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Material-Structure Integrated Design Optimization of GFRP Bridge Deck on Steel Girder

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Abstract:

Design optimization of fiber-reinforced polymeric (FRP) composite products is essential to facilitate their applications in engineering structures. For bridge structures, the main design optimization goals are the reduction of FRP material consumption and the structure weight, which aim to reduce the initial construction cost and achieve a longer bridge span. Compared with conventional steel-concrete composite bridges, FRP-steel composite bridges possess more design variables and more complex design process, which necessitate the simplified optimization models. This paper aims to propose a two-scale design optimization method for FRP bridge deck on the steel girder. The macro behavior of the pultruded FRP composite bridge deck is analyzed. Regarding the micro level, the equivalent properties of pultruded GFRP lamination are calculated by combining micromechanics and classical lamination theory (CLT). The above-mentioned macro pultruded GFRP bridge level and the micro fiber/resin level were bridged based on the assumption that the micro-component effective homogenized strain equals to the corresponding macro strain. The two-scale lamination optimization of pultruded GFRP bridge deck is finally achieved by finding optimized two-scale design variables that can achieve the minimum bridge weight or the lowest initial construction cost with all listed constraint requirements satisfied. A pultruded FRP deck supported on equally-spaced steel girders was selected as a case study to show how to obtain
the optimized two-scale parameters by using this proposed optimization method. The optimized results of
the top flange thickness, $t_u$, the bottom flange thickness, $t_l$, the web height, $h_w$, and the web thickness per
meter, $t_{wc}$, are 46.02 mm, 45.86 mm, 300.0 mm and 37.42 mm, respectively. Results also showed that the
optimized ratio of the $0^\circ$-lamina, $45^\circ$-lamina, and the $90^\circ$-lamina are 77.9%, 17.1%, 5.0%. The optimized
fiber volume fraction is 65.2%.

**Keywords:** Composite bridge girder; Pultruded GFRP bridge deck; Laminations; Multiscale
optimization.

1. Introduction

Fiber-reinforced polymer (FRP) composites have been greatly developed worldwide and have
become one of the most popular construction materials for repair and rehabilitation and new construction
[1–10]. Pultruded glass fiber reinforced polymer (GFRP) composites are great candidates for newly
constructed bridges decks. A variety of GFRP bridge deck applications are presented in [11]. Figure 1
shows a commonly used composite girder system which consists of the pultruded GFRP bridge deck and
the supporting steel girders. Noted that the the steel girder with a corrugated web [12] is also an
interesting surrogate. The pultruded GFRP bridge decks and steel girders can be connected using
adhesives or bolts [13].
Different from conventional isotropic construction materials like reinforced concrete and steel, GFRP composites are inhomogeneous and anisotropic, which require to be analyzed and designed on different scales, namely, the micro-scale and macro-scale. The importance of multi-scale analysis to determine the mechanical properties of GFRP materials has been pointed out in previous studies [14,15].

During the design stage of a GFRP bridge deck, engineers are not only interested in fulfilling the strength and serviceability requirements, which are the top design priorities, but also in satisfying these requirements with the least possible amount of materials that will result in a weight reduction of the structure and further achieve lower initial construction cost. Thus optimization techniques is very important in obtaining the best use of FRP material in bridge decks. The optimization tasks involve determining the optimal ratio of fiber reinforcements, the optimum fiber volume fractions and geometric variables in order to achieve the best design in both material and structure scales. In addition, the complexity of general pultruded GFRP bridge decks necessitates the development of simplified optimization models.

Most of the previous optimization work in the design of composite structures [16–20] focused on aerospace structures, but pultruded GFRP composites, commonly used in bridge decks, are quite different in nature with the composites used in aerospace structures [15], as can be reflected in Figure 2. These differences include: (i) the pultruded FRP laminations have a relatively poor quality, and (ii) the roving content is larger than fabrics, leading to an increase in the thickness of the unidirectional lamina (0°-lamina) of up to 5–15 times the laminas with other orientations.
A pilot investigation related to material-structure integrated design is presented in this paper. The macro behavior of the pultruded FRP composite bridge deck is analyzed. Regarding the micro level, the equivalent properties of pultruded GFRP lamination are calculated by combining micromechanics and classical lamination theory (CLT). The above-mentioned macro pultruded GFRP bridge level and the micro fiber/resin level were bridged based on the assumption that the micro-component effective homogenized strain equals to the corresponding macro strain. The two-scale lamination optimization of pultruded GFRP bridge deck is finally achieved by finding optimized two-scale design variables that can achieve the minimum bridge weight or the lowest initial construction cost with all listed constraint requirements satisfied. Also, a case study was presented to show how to obtain the optimized two-scale parameters by adopting the proposed optimization method in the last part of this paper.

2. Macro Behaviour of the Pultruded GFRP Composite Bridge Deck

GFRP composite bridge decks, together with the supporting steel girders, were subjected to longitudinal bending moment ($M^L$) and shear force ($Q^L$), as well as transverse bending moment ($M^T$) and shear force ($Q^T$). The following sections would describe the mechanical behaviors of bridge deck in
both the longitudinal and transverse directions under corresponding bending moment and shear force.

### 2.1 Macro Behavior in the Longitudinal Direction

Following assumptions were made to analyze the mechanical behavior of the composite girder along the longitudinal direction: (i) the shear connection stiffness is sufficient to ensure a full composite action between the GFRP bridge deck and the supporting steel girder; (ii) the longitudinal shear forces are fully resisted by the steel webs; (iii) the macro longitudinal stresses are uniformly distributed along the flange thickness considering the fact that the laminate thickness dimension is quite small relative to the total height of the steel girder; (iv) the flexural and shear resistances provided by discontinuous web along the longitudinal direction are neglected.

![Diagram of composite cross section](image)

**Figure 3.** Schematic of composite cross section

Due to the in-plane shear flexibility of the GFRP composite deck, the normal stress along the width of the deck is non-uniformly distributed, see Figure 3. The maximum stress in the deck occurs in the centerline of the web and stresses in the bridge deck away from the web lag behind [21]. Thus, the effective flange width, $b_{eff}$, is introduced in design practice to simplify the analytical procedure, as denoted in Figure 3. The effective flange width, $b_{eff}$, is defined as a reduced width of the deck over which the normal stresses are assumed to be uniformly distributed, and it is calculated [22] based on the premise...
that the stress resultant over the effective width should be equal to the stress resultant over the actual flange width, as defined in Eq. (1).

\[
b_{eff} = \frac{\int_{0}^{b} \sigma_f(x) \, dx}{\left(\sigma_f^\text{max}_f\right)}
\]  

(1)

where: \(\sigma_f^\text{max}_f\) is longitudinal normal stress in the flange of GFRP bridge deck; \(\left(\sigma_f^\text{max}_f\right)\) is the maximum longitudinal normal stress in the flange of GFRP bridge deck, and \(b\) is the center-to-center spacing of the steel girders.

The effective flange width of the GFRP bridge deck supported by the steel girders can be simply predicted [22] by using Eqs. (2) and (3) as follows:

\[
b_{eff} = Rb_{eff,s}
\]

(2)

\[
R = 1.025\left(1 - 0.0244\theta\right)
\]

(3)

where: \(b_{eff,s}\) is the effective width suggested by highway bridges design specifications [30, 31], and \(\theta\) is the degree of composite action between the GFRP composite bridge deck and the main girders. The longitudinal normal stresses at the top flange, \(\sigma_{f_u}^y\), and the bottom flange, \(\sigma_{f_b}^y\), can be calculated by Eqs. (4) and (5) as follows:

\[
\sigma_{f_u}^y = -\frac{M^f z_{f_{fu}}^f}{n I_v^y}
\]

(4)

\[
\sigma_{f_b}^y = -\frac{M^f z_{f_{fb}}^f}{n I_v^y}
\]

(5)

where: \(n\) is the elastic moduli ratio (modular ratio) between steel modulus \((E_s)\) and the longitudinal modulus of the GFRP composites deck \((E_f^y)\) and is expressed by Eq. (6):

\[
n = \frac{E_s}{E_f^y}
\]

(6)

\(z_{f_{fu}}^f\) and \(z_{f_{fb}}^f\) are the distances from the top and the bottom flanges of the GFRP deck to the neutral axis of the GFRP/steel composite girder, \(z_c\), respectively. Thus:
The distance between the neutral axis of the GFRP/steel composite girder and the bottom fiber of the steel girder, $z_c$, is calculated by the following equation:

$$ z_c = \frac{2A_sE_sz_s + b_{eff}E_f^y(t_u^3 + t_l^3 + 2t_uh_u + 2t_lt_l + 2u_h + 2h_lt_l)}{2(A_sE_s + b_{eff}t_uE_f^y + b_{eff}t_lt_l)} $$

(9)

The equivalent moment of inertia of the GFRP/steel composite girder $I_v$ could be calculated by Eq. (10).

$$ I_v = I_s + A_s(z_c - z_s)^2 + b_{eff}(t_u^3 + t_l^3)/(2n) + b_{eff}(t_u + t_l)(h_s + z_f - z_c)^2 / n $$

(10)

$$ z_f = \frac{t_l^2 + t_u^2 + 2t_uh_u + 2t_lt_l}{2(t_u + t_l)} $$

(11)

where: $h_s$ is the height of the steel beam; $t_l$ is the thickness of bottom flange; $t_u$ is the thickness of the top flange; $h_u$ is the web height of pultruded GFRP bridge deck; $A_s$ is the cross-sectional area of the steel beam, and $z_s$ is the distance between the neutral axis and the bottom fiber of the steel girder.

### 2.2 Macro Behavior in the Transverse Direction

The following assumptions were made to analyze the longitudinal mechanical behavior of the GFRP-steel composite girder: (i) the transverse shear force is fully resisted by the web of GFRP bridge deck; (ii) the transverse normal stress is uniformly distributed along with the top/bottom flange thickness.

The transverse normal stress in the top flange $\sigma_{ft}^x$ and bottom flange $\sigma_{fb}^x$, as denoted in Figure 4, could be calculated based on Eqs. (12)–(13).

$$ \sigma_{ft}^x = -\frac{M^Tz_{ft}^x}{I_f^x} $$

(12)

$$ \sigma_{fb}^x = \frac{M^Tz_{fb}^x}{I_f^x} $$

(13)

where: the transverse moment of inertia $I_f^x$ of pultruded GFRP bridge deck is:
Note positive and negative signs in Eqs. (12)–(13) represent tensile and compressive stresses, respectively.

The web thickness per meter $t_w$ along longitudinal direction is calculated by Eq. (15).

$$t_w = \frac{1000}{a} \sum_{i}^n t_{w(i)}$$

where, $a$ is the width of GFPP deck profile, and $t_{w(i)}$ is the thickness of web in each GFRP deck profile.

$z_{fu}$ in Eq. (11) and $z_{fl}$ in Eq. (12) respectively refers to the distances between the top/bottom flange of GFRP composite bridge deck and its neutral axis, and can be calculated by Eqs. (16) and (17), respectively:

$$z_{fu}^T = t_f + h_w + t_u - z_f^T$$

(16)

$$z_{fl}^T = z_f^T$$

(17)

where the height of the GFRP bridge deck neutral axis along the transverse direction, $z_f^T$, is given by Eq. (18):

$$z_f^T = \frac{1000(t_u t_f + t_u h_w) + h_u t_u (t_f + h_w / 2) + 500(t_f^2 + t_u^2)}{1000t_u + h_u + t_u + 1000t_f}$$

(18)

The shear stress, $\tau_{fu}$ in the web of the pultruded GFRP bridge deck is calculated by Eqs. (19):

$$\tau_{fu}^w = \frac{Q^T}{t_u h_w}$$

(19)

In order to guarantee a safe design, the GFRP bridge deck is assumed simply supported by steel girder. The transverse deflection of pultruded GFRP bridge deck can be conservatively predicted using Timoshenko beam theory [25]:

$$\delta_f^{(max)} = \frac{5M_f b^2}{48E_f I_f^r} + \frac{Q_f b}{4t_u h_u G_f^w}$$

(20)
where: $E_f$ and $G_{f}^{xy}$ are the elastic and in-plane shear moduli of the GFRP composite bridge deck in the transverse direction.

![Geometry symbols in YZ plane](image1)

(a) Geometry symbols in YZ plane

![Geometry symbols in XZ plane](image2)

(b) Geometry symbols in XZ plane

**Figure 4.** GFRP/Steel composite bridge girder parameters

### 3. Micro behavior of pultruded lamination

The reinforcements used for manufacturing the pultruded GFRP composite bridge deck described in this paper are composed of (i) unidirectional E-glass roving, and (ii) non-crimp (multi-warp knitted) fabrics [15]. In general, the laminations lay-up includes three different types of the lamina, namely, $0^\circ$-plies in the form of E-glass roving, and non-crimp E-glass fabrics with $90^\circ$ and $\pm 45^\circ$ orientations. Based on the classical lamination theory [26], the effective modulus of the pultruded laminate could be
estimated using Eqs. (21)–(23), assuming that the ratio of 0°, 45°, and 90° lamina to the total lamination are $\xi_0$, $\xi_{45}$, and $\xi_{90}$, respectively.

\[
E_f^x = \xi_0 \left[ \frac{E_1 - v_{12}^2 E_2}{1 - v_{12} v_{21}} \right] + \xi_{90} \left[ \frac{E_1 E_2 - v_{12}^2 E_2}{E_1 (1 - v_{12} v_{21})} \right] + \xi_{45} \frac{1}{(1 - v_{12} v_{21})} \left[ \frac{(E_1 + E_2 + 2v_{12} E_2 + 4(1 - v_{12} v_{21}) G_{12})^2 - 16 (v_{12} E_2)^2}{4 \left[ E_1 + E_2 + 2v_{12} E_2 + 4(1 - v_{12} v_{21}) G_{12} \right]} \right]
\]

(21)

\[
E_f^y = \xi_0 \left[ \frac{E_1 E_2 - v_{12}^2 E_2}{E_1 (1 - v_{12} v_{21})} \right] + \xi_{90} \left[ \frac{E_1 - v_{12}^2 E_2}{1 - v_{12} v_{21}} \right] + \xi_{45} \frac{1}{(1 - v_{12} v_{21})} \left[ \frac{(E_1 + E_2 + 2v_{12} E_2 + 4(1 - v_{12} v_{21}) G_{12})^2 - 16 (v_{12} E_2)^2}{4 \left[ E_1 + E_2 + 2v_{12} E_2 + 4(1 - v_{12} v_{21}) G_{12} \right]} \right]
\]

(22)

\[
G_f^{xy} = \xi_0 G_{12} + \xi_{90} G_{12} + \xi_{45} \left[ \frac{E_1 + E_2 - 2v_{12} E_2}{4(1 - v_{12} v_{21})} \right]
\]

(23)

where: $E_f^x$ is the effective elastic modulus of GFRP laminates in the longitudinal direction of the bridge; $E_f^y$ is the effective elastic modulus of GFRP laminates in the transverse direction of the bridge; $G_f^{xy}$ is the effective in-plane shear modulus of GFRP laminates.

The longitudinal modulus, $E_1$, transverse modulus, $E_2$, shear modulus, $G_{12}$, and Poisson’s ratio, $v_{12}$ of the lamina can be determined based on the modified mixture formulae [6]:

\[
E_1 = E_{f1} V_f + E_m V_m
\]

(24)

\[
E_2 = \frac{E_{f1} E_m \left[ V_f + \eta_2 V_m \right]}{E_m V_f + E_{f1} \eta_2 V_m}
\]

(25)

\[
\eta_2 = 0.2 + \frac{0.2}{1 - V_m} \left( 1 - \frac{E_m}{E_{f1}} \right) \left( 1 - \frac{E_{f1}}{3.5 E_m} \right) (1 + 0.22V_f)
\]

(26)

\[
G_{12} = \frac{G_f G_m (V_f + \eta_{12} V_m)}{G_m V_f + G_f \eta_{12} V_m}
\]

(27)

\[
\eta_{12} = 0.28 + \sqrt{\frac{E_m}{E_f}}
\]

(28)

\[
v_{12} = v_f V_f + v_m V_m
\]

(29)

where: $E_{f1}$ is the longitudinal elastic modulus of fiber, $E_{f2}$ is the transverse elastic modulus of fiber, $V_f$ is
the fiber volume fraction, \( v_f \) is the fibers’ Poisson’s ratio, \( E_m \) is the matrix elastic modulus, \( V_m \) is the resin volume fraction, \( v_m \) is the matrix’s Poisson’s ratio, \( G_f \) is the shear modulus of fibers, and \( G_m \) is the resin shear modulus.

The strength-based design method is accepted in many design practices, however, in this study, the variation of elastic moduli and ultimate strength of each lamina complicates the lamination optimization procedures. Thus, the strain-based design method is adopted in this paper.

By neglecting the curvature effects, the ultimate strain of each ply in the laminate is deemed to be the same based on First-Ply-Failure (FPF) analytical method [26]. The ultimate strain of each lamina can be obtained based on the micromechanics approach [6] using Eqs. (30)–(34).

\[
\varepsilon_{1u} = \frac{X_f}{E_1} \frac{X_f}{E_{f1}} = \varepsilon_f \tag{30}
\]

\[
\varepsilon_{2u} = \frac{X_c}{E_1} \frac{X_c}{E_{f1}} = \varepsilon_f \tag{31}
\]

\[
\varepsilon_{2u} = \frac{Y_T}{E_2} \frac{X_m \left( E_m V_f + E_f \eta_f V_m \right)}{SCF E_{f1} E_m \left( V_f + \eta_f V_m \right)} \tag{32}
\]

\[
\varepsilon_{2u} = \frac{Y_T}{E_2} \varepsilon_{mc} \left[ 1 - \left( 4V_f / \pi \right) \left( 1 - \frac{E_m}{E_f} \right) \right] \tag{33}
\]

\[
\gamma_{12} = \frac{S}{G_{12}} = S_m \left[ 1 + \left( V_f - \sqrt{V_f} \right) \left( 1 - \frac{G_m}{G_f} \right) \right] \left( \frac{G_m V_f + G_f \eta_f V_m}{G_f \left( V_f + \eta_f V_m \right)} \right) \left( \frac{G_f}{G_m} \right) \tag{34}
\]

When applying loads along the pultrusion direction, the ultimate strain of the \( 0^0, 90^0, \pm 45^0 \) lamina is \( \varepsilon_{1u}, \varepsilon_{2u}, \) and \( \gamma \), respectively. When the loads are applied perpendicular to the pultrusion direction, the ultimate strain of \( 0^0, 90^0, \pm 45^0 \) lamina is \( \varepsilon_{2u}, \varepsilon_{1u}, \) and \( \gamma \), respectively. Based on the “First-Ply-Failure” failure criterion, the ultimate strain of each lamina can be calculated using Eqs. (35)–(37). The ultimate strain variation as related to fiber volume fraction is shown in Fig. 5. These values were calculated using Eqs. (31)–(37) with material properties listed in Tables 1 and 2 [27].

\[
\varepsilon_u = \min \left( \varepsilon_{1u}, \varepsilon_{2u}, \gamma \right) = \varepsilon_{2u} \tag{35}
\]
\[ e^u = \min (e_1^u, e_2^u, e_3^u) = e_2^u \]  
\[ \gamma^u = \gamma_{12}^u \]  

**Table 1.** Mechanical properties of E-glass fibers[15]  

<table>
<thead>
<tr>
<th>Property</th>
<th>Value 1</th>
<th>Value 2</th>
<th>Value 3</th>
<th>Value 4</th>
<th>Value 5</th>
<th>Value 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitudinal modulus ( (E_f) )</td>
<td>74.0 GPa</td>
<td>74.0 GPa</td>
<td>0.20</td>
<td>30.80 GPa</td>
<td>2150 MPa</td>
<td>1450 MPa</td>
</tr>
<tr>
<td>Transverse modulus ( (E_t) )</td>
<td>74.0 GPa</td>
<td>74.0 GPa</td>
<td>0.20</td>
<td>30.80 GPa</td>
<td>2150 MPa</td>
<td>1450 MPa</td>
</tr>
<tr>
<td>Poisson's ratio ( (v_f) )</td>
<td>0.20</td>
<td>0.20</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>Shear modulus ( (G_f) )</td>
<td>30.80 GPa</td>
<td>30.80 GPa</td>
<td>1.24 GPa</td>
<td>1.24 GPa</td>
<td>1.24 GPa</td>
<td>1.24 GPa</td>
</tr>
<tr>
<td>Tensile strength ( (X_f) )</td>
<td>2150 MPa</td>
<td>2150 MPa</td>
<td>80 MPa</td>
<td>80 MPa</td>
<td>80 MPa</td>
<td>80 MPa</td>
</tr>
<tr>
<td>Compressive strength ( (X_c) )</td>
<td>1450 MPa</td>
<td>1450 MPa</td>
<td>120 MPa</td>
<td>120 MPa</td>
<td>120 MPa</td>
<td>120 MPa</td>
</tr>
<tr>
<td>Density ( (\rho) )</td>
<td>2560 kg/m³</td>
<td>2560 kg/m³</td>
<td>2560 kg/m³</td>
<td>2560 kg/m³</td>
<td>2560 kg/m³</td>
<td>2560 kg/m³</td>
</tr>
</tbody>
</table>

**Table 2.** Mechanical properties of epoxy resin [15]  

<table>
<thead>
<tr>
<th>Property</th>
<th>Value 1</th>
<th>Value 2</th>
<th>Value 3</th>
<th>Value 4</th>
<th>Value 5</th>
<th>Value 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulus ( (E_m) )</td>
<td>3.35 GPa</td>
<td>3.35 GPa</td>
<td>1.24 GPa</td>
<td>1.24 Gpa</td>
<td>1.24 Gpa</td>
<td>1.24 Gpa</td>
</tr>
<tr>
<td>Poisson's ratio ( (v_m) )</td>
<td>0.35</td>
<td>0.35</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>Shear modulus ( (G_m) )</td>
<td>1.24 GPa</td>
<td>1.24 Gpa</td>
<td>88 MPa</td>
<td>88 MPa</td>
<td>88 MPa</td>
<td>88 MPa</td>
</tr>
<tr>
<td>Tensile strength ( (X_m) )</td>
<td>80 MPa</td>
<td>80 MPa</td>
<td>80 MPa</td>
<td>80 MPa</td>
<td>80 MPa</td>
<td>80 MPa</td>
</tr>
<tr>
<td>Compressive strength ( (X_c) )</td>
<td>120 MPa</td>
<td>120 MPa</td>
<td>120 MPa</td>
<td>120 MPa</td>
<td>120 MPa</td>
<td>120 MPa</td>
</tr>
<tr>
<td>Shear strength ( (S_m) )</td>
<td>75 MPa</td>
<td>75 MPa</td>
<td>75 MPa</td>
<td>75 MPa</td>
<td>75 MPa</td>
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<tr>
<td>Density ( (\rho) )</td>
<td>1160 kg/m³</td>
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<td>1160 kg/m³</td>
<td>1160 kg/m³</td>
<td>1160 kg/m³</td>
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</tr>
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(a) Tension
4. Design values

In general, bridge structural members are exposed to harsh and changing environments such as moisture, salt-spray agents, freeze-thaw cycles, and large variations in both temperature and humidity [28–32]. Due to continuous exposure to such harsh environments, degradation in the mechanical properties of composites is expected to occur [28–32]. In this section, the assumption was made that the design strain equals to the product of ultimate strain and a reduction or a degradation coefficient. For the top flange of a pultruded GFRP composite bridge deck, we have:

\[
\begin{align*}
\epsilon_{cu}^{x} & \leq \epsilon_{u}^{d} \approx \chi_{d}^{c} \epsilon_{u}^{u} \\
\epsilon_{cu}^{y} & \leq \epsilon_{u}^{d} \approx \chi_{d}^{c} \epsilon_{u}^{u}
\end{align*}
\]  

(38)

for the bottom flange of pultruded GFRP bridge deck, we have:

\[
\begin{align*}
\epsilon_{cu}^{x} & \leq \epsilon_{u}^{d} \approx \chi_{d}^{t} \epsilon_{u}^{u} \\
\epsilon_{cu}^{y} & \leq \epsilon_{u}^{d} \approx \chi_{d}^{c} \epsilon_{u}^{u}
\end{align*}
\]  

(39)

and for the web of pultruded GFRP bridge deck, we have:
where: $c_{d}$, $d_{d}$, $s_{d}$ are the reduction (degradation) coefficients for GFRP materials in compression, in tension, and in shear, respectively.

The Chinese Technical Code for Infrastructure Application of FRP Composites (GB 50608-2010) [32] suggests that the design values are determined by dividing experimental ultimate strength by appropriate partial safety factors that account for material type, and the surrounding environment. The following equations can be used to calculate the reduction (degradation) coefficient:

$$\chi_{d} = \frac{\mu'_{u} - 1.645\sigma}{\mu''} \frac{1}{\gamma_{f}\gamma_{e}}$$  \hspace{1cm} (41)

where: $\mu'_{u}$ is the average material strength; $\sigma$ is the standard derivation of the test number; $\gamma_{f}$ is the partial safety factor to account for material type; $\gamma_{e}$ is the partial safety factor to account for environmental exposure.

In addition, the transverse deflection of the pultruded GFRP bridge deck should always be smaller than a limiting transverse deflection to ensure the stiffness requirement.

$$\delta_{f}^{z(max)} \leq \delta''$$  \hspace{1cm} (42)

where: $\delta_{f}^{z(max)}$ is the maximum transverse deflection of GFRP bridge deck under applied load, $\delta''$ is the limited transverse deflection based on the design requirement.

5. Bridging fiber/resin level to structure level

In this section, the micro fiber/resin scale is bridged to the macro-the GFRP/steel composite girder scale by assuming that the effective homogenized strain obtained from micro-component equals to macro-strain. Linking micro and macro longitudinal and transverse strains at the top flange of a pultruded GFRP bridge deck is achieved by using the following equations:

$$\sigma_{f}^{x} = \frac{M_{f}^{y}}{E_{f}^{x}} \approx \frac{\sigma_{f}^{x}}{E_{f}^{x}}$$  \hspace{1cm} (43)
Similarly, linking both micro and macro longitudinal and transverse strains at the bottom flange of a pultruded GFRP bridge deck is achieved by the following equations:

\[ \varepsilon_{fl}^{y} = \frac{\sigma_{fl}^{y}}{E_{f}^{y}} \approx \frac{M^{T}z_{fl}^{T}}{E_{f}^{y}l_{f}} \] (45)

\[ \varepsilon_{fl}^{x} = -\frac{M^{T}z_{fl}^{T}}{nI_{f}E_{f}^{y}} \] (46)

Eq. (48) shows how to link the micro and macro shear strains at the web of a pultruded GFRP bridge deck:

\[ \tau_{w}^{xy} = \frac{\tau_{w}^{xy}}{G_{f}^{xy}} = \frac{Q^{T}}{t_{w}h_{w}G_{f}^{xy}} \] (47)

6. Optimization equations for pultruded bridge decks

The main goals of multiscale optimization of GFRP bridge decks towards material-structure integrated design are to achieve: (i) the lightest weight to increase the bridge span while satisfying all design and manufacturing requirements, or (ii) the lowest cost for the economy and constructional convenience. Mathematically speaking, the multiscale optimization of GFRP bridge decks is to seek a minimum value of cost or weight by optimizing multiscale design variables within given allowed constrained functions determined by design and manufacturing requirements. In this paper, the multiscale lamination optimization of a pultruded GFRP bridge deck is achieved by finding an optimized two-scale design variable vector, \( x \), that drive the objective weight function, \( \Phi_{1} \), or the objective price function, \( \Phi_{2} \), to its lowest values while satisfying all constraint functions (\( \Phi^{1} \sim \Phi^{6} \)). The design variables, objective functions, and constraint functions will be explained in the following sections.

(1) Design Variables: Eq. (48) describes the two-scale optimization design variable vector, \( x \), including the thickness of the top flange, \( t_{u} \), the thickness of the bottom flange, \( t_{l} \), the height of the web,
274  $h_w$, the thickness per meter of the web, $t_w$, the ratio of $0^\circ$, $45^\circ$, $90^\circ$ lamina to the total laminate are $\xi_0$, $\xi_{45}$, and $\xi_{90}$, respectively, and the fiber volume fraction $V_f$.

275  

276  \[
  x = \begin{bmatrix} t_u, t_t, h_w, t_n, \xi_0, \xi_{90}, \xi_{45}, V_f \end{bmatrix}^T
\]  

277  (48)

278  (2) **Objective function:** The objective function $\phi_1$ related to the optimizing weight is given as follows:

279  

280  \[
  \phi_1 = 1000 \left( 1000 (t_u + t_t) + h_w t_n \right) \left( \rho_f V_f + \rho_m V_m \right)
\]  

281  (49)

282  where: $\rho_f$ is fiber density, and $\rho_m$ is the resin density.

283  The objective function $\phi_2$, related to the optimizing cost is given in Eqn. (50). It should be noted that the manufacturing cost is not included in this expression due to the fact that different manufacturers have different selling prices.

284  

285  \[
  \phi_2 = 1000 \left( 1000 (t_u + t_t) + h_w t_n \right) \left( \eta_f \rho_f V_f + \eta_m \rho_m V_m \right)
\]  

286  (50)

287  where: $\eta_f$ is the price of the fibers, and $\eta_m$ is the price of the matrix.

288  

289  (3) **Constraint functions:** In this study, a total of six constraint functions were specified as follows.

290  (i) **Constraint function $\Phi^1$ (strength requirement of the top flange):**

291  The longitudinal and transverse normal strains at the top flange of the GFRP deck should be smaller than corresponding design values of normal strains, i.e.,

292  \[
  \Phi^1 = \begin{cases} 
  |\vec{\varepsilon}^x| \leq \varepsilon^d_x \\
  |\vec{\varepsilon}^y| \leq \varepsilon^d_y
\end{cases}
\]  

293  (51)

294  (ii) **Constraint function $\Phi^2$ (strength requirement of the bottom flange):**

295  The longitudinal and transverse normal strains at the bottom flange of the GFRP deck should be smaller than corresponding allowable maximum normal strains, i.e.,

296  \[
  \Phi^2 = \begin{cases} 
  |\vec{\varepsilon}^x| \leq \varepsilon^d_x \\
  |\vec{\varepsilon}^y| \leq \varepsilon^d_y
\end{cases}
\]  

297  (52)
(iii) Constraint function $\Phi^3$ (strength requirement of the web):

The shear strain at the web of the GFRP deck should be smaller than allowable maximum shear strain, i.e.,

$$\Phi^3 = \left| \frac{1}{E_w} \right| \leq \gamma^d$$  \hspace{1cm} (53)

(iv) Constraint function $\Phi^4$ (stiffness requirement):

The transverse displacement of the GFRP deck should be smaller than the specified deflection, i.e.:

$$\Phi^4 = \delta^{(\text{max})}_f \leq \delta^w$$  \hspace{1cm} (54)

(v) Constraint function $\Phi^5$ (manufacturing requirement):

The fractions of different types of laminates should be within the specified ranges, which are determined by the pultrusion manufacture, i.e.:

$$\Phi^5 = \begin{cases} 
0.25 \leq V_f \leq 0.75 \\
\varepsilon^l \leq \varepsilon_0 \leq \varepsilon^h \\
\varepsilon^l \leq \varepsilon_{45} \leq \varepsilon^h \\
\varepsilon^l \leq \varepsilon_{90} \leq \varepsilon^h 
\end{cases}$$  \hspace{1cm} (55)

(vi) Constraint function $\Phi^6$ (geometrical requirement):

The thickness of the plates should be within the specified ranges to avoid local buckling occurring in the excessive thin plate, and to meet the manufacturing capabilities since each manufacturer can only produce the GFRP plate within the specific range of the thickness, i.e.:

$$\Phi^6 = \begin{cases} 
(t_u)^l \leq t_u \leq (t_u)^u \\
(t_l)^l \leq t_l \leq (t_l)^u \\
(t_w)^l \leq t_w \leq (t_w)^u \\
h_w^l \leq h_w \leq (h_w)^u 
\end{cases}$$  \hspace{1cm} (56)

7. Application to composite bridge girder

A composite bridge girder with a main span of 20.0 meters was selected for a case study. This bridge girder consists of GFRP bridge decks and I-shaped steel girders with equal center-to-center spacing of 3.0
meters. The GFRP composite deck is connected to steel girders using the bolted connector, and the degree of composite action between GFRP bridge deck and steel girder, \( \vartheta \) is specified as 0.72. The total height of the I-shaped steel girder is 1000 mm, the thickness of the top flange, bottom flange, and the web, are, 20.0mm, 25.0mm, and 20.0mm, respectively, and the width of both the top and bottom flanges is 400.0 mm. According to the Chinese bridge specifications [24], the design load was calculated as:

\[
S_{ud} = \sum_{i=1}^{m} \gamma_{Gi} S_{Gik} + \gamma_{Q1} S_{Q1k} + \varphi_{c} \sum_{j=2}^{n} \gamma_{Qi} S_{Qjk}
\]  

(57)

where: \( \gamma_{Gi} \), \( \gamma_{Q1} \), \( \gamma_{Qi} \) is the partial safety factor to dead load, vehicle load and live load excluding vehicle load; \( S_{Gik} \), \( S_{Q1k} \), \( S_{Qjk} \) represent the load effects, resulting from the dead load, vehicle load, and live load excluding vehicle load, respectively; and \( \varphi_{c} \) is the combination reduction parameter for the load effect resulting from the live load excluding vehicle load. Note that the design load \( S_{ud} \) can refer to different types of load effects, such as bending moment or shear force. In this study, the design loads include longitudinal bending moment \( M^L \), longitudinal shear force, \( Q^L \), transverse bending moment, \( M^T \), and transverse shear force, \( Q^T \), and they were computed based below equation:

\[
M^L = \frac{1.1\left[1.2\left(q_{s1} + 1.4(1+\mu)q_{Q1}\right) + 1.4bq_{Q2}\right]L^2}{8} + \frac{(1+\mu)q_{P1}L}{4}
\]  

(58)

\[
Q^L = \frac{1.1\left[1.2\left(q_{s1} + 1.4(1+\mu)q_{Q1}\right) + 1.4bq_{Q2}\right]L}{2} + \frac{1.2q_{P1}L}{2}
\]  

(59)

\[
M^T = \frac{1.1\left[1.2\left(q_{s1} + 1000q_{G2}\right) + 1120q_{Q2}\right]b^2}{8} + \frac{1.4(1+\mu)d_{P1}b}{4}
\]  

(60)

\[
Q^T = \frac{1.1\left[1.2\left(q_{s1} + 1000q_{G2}\right) + 1120q_{Q2}\right]b}{2} + \frac{1.4d_{P1}}{2}
\]  

(61)

Where \( q_{s1} \) and \( q_{s1} \) is the self-weight of GFRP deck along the longitudinal and transverse direction respectively; \( q_{s1} \) is the self-weight of steel girder; \( q_{G2} \) is the self-weight of paving, defined as 5 kN/m\(^3\); \( q_{Q1} \) is the line load of the vehicle, defined as 10.5 kN/m * b/3000; \( P_{Q1} \) is the concentration load of the vehicle, defined as 280kN*b/3000; \( \mu \) is impact coefficient, defined as 0.3.
The objective function of weight, $\phi_1$, is specified as:

$$\phi_1 = 1000 \left( 1000 (t_u + t_i) + h_u t_w \right) \left[ 1.28 \times 10^{-5} V_f + 3.25 \times 10^{-5} \left( 1 - V_f \right) \right]$$

while the objective function of price, $\phi_2$, is specified as:

$$\phi_2 = 1000 \left( 1000 (t_u + t_i) + h_u t_w \right) \left[ 2.56 \times 10^{-6} V_f + 1.16 \times 10^{-6} \left( 1 - V_f \right) \right]$$

The reduction (degradation) coefficient is specified as 0.43 based on Eqn. (41) as well as on experimental results of several durability tests [28–32]. The constraint functions for strength requirement $\Phi^l \sim \Phi^3$ thus can be presented as Eqs. (65)–(67).

$$\Phi^l = \begin{cases} [\bar{\sigma}^u] \leq 0.43 \varepsilon^u \\ [\bar{\sigma}^\ell] \leq 0.43 \varepsilon^\ell \end{cases}$$

$$\Phi^2 = \begin{cases} [\bar{\varepsilon}^u] \leq 0.43 \varepsilon^u \\ [\bar{\varepsilon}^\ell] \leq 0.43 \varepsilon^\ell \end{cases}$$

$$\Phi^3 = \left[ \bar{\gamma}^u \right] \leq 0.43 \varepsilon^u$$

The Chinese design specifications of highway bridges [24] recommended that the bridge deck transverse deflection should be smaller than the girder’s span ($b$) divided by 400 (i.e. $b/400$). The constraint functions for stiffness requirement thus should be expressed as:

$$\Phi^4 = \delta^{(\text{max})}_f \leq b / 400$$

The $0^\circ$-lamina of pultruded GFRP laminates is in the form of E-glass roving, while both the $90^\circ$- and $\pm45^\circ$-laminates are in the form of stitched E-glass fabrics. Due to the limitation of pultrusion manufacturing process, the contents of roving are much larger than fabrics for guaranteeing necessary pultrusion traction, making the content of $0^\circ$ lamina is much larger than the laminas with other angle orientations[15]. The minimum ratio of $0^\circ$ lamina is specified as 50% to guarantee necessary pultrusion traction, and the maximum ratio of $90^\circ$ and $\pm45^\circ$ lamina is set as 20% considering manufacture difficulties with larger fabric content. Then constrain functions for pultrusion manufacture requirement is
specified by Eq. (68).

\[ \Phi^t = \begin{cases} 
0.25 \leq V_f \leq 0.70 \\
0.5 \leq \xi \leq 0.95 \\
0.05 \leq \xi_{45} \leq 0.2 \\
0.05 \leq \xi_{90} \leq 0.2 
\end{cases} \]  \quad (68)

To avoid local buckling and consider manufacturing capabilities and limitations, a maximum height of the GFRP bridge deck is set to 300 mm, the maximum flange thickness is set as 50 mm, and the maximum web thickness per meter is assumed as 250 mm. The constraint functions for geometry requirements are specified as in Eq. (69) as follow:

\[ \Phi^e = \begin{cases} 
5 \leq t_u \leq 50 \\
5 \leq t_l \leq 50 \\
5 \leq t_w \leq 250 \\
50 \leq h_w \leq 300 
\end{cases} \]  \quad (69)

The optimization process was achieved by minimizing \( \phi_1 \) or \( \phi_2 \) under the constraint \( \Phi^t \cdot \Phi^e \) using constrained nonlinear minimization (fmincon) function in the MATLAB™ software [33]. The optimized two-scale parameters of this case study are listed in Table 3. It can be seen that the weight objective function \( \phi_1 \) and the price objective function \( \phi_2 \) also calculate the same results. This is mainly because that the stiffness requirement (constrain function \( \Phi^t \)) is most strict based on the specification of steel or concrete deck among all the constrained functions. The optimized top flange thickness \( t_u \), bottom flange thickness \( t_l \), web height \( h_w \), web thickness per meter \( t_w \) are 46.02 mm, 45.86 mm, 300 mm and 37.42 mm. Also, the optimized ratio of the 0°-lamina, the 45°-lamina, and the 90°-lamina are 77.9%, 17.1%, 5.0%. The optimized fiber volume fraction is 65.2%. The optimized parameters are the same in terms of price and weight optimization because the governing factor is the web height.

**Table 3. Optimized two-scale parameters of case study**

<table>
<thead>
<tr>
<th>Item</th>
<th>Unit</th>
<th>Price Optimization</th>
<th>Weight Optimization</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top flange thickness ( t_u )</td>
<td>\textit{mm}</td>
<td>46.02</td>
<td>46.02</td>
</tr>
<tr>
<td>Bottom flange thickness ( t_l )</td>
<td>( mm )</td>
<td>45.86</td>
<td>45.86</td>
</tr>
<tr>
<td>-----------------------------</td>
<td>------</td>
<td>-------</td>
<td>-------</td>
</tr>
<tr>
<td>Web height, ( h_w )</td>
<td>( mm )</td>
<td>300.0</td>
<td>300.0</td>
</tr>
<tr>
<td>Web thickness per meter, ( t_w )</td>
<td>( mm )</td>
<td>37.42</td>
<td>37.42</td>
</tr>
<tr>
<td>Ratio of 0^0 lamina, ( \xi_0 )</td>
<td>--</td>
<td>0.779</td>
<td>0.779</td>
</tr>
<tr>
<td>Ratio of 45^0 lamina, ( \xi_{45} )</td>
<td>--</td>
<td>0.171</td>
<td>0.171</td>
</tr>
<tr>
<td>Ratio of 90^0 lamina ( \xi_{90} )</td>
<td>--</td>
<td>0.050</td>
<td>0.050</td>
</tr>
<tr>
<td>Fiber volume fraction, ( V_f )</td>
<td>--</td>
<td>0.652</td>
<td>0.652</td>
</tr>
<tr>
<td>Price per square meter, ( \phi_1 )</td>
<td>( RMB )</td>
<td>2025.9</td>
<td>2025.9</td>
</tr>
<tr>
<td>Weight per square meter, ( \phi_2 )</td>
<td>( kg )</td>
<td>213.7</td>
<td>213.7</td>
</tr>
</tbody>
</table>

### 8. Conclusions

The optimization process described in this paper involves identifying the optimal ratio of reinforcements (roving and/or fabric), fiber volume fractions, in conjunction with geometrical variables in order to achieve the optimum design in both material and structure scales. In this paper, the macro behaviors of pultruded FRP bridge deck are analyzed based on the design specification of the highway bridge. The equivalent properties of the pultruded GFRP lamination are calculated by combining both micromechanics and classical lamination theory. The micro fiber/resin level is bridged to macro pultruded GFRP bridge level by assuming the effective strain homogenized from micro component equals to macro strain. The multiscale lamination optimization is achieved by finding optimized two-scale design parameters for minimizing bridge weight and/or materials and construction cost while satisfying all design parameters for the pultruded composite deck. The optimized two-scale parameters were obtained by solving the proposed multiscale optimization model, for a bridge with a main span of 20.0 meter and steel girders equal spacing of 3.0 meters. The optimized values of the top flange thickness, \( t_u \), the bottom flange thickness, \( t_l \), the web height, \( h_w \), and the web thickness per meter, \( t_w \), are 46.02 mm, 45.86 mm,
300.0 mm and 37.42 mm, respectively. Results also showed that the optimized ratio of the 0°-lamina, 45°-lamina, and the 90°-lamina are 77.9%, 17.1%, 5.0%. The optimized fiber volume fraction is 65.2%.

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