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Summary

The proposed migration algorithm is performed in the ray parameter-frequency domain. The shot-oriented data are first Fourier transformed to the frequency domain, and then double Radon transformed, with respect to the offset coordinate and the source coordinate. In this domain, the data are discretized according to the local dip of the discontinuity surfaces. The background velocity is estimated in the plane containing only the information from horizontal layers. The migration can be performed with the interval velocities, when three steps are required to produce a depth image: imaging, extrapolation, and coordinate shift. It can also be performed with the *RMS* velocity, when a single imaging step gives a time image of the whole data structure. Both options are tested on synthetic data and the results are promising.

Introduction

The common assumption in conventional migration is that the depth-velocity model is known, and this is used as an input for the migration. The model is usually subdivided into a number of horizontal strips of a small thickness Δz , each of (usually) uniform velocity. The data are then migrated from level z to $z+\Delta z$ in two steps: first, the sources and the receivers are downward continued to depth level $z+\Delta z$. Second, these data are imaged for time $t=0$. The migration is completed by repeating these two steps for all available data.

The proposed migration algorithm, performed in the ray parameter-frequency domain (p^o - p^s - f domain), offers a new approach to both imaging and extrapolation. The depth migration is carried out in three steps: imaging, extrapolation, and coordinate shift.

The imaging step is performed *prior* to the extrapolation step: from imaging, we learn about the spatial position of the discontinuity interface. With the extrapolation step the sources and the receivers are downward continued to the *known* interface. Before imaging the next interface, the origin of the coordinate system is shifted according to Snell's law, thereby avoiding the transformations back and forth from the ray parameter domain to the space domain at each migration step.

Due to the properties of the double transformed Radon domain (Vissinga, 1991), the data are discretized into extrapolation surfaces according to the local dip of the discontinuity interfaces. Figure 1 shows the extrapolation lines, which represent the extrapolation surfaces for a fixed frequency and velocity, for several dips. Velocity analysis is then performed in the surface containing the energy from the horizontally layered discontinuities only (plane $p^s=0$ in Figure 1), where the theoretical travel time curves fit the data much better than they do in conventional CMP gathers using hyperbolic assumption. Either interval velocities or *RMS*

velocities can be used for migration. The p^o - p^s - f migration can be layer-oriented, when all three steps are carried out with the interval velocities. The imaging can also be performed by using the *RMS* velocity for all the available data; then in a single imaging step the time image of the depth structure is obtained.

In this paper, we concentrate on the imaging, first, because this is where our method differs from conventional methods, and second, because it is the crucial step in the whole migration process. The theory is derived based on the assumptions that the Earth is acoustic and arbitrarily layered, but that within a common shot or a common receiver gather the interfaces can be regarded as plane.

Theoretical background

The scattered field $\hat{p}$ is represented as the field generated by secondary sources at the arbitrary interface S (Bleistein, 1984):

$$\hat{p}(x^r, x^s) = \iint_{x \in S} [\hat{G}_0(x^r - x) \partial_n \hat{p}(x, x^s) - \partial_n \hat{G}_0(x^r - x) \hat{p}(x, x^s)] dS, \quad (1)$$

where $\hat{G}_0$ is the Green's function, and ∂_n denotes the normal derivative pointing outwards. The explicit dependence of the Green's function and the scattered field on frequency is omitted. The three-dimensional vectors x^r , x^s , and x locate the receiver positions, the source positions, and the points at the discontinuity interface S respectively. The following analysis is restricted to two dimensions only, but can be extended to three dimensions (Fokkema and van den Berg, 1991). The horizontal coordinate then becomes x , and the vertical one $z(x)$. Rather than working with the absolute receiver coordinates, the offset coordinate, $x^o = x^r - x^s$, is introduced. The Radon transform is applied twice, with respect to the offset coordinates and the source coordinates. The resulting integrals are solved by applying the high frequency approximation and the stationary phase approximation. This leads to

$$\tilde{p}(p^o, p^s) = \int_{-\infty}^{\infty} E \exp[i2\pi f(2\Phi_0 z(x) - p^s x)] dx, \quad (2)$$

where E is the transformed and scaled amplitude, and $z(x)$ is the laterally changing distance from the source-receiver level (reference level) to the discontinuity interface. The phase function Φ_0 for the first interface is given by

$$\Phi_0 = \frac{1}{2} \left[\frac{1}{c_0^2} - p^{o2} \right]^{1/2} + \frac{1}{2} \left[\frac{1}{c_0^2} - (p^s - p^o)^2 \right]^{1/2}, \quad (3)$$

where p^o and p^s are the ray parameters resulting from the Radon transforms over x^o and x^s coordinates respectively, and c_0 is the interval velocity through the first layer.

Equations (2) and (3) form the basis for the p^o - p^s - f migration. The complete derivation of these equations can be found in Fokkema and van den Berg (1991).

Imaging

For a given value of p^s , such that $p^s \in D^s(\Phi_0)$, a real solution of p^o in equation (3) exists if

$$\frac{1}{2c_0} < \Phi_0 < \frac{1}{c_0}. \quad (4)$$

The acceptable p^o , p^s pairs for a fixed value of Φ_0 are located at the image domain $D^i(\Phi_0)$, which contains the energy reflected from the first interface. Depending on the value of Φ_0 , it describes any position of the reflected energy between one-way and two-way travel time. The imaging surfaces for a constant time, that is the imaging lines, are shown in Figure 2 for several values of Φ_0 .

With these restrictions on p^o , p^s values, equation (2) can be rewritten as

$$\tilde{P}(\Phi_0, p^s) = \int_{-\infty}^{\infty} E \exp[i2\pi f(2\Phi_0 z(x) - p^s x)] dx, \quad (5)$$

where

$$\tilde{P}(\Phi_0, p^s) = \tilde{P}(p^o, p^s), (p^o, p^s) \in D^i(\Phi_0). \quad (6)$$

To obtain the spatial position of the interface, we apply a single inverse Radon transform, without moving the source and the receivers, to the data contained at the imaging surface

$$E \exp[i4\pi f\Phi_0 z(x)] = \int_{p^s \in D^s} \tilde{P}(\Phi_0, p^s) \exp[i2\pi f p^s x] dp^s. \quad (7)$$

Transformed back to the time domain, the first arrivals from the interface are observed at

$$t = 2\Phi_0 z(x). \quad (8)$$

The depth image follows from scaling the data according to equation (8). Expressed in terms of ray-theoretical migration methods, this imaging procedure positions the events along the image rays rather than at the apex of diffractions (Larner et al., 1980). Since the events are already imaged to their proper lateral positions, simple depth conversion is sufficient. The next interface can be imaged after the data have been extrapolated down to the first interface, and the origin shifted to a new reference level.

Imaging with the RMS velocity

Equation (3) is valid for the first interface, since interval velocity through the first layer c_0 is used. It can be extended to any depth level, if *RMS* velocity is used instead of interval velocity. In this case, the imaging surface changes with depth, according to

$$\Phi_{RMS} = \frac{1}{2} \left[\frac{1}{c_{RMS}^2} - p^{o2} \right]^{1/2} + \frac{1}{2} \left[\frac{1}{c_{RMS}^2} - (p^s - p^o)^2 \right]^{1/2}. \quad (9)$$

Imaging performed with *RMS* velocities produces the time image of the depth structure. This procedure is equivalent to prestack time migration plus stack.

Data examples

The first synthetic data set with a single interface was created using finite differences in time domain, and subsequently transformed to double Radon domain. The squared amplitudes are summed along the time axis, and the resulting energy density is plotted as a function of p^o and p^s in Figure 3. The concentration of high amplitudes along three extrapolation lines indicates the presence of a horizontal segment and two dipping segments. The imaging was performed with the interval velocity, and the resulting depth image, shown in Figure 4, closely resembles the input model.

The second model consists of three interfaces: the middle one has the same geometry as the previous model, and the top and the bottom interface are horizontal. The energy density plot in Figure 5a, produced by stacking the data in the entire time window, indicates that the horizontal energy dominates in this model. When the time window for stack excludes the first interface (Figure 5b), the energy from dipping interfaces becomes apparent. The energy contained in the imaging domain, obtained by using the *RMS* velocity which was estimated in the $p^s=0$ plane, is shown in Figure 6. Finally, the image of the model as a function of one-way travel time is shown in Figure 7. The bottom interface suffers some propagation effects from the upper layer, but the overall image is correct.

Conclusions

Several features of this new migration algorithm make it attractive. It does not depend closely on the detailed input velocity model, unlike conventional prestack migrations. A good estimate of background velocity is easily obtained in the p^o - p^s domain, where the data are discretized according to the local reflector dip. The imaging is performed as a first step, defining the surface to which the sources and receivers need to be downward continued.

It can be organized either as a time migration, performed as a one-step imaging process with *RMS* velocities, or a depth migration, when the imaging of one layer with interval velocity is followed by extrapolation and coordinate shift.

The synthetic data examples indicate that the algorithm is promising. Further tests are being performed.

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Figure 1. The extrapolation curves in the p^o - p^s plane.

Figure 2. The imaging curves in the p^o - p^s plane.

Figure 3. The energy density distribution in the p^o - p^s plane, and the imaging curve.

Figure 4. The model and the migrated depth image.

Figure 5. (a) The energy density distribution in the p^o - p^s plane. The time window for stack was 0 msec - 1000 msec. (b) The energy density distribution in the p^o - p^s plane. The time window for stack was 280 msec - 1000 msec.

Figure 6. The image domain.

Figure 7. The model and the migrated time-space image of Figure 6. The vertical axis is one-way travel time, scaled using the chosen Φ_0 .