Abstract: When specifying the winding insulation of transformers, it is important to know the electrical stresses to which the winding can be exposed during fast transient oscillations. These oscillations occur during switching operations by circuit breakers, or when using gas insulated substations. Therefore, one of the priorities is to use a high-frequency transformer model capable to simulate fast transient oscillations in the windings. The model presented in this paper requires only information about the geometry of the winding and the core, as well the electrical and magnetic parameters for the used materials. In the model, frequency-dependent core and copper losses are included. Numerical computations are done with and without taking into account the core losses. Two types of measurements are done to verify the validity of the model. First, the voltage transients are measured and computed by applying a step impulse with a rise time of 50 ns. Then, the transformer is switched by a vacuum circuit breaker and the multiple reignitions are analysed. The results verify that the model is applicable to simulate the voltage distribution in transformer windings in a wide frequency range.

1 INTRODUCTION

Experience shows that very fast transient overvoltages (VFTO) are not only dangerous with their amplitude, but also with their rate of rise. When transformer or motor insulation is frequently exposed to fast transients like switching surges, it deteriorates and finally it may fail. The fact [1,2] is that overvoltages with lower amplitude and higher rate of rise can be as dangerous as overvoltages with higher amplitude. Transformers are normally protected by surge arresters. However, these devices limit only the amplitude of the overvoltages. In practice [2,3], zinc-oxide arresters with R-C snubbers (ZORC) are found to be an appropriate protection for medium voltage circuits. Because of the high price of these devices compared to the price of distribution transformers, utilities want to know how probable and how dangerous the occurrence of fast transients is. The study of very fast transients is also important for the design of the transformer insulation.

Traditionally we can distinguish two types of models: the first one is based on lumped parameters [4], while the others are based on the transmission line theory [5,6]. The advantage of the first type is that it can be easily implemented in simulation software, whilst the second model, which is purely mathematical, takes into account the frequency-dependent losses, but is not very easy to be implemented in a simulation software environment.

So far it is shown that the transmission line theory can be successfully applied to evaluate voltages along the windings of electrical machines and transformers [5-8]. Most of the time, it is considered that for high frequencies, the flux does not penetrate in the core and the iron core losses can be neglected accordingly. However, in [9] by measuring transformer impedance frequency characteristics for a short-circuited transformer and transformer under no load, it was demonstrated that the iron core does have significant influence up to approximately 100 kHz. In [10], it was reported that even up to 1 MHz, the iron core losses influence the frequency transients.

The present paper deals with the problem of evaluation of fast transient voltages in distribution transformer windings. The analysis is done for a 15 kVA single-phase test transformer with layer-type windings. The transformer has special measuring points at the 200th and 400th turn, which are accessible outside the transformer tank. To get insight of how the losses influence the distribution of fast transients, two types of computations have been performed. The first case takes into account only copper losses, whilst in the second case, voltage transients are computed by taking into account the frequency-dependent core losses. In both cases, the proximity effects are taken into account. The transformer is switched by a vacuum circuit breaker (VCB) and the voltage transients are measured at the same measuring points. The present model is able to simulate voltage distributions along the winding for a longer period of time. Within the observed time, voltage oscillations have a broad frequency range, from a few tens of kHz up to a few MHz.
2 THE MODEL

The origin of Multi Transmission Line Modelling is described through the theory of natural modes [11]. When a network of N coupled lines exists, and when $Z$ and $Y$ are the impedance and admittance matrices, which are the self and mutual impedances and admittances between the lines, then:

$$\frac{d^2V}{dx^2} = -ZV$$
$$\frac{d^2I}{dx^2} = -YZI$$  \hspace{1cm} (1)

where $V$ and $I$ are incident voltage and current vectors of the line. Note that $Z \neq Y^{-1}$. Applying the modal analysis, the system can be represented by the following two-port network:

$$\begin{bmatrix} I_s \\ I_r \end{bmatrix} = \begin{bmatrix} A & -B \\ -B & A \end{bmatrix} \begin{bmatrix} V_s \\ V_r \end{bmatrix}$$  \hspace{1cm} (2)

where:

$$A = Y_0S_Z \coth \left( \gamma l \right) S^{-1}$$

$$B = Y_0S_Z \cosh \left( \gamma l \right) S^{-1}$$  \hspace{1cm} (3)

In (2) and (3):

$L_s, L_r$ – are current vectors at the sending and the receiving end of the line,

$V_S, V_R$ – are voltage vectors at the sending and the receiving end of the line.

$Y_0 = Z^{-1}$ - characteristic admittance matrix,

$S, \gamma$ – eigen vectors and values of matrix $Z, Y$,

and

$l$ - length of the line.

Transformers are normally constructed with many turns, and therefore the above theory can be applied for a group of turns in order to avoid storing a large number of elements in the computer memory. One way is to observe some number of turns and the rest of the transformer to be represented by a terminating admittance [5,6]. Another way is to apply hybrid modelling [10]. This means that the total number of turns is split in a number of groups so that the number of a group of turns is far less than the number of turns in the transformer. The grouping should be done in such a way that the group of turns retains the information for the voltages and currents at the beginning and at the end of the observed group. If these parameters have the same value as they have on a turn-to-turn basis, than one can conclude that hybrid modelling is equivalent to modelling on a turn-to-turn basis.

For the observed case, the transformer winding is formed by layers, and therefore, the easiest way is to model the transformer on a layer-to-layer basis. When the voltages in each layer are known, the procedure can be applied for each layer and the voltages in the turns can be determined. The disadvantage of the terminal admittance model is that it is very difficult to provide the terminal admittance over a wide frequency range. Also, when a group of turns are terminated by the terminal admittance that represents the rest of the windings, the computed results are very sensitive to the variation of this admittance.

Fig. 1 shows the representation of the windings by transmission lines.

In the present case, the high-voltage winding is grounded, at the end. Therefore, to provide convergence during the computation process, the line is terminated by impedance $Z=1,e-9 \Omega$. Applying equation (2) to Fig. 1 results in the following equation:

$$\begin{bmatrix} L_{s1} \\ L_{s2} \\ \vdots \\ L_{sn} \\ L_{r1} \\ L_{r2} \\ \vdots \\ L_{rn} \end{bmatrix} = \begin{bmatrix} A' & -B' \\ -B' & A' \end{bmatrix} \begin{bmatrix} V_{s1} \\ V_{s2} \\ \vdots \\ V_{sn} \\ V_{r1} \\ V_{r2} \\ \vdots \\ V_{rn} \end{bmatrix}$$  \hspace{1cm} (4)

In (4), $A'$ and $B'$ are square matrices of n-th order calculated by (3). The following equations hold for Fig. 1:

$$L_{r1} = -L_{s2}, L_{r2} = -L_{s3}, \ldots, L_{rn} = \frac{V_{rn}}{Z}$$

$$V_{r1} = V_{s2}, V_{r2} = V_{s3}, \ldots, V_{rn-1} = V_{sn}$$  \hspace{1cm} (5)

By using these equations and making some matrix operations, equation (4) can be expressed as:
\[
\begin{bmatrix}
L_{S1} \\
0
\end{bmatrix} = \begin{bmatrix}
F \\
V_{S1} \\
0
\end{bmatrix}
\]

(6)

In (6), \(i = 1,2,3, \ldots, n\) and \(\mathbf{0}\) is a zero vector of order \(nx1\). When we observe the model on a layer-to-layer basis, then \(V_{Rh}=0\), hence (6) can be rewritten as:

\[
\begin{bmatrix}
V_{S1} \\
\mathbf{0}
\end{bmatrix} = \begin{bmatrix}
\mathbf{H}_{i-1} \\
\mathbf{H}_F
\end{bmatrix} \begin{bmatrix}
V_{S1} \\
\mathbf{0}
\end{bmatrix}
\]

(7)

where subscript \(i = 2,3, \ldots, n\), and \(\mathbf{H}_{i-1}\) is a vector of order \((n-1)x1\). \(\mathbf{H}_{i-1}\) represents the transfer function, the elements of which can be calculated from the elements of the \(\mathbf{F}\) matrix as:

\[
H_{i-1} = \frac{F_{i-1}^{-1}}{F_{i,1}^{-1}}
\]

(8)

The other sub-matrix \(\mathbf{H}_F\) does not have any physical meaning because a zero vector always multiplies it.

The voltages at the end of each layer can be calculated when the voltage at the input is known and the corresponding transfer functions are calculated. The time-domain solution results from the inverse Fourier transform:

\[
V_{S1}(t) = \frac{1}{2\pi} \int_{-W}^{W} \sin(\pi \omega/W) e^{(-b+j\omega)} \cos(\omega t) d\omega
\]

(9)

for \(i=2,3, \ldots, n\) and the \(V_{S1}(b+j\omega)=H_{i-1}(b+j\omega)V_{S1}(b+j\omega)\). If we divide the real and imaginary part of the integral function, and if we apply the property of evenness of the real part and oddness of the imaginary part with respect to \(\omega\), the following expression can be used [11]:

\[
V_{S1}(t) = \frac{2e^{bn}}{\pi} \int_{0}^{W} \frac{\sin(\pi \omega/W)}{\pi \omega/W} \cos(\omega t) d\omega
\]

(10)

In (10), the interval \([0,W]\), the smoothing constant \(b\) and the step frequency length \(d\omega\) must be chosen properly in order to arrive at an accurate time-domain response. The modified transformation requires the input function \(V_{S1}(t)\) to be filtered by an \(\exp(-bt)\) window function. To compute the voltages in separate turns the same procedure can be applied.

3 TRANSFORMER AND DATA

The propagation of fast surges in transformers depends on the geometrical design of the windings and the physical parameters of the insulation, like dielectric permittivity and magnetic permeability. Electrical and magnetic properties of the windings and core must also be taken into account. High-voltage transformers are normally designed with interleaved, concentric or layer windings.

The studied transformer is a single-phase 15 kVA, 6.6/0.07 kV/kV transformer with layer-type windings, which was specially built for the investigation of transient’s propagation. The transformer is equipped with special measuring points in the middle and at the end of the first layer of the transformer high-voltage side, and also at the end of the second layer. All measuring points can be reached from the outside of the transformer and measurements can be performed directly at the layers.

Fig. 2 shows the winding design of the studied transformer.

![Fig. 2 Description of the layers in a layer-type transformer winding](image)

The transformer is modelled on the layer-to-layer basis. The capacitances are calculated by assuming that the layers form cylindrical capacitors. So the capacitance matrix \(\mathbf{C}\) is formed in the following way:

\[C_{ii} - \text{is the capacitance of the layer} \ i \ \text{to ground and the sum of all other capacitances connected to the layer} \ i, \ C_{ij} - \text{is the capacitance between layers} \ i \ \text{and} \ j \ \text{taken with the negative sign} \ (i \neq j)\]

The investigation so far [12] has shown that the capacitances hardly change with the frequency. The capacitances to ground in the observed case are actually capacitances between the layers and the core. Because of the small surface of the layers these capacitances are very low and their value is around 1 pF. The static voltage distribution is strongly affected by the ground capacitances. However, for values less than 100 pF, the static voltage distribution can be considered as linear.

For very fast transients, especially for the first a few microseconds, it is a common practice to ignore the core and eddy current losses. The inductance matrix \(\mathbf{L}\) is evaluated as:

\[
\mathbf{L} = \frac{1}{V_y^2} \mathbf{C}^{-1}
\]

(11)
where the velocity of the wave propagation $v_s$ is calculated by

$$v_s = \frac{c}{\sqrt{\varepsilon_r}}. \quad (12)$$

In (12), $c$ is the speed of light in vacuum and $\varepsilon_r$ is the relative dielectric permittivity of the insulation. The losses were calculated from the inductance matrix $L$ and the capacitance matrix $C$ [4,5]. The impedance and admittance matrices $Z$ and $Y$ are then:

$$Z = \left( j\omega + \sqrt{\frac{2\omega}{\sigma\mu_0 d^2}} \right) L$$

$$Y = (j\omega + \omega \tan \delta) C \quad (13)$$

In (13), the second term in first equation corresponds to the Joule losses because of the skin effect in the copper conductor and the proximity effect. The second term in the second equation represents the dielectric losses. In (13), $d$ is the distance between layers, $\sigma$ is the conductor conductivity and $\tan \delta$ is the loss tangent of the insulation.

4 RESULTS AND VALIDATION

The measurements of fast transients for the particular measuring points are done in the following way. A pulse generator connected to the high-voltage transformer terminal provides maximum 300 V with a very short rate of rise. The full voltage is achieved within 50 ns. The transformer has special measuring points so that apart from the terminal voltage, measurements can be done at the end of the first layer, and at the end of the second layer. In Fig. 4, the measurements and the simulations of the voltage oscillations with and without iron core losses are presented.

In this way, the impedance matrix takes into account the frequency-dependent copper losses and the proximity effects. More general, the frequency-dependent core losses should be taken into account. Applying the Maxwell equations, explicit formulas for the self- and mutual impedances of coils have been derived [13,14]. In this way, the $Z$ matrix takes into account the frequency-dependent core and copper losses. For the studied case, the order of the $Z$ matrix is equal to the number of transformer layers. The tested transformer contains 15 layers with 200 turns/layer.

Fig. 3 Mutual inductance and resistance between the 1st and other coils of the studied transformer.

Fig. 4 Measured and computed voltage oscillations with and without taking into account the frequency-dependent core losses.

Fig. 5 Computed voltage oscillations with and without consideration of the frequency-dependent core losses at specific points.

The results, which do not take into account the frequency-dependent core losses are calculated by applying the impedance matrix according to (13). At the first local maximum, around 1 $\mu$s, the model with the core losses shows better matching with the measured
results. Since it was not possible to measure the voltages at each layer, and based on the validation of the model with the presented measurements, the computed voltages in the rest of the layers are compared with those computed by taking into account the frequency-dependent losses. In Fig. 5, computed voltage oscillations at specific layers are presented. For the first microsecond, the rate of rise of the voltages in both cases is almost similar.

Between 1 μs and 3 μs both lines show good agreement of the oscillation frequency and amplitude. Beyond 3 μs the difference becomes greater. Therefore, when the input excitation to the transformer is a complex surge with oscillations with different frequencies, and the surges are observed over a longer period of time, it is desirable to use a high-frequency transformer model that takes into account the frequency dependency of its parameters [12,15].

To show the validity of the transformer model for a wide frequency range, measurements of the multiple reignition overvoltages at the transformer high-voltage terminal, at the 200th and the 400th turn during switching of the transformer with a VCB have been done. The experimental test set-up is shown in Fig. 6. The capacitor applied at the source side keeps the voltage after switching stable. The voltage at the source side is measured by using a voltage transformer. An inductive load is used to provoke reignitions. The transformer is switched under a source voltage of 5 kV. The voltages are recorded at the measuring points simultaneously.

Fig. 7 shows the measured voltage at the terminal of the transformer. This voltage is used as an input voltage that is needed to determine the other line-end coil voltages. The calculated terminal voltage is produced by applying the inverse Fourier transform of the measured voltage, which is also a test that the inversion is correctly done. Fig. 8 and Fig. 9 represent the measured and computed voltage at the 200th turn and the 400th turn respectively.
The validity of the model is verified by measurements of fast transients produced during switching the transformer with a VCB. The method introduced here offers a possibility to predict the voltage distribution along the transformer winding when the transformer is excited by fast transient surges, which can be measured at the transformer terminal.

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7 REFERENCES