Influence of road pricing on network reliability

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Preface

The study presented in this master thesis investigates road pricing as an instrument to improve network travel time reliability. Nowadays, network reliability receives a lot of attention and becomes an important performance characteristic of road networks and transport services. Road pricing, a type of Intelligent Transport System (ITS), is a potentially efficient demand managing strategy. To our best knowledge, no researchers have advocated improving network travel time reliability as an objective of road pricing. Some researchers suggested evaluating road pricing from a network reliability point of view, which gives some incentives that road pricing could make contributions to network reliability. This study analyzes the influence of road pricing on network reliability.

This study has been conducted at the Transportation and Planning Section of the Faculty of Civil Engineering and Geosciences of Delft University of Technology. This study took six months, during this period I was supported by many people. Herein I’d like specially to express my sincere gratitude to my supervisor Prof. Dr. Ir. P.H.L. Bovy who provided scientific supports to me and gave me many valuable comments. He always stimulated me to think, to learn and to progress. I also would like to thank Prof. Dr. H.J. Van Zuylen who also provided many interesting new ideas. My special thanks should be devoted to my daily supervisor Michiel Bliemer who always encouraged me and helped me to solve a lot of technical difficulties in programming and many other aspects. I am also very thankful to Henk Taale, who always kindly supported me and gave me many good suggestions and comments. I’d like to thank Paul Wiggenraad who was helpful with the reporting and writing. Furthermore I’d like to express my gratitude to my parents, my sisters and my brothers, who always care me, love me and believe me. Last but certainly not least, I specially thank my thoughtful and affectionate husband, Huizhao, who always supports me, believes me and cheer me up. Certainly, without his full support and endurance, I was not able to successfully finish my master thesis within six months.

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Summary

Reliability is becoming a more and more important performance characteristic of road networks and transport services. Travel time reliability was found to be one of the most important factors for route choices of travelers. Drivers often complain that it is the unpredictability of travel time that they dislike most of a journey in a congested city or area. Network design and some traffic control measure or ITS, such as Advanced Traveler Information System (ATIS) and Incident Management (IM) have been proposed to improve road network reliability.

Road pricing has been advocated as a potentially powerful travel demand management (TDM) strategy capable of significantly influencing travel demand characteristics through the network. Generally, the policy objectives of road pricing could be summarized as follows: managing demand, optimizing congestion level, reducing environmental impacts, maximizing social welfare gains, raising revenues to recoup maintenance cost and construction cost, etc. To our best knowledge, no researchers have advocated improving network travel time reliability as an objective of road pricing. Some researchers suggested evaluating road pricing from a network reliability view. These suggestions give some incentives that road pricing could make contributions to network reliability.

This study investigates road pricing as an instrument to improve network travel time reliability, which is induced by link capacity variations and short-term origin-destination demand fluctuations. The network reliability is specified as a function of standard deviations of route travel times. It is assumed in this study that the road authority aims to optimize network reliability by setting tolls in a network design problem. Travelers are influenced by these tolls and make route and trip decisions by considering travel times and tolls (not taking the trip reliability into account).

Theoretical analyses using static models with elastic demand are performed to analyze the influence of road pricing on road network reliability. In the theoretical analyses, capacity variation is assumed as the major influencing factor on the network travel time unreliability. An important assumption is made in the theoretical analyses, being that
travelers make their route choices based on the average travel cost. Travel time and link capacity are stochastic variables, while route flow is deterministic for each charge level. The theoretical analyses start with the analysis on a single route network. Then a two-route network with one link charged is analyzed using deterministic user-equilibrium (DUE) and stochastic assignment. Finally theoretical analyses on a general network are performed and a formula to calculate route travel time unreliability is derived based on our assumptions mentioned previously. This formula is utilized in an application on a hypothetical five-link three-route network to investigate the impacts of road pricing on network reliability. The theoretical analyses show that road pricing may improve network reliability and that network reliability depends heavily on charge levels. Theoretical analysis shows that decreased OD demand and route flow switches, caused by implementing tolls, influence network reliability. Depending on the combination of the effects of decreased OD demand and route shifts, network reliability in case of elastic demand can either be improved when the effect of OD demand decreases is stronger than the effects of route shifts, or becomes worse when the influence of the route shifts is stronger than the influence of decreased OD demand.

Whereas the theoretical analyses are performed using static models and some assumptions had to be made, a dynamic simulation-based method using the dynamic model INDY for the five-link network is utilized to investigate the influence of road pricing on network reliability more appropriately. Using the dynamic simulation-based approach, capacity variations and short-term origin-destination demand fluctuations are considered as the most important factors affecting travel time variability. Fixed OD demand and elastic OD demand are taken into account respectively. Four cases are analyzed in the simulation-based analyses: the fixed demand case with capacity variations, the fixed demand case with demand fluctuations, the elastic demand case with capacity variations and the elastic demand case with demand fluctuations. With fixed demand, route shift is the major causal factor of the variability of network travel time reliability with respect to charge levels. The results from the simulation-based method show that in the fixed demand cases with the increase of the charge levels, route flow switches become less and less. Network reliability can be improved with tolls. With elastic demand, both decreased OD demand and route shifts play an important role in
network reliability. Decreased OD demand leads to positive effects on network reliability. Frequent route shifts is the essential cause of network travel time unreliability for the networks with tolling systems, and network reliability will be improved if there are less route flow switches. Combining the effects of decreased OD demand and route shifts, network reliability may either be improved or becomes worse, depending on the charge levels. The optimal charge from road authority’s view can be determined, aiming to optimize network reliability. With the optimal charge, people take longer but more reliable routes. It turns out that there is a trade-off between the average route travel time and the route travel time reliability. In a normal situation (without tolls), both capacity variations and demand fluctuations have great influence on travel time variability, since these two factors lead to stochastic route flow shifts. Demand fluctuations have greater influence on travel time variability than capacity variation does. Because demand fluctuations have direct influence on route flows, which leads to direct travel time variability.

The objective of optimizing network travel time reliability proposed in this study can contribute to the charging system design, which is a bi-level optimization problem. Road authority, as the leader, tries to optimize network reliability by setting tolls. Travelers, as the followers, try to minimize their own travel cost, which is the sum of travel time and tolls. The objective function can be used to optimize charge levels or search optimal links for charging for fixed charging systems and for variable charging systems. Only a small hypothetical network with a single OD pair has been analyzed in this study. For this small network, we conclude that road pricing can improve network travel time reliability. For more general networks, the objective of optimizing network reliability can be employed to determine the optimal charge level and to optimize the charging system design.
Notation and units

Sets

\( P^j \) set of routes from origin \( i \) to destination \( j \),
\( I \) set of origin nodes,
\( J \) set of destination nodes,

Indices

\( p \) route index,
\( i \) origin node index,
\( j \) destination node index,
\( a \) link index,
\( x \) observation index of capacity from a probability distribution,
\( y \) observation index of OD demand from a probability distribution,

Route variables

\( r_p \) route travel time unreliability of route \( p \), [minutes]
\( \sigma_p \) standard deviation of route travel time on route \( p \), [minutes]
\( q_p \) route flow on route \( p \), [veh/h]
\( q_p^* \) route flow on route \( p \) at equilibrium state, [veh/h]
\( z_p \) generalized cost of route \( p \), [minutes]
\( z_p^* \) route cost on route \( p \) at equilibrium state, [minutes]
\( t_{px} \) travel time on route \( p \), [minutes]
\( t_{fp} \) free flow travel time on route \( p \), [minutes]
\( \bar{t}_p \) average route travel time, obtained with mean capacities of all the links, [minutes];
\( s_p \) variance of travel time on route \( p \) after implementing road pricing, [min²]
\( s_p \) variance of travel time on route \( p \) before implementing road pricing, [min²]
\( f_p(C_{1x}, ..., C_{Ax}) \) function of capacities of all the links belonging to route \( p \), [veh/h]
\( f(-\theta_p^2) \) decreasing function of the square of charge on route \( p \), [veh²]

Link variables

\( C_{ax} \) stochastic link capacity, [veh/h]

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$C_a$ link capacity, [veh/h]
$\overline{C}_a$ mean value of link capacity, [veh/h]
$\sigma_a$ standard deviation of the link capacity distribution, [veh/h]
$t_{f0}$ free flow travel time on link $a$, [minutes]
$t_a$ link travel time, [minutes]
$l_a$ link length, [km]
$v$ travel speed, determined from speed-density function, [km/h]
$v_{\text{crit}}$ critical speed, [km/h]
$k_{\text{crit}}$ critical density, [veh/km]
$q_a$ link flow, [veh/h]

$f_a$ constant value for each link, equal to $\frac{\overline{C}_a}{\bar{d}}$, $\bar{d}$ is mean value of the standard distribution of capacity, see input parameters,

$Y_a$ link specific value, equal to $t_{f0} \cdot \alpha \left( \frac{q_a}{f_a} \right)^\beta$, $\alpha$, $\beta$ are parameters, [minutes]

$n$ number of observations of link capacities;

**OD variables**

$r_{ij}$ trip reliability between origin $i$ and destination $j$, [minutes]
$Q_{ij}$ OD demand between origin $i$ and destination $j$, [trips/h]
$Q_{ij}^0$ base OD demand between origin $i$ and destination $j$, [trips/h]
$Q_{ij}^*$ elastic OD demand between origin $i$ and destination $j$ after implementing road pricing, [trips/h]
$\sigma_{ij}$ standard deviation of OD demand distribution for OD pair $i$ and $j$, [trips/h]
$A_i$ production ability of zone $i$,
$X_j$ attraction ability of zone $j$,
$F_{ij}$ accessibility of $j$ from $i$, depends on the generalized OD travel costs,
$F_{ij}^0$ accessibility of $j$ from $i$ without tolls,
$F_{ij}^*$ accessibility of $j$ from $i$ after implementing road pricing,
$z_{ij}$ generalized cost from $i$ to $j$, [minutes]
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\[ z_{ij}^0 \]  generalized cost from i to j without tolls, [minutes];
\[ z_{ij}^\theta \]  generalized cost from i to j with a toll of \( \theta \), [minutes];
\[ m_{ij} \]  number of alternative routes from i to j;

**Network variables**

\( \tau \)  network travel time reliability, [minutes]
\( \bar{t} \)  average travel time per vehicle on a network level, [minutes]
\( Z \)  network performance indicator, [euros];

**Input parameters**

\( \mu \)  measure of average trip intensity in area,
\( VOT \)  value of time, [euros/h]
\( VOR \)  value of reliability, [euros/h]
\( \theta \)  charge, [euros]
\( \theta^* \)  optimal charge level, [euros]
\( \xi \)  constant in route travel time function, [vehicles]
\( \psi \)  constant in route travel time function, [veh/h]
\( \phi \)  constant for given observations of capacities, [h²/veh²]
\( \varphi \)  dimensionless constant for given observations of link capacities on both routes,
\( \pi \)  constant, [veh/h/euro]
\( \kappa \)  constant, [veh/h]
\( \eta_1 \)  constant for given observations of link capacities on route 1, [h⁴/veh⁵]
\( \eta_2 \)  constant for given observations of link capacities on route 2, [h⁴/veh⁵]
\( d \)  a normal probability distribution of capacity with mean \( \bar{d} \),
\( \bar{d} \)  mean value of the standard normal distribution of capacity, [veh/h]
\( s(\frac{1}{d^\beta}) \)  variance of the distribution function of \( \frac{1}{d^\beta} \)
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1. Introduction

The economy of a nation or region depends heavily upon an efficient and reliable transportation system to provide accessibility and promote the safe and efficient movement of people and goods. Reliability, however defined, provides a measure of the stability of the quality of service, which the transport system can offer to its users. Reliability can be achieved by network design in two ways. One is to over design network components. The other is to build in a degree of redundancy. Another possible way to improve reliability is to design network management systems that ensure smooth travel time under normal traffic fluctuations, with Intelligent Transport Systems (ITS).

Road pricing, a kind of ITS, has been advocated as a potentially powerful travel demand management (TDM) strategy capable of significantly influencing travel demand characteristics through the network. Generally, the policy objectives of road pricing can be summarized as follows: managing demand, optimizing congestion level, reducing environmental impacts, maximizing social welfare gains, raising revenues to recoup maintenance cost and construction cost, etc. The optimal charges obtained with different policy objectives are different. To our best knowledge, no researchers have advocated improving network travel time reliability as an objective of road pricing. Some researchers suggested evaluating road pricing from a network reliability view. These suggestions give some incentives that road pricing could make contributions to network reliability.

This study investigates road pricing as an instrument to improve network travel time reliability. Optimizing network travel time reliability is proposed as an objective of road pricing from road authority’s view, to determine the optimal charge for a charging system or to determine the optimal links for charging.

1.1 Review on reliability notion

Reliability is becoming a more and more important performance characteristic of road network and transport services. Reliability is generally defined as the ability of a system or process to perform a required function under given environmental and operational conditions and for a stated period of time. In the case of road networks, the required function can be described in terms of output variables like travel time, throughput, accessibility, equity, and so forth. Different functions may lead to different reliability measures (Bell and Cassir 2000).
Available research has put forward a number of performance criteria for reliability, such as connectivity reliability of a network, capacity reliability of a link or route, and travel time reliability of a link or route or network.

Connectivity reliability is defined as the probability that network nodes are connected (Iida and Wakabayashi 1989), it is suitable for abnormal situations, such as earthquakes, but there is an inherent deficiency in the sense that it only allows for two operating states: operating at full capacity or complete failure with zero capacity. This prevents the application to everyday situations where arcs are operating in-between these two situations.

Capacity reliability is defined as the probability that the network can accommodate a certain volume of traffic demand at a required service level (Chen, Yang et al. 2002), it is a measure to evaluate the performance of a degradable road network and is defined as the probability that the road network can accommodate a certain level of traffic demand.

Travel time reliability is defined as the probability that a trip between a given OD pair can be made successfully within a given time interval and a specified level-of-service (Asakura and Kashiwadani 1991; Bell 1999), it is useful to evaluate network performance under normal daily demand fluctuations. Asakura extends the notion of the travel time reliability to consider the capacity degradation due to deteriorated roads. Bell (2000) and Yin and Iida (2003) define another measure of network performance reliability as the utility of users when they are extremely pessimistic about the state of the network. It is defined as the total travel disutility of travelers, representing travelers’ dissatisfaction with levels of uncertainty associated with the supply of road networks.

In several recent surveys, travel time reliability was found to be one of the most important factors for route choices of travelers. Drivers often complain that it is the unpredictability of travel time that they dislike most of a journey in a congested city or area. The variability of travel time is more likely to affect traveler’s ability to estimate how long a journey would take before embarking on it. Greater uncertainty in these estimates would make travelers take preventive action, perhaps departing many minutes earlier than strictly necessary as they do not want to pay the penalty of a late arrival at their destination (Willumsen and Hounsell 2003). Different criteria of travel time reliability have been put forward in the research so far. Clark and Watling (2005) proposed the distribution of total network travel times of all drivers as a criterion of network reliability. Coefficient of Variation, which is calculated as the standard deviation divided by the mean, standard deviation of average travel time distribution, percentile travel time, etc are different measures of travel time reliability (Emam 2005).
Standard deviation of trip travel time is the most common measure of reliability, although the standard deviation of travel time will only imperfectly capture travelers’ preference. Most theories explaining aversion to unreliable travel are based on costs of unexpected arrival times at work (Bates, Polak et al. 2001), which are greater for being late than for being early. Using standard deviation to measure reliability assumes that negative and positive deviations have symmetric effects. For this reason, the difference between the 90th and 50th percentile travel times is more suitable to represent the travelers’ preferences.

Continuous variations in road way environment cause travel times to vary on the same facility during the same time period. The link capacity can degrade due to various factors, such as incidents, work zones, weather conditions, and natural/man-made disasters. The demand fluctuates within day and/or between days. These variations lead to unreliable travel time (Emam 2005).

Current research applies different assumptions of the distribution of stochastic link capacity, such as uniform distribution, normal distribution, exponential distribution (Lo and Tung 2003), gamma distribution (Brilon and Zurlinden 2003), weibull distribution (Brilon, Geistefeldt et al. 2005), etc. Depending on different characteristics of road facilities in different area, different link capacity distributions can be chosen. Brilon, Geistefeldt et al. (2005) have tested different types of distribution function based on the value of the likelihood function and found that Weibull distribution turned out to be the function that best fitted the observations.

Although little can be done about scale, frequency and predictability of the abnormal events (for instance accidents, incidents, flood, etc), particularly when natural disasters are concerned, it should be possible to design road networks so as to minimize the impacts of the disruptions such events can cause (Bell 2000). Also traffic control measures or ITS can contribute to network reliability. Chen, Skabardonis et al. (2003) showed that an Advanced Traveler Information System (ATIS) can make the travel time more predictable and can improve the travel time reliability improved on the freeway. Yin and leda (2003) found that Incident Management (IM) policy can improve the network performance reliability.

1.2 Review on road pricing

Road pricing, a charge for the use of a motor vehicle on public roads which affects the user’s choice of where and when to drive, since 1960’s, has been advocated as a potentially powerful travel demand management (TDM) strategy capable of significantly influencing travel demand characteristics through the network (Cain, Burris et al. 2001; Small, Winston et al. 2002). At the very beginning, road pricing is advocated to optimize the level of congestion.
The travelers, when entering a congested area, impose costs on other vehicles already on the road, however they don’t take into account this congestion externality. The idea is that motorists should pay for the additional congestion they create to others. Therefore certain types of road pricing are also called congestion pricing. With further attention of environmental costs, a road pricing that reflects the total social cost would internalize both congestion charge and the environmental externality, and would include a congestion charge and an environmental charge. The revenues from the price provide resources for investment of capacity expansion and can recoup partly the construction and maintenance costs. Therefore some researchers suggest a road charge should include the optimal congestion charge and the road damage charge and should cover the fixed operating cost and the variable maintenance cost (Santos 2004). Generally, the policy objectives of road pricing could be summarized as follows: managing demand, optimizing congestion level, reducing environmental impacts, maximizing social welfare gains, and raising revenues to recoup maintenance cost and construction cost.

To our best knowledge, no researchers have advocated improving network travel time reliability as an objective of road pricing. Some researchers such as (Brownstone and Small 2005) suggested to evaluate road pricing from a network reliability view. These suggestions give some incentives that road pricing could make contributions to network reliability.

The ideal optimal charge, based on existing research, from an economic view is determined by the marginal social cost. The idea of using marginal social cost, also called first-best pricing, is not new, as it is already known by many researchers as an efficient way, contributing to the social welfare. The implied equality of marginal benefits and marginal social costs secures that the social surplus is maximized. However, due to the practical difficulties, policy makers have to retreat to second-best pricing schemes that have lower transaction costs and are simpler for potential users to understand than the first-best marginal cost tolls (Safirova 2005), although the welfare gains from second-best policy are much smaller than the welfare gains that are possible with a complete set of first-best tolls and second-best is less efficient than the first-best scheme in reallocating traffic flows (Liu and McDonald 1999). The second-best pricing approach is dealing with constrained optimization problems. Depending on different policy objectives, the optimal charge can be derived differently. In the light of improving network travel time reliability, a new objective of road pricing from road authority’s view, aiming at optimizing network travel time reliability, is suggested to determine the optimal charge or optimal links for tolls for a charging system.
In this study, we focus on network travel time reliability, which is useful to evaluate the network performance under normal daily demand fluctuations and link capacity variations, ignoring the connectivity and capacity reliability. Standard deviation is selected from all the measures mentioned previously as the route travel time reliability indicator since it is the most common used measure and it’s meaningful to be able to give the range of travel time fluctuations. The argument about the selection of travel time reliability measure will be discussed more in detail in Chapter 2 of definition of travel time reliability. Stochastic capacity is assumed to following a normal distribution, which is based on the observations in the Netherlands (AVV, 1999) and (Minderhoud et al. 2000). Road pricing is motivated in this study as an instrument to improve network reliability, as other ITS measures like ATIS and IM. The analyses of the influence of road pricing on network travel time reliability is the core subject of this study. Optimizing network travel time reliability is proposed as a new objective of road pricing to determine the optimal charge level or optimal locations for charging. In the next Chapter 2, network travel time unreliability is defined and specified. Theoretical analyses of the impacts of road pricing on network travel time unreliability, using static models with elastic demand, are presented in Chapter 3. An illustrative example is given in Chapter 3 to show graphically network travel time unreliability changes with respect to different level of tolls. In theoretical analyses, link capacity variation is taken as the major causal factor of travel time unreliability. Theoretical analyses in Chapter 3 are performed based on an assumption that travelers make their route choice considering the average travel time cost. In Chapter 4, simulation-based approach using dynamic models is employed to investigate the influences of road pricing on travel time reliability. Fixed demand and elastic demand are taken into account separately. Link capacity variations and short-term origin-destination demand fluctuations are considered as the most important factors affecting travel time reliability. The contributions to network design problem, which is a bi-level optimization, are illustrated in Chapter 5. Some conclusions are drawn in Chapter 6.
2. Definition and specification of travel time reliability in road networks

Road pricing is suggested as an instrument to improve network travel time reliability, i.e. to reduce network travel time unreliability. In order to gain insights into the influences of road pricing on network reliability, firstly the notion of network travel time reliability should be motivated and defined on the basis of which an indicator for network travel time reliability should be specified to facilitate the analysis. As aforementioned, network reliability performance measures of total user disutility and total travel time have been suggested. In this study, we chose the standard deviation of route travel time as reliability measure and the path flow weighted standard deviation of path travel times as network reliability performance criteria since travel time is the most important factor that travelers take into account when they make decisions.

Route travel time unreliability

Generally actual (also called ideal, experienced or predictive) travel times and perceived travel times need to be distinguished. With perfect information, all travelers know the accurate travel times, which are deterministic. However, with incomplete information, travelers are assumed to perceive the travel times, e.g. based on historical experiences and make their choices based on the perceived travel times, which are on average equal to the actual travel times. In this study, depending on the application, the travel time we are talking about sometimes refers to the actual travel time, and sometimes to the perceived travel time. When the deterministic user equilibrium assignment is used, the travel time denotes actual travel time (for instance, in this study, theoretical analyses for a two-route network with DUE and application example in Chapter 3). In the dynamic simulation-based analyses (in Chapter 4), stochastic assignment is utilized. The travel time denotes perceived travel time.

In reality, many sources cause the travel times of users on certain routes to vary from time to time and day to day. Such sources are demand fluctuations, weather conditions like fog, rain, incidents, accidents, etc. The uncertainty of travel time always annoys travelers who suffer delays and may be penalized for being late for their work. A good description of the route travel time distributions could give travelers a general concept of the range of the route travel times and the maximum deviations of their expected travel times they would encounter. In current research, different travel time reliability measures have been used, such as Coefficient of Variation (CV) which is calculated as the standard deviation divided by the mean,
percentile travel time (90 percentile travel time), standard deviation, probability of making a trip successfully within a certain time interval, etc (Emam 2005). A traveler could multiply his/her average travel time by CV, and then add that product to his/her average travel time to get the time needed to be on-time about 85% of the time (Turner, Best et al. 1996). This effect is equivalent to the effect of using standard deviation since summing up the average travel time and standard deviation gives the 85 percentile travel time. In reality, travelers put different weights to the unexpected arrival times at work, which are greater for being late than for being early. Using standard deviation to measure reliability assumes that negative and positive deviations have symmetric effects. Thus, relatively, the difference between the 90th and 50th percentile travel times is more suitable to represent the travelers’ preferences. However in this study our focus is put on the analyses of the influence of road pricing on network reliability, not on selecting the best reliability measures. At least standard deviation is a well-accepted measure of variability (Chen, Skabardonis et al. 2003). Standard deviation of route travel time is such an indicator which can tell the travelers how large is the range that travel time deviates and at least within how many minutes they could have a guarantee of 85% (see FIGURE 1) for being on time for their work if the route travel time is normally distributed. Also standard deviation can reflect if the travel times are more concentrated with small deviation or more decentralized with large deviation.

FIGURE 1: percentile travel time with relation to standard deviation

% of observations less than the value

0% 2.5% 16% 50% 85% 97.5% 100%

-3σ -2σ -1σ mean 1σ 2σ 3σ travel time
The larger the standard deviation is, the more unreliable the travel time is, as illustrated in the FIGURE 2.

FIGURE 2 shows travel time distributions of a hypothetical network composed of two alternative routes connecting an OD pair. It is clear that the average travel time of the two routes are the same. But standard deviation of travel time on route 1 is smaller than that on route 2. Travel time on route 1 is more reliable than on route 2. Herein somebody may argue that with large standard deviation, the travel time still can be well predictable. The reliability of a trip depends on the variability as well as the predictability of the travel time (Bates, Polak et al. 2001). A trip that shows little variation in travel times is quite reliable. A large variability in travel times is not necessarily a problem, as long as the actual travel time can be well predicted. The reliability is poor when the variability is large and the predictability is small (Groep 2000). This leads us to a major problem. The question is: what is predictable? Since this is a very difficult problem, we focus on the influence of road pricing on network reliability, without addressing the predictability issue any further in this study. And in fact, we found that with large predictability, variability is very important to travelers. The larger the variability is (equivalent to the larger the standard deviation is), the less reliable the travel time is. There is a game, which is a good example implying that variability is very important. In this game, the travelers have to choose one of two alternative routes, A and B (see FIGURE3), in order to get from work place to home.

FIGURE3: example network
Travelers get the information (see TABLE 1) about the expected travel times of the alternative routes and the chances of getting them.

**TABLE 1: Route travel time information:**

<table>
<thead>
<tr>
<th>Route A</th>
<th>Route B</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 minutes with probability of 0.33</td>
<td>6 minutes with certainty</td>
</tr>
<tr>
<td>5 minutes with probability of 0.66</td>
<td></td>
</tr>
<tr>
<td>30 minutes with probability of 0.01</td>
<td></td>
</tr>
</tbody>
</table>

Many travelers make their decisions based on above information. The results show that more than 85% percent travelers choose route B because the travel time on route B is very reliable with no variability. However, in fact, the expected travel time of route A is smaller than route B, which is illustrated in **TABLE 2**.

**TABLE 2: expected travel time on route A**

<table>
<thead>
<tr>
<th>Expected travel time on route A</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.33<em>6+ 0.66</em>5+0.01*30=5.58 &lt; 6</td>
</tr>
</tbody>
</table>

Travelers prefer reliable routes with longer route travel time to the shorter route with large variability. The travel times on both routes are perfectly predictable, and variability plays the most crucial role in the decision making process. Standard deviation is such an indicator of reliability showing the travelers the variability of the travel time.

Therefore standard deviation of route travel time is selected as a route unreliability indicator, which is defined as formula (1).

\[ r_p = \sigma_p \]  

where,

- \( r_p \) — unreliability of route p, [minutes];
- \( \sigma_p \) — standard deviation of route travel time on route p, [minutes];

Standard deviation indicates the amount of extra travel time necessary to arrive on time in 85% of all the cases. This is comparable to arriving late at work three times a month.

**Trip travel time unreliability**

Trip travel time unreliability is a reliability indicator for a specific OD pair given that between each OD pair several alternative routes exist with their own travel time reliabilities. Trip
Travel time unreliability is important for users when they want to make a trip from one place to another. The trip travel time unreliability is defined on the basis of route travel time unreliability of all the routes belonging to this OD pair and use route flows as weights, see formula (2)

\[ r_{ij} = \sum_{p \in P^i} \frac{q_p r_p}{Q_{ij}} \]  

(2)

where,

- \( r_{ij} \) - trip travel time unreliability of OD pair \( i,j \), [minutes];
- \( q_p \) - traffic flow on route \( p \), [veh/h];
- \( Q_{ij} \) - OD demand of OD pair \( i,j \), [trips/h];
- \( P^i \) — route sets of OD pair \( i,j \);

**Road network unreliability**

Road network travel time unreliability is a useful measure to evaluate the road network performance as a whole. It should take into account all the road network users' experiences of travel time variability. Generally a road network consists of several OD pairs. Based on the trip-based unreliability, the road network travel time unreliability is formulated as:

\[ r = \sum_{i \in I} \sum_{j \in I} Q_{ij} r_{ij} = \sum_{i \in I} \sum_{p \in P^i} \frac{q_p r_p}{Q_{ij}} \]  

(3)

where,

- \( r \) - network travel time unreliability, [minutes];
- \( P^i \) — route sets of OD pair \( i,j \);

This definition provides a capability to assess network reliability, showing the general averaged travel time deviations for all the users through this road network. For a specific network, this network unreliability measure can be used to compare road network reliability performances under different conditions, such as with different link capacities, with different demand levels, with different toll systems and so forth. For comparison of network reliability between two different networks, the average network travel times need to be taken into account in the definition of reliability. Because a network (network 1) with 5 minutes average network travel time and 3 minutes standard deviation is less reliable than a network (network 2) with 20 minutes average network travel time and 4 minutes standard deviation. We can't say that network 1 is more reliable than network 2 since standard deviation of network 1 is
smaller than that of network 2. CV (Coefficient of Variability), here, is better to be the reliability indicator for the comparison between two different networks. In this study, we analyze the influence of road pricing on a certain network, thus the average network travel time is not included in the specification of the reliability.
3. Theoretical analysis of the impacts of tolls on road network reliability using static models

In this study, we firstly try to analyze the influence of road pricing from a theoretical viewpoint. Since some assumptions need to be made for the theoretical analyses, a simulation-based method is applied as well to analyze the impacts of tolls on the road network reliability and to derive the optimal charge level aiming to improve road network travel time reliability. This chapter theoretically analyzes the impacts of tolls on travel time reliability using static models and an application is presented.

Section 3.1 performs theoretical analyses using static models for different type of networks. Since the theoretical analyses are complex, we start with simple networks and then extend the analyses to more general networks. Subsection 3.1.1 analyzes the impacts of tolls on a single route network. In subsection 3.1.2, a two-route network is analyzed with one route charged, and different assignment techniques are applied. In Section 3.1.3, an analysis is presented for a general network with selected links charged and a general mathematical expression of network travel time reliability is derived. Network reliability appears to be a function of tolls, and an obvious relation between network reliability and tolls will be found. Finally, an application is presented in Section 3.2.

3.1 Theoretical analysis

In the theoretical analysis using the static models in this section, we analyze network reliability with the degradable links, which deals with the travel time variability with capacity variations.

Assumption 1: Link capacity is the major influencing factor of travel time unreliability. Other influencing factors, such as demand fluctuation, etc, are not taken into account.

As we reviewed previously, Brilon, Geistefeldt et al. (2005) have tested different types of distribution function for the road capacity of freeways and found that the Weibull distribution turned out to be the function that best fitted the observations. A normal distribution is also a good approximation of the link capacity distribution. Minderhoud, Botma et al. (1997) estimated capacity distributions of freeways based on several methods, such as The Product Limit Method, The Empirical Distribution Method and The Fundamental Diagram Method, and several real data analyses (AVV 1999) found that in the Netherlands generally the link
capacity follows a normal distribution and the standard deviation is approximately 10% of the mean capacity.

**Assumption 2:** All the link capacities follow normal distributions with standard deviation of 10% of the mean capacity.

**Assumption 3:** Travelers make their trip/route choices based on the average travel cost.

We assume that travelers don’t know the cost in advance, which is affected by stochastic capacities. Travelers make their trip/route choices based on the average travel cost, therefore the route flows are deterministic with each charge level. Travel times and link capacities are stochastic variables. The elastic OD demand is only dependent on the charge level in the network and is obtained with the mean capacities of the links in the network. The elastic OD demand is not a stochastic variable and won’t vary with stochastic capacities. For each charge level, with certain OD demand, the variability of stochastic travel times is caused by stochastic capacities only.

### 3.1.1 Single route network

We start with a simple network consisting of one OD pair and only one route. In this case, route travel time unreliability is equal to trip unreliability and also to the network unreliability. Link flow equals route flow in this case. The standard deviation of travel time with tolls will be calculated. For simplicity, we use a special case of the BPR function, in which the travel time and traffic flow on route p follow a linear relationship as given in function (4).

\[
t_{px} = t_f + \alpha \frac{q_p}{f_p(C_{1x}, ..., C_{Ax})}
\]  \hspace{1cm} (4)

where,

- \( x \) – observation index of capacity;
- \( t_{px} \) – travel time on route p for observation x, [minutes];
- \( t_f \) – free flow travel time on route p, [minutes];
- \( q_p \) – route flow on route p, [veh/h];
- \( C_{1x}, C_{2x}, ..., C_{Ax} \) – link capacities for observation x, where links 1, 2, ..., A are on route p, [veh/h];
- \( f_p(C_{1x}, ..., C_{Ax}) \) – traffic supply condition, which is a function of capacities of all the links belonging to route p, [veh/h];
- \( \alpha \) – dimensionless slope parameter of travel time function;
The theoretical analysis assumes stochastic variations of link capacities. Index \( x \) indicates a realization of the capacities from a stochastic distribution. It should be noticed that route flow \( q_p \) is not stochastic variable because we assume travelers don’t know the cost in advance. The OD demand is a function of average travel cost, but not the stochastic travel cost. Here we, still for simplicity, assume that travel demand \( Q \) will linearly decrease with the increase of travel cost \( z \), which is shown in equation (5):

\[
q_p = Q^0_{ij} - \gamma z_{ij}, \quad Q^0_{ij} \geq y z_{ij} \tag{5}
\]

where

\[
z_{ij} = \bar{z}_{ij} = t_p + \frac{\theta_p}{VOT}
\]

\( Q^0_{ij} \) – base demand between origin \( i \) and destination \( j \), [veh/h];
\( \gamma \) – demand function parameter, [veh/h^2];
\( z_{ij} \) – generalized cost from \( i \) to \( j \), [minutes];
\( \bar{z}_p \) – average route travel cost on route \( p \), [minutes];
\( t_p \) – average route travel time, obtained with mean capacities of all the links, [minutes];
\( VOT \) – value of time, [euros/h];
\( \theta_p \) – charge of route \( p \), which is the sum of the charges of all the links belonging to route \( p \), [euros];

\[
\text{then} \quad t_{px} = t_{fp} + \alpha \frac{t_{fp}}{f(C_{1x}, \ldots, C_{Ax})} (Q^0_{ij} - \gamma (t_{px} + \frac{\theta_p}{VOT})) \tag{6}
\]

\[
\Rightarrow t_{px} = \frac{\xi + f(C_{1x}, \ldots, C_{Ax})t_{fp} - \psi \theta_p}{f(C_{1x}, \ldots, C_{Ax}) + \psi} \tag{7}
\]

Where:

\[
\xi = \alpha \cdot Q^0_{ij} \cdot t_{fp}, \text{[vehicles]}
\]

\[
\psi = \alpha \cdot \gamma \cdot t_{fp}, \text{[veh/h]}
\]

\( f(-\theta_p) \) – decreasing function of charge \( \theta_p \), [minutes];

In Equation (7), the dependent variable is stochastic route travel time and the independent variables are stochastic link capacities and charge. It’s obvious from Equation (7) that tolls help to reduce the route travel time. This is because tolls lead to lower demand (Equation (5)).
and lower demand leads to lower flows and thus to travel times (Equation (7)). This relationship also can be shown graphically (FIGURE 4):

![Figure 4: relations between route travel time and toll](image)

**FIGURE 4: relations between route travel time and toll**

We are interested in the influences of tolls on the standard deviation of route travel time, viz the influences on the route unreliability. The variance of route travel time, from Equation (7), is related to the variance of link capacity and the charges. Since capacity is present in both the numerator and denominator of Equation (7), this relationship is not self-evident. The variance of the route travel time including road pricing is derived below:
\[ s_p = \sum x t_{px}^2 - \bar{t}_p^2 = \sum x \left( \frac{\xi + f(C_{1x}, ..., C_{Ax}) t_{fp} - \frac{\psi \theta_p}{VOT}}{f(C_{1x}, ..., C_{Ax}) + \psi} \right)^2 - \frac{\left( \sum x f(C_{1x}, ..., C_{Ax}) t_{fp} - \frac{\psi \theta_p}{VOT} \right)^2}{n} \]

\[
= \sum x \left( \frac{\xi - \frac{\psi \theta_p}{VOT} - \psi t_{fp}}{f(C_{1x}, ..., C_{Ax}) + \psi} \right)^2 - \frac{\left( \sum x \frac{\xi - \frac{\psi \theta_p}{VOT} - \psi t_{fp}}{f(C_{1x}, ..., C_{Ax}) + \psi} \right)^2}{n}
\]

\[
= \left( \xi - \frac{\psi \theta_p}{VOT} - \psi t_{fp} \right)^2 \left( \sum x \left( \frac{1}{f(C_{1x}, ..., C_{Ax}) + \psi} \right)^2 - \left( \frac{1}{n} \frac{1}{f(C_{1x}, ..., C_{Ax}) + \psi} \right)^2 \right)
\]

\[
= \phi f(-\theta_p^2), \quad \xi - \frac{\psi \theta_p}{VOT} - \psi t_{fp} > 0
\]

where,

\( s_p \) – variance of travel time on route \( p \) including road pricing, \([\text{min}^2]\)

\( n \) – number of observations of capacities;

\( \phi \) – constant for given observations of capacities, \([h^2/\text{veh}^2]\);

\( f(-\theta_p^2) \) – decreasing function of the square of charge on route \( p \), \([\text{veh}^2]\);

From the derivation of the variance of travel time (Equation(8)), the variance of travel time is a decreasing function of the square of charge on route \( p \), which means that charge on the route can decrease travel time variance. The travel time unreliability, i.e. standard deviation which equals: \( \sigma_p = \sqrt{s_p} \), therefore, is a linearly decreasing function of charge on route \( p \). The special case is when charge \( \theta \) equals zero, the standard deviation of travel time, from Equation(8), is surely larger than that with non-zero charge. A single route network with tolls is more reliable than the network without tolls, because with tolls less travelers make their trips, and less demand lead to shorter route travel time, thus to smaller travel time variability.

**FIGURE 5** shows graphically the derived findings:
This finding can be extended to a larger network, which consists of several OD pairs with each OD pair having several parallel routes (also with overlapping routes). But the precondition is that all the routes should be charged with the same level of tolls. With multiple routes, route shifts can happen due to the introduction of tolls. With the same charges for all the routes in the network, the route flows will surely be lower than that without tolls in the network. Lower route flows will lead to lower route travel time, lower route travel time will lead to lower standard deviations of route travel time, thus to lower unreliability. According to our network reliability definition, network travel time reliability will inevitably be improved after implementing road pricing.

### 3.1.2 Two routes network

In above subsection the impacts of tolls on a single route network with elastic demand and for the networks with all routes charged have been analyzed theoretically using simplified assumptions. This subsection will analyze the more complex networks with multiple routes where only selected routes are charged with elastic demand. For this type of situations, travel time reliabilities on the tolled routes are improved; however reliabilities on other routes may be worse due to traffic flow shifting. Therefore on the network level, the resulting reliability is uncertain after implementing road pricing. Below we are going to theoretically analyze this kind of situation with interrelations between routes using static models. We start with a network with one OD pair and two alternative routes between this OD pair (see **FIGURE 6**). Road pricing is implemented on one of these two routes. Note that there are no overlapping routes between this OD pair.

**FIGURE 5: relations between variance, standard deviation and charge**

\[
\text{Variance} \quad s_p \\
\text{\(\theta_p\)} \\
\]

\[
\text{s.d.} \quad \sigma_p \\
\text{\(\theta_p\)} \\
\]
Elastic OD demand, in this subsection, is calculated using the gravity model as a demand function, which in its general form states that the number of trips between an origin and a destination zone is proportional to three factors, which is mathematically summarized as follows:

$$ Q_{ij} = \mu A_i X_j F_{ij} $$  \hspace{1cm} (9)

where,
- $Q_{ij}$ - number of trips between OD pair $i,j$, [trips/h];
- $A_i$ - production ability of zone $i$;
- $X_j$ - attraction ability of zone $j$;
- $F_{ij}$ - accessibility of $j$ from $i$, depends on the generalized OD travel costs;
- $\mu$ - measure of average trip intensity in area.

After implementing road pricing on a route, we assume the production ability of zone $i$ and attraction ability of zone $j$ remain unchanged. The OD demand after implementing road pricing is only related to the OD travel costs. With tolls, the generalized travel cost will increase and the accessibility between this OD pair will become lower. It implies that the traffic demand between this OD pair will decrease. This is expressed as follows:

$$ Q'_{ij} = Q^0_{ij} \frac{F^0_{ij}}{F_{ij}} $$  \hspace{1cm} (10)

where,
- $Q^0_{ij}$ - base OD demand between origin $i$ and destination $j$, [trips/h];
- $Q'_{ij}$ - OD demand between origin $i$ and destination $j$ after implementing road pricing, [trips/h];
- $F^0_{ij}$ - accessibility of $j$ from $i$ without tolls;
- $F_{ij}$ - accessibility of $j$ from $i$ after implementing road pricing;

Therefore the OD demand after implementing road pricing depends on the accessibility, which is a function of the OD travel cost. Different mathematical forms of distribution functions $F$ have been proposed to express the accessibility, such as power function (Equation(11)), exponential function (Equation(12)), top-lognormal function (Equation(13)) and so on (Bovy and Bliemer 2003).
The exponential function is quite often applied in theoretical research. The top-lognormal function is used as an alternative of the lognormal function, especially for the unimodal model. In order to get a specific and accurate relationship between network reliability and tolls, deterministic user equilibrium and stochastic assignment techniques are applied respectively.

### 3.1.2.1 Deterministic User Equilibrium

Deterministic user-equilibrium assignment leads to a flow pattern in which no traveler can improve his travel time by unilaterally changing routes. (Wardrop 1952) showed that DUE conditions are equivalent to that the travel time on any used route must be equal to the travel time on any other used route between the same origin and destination and no greater than the travel time on any unused route. This principle is usually referred to as Wardrop’s first principle or simply Wardrop’s equilibrium. Using this assignment model, traffic flows will be assigned to the routes considering the travel time functions. Applying road pricing on route 1 and network reliability will be analyzed. Still the special case of BPR function is used as travel time function.

The generalized cost of tolled route 1 is:

\[
z_{1x} = t_f \left( 1 + \alpha \frac{q_l}{f_l(C_{1x},...,C_{Ax})} \right) + \frac{\theta_l}{VOT}
\]

where,

- \(z_{1x}\) — generalized costs of route 1 for observation x, [minutes];
- \(t_f\) — free flow travel time on route 1, [minutes];
- \(q_l\) — traffic flow on route 1, [veh/h];
- \(C_{1x}, C_{2x}, ..., C_{Ax}\) — link capacities for observation x, where links 1, 2, ..., A are on route 1, [veh/h];
- \(f_l(C_{1x},...,C_{Ax})\) — traffic supply condition on route 1, which is a function of capacities of all the links belonging to route 1, [veh/h];
- \(\theta_l\) — charge of route 1, which is the sum of the charges of all the links belonging to route 1, [euros];
VOT—value of time, [euros/h];

$\alpha$ — dimensionless slope parameter of travel time function;

Route flow $q$ is not a stochastic variable as noticed in (Minderhoud, Botma et al. 2000), because we made assumption that travelers don’t know the travel cost in advance.

Similarly the generalized cost of route 2 is:

$$z_{2x} = t_{f2} \left( 1 + \alpha \frac{q_2}{f_2(C_{1x}, \ldots, C_{Bx})} \right)$$  (15)

where,

$C_{1x}, C_{2x}, \ldots, C_{Bx}$ — link capacities for observation $x$, where links 1, 2, ..., B are on route 2, [veh/h];

$f_2(C_{1x}, \ldots, C_{Bx})$— traffic supply condition on route 2, which is a function of capacities of all the links belonging to route 2, [veh/h];

According to the DUE principle, the equilibrium is the state when the generalized costs of the two routes are equal, when the traffic flows on both routes are non-negative, as follows.

$$t_{f1} \left( 1 + \alpha \frac{q_1^*}{f_1(C_{1x}, \ldots, C_{Ax})} \right) + \frac{\theta_1}{VOT} = t_{f2} \left( 1 + \alpha \frac{q_2^*}{f_2(C_{1x}, \ldots, C_{Bx})} \right), \quad q_1^*, q_2^* \geq 0$$  (16)

where,

$q_1^*$ — equilibrium flow on route 1, [veh/h];

$q_2^*$ — equilibrium flow on route 2, [veh/h];

If one of the two routes is not used ($q_1^*$ or $q_2^*$ equals zero) due to the travel cost on this route is larger that that on the other route, the network is then equivalent to the single route network as we analyzed in subsection 3.1.1. We know that $q_2^*$ equals $(Q_{ij} - q_1^*)$, where $Q_{ij}^*$ is the elastic OD demand. Substitute $Q_{ij}$ and $q_2$ into function (16), we get route flow on route 1:

$$t_{f1} + t_{f1} \alpha \frac{q_1^*}{f_1(C_{1x}, \ldots, C_{Ax})} + \frac{\theta_1}{VOT} = t_{f2} + t_{f2} \alpha \frac{Q_{ij} - q_1^*}{f_2(C_{1x}, \ldots, C_{Bx})}$$

$$\Rightarrow q_1^*(\theta_1) = \frac{t_{f2} \alpha}{f_2(C_{1x}, \ldots, C_{Bx})} + t_{f2} \frac{Q_{ij} - q_1^*}{f_2(C_{1x}, \ldots, C_{Bx})}$$  (17)

where,
$\phi$ – dimensionless constant for given observations of link capacities on both routes,

\[ t_{f2} \phi \frac{1}{f_2(C_{1x},...,C_{8x})} = \frac{1}{t_{f1} \phi f_1(C_{1x},...,C_{8x}) + t_{f2} \phi f_2(C_{1x},...,C_{8x})}, \ \phi > 0; \]

$Q_j'(-\theta_i)$ – elastic OD demand which is a decreasing function of charge $\theta$, [trips/h];

$\pi$ – constant, equals $\frac{1}{\text{VOT}}$, $\pi > 0$, [veh/h/euro];

$\kappa$ – constant, equals $\frac{t_{f2} - t_{f1}}{t_{f1} \phi f_1(C_{1x},...,C_{8x}) + t_{f2} \phi f_2(C_{1x},...,C_{8x})}$, [veh/h];

It can be seen from equation (17) that route flow on tolled route 1 in the equilibrium state is a decreasing function of charge $\theta_i$ (which can be expressed as $q_1^* (-\theta)$), which indicates that route flow on tolled route will decrease after implementing road pricing and the higher the charge level is on the tolled route, the lower the traffic flow on the tolled route 1. From equation (17), flow on route 1 is influenced by two factors. One is due to the decreased OD demand, which is expressed as the first part of equation (17), the other is due to the route flow shifts to the non-charged route 2, which is expressed as the second part of the equation (17). While traffic flow on the untolled route 2 $q_2^*$ may increase or decrease due to decrease OD demand and route shifts. Mathematically, $q_2^*$ equals $(Q_j' (-\theta) - q_1^* (-\theta))$, which is not self-evident with relation to charge $\theta$ from Equation (18):

\[ q_2^* = Q_j' (-\theta) - (\phi Q_j' (-\theta) - \pi \theta_i + \kappa) \]

\[ = (1 - \phi) Q_j' (-\theta) + \pi \theta_i - \kappa \] (18)

In equation (18), the first part denotes the decreased OD demand and the second part denotes the route flow increase due to the route shifts. Route flow on non-charged route 2, depending on the charge level, may increase or decrease. If the effects of decreased OD demand are stronger than the effects of route shifts, route flow on non-charged route 2 will decrease, vice versa.

The route travel times at equilibrium state are derived:

\[ t_i = t_{f1} \left( 1 + \alpha \frac{q_1^* (-\theta)}{f_1(C_{1x},...,C_{8x})} \right) \] (19)
From Equation (19), at the equilibrium state, the travel time on the tolled route 1 is a decreasing function of charge $\theta$ since $q_1^*$ is a decreasing function of charge $\theta$. With tolls on the route, the travel time will decrease and the higher the charge is, the shorter the travel time on the tolled route is, until reaching the free flow travel time. This is reasonable that travelers pay extra money to save travel times. While the change of travel time on untolled route 2 (Equation (20)) is not evident with the relation to charge $\theta$. If decreased demand has stronger influence than route shifts have, the travel time on the untolled route 2 will decrease, vice versa.

Subsequently, the variances of travel times on each route and on the network level are derived as follows:

**Travel time variance on charged route 1:**

\[
s_1 = \sum_x \left( t_{f_1} \left( 1 + \alpha \frac{q_1^*(-\theta_i)}{f_1(C_{1x}, \ldots, C_{Ax})} \right) - \frac{\sum_x t_{f_1} \left( 1 + \alpha \frac{q_1^*(-\theta_i)}{f_1(C_{1x}, \ldots, C_{Ax})} \right)}{n} \right)^2
\]

\[
= \sum_x t_{f_1} \alpha q_1^*(-\theta_i) \left( \frac{1}{f_1(C_{1x}, \ldots, C_{Ax})} - \frac{\sum_x \frac{1}{f_1(C_{1x}, \ldots, C_{Ax})}}{n} \right)^2
\]

\[
= (t_{f_1} \alpha q_1^*(-\theta_i))^2 \sum_x \left( \frac{1}{f_1(C_{1x}, \ldots, C_{Ax})} - \frac{\sum_x \frac{1}{f_1(C_{1x}, \ldots, C_{Ax})}}{n} \right)^2
\]

\[
= \eta_1 q_1^*(-\theta_i)
\]

\[
= \eta_1 (\varphi Q_{\theta}(-\theta_i) - \pi \theta_i + \kappa)^2
\]

where,

\[s_1 — variance of travel time on route 1, \text{[min}^2\text{]}\]
Influence of Road Pricing on Network Reliability

The influence of road pricing on network reliability can be quantified through the following equation:

\[ t_f^2 \alpha^2 \sum_{x} \left( \frac{1}{f_i(C_{1x},...,C_{dx})} \sum_{x} \frac{1}{f_i(C_{1x},...,C_{dx})} \frac{n}{n} \right)^2 \text{[h/veh^2]} \]

The part \[ \sum_{x} \left( \frac{1}{f_i(C_{1x},...,C_{dx})} \sum_{x} \frac{1}{f_i(C_{1x},...,C_{dx})} \frac{n}{n} \right)^2 \] is a constant value since for each link the capacity follows a specific normal distribution with its own fixed mean capacity and standard deviation.

\( Q_{ij} \), depending on which distribution function is chosen, may be a power function or an exponential function of charge.

**TABLE 3** illustrates the derivative of the variance of travel time on charged route 1 with respect to charge \( \theta \). From the derivation, the derivative of \( s_1 \) is negative, which means that the variance of travel time on charged route 1 is a decreasing function of charge \( \theta \). The travel time on the charged route will become more reliable with tolls.

**TABLE 3: derivative of the variance of travel time on charged route 1**

\[
\frac{\partial s_1}{\partial \theta} = 2\eta q_i(-\theta) \frac{\partial q_i}{\partial \theta}
\]

where \( \eta > 0 \), since \( q_i(-\theta) > 0 \), and \( \frac{\partial q_i}{\partial \theta} < 0 \), then \( \frac{\partial s_1}{\partial \theta} < 0 \).

The improvement of travel time reliability on charged route 1 attributes to decreased demand and route flow shifts, as seen from equation (21).

**Travel time variance on non-charged on route 2:**

Similarly as above we derive:
Influence of Road Pricing on Network Reliability

\[ s_2 = (t_{f_2} \alpha q^*_2)^2 \sum_x \left( \frac{1}{f_2(C_{1x}, \ldots, C_{Rx})} \sum_x \frac{1}{n} \right)^2 = \eta_2 q^*_2 \]

where,

\[ \eta_2 \text{-- constant for given observations of link capacities on route 2, } [\text{h}^4/\text{veh}^2]; \]

The variance of travel time on non-charged route 2 is a complex function of charge (Equation(22)). Depending on the combined effects of decreased OD demand and route flow shifts, the variance of travel time on non-charged route 2 may decrease or increase, depending on the charge level.

Substituting equilibrium flow solution Equation(17) and(18), network travel time reliability (Equation(3)) is derived as shown in formula(23):

\[
\begin{align*}
    r &= \frac{q^* \sigma_1 + q^* \sigma_2}{Q_{Q}(-\theta_1)} = \frac{q^* \sqrt{s_1} + q^* \sqrt{s_2}}{Q_{Q}(-\theta_1)} \\
    &= \sqrt{\eta_1 q^* ((1 - \varphi) Q'_{Q}(-\theta_1) - \pi \theta_1 + \kappa)} + \sqrt{\eta_2 q^* ((1 - \varphi) Q'_{Q}(-\theta_1) + \pi \theta_1 - \kappa)} \frac{Q_{Q}(-\theta_1)}{Q_{Q}(-\theta_1)} \\
    &= \frac{\sqrt{\eta_1 q^* ((1 - \varphi) Q'_{Q}(-\theta_1) - \pi \theta_1 + \kappa)} + \sqrt{\eta_2 q^* ((1 - \varphi) Q'_{Q}(-\theta_1) + \pi \theta_1 - \kappa)}}{Q_{Q}(-\theta_1)}
\end{align*}
\]

From Equation(23), network travel time unreliability is somehow a complex function of the charge \( \theta \) such that different charge level \( \theta \) will lead to different levels of network unreliability. Decreased OD demand and route flow shifts are the influencing factors of network travel time variability. If the impacts of decreased OD demand are more significant than the impacts of route flow shifts, the network unreliability will be reduced by implementing tolls (see equation(23)). Therefore with appropriate charge level, when the decreased OD demand has stronger influence, network reliability can be improved. Network travel time unreliability depends heavily on the charge levels.

FIGURE 7 shows graphically the equation(23), illustrating the network unreliability with respect to charge level for this two-route network.
It can be seen from the FIGURE 7 that network reliability can be improved with tolls when the charge levels are low, when the decreased OD demand has stronger influences. An optimal charge level can be determined, with which the network unreliability is minimized.

3.1.2.2 Simple stochastic assignment

Stochastic methods of traffic assignment emphasize the variability in drivers' perceptions of costs and the composite measure they seek to minimize (generalized costs). It is assumed that the value of the generalized travel time that a traveler attaches to a route, known as the perceived or subjective generalized travel time, follows a probability distribution. In this assignment all travelers choose their perceived shortest path from the origin to the destination. The multinomial logit model is used to perform the traffic assignment on the same network. The proportion of traffic flows on each route can be calculated as follows:

\[
\beta_1^* = \frac{\exp(-\gamma z_1^*)}{\exp(-\gamma z_1^*) + \exp(-\gamma z_2^*)} \\
\beta_2^* = 1 - \beta_1^*
\]

(24)

where,

\( \beta_1^* \) — equilibrium traffic flow proportion on charged route 1, decreasing function of charge \( \theta_1 \), can be expressed as \( \beta_1^* (-\theta) \);
$\beta_2^* =$ equilibrium traffic flow proportion on non-charged route 2, increasing function of charge $\theta_1$, can be expressed as $\beta_2^* (+ \theta_1)$;

$\gamma =$ travel cost sensitivity parameter;

$z_1^*$ = equilibrium generalized cost of charged route 1, [minutes];

$z_2^*$ = equilibrium generalized cost of non-charged route 2, [minutes];

Therefore equilibrium traffic flows on both routes are:

$$q_1^* = Q_{ij}^* \beta_1^* = Q_{ij}^* \frac{\exp(-\gamma z_1^*)}{\exp(-\gamma z_1^*) + \exp(-\gamma z_2^*)}$$ (25)

$$q_2^* = Q_{ij}^* \beta_2^* = Q_{ij}^* \frac{\exp(-\gamma z_2^*)}{\exp(-\gamma z_1^*) + \exp(-\gamma z_2^*)}$$ (26)

Substitute Equation (25) and Equation (26) into Equation (23), the network unreliability is derived:

$$r = \sqrt{\frac{\eta_1 Q_{ij}^* \theta_1 \left( \frac{\exp(-\gamma z_1^*)}{\exp(-\gamma z_1^*) + \exp(-\gamma z_2^*)} \right)^2 + \sqrt{\eta_2 Q_{ij}^* \theta_1 \left( \frac{\exp(-\gamma z_2^*)}{\exp(-\gamma z_1^*) + \exp(-\gamma z_2^*)} \right)^2}}}{Q_{ij}^*}$$

Equation (27) is composed of two parts (see TABLE 4), the first part is from the contribution of travel time unreliability of charged route 1, which is decreasing with the increase of charge level (see TABLE 4). Travel time on charged route will become more reliable with tolls. The second part is from the contribution of travel time unreliability of non-charged route 2, which would decrease or increase depending on the charge level (see TABLE 4). Depending on the combination of the impacts of decreased OD demand and route flow shifts, network unreliability can either decrease when decreased OD demand exerts stronger influence or increase when route flow shifts have stronger impacts.
TABLE 4: explanation of equation (27)

<table>
<thead>
<tr>
<th>First part: contribution from charged route 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sqrt{\eta} Q_{ij}(-\theta) \beta_1^{+^2}(-\theta)$</td>
</tr>
<tr>
<td>$Q_{ij}(-\theta)$ is a decreasing function of charge $\theta$,</td>
</tr>
<tr>
<td>$Q_{ij}(-\theta) &gt; 0$,</td>
</tr>
<tr>
<td>$\frac{\partial \beta_1^{+^2}(-\theta)}{\partial \theta} = 2 \frac{\partial \beta_1(-\theta)}{\partial \theta} &lt; 0 \Rightarrow \beta_1^{+^2}(-\theta)$ is a decreasing function of charge $\theta$,</td>
</tr>
<tr>
<td>$\beta_1(-\theta) &gt; 0$,</td>
</tr>
<tr>
<td>thus $\sqrt{\eta} Q_{ij}(-\theta) \beta_1^{+^2}(-\theta)$ is a decreasing function of charge $\theta$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Second part: contribution from non-charged route 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sqrt{n_2} Q_{ij}(-\theta) \beta_2^{+^2}(+\theta)$</td>
</tr>
<tr>
<td>$Q_{ij}(-\theta)$ is a decreasing function of charge $\theta$,</td>
</tr>
<tr>
<td>$\beta_2^{+^2}(+\theta)$ is a increasing function of charge $\theta$,</td>
</tr>
<tr>
<td>thus $\sqrt{n_2} Q_{ij}(-\theta) \beta_2^{+^2}(+\theta)$ may decrease or increase depending on the influence of decreased OD demand and route flow shifts</td>
</tr>
</tbody>
</table>

In conclusion, travel time reliability on tolled route will surely be improved, which has been proven (see TABLE 4, the first part). The network unreliability is depending on the influence of decreased OD demand and route flow shifts, and is heavily counting on the charge level.

If $\gamma$ is infinite, then the stochastic assignment becomes an all-or-nothing assignment. If $\gamma$ decreases, which means the stochastic degree is higher, with the same charge level, network reliability will change differently, depending on the charge level.

If the alternative routes have the same perceived cost, the proportion of traffic flows on each route equals $\frac{1}{m_{ij}}$. Then network reliability could be derived as:

$$ r = \frac{Q_{ij}}{m_{ij}} \sum_{p \in P} t_{pj} f_p(C_{1x}, \ldots, C_{Ax}) $$

(28)

where,

$m_{ij}$—number of alternative routes from $i$ to $j$;

From Equation(28), $Q_{ij}$ is a decreasing function of charge and thus network unreliability $r$ is also a decreasing function of charge. Charges help to decrease network travel time unreliability, and the higher the charge level is, the more reliable the network is.
All above theoretical analyses come to the consistent findings that road pricing will surely improve travel time reliability on the charged route (see equation(21), TABLE 3 and TABLE 4). Network travel time reliability may decrease or increase, depending on the combined impacts of decreased OD demand and route flow shifts. With appropriate charge level, network reliability can be improved.

### 3.1.3 General network

Using above analyses, for simple networks, we found that network reliability is heavily dependent on the charge level. It's natural to think that for more complex networks, it might be similar that road pricing can improve network reliability, but there may be no absolutely decreasing relationship between the network unreliability and the pricing. Therefore we should try to find the optimal charges that minimize the network unreliability. It is the purpose of this subsection to derive the more general relationships between network reliability and the charge level for more general networks. Stochastic capacities are taken as the causal factor to the travel time variability. We assume travelers don’t know the travel cost in advance, thus the route choices are based on the average travel cost (see assumption 3). Route flows and link flows therefore, are not stochastic variables, which are calculated with the mean capacities of all the links in the network.

Firstly, the so-called BPR function is used to calculate the link travel time instead of the special BPR function which is a linear function of saturation ratio $q/C$ as we used before:

$$t_{ax}(\theta) = t_{fa} + a \alpha \left( \frac{q_a^\star(\theta)}{C_{ax}} \right)^\beta$$

(29)

where,

- $t_{ax}(\theta)$—stochastic link travel time, [minutes];
- $t_{fa}$—free flow travel time on link a, [minutes];
- $q_a^\star(\theta)$—link flow, not stochastic, [veh/h];
- $C_{ax}$—link capacity, [veh/h];
- $\alpha, \beta$—slope parameters;

In the more complex network, when overlapping routes exist, link flows are no longer equal to route flows, but equals the sum of route flows traversing the link:

$$q_a^\star(\theta) = \sum_{p, a \in p} q_p^\star(\theta)$$

(30)

where,
$q_p^*(\theta)$—equilibrium route flow on any route $p$, which includes link $a$, not stochastic, [veh/h];

According to assumption 2 that the capacities of all links follow normal distributions, which can be transformed into mathematical expressions:

$$C_{ax} = f_a \times d_x$$  \hspace{1cm} (31)

where,

$C_{ax}$—stochastic link capacity, with mean $\overline{C_a}$, [veh/h];

$d_x$—observation $x$ of a normal distribution of capacities with mean $\overline{d}$ and standard deviation $\sigma_d$, the standard normal distribution is expressed as $d \sim N(\overline{d}, \sigma_d^2)$, [veh/h];

$f_a$—constant for each link, equal to $\frac{C_a}{\overline{d}}$;

Therefore link capacity $C_a \sim N(\overline{C_a}, \frac{C_a^2}{\overline{d}^2} \sigma_d^2)$;

The route travel time is the sum of link travel times $T_a$ belonging to route $p$:

$$t_{px}(\theta) = \sum_{a \in p} t_{ax}(\theta)$$

$$= \sum_{a \in p} t_{fa} \left( 1 + \alpha \left( \frac{q_p^*(\theta)}{f_a d_x} \right)^\theta \right)$$  \hspace{1cm} (32)

where,

$t_{px}(\theta)$ — stochastic route travel time, [minutes];

The procedure of deriving route travel time variance is illustrated as follows:
The route travel time variance, which equals the sum of link travel time variance of composing links, is given in formula (34):

\[
 s_p = \left( \sum_{a \in p} t_{fa} \cdot \alpha \cdot \left( \frac{q_a^*(\theta)}{f_a} \right)^\beta \right)^2 + \sum_{x} \left( \frac{1}{d_x^\beta} - \frac{\sum_{x} \frac{1}{d_x^\beta}}{n} \right)^2
\]  

(34)

It can be seen from formula (34) that the variance of route travel time is related to the link free flow travel times, equilibrium link flows, capacity distributions and of course charge levels.

In the equation, \( \sum_{x} \left( \frac{1}{d_x^\beta} - \frac{\sum_{x} \frac{1}{d_x^\beta}}{n} \right)^2 \) is a constant value for a given standard distribution of link capacity, which is the variance of the distribution \( \frac{1}{d^\beta} \).

Let \( s\left( \frac{1}{d^\beta} \right) \) represent the variance of the distribution \( \frac{1}{d^\beta} \),

\[
s\left( \frac{1}{d^\beta} \right) = \sum_{x} \left( \frac{1}{d_x^\beta} - \frac{\sum_{x} \frac{1}{d_x^\beta}}{n} \right)^2
\]  

(35)
For each specific link, \( t_{fa} \cdot \alpha \cdot \left( \frac{q_{a}^*(\theta)}{f_a} \right)^\beta \) is a constant value for each charge level, because \( q_{a}^*(\theta) \) is deterministic for a charge level based on assumption 3, therefore use \( Y_a \) [minutes] to represent this link specific value,

\[
Y_a(\theta) = t_{fa} \cdot \alpha \cdot \left( \frac{q_{a}^*(\theta)}{f_a} \right)^\beta
\]  

(36)

Then the route travel time variance can be simplified as:

\[
s_p = \left( \sum_{a \in p} Y_a(\theta) \right)^2 \cdot \frac{1}{d^{\beta}}
\]  

(37)

With different toll levels, \( Y_a \) is different. With assumption 3 that travelers don’t know the travel cost in advance, equation(37) can be directly used to calculate route travel time unreliability which is induced by stochastic link capacity, knowing link free flow travel times, equilibrium link flows and capacity distribution of composing links. This equation is applied in the following example.

### 3.2 Application example using static DUE simulations

The analyses of above sections are theoretical and abstract. In this section, an example is presented to investigate the influences of road pricing on network travel time reliability, in which the derivation of route travel time unreliability (Equation(37))) from subsection 3.1.3 is utilized to calculate route travel time unreliability. A five-link three-route test network with overlapping for a single OD-pair with elastic demand is considered. The link numbers and travel time functions are shown in FIGURE 8:

![FIGURE 8: example network](image-url)
Three alternative routes are available from O to D, of which the numbers are illustrated in FIGURE 9. In this network, road pricing is implemented on link 5, which is the most congested link before implementing road pricing.

![Figure 9: route numbers in the five-link network](image)

From FIGURE 9, route 1 and route 3 experience a charge, and route 2 is toll free route. TABLE 5 gives the link free flow travel times and route free flow travel times. It can be seen that free flow travel times on all the three alternative routes are equal to 9 minutes. Without flows spreading on the routes, all the three routes are equally attractive to travelers.

**TABLE 5: free flow travel times of composing links and routes**

<table>
<thead>
<tr>
<th>Link 1</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Link 2</td>
<td>6</td>
</tr>
<tr>
<td>Link 3</td>
<td>2</td>
</tr>
<tr>
<td>Link 4</td>
<td>5</td>
</tr>
<tr>
<td>Link 5</td>
<td>3</td>
</tr>
</tbody>
</table>

(a): free flow travel times of links

<table>
<thead>
<tr>
<th>Route 1</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Route 2</td>
<td>9</td>
</tr>
<tr>
<td>Route 3</td>
<td>9</td>
</tr>
</tbody>
</table>

(b): free flow travel time of routes

The base OD demand $Q^0$ equals 100. Elastic demand is calculated before and after implementing road pricing. The power function (see equation (11)) with a value of parameter $\alpha$ equal 0.3 is employed to calculate the accessibility from origin to destination. Equation (10) is used to calculate the elastic OD demand for each charge level. The charge is ranging from 0 to 4 euros with a step of 0.5 euros.

In this network, the capacities of the 5 links follow the same normal distribution. According to Equation (37), $s\left(-\frac{1}{d^\alpha}\right)$ is a constant for all the three routes. In order to calculate the route travel time unreliability, only equilibrium link flows need to be obtained. A DUE assignment technique was used and MSA was utilized to perform the traffic flow assignment through the network.
The generalized travel cost from O to D is calculated using the flow weighted route travel costs. All the simulations are performed in Matlab, the program is given in Appendix I.

Results from simulations
The result of the relationship between network travel time unreliability and tolls from the Matlab simulations is illustrated in FIGURE 10:

From FIGURE 10, network unreliability decreases within the charge range of 0-0.1 euro. With charges higher than 0.1 euro, the network unreliability will increase with the rise of toll. For this test network, with the assumptions made in theoretical analyses, an optimal charge of 0.1 euro is determined, with which the network unreliability is minimized. From previous theoretical analyses, decreased OD demand and route flow shifts are the influencing factors of travel time variability. In the charge range of 0-0.1 euro, decreased OD demand has stronger influence on the travel time variability than route flows shifts do. After toll levels higher than 0.1, route shifts have stronger impacts on the network performance. Tolls indeed can reduce network unreliability and different charge level leads to different network reliability. An optimal charge can be obtained with which the network unreliability is minimized.
With charge levels higher than 3.2 euros, the network unreliability keeps constant, because there are no flows on the charged routes 1 and 3 any more, which can be seen from FIGURE 11. Tolls don’t have influence on the generalized OD cost and the OD demand keeps constant without decreasing. All travelers choose non-charged route 2 and there are route shifts between routes. Thus network travel time reliability keeps constant after charge level higher than 3.2 euros. Since DUE assignment was performed, this situation occurs that there are no travelers any more on the tolled routes when the charge is quite high. In reality, this will not be the case and stochastic assignment will give more realistic results.

OD demand will decrease with the rise of tolls, which can be seen from FIGURE 11. With increase of charge levels, no travelers choose route 1, which has more than 67% overlapping. Many travelers shift to the non-charged route 2, which can be seen from FIGURE 11 that the traffic flow on route 2 is increasing and the traffic flow on route 3 is decreasing with the rise of charge levels.

Due to route flow shifts, the travel time on the charged route 3 become shorter with tolls (see FIGURE 12). Travelers pay the tolls to save their travel times. Accordingly, travel time on the non-charged route 2 will increase due to more and more traffic take this route to avoid the tolls (see FIGURE 12). Although there are no flows on route 1, the travel time on route 1 still increase with the rise of charge levels (see FIGURE 12). Because route 1 has overlapping link (link 1) with route 2, the travel time on route 1 equals the sum of the travel times on link
1, 3 and 5. The flow increase on route 2 leads to the flow increase on link 1, flow increase on link 1 leads to travel time increase on link 1, thus to travel time increase on route 1. With charge levels higher than 3.2 euros, the travel time on charged route 3 equals to free flow travel time, which is 9 minutes since no flows on this route (see FIGURE 12). Travel times on route 1 and route 2 keep constant with the rise of charges because the flow on route 2 keeps constant.

FIGURE 12 route travel times

FIGURE 13 shows route travel costs (sum of route travel time and tolls) of the three routes with respect to charge levels. It is noticed within the charge range of 0-0.2 euro, the travel costs on all the three routes are the same. All the three routes are used by travelers, which is consistent with the DUE principle that all the used routes have equal travel costs. After the charge levels higher than 0.2, the used route 2 and used route 3 have equal travel costs. The unused route 1 always has larger travel cost than that of the used routes, which implies that DUE equilibrium is reached with the simulations. Route 1 and 3 experience the tolls, the travel costs on these two routes always increase with the rise of charge levels, although the travel times on these two routes are constant after the charge levels higher than 3.2 euros. The travel cost on non-charged route 2 equals the travel time on route 2.
From this application, OD demand will decrease and route shifts will happen after implementing road pricing on link 5. Decreased OD demand and route flow shifts cause network travel time variability. When the decreased OD demand has stronger influence on the network performance, network reliability can be improved. Based on our assumptions (see assumption 1, 2 and 3), tolls can help to improve network travel time reliability (see FIGURE 10) on this single OD pair network. In this application, the power function was chosen to calculate the accessibility for the OD pair. If different functions are chosen, elastic OD demand and traffic flow pattern will be different. Different optimal charge levels will be determined with different distribution functions.

In conclusion, tolls help to decrease the OD demand, which has positive impacts on network reliability. Route shifts plays an important role in network travel time variability. Depending on the combined effects of these two factors, network reliability may decrease or increase, depending on the charge levels. When decreased OD demand has stronger influence, network reliability will be improved.
4. Simulation-based analyses using dynamic flow approach

Chapter 3 performed theoretical analyses of the impacts of road pricing on network travel time unreliability using static models and a simulated application was presented. From theoretical analyses, OD demand and route flow shifts are the two factors influencing network travel time variability. Depending on the combined impacts of these two factors, network reliability may either be improved or become worse, depending on the charge levels. The application proved our proposition that road pricing can improve network travel time reliability (see FIGURE 10) when decreased OD demand has stronger effects. Aiming to maximize network reliability, an optimal charge level can be determined accordingly.

In the theoretical analyses, static models are utilized and three assumptions are made. We assumed that travelers make their choices based on the average travel cost. Travel times, link capacities are stochastic variables, while flows are deterministic for each charge level, which is obtained with the mean capacities of all the links in the network. The travel time variability is only caused by link capacities variations, and doesn’t influence route choices. In order to model the traffic pattern more appropriately, in this chapter, a simulation-based approach using dynamic models will be utilized on the five-link network which is the same network as in previous application in Section 3.2, to analyze the impacts of road pricing on network travel time unreliability and to find out if the results from the simulation-based approach using dynamic models are consistent with the findings from the static theoretical analyses.

In the simulation-based analyses, travel times, link capacities and traffic flows are all stochastic variables and interrelated with each other.

Assumption 4: Random fluctuations of link capacity or OD demand lead to random shifts in route choices.

Furthermore, not only capacity variations, but also short-term origin-destination demand fluctuations are considered as a factor affecting travel time unreliability. Network reliability will be analyzed with link capacity variations and OD demand fluctuations separately. The analyses are carried out for fixed OD demand case and elastic OD demand case in order to investigate the influences of OD demand on network reliability. In total four cases are analyzed using simulation-based approach and the scheme of the analyses are illustrated in FIGURE 14.
Section 4.1 introduces the methodology of performing dynamic simulation-based analyses. The DTA model INDY is introduced and the causal factors of network unreliability in case of dynamic networks are indicated. The methods adopted to deal with the random capacity and OD demand fluctuations and to calculate elastic demand are depicted in detail, since different methods are used than in theoretical analyses. The objective function from road authority’s view to optimize charge level is defined in this section as well. Section 4.2 presents the results from the simulations with fixed OD demand, only the route choices are considered. Section 4.3 presents the results considering elastic demand, trip choices and route choices are considered jointly. The comparison of the influences of capacity variations and demand fluctuations is performed in Section 4.4.

4.1 Traffic pattern prediction methodology

4.1.1 Introduction of INDY
The DTA (Dynamic Traffic Assignment) model INDY is used to perform the simulations. The INDY model is developed by Delft University of Technology, TNO-FEL, and TNO Inro. The dynamic INDY model predicts congestion and queues correctly, therefore predicts route choice and route travel time more accurately. INDY is capable of adding the road pricing system into the network, taking the pricing as a part of the travel cost. The utility function used in the assignment consists of two parts, the travel time and tolls.

The travel time, in INDY, is calculated according to the speed-density function. The link travel time is a quotient of link length and speed, and speed can be derived from density according to the speed-density function. The speed-density function used in INDY model is an adapted Smulders function (Bliemer 2004), which is shown in FIGURE 15:
Capacity is not used directly in the travel time function. In FIGURE 15, the critical density is determined by the capacity and critical speed (see TABLE 6). When we randomize link capacity in the input of INDY, the critical density will change accordingly, as shown in red arrows in FIGURE 15. Then different speed-density relationships will be determined with respect to randomized link capacities. Travel times will change accordingly.

**TABLE 6: relationship of capacity with speed-density function**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Formula</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Critical density: $k^{crit}$</td>
<td>$k^{crit} = \frac{C_a}{v^{crit}}$</td>
<td>$k^{crit}$: critical density, [veh/km]; $C_a$: link inflow/outflow capacity, [veh/h]; $v^{crit}$: critical speed, [km/h];</td>
</tr>
<tr>
<td>Link travel time: $t_a$</td>
<td>$t_a = \frac{l_a}{v}$</td>
<td>$t_a$: link travel time, [minutes]; $l_a$: link length, [km]; $v$: travel speed, determined from speed-density function, [km/h];</td>
</tr>
</tbody>
</table>

The capacity we randomized in the dynamic models is different from the capacity used in the static models. In INDY, inflow and outflow capacity are used when performing the traffic assignment and dealing with the queues on the road. In the version used in this study, inflow and outflow capacity are equal. We assume travel time variability is induced by capacity variations, which means inflow/outflow capacity variations.
A logit-based stochastic assignment technique was utilized. For the five-link network, the modeled period takes one hour and for each 5 minutes, a travel demand is given. Therefore 12 OD matrices (see TABLE 7) are given as input in INDY to perform the simulations.

**TABLE 7: OD matrix**

<table>
<thead>
<tr>
<th>Time period (minutes)</th>
<th>OD demand from O to D (veh/h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5000</td>
</tr>
<tr>
<td>6</td>
<td>5000</td>
</tr>
<tr>
<td>11</td>
<td>5000</td>
</tr>
<tr>
<td>16</td>
<td>5000</td>
</tr>
<tr>
<td>21</td>
<td>5000</td>
</tr>
<tr>
<td>26</td>
<td>5000</td>
</tr>
<tr>
<td>31</td>
<td>5000</td>
</tr>
<tr>
<td>36</td>
<td>5000</td>
</tr>
<tr>
<td>41</td>
<td>5000</td>
</tr>
<tr>
<td>46</td>
<td>5000</td>
</tr>
<tr>
<td>51</td>
<td>5000</td>
</tr>
<tr>
<td>56</td>
<td>5000</td>
</tr>
</tbody>
</table>

The outputs from INDY are stored aggregately every 5 minutes. The average travel time for one hour period is calculated using flow weighted travel time every 5 minutes. The calculations of travel time reliability and the analyses of the results take place in Matlab (see example program in Appendix II).

In this study, INDY is used as a plugin in OmniTRANS and all the simulations are performed in OmniTRANS, which has a friendly interface and can dynamically show the traffic conditions through the network in video. Jobs need to be created in OmniTRANS to give orders for performing certain commands and all the jobs are written in Ruby language.

### 4.1.2 Causal factors of network travel time unreliability

The sources leading to network travel time variability, and thus unreliability are manifold, such as capacity variations, demand fluctuations, traffic signal control settings, etc. In this study, capacity variations and short-term origin-destination demand fluctuations are taken into account as the most important factors affecting travel time unreliability. Here, the short-term OD demand is defined in the context of studying day to day or hour-to-hour (real time) fluctuations in demand against the long-term demand in which the temporal demand variations occur over a period, say months or years.

The causal diagram in the model is illustrated in FIGURE 16 to explain the how does network travel time unreliability occur.
Link inflow/outflow capacity and OD demand vary between days. According to FIGURE 16, Link capacity variations lead to link travel time variations, link travel time variations lead to route travel time variations, route travel time variations lead to route flow fluctuations and then to link flow variations. This iterative process is performed in INDY and stochastic link capacities lead to route travel time variability, thus route travel time reliability can be calculated from the output of INDY. According to our definition of network unreliability, for each toll levels with randomized link capacity, network unreliability can be calculated. With OD demand fluctuations, OD demand fluctuations directly influence route flows, then link flows, link travel times and route travel times. Tolls influence the elastic demand, which directly affects the route flows. In order to analyze the network unreliability, the task we have to fulfill is to model the link capacities and OD demand variations in the input of INDY instead of using fixed capacities and fixed OD demand as usually is assumed.

4.1.3 Modeling capacity variation:
The inflow/outflow capacities, when performing dynamic simulations, are assumed normally distributed as aforementioned when doing theoretical analyses (see assumption 3), and are independent from each other. The standard deviation, based on statistical observations (AVV 1999), is taken 10% of the mean capacity.
In order to model the normal distributions of link capacities, two methods are utilized. One is to generate enough random values of capacities (Monte Carlo method) for every link subject to a normal distribution with a standard deviation of 10% of the mean capacity. The other method is using Hermite polynomials to approximate the integral of a normal distribution, choosing several points with their weights from the normal distribution to represent the whole distribution. The first method needs a large sample of stochastic capacities to get more accurate representations of the capacity distributions and to get more accurate results, which are the results of the stochastic capacity. In order to investigate the accuracy of the results from the first method, Hermite polynomial approximation method is executed, which is statistically more correct, and efficient (less simulations are needed) when dealing with the OD demand fluctuations.

**Monte Carlo method**

For a specific link, the mean capacity \( \overline{C}_a \) is given and fixed. A random series of link capacities subject to a normal distribution for all the links are generated. In this study, 100 random values for each link are generated independently. The stochastic link capacity \( C_a \) is expressed as in formula (38):

\[
C_a \sim N(\overline{C}_a, (10\% \overline{C}_a)^2) \\
C_{ax} = \overline{C}_a + 10\% \overline{C}_a \varepsilon_x \\
\varepsilon \sim N(0,1)
\]

where,

- \( C_a \) - random link capacity, [veh/h];
- \( \overline{C}_a \) - mean value of link capacity, [veh/h];
- \( \varepsilon \) - random values following a standard normal distribution with mean zero and standard deviation of 1;

For each link, hundred \( \varepsilon \) values are generated in Matlab subject to a standard normal distribution. For this five-link network, 500 random values are generated in total. For different charge levels, the same random values are applied for each link. The capacities of connector links remain unchanged.

Jobs (see examples in Appendix III) are created in the OmniTRANS to deal with the link capacity variations and charge level variations. Several Ruby scripts are built to update the database of the link capacities after each simulation and the database of the tolls as well. The
framework of the iterative process of the program with capacity variations is illustrated in FIGURE 17:

From the scheme, tolls, mean capacities and base demand are needed as input in INDY to calculate the travel cost and route flows, based on which the elastic demand for each toll level is calculated. Then with the elastic demand, we randomize link capacities for 100 times for each charge level. 100 different route travel times and route flows will be obtained from the output of INDY. Based on the 100 route travel times and route flows, route travel time reliability and network reliability can be calculated for each charge level.
The elastic demand is calculated using price elasticity of demand, which is defined as the percentage change in demand per unit percentage change in cost. As literature (Akiyama and Noiri 2000; Cain, Burris et al. 2001) suggested, elasticity of -0.2 is used in this study. Herein the cost denotes the OD-trip generalized cost and demand means OD demand. From FIGURE 17, with each charge level, the constant link capacities are input and the trip generalized costs are calculated based on the output from INDY simulations. Using the elasticity of -0.2 and based on the demand without tolls, the elastic demand for each charge level can be derived.

The tolls are ranging from 0 to 8 euros with an increasing step of 0.5 euros. For the toll range of 0-0.5 euros, a step of 0.1 euro is used. Therefore a total of 2100 (21×100) dynamic equilibrium simulations need to be performed when modeling capacity variations with different charge levels.

**Hermite polynomial approximation method**

Hermite polynomial approximation method (see Appendix III) is used to approximate the average value of a function of which the independent variables follow normal distributions. In this study, the function denotes travel time function and the independent variables denote link capacities or OD demand. Using this method, a total of 5103 (21×243) simulations need to be performed, which are more than the simulations using the first method because for each charge level 243 (3^5, see Appendix IV) simulations need to be performed. Using Hermite polynomial approximation method, the more links are included in the network, the more simulations are needed.

4.1.4 Modeling demand fluctuation:

Day-to-day demand fluctuation is another influencing factor of travel time unreliability. Based on the analysis of data obtained from the Regiolab-Delft project, (Tu, Van Lint et al. 2005) found that the day-to-day demand fluctuation follows a normal distribution as well, of which the standard deviation is on average 20% of the mean demand.

**Assumption 5:** Short-term origin-destination demand follows a normal distribution with standard deviation of 20% of the mean demand.

This is expressed in formula (39):

\[ Q_y \sim N(\bar{Q}_y, (20\% \bar{Q}_y)^2) \]
\[ Q_{ij}^{el} = Q_{ij}^{0} + 20\% Q_{ij}^{0} \varepsilon, \]

where,
- \( Q_{ij}^{0} \) - OD demand between origin i and destination j, [trips/h];
- \( Q_{ij}^{el} \) - elastic OD demand between origin i and destination j after implementing road pricing, [trips/h];
- \( y \) - observation index of OD demand;

The same two randomization methods are used to model OD demand fluctuations as applied for the capacity distribution. 100 random values of OD demand subject to a normal distribution with a standard deviation of 20% are generated with the first method. With the second method, only three simulations are needed to be performed for each charge level. The mean capacities are applied when the OD demand fluctuates. Jobs are created to change the OD matrix after each simulation. The framework of the Ruby scripts with demand fluctuations is similar to the framework with capacity variations as shown in FIGURE 17.

The toll levels are kept the same as dealing with capacity variation and 2100 (21x100) simulations need to be performed when the first method is used. With the second method, only 63 (21x3) simulations need to be performed.

4.1.5 Objective function of road authority for network design

Based on a lot of surveys, it’s found that travelers put high values on travel times and reliability. (Small, Winston et al. 2002) find that travel time accounts for about two-thirds, and reliability one-third of the service quality differential between the free and express lanes. In spite of using a different reliability measure compared to ours, this finding potentially shows that both travel time and travel time reliability play important roles. In our study, however reliability is not included in the cost function. Travelers take only the travel time and tolls as the cost into consideration when they make route choice.

**Assumption 6:** Road authority aims to optimize network reliability by setting tolls in a network design problem. Travelers are influenced by these tolls and make route and trip decisions by considering travel times and tolls (not taking the trip reliability into account).

In the future study, reliability should be included into the cost function. A network design objective function from the road authority’s point of view, which is a combination of average travel time per vehicle in the network and network unreliability, is proposed. The function is given in formula(40):

\[ TU Delft \]

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Objective function

\[ \text{Min } Z = VOT \times \bar{t} + VOR \times r \] \hspace{1cm} (40)

where,

- \( Z \) – network performance indicator, [euros];
- \( VOT \) – value of time, equals 10, [euros/h];
- \( VOR \) – value of reliability, equals 24, [euros/h];
- \( \bar{t} \) – average travel time per vehicle on a network level, [minutes];
- \( r \) – network unreliability, [minutes];

The objective is to minimize the sum of the monetary values of travel time and travel time unreliability on a network level. A value of 10 euros per hour is chosen as the value of time in this study. A lot of research has been done to estimate the value of reliability. Since for different researches, different reliability measures are taken, different values of reliability have been found in the literature. (Small, Noland et al. 1999) found that for the average length of trip and for median household income, the value of travel time reliability per hour of standard deviation is 2.4 times the value of travel time per hour for normal time. Since their reliability measure is same as ours, we use a value of 24 euros of travel time reliability in our analysis.

We expect that with tolls and fluctuations in link capacities and demand, OD demand will decrease and route flows switch among alternative routes. Travelers shift to the least cost routes and travel time increases a lot on the non-charged routes. Network reliability will be influenced by the combined effects of decreased OD demand and route flow shifts.

### 4.2 Results of analyses with fixed demand

This section presents the results from simulations with fixed OD demand, considering capacity variations and short-term OD demand fluctuations respectively. The two randomization methods introduced in 4.1.3 to modeling capacity variations and demand fluctuations are both applied.

Subsection 4.2.1 presents results from capacity variation case and subsection 4.2.2 gives results from OD demand fluctuation case.

**TABLE 8** illustrates free flow travel times and other characteristics of links, such as capacity and critical speed for the five-link network. **TABLE 9** gives free flow travel times of the three routes. It can be seen that all the three routes are equally attractive to the users when there are
no flows through the routes since they have almost the same free flow travel times of 3.6 minutes.

TABLE 8: link characteristics

<table>
<thead>
<tr>
<th>Link</th>
<th>Free flow travel time (minutes)</th>
<th>Capacity (veh/h)</th>
<th>Speed at capacity (km/h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Link 1</td>
<td>1,7</td>
<td>2500</td>
<td>80</td>
</tr>
<tr>
<td>Link 2</td>
<td>2,4</td>
<td>2500</td>
<td>80</td>
</tr>
<tr>
<td>Link 3</td>
<td>0,6</td>
<td>2800</td>
<td>80</td>
</tr>
<tr>
<td>Link 4</td>
<td>1,9</td>
<td>2500</td>
<td>80</td>
</tr>
<tr>
<td>Link 5</td>
<td>1,2</td>
<td>2500</td>
<td>80</td>
</tr>
</tbody>
</table>

TABLE 9: free flow travel times of routes

<table>
<thead>
<tr>
<th>Route</th>
<th>Free flow travel time (minutes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Route 1</td>
<td>3,5</td>
</tr>
<tr>
<td>Route 2</td>
<td>3,6</td>
</tr>
<tr>
<td>Route 3</td>
<td>3,6</td>
</tr>
</tbody>
</table>

4.2.1 Results from fixed demand case considering capacity variations

FIGURE 18 shows the averaged total travel time and network unreliability with different charge levels using the method with 100 random values.

The total travel time \( (tt) \) is the sum of all the travelers’ travel times, which is expressed as

\[
(tt) = \sum_{i \in D} \sum_{p \in P^i} q_p t_p,
\]

where \( q_p, t_p, \) and \( P^i \) are route flow on route \( p \), route travel time and route

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sets of OD pair i j respectively. Since 100 series of link capacities are modeled for this network, 100 total travel times are obtained from the simulations. The average of the 100 total travel times is calculated and shown in FIGURE 18. Using the Hermite polynomials approximation, the average of total travel time is directly calculated. In the following analysis, the method with 100 random values is called method 1 and the method using the Hermite polynomials approximation is called method 2. The average total travel time and network unreliability using method 2 are presented in FIGURE 19.

FIGURE 19: total travel time and network unreliability with fixed demand (method 2)

The blue lines in both figures denote total travel time and the green lines denote the network unreliability. Small fluctuations of the total travel time and network unreliability are found within the charge range of 0-1 euro in FIGURE 18 and FIGURE 19. With higher charge levels, the two curves are very smooth. The fluctuations at the beginning are caused by the random value generation, because at the beginning capacity variations have significant influences on the network performance (see FIGURE 20). Random link capacity lead to random route travel times, random route travel times lead to frequent route flow shifts, thus to very high travel time variability (see FIGURE 21 and FIGURE 22). Small samples which seem not sufficient to represent the capacity distribution cause the fluctuations. When charge level is high, fluctuations in capacities don’t lead to big changes in route travel costs, thus route flows don’t switch too much. Travel time variability is very low (see FIGURE 21 and
FIGURE 22). With high charge levels, capacity variations have small influences. Small sample won’t cause fluctuations of total travel time and network unreliability.

With fixed demand, route flow shifts is the major influencing factor of travel time variability. FIGURE 20 shows the averaged total travel time obtained from method 1 and the total travel time with constant link capacities. Significant differences can be found within the charge range between 0-0.8 euros. As we explained previously, with low charge levels, fluctuations in capacities lead to frequent route flow shifts, thus to randomized travel times on all the routes. Therefore the average total travel time calculated from 100 random total travel times is quite different from the total travel time with constant link capacities. When the charge level is high, link capacity fluctuations don’t lead to frequent route flow shifts, the traffic pattern keeps almost constant with respect to stochastic capacities. Therefore the average total travel time with capacity variations is almost the same as total travel time obtained with constant capacities. This important finding explains the fluctuations at the beginning in FIGURE 18 and FIGURE 19, since capacity variations have great influences on the network performance and route flows switch frequently when the charges are low, the random values of capacities lead to significantly different outcomes and then lead to fluctuations. However when the charge levels are high, the random values of capacities don’t lead to too much different outcomes, the performance curves naturally turn out to be very smooth.
FIGURE 21: Path travel time standard deviations (fixed demand, method 1)

FIGURE 22: Path travel time standard deviations (fixed demand, method 2)

FIGURE 21 and FIGURE 22 show the standard deviations of route travel times with different charge levels. It can be seen that with low charge levels, standard deviations of all the three route travel times are high. High travel time variability is caused by frequent route flow shifts. With increase of charge levels, route flows don’t switch too much and standard deviations of route travel times become lower.
From FIGURE 18 and FIGURE 19, network unreliability (green line) is generally decreasing and total travel time (blue line) is increasing with the rise of charge levels. As can be seen from FIGURE 21 and FIGURE 22, the travel time unreliability on all the three routes are decreasing with the rise of charge levels. Since the OD demand is fixed, network travel time unreliability is decreasing as well.

The increase of total travel time can be explained by FIGURE 23 and FIGURE 24, which show the route travel times with respect to charge levels. From FIGURE 23 and FIGURE 24, the travel times on non-charged route 2 increases dramatically from 10 minutes without tolls to 27 minutes with a toll of 8 euros. The travel times on route 1 (have 67% overlapping) increase from 13 minutes without tolls to 18 minutes. With tolls, most travelers shift to the non-charged route 2 which can be seen from FIGURE 25 and FIGURE 26, thus the travel time on route 2 increase significantly. Although the traffic flow on route 1 is very low and almost keeps constant (see FIGURE 25 and FIGURE 26), the travel time on route 1 is still increasing (see FIGURE 23 and FIGURE 24). This has been explained in Chapter 3 in the application that route 1 has an overlapping link (link 1) with non-charged route 2. Since the traffic flow on route 2 is always increasing, which lead to travel time increase on the overlapping link (link 1), the travel time on route 1, which includes the travel time on link 1, will increase with the rise of tolls. Because more and more travelers on route 2 with higher travel times, the total travel time increases with the rise of tolls.

**FIGURE 23: Averaged path travel times (fixed demand, method 1)**
In FIGURE 23 and FIGURE 24, the travel time on charged route 3 finally converges to 4.9 minutes and keeps constant, which is not equal to the free flow travel time of 3.6 minutes. Because stochastic assignment is performed and logit model is utilized, the traffic flow on charged route 1 doesn’t reach zero (see FIGURE 25 and FIGURE 26) even with very high charge levels.
On the tolled route 1, route flow becomes less with the rise of tolls and travel time decreases, thus the travel time becomes more reliable with smaller standard deviation, which can be seen from FIGURE 21 and FIGURE 22. The travel time standard deviation on non-charged route 2 decrease as well with increase of route flows (see FIGURE 21 and FIGURE 22), which can be explained from three aspects. Firstly, with high charge levels, route flows don’t switch too much as we discussed already and keep almost fixed. Secondly, with nearly constant and high route flows at high charge level on untolled route 2, stochastic capacities don’t lead to significant changes of route travel time. Thirdly, the flow on route 2 exceeds the capacity when the charges are high, vehicles can’t move freely. Although the travel time becomes longer but the deviations of travel time are very small. The travel time standard deviations on all the three routes decrease with the rise of tolls, thus network unreliability decrease as well as shown in FIGURE 18 and FIGURE 19. Implementing tolls help to improve network reliability with fixed demand.

The changing of the significance of the influences of capacity variation can be further proved in FIGURE 27. FIGURE 27 shows the maximum, averaged and minimum total travel time from simulations using method 1. The same finding as from FIGURE 20 is obtained that the random values of capacities lead to quite different outcomes when the charge levels are low and don’t have significant influences with higher charge levels.
Comparing FIGURE 18 and FIGURE 19, the two randomization methods get the similar results. When the charge level is low, these two methods get different results of total travel time and network unreliability, but the difference is very small. For instance, when the charge is zero, method 1 get a network unreliability of 3.25 minutes and total travel time of 950 hours; method 2 get a network unreliability of 3.7 minutes and total travel time of 900 hours. When the charge level is high, these two methods get almost the same results since capacity variations don’t have significant influences. In spite of different capacities from method 1 and method 2, the same results are obtained with high charge levels.

In FIGURE 25, it can be found the averaged path flows obtained from stochastic capacity are different from that with constant capacities when charge levels are low. When the charge level becomes higher, the route flows obtained from both stochastic and constant capacities are almost the same, regardless of capacity variations. The explanation is still that capacity variations lead to significant variations of route travel times and route shifts when charge levels are low, and don’t have significant influences when charge levels are high.

As explained previously, travel time standard deviations of the three routes will decrease with increase of tolls. Tolls indeed can help to improve route travel reliability and thus network reliability. FIGURE 21 and FIGURE 22 show the path travel time standard deviations from simulations using method 1 and method 2 respectively. The fluctuations at the beginning are caused by the randomization of link capacities. The route travel time unreliability increases a
lot with small toll rise on route 1 with lower charges. This is caused by flow shifts from tolled path 3 to path 1. Charge levels have greater influences on the travel time standard deviation on path 1 than on other two routes. Since path 1 has 67% overlapping with other two routes, therefore is more sensitive with small changes in flow pattern than other two routes.

In FIGURE 23, it's found that the averaged path travel times of path 1 and path 2 with constant capacity are shorter than that with varied capacity when the charge levels are low. Oppositely, averaged path travel time of path 3 with constant capacity is longer than that with varied capacity.

Comparing FIGURE 23 and FIGURE 24, the curves in FIGURE 23 are smoother than that shown in FIGURE 24 when the charge levels are low. Generally, these two methods get similar results of path travel times.

Comparing FIGURE 25 and FIGURE 26, path flows obtained from the two methods are almost the same.

The objective defined in Section 4.1.5 is calculated for each charge level based on the results from two methods and shown in FIGURE 28 and FIGURE 29 separately. The two components of the objective function are presented in the figures as well. In both figures, the monetary component of average travel time increases and network unreliability component decreases with the increase of tolls. The trade-off between these two components has to be made and the sum of these two components is minimized at the charge level of 2 euros using method 1, and at the charge level of 2.5 euros using method 2. Based on the objective function, an optimal charge can be derived. The two figures appear a little bit different, an optimal charge of 2 euros is derived using method 1 and an optimal charge level of 2.5 euros is derived using method 2.
The optimal charge is derived aiming to improve travel time reliability. **FIGURE 30** presents path travel time distributions without tolls and **FIGURE 31** shows path travel time distributions with the optimal charge of 2 euros on link 5. The distribution of path travel times is calculated by counting the travel times belonging to a certain time step and calculating the probability of the travel times belonging to this time step.
It’s noticed that without tolls, route travel times spread with very large range. With the optimal charge level on link 5, the route travel time distributions become narrower and more concentrated with smaller deviations than that without tolls in the network. Route travel time reliabilities of all the three routes are improved a lot with optimal charge level on link 5.
Without tolls, the average values of travel times of the three routes are close to each other (see the quantified values in TABLE 10). It’s very logical that all the used routes have almost (due to stochastic assignment, they don’t need to be equal) the same travel costs. With the optimal charge level on link 5, the travel time on charged route 3 is far less than the travel times on the other two routes. Most travelers chose longer routes, but more reliable routes, avoiding the payment. The travelers, who pay the tolls, save their travel times and get a reliable travel time. TABLE 10 gives the quantified values of average route travel times and standard deviations without tolls and with optimal toll level. The travel time standard deviations decrease a lot with the optimal charge on all the three routes. Travel time on the charged route 3 becomes much shorter with optimal charge. Travel times on route 1 and non-charged route 2 increase with tolls.

TABLE 10: standard deviations and average route travel times with and without tolls

<table>
<thead>
<tr>
<th>Charge</th>
<th>Path</th>
<th>Route travel time standard deviation</th>
<th>Average route travel times</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>path 1</td>
<td>3.3</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td>path 2</td>
<td>3.3</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>path 3</td>
<td>3.1</td>
<td>13</td>
</tr>
<tr>
<td>2</td>
<td>path 1</td>
<td>1.3</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td>path 2</td>
<td>1</td>
<td>18</td>
</tr>
<tr>
<td></td>
<td>path 3</td>
<td>0.2</td>
<td>5</td>
</tr>
</tbody>
</table>

Conclusions of fixed demand case with capacity variations

This subsection presents the results from the fixed demand case, considering link capacity variations as influencing factor on travel time variability. With fixed demand, route flow shifts is the major influencing factor of the travel variability. With low level charges, capacity variations lead to frequent route flow shifts, thus high travel time variability and low network reliability. With higher charge levels, capacity variations don’t lead to significant changes in travel costs and route flows don’t switch too much. Travel times become more reliable. Road pricing can improve travel time reliability on the route level and the network level with fixed demand. Travelers are influenced by the tolls by route shifting to the non-charged route. Total travel time will increase dramatically with the rise of tolls. A trade-off between network reliability and average travel time per vehicle needs to be made to determine the optimal charge. The optimal charge level can be easily derived according to the objective function. The travel time reliability is indeed improved with the optimal charge from the travel time distributions comparison. With optimal charge, travelers on charged route have much shorter...
and reliable travel times. Most traveler chose longer routes, but more reliable routes without paying the tolls. The two methods are compared, and generally these two methods get similar results.

4.2.2 Results from fixed demand case considering demand fluctuations

This section presents the results from fixed OD demand case considering only demand fluctuations as the influencing factor of the travel time unreliability. The mean link capacities are applied in this case. Travel time variability is caused by the OD demand fluctuations, regardless of stochastic capacities. Demand fluctuations here refer to day-to-day demand fluctuations of the same hour in the daily operations of road networks, rather than the changes in demand patterns after major disasters. Since in this study, departure time choice is not considered, the within-day demand fluctuations are not taken into account. The same two methods described previously are used to analyze the impacts of road pricing on travel time reliability with different charge levels.

**FIGURE 32** shows the averaged total travel time and network unreliability from the simulations using method 1 and **FIGURE 33** presents the average of total travel time and network unreliability from simulations using method 2.

![Figure 32: averaged total travel time and network unreliability (fixed demand, method 1)](image-url)
FIGURE 33: average of total travel time and network unreliability (fixed demand, method 2)

In both figures, it can be seen that generally the total travel time increases with the rise of charge levels. Network unreliability decreases within the charge range of 0-5.5 euros, then increases with charge levels higher than 5.5 euros. It can be explained by the changes of travel time deviations of the three routes. FIGURE 34 and FIGURE 35 show that the travel time standard deviations (ttstd) on the three routes are decreasing with the increase of charges. Except non-charged route 2, with charges higher than 5.5 euros, the ttstd of route 2 will increases. With demand fluctuations, route flows will be directly influenced by OD demand fluctuations and change accordingly. Route flow shifts and route flow changes are the influencing factors of travel time variability, which is different from capacity variation case (with capacity variations, route shifts is the influencing factor of travel time reliability). With charge of 5.5 euros on link 5, the traffic flow on non-charged route 2 is 3400 veh/h (see FIGURE 36 and FIGURE 37), which is much higher than the capacity of 2500 veh/h. With charges higher than 5.5 euros, far more traffic is spreading on route 2, exceeding the capacity. Vehicles on non-charged route 2 can't move at all. Since we deal with fixed demand case, which is not realistic at all with high charge levels that most of the travelers are forced to the crowded route 2 without other choices. So the traffic situations obtained from INDY with higher charge levels (higher than 5.5 euros with flows on route 2 much higher than it's capacity) are not believable and not worthy of discussion. Although with OD demand fluctuations, which lead to flow fluctuations on non-charged route 2, the flows on route 2 with
Charges higher than 5.5 euros are always higher than the capacity of route 2. The density on route 2 exceeds the jam density and the speeds of all vehicles are assumed constant (minimum speed, for instance 10km/h). So there should be no travel time variability on route 2 with respect to demand fluctuations when the charge levels higher than 5.5 euros, instead of increasing as shown in FIGURE 34 and FIGURE 35. So when the traffic flows higher than the link/route capacities, the results from models are not worthy of discussion (from now on we only discuss the results with the range of 0-5.5 euros). Within the charge range of 0-5.5 euros, travel time standard deviations are decreasing with the rise of charge levels, because route flow shifts between the three routes become less and less with the increase of charge levels, The route travel times become more reliable. Network reliability, therefore, will be improved as shown in FIGURE 32 and FIGURE 33.

![Path travel time standard deviations (fixed method 1)](image)

FIGURE 34: Path travel time standard deviations (fixed method 1)
FIGURE 35: Path travel time standard deviations (fixed method 2)

With high charge levels, more travelers chose the non-charged route 2, flow on route 2 increases dramatically (see FIGURE 36 and FIGURE 37), thus the travel time on route 2 increases as well with the rise of charge levels (see FIGURE 38 and FIGURE 39). Therefore the total travel time will increase with the increase of tolls in the fixed OD demand case.

FIGURE 36: Averaged path flows (fixed demand, method 1)
FIGURE 37: Average of path flows (fixed demand, method 2)

FIGURE 38: Averaged path travel times (fixed demand, method 1)
FIGURE 39: Average of path travel times (fixed demand, method 2)

Because of logit model being used, the traffic flow on the charged route 3 and 1 doesn't reach to zero (see FIGURE 36 and FIGURE 37). Therefore the travel time on charged route 3 with very high level charges is not equal to the free flow travel time of 3.6 minutes (see FIGURE 38 and FIGURE 39).

Comparing FIGURE 32 and FIGURE 33, the two methods get different results. For instance, without tolls, method 1 gets a total travel time of 1250 hours and a network unreliability of 6.2 minutes. Method 2 gets a total travel time of 1050 hours and a network unreliability of 5.4 minutes. With a toll of 8 euros, method 1 gets a total travel time of 2500 hours and network unreliability of 2.6 minutes. While method 2 get a total travel time of 2200 hours and network unreliability of 3.4 minutes. This may be caused by the small numbers of points (3 points in this study) selected from the demand distribution when method 2 is applied. Only three points are selected, thus only three simulations are performed for each charge level. If larger number of points is chosen, the more accurate results can be obtained with method 2.

From FIGURE 38, generally the path travel times with constant demand are shorter than the averaged path travel times with varied demand. Comparing the average values of path travel times obtained from method 2, shown in FIGURE 39 with FIGURE 38, similar results are got from these two methods.

From FIGURE 36, when the fluctuated demand is modeled, the initial flows on non-charged route 2 and charged route 3 are nearly the same without tolls and more traffic chose route 1,
compared with the flow pattern obtained from constant demand. Comparing FIGURE 36 and FIGURE 37, it can be seen that almost the same results are got from these two methods.

FIGURE 40 shows the maximum, minimum and averaged total travel time obtained from the 100 simulations with random OD demand. It can be found that demand fluctuation always has significant influences on total travel time regardless of charge levels, which is different from capacity variations. This maybe explained from two aspects: one is due to that demand fluctuates with 20% variations, while capacities vary with only 10% variations. The other reason maybe that with capacity variations, stochastic capacities lead to route flow shifts. Route flow shifts lead to travel time variability. With higher charge levels, route flows don’t switch too much, thus the travel time don’t vary too much with high charge levels. However with demand fluctuations, although the route shifts lessly happen with the rise of charge levels, OD demand fluctuations directly influence the route flows. With increase in OD demand, route flows on the whole network will increase, vice versa. Thus demand fluctuations always have great influence on travel time variability.

![FIGURE 40: Maximum, minimum and averaged total travel time (fixed demand, method 1)](image)

The values of the objective function from method 1 and method 2 with different charge levels are presented graphically in FIGURE 41 and FIGURE 42 separately. In this case, the objective curve is more fluctuated. From method 1, the optimal charge is 2 euros and is the same as that obtained from the capacity variations. From method 2, an optimal charge of 2.5 is derived.
Route travel time distributions without tolls and with the optimal charge are shown in FIGURE 43 and FIGURE 44 to investigate the influence of optimal charge on network performance. It's found that the travel times on all the three routes vary with larger range without tolls than with optimal charge on link 5. With the optimal charge on link 5, the route
travel times on all the three routes become more concentrated with smaller standard deviations (see the quantified values in **TABLE 11**). Route reliability is improved on all the three routes. With optimal charge, travel time on charged route 3 becomes shorter (see **TABLE 11**) and travel times on charged route 1 and non-charged route 2 increase. Most travelers chose longer routes, but more reliable routes.

![Path travel time distributions without tolls (method 1)](image1)

**FIGURE 43: Path travel time distributions without tolls (method 1)**

![Path travel time distributions with optimal charge of 2 euros on link 5](image2)

**FIGURE 44: Path travel time distributions with optimal charge of 2 euros on link 5**
TABLE 11 shows the quantified values of route travel time standard deviations and average route travel times with and without tolls, as shown in FIGURE 43 and FIGURE 44.

### TABLE 11: standard deviations and average route travel times with and without tolls

<table>
<thead>
<tr>
<th>Charge</th>
<th>Path</th>
<th>Route travel time standard deviation</th>
<th>Average route travel times</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>path 1</td>
<td>7.2</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td>path 2</td>
<td>5.8</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>path 3</td>
<td>6</td>
<td>14</td>
</tr>
<tr>
<td>2</td>
<td>path 1</td>
<td>3.2</td>
<td>22</td>
</tr>
<tr>
<td></td>
<td>path 2</td>
<td>3.9</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>path 3</td>
<td>4.2</td>
<td>11</td>
</tr>
</tbody>
</table>

As we explained previously, with capacity variations, route flow shifts is the influencing factor of travel time variability. While in demand fluctuation case, both route flow shifts and OD demand fluctuations are the influencing factors of travel time variability. With higher charge levels, demand fluctuations have greater influence on travel time variability than capacity variations do. Capacity fluctuates with 10% variations and demand fluctuates with 20% variations. Due these reasons, the results from capacity variation case and demand fluctuation case are different, for instance the route travel times and route travel time unreliability. TABLE 12 compares the travel time standard deviations and the average route travel times obtained from with capacity variation case and demand fluctuation case.

### TABLE 12: comparison of the results from capacity variations and demand fluctuations with fixed demand

<table>
<thead>
<tr>
<th>Charge</th>
<th>Path</th>
<th>Affecting factors</th>
<th>Route travel time standard deviation</th>
<th>Average route travel times</th>
<th>CoV</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>path 1</td>
<td>capacity variation</td>
<td>3.3</td>
<td>13</td>
<td>25%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>demand fluctuation</td>
<td>7.2</td>
<td>16</td>
<td>45%</td>
</tr>
<tr>
<td></td>
<td>path 2</td>
<td>capacity variation</td>
<td>3.3</td>
<td>10</td>
<td>33%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>demand fluctuation</td>
<td>5.8</td>
<td>12</td>
<td>48%</td>
</tr>
<tr>
<td></td>
<td>path 3</td>
<td>capacity variation</td>
<td>3.1</td>
<td>13</td>
<td>24%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>demand fluctuation</td>
<td>6</td>
<td>14</td>
<td>43%</td>
</tr>
<tr>
<td>2.5</td>
<td>path 1</td>
<td>capacity variation</td>
<td>1.3</td>
<td>16</td>
<td>8%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>demand fluctuation</td>
<td>3.2</td>
<td>22</td>
<td>15%</td>
</tr>
<tr>
<td></td>
<td>path 2</td>
<td>capacity variation</td>
<td>1</td>
<td>18</td>
<td>6%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>demand fluctuation</td>
<td>3.9</td>
<td>20</td>
<td>20%</td>
</tr>
<tr>
<td></td>
<td>path 3</td>
<td>capacity variation</td>
<td>0.2</td>
<td>5</td>
<td>4%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>demand fluctuation</td>
<td>4.2</td>
<td>11</td>
<td>38%</td>
</tr>
</tbody>
</table>
The ratios of path travel time standard deviations and averaged path travel times are calculated to investigate the relative influences of capacity variations and demand fluctuations. The ratios are called Coefficient of Variance (CoV).

From Table 12, demand fluctuations always lead to higher standard deviations than capacity variations do. Without tolls, both capacity and demand fluctuations have great influence on travel time variability. 10% variations in capacity lead to about 25% variations in travel time. 20% variations in demand lead to about 45% variations in travel time. With optimal charge level, demand fluctuations lead to higher variations in travel time, about 20% variations in travel time. While capacity variations only lead to 6% variations in travel times. All these results are consistent with our previous analyses.

Conclusions of fixed demand case with demand fluctuations

In this subsection, the results from fixed demand case considering demand fluctuations are presented. Mean link capacities are applied and travel time variability is caused by the demand fluctuations. With demand fluctuations, route shifts and OD demand fluctuations are influencing factors of travel time variability. With higher charge levels, although route flows don't switch too much, demand fluctuations have direct influence on routes flows, which still cause large variability in travel times. This is the reason why with high charge levels, OD demand fluctuations have greater influences on travel time variability than capacity variations do. In normal situations (without tolls), both capacity variations and demand fluctuations have great impacts on travel time variability. The results presented in this subsection indicate that road pricing can improve travel time reliability on the route level and the network level due to less flow switches. An optimal charge can be determined according to the objective function and with the optimal charge, network reliability and route travel time reliability are improved a lot compared with non-toll situation. From two methods comparison, the two methods get similar results. In the following analyses with elastic demand, only method 1 is applied.

4.3 Results of analyses with elastic demand

Section 4.2 performed analyses of the results from simulations with fixed demand considering capacity variations and demand fluctuations as factors affecting on the travel time unreliability respectively. However fixed demand is not the case in reality when the charging systems are added into the network. In this section, elastic demand is taken into account. The analyses are divided into two parts, the first part considers capacity variations with constant
elastic OD demand and the second part considers demand fluctuations with mean link capacities.

Elastic demand is calculated using price elasticity of demand. In this study the price means the generalized trip cost, which is the sum of travel time and tolls. Demand elasticity of price of -0.2 is utilized to calculate the elastic demand. The formula for calculating the elastic demand is given:

\[ Q'_{ij} = Q^0_{ij} (1 - 0.2 \frac{z^\theta - z^0_{ij}}{z^0_{ij}}) \]  

where,
- \( Q'_{ij} \) — elastic OD demand between origin \( i \) and destination \( j \) after implementing road pricing, [trips/h];
- \( Q^0_{ij} \) — base OD demand between origin \( i \) and destination \( j \), without tolls, [trips/h];
- \( z^\theta_{ij} \) — trip generalized cost from \( i \) to \( j \) without tolls, [minutes];
- \( z^0_{ij} \) — trip generalized cost from \( i \) to \( j \) with a toll of \( \theta \), [minutes];

The elastic demand with respect to charge levels is presented in FIGURE 45:

With elastic demand, some travelers won’t make trips any more with tolls in the network. The route flow on the charged route is expected to decrease faster than that with fixed demand. Since the number of total travelers decrease, the total travel time will surely decrease with tolls, which is different from that with fixed demand.
4.3.1 Results from elastic demand case considering capacity variations

This subsection is going to present the results from elastic demand case considering link capacity variations. The same analyses are performed as in Section 4.2.1, the only difference is that the OD demand with each charge level changes as shown in FIGURE 45. In this case, decreased OD demand and route flow shifts are influencing factors of travel time variability. Method 1 is utilized in this analysis and the influences of demand can be investigated by comparing the results with fixed demand and the results with elastic demand presented in this section.

FIGURE 46 shows the averaged total travel time and network unreliability with elastic demand considering capacity variations.

FIGURE 46  Averaged total travel time and network unreliability (elastic demand)

It is found that both network unreliability and averaged total travel time is linearly decreasing with the increase of charge level within the charge range of 0-2.5 euros. When the charge levels are higher than 2.5 euros, both network unreliability and averaged total travel time will increase. This can be explained with FIGURE 47. From FIGURE 47, route travel time unreliability of the three routes decrease with the rise of charges within the charge range of 0-2.5 euros because route flows switch less with the increase of charges. With charges higher than 2.5, the traffic flow on the non-charged route exceeds the capacity (see FIGURE 48) and keeps increasing. Small changes in flows lead to great changes in the travel time. Since route
1 has overlapping link (link 1), small changes in flows on route 2 will also lead to great changes in travel time on link1, thus to great changes in travel time on route 1 (FIGURE 47).

The two curves in FIGURE 46 are totally different from that obtained with fixed demand (see FIGURE 18 and FIGURE 19). With fixed demand, we get the conclusions that network unreliability decreases and averaged total travel time increases with the rise of charge levels. This is mainly due to that with elastic demand, route flows, route travel times and route travel costs will be quite different from that with fixed demand.

From FIGURE 48 that the traffic flows on the charged route 3 decreases dramatically, this is due to the total demand in the network decreases and a part of flows from route 3 shifts to the toll free route 2. However flow on route 2 increases slowly, which is different from that in the fixed demand case (see FIGURE 25 and FIGURE 26) and is mainly caused by the elastic demand. Because logit model is used, the traffic flow on charge route 1 and 3 don’t reach zero with very high charge levels.
FIGURE 48: averaged path flows (elastic demand)

FIGURE 49 shows the averaged path travel times. It can be seen that capacity variation has great influences on the travel time of route 1. Due to the route shifts and decreased demand, travel time on route 3 decreases dramatically. When the charge levels are very high, the travel time on the charged route 3 doesn’t reach free flow travel time because with stochastic assignment, traffic flow on route 3 is not equal to zero. It is found on the non-charged route 2, when the charge is very high, the route travel time on route 2 will decrease. Since the traffic flow is much higher than the capacity of non-charged route 2 when the charge levels higher than 6 euros, the results obtained from models can be strange and are not worthy of discussion.
With elastic demand, the network performances appear differently from that with fixed demand as seen in above analyses. The values of the objective function and the two components with each charge level are calculated and shown graphically in **FIGURE 50**:

![Graph showing the relationship between charge on link 5 (euros) and objective function. The graph has three curves representing combined network performance, travel time component, and network unreliability component.](image)

**FIGURE 50: optimal charge (elastic demand)**

This result is quite attractive that all the three curves in the figure have the same relations with the charge level: decrease firstly and then increase with the rise of tolls. Both network travel time and network reliability can be optimized. The optimal charge of 2.5 euros is easily determined. The route travel time distributions without tolls and with the optimal charge on link 5 are presented in **FIGURE 51** and **FIGURE 52**.
Comparing these two figures, it's quite obvious that the route travel time distributions are more centralized with much smaller deviations with the optimal charge on link 5 than that without tolls. Without tolls, the three routes have close mean travel times and standard deviations (see quantified values in TABLE 13). With optimal charge on link 5, the average values of travel time on route 1 and non-charged route 2 are almost the same and are much
longer than the average of travel time on charged route 3. Travelers save their travel time by paying extra money. With the optimal charge, the three routes have almost the same travel time reliability (see TABLE 13).

**TABLE 13** shows the quantified values of route travel times and standard deviations without tolls and with optimal charge level. It's noticed that travel time standard deviations on the three routes decrease a lot with optimal charge and route travel time reliability is improved on the three routes. Travelers choosing charged route 3, save considerable travel times. Most travelers chose longer routes, but more reliable routes.

**TABLE 13: standard deviations and average route travel times with and without tolls**

<table>
<thead>
<tr>
<th>Charge</th>
<th>Path</th>
<th>Route travel time standard deviation</th>
<th>Average route travel times</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>path 1</td>
<td>3.6</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td>path 2</td>
<td>2.4</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td>path 3</td>
<td>2.15</td>
<td>14</td>
</tr>
<tr>
<td>2.5</td>
<td>path 1</td>
<td>0.25</td>
<td>17.6</td>
</tr>
<tr>
<td></td>
<td>path 2</td>
<td>0.3</td>
<td>17</td>
</tr>
<tr>
<td></td>
<td>path 3</td>
<td>0.1</td>
<td>5.2</td>
</tr>
</tbody>
</table>

**Conclusions of elastic demand case with capacity fluctuations**

This subsection presents the results from elastic demand case considering capacity variations only. Decreased OD demand and route flow shifts are the influencing factors of travel time variability. With low charge levels, less route flow shifts will occur with increase of charge levels; decreased OD demand has stronger influence on the travel time variability. Network reliability can be improved. With higher charge levels, small route flow changes on non-charged route 2 lead to great changes in travel times. Network unreliability will increase with charge levels higher than 2.5 euros. Both network travel time and network reliability can be optimized with tolls. Road pricing can improve network reliability and an optimal charge can be easily derived based on our objective function. With the optimal charge, the three routes have almost the same travel time reliability. With elastic demand, the results of the network performance are different from that with fixed demand. Because with elastic demand, route flows, route travel times and route travel costs will be quite different from that with fixed demand.
Comparing these two figures, it's quite obvious that the route travel time distributions are more centralized with much smaller deviations with the optimal charge on link 5 than that without tolls. Without tolls, the three routes have close mean travel times and standard deviations (see quantified values in TABLE 13). With optimal charge on link 5, the average values of travel time on route 1 and non-charged route 2 are almost the same and are much
longer than the average of travel time on charged route 3. Travelers save their travel time by paying extra money. With the optimal charge, the three routes have almost the same travel time reliability (see TABLE 13).

**TABLE 13** shows the quantified values of route travel times and standard deviations without tolls and with optimal charge level. It's noticed that travel time standard deviations on the three routes decrease a lot with optimal charge and route travel time reliability is improved on the three routes. Travelers choosing charged route 3, save considerable travel times. Most travelers chose longer routes, but more reliable routes.

**TABLE 13: standard deviations and average route travel times with and without tolls**

<table>
<thead>
<tr>
<th>Charge</th>
<th>Path</th>
<th>Route travel time standard deviation</th>
<th>Average route travel times</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>path 1</td>
<td>3.6</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td>path 2</td>
<td>2.4</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td>path 3</td>
<td>2.15</td>
<td>14</td>
</tr>
<tr>
<td>2.5</td>
<td>path 1</td>
<td>0.25</td>
<td>17.6</td>
</tr>
<tr>
<td></td>
<td>path 2</td>
<td>0.3</td>
<td>17</td>
</tr>
<tr>
<td></td>
<td>path 3</td>
<td>0.1</td>
<td>5.2</td>
</tr>
</tbody>
</table>

**Conclusions of elastic demand case with capacity fluctuations**

This subsection presents the results from elastic demand case considering capacity variations only. Decreased OD demand and route flow shifts are the influencing factors of travel time variability. With low charge levels, less route flow shifts will occur with increase of charge levels; decreased OD demand has stronger influence on the travel time variability. Network reliability can be improved. With higher charge levels, small route flow changes on non-charged route 2 lead to great changes in travel times. Network unreliability will increase with charge levels higher than 2.5 euros. Both network travel time and network reliability can be optimized with tolls. Road pricing can improve network reliability and an optimal charge can be easily derived based on our objective function. With the optimal charge, the three routes have almost the same travel time reliability. With elastic demand, the results of the network performance are different from that with fixed demand. Because with elastic demand, route flows, route travel times and route travel costs will be quite different from that with fixed demand.
4.3.2 Results from elastic demand considering demand fluctuations

In this section the results from simulations with elastic demand, considering demand fluctuations are analyzed. The OD demand fluctuates based on the elastic demand which has been given in FIGURE 45. Mean capacities of all the links are applied. In this case, decreased OD demand, demand fluctuations and route flow shifts are the influencing factors of travel time variability.

FIGURE 53 shows the averaged total travel time and network unreliability with elastic demand taking demand fluctuation as affecting factors on travel time unreliability.

![Graph showing averaged total travel time and network unreliability](image)

**FIGURE 53: averaged total travel time and network unreliability (elastic demand)**

It is noticed that the shapes and the trends of the two curves are very similar to that from the elastic demand case with only capacity variations as the influencing factor (see FIGURE 46). With charge levels less than 2.5 euros, travel time unreliability on the three routes decreases with the rise of charges (see FIGURE 54) because the route shifts become less. After charge level higher than 2.5, route flows on non-charged route 2 reaches the capacities of the composing links (see FIGURE 55). Small flow fluctuations will lead to significant changes in the travel time on this route, thus standard deviation of travel time on route 2 increases dramatically with charge levels higher 2.5 euros. Therefore the network unreliability increases accordingly as shown in FIGURE 53.
Due to logit model being used, traffic flow on charge route 1 and 3 don’t reach zero with very charge levels (see FIGURE 55). Therefore the travel time on charged route 3 is not equal to free flow travel time when charge levels are high (see FIGURE 56). The travel time on the charged route 3, as we expected, declines dramatically with the increase of charges (see FIGURE 56) since the traffic flow on route 3 decreases a lot (see FIGURE 55). Many people shift to the non-charged route and many people don’t make a trip any more. Travel time on
non-charged route 2 increases all the time with the rise of charges. It is noticed that with charges higher than 6 euros, the route travel time on route 2 will decrease. Since the traffic flow on route 2 is much higher than the capacity when the charge level higher than 6 euros, the results from the model can be strange and not worthy of discussions. Actually, in INDY, when the route flow exceeds the capacity, the speeds are assumed constant for all the vehicles. The route travel time on non-charged route 2 should be constant.

Recall that with fixed demand, capacity variations and demand fluctuations have different influences on the averaged total travel time and network unreliability (see FIGURE 18 and FIGURE 32). However with elastic demand, the influences of these two factors on network performances are similar (see FIGURE 46 and FIGURE 53). Thus it is indicated that elastic demand has significant influences on the network performances. It is noticed that the network unreliability is much higher than that caused by capacity variations (see FIGURE 46), because demand fluctuations with 20% variations have greater influences on the travel time than capacity variations with 10% variations on the one hand, demand fluctuations have direct influence on route flows on the other hand.

From FIGURE 54, it's noticed that the path travel time standard deviations are much higher than that from the capacity variations case (see FIGURE 47).

Comparing FIGURE 49 with FIGURE 56, it can be seen that demand fluctuations lead to higher averaged path travel times than capacity variation does. For instance, the averaged travel time on route 1 from capacity variation case without tolls is 16 minutes and the
averaged travel time on route 1 from demand fluctuation case without tolls is 16.9 minutes. For other routes the same can be found.

For elastic demand with demand fluctuations, the objective function and its two components with each charge level are shown in FIGURE 57:

![Combined network performance vs. charge on link 5 (euros)](image)

**FIGURE 57: optimal charge (elastic demand)**

Compared with FIGURE 50, although the values of the objective for different charge levels are higher than the values obtained from capacity variations, the same optimal charge level is derived and the shapes of these two curves are found similar to each other. We think the same optimal charge levels are obtained coincidently for this special network with single OD pair. Both network travel time and network reliability can be optimized.

The path travel time distributions without tolls and with the optimal charge level of 2.5 euros on link 5 are shown in FIGURE 58 and FIGURE 59 separately. Obviously, path travel time reliability is improved a lot when the optimal charge is tolled on link 5 with much smaller deviations. Travel time on the charged route 3 decreases considerable with optimal charge level. With the optimal charge, the travel time reliabilities of the three routes are almost the same.
TABLE 14 gives the quantified values of average route travel times and standard deviations as shown in FIGURE 58 and FIGURE 59.
From the quantified values in TABLE 14, route travel time standard deviations are reduced considerably with optimal charge. Travelers take longer routes, but more reliable routes. Travelers on charged route save a lot of travel times and get more reliable travel times.

**Conclusions of elastic demand case with demand fluctuations**

This subsection presents the results from elastic demand case considering demand fluctuations only. In this case, decreased demand, route flow shifts and demand fluctuations are influencing factors of travel time variability. From the results of analyses, network reliability can be improved with tolls due to less flow switches. Both network travel time and network reliability can be optimized. Elastic demand has significant influences on the network performances. Demand fluctuations lead to much higher network unreliability, path travel times and path travel time unreliability compared with capacity variations. This is mainly due to that demand fluctuates with 20% variation, while link capacities vary with 10% variation on one hand, and demand fluctuations have direct influence on route flows, which lead to travel time fluctuations, on the other hand. The optimal charge helps improve travel time reliability on both route level and network level. With the optimal charge, the three routes have almost the same travel time reliability.

---

**4.4 Comparison of influences of capacity variations and demand fluctuations**

The main task of this section is to compare the influences of capacity variations and demand fluctuations on the network performances. Based on the analyses in above section, we found that demand fluctuations lead to much higher network unreliability, path travel times and path travel time unreliability than capacity variations do. We made assumptions that link capacity distribution has a 10% standard deviation of the mean capacity. While demand distribution...
has a 20% standard deviation of the base demands. We are going to use CoV to compare the relative influences of capacity variations and demand fluctuations.

The ratios of path travel time standard deviations and averaged path travel times are calculated to investigate the relative influences of capacity variation and demand fluctuation. The ratios are called Coefficient of Variance (CoV). Based on FIGURE 47, FIGURE 49, FIGURE 54 and FIGURE 56, the path travel time standard deviations and averaged path travel times (from elastic demand case) with charge of 0 and 2.5 are illustrated in TABLE 15. The elasticity column presents capacity elasticity of travel time and demand elasticity of travel time.

**TABLE 15: comparison of the influence of capacity variations and demand fluctuations on the travel time variability**

<table>
<thead>
<tr>
<th>charge</th>
<th>path 1</th>
<th>affecting factors</th>
<th>route travel time standard deviation</th>
<th>route travel times</th>
<th>cov</th>
<th>elasticity</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td>capacity variation</td>
<td>3.6</td>
<td>16</td>
<td>23%</td>
<td>2.3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>demand fluctuation</td>
<td>7.6</td>
<td>16.9</td>
<td>45%</td>
<td>2.25</td>
</tr>
<tr>
<td></td>
<td></td>
<td>capacity variation</td>
<td>2.4</td>
<td>13</td>
<td>18%</td>
<td>1.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>demand fluctuation</td>
<td>6</td>
<td>13.2</td>
<td>45%</td>
<td>2.25</td>
</tr>
<tr>
<td></td>
<td></td>
<td>capacity variation</td>
<td>2.15</td>
<td>14</td>
<td>15%</td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>demand fluctuation</td>
<td>6.2</td>
<td>14.9</td>
<td>42%</td>
<td>2.1</td>
</tr>
</tbody>
</table>

|        |    | capacity variation | 0.25                                | 17.6              | 1%   | 0.1        |
|        |    | demand fluctuation | 2.9                                 | 17.6              | 16%  | 0.8        |
|        |    | capacity variation | 0.3                                 | 17                | 2%   | 0.2        |
|        |    | demand fluctuation | 2.7                                 | 17                | 16%  | 0.8        |
|        |    | capacity variation | 0.1                                 | 5.2               | 2%   | 0.2        |
|        |    | demand fluctuation | 1.8                                 | 5.6               | 32%  | 1.6        |

From **TABLE 15**, it can be seen that when charge is zero, 10% variations in capacity lead to about 18% variations in travel time, while 20% variations in demand lead to about 44% variations in travel time. The demand elasticity, which equals about 2.15, is larger than capacity elasticity, which equals about 1.8. With the optimal charge, we notice that CoV calculated from 10% capacity variations are almost zero, which means the travel times are perfectly predictable and capacity variations have very small influences on the travel time unreliability. Capacity elasticity is very small. In contrast, when the optimal charge is taken, demand fluctuations still have great influences on travel time unreliability with demand elasticity of about 1.

It’s noticed that capacity variations and demand fluctuations get almost the same averaged route travel times, although they have different range of variations.
Based on the above analyses, both capacity variations and demand fluctuations have great influences on the travel time variability when charge levels are low. With high charge levels, demand fluctuations has greater influences on network reliability than capacity variation does. Because demand fluctuations have direct influence on route flows, which lead to direct travel time fluctuations, although with less flow shifts when the charge levels are high.
5. Contribution to road network design

5.1 Introduction

From the theoretical analyses in Chapter 3 and simulation-based analyses in Chapter 4, we conclude that road pricing can improve network reliability with appropriate charge levels for this single OD pair network due to less route flow shifts. Road pricing is a possible instrument to improve network reliability. A road authority can optimize network reliability by setting tolls in a network design problem. Travelers are influenced by these tolls and make route and trip choices by considering travel times and tolls (not taking the trip reliability into account). The objective function proposed in this study is useful for a road authority to design optimal charging systems (determine optimal charge levels or optimal charging links). This network design problem is a kind of bi-level optimization. The road authority (upper level), as the leader, tries to optimize network travel time reliability by setting tolls. Travelers (lower level), as the followers, are influenced by these tolls and try to minimize their own travel cost, which is the sum of travel time and tolls (see FIGURE 60).

Although a vast, growing body of research on formulations and solution procedures for the problems of network design has developed in the past two decades (Yang and Bell 1998), most of the studies were conducted without considering impacts of uncertainties. It is well known in the literature that capturing uncertainty in transportation system evaluation is important for arriving at better planning decisions (Mahmassani 1984). (Barnhart 2000) presented a scenario-based stochastic model, called the average plan model, to incorporate
uncertainty into deterministic transportation planning models. The objective was to find a solution that is average in the sense that it is closer to the solution of very high probability events, as opposed to infrequent events (Yin and Le da 2005). Most of the efforts in the recent past have concentrated their efforts on studying reliability in terms of the uncertainty in link capacities (Chen, Yang et al. 2002; Lo and Tung 2003). In this study, both short-term origin-destination demand and link capacities are taken as random variables respectively with a known normal probability distribution in a network design problem.

5.2 Charging system design

The charging schemes depicted in FIGURE 60 can be fixed charging systems or variable charging systems. Fixed charging system means that the toll level keeps constant over time. Variable charging system means that the toll levels vary over time. With fixed charging systems, road authority can optimize toll levels or search optimal charging links aiming at optimizing network reliability. With variable charging systems, optimal charge levels for each time interval over time can be determined with the objective of optimizing network reliability. The contributions to the charging system design are depicted for fixed charging systems and variable charging systems respectively.

5.2.1 Fixed charging system design:

Contributions to the fixed charging system design can be categorized into two groups: single-link charging systems and multiple links charging systems (see FIGURE 61).

For single-link charging system, with the objective function proposed in this study aiming to optimize network reliability, two situations can be distinguished. One is determining the optimal charge level for a specific link; the other is searching the best location for tolls by comparing the network performance between the systems with different charging locations.
with their own optimal charge level. The application in this study in Chapter 4 (four cases in simulation-based analyses) is the trial belonging to the first situation to determine the optimal charge level for the specified link 5. The other links in the small network haven’t been tested. The optimizing process for the fixed charging systems is shown in **FIGURE 62** for better understanding:

**FIGURE 62: single link charging system design (fixed charging system)**

For multiple links charging systems with more than one link charged, different scenarios can be set up, compared and selected. The scenarios can be the combinations of different charging locations or for specified links, the combinations of different charge levels for these links. Still the network performances of each scenario are compared and the scenario with the minimum network performance indicator $Z$ (see formula(40)) is the best scenario.

**5.2.2 Variable charging system design:**

Optimal charging schemes for variable charging systems are also possible to be determined. The whole charging period can be divided into small time intervals and for each time interval, an optimal charge level can be derived. Since the charge level varies over time and the travelers will choose the time interval within which they want to make a trip, departure time
choice needs to be taken into account and the dynamic OD demand needs to be determined. The optimal charge for each time interval will be derived based on the dynamic OD demand considering departure time choice. This process is shown in FIGURE 63:

![Diagram](image)

**FIGURE 63: variable charging system design**

The design objective and the design process introduced in this study are aimed to optimize network travel time reliability from road authority’s view. The uncertainties of link capacities and short-term origin-destination demand are taken into account in the design process.
6. Conclusions and recommendations for future research

This chapter summarizes our work done for analyzing the influences of road pricing on network reliability, and the findings from theoretical analyses using static models and simulation-based analyses using dynamic models. Some general conclusions are drawn which not only hold for the hypothetic network analyzed in this study, but also for general networks. Some potential possibilities for future research are suggested.

6.1 Conclusions of theoretical analyses using static models with elastic demand

Theoretical analyses of the impacts of road pricing on network reliability with elastic demand were performed using static models. In the theoretical analyses, capacity variations are assumed as the major influencing factor on network travel time unreliability. An important assumption is made that travelers make their route choices based on the average travel cost.

From the theoretical analyses for a single-route network, it is obvious that tolls can help to reduce the route travel time due to the decreased OD demand and decreased route flow. Decreased route flows lead to smaller travel time variability. The route travel time and the route travel time unreliability are decreasing functions of tolls from the theoretical analyses on this single-route network. A single route network with tolls is more reliable than the network without tolls. This finding can be extended to a larger network with all the routes charged with the same level of tolls. With the same charges for all the routes in the network, the route flows will surely be lower than that without tolls in the network. Lower route flows will lead to lower route travel times, lower route travel times will lead to lower standard deviations of route travel times, thus to lower unreliability.

Theoretical analyses are performed for a two-route network with one link charged. Deterministic user-equilibrium and simple stochastic assignment techniques are utilized respectively.

From the theoretical analyses for a two-route network using DUE, route flow on the charged route will decrease due to decreased OD demand and route flow shifts to the non-charged route. Thus travel time on the charged route will decrease as well and travel time reliability on charged route will be improved with tolls. The travelers, paying the tolls, get shorter travel time and more reliable travel times. Route flow, route travel time and route travel time
unreliability on the non-charged route may decrease or increase, depending on the combined impacts of decreased demand and route flow shifts. Network travel time reliability can be improved when the decreased demand has stronger influence than route shifts do. Network travel time unreliability depends heavily on the charge levels. An optimal charge can be derived to optimize network reliability.

From the theoretical analyses for a two-route network using stochastic assignment, the same conclusions are drawn as using DUE. Travel time reliability on the tolled route will surely be improved with tolls due to decreased route flow, which is caused by decreased OD demand and route flow shifts. Network reliability may be improved or become worse, depending on the combined impacts of decreased OD demand and route flow shifts. With appropriate charge level, when the decreased demand has more significant impacts than route shifts do, network reliability can be improved.

For the theoretical analyses for general networks, link capacities are assumed independent of each other and follow a normal distribution. A general formula for calculating route travel time unreliability is obtained, which is efficient when knowing link free flow travel times, equilibrium link flows and capacity distributions with our assumptions made previously. This formula is used in the application.

A hypothetical five-link three-route network is tested using the formula obtained in the theoretical analyses for general networks. Tolls indeed can reduce network unreliability when the decreased demand has stronger influence on the travel time variability. Different charge level leads to different network reliability. An optimal charge can be obtained with which the network unreliability is minimized. More and more travelers shift to the non-charged route with the rise of charge levels. Traffic flow and travel time on the charged route will decrease. Theoretical analyses using static models show that road pricing can improve network reliability when the decreased demand has stronger influences on travel time variability. Network reliability depends heavily on the charge levels.

6.2 Conclusions of simulation-based analyses using dynamic models

Since in the theoretical analyses, some assumptions need to be made, the dynamic traffic assignment model INDY is utilized to analyze the influences of road pricing on network reliability more appropriately. Capacity variations and short-term origin-destination demand
fluctuations are considered as the most important factors affecting travel time unreliability. Link capacity is assumed following a normal distribution with a standard deviation of 10% of the mean capacity. OD demand is assumed following a normal distribution with 20% variations. Fixed OD demand and elastic OD demand cases are taken into account respectively.

6.2.1 Conclusions of fixed demand case with capacity variations

With fixed demand case considering only capacity variations for the five-link network, route flow shifts is the major influencing factor of travel time variability. With low level charges, capacity variations lead to frequent route flow shifts, thus high travel time variability and low network reliability. With higher charge levels, capacity variations don’t lead to significant changes in travel costs and route flows don’t switch too much. Travel times become more reliable. Road pricing can improve travel time reliability on the route level and the network level due to less flow switches. Total travel time will increase due to most travelers are forced to the non-charged route with serious congestion. A trade-off between network travel time and network reliability has to be made to determine the optimal charge. The optimal charge level can be directly derived according to the objective function. With optimal charge, travelers on the charged route have much shorter and reliable travel times. Most traveler chose longer routes, but more reliable routes without paying the tolls. The route travel time reliability of the three routes are improved, compared with that without tolls.

6.2.2 Conclusions of fixed demand case with demand fluctuations

With fixed demand case considering demand fluctuations, route flow shifts and demand fluctuations are influencing factors of travel time variability. With increase of charge levels, route flows don’t switch too much, thus travel times become more reliable and network reliability can be improved with tolls. With higher charge levels, although route flows don’t switch too much, demand fluctuations have direct influence on routes flows, which still cause large variability in travel times. This is the reason why with high charge levels, OD demand fluctuations have greater influences on travel time variability than capacity variation does. In normal situations (without tolls), both capacity variations and demand fluctuations have great impacts on travel time variability. An optimal charge can be easily derived based on the objective function. With optimal charge, travel time unreliability of the three routes is reduced.
6.2.3 Conclusions of elastic demand case with capacity variations

In this case, decreased OD demand and route flow shifts are influencing factors of travel time variability. With low charge levels, less route flow shifts will occur with increase of charge levels. Decreased OD demand has stronger influence on the travel time variability. Network reliability can be improved with the rise of tolls. With higher charge levels, small route flow changes on non-charged route 2 lead to great changes in travel times. Network unreliability will increase with charge levels higher than 2.5 euros. Both network travel time and network reliability can be optimized with tolls (see FIGURE 64). Road pricing can improve network reliability and an optimal charge can be easily derived based on our objective function. With the optimal charge, the three routes have more reliable travel times. With elastic demand, the results of the network performance are different from that with fixed demand. Because with elastic demand, route flows, route travel times and route travel costs will be quite different from that with fixed demand.

6.2.4 Conclusions of elastic demand case with demand fluctuations

In this case, decreased demand, route flow shifts and demand fluctuations are influencing factors of travel time variability. Network reliability can be improved with increase of tolls due to less flow switches. Both network travel time and network reliability can be optimized (see FIGURE 65). Demand fluctuations lead to much higher path travel times, path travel time variability and network unreliability. This is mainly due to that demand fluctuates with 20% variations, while link capacities vary with 10% variations on one hand, and demand fluctuations have direct influence on route flows, which lead to travel time
fluctuations, on the other hand. With the optimal charge, travel time reliability on the three routes are improved and the three routes get almost the same travel time reliability.

6.2.5 Conclusions of comparison of the influences of capacity variations and demand fluctuations

Without tolls, 10% variations in capacity lead to about 18% variations in travel time, while 20% variations in demand lead to about 43% variations in travel time. The demand elasticity, which equals about 2.15, is larger than the capacity elasticity, which equals about 1.8. With the optimal charge, capacity elasticity is very small, while demand elasticity is about 1. Both capacity variations and demand fluctuations have great influences on the travel time variability when the charge levels are low. Demand fluctuation has greater influences on the travel time variability than capacity variation does. Because demand fluctuations have direct influence on route flows, which lead to direct travel time fluctuations, although with less flow shifts when the charge levels are high.

6.2.6 General conclusions

With fixed demand, route shifts is the influencing factor of travel time variability.

With elastic demand, decreased OD demand and route flow shifts are the influencing factors of travel time variability.

Road pricing can improve network reliability due to less route flow switches.

Decreased OD demand has positive impacts on the travel time reliability.

Tolls can reduce travel time on the tolled route due to decreased route flow, which is caused by decreased OD demand and route flow shifts.

A trade-off needs to be made between network travel time and network reliability to derive the optimal charge. With the optimal charge, travelers take longer routes, but more reliable routes. Travelers, taking charged route, save considerable travel times and get more reliable travel times, which can be seen from FIGURE 66.
In normal situations (without tolls), both capacity variations and demand fluctuations have great influences on travel time variability. Demand fluctuations have greater influence on travel time reliability than capacity variations do. Because demand fluctuations have direct influence on route flows, which lead to direct travel time fluctuations, although with less flow shifts when the charge levels are high.

All the analyses are performed for the single OD pair network. With this network, we can conclude that road pricing can improve network reliability. For realistic networks with multiple OD pairs, road pricing may be a possible measure to improve network reliability.

The network design objective, aiming to optimize network reliability, can be used for all types of road networks to design optimal charging systems.

### 6.3 Conclusions of network design contributions

The road authority aims to optimize network reliability by setting tolls in a network design problem. Travelers are influenced by these tolls and make route and trip choices by considering travel times and tolls (not taking the trip reliability into account). The network design problem is a kind of bi-level optimization. The road authority, as the leader, tries to optimize network travel time reliability by setting tolls. Travelers, as the followers, are influenced by these tolls and try to minimize their own travel cost. The objective function proposed in this study is useful to optimize charge levels and search optimal charge locations in a network design problem, aiming to optimize network reliability. Uncertainties in capacities and OD demand are taken into account in the design process.
6.4 Recommendations for future research

In this study, only a small hypothetical network has been analyzed. A realistic network has not been tested with the described approach due to limited time. Future work can include a realistic network to test and investigate the influence of road pricing on network reliability. In the future, capacity variations and origin-destination demand fluctuations can be combined and simulated simultaneously to analyze the influences of road pricing on network reliability. In all analyses in this report, only single-link fixed charging systems are applied. In the future, different charge location can be tested to gain insight into how the location of pricing system influences network reliability and to determine the optimal link for tolls. Furthermore, multiple links can be charged at the same time and the optimal selected links charging systems can be derived.
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Appendix I Matlab Program of simulations in the application

% determine the best pricing for one OD with two routes, one routes is charged

% travel time function is BPR function

```matlab
clear all;
close all;
clc;
tic
a=0.15;
b=4;
r=0.45;
tf1=4;  % free flow travel time in minutes
tf2=6;  % in minutes
tf3=2;
tf4=5;
tf5=3;
Q=100;  % initial volume of OD
c=40;  % capacity
n=41;  % number of pricing interval
m=0.1;  % pricing interval
varc=1 ./((10'^l .5)'^(2*b));  % variance of capacity
edf=0.3;  % elastic demand efficient
for i=1:n
    p=(i-1)*m;  % road pricing
dt=100;  % t1 minus t1
b1=0.05834;
b2=0.46754;
b3=0.47412;
% initial volume for link
q1=Q*(b1+b2);
q2=Q*b3;
q3=Q*b1;
q4=Q*b2;
q5=Q*(b1+b3);
j=3;
while dt>0.001 & j>0 & j<32000
    % route 1 travel time: link1 + link3 + link5
    t1=tf1*(1+a*((q1./c)'^b))+tf3*(1+a*((q3./c)'^b))+tf5*(1+a*((q5./c)'^b));
    % route 2 travel time: link1 + link4
    t2=tf1*(1+a*((q1./c)'^b))+tf4*(1+a*((q4./c)'^b));
    % route 3 travel time: link2 + link5
    t3=tf2*(1+a*((q2./c)'^b))+tf5*(1+a*((q5./c)'^b));
    c1=t1+6*p;  % route 1 cost (link 5 pricing), p in euros
    c2=t2;
    c3=t3+6*p;  % route 3 cost (link 5 pricing)
    c10=min([c1 c2 c3]);
    cc=(c1*q3+c2*q4+c3*q2)./(q2+q3+q4);  % OD cost before pricing
    if i==1
        fcc0=cc0'^(edf);
    else
        fcc=12.192.'^(edf);
        end
    Q0=Q*fcc./fcc0;
    if j==2
        n=1;j;
        % (1-n)*v+n*Q  \rightarrow\ method of successive average
    else
        n=2;j;
    end
    if c1==c10
        b1=(1-n)*b1+rr;  % ratio of route 1 volume
        b2=(1-n)*b2;
        b3=(1-n)*b3;
    elseif c2==c10
        b1=(1-n)*b1;
        b2=(1-n)*b2+rr;
        b3=(1-n)*b3;
    else
        b1=(1-n)*b1;
        b2=(1-n)*b2;
    end
end
```

TU Delft
\[b_3 = (1 - \tau) b_3 + \tau; \% \text{ratio of route 3 volume}\]
\[
\begin{align*}
q_1 &= Q_0 (b_1 + b_2); \\
q_2 &= Q_0 b_3; \\
q_3 &= Q_0 b_1; \\
q_4 &= Q_0 b_2; \\
q_5 &= Q_0 (b_1 + b_3); \\
dt &= \max([\text{abs}(c_1 - c_2) \text{abs}(c_2 - c_3) \text{abs}(c_1 - c_3)]); \\
j &= j + 1;
\end{align*}
\]
\[
qq1(i) = q_1; \\
qq2(i) = q_2; \\
qq3(i) = q_3; \\
qq4(i) = q_4; \\
qq5(i) = q_5; \\
QQ(i) = Q_0; \\
cc(i) = c; \\
cc1(i) = c_1; \\
cc2(i) = c_2; \\
cc3(i) = c_3; \\
tt1(i) = t_1; \\
tt2(i) = t_2; \\
tt3(i) = t_3; \\
dp(i) = j; \\
dt(i) = dt; \\
\text{vart1}(i) = tf_1 a^2 q_1^{2b} \% \text{sd of link 1 travel time}; \\
\text{vart2}(i) = tf_2 a^2 q_2^{2b} \% \text{sd of link 2 travel time}; \\
\text{vart3}(i) = tf_3 a^2 q_3^{2b} \% \text{sd of link 3 travel time}; \\
\text{vart4}(i) = tf_4 a^2 q_4^{2b} \% \text{sd of link 4 travel time}; \\
\text{vart5}(i) = tf_5 a^2 q_5^{2b} \% \text{sd of link 5 travel time}; \\
varrt1(i) = tf_1 (q_1 + q_2 + q_3 + q_4 + q_5)^2 \% \text{route travel time variance}; \\
\text{varrt2}(i) = tf_2 (q_1 + q_4 + q_5)^2 \% \text{route travel time variance}; \\
\text{varrt3}(i) = tf_3 (q_2 + q_5)^2 \% \text{route travel time variance}; \\
\text{varrt4}(i) = tf_4 (q_1 + q_4)^2 \% \text{route travel time variance}; \\
\text{varrt5}(i) = tf_5 (q_1 + q_2)^2 \% \text{route travel time variance}; \\
\text{varrt}(i) &= \frac{(\sqrt{\text{varrt1}(i)} q_3 + \sqrt{\text{varrt2}(i)} q_4 + \sqrt{\text{varrt3}(i)} q_2)}{\sqrt{QQ(i)}}; \\
qt(i) &= \frac{(t_1 q_3 + t_2 q_4 + t_3 q_2)}{\sqrt{QQ(i)}}; \% \text{average travel time}.
\]

figure(1)
hold on;
x = 0:m:(n-1)*m;
figure(2)
hold on;
figure(3)
hold on;
figure(4)
hold on;
\"TU Delft\"
ylabel('route travel times','fontsize',12);
legend('Route 1','Route 2','Route 3');
grid on;

figure(5);
subplot(2,1,1),plot(x,varrt,'b-.');
xlabel('Pricing');
ylabel('Reliability');
grid on;
hold on;
subplot(2,1,2),plot(x,qq4,'b-',x,qq2,'r-.' ,x,qq3,'g-'' ,x,QQ,'k-');
legend('Route 1','Route 2','Route 3','OD');
xlabel('Pricing');
ylabel('Flow');
grid on;

figure(6);
hold on;
plot(x,varrt,'b-.' ,x,qt,'r-+','x,varrt+qt*2,'k-'' ,x,varrt+qt,'g-');
legend('Reliability','Total travel time','2tt+var','tt+var');
grid on;
xlabel('Pricing');
toc;
Appendix II Matlab Program Example for result analyses

clear all; 
close all; 
cic; 
disp('Load raw data'); 
dataraw=dlmread('full level charge.txt',''); 
dataraw1=dlmread('mean capacity full.txt',''); 

disp('Initial data'); 
% the number of path array 
datapath=unique(dataraw(:,3)); 
datanpath=size{datapath,1); % number of path 
datapath1=unique(dataraw1(:,3)); 
datanpath1=size{datapath1,1); 

% the number of rand capacity 
datacaty=unique(dataraw(:,8)); 
datancaty=size(datacaty,1); % number of capacity 

% the number of charge 
datachar=unique(dataraw(:,7)); 
datanchar=size(datachar,1); % number of charge 
datachar1=unique(dataraw1(:,7)); 
datanchar1=size(datachar1,1); 

% the time step 
datatmst=unique(dataraw(:,4)); 
datan tmst=size(datatmst,1); 
datatmst1=unique(dataraw1(:,4)); 
datan tmst1=size(datatmst1,1); 

disp('Processing data'); 
k=1; 
datanew1=[ ]; 
for r=1:datanpath1 
for i=1:datanchar1 
for j=1:datancaty 
idx=find(dataraw(:,3)==datapath{r} & dataraw(:,7)==datachar{i}... 
 & dataraw(:,4)~=datatmst{end}); 
ftemp=dataraw{idx,6)*3600; % path flow 
ttemp=dataraw{idx,5)/60; % in minutes 
datanew1{k,1}=datapath{r}; %path 
datanew1{k,2}=datachar{i}; %charge 
datanew1{k,3}=datacaty{j}; %capacity 
datanew1{k,4}=ttemp; %mean time 
datanew1{k,5}=ftemp; %flow 
k=k+1; 
clear idx ftemp ttemp 
end %capacity 
end %charge 
end %path 

save data_typical1 data*; 
load data_typical1;
k=1;  
datatotal=[];  
for i=1:datanpath % path  
    for j=1:datanchar % charge  
        idx=find(datanew(:,1)==datapath(i) & datanew(:,2)==datachar(j));  
        datatotal(k,1)=datapath(i);  
        datatotal(k,2)=datachar(j);  
        datatotal(k,3)=mean(datanew(idx,4));  
        k=k+1;  
    end  
end  

disp('Processing total travel time range');  
datatotal=[];  
for i=1:datanchar % capacity  
    for j=1:datanpath % charge  
        idx=find(datanew(:,2)==2 & datanew(:,1)==datapath(i) & datanew(:,3)==datachar(j));  
        tratemp=sum(datanew(idx,4).*datanew(idx,5))/60;  
        datatotal(k,1)=datachar(j);  
        datatotal(k,2)=datapath(i);  
        datatotal(k,3)=tratemp;  
        k=k+1;  
    end  
end  

disp('Processing the reliability');  
datatotal=[];  
for i=1:datanpath % path  
    for j=1:datanchar % charge  
        idx=find(datanew(:,1)==datapath(i) & datanew(:,2)==datachar(j));  
        maxtemp=max(datanew(idx,4).*datanew(idx,5));  
        mintemp=min(datanew(idx,4).*datanew(idx,5));  
        temp=mean(datanew(idx,4));  
        tsd=std(datanew(idx,4));  
        datatotal(k,1)=datapath(i);  
        datatotal(k,2)=datachar(j);  
        datatotal(k,3)=temp;  
        datatotal(k,4)=tsd;  
        datatotal(k,5)=maxtemp;  
        datatotal(k,6)=mintemp;  
        k=k+1;  
    end  
end  

disp('Plot some pictures');  
figure(1); % network  
hold on;  
for i=1:datanchar % charge  
    idx=find(datatotal(:,2)==datachar(i));  
    datatotalv(i)=sum(datatotal(idx,3).*datatotal(idx,5))/60;  
    datatotalsd(i)=sum(datatotal(idx,4).*datatotal(idx,5))./sum(datatotal(idx,5));  
end %charge  
[AX,H1,H2]=plotyy(datachar,datatotalv,datachar,datatotalsd,'plot');  
set(get(AX(1),'Ylabel'),'String','Total travel time (hours)');  
set(get(AX(2),'Ylabel'),'String','network unreliability (minutes)');  
set(H1,'LineStyle','-','linewidth',2);  
set(H2,'LineStyle','-.','linewidth',2);  
xlabel('Charge on link 5 (euros)','fontsize',12);  
grid on;  
figure(2); % different paths  
hold on;  
for i=1:datanpath % path
eval(sprintf('idx%.1d=find(datarel(:,1)==datapath(i));',i));
end %path
plot(datarel(idx1,2),datarel(idx1,4),'b-.');
plot(datarel(idx2,2),datarel(idx2,4),'r-+');
plot(datarel(idx3,2),datarel(idx3,4),'k-*');
xlabel('Charge on link 5 (euros)';'fontsize',12);
ylabel('Travel time standard deviation (minutes)';'fontsize',12);
legend('Path one','Path two','Path three');
grid on;
figure (3); %pathflow
hold on;
for i=1:datanpath % path
    eval(sprintf('idx%.1d=find(datarel(:,1)==datapath(i));',i));
end
plot(datarel(idx1,2),datarel(idx1,5),'b-.');
plot(datarel(idx2,2),datarel(idx2,5),'r-+');
plot(datarel(idx3,2),datarel(idx3,5),'k-*');
for j=1:datanpath1
    eval(sprintf('id%.1d=find(datanew1(:,1)==datapath1(j));',j));
end %path
plot(datanew1(idl,2),datanew1(idl,3),'m-.');
plot(datanew1(id2,2),datanew1(id2,3),'g-+');
plot(datanew1(id3,2),datanew1(id3,3),'c-*');
xlabel('Charge on link 5 (euros)';'fontsize',12);
ylabel('Averaged path flows (veh/h)';'fontsize',12);
legend('Path one-stochastic capacity','Path two-stochastic capacity','Path three-stochastic capacity','Path one-constant capacity','Path two-constant capacity','Path three-constant capacity');
grid on;
figure (4); %path travel time
hold on;
for i=1:datanpath % path
    eval(sprintf('idx%.1d=find(datarel(:,1)==datapath(i));',i));
end %path
for j=1:datanpath1
    eval(sprintf('id%.1d=find(datanew1(:,1)==datapath1(j));',j));
end %path
subplot(3,1,1);plot(datarel(idx1,2),datarel(idx1,3),'b-.');
hold on;
subplot(3,1,1);plot(datanew1(idl,2),datanew1(idl,4),'m-.');
title('Path one');
legend('varied capacity','constant capacity');
grid on;
subplot(3,1,2);plot(datarel(idx2,2),datarel(idx2,3),'r-+');
hold on;
subplot(3,1,2);plot(datanew1(id2,2),datanew1(id2,4),'g-+');
title('Path two');
legend('varied capacity','constant capacity');
grid on;
subplot(3,1,3);plot(datarel(idx3,2),datarel(idx3,3),'k-*');
hold on;
subplot(3,1,3);plot(datanew1(id3,2),datanew1(id3,4),'c-*');
title('Path three');
xlabel('Charge on link 5 (euros)';'fontsize',12);
legend('varied capacity','constant capacity');
grid on;
figure (5); % network
hold on;
for i=1:datanchar % charge
    idx=find(datarel(:,2)==datachar(i));
    datanetav(i)=sum(datarel(idx,3).*datarel(idx,5))/60;
end %charge
for j=1:datanchar1
    id=find(datanew1(:,2)==datachar1(j));
totra(i)=sum(datanew1(id,3).*datanew1(id,4))/60;
end
Influence of Road Pricing on Network Reliability

```matlab
% plot(datachar,datanetav,'k-*');
% plot(datachar1,totra,'r-+');
% xlabel('charge on link 5 (euros)','fontsize',12);
% ylabel('Total travel time (hours)','fontsize',12);
% legend('varied capacity','constant capacity');
% grid on;
%
% figure(6); % total travel time range
% hold on;
% for i=1:datanchar % charge
%  idx=find(datarel(:,2)==datachar(i));
%  datanetav(i)=sum(datarel(idx,3).*datarel(idx,5))/60;
%  id=find(datatra(:, 1 )==datachar(i));
%  datamax(i)=max(datatra(id,3));
%  datamin(i)=min(datatra(id,3));
% end %charge
% plot(datachar,datamax,'k-*');
% plot(datachar,datamin,'r-');
% plot(datachar,datanetav,'b-+');
% xlabel('Charge on link 5 (euros)','fontsize',12);
% ylabel('Total travel time (hours)','fontsize',12);
% legend('max tt','min tt','averaged tt');
% grid on;
%
% figure(7); %travel time distribution
% hold on;
% for i=1:datapath % path
%  eval(sprintf('idx%.1d=find(datadis(:,1)==datapath(i));',i));
% end % path
% subplot(3,1,1); hist(datadis(idx1,3),20);
% title('path one (tolled)');
% grid on;
% subplot(3,1,2);hist(datadis(idx2,3),20);
% ylabel('Probability','fontsize',12);
% grid on;
% title('path two (non-tolled)');
% subplot(3,1,3);hist(datadis(idx3,3),20);
% xlabel('Path travel time (minutes)','fontsize',12);
% title('path three (tolled)');
% grid on;
%
% figure(8); % network
% hold on;
% for i=1:datanchar % charge
%  idx=find(datarel(:,2)==datachar(i));
%  datanetav(i)=sum(datarel(idx,3).*datarel(idx,5))/60;
%  datanetsd(i)=sum(datarel(idx,4).*datarel(idx,5))./sum(datarel(idx,5));
%  objfun(i)=datanetav(i)/5000*10+datanetsd(i)*24/60;
% end %charge
% plot(datachar,objfun,'k-*');
% plot(datachar,datanetav/5000*10,'b-*');
% plot(datachar,datanetsd*24/60,'r+-');
% xlabel('Charge on link 5 (euros)','fontsize',12);
% ylabel('Objective function (euros)','fontsize',12);
% legend('combined netowrk performance','travel time component','network unreliability component')
```
Appendix III Ruby script example

```ruby
require "Otlindy.rb"

myrecord = Array.new
mylinknr = Array.new
mymode = Array.new
mytime = Array.new
mydirect = Array.new
myspeed = Array.new
myfreespeed = Array.new
mysatflow = Array.new
myspeedatcap = Array.new
newrecord = Array.new
my_array = [0.9104, 1.0135, ...]
myorig = Array.new
mydest = Array.new
mypath = Array.new
mytimein = Array.new
mypathcost = Array.new
myflow = Array.new
mycapacity = Array.new
mycharge = Array.new

# retrieve tolls from database events
sql = OtQuery.new("SELECT Toll FROM events.db")
sql.open
charge = sql.get("Toll")
sql.close

charge_max = 8
cap_max = 100
k = 0
while (charge < charge_max) do
  j = 0
  while (j < cap_max) do
    # retrieve link capacity from database
    sql = OtQuery.new("SELECT * FROM link3_2data1.db")
    sql.open
    tbl = OtTable.new("link3_1data1.db")
    tbl.open
    while (!tbl.eof?)
      tbl.delete
    end
    i = 0
    while (!sql.eof?) do
      R = 100*i + j
      mylinknr[0] = sql.get("linknr")
      mymode[0] = sql.get("mode")
      mytime[0] = sql.get("time")
      mydirect[0] = sql.get("direction")
      myspeed[0] = sql.get("speed")
      myrecord[0] = sql.get("capacity")
      myfreespeed[0] = sql.get("freespeed")
      mysatflow[0] = sql.get("satflow")
      myspeedatcap[0] = sql.get("speedatcap")
      newrecord[0] = myrecord[0] * my_array[R]
      tbl.append()
      tbl.set("linknr", mylinknr[0])
      tbl.set("mode", mymode[0])
      tbl.set("time", mytime[0])
      tbl.set("direction", mydirect[0])
      tbl.set("speed", Float(myspeed[0]))
      tbl.set("capacity", Float(newrecord[0]))
      tbl.set("freespeed", Float(myfreespeed[0]))
      tbl.set("satflow", Float(mysatflow[0]))
      tbl.set("speedatcap", Float(myspeedatcap[0]))
      tbl.post()
    end
    sql.next
    i = i + 1
  end
end

# close database
sql.close
```
sql.close

# Running INDY
indy=OtlIndy.new
indy.duration=[60,5]
indy.odMatrix=[1,1,1,1]

# Use of event-list
indy.readEvents=true
indy.valueOfTime=[10]
indy.aggregation=5 # minutes
indy.load=[1,1,1,1,1,1]
indy.timeStep=10 # seconds
indy.departureFractions=[1,1,1,1,1,1,1,1,1,1,1,1]
indy.extendLinks=false
indy.iterations=10
indy.pathIterations=15
indy.pathOverlap=0.99
indy.storePathResults=true
indy.showLogScreen=true
indy.execute

# Store pathcost
sql1=OtQuery.new("SELECT * FROM pathcost.db")
sql1.open

while (!sql1.eof?) do
  myorig[k] = sql1.get("Origin centroid")
  mydest[k] = sql1.get("Destination centroid")
  mypath[k] = sql1.get("PathNo")
  mytimein[k] = sql1.get("Timeinterval")
  mypathcost[k] = sql1.get("PathCost")
  myflow[k] = sql1.get("PathDepartureFlow")
  mycharge[k] = charge
  mycapacity[k] = j+1
  sql1.next
  k=k+1
end # (sql1.eof?)
sql1.close
j=j+1

charge=charge+0.5

if (sql1.eof?)
  table = OtTable.new("events")
  table.open
  while (table.eof?) do
    table.delete
  end
  table.append()
  #table.set(\"eventnr","linknr","direction","mode","type","time","Toll\",[1,1,2,1,5,1,#(charge)])
  table.set("EventID",1)
  table.set("Linknr",15)
  table.set("Direction",1)
  table.set("Time_min",1)
  table.set("Type",5)
  table.set("VehicleType",1)
  table.set("Maxspeed",)
  table.set("Lanes",)
  table.set("InflowCapacity",)
  table.set("OutflowCapacity",)
  table.set("Toll",charge)
  table.post()
  table.close
end
end
afile=File.open("06charge.txt","a")
i=0
while (i<k) do
  afile.print myorig[i],","mydest[i],","mypath[i],","mytimein[i],""mytimeout[i],","mypathcost[i],","myflow[i],","mycharge[i],","mycapacity[i],"in
  i=i+1
end
afile.close
Appendix IV Hermite approximation method

This method is used to approximate the average value of a function of which the independent variables follow normal distributions. In order to avoid Monte Carlo calculations, Hermite polynomials are utilized. Assume that a variable $x$ of a model has a normal probability distribution:

$$P(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-x_0)^2}{2\sigma^2}}$$

In this study, this variable $x$ is the link capacity or OD demand.

Calculating an average value of a function $f(x)$ can be done by randomly selecting values $x_i$ from the probability distribution and approximating

$$E(f) = \int_{-\infty}^{\infty} P(x) f(x) dx \approx \sum f(x_i)$$

Herein the function $f(x)$ could be travel times or flows. A reliable Monte Carlo requires that many $x_i$ are randomly chosen to represent both the normal distribution (there should be sufficient values chosen in the center of the distribution) and the possible non linear character of the function $f(x)$. However, there is a more clever way to do it by using Hermite polynomials. There is an approximation to the integral of a normal distribution with an arbitrary function (Abramowitz and Segun page 890):

$$\int_{-\infty}^{\infty} \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-x_0)^2}{2\sigma^2}} f(x) dx = \frac{1}{\sqrt{\pi}} \sum_{i=1}^N w_i f(x_i \sqrt{2} + x_0) + R_n$$

Where the rest term

$$R_n = \frac{n! \sqrt{\pi}}{2^n (2n)!} f^{(2n)}(\xi) \quad (-\infty < \xi < \infty)$$

has to be small.

The values of $x_i$ and the weights $w_i$ can be found in the table:

<table>
<thead>
<tr>
<th>$N$</th>
<th>$x_i$</th>
<th>$w_i$</th>
<th>$w_i/\sqrt{\pi}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>$\pm 0.70711$</td>
<td>0.88627</td>
<td>0.5000</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>1.18163</td>
<td>0.6666</td>
</tr>
<tr>
<td></td>
<td>$\pm 1.22474$</td>
<td>0.29541</td>
<td>0.1667</td>
</tr>
<tr>
<td>4</td>
<td>$\pm 0.52465$</td>
<td>0.80491</td>
<td>0.4541</td>
</tr>
<tr>
<td></td>
<td>$\pm 1.65068$</td>
<td>0.08131</td>
<td>0.0459</td>
</tr>
<tr>
<td>5</td>
<td>Etc.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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N denotes the number of points selected from the normal distribution. In this study, we have a more dimensional problem since the five-link network has five links, each link with a normal distributed capacity. 3 points from each link capacity distribution are selected in this study. It can be easily shown that $3^5$ simulations should be performed for each charge level to derive the average values of route travel times and route flows.

The standard deviations of route travel times are calculated using formula:

$$s_p = E(f^2) - E^2(f)$$
$$\sigma_p = \sqrt{s}$$