Department of Precision and Microsystems Engineering

Design of a compact AFM scanner

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Specialisation : Mechatronic System Design
Type of report : MSc. Thesis
Date : 20 June 2013
Abstract

The goal of this research is the design of a new type of Atomic Force Microscope (AFM), more specifically a new AFM scanner. An Atomic Force Microscope is a device used to measure the topography of small surfaces (the sample surface). The core component of an AFM is a probe tip mounted on a small cantilever which is moved relative to the sample surface. The AFM scanner is the part enabling the motion between the probe tip and the sample surface. The main research questions can be summarized as follows:

- How do the requirements of the AFM system translate to the requirements of the AFM scanner?
- Do the scanner concepts meet the requirements?
- Is the scanner concept valid as a real design?

A set of requirements in terms of size, accuracy and speed has been set up at AFM system level and is subsequently translated to requirements at AFM scanner level. An AFM scanner concept called the scanner tripod concept has been introduced, analyzed and evaluated against these requirements. The tripod scanner concept has proven to be flawed as the required accuracy can never be reached. A new concept, the orthogonal scanner concept, has been analyzed and evaluated in the same way and has turned out to meet all the AFM scanner requirements.

Subsequently, this concept has been further developed into a more detailed design including existing components in order to answer the final research question. Sensors, actuators, optics and an AFM probe holder have been selected and combined with the orthogonal scanner concept this has resulted in a final design. The final design has been analyzed and the AFM scanner specifications are evaluated. The selected components are found to impose limitations on the size of the scanner and on the roll angle requirement causing to be roughly a factor two above the required values. The remaining requirements in terms of measurement range and speed are all met.
Preface

This report is the final result of a graduation internship at the TNO optomechatronics department and the conclusion to a Master of Science of Mechanical Engineering at the TU Delft. More specifically at the department of Precision and Microsystems Engineering (PME) of the faculty 3mE. I’ve started this project with a literature survey in order to get properly acquainted with the Atomic Force microscope topic, the report of this survey can be found in Appendix A. The graduation project itself commenced from an idea of an AFM scanner concept from my supervisor, Stefan Kuiper, and the plan was to work out this idea to assess its feasibility. However during the course of my internship this idea turned out to have some fundamental flaws so a new concept needed to be invented and it was. This thesis deals with the most important questions during any design, what are the requirements and what needs to be done to meet them. I would like to thank my TNO supervisor Stefan Kuiper and my professor Rob Munnig Schmidt for all their support and advice throughout the course of the project. I would also like to thank Just Herder for his advice on attempting to save the first concept. Furthermore my thanks go out to my colleagues at TNO who have supported me during my internship and especially to Dorus de Lange for helping me out with some tedious Ansys calculations.
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Chapter 1

Introduction

1.1 Basics of atomic force microscopy

Atomic force microscopy is a type of microscopy where a mechanical probe is used as a sensing medium, as opposed to light used in conventional optical microscopes or electron beams as used in scanning electron microscopy. An atomic force microscope can be compared to feeling or touching an object while an optical microscope can be compared to looking at an object. An atomic force microscope uses a very sharp tip in order to measure the topography of a sample at a certain point on its surface. This only provides a single measurement value so to acquire an image of the sample it is necessary to scan the sample in such a way that an array of points can be measured. This is a time consuming process when compared to an optical microscope that provides an image instantaneously.

The basic working principle is schematically shown in Figure 1.1. The tip is attached to a cantilever moving with respect to the surface. As the tip moves along the surface topography a force will be exerted on the tip in the normal direction, due to contact forces, and in the lateral direction, due to friction forces, both causing the cantilever to bend. This "sliding" over the surface is known as contact mode. As the cantilever changes the angle of the laser beam on the cantilever changes causing a rotation of the reflected beam. This in turn translates the laser spot on a photo sensitive device (PSD). The PSD measures the translation of the laser spot. This gives a direct measure for the deflection of the cantilever. The high sensitivity of the PSD enables it to measure small deflections in the order of subnanometers [2].

1.1.1 Tip-sample interaction

The forces between the tip and the sample are being governed by the interaction of the atoms at the end of the tip and the atoms of the sample. Depending on the distance between the probe tip and the sample surface the tip is either attracted or repulsed. This effect has been graphed in Figure 1.1.2. As the tip approaches the sample from a distance there is no force interaction, signified by the horizontal line on the right side of the graph. When it enters the attractive force regime the tip experiences a force which pulls it in towards the sample surface. When the tip is moves past the attractive and into the repulsive regime the atoms on the probe tip and the sample surface are so close that they will repel each other. To keep the tip in the repulsive regime a force always needs to be applied.
Figure 1.1.1: Schematic of a general AFM, including the probe and the optical lever

Figure 1.1.2: Atomic Force as a function of the probe tip to sample distance
1.1.2 Modes of operation

There are three main modes of operation: contact mode, intermittent mode and non contact mode. In contact mode the tip is being forced in the repulsive regime which means that the tip is pushed on the sample surface. The tip is then moved over the sample surface.

The next mode is known as intermittent or tapping mode. This means that instead of sliding the probe over the surface the cantilever is being oscillated at a certain frequency and the tip "hits" the sample surface with the same frequency. As the contact time is very short the friction forces are smaller than while operating in contact mode.

The third mode is non contact mode. In this mode the probe tip is also oscillated but now at a small distance from the surface, in the attractive regime. The topography of the sample influences the amplitude and phase of the oscillation of the cantilever. This means that without making actual contact between the tip and the sample the topography can still be measured.

1.1.3 AFM Scanning pattern

In order to acquire a two-dimensional image of the sample surface the cantilever has to be scanned over the surface since only one point can be measured at a time. The most common way to do this is by scanning the sample line per line, at a constant velocity. In order to construct the image only the lines scanned in the same direction are recorded since the friction on the tip is direction and velocity dependent. If lines would be recorded in both directions the image will be distorted every two lines. This results in the triangular scanning pattern shown in Figure 1.1.3.

The speed of the tip needs be constant since the friction experienced by the probe tip is a velocity dependent effect. The triangular scanning pattern implies that the speed in the x direction is higher than in the y direction. The x axis is thus called the fast axis and the y axis the slow axis. It is also possible to scan in sinusoidal shapes, literature has shown Lissajous figure type scan patterns \[6\]. These are
based on superimposing sinus waves in x and y direction while keeping the tip speed constant. This eliminates the fast and the slow axis but parts of the sample surface are scanned multiple times before acquiring the complete image.

1.1.4 z control

In addition to moving in the $x,y$ plane the probe tip is also actuated in the $z$ direction. This is done by a feedback controller which either minimizes the forces between the tip and the sample surface or keeps these force at a constant level. The force on the probe tip can be estimated by measuring the cantilever bending displacement and this signal is used in the feedback loop. This ensures that the probe tip is in the correct regime (repulsive or attractive) corresponding with the chosen imaging mode (contact mode, tapping mode, non contact mode). When the controller can’t keep up with the topography a tracking error is induced. The goal of the controller is to keep the tracking error small and that the tip remains in the correct regime. When the tip is scanned relative to the sample with high speeds a fast $z$ controller is needed to maintain the probe force at the desired level and to keep the tracking error low. It is possible to use an AFM without $z$ feedback control, in constant height mode. This means that the tip sample forces are much higher since the cantilever bends to follow the topography of the sample as opposed to using feedback control. Very high scanning speeds can be achieved with constant height, given that the sample will not be damaged by the higher forces between the tip and the sample.
1.2 AFM scanners

A scanner is the mechanism used to move the probe tip relatively to the sample in both x, y and z directions. This could either mean that the sample itself is actuated and that the tip probe remains stationary or that the probe tip is actuated while the sample is fixed. The first method is called sample scanning (1) and the second is called probe scanning (2), schematically drawn in Figure 1.2.1.

There are many types of scanners available, three main types of scanners and their working principles will be discussed: the piezoelectric tube scanner, a tripod scanner and a serial scanner.

1.2.1 Piezoelectric tube scanner

The most common solution for an AFM scanner is the use of a so called piezo tube. This is an actuator shaped like a tube consisting of piezoelectric material. The converse piezoelectric effect results in expansion of the piezo material when applying a potential. Electrodes are applied on the tube in such a way that bending modes in x and y are excited in resulting in a translation of the end of the tube in x and y directions. The longitudinal mode is used to extend the tube in order to actuate the z direction. A piezo tube scanner can be used both as sample scanning and probe scanning. As the tube is hollow the laser beam used for the optical lever can be led through the piezo tube while probe scanning. A disadvantage of the piezo tube scanner is the scanner bow, since bending is used for the x and y directions there will also be a rotation at the end of the tube proportional to the lateral displacement.

1.2.2 Tripod scanner

Another solution is the tripod AFM scanner, a schematic example has been displayed in Figure 1.2.2. Three piezostacks placed perpendicular with respect to each other drive part in the middle which the cantilever is attached to. The longitudinal stiffness of a piezostack is much higher than the lateral stiffness, when driving the x-direction for example the actuator has to work against the lateral stiffness of the y and z piezostacks. If the piezostacks are not completely perpendicular the effect of this parasitic stiffness becomes worse as the longitudinal stiffness of the piezostacks now also has to be taken into account.
1.2.3 Serial scanner

An example of a serial scanner is shown in Figure 1.2.3. In this case the stage is on top of the z piezo stack which in its turn is attached to the x-piezo stack. The subassembly of the x and the z piezostacks is attached to the y piezostack and can move freely in the y direction. This type of scanner is called a serial scanner since the stiffnesses associated with the piezo stacks are in series. The disadvantage is that the actuated mass per piezo actuator is different, each piezo stack needs be dimensioned differently. When using a triangular scanning pattern as mentioned earlier this does not have to be an issue since there are separate slow and fast axises.

1.3 AFM compact tripod scanner concept

The proposed AFM scanner concept will now be introduced. It is slightly similar to a Stewart platform but more like a delta robot. The idea is that there are three slanted parallelograms attached to a top platform, placed at 120 degrees from each other. A schematic overview is drawn in Figure 1.3.1. The parallelograms, made up of rods and ball hinges, constrain the rotational degrees of freedom of the top platform. The bases of the parallelograms are attached to actuators which are allowed to translate in the vertical direction. The result is that three vertical...
displacements result in three $x, y, z$ displacements. There are also three position sensors aimed at the top platform of the scanner, directly aligned with the probe tip which is mounted somewhere on the top platform. As there are three sensors all at an angle with respect to each other the $x, y, z$ position of the probe can be derived from the sensor readouts. This configuration should allow for a compact and narrow AFM scanner. Assuming that the scanner will be quite small it will be difficult to construct tiny ball hinges. Instead the concept could be realized out of one piece of material. This monolithic piece would use flexure hinges instead of ball joints. The main benefit of using such hinges is that there are no moving parts so there are no friction or wear effects. If the material is sufficiently stiff and light high mechanical resonance frequencies will be achieved. The actuators could be piezostacks as the required stroke of the actuators is small and the required resolution is high. On top of that piezostacks are typically capable of high speed operation. The sensors could be capacitive sensors.
1.4 Scope of the thesis

This thesis deals with finding the requirements of the AFM scanner as defined by the requirements of the AFM system. Two concepts will be introduced and analyzed. The first concept has just been introduced in the previous section and will be further analyzed in Chapter 3. The second concept called the orthogonal scanner concept is discussed in Chapter 4. On top of an analysis the second concept is also turned into a more detailed design by selecting its components in Chapter 5.

1.4.1 Main research questions

The main research questions are listed below.

1. How do the requirements of the AFM system translate to the requirements of the AFM scanner?
2. Does the tripod scanner concept meet the requirements?
3. Does the orthogonal scanner concept meet the requirements?
4. Is the orthogonal scanner concept valid as a real design?
Chapter 2

Requirements

2.1 Introduction

This chapter will answer the first main research question: *How do the AFM requirements translate to those of the AFM scanner?* First the AFM system requirements will be explained. Subsequently the accompanying AFM system specifications will be stated in Section 2.2. These specifications are translated to the requirements of the AFM scanner in Section 2.3.

2.2 AFM requirements and specifications

The requirements for the AFM system are listed below.

**AFM system requirements:**

- The AFM will be mounted on a flexible platform so it must be compact
- The measurement must be sufficient for microchip feature inspection
- Low imaging time when compared to commercial AFMs
- Ability to measure large samples
- Accurate scanning and measurement to enable defect inspection

The whole AFM system is to be mounted on some kind of flexible platform, for instance on a robot arm. This constrains the size of the AFM, it should be as small as possible. Furthermore the range requirement states that the range should be about average when compared to other AFMs. The AFM should also be able to be used in industrial inspection applications, which means that it could be used inline. For this reason the imaging time should be low in order to establish a high throughput. For the same industrial applications the AFM will measure large samples, this means the probe will be scanned. The measuring accuracy needs to be sufficiently high, somewhere in the order of nanometers.

All these requirements lead to the specifications stated below.

**AFM system specifications:**

- Frontal surface area \((x,y)\) smaller than 15 x 15 mm
- Measurement range at least 10 x 10 x 2 µm
- Imaging time under 1 second
- Probe scanning AFM
- Measurement accuracy <1 nm
2.3 AFM scanner requirements

The AFM scanner requirements depend directly on the specifications of the AFM system. The scanner size requirement, based on the AFM specification of the size directly limits the allowed dimensions of the AFM scanner to 15 x 15 mm frontal surface area. The specified AFM scanning range can be translated directly into the required scanner stroke as the scanner has to actuate the probe within this range. The measurement accuracy of about 1 nm and the imaging time of 1 s will be translated into AFM scanner requirements.

2.3.1 Translating the AFM measurement accuracy to scanner roll angles

The AFM measurement accuracy is mainly defined by which sensor system is used and how it is applied. As mentioned in Section 1.3, there are three position sensors all positioned in line with the probe tip. However, due to the mounting of the AFM chip and the mounting of the sensors, the alignment is never perfect. This imperfection combined with rotations of the scanner platform cause Abbe errors as described below. It is to be expected that this Abbe error is the largest source of inaccuracy for the AFM scanner.

Abbe error

The Abbe error is caused by an offset between a measuring axis and an actuation axis. The tripod concept assumes that the position sensors are all aligned with the probe tip mounted on top of the top platform. The point where the sensor alignments intersect is called the Abbe point. An estimation will be made of the Abbe error in order to find the maximum allowable rotation of the platform. A schematic of the top platform and the Abbe point is given in Figure 2.3.1. In this case the Abbe error is given by Equation 2.1.

\[
e_{\text{abbe}} = \delta \ast \tan(\phi)
\]

It is assumed that the offset \( \delta \) is 0.5 mm and the allowable Abbe error is 1 nm. The allowable rotation is \( \phi = \arctan(1 \times 10^{-3}) = 2 [\mu rad] \).

2.3.2 Translating the imaging time to resonance frequencies

Lateral scanning

Scanning an image of 500 x 500 pixels is considered. This means 500 lines are to be scanned in or under 1 second, and the data of each line is only recorded in one scan direction. This results in a scan rate of 1 kHz. The scanner motion is described by triangular wave. In order to get a sharp triangular shape as a rule of thumb a frequency of about 10 times the harmonic frequency, i.e., the scan rate is needed. This means that the resulting lateral actuation bandwidth should be at least 10 kHz and the lateral resonance frequencies should also be over 10 kHz.

Vertical scanning

The vertical scanning motion is the result of a closed loop feedback control system attempting to track the sample topography. The lateral scanning speed multiplied by the vertical topography feature size gives the measurement time and the frequency content which is to be tracked. The accuracy requirement of < 1 nm sets the allowable tracking error. The tracking error is also a function of the sensitivity.
Figure 2.3.1: The Abbe error for the scanner top platform

function of the AFM system and in this way the required actuation bandwidth can be calculated. In this case with a tracking error under 1 nm, a scanning speed of 10 mm/s gives a minimum actuation bandwidth over 30 kHz. The vertical resonance frequencies of the scanner should be over 30 kHz.

2.4 Conclusion

The requirements for the AFM scanner have been set up which are necessary to meet the specifications of the AFM system. The size requirement can be translated 1:1 to a scanner requirement and the same goes for the measurement range requirement. The accuracy requirement has been found to pose a limitation on the allowable rotations of the scanner, they should be kept under 2 µrad. The imaging time requirement in combination with the tracking error requirement has been translated into minimum resonance frequencies for the scanner, 10 kHz and 30 kHz in lateral and vertical directions respectively.

All of the resulting AFM scanner requirements are summarized below:

Requirements:

- AFM scanner frontal area < 15x15 mm
- Scanner stroke lateral > 10 x 10 µm
- Scanner stroke vertical > 2 µm
- AFM scanner resonance frequencies > 10kHz in x,y direction and >30kHz in z direction
- Scanner rotations < 2 µrad
- The probe must be mounted on the scanner
Chapter 3

Analysis tripod concept

3.1 Introduction

In order to answer the second main research question "Does the tripod scanner concept meet the requirements?" an analysis will be performed on said concept. The concept in its current state has a number of design parameters which are yet to be filled in. Examples of design parameters are the length of the rods, the size of the platform, the angle of the rods, the rod thickness etc. The requirements are constraints on these parameters. Investigating the influences of the design parameters might lead to a set of parameters where the requirements are met. A number of sub-questions stemming from this and the main research question are formulated below.

Subquestions

1. How is the x, y, z displacement of the scanner related to the actuator strokes and how do the design parameters influence this?

2. Will the required stroke of 10x10x2 microns be achieved?

3. How do the design parameters influence the roll angles?

4. Are the scanner roll angles under the allowed 2 μrad at the maximum required lateral displacement of 5 microns?

5. What is the relation between the design parameters and the resonance frequencies?

6. Are the scanner secondary resonance frequencies above the required frequencies of 10 kHz horizontal and 30kHz vertical?

7. What are the most important design trade-offs, can all requirements be met simultaneously?

In order to answer the first two questions a kinematic analysis is performed in Section 3.2. The kinematical model assumes rigid links and ideal hinges in order to study the motion. To answer subquestions 3 and 4 the static deformations need to be taken into account which is done with the static model described in Section 3.3. The fifth and second to last questions are answered by the dynamical model in Section 3.4 using both analytical and finite element models. The final question will be answered in Section 3.5.
3.2 Kinematics analysis

A schematic overview of the tripod is shown in Figure 3.2.1. The geometry parameters are defined as distances $r$, $p$ and $l$. $r$ is the distance from the $z$-axis to the position of the actuator at the base. $p$ is the distance from the $z$-axis to the points where the rods are attached to the top platform. The length of the rods is $l$. Using these three parameters the reverse kinematical equations can be constructed in order to find the actuator displacements $u_1, u_2,$ and $u_3$ as a function of $x$, $y$ and $z$. First, one "arm" is considered as seen from the top. The arm is placed at an angle $\theta$ from the $y$ axis, as detailed in Figure 3.2.2. The parameters $r$, $p$ and $l$ are now called $r_1$, $p_1$ and $l_1$ to allow for different parameters per arm.

First the coordinates in $x$ and $y$ of the attachment point of the rod to the top platform ($x_a$ and $y_a$) are calculated using straightforward goniometry:

\[
x_a = r_1 \sin(\Theta) - p_1 \sin(\Theta_1) = (r_1 - p_1) \sin(\Theta_1)
\]
\[
y_a = r_1 \cos(\Theta) - p_1 \cos(\Theta_1) = (r_1 - p_1) \cos(\Theta_1)
\]  

Subsequently it is imagined that the top platform is translated in the $x$ and $y$ direction, these displacements are added to $x_a$ and $y_a$. Then the length of the rod $l_{1,xy}$ as if the rod were projected on the $xy$ plane is calculated:

\[
l_{1,xy} = \sqrt{(x_a + x)^2 + (y_a + y)^2} = \sqrt{((r_1 - p_1)\sin(\Theta_1) + x)^2 + ((r_1 - p_1)\cos(\Theta_1) + y)^2}
\]  

Now the cross-section A-A as denoted in Figure 3.2.3 is considered. The vertical length of the rod when the top platform has not been displaced is given by the constant $z_0$:

\[
z_{0,1} = \sqrt{r_1^2 - (r_1 - p_1)^2}
\]  

The cross-section is displayed in Figure 3.2.4.
Figure 3.2.2: Top view of the tripod (1)

Figure 3.2.3: Top view of the tripod (2)
Figure 3.2.4: Side view of the tripod

It is assumed that the \( x, y \) displacement is caused by a positive displacement \( u_1 \) of actuator 1. Finally the the total length of the rod can be calculated as a function of \( u_1, x \) and \( y \):

\[
\ell_1 = (z_{0,1} - u_1)^2 + \sqrt{((r_1 - p_1) \sin(\Theta_1) + x)^2 + ((r_1 - p_1) \cos(\Theta_1) + y)^2}
\]

\[
\ell_2 = (z_{0,1} - u_1)^2 + ((r_2 - p_2) \sin(\Theta_2) + x)^2 + ((r_2 - p_2) \cos(\Theta_2) + y)^2
\]

\[
(u_1) = \sqrt{\ell_1^2 - ((r_1 - p_1) \sin(\Theta_1) + x)^2 + ((r_1 - p_1) \cos(\Theta_1) + y)^2}
\]

Note that the displacement of the top platform in the \( z \) direction has not been taken into account yet. This is trivial since \( z \) can be directly added to \( u_1 \) which yields the final equation for \( u_1 \):

\[
u_1 = -\sqrt{\ell_1^2 - ((r_1 - p_1) \sin(\Theta_1) + x)^2 + ((r_1 - p_1) \cos(\Theta_1) + y)^2} + z_{0,1} + z \quad (3.5)
\]

Where \( z_{0,1} \) is given by Equation 3.3. It is assumed that actuators 2 and 3 are placed at angles \( \Theta_2 \) and \( \Theta_3 \), respectively and that the associated parameters for length of the rod etc. are \( l_2, l_3, p_2, p_3, r_2 \) and \( r_3 \). This gives the final reverse kinematic equations for the AFM scanner concept:

\[
u_1 = -\sqrt{\ell_2^2 - ((r_2 - p_2) \sin(\Theta_2) + x)^2 + ((r_2 - p_2) \cos(\Theta_2) + y)^2} + z_{0,1} + z \quad (3.6)
\]

\[
u_2 = -\sqrt{\ell_3^2 - ((r_3 - p_3) \sin(\Theta_3) + x)^2 + ((r_3 - p_3) \cos(\Theta_3) + y)^2} + z_{0,1} + z \quad (3.7)
\]

\[
u_3 = -\sqrt{\ell_3^2 - ((r_3 - p_3) \sin(\Theta_3) + x)^2 + ((r_3 - p_3) \cos(\Theta_3) + y)^2} + z_{0,1} + z \quad (3.8)
\]

### 3.2.1 Linearization of the kinematics

The forward kinematical equations are very complex to acquire analytically as not every set of \( x, y, z \) can be found as a function of the set \( u_1, u_2, u_3 \). The forward
kinematics can be approximated and for this purpose Equations 3.6, 3.7 and 3.8 have been linearized. The equations are now put in matrix form:

\[
\begin{bmatrix}
    u_1 - u_{1,\text{nom}} \\
    u_2 - u_{2,\text{nom}} \\
    u_3 - u_{3,\text{nom}}
\end{bmatrix}
= \mathbf{A} \cdot \begin{bmatrix}
    x - x_{\text{nom}} \\
    y - y_{\text{nom}} \\
    z - z_{\text{nom}}
\end{bmatrix}
\]

with

\[
\mathbf{A} = \begin{bmatrix}
    \frac{\partial u_1}{\partial x} & \frac{\partial u_1}{\partial y} & \frac{\partial u_1}{\partial z} \\
    \frac{\partial u_2}{\partial x} & \frac{\partial u_2}{\partial y} & \frac{\partial u_2}{\partial z} \\
    \frac{\partial u_3}{\partial x} & \frac{\partial u_3}{\partial y} & \frac{\partial u_3}{\partial z}
\end{bmatrix}
\]

(3.9)

\(u_{1,\text{nom}}, u_{2,\text{nom}}\) and \(u_{3,\text{nom}}\) are the nominal positions of actuators 1 to 3. \(x_{\text{nom}}, y_{\text{nom}}\) and \(z_{\text{nom}}\) are the nominal \(x,y,z\) positions. The elements of matrix \(\mathbf{A}\) can be calculated by partially deriving Equations 3.6, 3.7 and 3.8 with respect to \(x, y\) and \(z\):

\[
\frac{\partial u_1}{\partial x} = \frac{(r_1 - p_1) \sin(\Theta_1) + x_{\text{nom}}}{\sqrt{l^2 - ((r_1 - p_1) \sin(\Theta_1) + x_{\text{nom}})^2 + ((r_1 - p_1) \cos(\Theta_1) + y_{\text{nom}})^2}}
\]

(3.10)

\[
\frac{\partial u_1}{\partial y} = \frac{(r_1 - p_1) \cos(\Theta_1) + y_{\text{nom}}}{\sqrt{l^2 - ((r_1 - p_1) \sin(\Theta_1) + x_{\text{nom}})^2 + ((r_1 - p_1) \cos(\Theta_1) + y_{\text{nom}})^2}}
\]

(3.11)

\[
\frac{\partial u_1}{\partial z} = 1
\]

(3.12)

\[
\frac{\partial u_2}{\partial x} = \frac{(r_2 - p_2) \sin(\Theta_2) + x_{\text{nom}}}{\sqrt{l^2 - ((r_2 - p_2) \sin(\Theta_2) + x_{\text{nom}})^2 + ((r_2 - p_2) \cos(\Theta_2) + y_{\text{nom}})^2}}
\]

(3.13)

\[
\frac{\partial u_2}{\partial y} = \frac{(r_2 - p_2) \cos(\Theta_2) + y_{\text{nom}}}{\sqrt{l^2 - ((r_2 - p_2) \sin(\Theta_2) + x_{\text{nom}})^2 + ((r_2 - p_2) \cos(\Theta_2) + y_{\text{nom}})^2}}
\]

(3.14)

\[
\frac{\partial u_2}{\partial z} = 1
\]

(3.15)

\[
\frac{\partial u_3}{\partial x} = \frac{(r_3 - p_3) \sin(\Theta_3) + x_{\text{nom}}}{\sqrt{l^2 - ((r_3 - p_3) \sin(\Theta_3) + x_{\text{nom}})^2 + ((r_3 - p_3) \cos(\Theta_3) + y_{\text{nom}})^2}}
\]

(3.16)

\[
\frac{\partial u_3}{\partial y} = \frac{(r_3 - p_3) \cos(\Theta_3) + y_{\text{nom}}}{\sqrt{l^2 - ((r_3 - p_3) \sin(\Theta_3) + x_{\text{nom}})^2 + ((r_3 - p_3) \cos(\Theta_3) + y_{\text{nom}})^2}}
\]

(3.17)

\[
\frac{\partial u_3}{\partial z} = 1
\]

(3.18)
A useful solution is the default position as shown in the tripod scanner concept overview: \{u_{1,\text{nom}}, u_{2,\text{nom}}, u_{3,\text{nom}}, x_{\text{nom}}, y_{\text{nom}}, z_{\text{nom}}\} = 0. This holds because when the top platform is at \((0, 0, 0)\) the actuator displacements are also 0. Substituting this solution into Equation 3.9 as well as substituting Equation 3.10 into 3.18 gives:

\[
\begin{bmatrix}
u_1 \\ u_2 \\ u_3 
\end{bmatrix} = \mathbf{A} \ast \begin{bmatrix} x \\ y \\ z \end{bmatrix} \tag{3.19}
\]

\[
\frac{\partial u_1}{\partial x} = \frac{(r_1 - p_1)\sin(\Theta_1)}{\sqrt{l^2 - ((r_1 - p_1)\sin(\Theta_1))^2 + ((r_1 - p_1)\cos(\Theta_1))^2}} \tag{3.20}
\]

\[
\frac{\partial u_1}{\partial y} = \frac{(r_1 - p_1)\cos(\Theta_1)}{\sqrt{l^2 - ((r_1 - p_1)\sin(\Theta_1))^2 + ((r_1 - p_1)\cos(\Theta_1))^2}} \tag{3.21}
\]

\[
\frac{\partial u_1}{\partial z} = 1 \tag{3.22}
\]

\[
\frac{\partial u_2}{\partial x} = \frac{(r_2 - p_2)\sin(\Theta_2)}{\sqrt{l^2 - ((r_2 - p_2)\sin(\Theta_2))^2 + ((r_2 - p_2)\cos(\Theta_2))^2}} \tag{3.23}
\]

\[
\frac{\partial u_2}{\partial y} = \frac{(r_2 - p_2)\cos(\Theta_2)}{\sqrt{l^2 - ((r_2 - p_2)\sin(\Theta_2))^2 + ((r_2 - p_2)\cos(\Theta_2))^2}} \tag{3.24}
\]

\[
\frac{\partial u_2}{\partial z} = 1 \tag{3.25}
\]

\[
\frac{\partial u_3}{\partial x} = \frac{(r_3 - p_3)\sin(\Theta_3)}{\sqrt{l^2 - ((r_3 - p_3)\sin(\Theta_3))^2 + ((r_3 - p_3)\cos(\Theta_3))^2}} \tag{3.26}
\]

\[
\frac{\partial u_3}{\partial y} = \frac{(r_3 - p_3)\cos(\Theta_3)}{\sqrt{l^2 - ((r_3 - p_3)\sin(\Theta_3))^2 + ((r_3 - p_3)\cos(\Theta_3))^2}} \tag{3.27}
\]

\[
\frac{\partial u_3}{\partial z} = 1 \tag{3.28}
\]

### 3.2.2 Forward kinematics

Now the linearized forward kinematics can be constructed, they are found by reversing Equation 3.19 and inverting the \(\mathbf{A}\) matrix:

\[
\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \mathbf{A}^{-1} \ast \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \tag{3.29}
\]

### 3.2.3 Matlab example

These equations have been entered into a Matlab script to provide an example. The following parameters are used.

\[
\begin{align*}
  r &= 0.007 \text{[m]} \\
  p &= 0.004 \text{[m]} \\
  l &= 0.006 \text{[m]}
\end{align*} \tag{3.30}
\]
The rod angle can be calculated as \( \phi = \arccos\left(\frac{r - p}{l}\right) = 30^\circ \). These number represent a scanner which fits within the 15 x 15 mm required surface area with a height of approximately 5 mm. The input is an AFM scanning pattern in x and y of 10 lines in a space of 10 x 10 \( \mu \)m. The x axis is the fast axis and the y axis is the slow axis while the z coordinate is kept constant at 0. The results are calculated using both the nonlinear equations and the linearized matrix. The resulting graphs are shown in Figure 3.2.5.

The maximum peak to peak displacement of the actuators in this case is 8 micrometers, resulting in an approximate mechanical amplification vertical to lateral of \( \frac{8 \mu m}{8 \mu m} = 1.25 \). This amplification factor depends on the rod angle. If the rod angle is steeper the mechanical amplification will go up. The lateral displacement is independent of the vertical displacement of the scanner, for instance if the same scanning pattern is to be performed at a z height of 2 \( \mu \)m the only difference is that all three of the actuators need to perform an added 2 \( \mu \)m displacement.

3.2.4 Conclusion

The analytical relations between the parameters and the displacements of the actuators versus the displacement of the scanner platform have been established. The required scanner stroke of 10 x 10 x 2 \( \mu \)m can be reached within the given requirements. In the example of Subsection 3.2.3 the amplification between the vertical displacements of the actuator and the vertical stroke is about 1.25. This means that in this case actuators should be chosen with a stroke of at least 10 \( \mu \)m as 8 \( \mu \)m is needed for the lateral displacements and 2 \( \mu \)m is required for vertical stroke. The vertical to lateral mechanical amplification factor depends only on the rod angle. The required stroke can always be achieved given enough actuator stroke versus the mechanical amplification. The ideal design parameters from a kinematical point of view are a rod angle of almost 0 degrees (almost completely vertical) as this gives the most lateral displacement of the platform as function of a vertical actuator displacement. The vertical platform displacement will always be at a ratio of 1 : 1 with the vertical actuator displacement.
3.3 Statics Analysis

The next two subquestions are *How do the design parameters influence the roll angles?* and *Are the scanner roll angles under the allowed 2 µrad at the maximum required lateral displacement of 5 microns?*. To answer these questions a static model is constructed in order to estimate the deformations in the tripod.

The goal of this analysis to ascertain whether these displacements add up to a net rotation or roll angle of the top platform over the x or y axis. This rotation could cause an Abbe error, as explained in Subsection 2.3.1 and for this reason the roll angle should be kept under 2 µrad.

3.3.1 Scanner rotations

If the scanner is assumed to be ideal like in the kinematical model, assuming perfect hinges and infinitely stiff rods, the top platform’s rotational degrees of freedom are fully constrained. This makes it impossible for the platform to rotate. However the hinges and rods are not ideal which removes the assumption of fully constrained rotations. Instead the scanner is made out of one monolithic piece of material. In the simplest form there would simply be 6 solid rods attached to a top platform. If one of the actuator displaces the rods bend, and assuming that the size of the scanner is quite small and that the rods are thin enough the rods should be sufficiently weak to allow a motion of the platform very similar to that analyzed in the kinematics. There is one drawback and that is that a small rotation will be induced on the platform. A schematic side view drawing of the rods bending and the rotation of the top platform is shown in picture 3.3.1

3.3.2 Deformation analysis

It is assumed that the flexure rods are considered as springs with stiffnesses in both the lateral and the longitudinal direction or basically as 3D cantilever beams. The goal of this static model is to simulate the behavior of the flexure springs and hinges. To get an estimate of the stiffness it is assumed that the flexure springs are square aluminum rods, of which the stiffness in longitudinal and lateral directions can easily be calculated using Hooke’s law. In this case the lateral stiffness is equal
Figure 3.3.2: Analytical model schematic overview of parameter to the bending stiffness of the rod.

\[ k_{\text{longitudinal}} = \frac{EA}{L} \]  
(3.31)

\[ k_{\text{lateral}} = \frac{3EI}{L^3} \]  
(3.32)

The model is set up such that there are three legs, one on the left at an angle \( \alpha \) and two on the right at an angle \( \beta \). At a zero degrees angle the legs are vertical. The analytical equations behind the model are thoroughly explained in Appendix B. A vertical displacement input \( u_1 \) is applied at the single leg on the left and the resulting lateral, vertical and angular displacements of the rigid top platform are calculated. A schematic view of the analytical model is shown in Figure 3.3.2.

The nominal parameters used in the model are the same as used in the kinematical analysis, except for the rod cross section \( (h \times w) = (1 \times 1)[\text{mm}] \) and the rod distance \( d = 3[\text{mm}] \).

The input \( u_1 \)

The results of have been graphed using Matlab in order to show the effects of varying specific parameters. This means that all other parameters are kept constant. As the resulting horizontal displacement \( x \) can vary with the parameters the input \( u_1 \) is chosen such that the \( x \) displacement is at a constant 5 [\mu m]. The required input as a function of the rod angle \( \alpha \) is shown in 3.3.3, including the results from the previous kinematical analysis.

The graph results match closely for rod angles over 45 degrees. At lower angles the assumption of ideal hinges does not hold anymore because then the deformations of the rods are significantly larger and the assumptions of the kinematical model don’t hold anymore.

Model results

The rod angle parameter is varied versus the resulting roll angle \( \phi \) in Figure 3.3.4. As the required input to reach the correct horizontal displacement is included there
appears to be an optimum at 80 degrees. However, the lowest roll angle at reached at that point is still over 440 µrad which is at least a factor 200 higher than the requirement. On top of that the required vertical input is about 40 µm. At the nominal angle of 30 degrees the roll angle is 921 µrad.

Furthermore the distance between the rods on the right, parameter \( d \), is also varied versus the roll angle \( \phi \) in Figure 3.3.5. Increasing this distance has some effect in decreasing the roll angle but not enough to reach the required 2 µrad.

**Stiffness ratio**

The stiffness ratio of the rods is the ratio between the longitudinal stiffness and the lateral bending stiffness. For the current nominal parameters the stiffness ratio is 144. The ratio can be altered by making the rods longer or by modifying the rods, for example by applying notch hinges to the rods as shown in Figure 3.3.6. The variation of the stiffness ratio versus the roll angle is graphed in Figure 3.3.7. The horizontal axis is logarithmical. This graph shows that the roll angle can be lowered to within the requirement if the stiffness ratio can be increased far enough.

**Finite element results**

A model with identical parameters has been set up with FEM software. The difference with the analytical model is that the FEM model is in 3D instead of 2D. The rigid top platform is represented by a 4 mm thick top platform. Three cases have been calculated: the tripod scanner with square legs as in the analytical model, the tripod scanner with simple flexure notch hinges and finally the tripod scanner with extreme notch hinges. In all analyses the vertical input is chosen such that the lateral displacement is 5 µm. The roll angle is calculated by probing the vertical displacements at both ends of the platform and then applying the goniometrical equation: \( \phi = \tan^{-1}\left(\frac{z_2 - z_1}{l_{\text{platform}}}\right) \).
Figure 3.3.4: Analytical static model results: rod angle \( \alpha \) vs. roll angle \( \phi \)

Figure 3.3.5: Analytical static model results: \( d \) vs. roll angle \( \phi \)
Figure 3.3.6: Flexure notch hinges

Figure 3.3.7: Analytical static model results: stiffness ratio vs. roll angle $\phi$
The first case is displayed in Figure 3.3.8. The resulting roll angle $\phi$ is about 460 $\mu$rad. This is a factor two lower than predicted by the analytical model, this can be explained by the applied vertical input. In the 3D FEM case the required input turned out to be half the required input needed in the analytical model, 5 $\mu$m versus 10 $\mu$m.

The second case is displayed in Figure 3.3.9. The roll angle $\phi$ is about 360 $\mu$rad. The notch hinges are 1 mm in diameter and the thinnest part of the rod is now 0.2 mm. With respect to the previous case the roll angle is decreased by almost 20% but it is still way too high. The roll angle is reduced as the notch hinges affect the total stiffness ratio of the rods. The stiffness ratio is increased and this decreases the roll angle.

The second case is displayed in Figure 3.3.10. The notch hinges have been revolved around the rods, resulting in an even lower stiffness ratio. The roll angle $\phi$ is now about 60 $\mu$rad. This is still a factor 30 higher than the requirement.

3.3.3 Conclusion

The relations between the different design parameters and the roll angles have been investigated. The main cause of rotation of the platform is that the end of a thin rod is stiff not only in the longitudinal but also in the lateral direction and a torsional stiffness. A rod end of a hinged rod is ideally only stiff in the longitudinal direction. The ratio between the longitudinal and the lateral stiffnesses determines the amount of rotation induced on the platform. An increase in the rod angle $\alpha$ up to 80 degrees will decrease the roll angle but not enough to be within the requirement. The distance between a set of two rods when increased can also help to keep the roll angle down, but only to a certain point. The most effective method to reduce the roll angle is to adjust the stiffness ratio of the rods. However even when applying extreme notch hinges to the rods (Figure 3.3.10) the lowest roll angle found is 60 $\mu$rad. This is a factor 30 above the requirement.
Figure 3.3.9: FEM result of scanner with simple notch hinge flexure rods

Figure 3.3.10: FEM result of scanner with extreme notch hinge rods
3.4 Dynamics analysis

The following subquestions will be answered in this section:

- What is the relation between the design parameters and the resonance frequencies?

- Are the scanner resonance frequencies above the required frequencies of 10 kHz horizontal and 30 kHz vertical?

3.4.1 Modal Analysis

First the mode shapes and resonance frequencies will be considered in a modal analysis. The 6 rods which make up the tripod are represented by three rods, one rod for each set of two. The stiffness per rod is doubled. The top platform is considered as a rigid disc with a certain mass and rotational inertia. Considering these assumptions predictions can be made regarding the eigenmodes of the system and the corresponding eigenfrequencies. It is also assumed that the tripod is symmetrical around the z-axis every 120 degrees and that the rods are identical. This means that the stiffnesses are the same for each rod, only rotated by 120 degrees. The resulting eigenmodes must also be symmetrical in the same way.

The first four eigenmodes that can be expected on the basis of these assumptions are shown in Figure 3.4.1. They are the lateral and vertical translation modes as well as the roll and yaw modes.
Analytical stiffness calculation

The total vertical and horizontal stiffness can be approximated by setting up a total stiffness matrix describing the stiffness experienced by the top platform in x y and z directions. Based on this stiffness matrix and an approximation of the mass of the top platform the eigenfrequencies can be calculated. These matrices have been set up in Matlab and the used m-code is shown in Appendix D.

For a certain rod angle the total vertical stiffness will be higher than the lateral stiffnesses. The same modal mass can be assumed for all these eigenmodes so the corresponding eigenfrequency for vertical translation will therefore be higher than the other three eigenfrequencies. Depending on whether or not flexure notch hinges are applied to the rods either the total roll torsional stiffness or the lateral stiffness will be lower. The yaw mode can be expected to be in between the roll and lateral translation eigenmodes and the vertical translation eigenmodes.

A numerical example has been calculated using Matlab. The following parameters have been used:

\[
\begin{align*}
  r &= 0.006[m] \\
  p &= 0.004[m] \\
  l &= 0.005[m] \\
  d &= 0.004[m] \\
  w &= 0.001[m]
\end{align*}
\]

It is assumed that there is an aperture in the top platform of 6 mm in diameter. Other assumptions are that the top platform is fully rigid and that the material used for the rods is aluminium, with a Young’s modulus of \( E = 69 GPa \). The bases of the legs are assumed to be rigidly fixed to mechanical ground. The resulting total stiffnesses as calculated with the Matlab model in vertical direction is 79,5 \( \frac{N}{\mu m} \), in lateral (x) direction the total stiffness is 0,83 \( \frac{N}{\mu m} \).

FEM modal analysis

A finite element model has also been set up to evaluate the eigenmodes. First they have been drawn in Unigraphics NX 7.5 and subsequently imported into ANSYS. Using a static analysis similar to that in the previous section the stiffnesses in vertical and lateral direction have been established. These are found to be 58,8 \( \frac{N}{\mu m} \) in vertical direction and 2,38 \( \frac{N}{\mu m} \) in lateral direction. The vertical stiffness is in the same order of magnitude but the found lateral stiffness is much higher than the analytically calculated stiffness. This can be explained by the fact that the lateral stiffnesses of the rods are not equal in all directions as the rod’s cross section is a square but the analytical model does assume the same lateral stiffness in every direction.

Using the FEM model a modal analysis has been performed. The first six eigenmodes and frequencies are shown in Figures 3.4.2 to 3.4.7. Eigenmodes number 1 and 2 are lateral translation eigenmodes, each in one of the three directions of the tripod. The model is symmetrical but the mesh generated by the finite element software to solve the eigenmodes is not perfectly symmetrical. This could explain why the eigenfrequencies of the first two modes are the same but not exactly equal: 9,343 and 9,347 kHz. There are three eigenmodes each for each symmetry direction at identical frequencies. The third eigenmode is the yaw eigenmode, at 9,76 kHz.
Figure 3.4.2: FEM result: Tripod concept eigenmode 1, lateral translation

Figure 3.4.3: FEM result: Tripod concept eigenmode 2, lateral translation
Figure 3.4.4: FEM result: Tripod concept eigenmode 3, yaw

Figure 3.4.5: FEM result: Tripod concept eigenmode 4, roll
Figure 3.4.6: FEM result: Tripod concept eigenmode 5, roll

Figure 3.4.7: FEM result: Tripod concept eigenmode 6, vertical translation
The yaw mode should not pose a problem for the accuracy of the scanner as the tip of the cantilever is supposed to be centered on the platform. The next two modes are the roll modes. The eigenfrequencies are 42,044 and 42,053 kHz. This outcome is comparable to that of the lateral translation modes as the two frequencies are actually identical. The roll and yaw modes are unobservable modes as the sensors on the scanner will not be able to pick up the roll modes, instead an induced roll will affect the accuracy as is described in Section 3.4.4. Furthermore the last mode, the vertical translation has the highest associated eigenfrequency as was expected. This frequency is 48.3 kHz. There are more eigenmodes at higher frequencies which are not shown here.

Using \( \omega = \sqrt{k/m} \) an estimation can be made of the modal mass of the FEM solution. For the vertical and lateral translation modes it has been found to be about 0.5 gram.

**Analytical modal analysis**

The mass used in the analytical model is simply calculated by multiplying the volume of the top platform minus the volume of the aperture with the density of aluminium. Matlab gives the value of 0.55 gram which matches closely with the FEM estimation. The eigenfrequencies can be found by applying \( \omega_n = \sqrt{k_n/m_n} \) for each \( n^{th} \) diagonal elements of the stiffness and mass matrix. The roll and yaw modes are not estimated by the analytical model.

<table>
<thead>
<tr>
<th>Mode type</th>
<th>Frequency Matlab</th>
<th>Frequency Ansys</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lateral translation</td>
<td>6.21 kHz</td>
<td>9.35 kHz</td>
</tr>
<tr>
<td>Yaw</td>
<td>N/A</td>
<td>9.76 kHz</td>
</tr>
<tr>
<td>Roll</td>
<td>N/A</td>
<td>42.0 kHz</td>
</tr>
<tr>
<td>Vertical translation</td>
<td>60.9 kHz</td>
<td>48.3 kHz</td>
</tr>
</tbody>
</table>

The two most important modes, the translation in vertical and lateral direction are a over- and underestimated respectively but the orders of magnitude do match. This means that the analytical model is useful for a rough approximation.

### 3.4.2 Influence of the design parameters

The analytical model is used to quickly evaluate effects of changing the design parameters. Assuming the parameters used earlier it is interesting to see how changing for example the parameters \( \alpha \) or \( l \) influences the eigenfrequencies, shown in Figure 3.4.8 and Figure 3.4.9. The resonance frequencies should be as high as possible to raise the actuation bandwidth as high as possible and the ratio between the lateral and vertical translation modes should be proportional to the ratio of required actuation bandwidths.

The graph shows that the vertical translation eigenfrequency \( \omega_z \) is highest when the rods are fully vertical when \( \alpha = 0[^\circ] \). As the rod angle increases the lateral frequency \( \omega_x \) also increases while \( \omega_z \) decreases. This is explained by the longitudinal stiffness of the rods, when rods are more slanted the longitudinal stiffness also contributes to the total lateral stiffness. Decreasing the rod length leads to overall higher resonance frequencies. The rod length is an important factor in the longitudinal to lateral stiffness ratio of the rods. Another parameter is \( w \), the thickness of the rods. Increasing it should increase the stiffness in all directions and raise the eigenfrequencies. As shown in 3.4.10 this results in higher resonance frequencies in both vertical and lateral direction. In general the rod angle and length can be used to tweak the ratio between vertical and lateral resonance frequencies and the rod thickness to increase it.

To validate the found relations between the parameters and resonance another FEM model is set up. This time the rod angle is increased from 11.5 degrees to 30
Figure 3.4.8: Dynamics model result: $\alpha$ vs. the eigenfrequencies

Figure 3.4.9: Dynamics model result: $l$ vs. the eigenfrequencies

Figure 3.4.10: Dynamics model result: $w$ vs. the eigenfrequencies
degrees by changing \( r \) from 5 mm to 7 mm and the rod length \( l \) is slightly decreased from 6 to 5 mm. The corresponding analytical and finite element approximations of the vertical and lateral translation eigenmodes are shown below:

<table>
<thead>
<tr>
<th>Mode type</th>
<th>Frequency Matlab</th>
<th>Frequency Ansys</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lateral translation</td>
<td>4.87 kHz</td>
<td>13.3 kHz</td>
</tr>
<tr>
<td>Vertical translation</td>
<td>48.9 kHz</td>
<td>42.9 kHz</td>
</tr>
</tbody>
</table>

### 3.4.3 Actuator interface

The modal analysis so far is based on the assumption that the ends of the rods are attached to mechanical ground. In reality they are to be attached in some way to the actuators. It is assumed that the actuators are piezo stacks that need to be preloaded which means that the interface between the piezo actuators and the rods needs to be stiff with respect to mechanical ground. On top of that the stiffness also needs to be sufficient to maintain the overall stiffness from the top platform to the mechanical ground. An example interface is shown in Figure 3.4.11.

In this example there are thick cantilevers with applied notch hinges between the piezo actuators and the rods. To support the cantilevers and piezo stacks a solid frame has been implemented. The entire model is assumed to be made out of solid aluminium (including the piezo stacks) and the cantilevers are dimensioned as 4 x 4 x 2 mm blocks suspended by notch hinges.

These interfaces will affect the eigenmodes and eigenfrequencies found earlier, they will probably be lower as the total stiffness between the top platform and the mechanical ground will decrease. On top of that there will be additional undesired eigenmodes. This is because these cantilevers themselves also resonate at a certain frequency. To investigate the additional undesired eigenmodes another finite element model has been set up. The resulting eigenmodes and frequencies are listed here:
Undesired eigenmodes with frequencies between those of the lateral and vertical translation resonances are found. This is due to the interaction between the cantilevers, the piezo stacks and the rods. The undesired yaw mode, shown in Figure 3.4.12, is actually a second order torsional mode where the entire solid frame and the platform are twisting opposite to each other. The other undesired mode found is an additional roll mode, displayed in Figure 3.4.13. This eigenmode is caused by the resonance of the aluminium stacks which represent the piezo stacks which is transferred through the cantilevers to the rods.

These undesired modes will be problematic for the actuation bandwidth which is why their eigenfrequencies should be moved as far away from the primary translation mode frequencies as possible. In this example this could be done by substantially increasing the torsional stiffness around the $z$-axis of the frame and by selecting piezo stacks and interfaces that are stiff enough.

### 3.4.4 Frequency Response

It has been discovered that there are undesired eigenmodes, such as the roll and yaw modes and the parasitic eigenmodes due to the actuator interface. These eigenmodes might influence the accuracy of the AFM if they are excited since they can induce additional roll and yaw to the top platform hence inducing Abbe errors. In order to see whether these modes will be excited a frequency response analysis has been done for the finite element model of the scanner including the interface and the piezo actuators. A tool called ansys2bode has been used for this analysis. This tool uses the finite element Ansys results and loads them into Matlab in order to acquire the frequency response by using mode superposition. This analysis assumes a force as an input, in this case on the interface of one of the three actuators. There are five outputs, three translations and two rotations. The translations are defined as the mean displacement in the respective $x$, $y$ and $z$ directions. The rotations,
Figure 3.4.13: FEM example parasitic roll mode

Figure 3.4.14: Frequency response $x$, $y$ and $z$

denoted $\theta_x$ for the rotation around the x axis and $\theta_y$ for the y axis. The rotations are defined as the difference in $z$ of two nodes at opposite ends of the platform. This makes for a useful approximation of the angles, albeit not exactly the angle. The resulting magnitude Bode plots for are drawn in Figure 3.4.14 and Figure 3.4.15.

In the first Bode plot the first three lateral resonance frequencies are located at 7.8 kHz (lateral translation), 12.9 kHz (roll mode) and 22.1 kHz (parasitic roll mode). The frequency response is more or less the same in $x$ and $y$ since an actuator is chosen that moves the platform both in $x$ and $y$. The first two main vertical resonances are found to be at 23.0 kHz and 26.9 kHz, the first is a parasitic mode and the second is the vertical translation mode. This means that these eigenmodes do not meet the requirements. The second Bode plot shows the rotation frequency responses. The resonance frequencies correspond directly to those of the lateral displacement resonance frequencies. The first resonance at 7.8 kHz is supposed to be a pure lateral translation mode, however there evidently is also some roll associated with this mode, and this can also be seen in the mode shape, for example in Figure
3.4.2 The largest source of rotations is the third mode at 22.6 kHz, also shown in Figure 3.4.13 which is a parasitic roll mode.

3.4.5 Conclusion

The relation between the resonance frequencies of the scanner and its design parameters have been investigated. The resonance frequencies mainly depend on the total stiffness in each direction and the mass of the top platform of the scanner. It is found that changing the rod angle or the rod length changes the ratio between vertical and lateral resonance frequencies. In the case of the rod angle the vertical stiffness is being sacrificed for a higher vertical stiffness. If the rod length is increased both the vertical and lateral stiffness drop but very short rods also mean that both stiffnesses can be raised. Increasing the rod thickness also increases both stiffnesses and both resonance frequencies. A set of parameters has been found where the lateral translation eigenfrequency is 13.3 kHz and the vertical translation eigenfrequency is 42.9 kHz, both within the requirements.

Furthermore the effect of interfaces between the actuators and the tripod has been investigated. The effect is that the resonance frequencies overall are lower, 7.8 kHz lateral and 26.7 kHz vertical. On top of that additional undesired eigenmodes are introduced within the 30 kHz limit, at 16.9 kHz there is an undesired yaw mode and at 22.6 kHz an undesired roll mode occurs. These undesired eigenmodes can be moved to higher frequencies by adding extra stiffness to or lowering the mass of the components which induce them.

Subsequently a full frequency response has been simulated with use of the finite element model to investigate the effect of the dynamics on the scanner accuracy. Bode plots have been constructed for each of the degrees of freedom of the top platform of the scanner. The undesired roll mode shows a peak at 22.6 kHz. As this eigenmode is within the 30 kHz limit it will affect the accuracy of the scanner as it will be excited by the scanner motion.
3.5 Conclusion

The question to be answered here is:
What are the most important design trade-offs, can all requirements be met simultaneously?

The kinematics of the scanner are not the limiting factor. Even if the scanner has a very poor vertical to lateral mechanical amplification factor this can be compensated by selecting actuators with a higher stroke. In most cases the measurement range requirement can be met.

The static behavior of the tripod scanner concept is an issue as the lowest found roll angle is about 60 μrad. In very extreme cases it could be possible to get the roll angle within the required 2 μrad but to achieve this the stiffness ratio of vertical to lateral stiffness will become so high that the lateral stiffness will be very minimal. This is a constraint for the dynamic behavior as sufficient overall lateral stiffness is needed to get the lateral translation eigenfrequency over 10 kHz. On top of that the piezo interface also lower the overall stiffness.

This is the principal trade-off of the tripod scanner concept and within the design parameters it is impossible to find a set where both the roll angle and dynamical requirements are met.
3.6 Tripod concept improvements

The requirements of the roll angle and the resonance frequency can not be met at the same time. Fundamental improvements are needed to make this concept work. Two possible attempts to meet all of the requirements are presented. The first attempt is to investigate whether scaling up the entire scanner will be sufficient leading to the following question: Are the requirements met if the allowed dimensions are increased? The second idea is to add preloaded springs in such a way that zero stiffness of the scanner can be achieved and hopefully affect the roll angle. The second question that will be answered is: Can the requirements be met by statically balancing with preloaded springs?

3.6.1 Increasing total scanner size

The analytical model of Section 3.3 is used to graph the influence of increasing the outer diameter versus the roll angle $\phi$. In this analysis it is assumed that the rod thickness is the same and that the rod angle and top platform size are constant. This results in the graph shown in Figure 3.6.1

To reduce the roll angle under 2 $\mu$rad means that the total scanner size must be increased to over 6.8 meter. This is several orders of magnitude over the required specification and this is unacceptable.

3.6.2 Static balancing

The roll of the platform is caused by the ratio of vertical to lateral stiffnesses in the rods. By statically balancing the platform the apparent lateral stiffness can be lowered increasing the stiffness ratio. A single parallelogram can be modeled as shown in Figure 3.6.2

Adding a preloaded spring will statically balance the parallelogram. As the top moves in lateral direction effectively the spring will relax which introduces negative stiffness. If the spring is sufficiently preloaded this stiffness will cancel out the lateral stiffness of the parallelogram as displayed in Figure 3.6.3. This can affect the roll angle.
Figure 3.6.2: Parallelogram representation of the scanner rods

Figure 3.6.3: Zero stiffness parallelogram
Calculating the required preload

The lateral stiffness is 625 kN/m according to the ANSY FEM simulation. The rod length $L$ is 6 mm. The vertical offset of the spring $k$ is assumed to be 2 mm on both sides. The skewing angle of the parallelogram is denoted as $\psi$. The force due to the lateral stiffness is:

$$F_x = k_{lat} \cdot x = k_{lat} \cdot \sin(\psi) \cdot L \quad (3.34)$$

The spring force can be calculated as:

$$F_{spring} = F_{preload} - (k_{spring} \cdot \delta_{spring}) \cdot \sin(\psi) \quad (3.35)$$

Where the spring displacement is $\delta_{spring} = 2a+L-\sqrt{(2a+L \cdot \cos(\psi))^2 + (L \cdot \sin(\psi))^2}$

which ultimately yields

$$F_{preload} = (k_{spring} \cdot -\delta_{spring} + k_{lat} \cdot L) \quad (3.36)$$

Based on the term $k_{lat} \cdot L$ alone the preload is about 3750. This is in the order of kilonewtons, an order of magnitude higher than the internal forces. The vertical stiffness is about 20 MN/m, however 3.75 kN would cause unacceptable deformations in the order of hundreds of microns. Statically balancing the tripod concept is therefore not feasible using vertical preloaded springs.
Chapter 4

Analysis orthogonal scanner concept

4.1 Introduction

4.1.1 New concept

A new concept is needed. The fundamental flaw of the previous concept appeared to be the fact that the rods were loaded at an angle and not purely in a longitudinal direction. This is why the rods should be placed in such a way that the forces are along the longitudinal direction of the rods and orthogonal with respect to each other. The new concept is hence called the orthogonal scanner. An overview is shown in Figure 4.1.1.

The general idea behind this concept is that there are levers, shown as the red triangles, which transfer the vertical forces of the actuators into lateral forces to achieve lateral displacement. The actuators are vertically placed piezoelectric stack actuators just like the previous tripod scanner concept, shown in blue in Figure 4.1.1. The sensors are the same as in the previous concept. Three positions sensors are aligned with the probe tip, this means that the same rules apply for the Abbe error. The difference with the tripod concept is that there are now specific actuators for the lateral and vertical directions. This allows for a set of rods per actuation direction so the ratio between longitudinal and lateral stiffness will be highest. This should result in low roll angles. An alternative configuration is displayed in Figure 4.1.2 where the amount of levers is doubled which allows preloaded springs to be added opposite to the piezo actuators. This configuration can be made even more compact by inverting the levers as shown in Figure 4.1.3.

4.1.2 Subquestions

The subquestions apply in the same way as with the tripod scanner and are listed below. They will be answered in Sections 4.2 to ??.

1. How is the x, y, z displacement of the scanner related to the actuator strokes and how do the design parameters influence this?

2. Will the required stroke of 10x10x2 microns be achieved?

3. How do the design parameters influence the roll angles?

4. Are the scanner roll angles under the allowed 2 µrad at the maximum required lateral displacement of 5 microns?
Figure 4.1.1: The orthogonal scanner concept

Figure 4.1.2: Alternative orthogonal scanner concept
5. How do the design parameters influence the roll angles?

6. What is the relation between the design parameters and the resonance frequencies?

7. Are the scanner secondary resonance frequencies above the required frequencies of 10 kHz horizontal and 30 kHz vertical?

8. What are the most important design trade-offs, can all requirements be met simultaneously?

### 4.2 Kinematics analysis

The kinematics are parallel as with the tripod scanner concept. In this case every actuation direction has its own dedicated actuator. In the case of vertical translation the scanner displacement is equal to the $z$ actuator displacement. In the case of horizontal translation ($x$ and $y$) the displacement of the actuators are amplified by the width to height ratio of the levers. An overview is shown in Figure 4.2.1. A detailed of the lever is displayed in Figure 4.2.2 where the actuator displacement is noted as $u_{\text{lever}}$. The resulting lateral displacement is shown as $u_x$. The lever itself has width $a$ and length $b$. Assuming that the rotation of the lever $u_{\phi}$ is small the relation between $u_{\text{lever}}$ and $u_x$ can be stated as in Equation 4.1.

$$u_x = \frac{b}{a} u_{\text{lever}} \quad (4.1)$$

The range of the scanner is not fundamentally limited by the geometry. If the rotations of the levers are small there is a direct linear relation between the actuator displacements and the scanner displacements.
Figure 4.2.1: Orthogonal scanner overview

Figure 4.2.2: Lever schematic
4.3 Statics analysis

In terms of forces the levers transfer the actuator force into a lateral force and a moment on the horizontal rods. This moment will be transferred to the top platform causing it to roll. The length to width ratio of the vertical rods is low, in the order of 1:10, this means that the assumptions for an ideal rod are not true and that the end of the rods will also result in a moment on the top platform.

4.3.1 Analytical model

When comparing Figure 4.2.1 to Figure 3.3.2 the similarities are obvious. The same analytical can be used again although with some adjustments. A schematic overview of the revised analytical model for the orthogonal scanner is shown in Figure 4.3.1. The main difference with the previous analytical model is that instead of one input there are now three separate inputs, two translations and one rotation.

The nominal set of parameters is given below:

\[
\begin{align*}
L &= 0.003[m] \\
L_2 &= 0.010[m] \\
r &= 0.005[m] \\
p &= 0.003[m] \\
d &= 0.002[m] \\
h, w &= 0.001[m] \\
h_2, w_2 &= 0.001[m]
\end{align*}
\] (4.2)

The parameters \( h \) and \( w \) are the height and width of the horizontal rod and parameters \( h_2 \) and \( w_2 \) are the height of the vertical rods. The rod cross-section is assumed to be a square. The top platform is assumed to be rigid and the vertical rods are assumed to be clamped to mechanical ground. First off a number of test cases is run assuming that the input is a pure horizontal translation of 5 \( \mu m \).
Figure 4.3.2: Orthogonal scanner FEM model

length of the rods is varied. The outcome of the analytical for these cases is listed below:

<table>
<thead>
<tr>
<th>Parameters</th>
<th>( \phi ) analytical model</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L = 3[mm] ) ( L_2 = 10[mm] )</td>
<td>22,0 ( \mu )rad</td>
</tr>
<tr>
<td>( L = 3[mm] ) ( L_2 = 6[mm] )</td>
<td>49,9 ( \mu )rad</td>
</tr>
<tr>
<td>( L = 6[mm] ) ( L_2 = 6[mm] )</td>
<td>64,9 ( \mu )rad</td>
</tr>
</tbody>
</table>

A FEM model has also been set up to validate the results of the analytical model. The model itself is a MAPLE file shown in Appendix E. An example is shown in Figure 4.3.2. The input is the same as with the analytical model, 5 \( \mu \)m pure horizontal translation. The FEM analyses are run two times. The out of plane rods, the horizontal rods in the y direction, are not fixed to mechanical ground in the first run but they are in the second run. The analytical model does not take these out of plane rods into account.

The previous results of the analytical model as well as the FEM results are displayed below:

<table>
<thead>
<tr>
<th>Parameters</th>
<th>( \phi ) analytical</th>
<th>( \phi ) FEM</th>
<th>( \phi ) FEM out of plane rods</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L = 3[mm] ) ( L_2 = 10[mm] )</td>
<td>22,0 ( \mu )rad</td>
<td>22,9 ( \mu )rad</td>
<td>22,0 ( \mu )rad</td>
</tr>
<tr>
<td>( L = 3[mm] ) ( L_2 = 6[mm] )</td>
<td>49,9 ( \mu )rad</td>
<td>47,8 ( \mu )rad</td>
<td>43,0 ( \mu )rad</td>
</tr>
<tr>
<td>( L = 6[mm] ) ( L_2 = 6[mm] )</td>
<td>64,9 ( \mu )rad</td>
<td>57,9 ( \mu )rad</td>
<td>57,5 ( \mu )rad</td>
</tr>
</tbody>
</table>
The analytical results match the finite element modeling results. The effect of the out of plane rods is always slightly decreasing the roll angle. This can be expected as these rods add extra torsional stiffness to the platform.

The rotation and vertical displacement of the lever are taken into account as well as shown earlier in Figure 4.2.2. The nominal values for the lever dimensions are assumed to be $a = 4.0[mm]$ and $b = 5.0[mm]$. The relation between the lever input and the horizontal displacement has already been found in Section 4.2. The rotation can be calculated by using $u_\phi = \arcsin\left(\frac{u_\text{lever}}{a}\right)$. And similarly the vertical translation can be found with $u_z = b - b \cdot \cos(u_\phi)$. In the case where $u_x = 5.0[mm]$ the lever input $u_\text{lever} = 4.0[mm]$ resulting in $u_\phi = 1.0[\mu\text{rad}]$ and $u_z = -2.5[\text{mm}]$. Again a comparison is made with a FEM model which is shown in Figure 4.3.3. It can be seen that the lever in the FEM model is in fact inverted but with a negative lever input this still results in the same rotation and vertical translation. The lengths of the vertical rods are varied similarly to before. The results are listed below:

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$\phi$ analytical</th>
<th>$\phi$ FEM</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L = 3[mm]$</td>
<td>$L_2 = 10[mm]$</td>
<td>-82.1 $\mu$rad</td>
</tr>
<tr>
<td>$L = 3[mm]$</td>
<td>$L_2 = 6[mm]$</td>
<td>-28.4 $\mu$rad</td>
</tr>
<tr>
<td>$L = 6[mm]$</td>
<td>$L_2 = 6[mm]$</td>
<td>+123.1 $\mu$rad</td>
</tr>
</tbody>
</table>

The results do not match as nicely as they did in the case of the pure horizontal
translation. Nonetheless the orders of magnitude are roughly the same and the same trend can be seen. When varying the length of the vertical rods there is a crossover point where the roll angle $\phi$ goes from negative to positive. If the length is set exactly at this point the theoretical roll angle should be 0. This turning point can be explained by thinking of the total roll as a superposition or linear combination of the results of a pure horizontal translation input and that of the rotation input. The horizontal translation $u_x$ results in a positive roll angle, as has been established at the beginning of this Section. The main cause of this rotation is the ratio between lateral and longitudinal stiffness in the vertical rods. This effect is shown in Figure 4.3.4. The effect of a rotation $u_\phi$ is a negative roll angle as the moment on the end of the rod results in an opposite moment at the base of the horizontal rod as drawn in Figure 4.3.5.

The variation of the vertical rod length $L_2$ has been graphed versus both the roll angle and the lateral displacement in Figure 4.3.6. The analytical predicts that at roughly $L_2@ = 5$ [mm] the roll angle should be 0. Interpolation of the FEM result gives roughly $L_2 = 7$ to 8 mm. This is a substantial discrepancy of about 50 percent and can be explained by the deformations in the lever structure as well as the deformations in the top platform. The analytical model assumes these parts while the FEM model does not. The analytical prediction is useful as a starting point to find the cross over point by using multiple FEM iterations. The graph also shows that the lateral displacement decreases as $L_2$ decreases.

All the parameters have been varied using the analytical model to find further improvements, the input $u_{lever}$ has been such that the lateral displacement is constant at 5 $\mu$m. The nominal value of $L_2$ has been changed to 5 mm, at the theoretical cross over point. The results are shown in Appendix F. The outcome is that the
set of parameters $L, L_2, r, d, h, w, h_2, w_2$ are already on or near their optimal values and that increasing parameter $p$ has a further decreasing effect on the roll angle.

The sensitivities of the roll angle versus the parameter variation at the cross over point have also been established from the graphs. They are listed below with the corresponding nominal points at which they have been found. The sensitivity is given in $\mu$rad/mm.

<table>
<thead>
<tr>
<th>Parameter value [m]</th>
<th>Sensitivity $[\mu$rad/mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L = 0.003$</td>
<td>33</td>
</tr>
<tr>
<td>$L_2 = 0.005$</td>
<td>44</td>
</tr>
<tr>
<td>$r = 0.005$</td>
<td>10</td>
</tr>
<tr>
<td>$p = 0.003$</td>
<td>1,1</td>
</tr>
<tr>
<td>$d = 0.002$</td>
<td>15</td>
</tr>
<tr>
<td>$h = 0.001$</td>
<td>170</td>
</tr>
<tr>
<td>$w = 0.001$</td>
<td>65</td>
</tr>
<tr>
<td>$h_2 = 0.001$</td>
<td>200</td>
</tr>
<tr>
<td>$w_2 = 0.001$</td>
<td>70</td>
</tr>
</tbody>
</table>

The conclusion that can be drawn is that the manufacturing tolerances for $h$ and $h_2$ should be in the micrometer range. A difference of 1 $\mu$m for $h_2$ would mean a roll angle difference of 0,2 $\mu$rad.

4.3.2 FEM optimization

The objective of this FEM optimization is to get the roll angle $\phi$ as close to zero as possible. For this reason the configuration with only two levers shown earlier in Figure 4.1.1 is less suitable than the configurations with four levers. This is because a vertical displacement of the platform will also induce a roll angle due to the asymmetry of the two levers. A FEM model has been set up to resemble Figure 4.1.3 except there are only two levers on opposite ends of the platform. The other two levers are added later on in this analysis, instead the horizontal rods perpendicular to the actuation direction are assumed to be fixed to mechanical ground. The levers are now using notch hinges with a diameter of 1,0 mm and the thinnest cross-section is 0,2 mm. The bottom end of the notch hinges are fixed to mechanical ground. The length of the vertical rods is set to 5 mm as was found to
be the optimal value according to the analytical model. The horizontal center to vertical rod distance \( p \) has been raised to 5 mm. The result is displayed in Figure 4.3.7.

The lever input \( u_{\text{lever}} \) is 7.5 \( \mu \text{m} \), this is needed to raise the lateral displacement towards 5 \( \mu \text{m} \). The resulting roll angle is -8.3 \( \mu \text{rad} \). Using the same FEM model the length of the vertical rods has been varied from 5.0 mm to 4.1 mm. The resulting roll angles and lateral displacements are listed in the table below.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>( L_2 )</th>
<th>( \phi ) FEM analysis</th>
<th>( x ) FEM analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L_2 = 5.0 \text{ mm} )</td>
<td>-8.3 ( \mu \text{rad} )</td>
<td>4.7 ( \mu \text{m} )</td>
<td></td>
</tr>
<tr>
<td>( L_2 = 4.8 \text{ mm} )</td>
<td>-6.2 ( \mu \text{rad} )</td>
<td>4.7 ( \mu \text{m} )</td>
<td></td>
</tr>
<tr>
<td>( L_2 = 4.6 \text{ mm} )</td>
<td>-4.3 ( \mu \text{rad} )</td>
<td>4.6 ( \mu \text{m} )</td>
<td></td>
</tr>
<tr>
<td>( L_2 = 4.4 \text{ mm} )</td>
<td>-2.3 ( \mu \text{rad} )</td>
<td>4.6 ( \mu \text{m} )</td>
<td></td>
</tr>
<tr>
<td>( L_2 = 4.2 \text{ mm} )</td>
<td>( \sim -6 ) nrad</td>
<td>4.5 ( \mu \text{m} )</td>
<td></td>
</tr>
<tr>
<td>( L_2 = 4.1 \text{ mm} )</td>
<td>+1.8 ( \mu \text{rad} )</td>
<td>4.5 ( \mu \text{m} )</td>
<td></td>
</tr>
</tbody>
</table>

The roll angle result for \( L_2 = 4.2 \text{ mm} \) could not be very accurately measured using the FEM software but it has been established that it is in the nanorad order of magnitude. This means that the roll angle is well within the required 2 \( \mu \text{rad} \). The realised lateral displacement is only 4.5 \( \mu \text{m} \) which is about 10 percent to low but this can be compensated by raising the input.

A final FEM model is made using the same parameters but now including all four levers. The vertical rod length is set to 4.2 mm. The resulting roll angle is found to be -2.8 \( \mu \text{rad} \). The roll angle is higher is due to the deformations in the levers,
more specifically due to the torsional stiffness of the lever. This is compensated by lowering $L_2$ to 4.0 mm. The final FEM result is shown in Figure 4.3.8.

The input required to reach a lateral $x$ displacement of $5 \mu m$ was found to be $u_{lever} = 10 \mu m$. This can be explained by the lever ratio and displacements in the lever. The final roll angle $\phi$ is $-0.63 \mu rad$. It can be concluded that the roll angle is within the requirements.

### 4.3.3 Vertical displacement

The same FEM model as used in Figure 4.3.8 is now changed to investigate the roll angle due to vertical displacements. The bottom side of the vertical rods is moved up by $5 \mu m$, the result is shown in Figure 4.3.9. The resulting roll angle is nearly zero $\mu rad$ due to the symmetry of the structure and the realized vertical displacement is about $4.4 \mu m$.

### 4.4 Dynamics analysis

The dynamical behaviour of the orthogonal scanner is similar to that of the tripod concept (Section 3.4). The main eigenmodes are the same: lateral translation modes, vertical translation, a yaw modes and roll modes. The final FEM model from Section 4.3 is now utilized in a modal analysis to find the corresponding resonance frequencies. The first five modes are listed below. The lateral translation eigenmode is displayed in Figure 4.4.1 and the vertical translation mode is shown in Figure 4.4.2.
Figure 4.3.9: Orthogonal scanner FEM model vertical displacement
The lateral eigenmodes are required to have eigenfrequencies over 10 kHz which they are at 12.3 kHz. The vertical mode is well over the required 30 kHz at 36.5 kHz. The remaining modes should not be a problem as they are found to have resonances at 43 kHz and over.
4.5 Conclusion

Can all requirements be met simultaneously?

The current configuration of the orthogonal scanner concept meets all the set requirements as long as a number of conditions are met. The first condition is that the vertical actuators used for the lateral displacement of the platform have a total stroke of 20 µm, that is 10 µm in both directions. The z actuator should have a stroke of about 5 µm. It must be noted that these values were acquired under the assumption that the actuator acts at the outer edge of the lever and that the lever ratio is about 4:5. In a more realistic case that the actuators acts more to the middle of the lever the actuator stroke requirement is lowered significantly. Another condition is the manufacturing accuracy of the scanner. In order to find the balance at which the roll angle of the platform is at minimum the thickness of the rods, vertical as well as horizontal, should be machined with tolerances in the µm range as the sensitivities of the design parameters to the roll angle has been discovered to be highest for the rod thicknesses. The third condition is that the structure supporting the orthogonal scanner is stiff enough such that no additional deformations are introduced that might add to the roll angle.
Chapter 5

Detailed scanner design

5.1 Introduction

In this chapter the scanner concept will be turned into a real design while so far it has only been a concept. All the necessary components are selected to turn the orthogonal scanner concept into a working AFM, such as the sensors in Section 5.2, the actuators in Section 5.3, the AFM chip holder in Section 5.4 and the optics for the optical lever in Section 5.5. Furthermore all these components are integrated into a design based on the orthogonal scanner concept in Section 5.6. The question that is to be answered in this chapter is Is the orthogonal scanner concept valid as a real design?

5.2 Sensors

The sensors are a very important part of the orthogonal scanner concept. In total three sensors are aligned with the AFM probe in order to enable accurate triangulation sensing, meaning the three degrees of freedom $x$, $y$ and $z$ can be deducted from three off-axis measurements. The sensor resolution is required to be nanometers or less. The range of the sensors should be similar to the actuator stroke. A number of options are available as sensors: interferometers, strain gages, capacitive sensors. The strain gauges can not be aligned with the AFM probe as they need to be attached to some part of the scanner to measure the strain. Interferometers are not very suitable either as a lot of optics and three laser sources are required. This leaves capacitive sensors, this type of sensors is actually well-suited for application in the orthogonal scanner concept. Capacitive sensor probes are available in many sizes including compact probes and the measuring range and resolution are within the requirements. Some of the smallest capacitive sensor probes available are from Lion Precision\(^1\), measuring 3 mm in diameter. Physik Instrumente\(^2\) also supplies capacitive sensor probes but starting at 8 mm diameter. The Lion Precision 3 mm capacitive sensor probes are listed below and displayed in Figure 5.2.1.

<table>
<thead>
<tr>
<th>Vendor</th>
<th>Type</th>
<th>Dimensions</th>
<th>Resolution</th>
<th>Range</th>
<th>Near Gap</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lion Precision</td>
<td>C3S-0.8</td>
<td>Ø3×18 mm</td>
<td>1.5 nm @ 15kHz</td>
<td>25 µm</td>
<td>75 µm</td>
</tr>
<tr>
<td>Lion Precision</td>
<td>C3R-0.8</td>
<td>Ø3×15 mm</td>
<td>1.5 nm @ 15kHz</td>
<td>25 µm</td>
<td>75 µm</td>
</tr>
</tbody>
</table>

The measurement range and resolution vary depending on the sensor module and calibration but this is not within the scope of this research. The listed values are

\(^1\)http://www.lionprecision.com/capacitive-sensors/probes3mm.html
\(^2\)http://www.physikinstrumente.com/
based on a "fine calibration" using a Lion Precision CPL190 or CPL290 module. The near gap of 75 µm is the distance the probe surface needs to be mounted relative to the surface that is to be measured. The C3S and C3R are very similar, the only difference being their length and the way their wires are attached. The smallest probe is the most suitable.

The selected probe for use in the orthogonal scanner concept is the Lion Precision C3R-0.8.

5.3 Actuators

In total three linear actuators need to be selected. Two for the lateral displacement of the scanner and one for the vertical displacement. As mentioned earlier in this report the most suitable actuators are piezoelectric stack actuators because they are very compact and are typically capable of high speed operation. The required strokes are in the order of 10 µm for the lateral scanning direction and around 5 µm for the vertical direction.

5.3.1 Lateral scanning actuators

A number of eligible piezoelectric stack actuators are listed below. They are taken from the catalogs of Physik Instrumente\(^3\) (P.I.) and Noliac\(^4\). The listed resonance frequencies are the unloaded free resonance frequencies of the piezostacks.

<table>
<thead>
<tr>
<th>Vendor</th>
<th>Type</th>
<th>Dimensions</th>
<th>Stiffness</th>
<th>Nominal stroke</th>
<th>Resonance frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>P.I.</td>
<td>P882.31</td>
<td>3×2×13.5 mm</td>
<td>16 N/µm</td>
<td>~ 13 µm</td>
<td>90 kHz</td>
</tr>
<tr>
<td>P.I.</td>
<td>P883.31</td>
<td>3×3×13.5 mm</td>
<td>24 N/µm</td>
<td>~ 13 µm</td>
<td>90 kHz</td>
</tr>
<tr>
<td>Noliac</td>
<td>NAC2001</td>
<td>2×2×12 mm</td>
<td>N/A</td>
<td>~ 12.4 µm</td>
<td>N/A</td>
</tr>
<tr>
<td>Noliac</td>
<td>NAC2001</td>
<td>2×2×14 mm</td>
<td>N/A</td>
<td>~ 14.8 µm</td>
<td>N/A</td>
</tr>
</tbody>
</table>

The specifications of all four of these piezostacks are very similar, the differences are in dimensions and stiffness. Noliac does not supply specific stiffness data for each length of the NAC2001 stack but it is specified that the stiffness is between 8.0 and

\(^3\)http://www.physikinstrumente.com/
\(^4\)http://www.noliac.com
For this application in the orthogonal scanner concept the stiffness should be as high as possible because the scanner resonance frequencies increase with the stiffness. The piezoelectric stack actuator selected for use as lateral scanning actuator is therefore the P883.31 the second in the list.

### Vertical scanning actuators

The vertical scanning actuator is allowed to have a larger surface area than the actuators used for the lateral scanning direction and the required stroke is lower while the required speed is higher. The following piezoelectric actuators are suitable for the orthogonal scanner.

<table>
<thead>
<tr>
<th>Vendor</th>
<th>Type</th>
<th>Dimensions</th>
<th>Stiffness</th>
<th>Nominal stroke</th>
<th>Resonance frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>P.I.</td>
<td>P885.11</td>
<td>5×5×9 mm</td>
<td>100 N/μm</td>
<td>~ 6.5 μm</td>
<td>135 kHz</td>
</tr>
<tr>
<td>Noliac</td>
<td>NAC2013</td>
<td>5×5×8 mm</td>
<td>N/A</td>
<td>~ 8.6 μm</td>
<td>N/A</td>
</tr>
<tr>
<td>Noliac</td>
<td>NAC2013</td>
<td>5×5×10 mm</td>
<td>N/A</td>
<td>~ 11.4 μm</td>
<td>N/A</td>
</tr>
</tbody>
</table>

The piezostack supplied by P.I. has the option to add an aperture through the stack. This will prove to be necessary in Section 5.5. Hence, the actuator selected for the vertical scanning motion is the P885.11 piezostack.

### Actuator preload

For dynamic operation of the piezosstacks a preload is necessary. The specified preload by P.I. is 36 MPa. For the lateral (x and y actuators this means a force of about 324 N and for the z piezostack the preload force is roughly 900 N. In the latter case the actuator can be preloaded by moving the scanner upwards with a certain offset sufficient to induce the required stress of 36 MPa since the required vertical scanner stroke is about 2 μm and the scanner stiffness in z direction is high. In the case of the lateral scanner displacement the scanner stiffness is lower which means that for preloading those piezostacks extra stiffness needs to be added such as a preload spring as depicted in Figure 4.1.2. This spring is preloaded, ideally in such a way that the scanner platform has a lateral offset of -5 μm, at the edge of its operating range.

### Lateral reinforcements

In the current configuration (Figure 4.1.2) of the orthogonal scanner concept the vertical rods are directly attached or pressed on to the z piezostack. During lateral motions of the scanner platform lateral forces and moments will be transferred to the piezostack. A piezostack is only allowed a positive load in longitudinal direction, a compression load. A countermeasure is required to divert the lateral forces, a lateral reinforcement. The solution is to add leaf springs as drawn in Figure 5.3.1. The thickness of the leafsprings is set at 0.5 mm.

### AFM probe

The AFM probe is a chip of a standard size of 1.6 × 3.2 mm and needs to be fixed in place. It must also be actuated by a high speed dither piezoelectric actuator to enable intermittent and non contact AFM measuring modes that require high frequency oscillations of the probe cantilever. A standard AFM probe holder including the piezostack is shown in Figure 5.4.1. Specifically this is the DAFMCH probe holder from Bruker AFM probes[3]. It consists out of a base platform containing

---

the connections for the piezoelectric actuator, the dither piezo itself and a spring loaded clamp keeping the probe chip in place. Figure 5.4.1 also shows a schematic view of the dither piezo, its dimensions are about $4.0 \times 5.0$ mm and about 2.0 mm in height. For the purpose of the scanner design the dimensions of this dither piezostack will be assumed. The specifics of the connections and the clamp are not in the scope of this thesis. As the largest dimension is 5.0 mm it can be concluded that the top platform of the scanner needs to be at least 10.0 mm to allow the probe tip to be coincident with the central $z$ axis of the scanner.

5.5 Optical lever

Optics are required for the optical lever that measures the deflection of AFM cantilever. For the purpose of this design it is assumed that a laser beam is being generated below the scanner and delivered vertically through the scanner. An aperture is required through the $z$ piezostack and the scanner. A lens will be used in order the keep the laser beam directed at the probe as drawn in Figure 5.5.1. The idea is that the reflecting part of the probe cantilever is in the focal point of the lens and that as the scanner moves laterally the beam will still be pointed at the probe. The lens could for example be a $4.0$ mm plano-convex lens from Edmund Optics with a focal distance of 4.0 mm. The returning beam coming from the focal point will travel back parallel to the original beam and on to a PSD. Assuming the use of this particular lens and an estimation of the rotation of the cantilever the required size of the aperture through the scanner can be calculated by adding the offsets of the incoming and outgoing beam. The conclusion drawn from this is that the aperture should be at least 2.0 mm wide, assuming an incoming offset of -0.8 mm.

Figure 5.4.1: Bruker DAFMCH probe holder and schematic view of dither piezostack, probe holder and clamp

Figure 5.5.1: Detailed view of the cantilever and lens setup
and an outgoing beam offset of ±0.8 mm.

According to Sarid [7] the deflection angle of the cantilever can be approximated by \( \theta = \frac{2}{3} \frac{z}{l_c} \), where \( z \) is the vertical displacement of the cantilever and \( l_c \) is the cantilever length. Assuming \( z = 1 \mu m \) and \( l_c = 50 \mu m \) this results in an angle \( \theta \) of about 0.06°. The nominal cantilever angle is 11.0°. A simple model has been set up using the law of reflection to investigate the required aperture. The graph in Figure 5.5.2 shows three lines for cantilever angles of respectively 10.5°, 11.0° and 11.5°. The outgoing offset as shown in Figure 5.5.1 is plotted versus the incoming offset. At an incoming beam offset of about -1.6 mm, meaning left of the optical axis, the beam will be returning roughly at the center of the aperture. Vice versa if the incoming beam is at the center the outgoing beam offset will be about 1.6 mm. The relation is non linear because a tangent relation exists between the angle of reflection and the outgoing offset but in the limited range of the graph the distance between in and outgoing beams remains at about 1.6 mm. The sensitivity of the outgoing position of the beam as a function of cantilever deflection can be estimated by measuring the distance between the top and bottom line of the graph resulting in a sensitivity of about 0.1 mm per degree of rotation.

5.6 Final Design

All the components selected in the previous have been imported into Unigraphics NX. The actuators and sensors CAD models were provided by their respective suppliers and an approximate cad model of the piezo holder has been made for illustrative purposes. The lens mount and lens have not been included, the top platform has been dimensioned to accommodate it. Because of the size of the dither piezostack the top platform has to measure at least 10 \( \times \) 10 mm and next to the platform the triangulation sensors, measuring 03\( \times \)15 mm, need to be mounted at an angle relative to the platform the total surface area of 15 \( \times \) 15 mm can not be met. Through a number of design iterations this requirement has been stretched to 26 \( \times \) 26 mm needed to gain the space required to fit the capacitive sensors between the levers. An overview of the complete CAD model is shown in Figure 5.6.1 and a
cross-section of the same model is shown in Figure 5.6.2.

The levers are narrower and taller than in the original orthogonal scanner concept, this was necessary to make room for the capacitive sensors. A support structure has been designed to hold the actuators and sensors. The sensors are aligned with the probe tip.

5.6.1 Statics

The CAD model has been imported to ANSYS and a number of iterations similar to those in Subsection 4.3.2 have been performed in order to find the most optimal vertical rod length: \( L_2 = 6.8 \text{ mm} \). The piezostacks are not included in this analysis. The final realized parameters as shown in Figure 4.3.1 are listed below.

\[
L = 0.0030[m]
\]
\[
L_2 = 0.0068[m]
\]
\[
r = 0.005[m]
\]
\[
p = 0.003[m]
\]
\[
d = 0.002[m]
\]
\[
h, w = 0.001[m]
\]
\[
h_2, w_2 = 0.001[m]
\]

The resulting roll of the top platform is shown in Figure 5.6.3. The final roll angle is \( \phi = 5.4 \mu \text{rad} \). The realized displacement in lateral direction is 5 \( \mu \text{m} \) as
shown in Figure 5.6.4 while the lever input is 4 µm. This means that the required actuator stroke is slightly lower as the lever input is assumed to act on the end of the lever. It can be safely assumed that a piezo displacement of 4 µm will result in a 5 µm lateral scanner displacement.

5.6.2 Dynamics

The same FEM model has also been used to find the eigenmodes of the scanner. The first six modes are displayed in Figures 5.6.5 to 5.6.10. The first two eigenmodes are the lateral translation modes in the $x$ and $y$ directions. The third eigenmode is the yaw mode, the rotational mode around the $z$ axis. The fourth mode is the vertical translation mode. Finally the fifth and sixth modes are eigenmodes where the levers resonate. The corresponding resonance frequencies are listed in the table below.

<table>
<thead>
<tr>
<th>Mode number</th>
<th>Mode type</th>
<th>Resonance frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,2</td>
<td>Lateral translation ($x$ and $y$)</td>
<td>7.7 kHz</td>
</tr>
<tr>
<td>3</td>
<td>Yaw</td>
<td>15.3 kHz</td>
</tr>
<tr>
<td>4</td>
<td>Vertical translation ($z$)</td>
<td>22.8 kHz</td>
</tr>
<tr>
<td>5,6</td>
<td>Lever mode</td>
<td>24.4 kHz</td>
</tr>
</tbody>
</table>

The resonance frequencies are significantly lower than those encountered in Section 4.4. The required 10 kHz for the lateral and translation and 30 kHz for vertical translation are not met. The decrease in frequency can be explained due to the increased length of the vertical rods which causes a decrease in stiffness both lateral and vertical while the mass is still the same. This analysis did not include the piezostacks however, these will add in extra stiffness which will increase the
Figure 5.6.3: Static FEM analysis of the final design $z$ displacement result

Figure 5.6.4: Static FEM analysis of the final design lateral displacement result
Figure 5.6.5: FEM eigenmode 1 of the final design

Figure 5.6.6: FEM eigenmode 2 of the final design
Figure 5.6.7: FEM eigenmode 3 of the final design

Figure 5.6.8: FEM eigenmode 4 of the final design
Figure 5.6.9: FEM eigenmode 5 of the final design

Figure 5.6.10: FEM eigenmode 6 of the final design
resonance frequencies. Piezostacks are very complex to model so in the FEM model they are approximated by linear springs. The stiffness of these springs is the stiffness given by the piezo manufacturer as displayed in Section 5.3. The stiffnesses are $24 \, \text{N/µm}$ for the lateral actuator piezostacks and $100 \, \text{N/µm}$ for the vertical actuator. The resulting eigenmodes are shown in Figures 5.6.11 to 5.6.14.

Figure 5.6.11: FEM eigenmode 1 of the final design including linear springs
Figure 5.6.12: FEM eigenmode 2 of the final design including linear springs

Figure 5.6.13: FEM eigenmode 3 of the final design including linear springs
Figure 5.6.14: FEM eigenmode 4 of the final design including linear springs
5.7 Conclusion

The orthogonal scanner concept has been turned into a real world design including all the critical components for operation. Suitable sensors, actuators, a probe holder and optics have been selected. The final orthogonal scanner design covers a surface area of 26 × 26 mm, substantially larger than the required 15 × 15 mm. The main cause of this is the space that is needed to fit in the capacitive triangulation sensors. Furthermore using FEM analysis the kinematic and static properties of the scanner have been investigated. The ratio of vertical piezostack displacement versus lateral scanner displacement has been found to be 4:5. Given that the chosen piezostacks used for the lateral scanning have a nominal stroke of about 13 µm a scanning range of 16 × 16 µm can be achieved well over the required 10 × 10 µm. The vertical stroke is at a one to one ratio with the stroke piezostack used for the vertical scanning motion, measuring about 6.5 µm. The required roll angle is 2 µrad and the final realized roll angle is found to be 5.4 µrad, this is over factor 2 higher. The lateral translation resonance frequency is about 9.8 kHz and the vertical translation resonance frequency is about 30.4 kHz, both very close to the required 10 and 30 kHz, respectively. In order to evaluate whether the final scanner design is still useful the scanner specifications need to be translated back to AFM specifications. The scanner requirements and specifications are listed below.

<table>
<thead>
<tr>
<th>Scanner requirements</th>
<th>Final design specifications</th>
</tr>
</thead>
<tbody>
<tr>
<td>AFM scanner frontal area &lt; 15 × 15 mm</td>
<td>26 × 26 mm</td>
</tr>
<tr>
<td>Scanner stroke lateral &gt; 10 × 10 µm</td>
<td>16 × 16 µm</td>
</tr>
<tr>
<td>Scanner stroke vertical &gt; 2 µm</td>
<td>6.5 µm</td>
</tr>
<tr>
<td>Scanner roll angle &lt; 2 µrad</td>
<td>5.4 µrad</td>
</tr>
<tr>
<td>Scanner resonance frequency &gt; 10kHz in x,y</td>
<td>9.8 kHz</td>
</tr>
<tr>
<td>Scanner resonance frequency &gt; 30kHz in z</td>
<td>30.4 kHz</td>
</tr>
</tbody>
</table>

AFM Specifications

The specifications that require translation are the roll angle and the resonance frequencies. This is done by reversing the calculations made earlier in Section 2.3. First the Abbe error can be calculated using Equation 2.1 since the roll angle is now known. The assumed offset δ is the alignment error between the measurement surface of the capacitive sensor measurement are and the probe tip. It was assumed to be about 0.5 mm and in that case the Abbe error would amount to \( e_{\text{abbe}} = 2.7 \text{nm} \). The Abbe error will scale linearly with the the offset δ. Also the accuracy of the triangulation sensors is in the order of 1.5 nm. Therefore it is concluded that measuring accuracy will be in the order of about 3 nm. The imaging time depends on how fast the scanner can be operated which in turn depends on the tracking speed in z and the scanning speed in x, y. As the scanner specifications are very close to the required values it can be concluded that the original AFM specifications are valid. Hence the imaging time is under one second and the tracking accuracy will be in the nanometer range. The resulting AFM system specifications are listed below.

<table>
<thead>
<tr>
<th>AFM final design system specifications</th>
</tr>
</thead>
<tbody>
<tr>
<td>▶ Frontal surface area ((x,y)) 26 x 26 mm</td>
</tr>
<tr>
<td>▶ Measurement range 16 x 16 x 6 µm</td>
</tr>
<tr>
<td>▶ Imaging time &lt; 1 second</td>
</tr>
<tr>
<td>▶ Probe scanning AFM</td>
</tr>
<tr>
<td>▶ Measurement accuracy ~ 3 nm</td>
</tr>
<tr>
<td>▶ Nanometer tracking accuracy</td>
</tr>
</tbody>
</table>
Chapter 6

Conclusion

6.1 Main research questions

The main research questions are summarized again:

1. *How do the requirements of the AFM system translate to the requirements of the AFM scanner?*

2. *Does the tripod scanner concept meet the requirements?*

3. *Does the orthogonal scanner concept meet the requirements?*

4. *Is the orthogonal scanner concept valid as a real design?*

6.2 Requirements

The first research question has been answered in Chapter 2. The accuracy requirements have been translated to a maximum scanner roll angle of $2 \mu$rad. The imaging time requirements were translated to minimum scanner resonance frequencies of 10 kHz lateral and 30 kHz vertical. The found scanner requirements are listed below:

Requirements:

- AFM scanner frontal area $< 15 \times 15$ mm
- Scanner stroke lateral $> 10 \times 10 \mu$m
- Scanner stroke vertical $> 2 \mu$m
- AFM scanner resonance frequencies $> 10$kHz in $x,y$ direction and $>30$kHz in $z$ direction
- Scanner rotations $< 2 \mu$rad
- The probe must be mounted on the scanner

These requirements remain valid throughout the research for both the tripod scanner concept and the orthogonal scanner concept. If a scanner concept meets these requirements it means that the scanner subsystem is not the bottleneck for the AFM system specifications.
6.3 Tripod scanner concept

Chapter 3 answers the second question. The tripod scanner concept is introduced and analyzed. In terms of size and measurement range the concept holds. The lowest found roll angle was only 60 $\mu$rad. The main trade-off of the tripod scanner concept however is the roll angle versus the eigenfrequencies. It has been shown that when the stiffness ratio of the rods is high enough to enable a relatively low roll angle it means that the total lateral stiffness is too low for the horizontal translation eigenmode to meet the requirement and vice versa. The conclusion is drawn that the tripod scanner concept will never be feasible within the current set of requirements. A number of improvements are proposed but none of these improvements makes the tripod scanner concept feasible.

6.4 Orthogonal scanner concept

In Chapter 4 a new concept has been introduced. Aligning the rods with the actuation direction means that there is no trade off between stiffness ratio and eigenfrequencies. The levers of this concept ensure that the lateral scanner stroke can be directly related to the corresponding actuator stroke. By tuning the ratio of the horizontal to the vertical rods the roll angle $\phi$ can be as low as 0.6 $\mu$rad. The dynamical behavior also meets the requirement with a lateral translation eigenfrequency of 12.3 kHz and a vertical translation eigenfrequency of 36.5 kHz. It is concluded that the orthogonal scanner concept meets all the requirements as long as the actuators can provide enough stroke and that the rods themselves can be manufactured accurately enough.

6.5 Final design

The final research question is answered in Chapter 5. Sensors, actuators, the AFM probe chip holder and optics have been selected to make the concept into a feasible real design. The concept has been dimensioned in such a way that all components will fit and are properly aligned. The ratio of the horizontal to vertical rod lengths has been tuned to find the lowest roll angle. The specifications of the AFM scanner design have been evaluated and are listed below:

<table>
<thead>
<tr>
<th>Scanner requirements</th>
<th>Final scanner specifications</th>
</tr>
</thead>
<tbody>
<tr>
<td>AFM scanner frontal area &lt; 15 × 15 mm</td>
<td>26 × 26 mm</td>
</tr>
<tr>
<td>Scanner stroke lateral &gt; 10 × 10 $\mu$m</td>
<td>16 × 16 $\mu$m</td>
</tr>
<tr>
<td>Scanner stroke vertical &gt; 2 $\mu$m</td>
<td>6.5 $\mu$m</td>
</tr>
<tr>
<td>Scanner roll angle &lt; 2 $\mu$rad</td>
<td>5.4 $\mu$rad</td>
</tr>
<tr>
<td>Scanner resonance frequency &gt; 10kHz in $x,y$</td>
<td>9.8 kHz</td>
</tr>
<tr>
<td>Scanner resonance frequency &gt; 30kHz in $z$</td>
<td>30.4 kHz</td>
</tr>
</tbody>
</table>

Because of the size of the sensors the front surface area requirement had to be exceeded. The dimensions of the scanner are different than with the orthogonal scanner concept itself, this has resulted in different specifications for roll angle and for the eigenfrequencies. The lowest roll angle found was about 5 $\mu$rad and the eigenfrequencies are now just within requirements. Subsequently the final scanner specifications have been translated back to AFM system specifications:
AFM orthogonal scanner final design system specifications

▷ Frontal surface area \((x,y)\) 26 x 26 mm
▷ Measurement range 16 x 16 x 6 \(\mu\)m
▷ Imaging time < 1 second
▷ Probe scanning AFM
▷ Measurement accuracy \(\sim 3\) nm
▷ Nanometer tracking accuracy

6.6 Recommendations

A number of recommendations can be made for future improvements and research of the orthogonal scanner design.

The critical aspects of this design appeared to be the sensors which are actually slightly too large for the concept. It would be a great idea to investigate custom capacitive sensors, for example with a diagonal edge so that it could be fitted vertically into the scanner.

Furthermore some research might be necessary for the interface between the piezo stacks and the levers. In the current design they are simply clamped together which might be a good solution. However if the rotations of the levers and their moments are an issue for the piezo stacks it could be a good idea to fit them with rounded tops which should decrease the moment but introduce some kind of point contact between the piezo stack and the lever. Another issue is the way in which the scanner will be manufactured. Currently it is modeled as a monolithic piece of aluminium but it could turn out to be necessary to split up the scanner into several components. Electrical Discharge Machining (EDM) is would be a good way to machine a single block of aluminum into an orthogonal scanner as the machining tolerances are excellent and parts can be machined very deeply with a thin wire. Even so, not all of the cavities, especially the one between the top platform and the lateral reinforcement, can be reached. A smart division into several parts which could be bolted or glued together should make it possible to manufacture this scanner. It is important that the stiffness is maintained as much as possible so that the eigenfrequencies remain high.
Appendix A

Literature survey

A.1 Introduction

A literature survey has been performed to investigate the potential of an Atomic Force Microscope (AFM), or more specifically the considerations to realize a miniaturized AFM scanner device are investigated. The mechanism governing the behavior of the AFM scanner is described in relation to the AFM performance. Specific attention is paid to both high speed and metrological applications of AFM scanners, which are based on fundamentally different design principles. For these two types of applications some examples from recent literature are discussed in Sections A.3 and A.4. The specifications of different AFM scanners will be quantitatively compared in order to establish which scanner principles are suited best for either high speed or metrological purposes. Furthermore, the compatibility of the principles of high speed and metrological AFMs will be examined and assessed whether they could also be feasible on a smaller scale. Finally, a conclusion is drawn on the potential of realizing an AFM scanner which is both fast and metrological in Section A.5.

A.2 AFM Scanner specifications

First some important specifications of an AFM scanner are listed below. These are not to be confused with general AFM specifications or specifications of the other subsystems. An overview of the different examples of specifications is shown in Figure A.2.1.

**Lateral stroke**

The lateral stroke of the scanner is the stroke of the actuators acting in the $x,y$ directions. Assuming that the measurement of the sample surfaces takes place on the $x,y$-plane the lateral stroke defines the usable measurement area. This area can vary from $300 \times 200 \text{ nm} \ [8]$ up to $1000 \times 1000 \text{ µm} \ [9]$. In serial scanners the $x$ and $y$ actuators are usually dimensioned asymmetrically [10], meaning that one is the fast axis and the other the slow axis and the actuators are not identical. Symmetrical scanners can have identical actuators in the $x$ and $y$ directions resulting in a square measurement area. On top of the scanning direction can be rotated since there is no direction bias.

**Vertical stroke**

In the vertical $z$ direction the stroke is equal to that of the $z$-actuator. The height of the topographical features on the sample surfaces determines the required vertical...
stroke as well as the out of plane tilt of the sample, which occurs when the sample is mounted or if the probe is tilted with respect to the sample. For example, when a sample is tilted about 6 degrees the height different over 10 µm is already about 1 µm. Even when the features to be measured are in the sub-µm range a vertical stroke of some µm is required. The stroke can vary from several µm down to the 100 nm range.

Resonance frequencies

The resonance frequencies of the scanner in both the lateral and vertical direction are determined by the modal stiffnesses and masses of the AFM scanner. If the system is excited at a frequency near or on the resonance frequency it will start to resonate with an increasing amplitude meaning the forces will increase which may damage the scanner.

The following specifications are at system level.

Scan rate

The scan rate of an AFM is the frequency of the lateral fast axis, and also the number of lines that can be scanned per second. For example when scanning a 10 x 10 µm sample surface with 512x512 pixels the number of lines that need to be scanned for one picture is 512. If the required number of images per second is 3 fps this means that a scan rate is required of about 3 * 512 = 1,544 kHz.

Tip speed

While scanning a given area with a given scan rate the probe tip speed can be calculated as the scan rate multiplied by the scan range in the fast axis direction. As scan rates can become high the tip speed can reach up to the mm/s range.
A.2.1 Definition of terms

Some terms associated with measurements in general are defined below, as they can sometimes be a cause for ambiguity and sometimes cause confusion as they are not unequivocally used in literature.

**Precision and accuracy** The precision of a measuring instrument is a measure for the noise or random errors. By averaging multiple measurements the precision might be improved, however the noise cannot be compensated by calibration as it is an unpredictable or stochastic error. The accuracy is a measure for a certain static offset from the true value, caused by systematic errors which means they are deterministic. Averaging does not affect the accuracy but the offset can be canceled out by calibration.

**Repeatability and reproducibility** The degree to which different measurements on the same instrument produce similar results is the repeatability. Reproducibility the same thing but for the same instrument in different environments.

**Measurement Uncertainty** The measurement uncertainty is the result of the systematic and random errors after corrections. This value reflects the probable deviation from the true value up to a certain confidence level. The main difference between error and uncertainty is that the error is the numerical difference between the true value and the measurand (deterministic) and that the uncertainty is a measure for the probability the measurand is near the true value (probabilistic).

**Resolution** A clear distinction needs to be made between spatial resolution and grid resolution. The spatial resolution is not the smallest distance measurable but the smallest change in distance or distance step. For example, a nanometer resolution interferometer can not directly measure objects at nanometer size but it can measure nanometer displacement of objects of larger size. Furthermore the grid resolution is the resolution of the grid measured by the AFM. This resolution is in arbitrary units and indicates the number of measurement points are in the grid, e.g. $512 \times 512$. Combined with the range the grid distance can be calculated. However, even on a very rough grid with 1000 nm grid distance it could be that the spatial resolution of the instrument is in the subnanometer range.

A.2.2 Difference between high speed and metrological AFM scanning

A clear distinction needs to be made between high speed and metrological AFMs as they will be discussed in the upcoming sections.

A high speed AFM has the goal to scan a sample as quickly as possible. For example, this can be necessary when trying to image biological processes of nanometer scale at real time. Also when used for industrial measuring purposes, such as in-line inspection a high speed AFM is required in order not to delay the rest of the production line.

AFMs used for metrology are used for a different purpose, their goal is to measure with the highest possible precision and accuracy. This is typically done by scanning point for point instead of lines and stabilizing the cantilever and the probe tip at each measurement in order to rule out all dynamical error sources. In this way the measuring results can be used as a standard in national metrology institutes.
A.3 High-speed AFM Scanners

A.3.1 Introduction

This section provides a summary of current state of the art AFM high-speed scanner designs. The working principles have previously been explained and now some recent examples will be shown. The goal is to establish the specifications for state of the art scanners and to investigate what mechanisms enable these specifications.

A.3.2 Recent AFM scanners

Nine recently developed scanners are discussed in this section. They are evaluated using the specifications listed in Subsection A.2. The full detailed specifications are listed in Table A.1. They are listed in chronological order to get an idea of AFM scanner development through the years.

Video-rate scanner (Rost 2005)

The first scanner discussed is the video-rate scanner as demonstrated by Rost in 2005. It’s not strictly an AFM scanner but an Scanning Tunneling Microscope (STM) scanner. The potential for AFM use is explicitly mentioned. Using a shear piezo stage with resonances in the 100 kHz range a high scan rate of over 10 kHz is achieved. The focus of this research lies mainly on the control electronics and less on the mechanics. The disadvantage of this type of scanner is the limited range of 300x200 nm and resulting image resolution.

Figure A.3.1: video-rate scanner by Rost, 2005

Tuning fork scanner (Picco 2006)

Picco et al. have applied a tuning fork in 2006 allowing for high scanning rates, up to 40 kHz. The concept is that the resonance frequency of the tuning fork is used as the fast axis of the scanning motion which enables a very high sinusoidal scan pattern. The measurements were performed in constant height mode resulting in high forces on the sample but eliminating the need for feedback control on the z-axis. The result is that this scanner is only applicable in very specific situations, for instance on very hard surfaces able to resist the high forces.

Symmetric flexure scanner (Schitter 2006)

Schitter designed a new type of scanner in 2006. Utilizing flexures as hinges a rigid scanner with a substantial range was created while minimizing crosstalk
between the x and y actuators. From a kinematic point of view the z actuator is parallel to the x and y actuators, which in turn are in a serial configuration. The first mechanical resonances are at 22 kHz and 40 kHz for the lateral and vertical directions respectively. The range is 13x13 µm which is a huge when compared to the previous two scanners. The scanning rate is lower than previous, just over 1 kHz.

Serial scanner (Ando 2008)

A scanner was developed by Ando et al.\citep{ando2008} in 2008 which is completely serial kinematic. It is comprised of piezo actuators and flexure hinges. There are separate stages which eliminate crosstalk between the x, y and z actuators. This scanner has the disadvantage is that the scan range is asymmetrical due to the differing strokes of the x and y actuators (1 x 3 µm). The scanner is stiff in the z direction resulting in a z resonance frequency of 171 kHz, the resulting scan rate is 3.2 kHz.

Dual actuated scanner (Schitter 2008)

An AFM was built by Schitter et al.\citep{schitter2008} in 2008 which is dual actuated, the probe is scanned in x, y and z directions and the sample is scanned in the z direction. The probe scanner is a conventional piezo tube scanner and the sample is actuated with a piezo stack. The advantage of this AFM is that it is relatively fast and that the range is quite large, 30 x 30 µm. The achieved scan rate is 3 Hz while scanning a range of 25 µm.
A piezoelectric tube scanner was developed by Yong et al. in 2010 with 12 electrodes which allow for actuation and sensing simultaneously. The range is 10 x 10 μm and the scanning rate is 31 Hz.

Three axis serial scanner (Kenton 2011)

Kenton et al. have built and designed a three axis serial AFM scanner, but in a different orientation than Ando’s scanner. This scanner is smaller and the resonance frequencies are of the same order. The range of this scanner is 9 x 9 μm. A number of different control algorithms, open-loop, closed-loop and feed forward have been implemented resulting in a maximum scan rate of 7 kHz when using feedforward control. However there is some significant image distortion when scanning at maximum speed.

Compact serial scanner (Wadikhaye 2012)

A compact scanner is presented by Wadikhaye in 2012. The intended resonances as calculated through finite element analysis are twice as high as the actual achieved resonance frequencies. The achieved scan rate is 150 Hz.
Bruker Dimension Fastscan

Unlike the aforementioned academically developed AFM the Bruker Dimension Fastscan has been commercially developed by Bruker AXS and counts as one of
the industry’s benchmarks in terms of high speed atomic force microscopy. Not all specifications have been made public but it is known to be able to scan with a scan rate of over 100 Hz. The scanner is flexure piezo based.

Comparison

The specifications of all of the aforementioned AFM scanners are summarized in Table A.1. It should be noted that precision and accuracy as well as noise levels are not stated. When building a metrological AFM as discussed in the next section this would make sense to include this data but all aforementioned AFM scanners are used solely for imaging purposes which does not necessarily require quantitative data.

A.3.3 Conclusion

The highest scan rates achieved today are in the order of a few kHz with mechanical resonances up to several tens of kHz. This excludes the use of tuning forks and shear piezo actuators. Nearly all high speed AFMs mentioned in this chapter are sample scanners as opposed to probe scanners. The ones with an achieved scan rate in the kHz are constructed using piezo stack actuators and flexure hinges, which appears to be a good solution to achieve a stiff and fast scanner. None of the AFMs delivers real quantified data, instead images are generated which are usually software improved since high speed scanning often results in distorted images.
### AFM Scanner Comparison

<table>
<thead>
<tr>
<th>Author</th>
<th>Year</th>
<th>Location</th>
<th>Scanner Type</th>
<th>Actuator Type</th>
<th>Configuration</th>
<th>Cantilever</th>
<th>Stroke xy</th>
<th>Stroke z</th>
<th>Resonance xy</th>
<th>Resonance z</th>
<th>Control (lat.)</th>
<th>Closed/openloop</th>
<th>Control B/W</th>
<th>Sample freq</th>
<th>Scan Rate</th>
<th>Frame Rate</th>
<th>Image Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rost et al. [7]</td>
<td>2005</td>
<td>Leiden, Holland</td>
<td>Video-rate scanner</td>
<td>Shear-mode piezo stacks</td>
<td>Sample scanning</td>
<td>actually STM</td>
<td>300x200 nm</td>
<td>200 nm</td>
<td>64 kHz</td>
<td>&gt;100 kHz</td>
<td>Analog control</td>
<td>Closed loop</td>
<td>150 kHz</td>
<td>n/a</td>
<td>10.2 kHz</td>
<td>80 / 200 fps</td>
<td>128x128 / 256x32</td>
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<td>Picco et al. [12]</td>
<td>2006</td>
<td>Bristol, United Kingdom</td>
<td>Tuning fork scanner</td>
<td>Piezo tube, flexure stage</td>
<td>Sample scanning</td>
<td>Infinitesima</td>
<td>1 x 2 μm</td>
<td>constant height mode</td>
<td>32 kHz (tuning fork)</td>
<td>unknown</td>
<td>Closed loop</td>
<td>100x100</td>
<td>32 kHz, 100 kHz</td>
<td>1280 fps</td>
<td>4 fps</td>
<td>256x256</td>
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<tr>
<td>Schitter et al. [13]</td>
<td>2006</td>
<td>Santa Barbara, US</td>
<td>Symmetric flexure scanner</td>
<td>Flexure hinged piezos</td>
<td>Sample scanning</td>
<td>unknown</td>
<td>1 x 2 μm</td>
<td>4.3 μm</td>
<td>22 kHz</td>
<td>40 kHz</td>
<td>Analog PID</td>
<td>Closed loop</td>
<td>approx. 5-6 kHz</td>
<td>unknown</td>
<td>unknown</td>
<td>1030 Hz</td>
<td></td>
</tr>
<tr>
<td>Ando et al. [10]</td>
<td>2008</td>
<td>Kanazawa, Japan</td>
<td>serial scanner</td>
<td>Flexure hinged piezos</td>
<td>Sample scanning</td>
<td>Si nitride</td>
<td>1 x 3 μm</td>
<td>&gt;30 x 30 μm</td>
<td>6.35 kHz</td>
<td>PID, FF</td>
<td>Closed loop</td>
<td>100 kHz</td>
<td>unknown</td>
<td>unknown</td>
<td>3.2 kHz</td>
<td>32 fps</td>
<td>100x100</td>
</tr>
<tr>
<td>Schitter et al. [11]</td>
<td>2008</td>
<td>Delft, Holland</td>
<td>Dual actuated</td>
<td>Piezo tube, z stack</td>
<td>Probe scanning</td>
<td>Si cantilever</td>
<td>2 μm</td>
<td>&gt;6 μm</td>
<td>171 kHz</td>
<td>80 kHz</td>
<td>PI</td>
<td>Closed loop</td>
<td>80 kHz</td>
<td>unknown</td>
<td>unknown</td>
<td>11 Hz</td>
<td>32 fps</td>
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<tr>
<td>Yong et al. [?9]</td>
<td>2010</td>
<td>Newcastle, Australia</td>
<td>12-electrode Piezo</td>
<td>ContAl</td>
<td>Sample scanning</td>
<td>ContAl</td>
<td>unknown</td>
<td>0.6 μm</td>
<td>499 Hz, unknown</td>
<td>40 kHz</td>
<td>Closed loop</td>
<td>Closed loop</td>
<td>unknown</td>
<td>unknown</td>
<td>31 Hz</td>
<td>unknown</td>
<td>256 x 256</td>
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<tr>
<td>Kenton and Leang [14]</td>
<td>2011</td>
<td>Reno, Nevada, US</td>
<td>3-axis serial kinematic</td>
<td>Flexure hinged piezos</td>
<td>Sample scanning</td>
<td>Vista Probes T300</td>
<td>unknown</td>
<td>9 x 9 μm</td>
<td>2 μm</td>
<td>5.96 kHz, 25.9 kHz</td>
<td>13 kHz</td>
<td>PID, RC, FF</td>
<td>Closed loop</td>
<td>unknown</td>
<td>unknown</td>
<td>2.7 kHz</td>
<td>32 fps</td>
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<tr>
<td>Wadikhaye [15]</td>
<td>2012</td>
<td>Newcastle, Australia</td>
<td>Compact serial kinematic</td>
<td>Flexure hinged piezos</td>
<td>Sample scanning</td>
<td>NTEGRA cantilever</td>
<td>unknown</td>
<td>8 x 6 μm</td>
<td>3 μm</td>
<td>10 kHz, 7.5 kHz</td>
<td>2.7 kHz</td>
<td>standard NTEGRA control</td>
<td>Closed loop</td>
<td>unknown</td>
<td>2.7 kHz</td>
<td>unknown</td>
<td>256x256</td>
</tr>
<tr>
<td>Bruker AXS</td>
<td>2012</td>
<td>Reno, Nevada, US</td>
<td>Dimension Fastscan</td>
<td>Flexure hinged piezos</td>
<td>Sample scanning</td>
<td>Vista Probes T300</td>
<td>unknown</td>
<td>30 x 30 μm</td>
<td>3 μm</td>
<td>64 kHz</td>
<td>&gt;50 kHz</td>
<td>standard NTEGRA control</td>
<td>Closed loop</td>
<td>unknown</td>
<td>2.7 kHz</td>
<td>unknown</td>
<td>256x256</td>
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</table>

Table A.1: A quantitative comparison of several recently developed AFM scanners.
A.4 Metrological AFM Scanners

A.4.1 Introduction

Metrological AFMs (mAFM) have the ability to obtain accurate quantitative data from a sample. They are the most accurate AFMs which are used to calibrate normal AFMs for example. They are different from regular AFMs in the fact that the sensors used for the position of the scanner are laser interferometers as well as that they are designed to all external error sources, such as thermal drift, external vibrations air turbulence etc. The result is that the measured values can be traced back to the SI meter with low uncertainty.

A.4.2 Traceability

The SI meter is realized by a femtosecond frequency comb [16], for instance used at national metrology institutes (NMI). In the traceability chain the frequency comb is only step away from the definition of the SI meter and it is the most accurate and precise length measuring device available. Each step further away from the definition of the meter adds to a certain amount of measuring uncertainty. This is schematically shown in Figure A.4.1.

![Figure A.4.1: Pyramid of traceability, source: Ducourtieux, 2011 [4]](image)

A metrological AFM uses laser interferometers as position sensors as opposed to capacitive sensors, strain gages or other sensors. The wavelength of a laser (about 630 nm) is used as a measure for the distance. The wavelength of the laser can be calibrated by using a frequency comb or an $I_2$ stabilized Helium Neon laser of which the wavelength is very accurately known. The next link in the traceability chain is the mAFM. The measurement uncertainty of the mAFM is larger than that of a calibrated interferometer since external factors such as noise and external vibrations also start playing a role. A typical metrological AFM has a measurement uncertainty of about $\sim 1$ nm. A step further down the pyramid are regular AFMs. They can be calibrated using calibration gratings for example but they will never reach the same level of measurement uncertainty.
A.4.3 Error sources

There are two types of errors: systematic errors and random errors. Systematic errors affect the system accuracy but when properly quantified can be calibrated/compensated for. Random errors affect the repeatability of the measurement and can’t be compensated for but diminished by averaging. A number of example AFM error sources are listed below. Some uncertainties as adapted from literature [4], [17] are included:

**Systematic errors:**
- Abbe errors: $0.1 \sim 0.4$ pm (offset: 0.5 mm)
- Interferometer laser wavelength: $\sim 52$ pm
- Interferometer non linearity: $\sim 0.1$ nm
- Thermal drift: $\sim 0.1$ nm h$^{-1}$ for $\sim 3$ mK h$^{-1}$
- Non-orthogonality (crosstalk)
- Probe tip wear

**Random errors:**
- Interferometer resolution limit: $\sim 22$ pm
- Air turbulence: $\sim 15$ nm (unshielded) $\sim 0.1$ nm (shielded)
- Thermal noise
- Electrical noise
- External vibrations/forces

The total influence of all these errors can be calculated using a dynamic error budget (DEB). Such a budget lists all error including a weighing factor and with the total measuring uncertainty can be calculated. An example error budget is displayed in Figure A.4.2.

![Figure A.4.2: Example error budget](source: Chen, 2003 [5])

| Error (vi) | Estimated Uncertainty | Type | Distribution | Standard Uncertainty | $|\hat{\sigma} - \sigma_i|/|\sigma_i|$ | vi |
|-----------|-----------------------|------|--------------|----------------------|---------------------------------------------|----|
| Laser Interferometer | | | | | | |
| Wavelength($\lambda_0$) | $1.27 \times 10^3$ mm | B | $\mathcal{W}/\mathcal{W}$ | $0.73 \times 10^3$ mm | 1.5798 L/m | 1.15 nm | 0.012x10$^5$ L/m | (50) |
| Non-linearity($\kappa$) | $2$ mm | B | $\mathcal{W}/\mathcal{W}$ | $1.15$ mm | 1 | 1.15 nm | (50) |
| Relative Index of Refractive | Air | | | | | | |
| Fusibility | | | | | | | |
| Temperature(T) | $303 \pm 0.3$ C | B | $\mathcal{W}/\mathcal{W}$ | $0.173 \pm 10^{-6}$ C | 4.35x10$^{-1}$ C | 2.68x10$^{-1}$ L | 8.5x10$^{-1}$ L | (50) |
| Air Pressure(p) | $50$ Pa | B | $\mathcal{W}/\mathcal{W}$ | $28.87$ Pa | 1 | 1 | 0.074x10$^5$ L | (50) |
| Relative Humidity | $10 \%$ | B | $\mathcal{W}/\mathcal{W}$ | $5.77 \%$ | 8.5x10$^{-1}$ L | 8.5x10$^{-1}$ L | (50) |
| CTE($\alpha$) | $1 \times 10^{-6}$ C$^{-1}$ | B | $\mathcal{W}/\mathcal{W}$ | $0.57 \times 10^{-6}$ C$^{-1}$ | 0 | 0 | 0 | (50) |
| Sample Temperature(T) | $0.3$ C | B | $\mathcal{W}/\mathcal{W}$ | $0.173$ C | 2.55x10$^{-1}$ L | 4.41x10$^{-1}$ L | (50) |
| Mechanical structure | | | | | | | |
| A | | | | | | | |
| C | | | | | | | |
| Air 1.23 mm | B | $\mathcal{W}/\mathcal{W}$ | $0.36$ mm | 8.36 mm | (12.5) |
| 28.92 mm | B | $\mathcal{W}/\mathcal{W}$ | $8.35$ mm | 8.35 mm | (12.5) |
| Dead Path | | | | | | | |
| 0.74 mm | B | $\mathcal{W}/\mathcal{W}$ | $0.21$ mm | 0.21 mm | 12.5 |
| Stage alignment ($\theta$) | $0 \degree$ | B | $\mathcal{W}/\mathcal{W}$ | 0.577 | 0 | 12.5 |
| Abbe error ($\beta$) | $0.5 \degree$ | B | $\mathcal{W}/\mathcal{W}$ | 0.2886 | 1 mm | 1.4 mm | 12.5 |
| Pitch Measuring ($N_{p}$) | | | | | | | |
| $\mu(N_{p})$ | A | $\mathcal{W}/\mathcal{W}$ | 0.077 mm | 0.077 mm | 15 | 15 |
A.4.4 Recent metrological AFM scanners

Five metrological AFMs are being compared:

Comparison

<table>
<thead>
<tr>
<th>Author:</th>
<th>Chen and Jaeger</th>
<th>Misumi et al</th>
<th>Eves, B.</th>
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<tr>
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<td>2006</td>
<td>2009</td>
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<tr>
<td>Location:</td>
<td>Ilmenau, Germany</td>
<td>Tsukuba, Japan</td>
<td>Ottawa, Canada</td>
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<tr>
<td>Scanner:</td>
<td>Traceable AFM</td>
<td>Japan NNI mAFM</td>
<td>Large volume mAFM</td>
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<tr>
<td>Actuator type:</td>
<td>Piezo (x,y,z)</td>
<td>Piezo (x,y), Piezotube (z)</td>
<td>Flexures</td>
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<td>Motion guides:</td>
<td>Flexures</td>
<td>Flexures</td>
<td>Flexures</td>
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<tr>
<td>Sensor type:</td>
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<tr>
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<tr>
<td>Sensor sample rate:</td>
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<td>170 Hz</td>
<td>25 kHz</td>
</tr>
<tr>
<td>Uncertainty:</td>
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<td>3 nm</td>
<td>1 nm (expected)</td>
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<td>Stroke x,y:</td>
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<table>
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<th>Werner, C.</th>
<th>Ducourtieux and Poyet</th>
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<tr>
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<td>2011</td>
</tr>
<tr>
<td>Location:</td>
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<td>Trappes, France</td>
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<td>Diff. interferometer mAFM</td>
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<td>Flexures (stiffness compensated)</td>
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<td>Resonance z:</td>
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Table A.2: A quantitative comparison of metrological AFMs.

A.4.5 Conclusion

The measurement uncertainty achieved by current metrological AFMs is in the order of 0.5 to 1 nm. All mAFMs use laser interferometers to measure the absolute position of the scanner stage. Sometimes auxiliary capacitive sensors are used for control purposes but always backed up by interferometers. Using differential plane interferometry thermal drifts can be taken into account. Furthermore the metrological frames supporting the AFMs have high masses and stiffness to ensure thermal stability of the metrology loop and to decrease sensitivity for external vibrations. The most important causes for uncertainty are interferometer non-linearity and change in the refraction coefficient caused by air turbulence, both in the order of magnitude of 0.1 nm. Imaging times for metrological AFMs can take up to 11 hours (1000 x 1000 points grid) due to stopping and stabilizing the cantilever at each point as opposed to continuous constant speed scanning.
A.5 Conclusion

A metrological AFM scanner able to perform high speed measurements seems to be a contradiction in itself. There is a trade-off between speed and accuracy, for instance fast sensors are less accurate. However there are also favorable effects; thermal drift for example, a slow process, is less of an issue when scanning at high speed. For both metrological and high speed purposes probe scanning is not recommended. Sample scanning has the advantage that the probe cantilever remains stationary which means that the optics for the optical lever and positions sensors can be designed to be stationary as well. The only disadvantage is that it imposes a limit on the sample size and mass. The obvious solution for a high speed metrological AFM scanner would be a flexure hinged piezo stage with laser interferometers. Piezo stacks are required for the high mechanical bandwidths and laser interferometers for the traceability of the measurements. There is a difference between measuring position in an absolute manner, i.e. with respect to a metrology frame, or relatively i.e. within a local coordinate system in relation to the tip position for instance. But even for relative position measurements it is important to minimize drift within the metrology loop.
Appendix B

Analytical static model

This appendix attempts to clarify the analytical static model which has been used to predict the roll angles for both the tripod and the orthogonal scanner concept. First an example of a rigid body with two vertical legs:

The static equations are set up for the sum of forces in the x and z directions, $F_x$ and $F_z$ as well as the sum of moments $M$. First a stiffness matrix $K_1$ is set up which defines the relation between the forces and displacements:

$$K_1 = \begin{bmatrix} \frac{3EI_{xx}}{L} & 0 & -\frac{2EI_{xx}}{L} \\ 0 & \frac{EA}{L} & 0 \\ -\frac{2EI_{zz}}{L^2} & 0 & \frac{EI_{zz}}{L} \end{bmatrix}$$ (B.1)

$K_2$ is identical to $K_1$ since both legs are assumed the same. The same goest

$$\begin{bmatrix} F_{x1} \\ F_{z1} \\ M_1 \end{bmatrix} = K_1 \begin{bmatrix} x_1 - u_{x1} \\ z_1 - u_{z1} \\ \phi_1 \end{bmatrix}$$ (B.2)
\[ \begin{bmatrix} F_{x_1} \\ F_{z_1} \\ M_1 \end{bmatrix} = K_2 \begin{bmatrix} x_1 - u_{x_2} \\ z_1 - u_{z_2} \\ \phi_2 \end{bmatrix} \quad (B.3) \]

The force balance is included, accounting for the rigid link between the two legs.

\[ \Sigma F_x = 0 \quad \rightarrow \quad F_{x_1} + F_{x_2} = 0 \quad (B.4) \]
\[ \Sigma F_z = 0 \quad \rightarrow \quad F_{z_1} + F_{z_2} = 0 \quad (B.5) \]
\[ \Sigma M = 0 \quad \rightarrow \quad M_1 - F_{z_2}(a + b) = 0 \quad (B.6) \]

The final requirement for solving the equations is to define the displacements in terms of the general coordinates \( x, z \) and \( \phi \). Also some of the tripod specific parameters are plugged in. These parameters are \( p \), the distance between the attachment point of the rod and the center of the tripod, \( w \) and \( h \), the width and depth of the rods. The rods are assumed to be square hence \( h \) is equal to \( w \). Furthermore all input variables \( u \) besides \( u_{z_1} \) are assumed to be zero and are eliminated.

\[
\begin{align*}
x_1 &= x \\
x_2 &= x \\
z_1 &= z + \phi a \\
z_2 &= z - \phi b \\
\phi_1 &= \phi \\
\phi_2 &= \phi 
\end{align*}
\]

\[ a = p \\
b = \frac{p}{2} \\
I_{xx} = \frac{1}{12} w h^3 \\
A = w h \quad (B.7) \]

Now all the previous equations are solved for \( x, z \) and \( \phi \):

\[
\begin{align*}
x &= \frac{u_{z_1} 9pL}{27p^2 + 2h^2} \\
z &= \frac{u_{z_1} 9p^2 + h^2}{27p^2 + 2h^2} \\
\phi &= \frac{18p}{27p^2 + 2h^2} 
\end{align*}
\]

(B.9)

It can be observed that only the parameters \( h, p \) and \( L \) remain in the equation. \( p \) is still there since it is a direct measure for the length of the platform. Obviously a longer platform will have smaller rotation when pushed on one of its ends. \( h \) is a defining factor for the stiffness of the rods. The length of the rods \( L \) only has an influence on the \( x \) displacement of the platform. Because the rods and stiffnesses are equal the stiffness parameters cancel out each other in terms of displacements. Parameters such as the Young’s modulus do influence the internal forces. This solution will be verified by a FEM example. The following values are assumed:
\[ E = 69 \text{ GPa} \]
\[ p = 0.01 \text{ [m]} \]
\[ w = 0.001 \text{ [m]} \]
\[ h = 0.001 \text{ [m]} \]
\[ L = 0.01 \text{ [m]} \]  \hspace{1cm} (B.10)

Substitution into [B.9] provides the following results:

\[ x = 3.331 \text{ [\mu m]} \]
\[ z = 3.334 \text{ [\mu m]} \]
\[ \phi = 0.6662 \text{ [mrad]} \]  \hspace{1cm} (B.11)

Now the same parameters are used in a simple ANSYS FEM model:

Using the z displacement solution data found in Figure B.0.2 it can be seen that the z displacement itself matches the answer found earlier and the angle \( \phi \) can be calculated:

\[ \phi = \arctan\left( \frac{10.344 + 0.327}{0.015} \right) = 0.711 \text{ [mrad]} \]  \hspace{1cm} (B.12)

Although the numbers do match reasonably this is not a very convincing validation since if a bar of with a length of 15 [mm] is moved up at one end by about 10 [\mu m] roughly the same number is found for the rotation: 0.6667 [mrad]. Now an extra leg will be added and modeled similarly, the schematic is shown below:

The two legs on the right are placed at a distance of \( d \) with regards to each other. The main difference is that there are now an extra stiffness matrix \( K_3 \) as...
well as extra local coordinates $x_3$, $z_3$ and $\phi_3$. This results in a new force balance equation.

\[
\begin{align*}
\sum F_x &= 0 \quad \rightarrow \quad F_{x1} + F_{x2} + F_{x3} = 0 \quad \text{(B.13)} \\
\sum F_z &= 0 \quad \rightarrow \quad F_{z1} + F_{z2} + F_{z3} = 0 \quad \text{(B.14)} \\
\sum M &= 0 \quad \rightarrow \quad M_1 - F_{z2}(a + b - \frac{d}{2}) - F_{z3}(a + b + \frac{d}{2}) = 0 \quad \text{(B.15)}
\end{align*}
\]

Again solving for $x$, $z$ and $\phi$:

\[
\begin{align*}
x &= \frac{u_{z1} 3L(-3p + d)}{2h^2 + 3d^2 + 27p^2 - 18pd} \\
z &= \frac{u_{z1} 9p^2 - 12pd + 3d^2 + h^2}{2h^2 + 3d^2 + 27p^2 - 18pd} \\
\phi &= -\frac{u_{z1} 6(-3p + d)}{2h^2 + 3d^2 + 27p^2 - 18pd} \quad \text{(B.16)}
\end{align*}
\]

Substitution of the same parameters including the new parameter $d = 0.002 \ [\text{m}]$ into (B.16) provides the following results:

\[
\begin{align*}
x &= 3.568[\mu\text{m}] \\
z &= 2.859[\mu\text{m}] \\
\phi &= 0.7137[\text{mrad}] \quad \text{(B.17)}
\end{align*}
\]

A similar configuration has been modeled with ANSYS:

The $z$-displacement is clearly lower than in Figure B.0.2 as predicted by the analytical, and the angle $\phi$ is larger: $\phi = 0.751[\text{mrad}]$. This means that adding a leg to decrease rotation of the top platform actually makes it worse. This effect is shown by varying the parameters $h$ and $d$ in equations B.9 and B.16.
The d parameter in the FEM model is doubled from 0.002 to 0.004 [m]. According to the analytical model $\phi = 0.7684 \text{[mrad]}$ and $z = 2.310344828e-5 \text{[\mu m]}$. The FEM results are shown in Figure B.0.5. The angle $\phi$ is about 0.762 [mrad] which as well as the z displacement matches closely with the analytical result.
Figure B.0.5: FEM model of tripod with three legs while parameter d is doubled
Appendix C

Tripod scanner static model
MAPLE file

This is the maple file used to find the analytical relations of the static behavior of the tripod scanner concept.
\( \begin{align*}
\mathbf{a} &= p; \\
\mathbf{b} &= \mathbf{b}_0; \\
I_{xx} &= \frac{(w^3 h^3)}{12}; \\
I_{yy} &= \frac{(h w^3)}{12}; \\
A &= \frac{w^3}{4}; \\
T_{\text{soft}} &= 1; \\
K_1 &= \text{2-inverse} \\
&= \frac{l^3}{(3 E I_{xx})} \begin{bmatrix}
0 & -\frac{l^2}{2 E I_{xx}} T_{\text{soft}} \\
0 & \frac{L}{E A} \\
-\frac{l^2}{2 E I_{xx}} T_{\text{soft}} & 0 \\
0 & \frac{L}{E I_{xx}}
\end{bmatrix}; \\
K_2 &= \text{2-inverse} \\
&= \frac{l^3}{(3 E I_{xx})} \begin{bmatrix}
0 & -\frac{l^2}{2 E I_{xx}} T_{\text{soft}} \\
0 & \frac{L}{E A} \\
-\frac{l^2}{2 E I_{xx}} T_{\text{soft}} & 0 \\
0 & \frac{L}{E I_{xx}}
\end{bmatrix}; \\
K_3 &= K_2; \\
\mathbf{d}_{\text{displacements}} &= \begin{bmatrix} \mathbf{x}_1 \\
\mathbf{z}_1 \\
\mathbf{\phi}_1 \\
\mathbf{x}_2 \\
\mathbf{z}_2 \\
\mathbf{\phi}_2 \\
\mathbf{x}_3 \\
\mathbf{z}_3 \\
\mathbf{\phi}_3 \end{bmatrix}; \\
\mathbf{d}_{\text{forces}} &= \text{eval} (K_1 \mathbf{d}_{\text{displacements}}); \\
\mathbf{d}_{\text{forces}} &= \text{eval} (K_2 \mathbf{d}_{\text{displacements}}); \\
\mathbf{d}_{\text{forces}} &= \text{eval} (K_3 \mathbf{d}_{\text{displacements}}); \\
\mathbf{f}_{x1} &= \mathbf{f}_{x1}[1]; \\
\mathbf{f}_{x2} &= \mathbf{f}_{x2}[1]; \\
\mathbf{f}_{x3} &= \mathbf{f}_{x3}[1]; \\
\mathbf{f}_{z1} &= \mathbf{f}_{z1}[2]; \\
\mathbf{f}_{z2} &= \mathbf{f}_{z2}[2]; \\
\mathbf{f}_{z3} &= \mathbf{f}_{z3}[2]; \\
\mathbf{m}_{x1} &= \mathbf{m}_{x1}[3]; \\
\mathbf{m}_{x2} &= \mathbf{m}_{x2}[3]; \\
\mathbf{m}_{x3} &= \mathbf{m}_{x3}[3]; \\
\mathbf{x}_1 &= \frac{x}{\cos(\alpha)}; \\
\mathbf{x}_2 &= \frac{x}{\cos(\beta)}; \\
\mathbf{x}_3 &= \frac{x}{\cos(\beta)}; \\
\mathbf{\phi}_1 &= \mathbf{\phi}_1; \\
\mathbf{\phi}_2 &= \mathbf{\phi}_2; \\
\mathbf{\phi}_3 &= \mathbf{\phi}_3;
\end{align*} \)
\[ z_1 = \frac{z}{\cos(\alpha)} + \phi u; \]
\[ z_2 = \frac{z}{\cos(\beta)} - \phi \left( b - \frac{d}{2} \right); \]
\[ z_3 = \frac{z}{\cos(\beta)} - \phi \left( b + \frac{d}{2} \right); \]

\[ eq1 := Fz1 + Fz2 + Fz3 = 0; \]
\[ eq2 := Fx1 + Fx2 + Fx3 = 0; \]
\[ eq3 := M1 - Fz2 \left( a + b - \frac{d}{2} \right) - Fz3 \left( a + b + \frac{d}{2} \right) = 0; \]

\[ S := \text{solve}\{ eq1, eq2, eq3 \}, \{ z, x, \phi \} \]

\[ S[1], S[2], S[3]; \]

\[ \phi = \frac{36 \cos(\alpha) u \rho}{8 \ h^2 \cos(\alpha) + 12 \ d^2 \cos(\alpha) + 36 \ p^2 \cos(\alpha) + 5 \ h^2 \cos(\beta) + 6 \ d^2 \cos(\beta)} \]
\[ x = -\left(54 L \cos(\alpha)^2 \cos(\beta) u \rho \right) / \left( \left(8 \ h^2 \cos(\alpha) + 12 \ d^2 \cos(\alpha) + 36 \ p^2 \cos(\alpha) + 18 \ p^2 \cos(\beta) - 5 \ h^2 \cos(\beta) + 6 \ d^2 \cos(\beta) \right) \left( \cos(\beta) + 2 \cos(\alpha) \right) \right) \]
\[ z = \frac{\cos(\beta) u \rho \cos(\alpha)}{\cos(\beta) + 2 \cos(\alpha)} \]
Appendix D

Dynamics Matlab model

This m-file has been used to estimate the eigenfrequencies of the tripod scanner concept as a function of the design parameters.

```matlab
% tripod parameters
p = 0.004;
r = 0.006;
l = 0.005;

% stiffness parameters
E = 69E9;
w = 0.001;

z0 = sqrt(l^2-(r-p)^2);
I = (w*w^3)/12;
A = w*w;
L = l;
k_lat = (3*E*I)/L^3;
k_long = E*A/L;

for i = 1:3
    theta = (120*i-60)/180*pi;
    % euler angles
    rx = -atan2(((r-p)*cos(theta)),z0);
    ry = atan2(((r-p)*sin(theta)),z0);
    rz = pi-theta;
    % transformation matrix
    T = [ cos(ry)*cos(rz), -cos(ry)*sin(rz), sin(ry); ...
         cos(rx)*sin(rz) + cos(rz)*sin(rx)*sin(ry), cos(rx)*cos(rz)*sin(ry) + cos(rz)*sin(rx), -cos(ry)*cos(rx); ...
         sin(rx)*sin(rz) - cos(rx)*cos(rz)*sin(ry), cos(rz)*sin(rx) + cos(rx)*sin(ry)*sin(rz), cos(rx)*cos(ry)];
    Tx = [1 0 0;
         0 cos(rx) sin(rx);
         0 -sin(rx) cos(rx)];
    Ty = [cos(ry) 0 -sin(ry);
         0 1 0;
         sin(ry) 0 cos(ry)];
    Tz = [cos(rz) sin(rz) 0;
```
\[-\sin(rz) \cos(rz) 0; \\
0 0 1\];

\begin{verbatim}
switch i
    case 1
        T1 = T;
    case 2
        T2 = T;
    case 3
        T3 = T;
end
end
\end{verbatim}

%stiffness matrices
K = [k_lat 0 0; \\
0 k_lat 0; \\
0 0 k_long];

K1 = T1'*K*T1;
K2 = T1'*K*T1;
K3 = T2'*K*T2;
K4 = T2'*K*T2;
K5 = T3'*K*T3;
K6 = T3'*K*T3;

K_total = K1 + K2 + K3 + K4 + K5 + K6;

%estimate mass
h = 0.004;
rho = 2700;
V = \pi*(0.005)^2*h;
V_aperture = \pi*(0.003)^2*h;
m = (V - V_aperture)*rho;

%eigenfrequencies
omega_x = sqrt(K_total(1,1)/m)/(2*pi);
omega_y = sqrt(K_total(2,2)/m)/(2*pi);
omega_z = sqrt(K_total(3,3)/m)/(2*pi);
Appendix E

Orthogonal scanner static model MAPLE file

This is the maple file used to find the analytical relations of the static behavior of the orthogonal scanner concept.
\[ I_{xx} = d w \frac{h^3}{12}; \]
\[ I_{yy} = d h \frac{w^3}{12}; \]
\[ A = d w h; \]
\[ I_{xx}^2 = d w^2 \frac{h^2}{12}; \]
\[ I_{yy}^2 = d h^2 \frac{w^2}{12}; \]
\[ A^2 = d w^2 h^2; \]
\[ K_1 = d^2 \frac{1}{E I_{xx}} \]
\[ K_2 = d^2 \frac{1}{E I_{xx}^2} \]
\[ K_3 = d^2 \frac{1}{E I_{xx}^2} \]
\[ \text{displacements1} = x_1 \phi_1 \]
\[ \text{displacements2} = x_2 \phi_2 \]
\[ \text{displacements3} = x_3 \phi_3 \]
\[ \text{forces1} = \text{evalm} K_1 \text{displacements1} \]
\[ \text{forces2} = \text{evalm} K_2 \text{displacements2} \]
\[ \text{forces3} = \text{evalm} K_3 \text{displacements3} \]
\[ F_{x1} = \text{forces1}^T \]
\[ F_{z1} = \text{forces1}^T \]
\[ M_1 = \text{forces1} \]
\[ F_{x2} = \text{forces2}^T \]
\[ F_{z2} = \text{forces2}^T \]
\[ M_2 = \text{forces2} \]
\[ F_{x3} = \text{forces3}^T \]
\[ F_{z3} = \text{forces3}^T \]
\[ M_3 = \text{forces3} \]
\[ e_{1} = C_{F_{x1}} K F_{x1} + C_{F_{z2}} F_{z2} + C_{F_{z3}} F_{z3} = 0; \]

\[ e_{2} = d F_{z1} + C_{F_{x2}} F_{x2} + C_{F_{x3}} F_{x3} = 0; \]

\[ e_{3} = d M_{1} + C M_{2} + C M_{3} K F_{x1} + r K F_{x2} + d K F_{x3} + p C F_{z2} + p K F_{z3} = 0; \]

\[ S = \text{solve}(e_{1}, e_{2}, e_{3}, z, x, \phi). \]
Appendix F

Orthogonal scanner static model parameter variations

Six graphs have been made varying L, L2, p, r, w,h and w2,h2. In the last two cases the width w is equal to the depth h and for this reason varied as only one parameter.
Bibliography


