Experimental demonstration of phase retrieval from a single defocused intensity measurement in the approximation of small aberrations

Alessandro Polo,* Silvania F. Pereira, and Paul H. Urbach

Optics Research Group, Delft University of Technology, 2628 CJ Delft, The Netherlands

*a.polo@tudelft.nl

Abstract: We analytically demonstrate that Phase Retrieval (PR) from defocused intensity measurements is achievable using a single measurement plane. Following this approach, the predicted plane is found at a defocus distance of 4π optical unit. Experimental results confirm the theoretical predictions.

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1. Introduction

In the semiconductor industry, Next-Generation Lithography (NGL) [1] needs an accurate characterisation of the wavefront in the optical system due to the critical requirements for the specification of aberrations. Although the optics manufacturer will deliver a well-optimised optics, the final user will need to balance the optical aberration that may vary in time due to thermal heating of the mirrors [2]. For this reason, the development of a robust and fast aberration measurement technique for NGLs is attractive. Phase retrieval (PR) can be used for this task since it aims to estimate the aberrations of an optical system from a set of defocused intensity measurements, usually from a point source. Especially in astronomy, several researches have proved its benefits, especially in the characterisation of the aberrations of telescopes [3]. However, the use of several defocused measurement planes requires a high precision scanning system and causes computation loads and therefore long measurement time. In order to boost the performance of the algorithm without loosing accuracy in the retrieved phase measurement, it could be attractive to use only a single intensity measurement plane. Here, we show that, in the limit of small aberrations [4] one can derive an analytical model that identifies the optimum measurement plane to achieve accurate phase retrieval. Simulations are carried out and experimental results verify the predictions.

2. Optimum measurement plane

In this section, the analytical approach to identify the optimum measurement plane is derived. Let us consider the field distribution in the complex exit pupil (Fig. 1):

$$P(\vec{\xi}; \alpha_n^m) = A(\vec{\xi}) \exp \left[i\phi(\vec{\xi}) \right] = A(\vec{\xi}) \exp \left[i \sum_n^N \sum_{m=0}^n \alpha_n^m Z_n^m(\vec{\xi}) \right]. \quad (1)$$

Here, $A(\vec{\xi})$ is the amplitude (assumed to be constant in the pupil and zeros outside of it) at the exit pupil coordinate $\vec{\xi}$ and $\phi(\vec{\xi}; \alpha_n^m)$ is the aberration function described by a set of coefficients α_n^m in the Zernike orthonormal base Z_n^m . The through-focus field distribution in a plane perpendicular to the optical axis and at a distance z from the focal point is given by the operator $G_z[P]$ [5]:

$$U_z(\mathbf{r}_\perp; \alpha_n^m) = \mathcal{F}^{-1} \left\{ \mathcal{F} \left[\exp \left[i \frac{k}{2f} \mathbf{r}_\perp \right] \mathcal{F} \left[P(\vec{\xi}; \alpha_n^m) \right] \right] \exp^{ik_z z} \right\} / i\lambda f = G_z[P(\vec{\xi}; \alpha_n^m)] \quad (2)$$

where $\mathbf{r}_\perp$ is a point in the mentioned plane, $\mathcal{F}$ is the 2-D Fourier Transform and $k_z = \sqrt{k^2 - k_x^2 - k_y^2}$ is the z -component of the wave vector $\vec{k}$ and f is the focal distance.

Fig. 1: Graphical representation of a point source through focus scanning system

Fig. 2: Axial intensity distribution (red dashed line); minimum contribution of the interference term interference (blue solid line)

Assuming small aberrations we linearise Eq. (2), to obtain:

$$U_z(\mathbf{r}_\perp; \alpha_n^m) \simeq G_z[A(\vec{\xi})] + iG_z[A(\vec{\xi})\phi(\vec{\xi}; \alpha_n^m)], \quad (3)$$

and considering only the term in the field intensity that is linear in the phase:

$$|U_z(\mathbf{r}_\perp)|^2 \simeq |G_z[A(\vec{\xi})]|^2 + 2\text{Re} \left\{ iG_z[A(\vec{\xi})]G_z[A(\vec{\xi})\phi(\vec{\xi}; \alpha_n^m)]^* \right\} + O(\phi^2) \quad (4)$$

Therefore, the through-focus intensity distribution is expressed by a term $|G_z[A(\vec{\xi})]|^2$ that is independent of the aberrations and that identifies uniquely the optical system plus an interference term that carries the phase information [6]. The latter can be used to evaluate how a particular Zernike polynomial $Z_n^m(\vec{\xi})$ contributes to the total intensity distribution $|U_z(\mathbf{r}_\perp)|^2$ for different through-focus planes. In Fig. 2 the value of the minimum contribution of the interference term for a set of 36 Zernike coefficients and the axial intensity distribution against the defocus distance $u = 2\pi z/\lambda(NA)^2$ is shown. From this result we can see that the interference term has its minimum value in the geometrical focus of the optical system and that reaches its maximum value at a distance close to $u = 4\pi$ (i.e. where the intensity distribution along the optical axis has its first zero). This result demonstrates analytically that in case of a complete arbitrary phase retrieval problem from one single measurement plane, the intensity distribution measured in such a region has the biggest contribution of the exit pupil phase aberrations. Hence, this plane identifies the defocus position where the phase retrieval algorithm can be performed efficiently.

3. Experiment

The theoretical predictions were also experimentally validated. We use a deformable mirror to induce known aberrations in a spherical lens (0.1 NA). The wavelength of the experiment is $\lambda=638$ nm. We collect the intensity measurement at 2 different measurement planes (i.e. $u = 0$ and $u = 4.2\pi$) with a NA=0.4 microscope objective and a CCD camera. We generate three different Zernike polynomial distributions, namely astigmatism ($\alpha_2^{-2}=0.05\lambda$), coma ($\alpha_3^{-1}=0.05\lambda$) and 2nd order astigmatism ($\alpha_4^{-2}=0.05\lambda$). We performed the phase retrieval by optimising for the first 8 Zernike's orders [7]. The phase retrieval results were compared with an independent phase measurement done with a Shack-Hartmann wavefront sensor. The RMS deviations of the retrieved phase distributions from the corresponding one measured by the SH-WFS for the respective out-of-focal measurement planes are listed in Table 1.

The retrieved phase aberrations from the intensity distributions at $u = 4.2\pi$ defocus show the best agreement and are comparable with the accuracy of the SH-WFS used as a reference measurement. On the contrary, the intensity distributions at the geometrical focus ($u = 0$) give much less accurate results for the retrieved phase distributions, thus confirming the predictions of the theory.

Fig. 3: Through-focus intensity measurements.

Table 1: Phase retrieval RMS wavefront deviations for different aberrations and different measurement planes.

	$u = 0$	$u = 4.2\pi$
RMS $\alpha_2^{-2}(\lambda)$	0.26	0.01
RMS $\alpha_3^{-1}(\lambda)$	0.07	0.01
RMS $\alpha_4^{-2}(\lambda)$	0.21	0.02

4. Conclusion

In conclusion, we have identified analytically, by using the linear approximation which is valid for small aberrations, an optimal plane of measurement for the focused field phase retrieval technique, using one single intensity measurement plane. Results from numerical simulation were experimentally validated using 638 nm coherent visible light and a deformable mirror as a device to introduce known aberrations in the optical system. The phase retrieval was performed for one single intensity measurement in the geometrical focus and for the optimal plane as predicted theoretically. The results from the predicted optimal plane of measurement were in excellent agreement with an independent measurement done with a Shack-Hartmann wavefront sensor. This demonstrates that phase retrieval can be performed in a more optimised way once the right plane is chosen. The use of one single intensity plane for phase retrieval is very attractive for applications like fast adaptive optics system.

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