**Introduction:**

In fractured reservoirs, much of the oil is stored in low permeable matrix blocks that are surrounded by a high permeability fracture network. Therefore, production from fractured reservoir depends on the transfer between fracture and matrix, which is critically dependent on the matrix fracture interaction and as a result on the transfer rate between matrix and fracture (Chahardowli et al. 2007). When a strongly or partially water-wet matrix is surrounded by an immiscible wetting phase in the fracture, spontaneous imbibition is the most important production mechanism (Chahardowli et al. 2013). The dominant force in the transfer between matrix and fracture can be either capillary forces or gravity forces (Chahardowli and Bruining 2012). Mutual solvent (diethyl ether) could improve the ultimate recovery and the imbibition rate in partially and completely water wet core samples. The main recovery mechanisms are the wettability change of the partially water-wet cores and oil swelling and the oil viscosity reduction in both partially and completely water-wet cores. (Chahardowli et al., 2013).

We derive a simplified model for imbibition of water that contains solvents that are both soluble in water and oil can improve the recovery. We consider a cylindrical core that is initially filled with oil. The completely water-wet core does not contain water. However, the partially water-wet core contains connate water in addition to the oil. The core is immersed in a water bath (Amott-cell). Water penetrates in the core and releases some oil, which rises to the top and can be monitored. The model consists of a two phase flow problem and includes interchange of the solvent between the oil and the water phase. The model includes gravity and capillary effects. The boundary conditions are pressure continuity between the boundary of the core and the fluid surrounding the core. The model can quantify the amount of oil that is capillarity displaced from the core both due to imbibition and molecular diffusion.

**Classical mathematical Model:**

In a spontaneous imbibition of Mutual solvent in the porous media, simplified governing equations are:

\[
\begin{align*}
\phi \frac{\partial v_o}{\partial t} + \nabla \cdot \left( V_{oh}(P_a, V_o, V_o) u_o(P_a, V_o, V_o) \right) = 0, \\
\phi \frac{\partial v_a}{\partial t} + \nabla \cdot \left( V_{aw}(P_a, V_o, V_o) u_a(P_a, V_o, V_o) \right) = 0, \\
\nabla \cdot (u_a(P_a, V_o, V_o) + u_o(P_a, V_o, V_o)) = 0.
\end{align*}
\]

Where Darcy velocities for the aqueous phase and the oleic phase are defined respectively as

\[
\begin{align*}
u_a &= -\lambda_a(V_o, V_o)(\nabla P_a + \rho_a(V_o, V_o)g), \\
u_o &= -\lambda_o(V_o, V_o)(\nabla P_a + \nabla P_e(V_o, V_o) + \rho_a(V_o, V_o)g).
\end{align*}
\]

**Implementation of dynamic effects in the model:**

To consider the delayed imbibition in the model, we used the concept of non-equilibrium and dynamic capillary pressure and relative permeability effects. We formulated the extended model that includes the dynamic constitutive relations. The improved model is

\[
\begin{align*}
\phi \frac{\partial t \psi_o}{\partial t} + \nabla \cdot \left( V_{oh}(P_a, \psi_o, \phi_o) u_o(P_a, \psi_o, \phi_o) \right) + \tau_1 \nabla \cdot (\nabla \left( V_{oh}(P_a, \psi_o, \phi_o) u_o(P_a, \psi_o, \phi_o) \right)) = 0, \\
\phi \frac{\partial t \psi_a}{\partial t} + \nabla \cdot \left( V_{aw}(P_a, \psi_o, \phi_o) u_a(P_a, \psi_o, \phi_o) \right) + \tau_2 \nabla \cdot (\nabla \left( V_{aw}(P_a, \psi_o, \phi_o) u_a(P_a, \psi_o, \phi_o) \right)) = 0.
\end{align*}
\]

Where equations (3-5) are written as a function of new primary variables of \( \phi_o, \psi_o \). Moreover, the primary variables are defined as

\[
\phi_o = V_o + \tau_1 \frac{\partial t}{\partial} V_o, \quad \psi_o = V_a + \tau_2 \frac{\partial t}{\partial} V_a
\]

**References:**

