Report LR-376

COMPARISON BETWEEN EMPIRICAL AND CALCULATED STRESS INTENSITY FACTORS OF HOLE EDGE CRACKS

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SUMMARY

Digitized data on crack growth of corner cracks at holes in a plate loaded in tension were published by Grandt and Macha. Empirical stress intensity factors were derived from these data. The empirical values were compared to stress intensity factors obtained by interpolation between calculated data of Raju and Newman. A satisfactory agreement was found.

NOTATIONS

- a crack length of through crack
- crack depth of quarter-elliptical crack
- c crack length of quarter-elliptical crack
- $F_{1/2}$ correction factor for 1 corner crack instead of 2 symmetric corner cracks
- Δl crack extension along curved crack front
- N number of cycles
- $\sqrt{Q}$ shape factor
- R hole radius
- t plate thickness
- W plate width
- θ polar angle

Units: 1000 psi = 6.90 MPa

1000 psi $\sqrt{\text{in}}$ = 1.099 MPa $\sqrt{\text{m}}$

1 inch = 25.4 mm

1. INTRODUCTION

Grandt and Macha [1] have digitized crack front profiles of quarter-elliptical fatigue cracks of a test program carried out by Snow [2]. In this program plate specimens with an open hole were loaded in tension. The specimens were made from transparent polymethylmethacrylate (PMMA), and Snow developed an ingenious method to make direct observations of the crack by time lapse photographs. A corner crack was initiated at the edge of the hole, and Snow measured the crack depth (a) along the hole and the crack length (c) along the specimen surface. Grandt and Macha reexamined Snow's films and produced co-ordinates of successive crack front profiles in digital format. About this more complete description of the crack shapes they say "it is believed that this crack shape information will be of interest to those interested in modelling corner cracks by analytical and numerical procedures". Grandt and Macha selected one interesting crack for which full tabulated data are presented in [1]. They also present baseline fatigue crack growth data for through cracks in the same material.

The purpose of the present report is to derive empirical K-factors from the crack growth data in [1] and to compare the results to calculated K-factors. Such factors were published by Raju and Newman [3] who employed the finite-element method. Interpolations between the Raju/Newman data are necessary in order to agree with the crack shapes observed in the tests. Moreover some other geometry corrections have to be considered.

2. EMPIRICAL K-FACTORS DERIVED FROM THE GRANDT/MACHA DATA

In [1] crack front profiles are documented by presenting co-ordinates (x,y) of 9 points of each crack front. This is done for a large number of successive crack front positions, which allows an easy reproduction of the development of the corner crack at the hole of a plate specimen. Data on the specimen and the fatigue test, and a number of the selected crack front profiles are presented in Figure 1. The empirical K-factors were derived in two steps:

- (1) The crack growth rate ($\Delta l/\Delta N$) along the crack front was calculated from distances between successive crack front profiles.
- (2) The K-factor was then obtained from $\Delta l/\Delta N$ by applying the baseline crack growth data.

The first step was made in a most simple way. A picture of the crack front profiles was plotted by the computer to a magnification of 25 times real size. Because there are data in [1] on many crack front profiles the distance (Δl) between these profiles is sometimes rather small. This can lead to inaccuracies on the Δl -measurements. So a smaller number of crack front profiles was selected to arrive at about equidistant profiles. The selected crack fronts are shown in Figure 1. Distances between these crack fronts were measured with a ruler in a direction estimated to be perpendicular to the crack front, see the method indicated in Figure 2. The local crack growth rate was calculated as $\Delta l/N$, where N is the number of cycles covered between two successive crack front positions. Results are compiled in Appendix A. As a result of the 25 x magnification most Δ -values were in the range of 10 mm (0.4") to 25 mm (1"), which implies a high accuracy of the measurements. The Δ -measurement according to the method of Figure 2 can only be applied to 7 of the 9 points of a crack front profile (points B to H in Figure 3). For the end points of the crack front (points A and J in Figure 3) Δl was supposed to be equal to Δc and Δa , where c and a are semi-axis of the elliptical approximation of the crack front profile.

In the second step the corresponding K-values were calculated by adopting the Paris relation for the baseline crack growth data as presented in [1] and reproduced here in Figure 4.

$$da/dN = 0.6949 * 10^{-22} * (\Delta K)^{6.0954} \quad (1)$$

(da/dN in inch/cycle and K in $\text{psi}\sqrt{\text{in}}$). The scatter shown by Figure 4 implies a source of inaccuracies to be considered later.

The value of $\Delta l/\Delta N$ obtained in the first step is substituted for da/dN in Eq. (1) and ΔK can then be calculated.

For the comparison of the empirical K-values to calculated results of Raju and Newman it is necessary to indicate the corresponding values of a, c and the polar angle θ , which locate a point along the crack front by:

$$x = c \cos \theta \quad (2a)$$

$$y = a \sin \theta \quad (2b)$$

For each crack front the values of a and c were obtained by requiring that the elliptical equation $(x/c)^2 + (y/a)^2 = 1$ should pass through the two points B and H (see Figure 3). Points A and J were not used for this purpose because sometimes the crack front at the material surface is lagging behind. For each crack front the values of a and c thus obtained are presented in Appendix A. It turns out that a/c increases from 1.15 for the first crack front to 1.82 for the last one. Values of θ for each point of a crack front are obtained with Eq. (2a) (results also in Appendix A).

For the comparison with the Raju/Newman data it should be realized that the calculated K-values are derived from average crack growth rates $\Delta a/\Delta N$ between two successive crack front profiles. For that reason an "average" elliptical crack between the two crack fronts was considered. This was handled by taking the average a- and c-value of the two crack front profiles involved (see Appendix A). The empirical K-values are presented in Figure 5.

Note: Because all units in [1] are in inches and pounds the data in the present document are not converted into modern units.

3. K-FACTORS DERIVED FROM CALCULATED RESULTS OF RAJU AND NEWMAN

Raju and Newman [3] calculated stress intensity factors by a 3-dimensional finite-element technique for two symmetrical corner cracks at the edge of a hole in a plate loaded in tension. The corner cracks had a quarter-elliptical shape. The geometry ratio's for which calculations were made are presented in Figure 6. Assuming that the cracks of Figure 1 can be approximated by a quarter ellipse, the geometry ratio's of the cracks in Figure 5 are still different from the values adopted by Raju and Newman (Figure 6). Consequently for a comparison between empirical K-values and calculated K-values interpolations have to be made between the Raju/Newman data. The interpolation problem was analysed in [4] where it was concluded that fairly accurate interpolated results can be obtained. The interpolation procedures adopted here are summarized in Appendix B.

In addition to the interpolation two corrections have to be considered:

- (1) A width correction factor
- (2) A correction factor for having one corner crack (Grandt/Macha data) instead of two symmetrical corner cracks (Raju/Newman data).

The width correction will be ignored here because the W/R-values are large for both data sets. As a result the width correction factor will be close to 1, and differences between the factors for the two cases will be very small. The second correction is applied here, although it is a fairly small factor. For this correction Shah [5] proposed to replace a quarter-elliptical crack by a through crack (length c') with the same area:

$$c' t = \frac{\pi}{4} a c \quad (3)$$

Shah then assumes that the ratio ($F_{1/2}$) between K-factors for one hole-edge crack and two hole-edge cracks is the same as for cracks at a hole in an infinite plate (Bowie problem). For the latter case a more accurate approximation was recently proposed [6]:

$$F_{1/2} = \sqrt{\frac{2 + c'/R}{2 + 2 c'/R}} \left[1 + \frac{0.2 c'/R}{(1 + c'/R)^3} \right] \quad (4)$$

For the cracks considered here c' was calculated with Eq. (3), which was substituted in Eq. (4). This factor was then applied to the interpolated data. $F_{1/2}$ -values were in the range of 0.997 (negligible correction) for the smallest crack to 0.929 (7 % reduction) for the largest crack of Figure 1. The final K data thus obtained are plotted in Figure 5.

4. DISCUSSION

Empirical K-values

The results in Figure 5 show two trends:

- (1) Along the crack front a slight increase of K is found when going from the plate surface ($\theta = 0$) to the bore of the hole ($\theta = \pi/2$).
- (2) When the crack is growing a small increase of K is observed. The average values of ΔK for the smallest crack ($a/t = 0.20$) and the largest crack ($a/t = 0.89$) are 580 and 860 $\text{psi}\sqrt{\text{in}}$ respectively.

The general picture of the empirical data is rather consistent. Possible inaccuracies can be due to deviations of the baseline data equation (Eq.1).

Calculated K-values

The results in Figure 5 indicate that the variation of K along the crack front is fairly small, except for $\theta = \pi/2$ (near the hole) where a drop of K is found for all cracks. This drop was already noticed in [3].

The increase of K when the crack becomes larger is smaller than for the empirical data. Average values for the smallest and the largest crack in Figure 5 are 540 and 760 $\text{psi}\sqrt{\text{in}}$ respectively.

Comparison between empirical and calculated K-values

As a first general observation of Figure 5 it should be said that the agreement between empirical and calculated results is satisfactory. The agreement becomes less good for the largest cracks. It should be noted that the two largest cracks with $a/t = 0.81$ and $a/t = 0.89$ are outside the Raju/Newman databank. An extrapolation was necessary for these two cracks.

Looking to the results in more detail some small but systematic differences become apparent. For larger cracks the calculated K -values show a weak minimum at $\theta = 0.625$, which is not observed for the empirical results. It is believed that small deviations from the elliptical shape can already cause such differences.

Another problem is the free-surface effect. The lagging behind of the crack front at the surface can be associated with differences between plane stress and plane strain conditions. It is possible that this will affect the crack growth resistance. It also can lead to differences in effective K -values due to different amounts of crack closure. The fairly sharp drop of the calculated K -value at $\theta = \pi/2$ is more difficult to understand. Anyhow, this feature is not evident from the empirical data.

The overall agreement between empirical and calculated K -values confirm the usefulness of the Raju/Newman data for predicting fatigue crack growth of corner cracks. On the other hand it also indicates that empirical K -factors can be obtained by this type of fatigue tests.

5. CONSLUSION

Digitized data on crack front profiles of corner cracks at holes were presented by Grandt and Macha for the purpose of modelling analytical or numerical procedures. Empirical stress intensity factors were derived from these data. Stress intensity factors for the same cracks were also obtained by interpolation between data, calculated by Raju and Newman. A satisfactory agreement was found between the empirical and the calculated stress intensity factors.

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Appendix A: Empirical ΔK -values

Values of Δl (Fig. 2) were measured in mm on a 25 times magnified plot. These values were converted to inches, real size on fracture surface, and divided by ΔN to obtain the average crack growth rate between successive crack fronts. The corresponding ΔK -values were then obtained with Eq. (1). The results are presented in Table A1 and plotted in Figure 5 as a function of the polar angle. Each crack growth interval between two successive crack fronts is associated with the average a and c of the interval. The polar angle θ applies to points of the first crack front of the interval (Fig. 3). An evident averaging procedure for θ is not easily indicated, but is also thought to be unnecessary because it would produce minor changes of θ only, which would not affect the comparison to the calculated ΔK -values.

Appendix B: Interpolated ΔK -values

Several interpolation procedures were analysed in [4]. For the present case (a plate with an open hole loaded in tension) it was recommended to interpolate first with a/c as a variable and a/t being constant to arrive at the correct value of c . The second step should be to interpolate with a/t as a variable, and c being constant to the desired value of a/t . Both steps should be done for the two R/t -values (0.5 and 1). The last step is to interpolate to the applicable R/t with R/t as a variable and c and a/t being constant. Moreover, it was analysed which interpolation functions should give the most accurate results. If the recommendation of [4] are followed it turns out that K -data for $a/c > 2$ are required, which are not available in the Raju/Newman databank. According to the analysis in [4] reasonably accurate interpolation results will still be obtained by:

(1) Linear interpolation between $K/S\sqrt{\pi a/Q}$ data of the Raju/Newman databank with a/c as a variable and $a/t = \text{constant}$ to arrive at data for applicable a/c value. This should be done for two adjacent a/t -values with the applicable a/t in between.

(2) A linear interpolation between the results of step (1) with a/t as a variable to arrive at data for the applicable a/t .

(3) Steps (1) and (2) should be done for $R/t = 0.5$ and $R/t = 1$. Before the third step is made the $K/S\sqrt{\pi a/Q}$ results of step (3) are first converted into

$K/[S\sqrt{\pi a/Q} \{ \sin^2 \theta + (a/c)^2 \cos^2 \theta \}^{1/4}]$ because this improves the interpolation accuracy of the last step. The interpolation then occurs with $\sqrt{R/t}$ as a variable in order to arrive at results for the applicable R/t ($= 0.529$). From these results K can be recalculated. The final results are presented in Table B1. It includes the correction factor $F_{1/2}$ of Eq. (4). The data are plotted in Figure 5.

The justification to use $\sqrt{R/t}$ as a variable in step (3) could not be supported in [4] by calculated data. However, an analogy to thickness effects for semi-elliptical surface cracks suggested the $\sqrt{R/t}$ procedure. In the present case a linear interpolation with R/t as a variable would give almost the same results because for the specimen $R/t = 0.529$, which is fairly close to $R/t = 0.5$.

It should be pointed out that the analysis of interpolation methods in [4] was based on a continuous $\sqrt{Q}$ definition instead of the "discontinuous" $\sqrt{Q}$ definition adopted by Raju and Newman. In the discontinuous definition different equations apply to $a/c \leq 1$ and to $a/c > 1$. Very accurate approximations of Raju

and Newman are:

$$Q = \left[1 + 1.464 (a/c)^{1.65} \right]^{\frac{1}{2}} \text{ for } a/c \leq 1$$

$$Q = \left[1 + 1.464 (c/a)^{1.65} \right]^{\frac{1}{2}} \text{ for } a/c > 1$$

It was shown in [7] that a discontinuous definition is not necessary, while it is even undesirable for interpolations. The equation (complete integral of the second kind)

$$E(k) = \sqrt{Q} = \int_0^{\pi/2} \sqrt{1 - k^2 \sin^2 \varphi} \, d\varphi$$

$$k^2 = 1 - (a/c)^2$$

can be used for any value of a/c with a very accurate approximation:

$$\sqrt{Q} = \frac{\pi}{2(1+m)} \left(1 + \frac{m^2}{4} + \frac{m^4}{64} \right) \text{ with } m = \frac{1 - a/c}{1 + a/c}$$

The Raju/Newman data for $a/c = 2$ were corrected for the continuous $\sqrt{Q}$ definition, which is a simple procedure in view of the relation:

$$[\sqrt{Q(a/c > 1)}] = \frac{a}{c} [\sqrt{Q(c/a)}]$$

It implies that the $K/\Delta S \sqrt{\pi a/Q}$ data for $a/c = 2$ as presented in [3] have to be multiplied by 2.

N (cycles)	Axis of ellipse (in)		Average axis (in)		and polar angle $\theta' = \theta/(\pi/2)$ at A to J (Fig. 3)																	
	c	a	c	a	Point A		B		C		D		E		F		G		H		J	
					ΔK	θ'	ΔK	θ'	ΔK	θ'	ΔK	θ'	ΔK	θ'	ΔK	θ'	ΔK	θ'	ΔK	θ'	ΔK	θ'
4000	0.109	0.125	0.115	0.143	528	0	526	.086	540	.173	543	.229	561	.326	587	.484	628	.683	636	.835	642	1
8000	0.120	0.161	0.130	0.183	606	0	597	.116	608	.184	617	.299	631	.435	640	.570	660	.697	682	.872	693	1
11000	0.139	0.204	0.150	0.226	621	0	613	.133	621	.203	640	.318	653	.448	665	.581	677	.701	683	.824	693	1
14000	0.161	0.247	0.177	0.279	660	0	656	.142	658	.237	662	.344	680	.482	705	.608	729	.765	732	.889	739	1
17000	0.193	0.311	0.208	0.345	694	0	695	.145	695	.253	714	.384	724	.459	733	.564	749	.706	784	.840	798	1
19000	0.222	0.379	0.236	0.413	723	0	725	.121	744	.251	756	.373	756	.462	780	.587	800	.722	823	.855	837	1
20500	0.250	0.447	0.265	0.471	782	0	780	.099	773	.181	782	.263	797	.405	789	.574	814	.696	832	.851	842	1
21500	0.280	0.494	0.292	0.519	744	0	760	.093	774	.230	792	.362	818	.484	828	.610	854	.731	845	.827	879	1
22300	0.303	0.543	0.316	0.576	831	0	820	.104	831	.172	849	.276	844	.382	846	.504	849	.623	882	.754	919	1
22900	0.329	0.591	0.343	0.619	836	0	822	.111	839	.193	847	.288	850	.448	873	.549	882	.654	889	.785	942	1
23000	0.356	0.647																				

For points A and J the value of $\theta' = \theta/(\pi/2)$ is set equal to 0 and 1 respectively.

Table A1: Empirical ΔK -values

Axis (mm) c a		$\Delta K \quad (\text{psi } \sqrt{\text{in}})$										
		$\theta=0$	0.125	0.25	0.375	0.5	0.625	0.75	0.833	0.917	0.958	1.0
0.115	0.143	509	516	516	521	531	543	560	574	584	566	466
0.130	0.183	575	580	576	577	580	586	597	607	617	599	497
0.150	0.226	627	629	623	620	620	622	632	642	655	639	534
0.177	0.279	672	673	663	658	655	656	666	679	698	687	581
0.208	0.345	709	707	694	685	679	679	691	709	737	733	629
0.236	0.413	755	749	730	714	703	698	708	726	758	758	657
0.265	0.471	786	776	751	731	716	709	719	739	776	782	685
0.292	0.519	806	792	763	739	722	714	724	747	790	801	708
0.316	0.567	822	805	771	742	723	713	724	749	796	813	725
0.343	0.619	835	814	776	742	720	709	720	749	802	825	742

Table B1: Calculated ΔK -value-, including the $F_{1/2}$ correction.

Figure 1: Selected crack front profiles and data on specimen and fatigue test

Figure 2 : Determination of average crack growth rate between two successive crack fronts.

Figure 3 : Ellipse axes a and c follow from the co-ordinates of B and H

Figure 4. Baseline fatigue crack growth data for polymethylmethacrylate (PMMA) (reproduced from [1])

$\Delta K \text{ (psi}\sqrt{\text{in}})$

Figure 5: Comparison between empirical and predicted ΔK -values for increasing crack size

$\Delta K (\text{psi}\sqrt{\text{in}})$

Figure 5 (continued)

Raju and Newman calculated K for:

$a/c = 0.2$	1	2
$a/t = 0.2$	0.5	0.8
$R/t = 0.5$	1	
11 values of θ		

Figure 6: Ratio's of dimensions covered by the finite - element calculations of Raju and Newman [3].