Extension and Verification of Sequentially Linear Analysis to
Solid Elements

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Preface

In this Master Thesis the results are presented of my research of the extension and verification of the Sequentially Linear Analysis (SLA) to solid elements. SLA is an alternative for incremental-iterative solution schemes to model the nonlinear fracture behavior of quasi-brittle materials. It is an attractive method since it avoids the well known convergence and bifurcation problems that are often encountered when using incremental-iterative schemes such as Newton-Raphson.

This study has been conducted in order to obtain my Master’s degree in Civil Engineering at the Delft University of Technology, from September 2010 till March 2011. The main objective of this research was to see how the Sequentially Linear Analysis approach could be extended to solid elements, so that it could be used for three-dimensional fracture problems as well. Although three-dimensional geometries such as masonry structures have been analyzed before using SLA, it was always restricted to two-dimensional finite elements only (shell elements). Therefore, first a theoretical constitutive model has been developed that served as the starting point. Implementation in DIANA was the major second step from which I could start the third and last step: the verification on various fictive and real cases. Most attention was dedicated to the verification and physical interpretation of a real reinforced concrete slab.

This report consists of a general paper in which the new and most innovative aspects of this study are presented and various appendices that are used to show intermediate results. Even though most of these intermediate results were fictive, they were of great importance to verify the functionality of the new 3D SLA-code.

To me, this research topic offered a nice challenge and the variety of aspects made it interesting to work on. Furthermore, the innovative character of this study offered a great motivation, since it kept me intrinsically interested on how the method would perform for solid elements. Hence, I would like to thank Max Hendriks and Jan Rots for offering me this possibility.

Moreover, I would like to thank all members of my graduation committee for their time and comment. In particular, I thank Anne van de Graaf and Max Hendriks for their useful tips and guidance.

Finally, I would like to thank my family and my girlfriend for their support and understanding.

Lars Voormeeren

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Abstract
When analyzing three-dimensional problems with nonlinear finite element analysis (NLFEA) often problems are encountered such as bifurcation and divergence of the solution. In particular, cases subjected to tension softening tend to encourage the emergence of multiple equilibrium paths. In order to overcome these problems the alternative Sequentially Linear Analysis (SLA) method has been developed for three-dimensional solid elements. Here, a series of linear analyses are used to model the nonlinear behavior of the structure. By directly specifying a damage increment in each linear analysis, extensive iterations within the load or displacement increment can be avoided. In this paper the derivation of the 3D SLA method will be extensively treated. Furthermore, the strengths of the SLA method will be demonstrated by verification study on a tested reinforced concrete slab. The results are critically evaluated, interpreted and compared to results from the experiment and the incremental-iterative Newton-Raphson method. It appeared that the Sequentially Linear Analysis is able to properly capture the quasi-brittle behavior of the reinforced concrete slab.

Keywords: sequentially linear analysis; three-dimensions; solid elements; experiment; concrete cracking; reinforced concrete slab; tension softening; derivation orthotropic stress-strain relation; shear retention.
1 Research significance

Over the last decades computational modeling of structures has become more and more common. Various advanced numerical solution procedures exist which allow analysis of the most complex structures. Most notably, nonlinear finite element analysis (NLFEA) is nowadays often used as an engineering tool and is an essential part of various design processes. These analyses are used to assess the safety of a structure and serve as a guideline to improve the design. However, some problems can be encountered using these techniques, in particular in cases considering the fracture of brittle materials such as concrete. Strong softening and snap-back behavior due to brittle cracking still form a big challenge for nonlinear finite element techniques, since negative tangential stiffness could lead to divergence and singularity. Even though methods like arc-length control are available to handle such behavior, they are generally not easy to use. The results obtained depend strongly on the experience of the practicing engineer and often differ from experimental results. Especially when three-dimensional cases are considered, this lack of agreement becomes clearly apparent. Hence, there is a need for a more reliable and straightforward method.

It has been demonstrated that the Sequentially Linear Analysis (SLA) approach is an attractive alternative to simulate two-dimensional fracture problems, since by directly specifying damage increments it avoids the well known convergence problems [3]. As a result no iterations within the load or displacement increments are needed. This ‘event-by-event’ strategy consists of a series of linear-elastic analyses and through reduction of the stiffness and strength, the critical integration point is damaged. Therefore, one discretizes the constitutive stress-strain softening relation by a saw-tooth curve of positive slopes. Consequently, convergence issues are entirely avoided since the secant stiffness is always positive.

Although the SLA approach has proven its robustness and promising potential already for two-dimensional cases, it has not yet been developed for three-dimensional stress-strain states. This innovative study is therefore focused on the extension of the Sequentially Linear Analysis method to solid elements, as an alternative to address the difficulty of nonlinear fracture behavior. The research question can be formulated as:

‘How can the Sequentially Linear Analysis approach be extended to solid elements so that it can be used for three-dimensional fracture problems?’

This paper is organized as follows. First, the Sequentially Linear Analysis approach will be discussed in Section 2. An outline of the general procedure will be treated, as well as the derivation of the orthotropic stress-strain relation for 3D and the discretization of the constitutive relation by a saw-tooth. In Section 3 the force transfer in reinforced concrete slabs will be covered, to help interpreting the obtained results. Topics such as the shear failure mechanisms of reinforced concrete slabs are addressed here. Subsequently, finite element analyses on the concrete reinforced slab are presented in Section 4. Results are interpreted and discussed, in order to make a comparison between SLA, Newton-Raphson and the experiment. Finally, in Section 5 conclusions are drawn and recommendations are given for further research.
2 Sequentially Linear Analysis for solid elements

The Sequentially Linear Approach is based upon the concept of directly capturing fracture damage at micro-scale, while modeling the structural response at macro-scale [4, 21]. In this way one can track the development of brittle cracks at micro-scale and see how they affect the response of the structure. By capturing these fracture ‘events’ directly through damage increments, there is no need to iterate around them like in the Newton-Raphson solution procedure. In order to do this, SLA approximates the constitutive stress-strain relationship using a series of saw-teeth with positive secant stiffnesses. In each linear-elastic analysis a critical event takes place and at this location the strength and stiffness are reduced according to the above mentioned saw-tooth curve. In general the analyses, with reduced secant stiffnesses, are continued until the global structure fails (by predefining the maximum number of analyses). Since only linear-elastic analyses are performed, iterations are completely avoided and convergence problems cannot occur.

Another interesting feature of the SLA is that it circumvents bifurcation problems [22]. In contrast to the Newton-Raphson method, where multiple integration points are allowed to crack simultaneously within a load increment, SLA allows only one integration point to crack at a time, i.e. movement from the elastic to the softening branch. Particularly in cases where many integration points crack simultaneously, the switch to a negative tangential stiffness often causes multiple equilibrium paths to occur. Despite this loss of uniqueness the solution could still converge, although it may not automatically pick up the lowest equilibrium path.

2.1 General procedure for proportional loading

Before one can start a Sequentially Linear Analysis, one should first discretize the structure using finite elements (FE) and assign initial material properties to the elements. In case of proportional loading and tensile failure only, the general procedure is quite straightforward and easy to comprehend:

- Apply a reference proportional load $F$ and calculate the principal stresses by linear-elastic analysis.
- Determine the critical integration point, which is defined as the integration point for which the quotient of its maximum principal stress, $\sigma_1$, and its current tensile strength, $f_t^{+}$ is largest.
- Calculate the critical load multiplier that belongs to the critical integration point by $\lambda_{\text{crit}} = \frac{f_t^{+}}{\sigma_1}$.
- Scale the reference load $F$ proportionally using the critical load multiplier, $\lambda_{\text{crit}}$, and determine the new stress-strain state.
- Apply a damage increment by reducing the stiffness (Young’s modulus) and (tensile) strength of the critical integration point according to the saw-tooth discretization of the tensile constitutive relationship.
- Repeat this cycle of steps continuously by updating the material properties of a single integration point after each cycle.
- By consecutively linking the results of each cycle, one can find the nonlinear response of the structure.
Things become more difficult in case of non-proportional loading, for instance where self-weight and prestress load of a concrete beam are considered [5]. The general procedure still holds, although the calculation of the critical load multiplier becomes rather complicated. However, this study is restricted to proportional loading and tensile failure only.

2.2 Crack model

Extending SLA to solid elements requires the derivation and definition of a new approximation of the existing crack model. The model is based on a continuum fracture model, namely the total strain smeared crack concept [20]. In a smeared crack model a crack is smeared out over the element area. In this way, one can preserve the topology of the original finite element and no restrictions are imposed with respect to the orientation of the crack planes. Cracks can occur everywhere in the structure and they are well distributed as is the case in reinforced concrete structures. Here we assume a fixed crack model that preserves a fixed orientation of the crack during the entire process. This means that upon crack formation the transformation matrix \( T \), which relates the local and global crack strains, is fixed. Furthermore, the smeared concept is based on the total strain model. This includes the strain due to cracking as well as the strain of the material between the cracks, i.e. \( \epsilon_{\text{tot}} = \epsilon_{\text{cr}} + \epsilon_{\text{el}} \).

2.2.1 Constitutive stress-strain relation

It is assumed that before any damage will occur, i.e. in the uncracked stage, the material will act isotropic. Hooke’s law assumes material isotropy and the relation between strains and stresses is expressed in the following compliance matrix:

\[
\begin{bmatrix}
\epsilon_{xx} \\
\epsilon_{yy} \\
\epsilon_{zz} \\
\gamma_{xy} \\
\gamma_{yz} \\
\gamma_{zx} \\
\end{bmatrix} = \frac{1}{E} \begin{bmatrix}
1 & -\nu & -\nu & 0 & 0 & 0 \\
-\nu & 1 & -\nu & 0 & 0 & 0 \\
-\nu & -\nu & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 2(1+\nu) & 0 & 0 \\
0 & 0 & 0 & 0 & 2(1+\nu) & 0 \\
0 & 0 & 0 & 0 & 0 & 2(1+\nu) \\
\end{bmatrix} \begin{bmatrix}
\sigma_{xx} \\
\sigma_{yy} \\
\sigma_{zz} \\
\sigma_{xy} \\
\sigma_{yz} \\
\sigma_{zx} \\
\end{bmatrix}
\]  

(1)

where \( E \) is the Young’s modulus and \( \nu \) represents Poisson’s ratio. From moment equilibrium it can be derived that not all stress components are independent.

\[
\sigma_{xy} = \sigma_{yx} \\
\sigma_{yz} = \sigma_{zy} \\
\sigma_{zx} = \sigma_{xz}
\]  

(2)

The first index refers to the plane in which the stress occurs, which is defined normal to this direction, while the second indicates the positive stress direction. In order to find an orthotropic expression for the stiffness matrix, the influence of various uniaxial stresses on the strain terms is determined. For instance, uniaxial stress \( \sigma_{xx} \) results in the following strain components:

\[
\begin{align*}
\epsilon_{xx} &= \frac{\sigma_{xx}}{E_x} \\
\epsilon_{yy} &= -\nu_{yx} \frac{\sigma_{xx}}{E_x} \\
\epsilon_{zz} &= -\nu_{zx} \frac{\sigma_{xx}}{E_x}
\end{align*}
\]  

(3)

Here, \( \nu_{ij} \) is the Poisson’s ratio due to a contraction in direction \( i \) when an extension is applied in \( j \)-direction. Substitution in the compliance relation (1) results in:
Due to symmetry of the stress and strain tensor, the six Poisson’s ratios are not all independent. This implies that there are not twelve independent variables, but only nine: 3 Young’s moduli, 3 shear moduli and 3 Poisson’s ratios. From the previous three-dimensional compliance matrix it can be seen that the Poisson’s ratios are related through:

\[
\frac{\nu_{yx}}{E_x} = \frac{\nu_{xy}}{E_y}, \quad \frac{\nu_{zx}}{E_x} = \frac{\nu_{xz}}{E_z}, \quad \frac{\nu_{yz}}{E_z} = \frac{\nu_{zy}}{E_y}
\]

(5)

When writing the previous compliance relation in matrix-vector format:

\[
\epsilon = C\sigma
\]

(6)

By inverting the compliance matrix \( C \) the elastic stiffness matrix \( D \) is obtained:

\[
D = C^{-1}
\]

(7)

After substitution of (5) in (7) the constitutive stress-strain relation for the three-dimensional situation is given by \( D \):

\[
F = \begin{bmatrix}
\frac{(\nu_{yx}^2 E_y - 1) E_x}{E_x} & -(\nu_{xy}^2 + \nu_{yx}^2 E_z E_x) E_y & -(\nu_{xy}^2 E_z + \mu_{xx} E_y) E_z & 0 & 0 & 0 \\
-(\nu_{zx}^2 E_z + \nu_{zy}^2 E_x E_y) & (\nu_{zx}^2 E_y - 1) E_y & -(\nu_{zy}^2 + \nu_{zx}^2 E_z E_y) E_x & 0 & 0 & 0 \\
-(\nu_{zy}^2 E_z + \nu_{xz}^2 E_x E_y) & -(\nu_{yx}^2 E_x + \nu_{xz}^2 E_x E_y) E_z & (\nu_{yx}^2 E_x - 1) E_z & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{\nu_{zx}}{F_{xx}} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{\nu_{zy}}{F_{yy}} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{\nu_{zy}}{F_{zz}}
\end{bmatrix}
\]

(8)

in which \( F \) is defined as:

\[
F = \frac{E_x}{E_x E_y E_z} - \frac{E_y}{E_x E_y E_z} + \nu_{xy}^2 E_x E_y + \nu_{yx}^2 E_x E_y E_z + \nu_{zy}^2 E_z E_y E_x + 2\nu_{xy} \nu_{yz} \nu_{zx} E_x E_y E_z
\]

(9)

Recall that before any damage will occur, i.e. uncracked stage, the material acts isotropic, which means:

\[
E_x = E_y = E_z = E_0
\]

(10)

\[
\nu_{xy} = \nu_{yx} = \nu_{yz} = \nu_{zx} = \nu_{zy} = \nu_0
\]

(11)

\[
G_{xy} = G_{yz} = G_{zx} = G_0 = \frac{E_0}{2(1 + \nu_0)}
\]

(12)

where the subscript \( 0 \) indicates the initial material properties.
As soon as the principal stress violates the tensile strength in an integration point, a crack will be initiated perpendicular to the direction of the principal stress $\sigma_1$. Now the isotropic formulation does no longer hold. Hence, an orthotropic stress-strain relation should be formulated in a local three-dimensional $n, s, t$-coordinate system. However, this smeared fixed crack model is based on the assumption that all axes of orthotropy are fixed upon crack initiation, i.e. the tangential $s$- and $t$-directions as well. In this definition $n$ is the direction aligned with the $\sigma_1$ direction, thus normal to the primary crack plane. The $s, t$-plane covers the tangential crack directions, for which the directions are aligned with the principal stress directions $\sigma_2$ and $\sigma_3$ respectively. In order to form a right-handed coordinate system, $\sigma_1 \geq \sigma_2 \geq \sigma_3$ holds by definition. Note that in total three crack planes can originate per integration point, as secondary cracking is allowed in solid elements. See figure 1(b).

![Figure 1](image)

**Figure 1:** (a) The principal stress directions in the critical integration point before cracking. (b) Upon crack initiation, the three axes of the orthotropic $n, s, t$-coordinate system are fixed. The $n$-axis is aligned with $\sigma_1$ and is perpendicular to the primary crack plane.

Now, the orthotropic stress-strain relationship changes to the local $n, s, t$-coordinate system:

$$
\begin{bmatrix}
\sigma_{nn} \\
\sigma_{ss} \\
\sigma_{tt} \\
\sigma_{nt} \\
\sigma_{st} \\
\sigma_{tn}
\end{bmatrix} =
D
\begin{bmatrix}
\epsilon_{nn} \\
\epsilon_{ss} \\
\epsilon_{tt} \\
\gamma_{ns} \\
\gamma_{st} \\
\gamma_{tn}
\end{bmatrix}
$$

(13)

Here, $D$ represents the orthotropic stiffness matrix and has been derived for three-dimensions:

$$
F = \begin{bmatrix}
    (\nu^2_{st} \frac{E_t}{E_s} - 1)E_n & -(\nu_{ns} + \nu_{st} \nu_{tn} \frac{E_t}{E_s})E_n & -(\nu_{ns} \nu_{st} + \nu_{tn} \frac{E_t}{E_s})E_n & 0 & 0 & 0 \\
    -(\nu_{ns} \nu_{st} + \nu_{tn} \frac{E_t}{E_s})E_s & (\nu^2_{st} \frac{E_s}{E_t} - 1)E_s & -(\nu_{nt} \nu_{ns} + \nu_{st} \frac{E_s}{E_t})E_s & 0 & 0 & 0 \\
    -(\nu_{nt} \nu_{ns} + \nu_{st} \frac{E_s}{E_t})E_t & -(\nu_{nt} \nu_{ns} + \nu_{st} \frac{E_s}{E_t})E_t & (\nu^2_{ns} \frac{E_t}{E_s} - 1)E_t & 0 & 0 & 0 \\
    0 & 0 & 0 & \sigma_{ns} & 0 & 0 \\
    0 & 0 & 0 & 0 & \sigma_{st} & 0 \\
    0 & 0 & 0 & 0 & 0 & \sigma_{tn}
\end{bmatrix}
$$

(14)

where $F$ is defined as:
\[ F = \frac{E_n E_s E_t}{E_n E_s E_t + \nu_{ns}^2 E_n^2 E_t + \nu_{ts}^2 E_t^2 E_n + \nu_{st}^2 E_s^2 E_n + 2 \nu_{ns} \nu_{st} \nu_{tn} E_n E_s E_t} \]  

where \( E_n \), \( E_s \) and \( E_t \) are the newly defined Young’s moduli, due to a change in stiffness. Once again, \( \nu_{ij} \) is the Poisson’s ratio due to a contraction in direction \( i \) when an extension is applied in \( j \)-direction. The previous orthotropic expression can be written in a more compact form:

\[ \sigma_{nst} = D \epsilon_{nst} \]  

In order to transform the local \( n,s,t \) stress-strain relation into the global \( x,y,z \)-coordinate system, standard transformation matrices for three-dimensional problems can be used.

\[ \sigma_{xyz} = T^{-1}_s D T^T \epsilon_{xyz} \]  

It has been proven [6] that the inverse of stress transformation matrix is equal to the transpose of the strain transformation matrix.

\[ T^{-1}_s = T^T_s \]  

Finally, by substitution of the previous equation the stresses and strains belonging to the global coordinate system can be calculated with:

\[ \sigma_{xyz} = T^T_s D T \epsilon_{xyz} \]  

Equation (19) is the constitutive stress-strain relation that accounts for material orthotropy and has been used for implementation in finite element program DIANA.

### 2.2.2 Poisson behavior

After the first crack appeared, the Young’s modulus normal to the crack plane, \( E_n \) and the tensile strength \( f_{tn} \) will be reduced according to the predefined saw-tooth curve (see subsection 2.3). Due to the occurrence of secondary cracks the material properties in the tangential \( s,t \)-directions might change as well, which results in reductions of \( E_s, E_t, f_{ts} \) and \( f_{st} \). The Poisson’s ratios are assumed to be reduced at an equal rate as the corresponding E-moduli [4]. Therefore:

\[ \nu_{sn} = \nu_{tn} = \nu_0 \cdot \frac{E_n}{E_0} \quad \nu_{ns} = \nu_{ts} = \nu_0 \cdot \frac{E_s}{E_0} \quad \nu_{nt} = \nu_{st} = \nu_0 \cdot \frac{E_t}{E_0} \]  

Recall that \( \nu_{ij} \) is the Poisson’s ratio due to a contraction in direction \( i \) when an extension is applied in \( j \)-direction. Thus, when a crack completely opened in the \( n \)-direction \( (E_n = 0) \), increasing the crack strain in \( n \)-direction does not cause a Poisson effect in the \( s \) or \( t \)-direction, and vice versa.

### 2.2.3 Shear behavior

As a result of cracking the shear modulus is affected as well. Since shear forces can have a significant contribution to the bearing capacity of the structure, it is important to incorporate them properly. Especially in cases where redistribution of stresses takes place, large shear stresses at the crack faces can occur. In order to take the effect of
crack opening into account, the shear modulus is often reduced with a constant shear retention factor $\beta$:

$$G_{ns} = G_{st} = G_{tn} = \beta G_0$$

(21)

However, care should be taken at choosing the $\beta$-value, since a seemingly small constant shear retention factor of for instance 1 percent can result in unexpected large shear forces. Due to large shear planes the structure can bear great loads, even when the crack is almost fully opened. Of course this is not always correct. Therefore, a variable shear retention approach, which depends on the damage that occurred in the integration point, seems to be a more appropriate choice. In this way, completely opened cracks cannot transfer shear forces anymore. Note that this is not the case when a constant shear retention is adopted, where the crack always can transmit shear forces/stresses along its surface. In this study to three-dimensional SLA cases, the variable shear retention is assumed to reduce the shear stiffness at a rate equal to the minimum of the corresponding Young moduli [5]:

$$G_{ns} = \frac{\min(E_n, E_t)}{2 \left(1 + \nu_0 \frac{\min(E_n, E_t)}{E_0}\right)}$$

$$G_{st} = \frac{\min(E_s, E_t)}{2 \left(1 + \nu_0 \frac{\min(E_s, E_t)}{E_0}\right)}$$

$$G_{tn} = \frac{\min(E_n, E_t)}{2 \left(1 + \nu_0 \frac{\min(E_n, E_t)}{E_0}\right)}$$

(22)

Consequently, the amount of variables in constitutive stress-strain relation are now reduced to only three, namely the three Young’s moduli: $E_n$, $E_s$ and $E_t$.

### 2.3 Saw-tooth laws for concrete in tension

The SLA method requires a discretization of the constitutive stress-strain relation. Hereby damage increments are used to reduce the strength and stiffness properties of the critical integration point, according to a predefined saw-tooth model. By defining damage factors for the Young’s moduli and the tensile strengths, one can determine the material properties for the next damage state. In case of discretization of the direction normal to the crack, the damage factors for the reduction of the Young’s modulus and tensile strength are specified by $d_{n,i}$ and $r_{n,i}$ respectively. Here, $n$ is the direction and $i$ is the number of the damage increment. Discretization of the tangential $s,t$ material properties can be done in a similar manner.

$$E_{n,i} = (1 - d_{n,i})E_0$$

$$f_{t,n,i} = (1 - r_{n,i})f_t$$

(23)

<table>
<thead>
<tr>
<th>Damage increment $i$</th>
<th>$d_{n,i}$</th>
<th>$r_{n,i}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>m</td>
<td>$\sim 1.0$</td>
<td>$\sim 1.0$</td>
</tr>
</tbody>
</table>

Table 1: Discretization saw-tooth model in $n$-direction.
A commonly used saw-tooth model is the so-called ‘ripple’ approach. This formulation can be applied to any stress-strain relation and is derived by specifying an envelope around the tensile softening curve of magnitude $2p_f t$. Here, $p$ represents the range percentage that controls the fineness of the saw-tooth approximation [2, 3]. In order to preserve the dissipated energy, the area under the saw-tooth curve must equal the fracture energy $G_f/h$. In case of linear tension softening, the ultimate strain at which the cracks are fully opened can then be defined by:

$$\epsilon_u = \frac{2G_f}{f_y h} \quad (24)$$

However, it was found that in the aforementioned procedure the ultimate strain and fracture energy not always matched with the values of the original softening curve. Therefore, an improvement had been proposed for generating the saw-tooth curve with constant bandwidth. Now, the upper and lower band are allowed to vary in magnitude (see $p_u$ and $p_l$ in figure 2).

Figure 2: Stress-strain curve for linear tension softening and the corresponding saw-tooth diagram.

3 Shear in reinforced concrete slabs

In case of distributed surface or line loads that are supported by beams or walls, the reinforced concrete slab behavior will be dominated by bending. Shear stresses are well distributed along the slab and generally shear will not govern the design. However, in case of concentrated loads, shear behavior can be critical and should be considered. High shear stresses can occur around the load and supports and failure through shear forces is possible. Especially in slab-column connections, shear stresses are often more critical than bending stresses and govern the design of the slab. In this section, first the mechanisms of shear transfer are reviewed. Secondly, the important parameters that influence the shear capacity are elaborated in Subsection 3.2. Finally, the two shear failure mechanisms in slabs are treated in Subsection 3.3, namely wide beam shear failure (one-way shear) and punching shear failure (two-way shear).
3.1 Shear-carrying mechanisms

In general, the shear transfer in concrete slabs without transverse reinforcement is rather complex. Due to the three-dimensional nature of the problem, identification of the different shear-carrying mechanisms can be extremely difficult. However, in this section the most important shear-carrying mechanisms are reviewed, based on [14, 15].

Uncracked concrete - In uncracked concrete the shear force is carried by inclined principal tensile and compressive stresses, which can be illustrated by the elastic stress trajectories [15]. The amount of shear force generated in the compression zone is determined by the depth of the compression zone. Particularly in slender slabs without axial compression, the contribution of the compression zone to the shear capacity is limited. However, as soon as cracking initiates a few important shear-carrying mechanisms can develop, as illustrated in figure 3.

Cantilever action - Cantilever action can develop between two flexural cracks without the need of developing compression or tension forces through the flexural cracks [15], see figure 3(b). The concrete between the flexural cracks can be visualized as cantilever beams fixed in the upper compression zone (described by Kani [8]). However, it requires the compression chord (strut) to be inclined in order to carry the shear force. The shear strength of the cantilever action is limited by tensile strength of the concrete at the location of the inclined tension tie. After reaching the tensile strength in the tie, the cantilever action is diminished by the horizontal propagation of the crack.

Aggregate interlock - According to [15] cracked concrete can possess a significant capacity to transfer shear forces, depending on the crack opening and the roughness of the cracks. Relative sliding between the lips of the cracks is required to activate the aggregate interlocking. The amount of shear force that can be transferred by aggregate interlock depends on the shape of the crack and the roughness of the lips. Hence, it is a function of the maximum aggregate size of the concrete. The shear mechanism of aggregate interlock is illustrated in figure 3(c).

Dowel action - Flexural reinforcement can generate another shear-carrying mechanism by doweling action [18], see figure 3(d). In case of shear displacement along an inclined crack, part of the shear force will be transferred by the flexural reinforcement. This might cause concentrated tensile stresses at the concrete cover and the dowel shear capacity.
is now limited by the tensile strength of the concrete. Generally, the contribution of dowel action to the shear resistance is low. However, in cases with large amounts of flexural reinforcement this mechanism can generate fairly high shear forces, especially when distributed in more than one layer. Nevertheless, in members with transverse reinforcement the dowel action is more significant, since the flexural reinforcement is better fixed within the tightly bend stirrup surrounding it.

**Concrete in tension** - After reaching the peak tensile strength concrete still possesses residual capacity to carry tensile stresses [16]. In particular for very small crack widths the shear contribution of concrete in tension is significant. Due to the gradual development of micro-cracks, the tensile strength reduces gradually as well. As the strain increases, debonding of the matrix and aggregates causes the micro-cracks to form a single macro-crack. The typical exponential stress-strain softening curve of concrete exemplifies this behavior. As soon as the macro-crack widely opens up, the concrete loses its ability to carry tensile stresses. This shear mechanism is illustrated in figure 3(e).

**Arching action** - After development of a critical shear crack the concrete member still possess shear-carrying capacity by arching action [14]. Partly this is due to the formation of a direct strut, as a result of aggregate interlock in the critical shear crack. The inclined strut has a limited strength and depends on several parameters such as the maximum aggregate size and the crack width. Another part is due to the formation of an elbow-shaped strut in order to avoid further cracking. This mechanism is limited by the concrete tensile strength. Actually, arching action can be interpreted as a combination of both, as shown in figure 3(f).

### 3.2 Important parameters

From Subsection 3.1 it already became clear that there are a lot of parameters influencing the shear capacity of concrete members. The most important ones are reviewed here.

**Reinforcement ratio** - The amount of longitudinal reinforcement is a very important parameter in the shear capacity of reinforced concrete slabs [11]. The reinforcement ratio determines the strains in the reinforcement. Hence, the depth of the compression zone and the axial concrete strains are directly affected. Generally, higher reinforcement strains correspond to larger crack widths and the aggregate interlocking mechanism loses its strength. Moreover, the reinforcement ratio will influence the dowel capacity of the longitudinal bars. Particularly multiple layers of reinforcement strongly increase the dowel action.

**Maximum aggregate size** - The aggregate interlocking mechanism plays an important role in the shear behavior of reinforced concrete slabs. Particularly for reinforced concrete members without transverse reinforcement, aggregate interlock is the dominant mechanism of shear transfer [23]. Hence, choosing the maximum aggregate size is a delicate matter. In general, increasing the size of the coarse aggregate results in rough cracks that are able to carry higher shear loads. Reduction of the maximum aggregate size decreases the shear strength.

**Concrete strength** - The shear strength of reinforced concrete members increases when concrete with higher material strength is used [9]. The tensile strength of the concrete is often governing for the shear strength. However, usually the compressive strength is included in the design formulas, because experimental tests show less scatter on compression specimen.
Size effect - Due to the size effect the shear strength of large concrete members without transverse reinforcement is found less than smaller members [16]. The generally accepted explanation for this size effect is that larger members show larger diagonal crack widths, which in turn reduces the residual tensile stresses and the amount of aggregate interlock. However, some disagreement exists about how to quantify each mechanism.

Axial force - In general, axial compression or prestressing increases the stiffness of the members. Therefore, the diagonal crack width reduces and aggregate interlocking enhance the concrete shear strength. Furthermore, the depth of the uncracked compression zone is increased by axial compression and so is the shear capacity [16]. Logically, axial tension decreases the shear strength.

Shear span-to-depth ratio - The shear span-to-depth ratio $a/d$ shows the relation between the support and the loading point $a$ to the effective depth $d$ of the concrete member. The shear strength increases as the shear span-to-depth ratio increases. Since deep beams with a ratio of less than 2.5 can transmit a large portion of shear directly to the support by an inclined compression strut, the average shear stress at failure becomes larger [16].

Several tests performed by Leonhardt and Walther show the sensitivity of the shear strength to the $a/d$ ratio [14]. This led to the conclusion that for small values of $a/d$ (test B2) the cracks do not propagate through the inclined compression strut. Here the flexural strength of the concrete member is governing. However, for larger values of $a/d$ the critical crack does develop through this inclined strut, decreasing the shear capacity of the member (test B4 and B6). Generally, for $a/d$-values between 1.0 - 3.0, arching action is the dominant shear mechanism. In case of slender concrete members with very large shear span-to-depth ratios (test B10/1) failure occurs by flexure instead of shear. The critical diagonal crack cannot be formed before the flexural strength is reached.

![Figure 4: Influence of the shear span-to-depth ratio][14]. (a) Tests B2, B4, B6 and BP10/1 by Leonhardt and Walther show the influence of the $a/d$ ratio on the compressive strut position and the cracking pattern. (b) Comparison between the actual shear strength and failure load according to the plasticity theory.
3.3 Shear failure mechanisms

One of the test cases for the 3D SLA method concerns the study of a concentrated load applied near the support of a reinforced concrete slab, see Subsection 4.1. The concentrated load should model the traffic loads generated by vehicles. Since the load transmission in reinforced concrete slabs is rather complex, particularly in cases without stirrups, it is important to understand the different failure mechanisms well. Given the topology of the slab and the loading condition, shear failure mechanisms are most plausible. In shear failure mechanisms without transverse reinforcement inclined cracks form, before or after flexural cracks are initiated nearby. After opening of the crack, the force transfer changes significantly in order to establish a condition of static equilibrium. In this subsection, the two shear failure mechanisms for slabs are reviewed: one- and two-way shear failure. In case of concentrated loads the shear strength is governed by the more severe of the two [17]. However, in general one-way shear failure predominates when beams are located between columns that support the slabs. Two-way shear failure predominates in case of flat beamless slabs supported by columns [19].

3.3.1 One-way shear - Wide beam shear failure

The one-way shear failure mechanism is based on the assumption that the slab fails due to a critical shear crack over the entire width of the slab, as illustrated in figure 5(a). Therefore the slab can be interpreted and treated as a wide beam of which the critical section is located a distance \(d\) from the applied load [17]. The parameter \(d\) is defined as effective depth, i.e. the distance from the outer compression fiber to the center of the longitudinal reinforcement. The assumed critical section passes through the critical diagonal tension crack.

![Figure 5](image)

Hence, the wide beam shear capacity of a slab can be calculated using the conventional beam formulas. In case of concentrated loads the width \(b\) is then replaced by the effective width \(b_{eff}\). In practice, the effective width is often calculated by assuming load spreading under a 45 degree angle from the loading position. Therefore the effective width depends on the distance between the loading position and the support. If the shear span decreases, the effective width will decrease as well, resulting in a lower shear capacity. In figure 6 the effective width of the analyzed reinforced concrete slab is illustrated. The dark gray
elements indicate the line supports, while the light gray elements show the concrete. The concentrated load is presented by the orange square.

![Figure 6: Effective width due to load spreading.](image)

### 3.3.2 Two-way shear - Punching shear failure

Another commonly known failure mechanism is two-way shear failure, also known as punching shear [17]. Here, the reinforced slab fails due to high shear stresses in a relatively small area around the concentrated load. Punching shear failure occurs along a truncated pyramid of concrete, caused by a clearly visible critical diagonal crack. The critical punching shear surface sloping outwards in all directions from the loading area. The region associated with punching shear is assumed to be equal to $d/2$ from the perimeter of the applied load, where $d$ is the effective slab thickness. See figure 5(b).

According to Muttoni and Schwartz [13], the opening of a critical shear crack causes the strength of the shear-carrying inclined compression strut to reduce and eventually this will lead to the punching shear failure. The shear strength of a concrete member without transverse reinforcement is therefore governed by the width and the roughness of the inclined shear crack. The roughness is again depending on the maximum aggregate size. The width of the critical crack is assumed to be proportional to the rotation and the effective depth of the slab. Tests by Kinnunen and Nylander [13] to the punching behavior of slabs, varying the amount of flexural reinforcement, showed a general relationship between the rotation and the load:

- In slabs with low reinforcement ratios of about $\rho = \pm 0.5\%$ flexural yielding governs the behavior. The reinforcement yields and the strength of the reinforced concrete slab is limited by its flexural capacity. Punching failure can only occur after significant plastic deformations.

- Intermediate reinforcement ratios ($\rho = \pm 1.0\%$) cause the reinforcement to yield in the vicinity of the concentrated load. However, punching shear failure occurs before the entire reinforcement was able to yield. In this case, the strength of the slab is limited by shear capacity instead.

- In case of high reinforcement ratios ($\rho = \pm 2.0\%$) brittle failure occurs directly through punching shear. No yielding of the reinforcement takes place. Therefore, the shear strength is governing.
4 Three-dimensional analyses reinforced concrete slab

In order to see how the newly developed SLA-code performs when applied to large three-dimensional structures, a reinforced slab has been investigated. This reinforced slab subjected to a concentrated load takes part in a broader study of shear capacity of concrete bridges under increasing traffic loads. In general, it appears to be quite difficult for incremental-iterative solution schemes to obtain convergence in structures consisting of solid elements. Consequently, the results of these numerical solution schemes showed poor agreement with the experimental results [1]. Hence, there is a need for a more accurate solution procedure that avoids convergence problems. SLA offers a relatively simple and comprehensive method for dealing with nonlinear fracture behavior, which governs a reinforced slab subjected to a concentrated load. This section first treats the experimental setup and results, after which in Subsection 4.2 the finite element model will be introduced. Subsequently, the SLA results are presented, discussed and compared to the experimentally obtained results in Subsection 4.3. Finally, comparison with the incremental-iterative Newton-Raphson method has been made in Subsection 4.4.

4.1 Experiment

Over the last decades traffic loads have increased heavily. Due to this phenomenon, various existing bridges are under investigation. The research program covers several studies of the shear capacity of reinforced concrete bridges. Ten slabs are tested and evaluated at the Stevin Laboratory at the Delft University of Technology, each having different reinforcement ratios and loading positions [10]. In this work slab S1T1 has been selected for comparison with finite element modeling, see figure 7(a).

Figure 7: (a) Experimental setup and the (b) Details of the supports and loading conditions [10].

This experiment has been performed on a concrete slab, reinforced in both longitudinal and transversal direction. However, shear reinforcement (stirrups) was absent. HEM300 beams are located along the short edges of the slab to provide continuous supports; see appendix A for the experimental setup. The edge, close to the loading position, is simply supported, while the opposite edge is partially clamped by three prestressed Dywidag bars that restrict rotations. To avoid concentrated stresses at the constraints, continuous plywood and felt layers are placed in between the concrete slab and the steel profiles, illustrated in figure 7(b). The concrete slab is subjected to a concentrated load.
The load is applied by a hydraulic jack, that controls the deformation at a constant rate. The loading position is symmetrical with respect to the transversal direction. However, in longitudinal direction it is asymmetric since it is close to the simple support. The hydraulic jack transmits its force through a steel plate of 200 x 200 x 20mm. The slab has dimensions of 5000 x 2500mm and a thickness of 300mm, which gives it a shear span-to-depth ratio of approximately:

\[
\frac{a}{d} = \frac{a}{h - c - \frac{1}{2}d} = \frac{600\text{mm}}{265\text{mm}} = 2.3
\]  

(25)

Based on the discussion in Subsection 3.2, arching action can be expected to be the dominant shear-carrying mechanism. The longitudinal reinforcement at the top consists of 11 \( \varnothing 10/250 \) in the zone subjected to positive moments over a length of 2300mm from the simple support and 21 \( \varnothing 20/125 \) in the zone subjected to negative moments over a length of 3000mm from the fixed support. Longitudinal reinforcement at the bottom consists of 21 \( \varnothing 20/125 \) and the transversal reinforcement comprises 21 \( \varnothing 10/250 \) for the bottom, as well as for the top. A concrete cover of 25mm is applied throughout the slab. Geometry and reinforcement are displayed in figure 8(a).

![Figure 8](image)

Figure 8: (a) Reinforcement plan and (b) the experimentally found load-displacement diagram [10].

The load-displacement diagram in figure 8(b) of the experimental results shows a peak load of 954kN. After reaching the peak load the slab was unloaded again, explaining the downward-sloping post-peak branch of the load-displacement curve. At 700kN a flexural crack appeared at the front face [10]. At the bottom flexural cracks occurred throughout the slab. However, the main cracks appeared around the loading position and ran towards and away from the supports. At failure the crack width at the front face was measured to be 1.7-1.8mm. The failure mode of the reinforced concrete slab appeared to be a combination of one-way shear (wide beam failure) and two-way shear (punching shear) after significant flexural cracking. The load spread under an angle towards the simple support and could not force failure through a clear truncated pyramid of concrete. Punching shear could not develop fully around the load, just partly. After failure it was noticed that the loading plate sank in the top face of the concrete during the loading process. The pictures in figures 9 and 10 show the most significant cracks in the structure.
4.2 Finite element model

After the implementation of the newly developed 3D SLA-code in finite element program DIANA, the finite element model of the slab could be developed. Due to asymmetry in longitudinal direction, the size of the model could not be significantly reduced. However, the part between (simple) support 2 and support 3 (Dywidag bars) that represents the partially fixed support could be omitted. Instead, the side nodes of the elements on top of support 2 were constrained in longitudinal \( x \)-direction to simulate a (fully) fixed support. This assumption is justified since preventing any rotation will not strongly affect the magnitude of the moment near the simple support. Especially because the amount of reinforcement is chosen in such a way that yielding in the longitudinal reinforcement should not occur. The concrete has been meshed with three-dimensional twenty-node solid brick elements, based on quadratic interpolation and Gauss integration. The adopted integration scheme was \( 2 \times 2 \times 2 \). The steel plate that transfers the load of the hydraulic jack to the concrete slab has also been modeled by 20-node brick elements with an \( 2 \times 2 \times 2 \) Gauss integration scheme. The same holds for the steel HEM300 profiles that support the structure. In total the finite element model counts 463 elements, divided in two layers over the height of the slab. The mesh and constraints are shown in figure 11. This resulted in a computation time of 10 hours to perform 33000 linear analyses, which comes down to 1.05 seconds per analysis. Unfortunately, due to increasing computation times and the timespan of this study, no finer mesh could be adopted. For example, a mesh consisting of 2084 elements divided over three layers resulted in a computation time of approximately 9 days (5.10 seconds/analysis).
Figure 11: (a) The adopted mesh of 463 elements and (b) the loading and boundary conditions of the reinforced concrete slab.

Since the HEM300 beams are modeled with a rectangular cross-section of 340x100mm, the Young’s Modulus of the steel has been adjusted to match the exact bending stiffness \( EI \) of the original HEM300 profile. The layers of felt and plywood, that prevent stress concentrations at the support, are modeled as a spring through interface elements. These plane interface elements are 8+8 node interface elements that are used in three-dimensional configurations, based on quadratic interpolation. A Gaussian 2 x 2 integration has been selected. The steel reinforcement bars are modeled by embedded reinforcement. Two integration points per particle based on Gauss integration scheme are adopted along the axis of the element. Furthermore, perfect bond is assumed, which implies that the strains in the reinforcement are computed from the displacement field of the mother elements.

The material properties of the concrete are derived from experimental specimen tests, for which a maximal aggregate diameter of \( d_{\text{max}} = 16\text{mm} \) has been used [7]. From compressive tests on cubic specimens the compressive strength is determined using \( f_c = 0.83 \cdot f_{c,\text{cube}} \). The tensile strength followed from splitting tests and was calculated by \( f_t = 0.9 \cdot f_{sp} \). The initial Young’s Modulus is estimated and the tensile fracture energy is scaled by the ratio between mean and characteristic compressive strength and is calculated with:

\[
G_f = G_f^0 \left( \frac{f_{cm}}{f_{cm0}} \right)^{0.7} \left( \frac{f_{ck}}{f_{cm}} \right). 
\]

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<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
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<td>31601</td>
<td>N/mm(^2)</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>( \nu )</td>
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<td>-</td>
</tr>
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<td>Tensile strength</td>
<td>( f_t )</td>
<td>2.78</td>
<td>N/mm(^2)</td>
</tr>
<tr>
<td>Compressive strength</td>
<td>( f_c )</td>
<td>29.63</td>
<td>N/mm(^2)</td>
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<tr>
<td>Tensile fracture energy</td>
<td>( G_f )</td>
<td>0.06418</td>
<td>Nmm/mm(^2)</td>
</tr>
</tbody>
</table>

Table 2: Material properties concrete [7].

A variable shear retention relation was used to model the aggregate interlock. For concrete in compression a linear elastic relation was adopted, while for the tensile softening curve linear softening behavior was chosen. This linear softening curve has been discretized by a saw-tooth for the Sequentially Linear Analysis, illustrated in figure 12(a). The area under the softening diagram is depending on the tensile fracture energy \( G_f/h \) and is thus depending on an estimation of the crack bandwidth \( h \). In DIANA and in simple a priori models the crack bandwidth in solids is assumed to be equal to \( h = \sqrt{V} \), where \( V \) is the volume of the finite element. This assumption makes sense in regular meshes, with low aspect ratios of the brick elements, where cracks run along the meshing lines. However, particularly in rectangular solid element meshes the cracks not always run parallel to the mesh lines. In fact, they sometimes tend to propagate in an inclined direction and a more accurate estimation of the crack bandwidth seems necessary [12].
That is why in this study the crack bandwidth $h$ has been varied to show its influence on the solution. Note that by adjusting the crack bandwidth the tensile fracture energy is implicitly changed as well:

$$h = \alpha \sqrt{V}$$  \hspace{1cm} (26)

Here, $\alpha$ is a variable that affects the size of the crack bandwidth and thus the tensile fracture energy $G_f/h$.

Figure 12: Saw-tooth discretization of (a) the tensile linear softening diagram and (b) the reinforcement hardening diagram.

Since the mechanical properties of the steel reinforcement bars were not available at the time of this study, they have been estimated in [7]. The adopted constitutive stress-strain relation of the embedded reinforcement steel accounts for hardening. A saw-tooth has been defined for the hardening plasticity of the reinforcing steel, as shown in figure 12(b). The steel HEM300 profiles and the loading plate are assumed to behave linear-elastic.

<table>
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<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
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<td>Yield strength reinforcement steel</td>
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<td>560 N/mm$^2$</td>
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<tr>
<td>Ultimate strength reinforcement steel</td>
<td>$f_{su}$</td>
<td>600 N/mm$^2$</td>
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<tr>
<td>Young’s modulus steel loading plate</td>
<td>$E_{eq}$</td>
<td>200000 N/mm$^2$</td>
<td></td>
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Table 3: Material properties steel [7] and the interface stiffnesses.

Another model parameter of interest was the interface stiffness. Its influence on the load-displacement turned out to be significant, since the slope of the curve greatly depends on the chosen value for the normal stiffness. The mechanical properties of the felt and plywood provided by the manufacturer combined with strong nonlinear behavior, resulted in poor agreement with the experiment. Consequently, a linear-elastic behavior of the interface elements has been adopted and the properties of the felt and plywood have been tuned by back-analysis. The interface stiffnesses in table 3 turned out to give the best agreement to the experimental results. A small tangential stiffness has been chosen to avoid membrane action. Finally, a total strain fixed smeared crack model was adopted to account for the tensile fracture behavior of the concrete. Similar to the experiment the finite element analysis has been performed using displacement control.
In summary, only two material properties will be varied in the Sequentially Linear Analyses performed in Subsection 4.3, namely the tensile fracture energy $G_f/h$ and the interface stiffness $k_n$.

4.3 Numerical results SLA

In this subsection the numerical results are presented, compared and discussed. First, the influence of the tensile fracture energy and the interface stiffnesses are evaluated by varying the magnitude of these material properties in 4.3.1 respectively 4.3.2. Secondly, the influence of adopting a constant shear retention is examined in 4.3.3. Finally, the failure mechanism and cracking behavior are investigated by interpretation of visual output in 4.3.4.

4.3.1 Influence of tensile fracture energy on numerical results

The first comparison between the experimental results and the three-dimensional Sequentially Linear Analysis showed a fairly good agreement. The load-displacement diagram, as shown in figure 13(a), indicated a clear peak load and showed decreasing post-peak behavior. Moreover, the slope of the load-displacement diagram was well estimated by the chosen interface stiffnesses. However, the results appeared to be sensitive to the amount of tensile fracture energy adopted [24]. In general, the a priori assumption for the crack bandwidth $h = \sqrt{{V}}$ seemed too simple, since the crack bandwidth is greatly depending on the mesh shape and the inclination of the crack in the brick elements. Hence, to emphasize the importance of the fracture energy on the structural behavior, three different $\alpha$-values (1.0, 1.5 and 2.0) for $h = \alpha \sqrt{{V}}$ have been considered. Generally, the structure shows a stiffer response and higher peak loads in case of a higher fracture energy, i.e. it behaves more ductile. Consequently, an increase in the crack bandwidth $h$ as proposed here implies a reduction of the fracture energy $G_f/h$, which in turn results in a lower ultimate strain $\epsilon_u$ (ductility) of the concrete. The best agreement with the experiment has been found for $\alpha = 1.5$, for which a peak load of 966kN has been derived. This is illustrated in figure 14(a) and 14(b). Comparison with the load-displacement diagram of the experiment shows an excellent agreement and the 3D SLA solution only deviates 1.3% from the experimentally obtained peak load of 954kN.

Figure 13: Load-displacement curves for (a) $\alpha = 1.0$ and (b) $\alpha = 2.0$. 
Figure 14: (a) The load-displacement curve for $\alpha = 1.5$ shows the best agreement with the experiment. (b) Displaying only the highest value of each branch results in a smoother curve.

This reduction of the fracture energy is partly caused by the aforementioned assumption that cracks in rectangular solid element meshes not always run along the meshing lines. Especially in the case of the reinforced slab subjected to a concentrated load, one expect shear crack propagation in an inclined direction. For example, in case of an inclined crack of 45 degrees in a cubic element, the crack bandwidth $h$ is clearly underestimated and should be multiplied by a factor $\sqrt{2}$. Moreover, this value also strongly depends on the shape of the element in the finite element mesh. Another clarification why the crack bandwidth should be increased, originates from the adopted linear softening curve. In case of exponential softening the negative slope after reaching the tensile strength is steeper than in case of linear tension softening. Therefore, one normally reduces the adopted amount of tensile fracture energy by a factor $(\frac{1}{3} - \frac{1}{2})$ to simulate the slope of the real exponential softening curve of concrete, see figure 15. A final explanation for correcting the crack bandwidth could be due to an overestimation of the fracture energy by the guidelines.

Figure 15: Reduction of the fracture energy to simulate the negative slope of the exponential softening curve of concrete.

4.3.2 Influence of interface stiffness on numerical results

The numerical SLA solution appeared to be sensitive to the adopted stiffness of the felt and plywood layer. In particular, the interface stiffness in normal direction greatly affected the accuracy of the solution. At first, the nonlinear behavior of the interface elements was evaluated on the basis of the product data specifying the mechanical properties of the felt and plywood. However, these appeared to result in a poor agreement
with the experiment. Therefore, linear elastic behavior was assumed for which the normal stiffness has been tuned by means of back-analysis. In order to show the importance of the adopted normal interface stiffness on the numerical solution, the same analysis has been performed using now $k_n = 1.0 \, N/mm^3$ instead of the $k_n = 2.5 \, N/mm^3$ from figure 14(a).

Figure 16: A reduced normal interface stiffness of $k_n = 1.0 \, N/mm^3$ shows less agreement with the experiment.

Clearly, a reduced normal interface stiffness results in larger deflections, i.e. the slab behaves more ductile. The slope of the load-displacement diagram decreased considerably. Apparently, the slope largely depends on the adopted value for $k_n$. In this case, the peak load was less affected by the change in the stiffness as it was approximated by 944kN.

4.3.3 Influence of shear retention relation on numerical results

A part of this study covered the investigation of the shear retention relation on the numerical results. It turned out that the choice of shear retention factor strongly affected the structural behavior and therefore needs to be considered carefully. In order to show the importance, first an analysis has been performed with a constant shear retention factor of 1%. Subsequently, the same analysis has been performed using the variable shear retention relation mentioned in subsection 2.2.3 ‘shear behavior’.

Figure 17: (a) Variable shear retention versus (b) constant shear retention of 1%.
It becomes clear from the load-displacement curves in figure 17 that the variable shear retention results in significantly better agreement with the experiment compared to the constant shear retention. The constant shear retention factor of 1 percent enables the slab to carry loads far exceeding the peak load of the experiment. Both analyses have been performed for 32000 steps. While the variable shear retention showed a clear peak and post-peak behavior, the case with constant shear retention shows no sign of reaching a peak. Even, after 64000 steps the load-displacement curve keeps climbing upward. Apparently, a small constant shear retention factor can already resist high shear forces. Due to large shear planes the effect of aggregate interlock becomes dominant and enhances the shear-carrying capacity of the slab significantly. The variable shear retention approach seems the better choice, since the peak load is well estimated and the slab shows a clear decreasing post-peak behavior.

4.3.4 Failure mechanism and crack behavior

In this subsection the failure mechanism of the reinforced concrete slab has been investigated with the help of various vector and contour plots. First, the redistribution capacity of the slab has been evaluated. As the load increases, significant redistribution of stresses takes place. Of particular interest are the reaction forces in the continuous felt and plywood layer. These can be presented by vector plots of the interface tractions at the simple support, as shown figure 18. Since the interpretation is meant qualitative, it was not necessary to adopt a constant (color) scale. The numbers are referring to the points indicated in the load-displacement diagram of figure 17(a).

![Vector plots of the interface tractions at the indicated points in figure 17(a).](image)

Figure 18: Vector plots of the interface tractions at the indicated points in figure 17(a).

At the beginning of the loading procedure the interface tractions show a fairly uniform distribution along the width of the slab. However, as the load increases, significant redistribution of the tractions takes place that suggests the growing concentration of reaction forces at the middle of the support line. At the peak load, point 4 in figure 17(a), there is a clear concentration of tractions in the middle indicating the effective width of the slab. Since linear elastic interface stiffnesses are assumed in both compression and tension, the normal tractions even show some tension at the edges. Apparently, the edges of the reinforced concrete slab are lifted up due to bending, while the inner part accounts for the force transfer to the support. By means of the vector plots in figure 18 the effective width is estimated at $b_{eff} = 1850\, \text{mm}$. This implies that the load spreads towards the support under an angle of approximately 60°, shown in figure 19.
In practice, the load is assumed to spread under a 45 degrees angle, resulting in a more conservative \( b_{eff} = 1100\text{mm} \). Clearly, the effective width is then considerably underestimated. The same conclusion holds for the bearing capacity predicted by the shear equations of the Dutch Code, which was 539kN while both the experiment and the Sequentially Linear Analysis showed significant higher peak loads (954kN respectively 966kN). This difference is due to the fact that the shear equations in the Dutch Code are mostly derived from tests on beams without transverse reinforcement, whereas the results obtained on slabs are influenced by considerable redistribution because of larger widths and the effect of transverse reinforcement [25].

In order to investigate the cracking behavior of the concrete slab, vector plots of the principal tensile strains are made. In figure 20, the concrete slab has been viewed from the bottom face and from two side faces. The color scales of the vector plots are indicated in the softening curve of an average sized element. Soon after the uncracked stage, a straight longitudinal flexural crack starts to develop at the bottom face from the loading position towards the simple support. The crack appears at the front face of the slab, which corresponds to what has been observed during the experiment. However, in the numerical analysis this crack occurred at 400kN, while in the experiment it was noticed at 700kN. This difference could be partly due to inaccuracy of the observations made during the test. Moreover, the crack started to propagate in the opposite direction as well, towards the fixed support. This agrees to the global crack behavior as observed in the experiment [10].
The principal tensile strain $\epsilon_1$ at the peak load, point 4 in figure 17(a), illustrates the locations of the cracks in figure 21(a). Note, that in figure 21 the slab is viewed from the bottom and from the side. As can be seen from the color scale, the principal tensile strains are far exceeding the ultimate strain of an average sized concrete element. The largest strains are present in the direct vicinity of the concentrated load. However, the cracks are more developed in the direction of the fixed support. As mentioned in Subsection 4.1, arching action is expected to be dominant for a shear span-to-depth ratio of 2.3. In order to check whether this is true, a vector plot of the principal compression stresses at the peak load, $\sigma_3$, has been made. Figure 21(b) indeed indicates the presence of a compression strut extending from the concentrated load towards the simple support. As described in Subsection 3.1 arching action can be interpreted as the combination of a direct strut enabled by aggregate interlock and an elbow-shaped strut enabled by the tensile strength [15]. Here, it seems that the direct strut is more dominant, since no clear tension ties can be observed in figure 21(a) above the compression strut. Apart from this, it should be noted that the compressive stresses hardly ever exceeded the compressive strength of the concrete and therefore crushing did not occur.
In order to be sure that the slab does not fail due to its limited flexural capacity, the amount of yielding at the reinforcement steel should be checked. The reinforcement ratio of the slab in longitudinal direction is $\rho_l = 0.996\%$. According to Kinnunen and Nylander this would mean that some yielding of the reinforcement could be present in the direct vicinity of the load [13]. From the contour plots of the reinforcement strains in figure 22 it indeed can be observed that some yielding occurs at the peak load (red). However, this is in the transversal reinforcement. Even the top reinforcement showed little yielding at the front face, indicating the opening of the longitudinal flexural crack at the front face over the full height of the slab. At the peak load of 966kN the width of this crack is about 1.9mm at the bottom and 0.5mm at the top. This is in excellent agreement with the experimentally found bottom crack width of 1.7 - 1.8mm.

In case of intermediate reinforcement ratios, punching shear failure would occur before yielding of the entire slab reinforcement, as described in Subsection 3.2.2. However, the punching shear capacity changes significantly when the concentrated load is located close to a continuous support, as is the case here. The punching shear strength improves
considerably and the truncated cone around the concentrated load can therefore not fully develop. From the contour plots of the bottom face it indeed becomes clear that the principal tensile strains are less developed at the side of the continuous support. Figure 23 shows the results at indicated points 4 (peak load) and 5 in figure 17(a). The color scale of the contour plots is held constant in order to see the development of the tensile strain area and is indicated in the softening curve of an average sized element.

Figure 23: Contour plots of the principal tensile strain on the bottom face at the indicated points 4 and 5 in figure 17(a) showing the distribution of strains (a) at the peak load and (b) after the peak load. (c) The color scale indicates strains far beyond the ultimate strain of the concrete.

From the previously shown plots it is still hard to tell whether the slabs fails due to punching shear or by wide beam shear failure. No clear critical diagonal cracks have been observed around the concentrated load, that would suggest pure punching failure. Furthermore, no critical section across the entire width of the slab was visible, suggesting wide beam failure. The fact that neither punching shear nor wide beam shear failure occurs can be confirmed by the deformed mesh shape. See figure 24.

Figure 24: (a) Deformed structure at the peak load. Cross-sections of the deformed mesh (b) in longitudinal direction and (c) in transversal direction.

Hence, it seems most likely that a combination of both caused the concrete slab to fail. The same conclusion was drawn for the experiment. Anyhow, the three-dimensional nature of the problem and the combined flexural and diagonal cracks, make it difficult to determine the failure mechanism. However, the global cracking behavior predicted by the Sequentially Linear Analysis seemed to correspond well with the observations made during the experiment.
4.4 Newton-Raphson comparison

In order to examine how the Sequentially Linear Analysis compares to conventional numerical solution schemes, a finite element Newton-Raphson model had been developed. The Newton-Raphson (N-R) analysis has been performed using the same finite element mesh, loading and boundary conditions and model parameters as previously described for SLA. Despite this, still some model changes were required to avoid divergence of the solution. Since the computation times of the N-R analysis were not governing, a Gaussian 3 x 3 x 3 integration scheme could be adopted for the 20-node brick elements. Note that these elements were used to model the concrete, HEM300 beams and the steel loading plate. For the plane interface elements the default 4 x 4 Newton-Cotes integration scheme was chosen. The load of 40mm was applied using displacement control, with a force and energy tolerance of 1% respectively 0.1% set as convergence norm. In order to avoid convergence problems the steps sizes of the load application had to be chosen quite carefully. The analysis started off with 100 load steps of 0.00375mm, followed by 50 steps of 0.005mm and finishing with 50 steps of 0.0075. Per step 25 iterations were allowed.

Unfortunately variable shear retention was not available for N-R analyses, hence a constant shear retention factor of 1% was assumed as this resulted in the best agreement. Moreover, the used version of finite element program DIANA is unable to reduce the Poisson ratio as cracks are opening up and therefore a constant value was assumed. Given that the slab is subjected to significant flexural cracks, the lateral contraction should gradually decrease. To emphasize the importance of this reduction, which is an essential part of the SLA-code, the load-displacement curves are presented for one analysis performed with $\nu = 0.15$ and one with $\nu = 0.0$. Finally, the tensile fracture energy for one analysis has been reduced according to the SLA results to determine whether a crack bandwidth of $h = 1.5\sqrt{V}$ benefits the N-R solution as well.

![Load-displacement curves](image)

Figure 25: Load-displacement diagram of SLA shows excellent agreement with the experiment in comparison to various Newton-Raphson analyses.

34
In general, the Newton-Raphson solutions reveal a poor agreement with the experiment. No clear peaks could be derived from the load-displacement diagram and they all show stiffening instead of decreasing post-peak behavior. The assumption to choose $\nu = 0.0$ seems to improve the results slightly, however the agreement remains poor. As expected, reduction of the tensile fracture energy would cause a decrease in stiffness, leading to even lower local peaks. Also the crack behavior showed little resemblance, as the longitudinal crack at the front face could not be identified in the post-processing. This poor behavior can partly be explained by the adoption of a constant shear retention, as in case of SLA this assumption lead to an overshoot/stiffening as well. This simplification with respect to the shear retention causes large displacements and significant ductile behavior. However, the solution seemed very sensitive to chosen step sizes, since the equilibrium path was directly affected by it. Clearly, bifurcation is a major weakness of the N-R method and solution paths should initially be distrusted. Even experienced users have trouble choosing right step sizes. SLA circumvent these problems by the scaled loading procedure in which only one integration point can become critical at the time [22]. Moreover, the convergence rates of the N-R solutions were about 93% for the cases of $\nu = 0.0$ and 55% for the analysis adopting $\nu = 0.15$. This means that convergence could not be reached at all load steps. In fact, the force norm could hardly ever be met.

Even though exact comparison with SLA could not be established, because of difference in shear retention approach, this example did demonstrate the problems involved when using a Newton-Raphson scheme for 3D. Unlike N-R, the Sequentially Linear Analysis requires little amount of modeling parameters to capture three-dimensional fracture behavior and does not suffer from bifurcation and convergence problems.

5 Conclusions and recommendations

Since in practice many problems are encountered while using three-dimensional numerical procedures, like arc-length and Newton-Raphson, the need for a more robust method was imperative. This study offered a promising alternative for three-dimensional applications, to directly capture quasi-brittle fracture behavior by applying damage increments. Verification of the results obtained by the newly developed 3D Sequentially Linear Analysis leads to the conclusions discussed in the next subsection. Recommendations for further research are given in Subsection 5.2.

5.1 Conclusions

- The Sequentially Linear approach is able to properly capture quasi-brittle fracture behavior for three-dimensional cases using solid elements. In general, the SLA results show good agreement with the results obtained by the experiment. The method avoids convergence and bifurcation problems, which were encountered in the NLFE-analysis. SLA approaches the load-displacement curve of the experiment quite well and the peak load is well-estimated with a deviation of 1.5%. Moreover, the global cracking behavior is properly simulated by the SLA results, indicating a clear longitudinal crack at the front face.

- The SLA results appeared to be sensitive to the adopted amount of tensile fracture energy. In rectangular meshes the assumption to estimate the crack bandwidth $h$ by $\sqrt{V}$ seemed to be too simple. This resulted in a overestimation of the peak load and the corresponding deflection of the slab. In order to limit its peak value a
reduction of the tensile fracture energy was proposed, by varying the magnitude of the crack bandwidth with a factor $\alpha$, thus $\alpha \sqrt{V}$. An $\alpha$-value of 1.5 turned out to give the best agreement with the load-displacement curve of the experiment, given this particular mesh.

- The way of modeling the aggregate interlock seemed to greatly affect the results. A constant shear retention appeared to be too simple and was not able to properly simulate the shear behavior of the reinforced concrete slab. The variable shear retention on the other hand, resulted in a better agreement with the experiment. The load clearly reaches a peak value and the global trend of load-displacement diagram after the peak is decreasing.

- The bearing capacities according to the experiment and the Sequentially Linear Analysis, were significantly higher than the prediction based on the shear equations of the Dutch Building code. The practical assumption of a 45 degrees load spreading appeared to be quite conservative in comparison to the SLA results, which in turn resulted in a considerable underestimation of the effective width.

- In contrast to SLA, the Newton-Raphson results displayed a poor agreement with the experiment. No clear peak was visible and convergence could not always be achieved. Since variable shear retention was not incorporated in the used Newton-Raphson code, a constant shear retention of 1% was adopted. This is one of the reasons for the odd post-peak behavior.

- Since SLA is based on a ‘event-by-event’ strategy, only one integration point can be damaged at the time (per linear analysis). In particular, in finite element models with fine meshes, the total number of integration points that should be damaged before failure is enormously. Hence, obtaining a solution is a very time-consuming job and especially in three-dimensional applications using solid elements this problem is pronounced.

5.2 Recommendations

This study was the first step for SLA in the direction of three-dimensional problems and it already offered some interesting and promising results. However, the SLA method could be improved even further by instead of immediately fixing the tangential $s-$ and $t$-directions after primary crack initiation, keeping them variable until secondary cracking in the integration point occurs. Furthermore, the finite element model of the concrete slab could be evaluated again using the newly provided data with respect to the mechanical properties of the reinforcement and felt/plywood. Experimental specimen tests to determine the tensile fracture energy of the concrete, combined with the implementation of a more accurate way of calculating the crack bandwidth could enhance the results even more.

Other future research should aim on the extension of 3D SLA-code to cases subjected to non-proportional loading, a topic that is heavily under discussion at the moment. Moreover, extending the SLA-code to limit the compressive stresses could improve the SLA results for solid elements even further. Another field of research should focus on how to solve the long computation times, which clearly troubles 3D applications. A smarter solver or damaging multiple integration points per linear analysis could offer the solution. However, care must be taken by cracking multiple integration points simultaneously, as this is the origin of bifurcation problems.
References


Experimental setup

Figure 26: Experimental setup.
Appendix A

Three-dimensional constitutive relation

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A Three-dimensional constitutive relation

A.1 General derivation 3D orthotropic stress-strain relation

In order to derive the orthotropic constitutive stiffness matrix for solid elements, the simplest constitutive model defined by Hooke will be used. Hooke’s law assumes material isotropy and the relation between strains and stresses is expressed in the following compliance matrix:

\[
\begin{bmatrix}
\epsilon_{xx} \\
\epsilon_{yy} \\
\epsilon_{zz} \\
\gamma_{xy} \\
\gamma_{yz} \\
\gamma_{zx}
\end{bmatrix} = \frac{1}{E} \begin{bmatrix}
1 & -\nu & -\nu & 0 & 0 & 0 \\
-\nu & 1 & -\nu & 0 & 0 & 0 \\
-\nu & -\nu & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 2(1 + \nu) & 0 & 0 \\
0 & 0 & 0 & 0 & 2(1 + \nu) & 0 \\
0 & 0 & 0 & 0 & 0 & 2(1 + \nu)
\end{bmatrix} \begin{bmatrix}
\sigma_{xx} \\
\sigma_{yy} \\
\sigma_{zz} \\
\sigma_{xy} \\
\sigma_{yz} \\
\sigma_{zx}
\end{bmatrix}
\]

where \( E \) is the Young’s modulus and \( \nu \) represents the Poisson’s ratio. Here it is assumed that the shear stress components are not independent.

\[
\sigma_{xy} = \sigma_{yx} \quad \sigma_{yz} = \sigma_{zy} \quad \sigma_{zx} = \sigma_{xz}
\]

The first indice refers to the plane in which the stress occurs, which is defined normal to this direction. While the second indicates the positive stress direction.

In order to find the orthotropic expression for the stiffness matrix, the influence of various uniaxial stresses on the strain terms is considered. Uniaxial stress \( \sigma_{xx} \) results in the following strain components:

\[
\epsilon_{xx} = \frac{\sigma_{xx}}{E_x} \quad \epsilon_{yy} = -\nu_{yx} \frac{\sigma_{xx}}{E_x} \quad \epsilon_{zz} = -\nu_{zx} \frac{\sigma_{xx}}{E_x}
\]

Uniaxial stress \( \sigma_{yy} \) results in strain components:

\[
\epsilon_{xx} = -\nu_{xy} \frac{\sigma_{yy}}{E_y} \quad \epsilon_{yy} = \frac{\sigma_{yy}}{E_y} \quad \epsilon_{zz} = -\nu_{zy} \frac{\sigma_{yy}}{E_y}
\]

Figure 27: Stress components in a three-dimensional continuum.

In order to find the orthotropic expression for the stiffness matrix, the influence of various uniaxial stresses on the strain terms is considered. Uniaxial stress \( \sigma_{xx} \) results in the following strain components:

\[
\epsilon_{xx} = \frac{\sigma_{xx}}{E_x} \quad \epsilon_{yy} = -\nu_{yx} \frac{\sigma_{xx}}{E_x} \quad \epsilon_{zz} = -\nu_{zx} \frac{\sigma_{xx}}{E_x}
\]

Uniaxial stress \( \sigma_{yy} \) results in strain components:

\[
\epsilon_{xx} = -\nu_{xy} \frac{\sigma_{yy}}{E_y} \quad \epsilon_{yy} = \frac{\sigma_{yy}}{E_y} \quad \epsilon_{zz} = -\nu_{zy} \frac{\sigma_{yy}}{E_y}
\]
Finally, uniaxial stress \( \sigma_{zz} \) gives strain components:

\[
\epsilon_{xx} = -\nu_{xz} \frac{\sigma_{zz}}{E_z}, \quad \epsilon_{yy} = -\nu_{yz} \frac{\sigma_{zz}}{E_z}, \quad \epsilon_{zz} = \frac{\sigma_{zz}}{E_z}
\]  

(31)

Here, \( \nu_{ij} \) is the Poisson’s ratio due to a contraction in direction \( i \) when an extension is applied in \( j \)-direction. The compliance relation can now be written as:

\[
\begin{bmatrix}
\epsilon_{xx} \\
\epsilon_{yy} \\
\epsilon_{zz} \\
\gamma_{xy} \\
\gamma_{yz} \\
\gamma_{zx}
\end{bmatrix} =
\begin{bmatrix}
\frac{1}{E_x} & -\nu_{xz} & \frac{1}{E_z} & 0 & 0 & 0 \\
-\nu_{xy} & \frac{1}{E_y} & \frac{1}{E_z} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{G_{xy}} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{G_{yz}} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{G_{zx}} \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\sigma_{xx} \\
\sigma_{yy} \\
\sigma_{zz} \\
\sigma_{xy} \\
\sigma_{yz} \\
\sigma_{zx}
\end{bmatrix}
\]  

(32)

where the Young’s and shear moduli should never fail to preserve its thermodynamic laws since \( E > 0 \) respectively \( G > 0 \). The same holds for the Poisson’s ratios for which the following conditions are formulated:

\[
\nu_{2xy}^2 < \frac{E_x}{E_y}, \quad \nu_{2yz}^2 < \frac{E_y}{E_z}, \quad 2\nu_{2xy}\nu_{2yz}E_zE_y < 1 - \nu_{2xy}^2 E_yE_z - \nu_{2yz}^2 E_xE_z \leq 1
\]  

(33)

The previous compliance relation can be written in vector format as:

\[
\epsilon = C\sigma
\]  

(34)

By inverting the compliance matrix \( C \) the elastic stiffness matrix \( D \) is obtained.

\[
D = C^{-1}
\]  

(35)

In a three-dimensional situation the constitutive stress-strain relation can now be expressed using the elastic stiffness matrix:

\[
F = \begin{bmatrix}
(\nu_{2yz}\nu_{2zx} - 1)E_x & -\nu_{2yz}\nu_{yz}E_x & -\nu_{2yz}\nu_{yz}E_x & 0 & 0 & 0 \\
-(\nu_{2zx}\nu_{2zy} + \nu_{2zy})E_y & (\nu_{2zx}\nu_{2zy} - 1)E_y & -(\nu_{2zx}\nu_{2zy} + \nu_{2zy})E_y & 0 & 0 & 0 \\
-(\nu_{2zy}\nu_{2zx} + \nu_{2zx})E_z & -(\nu_{2zy}\nu_{2zx} + \nu_{2zx})E_z & (\nu_{2zy}\nu_{2zx} - 1)E_z & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{G_{xy}}{E_y} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{G_{yz}}{E_z} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{G_{zx}}{E_z}
\end{bmatrix}
\]  

(36)

where \( F \) is defined as:

\[
F = \frac{1}{\nu_{2yz}\nu_{yz}\nu_{2zx} + \nu_{2zx}\nu_{2zy} + \nu_{2zy}\nu_{2zx} + \nu_{2zx}\nu_{2zy} + \nu_{2yz}\nu_{2yz} - 1}
\]  

(37)

Due to symmetry of the stress and strain tensor, the six Poisson’s ratios are not all independent. This implies that there are not twelve independent variables, but nine: 3 Young’s moduli, 3 shear moduli and 3 Poisson’s ratios. From the previous three-dimensional compliance matrix it can be noted that the Poisson’s ratios are related through:
Substitution of these equations in the new orthotropic stiffness matrix:

\[
F = \begin{bmatrix}
   \frac{\nu_{yz}}{E_z} - 1 & -(\nu_{xy} + \nu_{yz} \nu_{xz}) \frac{E_z}{E_x} & -\nu_{xz} \nu_{yz} \frac{E_z}{E_x} & 0 & 0 & 0 \\
-\nu_{xz} \nu_{yz} + \nu_{xy} \frac{E_z}{E_y} & \frac{\nu_{xy}^2}{E_y} - 1 & -(\nu_{xy} \nu_{yz} + \nu_{xz} \frac{E_z}{E_y}) & 0 & 0 & 0 \\
-\nu_{xz} \nu_{yz} + \nu_{xy} \frac{E_z}{E_y} & -(\nu_{xz} \nu_{yz} + \nu_{xy} \frac{E_z}{E_y}) & \frac{\nu_{xy}^2}{E_y} - 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{G_{xy}}{E_y} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{G_{yz}}{E_z} \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{G_{zx}}{E_z}
\end{bmatrix}
\]

in which \( F \) has changed to:

\[
F = \frac{E_x E_y E_z}{E_z E_y E_z + \nu_{xy}^2 E_z^2 E_x + \nu_{yz}^2 E_y^2 E_z + \nu_{xz}^2 E_x^2 E_y + 2 \nu_{xy} \nu_{yz} \nu_{xz} E_x E_y E_z}
\]

### A.2 3D orthotropic stress-strain relation for SLA

As soon as the principal stress violates the tensile strength, a fixed crack will be initiated perpendicular to the direction of the greatest principal stress \( \sigma_1 \). After crack initiation in this so-called critical integration point, all three crack directions are fixed. Hence, an orthotropic stiffness matrix is formulated in a local 3-dimensional \( n,s,t \)-coordinate system. In this definition \( n \) is the direction aligned with the \( \sigma_1 \) direction, thus normal to the crack plane. The \( s,t \)-plane covers the tangential crack directions, for which the directions are aligned with the principal stress directions \( \sigma_2 \) and \( \sigma_3 \) respectively. This is done in order to form a right-handed coordinate system. The orthotropic stress-strain relationship can be written as:

![Figure 28: Orthotropic coordinate system in case of smeared fixed cracking. N-axis is perpendicular to the crack](image-url)
\[
\begin{bmatrix}
\sigma_{nn} \\
\sigma_{ss} \\
\sigma_{tt} \\
\sigma_{ns} \\
\sigma_{st} \\
\sigma_{tn}
\end{bmatrix} = \mathbf{D}
\begin{bmatrix}
\epsilon_{nn} \\
\epsilon_{ss} \\
\epsilon_{tt} \\
\gamma_{ns} \\
\gamma_{st} \\
\gamma_{tn}
\end{bmatrix}
\] (41)

\[D\]

is elaborated as:
\[
F = \begin{bmatrix}
(v_n^2 E_n - 1)E_n & -(v_n + v_{nt} E_n/E_t)E_n & -(v_n + v_{nt} E_n/E_t)E_n & 0 & 0 & 0 \\
-(v_n + v_{ns} E_n/E_s)E_s & (v_n^2 E_n - 1)E_s & -(v_n + v_{ns} E_n/E_s)E_s & 0 & 0 & 0 \\
-(v_{tn} + v_{ns} E_s/E_n)E_n & -(v_{tn} + v_{ts} E_t/E_s)E_n & (v_n^2 E_n - 1)E_t & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & G_n & 0 \\
0 & 0 & 0 & 0 & 0 & G_t \\
0 & 0 & 0 & 0 & 0 & G_t
\end{bmatrix}
\] (42)

in which \(F\) is defined as:
\[
F = \frac{E_n E_s E_t}{E_n E_s E_t + v_n^2 E_n^2 E_t + v_{nt} E_n^2 E_s + v_{ns} E_n^2 E_s + 2v_n v_{nt} E_n E_s E_t + 2v_n v_{ts} E_n E_s E_t}
\] (43)

where \(E_n, E_s\) and \(E_t\) are the newly defined Young's moduli, due to a change in stiffness. It is assumed that before any damage will occur, i.e. uncracked stage, the material will act isotropic, which means:
\[
\nu_0 = \nu_{ns} = \nu_{nt} = \nu_{sn} = \nu_{st} = \nu_{tn} = \nu_{ts}
\] (44)

\[
E_0 = E_n = E_s = E_t
\] (45)

\[
G_0 = G_{ns} = G_{st} = G_{tn} = \frac{E_0}{2(1 + \nu_0)}
\] (46)

where the subscript \(0\) indicates the initial values of the material properties.

After the first crack sets in the Young’s modulus normal to the crack plane, \(E_n\) will be reduced, as well as the tensile strength in this direction. This is in line with the concept of applying damage increments, conform the predefined saw-tooth curve. Due to a change in crack direction or the occurrence of secondary cracks the material properties in the tangential s,t-directions might decrease as well, which results in reductions of \(E_s, E_t, f_{ts}, \) and \(f_{tt}.\) The Poisson’s ratios will be reduced at an equal rate as the corresponding E-moduli. Therefore:
\[
\nu_{sn} = \nu_{tn} = \nu_0 \cdot \frac{E_n}{E_0} \quad \nu_{ns} = \nu_{st} = \nu_0 \cdot \frac{E_s}{E_0} \quad \nu_{nt} = \nu_{st} = \nu_0 \cdot \frac{E_t}{E_0}
\] (47)

Recalling that \(\nu_{ij}\) is the Poisson’s ratio due to a contraction in direction \(i\) when an extension is applied in \(j\)-direction. Thus, when a crack completely opened in the \(n\)-direction \((E_n = 0)\), increasing the crack strain in \(n\)-direction does not cause a Poisson effect in the \(s\)-direction, and vice versa. Moreover, as a result of cracking the shear modulus is affected as well. Since shear forces can have a significant contribution on the
bearing load, through shear transfer along cracks, it is important to incorporate them well. For the three-dimensional case the shear stiffness is assumed to reduce at a rate equal to the minimum of the corresponding E-values:

\[
G_{ns} = \frac{\min(E_n, E_s)}{2 \left(1 + \nu_0 \frac{\min(E_n, E_s)}{E_0}\right)} \quad G_{st} = \frac{\min(E_s, E_t)}{2 \left(1 + \nu_0 \frac{\min(E_s, E_t)}{E_0}\right)} \quad G_{tn} = \frac{\min(E_n, E_t)}{2 \left(1 + \nu_0 \frac{\min(E_n, E_t)}{E_0}\right)}
\]

(48)

where the thermodynamic constraint of \(G > 0\) is met, since \(E > 0\) and \(0 \leq \nu < 0.5\). Consequently, the amount of variables in orthotropic stiffness matrix are now reduced to only three and consists of the three Young’s moduli: \(E_n, E_s\) and \(E_t\). An important check is to see whether the orthotropic expression of (42) and (43) will result in the isotropic elastic stiffness matrix, when equations (45),(47) and (48) are substituted:

\[
D_e = \frac{E_0}{(1 + \nu_0)(1 - 2\nu_0)} \begin{bmatrix}
1 - \nu_0 & \nu_0 & 0 & 0 & 0 \\
\nu_0 & 1 - \nu_0 & \nu_0 & 0 & 0 \\
\nu_0 & \nu_0 & 1 - \nu_0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{2}(1 - 2\nu_0) & 0 \\
0 & 0 & 0 & 0 & \frac{1}{2}(1 - 2\nu_0)
\end{bmatrix}
\]

(49)

It indeed can be noted that it equals Hooke’s well known expression for the elastic stiffness matrix of isotropic materials. Moreover, inverting equation (49) results again in the compliance matrix \(C\) of (27). Consequently, the discretization of the constitutive relation seems valid and consistent. The previous stress-strain relation (41) can be displayed in a more compact form:

\[
\sigma_{nst} = D\epsilon_{nst}
\]

(50)

Using the standard transformation matrices for 3-dimensional problems, the local stress-strain relation can be transformed to the global x,y,z-system.

\[
\sigma_{xyz} = T^-1_{\sigma}DT_e \epsilon_{xyz}
\]

(51)

It has been proven, by Asaye Chemeda Dilbo in his Master Thesis, that the inverse of stress transformation matrix is equal to the transpose of the strain transformation matrix.

\[
T^-1_{\sigma} = T_e^T
\]

(52)

Substitution gives:

\[
\sigma_{xyz} = T_e^T DT_e \epsilon_{xyz}
\]

(53)
A.3 Discretization constitutive relation for SLA

In the saw-tooth model one will discretize the constitutive stress-strain relation using damage increments. Hereby the strength and stiffness properties of the critical integration point will be reduced using a predefined saw-tooth model. By defining damage factors for the Young’s moduli and the tensile strengths, one can determine the new discretized material properties. In case of discretization of the direction normal to the crack, the damage factors \( d_{n,i} \) and \( r_{n,i} \) should be considered. Here, \( n \) is the direction and \( i \) is the number of the damage increment. Discretization of the tangential \( s,t \) material properties can be done in a similar manner.

\[
E_{n,i} = (1 - d_{n,i})E_0 \quad f_{t,n,i} = (1 - r_{n,i})f_t
\]

<table>
<thead>
<tr>
<th>Damage increment ( i )</th>
<th>( d_{n,i} )</th>
<th>( r_{n,i} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>:</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>m</td>
<td>( \sim 1.0 )</td>
<td>( \sim 1.0 )</td>
</tr>
</tbody>
</table>

Table 4: Discretization saw-tooth model in \( n \)-direction.

A commonly used saw-tooth model is the so-called ‘ripple’ approach. This formulation can be applied on any stress-strain relation. Furthermore, it preserves the fracture energy \( G_f/h \) and one can control the accuracy, since the bandwidth can be varied using a range percentage of the tensile strength (see \( p_u \) and \( p_l \) in the next figure).

Figure 29: Stress-strain curve for linear tension softening and the corresponding saw-tooth diagram.
Appendix B

Single element pull test

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April 5, 2011
B Single element pull test

B.1 Introduction

In this appendix two different numerical methods, the Modified Newton-Raphson and the Sequentially Linear Analysis (SLA) method, are used to examine the three-dimensional nonlinear fracture behavior of a fictive single element model. It has first been analyzed for the Modified Newton-Raphson method, using a total strain smeared fixed crack model. Since the Sequentially Linear Analysis, that avoids well known convergence problems, is a promising alternative for the conventional Newton-Raphson method an extension of the SLA method to solid elements has been developed. Implementation in Diana has been achieved by adjustment and extension of the existing Diana code for SLA. Starting point was of course the development of the new orthotropic stress-strain relation for three-dimensional solids. After solving (most of) the program errors the first results were drawn for the single element pull test. The output served as a verification and comparison has been made with the commonly used Newton-Raphson method. The results gave insight in validity of the Sequentially Linear Analysis method and possible programming errors could be detected, which when clarified helped adjusting the model.

B.2 3D Single element testcases

In order to create the simplest 3D-model, that at the same time covers nonlinear fracture behavior, there has been chosen for an single element pull test of which the top is subjected to a tensile surface load. This will cause the linear solid element to crack in its 8 integration points. The solid brick element has a fictive width and depth of 10 mm and is 10 mm high. The boundary conditions allow Poisson effects to take place, which are depicted in the following figure:

![Figure 30: The boundary conditions of the 3-dimensional single element pull test.](image)

Here we will consider the mode 1 linear tension softening curve.
The material and physical properties of the concrete brick are summarized in the following table:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus concrete</td>
<td>E</td>
<td>31000</td>
<td>MPa</td>
</tr>
<tr>
<td>Poisson’s ratio concrete</td>
<td>$\nu$</td>
<td>0.15</td>
<td>-</td>
</tr>
<tr>
<td>Tensile strength concrete</td>
<td>$f_t$</td>
<td>2.4</td>
<td>MPa</td>
</tr>
<tr>
<td>Fracture energy (Mode 1 Linear Tension softening)</td>
<td>$G_f$</td>
<td>0.08</td>
<td>N/mm²</td>
</tr>
<tr>
<td>Shear retention factor</td>
<td>$\beta$</td>
<td>0.001</td>
<td>-</td>
</tr>
<tr>
<td>Young’s modulus reinforcement steel</td>
<td>E</td>
<td>210000</td>
<td>MPa</td>
</tr>
<tr>
<td>Tensile strength reinforcement steel</td>
<td>$f_t$</td>
<td>440</td>
<td>MPa</td>
</tr>
<tr>
<td>Compressive strength reinforcement steel</td>
<td>$f_c$</td>
<td>440</td>
<td>MPa</td>
</tr>
<tr>
<td>Load</td>
<td>$u_{upper\ nodes}$</td>
<td>0.9E-01</td>
<td>mm</td>
</tr>
</tbody>
</table>

Table 5: Model parameters.

The constitutive stress-strain relationships are discretized using the Ripple approach. The so-called sawtooth curve is determined by amount of teeth assigned to them.

Figure 32: Discretization of the constitutive stress-strain relation of the concrete.

The elastic-plastic behavior of the reinforcement steel is discretized using approximately the same bandwidth (20%) as used for concrete.
B.2.1 Single element pull test - Constrained upper nodes and Poisson zero

In the first analysis the upper nodes (3, 4, 7 and 8) were constrained in order to be sure the specimen would deform vertically while the boundary conditions are chosen in such a way that Poisson effects still can take place. However, to keep things simple the Poisson’s ratios in this first analysis are neglected and set to zero. Since a smeared total crack strain model is adopted to simulate the cracking behavior, significant strains will be expected halfway the element where the crack initiates. The first analysis has been done using the Modified Newton-Raphson method, an incremental-iterative numerical method. Subsequently, after some simple adjustments in the .dat and .com file, the 3D-analysis was performed using the Sequentially Linear Analysis (SLA). The parameters for both models are presented in the next table.

<table>
<thead>
<tr>
<th>Convergence criterium</th>
<th>Modified Newton-Raphson</th>
<th>Sequentially Linear Analysis (SLA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load application</td>
<td>Displacement control</td>
<td>Displacement control</td>
</tr>
<tr>
<td>Load step size</td>
<td>0.005(200)</td>
<td>1.0(200 Linear Analyses)</td>
</tr>
<tr>
<td>Saw-tooth model</td>
<td>None</td>
<td>Automatic ripple, 12 teeth</td>
</tr>
<tr>
<td>2D-plane stress element</td>
<td>HX24L, linear solid brick</td>
<td>HX24L, linear solid brick</td>
</tr>
<tr>
<td>Integration scheme</td>
<td>([n_\xi = 2, n_\eta = 2, n_\zeta = 2])</td>
<td>([n_\xi = 2, n_\eta = 2, n_\zeta = 2])</td>
</tr>
</tbody>
</table>

Table 6: Model parameters - 3D solid with total strain smeared cracking.

Comparison of the load-displacement curves shows an excellent agreement between both methods. The ‘saw-tooth’ softening curve, which has been constructed using the ripple-approach, was generated automatically for a constant bandwidth by 12 teeth. In this case this number suffices, since the results deviate little from the Newton-Raphson results. However, by increasing the amount of teeth, an even more accurate solution can be found. One should keep in mind that this will increase the amount of linear analyses and therefore the total computational time as well.

One of the problems faced during the programming of this ‘relatively’ simple model, was the way in which Diana calculates strainfields. By default Diana applies 15 incompatible enhanced assumed strain modes (\(EAS = 15\)). This is due to the fact that...
low order elements such as the four-node quadrilaterals and the eight-node brick perform poorly in bending dominated problems, in case the strain interpolation is derived directly from the geometrical interpolation. Hence, one uses ‘assumed strain concepts’ to improve the bending behavior. By using the option \textit{NOCSHE} all assumed strain options are suppressed and the strain interpolation follow from the geometrical interpolation, which in case of a single element pull test is the only correct way.

\textbf{B.2.2 Single element pull test - Constrained upper nodes}

In the second analysis the upper nodes (3,4,7 and 8) were still constrained. However, the Poisson effects are taken in consideration in this analysis to see how the SLA-code would handle Poisson ratios. Due to the extension the specimen contracts in the tangential directions, which agrees with the Newton-Raphson model and common sense. Hardly any change in the load-displacement diagram is visible.
B.2.3 Single element pull test - Unconstrained upper nodes

The third and last unreinforced analysis was performed for a single element for which the upper nodes (3, 4, 7 and 8) are unconstrained. This means that the element is allowed to deform freely and therefore stress peaks at the constraints are prevented.

Figure 36: The boundary conditions of the three-dimensional single element pull test.

As can be seen from the load-displacement curve constraining the upper nodes barely affected the results.

Figure 37: Load-displacement curves for Modified Newton-Raphson and SLA - unconstrained upper nodes.
B.2.4 Single element pull test - One bar embedded reinforcement

In order to see whether the three-dimensional SLA-code could handle reinforcement, a simple single element pull test with embedded reinforcement was designed. To start easy, the model was limited to one bar placed in the middle of the element. Perfect bond is assumed, which implies that the strains in the reinforcement are computed from the displacement field of the mother elements. Moreover, embedded reinforcement does not occupy any space and does not contribute to the weight of the structure. It purely adds stiffness to the element. The boundary conditions are again chosen in such a way that it allows Poisson effects to take place.

![Figure 38: The boundary conditions of the three-dimensional single element pull test with embedded reinforcement (1 bar).](image)

After discretization of the elastic-plastic constitutive relation of the reinforcement steel, using an equal bandwidth as for concrete, the first 3D SLA analysis with embedded reinforcement has been performed. The load-displacement diagram shows a good resemblance with the Modified Newton-Raphson results.
The peak of load-displacement curve exceeds the load bearing capacity of the steel. At first, this seems a bit odd. However, since perfect bond between the steel and concrete is assumed the strains in the reinforcement steel are solely based upon the strains in the concrete. By the principle of superposition the load-displacement curves of both materials can be added to obtain the load-displacement diagram of the structure. This is visualized in the following figure. The blue line is a summation of the red (concrete) and the green (steel) one.

Figure 39: Load-displacement curves for Modified Newton-Raphson and SLA - Embedded reinforcement (1 bar).

Figure 40: Summation of the load-displacement curves of the materials result in the load-displacement curve of the structure.
B.2.5 Single element pull test - Four bars embedded reinforcement

The last check is to see whether the three-dimensional SLA-code could handle multiple reinforcement bars embedded in the element. Therefore a model has been developed containing 4 embedded reinforcement which are equally spread over the element. Again, perfect bond is assumed between the concrete and the steel.

Figure 41: The boundary conditions of the 3-dimensional single element pull test with embedded reinforcement (4 bars).

Once again the load-displacement curve barely deviate from the Modified Newton-Raphson results. This implies that SLA is able to cope with embedded reinforcement without any problem. Especially when larger models are evaluated such as reinforced concrete slabs, the use of embedded reinforcement is a must.

Figure 42: Load-displacement curves for Modified Newton-Raphson and SLA - Embedded reinforcement (4 bars).
B.3 Results

The previous load-displacement curves showed an excellent agreement with the widely used Modified Newton-Raphson Method. Even though the model is simple, it proved that the Sequentially Linear Analysis could handle three-dimensional fracture problems as well. Therefore finally acquiring these results after 3D implementation and solving lots of program errors was a big breakthrough. It shows the potential of the SLA method and give confidence in future 3D SLA results.
Appendix C

Notched beam

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April 5, 2011
C  Notched beam

C.1 Introduction

After the single element pull test, it was time to put the 3D SLA-code to a more difficult test. Hence, in this appendix two different numerical methods, the Modified Newton-Raphson and the Sequentially Linear Analysis (SLA) method, are used to examine the nonlinear fracture behavior of a fictive notched beam. The beam has been analyzed first for the two-dimensional case, using both discrete and smeared fixed cracking. Secondly the beam has been analyzed three-dimensionally for the Modified Newton-Raphson method, using the total strain smeared fixed crack model. Finally, the Sequentially Linear Analysis for the three-dimensional case has been performed using solid elements. The results of both methods have been examined and a comparison has been made, which would show once again the high potential of the SLA.

C.2 2D modeled notched beam

In order to create a finite element model, that at the same covers nonlinear brittle fracture behavior, there has been chosen for a concrete notched beam. The statically determined beam is subjected to a concentrated load at the midspan which will cause a vertical crack just above the notch. The notch has a width of 5mm and is 50mm high. The beam has a length of 450mm and a height of 100mm. In order to create a finite element model with a single element over the thickness, a fictive value of 10mm has been adopted. This way the interpretation of the results was considerably easier.

![Figure 43: Notched beam subjected to a displacement load [mm].](image)

Instead of using the exponential softening curve, a mode 1 linear tension softening curve is considered.

![Figure 44: Mode 1 Linear tension softening.](image)

The material and physical properties of the concrete beam are summarized in the following table:
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus</td>
<td>E</td>
<td>31000</td>
<td>MPa</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>ν</td>
<td>0.15</td>
<td>-</td>
</tr>
<tr>
<td>Tensile strength</td>
<td>f_t</td>
<td>2.4</td>
<td>MPa</td>
</tr>
<tr>
<td>Fracture energy (Mode 1 Linear Tension softening)</td>
<td>G_f</td>
<td>0.08</td>
<td>N/mm²</td>
</tr>
<tr>
<td>Shear modulus after cracking</td>
<td>G_cr</td>
<td>0.001</td>
<td>MPa</td>
</tr>
<tr>
<td>Dummy stiffness normal direction</td>
<td>D_n</td>
<td>1.0·10⁴</td>
<td>N/mm³</td>
</tr>
<tr>
<td>Dummy stiffness tangential direction</td>
<td>D_t</td>
<td>1.0·10⁴</td>
<td>N/mm³</td>
</tr>
<tr>
<td>Beam and interface thickness</td>
<td>t</td>
<td>10</td>
<td>mm</td>
</tr>
<tr>
<td>Load</td>
<td>u_{middle}</td>
<td>-1.0</td>
<td>mm</td>
</tr>
</tbody>
</table>

**Table 7: Model parameters.**

### C.2.1 Discrete crack model

In case of the discrete crack model a vertical crack is anticipated at the midspan, right above the notch. Here, line interface elements have been used to simulate the cracking behaviour of the beam. The first analysis has been done using the Modified Newton-Raphson method, an incremental-iterative numerical method. Subsequently, after some simple adjustments in the .dat and .com file, an analysis was performed using the Sequentially Linear Analysis (SLA). The parameters for both models are presented in the next table.

<table>
<thead>
<tr>
<th>Convergence criterium</th>
<th>Modified Newton-Raphson</th>
<th>SLA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load application</td>
<td>Displacement control</td>
<td>Displacement control</td>
</tr>
<tr>
<td>Load step size</td>
<td>0.005(4) 0.02(49)</td>
<td>1.0(600 Linear Analyses)</td>
</tr>
<tr>
<td>Saw-tooth model</td>
<td>None</td>
<td>Automatic ripple, 20 teeth</td>
</tr>
<tr>
<td>2D-plane stress element</td>
<td>T6MEM, Triangular linear</td>
<td>T6MEM, Triangular linear</td>
</tr>
<tr>
<td>Integration scheme</td>
<td>[\eta_c = 1]</td>
<td>[\eta_c = 1]</td>
</tr>
<tr>
<td>Interface element</td>
<td>Linear line 4 nodes, L8IF</td>
<td>Linear line 4 nodes, L8IF</td>
</tr>
<tr>
<td>Elementsize above notch</td>
<td>10mm</td>
<td>10mm</td>
</tr>
</tbody>
</table>

**Table 8: Model parameters - 2D plane stress discrete cracking**

Comparison of the load-displacement curves shows an excellent agreement between both methods. The ‘saw-tooth’ softening curve, which has been constructed using the ripple-approach, was generated automatically for a constant bandwidth by 20 teeth. In this case this number suffices. However, by increasing the amount of teeth, a more accurate solution can be found. One should keep in mind that this will increase the amount of linear analyses and therefore the total computational time as well.
C.2.2 Smeared fixed crack model

In a smeared crack model a crack is smeared-out over an area, hence the name. Therefore cracks can occur everywhere in the structure and they are well distributed. Here a fixed crack model is assumed that preserves a fixed orientation of the crack during the entire numerical process. This means that upon crack formation the transformation matrix $N$, which relates the local and global crack strains, is fixed. Furthermore, the smeared concept is based upon the total strain model. This includes the strain due to cracking as well as the strain of the material between the cracks. Again, the first analysis has been done using the Modified Newton-Raphson method. Subsequently, after some simple adjustments in the .dat and .com file, an analysis was performed using the Sequentially Linear Analysis (SLA). The parameters for both models are presented in the next table.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Modified Newton-Raphson</th>
<th>SLA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Convergence criteria</td>
<td>Energy norm $10^{-5}$</td>
<td>None</td>
</tr>
<tr>
<td>Max iterations</td>
<td>50</td>
<td>None</td>
</tr>
<tr>
<td>Load application</td>
<td>Displacement control</td>
<td>Displacement control</td>
</tr>
<tr>
<td>Load step size</td>
<td>0.005(4) 0.02(49)</td>
<td>1.0(400 Linear Analyses)</td>
</tr>
<tr>
<td>Saw-tooth model</td>
<td>None</td>
<td>Automatic ripple, 20 teeth</td>
</tr>
<tr>
<td>2D-plane stress element</td>
<td>Q8MEM, Quadrilateral lin.</td>
<td>Q8MEM, Quadrilateral lin.</td>
</tr>
<tr>
<td>Integration scheme</td>
<td>$[n_\xi = 4, n_\eta = 1]$</td>
<td>$[n_\xi = 4, n_\eta = 1]$</td>
</tr>
<tr>
<td>Element size above notch</td>
<td>10mm</td>
<td>10mm</td>
</tr>
</tbody>
</table>

Table 9: Model parameters - 2D plane stress smeared fixed cracking.
Figure 46: 2D Smeared cracking - Load-displacement curves for Modified Newton-Raphson and SLA.

Right above the notch there has been chosen for an integration scheme $4 \times 1; 4$ integration points vertically distributed in the middle of the element. Comparison of the load-displacement curves shows again an excellent agreement between the methods. Once more, the ‘saw-tooth’ softening curve was generated for 20 teeth.

Figure 47: Deformed shape notched beam and the maximum principal stress for the first load step.

Figure 48: Maximum principal stress for load step 10 and 53 (last) respectively.
C.3 3D modeled notched beam

C.3.1 Smeared fixed crack model

The notched beam is now modeled using solid brick elements. Because of simplicity there has been chosen for a fictive 1 element 10 mm thick beam. Since the load is applied at the midspan, element edges should be avoided at this location. Hence, CHX60, 20-node quadratic brick nodes are chosen and the load is applied at the midnodes. Furthermore, the default integration scheme, 3 x 3 x 3, has been used for all elements.

<table>
<thead>
<tr>
<th>Convergence criterium</th>
<th>Modified Newton-Raphson</th>
<th>SLA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max iterations</td>
<td>100</td>
<td>None</td>
</tr>
<tr>
<td>Load application</td>
<td>Displacement control</td>
<td>Displacement control</td>
</tr>
<tr>
<td>Load step size</td>
<td>0.005(4) 0.02(49)</td>
<td>1.0(2000 Linear Analyses)</td>
</tr>
<tr>
<td>Saw-tooth model</td>
<td>None</td>
<td>Automatic ripple, 15 teeth</td>
</tr>
<tr>
<td>3D-solid brick element</td>
<td>CHX60, 20-node quadratic</td>
<td>CHX60 20-node quadratic</td>
</tr>
<tr>
<td>Integration scheme</td>
<td>([n_\xi = 3, n_\eta = 3, n_\zeta = 3] )</td>
<td>([n_\xi = 3, n_\eta = 3, n_\zeta = 3] )</td>
</tr>
<tr>
<td>Element size above notch</td>
<td>16,7mm</td>
<td>16,7mm</td>
</tr>
</tbody>
</table>

Table 10: Model parameters - 3D solid brick smeared fixed cracking.

Since the SLA method is developed for the three-dimensional case, the analysis could be performed for both SLA and the Modified Newton-Raphson method. However, the latter process was subjected to some convergence problems. This is clearly visible in the obtained load-displacement diagram by the post-peak softening behaviour. Instead of declining to zero the tensile strength increases and decreases randomly. Due to the fact that in the Modified Newton-Raphson theory multiple integration points are allowed to crack at the same time, bifurcations points can occur. This means that more than one equilibrium states exists. This loss of uniqueness can be characterized by sudden jumps in the load-displacement curve, since it will not automatically pick the lowest equilibrium path. Especially softening models are sensitive to multiple equilibrium states.

The sequentially linear analysis has been performed once with a constant shear retention factor of 1 percent and once with a variable shear retention factor, which are computed by:

\[
G_{ns} = \frac{\min(E_n, E_s)}{2 \left(1 + \nu_0 \frac{\min(E_n, E_t)}{E_0}\right)} \quad G_{st} = \frac{\min(E_s, E_t)}{2 \left(1 + \nu_0 \frac{\min(E_n, E_t)}{E_0}\right)} \quad G_{tn} = \frac{\min(E_n, E_t)}{2 \left(1 + \nu_0 \frac{\min(E_n, E_s)}{E_0}\right)}
\]

(55)

It is clear from the graphs that the variable shear retention approach results in a fairly better agreement with what is expected than the constant retention case. Apparently, a seemingly small constant shear retention factor of 1 percent can cause unexpected large shear forces. Due to large shear planes the structure can bear great loads, even when the crack is almost fully opened. Hence, hardening is visible. Of course this is not correct in this simple case. Therefore the variable approach seems to be the only right choice here, since widely opened cracks cannot transfer shear forces anymore. However, this is not the case with constant shear retention factor, where residual shear stresses are always present.
Figure 49: 3D Smeared cracking - Load-displacement curve for Modified Newton-Raphson and SLA with constant shear retention.

Figure 50: 3D Smeared cracking - Load-displacement curve for Modified Newton-Raphson and SLA with variable shear retention.

C.4 Results

By plotting the previous load-displacement curves in one diagram, it can be seen that the various models show an excellent agreement. The peak load is well estimated for all models, that has a value of approximately $F_t = 170\, \text{N}$. As mentioned, the post-peak behaviour of the 3D smeared total strain model possibly suffers from bifurcation problems, explaining the odd softening curve.

It is remarkable to see that the peak of the 3D SLA calculation is a bit sharper than the other 2D and 3D N-R curves. An explanation for this could be that the Newton-Raphson and other SLA calculations have been performed with a constant shear retention factor of 1 percent. This might have caused a slightly more ductile behavior.
Figure 51: Load-displacement curves for notched beam, modeled 2D and 3D using discrete and smeared cracking.

The conformity of the results gave confidence in the validity of the SLA-code for solid elements. However, in order to make definite statement about the promising potential of the three-dimensional SLA-code more models should be verified. Anyhow, the notched beam served as the first adequate verification.
Appendix D

Implementation

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April 5, 2011
D Implementation

D.1 Introduction

After development of the constitutive relation for the three-dimensional Sequentially Linear Analysis, it was time for the implementation in finite element program DIANA. However, this is easier said than done. The currently used SLA-code in DIANA is programmed by numerous Fortran files. Therefore the first step of the implementation was getting to know the Fortran language. It took a while before I could read and understand the SLA subroutines that were programmed in Fortran. As soon as I was finished studying Fortran, it was time to learn how the SLA-code was structured. Anne pointed me in the right direction from which I could start the extension to solid elements. Some subroutines had to be adjusted, such as the switch routines that call the subsequent subroutines. Others had to be written from scratch on, since these subroutines simply did not exist yet. The latter one was by far the most time-consuming and difficult job.

The single element pull test was developed in order to check the functionality of the newly developed 3D SLA-code. Built-in checks in the code enabled me to see what went wrong and programming errors could be solved adequately. From time to time Anne guided me in the right direction and gave me useful tips, which finally enabled me to get it working properly.

D.2 Overview subroutines

As mentioned, some subroutines had to be adjusted, while others had to be created. The following table gives an overview of the subroutines involved to get the SLA-code working for solid elements.

<table>
<thead>
<tr>
<th>Fortran file</th>
<th>Adjustment</th>
<th>Newly written</th>
</tr>
</thead>
<tbody>
<tr>
<td>inslw.f</td>
<td>Yes</td>
<td>-</td>
</tr>
<tr>
<td>inisol.f</td>
<td>-</td>
<td>Yes</td>
</tr>
<tr>
<td>dlso.f</td>
<td>Yes</td>
<td>-</td>
</tr>
<tr>
<td>lambip.f</td>
<td>Yes</td>
<td>-</td>
</tr>
<tr>
<td>lamsol.f</td>
<td>-</td>
<td>Yes</td>
</tr>
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<td>lswit2.f</td>
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<td>matso2.f</td>
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<tr>
<td>sesol2.f</td>
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<td>Yes</td>
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</tbody>
</table>

Table 11: Subroutines involved for implementation.

Each subroutine has a specific function in the SLA-code and the content of each one will be reviewed briefly:

inslw.f: This switch routine calls the subroutine that corresponds to the current element type. For solid brick elements, it will call the newly developed subroutine inisol.f, that checks the material properties and either read or set up a saw-tooth softening law.

inisol.f: This subroutine initializes the saw-tooth law for solid elements.

dlso.f: This subroutine is adjusted to store the stiffness matrix at the right location.
**lambip.f**: This switch routine calls the appropriate subroutine that determines the admissible load factors for the current integration point. In case of solid elements it calls subroutine `lamsol.f`.

**lamsol.f**: This subroutine calculates the admissible load factors for the current integration point.

**lswit2.f**: `lswit2.f` is a switch routine that calls the subroutines that update the damage level indicators and set-up the new constitutive matrix for the critical integration point. For solid elements it calls subroutines `damsol.f` and `matso2.f`.

**damsol.f**: `damsol.f` updates the damage level indicators of the critical integration point that is located in the solid element.

**matso2.f**: This subroutine updates the material properties of the critical integration point in the solid element.

**sesol2.f**: `sesol2.f` sets up the stress-strain relation matrix $SE$ for solid elements and returns it to `matso2.f` that will put it on the filos file.
D.3 SLA-code for solid elements

D.3.1 New subroutines

```
SUBROUTINE INISOL
C..................................................Copyright (c) 2008-2009 TU DELFT
C
C.............................................
C PURPOSE: ...
C
C ARGMUMENTS:
C
C.............................................
C DESCRIPTION:
C
C PROGRAMMED BY A.V. VAN DE GRAAF AND L.O. VOORMEEREN
C
C Author: A.V. van de Graaf and L.O. Voormeeren
C
C Revision: 0.1
C
C Date: 2010/11/11
C
C..................................................
C
INTEGER MTEETH, MDMIN, MSTATE, MIP
PARAMETER (MTEETH=250, MDMIN=2, MSTATE=12, MIP=447)
C
INTEGER INO
LOGICAL XSGTCX, XSGTC, XSGTB, XIST
DOUBLE PRECISION RSMU
CHARACTER*6 TPSLWL, TOTCRK, CSHAPE, SANSNOOD
DOUBLE PRECISION EPSUL, YOUNGS, GFL, AREA, FLIMLT, CRACKB, FO,
$ TENSTR(MDMIN), SAWBA(=MTEETH-1)),
$ YOUNG(MDMIN), USRSTA(MSTATE), EPSI, SIFI,
$ COMPR(MDMIN), OVSPLP(MIP), VSLUM
INTEGER IDUM, MTEETH, IDUM, IP, MIP, DAMIND(MDMIN),
$ LSANS, MATIX, I
LOGICAL LTSANS, FIXED, ROTATE, MATCHK,
$ ERRFLG, SANCHK
C
C PERFORM SOME INITIALISATION
TSANLW = ... LTSANL = XSGTCX( 'MATERI/TSANLW', TSANLW )
CALL GTC( 'MATERI', MATIX, 1 )
C
CALL LOGPRI( 'DIT IS DE EERSTE CONTROLE', 8 )
C
C DETERMINE WHICH CRACK MODEL IS BEING USED
IF ( XSGTCX( 'MATERI/CRAK ', IDUM, ) ) THEN
C CRACK MODEL BASED ON STRAIN DECOMPOSITION HAS NOT BEEN
C IMPLEMENTED, SO STOP PROGRAM EXECUTION
CALL ERRMS( 'NLIN41', 100, 0, 0, 0, 0, 0 )
C
ELSE IF ( XSGTCX( 'MATERI/CTOPL ', TOTCRK ) ) THEN
C TOTAL STRAIN CRACK MODEL
C
C FIXED OR ROTATING CRACK MODEL?
FIXED = ( TOTCRK.EQ. 'FIXED' )
ROTATE = ( TOTCRK.EQ. 'ROTA' )
IF ( NOT. ( FIXED .OR. ROTATE ) )
$ CALL PRGERR( 'INISOL', 2 )
c
rotating crack model not yet implemented --> stop
IF ( ROTATE ) CALL PRGERR( 'INISOL', 500 )
C
C TENSILE BEHAVIOUR
IF ( LTSANL ) THEN IF ( TSANL .EQ. 'TABLE' )
```

70
$THEN
C    READ AND CHECK SAW-TOOTH LAW TABLE
$    IF ( .NOT. XSGTC( 'MATERI/TSAWS', SAWCHK, 1 ) )
$      SAWCHK = .FALSE.
C    IF ( SAWCHK ) THEN
$      IF ( .NOT. XSGTC( 'MATERI/TSAWER', ERRFLG, 1 ) )
$        CALL PRGERR( 'INISOL', 20 )
$      ELSE
$        ERRFLG = .FALSE.
C      END IF
C
C    IF ( .NOT. SAWCHK ) THEN
C      READ AND CHECK SAW-TOOTH LAW FROM INPUT TABLE
$      IF ( XSGTLB( 'MATERI/TSAWD', SAWDIA, LSAMD, $      2*(MTEETH+1) ) ) THEN
$        DO 10, I = 1, (LSAMD/1)
$          EPSI = SAWDIA(I-1)
$        SIGI = SAWDIA(I)
$        IF ((EPSI .LE. 0.D0) .OR. (SIGI .LE. 0.D0))
$          THEN
$            IF ( EPSI .LE. 0.D0 )
$              CALL ERPMG( 'NLIN41', 157, MATIDX, 0, 0,
$                0, 'TSAMD' )
$            IF ( SIGI .LE. 0.D0 )
$              CALL ERPMG( 'NLIN41', 157, MATIDX, 0, 0,
$                0, 'TSAMD' )
$            ERRFLG = .TRUE.
$            GO TO 11
$        END IF
$      10 CONTINUE
C      ELSE
C      ERROR EXIT
$      ERRFLG = .TRUE.
$      CALL ERPMG( 'NLIN41', 127, MATIDX, 0, 0, 0,
$        'TSAMD' )
$      END IF
C      ELSE
C      ERRFLG = .TRUE.
$      CALL ERPMG( 'NLIN41', 156, MATIDX, 0, 0, 0,
$        'TSAMD' )
$      END IF
C      ELSE
C      CALL PTL( 'MATERI/TSAWS', .TRUE., 1 )
C      CALL PTL( 'MATERI/TSAWER', ERRFLG, 1 )
C      ELSE
C      IF ( .NOT. ERRFLG ) CALL GTLB( 'MATERI/TSAWD', $        SAWDIA, LSAMD, 2*(MTEETH+1) )
C    END IF
C
C    IF ( .NOT. ERRFLG ) CALL PTL( 'TSAMD', SAWDIA, LSAMD )
C
C    ELSE
C    READ MATERIAL PARAMETERS AND GENERATE SAW-TOOTH LAW
C    CALL LOGPRI( 'DIT IS DE TWEEDE CONTROLE', 0 )
C    ERRFLG = .FALSE.
IF ( LTSWL ) THEN
  IF ( .NOT. XSGTC( 'MATERI/THAWCHK', MATCHK, 1 ) )
    MATCHK = .FALSE.
  IF ( MATCHK ) THEN
    IF ( .NOT. XSGTC( 'MATERI/THAWERR', ERFFLG, 1 ) )
      CALL PRGERR( 'INISOL', S3 )
  END IF
END IF

C
C... CHECK MATERIAL PARAMETERS IF NOT DONE BEFORE
C
C... GET AND CHECK SAW-TOOTH MODEL TO BE APPLIED
C... temp. solution to use SAMMOD in case of SAMLAW as well as TSAMLAW
IF ( LTSWL ) THEN
  IF ( XSGTC( 'MATERI/SAMMOD', SAMMOD ) ) THEN
    IF ( ( SAMMOD .NE. 'RIPPLE' ) .AND. (
      ( SAMMOD .NE. 'rippLe' ) .AND. (
      ( SAMMOD .NE. 'rippLeS' ) )
    )
      CALL ERRPGR( 'NLINII', 163, MATIDX, 1, 0, 0, 'SAMMOD'

    ELSE
      to do: print warning instead of abort
      CALL PRGERR( 'INISOL', 582 )
    END IF
  ELSE
    still to be implemented... ---> stop
    CALL PRGERR( 'INISOL', 563 )
  END IF
C
  CALL LOGPRI( 'dit is de dendo controle', 0 )
C
C... GET AND CHECK TENSILE SOFTENING CURVE SHAPE
IF ( XSGTC( 'MATERI/TENCHY', CSHAPE ) ) THEN
  IF ( ( CSHAPE .NE. 'LINEAR' ) .AND.
    ( CSHAPE .NE. 'EXPOSE' ) ) THEN
    ERFFLG = .TRUE.
    to do: print warning instead of abort
    CALL PRGERR( 'INISOL', 584 )
  END IF
ELSE
  CSHAPE = 'LINEAR'
END IF
C
C... GET AND CHECK NUMBER OF SAW-TEETH
C... temp. solution to use NTEETH in case of SAMLAW as well as TSAMLAW
IF ( LTSWL ) THEN
  IF ( XSGTC( 'MATERI/NTEETH', NTEETH, 1 ) ) THEN
    to do: print warning instead of abort
    CALL PRGERR( 'INISOL', 585 )
  ELSE
    to do: print warning instead of abort
    CALL PRGERR( 'INISOL', 586 )
  END IF
ELSE
  still to be implemented... ---> stop
  CALL PRGERR( 'INISOL', 567 )
END IF
C
C... GET AND CHECK TENSILE STRENGTH
IF ( XSGTC( 'MATERI/TENSTR', FLIMIT, 1 ) ) THEN

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IF ( FLIMIT .LE. 0.0 ) THEN
  ERRFLG = .TRUE.
END IF

C... to do: print warning instead of abort
CALL PRGERR( 'INISOL', 506 )

ELSE

C... to do: print warning instead of abort
ERRFLG = .TRUE.
CALL PRGERR( 'INISOL', 509 )
END IF

C... GET AND CHECK ULTIMATE STRAIN OR FRACTURE ENERGY
IF ( ( CSHAPE .EQ. 'LINEAR' ) .AND. 
  $XSGTC( 'MATERI/EPSSUL', EPSULT, 1 ) ) THEN
  CALL GT( 'MATERI/YOUNG', YOUNG, 1 )
  IF ( EPSULT LT. ( FLIMIT / YOUNG ) ) THEN
C... SNAP BACK AT CONSTITUTIVE LEVEL NOT SUPPORTED
    ERRFLG = .TRUE.
    CALL PRGERR( 'INISOL', 510 )
  END IF
ELSE
  IF ( XSGTC( 'MATERI/GFI', GFI, 1 ) ) THEN
    IF ( GFI LE. 0.0 ) THEN
C... to do: print warning instead of abort
      ERRFLG = .TRUE.
      END IF
    END IF
  ELSE
    IF ( XSGTC( 'MATERI/GFI', GFI, 1 ) ) THEN
      IF ( GFI LE. 0.0 ) THEN
C... to do: print warning instead of abort
        ERRFLG = .TRUE.
      END IF
    END IF
  END IF
END IF

C... IF ( LTSAWL ) THEN
  CALL PTL( 'MATERI/TMACHK', .TRUE., 1 )
  CALL PTL( 'MATERI/TMERR', ERRFLG, 1 )
END IF

C... END IF
C... CALL LOGPR( 'DIT IS DE VIERDE CONTROLE', 8 )
C...

C... GET MATERIAL PARAMETERS IF NOT DONE YET
IF ( MATCHK .AND. NOT. ERRFLG ) THEN
  c... temp. solution to use SAMMOD in case of TSAWL as well as TSAMLM
  IF ( LTSAWL ) THEN
    CALL GTCH( 'MATERI/SAMMOD', SAMMOD )
  ELSE
    CALL PRGERR( 'INISOL', 510 )
  END IF

  IF ( NOT. XSGTC( 'MATERI/SGECK', CSHAPE ) )
$   CSHAPE = 'LINEAR'
   CALL GT( 'MATERI/TENSY', FLIMIT, 1 )
   IF ( ( NOT. ( CSHAPE .EQ. 'LINEAR' ) ) .AND. 
$   XIST( 'MATERI/EPSSUL' ) ) THEN
      CALL GT( 'MATERI/GFI', GFI, 1 )
   ELSE
      CALL GT( 'MATERI/NTEETH', EPSULT, 1 )
  END IF
  CALL GT( 'MATERI/NTEETH', NTEETH, 1 )
END IF

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END IF
C...
C... GENERATE SAW-TOOTH MODEL IF NO ERRORS WERE FOUND
C...
IF (.NOT. ERRFLG) THEN
C...
DETERMINE AREA UNDER SOFTENING CURVE
C...
$ XIST( 'MATERIAL/EPSULT' ) ) THEN
IF (.NOT. XSGTLB( 'MATERIAL/Crackb', Crackb, 1)) THEN
NIP = INQ(' INTPT', 'DIM')
CALL GTC( 'DVOLIP', DVOLIP, NIP)
VOLUME = RSM( DVOLIP, NIP, )
Crackb = (1 DB/H DB)*VOLUME**2 (H DB/H DB)
C call privil( Crackb, 'crackb' )
C call privil( volume, 'volume' )
C call privil( nip, 'nip' )
END IF
C...
negative crack bandwidth should be detected
C...
earlier -> still to be done
IF ( Crackb ) .GT. 0. DB) THEN
AREA = GFI / Crackb
ELSE
CALL PRGERR( 'INISOL', 50 )
END IF
ELSE
AREA = 0. DB * EPSULT * FLIMIT
END IF
CALL GTC( 'MATERIAL/YOUNG', YOUNGB, 1 )
c call logpri( 'sawmod = 'sawmod, 0 )
c call privil( 'nneck = 'nneck' )
c call logpri( 'cshape = 'cshape, 0 )
c call privil( area, 'area' )
c call privil( flimit, 'flimit' )
CALL GENGAW( SAMMOD, NTEETH, CSHAPE, AREA, FLIMIT,
$ END IF
C
C...
PUT INITIAL YOUNG'S MODULI AND TENSILE STRENGTHS AT I.P.
C...
LEVEL AND INITIALISE DAMAGE LEVEL INDICATORS
C call privil( errflg, 'errflg' )
IF (.NOT. ERRFLG) THEN
CALL GTC( 'MATERIAL/YOUNG', YOUNGB, 1 ) CALL RESET( YOUNGB, YOUNGB, DAMIN )
IF ( LTAWA ) THEN
IF (.NOT. XSGTLB( 'TSAMID', SAMDIA, LDUM,
$ END IF
FB = SAMDIA(2)
CALL RESET( FB, TENST, DAMIN )
CALL RESET( 0, DAMINT, DAMIN )
NIP = INQ(' INTPT', 'DIM') CALL PSHIR CALL MD( 'INTPT' ) DO 100, IP = 1, NIP CALL IXDIR( IP)
CALL RESET( 0, USRSTA, MSTATE )
USRSTA(I) = YOUNGB(I) USRSTA(I) = YOUNGB(I) USRSTA(I) = YOUNGB(I) CALL PTL(' YOUNG', YOUNG, DAMIN )
CALL PTL( 'TENSTR', TENSTR, MDAMIN )
CALL PTL( 'DAMIND', DAMIND, MDAMIN )
CALL PTL( 'USRSTA', USRSTA, MSTATE )
188 CONTINUE
   CALL POPDR
   END IF
C
   END IF
C
   CALL LOGPRI( 'DIT IS DE VIJFDE CONTROLE', 0 )
C
   ELSE
C
   ....
   NO CRACK MODEL FOUND
   CALL ERMSG( 'NLINOM', 187, MATIDX, 0, 0, 0, ' ' )
C
   END IF
C
   CALL LOGPRI( 'DIT IS DE LAATSTE CONTROLE', 0 )
C
   END

Figure 52: New subroutine inisol.f
SUBROUTINE LAMSOI( EMPSOI, INFSOI, LAMBDA, SELIP, WRDATA )
C..................Copyright (c) 2008 TU Delft
C
C... PURPOSE:
C... Calculate admissible load factor set for current i.p.
C... Assume proportional loading conditions.
C...
C... ARGUMENTS:
C... EMPSOI logical out empty solution set?
C... INFSOI logical(2) out [1] lower bound at minus infinity?
C... [2] upper bound at plus infinity?
C... LAMBDA double(2) out [1] (finite) value of lower bound
C... [2] (finite) value of upper bound
C... SELIP logical out may current i.p. be selected as
C... critical i.p.?
C... WRDATA logical in write data of current i.p. to file?
C...
C... PROGRAMMED BY A.V. VAN DE GRAAF AND L.O. VOOMEREEN
C... Author: A.V. van de Graaf and L.O. Voormeerem
C... Revision: 0.1
C... Date: 2018/11/15
C
C... let op: in deze subroutine wordt uitgegaan van een “constant
C... stress cut-off” als criterium voor schaduwberekening
C
DOUBLE PRECISION PI
PARAMETER ( PI=3.141592653D0 )
INTEGER MSTR, NDAMIN, MSTATE, NSHR, NCOMP
PARAMETER ( MSTR=6, NDAMIN=4, MSTATE=12, NSHR=4, NCOMP=6 )
DOUBLE PRECISION LAMBDA(2)
LOGICAL EMPSOI, INFSOI(2), SELIP, WRDATA
INTEGER ING
LOGICAL XSGLTB, XSGLC, XIST
INTEGER ITYP
COMMON /DSTYPV/ ITYP(10)
EQUIVALENCE ( MSTR, ITYP(4) )
DOUBLE PRECISION ALLSIG(2*MSTR), SIGAXY(MSTR), SIGBXY(MSTR),
$ C, TENSTR(NDAMIN), ANGLE, USRSTA(MSTATE),
$ S, T(6,8), SIGANT(MSTR), SIGNT(MSTR),
$ LAMBDA1(), LAMBDA2(), LAMBDA3(), LAMBDA4(),
$ OMSTR(NDAMIN), SIGPR(2), LAMBDA5(), SIG(1),
$ QIMPS(1,1),
$ INTEGER MTH, I, J, DAMIN(NDAMIN), NTEETH, DECISI,
$ CRTOIR, LAMDIR
LOGICAL INFB(1), INFB(2), INFB(3), INFPI2(), PROPLD, CRACKD, LSAWL, LSTWL, LCSWL, CONLOD,
$ INFIB2()
CALL LOGPI( 'LAMSOI start', 0 )
C... PERFORM INITIALISATION
CONLOD = .FALSE.
EMPSOI = .FALSE.
SELIP = .TRUE.
CRTOIR = -1
DAMING = 0
CALL RSET( 0, DB, TENSTR, NDAMIN )
LTSWL = XIST( 'MATER/TSWL' )
c call privc( ltswl, 'ltswl' )
if ( .WDATA ) call gtc( 'USRSTA', USRSTA, MSTATE )
c
C... PROPORTIONAL LOADING CONDITIONS OR NOT?
call gtc( './JOBIN/MODEL/PROPLO', PROPLO, 1 )
c... non-proportional loading has not been restored yet --> stop
C... Mind you: perhaps IMPOIS needs to be called more than once for
C... non-proportional loading. This is relevant if you first try to
C... load the structure with the constant loads and only once this
C... is accomplished continue loading with the unit loads.
if (.NOT. PROPLO ) then
  if (.NOT. XSTC( './JOBIN/MODEL/CONLOD', CONLOD, 1 ) )
    call prgerr( 'LAMSL', 1 )
  end if
end if
c call privc( conlod, 'conlod' )
c call logpri( 'LAMSL THREEDE CONTROLE', 0 )
c
C... GET ALL STRESS COMPONENTS AND PUT THEM IN ARRAYS:
c... SIGXY CONTAINS STRESS COMPONENTS DUE TO CONSTANT LOAD(S)
c... SIGXY CONTAINS STRESS COMPONENTS DUE TO UNIT LOAD(S)
if ( XSTLB( 'SIG', ALLSIG, NITM, 2*NSTR ) ) then
  if ( NITM .EQ. 2*NSTR ) then
    c... THE ORDER OF STRESS COMPONENTS IN ALLSIG IS AS FOLLOWS:
c... STRESS CMP. 1 TO NSTR ARE DUE TO CONSTANT LOAD(S)
c... STRESS CMP. (NSTR+1) TO (2*NSTR) ARE DUE TO UNIT LOAD(S)
c
C... ALWAYS COPY STRESS COMPONENTS DUE TO UNIT LOAD(S)
do 10 j = 1, NSTR,
  if ( (.NOT. PROPLO ) .AND. CONLOD ) then
    c... IF FOR NON-PROPORTIONAL LOADING, ONLY THE CONSTANT
    c... LOADS NEED TO BE CONSIDERED ASSIGN THOSE STRESSES
    c... AS IF THEY WERE TO UNITS LOADS
    SIGXY(J) = ALLSIG(J)
  else
    SIGXY(J) = ALLSIG(NSTR+J)
  end if
10 continue
  c call privc( sigxy, nstr, 'sigxy' )
c
C... IN CASE OF NON-PROPORTIONAL LOADING ALSO COPY STRESS
C... COMPONENTS DUE TO CONSTANT LOAD(S) IF NOT ONLY THE
C... CONSTANT LOADS NEED TO BE CONSIDERED
  if ( (.NOT. PROPLO ) .AND. (.NOT. CONLOD ) ) then
    do 20 i = 1, NSTR
      SIGXY(I) = ALLSIG(I)
  continue
20 else
    call rset( 0 DO, SIGAXY, NSTR )
  end if
  c call privc( sigaxy, nstr, 'sigaxy' )
c
c else
  call prgerr( 'LAMSL', 2 )
end if
c else
  call prgerr( 'LAMSL', 3 )
end if
c c call logpri( 'LAMSL DERDE CONTROLE', 8 )
C... GET DAMAGE LEVEL INDICATORS
IF (.NOT. XSGT( 'DAMIND', DAMIND, NDAMIN ) )
$ CALL PRGERR ( 'LAMSOL', 4 )
c call prlvc( damind, ndamin, 'damind' )
C... GET CURRENT MATERIAL STRENGTHS
IF ( LTSAWL ) THEN
IF (.NOT. XSGT( 'TENSTR', TENSTR, NDAMIN ) )
$ CALL PRGERR ( 'LAMSOL', 5 )
c call prlvc( tenstr, ndamin, 'tenstr' )
END IF
C... IS CURRENT INTEGRATION POINT CRACKED OR NOT?
CRACKD = ( DAMIND(i) .NE. 8 )
c call prlvc( crack, 'crack' )
C... USE STRESSES IN PRINCIPAL DIRECTIONS TO DETERMINE THE
C... ADMISSIBLE LOAD FACTOR SET
IF ( (PROPLO .OR. ( .NOT. PROPLO ) .AND. CONLOD ) ) THEN
C... IN CASE OF UNIT LOADS ONLY, THIS IS RELATIVELY EASY AS
C... THE PRINCIPAL DIRECTIONS WILL NOT DEPEND ON LAMBDA
ERRFLG = .FALSE.
call mgsol ( SIGBX, SIG, DIRM5, .TRUE., ERRFLG )
c call prlvc( sign, 3, 'sign' )
c call prlvc( dirm5, 3, 3, 'dirm5' )
C... CONSIDER TENSILE FAILURE CRITERION?
IF ( LTSAWL ) THEN
STORE SOLUTION SET IN INFIBI AND LAMBDI
IF ( (SIGN1(i)).GT.0.0 ) THEN
INFIBI(i) = .FALSE.
LAMBDI(i) = TENSTR(i) / SIGN1(i)
END IF
IF ( (SIGN1(i)).LT.0.0 ) THEN
INFIBI(i) = .FALSE.
LAMBDI(i) = TENSTR(i) / SIGN3(i)
END IF
END IF
C... CONSIDER COMPRESSIVE FAILURE CRITERION?
C... STORE SOLUTION SET IN INFIB2 AND LAMBD2
IF ( (SIGN2(i)).LT.0.0 ) THEN
INFIB2(i) = .FALSE.
LAMBD2(i) = COMSTR(i) / SIGN2(i)
END IF
IF ( (SIGN2(i)).GT.0.0 ) THEN
INFIB2(i) = .FALSE.
LAMBD2(i) = COMSTR(i) / SIGN2(i)
END IF
END IF
C ELSE
C... IN THIS CASE THE SITUATION IS MORE COMPLICATED AS THE
C... PRINCIPAL DIRECTIONS ARE DEPENDENT ON LAMBDA
C... to be completed
C CALL PRGRerr( 'LAMBSOL', 800 )
C END IF
C call privc( infib2, 2, 'infib2' )
C call privc( lambda2, 2, 'lambda2' )
C... DETERMINE INTERSECTION OF THE TWO SOLUTION SETS
C CALL INTRESE( INFI1, LAMBD1, INFI2, LAMBD2, EMPSOL, INFBW, 
$ LAMDA, DECISI )
C IF ( DECISI .EQ. 0 ) THEN
C CALL PRGRerr( 'LAMBSOL', 58 )
C ELSE IF ( DECISI .EQ. 1 ) THEN
C DAMC = 1
C ELSE IF ( DECISI .EQ. 2 ) THEN
C DAMC = -1
C END IF
C call privc( infbw, 2, 'infbw' )
C call privc( lambda, 2, 'lambda' )
C call privc( decisi, 'decisi' )
C IF ( WRDATA ) THEN
C... CALCULATE PRIMARY CRACK ORIENTATION
C CALL PTL( 'DIRCRK', DIMS, 9 )
C CDIR = 1
C END IF
C ELSE
C IF ( .NOT. proplo ) CALL PRGRerr( 'LAMBSOL', 980 )
C... TRANSFORM STRESS VECTORS TO N,S,T COORDINATE SYSTEM
C... IF ( .NOT. SUGIT( 'ISISNE', DIMS, 9 ) )
C$ CALL PRGRerr( 'LAMBSOL', 13 )
C call TRMATX( NSTR, NCOMP, NSHR, DIMS, T )
C call privat( T, 6, 6, 'T' )
C CALL RAB( T, 0, 0, SIGXY, 1, SIGNT )
C call privc( sight, 6, 'sight' )
C CALL LOGPRI( 'LAMBSOL VIERDE CONTROLE', 0 )
C... DETERMINE SOLUTION SET IN N-DIRECTION
C CALL LSET( 'TRUE', INFI1, 2 )
C CALL RSET( 6 DO, LAMBD1, 2 )
C IF ( SIGNT(1) .NE. 0.0D0 ) THEN
C... CONSIDER TENSILE FAILURE CRITERION?
C... IF ( LTSWL ) THEN
C... STORE SOLUTION SET IN INFI1 AND LAMBDA
C... IF ( LTSWL ) THEN
C... NTEETH = ( INQ( '.../TSAND', 'DIM' ) / 2 ) + 1
C END IF
C IF ( ABS(DAPMND(1)) .LT. NTEETH ) THEN
C IF ( SIGNT(1) .GT. 0.0D0 ) THEN


INFIB1() = .FALSE.
LAMBDO1() = TENSTR(1) / SIGBNT(1)
ELSE
INFIB1() = .FALSE.
LAMBDO1() = TENSTR(1) / SIGBNT(1)
ENDIF
ENDIF
C
END IF
C
call logpri( 'solution set in direction N', 0 )
C
Determine solution set in S-direction
CALL LSET( .TRUE., INFIB2, 2 )
CALL RSET( 0.D0, LAMBD2, 2 )
C
IF( SIGBNT(2) .NE. 0.0D0 ) THEN
C
Consider tensile failure criterion?
IF( LTSAWL ) THEN
C
Store solution set in INFIB2 and LAMBD2
IF( LTSAWL ) THEN
NTEETH = INT( '(/TSAWL', 'DIM' ) / 2 ) + 1
ENDIF
IF( ABS(DAMP12L) .LT. NTEETH ) THEN
INFIB2() = .FALSE.
LAMBD2() = TENSTR(2) / SIGBNT(2)
ELSE
INFIB2() = .FALSE.
LAMBD2() = TENSTR(2) / SIGBNT(2)
ENDIF
ENDIF
ENDIF
C
END IF
C
call logpri( 'solution set in direction S', 0 )
C
Determine intersection solution sets N AND S DIRECTION
CALL LSET( .TRUE., INFIB1, 2 )
CALL RSET( 0.D0, LAMBD1, 2 )
C
CALL INTSIE( INFIB1, LAMBD1, INFIB2, LAMBD2, EMP50L, $ INFIB3, LAMBD3, DECISI )
IF( DECISI .EQ. 0 ) THEN
CALL PRGERR( 'LAMBSOL', 208 )
ELSE IF( DECISI .EQ. 1 ) THEN
LAMBO1 = 0
ELSE IF( DECISI .EQ. 2 ) THEN
LAMBO1 = 2
ENDIF
C
call privi( 'landir', 'landir' )
C
Determine solution set in T-direction
CALL LSET( .TRUE., INFIB4, 2 )
CALL RSET( 0.D0, LAMBD4, 2 )
C
IF( SIGBNT(3) .NE. 0.0D0 ) THEN
C
Consider tensile failure criterion?
IF( LTSAWL ) THEN

80
C...
STORE SOLUTION SET IN INFB4 AND LAMBDA4
IF ( LTSAWK ) THEN
    NTEETH = ( INQ( '../TSAWK', 'DIM' ) / 2 ) - 1
END IF
IF ( ABS(DAMIND()) .LT. NTEETH ) THEN
    IF ( SIGNT() .GT. 0.00 ) THEN
        INFB4() = .FALSE.
        LAMBDA4() = TENSTR(3) / SIGNT(3)
    ELSE
        INFB4() = .FALSE.
        LAMBDA4() = TENSTR(3) / SIGNT(3)
    END IF
END IF
END IF
CALL LOGPRI( 'solution set in direction T', 0 )
C ...
SUPERPOSE SOLUTION SETS (N AND S) AND T DIRECTION
IF ( .NOT. EMP5OL ) THEN
    CALL INTRSR( INFB3, LAMB3, INFB4, LAMB4, EMP5OL, $ INFBOU, LAMBDA, DECISI )
    IF ( DECISI .EQ. 0 ) THEN
        CALL PRGERR( 'LAM5OL', 298 )
    ELSE IF ( DECISI .EQ. 1 ) THEN
        CRTOIR = LAMDIR
        ELSE IF ( DECISI .EQ. 2 ) THEN
            CRTOIR = 3
    END IF
C CALL PRIV( 'decisi', 'decisi' )
C CALL PRIVC( CRTOIR, 'crtoir' )
C CALL PRIVC( INFBOU, 2, 'INFBOU' )
C CALL PRIVC( LAMBDA, 2, 'LAMBDA' )
END IF
C ...
IF ( WRDATA ) THEN
    IF ( CRTDIR .EQ. 1 ) CALL PRGERR( 'LAM5OL', 168 )
    CALL PTL( 'REDIR', CRTDIR, 2 )
END IF
C CALL LOGPRI( 'LAM5OL end', 0 )
END
SUBROUTINE DAMSOL
C
C...----------------------------------------------------------------------
C... PURPOSE:
C... Update damage level indicators and mechanical properties of
C... critical i.p. which is located in a solid element.
C...
C... ARGUMENTS:
C...
C... DESCRIPTION:
C...
C... PROGRAMMED BY A.V. VAN DE GRAAF AND L.O. VOORMEELEN
C... Author: A.V. van de Graaf and L.O. Voormeelen
C... Revision: 0.1
C... Date: 2010/11/15
C...----------------------------------------------------------------------
C
INTEGER NDAMIN, MTEETH, MSTATE, DAMINC
LOGICAL XSGTC, XSGTLB, XIST

DOUBLE PRECISION SAMXIA(2*(MTEETH+1)), EPS, SIG, YOUNG(NDAMIN),

PARAMETER ( NDAMIN=3, MTEETH=250, MSTATE=12, DAMINC=1 )

LOGICAL LTSWAL

CALL LOGPRI( 'DAMSO start', 0 )
LTSWAL = XIST( 'MATER/TSWAL' )
call privc( ltswal, 'ltswal' )

GET DAMAGE LEVEL INDICATORS AND UPDATE INFORMATION
IF ( .NOT. XSGTC( 'DAMIND', DAMIND, NDAMIN ) )
$ CALL PRGERR( 'DAMSO', 1 )
IF ( .NOT. XSGTC( 'REDDIR', DIR, 1 ) )
$ CALL PRGERR( 'DAMSO', 2 )
IF ( .NOT. XSGTC( 'USRSTA', USRSTA, MSTATE ) )
$ CALL PRGERR( 'DAMSO', 3 )

CALL LOGPRI( 'DAMSO THEEDE CONTROLE', 0 )

UPDATE CRITICAL DAMAGE LEVEL INDICATOR
IF ( (DAMIND(DIR) / DAMINC) .GE. 0 ) THEN
DAMIND(DIR) = DAMIND(DIR) + DAMINC
ELSE
CALL ERRMS( 'MIND', 169, 0, 0, 0, 0, 0 )
DAMIND(DIR) = -1 * ( DAMEND(DIR) - DAMINC )
END IF

call privc( damind, ndamin, 'damind' )

CALL LOGPRI( 'DAMSO DERDE CONTROLE', 0 )

UPDATE TENSILE PROPERTIES
IF ( LTSWAL ) THEN
IF ( .NOT. XSGTLB( 'TSWAL', SANDIA, LDUM, 2*(MTEETH+1) ) )
$ CALL PRGERR( 'DAMSO', 21 )
END IF
IF ( ABS(DAMIND(DIR)) .GT. (LDUM / 2) )
$ CALL PRGERR( 'DAMSO', 22 )
CALL GTC( 'TENH', TENSTR, NDAMIN )
SIG = SANDIA(ABS(DAMIND(DIR))+1)
Figure 54: New subroutine damsol.f
SUBROUTINE MATSO2
C-----------------------------Copyright (c) 2008-2009 TU DELFT
C...
C.. PURPOSE:
C...
C.. ARGUMENTS:
C...
C.. DESCRIPTION:
C...
C.. PROGRAMMED BY A.V. VAN DE GRAAF AND L.O. VOORMEEREN
C.. Author: A.V. van de Graaf and L.O. Voormeeren
C.. Revision: 0.1
C.. Date: 2018/11/18
C-----------------------------
C
INTEGER NDMIN
PARAMETER ( NDMIN=3 )

LOGICAL XSGTC, XSGTCH

CHARACTER*6 SHRCRV
DOUBLE PRECISION YOUNGB, YOUNG(3), POIS0, G(3),
$ POISON(3), DIPCRC(3,3), SE(3,3), BETA,
$ YKMIN(3), POISRD(3)

INTEGER TAUCCI
LOGICAL SHRFLG

CALL logpri ( 'MATSO2 start', 0 )

GET INITIAL AND UPDATED YOUNG'S MODULI
CALL GTC( '.../MATERI/YOUNG', YOUNG, 1 )
CALL GTC( '.../MATERI/POISO', POISO, 1 )

CALL privec( young, ndmin, 'young' )

DETERMINE POISSON'S RATIO IN BOTH DIRECTIONS
CALL GTC( '.../MATERI/POISO', POISO, 1 )
POISON() = POISO * ( YOUNG() / YOUNGB )

POISON() = POISO * ( YOUNG() / YOUNGB )

CALL privec( poison, ndmin, 'poisson' )

misschien nog thermodynamische constraints checken??

DETERMINE SHEAR RETENTION RELATION
SHRFLG = XSGTCH( '.../MATERI/SHRCRV', SHRCRV )
IF ( .NOT. SHRFLG ) THEN
CALL GTC( '.../MATERI/TAUCRI', TAUCCI, 1 )
IF ( TAUCCI .EQ. 1.0 ) THEN
SHRCRV = 'CONT2'
ELSE IF ( TAUCCI .EQ. 2.0 ) THEN
SHRCRV = 'VARIAB'
END IF
END IF

CALL logpri ( 'shrcrv = ' /shrscrw, 0 )
IF ( SHRCRV .EQ. 'CONST2' ) THEN

DETERMINE SHEAR RETENTION
IF ( .NOT. XSGTC( '.../MATERI/BETA', BETA, 1 ) ) BETA = 1.0

CALL privec( beta, 'beta' )
G1() = BETA * ( YOUNG0 / ( 1.0 + POISO)))
G2() = BETA * ( YOUNG0 / ( 1.0 + POISO)))
ELSE IF ( SHRCRV .EQ. 'VARIAB' ) THEN

END IF
C... VARIABLE SHEAR RETENTION (STEPWISE REDUCTION OF G)
    YGNMIN(:,i) = MIN(YOUNG(:,1), YOUNG(:,i))
    POISRD(:,i) = POISO * ( YGNMIN(:,i) / YOUNG )
    G(:,i) = YGNMIN(:,i) / ( 2.00 * ( 1.00 + POISRD(:,i) ) )
    YGNMIN(:,i) = MIN(YOUNG(:,2), YOUNG(:,i))
    POISRD(:,i) = POISO * ( YGNMIN(:,i) / YOUNG )
    G(:,i) = YGNMIN(:,i) / ( 2.00 * ( 1.00 + POISRD(:,i) ) )
    YGNMIN(:,i) = MIN(YOUNG(:,3), YOUNG(:,i))
    POISRD(:,i) = POISO * ( YGNMIN(:,i) / YOUNG )
    G(:,i) = YGNMIN(:,i) / ( 2.00 * ( 1.00 + POISRD(:,i) ) )
END IF
C    call privec( g,j,'g' )
C C... GET PRIMARY CRACK ORIENTATION
    IF (.NOT. XSGTC('DIRCRA', DIRCRA, 9 ) )
      $ CALL PRERR('MATSO2', )
C C... CALCULATE NEW STRESS-STRAIN MATRIX AND PUT IT ON THE FILOS FILE
    CALL SESOL2( YOUNG, POISON, 3, DIRCRA, SE )
    CALL PTL('3E', SE, 36 )
    C    call primat( se, 6, 6, 'se' )
    C    call logpri('MATSO2 end', 0 )
END

Figure 55: New subroutine matso2.f
SUBROUTINE SESOL2( YOUNG, POISON, G, DIRCRA, SE )

C.......... Copyright (c) 2008 TU Delft
C
C.. PURPOSE:
C
C.. ARGUMENTS:
C.. YOUNG double(3) in Young's moduli in all directions
C.. POISON double(3) in Poisson's ratio in all directions
C.. G double(3) in shear modulus
C.. DIRCRA double(3,3) in primary crack orientation
C.. SE double(6,6) out stress-strain relation matrix
C
C.. DESCRIPTION:
C.. This subroutine returns stress-strain relation matrix SE
C.. which defines the relation between stresses (sig) and strains
C.. (eps) for a solid elements after cracking:
C
C.. (sig) = [SE](eps)
C
C.. Assuming an orthotropic formulation.
C
C.. PROGRAMMED BY A.V. VAN DE GRAAF AND L.O. VOORMEEREN
C.. Author: A.V. van de Graaf and L.O. Voormeeren
C.. Revision: 6.1
C.. Date: 2018/11/18
C
C INTEGER NSHA, NCMP, NSR
PARAMETER ( NSHA=4, NCMP=4, NSR=6 )
C
DOUBLE PRECISION YOUNG(3), POISON(3), G(3), DIRCRA(3,3),
SE(6,6)
C
DOUBLE PRECISION F, NUNS, NUNT, NUNS, NUNT, NUTN, NUTS, EN,
$ ES, ET, C, S, TEPS(6,6), SENST(6,6), WDOM(6),
$ PRT1, PRT2, PRT3, PRT4, PRT5, PRT6
C
C call logpr( 'SESOL2 start', 0 )
EN = YOUNG(1)
ES = YOUNG(2)
ET = YOUNG(3)

NSHA = POISON(1)
NUNS = POISON(2)
NUNT = POISON(3)
NUTN = POISON(4)
NUTS = POISON(5)
NUST = POISON(6)

PRT1 = EN * ES * ET
PRT2 = ( NUNS ** 2.0 ) * ( EN ** 2.0 ) * ET
PRT3 = ( NUNS ** 2.0 ) * ( ET ** 2.0 ) * ES
PRT4 = ( NUST ** 2.0 ) * ( ES ** 2.0 ) * EN
PRT5 = 2.0 * NUNS * NUST * NUTN * EN * ES * ET
PRT6 = ( 1.0 * PRT1 ) + ( PRT2 ) + ( PRT3 ) + ( PRT4 ) + ( PRT5 )
F = ( PRT1 ) / ( PRT6 )
CALL RSET( 0.000, SE, 0*6 )
CALL RSET( 0.000, SE, 0*6 )
C
C.. DEFINE STIFFNESS MATRIX IN N.S.T.AXES SYSTEM
SENS(1,1) = F * ( NUST ** 2.0 ) * ( ES / ET ) * 1.00 * EN
SENS(1,2) = F * -1.00 * ( NUNS + NUST * NUTN * ( ES / EN ) ) * EN

86
SENST(1,3) = F * (2.0 - ( NUNS + NUST + NUTN * ( ET / EN ))) * EN
SENST(1,4) = F * (2.0 - ( NUTN + NUST + NUNS * ( EN / ET ))) * ES
SENST(2,3) = F * (2.0 - ( NUST + NUTN * ( ET / EN ))) * EN
SENST(2,4) = F * (2.0 - ( NUSN + NUST + NUTN * ( EN / ET ))) * ES
SENST(3,3) = F * (2.0 - ( NUST + NUSN * ( ET / EN ))) * EN
SENST(3,4) = F * (2.0 - ( NUST + NUSN * ( ET / EN ))) * ES

C call primat( sensst, 6, 6, 'sensst' )
C C... TRANSFORM STIFFNESS MATRIX TO ELEMENT AXES SYSTEM
C... DEFINE TRANSFORMATION MATRIX
C C... PERFORM TRANSFORMATION OF STIFFNESS MATRIX
C CALL RATBA( TEPS, 6, 5, SENST, 5E, MDUM, 0 )
C C call logpri( 'SESOL2 end', 0 )

END

Figure 56: New subroutine sesol2.f
D.3.2 Adjusted subroutines

```
C
250 CONTINUE
C...
   CALL INISOL
   RETURN
C
```

Figure 57: Adjustments subroutine inslw.f

```
Clov --------------------- begin ---------------------
   INTEGER MIP
   PARAMETER ( MIP=847 )
Clov --------------------- end ---------------------
```

```
Clov --------------------- begin ---------------------
   INTEGER NIP, IP
   LOGICAL SAWELM
Clov --------------------- end ---------------------
```

```
Clov --------------------- begin ---------------------
   C...
   STORING SE HAS BEEN MOVED TO THE END OF THIS SUBROUTINE
   C...
   BECAUSE DIRECTORY 'INTPT' DOESN'T EXIST AT THIS POINT YET
   C...
   (IT IS DIMENSIONED IN SUBROUTINE NIMX3D, NITP3D, NITE3D)
Clov --------------------- end ---------------------
```

```
Clov --------------------- begin ---------------------
   C...
   STORE SE AT RIGHT LOCATION
   SAWELM = ( XIST( 'MATERI/TSAWEL' )
   IF ( SAWELM ) THEN
   C...
   SLA ELEMENT -> STORE SE AT IP LEVEL
   NIP = INQ ( 'INTPT', 'DIM' )
   IF ( NIP.GT. MIP ) CALL PRGERR( 'DLSO', 1 )
   CALL PSHDR
   CALL CWD( 'INTPT' )
   DO 300, IP = 1, NIP
   CALL IDXIR( IP )
   CALL PTL ( 'SE', SE, 36 )
   300 CONTINUE
   CALL POPDIR
   ELSE
   C...
   NON-SLA ELEMENT -> STORE SE AT ELEMENT LEVEL
   CALL PTL( 'SE', SE, 36 )
   END IF
Clov --------------------- end ---------------------
```

Figure 58: Adjustments subroutine dlso.f
C...
    SOLID
    CALL LAMSOL( EMPSOL, INFBOU, LAMBDA, SELIP, WRDATA )
    RETURN

Figure 59: Adjustments subroutine lambip.f

C
    250 CONTINUE
C...
    SOLID
    CALL DAMSOL
    CALL MATS02
    RETURN

Figure 60: Adjustments subroutine lswit2.f
Appendix E

Computation times

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April 5, 2011
E Computation times

E.1 Introduction

One of the problems encountered in the Sequentially Linear Analyses was possibility of long computation times. In particular, in finite element models with fine meshes, the total number of integration points that should be damaged before failure is enormous. Hence, obtaining a solution is a very time-consuming job and especially in three-dimensional applications using solid elements this problem is pronounced.

E.2 Computation details SLA - Reinforced concrete slab

The Sequentially Linear Analysis of the reinforced concrete slab that has been performed in this research, suffered from this problem as well. In total the finite element model counted 463 elements, divided in two layers over the height of the slab. Unfortunately, due to increasing computation times and the timespan of this study, no finer mesh could be adopted. The mesh is shown in the next figure.

![Figure 61: The adopted mesh of 463 elements.](image)

An interesting statistic is to see how the total amount of damage increments are distributed over the concrete and reinforcement. In other words, how many analyses have been performed to crack the concrete and how many caused yielding of the reinforcement. The peak load was reached after 15891 linear analyses and in total 33000 analyses have been executed. Yielding occurred for the first time after 3753 analyses. The next table summarizes the results:

<table>
<thead>
<tr>
<th>Steps</th>
<th>Concrete</th>
<th>Reinforcement</th>
<th>Distribution [%]</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 - 3752</td>
<td>3752</td>
<td>0</td>
<td>100.00 - 0.00</td>
<td>First yielding in step 3753</td>
</tr>
<tr>
<td>1 - 15891</td>
<td>15857</td>
<td>34</td>
<td>99.79 - 0.21</td>
<td>Some yielding till peak load</td>
</tr>
<tr>
<td>1 - 33000</td>
<td>32915</td>
<td>84</td>
<td>99.75 - 0.25</td>
<td>More yielding after peak</td>
</tr>
</tbody>
</table>

Table 12: Distribution damage increments concrete versus reinforcement.

Before reaching the peak load, only 34 damage increments were used for yielding of the Gaussian integration points in the embedded reinforcement. It appeared that yielding of the reinforcement mostly occurred in the transversal bottom reinforcement, at the location of the longitudinal crack towards the simple support. Even in the top reinforcement some yielding occurred, as shown in figure 62.
After the peak some more yielding occurred, but not anything substantial. Nearly all linear analyses were performed in order to damage the integration points of the concrete. This result was not surprising, since the amount of longitudinal reinforcement was chosen such that it would not yield. In total 32915 damage increments were dedicated to the concrete. The adopted integration scheme in the 20-node brick elements was 2 x 2 x 2, which means 8 integration points per element. It should be noted that there are three crack planes per integration point. The linear softening curve was discretized by saw-tooth of 12 teeth. A simple calculation shows how many of the 396 concrete elements were fully damaged after 33000 analyses:

\[ n = \frac{32915}{8 \cdot 12 \cdot 3} = 114 \]  

E.3 Computation time SLA - Reinforced concrete slab

The fineness of the adopted mesh mostly depended on the computation time of the Sequentially Linear Analysis procedure. Since numerous variations were assessed in this research, it was important to restrict the computation time to a matter of hours instead of days. This resulted in the mesh shown in figure 61, consisting of 463 elements. Changing the default 3 x 3 x 3 integration scheme of the quadratic 20-node brick elements to a 2 x 2 x 2 scheme, reduced the total computation time of 2 days to 10 hours. In total, it took 9 hours and 40 minutes to perform the 33000 linear analyses. Consequently, performing one step requires:

\[ t = \frac{9 \cdot 3600 + 40 \cdot 60}{33000} = 1.05s \]  

As it was difficult to identify which building block in the numerical process was most time-consuming, a finer mesh has been adopted consisting of 2084 elements with three elements over the height.
Again a $2 \times 2 \times 2$ integration scheme has been selected. The time required for 1000 analyses was 1 hour and 25 minutes, what comes down to a computation time for one linear analyses of:

$$t = \frac{1 \cdot 3600 + 25 \cdot 60}{1000} = 5.1 \text{s}$$

(58)

Clearly, the amount of linear analyses necessary to obtain failure is heavily increased. Therefore the total computation time increases even more, which is roughly:

$$t = \frac{2084}{463} \cdot 33000 \cdot 5.1 = 750000 \text{s}$$

(59)

In other words, it approximately takes 8 days and 18 hours to obtain a solution. Of course, this is inconvenient for daily practice and the problem needs to be solved in the future. Therefore, it is important to first identify the most time-consuming routines in the numerical process. By adding ‘log 9’ in the .com file it was possible to monitor the computation times of the building blocks. As can be seen from figure 64 the building block ‘SOLVE’ practically requires all of the computation time (5 seconds). This makes sense, since all large calculations are taking place in this building block. Solving and assembling the enormous stiffness matrices more efficiently by a smarter or more powerful solver, would result in an major improvement of the computation time.
Figure 64: .out file monitoring the computation times of the building blocks.