in each direction, which is smaller than the typical distance between detected structures; see § 4.2). Approximately 85 % of the detected structures can be tracked over one or more time steps in this way. The other structures must be considered spurious, since the real coherent structures should reappear at least once due to the high temporal resolution. To reduce the noise in the final trajectories, a regression filter is applied, in which a second-order polynomial in time is fitted to each spatial coordinate over a window of 31 time steps (that is 31 ms corresponding to $\Delta t^+ = 15$ or $0.44\delta/U_c$). For this one-dimensional filter the cut-off wavelength is approximately equal to the window (see the appendix in the paper by Elsinga *et al.* 2010). A smaller window of 11 time steps was also tried yielding similar results, but with more noise.

For further analysis we consider here only the clearly detectable and traceable structures, which are not too close ($>0.07\delta$) to the borders of the measurement domain as the structure detection method by cross-correlation may not be sufficiently accurate there. Hence, only events at a distance between 0.11δ and 0.30δ from the wall are considered. Reliable detection is established by requiring that the cross-correlation coefficient during structure detection is above 0.5 (averaged over the trajectory), while traceability requires that a structure can be traced over more than 100 time steps corresponding to a convection distance of $\sim 1.2\delta$.

Some examples of the resulting trajectories are given in figure 2, with superposed the corresponding detected vortex structures at its start and end. The first structure (figure 2a) is a typical arch vortex in the lower part of the measurement domain (its legs are cut at the edge of the measurement domain). In the other two examples (figures 2b and 2c), the detected event is a spanwise element connected to a larger vortex element, which extends beyond the measurement domain and is inclined with respect to the wall. The spanwise vortex elements in figures 2(a) and 2(b) do not display an appreciable change over time, while in figure 2(c) the spanwise element starts as a thin bridge between to neighbouring taller structures and then thickens over time. The diameter increases by about a factor of two over this distance, but in the figure it appears to be more due to a 60 % increase in Q inside this vortex causing the Q iso-surface to move outward. The final result of the outlined tracking procedure is a total of 392 of such spanwise vortex trajectories (figure 3).

3.2. Tracking bulges

The average convection velocity depends on the spatial scale of the considered flow structure, as mentioned before, which likely applies to the variation in convection velocity as well. In order to explore scale dependence, a second-order spatial regression filter with a kernel size of 0.76δ , 0.28δ and 0.17δ in the streamwise, spanwise and wall-normal direction was applied to the measured velocity fields. The low-pass filtering brings forward the large-scale wall-normal vortices, as described in Elsinga *et al.* (2010), which are believed to be representative of large-scale bulges (Falco 1977). Unfortunately the present measurement domain is restricted in the wall-normal direction to $z/\delta < 0.30$. This does not allow us to observe the complete bulge structure and its head in particular, which is typically found at distances greater than 0.5δ from the wall. Therefore, the large-scale wall-normal swirling motion associated

FIGURE 3. Trajectories of 392 detected spanwise vortex elements in oblique (a), top (b) and side view (c). The spatial coordinates (Δx , Δy , Δz) are defined relative to the initial position of the vortex.

FIGURE 4. Large-scale wall-normal vortex template derived from the LSE conditional eddy. The grey scale indicates the wall-normal swirling strength normalized by the peak value.

with the neck of the bulge, which does reach down into the present measurement volume, is tracked instead.

A tracking of these large-scale wall-normal vortices is performed using a template derived from a LSE of the flow associated with a wall-normal swirl event in the filtered velocity fields. The (unsigned) wall-normal swirling strength $\lambda_{ci,z}$ is used here as the event E in (3.2). It is the imaginary part of the eigenvalue of the reduced velocity gradient tensor, J_{uv} , and indicates swirling motion in the plane parallel to the wall. The resulting LSE conditional eddy does not contain an important variation over the height of the measurement domain (see figure 11 in Elsinga *et al.* 2010) making it impossible to follow its motion in the wall-normal direction. Therefore, these large-scale wall-normal swirl events are tracked only in the two dimensions of the streamwise–spanwise plane at $z/\delta = 0.2$. The template used is shown in figure 4. Otherwise the procedure is the same as before (§ 3.1) resulting in 150 tracks (figure 5). Owing to the spatial filtering and the relatively large size of the template both affecting the data near the edges of the measurement domain, the large-scale vortices can only be traced with a high level of confidence over a streamwise distance of just 0.4δ around the centre of the measurement volume.

FIGURE 5. Trajectories of 150 detected large-scale wall-normal vortex elements representing bulges. The spatial coordinates are relative to the initial position of the vortex.

4. Dispersion of vortices

4.1. Convection of individual vortices

The spanwise vortex element tracks (figure 3) reveal a significant spreading relative to the mean convection. The average convection velocity of similar structures in a boundary layer and channel flow has already received considerable attention in the literature (e.g. Kim & Hussain 1993; Krogstad *et al.* 1998; Pirozzoli *et al.* 2008; Del Alamo & Jimenez 2009), and the tracking results just lend additional support for the earlier observations. The average convection velocity inferred from the tracks is constant over the considered time period and equal to $0.78U_e \pm 0.004U_e$ in the streamwise direction (equal to the local mean flow velocity at a distance of 0.28δ from the wall), $0.006U_e \pm 0.002U_e$ in the wall-normal direction and zero in the spanwise direction ($\pm 0.003U_e$). The quoted uncertainty was estimated as the ratio of the r.m.s. convection velocity divided by the square root of the number of tracks. In this analysis the distribution of convection velocities was assumed to be Gaussian. As already mentioned in the introduction, the mean is not the main interest here, but rather the variations around the average.

The spreading of structures is expressed in terms of the r.m.s. of the trajectory position as a function of time, which is shown in figure 6. The effect of experimental noise was removed from these profiles by a procedure outlined in the appendix. The remaining statistical uncertainty on the r.m.s., which is due to the finite number of samples, can be estimated as one over the square root of two times the number of tracks. This is again in the assumption of a Gaussian distribution. For all components, the resulting uncertainty is 3.6% of the r.m.s. value. The results show that the r.m.s. of the streamwise position grows fastest followed by the spanwise and wall-normal position.

Moreover, the curves are reminiscent of dispersion for infinitesimal particles in homogenous turbulence, which for the streamwise component is given by (Taylor 1921; Tennekes 1979):

$$\sigma_x^2(t) = 2\overline{u_c'^2}T_L[t - T_L(1 - e^{-t/T_L})] \quad (4.1)$$

where σ_x is the r.m.s. of the streamwise position, u_c' is the streamwise convection velocity fluctuation, T_L is the Lagrangian integral time scale and t is time. Indeed (4.1) fits the results well as shown in figure 6. The fitted values used for the Lagrangian time scale and the r.m.s. convection velocities are listed in table 2. It appears that the Lagrangian time scale for the streamwise dispersion is larger than for the other two directions, which suggests changes in the streamwise convection velocity of a given structure are relatively slow. Furthermore, when looking at the r.m.s. convection

FIGURE 6. (Colour online) Spreading of the vortex trajectories in figure 3, as represented by the r.m.s. of the relative positions Δx , Δy , Δz versus time (symbols). The dashed black lines are fits of Taylor's (1921) model for turbulent dispersion of infinitesimal particles (4.1), which suggest the observed spreading of the vortices can be described well by such a dispersion process up to at least $\Delta t U_e / \delta = 1.2$.

Direction	T_L [δ/U_e]	r.m.s. u'_c, v'_c, w'_c [U_e]	r.m.s. u', v', w' [U_e]
Streamwise, σ_x	1.0	0.070	0.071
Spanwise, σ_y	0.6	0.060	0.050
Wall-normal, σ_z	0.6	0.035	0.038

TABLE 2. Fit parameters Lagrangian integral time scale T_L and r.m.s. convection velocity components u'_c, v'_c, w'_c in the dispersion model (4.1) for the spanwise vortex elements. For comparison the r.m.s. flow velocities u', v', w' measured at a distance of 0.20δ from the wall are included.

velocity fluctuations (table 2), it is found that the streamwise and wall-normal components of the model fit are only slightly lower than the measured r.m.s. flow velocity fluctuations. The spanwise component, however, is $\sim 20\%$ larger than the flow velocity fluctuations, meaning that the spanwise vortex elements experience greater movements along its axis than what may be expected based on the flow velocity alone. The goodness of the fit in figure 6 demonstrates that the motion of spanwise vortices in a boundary layer can be considered as a turbulent diffusion process up to at least a convection distance of 1.8δ (the size of the present measurement domain). In this diffusion model the fluctuating convection velocities are close, but not identical, to the flow velocity fluctuations. To some extent this may be expected, since the characteristic time scale for local flow topology evolution is much larger (a convection distance of $\sim 11\delta$, Elsinga & Marusic 2010). Hence, the topology and spanwise vortices are not expected to undergo significant changes within the present observation length, and consequently they experience predominantly convection, which is what has been observed.

In addition, the instantaneous local flow velocity at the track location (u, v, w) has been determined and compared with the track velocity (u_c, v_c, w_c). The differences ($u_c - u$), ($v_c - v$) and ($w_c - w$) have been averaged over a time interval of $0.7\delta/U_e$ in order to reduce measurement uncertainties. The averaging interval was chosen

FIGURE 7. Histograms of the difference between the track velocity (u_c , v_c , w_c) and the local flow velocity at the track location (u , v , w).

close to the Lagrangian integral time scale T_L , after which significant changes in convection velocity start to occur. The resulting histograms of these normalized velocity differences are shown in figure 7 and reveal some important variation, which is further quantified by their r.m.s. values. These are $0.026U_e$, $0.064U_e$ and $0.026U_e$ for the streamwise, spanwise and all-normal component, respectively, each with a statistical uncertainty of 3.6%. From the above results it is concluded that, although the statistical properties (i.e. mean and r.m.s.) of the spanwise vortex convection velocity and the flow velocity are quite close, this does not mean that the instantaneous convection and flow velocities are always the same.

The magnitudes of the velocity differences ($u_c - u$), ($v_c - v$) and ($w_c - w$) are comparable to the Kolmogorov velocity scale, $0.013U_e$ at the present height in the boundary layer, or $u_\tau = 0.041U_e$. This observation seems consistent with the work of Holzner & Lüthi (2011), who considered the velocity of the turbulent non-turbulent interface relative to the local flow velocity. Like the present vortical structures, this interface is associated with small-scale turbulent motions (Westerweel *et al.* 2009), and it typically moves at speeds of the order of the Kolmogorov velocity scale relative to the local flow velocity (Holzner & Lüthi 2011).

4.2. Relative velocity of vortices

The detection and tracking approach offers the opportunity to explore more complex statistics involving multiple vortices. We make a first attempt at this by considering the relative velocity between two neighbouring spanwise vortex elements, in what may be considered part of a packet (Adrian 2007). The current understanding is that vortices within a packet would travel at the same velocity, which is something that can be easily verified. Even if the vortices are not part of a packet, their relative velocity will suggest a time scale for their interaction.

Here, streamwise-aligned vortices are taken into consideration, which are required to be at a relative position of ± 0.38 and ± 0.28 in the spanwise and the wall-normal direction, respectively. The spanwise range is chosen to match the width of the detection template (figure 1) and is similar to the width of a low-speed region, or packet, in the outer layer (see, for example, Tomkins & Adrian 2003). The wall-normal range is limited by the height of the measurement domain. Given these

FIGURE 8. Histogram of the separation distance, Δx_{sep} , between consecutive streamwise aligned spanwise vortex structures.

restrictions, figure 8 shows a histogram of the downstream streamwise distance to the next detected spanwise vortex element. The plot reveals a distinct peak for separation distances around 0.25δ corresponding to 140 wall units, which is consistent with our earlier observation of the typical spacing between consecutive vortex structures in a packet (Adrian, Meinhart & Tomkins 2000; Elsinga *et al.* 2010). For further analysis we consider only separation distances corresponding to the idealized or typical packets, and therefore require the streamwise spacing to be within the range $0.1 < \Delta x/\delta < 0.3$. This leaves a total of 86 spanwise vortex pairs.

The 86 pairs may seem to represent only a small fraction of the full dataset containing 392 spanwise vortices, but in fact it corresponds to 172 individual vortices, 130 of which are unique. This corresponds to 33% of the entire set, which is significant. Moreover, only idealized packets of detectable spanwise vortex elements are considered, and it should not be expected that all vortices fit this model perfectly. Out of the mentioned 130 distinct vortices, 88 contribute to only a single pair, while 42 are part of two pairs meaning there is one vortex upstream as well as one downstream and the packet consists of at least 3 vortices. At relatively low Reynolds numbers and in the outer region of the boundary layer, packets typically contain two or three vortices (Adrian *et al.* 2000). Hence, it is expected that when a particular vortex is indeed part of a packet, it will often be accompanied by only one or two other vortices. This seems consistent with our findings.

Histograms of the relative convection velocity are presented in figure 9. The relative velocity in this case is the difference in convection velocity between the downstream and the upstream vortex averaged over a $0.7\delta/U_e$ time interval. The averaging was performed to reduce measurement uncertainties with the averaging interval chosen close to the Lagrangian integral time scale T_L (table 2), after which significant changes in convection velocity start to occur. Even though the histograms in figure 9 are not fully converged, some basic observations can be made.

The results show that the relative velocity in the streamwise and wall-normal direction is small compared with the relative flow velocities, which means they move more coherently, adhering to the packet idea. This observation is supported further by the corresponding r.m.s. values (table 3). Clearly the r.m.s. of the vortex

FIGURE 9. (Colour online) Histogram of the relative convection velocity of a spanwise vortex structure with respect to another structure, which is located between 0.1δ and 0.3δ upstream. The solid lines are p.d.f.s of relative flow velocity for the same separation distance, but considering all points independent of the presence of any flow structure. The area of the p.d.f. is scaled to match the area under graph with that of the histogram.

Direction	r.m.s. $\Delta u'_c, \Delta v'_c, \Delta w'_c [U_e]$	r.m.s. $\Delta u', \Delta v', \Delta w' [U_e]$
Streamwise	0.079 ± 0.006	0.095 ± 0.000
Spanwise	0.078 ± 0.006	0.064 ± 0.000
Wall-normal	0.036 ± 0.003	0.051 ± 0.000

TABLE 3. The r.m.s. of the relative convection velocity of a spanwise vortex structure with respect to another structure, which is located between 0.1 and 0.3δ upstream, $(\Delta u'_c, \Delta v'_c, \Delta w'_c)$. For comparison the r.m.s. relative flow velocities, $(\Delta u', \Delta v', \Delta w')$, measured at the same separation distance are included. The reported uncertainty is due to the limited number of samples and is estimated as the r.m.s. value divided by the square root of twice the ensemble size.

relative convection velocity is smaller than r.m.s. of relative flow velocities for these components. This difference is larger than the statistical uncertainty. In contrast, the r.m.s. convection velocity is higher in the spanwise direction suggesting the vortices move more in their axial direction than what may be expected simply from the relative flow velocities. The latter is consistent with the r.m.s. spanwise convection velocity of a single spanwise vortex element being larger than the r.m.s. spanwise flow velocity (table 2). Further note that the r.m.s. relative flow velocities is approximately 30 % higher than the r.m.s. flow velocities in a single point (compare tables 3 and 2).

In a few cases the relative convection velocity can still be quite large, reaching values down to almost $-0.2U_e$ in the streamwise direction (figure 9). Over the $0.7\delta/U_e$ time interval this corresponds to a decrease in separation distance of 0.14δ , which is significant considering the typical initial spacing of $\sim 0.2\delta$. The initial gap will be completely closed, forcing an interaction between the two vortices, in a time period of $1.0\delta/U_e$ (assuming the convection velocities remain constant). This time scale is an order of magnitude lower than the characteristic time for the average local flow topology dynamics (Elsinga & Marusic 2010). Hence, there exists a very wide range

Direction	T_L [δ/U_e]	r.m.s. u'_c, v'_c [U_e]
Streamwise, σ_x	0.6	0.056
Spanwise, σ_y	0.6	0.028

TABLE 4. Model parameters Lagrangian integral time scale T_L and r.m.s. convection velocity components u'_c, v'_c, w'_c in the dispersion model (4.1) for the large-scale wall-normal vortices.

of time scales for coherent structure dynamics, where the fastest processes may already happen within a convection distance of one boundary layer thickness.

A visual example of two spanwise vortex elements approaching each other is given in figure 10. This pair was identified as having a relative convection velocity less than $-0.12U_e$ in the streamwise direction. Initially (figure 10a) their streamwise separation is 0.22δ where the upstream vortex is located at $x/\delta = 0.27$ and $z/\delta = 0.28$, while the downstream vortex is at $x/\delta = 0.49$ and $z/\delta = 0.20$. As these spanwise vortex elements approach, the upstream vortex (in red) is seen to first detach from the larger wall-normal vortex element on its right at $y/\delta = 1.0$ (figure 10a–c). This detachment is resulting from the detected spanwise vortex element convecting at a faster rate, which causes a distortion of its connection to the larger wall-normal vortex element. At the same time, the downstream vortex also changes shape. The small spanwise vortex element, which is part of the larger scale structure (in green), reduces in width (figure 10a–c). It is seen that these vortices indeed catch up after approximately $1.0\delta/U_e$ and merge (figure 10e). This example serves only to demonstrate that it is possible (although rare) for vortices to interact on such a short time scale.

4.3. Convection of bulges

The mean convection velocity of the bulges, obtained from the large-scale wall-normal vortex tracks (figure 5), is $0.74U_e \pm 0.005U_e$, which is 5 % lower than the local mean velocity. It seems to be consistent with trends reported in Del Alamo & Jimenez (2009) for the large scales of motion in the outer region of a turbulent channel.

The dispersion of the tracks is again expressed in terms of the r.m.s. relative track position over time (figure 11). The statistical uncertainty on the r.m.s. values in this case is estimated at 5.8 %. The spreading rate of the large-scale wall-normal vortex trajectories is lower than for the smaller scale spanwise vortex elements (compare with figure 6), which can be further quantified by comparing them to the dispersion model (4.1). The range over which there is reliable data is limited, and therefore the model has not been fit to the data in this case, but rather the model parameters were determined by other means. Specifically, the Lagrangian time scale is assumed to be $0.6\delta/U_e$ (after the results in table 2) and the fluctuating convection velocities have simply been taken equal to the flow fluctuations in the filtered velocity fields (values are listed in table 4). The present results seem to follow the dispersion model over the observation range.

5. Prediction of space–time correlations

The present results suggest that the flow structures mainly convect without significant change over a time of $1.2\delta/U_e$. This observation can be used to predict the space–time correlation of turbulent signals, as shown below.

FIGURE 10. Time series of two spanwise vortex elements approaching each other, which finally results in their merging. The vortical structures are represented by iso-surfaces of constant positive Q . The upstream vortex is coloured red and the downstream vortex shown in green. The blue crosses mark the locations of the two detected spanwise vortex elements. In the frames (e) and (f) the structures have merged. The elapsed time relative to the first frame (a) is $\Delta t U_e / \delta = 0.24$ (b), 0.56 (c), 0.74 (d), 0.99 (e) and 1.46 (f) respectively.

Consider a turbulent signal containing the turbulent eddies. From here on, we use the streamwise velocity distribution $u(\mathbf{x}, t)$ as an example, but other quantities (such as the invariants of the velocity gradient tensor) are equally possible. Then the eddy shape can be characterized statistically by the instantaneous auto-correlation

FIGURE 11. (Colour online) Spreading of the large-scale wall-normal vortex trajectories in figure 5, as represented by the r.m.s. of the relative positions Δx , Δy , Δz versus time (symbols). The dashed black lines are Taylor's (1921) model for turbulent dispersion of infinitesimal particles (4.1).

function $A_{uu}(s)$:

$$A_{uu}(s) = \frac{1}{N} \sum_{\mathbf{x}, t} u(\mathbf{x}, t) u(\mathbf{x} + s, t) \quad (5.1)$$

where N is the number of samples. Assuming that the eddies do not evolve in time and are simply convected by a constant velocity $\mathbf{u}_c$, then $u(\mathbf{x}, t) = u(\mathbf{x} + \mathbf{u}_c \Delta t, t + \Delta t)$, which can also be written as a convolution:

$$u(\mathbf{x}, t + \Delta t) = \int u(\mathbf{x} - \mathbf{r}, t) \delta(\mathbf{r} - \mathbf{u}_c \Delta t) d\mathbf{r} \quad (5.2)$$

where δ is the Dirac delta function. The hairpin tracking results, however, indicate that the convective velocity is not a constant. Allowing for these variations in the convective trajectories $\mathbf{r}(\Delta t)$ according to a p.d.f. $P(\mathbf{r}, \Delta t)$ (still assuming a pure convection without any eddy evolution), the expected velocity distribution at time $t + \Delta t$ is given by

$$u(\mathbf{x}, t + \Delta t) = \int u(\mathbf{x} - \mathbf{r}, t) P(\mathbf{r}, \Delta t) d\mathbf{r} \quad (5.3)$$

or in discrete form

$$u(\mathbf{x}, t + \Delta t) = \sum_{\mathbf{r}} u(\mathbf{x} - \mathbf{r}, t) P(\mathbf{r}, \Delta t) \quad (5.4)$$

with

$$\sum_{\mathbf{r}} P(\mathbf{r}, \Delta t) = 1. \quad (5.5)$$

Subsequently, the space–time correlation $R_{uu}(s, \Delta t)$ for non-evolving eddies can be estimated as

$$\begin{aligned}
 R_{uu}(s, \Delta t) &= \frac{1}{N} \sum_{\mathbf{x}, t} u(\mathbf{x}, t) u(\mathbf{x} + \mathbf{s}, t + \Delta t) \\
 &= \sum_{\mathbf{r}} \frac{1}{N} \sum_{\mathbf{x}, t} u(\mathbf{x}, t) u(\mathbf{x} + \mathbf{s} - \mathbf{r}, t) P(\mathbf{r}, \Delta t) \\
 &= \sum_{\mathbf{r}} A_{uu}(\mathbf{s} - \mathbf{r}) P(\mathbf{r}, \Delta t).
 \end{aligned} \tag{5.6}$$

The space–time correlation is a convolution of the instantaneous auto-correlation A_{uu} with the p.d.f. $P(\mathbf{r}, \Delta t)$ representing a statistical description of the convection and spreading rate of the eddies (figure 6). The function $P(\mathbf{r}, \Delta t)$ will be approximated by a Gaussian distribution with the r.m.s. given by the dispersion model (tables 2 and 4). A comparison with the actually measured distribution function suggests that the approximation is appropriate.

Equation (5.6) has been evaluated to predict the space–time correlation R_{QQ} for the second invariant of the velocity gradient tensor Q (figure 12). The distribution $P(\mathbf{r}, \Delta t)$ for the spanwise vortex elements (table 2) was used in this case. The prediction is found to follow both the broadening and the decrease of the correlation peak with time, which further supports the underlying assumptions of a passive eddy convection. The same can be said for the space–time correlation R_{uu} for the fluctuating streamwise velocity component (figure 13). The fluctuating velocity is typically a large-scale quantity, as can be seen from the correlation peak width (compare also with the correlation peak width of Q , which is associated with the small scales). Therefore, the distribution $P(\mathbf{r}, \Delta t)$ for the large-scale structures (table 4) was used to predict R_{uu} yielding a more accurate result compared with using the spreading rate for the smaller-scale spanwise vortex elements. The wall-normal convection for the large-scale structures was not available and was estimated by the r.m.s. flow fluctuations in the filtered velocity fields ($w'_c = 0.022U_e$).

The decrease in the correlation peak height over time was used in the past to validate or disprove Taylor’s hypothesis. The present results indicate that this decrease can be attributed to the dispersion of flow structures rather than any actual change in topology. Therefore, the hypothesis may still be useful in studying certain aspects of turbulence (e.g. the nature of the flow topology does not change significantly), even though the correlation peak may drop considerably. This drop will be stronger for the small scales, first of all because their correlation peak is narrow compared with the spreading rate of the flow structures, but also due to this spreading rate being larger than for the large scales.

A word of caution may be added to the last point. Recently, Schröder *et al.* (2011) have suggested that specific, strong events can undergo visible topological change within a time period of $0.88/U_e$. While their reported time evolution of conditional averages for strong vorticity and ejection events reveal only slight changes in the inclination angle, which they explained by the variation in convection velocity with wall-normal distance (consistent with what we propose here), a rapid transformation is reported for the strong sweep events. Such strong sweep events are rare, however, and contribute only marginally to the overall correlations presented in this paper. The same may be said for the few cases where the relative velocity of neighbouring structures is large (§ 4.2).

FIGURE 12. Predicted (red) and measured (blue) space-time auto-correlation $R_{QQ}(\Delta x, \Delta y, \Delta z, \Delta t)$ of the second invariant of the velocity gradient tensor Q , plotted along $\Delta y = \Delta z = 0$. The prediction is based on the instantaneous auto-correlation function $R_{QQ}(\Delta x, \Delta y, \Delta z, \Delta t = 0)$ and uses the observed spreading rate of the flow structures (figure 6) to predict the temporal development.

FIGURE 13. Predicted (red) and measured (blue) space-time auto-correlation $R_{uu}(\Delta x, \Delta y, \Delta z, \Delta t)$ plotted along $\Delta y = \Delta z = 0$. The prediction is based on the instantaneous auto-correlation function $R_{uu}(\Delta x, \Delta y, \Delta z, \Delta t = 0)$ and uses the observed spreading rate of the large-scale flow structures (figure 11) to get the temporal development.

6. Conclusions

Spanwise vortex elements and large-scale wall-normal vortices, representing bulges, were tracked in time in the outer layer of a turbulent boundary layer. The following points summarize the main conclusions.

(i) The average spanwise vortex element trajectory corresponds to the local average flow velocity, while the r.m.s. track position can be characterized as dispersion of infinitesimal particles in homogenous turbulence (4.1). The magnitude of the

Direction	r.m.s. before filter [—]
Streamwise, $\Delta x/\delta$	0.007
Spanwise, $\Delta y/\delta$	0.010
Wall-normal, $\Delta z/\delta$	0.006

TABLE 5. Noise in the spanwise vortex element tracks before temporal filtering has been applied.

fluctuating convection velocity in this model is close, but not identical, to the magnitude of the velocity fluctuations at the present height in the boundary layer.

(ii) The instantaneous convection velocity of the spanwise vortex elements, however, does not strictly follow the instantaneous local flow velocity. The variation of the velocity difference appears to be of the order of the Kolmogorov velocity scale or the wall friction velocity.

(iii) The large-scale wall-normal vortices move on average at a speed below the local average flow velocity. The r.m.s. track position can again be characterized as turbulent dispersion of infinitesimal particles, like for the smaller scale structures. The fluctuating convection velocities, however, are lower for the large-scale structures.

(iv) The relative velocity of two (nearly) streamwise aligned, spanwise vortex elements was determined. The majority of such vortex pairs have a negligible velocity difference indicating that they move coherently. In some cases, however, the structures approach each other at speeds exceeding $0.2U_e$, which suggests they interact on a time scale of the order of $1\delta/U_e$ (an order of magnitude lower than the time characteristic for the average local flow topology dynamics).

(v) Assuming the vortical structures predominantly convect without interaction or significant topological change, the space–time correlations for different flow variables can be predicted based on their instantaneous auto-correlation and their spreading rate (i.e. the p.d.f. of their track position in time). This was illustrated for two flow variables: the streamwise fluctuating velocity, u , and the second invariant of the velocity gradient tensor, Q . The prediction describes well the decrease and broadening of the correlation peak in time (at least up to the present observation time, $1.2\delta/U_e$). These temporal changes in the correlation must, therefore, be attributed mostly to the dispersion of flow structures rather than any actual change in topology.

Appendix. Noise estimation and correction

The vortex tracks naturally contain some noise, which will affect the measured r.m.s. of the relative track positions. The noise correction outlined in this appendix has been used to produce figure 6.

In order to estimate the noise level, the r.m.s. profiles for the spanwise vortex elements are plotted in figure 14 for the case where no temporal filtering has been applied to smooth the tracks. Apart from the filtering, there is no difference with respect to tracking procedure outlined in § 3.1. The first independent data point away from $\Delta t = 0$ reveals values for the r.m.s., which are almost an order of magnitude higher compared with what is expected, when assuming the initial spreading of tracks is solely due to variations in the local instantaneous flow velocity. Therefore, the r.m.s. of the relative position in this point is attributed to the noise in the unfiltered vortex tracks. The r.m.s. error in the relative position relates to the r.m.s. error of the absolute track position through a division by $\sqrt{2}$. The resulting values are listed in table 5.

FIGURE 14. (Colour online) The r.m.s. of the relative track positions without temporal filtering (only independent samples shown).

FIGURE 15. (Colour online) The r.m.s. of the relative track positions with temporal filtering before (solid lines without symbols) and after noise correction (with symbols).

The r.m.s. noise on the track position is less than a grid spacing (0.018δ) and largest for the spanwise direction. The latter is likely due to the elongation of the spanwise vortex structure in that direction (figure 1), which causes in a broader correlation peak in the structure detection stage. This results in less accurate peak localization.

The effect of noise on the complete r.m.s. profiles has been estimated taking noise-free tracks and adding to it white noise with the amplitude according to table 5. The noise-free tracks are a linear model after the measured spanwise vortex element tracks (figure 3), in which straight lines connect the initial and the final point. After applying the temporal filter, the r.m.s. profiles are compared. The difference between the case with and without noise is then the contribution of the noise to the r.m.s. profiles. This has been subtracted from the originally measured r.m.s. profiles. From figure 15 it

Direction	r.m.s. before filter [—]
Streamwise, $\Delta x/\delta$	0.019
Spanwise, $\Delta y/\delta$	0.008

TABLE 6. Noise in the large-scale wall-normal vortex element tracks before temporal filtering has been applied.

can be seen that the correction is small and can hardly be noticed for the streamwise component. Furthermore, it mainly affects the r.m.s. in the region $\Delta t U_e/\delta < 0.5$.

The noise level in the unfiltered tracks for the large-scale wall-normal vortices has been determined in a similar way. Results are presented in table 6 and are again used to correct the profiles of r.m.s. relative track position.

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