Determination of the Optimal Pipe Configuration and Diameters for a District Heating System

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Chapter 1

Abstract

In this report a district heating system is analyzed at the Tata steel site. A case study is made for a number of buildings, which are currently heated with a gas boiler. A computer model is developed to calculate the optimal piping path and diameters for such a network. The heat losses of this network are calculated with the complex potential method. The payback period was found to be around 5 years, depending on the economic conditions.
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Chapter 2

Introduction

Tata steel has set a goal of reducing the energy consumption of the company by 2 % each year from 2010 to 2030. Every division of Tata steel has to meet this goal. One of these divisions is site facilities. Site facilities is the division where all the offices are maintained and controlled. As such, the energy consumption of the offices buildings on site are accounted for by site facilities.

The process of steel making requires a tremendous amount of energy. Therefore Tata steel is by far the greatest energy consumer in the Netherlands. However a lot of this energy is “wasted” by dumping it to the environment. Meanwhile natural gas is burned to heat the office buildings. This waste is well known at Tatasteel. Therefore several studies have been made at Tata Steel to use this waste heat. One of the potential uses for this waste heat, is to heat the buildings with this energy. Therefore several studies have been conducted to determine the viability of such practice.[32] [34] The primary problem in these studies found, was always cost. The investment cost are very high for a district heating system, and these projects never made it past the development stage.

Although a large district heating network was not considered economic viable, smaller localized networks were not investigated. Smaller networks are potentially cheaper and can therefore be an interesting alternative to the natural gas boilers currently used for heating. The aim of this report is to determine the economic viability of such smaller networks.

2.1 Network analysis

At the Tata steel site the office buildings are generally close to the the production process. This means most of the offices are scattered across the terrain.
Figure 2.1: The buildings in the 3D and 4D area that are currently heated with gas boilers.

For district heating this is generally not favorable, and this is the main reason why a large network at the site is too expensive. A notable exception are the buildings in the 3D and 4D region. In this area the offices are closer too each other compared to the other regions on the site. To determine the viability of a smaller network on the site, this is the best place to start an analysis.
Chapter 3

Analyses of physical and economical relations

3.1 Introduction

To design an optimum district heating system, a computer model is developed. The goal of the computer model is to produce the most economic district heating pipe route and diameters. The model will determine the optimum based on the pipe and pump cost, and the cost of the electricity the pump uses. Heat losses are not taken into account, because the network is powered by waste heat. Higher heat losses will result in slightly higher cost, because the equipment size will become larger, but this is assumed to be negligible. The heat losses of the network are calculated in chapter 7. The objective function is solved with an nonlinear algorithm. The choice of the correct algorithm is discussed in chapter 4.

3.2 Physical relations

In a district heating system there are several physical phenomena. One of the more important physics in district heating systems is pipe friction. In the past several correlations have been developed. One of the most common methods to calculate these losses, is the Darcy-Weischbach friction equation. The Darcy-Weischbach friction equation is expressed in the following formula:

\[ \Delta P = f_d \left( \frac{L}{D} + \xi \right) \frac{\rho * V^2}{2} \]  

(3.1)

In this equation \( f_d \) is the Darcy friction factor, \( D \) is the pipe diameter and \( L \) the pipe length. The Parameter \( \xi \) is to model local losses, such as valves, bends in the pipes etc. \( \rho \) and \( V \) are the density and velocity of the fluid.
Depending on the Reynolds number the flow is either Laminar, Turbulent or Transient. For each type of flow there are different relations for the Darcy friction factor $f_d$.

For Laminar flow with $Re < 2300$ the so called Hagen-Poiseuille equation holds true:

$$f_{lam} = \frac{64}{Re}$$  \hfill (3.2)

For turbulent flow there are several different relations. The most recent improvement on existing relation, which is verified for $4 \times 10^3 < Re < 1 \times 10^8$, is given by the following relation [26].

$$f_{turb} = \frac{0.2479 - 0.0000947 \times (7 - \log_{10}(Re))^4}{[\log_{10}(k/3.615 + \frac{7.366}{Re^{0.9142}})]^2}$$  \hfill (3.3)

In this equation $k$ is the relative roughness $k = \frac{\epsilon}{D}$. The transient region is up to this day a not well understood region. There are several relations in use, but most have a lot of uncertainties in the used approximations. To avoid numerical problems in the Transient region, a smoothing function, which connects the laminar and turbulent region, is used.

It must be noted that final results will probably be in the turbulent region. The Reynolds number will likely be very high, typically $Re > 10^5$. The laminar and transient region still need to be modeled, because the program can explore solutions in these regions. These regions therefore only serve the purpose of avoiding numerical problems and infeasible solutions.

### 3.2.1 Smoothing function

The step between the laminar and turbulent region is a mathematical singularity. Numerical programs have problems evaluating singularities. To overcome this problem the two functions are smoothed. When this is not properly done, a program may not reach the optimum solution, or even no solution at all.

The smoothing is achieved with a so called smooth function. The function used here to connect the two functions is a sigmoid function. This is a mathematical S-shape, which is differentiable for all real inputs. This means that there are no numerical problems when evaluation this function.

$$S = \frac{1}{1 + e^{-t}}$$  \hfill (3.4)

When the sigmoid function is used to connect the friction function it has to be rewritten to:

$$\sigma(Re) = \frac{1}{1 + e^{(Re-Re_0)/a}}$$  \hfill (3.5)
In this function \( \alpha \) is the smoothing parameter. The value of \( \alpha \) is chosen based on the “smoothness” of the function.

The two friction functions can now be connected with the following relation:

\[
f_n = (1 - \sigma) \ast f_{lam} + \sigma \ast f_{turb}
\]  

(3.6)

The results of the smoothing function can be seen in figure 3.1b. This function is valid from \( 0 < Re < 1 \times 10^8 \). It is assumed that Reynolds numbers higher than this are not a valid solution to the problem. Because the computer model can have an intermediate solution with Reynolds numbers higher than \( 10^8 \). Therefore it is assumed that \( f_d \) for \( Re > 1 \times 10^8 \) is equal to \( f_d \) at \( Re = 1 \times 10^8 \).

### 3.3 Economic analyses

The main purpose of this model is to minimize cost of the district heating network. The cost span the entire lifetime of the network. There are two types of cost, capitalized cost and variable cost. The capital cost include the cost of the network construction, the pipe cost and the investment cost of the pump. The variable cost are the electricity cost, maintenance cost, and the cost of the lost heat.

The investment cost of district heating systems are very large. It is therefore important to accurately predict the cost of the pipes. There are several examples in the literature. Depending on the source this is either a linear approximation, or a polynomial function. In general polynomial function is preferred over the linear, because a polynomial function can approximate data more accurate. [21][33][27]

\[
C_{pipes} = A_c + B_c \ast d_{pipe} + C_c \ast d_{pipe}^2
\]  

(3.7)

The construction cost can be approximated with the same analogy.

\[
C_{cons} = D_c + E_c \ast d_{pipe} + F_c \ast d_{pipe}^2
\]  

(3.8)

The constant \( F_c \) and \( C_c \) cause the pipes to have a cost higher then zero when the diameter is zero. This means that even if there is no pipe, because the diameter is zero, the imaginary pipe still has cost. This is of course not realistic when evaluating the system. To have a more accurate prediction of the cost the function of the pipe and construction cost, they are connected to the origin in a similar way the laminar and turbulent flow are connected to each other (see section 3.2.1 for more information). With this correction
Figure 3.1: Smoothing function vs Moody diagram. The end result will likely be outside this region.
the cost are indeed zero when the pipe diameter is zero as shown in figure 3.2.

The investment cost of the pump is mainly dependent on the power of the pump. To estimate pump cost a linear relation between the pump power and cost can be developed. The accuracy of this approximation can be seen in figure 3.6. The power of the pump is dependent of the flow. The relation that is used is shown in equation 3.9. The cost can be accurately approximated with a linear function.[2] [25]
Figure 3.4: Cost of a PE pipe with various insulation thickness. The trend line is chosen for an insulation thickness of 50mm. Notice how the insulation thickness has very little influence on the total cost of the pipe.[25]

Figure 3.5: Cost of an underground pipeline. Duplex stands for stainless steel and CS for Carbon Steel. Because this data is based on a total pipe length of 10km, the trend line is plotted on the duplex high estimate.
Table 3.1: Parameters of the different sources for the insulated pipeline

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>A</td>
<td>18</td>
<td>€</td>
<td>46.703</td>
<td>€</td>
</tr>
<tr>
<td>B</td>
<td>291</td>
<td>€/m</td>
<td>46.829</td>
<td>€/m</td>
</tr>
<tr>
<td>C</td>
<td>229</td>
<td>€/m²</td>
<td>3114.9</td>
<td>€/m²</td>
</tr>
<tr>
<td>D</td>
<td>287</td>
<td>€</td>
<td>127.52</td>
<td>€</td>
</tr>
<tr>
<td>E</td>
<td>310</td>
<td>€/m</td>
<td>4796.1</td>
<td>€/m</td>
</tr>
<tr>
<td>F</td>
<td>1275</td>
<td>€/m²</td>
<td>1703.1</td>
<td>€/m²</td>
</tr>
</tbody>
</table>

Figure 3.6: Pump price from different sources
\[ P = \frac{q_e \Delta P}{\eta} \quad (3.9) \]

\[ C_{\text{pump}} = C_{p,a} + C_{p,b} \times P_{\text{pump}} \quad (3.10) \]

The cost of the electricity of the pump can be determined as a function of the pumping power, electricity price and pumping operation time.

\[ C_{\text{elec}} = C_e \times P \times t \quad (3.11) \]

Normally the cost of heat losses are also included. Since the heat source in this case is waste heat, the heat can be considered “free”. Instead of determining the cost of the heat losses, the insulation thickness is based on the temperature requirement of the buildings, so that in each case the demand of the heat can be met.

The total cost can be expressed as

\[ Total_c = C_{\text{pipes}} + C_{\text{cons}} + C_{\text{pump}} + C_{\text{elec}} \quad (3.12) \]

The influence of each cost parameter is displayed in figure 3.7. The figure shows that smaller pipes lead to lower investment cost of the pipeline, but also lead to greater cost of electricity used. The increased pumping power also increases the pump investment cost. The optimum pipe is therefore neither too small, nor too large.

### 3.4 Constrains

To solve the equations proposed in earlier chapters, a computer model is developed. the model is a constrained linear system of nonlinear equations.

The purpose of the model is to find the minimum cost of the the total pipe network. The objection function can be seen in equation 3.12. The goal of the computer program is minimize this cost function for the total system. The system has several hydraulic constrains.

\[ qv_1 = \sum_{i=2}^{N_c} q_{vi} \quad (3.13) \]

\[ \sum_{j=2}^{N_c} q_{vij} + q_{vi} = 0 (i = 1, 2, 3, ..., N_v) \quad (3.14) \]

In these constrains the \( qv_1 \) is the volume flow from the source, and \( q_{vi} \) is the volume flow from the buildings. The variable \( q_{vij} \) is the volume flow in the pipes.
Figure 3.7: Plot of the various cost for a pipe of 100m length and a flow of $10 \times 10^{-3} \text{m}^3\text{s}^{-1}$. These values are chosen randomly, but these values are of the same order of magnitude that can be expected in a district heating network.
The last hydraulic constrain is that the flow cannot go from a lower to a higher pressure. To allow the program to solve an simpler problem, the constrained is replaced with a penalty function. A penalty function is an equation that has a value higher than zero when the constrains are violated. The optimization program will therefore avoid having an infeasible solution as optimal [13]. The penalty function for this problem is shown in equation 3.15.

\[ \text{Penalty} = \beta \times q_v \times \text{violation} \]  

The parameter \( \beta \) is an arbitrary high number to ensure a significant impact on the cost of the design when the constrains are violated. Usually the maximum cost of the total network is used for \( \beta \) [14] (maximum network cost are the cost when every pipe in the network has the maximum available diameter). In this case the value of \( \beta \) was set at 2 million euro, which produced satisfying results. When the constrain is not met the violation will be a positive number, and when the constrains are satisfied, the violation is a zero.
Chapter 4

Computer Model

In the previous chapters the economic and physical relations were obtained. In essence this is an optimization problem. To solve these kind of problems different methods have been proposed in the past. One of the most common methods to solve this problem is by (non)linear programming.

Linear programming was one of the most important uses of computer power in the modern times. It was originally used in WW2 to reduce the cost of the military through optimization. Since then a lot of algorithms have been developed to solve more complex problems. Each algorithms have their own strengths and weaknesses. To choose the right algorithm for the right job can be a complex task. To select the best algorithm possible a number of algorithms are evaluated.

Non-linear programming has, in contrary to Linear programming, non-linear objective functions, and/or constrains. This results in much more complex algorithms. Because the objective function presented in chapter 3.3 is nonlinear, the problem cannot be solved with linear programming. This means the more complex field of Non-linear programming is required.

Most complex algorithms are based on simpler ones. For this reason, the algorithms that cannot be used because of obvious limitations are still briefly discussed to get a deeper understanding of the underlying mathematics. This is necessary to understand how the more complex algorithms work.

4.1 Algorithms in nonlinear programming

The problem described in the earlier chapters is a Nonlinear Program (NLP) optimization problem. In general a NLP can be described as:
min \( f(x) \)
\[ c(x) \leq 0 \text{ for inequality constrains} \] (4.1)
\[ c(x) = 0 \text{ for equality constrains} \]

Where \( x \) is moving in a continuous way in the feasible set \( X \) that is defined by the inequality and equality constrains. To solve these kind of problems different methods have been developed.

To reach a optimal solution there are optimization conditions. For more advanced methods these often involve the Hessian matrix and the Lagrange multiplier. The Hessian matrix for a function \( f(x_1, x_2, \ldots, x_n) \) is defined as:

\[
H(f)_{ij}(x) = D_iD_jf(x) = \begin{pmatrix}
\frac{\delta^2 f}{\delta x_1 \delta x_1} & \frac{\delta^2 f}{\delta x_1 \delta x_2} & \cdots & \frac{\delta^2 f}{\delta x_1 \delta x_n} \\
\frac{\delta^2 f}{\delta x_2 \delta x_1} & \frac{\delta^2 f}{\delta x_2 \delta x_2} & \cdots & \frac{\delta^2 f}{\delta x_2 \delta x_n} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\delta^2 f}{\delta x_n \delta x_1} & \frac{\delta^2 f}{\delta x_n \delta x_2} & \cdots & \frac{\delta^2 f}{\delta x_n \delta x_n}
\end{pmatrix} \quad (4.2)
\]

The Hessian matrix is related to the Jacobian matrix in the following fashion:

\[ H(f)(x) = J(\nabla f)(x) \quad (4.3) \]

The Lagrange multiplier is also often used in optimization. For a function \( f(x_n) \) constrained by \( g(x_n) \) the Lagrangian is defined as:

\[
\frac{\delta f}{\delta x_k} + \lambda \frac{\delta g}{\delta x_k} = 0 \quad (4.4)
\]

For \( k = 1, \ldots, n \), where the constant \( \lambda \) is the Lagrange multiplier. [3]

### 4.2 Unconstrained Optimization

Although the district heating network problem is a constrained problem, unconstrained algorithms are still discussed here. This is mainly because the constrained algorithms are based unconstrained problems. The methods used in unconstrained and constrained optimization can be divided in several groups.

#### 4.2.1 Interval reduction

The first group of optimization algorithms is based on interval reduction. As the name implies these methods reduce the interval of the function, until
Figure 4.1: Example of the Golden Search algorithm. In figure (a) the function value of \( f(x_4) > f(x_2) \) and \( f(x_3) > f(x_1) \). This means the minimum is on the interval \([x_1, x_4]\). In figure b the next interval of the search is \([x_2, x_3]\). These intervals are reduced until the minimum is found. \[29\]

the optimum of that function is found. These algorithms generally solve single variable problems only. The algorithms are typically very robust, but also rather slow. These algorithms are generally restricted to single variable problems only. Prime examples are the Bisect search algorithm and the Golden Section Search method.

The Bisect method halves the search interval of each iteration and uses the derivative of the two points to determine the search direction. Compared to more advanced methods the halving of the search interval is a slow method to reach the optimum.

The Golden search section works similar to the Bisect search, but uses the function value instead of its derivative. This results in smaller steps, and therefore more iterations, but each iteration has only one function evaluation. The Golden search method owns its name to the Golden ratio \( \varphi = \frac{1+\sqrt{5}}{2} \) which is the ratio of the step size of the next iterative point. An example of this algorithm is shown in figure 4.1. Both the bisect and Golden search method are rather slow compared to other methods, but they are very robust. As long as the function is continuous these methods will always find a solution.

### 4.2.2 Interpolation

More advanced methods use interpolation along with interval reduction. Examples of these methods are the Quadratic and Cubic Interpolation and Newton’s method. Quadratic interpolation uses the interval points to fit a parabolic function between the end points. The parabola is then used to
determine the next iteration point. This method is usually used in conjunction with the golden search method, because sometimes a parabola cannot be constructed. The Golden search method is then used to determine the next interval. This approach is called Brent’s method. An example of this is shown in figure 4.2.

Cubic interpolation is similar to Quadratic interpolation. It constructs a function based on the endpoints, but in this case instead of a parabola, a cubic polynomial is formed. The advantage compared to Quadratic interpolation, is that this method converges faster. The price to pay for this advantage is that this method also requires the derivatives of the function. The equation to determine the next iteration is shown in equation number 4.5

\[
x_k = b - (b - a) \frac{f'(b) + v - u}{f'(b) - f'(a) + 2v} \\
u = f'(a) + f'(b) - 3 \frac{f(a) - f(b)}{a - b} \\
v = \sqrt{u^2 - f'(a)f'(b)}
\]

(4.5)

Newton’s method uses the Newton-Raphson iterative formula to predict the next iteration step. Newton’s method Convergence speed is comparable
to the Cubic interpolation method. The modified Newton-Raphson formula used in this method is shown in equation 4.6. This algorithm can easily be adapted to more complex problems as later chapters show. While the Newton Raphson formula uses the function value and it’s derivative, the Newton method uses the derivative and second derivative to determine the next iteration step.

\[ x_{k+1} = x_k - \frac{f'(x)}{f''(k)} \]  

\(4.6\)

### 4.2.3 Derivative free

Sometimes derivative information is not available. While it is possible to numerically evaluate the functions, other methods have also been developed. The most well known methods to solve these problems are the Nelder and Mead method and Powell’s method.

Nelder and Mead method is also known as the downhill simplex method. Nelder and mead uses a simplex, which is a polytopes of \(N + 1\) vertices’s with \(N\) the number of dimensions. The simplex is extrapolated based on the objective function evaluation in each point. The simplex is then either reflected, contracted or expanded, based on the function values returned by these methods. This is shown in figure 4.3. This method is a standard implement in many software packages, including Matlab.

Powell’s method, also called Powell’s conjugate direction method, has slightly different approach. Powell’s method doesn’t use a simplex, but has \(N\) amount of step directions. These are then used to determine the next search direction. The step size is determined with exact line search.[28]

### 4.2.4 Derivative methods

When derivative information is available it is generally excepted that using this information results in a faster computational time. To use this characteristic several methods have been developed. The most well known algorithms are discussed here.

The steepest descend algorithm uses, like it’s name implies, the steepest direction to determine the next iteration. To calculate the the search direction the gradient of the function is used. Thats why this method is sometimes called the gradient method. The search direction is always perpendicular tot the function. The perpendicular search will result in a zig- zag search pattern when the solutions of different function evaluations are relatively close
Figure 4.3: The Nelder and Mead simplex for a two variable problem. The fitness of each solution is analyzed in each step, and W is the worst solution. C is the center between the other two solutions. The algorithm then produces a reflection, as shown in the upper right corner. When this does not improve the results, the other options are explored. This is repeated until the optimum is found. [4]
Figure 4.4: Comparison between the Newton method, steepest descent an Conjugate Gradient method. The steepest decent algorithm clearly shows zig-zag behavior, while the Conjugate Gradient method has not. Newtons method converges the fastest. [5]

to each other. This results in a slower computational time when the function is close to a minimum. The search gradient is expressed in equation 4.7.

$$\min_{r \in \mathbb{R}^n} \frac{\nabla f(x)^T r}{||r||}$$

(4.7)

To improve the zig-zag behavior of the steepest decent algorithm, the conjugate gradient method was developed. The search direction is modified by a combination of the earlier directions. This will generally result in a faster computational time. The expansion on the original function is shown in equation 4.8.

$$x_{k+1} = x_k + \lambda r_k$$
$$r_k = -\nabla f_k + \beta_k r_{k-1}$$
$$\beta_k = \frac{\nabla f_k^T (\nabla f_k - \nabla f_{k-1})}{r_k^T (\nabla f_k - \nabla f_{k-1})}$$

(4.8)

Newton’s method, which was already discussed for single variable problems, was also adapted for multi variable problems. To implement Newton’s method for multi variable problems the Hessian matrix is used. When the gradient of the function is zero, and the Hessian is positive and invertible, an optimum is found. The drawback of this method is the Hessian matrix itself. Sometimes it is not positive or invertible, which means the function can not be evaluated. This characteristic results in a less robust algorithm.
To improve Newton’s method, the so-called Quasi-Newton method was implemented. Just like the Newton method this one is based on the Hessian matrix. It differs from the Newton method by estimating the Hessian matrix, instead of computing it at every iteration. The Quasi-Newton method’s main advantage lies in the fact that the Hessian matrix problems of the original Newton method can largely be avoided. The most used formula to update the Hessian is the Broyden-Fletcher-Goldfarb-Shano (BFGS) method. The Hessian approximation of this method is shown in equation 4.10. In this equation $B_k$ is the approximated Hessian, and $M_k$ the search direction of that matrix. [18]

\[
B_{k+1} = B_k + \frac{y_ky_k^T - B_kr_kr_k^TB_k}{y_k^T r_k} r_k
\]

\[
M_{k+1} = M_k + \frac{(r_k - M_ky_k)(r_k - M_ky_k)^T - M_ky_ky_k^TM_k}{y_k^T r_k} + \frac{y_k^TM_ky_kr_kr_k^T}{(y_k^T r_k)^2}
\]

\[
y_k = B_kr_k
\]

\[
r_k = x_{k+1} - x_k
\]

\[f_k + (1 - \alpha)\lambda\nabla f_k^T r_k < f(x_k + \lambda r_k)
\]

\[f(x_k + \lambda r_k) < f_k + \alpha\lambda\nabla f_k^T r_k
\]
actual minimization is done through other methods. Usually the steepest
descent direction or linear approximation.

\[
\min_{||r||<\delta} m_k(x_k + r) \\
m_k(x_k + r) = f(x_k) + \nabla f(x_k)^T r + \frac{1}{2} r^T H_f(x_k) r
\] (4.12)

4.3 Constrained Optimization

Constrained optimization can roughly be divided in two distinctive cate-
gories. The first converts the constrained problem to an unconstrained prob-
lem. This is achieved by embedding the constrains in the objective function.
This problem can then be solved with the algorithms mentioned in the uncon-
strained section. To solve these kind of problems special algorithms have also
been developed, most notably the Penalty method and the Barrier Method.

The second method is to restrict the search directly to the feasible re-
gion only. The Gradient Projection method and the Sequential quadratic
programming are prime examples of these methods.

An important aspect in constrained optimization is whether or not the
function has actually reached the optimal point. This is a much more com-
plex affair in the constrained optimization compared to the other types of
optimization. The Karush-Kuhn-Tucker(KKT) conditions are the excepted
method to determine whether or not the optimum is reached. The KKT
conditions use the auxiliary Lagrangian function:

\[
L(x, \lambda) = f(x) + \sum \lambda_{g,i} g_i(x) + \sum \lambda_{h,i} h_i(x) 
\] (4.13)

The KKT conditions are:

\[
\begin{align*}
\nabla_x L(x, \lambda) &= 0, \\
\lambda_{g,i}(x) &= 0 \forall i, \\
\begin{cases}
g(x) &\leq 0 \\
h(x) &= 0 \\
\lambda_{g,i} &\geq 0
\end{cases}
\end{align*}
\] (4.14)

The penalty method was already briefly discussed in chapter 3.4. With
this method the problem is rewritten to an unconstrained problem. The con-
strains are written as penalties in the objective function. When the program
violates the constrains the penalty function increases the objective function
value. The drawback is that the program can still violate constrains and can thus produce infeasible designs. When solutions are close to, or on the boundary of the problem these methods can also produce unsatisfying results. These problems can be mitigated by choosing a penalty that has a value neither to large, nor to small. An example of a penalty function can be seen in equation 4.15.[31]

\[ p_\mu(x) = \mu\left(\sum_{i=1}^{P} \max\{g_i(x), 0\} + \sum_{i=P+1}^{m} |g_i(x)|\right) \quad (4.15) \]

The Barrier method works similar to the penalty function. The Barrier function makes a barrier at the defined constrains. The drawback is that barriers can only be constructed for inequality constrains. Barrier methods cannot, in contrary to the penalty function, leave the feasible region. This is an advantage that will generally result in a shorter computational time. An example of an barrier function is given in equation 4.16

\[ b_\mu(x) = -\mu \sum_{i=1}^{P} \frac{1}{g_i(x)} \quad (4.16) \]

The gradient projection method is a modified steepest descend method. At every step the new direction is modified by projecting the gradient to the active constraints. This projection is carried on with an projection matrix, that uses the Jacobian matrix of the active constrains.[6]

\[ x_{k+1} = P[x_k - \alpha_k \nabla f(x_k)] \]
\[ [P(x)]_i = \text{mid}\{x_i, l_i, u_i\} \quad (4.17) \]

The sequential quadratic program is in essence a modified Newton method. The main difference is the application of the KKT conditions. With this method this means the gradient of the Lagrangian should be zero. The quadratic approximation equation which is the simplest form of the SQP algorithm is shown in equation 4.18.[30]

\[ q_k(d) = \nabla f(x_k)^T d + \frac{1}{2} d^T \nabla^2 \mathcal{L}(x_k, \lambda_k) d \quad (4.18) \]

The SQP algorithm is currently one of the best algorithms available to solve non-linear problems. Compared to other algorithms it is usually the fastest to converge.
4.4 Matlab Algorithms

To solve the district heating problem, the mathematical problem is implemented in Matlab. Matlab has standard build in of the methods mentioned here. The solver fmincon can work with an Interior-point, SQP, a trust region reflective and active set.

The interior point algorithm needs some explanation. The interior point methods use an improved barrier method, so the constrains are modified into a barrier function. The interior point method from Matlab either uses a direct step (also known as a Newton step) which satisfies the KKT conditions, or a Conjugate gradient step, using a trust region. The main advantage of this algorithm is that of robustness and accuracy. The drawback of this method is that it is rather slow. The direct step has the following Hessian

\[ H = \nabla^2 f(x) + \sum_i \lambda_i \nabla^2 g_i(x) + \sum_j \lambda_j \nabla^2 h_j(x) \]  

(4.19)

And the direct step is constructed with:

\[
\begin{pmatrix}
H & 0 & J_h^T \\
0 & S \Lambda & 0 & -S \\
J_h & 0 & I & 0 \\
J_g & -S & 0 & I
\end{pmatrix}
\begin{pmatrix}
\delta x \\
\delta s \\
-\delta y \\
-\delta \lambda
\end{pmatrix}
= -
\begin{pmatrix}
\nabla f - J_h^T y - J_g^T \lambda \\
S \lambda - \mu e \\
h \\
g + s
\end{pmatrix}
\]  

(4.20)

This equation is the result of solving the KKT conditions using a linearized Lagrangian. In these equations \( s \) is the slack variable.

The conjugate gradient step used in the interior point method is defined as:

\[ \nabla_x L = \nabla_x f(x) + \sum_i \lambda_i \nabla g_i(x) + \sum_j y_j \nabla h_j(x) = 0 \]  

(4.21)

The step taken \((\delta x, \delta s)\) to approximately solve

\[ \min_{\delta x, \delta s} \nabla^T f(x) + \frac{1}{2} \delta x^T \nabla^2_{xx} L \delta x + \mu e^T S^{-1} \Lambda \delta s \]  

(4.22)

with the linearized constraints

\[ g(x) + J_g \delta x + \delta s = 0, \ h(x) + J_h \delta x = 0 \]  

(4.23)

The interior point algorithm was favored instead of the SQP method. Although the SQP method is in theory faster, it is also less robust method compared to the Matlab interior-point algorithm. The computational time
was not significant enough to justify the use of a less robust method. For more complex district heating systems it can be an interesting option to consider, since the computational time can reach unacceptable values. [19]

The program is set up to solve the algorithm shown in figure 4.5.

### 4.5 Example of the program

To get a good impression of how the program works a small example is made here.

The start of the program is the input of all the economic data presented in earlier chapters. These parameters need to be set correctly according to the latest economic data available. Since the final answer will very much depend on these factors, great care has to be taken to select the correct input.

When the economic data input is completed it is time to set the system boundaries and constrains. In this example the system presented in figure 4.6 and table 4.1. This data is then used to construct the fmincon problem. The fmincon solver is specified in equation 4.24. In this equation $c$ and $ceq$ are the nonlinear inequality and equality constrains. $A \& b$ are the linear
inequality matrix and vector, and $A_{eq}$ & $b_{eq}$ are the linear equality matrix and vector. The $lb$ and $ub$ are the lower and upper bounds of the problem.

In the pipe network problem there are no linear inequality constrains. The nonlinear constrains are modeled as an penalty function, so the fmincon solver also doesn’t have nonlinear constrains. The only constrains left are the linear equality matrix and vector. The equality matrix and vector are represented in table 4.2.

\[
\min_{x} f(x) \text{ such that } \begin{cases} 
    c(x) \leq 0 \\
    ceq(x) = 0 \\
    A \ast x \leq b \\
    A_{eq} \ast x = b_{eq} \\
    lb \leq x \leq ub
\end{cases}
\] (4.24)
Figure 4.6: Example problem of a district heating network
Chapter 5

Steam

One critical component of a district heating system is a heat source, from which the heat can be transported to the users. At Tatasteel there are plenty of steam pipelines that can produce the necessary heat for a district heating system. The heat source that is considered optimal should meet certain criteria to be considered optimal for a district heating system.

5.1 Steam source

At the site of Tata steel there are numerous sources of steam. Steam is used in a lot of processes, so there are a plenty of steam pipes available choose from. Not every pipe is deemed fit for district heating. There are numerous different pressures, and different sources. To select the best source of steam, the steam should meet certain criteria.

5.1.1 Steam criteria

The correct choice of the steam source depends on different factors. The first factor is that the steam should be produced with waste heat. Although most steam is produced with waste heat, there are also a lot of natural gas fired steam boilers. These boilers are usually at places where there is no waste steam, or they serve as a backup/auxiliary equipment, to meet the demand of critical processes at all times. It would also be a waste to use high pressure steam to heat the buildings.

Another important factor is the distance from the steam to the buildings that need to be heated. Larger distances mean higher transportation cost. It is therefore important to look for steam pipes that are close to the place where the heat is needed.
When looking at the buildings in the region 3D and 4D the steam pipe closest to the buildings is the one from the Oxystaalfabriek 2 to Centrale 1. The steam from this pipeline is produced with waste heat from either the powerplant centrale 1 or the Oxystaalfabriek. The steam is also available year round. When the Oxysteelplant produces more steam than is used in the factory, this steam is transported to the Dolomiet Steenfabriek. When the Oxystaalfabriek requires steam, Centrale 1 produces this. This results in a constant steam flow in the pipeline of the Dolomietsteenfabriek. This steam has a pressure of 15 bar. The temperature of the saturated steam at 15 bar is $198^\circ C$, which is high enough to directly use for space heating. The temperature is not so high that it would be interesting for other processed, since this usually requires higher pressures and/or temperatures. It can therefore be considered the best source available to heat these buildings.

### 5.2 Heating medium

The waste steam can be used in various ways to heat the buildings. The steam can be used both directly and indirectly to heat the buildings in situ. When used directly the steam can be tapped of the main steam pipe line, and the steam pressure is enough to distribute the steam in the network. At Tata steel this has been done in the past with office buildings close to a steam pipe line. The main drawback of this approach is that the investment cost of a steam pipeline is very high.
Figure 5.2: Steam flow from 2004 to December 2013 in the pipeline of the Dolomiet Steenfabriek. The graph indicates the increased use in recent years.
Figure 5.3: Flow steam every week from central 1 too the OXY Steel plant in 2013 [tons/h]
Figure 5.4: The difference between direct and indirect heating
A frequently used alternative for steam is thermal oil. Thermal oil can handle high temperatures, just like steam. Because the thermal oil is transported in the liquid phase, the density of the fluid is a lot higher than steam. Therefore the pipelines can be made of a smaller diameter. This leads to significant savings in pipeline investment cost. The drawback of thermal oil is that it needs more equipment and the cost of the thermal oil. The extra equipment needed for a system with thermal oil include a heat exchanger, and a pump. Thermal oils are also hazardous to the environment. To prevent environmental damage when a leak occurs, special care has to be taken. This will increase the cost of the system. Thermal oil is usually a flammable substance, which can lead to unsafe situations.

Instead of thermal oil, liquid water can also be used. Liquid water can’t handle temperatures as high as steam or thermal oil, but buildings usually don’t require very high temperatures to heat them. The advantage of liquid water over steam is the smaller pipe diameters. The pressure of the liquid water can also be lower than steam. Water is also environmental friendly and cheap. The main drawback in a district heating system is the potential threat of freezing when there is a pump failure. Ice expands, which can cause damage to the equipment. Steam lacks this problem, because the specific volume of ice is significantly smaller compared to steam, although the condensation lines can suffer from freeze damage. Thermal oil usually has a freezing point in low enough to prevent the freezing problems. For above ground pipelines a anti-freeze compound like water-glycol mixtures can prevent freezing. However, most district heating pipes using water are buried underground, where the temperature is high enough to prevent pipeline freezing.

To get an accurate impression of the equipment size the enthalpy available per volume is calculated. Because the maximum velocity is usually the limiting factor in equipment this is a good indicator for the actual equipment size. In the calculation this it is assumed that the heating fluid is cooled down to 70°C or, in the case of steam, to environmental pressure.

When water is used for the distribution it can be heated up to 100°C. The average $C_p$ in this case is 4.2kJ/kg and the density is 969kg/m$^3$. [24] The thermal oil can be heated up safely to 250°C, but the steam that is available has a temperature of 198°C. Because a heat exchanger always results in a temperature drop it is assumed that the thermal oil is heated to 190°C. The $C_p$ of the thermal oil is 2kJ/kg and the density 882kg/m$^3$. [15] Steam can be expanded and condensed from 15 bar to atmospheric pressure. The enthalpy of saturated steam at 15 bar is 2789.88kJ/kg and saturated water at 1 bar has an enthalpy of 417.51kJ/kg. The density of steam at 15 bar is
The fluids are compared in table 5.1.

When comparing this data it is shown that steam has the highest heat capacity, because it is the only fluid which changes phase. The heat released when condensing is very high, however the low density of steam results in the worst energy density per unit of volume. The energy density per unit volume is what will ultimately determine the size of the equipment. The size of the equipment will be the biggest influence on the investment cost. Thermal oil has the best equipment size, however the higher temperatures will result in more expensive pipelines, since most pipelines are not capable of handling temperatures higher than 110°C[8]. In addition it is a toxic compound. Because of these drawbacks water is the preferred heating medium.
Chapter 6

Implementation of the model

The mathematical program was used to determine the optimal route of a district heating pipeline on the Tata steel site. The offices in situ are: 3D-06, 3D-08, 3D-10, 4D-08, 4D-10 and 4D-11. All the office buildings are modeled as consumption nodes. The buildings 3D-06 and 3D-08 are modeled as one node, because there very close to each other. In this model there are no fictive nodes, because all the pipes can be constructed in a straight line. The sketch of this setup can be found in figure 6.2.

The mass flow required for each building is based on the maximum power that is needed to heat the building. The heating systems are classic 90-70 systems, so it is assumed that a $\Delta T$ of 20$^\circ$C is enough to heat the buildings. The Mass flow that is required can be seen in table 6.1. The pipe length and local friction losses can be found in table B.1

When this is used in the matlab model with the data presented in [21] this results in the plot represented in appendix B. The calculation is done multiple times with different economic data sets available. The final result of the data supplied by Tatasteel resulted in a total cost of €805.000. The investment cost were €754.000, while the yearly energy bill of the pump is €1217.

<table>
<thead>
<tr>
<th>Building</th>
<th>Power $[kW]$</th>
<th>Massflow $[kg/s]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3D-06</td>
<td>390</td>
<td>4.67</td>
</tr>
<tr>
<td>3D-08</td>
<td>85</td>
<td>1.01</td>
</tr>
<tr>
<td>3D-10</td>
<td>350</td>
<td>4.19</td>
</tr>
<tr>
<td>4D-08</td>
<td>230</td>
<td>2.75</td>
</tr>
<tr>
<td>4D-10</td>
<td>40</td>
<td>0.48</td>
</tr>
<tr>
<td>4D-11</td>
<td>450</td>
<td>5.38</td>
</tr>
</tbody>
</table>

Table 6.1: the buildings with the required power and massflow
Figure 6.1: Overview of the buildings. The blue circles indicate the possible steam pipeline source. The red circles show the buildings that must be heated.
Figure 6.2: situation sketch derived from figure 6.1
When comparing the different outputs, it can be seen that the economic data has great influence on the final output. The data that is used can not only result in different pipe diameters, but can also influence the optimal pipe path. When comparing the results from the shortest route with the optimal route, a saving of 5% can be achieved over the entire lifetime of the pipeline. This is similar to what was found in the literature. [14] This is very significant, since cost are usually the most important reason why district heating can not be implemented.
Chapter 7

Heat loss pipes

To determine the heat loss of the pipeline a second computer model was developed. The purpose of this model is to determine the minimum isolation thickness that is required to ensure that for every consumer the minimum temperature that is required to heat the building is met.

The cost of the heat loss were not included in the optimization problem for the pipe network. Because the district heating network is heated with waste heat, this heat can be considered “free”. Free is of course relative, because a larger heat loss in the pipe system will result in a bigger equipment, which will ultimately lead to increased cost. Because this district heating network is rather short compared to other district heating networks, the assumption of free heat is considered acceptable.\cite{21,33}

Every district heating system has a critical consumer. This consumer is located at a critical spot. This can either be critical in terms of pressure and/or temperature. The pressure constrains were met in the previous chapters, so only the temperature is analyzed here. The consumer in this network is node number 1. Node 1 is the node that is located at the end of the piping route and has therefore the longest route, and will thus lose more heat then other routes. This route is therefore analyzed. \cite{27,20}

7.1 Single pipe

To determine the heat lost in the pipe network, a simple computer program is developed. This program divides the pipes in elements and determines the lost heat per element. The sketch of this approach can be seen in figure 7.1.\cite{23}

To calculate the temperature $T_e$ the following relations can be developed
When $T_f$ is replaced with $T_f = (T_0 + T_e)/2$ the equation becomes:

$$\dot{M}C_p(T_0 - T_e) = ((T_0 + T_e)/2 - T_s)UA$$

The only unknown variable in this equation is $T_e$. The heat transfer problem can be solved when the equation is rewritten to explicitly state $T_e$. When doing this the equation becomes:

$$T_e = \frac{T_s + T_0(M\dot{C}_p/C_p - 1/2)}{M\dot{C}_p/C_p - 1/2}$$

The pipe is an insulated pipe as shown in figure 7.2. The total heat transfer from the pipe $UA$ can be calculated with $UA = 1/R$. $R$ is calculated with:

$$R = R_1 + R_2 + R_3 + R_4 + R_5$$

The $R_1$ is the resistance of the convective layer inside the tube. $R_2$ represents the pipe wall resistance to heat transfer, and $R_3$ is the conductive resistance of the Insulation. $R_4$ is the outer wall resistance to heat transfer, and $R_5$ is the resistance to the outside of the pipe. (see figure 7.2)
In this equation $dh$ is the burial dept of the pipe centerline, $r_o$ the outer diameter of the pipe, $k_s$ the conduction coefficient of the soil and $\Delta x$ the length of the pipe section.\[24\]

The convective heat transfer inside the tube $R_1$ can be found in the literature [24]. The method used here determines the Nusselt number with a correlation. The Nusselt number is then used to calculate the heat transfer coefficient. The correlation to determine the Nusselt number in a pipe is the Gnielinski’s formula:

$$Nu_d = \frac{(f/8)(\text{Re}_d - 1000)Pr}{1 + 12.7(f/8)^{0.5}(Pr^{2/3} - 1)}$$ (7.6)

$$h_c = \frac{Nu_d \ast k}{L}$$ (7.7)

With these equations the resistance $R_1$ can be calculated by the following relation:

$$R_1 = \frac{1}{h_cA_1}$$ (7.8)

The Area in this equation is the inside hydraulic area of a pipe segment. $R_2$ and $R_4$ is the heat resistance of the pipe and $R_3$ is the heat resistance of the insulation.

$$R_{2,3,4} = \frac{L_k}{kA}$$ (7.9)
In this equation \( L_k \) is the thickness of the insulation/wall, \( k \) is the conductivity of the insulation/wall and \( A \) is the mean logarithmic area. The logarithmic area can be calculated with:

\[
A_m = \frac{A_{\text{outer}} - A_{\text{inner}}}{\ln(A_{\text{outer}}/A_{\text{inner}})} \tag{7.10}
\]

### 7.1.1 Example single pipe problem

For example the resistances are calculated for the largest pipe in the system. This is the pipe from the source. The pipe has an internal diameter of 0.12[m] and has a flow of 18.76[kg/s]. The insulation thickness is assumed to be 50mm thick, while the pipe thickness is assumed to be 10mm thick. The length of the pipe section is assumed to be 0.1[m] With these parameters it is possible to calculate all the resistances.

The area’s that are required for the heat transfer problem are:

\[
A_1 = \pi d_i L = \pi \times 0.12 \times 0.1 = 0.0377 m^2 \tag{7.11}
\]

\[
A_2 = \pi (d_i + 2t) L = \pi \times (0.12 + 2 \times 0.01) \times 0.1 = 0.044 m^2 \tag{7.12}
\]

\[
A_3 = \pi (d_i + 2t + 2t_{\text{ns}}) L \\
= \pi \times (0.12 + 2 \times 0.01 + 2 \times 0.05) \times 0.1 \\
= 0.0754 m^2 \tag{7.13}
\]

\[
A_4 = \pi (d_i + 2t + 2t_{\text{ns}} + 2t_{\text{o}}) L \\
= \pi \times (0.12 + 2 \times 0.01 + 2 \times 0.05 + 2 \times 0.01) \times 0.1 \\
= 0.0817 m^2 \tag{7.14}
\]

With the area’s known of all the section’s it is possible to calculate the logarithmic area of the section’s.

\[
A_{m,2} = \frac{A_2 - A_1}{\ln(A_2/A_1)} = \frac{0.044 - 0.0377}{\ln(0.044/0.0377)} = 0.0408 m^2 \tag{7.15}
\]

\[
A_{m,3} = \frac{A_3 - A_2}{\ln(A_3/A_2)} = \frac{0.0754 - 0.044}{\ln(0.0754/0.044)} = 0.0583 m^2 \tag{7.16}
\]
\[ A_{m,4} = \frac{A_4 - A_3}{\ln(A_4/A_3)} = \frac{0.0817 - 0.0754}{\ln(0.0817/0.0754)} = 0.0785 \text{m}^2 \quad (7.17) \]

To calculate \( R_1 \) the Nusselt number is required. For the Nusselt number calculation, it is assumed that the Prandtl number is 2 and the conduction coefficient is 0.676[W/mK][24]. The Darcy friction factor is 0.0221 with a Reynolds number of 5.85 * 10^5, with the data from the model in section .

\[
N_{ud} = \frac{(f/8) * (Re_d - 1000) * Pr}{1 + 12.7 * (f/8)^{0.5} * (Pr^{2/3} - 1)}
= \frac{(0.0221/8) * (5.85 * 10^5 - 1000) * 2}{1 + 12.7 * (0.0221/8)^{0.5} * (2^{2/3} - 1)}
= 2.32 \times 10^4 \quad (7.18)
\]

\[
h_c = \frac{N_{ud} * k}{L} = \frac{2.32 \times 10^4 \times 0.676}{0.1} = 1.67 \times 10^4 \text{W/m}^2 \text{K} \quad (7.19)
\]

With these equations the resistance \( R_1 \) can be calculated by the following relation:

\[
R_1 = \frac{1}{h_c * A_1} = \frac{1}{1.67 \times 10^4 \times 0.0377} = 0.0017 \text{K/W} \quad (7.20)
\]

\( R_2, R_3 \) and \( R_4 \) can be calculated. It is assumed the conduction coefficient of the pipe material is 1W/mk and 0.03W/mk for the insulation.

\[
R_{2,3,4} = \frac{L_k}{k * A}
R_2 = \frac{t_i}{k * A_{m,2}} = \frac{0.01}{1 \times 0.0408} = 0.25 \text{K/W}
R_3 = \frac{t_{ins}}{k * A_{m,3}} = \frac{0.05}{0.03 \times 0.0583} = 28.6 \text{K/W}
R_4 = \frac{t_o}{k * A_{m,4}} = \frac{0.01}{1 \times 0.0785} = 0.127 \text{K/W} \quad (7.21)
\]

The last resistance that has to be calculated is \( R_5 \). For this calculation it is assumed the conduction coefficient of the ground is 1.3[W/mK], and the center of the pipe is 1 meter below the ground surface.

\[
R_5 = \frac{\ln(2dh/r_o)}{2 * \pi * k_s * \Delta x} = \frac{\ln(2 * 1/(0.13))}{2 * \pi * 1.3 * 0.1} = 3.35 \text{K/W} \quad (7.22)
\]
The total resistance is the sum of all the individual resistances, which becomes:

\[ R = R_1 + R_2 + R_3 + R_4 + R_5 = 0.0017 + 0.25 + 28.6 + 0.127 + 3.35 = 32.3 \text{K/W} \]  

(7.23)

When the different resistances are compared it clearly shows the biggest influence on the heat transfer problem are the insulation of the pipe and the resistance of the ground. These two combined are 99\% of the heat transfer resistance. With \( R \) known the total heat transfer coefficient can be calculated:

\[ U A = 1/R = 1/32.3 = 0.0309 \]  

(7.24)

These results can be used to calculate the total heat loss of the 150m pipe from the source. The length of a pipe section was assumed to be 0.1m, which means that equation 7.3 is iterated 1500 times. With a \( C_p \) of 4.18\,[kJ/kgK] and a \( \rho \) of 960\,[kg/m^3] the end temperature of the pipe is 89.95\,[\circ C]. Because the temperature drop is very small, the heat loss can also be approximated by calculating the heat transfer per meter pipe. With the data readily available the \( q \) can be calculated with:

\[ q = U A \Delta T = \frac{0.0309}{0.1} \times (90 - 8) = 25.36 \text{W/m} \]  

(7.25)

7.2 Double Pipe

In district heating systems it is common to that the supply and return pipe are relatively close to each other, which means they can influence each other. To analyze the effect of the two pipes a different calculation method, based on a potential function is developed.

7.2.1 Assumptions for the double pipe problem

In the single pipe problem the main resistance through heat transfer are the insulation and the ground around the pipe. In the double pipe problem the resistance of the convective layer inside the tube and the resistance of the inner and outer pipe are therefore neglected.

In this problem the potential function is used. To simplify this problem the pipes are simplified to ideal points. It is also assumed the temperature is equal on the outer surface of the pipes. Of course the sides of the pipes facing
each other will be slightly hotter, but the conduction coefficient of the soil is much higher than the conduction coefficient of the insulation. A common method of determining if this is a valid assumption is the Biot number. If the Biot number is $Bi < 0.1$ the problem can be simplified to a lumped thermal model.[24] The Biot number is defined as the internal resistance divided by the external resistance. For the pipe problem this means the conductivity of the insulation divided by the conductivity of the soil. This is $0.03/1.3 = 0.023$. This means the lumped thermal model is a valid assumption.

To solve the double pipe problem, a differential equation has to be solved. The differential equation to the heat transfer problem can be found in the literature: [24]

$$\rho c \frac{\partial T}{\partial t} = k \nabla^2 T + \dot{Q}_v'''$$  \hspace{1cm} (7.26)

To simplify the problem it is assumed that there is a steady state situation, which means there are no changes in time. The differential equation 7.26 simplifies to:

$$\dot{Q}_v''' = -k \nabla^2 T$$  \hspace{1cm} (7.27)

The problem presented in equation 7.27 can best be solved in cylindrical coordinates. In cylindrical coordinates, $r$ is the radius, $\phi$ is the angle and
\[ z \text{ is the depth coordinate. When the equation is rewritten in cylindrical coordinates the result is:} \]
\[
\nabla^2 = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2}{\partial \phi^2} + \frac{\partial^2}{\partial z^2} \tag{7.28}
\]

The problem was assumed to be 2D only. This means there is no deviation in the axial direction, or \( \frac{\partial}{\partial z} = 0 \). This simplifies the equation to:
\[
\nabla^2 = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2}{\partial \phi^2} \tag{7.29}
\]

One of the Laplace equation characteristics is that of the superposition principle. This means the problem is linear, and the solutions of simpler problems can be combined to create the solution of a more complex problem. To take advantage of this principle the pipe can be modeled as an ideal source. This means the heat transfer is equal in every direction, or \( \frac{\partial}{\partial \phi} = 0 \). Equation 7.30 is the result of this simplification.
\[
\nabla^2 = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} \right) \tag{7.30}
\]

With equation 7.30 it is possible to solve equation 7.27. The result is:
\[
T(x, y) = -\frac{q}{2\pi k} \ln(r) \tag{7.31}
\]

In this equation \( q \) is the strength of the source, \( T \) is the temperature of the soil at point \( x, y \), \( k \) is the conduction coefficient of the soil and \( r \) is the distance from the source \( q \).

To ease the calculation in the Matlab program, the cylindrical coordinates are substituted with polar coordinates. Polar coordinates allow for a simpler equation to be solved. It is not absolutely necessary to solve the problem, but it was appropriate to reduce the chance of error when typing the code.
\[
T(x, y) = -\frac{q}{2\pi k} \ln|z| \tag{7.32}
\]

In this equation \( z \) is defined as:
\[
z = x + iy \tag{7.33}
\]
And \( i \) is defined as:
\[
i^2 = -1 \tag{7.34}
\]

Because there are two pipes, there are also two sources. Because these sources are spaced apart, at least one is not in the origin. For a source that
is not located at point $(0, 0)$ the parameter $a_n$ is introduced. Where $a_n$ is the distance from the origin of the system. With the introduction of the new parameter the equation 7.32 becomes:

$$T(x, y) = -\frac{q}{2\pi k} \ln|z - a_n|$$  \hspace{1cm} (7.35)

With $a_n$ defined as:

$$a_n = x_n + iy_n$$  \hspace{1cm} (7.36)

The boundary condition of this problem is that at $y = 0$ the temperature is equal to the surface temperature, so:

$$T(x, 0) = T_0$$  \hspace{1cm} (7.37)

To satisfy the boundary conditions of equation 7.37 the method of images is used. [22] With this method an imaginary source of equal strength, but of opposite strength, is placed at the opposite side of a wall, which is illustrated in figure 7.4.

$$T(x, y) = \frac{q}{2\pi k} (ln|z - \bar{a}_n| - ln|z - a_n|)$$  \hspace{1cm} (7.38)

With the temperature $T_0$ as the ground temperature:

$$T(x, y) = T_0 + \frac{q}{2\pi k} \frac{ln|z - \bar{a}_n|}{|z - a_n|}$$  \hspace{1cm} (7.39)
When this problem is explicitly written for two pipes, it becomes:

$$T(x, y) = T_0 + \frac{q_1}{2\pi k} \ln \left| z - \frac{a_1}{a_1} \right| + \frac{q_2}{2\pi k} \ln \left| z - \frac{a_2}{a_2} \right|$$ \hspace{1cm} (7.40)

Equation 7.40 is plotted in figure 7.6. Where it shows the temperature distribution in the ground. The strength of the sources $q_1$ and $q_2$ are calculated with equation 7.48.

Between the fluid and the ground there is also pipe insulation. The equation necessary to deal with these resistance can be found in the literature[24]. This is similar to the equations used to model the single pipe. Here only the insulation resistance is used, because the resistance of the pipes and convection layer were found to be neglectable.

$$T_n - T_0 = \frac{q_n}{2\pi k_n} \ln \left( \frac{r_{no}}{r_{ni}} \right)$$ \hspace{1cm} (7.41)

In this equation $k_n$ is the conductivity of the insulation in pipe $n$. $r_{no}$ and $r_{ni}$ are the outer and inner radius of the pipe. When equation 7.40 is written for the temperature in pipes 1&2, and equation 7.41 is substituted in both, the result is:

$$T_1 = T_0 + \frac{q_1}{2\pi k_1} \ln \left( \frac{r_{1o}}{r_{1i}} \right) + \frac{q_2}{2\pi k} \ln \left| \frac{a_1 - a_2}{a_1 - a_2} \right|$$ \hspace{1cm} (7.42)

$$T_2 = T_0 + \frac{q_2}{2\pi k_2} \ln \left( \frac{r_{2o}}{r_{2i}} \right) + \frac{q_1}{2\pi k} \ln \left| \frac{a_2 - a_1}{a_2 - a_1} \right|$$ \hspace{1cm} (7.43)

These equations can be substituted in each other to calculate $q_1$ and $q_2$. The result of this substitution is:

$$q_1 = - \left( \frac{T_0 - T_2 \ln \left| \frac{a_1 - a_2}{a_1 - a_2} \right| + T_1 - T_0 \ln \left( \frac{r_0}{r_i} \right)}{2\pi k_1 \ln \left| \frac{a_1 - a_2}{a_1 - a_2} \right| + 2\pi k_{ins} \ln \left( \frac{r_0}{r_i} \right)} \right)$$ \hspace{1cm} (7.44)

while $q_2$ can be calculated with:

$$q_2 = \frac{T_2 - T_0 - \frac{q_1}{2\pi k} \ln \left| \frac{a_2 - a_1}{a_2 - a_1} \right|}{\ln \left( \frac{r_0}{r_i} \right)}$$ \hspace{1cm} (7.45)
7.2.2 example problem double pipe

For the calculation of the double pipe problem the same values are used as the single pipe example. In the double pipe problem the pipe thickness is neglected. The inner radius is therefore 0.07\,m and the outer radius is 0.12\,m. The insulation conductivity is again 0.03\,W/mK and the conductivity of the ground is 1.3\,W/mK. The position of the pipes has to be written in polar coordinates. The center is taken between the two pipes and the surface is set at \( y = 0 \) this is illustrated in figure 7.5. The coordinates of pipes can therefore be expressed as:

\[
\begin{align*}
a_1 &= -0.5 - i \\
a_2 &= 0.5 - i
\end{align*}
\]  \hspace{1cm} (7.46)

Because the pipes are symmetric the following holds

\[
\begin{align*}
\ln \frac{|a_2 - \bar{a}_1|}{|a_2 - a_1|} &= \ln \frac{|a_1 - \bar{a}_2|}{|a_1 - a_2|} \\
\ln \left| -0.5 - i - (0.5 + i) \right| &= 0.8047
\end{align*}
\]  \hspace{1cm} (7.47)

This can now be substituted in equation 7.48.
\[ q_1 = -\left( \frac{8 - 70}{2\pi \ast 1.3} 0.8047 + \frac{90 - 8}{2\pi \ast 0.03} ln(0.12/0.07) \right) / \left( \left( \frac{1}{2\pi \ast 1.3} 0.8047 \right)^2 - \left( \frac{1}{2\pi \ast 0.03} ln(0.12/0.07) \right)^2 \right) = 28.05W/m \]  

(7.48)

and \( q_2 \) becomes:

\[ q_2 = \frac{\frac{T_2 - T_0}{2\pi k} ln\left(\frac{a_2-a_1}{a_2-a_1}\right)}{\frac{1}{2\pi k_{ins}} ln\left(\frac{r_0}{r_i}\right)} \]

\[ = \frac{70 - 8 - \frac{28.05}{2\pi k} 0.8047}{\frac{1}{2\pi \ast 1.3} ln \left(0.12/0.07\right)} = 20.85W/m \]  

(7.49)

The final result is the hot pipe loses 28.05W/m and the cold pipe loses 20.85W/m.

### 7.3 comparison results single and double pipe

In this chapter the the heat loss of both a single pipe and double pipe has been evaluated. In section 7.1.1 the heat loss of a single pipe was determined at \( q \) of 25.36[W/m]. For the double pipe heat loss calculation the same parameters were used. With these parameters the heat loss \( q \) of the hot pipe was found to be 28.05[W/m]. This is an increase of 10.6% when comparing the results to the single pipe heat loss. This increase is quite significant. It can thus be concluded the influence of the cold pipe next to the hot pipe can not be neglected in critical areas.
Figure 7.6: contour plot of function 7.40 which shows the temperature distribution in the soil.
Chapter 8
Payback time analyses

To determine whether or not a district heating network is an interesting option, the payback time for such a system should be determined. Currently the Buildings are all equipped with a CV installation powered by natural gas. The largest cost of such a system is the natural gas consumption, while in a district heating system the largest cost is the investment. For an accurate comparison the natural gas consumption should be calculated.

8.1 Natural gas consumption

The energy consumption of a building at Tata steel is not metered. Because there is no data available of the actual gas consumption, the cost of the natural gas has to be calculated with another method. A common method to determine the usage of heating is the method of degree days.

A degree day is defined as the outside temperature relative to the base temperature. The base temperature is 18°C. When the outside temperature drops below this value it is counted for as a degree day. For example if the average outside temperature is 10°C that counts for \( 18 - 10 = 8 \) degree days.

For Wijk aan Zee the average number of degree days from 2004-2013 was 2802.1 based on the Data supplied by the KNMI. [9]

The Heating demand can be determined with the following equation

\[
q \text{[kWh]} = UA \times \text{degreedays} \times 24
\]

(8.1)

The term \( UA \) depends on the building surface \( A \) and conduction coefficient \( U \). These factors are determined in Appendix A.

The total cost of natural gas can be calculated with:

\[
\text{Cost}_{\text{gas}} = \frac{q}{\text{LHV}_{\text{gas}}} \times \text{Price}_{\text{gas}}
\]

(8.2)
Table 8.1: Natural gas evaluation

<table>
<thead>
<tr>
<th>building</th>
<th>UA</th>
<th>Energy demand ([\text{kWh/year}])</th>
<th>Natural Gas consumption ([\text{m}^3/\text{year}])</th>
<th>Cost [(\text{€/year})]</th>
</tr>
</thead>
<tbody>
<tr>
<td>3D-06</td>
<td>-</td>
<td>1,708,200</td>
<td>184,559</td>
<td>60,905</td>
</tr>
<tr>
<td>3D-08</td>
<td>2.81</td>
<td>188,750</td>
<td>20,393</td>
<td>6,730</td>
</tr>
<tr>
<td>3D-10</td>
<td>11.7</td>
<td>784,558</td>
<td>84,769</td>
<td>27,973</td>
</tr>
<tr>
<td>4D-08</td>
<td>7.7</td>
<td>515,586</td>
<td>55,706</td>
<td>18,383</td>
</tr>
<tr>
<td>4D-10</td>
<td>1.3</td>
<td>89,667</td>
<td>9,688</td>
<td>3,197</td>
</tr>
<tr>
<td>4D-11</td>
<td>15</td>
<td>1,008,756</td>
<td>108,989</td>
<td>35,966</td>
</tr>
<tr>
<td>total</td>
<td>-</td>
<td>4,295,547</td>
<td>464,105</td>
<td>153,155</td>
</tr>
</tbody>
</table>

The LHV of the natural gas used on the Tata Steel site has a value of 33.32MJ/m\(^3\) which is equal to 9.26KWh/m\(^3\). The results for this analysis is shown in table 8.1\(^1\).

### 8.2 Payback time

The computer program of the pipe network shows that the operational cost of the network are very small. Electricity consumption is around €1,000 a year, depending on the configuration of the network. With the model using the data of Tata steel the electricity cost are €1,217. This means the district heating system has the potential of saving over €150,000 a year. The investment cost of a district heating system are however high. Using the data provided by Tata steel, the investment cost are €754,297. This will result in a payback time of \(\frac{754,297}{\text{153,155–1217}}\) = 4.95 years, which is quite long. It will become more interesting when the Boilers of the buildings need to be replaced. According to the DACE[25] the cost of a boiler is around 200[€/kW]. The combined power of the boilers is 1.54MW. This means the total cost of the boilers in the buildings is \(1,540 \times 200 = \text{€308,000}\). The payback period in this situation is reduced to \(\frac{754,297–308,000}{153,155–1217}\) = 2.9 years.

Tata steel policy only allows project to start when the payback period is 1 year. When it is two years, the investment is a debatable subject, but projects that require longer payback periods are generally not started.

\(^1\)The boiler in building 3D-06 supplies hot water for showers. Instead of using degree days it is assumed the boiler has a load factor of 0.5
8.3 Conclusion and Recommendations

The use of district heating is growing in the world. The potential savings can be quite high when compared to traditional natural gas installations. The main drawback of these systems has always been, and still is investment cost. The use of computer models can reduce the cost of a district heating system considerably when compared to rule of thumb designs. Studies show that optimal diameter calculation can reduce the price by 17% [27]. The use of network optimization in conjunction with pipe optimization can further reduce the cost by 5%.

However Tata’s policy of a payback time of less then 2 years is to short to make the district heating network that was analyzed economic viable. The district heating network that is examined in this study has a payback period of about 5 years depending on the economic situation. For a long term investment it is certainly an interesting option that can contribute to minimize the energy consumption of the site.

The network that was analyzed is the best case scenario on the Tata Steel site. Most other office buildings are further apart, which will lead to even higher cost. Unless a lot of office buildings are build in the neighborhood of other buildings, district heating networks will be too expensive when compared to other heating methods.

Currently there are plans for Tata steel to supply heat for residences for the surrounding town’s. Perhaps when this happens, a case can be made for district heating on the site.

It should be noted that in some cases the government subsidizes the construction of district heating networks. When a large number of boilers needs to be replaced and the government is willing to subsidize a significant part of the cost, perhaps the payback time is reduced to less then two years.
Bibliography


Appendices
Appendix A

Buildings energy consumption

The first step in determining a district heating system is the energy requirements of the buildings that can be heated. The buildings in situ are 3D-06, 3D-08, 3D-10, 3D-08, 4D-10 and 4D11. These buildings were selected, because these buildings are relative close to each other, which results in lower transportation cost.

When buildings are renovated the insulation of the building is usually improved. This means that the old installation is over dimensioned. To avoid a district heating design that is also greatly over dimensioned a detailed calculation of the building energy requirements is required. for some buildings these calculations were already done by other people [17] [16]. To check if the heating system is not larger than needed, a simplified calculation sheet is developed in excel according to the recent standards in heat loss calculations.[11] [12] [10]

A.1 Method

The method presented in the standards, calculates the heat loss based on different factors. These factors take in account the shape of the building, the thickness and effectiveness of the insulation.

A.2 Transmission losses

The transmission losses are the losses which occur through convection and conduction. Radiative heat transfer is not modeled, because the temperature difference between the outside of the building and the outside air are not very large. Therefore these losses are usually ignored. The transmission losses can be calculated with equation A.1
\[ \Phi_t = (H_{t,ie} + H_{t,ia} + H_{t,ib} + H_{t,ig}) \times (\theta_i - \theta_e) \quad (A.1) \]

The variables \( H \) in equation A.1 are the resistance of heat transfer multiplied with the area of that parameter. For example \( H_{t,ie} \) is the transmission through the outside wall, and can be calculated with equation A.2. This equation includes the window.

\[ H_{t,ie} = \sum_k (A_k \times f_k \times (U_k + 0.1)) \quad (A.2) \]

The parameter \( f_k \) is 1. To determine the parameter \( U_k \) a more complex relation is used.

\[ U_{wall} = \frac{1}{R_c + R_{si} + R_{se}} \quad (A.3) \]

\[ f = \frac{A_{frame}}{A_{glass}} \quad (A.4) \]

\[ U_{glass} = (1 - f)/(R_c + R_{si} + R_{se}) + f \times U_{frame} + U_{toe} \quad (A.5) \]

Because most office buildings at Tata steel are more or less rectangular buildings, where every space is heated, the calculation can be simplified greatly. The values \( R_{si} \) and \( R_{se} \) are 0.13 and 0.04 for the walls and 0.1 and 0.04 for the roof. \( R_c \) depends on the wall construction and is calculated with equation A.8.

### A.3 Heat losses through the ground

The stationary heat losses though the ground can be expressed as:

\[ L_S = a(A_{T,rand}U_{vl+gr;rand} + A_{T,midden}U_{vl+gr;midden} + \sum_j P_j \Psi_{gr;j}) + \sum_i P_i (\Psi_{e;j} + 180\epsilon) \quad (A.6) \]

In this equation \( L_S \) is the heat loss coefficient in W/K, 
\( a \) is an constant of 0.6 for heating, 
\( A_{T,rand} \) the area of the floor 5 meters from the wall and closer, 
\( A_{T,midden} \) the area of the floor 5 meters from the wall and further, 
\( U_{vl+gr;rand} \) is the transmission coefficient of the ground up to 5 meters from the wall, 
\( U_{vl+gr;midden} \) is the transmission coefficient of the ground of the ground in the center.
\[ P_i \] is the \( i \text{th} \) part of the floor circumference, 
\[ \Psi_{gr} \] is the linear transmission coefficient to the ground and is a constant of \(-0.1W/(mK)\) 
\[ \Psi_e \] is the linear transmission coefficient to the outside air and is a constant of \(0.9W/(mK)\) 
\( \epsilon \) is the area of the ventilation gaps in \(m^2/m\) this is a constant of \(0.0012m^2/m\). 
\( U_{vl+gr} \) can be calculated with the following equation:
\[
U_{vl+gr} = \frac{1}{R_{c;vl} + (D_{gr} - h_{kr})/\lambda_{gr} + h_{kr}/\lambda_{eq} + R_{si}} \tag{A.7}
\]
\( R_{c;vl} \) is the heat resistance of the floor in \(W/(m^2K)\), 
\( D_{gr} \) is the calculation thickness of the layer of dirt. This is \(10m\), 
\( h_{kr} \) is the height of the crawl space in \(m\), 
\( \lambda_{gr} \) is the transmission coefficient of the ground. This is taken as \(2.0W/(mK)\), 
\( \lambda_{eq} \) is the equality transmission coefficient of the crawl space. This is defined as \(3W/(mK)\), 
\( R_{si} \) is the heat transfer coefficient at the heat inflow in \(m^2K/W\), 
\( R_{si} \) and \( R_{se} \) can be determined from other literature[12]. For a normal building they are both \(0.17\)
\[
R_{c;vl} = \frac{\sum R_m + R_{si} + R_{se}}{1 + \alpha} - R_{si} - R_{se} \tag{A.8}
\]
\( R_m \) is the heat transfer coefficient of the construction layer \((m^2K)/W\). Which is calculated with:
\[
R_m = \frac{d}{\lambda} \tag{A.9}
\]

The other values that are necessary to complete the calculation are listed below:
\[
\alpha = R_T * \Delta U \tag{A.10}
\]
\[
R_T = R_{si} + \sum_i (R_{m;i}) + R_{se} \tag{A.11}
\]
\[
\Delta U = \Delta U'' + \Delta U_a + \Delta U_{fa} + \Delta U_r + \Delta U_w \tag{A.12}
\]
A.4 Ventilation and infiltration

In addition to the transmission losses, there are also losses through ventilation and infiltration. Ventilation losses occur because people need fresh air. This air has the outside temperature, and must therefore be heated to the conditions of the office. Infiltration is the heat lost through the gaps in the construction of the building. The losses of the ventilation can be accurately predicted by the following relation:

\[ H_v = 1200 \times q_v \times f_v \quad \text{(A.13)} \]

And the infiltration losses can be calculated with:

\[ H_i = 1200 \times q_{i,k} \times \frac{V}{3600} \quad \text{(A.14)} \]

The factor \( q_{i,k} \) is dependent on the construction of the building and is derived from the construction properties. The factor \( q_v \) can be approximated with:

\[ q_v = A_{\text{floor}} \times q_{\text{min}} \quad \text{(A.15)} \]

Where \( q_{\text{min}} \) is the minimum ventilation requirement for the buildings.

A.5 Total heat losses

The total heat losses are the sum of the heat losses mentioned in the previous sections. The result of this is:

\[ Q_{\text{total}} = H_i + H_v + (U_{\text{wall}}A_{\text{wall}} + U_{\text{window}}A_{\text{window}} + U_{\text{roof}}A_{\text{roof}} + \sum \Psi + L_S) \times (T_{\text{in}} - T_{\text{out}}) \quad \text{(A.16)} \]

A.6 Example calculation

The heat loss of the building can be determined with above method. The building that is analyzed here is building numer 4D-10. This building is a one floor building with the dimensions \( L \times B \times H \) of 12.17 \( \times \) 11.82 \( \times \) 3.75. The floor and roof area are 12.17 \( \times \) 11.82 = 144m². The wall area is 2 \( \times \) 12.17 \( \times \) 3.75 + 11.82 \( \times \) 3.75 = 180m², of which 45m² is window area.

The U values of the roof, walls and glass are to be determined first in the heat loss calculation. The U value of the wall can be calculated with equation A.23. As mentioned before the factors \( R_{si} \) and \( R_{se} \) are 0.13 and 0.04 respectively. The walls of the buildings have 3 layers, one brick, one air and
one limestone. The layers are 0.1m, 0.08m and 0.1m thick. The conduction coefficient of brick and limestone are $1.5\text{W/mK}$ and $1.52\text{W/mK}$. The resulting $R_m$ of these layers are $0.1/1.5 = 0.067$ for the brick and $0.1/1.52 = 0.66$ for the limestone. The $R_m$ of the air is 0.18 in this situation. With these values the $R_T$ value can be calculated:

$$R_T = R_{si} + \sum_i (R_{m;i}) + R_{se} = 0.13 + 0.67 + 0.18 + 0.66 + 0.04 = 0.48[\text{m}^2\text{K/W}]$$  \hspace{1cm} (A.17)

The parameter $\Delta U_a$ can be determined with the parameter $\Delta U''$. in the following relation:

$$\Delta U_a = \Delta U'' \times (R_{m;insulation}/R_T) = 0.04 \times (0.18/0.48) = 0.015[\text{W/m}^2\text{K}]$$  \hspace{1cm} (A.18)

The parameters $\Delta U_{fa}$ and $\Delta U_r$ are assumed to be zero, which only leaves parameter $\Delta U_w$:

$$\Delta U_w = 0.05 \times R_T = 0.05 \times 0.48 = 0.024[\text{W/m}^2\text{K}]$$  \hspace{1cm} (A.19)

With these parameters known it is possible to determine $\Delta U$:

$$\Delta U = \Delta U'' + \Delta U_a + \Delta U_{fa} + \Delta U_r + \Delta U_w = 0.04 + 0.015 + 0 + 0 + 0.024 = 0.079[\text{W/m}^2\text{K}]$$  \hspace{1cm} (A.20)

$\alpha$ can now be calculated with the relation:

$$\alpha = \Delta U \times R_T = 0.079 \times 0.48 = 0.038[-]$$  \hspace{1cm} (A.21)

The data needed to calculate the parameter $R_{c,vi}$:

$$R_c = \frac{\sum R_m + R_{si} + R_{se}}{1 + \alpha} - R_{si} - R_{se} = \frac{0.06 + 0.18 + 0.066 + 0.13 + 0.04}{1 + 0.038} - 0.13 - 0.04 = 0.29[\text{m}^2\text{K/W}]$$  \hspace{1cm} (A.22)

The parameters needed for the $U$ value of the wall are now all known:

$$U_{wall} = \frac{1}{R_c + R_{si} + R_{se}} = \frac{1}{0.29 + 0.13 + 0.04} = 2.15[\text{W/m}^2\text{K}]$$  \hspace{1cm} (A.23)
The roof acts just like a wall. Only composition and parameters are different. For the roof the U value is 1.2[W/m²K]. The U value of the windows can be calculated with equation A.5. The windows have the dimension 1.8 * 1.5 = 2.79m² and the frame has three vertical and two horizontal jambs. The combined area of these jambs, which have a thickness of 0.025m, is (2 * 1.8 + 3 * 1.5) * 0.025 = 0.215m². This leads to an f value of:

\[ f = \frac{A_{\text{frame}}}{A_{\text{window}}} = \frac{0.215}{2.79} = 0.076[-] \] (A.24)

The U value of the glass is 3, and of the frame it is 7. \( U_{\text{toe}} \) has a value of 0.1. The combined heat transfer coefficient of the window can now be calculated with equation A.5

\[
U_{\text{glass}} = \frac{(1 - f)}{(R_c + R_{si} + R_{se}) + f \cdot U_{\text{frame}} + U_{\text{toe}}}
= \frac{(1 - 0.076)}{(0.03 + 0.13 + 0.04) + 0.076 \cdot 7 + 0.1} = 2.6[W/m²K]
\] (A.25)

The heat loss through the ground need a different method compared to the others. \( D_{gr} \) is 10[m], \( \lambda_{gr} \) is 1.3[W/mK], \( h_{kr} \) is 0.5[m], \( \lambda_{eq} \) is 3[W/mK] and \( R_{si} \) and \( R_{se} \) are both 0.17. The floor is constructed with concrete of 0.4[m] thick and a \( \lambda \) of 0.8[W/mk]. The \( R_m \) of concrete is therefore 0.4/1.5 = 0.27With these coefficients it is possible to calculate \( R_T \):

\[
R_{T;floor} = \sum R_m + R_{si} + R_{se} = 0.27 + 0.17 + 0.17 = 0.61[m²K/W]
\] (A.26)

The parameters \( \Delta U_a, \Delta U_{fa} \) and \( \Delta U_r \) are assumed to be zero, which only leaves parameter \( \Delta U_w \):

\[
\Delta U_w = 0.05 \cdot R_T = 0.05 \cdot 0.61 = 0.03[W/m²K]
\] (A.27)

With these parameters known it is possible to determine \( \Delta U \):

\[
\Delta U = \Delta U'' + \Delta U_a + \Delta U_{fa} + \Delta U_r + \Delta U_w = 0.04 + 0 + 0 + 0 + 0.03 = 0.07[W/m²K]
\] (A.28)

\( \alpha \) can now be calculated with the relation:

\[
\alpha = \Delta U \cdot R_T = 0.07 \cdot 0.61 = 0.043[-]
\] (A.29)

The data needed to calculate the parameter \( R_{c,vt} \):
\[
R_{cvl} = \frac{\sum R_m + R_{si} + R_{se}}{1 + \alpha} - R_{si} - R_{se} \\
= \frac{0.27 + 0.17 + 0.17}{1 + 0.043} - 0.17 - 0.17 = 0.245\text{[m}^2\text{K}/\text{W}] 
\]  
(A.30)

The parameter \(U_{vl+gr}\) can now be determined with the following relation.

\[
U_{vl+gr} = \frac{1}{R_{cvl} + (D_{gr} - h_{kr})/\lambda_{gr} + h_{kr}/\lambda_{eq} + R_{si}} \\
= \frac{1}{0.0245 + (10 - 0.5)/1.4 + 0.5/3 + 0.17} = 0.127\text{[W/m}^2\text{K]} 
\]  
(A.31)

The surface area of the floor was 144\text{[m}^2\text{]} This area is divided into two parts. The area from the wall to 10m and the area in the middle. They are 140\text{[m}^2\text{]} and 4\text{[m}^2\text{]} respectively.

\[
L_S = a(A_{T;rand}U_{vl+gr;rand} + A_{T;midden}U_{vl+gr;midden} + \sum_j P_j \Psi_{gr;j}) + \sum_i P_i(\Psi_{e;j} + 180\epsilon) \\
= 0.6(140 * 0.127 + 4 * 0.127 + 48 * -0.1) + 48(0.9 + 180 * 0.0012) = 61.6\text{[W/K]} 
\]  
(A.32)

With the transmission losses known only the Ventilation and infiltration losses need to be calculated. For the ventilation losses the parameter \(q_v\) and \(f_v\) has to be known. According to the ISSO standard, a office needs 1.3 * 10^{-3}\text{[m}^3\text{/sm}^2\text{floor]} The parameter \(f_v\) is 1, because there is no heat recovery. This means the ventilation losses are:

\[
H_v = 1200 * q_v * f_v = 1200 * 1.3 * 10^{-3} * 144 * 1 = 224\text{[W/K]} 
\]  
(A.33)

The last loss that needs to be calculated is the infiltration losses. The ISSO standard this building loses 0.0026\text{[m}^3\text{/sm}^2\text{side]} of air. This means the infiltration losses are:

\[
H_i = 1200 * 0.0026 * 180 = 561\text{[W/K]} 
\]  
(A.34)

The total heat losses can now be determined:

\[
Q_{total} = H_i + H_v + (U_{wall}A_{wall} + U_{window}A_{window} \\
+ U_{roof}A_{roof} + \sum \Psi + L_S) * (T_{in} - T_{out}) \\
= 561 + 224 + ((2.15 * 135 * 2.62 * 45 \\
+ 1.23 * 144 + 7 + 61.6) * (20 - 10) = 43\text{[kW]} 
\]  
(A.35)
Table A.1: Power requirement of the buildings

<table>
<thead>
<tr>
<th>Building</th>
<th>Calculated power[kW]</th>
<th>Installed power[kW]</th>
<th>UA[kW/K]</th>
</tr>
</thead>
<tbody>
<tr>
<td>3D-10</td>
<td>350</td>
<td>1164</td>
<td>11.7</td>
</tr>
<tr>
<td>4D-08</td>
<td>350</td>
<td>230</td>
<td>7.7</td>
</tr>
<tr>
<td>4D-10</td>
<td>43</td>
<td>40</td>
<td>1.3</td>
</tr>
<tr>
<td>4D-11</td>
<td>535</td>
<td>450</td>
<td>15</td>
</tr>
</tbody>
</table>

The $UA$ value can now also be calculated. This value is used in the determination of the building energy consumption.

$$UA = \frac{Q_{Total}}{\Delta T} = \frac{43}{30} = 1.43[kW/K]$$  \hspace{1cm} (A.36)

A.7 Calculated heat loss

When this same method is used to determine the heat loss of the other buildings the result is in table A.1. It shows the calculated power is slightly higher than the installed power. The main reason for this is the ventilation losses and infiltration losses. These are responsible for roughly 2/3 of the heat losses. The exception is building 3D-10. This building has a greatly over dimensioned boiler, which is well known at Tata. Apparently the offices don’t report a lack of heating power. Therefore the district heating network size is determined for the installed power, with the exception of building 3D-10.
Appendix B

Computer Model results
Figure B.1: shortest route

(a) Nodal data

<table>
<thead>
<tr>
<th>Node</th>
<th>flow ($1 \times 10^{-3} m^3/s$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.48</td>
</tr>
<tr>
<td>2</td>
<td>5.38</td>
</tr>
<tr>
<td>3</td>
<td>2.75</td>
</tr>
<tr>
<td>4</td>
<td>5.67</td>
</tr>
<tr>
<td>5</td>
<td>4.48</td>
</tr>
</tbody>
</table>

(b) Pipe data

<table>
<thead>
<tr>
<th>Pipe</th>
<th>Length [m]</th>
<th>ξ [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>70</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>90</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>270</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>180</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>180</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>100</td>
<td>5</td>
</tr>
<tr>
<td>7</td>
<td>180</td>
<td>5</td>
</tr>
<tr>
<td>8</td>
<td>200</td>
<td>5</td>
</tr>
<tr>
<td>9</td>
<td>150</td>
<td>5</td>
</tr>
</tbody>
</table>

Table B.1: Data for the pipe network
Figure B.2: economic optimum route with data supplied by [21]

<table>
<thead>
<tr>
<th>Pipe</th>
<th>Flow $[1 \times 10^{-3} \text{m}^3/\text{s}]$</th>
<th>Diameter [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0.48</td>
<td>0.0276</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>5.86</td>
<td>0.0918</td>
</tr>
<tr>
<td>6</td>
<td>2.75</td>
<td>0.0645</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>4.48</td>
<td>0.0806</td>
</tr>
<tr>
<td>9</td>
<td>18.76</td>
<td>0.1608</td>
</tr>
</tbody>
</table>

Table B.2: results with data from [21]. Total cost is €2.85 * 10^5 based on a 40 year period
<table>
<thead>
<tr>
<th>Pipe</th>
<th>Flow $[1 \times 10^{-3} \text{m}^3/\text{s}]$</th>
<th>Diameter $[\text{m}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0.48</td>
<td>0.0187</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>5.86</td>
<td>0.067</td>
</tr>
<tr>
<td>6</td>
<td>2.75</td>
<td>0.046</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>0.0588</td>
</tr>
<tr>
<td>8</td>
<td>4.48</td>
<td>0.058</td>
</tr>
<tr>
<td>9</td>
<td>18.76</td>
<td>0.118</td>
</tr>
</tbody>
</table>

Table B.3: results DACE. Total cost are $£8.6 \times 10^5$ based on a 40 year period.

<table>
<thead>
<tr>
<th>Pipe</th>
<th>Flow $[1 \times 10^{-3} \text{m}^3/\text{s}]$</th>
<th>Diameter $[\text{m}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>18.76</td>
<td>0.120</td>
</tr>
<tr>
<td>2</td>
<td>18.28</td>
<td>0.118</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>13.8</td>
<td>0.101</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>5.57</td>
<td>0.066</td>
</tr>
<tr>
<td>7</td>
<td>4.48</td>
<td>0.058</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table B.4: results DACE for the shortest route. Total cost are $£9.21 \times 10^5$ based on a 40 year period.

<table>
<thead>
<tr>
<th>Pipe</th>
<th>Flow $[1 \times 10^{-3} \text{m}^3/\text{s}]$</th>
<th>Diameter $[\text{m}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0.48</td>
<td>0.023</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>5.86</td>
<td>0.0788</td>
</tr>
<tr>
<td>6</td>
<td>2.75</td>
<td>0.0551</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>4.48</td>
<td>0.0693</td>
</tr>
<tr>
<td>9</td>
<td>18.76</td>
<td>0.1375</td>
</tr>
</tbody>
</table>

Table B.5: Results of the data supplied by Tata Steel. The total cost are $8.05 \times 10^5$ based on a 40 year period.
Figure B.3: Cost distribution of the piping network based on the data supplied by TataSteel. The data can be found in table B.6.

<table>
<thead>
<tr>
<th>Item</th>
<th>Cost [€]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Piping</td>
<td>332,110</td>
</tr>
<tr>
<td>Construction</td>
<td>419,650</td>
</tr>
<tr>
<td>Electricity</td>
<td>48,684</td>
</tr>
<tr>
<td>Pump</td>
<td>2537</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>802,982</strong></td>
</tr>
</tbody>
</table>

Table B.6: Cost based on the TataSteel data and based on a 40 year period.
Appendix C

Matlab Computer code

C.1 optimum configuration

clear all
close all
clc

%massflow nodes
qv2=0.48;
qv3=5.38;
qv4=2.75;
qv5=5.67;
qv6=4.48;
qv1=qv2+qv3+qv4+qv5+qv6;

%Pipe data
n=9; %number of pipes
%Pipe length
L=[70;90;270;180;180;100;180;200;150;];

%friction data
xi=[5;5;5;5;5;5;5;5;5;];

%Optimization data
A=[]; %linear inequality constrain matrix
b=[]; %linear inequality constrain vector
beq=[qv1,qv2,qv3,qv4,qv5,qv6]; %linear equality constrain vector
Aeq=[0 0 0 0 0 0 0 0 1;
     1 -1 -1 0 0 0 0 0 0;
     0 1 0 -1 -1 0 0 0 0;
     0 0 0 1 0 -1 -1 0 0;
     0 0 1 0 1 1 0 1 1;
     0 0 0 0 0 0 1 -1 0]; %linear equality constrain matrix
LB=[−qv1 −qv1 −qv1 −qv1 −qv1 −qv1 −qv1 −qv1]; % lower bound constrains  
UB=[qv1 qv1 qv1 qv1 qv1 qv1 qv1 qv1]; % upper bound constrains  
nonlincons=[];  
% different starting points  
qt0=[0;−qv2;0;0;−qv1;qv6;0;0]; % starting condition  
% qt0=[0;−qv2;0;0;−qv1;−qv1;0;0];  
% qt0=[0;0;0;0;0;0;0;0];  
% qt0=[qv1;qv1;0;0;−qv4;0;−qv6;0];  
f=@(qt)optimum_diam_imp2(qt,L,xi,n); % function call  
options= optimset('MaxFunEvals',10000,'MaxIter',1000,'Algorithm','interior-point','TolCon',1e−12,'Tolfun',1e−12,'TolX',1e−12);  
[Qt] = fmincon(f,qt0,A,b,Aeq,beq,LB,UB,nonlincons,options)  
[cost,D]=optimum_diam_imp2(Qt,L,xi,n)  

C.1.1 Optimum diameter

function [obj,D] = optimum_diam_imp2(qt,L,xi,n)  
syms d_pipe  
d=zeros(1,n);  
cost=zeros(1,n);  
dp=zeros(1,n);  
for k = 1:n  
if (qt(k))==0  
    cost(k)=0;  
end  
if abs(qt(k))>0  
y = @(d_pipe)diameter_cost_optimal(d_pipe,qt(k)*1e−3,L(k),xi(k));  
[d(k),cost(k)] = fminsearch (y,1e−20);  
[cost(k),dp(k)]= diameter_cost_optimal(d(k),qt(k)*1e−3,L(k),xi(k));  
end  
end  

% pressure check  
P=zeros(1,6);  
P(1)=1e6;  
if qt(1)>0  
P(2)=P(1)−abs(dp(1));  
end  
if qt(2)>0  
P(3)=P(1)−abs(dp(2));  
end  
if qt(4)>0
\begin{verbatim}
P(5) = P(1) - \text{abs}(dp(4));
end

if qt(3) > 0 && qt(3) > qt(6)
P(4) = P(2) - \text{abs}(dp(3));
elseif qt(6) > 0 && qt(6) > qt(3)
P(4) = P(5) - \text{abs}(dp(6));
end

if qt(5) > qt(7)
P(6) = P(3) - \text{abs}(dp(5));
else
P(6) = P(5) - \text{abs}(dp(7));
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Penalty cost
if qt(1) > 0 && P(2) > P(1) || qt(1) < 0 && P(2) < P(1)
c(1) = \text{abs}(qt(1));
else
c(1) = 0;
end

if qt(2) > 0 && P(3) > P(1) || qt(2) < 0 && P(3) < P(1)
c(2) = \text{abs}(qt(2));
else
c(2) = 0;
end

if qt(3) > 0 && P(4) > P(2) || qt(3) < 0 && P(4) < P(2)
c(3) = \text{abs}(qt(3));
else
c(3) = 0;
end

if qt(4) > 0 && P(5) > P(1) || qt(4) < 0 && P(5) < P(1)
c(4) = \text{abs}(qt(4));
else
c(4) = 0;
end

if qt(5) > 0 && P(6) > P(3) || qt(5) < 0 && P(6) < P(3)
c(5) = \text{abs}(qt(5));
else
c(5) = 0;
end

if qt(6) > 0 && P(4) > P(5) || qt(6) < 0 && P(4) < P(5)
\end{verbatim}
\[ c(6) = \text{abs}(qt(6)); \]
\text{else}\n\[ c(6) = 0; \]
\text{end}\n
\text{if} \ qt(7) > 0 \text{ && } P(6) > P(5) || qt(7) < 0 \text{ && } P(6) < P(5) \text{ then}\n\[ c(7) = \text{abs}(qt(7)); \]
\text{else}\n\[ c(7) = 0; \]
\text{end}\n
Qt = qt;\nobj = \text{sum}(	ext{cost}) + 2e5 \cdot \text{sum}(c);\nD = d;\n
\textbf{C.2 Economic data}\n
% optimum pipe diameter
\textit{function} [\text{Total}_c, dp] = diameter\_cost\_optimal(d\_pipe, qt, L, xi)\n
% Physical and economical data
P1 = 1e6; \text{ Pa}\nT1 = 110; \text{ degrees celcius}\nT2 = 70; \text{ degrees celcius}\nTa = 10; \text{ ambient temperature celcius}\nrho = 934.8; \text{ kg/m}^3\nmu = 0.226e^{-6}; \text{ m}^2/s\nt = 8760; \text{ operating time [h]}\ni = 0.1; \text{ interest rate}\nec = 7.5e-5; \text{ electricity price [\text{euro}/Wh]}\nhc = 7e-5; \text{ cost of heat [\text{euro}/Wh]}\nCpa = 1540; \text{ [euro]}\nPn = 0.75; \text{ pump efficiency}\nk = 0.4e-3; \text{ pipe roughness [m]}\nPc = 18; \text{ [euro]}\nPb = 291; \text{ [euro]/m}\nC = 229; \text{ [euro]/m}^2\n
% pipe insulation properties
\textbf{labda} = 0.0025; \text{ conduction coefficient [W/mK]}\n
% pipe cost coefficients polynomial
A = 18; \text{ [euro]}\nB = 291; \text{ [euro]/m}\nC = 229; \text{ [euro]/m}^2
% construction cost polynomial
D=287; %[euro]
E=310; %[euro/m]
F=1275; %[euro/m^2]

% friction calculation
V=abs(qt)/(d_pipe*d_pipe*pi*0.25); %[velocity m/s]
Re= (V*d_pipe)/mu; %Reynolds number

% calculation of darcy friction factors
sigma_fd = 1./(1+exp(-(Re-3000)/450)); %smoothing parameter
fd_lam=64/Re;
fd_turb=(0.2479-0.0000947*(7-log10(Re)^4)/(log10(k/3.645+7.366/Re^0.9142))^2); %for Re>4000
if Re < 1e8
fd=(1-sigma_fd)*fd_lam+sigma_fd.*fd_turb;
else
Re_calc=1e8;
fd=(0.2479-0.0000947*(7-log10(Re_calc)^4)/(log10(k/3.645+7.366/Re_calc^0.9142))^2;
end
dp=(fd*L/d_pipe+xi)*rho*V^2/2;

% cost calculation
sigma_cost= 1./(1+exp(-(d_pipe-1e-2)/2e-3)); %smoothing parameter
P=abs(qt)*dp/Pn;
Pipe_c=(A+B*d_pipe+C*d_pipe*d_pipe)*L;
Con_c=(D+E*d_pipe+F*d_pipe*d_pipe)*L;
Pump_c=Cpb*P*PPL/PL;
elec_c=ec*P*t*PPL;
Total_c=(Pipe_c+Con_c)*sigma_cost+Pump_c+elec_c;