Stellingen

behorende bij het proefschrift

Wireless Channel Modeling and Code Division Multiple Access for Indoor Communications

H. Nikookar

Delft, 3 oktober 1995
1. It is interesting to note how little we know about radio propagation inside buildings, the environment in which we spend most of our time.

Het is interessant om te merken hoe weinig we weten over radio propagatie binnen gebouwen, de omgeving waarin we het grootste deel van onze tijd doorbrengen.

2. Although research and development in the field of wireless communications are increasing, considering the amazing growth of the field, they are in the infancy state and more investigation endeavors are required in this vast fertile technological field.

Hoewel onderzoek en ontwikkeling op het gebied van draadloze communicatie toenemen, de enorme groei op dit gebied in aanmerking nemend, staat het nog in de kinderschoenen en zijn meer onderzoeksinspanningen op dit vruchtbare gebied nodig.

3. A realistic indoor wireless channel model should take into account these factors: the random number of multipath components in each impulse response, correlation between adjacent multipath components, spatial correlation between impulse response profiles, path loss, dependence of the parameters on the shape, size and construction of the building, as well as frequency and temporal variations of the channel. Furthermore, it should derive its parameters from actual field measurements rather than basing them on simplified theory.


Een realistisch "indoor wireless channel" model moet rekening houden met deze factoren: het willekeurige aantal multipad componenten in elke impulstreponie, de correlatie tussen naaste multipad componenten, ruimtelijke correlatie tussen impulsresponsieprofielen, propagatieverlies, afhankelijkheid van de parameters van de vorm, grootte en constructie van het gebouw, en ook van de frequentie en tijdelijke variaties van het kanaal. Verder zou het de parameters moeten afleiden van werkelijke veldmetingen, in plaats van ze te baseren op versimpelde theorie.

4. Besides from the power control problem, the performance of the DS-CDMA technique is mainly limited by the multiple access interferences. Application of the efficient signal processing methods, offering a satisfactory compromise of complexity and performance, is highly suggested to surmount these shortcomings.

Afgezien van het probleem van vermogenscontrole, wordt de prestatie van de DS-CDMA techniek voornamelijk beperkt door de multiple access interferenties. Toepassing van efficiënte signaalverwerkingsmethoden, die een aanvaardbare compromis bieden tussen complexiteit en prestatie, wordt warm aanbevolen om deze tekortkomingen te overwinnen.

5. Whichever multiple access technique is employed, the ultimate performance limitation is the system's susceptibility to interference.


Welke "multiple access" techniek ook wordt gebruikt, de uiteindelijke prestatiegrens is de gevoeligheid van het systeem voor interferentie.

6. He uses statistics as a drunken man uses lampposts for support rather than illumination.


Hij gebruikt statistiek zoals een dronken man een lantaarnpaal, voor ondersteuning in plaats van verlichting.
7. In the engineering curriculum the amount of the knowledge we acquire is not important, what is notable is the scientific, well organized approach, that we learn to curb our envisaged problems.

In het technisch universitair onderwijsprogramma is de hoeveelheid kennis die we verkrijgen niet belangrijk, wat belangrijk is, is de wetenschappelijk verantwoorde en goed georganiseerde aanpak die we leren om onze problemen op te lossen.

8. Just like a simulation program performed before construction of an engineering product, those who ratify the rules should inspect the consequences of the regulations they are sending to effect, in both the transient and steady state terms, in order to minimize the possible adverse effects of those rules, in the cases which have not been noted at the time of rule making and which may take a long time to provide a feed back to the authorities to modify.

Net als een simulatie programma wordt uitgevoerd voordat met de constructie van een technisch produkt wordt begonnen, net zo, zouden zij die wetten ratificeren, de consequenties moeten onderzoeken van de regels die zij invoeren, voor zowel de overgangs- als evenwichtssituatie, om de mogelijk negatieve effecten van die regels te minimaliseren, voor die gevallen die niet werden opgemerkt tijdens het ontwerpen van de wet en welke een lange tijd nodig kunnen hebben om terugkoppeling te geven aan de makers voor aanpassing.

9. Don’t notice to which state of life trajectory you are. Try to follow the best way from that state to your target. With this approach your route to your aim is optimal.

Let niet op de toestand van het levenspad waarin je nu bent. Probeer de beste weg te volgen van die toestand naar je doel. Met deze aanpak is je route naar je doel optimaal.

10. Life is a "Markov II" process. Our afterlives depend on what we have done in this world.

Leven is een "Markov II" proces. Ons leven na het hiernamaals hangt af van wat we gedaan hebben in deze wereld.
Wireless Channel Modeling
and Code Division Multiple Access
for Indoor Communications
On the cover: A graphical description of phase initiation and updating for the two phase models of the indoor radio propagation channels (explained in chapter 4).
Wireless Channel Modeling and Code Division

Multiple Access for Indoor Communications

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To My Father, Mother, Sisters and Brother
SUMMARY

Over the past few years wireless communication has witnessed a prominent growth. Worldwide research and development in radio communications are increasing exponentially, resulting in the vast availability of wireless products in the market. Digital communication, initially used in the military applications, due to resistance to jamming, and secrecy, is now under way to the commercial side. Supremacy of the performance of the digital communication systems to the analog, accompanied with the technological advances in the solid-state microelectronics, have bolstered this trend. Employment of digital techniques in wireless communications, providing a good quality and cost, as well as service to a large number of users, has a promising landscape in the realization of tetherless Personal Communication Services (PCS). Wireless indoor communication is the transmission of voice and data to people on the move inside buildings, residential areas, supermarkets, shopping malls, and so on. It also comprises the link between portable computers, vision systems, cash registers, etc. working inside a building, as well as communication between fixed base stations and robots in motion in manufacturing or factory environments. Indoor radio communication plays a key role in the implementation of PCS.

Wireless indoor communications are subject to multipath fading, which seriously degrades the performance of the system. Modeling of the indoor propagation channel enables proper design of the transmitter and receiver in order to reduce the effect of fading. Therefore, detailed characterization of the radio propagation channel is a major requirement for the successful design of indoor wireless communication systems. In the first part of this dissertation, which comprises chapters 1-4, statistical modeling and characterization of the indoor radio propagation channel is carried out. The results are based on mathematical analysis, simulation and measurement. The large empirical data base used in this dissertation consists of 12000 impulse response profiles of the channel, collected at two dissimilar office environments. On the other hand, because of the
high rate of growth of wireless communication, it becomes necessary to use the spectrum more effectively. Spread spectrum is a technique that not only provides efficient use of the spectrum (by allowing additional users to utilize the same band as the other users), but also combats fading of the indoor channel. In the second part of this thesis, (chapters 5-7), the performance of the spread spectrum modulation techniques is evaluated in an indoor communication system using the above mentioned database.

In chapter 1 the mathematical modeling of the indoor radio propagation channel is carried out and the method of the impulse response is described. In chapter 2 details of the measurement campaign leading to the large database of indoor wireless propagation channel are explained and the measurement plan, procedure, and technique are expressed. In chapter 3 statistical modeling of signal amplitude fading in indoor radio propagation channels is carried out. Amplitude fading in the indoor multipath environment may follow different distributions depending on the area covered by measurements, the presence or absence of dominating strong components and some other conditions. The major candidate distributions are described and elaborate tests are used to examine the goodness of fit of these distributions over amplitudes of the empirical large database. It is known that the performance of digital communication systems operating in multipath environments is very sensitive to the statistical properties of the received signal's phase. Therefore, in chapter 4, results of the comprehensive modeling effort to describe variations of received signal phase for digital transmission within buildings are reported. To this end, two phase models are proposed and studied in great detail. The performance of these models are evaluated by means of extensive computer simulations and by utilization of the database.

In chapter 5 direct sequence spread spectrum technique is studied in the multipath fading indoor radio channel, and its performance is evaluated using the measured data. Multicarrier modulation technique, which uses the spectrum more efficiently, is investigated in chapter 6. In this parallel transmission technique the spectra of subchannels overlap each other while satisfying orthogonality, giving rise to the spectral efficiency. The advantages of this technique, in particular to combat the frequency selectivity of the channel, are explained. The performance of multicarrier transmission is evaluated using realistic indoor channels. In chapter 7 the two techniques (spread spectrum and multicarrier modulations) are combined to achieve high spectral efficiency and capacity. Again the performance is assessed over the measured channels. In chapter 8 concluding remarks are given and recommendations for future work are provided.
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CHAPTER 1

INTRODUCTION TO MATHEMATICAL MODELING OF THE INDOOR RADIO PROPAGATION CHANNELS

Analysis and design of indoor wireless communication systems are based on the knowledge of indoor propagation characteristics. Radio propagation mechanisms in the mobile and particularly in the indoor environments are complex. Radio signals propagated inside buildings undergo attenuation by walls, roofs and objects. In general, because of the reflection, refraction and scattering of the transmitted wave, the signal arrives in the receiver through different paths having different amplitudes and phases. In wide band pulse transmission this causes the delayed and attenuated echoes of each transmitted pulse, which in the digital communication systems results into intersymbol interference and eventually limits the rate of the wireless transmission in indoor environments. In narrow band transmission the indoor medium causes fluctuations in magnitude and phase of the received signal.
Chapter 1  Introduction to Mathematical Modeling of Indoor Radio Propagation

Multipath fading is a severe problem in the indoor communication systems. By well characterization of the channel the adverse effect of the fading can be diminished. The characterization of the indoor radio propagation channel is based on the mathematical model of the channel which can be carried out in the time or frequency domain. In this chapter these mathematical models of the indoor radio propagation channels are described. In section 1.1 we put special emphasis on the time domain impulse response modeling of the channel, since the multipath medium can fully be described by its time and space varying impulse responses. Accordingly, this method will be used for the statistical modeling and performance evaluation of indoor communication systems throughout the dissertation. In section 1.2 the frequency domain modeling of the indoor channel is explained. Representation of the channel with the poles of an autoregressive process is mentioned. In order to make this chapter complete, other methods of modeling of the indoor propagation channels are also discussed briefly. To this end, in section 1.3 the modeling of the indoor radio propagation channel with the methods like ray tracing, finite difference approach of the Maxwell equations, as well as a signal processing technique is mentioned. Conclusions are given in section 1.4. Finally, section 1.5 presents the scope and overview of this dissertation.

1.1  TIME DOMAIN MODELING

1.1.1 The Impulse Response Approach

In general, the indoor radio propagation channel due to the motion of portable, people and equipments is time varying. The complicated random and time varying indoor radio channel at each point in the three dimensional space can be modeled as a linear time-varying filter with the impulse response given by [1]

\[ h(t, \tau) = \sum_{n=0}^{N(\tau)-1} a_n(t) \delta[\tau - \tau_n(t)] e^{j\theta_n(t)} \]  \hspace{1cm} (1.1)

where \( t \) and \( \tau \) are the observation time and application of the impulse, respectively, \( N(\tau) \) is the number of multipath components, \( a_n(t) \), \( \tau_n(t) \), and \( \theta_n(t) \) are the random time-varying amplitude, time of arrival, and phase sequences respectively, and \( \delta \) is the Dirac delta function. The channel is completely characterized by these path variables. This mathematical model is a wideband model that can be used to obtain the response of the channel to the transmission of any signal. Analysis of the time varying filter model of (1.1) is difficult, therefore, studying the variations of the indoor
channel can be divided into two categories. i) Variations due to motion of the portable antenna in the static environment (spatial variations), and ii) variations caused by motion of people and equipment in the environment when both transmitting and receiving antennas are stationary (temporal variations). These two variations are different, however, both are present in portable radio telephone applications [2]-[3]. For the applications such as fixed wireless data communication only ii) is important.

Analysis and measurement of the indoor radio propagation channels are reported in [1]-[20]. In [2] and [3] temporal variation of the indoor radio propagation channel is studied and the effect of different antenna separations, different degrees of motion of people in the indoor environment and around the transmitter and receiver antennas as well as the correlation properties of signals are investigated based on an extensive empirical data base of 192 one-minute recording of CW envelope fading waveforms between two stationary antennas. The investigation of temporal variation of the channel shows deep fades up to 20 dB below the mean level [1],[8]. The results of the study of time variation of the indoor channel is important in the analysis and design of fixed wireless data communication applications (e.g., wireless computer communications with both terminals stationary) working in indoor environments.

In the study of spatial variations of the indoor channel a static environment is considered and the effect of variation of the position of the portable is investigated. The stationary indoor medium can be accessed during the "quiet" hours (e.g., during the nights or weekends) of the environment, where there are few, if any, motion of people and equipments in the medium. As will be described in detail in the next chapters of this dissertation, the measurements, modeling and performance evaluation of indoor communications systems have been performed based on the spatial variation of the channel. However, the ideas and methods with proper modifications, are applicable to the other type of variation of the channel. In the stationary indoor channel the time invariant version of (1.1) can be used to describe the channel as

\[ h(t) = \sum_{n \neq 0}^{N-1} a_n \delta(t - \tau_n) e^{j \theta_n} \]  

(1.2)

This model was first suggested by Turin [21] to describe the multipath fading mobile channel, and has been successfully used for the characterization [22],[23] and simulating [24] of the urban radio channel. As will be elucidated in the following chapters the mathematical model of (1.2) plays a dominant role in the statistical characterization of indoor radio propagation channels as well as
Chapter 1  Introduction to Mathematical Modeling of Indoor Radio Propagation

The performance evaluation of communication systems working inside buildings. Meanwhile, this mathematical model is a wide band model, which has the advantage that because of its generality it can be used to obtain the response of the channel \( y(t) \) to any transmitted signal \( s(t) \) by the convolution integral and by adding the noise,

\[
y(t) = \int_{-\infty}^{\infty} s(x) h(t-x) \, dx + n(t)
\]  

(1.3)

where \( n(t) \) is the lowpass complex value additive Gaussian noise. With the above model if \( \text{Re}(s(t) \cdot \exp(j\omega_0 \phi)) \) is sent, where \( s(.) \) is a lowpass waveform and \( \omega_0 \) is the carrier frequency, the received signal \( \text{Re}(p(t) \cdot \exp(\omega_0 \phi)) \) becomes \([1],[22]\)

\[
p(t) = \sum_{k=0}^{N-1} a_k s(t-\tau_k) e^{j\theta_k} + n(t)
\]  

(1.4)

In reality as a portable moves the received signal changes with space. Accordingly, for each position of the portable an impulse response is considered. These impulse response profiles are shown in Fig. 1.1. The adjacent impulse response profiles in space have similar characteristics since the reflectors and scatterers producing the multipath remain nearly the same for short distances.

1.1.2 The Narrow-Band Model

By transmission of a constant envelope signal (i.e., \( |s(t)| = 1 \)), and ignoring the noise, the signal of (1.4) at a point in space is

\[
A e^{j\phi} = \sum_{k=0}^{N-1} a_k e^{j\theta_k}
\]  

(1.5)

This vector addition of the multipath components is represented in Fig. 1.2. According to (1.5) the received signal \( y(t) \) is written as

\[
y(t) = \text{Re} [A e^{j\phi} e^{j\omega_0 \phi}] = A \cos(\omega_0 t + \phi)
\]  

(1.6)
Figure 1.1 Sequence of adjacent impulse response profiles in space.

In this case the effect of multipath fading of the channel on the transmitted signal is given by a change in amplitude by a factor

\[
A = \sqrt{\left( \sum_{i=0}^{N-1} a_i \cos \theta_i \right)^2 + \left( \sum_{i=0}^{N-1} a_i \sin \theta_i \right)^2} \tag{1.7}
\]

and in phase by

\[
\phi = \tan^{-1} \left( \frac{\sum_{i=0}^{N-1} a_i \sin \theta_i}{\sum_{i=0}^{N-1} a_i \cos \theta_i} \right) \tag{1.8}
\]

Sampling the channel's impulse responses frequently enough, one should be able to generate narrow band CW fading results for the receiver in motion, using the wide band impulse response model [1].
1.2 FREQUENCY DOMAIN MODELING

Another approach for the modeling of the indoor radio propagation channel is using the frequency domain. The main advantage of the modeling in the frequency domain is that it requires less parameters for characterization of the channel than time domain methods. Full description of the frequency domain modeling of indoor channels is reported in [14]-[20]. The basic idea of this technique is that the frequency response of the indoor channel at each point, $H(f)$, is modeled by an auto-regressive (AR) process. Accordingly, it is determined by the output of a linear filter with transfer function $G(z)$ having $p$ poles, that is

$$G(z) = \frac{1}{\prod_{i=1}^{p} (1-p_i z^{-1})}$$

(1.9)

when excited by a white noise process with the variance [18]

$$\sigma^2 = R(0) - \sum_{i=1}^{p} a_i R(i)$$

(1.10)

where $a_i$'s are the parameters of the model and $R(.)$ is the auto correlation function of the frequency response [16].
\[ R(i) = \int_{-\infty}^{\infty} H(f)H^*(f+i\Delta f)df \]  

(1.11)

and \( \Delta f \) is the sampling frequency. With this method only \( p \) poles are required to determine the frequency response of the channel. Based on the frequency domain measurements in the 0.9-1.1 GHz frequency band, a second order \( (p=2) \) AR model is reported for the characterization of the indoor channel [15],[18]. The geometry of the poles is important. The delay associated with a pole is determined by the angle of that pole and the sampling frequency \( \Delta f \). The closeness of a pole to the unit circle represents significant power at the corresponding delay.

1.3 OTHER METHODS

1.3.1 The Ray Tracing Model

Another method of the modeling of the indoor radio propagation channel is the ray tracing algorithm. This method has been widely used for analyzing propagation in outdoor and indoor areas [25]-[30]. In this method the emitted energy of the transmitter antenna is traced geometrically and surfaces which are illuminated are determined. Accordingly, each illuminated surface is replaced by an image transmitter such that the radiation from the image represents the energy reflected from the source. These images are then used as objects for second round reflections and are considered in turn by ray tracing to determine the surface they illuminate, and the process is repeated in the same way. The images are therefore in a tree graph. Only the rays which leave the source and reach the destination are treated in this method. For an area with \( M \) surfaces the number of \( i \)-reflection images of a source is \( M(M-1)^{i-1} \), [25]. Therefore, the complexity of this method increases exponentially with the complexity of the area. Using the high computational capacity of the modern computers this method can be employed for the deterministic prediction of the radio propagation characteristics inside the buildings. The accuracy of the ray tracing algorithm depends mainly on the ratio of the wavelength to the dimensions of the scatterers and the volume of interest [25]. The main feature of the ray tracing technique is that it considers the precise location of the transmitter and receiver in the propagation process, an issue which is not addressed in the statistical approach.
1.3.2 The Finite Difference Method

Prediction of indoor radio propagation characteristics can be carried out by solving the Maxwell equations numerically using the finite difference technique in time domain [31]. With this model the Maxwell's equations are approximated by a set of finite difference equations, where emitting source and the initial conditions are taken into consideration. By doing so, the electric field in time and at the portable position is calculated. The complexity of this technique depends mainly on the size of the area under consideration and the frequency [31]. For large areas and higher frequencies a large computational capacity and computer memory are required. Application of finite difference time domain method in indoor wireless channel is reported in [32].

1.3.3 The Signal Processing Method

Another method for the modeling and estimation of the propagation characteristics of the indoor channel is a signal processing approach [33]. Basically, in this method the transfer function \( H(f) \) of the channel (i.e., the Fourier transform of (1.2)), with

\[
H(f) = \int_{-\infty}^{\infty} h(t) e^{-j2\pi ft} dt = \sum_{n=0}^{N-1} a_n e^{-j(2\pi ft_n - \theta_n)}
\]  

(1.12)

is sampled at the discrete frequencies \( \omega_k = 2\pi f_k \), i.e.,

\[
H(\omega_k) = \sum_{n=0}^{N-1} a_n e^{-j(\omega_k t_n - \theta_n)}
\]  

(1.13)

The samples of the transfer function are added by white Gaussian noise and they are represented now as

\[
T_k = H_k + \nu_k \quad k = 1, 2, \ldots, K
\]  

(1.14)

where \( \nu_k \) is the noise term for the \( k \)th sample. In vector form, (1.14) may be written as [34]

\[
T = S a + \nu
\]  

(1.15)

where,
\[ T = [T_1 \ T_2 \ ... \ T_K]' \]
\[ S = [S_0 \ S_1 \ ... \ S_{N-1}] \]
\[ S_n = [1 \ e^{-j\omega t_0} \ e^{-j2\omega t_0} \ ... \ e^{j(K-1)\omega t_0}]' \]
\[ a = [a_0 e^{i\theta_0} \ a_1 e^{i\theta_1} \ ... \ a_{N-1} e^{i\theta_{N-1}}]' \]
\[ v = [v_1 \ v_2 \ ... \ v_K]' \]

where \(^T\) denotes the transpose. In this method the eigenanalysis of the covariance matrix \( P \), (i.e., \( P=E[T.T^\dagger] \)), determines the parameters of the channel (the number of multipath components, the amplitude, phase, and arrival time of the paths). In other words, the channel is modeled by the \( K \) principal eigenvalues of the covariance matrix \( P \). The relationship between the eigencomponents and the parameters of the channel as well as the application of this method to the simulated and measured indoor radio channels is reported in [33]-[34]. Although the simplicity of this model is an advantage, the model has the problem of ill-conditioning of the covariance matrix as well as the limitation of white Gaussian noise assumption in the modeling procedure.

1.4 CONCLUSIONS

In this chapter the mathematical models for indoor radio propagation channels were discussed. Since the multipath medium is fully described by its impulse responses, special emphasis was exerted on the modeling of the indoor channel based on the time domain impulse response method. However, the other methods of modeling of the channel (i.e., frequency domain, ray tracing, finite difference method of the Maxwell’s equations as well as a signal processing approach) were also mentioned. The impulse response approach has been used for the measurements of indoor radio propagation channels (which is described in the next chapter), and as will be seen in the other chapters, it will be utilized for the statistical characterization and modeling of indoor channels as well as the performance evaluation of spread spectrum and orthogonal frequency division techniques in the wireless in-building communication systems.
1.5 SCOPE OF THE THESIS

The objective of this dissertation is two fold: i) Mathematical modeling of the indoor radio propagation channel, and ii) Application of the model in the investigation of the direct sequence code division multiple access [35] and multi-carrier techniques as well as the combination scheme. Accordingly, the thesis comprises two parts. In the first part, which contains chapters 1-4, statistical modeling and characterization of the wireless indoor propagation channel is carried out. To this end, in this chapter impulse response modeling of the indoor channel was described. In chapter 2 details of the measurement effort leading to the large data base of the indoor radio propagation channel are explained. This data base has been used in the whole of this dissertation. In chapter 3 statistical modeling of signal amplitude fading in the wireless indoor propagation channel is presented. Results of comprehensive modeling effort to describe signal phase for digital transmission inside buildings are reported in chapter 4.

In the second part of the dissertation, which entails chapters 5-7, direct sequence spread spectrum multiple access as well as efficient multi-carrier modulation scheme is studied in indoor communication systems. In this regards, in chapter 5 performance of the direct sequence code division multiple access technique is assessed over the large data base of the indoor propagation channel. Multi-carrier transmission in indoor radio communication systems is investigated in chapter 6 and its performance is evaluated over realistic indoor channels. The combination of these two techniques for achieving advantages of both methods, is studied in chapter 7 and its performance is assessed using the empirical indoor propagation data. In chapter 8 conclusions are given and recommendations for the future works are provided.
REFERENCES OF CHAPTER 1


References of Chapter 1


CHAPTER 2

MEASUREMENTS OF INDOOR RADIO PROPAGATION CHANNELS

Indoor wireless communication is the transmission of voice and data to people on move inside buildings, residential areas, supermarkets, shopping malls, etc. It also comprises the link between portable computers, vision systems, cash registers, etc. working inside buildings as well as communication between fixed stations and robots in motion in manufacturing or factory environments. Successful design of indoor wireless communication systems requires detailed knowledge of radio propagation inside buildings. In order to study the indoor radio propagation thoroughly, in an elaborate measurement campaign an extensive multipath indoor propagation data base has been set up. The purpose of these measurements was characterization and modeling of indoor wireless propagation channels based on statistical analysis of the impulse responses of the channels. These measurements include 12 000 impulse response profiles of indoor radio
channels in two office buildings, which is the largest in its kind.\(^1\)

In this chapter details of these measurements are described. In sections 2.1 to 2.3 the plan, sites and technique of the measurements are explained, respectively. Discrete-time method of impulse responses is expressed in section 2.4 and in section 2.5 the specifications of final data base as well as the main statistical characteristics of these indoor wireless propagation channels are mentioned. As we will see in the next chapters, this large empirical data base is an invaluable asset for statistical modeling of indoor radio propagation channels, evaluation of performance of spread spectrum techniques in indoor multipath environments, as well as for developing of a general purpose indoor radio propagation simulator.

2.1 MEASUREMENT PLAN

The goal of original measurements was to investigate "small-scale", "mid-scale" and "large scale" variations in the wide-band statistics of the channel. Small-scale variations imply changes in the statistics when the moving antenna's position changes by a few centimeters. By mid-scale variations we mean changes in statistics when the environment of the moving antenna changes (i.e., from a room to hallway), with antenna separation remaining unchanged. The channel's structure may change drastically when the transmitter-receiver antenna separation increases, among other reasons due to increase in the number of intervening obstacles. Gross changes in statistics due to changes in antenna separations are labelled "large-scale variations". These three types of variations are fully described in [2]-[3].

In a typical wireless indoor radio communication system two-way transmission takes place between a fixed station and a moving unit. Correspondingly, the following measurement plan was devised: four transmitter-receiver antenna separations of 5, 10, 20 and 30 meters were considered. For each antenna separation several places for the fixed antenna (base unit) were selected. The selection has been made on the basis of what is considered to be typical positions for base antenna in future systems. For each antenna separation a total of 20 small 1.5 to 2 meter areas (locations) were selected for the moving (portable unit) antenna position. The selections have

\(^1\) The ideas and plan of measurements were devised by Prof. H.Hasbemi from Sharif University of Technology, Tehran, Iran on his sabbatical leave at NovAtel Communications Ltd., Calgary, Canada. Measurements in the first building were carried out by D.Tholl of NovAtel (currently with Northern Telecom, Calgary), and in second building by G.Morrison of TR Labs, Calgary. Software assistance was provided by D.Lee and D.Ehman of NovAtel. Main body of this chapter has been written using documents of the measurements [1]-[2].
been made on the basis of good variation of typical conditions within buildings. Each location was carefully chosen, with both line-of-sight (LOS) and non-line-of-sight (NLOS) or obstructed topographies included. For large antenna separations, the number of locations with obstructed paths between transmitter and receiver has been higher than the corresponding for small antenna separations, consistent with conditions encountered in real-life indoor wireless communication systems. For each location 75 frequency response estimates of the channel were recorded by changing the portable antenna in steps of 2 cm using a step motor. The measurement scenario is illustrated in Fig. 2.1, [2].

2.2 MEASUREMENT SITES

The above measurements were carried out at two dissimilar office buildings. The first building (Building A) was the NovAtel Communications Ltd. Corporate Office located at 1020, 64th Ave. NE Calgary, Alberta, Canada. This is a three story facility containing hard partitioned offices, hallways, laboratory space in addition to a number of soft partitioned cubicles. Measurements were performed on the first and third floors, with both antennas located in the same floor. These floors are dissimilar in the sense that first floor consists of a lobby, a library, several laboratory spaces and some office and cubicles, while the third floor consists mainly of cubicles and office spaces. The construction of Building A is [1]

- Inner walls
  406 mm metal stud spacing
  13 mm gypsum covering

- Outer walls
  254 mm concrete slab
  windows around perimeter from 1.1 to 2.5 m in height

- Floors
  254 mm carpeted concrete slab

- Suspended ceiling
  standard fire retardant material

The second building (Building B) was the Alberta Government Telephone (AGT) Tower located at 411, 1st street SE Calgary, Alberta. This is a high rise building containing offices and
Figure 2.1 The scenario of measurements of indoor radio propagation channels, [2].
cubicles with some laboratory space. Measurements took place in 19th floor with transmitter and receiver in the same floor. The floor consists mainly of hard-partitioned hallways and soft-partitioned cubicles with standard office furniture. The construction of Building B is [1]

- Inner walls
  - 406 mm metal stud spacing
  - 13 mm gypsum covering
- Outer walls
  - 254 mm concrete slab
  - windows around perimeter from 1.1 to 2.7 m in height
- Floors
  - 254 mm carpeted concrete slab
- Suspended ceiling
  - standard fire retardant material

According to the scenario of the measurements at each antenna separation (i.e., 5, 10, 20 and 30 meters), 20 locations have been considered. Since geometry of the buildings made it impossible to pick up 20 portable locations per fixed site (base), the base sites were also varied for each antenna separation [2]. This has the added advantage of making the results more general, as compared to a scenario in which a single base station is selected for all the measurements. The transmitter and receiver in each building were chosen carefully to be representative of typical base and portable locations in indoor wireless communication systems [2].

2.3 MEASUREMENT TECHNIQUE

The measurements were performed using a vector network analyzer measuring the frequency response of the indoor propagation channel between two discone antennas. The idea of measuring the frequency response of the indoor channel is relatively new [4]. In this method the channel is excited with tones over a wide range of frequencies. The attenuation and phase shift of each frequency component (caused by propagation medium) is measured. The components used for carrying out frequency response measurements are reported in [1]. It includes HP8753A Vector Network Analyzer, HP85046A S-parameter test set, Mini-circuits ZLF2000 Amplifier, about 40 meters of RG400 coaxial cable, 2 stage Mini-circuits MAR-2 Monolithic Amplifier, 2
discone antennas, stepper motor with antenna positioning cart and an IBM 486 personal computer.

The network analyzer sweep frequency band in the measurements was 900 MHz to 1300 MHz in 500 KHz steps (801 points). The time required was 400 msec. per sweep with 10 sweeps being averaged per measurements, making actual time for each measurement (i.e., for each frequency response) 4 seconds [2].

The transmitter and receiver antennas were vertically polarized with a radiation pattern very similar to that of λ/2 dipole. The maximum gain in the horizontal plane was approximately 2 dBi. The return loss of the antennas exceeded 13 dB over the 400 MHz bandwidth used in these measurements. Base antenna heights were 2.5 m for Building A and 1.7 m for Building B. Portable antenna height was 1.2 m for both buildings. It is important to note that the fixed (base) unit was the receiving antenna in this case, placed close to the network analyzer, and the moving (portable) antenna was transmitting. Each antenna was connected to one port of the network analyzer by cable. Hardware of the measurements is well documented in [1]. Photographs of all the base antenna positions as well as mobile locations have been taken and collected in an album.

As explained in chapter 1 due to motion of people and equipment indoor propagation channels are in general, time varying. Since the purpose of the measurements was to investigate channel's variations in space (not in time), it was essential to keep the channel stationary during the measurements. Therefore, all measurements were carried out at nights or weekends when there were few, if any, other personnel in the vicinity of measurement set up.

Since a large number of measurements was planned, in order to gain confidence in the relatively new measurement technique described above, prior to measurements a comparison with other method of obtaining impulse response of the channel i.e., the Sliding Correlator Method, was made [2]. This method is a spread spectrum technique based on correlation properties of pseudo-random binary sequences. It has been used in measuring the impulse responses of the indoor radio channels [5]-[7]. The comparisons involved obtaining many impulse response estimates under identical conditions using these two measurement techniques. The results were very similar\(^2\). Details are reported in [1] and [8].

\(^2\) The sliding correlator measurements were carried out by a team at Communication Research Center in Ottawa headed by Dr. R.J.C.Baltitude.
The 12000 measured frequency response data were windowed by a Minimum 3-term Blackman-Harris window [9]. The window is

\[ w(n) = a_0 - a_1 \cos\left(\frac{2\pi}{N-1} n\right) + a_2 \cos\left(\frac{2\pi}{N-1} 2n\right) \quad n = 0, 1, \ldots, N \]  

(2.1)

where, \( a_0 = 0.42323 \), \( a_1 = 0.49755 \) and \( a_2 = 0.07922 \). After windowing the frequency domain data were zero padded to the next highest number of points which is a power of two (1024 in this case). The zeros were padded at either end of the spectrum and the frequency data were converted to time domain by inverse fast Fourier transform algorithm. Windowing in frequency domain is equivalent to interpolation in time domain. Resolution of the impulse response estimates is reciprocal of the bandwidth swept (2.5 nsec.) multiplied by the additional width of window. A resolution of 5 nsec. for the impulse response data was therefore obtained [1]-[2].

The narrow-band measurements have also been carried out with a CW tone at 915 MHz and were sampled 1601 times over 30 seconds. Four profiles have been gathered at two locations for each distance in both buildings with both antennas stationary at a busy time with people moving freely. These data have been processed to observe the Doppler spectrum of the channel. A complete description of the measurement procedure, providing information about measurement locations, hardware and techniques of the processing data can be found in [2] and details in [1]. Meanwhile, a later addition of the data base of 12000 impulse responses is a 215 one minute recording of the channel's temporal variations. The results of these extensive measurements and analysis of temporal variations of indoor radio propagation channel are reported in [11] and [12]. These measurements supplement the mentioned available spatial variation model of indoor radio propagation channel. In other measurement campaign another data base of frequency response estimates in the infrared range has been set up [13]. These frequency responses are good complements to the large data base of indoor radio propagation channels, explained in the above sections, to simulate, design and evaluate future wireless in-building (radio and optics) communication systems [10].
2.4 THE DISCRETE-TIME IMPULSE RESPONSE METHOD

A convenient model for characterization of multipath propagation channels is the discrete-time impulse response model [2],[14]-[16]. In this model the time axis is divided into small time intervals called "bins". Each bin is assumed to contain either one multipath component or no multipath component. The possibility of more than one path in a bin is excluded. A reasonable bin size is the resolution of the specific measurement, since two paths arriving within a bin cannot be resolved as distinct paths. Using this model, each impulse response can be described by a sequence of "0"s and "1"s, (the path indicator sequence), where a "1" indicates the presence of a path in a given bin and a "0" represents the absence of a path in that bin. To each "1", an amplitude and a phase value are associated. All the data were reduced accordingly using a bin size of 5 nsec. (resolution of measurements). Each impulse response function contains 100 bins corresponding to an excess delay of 500 nsec. The probability of receiving longer-delayed components was found to be negligible. It is important to note that since absolute timing information is essential for a number of applications, excess delay is measured relative to the delay of the direct path between transmitter and receiver (16.7, 33.3, 66.7 and 100 nsec. for antenna separations of 5, 10, 20 and 30 meters, respectively). The first arriving component will, therefore, have an excess delay greater than 0 if it does not arrive over a direct path [2]. In Fig. 2.2 sequences of "0"s and "1"s of a discrete-time impulse response are depicted.

![Graphical presentation of discrete-time impulse response method](image)

**Figure 2.2** Graphical presentation of discrete-time impulse response method.
2.5 THE DATA BASE

The measurement plan and procedure of the large data base of indoor radio propagation channel were explained in the previous sections. The results of these measurements are two sets of 6000 data files for each building. As mentioned in section 2.3 the frequency domain data were converted to time domain by inverse fast Fourier transform (IFFT). Accordingly 12000 time domain data files are available (2 buildings × 4 antenna separations × 20 locations × 75 profiles = 12000). The following naming method was used for the data files: An eight character alphanumeric code was assigned to each one of 12000 data files. The first letter which is A or B indicates the building (i.e., "A" for Building A and "B" for Building B). The next two numbers indicate transmitter-receiver antenna separation in that building (these are 05, 10, 20 and 30 corresponding to 5, 10, 20 and 30 meters, respectively). The following two numbers show the location number at that antenna separation (which starts from 01 and ends to 20 and means locations 1 to 20), and finally the last two numbers reveal the profile number at that location (referring to measurement plan it is from 01 to 75). Meanwhile since each profile has the measured information of amplitudes, time of arrivals and phases of multipath components, the last character of the name of each profile (which is A, T or P) indicates that data relate to amplitude, time of arrival or phase of multipath components at that profile. In order to explain the method of naming of each profile clearly we show it by the following example. Consider the following profiles

A050101A  A050101T  A050101P

It means the amplitude, time of arrival and phase of impulse response profile of Building A with transmitter-receiver antenna separation of 5 meters, in location number 1 and first position of portable at that location. In Figs. 2.3-2.6 some typical profiles of impulse responses are shown. It should be mentioned that all path strengths are relative to the transmission. In Table 2.1 a discrete-time version of the above mentioned impulse response profile is depicted. Each "0" in this table means there is no path component in that bin. Considering amplitude, arrival time and phase profiles for each impulse response, the total number of profiles is 3 × 12000 = 36000. This large data base has been analyzed in order to characterize the channel's impulse response [2]-[3]. The mathematical model of the channel has been employed (see subsection 1.2.1). In this section we briefly mention the result of the analysis of the number of path components and modeling of the time of arrival sequences which have been reported in [2]-[3] and [17]-[20].
Figure 2.3  Several adjacent impulse responses. Building A, antenna separation 5 meters, location 1, profile numbers 1 to 4.

Figure 2.4  Several adjacent impulse responses. Building A, antenna separation 5 meters, location 7, profile numbers 12 to 15.
Figure 2.5  Several adjacent impulse responses. Building B, antenna separation 20 meters, location 1, profile numbers 1 to 4.

Figure 2.6  Two impulse response profiles at 6th and 7th position of portable with antenna separation 20 meters of Building B. (left) Magnitude  (right) Phase.
### Table 2.1 Discrete-time impulse response of the first profile in location one with antenna separation 5 meters of Building A.

<table>
<thead>
<tr>
<th>Time of Arrival Sequence of</th>
<th>Amplitudes of Multipath Components (dB)</th>
<th>Phase of Multipath Components (radians)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multipath Components</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(multiplied by Time)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The number of multipath components in each profile of the data base has been calculated. Results show that the number of path components is a normal (Gaussian) random variable with the mean and standard deviation increasing with transmitter-receiver antenna separation [2]. Study of probability of occupancy (i.e., probability of having a multipath component) with an excess delay greater than 400 nsec. is negligible in the buildings where measurements were made [2].
Distribution of the arrival time sequence has also been investigated in [2]-[3] and [18]. As a first model Poisson distribution for the sequence of path arrival times may be considered. This distribution is often addressed when certain events occur with complete randomness. If \( I \) denotes the number of paths occurring in a given time interval \( T \), Poisson distribution will be

\[
P(I = i) = \frac{\mu^i e^{-\mu}}{i!}
\]

(2.2)

where \( \mu = \int_T \lambda(t)dt \) and \( \lambda(t) \) is mean arrival rate at time \( t \). For a stationary process \( \lambda(t) \) is constant and \( E[I]=\text{Var}(I)=\lambda \). Analysis of time of arrival of multipath components of the data base has shown that standard Poisson model does not provide a good fit. A more realistic model is "modified Poisson" or \( \Delta-K \) model. This model which takes into account the clustering properties of multipath components was first suggested by G.L.Turin [21] and was successfully used in analysis [15] and simulation [16] of mobile radio propagation channels. In standard Poisson model path components are in complete randomness, however, in modified Poisson process, occurring of a path will change the probability of having another one. According to this model process starts with a pure Poisson. If a path exists at time \( t \), then the process will switch to another Poisson process with parameter \( K\lambda(t) \) and if there is no further path in the interval \([t, t+\Delta]\) it comes back to its initial state at the end of the interval. This model is described by a series of transitions between two states. with \( K=1 \) or \( \Delta=0 \) this process reverts to standard Poisson process. Application of discrete version of this process to the path arrival times of data base of indoor radio propagation channel shows a very good fit [2]-[3] and [18]. In these references the optimal value of \( K \) of this process for all antenna separations of both buildings are reported. Good fit of \( \Delta-K \) model in describing time of arrival of path components is mainly due to nonrandomness of local structure, which means multipath components are occurring in groups. Another justification is that modified Poisson due to its nature, uses more information of the data compared with pure Poisson, i.e., the \( \Delta-K \) model uses empirical probabilities associated with individual small intervals \( \Delta \), while standard Poisson process model uses the total probability associated with a larger interval \( T > \Delta \),[2]-[3]. More details about the modified Poisson process can be found in [15].

Another parameter of interest of indoor radio propagation channels is RMS delay spread. The RMS delay spread is defined as [3]
\[ \tau_{rms} = \sqrt{\frac{\sum_k (\tau_k - \bar{\tau} - t_a)^2 a_k^2}{\sum_k a_k^2}} \]  

(2.3)

where

\[ \bar{\tau} = \frac{\sum_k (\tau_k - t_a) a_k^2}{\sum_k a_k^2} \]  

(2.4)

\( t_a \) is the arrival time of the first path in a profile and \( \bar{\tau} \) is the mean excess delay. \( \tau_{rms} \) is the second root of the second central moment of power delay profile. RMS delay spread shows the multipath spread of propagation channel and its potential intersymbol interference (ISI). Paths with long delays and strong amplitudes contribute in \( ms \) delay spread remarkably. It has been shown that performance of communication systems working in multipath environment is very sensitive to the value of \( ms \) delay spread and the maximum rate of data transmission in indoor environment is a few percent of \( 1/\tau_{rms} \) , [20] and [22]. Using the explained large empirical data base of impulse response estimates, the \( ms \) delay spread for the individual impulse responses have been analyzed [17],[19]-[20]. The mean value of \( ms \) delay spread is in the range of 20-30 nsec. RMS delay spread increases with transmitter-receiver antenna separations. Distribution of \( \tau_{rms} \) of the data base shows a good fit to normal distribution. Mean of \( \tau_{rms} \) for each location has great linear dependence with average path loss of that location. Quantified spatial correlations of \( \tau_{rms} \) is also reported in [20]. In Table 2.2 multipath statistics of the data base are seen.
Table 2.2 Multipath statistics of indoor radio propagation data base, [2].

<table>
<thead>
<tr>
<th>Building A</th>
<th>antenna separation (m)</th>
<th>mean of no. of path components</th>
<th>std. of no. of path components</th>
<th>mean of rms delay spread (nsec.)</th>
<th>std. of rms delay spread (nsec.)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5</td>
<td>10</td>
<td>20</td>
<td>30</td>
<td>16.9</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>4.2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Building B</th>
<th>antenna separation (m)</th>
<th>mean of no. of path components</th>
<th>std. of no. of path components</th>
<th>mean of rms delay spread (nsec.)</th>
<th>std. of rms delay spread (nsec.)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5</td>
<td>10</td>
<td>20</td>
<td>30</td>
<td>17.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3.9</td>
</tr>
</tbody>
</table>

2.6 CONCLUSIONS

In this chapter details of the measurements of indoor wireless propagation channels leading to the large data base of impulse response estimates of the channel were described. Reduction of the data to discrete-time version was explained. Major achievement of these measurements is the amount of data (6000 profiles per building), which is the largest in its kind. Results of statistical analysis of the time of arrivals, number of path components and rms delay spread of the data were mentioned. Using these data base in the next chapter we focus our attention on studying of paths' amplitudes and we will investigate the best distribution that describes the signal amplitude fading of indoor radio propagation channel.
REFERENCES OF CHAPTER 2


CHAPTER 3

MODELING OF AMPLITUDES

As explained earlier, in indoor wireless propagation we have multipath fading components. These multipath components are due to reflection, refraction and scattering of the propagated waves and are caused by obstacles, people, equipments, roofs, walls, etc. in indoor environment. Multipath fading seriously degrades the performance of communication systems. Unfortunately, little can be done to eliminate multipath disturbances. However, if the multipath medium is well characterized, transmitter and receiver can be designed to match the channel and to reduce the effect of these disturbances. In this chapter we focus our attention on amplitude fading of indoor radio propagation channels and provide a statistical model for signal amplitude fading inside buildings. Amplitude fading in indoor multipath environment may follow different distributions depending on area covered by measurements, presence or absence of a dominating strong component, and some other conditions. Major candidate distributions are described in this chapter. Elaborate tests based on Kolmogrov-Smirnov and Wilcoxon procedures are used to

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examine the goodness of fit of these distributions over amplitudes of measured data base of 12000 impulse responses of indoor radio propagation channels. Accordingly, the best distribution to model the signal amplitude fading in indoor is determined. Statistical properties of multipath amplitude components are investigated.

In indoor radio channel at a fixed delay the signal, \( re^{j\theta} \), is the vector summation of signals, \( r_i e^{j\theta_i} \), which are arriving at the receiver through different ways, i.e.,

\[
re^{j\theta} = \sum_{i=1}^{n} r_i e^{j\theta_i}, \tag{3.1}
\]

where, \( n \) is the number of path components. In multipath wireless propagation channel, if the difference between delay of paths is very smaller than inverse of transmission bandwidth, multipath components cannot be resolved as distinct pulses. These subpaths are added vectorially and the summation envelope is seen. Therefore, envelop is a random variable. Different distributions for this random variable are explained in the following section.

3.1 DIFFERENT DISTRIBUTION FUNCTIONS

3.1.1 The Rayleigh Distribution

Referring to Equation (3.1) a common assumption that can be considered is that \( r_i \) s be nearly equal. This assumption can be justified in small areas and in the absence of line of sight (LOS) component. Therefore, (3.1) is written

\[
re^{j\theta} = r' \sum_{i=1}^{n} e^{j\theta_i}, \tag{3.2}
\]

where, \( r'=r_1=r_2=\ldots=r_n \). The phase \( \theta_i \) is very sensitive to the path length and changes by \( 2\pi \) as the path length changes by a wavelength. Therefore, phases have uniform distribution between \( [0,2\pi] \). Considering this distribution for phases in (3.2), the joint distribution of \( r \) and \( \theta \) will be the product of uniform\([0,2\pi]\) distribution with Rayleigh distribution described by
Different Distribution Functions

\[ f(r) = \frac{r}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}} u(r) \]  \hspace{1cm} (3.3)

where, \( \sigma^2 = \Sigma r_i^2 \) \( (i=1,2,\ldots,n) \), and \( u(.) \) is the unit step function. The mean and variance of Rayleigh distribution are \( \sqrt{(\pi/2)}\sigma \) and \( (2-\pi/2)\sigma^2 \), respectively. The assumption of approximately equal \( r_i \) s in (3.2) may be unrealistic in practice. If these components are not equal but each individual component does not have a main contribution in received power (i.e., \( r_i^2 < \Sigma r_i^2 \)), and when the number of components \( n \) is large, the envelope \( r \) in (3.1) will have Rayleigh distribution described by (3.3).

### 3.1.2 The Lognormal Distribution

Suppose \( x \) is a normal (Gaussian) random variable with mean \( \mu \) and variance \( \sigma^2 \). If \( x = \ln r \), it can be shown [1] that \( r \) has lognormal distribution as

\[ f(r) = \frac{1}{\sigma r \sqrt{2\pi}} \exp[-\frac{(\ln r - \mu)^2}{2\sigma^2}] u(r) \]  \hspace{1cm} (3.4)

where, \( u(.) \) is the unit step function. A justification for using this distribution is when the fading is modeled as a multiplicative process. In this case, logarithms are added and according to Central Limit Theorem (CLT), the distribution will be lognormal.

### 3.1.3 The Nakagami Distribution

In the Rayleigh distribution we saw that the length of scatter vectors are nearly equal and their phases are random, (see (3.2)). In Nakagami distribution which is also called \( m \) distribution, the length of scatter vectors is kept random. Therefore, this distribution is more realistic than Rayleigh distribution. The probability density function (pdf) of Nakagami distribution is [2]

\[ f(r) = \frac{2m^m r^{2m-1}}{\Gamma(m) \Omega^m} e^{-\frac{mr^2}{\Omega^2}} u(r) \]  \hspace{1cm} (3.5)

where \( \Omega = \text{E}[r^2], \ m = [\text{E}(r^2)]^2/\text{Var}(r^2) \geq 0.5, \ r = |\Sigma r_i \exp(j\theta_i)|, \ u(.) \) is the unit step function, \( \Gamma(.) \) is the Gamma function defined as
\[
\Gamma(m) = \int_0^{\infty} t^{m-1} e^{-t} dt \quad m \geq 0
\] (3.6)

and \( E[.] \) is mathematical expectation. Nakagami distribution for \( m=1 \) reduces to Rayleigh and reverts to one sided Gaussian distribution for \( m=1/2 \). It also can approximate lognormal distribution under ceratin conditions [3].

3.1.4 The Rician Distribution

Referring to the Rayleigh distribution if we assume one of the components \((r_k, \theta_k)\) is fixed and other scatter vectors are random in amplitude and phase, (3.1) is rewritten as

\[
re^{j\theta} = ae^{j\theta_0} + \sum_{i=1, i \neq k}^n r_i e^{j\theta_i}
\] (3.7)

and probability density function of \( r \) will be [4]

\[
f(r) = \frac{r}{\sigma^2} \exp\left(-\frac{r^2 + \alpha^2}{2\sigma^2}\right) I_0\left(\frac{\alpha r}{\sigma^2}\right) u(r)
\] (3.8)

which is called Rician distribution. In (3.8), \( \alpha \) is a fixed vector, \( u(.) \) is the unit step function, \( \sigma^2 \) is power of scatter Rayleigh components, and \( I_0(.) \) is the modified Bessel function of the first kind and zeroth order

\[
I_0(x) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{x \cos \phi} d\phi
\] (3.9)

The Rician distribution occurs when a strong path exists in addition to the low level scattered path. The strong or fixed path component is line of sight path or the path that has much less attenuation with respect to other components. As a special case when \( \alpha = 0 \) (or \( \alpha^2 / 2 \sigma^2 < r^2 / 2 \sigma^2 \)) Rice distribution in (3.8) becomes Rayleigh distribution. Meanwhile, when the fixed vector has much more power than scatter components, \( r \) in (3.7) is approximately Gaussian. That is, in this case, the Rician distribution is approximated by a Gaussian distribution [5].
3.1.5 The Suzuki Distribution

This distribution which is also called mixed distribution, is a combination of Rayleigh and lognormal distributions and was first suggested by Suzuki [6] in modeling and simulation of mobile radio propagation channel. The pdf of this distribution is

\[ f(r) = \int_0^\infty \frac{r}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}} \frac{1}{\sigma \sqrt{2\pi \zeta}} e^{-\frac{(\ln \sigma - \mu)^2}{2\zeta^2}} d\sigma \]  \hspace{1cm} (3.10)

where, \( \mu \) and \( \zeta \) are the mean and standard deviation of the lognormal distribution. A theoretical explanation for application of this distribution in indoor channels is reported in [5]. One or more relatively strong signals arrive at the general location of the portable. The main wave, which has a lognormal distribution due to multipath reflections, is broken up into subpaths at the portable site due to scattering by local objects. Each subpath is assumed to have approximately equal amplitudes and random uniformly distributed phases. Furthermore, they arrive at the portable with approximately the same delay. The envelope sum of these components has Rayleigh distribution with lognormally distributed parameter \( \sigma \), giving rise to the mixture distribution of (3.10), [5].

3.1.6 The Weibull Distribution

The pdf of this distribution is written as

\[ f(r) = \frac{\beta}{\gamma} \left( \frac{r}{\gamma} \right)^{\beta-1} e^{-\left( \frac{r}{\gamma} \right)^\beta} \] \hspace{1cm} (3.11)

where \( \beta \) and \( \gamma \) are shape and scale factors, respectively, and again \( u(.) \) is the unit step function. This distribution reduces to exponential distribution with \( \beta = 1 \), and to Rayleigh distribution with \( \beta = 2 \). The cumulative density function (CDF) of this distribution is

\[ F(r) = \left[ 1 - e^{-\left( \frac{r}{\gamma} \right)^\beta} \right] u(r) \] \hspace{1cm} (3.12)

Different distribution functions were explained in the above section. These distributions are used in statistical modeling of amplitudes in indoor radio propagation channel. A comprehensive tutorial-survey of application of these distributions in indoor radio propagation is
reported in [5]. In this wide-ranging reference distribution of amplitudes, phases as well as other relevant concepts of indoor wireless propagation are well described. In order to make our study complete, a brief survey of application of the explained distributions in indoor is provided. Details can be found in [5].

Rayleigh distribution is reported a good distribution for amplitudes of wide-band data collected in five factory environments with heavy clutter situations [7]. Analysis of this data shows that over "certain range of signal amplitudes" Rician distribution shows good fit [7]. CW measurements in these factory environments have shown that small scale fading is Rayleigh [8]. Limited data of wide band propagation inside a building has shown a good fit for Rayleigh distribution [9]. Analysis of temporal fading data reported in [10] shows the Rician distribution well fits the data even in the absence of LOS. In [8] temporal fading measurements in factory environments are reported to be Rice. CW measurements inside a university building [11] and in office environment [12] have shown that with the presence of LOS path between transmitter and receiver, the amplitudes are Rician distributed. Application of Nakagami distribution in large areas of indoor radio propagation data is reported in [13]. It is reported that the other tested distributions (lognormal and Suzuki) show better fit to the data. The good lognormal fit to local data with small number of profiles in each location at several factory environments [14]-[15], for CW fading data of obstructed factory paths in [8] and for limited wide-band data at several college buildings [16], is reported. Large scale variations of data measured at 900 MHz, 1800 MHz and 2.3 GHz for transmission into and within buildings show good lognormal fit [17]. A review of indoor radio propagation literatures shows that Suzuki distribution has been generally neglected [5]. This might be due to complexity of data reductions. Application of this distribution over CW data collected with transmitter located outside and the receiver placed inside different floors of a building is reported in [13]. Weibull distribution has been applied to amplitude data of mobile radio [18]. Narrowband measurements at 910 MHz in several laboratories showed that Weibull distribution described fading in the period of movement [19].

As is seen the distribution of signal amplitudes is not conclusive. Different researchers have reported different results. Although each result may be justifiable based on conditions governing the measurements, a consistent model presenting the amplitude distribution under a diversified set of conditions is unavailable [5]. In the following we analyze the mentioned distributions over amplitude fading of 12000 impulse response profiles of indoor radio propagation channels and determine the best distribution that fits the amplitudes’ data. These indoor channels
were explained in detail in chapter 2. In order to check the suitability of the above distributions, we test them with the distribution of the signal amplitudes' fading. This procedure is usually addressed as curve fitting. To this end, different curve fitting techniques are explained in the following section.

3.2 CURVE FITTING TECHNIQUES

Basically the first way of performing curve fitting for a set of measured data is to draw the cumulative distribution function of the measured data and compare with the different distributions which were explained in previous section. If there is a remarkable difference, that distribution can simply be rejected. In this method of curve fitting, usually the curve is sketched which changing the scale of axes in order to make it a straight line. In this case each deviation from linearity can be discerned easily. Suppose the sorted values of a set of measured data are \( x_{(1)}, x_{(2)}, x_{(3)}, \ldots, x_{(N)} \), where subscript \( (i) \) is used to denote the sorted value of data. The empirical cumulative distribution function (ECDF) is

\[
F(x_{(k)}) = \frac{k}{N} \quad k = 0, 1, 2, \ldots, N
\]  

(3.13)

where \( k \) is the number of samples which is equal or less than \( x_{(k)} \). Suppose the theoretical probability density function under consideration is \( g(r) \), and theoretical cumulative density function is \( G(\cdot) \), therefore,

\[
G(z) = \int_{-\infty}^{z} g(r) dr
\]  

(3.14)

Now if the sketch of \( x_{(i)} \) versus \( G^{-1}[F(x_i)] \) in linear scale is a line which passes through the origin, then \( G(\cdot) \) will be a good distribution for the data under study. In this method of curve fitting each deviation from linearity indicates where deviation from hypothetical model has occurred.

Regarding to this method of curve fitting two points should be mentioned: First, the hypothetical distribution of \( G(\cdot) \) should be completely specified (i.e., its parameters should be estimated in advance), and second, usually the calculation of inverse function of (3.14) is not simple, and sometimes there would be no closed form.
3.2.1 The Kolmogrov-Smirnov Test

The Kolmogrov-Smirnov test is a measure to test the suitability of a hypothetical (theoretical) distribution which fits the empirical distribution with a certain confidence level. Suppose $X$ is a random variable which has a continuous distribution function and let $x_{(1)}, x_{(2)}, x_{(3)}, \ldots, x_{(N)}$ be an ordered set of samples. The empirical cumulative distribution function is written as

$$F_N(x) = \begin{cases} 
0 & x < x_{(1)} \\
\frac{k}{N} & x_{(k)} \leq x < x_{(k+1)} \text{ and } k=1,2,\ldots,N-1 \\
1 & x \geq x_{(N)}
\end{cases}$$  \hspace{1cm} (3.15)

From (3.15) it is seen that ECDF has stepwise form and $N$ jumps each of height $1/N$ occurring at the points of samples. The parameter of Kolmogrov-Smirnov (K-S) test is introduced as

$$D_N = \max_x | F(x) - F_N(x) |$$  \hspace{1cm} (3.16)

where, $F_N(x)$ is ECDF and $F(x)$ is the hypothetical distribution function under study. $D_N$ is the maximum distance of theoretical from empirical distribution, and shows how close $F(x)$ is to $F_N(x)$. The distribution of $D_N$ depends on $N$ or size of samples, but it is independent of hypothetical distribution. This property is sometimes referred as distribution free statistics, [20]. For large $N$ the probability distribution of $D_N$ is [21]

$$\lim_{N \to \infty} \text{Prob} \left[ D_N < \frac{c}{N} \right] = 1 - 2 \sum_{i=1}^{\infty} (-1)^{i-1} e^{-2i^2c^2}$$  \hspace{1cm} (3.17)

In [22] Kolmogrov has provided a set of recursive equations to calculate the above probability for limited $N$. Massey [23] obtained recursive relations with a system of difference equations to calculate the above probability. In Table 3.1 the 95% and 99% points of $D_N$ for different $N$ are shown [24]. Equation (3.17) can also be written as

$$\text{Prob} \left[ D_N < \epsilon_{N, \alpha} \right] = 1 - \alpha$$  \hspace{1cm} (3.18)
Table 3.1  Kolmogrov-Smirnov statistic's 95% and 99% points, (from [24]).

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<th>N</th>
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For large \( N \) and for \( 1-\alpha \) equals 95% and 99% the values of \( \varepsilon_{N,\alpha} \) are \( 1.3851/\sqrt{N} \) and \( 1.6276/\sqrt{N} \), respectively [21]. In fact for large \( N \) the above asymptotical values can be expressed by [1],[24]

\[
\varepsilon_{N,\alpha} = \sqrt{\frac{1}{2N} \ln \frac{\alpha}{2}}
\]  

Calculating the value of \( \varepsilon_{N,\alpha} \) and with confidence level of \( 1-\alpha \) we can test the suitability of the theoretical distribution \( F(x) \) to fit the empirical distribution \( F_N(x) \). One other application of the K-S test is obtaining confidence level for an unknown distribution \( F(x) \). In this case, first based on the sample size and confidence level \( 1-\alpha \) the value of \( \varepsilon_{N,\alpha} \) is determined. Using (3.18) we have

\[
1-\alpha = \text{Prob} \left[ \max_x \left| F_N(x) - F(x) \right| < \varepsilon_{N,\alpha} \right]
\]  

or

\[
1-\alpha = \text{Prob} \left[ F_N(x) - \varepsilon_{N,\alpha} < F(x) < F_N(x) + \varepsilon_{N,\alpha} \right]
\]  

If we assume
\[ F_L(x) = \begin{cases} 
0 & F_N(x) - \epsilon_{N, \alpha} < 0 \\
F_N(x) - \epsilon_{N, \alpha} & F_N(x) - \epsilon_{N, \alpha} > 0 
\end{cases} \] (3.22)

and

\[ F_U(x) = \begin{cases} 
F_N(x) + \epsilon_{N, \alpha} & F_N(x) + \epsilon_{N, \alpha} < 1 \\
1 & F_N(x) + \epsilon_{N, \alpha} > 1 
\end{cases} \] (3.23)

then the two stepwise functions \((F_L(x)\) and \(F_U(x)\)) determine the confidence band of \(100(1-\alpha)\)% for the unknown distribution \(F(x)\), [24] and [25]. Another application of the K-S test is estimating the size of samples in order to test the fitness of a hypothetical distribution with a certain error, (see (3.19)).

3.2.2 The Wilcoxon Test

This test is sometimes called Signed Rank Test. The criterion which is used in this test is, [26]

\[ \omega^2 = \int_{-\infty}^{\infty} \left[ F(x) - F_N(x) \right]^2 dF(x) \] (3.24)

where \(F(x)\) and \(F_N(x)\) are theoretical and empirical cumulative distribution functions, respectively. It has been shown that the above criterion has a good efficiency when the number of samples is small [6]. If \(F(x)\) is continuous, \(\omega^2\) in (3.24) can be written as, [27]

\[ \omega^2 = \frac{1}{2N^2} + \frac{1}{N} \sum_{i=1}^{N} \left[ F(x_i) - \frac{2i-1}{2N} \right]^2 \] (3.25)

where \(N\) is the sample size and \(x_i\) is one of the samples. The limiting distribution of \(N\omega^2\), that is

\[ a(z) = \lim_{N \to \infty} \left[ \text{Prob} \left( N\omega^2 \leq z \right) \right] \] (3.26)

is shown in Table 3.2, [27].

The theory of Wilcoxon test can be found in many statistics books, and is not repeated
here. But, the procedure of using this test in indoor channel is described by an example. Suppose we want to compare the goodness of fit of distributions "A" and "B" for a set of data from propagation measurements in indoor environment. According to Wilcoxon test, first we calculate the value of $N\omega^2$ of the two distributions and then find the differences between them. The absolute values of these differences are ordered in increasing manner and then the sum of the ranks corresponding to the negative values, which we call $w$, is determined. These are shown in Table 3.3.

We are interested in the probability $p$ as

$$p = \frac{\text{number of cases } W \text{ is less than or equal to } w}{\text{total number of possible cases}}$$  \hspace{1cm} (3.27)

Since each rank, with probability of $\frac{1}{2}$, can be positive (or negative), the total number of cases is $2^N$. The number of cases $W$ is less than or equal to $w$ has been gathered in the tables of this test. In Table 3.3 the values of $N\omega^2$ for different distributions of "A" and "B" and their differences are seen. Ranks are shown in the last column of this Table. Ranks associated with negative values are indicated by * in Table 3.3. For this set of data the sum of $W$'s is 34. If $m=N+1$, we have [27]

$$P(W < w) = \frac{A_0(w) - A_1(w-m) + A_2(w-2m) - A_3(w-3m) + \ldots}{2^N}$$  \hspace{1cm} (3.28)

The values of $A_i(k)$ in (3.28) are given in tables of the test and are reproduced in Table 3.4, [27]. The value of $A_i(k)$ for negative $k$ is zero. Using Table 3.4 for the above example we have

$$p(w_8 < 34) = \frac{A_0(34) - A_1(34-9) + A_2(34-18) - A_3(34-27)}{2^8} = \frac{3725 - 5218 + 1878 - 131}{256} = 0.9922$$  \hspace{1cm} (3.29)

It means that with confidence level 99.22% distribution "A" is more suitable to fit the measured data than distribution "B". In Table 3.3 also the goodness of fit of the distributions "B" and "C" for measured data with Wilcoxon test is shown. In this case the above mentioned process is repeated for the bottom columns of the table. The sum of the ranks corresponding to negative differences is 33 and therefore, the probability $p(W<33)$ is calculated as 0.9883. This means that distribution "B" with confidence level of 98.83% fits better than distribution "C" for the measured data.
Table 3.2 Limiting distribution of $N w^2$ of the Wilcoxon Test, [27].

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Table 3.3  An example of application of the Wilcoxon test over measured data of indoor radio propagation channel. "A" = Lognormal, "B" = Nakagami, and "C" = Rayleigh distribution.

| $N \omega^2(A)$ | $N \omega^2(B)$ | $\Delta = N \omega^2(A) - N \omega^2(B)$ | $|\Delta|$ | Rank |
|-----------------|-----------------|--------------------------------|---------|------|
| .0077           | .0775           | -.0698                        | .0698   | 3*   |
| .0621           | .4215           | -.3594                        | .3594   | 8*   |
| .0328           | .1892           | -.1564                        | .1564   | 5*   |
| .0446           | .3934           | -.3488                        | .3488   | 7*   |
| .2099           | .2238           | -.0139                        | .0139   | 1*   |
| .1893           | .1723           | .0160                         | .0160   | 2    |
| .0039           | .0863           | -.0824                        | .0824   | 4*   |
| .0320           | .2060           | -.1740                        | .1740   | 6*   |

| $N \omega^2(B)$ | $N \omega^2(C)$ | $\Delta = N \omega^2(B) - N \omega^2(C)$ | $|\Delta|$ | Rank |
|-----------------|-----------------|--------------------------------|---------|------|
| .0775           | .3108           | -.2333                        | .2333   | 6*   |
| .4215           | .5918           | -.1703                        | .1703   | 3*   |
| .1892           | .1483           | .0409                         | .0409   | 2    |
| .3934           | .6005           | -.2071                        | .2071   | 5*   |
| .2238           | 1.786           | -.1562                        | 1.562   | 8*   |
| .1733           | 1.474           | -.1301                        | 1.301   | 7*   |
| .0863           | .2675           | -.1812                        | .1812   | 4*   |
| .2060           | .1745           | .0315                         | .0315   | 1    |

* refer to text
3.2.3 The $\chi^2$ Test

Another measure for goodness of fit is the Chi square test. This test compares the observed sample histogram with the expected frequencies which are computed from theoretical distribution function. The criterion of this test is \[ \chi^2 = \sum_{all \ samples} \frac{(observed - expected)^2}{expected} \] (3.30)

The critical values of $\chi^2$ test are shown in Table 3.5, [28]. According to this test, first the value of $\chi^2$ is calculated and then is compared with the critical value of the test with the desired confidence level and the degree of freedom (number of samples - 1). If the value of $\chi^2$ is less than the critical value, with that confidence level the theoretical distribution cannot be rejected for the data. (i.e., accept the distribution for these data samples).

3.2.4 The Mean Square Error Test

The criterion of mean square error test is

\[ \eta^2 = \int_{-\infty}^{\infty} [F(x) - F_N(x)]^2 \, dx \] (3.31)

again, $F(x)$ and $F_N(x)$ are hypothetical and empirical distribution functions, respectively. According to (3.31) $\eta^2$ is the average of squared difference between $F(x)$ and $F_N(x)$. The closer the $F(x)$ to $F_N(x)$, the smaller the mean square error and the better the fitness of theoretical to empirical to empirical distribution. The limiting value of $\eta^2$ is not available.
Table 3.4  The values of $A_j(k)$ in the Wilcoxon test, \( (\text{from [27]}) \).

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### Table 3.5 The critical values of $\chi^2$ test, [28].

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![Image of a graph showing the power of the K-S Test, [25].](image-url)

**Figure 3.1** Power of the K-S Test, [25].
3.2.5 Comparison of the Tests

Before comparing different tests let us define the quantity of power of the test. Suppose $F(x)$ and $F_N(x)$ are theoretical and empirical distribution functions, respectively, and assume the distance between these two functions is $\Delta$. The power of test is defined as the lower bound on the probability of rejection of theoretical distribution. For the K-S test it is shown that the power of the test is [25]

$$1 - \frac{1}{2\pi} \int_{\frac{2(\Delta/\sqrt{N} + \epsilon_{N,\alpha})}{2(\Delta/\sqrt{N} - \epsilon_{N,\alpha})}} e^{-\frac{t^2}{2}} dt$$  \hspace{1cm} (3.32)

In Fig. 3.1 the power of the K-S test for two values of $\alpha$ (1% and 5%) is shown [25]. Using Fig. 3.1 and with confidence level of 95% and the power of test of 50% the size of samples can be determined. In Table 3.6 different sample sizes and minimum distance for power of test of 0.5 for $\chi^2$ and K-S tests are seen, [25]. In the comparison of K-S and $\chi^2$ tests it is seen that K-S has less detectable differences. This means that with the K-S test and with power of test of 50% lower deviations in cdf is detectable than $\chi^2$ test. In the comparison of K-S and $\chi^2$ tests it should be noted that $i)$ the power of $\chi^2$ test is unknown in closed form, however, for K-S test it is determined by (3.32), $ii)$ if the size of samples is small $\chi^2$ test may not be used, $iii)$ the K-S test needs less computations than $\chi^2$ test and therefore, it can be carried out faster, $iv)$ in the K-S test the distribution of population should be continuous but, such an assumption is not necessary for the $\chi^2$ test, and $v)$ as mentioned in section 3.2.1, the K-S is distribution free for finite $N$ but, $\chi^2$ test does not have this characteristic, (i.e., the $\chi^2$ statistic becomes approximately distribution free for $N \to \infty$). Comparing the K-S test with the wilcoxon test it should be emphasized that Wilcoxon test is usually used when number of samples is small.

To this end, we use the K-S statistic for curve fitting of the data of amplitudes of indoor radio propagation channels. As mentioned, this test has more power, i.e., the probability of avoidance of error due to acceptance of hypothetical distribution, when this distribution really does not fit the data, is higher. Moreover, the K-S test is distribution free and since the theoretical distributions under study (i.e., lognormal, Rayleigh, Nakagami, Rice, Suzuki and Weibull) are continuous this criterion well serves the test. Availability of critical values of K-S test is another feature of this test. As we explained earlier, in each location (small area) of indoor
Table 3.6 Different sample sizes and minimum distances for power of the test of 0.5 for $\chi^2$ and K-S tests, [25].

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In the radio propagation data base we have 75 profiles. Combining the data sample of each excess delay with adjacent bins, the number of samples under study will be more than 300 (this will be addressed later). With this sufficient number of samples, it is not necessary to use Wilcoxon test. Another point which should be mentioned here is that with the K-S test and other mentioned tests fitness of a theoretical distribution with the empirical distribution function is examined. This issue slightly differs from hypothesis testing. In the classical hypothesis testing parameters of the distribution are tested. However, in the mentioned tests the suitability of a hypothetical distribution function, whose parameters have already been estimated, with the empirical distribution function is examined and not estimation of its parameters.

3.2.6 Estimation of Parameters of Theoretical Distributions

We saw in section 3.2.1 that in carrying out the K-S test, first $D_N$ should be computed. In order to specify $D_N$, the distribution of $F_N(x)$ and $F(x)$ should be determined in advance. The
empirical distribution \( F_M(x) \) is calculated from the data (see (3.15)), and sketched as a stepwise curve. The theoretical distributions (explained in section 3.1) should be determined and therefore, their parameters have to be estimated, prior to applying the K-S test. The method we use in estimation of parameters is the moment method. One of the main advantages of this method is its simplicity. Other methods of estimation (such as maximum likelihood or minimum variance) could also be used in this regards. However, they lead to complex relations. On the other hand, when the size of the samples is large, the estimates of these methods will be very close to each other. In the moment method, theoretical moments of distributions are obtained and equated with the moments estimated from data, and then parameters of \( F(x) \) are calculated.

For the Rayleigh distribution, (see (3.3)), we have

\[
E[x] = \sqrt{\frac{\pi}{2}} \sigma \quad (3.33)
\]

The mean of samples \( S \) is

\[
S = \frac{1}{N} \sum_{i=1}^{N} x_i \quad (3.34)
\]

and its variance \( V \)

\[
V = \frac{1}{N-1} \left( \sum_{i=1}^{N} x_i^2 - NS^2 \right) \quad (3.35)
\]

Therefore, for Rayleigh distribution we have \( \sigma = \sqrt{(2/\pi)S} \). Regarding Rice distribution it can be shown that [27]

\[
S = \sqrt{\frac{\pi}{2}} \sigma \exp\left(-\frac{\alpha^2}{2\sigma^2}\right) H\left(\frac{3}{2}, 1, \frac{\alpha^2}{2\sigma^2}\right) \quad (3.36)
\]

where \( H(\ldots, \ldots) \) is the confluent hypergeometric function and is expressed by the following power series [29]

\[
H\left(\frac{3}{2}, 1, r^2\right) = 1 + \frac{3}{1} r^2 + \frac{3 \cdot 5}{1 \times 2} r^4 + \frac{3 \cdot 5 \cdot 7}{1 \times 2 \times 3} r^6 + \ldots \quad (3.37)
\]

If we assume \( z \sigma^2 = \alpha^2 / 2\sigma^2 \), then equation (3.37) is written as
\[ H(\frac{3}{2}, 1, z) = \sum_{k=0}^{\infty} \frac{\Gamma(k + \frac{3}{2})}{\Gamma(\frac{3}{2})} \frac{z^k}{(k!)^2} \]  

(3.38)

For the Rice distribution if \( T(1/N) \sum_{i=1}^{N} X_i^2 \) \((i=1,2,...,N)\), we can write

\[ \sigma^2 = \frac{T}{2 \left( 1 + \frac{\alpha^2}{2 \sigma^2} \right)} \]  

(3.39)

Substituting (3.39) in (3.36) yields

\[ S = \frac{1}{2} \sqrt{\frac{\pi T}{1 + z}} e^{-z} H\left(\frac{3}{2}, 1, z\right) \]  

(3.40)

or

\[ g(z) = \frac{e^{-z}}{\sqrt{1 + z}} \sum_{k=0}^{\infty} \frac{\Gamma(k + \frac{3}{2})}{\Gamma(\frac{3}{2})} \frac{z^k}{(k!)^2} - \frac{2S}{\sqrt{\pi T}} = 0 \]  

(3.41)

In order to find the parameters of Rice distribution equation (3.41) is solved and accordingly the value of \( \sigma \) is obtained and eventually \( \alpha \) is calculated. Solving (3.41) is carried out numerically with the Newton-Raphson method [30]

\[ z_{n+1} = z_n - \frac{g(z_n)}{g'(z_n)} \]  

(3.42)

After some manipulations we have

\[ z_{n+1} = z_n - \frac{\sum_{k=0}^{\infty} a_k z_k - e^z \sqrt{1 + z} \frac{2S}{\sqrt{\pi T}}}{\sum_{k=0}^{\infty} \left( k - \frac{1.5 + z}{1 + z} \right) a_k z_k} \]  

(3.43)

where
\[ a_k = \frac{\Gamma\left(k+\frac{3}{2}\right)}{\Gamma\left(\frac{3}{2}\right)} \frac{1}{(k!)^2} \quad (3.44) \]

Now we turn our attention to the estimating of the parameters of the Nakagami distribution. In this case we have [26]

\[ S = \frac{\Gamma\left(m+\frac{1}{2}\right)}{\Gamma(m)} \sqrt{\frac{\Omega}{m}} \quad (3.45) \]

Therefore,

\[ S = \frac{\Gamma\left(m+\frac{1}{2}\right)}{\sqrt{\Omega} \sqrt{m \Gamma(m)}} \quad (3.46) \]

and

\[ \sqrt{\Omega} = \sqrt{\text{Var}(r) + \bar{r}^2} \quad (3.47) \]

Equation (3.46) is solved numerically and parameter \( m \) of the Nakagami distribution is determined.

For the lognormal distribution estimation of parameters is carried out easily. The parameters of this distribution are mean and variance which are simply determined from (3.34) and (3.35).

Using the Rayleigh and lognormal distributions parameters of the Suzuki distribution are also determined. In Suzuki distribution integration should be done. The Suzuki Cumulative distribution function is written as

\[ F(r) = \int_0^r (1-e^{-\frac{r^2}{2\sigma^2}}) \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(\ln \sigma - \mu)^2}{2\zeta^2}} d\sigma \quad (3.48) \]

In this equation integration is performed by standard trapezoidal method [30].

Till now estimation of parameters was in linear scale. In linear scale, paths with large attenuation have little contribution in calculation of the mean and standard deviation of the samples. If the dynamic range of amplitudes is larger than 20 dB, contribution of weak amplitudes for curve fitting on a linear scale is insignificant, i.e., it makes curve fitting not so good for small
amplitude samples. In order to overcome this problem, estimation of parameters is carried out on a logarithmic scale. Therefore, instead of the Rayleigh distribution we have lograyleigh distribution as

\[
f(r) = \frac{a}{2\sigma^2} \exp\left[a r - \frac{1}{2\sigma^2} e^{ar}\right]
\]  

(3.49)

where, \(a = \ln 10/10\). The cdf will be

\[
F(r) = 1 - \exp\left(-\frac{1}{2\sigma^2} e^{ar}\right)
\]  

(3.50)

and it can be shown [31]

\[
\sigma = 0.9436 \ e^{-\frac{a\bar{r}}{2}}
\]  

(3.51)

For normal distribution we have

\[
f(r) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(r-\mu)^2}{2\sigma^2}}
\]  

(3.52)

and

\[
F(r) = \frac{1}{\sigma \sqrt{2\pi}} \int_{-\infty}^{r} e^{-\frac{(t-\mu)^2}{2\sigma^2}} dt
\]  

(3.53)

where

\[
\bar{r} = \mu
\]  

(3.54)

In a same manner for lognakagami distribution the pdf is [26]

\[
f(r) = \frac{2m^m}{a \Gamma(m) \Omega^m} \exp\left[m \left(\frac{2r}{a} - \frac{1}{\Omega} e^{\frac{2r}{a}}\right)\right]
\]  

(3.55)

where again \(a = \ln 10/10\). For this distribution we also have

\[
\bar{r} = E[r] = \frac{a}{2} \left[ \psi(m) - \ln\left(\frac{m}{\Omega}\right) \right]
\]  

(3.56)

and
\[ E[r^2] = a^2 \left( \psi(m) - \ln \left( \frac{m}{\Omega} \right) \right)^2 + \psi'(m) \]  \hspace{1cm} (3.57)

In (3.58) and (3.59), \( \psi(m) \) and \( \psi'(m) \) are Digamma and Trigamma functions and are written

\[ \psi(m) = \frac{d \ln \Gamma(m)}{dm} = \frac{\Gamma''(m)}{\Gamma(m)} \]  \hspace{1cm} (3.58)

and

\[ \psi'(m) = \frac{d \psi(m)}{dm} = \frac{\Gamma''(m)}{\Gamma(m)} - \left( \frac{\Gamma'(m)}{\Gamma(m)} \right)^2 \]  \hspace{1cm} (3.59)

The following expressions are used for the Digamma and Trigamma functions [29]

\[ \psi(m) = -0.577 + \sum_{n=0}^{\infty} \left( \frac{1}{n+1} - \frac{1}{n+m} \right) \]  \hspace{1cm} (3.60)

and

\[ \psi'(m) = \sum_{n=0}^{\infty} \left( \frac{1}{n+m} \right)^2 \]  \hspace{1cm} (3.61)

Convergence of (3.60) and (3.61) is not too fast, however, with consideration of first several hundred terms an accuracy of three decimal points is achieved. Considering (3.56) and (3.57) we will have

\[ S' = a^2 \left( \psi(m) - \ln \left( \frac{m}{\Omega} \right) \right) \]  \hspace{1cm} (3.62)

and

\[ V' = \left( \frac{a}{2} \right)^2 \psi'(m) \]  \hspace{1cm} (3.63)

where \( S' \) and \( V' \) are mean and variance in logarithmic scale, respectively. Equation (3.63) is solved numerically and following to obtaining \( m \), the value of \( \Omega \) is determined as

\[ \Omega = m e^{\left( \frac{2S'}{a} - \psi(m) \right)} \]  \hspace{1cm} (3.64)

Regarding the Rice distribution since there are no explicit expressions for moments in log scale, calculations are not carried out in logarithmic scale.
For logweibull distribution by referring to (3.11) we have

\[ f(r) = \frac{1}{\gamma} \left( \frac{\ln r}{\gamma} \right)^{\beta-1} \exp \left[ -\left( \frac{\ln r}{\gamma} \right)^{\beta} \right] \] (3.65)

where \( \beta \) and \( \gamma \) are the same as those used in (3.11).

### 3.3 APPLICATION OF THE TESTS OVER MEASURED AMPLITUDE DATA

In this section we describe the method of processing the amplitudes of measured data of indoor radio propagation channels and applying the explained tests. To this end, we briefly mention the method of discrete-time impulse response and then discuss the method of combining the data and application of the tests over local and global areas.

As mentioned in detail in chapter 2, the method that is used in studying the indoor radio propagation channel is the discrete-time impulse response method. In this model the time axis is divided into small portions, called bins. Each bin contains either one component or no component. The possibility of more than one component in each bin is excluded. Using this method for each bin with a component a "1" is considered and a "0" otherwise. Accordingly, each impulse response is modeled as a sequence of "1"s and "0"s. The binwidth of measurements was 5 nsec. For each "1" in a bin there are two components (amplitude and phase), amplitudes are used in this chapter. The main advantage of the discrete-time impulse response is its convenience in analysis and simulation.

#### 3.3.1 Local Amplitude Distributions

As explained in chapter 2 each location of measurements contains 75 profiles. Therefore, for each bin (i.e., for each excess delay) maximum number of samples is 75. To increase the number of samples, data of 5 adjacent bins were combined. This is justified since amplitude components in adjacent bins have similar characteristics. Therefore, curve fitting was performed for portions of the excess delay axis which are 25 nsec wide and are centered at excess delays of 25, 50, 75, 100, ..., 425, 475 nsec. For each portion of the excess delay axis and each theoretical distribution the Kolmogrov-Smirnov test was performed with a confidence level of 90%. In [32] it is mentioned that the standard critical values of the K-S test when parameters of the theoretical distributions are estimated from the data are somehow conservative. That is, the actual
significance level would be lower than that given by Table 3.1. In this reference results of Monte Carlo calculation for critical values of the K-S test in this case have been reported. Using the critical values of the K-S test in [32], goodness of fit of all distributions described in section 3.1 was examined. Parts of the results of this investigations are shown in Figs. 3.2 - 3.3 as well as Appendix A.1. Maximum distance of each theoretical distribution from empirical distribution as well as the K-S critical values are seen on each figure. From this research it was found that Rice, Suzuki and Weibull did not provide good fit to most of the data. Therefore, in the following the results of curve fitting for other three distributions are reported. Samples of this curve fitting diagrams are reproduced in Figs. 3.4 - 3.6 and Appendix A.1. As mentioned earlier, because the dynamic range of the amplitudes is greater than 20 dB, curve fitting was carried out on log scale. To show the difference of curve fitting in linear and log scale, result of goodness of the Nakagami distribution on linear and logarithmic scales is depicted in Fig. 3.7. As is expected, the results of curve fitting on the two scales are not the same, and supremacy of logarithmic scale in curve fitting is obvious from this figure.

Samples of the results of curve fitting of amplitudes are reproduced in Table 3.7 as well as Appendix A.2. In these tables a "1" indicates that the data passed K-S test for the corresponding theoretical distribution and "0" means that it did not pass the test.

The amount of data analyzed provides statistically significant results. A total of 160 locations for both buildings (20 locations for each one of 4 antenna separations at each one of two buildings) were tested. For each location 19 portions of the excess delay axis were considered. Overall more than 3000 curves were therefore tested. Results of the local curve fitting (a few of which are reflected in Figs. 3.2 - 3.7 and Tables 3.7 - 3.8) are summarized in Table 3.9, which shows that with confidence level of 90% in 75.9% of cases the lognormal distribution passes the Kolmogrov-Smirnov test. These numbers are 36.3% and 12.6% for Nakagami and Rayleigh distributions, respectively [33]-[34]. It should be noted the sum of the above three figures is greater than unity. This is because in some excess delays two or all three distributions passed the K-S test. Based on this extensive research it is concluded that lognormal distribution is the best candidate to describe amplitude fading over local areas [34]-[35]. In comparison of the Nakagami and Rayleigh distributions, on the average Nakagami has passed the test twice the Rayleigh. Therefore, it better describes the distribution of amplitudes of indoor fading channel than Rayleigh. This is due to more parameters that Nakagami distribution has, (see (3.5)), i.e., Rayleigh distribution is a special case of Nakagami distribution with \( m=1 \).
Figure 3.2  Empirical and theoretical local amplitude distributions. Building B, antenna separation 20 meters, location 5, excess delay 175 nsec.

Figure 3.3  Empirical and theoretical local amplitude distributions. Building B, antenna separation 20 meters, location 5, excess delay 75 nsec.
Figure 3.4 Empirical and theoretical local amplitude distributions. Building A, antenna separation 5 meters, location 1, excess delay 100 nsec.

Figure 3.5 Empirical and theoretical local amplitude distributions. Building A, antenna separation 10 meters, location 5, excess delay 175 nsec.
Figure 3.6 Empirical and theoretical local amplitude distributions. Building B, antenna separation 30 meters, location 14, excess delay 25 nsec.

Figure 3.7 Curve fitting on linear and logarithmic scales. Building A, antenna separation 5 meters, location 1, excess delay 225 nsec.
Table 3.7  Local amplitude distributions. Building A, antenna separation 5m, locations 6-10. "1" means the distribution passed the K-S test and "0" means it did not.

<table>
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<tr>
<th>Excess Delay (Nsec.)</th>
<th>25</th>
<th>50</th>
<th>75</th>
<th>100</th>
<th>125</th>
<th>150</th>
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Table 3.8  Local amplitude distributions. Building B, antenna separation 20m, locations 1-5. "1" means the distribution passed the K-S test and "0" means it did not.

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</table>
Table 3.9 Percentage of locations for both buildings that passed the K-S test with confidence level of 90%.

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<th>Excess Delay (Nsec.)</th>
<th>25</th>
<th>50</th>
<th>75</th>
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<th>125</th>
<th>150</th>
<th>175</th>
<th>200</th>
<th>225</th>
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<th>Average</th>
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</thead>
<tbody>
<tr>
<td>Rayleigh Distribution</td>
<td>7.5%</td>
<td>11.7%</td>
<td>13.3%</td>
<td>3.3%</td>
<td>10.8%</td>
<td>20.8%</td>
<td>15.0%</td>
<td>15.0%</td>
<td>14.2%</td>
<td>15.0%</td>
<td>12.6%</td>
</tr>
<tr>
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<td>73.3%</td>
<td>79.2%</td>
<td>77.5%</td>
<td>75.8%</td>
<td>80.8%</td>
<td>80.0%</td>
<td>80.8%</td>
<td>80.8%</td>
<td>74.2%</td>
<td>56.7%</td>
<td>75.9%</td>
</tr>
<tr>
<td>Nakagami Distribution</td>
<td>40.0%</td>
<td>44.2%</td>
<td>43.3%</td>
<td>53.3%</td>
<td>40.0%</td>
<td>53.3%</td>
<td>30.0%</td>
<td>24.2%</td>
<td>14.2%</td>
<td>20.0%</td>
<td>36.3%</td>
</tr>
</tbody>
</table>

One justification of suitability of lognormal distribution for statistical modeling of amplitudes in indoor fading channel is that because of limitation of resolution of measurement equipments, paths with deep fading are lost in adjacent fade paths. Therefore, it seems Rayleigh and Nakagami distributions which are normally used for curve fitting of deep fading, are not suitable in this case. Good lognormal fit to the local amplitude data is in consistency with the results reported in [8], [14]-[16], using limited data.

3.3.2 Global Amplitude Distributions

According to the measurement plan, in global areas 8 sets of antenna separations are available (4 antenna separations in each building). In order to determine the best distribution expressing global amplitude distributions, over 8 areas, Wilcoxon test with criteria given by (3.24) was used. The procedure of application of this test was described by an example in section 3.2.2. In this example distribution "A" stands for lognormal, distribution "B" stands for Nakagami and "C" stands for Rayleigh distribution. Typical theoretical and empirical global distributions are shown in Figs. 3.8 - 3.9 and Appendix A.3. The $\omega^2$ values (refer to (3.24)) for both buildings, which explains the goodness of fit, for global distributions of amplitudes of both buildings and different antenna separations are reproduced in Tables 3.10 - 3.11 as well as Appendix A.4. It is observed that for most of cases the lognormal distribution provides smaller $\omega^2$ as compared to the Nakagami and Rayleigh distributions. In global areas, and each 19 portions of excess delay, there are 8 values of $\omega^2$ for each distribution. Detailed result of application of the Wilcoxon test on these $\omega^2$ values are shown in Table 3.12.
Figure 3.8 Empirical and theoretical global amplitude distributions. Building A, antenna separation 20 meters, excess delay 100 nsec.

Figure 3.9 Empirical and theoretical global amplitude distributions. Building B, antenna separation 5 meters, excess delay 375 nsec.
### Table 3.10 The value of $\omega^2$ (Wilcoxon Test) for global amplitude distributions. Building A, antenna separation of 10 meters.

<table>
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<th>150</th>
<th>175</th>
<th>200</th>
<th>225</th>
<th>250</th>
<th>275</th>
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<td>.3080</td>
<td>.2911</td>
<td>.2642</td>
<td>.3037</td>
<td>.4916</td>
<td>.5918</td>
<td>.2597</td>
<td>.5034</td>
<td>.4080</td>
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<tr>
<td>Lognormal Distribution</td>
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<td>.0028</td>
<td>.0181</td>
<td>.0139</td>
<td>.0157</td>
<td>.0163</td>
<td>.0385</td>
<td>.0621</td>
<td>.0284</td>
<td>.1441</td>
<td>.0518</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
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<th>300</th>
<th>325</th>
<th>350</th>
<th>375</th>
<th>400</th>
<th>425</th>
<th>450</th>
<th>475</th>
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</thead>
<tbody>
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<td>.2254</td>
<td>.3738</td>
<td>.0821</td>
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<td>Lognormal Distribution</td>
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<td>.0820</td>
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<td>Nakagami Distribution</td>
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<td>.1257</td>
<td>.1394</td>
<td>.0720</td>
<td>.0479</td>
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### Table 3.11 The value of $\omega^2$ (Wilcoxon Test) for global amplitude distributions. Building B, antenna separation of 30 meters.

<table>
<thead>
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<th>100</th>
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<th>225</th>
<th>250</th>
<th>275</th>
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<tbody>
<tr>
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<td>.8852</td>
<td>.6129</td>
<td>.5493</td>
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<td>.2343</td>
<td>.3028</td>
<td>.1745</td>
<td>.1106</td>
<td>.1546</td>
<td>.2608</td>
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<tr>
<td>Lognormal Distribution</td>
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<td>.0182</td>
<td>.0084</td>
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<td>.0118</td>
<td>.0320</td>
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<td>.0330</td>
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<td>Nakagami Distribution</td>
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<td>.1959</td>
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<td>.1524</td>
<td>.1325</td>
<td>.1018</td>
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<table>
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<th>Excess Delay (Nsec.)</th>
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<th>325</th>
<th>350</th>
<th>375</th>
<th>400</th>
<th>425</th>
<th>450</th>
<th>475</th>
</tr>
</thead>
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<tr>
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<td>.3832</td>
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</tr>
<tr>
<td>Lognormal Distribution</td>
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<td>.0319</td>
<td>.0497</td>
<td>.1075</td>
<td>.2276</td>
<td>.0447</td>
<td>.1395</td>
<td>.1615</td>
</tr>
<tr>
<td>Nakagami Distribution</td>
<td>.2104</td>
<td>.1173</td>
<td>.1500</td>
<td>.2036</td>
<td>.3503</td>
<td>.1173</td>
<td>.2371</td>
<td>.2579</td>
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</table>
In Table 3.13 final averaged result of application of this test on global amplitude distributions is shown. According to this table with average confidence level of 95.83% the lognormal distribution is better than Nakagami. The test also shows that with an average confidence level of 92.76% Nakagami is better than Rayleigh. It is therefore concluded that lognormal distribution is also the best candidate for describing of amplitudes over global areas [33]-[35].

Determination of lognormal distribution for signal amplitude fading is a major achievement. Application of elaborate tests added with enormous number of data has increased the validity of this model for signal amplitude fading inside buildings. As we will explain in chapter 8 this result can be used in design, performance evaluation as well as simulation of indoor wireless communication systems.
### Table 3.13 Final result of global amplitude distributions with the Wilcoxon test.

<table>
<thead>
<tr>
<th>Average Confidence level</th>
<th>Average Confidence level</th>
<th>Excess Delay</th>
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</thead>
<tbody>
<tr>
<td>Nakagami better than Rayleigh</td>
<td>lognormal better than Nakagami</td>
<td>(nsec.)</td>
</tr>
</tbody>
</table>

95.83%  
92.76%  
All excess delays

### 3.4 STATISTICAL CHARACTERISTICS OF AMPLITUDES

In this section statistical characteristics of amplitudes are explained. In section 3.3.1 the details of combining data samples of adjacent bins were explained. In order to determine the statistical characteristic of amplitudes the average of combined data samples of these adjacent bins for all profiles, all antenna separations of both buildings have been determined. Parts of these results are shown in Tables 3.14, 3.15 and Appendix A.5. In these tables the mean and standard deviation of amplitudes for different excess delays are depicted. The mean and standard deviation of these means and standard deviations (over all locations of each antenna separation) have also been shown in the last two rows of these tables. The mean values of the amplitudes relates to the power of transmitter and increases with increase of power of the transmitter. However, standard deviation of averaged amplitudes are more important, and show the deviation from mean. Parts of the figures of mean and standard deviation of local amplitudes are sketched in Figs. 3.10 - 3.13 and Appendix A.6. From these figures the variation of mean of standard deviation of local amplitudes versus excess delay is quite clear. It is seen that standard deviation of amplitudes of multipath components decreases with increasing excess delay and has a range of 1 - 4.5 dB. One justification for this behavior is as follows. The multipath signals are arrived at the receiver. Because the long delay signals are reflected and scattered from many paths, they have less deviation around their mean when compared to signals with small excess delays. In the latter, reflected signals may have drastic deviations around their mean or relative to line of sight received signal. Referring to Figs. 3.10 - 3.13 and figures of Appendix A.6, it is seen that mean
amplitudes of multipath components decrease almost linearly with increased excess delay.

Similar procedure for mean and standard deviation of amplitudes in global areas (i.e., all antenna separations of each building) have been carried out and depicted in Figs. 3.14 - 3.15 and for both buildings in Fig. 3.16. Again in this investigation the data of 5 adjacent bins in different profiles were combined and mean and standard deviations have been calculated. These results show the range of variation of standard deviations in global areas is 1-4 dB. Meanwhile, as in local areas, the standard deviation decreases with increasing excess delay (bin number). In fact standard deviations for the amplitude of initial components are higher than those of the latter one. As explained for local areas, this is because there is greater deviation among the initial paths of two different local areas, e.g., the impulse response function at one location may contain a strong line of sight component at 0 excess delay while the other may have a severely attenuated component at the same delay [35].

\[\text{Figure 3.10} \quad \text{Mean of amplitudes versus excess delay. Averaged over all locations with antenna separation 5 meters of Building A.}\]
Figure 3.11  Mean of amplitudes versus excess delay. Averaged over all locations with antenna separation 30 meters of Building B.

Figure 3.12  Standard deviation of amplitudes versus excess delay. Averaged over all locations with antenna separation 5 meters of Building A.

Figure 3.13  Standard deviation of amplitudes versus excess delay. Averaged over all locations with antenna separation 30 meters of Building B.
Figure 3.14 Standard deviation of amplitudes versus excess delay. Averaged over all antenna separations of Building A.

Figure 3.15 Standard deviation of amplitudes versus excess delay. Averaged over all antenna separations of Building B.

Figure 3.16 Standard deviation of amplitude versus excess delay. Averaged over all antenna separations of both buildings.
Table 3.14  Mean of amplitudes at different excess delays. Different locations of Building A, antenna separation of 20 meters. All values in dB.

<table>
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<th>75</th>
<th>100</th>
<th>125</th>
<th>150</th>
<th>175</th>
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<th>225</th>
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<td>-113.3</td>
<td>-114.2</td>
<td>-112.3</td>
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</table>

| Av.       | -89.1 | -91.9 | -95.8 | -99.6 | -103.4 | -106.7 | -108.5 | -110.6 | -112.3 | -113.6 |
| Std.      | 5.1   | 4.1   | 3.8   | 2.9   | 2.6    | 2.4    | 3.9    | 3.2    | 3.2    | 2.7    |

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| Std.      | 2.9   | 3.1   | 8.4   | 2.9   | 3.6   | 1.9   | 3.3   | 3.1   | 3.2   |

* 0 means no component presents for calculating of mean and variance at that delay.
Table 3.15  Standard deviation of amplitudes at different excess delays. Different locations of Building A, antenna separation of 20 meters. All values in dB.

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Av.  1.43  1.31  1.21  1.10  1.66  1.06  .84  .89  1.54
Std.  .89  .80  .51  .41  .70  .53  .49  .57  .50

* 0 means no component presents for calculating of mean and variance at that delay.
3.5 CONCLUSIONS

In this chapter statistical analysis of signal amplitudes in indoor radio propagation channel was carried out. Candidate distributions of amplitudes were explained and different tests were described and compared for goodness of curve fitting. The data base of 12 000 profiles of impulse responses of indoor radio propagation channels, explained in chapter 2, were used. Extensive curve fitting for the distribution of individual multipath components' amplitudes using Kolmogrov-Smirnov and Wilcoxon procedures showed that amplitude fading is lognormal for both local and global data.

For local data, the lognormal, Nakagami and Rayleigh distributions passed the Kolmogrov-Smirnov test with 90% confidence level for 76%, 36% and 13% of cases, respectively. For global data the Wilcoxon test showed that with 96% confidence lognormal is better than Nakagami distribution and with 93% confidence Nakagami fits the data better than Rayleigh distribution.

Statistical characteristics of signal amplitude fading were studied. For local and global data, standard deviations were in the 1-4.5 dB range decreasing with increased excess delay. Mean amplitudes of multipath components (in dB) decrease almost linearly with increasing excess delay.

Determination of lognormal distribution for signal amplitude fading of indoor radio propagation channels is extremely useful. It can be used in the design, performance assessment as well as simulation of indoor wireless communication systems.
REFERENCES OF CHAPTER 3


References of Chapter 3


CHAPTER 4

MODELING OF PHASES

In chapter 1 mathematical modeling of indoor radio propagation channel was discussed. As we saw, in multipath fading channel sequences of path amplitudes, time of arrivals and phases are present. Statistical modeling of time of arrivals and amplitudes was explained in chapter 2 and 3, respectively. On the other hand since performance of digital communication systems, specially working in indoor environments, is very sensitive to statistical properties of received signal's phase and since to date, no empirically driven model for signal phase has been reported in the literature\(^1\), in this chapter we investigate the phase sequences of multipath components of indoor wireless propagation channels and provide models for these phases. In [2] the probability density of received signal phase of a land mobile satellite channel is reported to be approximately Gaussian. In [3] - [6] phase of indoor (mobile) radio channels are studied. Some simplifying

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\(^1\) This is probably due to difficulties associated with measuring the phase of individual components, i.e., recording of signal phase is incompatible with some measurement techniques [1].

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assumptions have been made in these investigations. For example in [3] and [4] it was assumed that in a length of 1 meter in space, all multipath components with the same delay are caused by reflection from the same fixed scatterer. In [5] and [6] it was assumed that for small spatial separations the angle of arrival of each multipath component remains the same with respect to the direction of motion of the vehicle (portable).

In this chapter phase modeling of indoor radio channel is carried out based on the large multipath propagation data base of 12000 profiles of impulse responses of the channels, which was described in chapter 2. To this end, following a background of the issue in section 4.1, deterministic and random phase increment models are explained in sections 4.2 and 4.3, respectively. These models are used with the large data base in section 4.4. In section 4.5 comparison of the results of the two models with each other and with those obtained from empirical data is carried out. Algorithms for these phase models are explicated in section 4.6 and finally conclusions are given in section 4.7.

4.1 BACKGROUND

Basically, in multipath environments the signal phase is critically sensitive to path length and changes by a factor of $2\pi$ as the path length changes by a wavelength (30 cm at 1 GHz). Considering the geometry of the paths, moderate changes (in order of meters) in the position of portable results in a great change in phase. When one considers an ensemble of points, therefore, it is reasonable to expect uniform distribution; i.e., on a global basis, $\theta_k$, (see chapter 1, section 1.1), has a Uniform$[0,2\pi)$ distribution. This phenomenologically reasonable assumption can be taken as a fact with no need for empirical verification. For small sampling distances, however, great deviations from uniformity may occur. Furthermore, phase values are strongly correlated if the channel's impulse response is sampled at the sampling rate (tens to hundreds of kilobits per second), [1]. Phase values at a fixed delay for a given site are therefore correlated [1]. Adjacent detectable multipath components of the same profile, on the other hand, have independent phases since their excess range (excess delay multiplied by speed of light) is longer than a wavelength, even for very high resolution (a few nanoseconds) measurements [1].

Taking above into consideration it is correct to say that the absolute phase value of a multipath component at a fixed point in space is not important; emphasis of the modeling should be placed on changes in phase as the portable moves through the channel. Let $\theta_k$ denote the
phase of a multipath component at a fixed delay for profile number \(k\) where \(k=1,2,...\) numbers contiguous points in space at a given site. Equivalently, \(\theta_k\) may denote the phase of multipath component occupying a given bin (the discrete-time impulse response model) at spatial point \(k\), (see section 2.4). For the first profile in a sequence \((k=1)\), \(\theta_k\) is assumed to have a \(\text{Unif}(0,2\pi)\) distribution. Subsequent phase values are assumed to follow the following relation, \([1]\)

\[
\theta_k = \theta_{k-1} + \varphi\left(\frac{s_k}{\lambda}\right) \quad k=2,3,...
\]

(4.1)

where \(s_k\) is the spatial separation between \((k-1)\)st and \(k\)th profiles, \(\lambda\) is the wavelength, and \(\varphi(s_k/\lambda)\) is a phase increment. On a sequence of spatially separated profiles, the chain of values defined by (4.1) is interrupted when a path in a given excess delay time (or at a given bin) ceases to exist. A new chain of values (with uniformly distributed first component) starts if a path with the same excess delay appears at a later profile.

Appropriate choices for \(\varphi(s_k/\lambda)\) will impose the necessary spatial correlation on phase values. Using this approach two models for this phase increment are considered.

### 4.2 THE DETERMINISTIC PHASE INCREMENT MODEL

In the deterministic phase model (Model I) changes in the phase value of a multipath component at a fixed delay when the portable moves through space is not random; i.e., knowing \(\theta_j\) and \(\varphi(s_k/\lambda)\), \(\theta_k\) \((k=2,3,...)\) can be calculated deterministically.

As mentioned before, to use this model some simplifying assumptions are made by investigators in order to reduce the degree of randomness of the channel. In one such application, it was assumed that in a length of one meter in space, all multipath components with the same delay are caused by reflection from the same fixed (but randomly located) scatterer, [3]–[4]. In this simulation application the initial phase was generated according to a \(\text{Unif}(0,2\pi)\) distribution. Other spatially separated phases (with the same delay) were obtained by adding \(\varphi(s_k/\lambda)\) to the previous phase. The phase increment was calculated using the single scatterer and local geometry. This is a one-hop model which excludes multiple reflections. This phase model was used in a simulation package for predicting the impulse response of open plant and factory environments.

In two other simulation applications, the deterministic phase model has been used for the mobile and indoor channels [5],[6]. In both applications it was assumed that the angle of arrival
of the \( n \)th multipath component with respect to the direction of motion of the vehicle (portable), \( \psi_n \) remains the same for small spatial separations. Therefore,

\[
\varphi\left(\frac{s_k}{\lambda}\right) = \frac{2\pi s_k}{\lambda} \cos \psi_n
\]  \hspace{1cm} (4.2)

For the mobile channel \( \psi_n (n=1,2,3,...) \) were generated according to a uniform distribution. The power spectra of simulated CW data generated using a wide-band channel simulator (reported in [7]) showed better agreement with theory [5], when compared with spectra obtained using the random phase increment model (which is described in next section). For the indoor channel, \( \psi_n (n=1,2,3,...) \) were estimated with a 5° resolution based on measurements and by using a Fourier transform method (details are reported in [6]).

In the deterministic phase increment model we note that the locus of all scatterers contributing to a single multipath component with a fix excess delay is an ellipse with the transmitter and receiver positioned at its focal points, as illustrated in Fig. 4.1. Different con-focal ellipses correspond to different excess delays. Scatterers are distributed uniformly on the perimeter of each ellipse. Considering the width \( \tau_0 \) (5 nsec. for each bin) in discrete-time impulse response model, the distance between two consecutive ellipses is \( c\tau_0/2 \), where \( c \) is the speed of light.

\[\text{Figure 4.1 The confocal ellipses. Transmitter and receiver are located on focal centers of ellipses. For each excess delay if there is a path component, an ellipse is considered.}\]
Referring to Fig. 4.1 the portable moves from right focus of ellipses in the direction of the positive main axis of ellipses and paves the sampling distance \( D \), (which is 2 cm, refer to section 2.1). Considering the maximum excess delay of \( N \tau_0 \) and bin resolution of 5 nsec., the maximum number of confocal ellipses is \( N=100 \). Let \( M \) denote the number of scatterers distributed on each ellipse, \( S_{n,m} \) the \( m \)th scatterer on \( n \)th ellipse, \( 1 \leq m \leq M \), and \( 1 \leq n \leq N \). At each profile (position of portable) and in a fixed excess delay \( M \) rays are received through \( M \) scatterers. As mentioned earlier, some ellipses do not exist because there is no path component at the corresponding bin in the discrete-time impulse response. Therefore \( N \) is a random variable and consequently the impulse response of the channel (see subsection 1.1.1) at the \( k \)th position of portable is written as

\[
h_k(t) = \sum_{n=1}^{N_k} a_{k,n} \delta(t-\tau_{k,n}) e^{j\theta_{k,n}}
\]

(4.3)

where \( N_k \) is the number of multipath components, \( \{a_{k,n}\} \), \( \{\tau_{k,n}\} \) and \( \{\theta_{k,n}\} \) are random amplitude, time of arrival and phase sequences of \( n \)th multipath at the \( k \)th profile, respectively.

Now suppose that the portable is in its initial position, which is right focus of confocal ellipses, (Fig. 4.2). The amplitude and phase of the \( n \)th multipath signal due to \( n \)th ellipse is:

\[
R_{0,n} e^{j\phi_{0,n}} = \sum_{m=1}^{M} \alpha_{0,n,m} e^{j\theta_{0,n,m}}
\]

(4.4)

where \( \alpha_{0,n,m} \) and \( \theta_{0,n,m} \) are real attenuation coefficient and phase change with respect to LOS of reflected path, of \( m \)th scatterer located on \( n \)th ellipse when portable is in its initial position, respectively. \( M \) is number of scatterers on each ellipse, \( R_{0,n} \) and \( \phi_{0,n} \) are path amplitude and phase of signal at \( n \tau_0 \) excess delay at initial position of portable. The resultant signal in the first profile (first position of portable), due to all path components (ellipses) is

\[
R_0 e^{j\phi_0} = \sum_{n=1}^{N_0} R_{0,n} e^{j\phi_{0,n}} = \sum_{n=1}^{N_0} \sum_{m=1}^{M} \alpha_{0,n,m} e^{j\theta_{0,n,m}}
\]

(4.5)

where \( N_0 \) is the number of path components in initial position of portable (profile). Due to major property of ellipse, the delay (phase) due to scatterers on each ellipse are the same, i.e.

\[
\theta_{0,n,m} = \theta_{0,n,m+1} & \theta_{0,n} \quad m=1,2,...,M-1 \quad n=1,2,...,N_0
\]

(4.6)

Therefore (4.5) will be
Figure 4.2  Phase updating at a fixed excess delay in model I.

\[ R_0 e^{j\Phi_0} = \sum_{n=1}^{N_0} e^{j\theta_{0,n}} \left( \sum_{m=1}^{M} \alpha_{0,n,m} \right) \]  \hspace{1cm} (4.7)

Let us suppose

\[ \alpha_{0,n} = \sum_{m=1}^{M} \alpha_{0,n,m} \]  \hspace{1cm} (4.8)

so

\[ R_0 e^{j\Phi_0} = \sum_{n=1}^{N_0} \alpha_{0,n} e^{j\theta_{0,n}} \]  \hspace{1cm} (4.9)

When the portable goes to the next position, with a sampling distance \( D \), at each excess delay the signal is

\[ R_{1,n} e^{j\Phi_{1,n}} = \sum_{m=1}^{M} \alpha_{1,n,m} e^{j\theta_{1,n,m}} \]  \hspace{1cm} n=1,2,...,N_1 \hspace{1cm} (4.10)

and its amplitude as
The Deterministic Phase Increment Model

\[ R_{1,n} = \left[ \left( \sum_{m=1}^{M} \alpha_{1,n,m} \cos \theta_{1,n,m} \right)^2 + \left( \sum_{m=1}^{M} \alpha_{1,n,m} \sin \theta_{1,n,m} \right)^2 \right]^{\frac{1}{2}} \]  

(4.11)

and phase

\[ \Phi_{1,n} = \tan^{-1} \left( \frac{\sum_{m=1}^{M} \alpha_{1,n,m} \sin \theta_{1,n,m}}{\sum_{m=1}^{M} \alpha_{1,n,m} \cos \theta_{1,n,m}} \right) \]  

(4.12)

Referring to Fig. 4.2 the phase of component at nth excess delay, in the first profile (portable position), due to nth scatterer is

\[ \theta_{1,n,m} = \theta_{0,n} + \frac{2\pi}{\lambda} \left( |TS_{n,m}P_1| - |TS_{n,m}P_0| \right) \]  

(4.13)

The resultant received signal at first position of portable is

\[ R_1 e^{j\Phi_1} = \sum_{n=1}^{N_1} R_{1,n} e^{j\Phi_{1,n}} = \sum_{n=1}^{N_1} \sum_{m=1}^{M} \alpha_{1,n,m} e^{j\theta_{1,n,m}} \]  

(4.14)

In a same manner at nth delay and kth profile we have

\[ R_{k,n} e^{j\Phi_{k,n}} = \sum_{m=1}^{M} \alpha_{k,n,m} e^{j\theta_{k,n,m}} \quad n = 1, 2, ..., N_k \]  

(4.15)

and the phase is

\[ \theta_{k,n,m} = \theta_{k-1,n,m} + \frac{2\pi}{\lambda} \left( |TS_{n,m}P_k| - |TS_{n,m}P_{k-1}| \right) \]  

(4.16)

Let us show the second term of (4.16) as

\[ \Delta \theta_{k,n,m} = \frac{2\pi}{\lambda} \left( |TS_{n,m}P_k| - |TS_{n,m}P_{k-1}| \right) \]  

(4.17)

Therefore, (4.16) is written

\[ \theta_{k,n,m} = \theta_{k-1,n,m} + \Delta \theta_{k,n,m} \]  

(4.18)
Referring to Fig. 4.2, \( T \) is the left focal center of confocal ellipses, \( P_k \) and \( P_{k-1} \) are position of portable at \( k \)th and \((k-1)\)st sampling distances, \( S_{n,m} \) is the \( m \)th scatterer on \( n \)th ellipse, and \( \lambda \) is wavelength. The equation of \( n \)th ellipse (i.e., ellipse corresponding to the \( n \)th excess delay) is

\[
\frac{x^2}{p_n^2} + \frac{y^2}{q_n^2} = 1
\]

(4.19)

where \( p_n \) and \( q_n \) are half of the major and minor axes of the \( n \)th ellipse, respectively. Considering the distance between transmitter and receiver as \( 2r \) and \( x \) axis position of \( m \)th scatterer as \( x_m \) the value of \( \Delta \beta_{k,n,m} \) in (4.17) is calculated

\[
\Delta \beta_{k,n,m} = \frac{2\pi}{\lambda} \left[ \sqrt{(r+kD-x_m)^2 + q_n^2 - c_n^2 x_m^2} - \sqrt{(r+kD-D-x_m)^2 + q_n^2 - c_n^2 x_m^2} \right]
\]

(4.20)

where \( c_n = q_n / p_n \) and \( D \) is sampling distance.

Equation (4.18) can also be written with reference to the phase of the initial profile as

\[
\theta_{k,n,m} = \theta_{0,n} + \beta_{k,n,m}
\]

(4.21)

where

\[
\beta_{k,n,m} = \frac{2\pi}{\lambda} \left( |TS_{n,m}P_k| - |TS_{n,m}P_{k-1}| \right)
\]

(4.22)

Using (4.15) and similar to (4.12) phase of the path component at \( n \)th excess delay and \( k \)th profile is

\[
\phi_{k,n} = \tan^{-1} \left[ \frac{\sum_{m=1}^{M} \alpha_{k,n,m} \sin \theta_{k,n,m}}{\sum_{m=1}^{M} \alpha_{1,n,m} \cos \theta_{k,n,m}} \right]
\]

(4.23)

Now the phase increment at \( n \)th excess delay and \( k \)th profile which is \( \phi_{k,n} - \phi_{k-1,n} \) is calculated as

\[
\phi^{(I)}_{k,n} = \tan^{-1} \left[ \frac{\sum_{m_1=1}^{M} \sum_{m_2=1}^{M} \alpha_{k,n,m_1} \alpha_{k-1,n,m_2} \sin(\theta_{k,n,m_1} - \theta_{k-1,n,m_2})}{\sum_{m_1=1}^{M} \sum_{m_2=1}^{M} \alpha_{k,n,m_1} \alpha_{k-1,n,m_2} \cos(\theta_{k,n,m_1} - \theta_{k-1,n,m_2})} \right]
\]

(4.24)

where superscript \((I)\) is used to indicate the phase increment of Model \( I \).
The phase of path component at \( n \)th excess delay (4.23) can also be written as (see Appendix B)

\[
\phi_{k,n} = \theta_{0,n} + \zeta_{k,n}
\]

(4.25)

where

\[
\zeta_{k,n} = \tan^{-1} \left( \frac{\sum_{m=1}^{M} \alpha_{k,n,m} \sin \beta_{k,n,m}}{\sum_{m=1}^{M} \alpha_{k,n,m} \cos \beta_{k,n,m}} \right)
\]

(4.26)

Now the resultant signal at \( k \)th profile is

\[
R_k e^{j \Phi_k} = \sum_{n=1}^{N_k} R_{k,n} e^{j \phi_{k,n}}
\]

(4.27)

and its amplitude as

\[
R_k = \left[ \sum_{n=1}^{N_k} R_{k,n} \cos (\phi_{k-1,n} + \phi^{(f)}_{k,n}) \right]^{1/2} + \left[ \sum_{n=1}^{N_k} R_{k,n} \sin (\phi_{k-1,n} + \phi^{(f)}_{k,n}) \right]^{1/2}
\]

(4.28)

In the deterministic phase increment model \( M \) independent scatterers are uniformly distributed over perimeter of each confocal ellipse. In a fixed excess delay phase of first component is generated according to Unif(0,2\( \pi \)) distribution and when portable moves, if there is a path component in next profile phase is updated as

\[
\phi_{k,n} = \phi_{k-1,n} + \phi^{(f)}_{k,n}
\]

(4.29)

This process of phase updating continues in each excess delay until component ceases to exist. In this case new scatterers are uniformly generated on perimeter of the same ellipse (same excess delay). It should be emphasized that in this model of phase, the initial positions of scatterers on each ellipse are random, but updating of phase sequences due to those scatterers is deterministic (i.e., knowing \( \theta_{0,n} \), \( S_{n,m} \) and \( \alpha_{k,n,m} \) the value of \( \zeta_{k,n,m} \) in (4.26) or equivalently the value of \( \phi^{(f)}_{k,n} \) in (4.24) is determined). This is why we call this model as deterministic phase increment. As a special case suppose \( M=1 \), (i.e., one scatterer on each ellipse), therefore, (4.24) reduces to
\[ \phi_{k,n}^{(f)} = \theta_{k,n,1} - \theta_{k-1,n,1} = \Delta \phi_{k,n,1} = \frac{2\pi}{\lambda} \left( |TS_{n,1}^{k-1} P_k| - |TS_{n,1}^{k} P_{k+1}| \right) \] (4.30)

and subsequently phase updating (4.29) will be

\[ \phi_{k,n} = \phi_{k-1,n} + \Delta \phi_{k,n,1} \] (4.31)

which is quite clear from Fig. 4.2.

Referring to Fig. 4.3 when the distance between portable and mth scatterer on nth ellipse is much more than sampling distance \(D\), parallel wave approximation form of (4.16) may be considered

\[ \theta_{k+1,n,m} = \theta_{k,n,m} + \frac{2\pi}{\lambda} D \cos \psi_{k,n,m} \] (4.32)

where \(\psi_{k,n,m}\) is the angle between the line connecting the mth scatterer on the nth ellipse to the \(k\)th position of portable. It is clear in this case \(\Delta \phi_{k,n,m}\) is

\[ \Delta \phi_{k,n,m} = \frac{2\pi}{\lambda} D \cos \psi_{k,n,m} \] (4.33)

and \(\phi_{k,n,m}\) as

\[ \phi_{k,n,m} = \frac{2\pi}{\lambda} D \cos \psi_{k,n,m} \] (4.34)

It should be emphasized that this model of phase is a comprehensive one and no simplistic assumptions, such as one-hop reflection for each multipath component (which is caused by the single scatterer and remains the same with the motion of portable), have been made.

\[\text{Figure 4.3 Parallel wave approximation.}\]
4.3 THE RANDOM PHASE INCREMENT MODEL

In random phase increment model (Model II), phase increment is a random variable, i.e., starting with a Unif[0,2π) initial phase, each subsequent value is obtained by adding a random phase increment to the previous phase value. Suppose \( \phi_{k,n} \) and \( \phi^{(II)}_{k,n} \) are phase and phase increment components at \( n \)th excess delay and \( k \)th profile, respectively. Path phase at \((k+1)\)st profile and \( n \)th delay is written as

\[
\phi_{k+1,n} = \phi_{k,n} + \phi^{(II)}_{k,n}
\]  

(4.35)

where superscript (II) is used to indicate the model II. Phase updating (4.35) can also be written with reference to the initial path phase \( \phi_{0,n} \), i.e.,

\[
\phi_{k+1,n} = \phi_{0,n} + \sum_{i=0}^{k} \phi^{(II)}_{i,n}
\]  

(4.36)

By denoting

\[
\zeta_{k,n} = \sum_{i=0}^{k} \phi^{(II)}_{i,n}
\]  

(4.37)

phase updating relation will be

\[
\phi_{k+1,n} = \phi_{0,n} + \zeta_{k,n}
\]  

(4.38)

Generally the parameters of probability distribution of this increment are functions of \( s_k/\lambda \) (\( s_k \) is the spatial separation between \((k-1)\)st and \( k \)th profile, and \( \lambda \) is wavelength). As an example \( \phi^{(II)}_{s_k/\lambda} \) can be assumed a Gaussian random variable with zero mean and standard deviation of \( \sigma_{s/\lambda} \). By making \( \sigma_{s/\lambda} \) an increasing function of \( s/\lambda \) (or \( s \), for a fixed \( \lambda \)), the degree of correlation between \( \phi_{k-1,n} \) and \( \phi_{k,n} \) can be controlled. For \( s=0 \), \( \sigma_{s/\lambda}=0 \), and \( \theta_{k-1}=\theta_k \) (assuming space-invariant channel). The correlation between \( \phi_{k-1,n} \) and \( \phi_{k,n} \) decreases as \( \sigma_{s/\lambda} \) increases, until they become uncorrelated [1].

In our phase increment model we assume \( \phi^{(II)}_{s_k/\lambda} \) is zero mean Gaussian random variable with functional form of standard deviation as
\[ \sigma_{s/\lambda} = \sigma_{\text{max}} \left( 1 - e^{-b \frac{s}{\lambda}} \right) \]  \hspace{1cm} (4.39)

where \( s \) is the spatial separation, \( \lambda \) is wavelength and \( b \) is a proper constant. This method with the same functional form of phase increment (Gaussian distribution of \( \Phi(s/\lambda) \)), has been previously applied in simulation of the phase components of a wideband mobile radio channel [7].

Just like the first model, in each given excess delay, this process is interrupted when a path component ceases to exist. A new \( \text{Unif}(0, 2\pi) \) distributed phase is generated if a path with the same excess delay appears at a later profile, and random updating starts.

### 4.4 RESULTS OF MODELING

In this section the two discussed phase models are used with large empirical wideband impulse response data base, described in chapter 2. Performances of these two models are evaluated by means of extensive computer simulations.

#### 4.4.1 Analysis

In order to study the suitability of the two phase models, we investigated narrowband CW fading based on wideband models. For all locations and antenna spacings of both buildings, phase profiles were generated, (each profile contains the maximum number of 100 path phase components). The magnitude of the phasor summation of all measured amplitude and simulated phase path components in a profile was then computed to give the CW envelope waveform for a particular position of the portable in each location. Cumulative distribution function (CDF), and second order statistics, such as level crossing rate (LCR)\(^2\), average duration of fade (ADF)\(^3\) of CW fading of simulated and empirical data were calculated. Moreover, for further study of efficiency of these models Doppler spectra of simulated and measured data were determined. In the following, results of simulation for each models are explained separately.

---

\(^2\) Level crossing rate is the expected rate at which a given level is crossed with positive slope [8].

\(^3\) Average duration of fade (ADF) is length of time where the signal remains below a given level once it crosses that level with a negative slope [8].
4.4.2 Results of The Model I

As explained in section 4.2, in deterministic phase increment model, actual reflection refraction and scattering of propagated signal are modeled by the reflection from \( M \) independent uniformly distributed random scatterers around confocal ellipses, for each path. Meanwhile reflection coefficients corresponding to scatterers were assumed to be identically independent and uniform \([0,1]\) distributed.

In order to determine how many scatterers to be considered on each ellipse, one, two, five and ten scatterers were examined. CW fading waveforms, Doppler spectra and their CDFs were compared with empirical curves. Parts of this comparison are shown in Fig. 4.4 and Fig. 4.5. It should be noted that in applying the deterministic phase increment model if the number of scatterers on each ellipse \( M \) is even, half of scatterers are distributed on the upper half-ellipse and half of them on the lower half-ellipse. If \( M \) is odd, one half-ellipse will have one more scatterer than the other half-ellipse. Extensive comparison of results from consideration of several scatterers showed assuming \( M=5 \) scatterers (3 scatterers on upper half-ellipse and 2 scatterers on lower half-ellipse), is reasonable.

In order to analyze the model, for each location a number of profiles were generated between each of 75 wideband profiles. The number of profiles generated \( L \), changes the sampling distance to \( D/L \). In generating new profiles, due to the high correlation of amplitudes in a sampling distance \( D \) in each excess delay \([9]\), the amplitudes of last profile were considered, but, the phase of each component was calculated based on model \( I \). This investigation was done for 1, 3, 4 and 19 profiles generated in between each 75 profiles of each location, i.e., 1 cm, 5mm, 4mm and 1mm spacing of the portable. For saving space only part of the results are shown in Figs. 4.6 and 4.7. Comparison of the results showed consideration of 2 cm sampling distance is reasonable \([10]\).

Moreover, in order to analyze the effect of the model exactly, the total area covered by portable in each location (which is 75 \( \times \) 2 cm = 1.5 m), is divided to 3 tracks each track with coverage of 50 cm, as well as, 5 tracks each track with coverage of 30 cm (which is about one wavelength for 1 GHz). To save space only parts of these figures showing first and second order statistics of CW fading of empirical data and simulation, are reproduced in Fig. 4.8 for Building
Figure 4.4  CW fading waveforms (relative to the mean level). Building B, antenna separation of 10 m, location 1, with 1, 2, 5 and 10 scatterers on each ellipse of Model I.

Figure 4.5  CDF of CW waveforms of empirical data and Model I for Building B, antenna separation of 10 m, location 1, with 1, 2, 5 and 10 scatterers.
Figure 4.6 CW fading waveforms, (relative to mean) of empirical data and Model 1. Building B, antenna separation of 10 m, location 1, with 1mm, 5mm, 1 cm, and 2 cm sampling distances.

Figure 4.7 CDF of CW waveforms of empirical data and Model 1 with 5 scatterers. Building B, antenna separation of 10 m, location 1, with 1mm, 5 mm, 1 cm and 2 cm sampling distances.
Figure 4.8 Data of all antenna separations of Building A combined. CDF, LCR, ADF and Doppler spectra of CW fading of empirical data and Model I along whole 150 cm. ($f_m = v/\lambda$, $v=25$ cm/s).
Figure 4.9 Data of all antenna separations of Building B combined. CDF, LCR, ADF and Doppler spectra of CW fading of empirical data and model II along 3rd 30cm (3rd wavelength) track. Velocity assumed 25cm/s.
Figure 4.10 Data of all antenna separations of the two buildings combined. CDF, LCR, ADF and Doppler spectra of CW fading of empirical data and Model I along whole 150 cm. Velocity assumed 25 cm/s.
Figure 4.11 Data of all antenna separations of Building A combined. CDF, LCR, ADF and Doppler spectra of CW fading of empirical data and Model II along 4th 30 cm (4th wavelength) track. Velocity assumed 25 cm/s.
Figure 4.12 Data of all antenna separations of Building B combined. CDF, LCR, ADF and Doppler spectra of CW fading of empirical data and Model II along 2nd 30 cm (2nd wavelength) track. Velocity assumed 25cm/s.
Figure 4.13 Data of all antenna separations of both buildings combined. CDF, LCR, ADF and Doppler spectra of CW fading of empirical data and Model II along whole 150 cm. Velocity assumed 25 cm/s.
Figure 4.14  Comparison of the statistics of fading waveforms of Model I and II with empirical data. Building B, data of all antenna separations combined. Over a) 3rd wavelength track  b) 4th wavelength track.
A and in Fig. 4.9 for Building B. In Fig. 4.10 these results when all data of the two buildings are combined are shown. Exact study of plots, parts of them depicted in Figs. 4.8 - 4.10, indicates that first and second order statistics of the CW fading waveforms of Model I has a very good agreement with those obtained from measurement [10]-[11]. Furthermore, it should be noted that due to large number of data (profiles), which were examined, the validity of this model is well-confirmed.

4.4.3 Results of The Model II

In random phase increment model zero mean Gaussian increments with the functional form of standard deviation mentioned in (4.39) were generated. The values of $\sigma_{\text{max}}$ and $b$ were determined according to the simulation experiment of this model related to urban (mobile) propagation channel [7], which are $10^6$ and 1, respectively. In [7] the parameters $\sigma_{\text{max}}$ and $b$ were obtained through simulation experiment. In a fixed excess delay first $\theta_1$ was set to $\pi$ and $\theta_k$, $k=1,2,\ldots, 10000$ by relation (4.1), where $\theta_{k-1}$ was generated as above. The objective was to obtain a correlation function for sequence \{\theta_k\}, \((k=1,2,\ldots,10000)\) which becomes negligible at about one in this section we want to compare these two phase models with each other and wavelength, while maintaining a $\text{Unif}(0,2\pi)$ distribution for the phase sequence over 10000 samples [7].

Again for the exact analysis of this model the total area covered by portable in each location is divided to 3 tracks each track with coverage of 50 cm, as well as, 5 tracks with each track coverage of one wavelength (30 cm). CW fading, CDF, LCR, ADF and Doppler spectrum of empirical and simulated model have been investigated. First and second order statistics of CW fading data (empirical and simulation ), when all data of a building are combined are shown in Fig. 4.11 for Building A and in Fig. 4.12 for Building B. In Fig. 4.13 these curves when all data of two buildings are combined are shown. Detailed study of fading statistics generated by this model provides excellent agreement with empirical data [10]-[11]. Again, due to very large number of profiles, which were examined, the validity of this model is well-confirmed.
4.5 COMPARISON

4.5.1 Comparison With Empirical Data

Detailed comparison of the results of these two models and empirical data were carried out. As mentioned earlier besides from study of performance of the models over whole coverage area, in order to analyze the model exactly, the total area covered by the portable in each location is divided to 5 tracks each with coverage of one wavelength. In Fig. 4.14, CDF, LCR and ADF of CW fading signal of simulated models (I and II) with each other and with those of measured data for Building A and Building B are shown. A good agreement between these models and empirical data is seen.

In order to compare the results of two models with empirical data more, we compare the path phase distribution of each model with those of measured data. Equivalent to the models given, we consider one bin and assume a path is present in that bin over all profiles. In Model I, according to (4.25) and (4.26), \( \theta_{0,n} \) is a uniform \([0,2\pi)\) distributed random variable and \( \zeta_{k,n} \) is a deterministic increment. The probability density function of \( \zeta_{k,n} \) will be a Dirac delta function located at the value of \( \zeta_{k,n} \). Therefore, the resultant path phase has a pdf which is the convolution of Unif\([0,2\pi)\) with \( \delta(\zeta_{k,n}) \) and this is a uniform \([0,2\pi)\) function located at the value of \( \zeta_{k,n} \). Removing modulo \( 2\pi \), the pdf of \( \phi_{k,n} \) will be uniform.

Referring to (4.36)-(4.38), in Model II initial phase \( \phi_{0,n} \) has uniform \([0,2\pi)\) distribution, and \( \zeta_{k,n} \) is the summation of several independent zero mean Gaussian increments with functional form of standard deviation as (4.39). Referring to (4.37), \( \zeta_{k,n} \) will have the zero mean and variance as

\[
\sigma_\zeta^2 = \sum_{i=0}^{k} \sigma_{max}^2 \left( 1 - e^{-\frac{\delta_i}{\lambda}} \right)^2
\]  

(4.40)

which can be written as (see Appendix C)
\[ \sigma^2_c = \sigma_{\text{max}}^2 \left[ k + 1 + \frac{1 - e^{-\frac{\lambda}{bD}}}{1 - e^{-\frac{\lambda}{bD}}} - 2 + \frac{1 - e^{-\frac{\lambda}{bD}}}{1 - e^{-\frac{\lambda}{bD}}} \right] \] (4.41)

Therefore, if \( k \) is large we can assume \( \zeta_{kn} \) be a zero mean Gaussian random variable with variance of (4.41). Now considering (4.38), due to independence of \( \phi_{0,n} \) and \( \zeta_{kn} \), the probability density function of \( \Phi_{kn} \) will be the convolution of Unif[0,2\( \pi \)] of \( \phi_{0,n} \) with Gaussian distribution of \( \zeta_{kn} \). The final result is (refer to Appendix D)

\[ P[\Phi_{k,n} < \xi] = \frac{1}{2\pi} \int_{-\pi}^{\xi} \{ Q(\frac{\xi - \pi}{\sigma}) - Q(\frac{\xi + \pi}{\sigma}) \} \, dy \quad [\text{mod}(2\pi)] \] (4.42)

where

\[ Q(y) = \frac{1}{\sqrt{2\pi}} \int_{y}^{\infty} e^{-\frac{u^2}{2}} \, du \] (4.43)

In Fig. 4.15 and Fig. 4.16 cumulative distribution functions of path phases of two models as well as empirical data are shown. A good consistency between the distribution of path phases of these models and measured data is seen.

### 4.5.2 Comparison of The Two Models

#### 4.5.2.1 Increasing the Number of Scatterers of Model I

In this section we want to compare these two phase models with each other and particularly when number of scatterers of Model I gets large. Results of carrying out simulation with increasing the number of scatterers for different excess delays, antenna separations and positions of portable are shown in Figs. 4.17 - 4.20 and Tables 4.1 - 4.4. From these figures and tables it is clear that as the number of scatterers increases the distribution of phase increments gets closer to the normal distribution. Extensive simulation shows with 5 scatterers or more distribution of phase increments has a very good agreement with normal distribution of phase increments of Model II. The mean of simulated phase increments is not zero and it decreases with increasing excess delay and increases with increasing antenna separations and is nearly
Figure 4.15  CDF of path phases of empirical data and Models I and II. Building A, antenna separation 30 m, location 15.

Figure 4.16  CDF of path phases of empirical data and Models I and II. Building B, antenna separation 10 m, location 10.

independent of the number of scatterers. Standard deviation of phase increments decreases with increasing the number of scatterers and is nearly independent of the antenna separation and position of portable [12].
Non-zero mean of phase increments in model I is completely justified by referring to Fig. 4.2. From this figure it is clear that $\Delta \beta_{k,n,m}$ which corresponds to the difference $|TS_{n,m}P_k| - |TS_{n,m}P_{k-1}|$ is positive if $x_m \in [p_n, r+(k-1/2)D]$ and negative if $x_m \in [r+(k-1/2)D, p_n]$. Now because $x_m$ is assumed to be distributed uniformly between $[p_n, p_n]$, the probability of $x_m \in [p_n, r+(k-1/2)D]$ is greater than the probability of $x_m \in [r+(k-1/2)D, p_n]$. Therefore, the mean of $\Delta \beta_{k,n,m}$ and subsequently the mean of phase increment is not zero [13].

![Graph showing CDF of phase increments in Model I](image)

**Figure 4.17** CDF of phase increments in Model I. Solid line (simulation), dotted curve (Gaussian distribution), 1 scatt., ant. sep. 10 m, excess delay 25 ns., profile no. 30.

As explicated earlier, in Model II a random Gaussian increment is added to the phase of previous path component. This phase increment has the functional form of standard deviation as (4.39). From simulation we see the phase increments in the deterministic model have the non-zero mean Gaussian distribution. Non-zero mean of phase increments is not important, because we can attribute this non-zero mean to the random initial phase of $\Phi_{0,n}$, so the increments of phase will have the zero-mean Gaussian distribution. From entries of Tables 4.1 - 4.4 the average value of standard deviation of phase increments with five scatterers around ellipses is 5.8°.
Figure 4.18 CDF of phase increments in Model I. Solid line (simulation), dotted curve (Gaussian distribution), 3 scatt., ant. sep. 10m, excess delay 25 ns, profile no. 30.

Figure 4.19 CDF of phase increments in Model I. Solid line (simulation), dotted curve (Gaussian distribution), 5 scatt., ant. sep. 10m, excess delay 25 ns, profile no. 30.

Now we want to compare this standard deviation with one of model II. However, it should be stressed that the standard deviation of phase increments in model II is not constant and changes with position of the portable, (see (4.39)). Therefore, in order to compare the standard deviation of phase increments of the two models we evaluate the average of standard deviation of phase
Figure 4.20  CDF of phase increments in Model I. Solid line (simulation), dotted curve (Gaussian distribution), 10 scatt., ant. sep. 10m, excess delay 25 nsec, profile no. 30.

increments of model II. This can be done by averaging (4.39), over whole distance of a location as

\[
\bar{\sigma} = \frac{1}{L_t} \int_0^{L_t} \sigma_{max} \left( 1 - e^{-\frac{x}{\lambda}} \right) dx
\]  

(4.44)

where \( L_t \) is the total distance making average on (i.e., \( KD=5\lambda=150 \) cm). The numerical value of (4.44) is 6.32° which is in agreement with the average of standard deviation obtained from simulation of model I.

4.5.2.2  Computational Efficiency of Each Model

Before comparing these models it should be emphasized again that in both models chains of updated phases are interrupted when a path in a given excess delay ceases to exist. A new chain of phase values, with uniformly distributed first component starts if a path with the same excess delay appears at a later profile. Referring to average correlation coefficient of signal amplitudes for both buildings, reported in [9], and by considering correlation coefficient of 0.3 as a threshold of correlation between amplitudes, in a fixed excess delay path amplitudes of up to
**Table 4.1**  Mean and standard deviation of phase increments in model 1 with different number of scatterers. Antenna separation 5m, excess delay 25 nsec, profile no. 30.

<table>
<thead>
<tr>
<th>phase increment (°)</th>
<th>no. of scatt.</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>std</td>
</tr>
<tr>
<td>6.24</td>
<td>12.36</td>
</tr>
<tr>
<td>6.42</td>
<td>8.09</td>
</tr>
<tr>
<td>6.81</td>
<td>5.95</td>
</tr>
<tr>
<td>6.69</td>
<td>4.43</td>
</tr>
<tr>
<td>6.93</td>
<td>2.79</td>
</tr>
<tr>
<td>6.83</td>
<td>1.40</td>
</tr>
</tbody>
</table>

**Table 4.2**  Mean and standard deviation of phase increments in model 1 with different number of scatterers. Antenna separation 5m, excess delay 50 nsec, profile no. 30.

<table>
<thead>
<tr>
<th>phase increment (°)</th>
<th>no. of scatt.</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>std</td>
</tr>
<tr>
<td>4.40</td>
<td>11.89</td>
</tr>
<tr>
<td>3.35</td>
<td>8.07</td>
</tr>
<tr>
<td>4.35</td>
<td>5.97</td>
</tr>
<tr>
<td>3.70</td>
<td>4.45</td>
</tr>
<tr>
<td>4.00</td>
<td>2.72</td>
</tr>
<tr>
<td>4.16</td>
<td>1.43</td>
</tr>
</tbody>
</table>
Table 4.3  Mean and standard deviation of phase increments in model I with different number of scatterers. Antenna separation 10m, excess delay 50 nsec, profile no. 3.

<table>
<thead>
<tr>
<th>phase increment (°)</th>
<th>no. of scatt.</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>std</td>
</tr>
<tr>
<td>3.84</td>
<td>11.62</td>
</tr>
<tr>
<td>4.38</td>
<td>7.36</td>
</tr>
<tr>
<td>3.65</td>
<td>6.21</td>
</tr>
<tr>
<td>3.89</td>
<td>4.33</td>
</tr>
<tr>
<td>4.50</td>
<td>2.54</td>
</tr>
<tr>
<td>4.24</td>
<td>1.41</td>
</tr>
</tbody>
</table>

Table 4.4  Mean and standard deviation of phase increments in model I with different number of scatterers. Antenna separation 30m, excess delay 50 nsec, profile no. 30.

<table>
<thead>
<tr>
<th>phase increment (°)</th>
<th>no. of scatt.</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>std</td>
</tr>
<tr>
<td>11.48</td>
<td>11.25</td>
</tr>
<tr>
<td>11.93</td>
<td>6.94</td>
</tr>
<tr>
<td>11.35</td>
<td>5.36</td>
</tr>
<tr>
<td>11.32</td>
<td>4.27</td>
</tr>
<tr>
<td>11.24</td>
<td>2.85</td>
</tr>
<tr>
<td>11.51</td>
<td>1.22</td>
</tr>
</tbody>
</table>
10 cm are correlated. Now we assume that in each fixed excess delay on the average \( K_f = 10 \text{ cm} / 2 \text{ cm} = 5 \) path components are correlated and accordingly on the average after each \( K_f \) times of phase updating the sequence is interrupted and new phase components with new scatterers are generated. In the random phase increment model, the phase of the first component is selected according to \( \text{Unif}[0, 2\pi] \) and is updated for \((K_f - 1)\) next components. For Model II let us show the number of computations for generating each uniform distributed initial phase as \( G_2 \) and the number of computations for updating each path phase component as \( U_2 \). The average number of computations for all excess delays and all profiles will be

\[
NC_{\text{model II}} = [G_2 + (K_f - 1)U_2] \mathcal{N} \frac{K}{K_1}
\]  
(4.45)

where \( \mathcal{N} \) is the average number of multipath components in each profile. In Model II phase updating is generating normal phase increments. Generating of normal increments is carried out with the Box-Muller method using uniform random variables \([14]\), which needs two times of computation of uniform random variables (i.e., \( U_2 = 2G_2 \)). Therefore (4.45) will be

\[
NC_{\text{model II}} = [G_2 + (K_f - 1)2G_2] \mathcal{N} \frac{K}{K_1} = (2K_1G_2 - G_2) \mathcal{N} \frac{K}{K_1}
\]  
(4.46)

In Model I, phase of first path is selected according to \( \text{Unif}[0, 2\pi] \) and updated based on (4.29). In this regards the number of computations for generating \( M \) scatterers on each ellipse and for generation of \( M \) attenuation coefficients \((a_{K_f, n, m})\) should be considered added to the number of computations for updating \((K_f - 1)\) path phases, (based on (4.24)). Let us show the number of computations for generating initial phases as \( G_1 \) and the number of computations for updating phase in Model I with \( U_1 \), the average total number of computations for all excess delays of all profiles in model I is

\[
NC_{\text{model I}} = [G_1 + MG_1 + MG_1 + (K_f - 1)U_1] \mathcal{N} \frac{K}{K_1} = [(1 + 2M)G_1 + (K_f - 1)U_1] \mathcal{N} \frac{K}{K_1}
\]  
(4.47)

The first term inside bracket in (4.47) is for generating initial phases, second term for generating of \( M \) uniformly distributed scatterers around an ellipse, third term for generating of \( M \) uniformly distributed attenuation coefficients and fourth term for updating of phase for the average number of \((K_f - 1)\) profiles.

For comparing the efficiency of these two models we define the ratio of the number of computations of two models
\[
\frac{t \cdot NC_{\text{model } II}}{NC_{\text{model } I}} = \frac{2K_1 - 1}{1 + 2M + (K_1 - 1) \frac{U_1}{G_1}}
\]  
(4.48)

Denoting \( U_j / G_j \Delta \rho \), and because the number of computations for calculating deterministic phase increment \( U_j \) depends on the number of scatterers, (4.48) is written as

\[
t = \frac{2K_1 - 1}{1 + 2M + (K_1 - 1) \rho(M)}
\]  
(4.49)

When the number of scatterers in the deterministic phase increment model gets larger \( U_j \) and consequently \( \rho(M) \) increases and accordingly \( t \) gets smaller, which implies that Model II is better. The ratio \( t = 1 \) could be obtained with the value of \( M_0 \) as

\[
M_0 = (K_1 - 1) \left( 1 - \frac{1}{2} \rho(M_0) \right)
\]  
(4.50)

With \( M_0 \) scatterers of model I, the performance of two models is the same. As explained earlier for generating uniform random variables the linear congruential method is used, [14]. This is a fast method requiring only a few operations per call. Inspection of number of computations of \( U_j \) for up to \( M = 10 \), shows \( \rho \) is near to unity. Subsequently using (4.50), \( M_0 \) is 2. Therefore, by considering one scatterer on each confocal ellipse, the deterministic phase increment model is faster than the random phase increment model, and with more than 2 scatterers the random phase increment is faster (more efficient) than the deterministic phase increment model. With 2 scatterers the performance of two models is the same. This is shown in Fig. 4.21. Referring to this figure below the horizontal line \( t = 1 \) Model II is faster and above this line Model I is more efficient. With 5 scatterers of model I, the exact value of \( t^I \) is 1.67, which means random phase increment model is 1.67 times faster (more efficient) than deterministic phase model [12]-[13].

Based on these results and as we will see in chapter 8, in simulation of phases in indoor radio propagation channel it is recommended to invoke the random phase increment model since it is computationwise more efficient.
Figure 4.21 Performance comparison of Model II with respect to Model I, versus different number of scatterers. For $M=2$, both methods have the same performance.

4.6 ALGORITHMS FOR THE TWO MODELS

4.6.1 Model I

In the deterministic phase increment model first initial parameters such as antenna separation, sampling distance, number of scatterers, wavelength, etc. are entered the program. As explained in section IV, in Model I for the first position of a portable, which is assumed to be on right focal center of confocal ellipses, maximum number of $N$ confocal ellipses are generated. Generation of ellipses depends on the existence of a path component at that excess delay. Referring to the equation of ellipses (4.19), with the transmitter and receiver located at $(-\sqrt{p_n^2-q_n^2},0)$ and $(\sqrt{p_n^2-q_n^2},0)$, respectively, the distance between two consecutive ellipses (assuming path components exist in $n$th and $(n+1)$st bins) is $c\tau_0/2$, ($\tau_0$ is binwidth and $c$ speed of light). Therefore,

$$p_{n+1} = p_n + \frac{c\tau_0}{2}$$  \hspace{1cm} (4.51)

Due to confocality of ellipses we have

$$q_{n+1} = \sqrt{q_n^2 + p_n c\tau_0 + \frac{1}{4}c^2\tau_0^2}$$  \hspace{1cm} (4.52)

We assume that with maximum displacement of the portable, which is $KD$, it does not exit the
first ellipse. Therefore, by supposing antenna separation $2r$, $p_1$ of the first ellipse will be

$$p_1 = r + KD$$  \hspace{1cm} (4.53)$$

and $q_1$ is

$$q_1 = \sqrt{KD(KD + 2r)}$$  \hspace{1cm} (4.54)$$

After generating of the ellipses, a number of $M$ identical independent reflectors are uniformly distributed over the perimeter of each existing ellipse. Meanwhile, corresponding to each scatterer a $\text{Unif}[0,1]$ random attenuation coefficient is generated. It should be emphasized that scatterers on the ellipse at excess delay $i\tau_0$, (i.e., $i$th ellipse), and scatterers on $j$th ellipse ( $j \neq i$ ), are independently generated. This is because path components at different excess delays are due to independent obstacles. Meanwhile, as mentioned earlier, regarding to each bin which has a component, a $\text{Unif}[0,2\pi]$ initial phase is considered.

Now for the next profile of the portable and each fixed excess delay, first it is decided on existence of a path component. If a component exists on this and previous profile, the phase of that component is updated based on (4.24) and (4.29) or (4.25) and (4.26), assuming the previous scatterers around the ellipse. This process of updating terminates when a path at the chain of path components at that fixed excess delay no longer exists. The process restarts with generating of new random independent scatterers on the same ellipse at that excess delay upon existence of a path component. If the path component exists on this profile but not on previous profile, $M$ new scatterers are uniformly distributed over the perimeter of the ellipse at that specified excess delay. Meanwhile, the initial phase of that component is uniformly selected according to $\text{Unif}[0,2\pi]$. This process is repeated for all other excess delays. The algorithm terminates after performing the mentioned process over all profiles (positions) which portable has paved. In Fig. 4.22 a graphical description of this algorithm is depicted.

4.6.2 Model II

In the random phase increment algorithm first the initial parameters are entered. In the first position of the portable, for each bin with a path component a $\text{Unif}[0,2\pi]$ initial phase is generated. In the next profile for each bin with a path component, if the bin with same delay in previous profile has the path component, a zero mean Gaussian phase increment with standard
\[ \begin{align*}
    k=1 & \quad x & x & x & 0 & x & \ldots & x \\
    k=1 & \quad G_i & G_i & G_i & G_i & G_i & \ldots & G_i \\
    k=1 & \quad 1 & 1 & 1 & 1 & 1 & \ldots & 1 \\
    k=2 & \quad x & 0 & x & x & x & \ldots & 0 \\
    k=2 & \quad U_i & U_i & G_i & U_i & U_i & \ldots & 1 \\
    k=2 & \quad 1 & 1 & 1 & 1 & 1 & \ldots & 1 \\
    k=3 & \quad 0 & x & x & x & 0 & \ldots & x \\
    k=3 & \quad G_i & U_i & U_i & U_i & U_i & \ldots & G_i \\
    k=3 & \quad 1 & 1 & 1 & 1 & 1 & \ldots & 1 \\
    k=4 & \quad x & x & x & 0 & 0 & \ldots & x \\
    k=4 & \quad G_i & U_i & U_i & U_i & U_i & \ldots & U_i \\
\end{align*} \]

\( k \) : profile number
\( x \) : indicator of presence of a path component
\( 0 \) : indicator of absence of a path component
\( G_i \) : \( i = 1 \) generating of scatterers around ellipses, Model I, \( i = 2 \) generating of new initial phases, Model II
\( U_i \) : updating of phase, \( i = 1 \) Model I, \( i = 2 \) Model II

Figure 4.22 Description of algorithms of the two phase models.

deviation denoted by (4.39), is added to the previous phase, (see (4.35)). If the bin in the current profile has a path component, but not the bin with the same excess delay in previous profile, a new \( \text{Unif}[0,2\pi] \) phase is generated for that bin in the current profile. In fact in each excess delay the above process is interrupted when a path component ceases to exist. A new \( \text{Unif}[0,2\pi] \) distributed phase is generated if a path with the same delay appears at later profile, and random updating starts. This process is repeated for all \( N \) excess delays and all \( K \) positions of portable. A graphical description of this algorithm is depicted in Fig. 4.22.
4.7 CONCLUSIONS

In this chapter two novel phase models for indoor radio propagation channel were studied in great detail. In Model I, the phase of each path is updated deterministically using several random independent scatterers for each multipath component. In Model II the phase of each path is updated with random independent Gaussian increments whose standard deviations change with distance. Performance of these models were evaluated by means of extensive computer simulations and based on large data base of 12000 profiles of impulse response of indoor radio propagation channel gathered at two dissimilar office buildings. First and second order statistics of narrowband CW fading waveforms were obtained using simulated phases and compared to those of empirical data. It was shown that Model I with 5 independent scatterers for each path as well as Model II provides results consistent with measured data. Multiple reflectors are considered explicitly in Model I, while in Model II it is implicitly considered. Moreover, these two phase models were compared with each other. Statistical properties of phase increments in Model I particularly when number of scatterers increases were studied, simulated and compared with Model II. It was shown that with 5 scatterers, the phase increments of Model I have a good fit to normal distribution of phase increments of Model II. Standard deviation of theses phase increments has a good agreement with average standard deviation of phase increments of Model II. Comparison of these models was carried out from computational complexity. It was shown that with this number of scatterers, Model II is 1.7 times faster (more efficient) than Model I. Therefore, it is recommended to employ Model II in simulation of phases in indoor radio propagation channels. The more the number of scatterers of Model I, the more efficient the Model II. Results of this research can be used in design, simulation and performance evaluation of indoor wireless communication systems.
REFERENCES OF CHAPTER 4


CHAPTER 5

DIRECT SEQUENCE SPREAD SPECTRUM
IN INDOOR RADIO COMMUNICATION
SYSTEMS

With an annual growth rate of 40% per year, wireless (mobile) communication is the fastest growing sector of the communication industry. Worldwide research and development in wireless communication is growing rapidly. With this high rate of growth, it becomes necessary to use spectrum more effectively. Spread spectrum is a technique for efficiently using spectrum by allowing additional users to use the same band as other users. Due to resist of spread spectrum to jamming as well as providing privacy and low probability of intercept, its initial applications have been in military communications; however, it is now on the commercial developments, and recently there has been a great interest in understanding both the capabilities and limitations of spread spectrum for commercial applications. Indeed, new opportunities such as Personal
Communication Services (PCS) and digital cellular radios have created the need for research on how spread spectrum techniques can be employed for efficient use in mobile and indoor environments as a code division multiple access (CDMA) scheme [1]. For instance direct sequence code division multiple access (DS-CDMA) has been proposed for enhancing the capacity of North American Cellular Mobile Telephone System [2] and is under consideration in Europe as well as Japan [1].

Spread spectrum is a means of transmission in which the signal occupies a bandwidth in excess of the minimum necessary to send information. The spreading of bandwidth is performed by means of a code which is independent of data, and a synchronized reception with the code at the receiver is used for recovery of the data.

There are several methods to spread the spectrum, i.e., "direct sequence" in which a fast pseudo-randomly generated sequence directly causes phase transitions in the carrier containing data, "frequency hopping" in which the carrier is caused to shift frequency in a pseudo-random manner, and "time hopping" in which burst of signal are initiated at pseudo-random times. Also hybrid combinations of these methods are frequently used [1]. As will be explained in section 5.2 we restrict ourselves to direct sequence spread spectrum technique.

A large body of literature exists on the analysis of spread spectrum multiple access (SSMA) technique, which is also called code division multiple access (CDMA). Much of this literature, such as initial work of Cooper and Nettleton [3], considers frequency hopping CDMA. Here we mention some of the existing literature on the analysis of direct sequence code division multiple access system. A tutorial theory of spread spectrum communications appeared in [4]. In [5], Turin examined the structure of a number of spread spectrum antimultipath receivers for mobile digital radio, where he limited himself to the behavior of the single transmitter-receiver pair. The behavior of groups of transmitter-receiver pairs working simultaneously in the same band (i.e., multiple access) by an asynchronous direct sequence CDMA system using DPSK is discussed in [6]. As we will see in section 5.2 many analyses of DS-CDMA have resulted in Gaussian approximations, meanwhile various upper and lower bounds on the bit error probability for additive white Gaussian noise (AWGN) channels are reported in [7]-[10]. Geraniotis and Pursley in [11] and [12] have approximated error probability for single user direct sequence spread spectrum systems that use coherent and noncoherent detection, respectively. This analysis has been extended to DS-CDMA for a multipath rejection receiver [13]. Capacity of code division multiple access system is discussed by Gilhousen et. al. [14], Xiang [15] and Turin [6]. The
characteristics of spread spectrum that make it advantageous for mobile communications as well as the spread spectrum overlay, in which a code division multiple access shares a frequency band with narrow band users, is analyzed in [16]. Rappaport and Milstein [17] have examined the effect of user distribution on the performance of a direct sequence CDMA cellular system. In [18] the performance of direct sequence cellular CDMA in the presence of path-loss, shadowing, fading, multiple access interference and noise is determined. The analysis and simulation of the performance and the capacity of cellular CDMA mobile radio system in reverse and forward links are reported in [19] and [20], respectively. In [21] and [22] the performance of direct sequence spread spectrum scheme over the indoor multipath fading channel is reported. Performance of DS-SS technique over Rician fading channel is evaluated in [23] and simulated in [24]. Bit error performance of direct sequence spread spectrum land mobile satellite system with shadowed Rician fading is analyzed in [25]. Capacity and spectrum efficiency of an indoor wireless DS-CDMA system operating in the 2.4 GHz band using measured delay profiles are reported in [26].

In this chapter we investigate the direct sequence spread spectrum technique in multipath indoor wireless communication systems. In this regards first in section 5.1 the main capabilities of direct sequence spread spectrum method are studied in detail, and then in section 5.2, the DS-CDMA technique in indoor radio systems is discussed and the system model, the transmission channel, the receiver and the methods of assessment of the performance of the system are explained. In section 5.3 the performance of DS-CDMA in terms of average probability of error versus signal-to-noise ratio is evaluated over the measured large data base of indoor radio propagation channels, which were described in chapter 2. Conclusions are given in section 5.4.

5.1 DIRECT SEQUENCE SPREAD SPECTRUM CAPABILITIES

In a direct sequence spread spectrum multiple access communication system several signals simultaneously occupy the same channel. Each of the signals employs a sequence which is selected to have certain desirable correlation properties. For multiple access communications the primary goal is to be able to separate the spread signals at the receiver even though they occupy the same bandwidth at the same time. This problem is considered in this section.

For the purpose of illustrating the basic concepts of DS-CDMA, first we consider a system in which the spreading of spectrum is performed by a random process, described in [27]. This gives us the insight of basic concepts of spread spectrum communication systems. Suppose \( v(t) \) is a zero mean Gaussian random process with the autocorrelation function shown in Fig. 5.1.
Figure 5.1  Autocorrelation of the process \( v(t) \).

\[ R_v(\tau) = e^{-|\tau|/4T_o} \Pi\left(\frac{\tau}{2T_o}\right) \]  \hspace{1cm} (5.1)

where \( \alpha \) is a constant and \( \Pi(\tau/2T_o) \) is a rectangular waveform with amplitude 1 and duration of \( 2T_o \). Now let \( s(t) \) be a binary data signal which consists of a series of rectangular pulses, each with duration \( T \) and amplitude \( \pm A \), (see Fig. 5.2). For, \( t \) in the interval of \([0, T]\), \( s(t) \) may be \( A p(t) \) or \(-A p(t)\), where \( p(t) \) is a unit amplitude rectangular pulse with the duration of \( T \). It is assumed that \( T > T_o \) (i.e., the bandwidth of \( v(t) \) is much greater than the bandwidth of data signal). First let us consider a noiseless baseband channel. More complicated models will be considered later.

The spread signal \( s(t)v(t) \) is transmitted through the channel and in the receiver it is decided whether \( s(t) = A \) or \( s(t) = -A \) is transmitted. The baseband receiver is a correlation receiver, i.e.,

\[ Z = \int_{0}^{T} s(t)v(t)dt = \int_{0}^{T} s(t)v^2(t)dt \]  \hspace{1cm} (5.2)

If \( s(t) \) is \( \pm A \) in the interval \([0,T]\), then

Figure 5.2  Binary data signal.
\[ Z = (-1)^m A \int_0^T v^2(t) \, dt \quad m=0,1 \] (5.3)

If we consider the process \( v(t) \) be ergodic, i.e., its time and ensemble averages are the same, we have [27]

\[ \frac{1}{T} \int_0^T v^2(t) \, dt = E[v^2(t)] = R_v(0) = 1 \] (5.4)

where \( E[.] \) is the expected value. Therefore (5.3) becomes

\[ Z = (-1)^m A T \quad m=0,1 \] (5.5)

So, in the absence of noise the transmitted information is extracted from \( s(t)v(t) \). As it is seen from (5.5) the value of \( Z \) is \( +AT \) or \( -AT \). Therefore, the decision is made upon the sign of \( Z \) in the receiver. If \( Z > 0 \) then the receiver decides positive pulse was sent and if \( Z \leq 0 \) it is decided the negative pulse was sent. Note, the receiver must know \( v(t) \) exactly, and this is the reason why this model is not of interest for practical situation. In practice \( v(t) \) is usually a pseudorandom waveform derived from a pseudorandom sequence. In obtaining (5.5) no interfering signals from other transmitters in the multiple access system, or due to multipath were considered. The effect of such interferences are considered in section 5.2. Now we study the major capabilities of the above spread spectrum system.

### 5.1.1 Multiple Access Capability

Suppose we have \( K \) transmitters. The \( k \)th transmitter has the process of \( v_k(t) \). Again it is assumed that \( v_k(t) \) is a zero mean stationary Gaussian process with autocorrelation function \( R_v(\tau) \), of (5.1), and the processes are assumed to be mutually independent. Therefore the received signal will be

\[ r(t) = \sum_{k=1}^K s_k(t)v_k(t) \] (5.6)

The output of \( i \)th correlator is
\[ Z_i^T = \int_0^T r(t) v_i(t) dt = \int_0^T \left( \sum_{k=1}^K s_k(t) v_k(t) \right) v_i(t) dt \]  

which can be written as

\[ Z_i = \sum_{k=1}^K \int_0^T s_k(t) v_k(t)v_i(t) dt \]  

or

\[ Z_i^T = \int_0^T s_i(t) v_i^2(t) dt + \sum_{k=1, k \neq i}^K \int_0^T s_k(t) v_k(t)v_i(t) dt \]  

The first term in (5.9) is \((-1)^m A T\) (refer to (5.4) and (5.5)). In the interval \(0 < t < T\) the amplitude of \(s_k(t)\) is \(+A\) or \(-A\). We show its amplitude by \(\gamma A\), where \(\gamma\) is +1 or -1. Now (5.9) can be written as [27]

\[ Z_i = (-1)^m A T + A \sum_{k=1, k \neq i}^K \gamma \int_0^T v_k(t)v_i(t) dt \]  

It was assumed that the processes are uncorrelated, therefore, for \(i \neq k\)

\[ \int_0^T v_i(t)v_k(t) dt = 0 \]  

so

\[ Z_i = (-1)^m A T \]  

Which is the same as (5.5). The receiver makes decision on the sign of \(Z_i\). Note, as long as the processes are completely uncorrelated, and there is no noise in the channel, the other signals in the system do not interfere with the transmission of \(i\)th signal. This shows the multiple access capability of the spread spectrum technique.

### 5.1.2 Anti-multipath Capability

In the case of multipath, the received signal at the output of the channel will be as follows
\[ r(t) = \sum_{i=0}^{N-1} a_i s(t-\tau_i) e^{j\theta_i} v(t-\tau_i) \]  
(5.13)

\( a_i, \tau_i \) and \( \theta_i \) are random amplitudes, time delays and phases, respectively, and \( N \) is the number of multipath components. The decision variable is

\[ Z = \int_0^T r(t)^* v(t) \, dt = \int_0^T \sum_{i=0}^{N-1} a_i s(t-\tau_i) e^{j\theta_i} v(t-\tau_i) v(t) \, dt \]
(5.14)

which can be written as

\[ Z = \sum_{i=0}^{N-1} a_i \int_0^T s(t-\tau_i) e^{j\theta_i} v(t-\tau_i) v(t) \, dt \]
(5.15)

Let us assume \( a_0 = 1, \tau_0 \) and \( \theta_0 \) are zero. We then find

\[ Z = \int_0^T s(t)^2 v(t) \, dt + \sum_{i=1}^{N-1} \int_0^T s(t-\tau_i) v(t-\tau_i) e^{j\theta_i} \, dt \]
(5.16)

The first term in (5.16) is \((-1)^m T\), and second term contains the autocorrelation of the process \( v(t) \), i.e.,

\[ Z = (-1)^m A T + \sum_{i=1}^{N-1} a_i (-1)^m A R_v(\tau_i) e^{j\theta_i} \quad m=0,1 \]
(5.17)

If we assume \( \tau_i \geq T_\theta \), referring to (5.1), \( R_v(\tau_i) \) would be zero. Therefore (5.17) reduces to (5.5). Again the multipath signal does not interfere with the output of the correlation receiver as long as \( \tau_i > T_\theta \).

5.1.3 Anti-Interference Capability

Spread spectrum technique has the capability to combat interference. Suppose the received signal contains a dc, which is added to the desired signal

\[ r(t) = s(t) v(t) + a_0 \]
(5.18)

where \( a_0 \) is a dc signal. At the output of the correlation receiver we have
\[ Z = \int_0^T s(t)v^2(t)\,dt + \alpha_0 \int_0^T v(t)\,dt \]  \hspace{1cm} (5.19)

or this can be written as

\[ Z = (-1)^m A_T + \alpha_0 T \left[ \frac{1}{T} \int_0^T v(t)\,dt \right] \]  \hspace{1cm} (5.20)

The term in the bracket of (5.20) is mean of \( v(t) \), which is assumed be zero \( (T > T_0) \). So (5.20) reduces to (5.5). If the interference signal is in the form of an ac signal we write

\[ r(t) = s(t)v(t) + \alpha_0 \cos \omega_0 t \]  \hspace{1cm} (5.21)

Now the signal at the output of the receiver is

\[ Z = (-1)^m A_T + \alpha_0 T \left[ \frac{1}{T} \int_0^T v(t)\cos \omega_0 t\,dt \right] \]  \hspace{1cm} (5.22)

With a similar argument, the term in the bracket of (5.22) is zero, so it reduces to (5.5). In all cases discussed above, the interfering signal has a negligible correlation with \( v(t) \), therefore, produces no change at the output of correlation receiver. Meanwhile, we note that the actual shape of the autocorrelation function of (5.1) is not important. All that is important is the autocorrelation to be zero for \( \tau \geq T_0 \). The integrator smooths the interfering signals but produces a large peak in the response to the desired signal since it is matched to the rectangular data pulse[27].

5.2 DS-CDMA TECHNIQUE

5.2.1 System Model

In the binary direct-sequence code division multiple access (DS-CDMA) technique a baseband signal of the form

\[ c_k(t) = \sum_{i=-\infty}^{\infty} c_{k,i} P_{T_c}(t - iT_c) \]  \hspace{1cm} (5.23)

is used as a spreading signal. The sequence \( c_{k,i} \) is a periodic sequence of elements \( \{+1,-1\} \) and
$P_{T_c}(t)$ is a time limited signal, limited to $[0,T_c]$, with unit energy; i.e.,

$$\frac{1}{T_c} \int_0^{T_c} P_{T_c}^2(t) \, dt = 1$$  \hspace{1cm} (5.24)

A good example of $P_{T_c}(t)$ is a rectangular pulse of amplitude 1 and duration $T_c$

$$P_{T_c}(t) = \Pi \left( \frac{t}{T_c} \right)$$ \hspace{1cm} (5.25)

Usually $P_{T_c}(t)$ is called chip waveform and $T_c$ is called chip duration. Binary data is explained as

$$b_k(t) = \sum_{l=-\infty}^{\infty} b_{k,l} P_T(t-lT)$$ \hspace{1cm} (5.26)

where $b_{k,l} \in \{-1,1\}$ is the binary data of the $k$th user at $l$th interval $T$. The $P_T(t)$ has the shape like $P_{T_c}(t)$, rectangular with amplitude 1, but it has a duration of $T > T_c$. The ratio of $T/T_c = M$ is called processing gain and is usually expressed in dB. The base-band spread spectrum signal is

$$y_k(t) = c_k(t) \cdot b_k(t)$$ \hspace{1cm} (5.27)

$c_{ki}$ in (5.23) is a periodic sequence with the period of $M$, i.e., $c_{ki+lM} = c_{ki}$. In the frequency domain the spectrum of the spreading signal is convolved with the spectrum of the data. Considering $\text{sinc}$ squared shapes for power spectra of spreading signal and data, the spread spectrum bandwidth, $B_{ss}$, is given by

$$B_{ss} = \frac{1}{T_c} + \frac{1}{T} = \frac{1}{T_c} = \frac{M}{T}$$ \hspace{1cm} (5.28)

Therefore, the bandwidth of the spread signal is $M$ times of the bandwidth of the data.

The transmitted signal of the $k$th user in binary DS-CDMA is

$$s_k(t) = A c_k(t) b_k(t) \cos(\omega_c t + \varphi_k)$$ \hspace{1cm} (5.29)

where $\omega_c$ is the carrier frequency, $A^2/2$ is the transmitted power and $\varphi_k$ is the phase of $k$th user. In general the phase sequences $\varphi_k$ are not the same because in practice the transmitters are not synchronous. The signal of $k$th user is transmitted through the channel which is described in the following.
5.2.2 Transmission Channel and the Receiver

As the first case let us consider propagation delays for various signals only. Therefore, the received signal \( r(t) \), output of the channel, will be

\[
r(t) = \sum_{k=1}^{K} s_k(t-\tau_k) + n(t)
\]

(5.30)

where \( \tau_k \) is the time delay associated with \( k \)th signal and \( n(t) \) is the additive white Gaussian noise (AWGN) with two sided spectral height of \( N_0/2 \). Using (5.29) and (5.30), we have

\[
r(t) = \sum_{k=1}^{K} A c_k(t-\tau_k) b(t-\tau_k) \cos(\omega_c t - \omega_k \tau_k + \phi_k) + n(t)
\]

(5.31)

If we assume \( \phi_k = \omega_k \tau_k \), (5.31) becomes

\[
r(t) = \sum_{k=1}^{K} A c_k(t-\tau_k) b_k(t-\tau_k) \cos(\omega_c t + \phi_k) + n(t)
\]

(5.32)

For the receiver matched to \( i \)th signal we assume without loss of generality that \( \phi_i = 0 \) and \( \tau_i = 0 \), meanwhile we assume \( 0 \leq \tau_k < T \) and \( 0 \leq \phi_k < 2\pi \), \( k = 1, 2, ..., K \). The output of the \( i \)th correlator receiver is

\[
Z_i = \int_{0}^{T} r(t) c_i(t) \cos \omega_c t \, dt
\]

(5.33)

or

\[
Z_i = \int_{0}^{T} \sum_{k=1}^{K} A c_k(t-\tau_k) b_k(t-\tau_k) \cos(\omega_c t + \phi_k) c_i(t) \cos \omega_c t \, dt + \int_{0}^{T} n(t) c_i(t) \cos \omega_c t \, dt
\]

(5.34)

The second term of (5.34) is due to noise, and is written as

\[
n_i = \int_{0}^{T} n(t) c_i(t) \cos \omega_c t \, dt
\]

(5.35)

If the carrier frequency \( \omega_c \) is much greater than the reciprocal of chip duration \( T_c \), which is
usually the case, we can filter out the double carrier frequency terms of (5.34), so

$$Z_i = \int_{0}^{T} A c_i^2(t) b_i(t) \cos^2 \omega_c t \, dt + \sum_{k=1, k \neq i}^{K} \int_{0}^{T} A c_k(t-\tau_k) b_k(t-\tau_k) c_i(t) \cos \omega_c t \cos (\omega_c t + \phi_k) \, dt + n_i$$  \hspace{1cm} (5.36)$$

By substituting of \(\cos^2 \omega_c t\) with \((1+\cos2\omega_c t)/2\) in the first term of (5.36) and introducing [27]

$$f_{k,j}(\tau_k) = \int_{0}^{\tau_k} b_k(t-\tau_k) c_k(t-\tau_k) c_j(t) \, dt$$  \hspace{1cm} (5.37)$$

and

$$f_{k,j}^I(\tau_k) = \int_{\tau_k}^{T} b_k(t-\tau_k) c_k(t-\tau_k) c_j(t) \, dt$$  \hspace{1cm} (5.38)$$

(5.36) becomes

$$Z_i = \frac{1}{2} A \int_{0}^{T} b_i(t) \, dt + \sum_{k=1, k \neq i}^{K} \frac{A}{2} [f_{k,j}(\tau_k) + f_{k,j}^I(\tau_k)] \cos \phi_k + n_i$$  \hspace{1cm} (5.39)$$

Since \(b_k(t)\) is a signal of the form given by (5.26) then \(b_k(t-\tau_k) = b_{k,-1}\) for \(0 \leq \tau_k < T\) and for \(\tau_k \leq t < T\). Therefore, for \(0 \leq \tau_k < T\) eq. (5.37) implies that

$$f_{k,i}(\tau_k) = b_{k,-1} \int_{0}^{\tau_k} c_k(t-\tau_k) c_i(t) \, dt = b_{k,-1} R_{k,i}(\tau_k)$$  \hspace{1cm} (5.40)$$

where

$$R_{k,i}(\tau_k) = \int_{0}^{\tau_k} c_k(t-\tau_k) c_i(t) \, dt$$  \hspace{1cm} (5.41)$$

and similarly

$$f_{k,i}^I(\tau_k) = b_{k,0} \int_{\tau_k}^{T} c_k(t-\tau_k) c_i(t) \, dt = b_{k,0} R_{k,i}^I(\tau_k)$$  \hspace{1cm} (5.42)$$

where
\[ R'_{k,i}(\tau_k) = \int_{\tau_k}^{T} c_k(t-\tau_k) c_i(t) \, dt \quad 0 \leq \tau_k < T \quad (5.43) \]

Substituting (5.41) and (5.43) in (5.39) results into

\[ Z_i = \frac{AT}{2} b_{i,0} + \sum_{k=1, k \neq i}^{K} \frac{A}{2} [b_{k,-1} R_{k,i}(\tau_k) + b_{k,0} R'_{k,i}(\tau_k)] \cos \phi_k + n_i \quad (5.44) \]

\( R_{k,i}(\cdot) \) and \( R'_{k,i}(\cdot) \) in (5.41) and (5.43) are called continuous time partial cross correlations. They were introduced by Pursley [28] and used in the analysis of binary direct sequence CDMA in [27]. The second term in (5.44) is the multiple access interference at the output of \( i \)-th receiver due to \( k \)-th \((k \neq i)\) signal. We show this interference as

\[ I_i = \frac{1}{T} \frac{A}{2} \sum_{k=1, k \neq i}^{K} [b_{k,-1} R_{k,i}(\tau_k) + b_{k,0} R'_{k,i}(\tau_k)] \cos \phi_k \quad (5.45) \]

Now (5.44) is written as

\[ Z_i = \frac{1}{2} AT [b_{i,0} + I_i] + n_i \quad (5.46) \]

It is clear that the decision variable contains two undesired components, i.e., multiuser interference and noise. Meanwhile, note that if \( K = 1 \), we have no multiple access system and the second term in (5.46) vanishes.

Now we consider the multipath fading channel of indoor radio communication systems. The block diagram of DS-CDMA in indoor system is shown in Fig. 5.3. According to this figure \( K \) independent portables (users) transmit their data over different channels. The equivalent complex impulse response of each channel was described in chapter 1. The signal of each user (5.29) is convolved with the impulse response of its multipath fading channel described by (4.3). (also see chapter 1). Therefore, the received signal \( r(t) \) is

\[ r(t) = \sum_{k=1}^{K} \sum_{n=0}^{N_t-1} A_{k,n} b_k(t-\tau_{k,n}) c_k(t-\tau_{k,n}) \cdot \cos(\omega t-\omega_{k,n}\tau_{k,n} + \varphi_{k,n} + \theta_{k,n}) \cdot n(t) \quad (5.47) \]

where \( a_{k,n}, \tau_{k,n}, \theta_{k,n} \), are the random amplitude, time of arrival and phase sequences, respectively,
Figure 5.3  Block diagram of DS-CDMA in indoor.

and $N_k$ is the number of path components of the $k$th impulse response of the indoor propagation channels. $n(t)$ is additive white Gaussian noise with double side spectral height of $N_0/2$. The output of the matched filter matched to the $i$th signal is

$$Z_i(t) = \int_0^T r(t) c_i(t) \cos(\omega_c t) \, dt$$  \hspace{1cm} (5.48)$$

Denoting the matched filter's output due to $m$th multipath at time $t=T+\tau_{im}$ as $Z_i^{(m)}$

$$Z_i^{(m)} = \frac{AT}{2} [a_{i,m} b_{i,0} + I_i] + n_i$$  \hspace{1cm} (5.49)$$

where $b_{i,0}$ is the current information being transmitted from $i$th user and $b_{i,-1}$ is its preceding bit. $I_i$ is the multipath/mutual multiple access interference, and entails two types of interferences. These are the self multipath interference ($I_{SMP}$) and the mutual multiple access multipath interference ($I_{MAMP}$), i.e.,

$$I_i = I_{SMP} + I_{MAMP}$$  \hspace{1cm} (5.50)$$

where
\[ I_{SMP} = \frac{1}{T} \sum_{n=1}^{N_t-1} a_{i,n} \cos(\phi_{i,n} + \theta_{i,n}) \cdot \left[ b_{l,-1} R_{i,l}(\tau_{i,n}) + b_{l,0} R'_{i,l}(\tau_{i,n}) \right] \]  \hspace{1cm} (5.51)

and

\[ I_{MAQP} = \frac{1}{T} \sum_{k=1}^{K} \sum_{n=0}^{N_t-1} a_{k,n} \cos(\varphi_{k,n} - \phi_{k,n} - \theta_{k,n}) \cdot \left[ b_{k,-1} R_{k,l}(\tau_{k,n}) + b_{k,0} R'_{k,l}(\tau_{k,n}) \right] \]  \hspace{1cm} (5.52)

Just like (5.41) and (5.43), \( R_{k,l} \) and \( R'_{k,l} \) are partial cross correlations of spreading code sequences and in this case they are [27]-[28]

\[ R_{k,l}(\tau) = \int_{\tau_{i,m}}^{\tau + \tau_{i,m}} c_k(t - \tau) c_l(t) dt \quad \tau_{i,m} \leq \tau \leq T + \tau_{i,m} \]  \hspace{1cm} (5.53)

and

\[ R'_{k,l}(\tau) = \int_{\tau + \tau_{i,m}}^{\tau + \tau_{i,m}} c_k(t - \tau) c_l(t) dt \quad \tau_{i,m} \leq \tau \leq T + \tau_{i,m} \]  \hspace{1cm} (5.54)

In (5.49) \( n_i \) is the term due to noise and is a zero mean Gaussian random variable

\[ n_i = \int_{\tau_{i,m}}^{T + \tau_{i,m}} n(t) c_i(t) \cos(\varphi_k t) dt \]  \hspace{1cm} (5.55)

Its variance is calculated as

\[ E[n_i n_k] = E\left[ \int_{0}^{T} \int_{0}^{T} n(t_1)n(t_2)c_i(t_1)c_k(t_2)\cos \varphi_k t_1 \cos \varphi_l t_2 dt_1 dt_2 \right] \]  \hspace{1cm} (5.56)

since \( n(t) \) is AWGN we have

\[ E[n_i n_k] = \int_{0}^{T} \int_{0}^{T} \frac{N_0}{2} \delta(t_1-t_2) E[c_i(t_1)c_k(t_1)] \cos \varphi_k t_1 \cos \varphi_l t_2 dt_1 dt_2 \]  \hspace{1cm} (5.57)

Therefore
\[ \text{Var}(n_i) = E[n_i^2] = \frac{N_0 T}{4} \]  

(5.58)

Till now, we discussed DS-CDMA technique in the multipath indoor fading channel. We specified the multiuser and noise disturbances on the system. Before studying the performance of this technique we are in the position to mention the major points regarding to direct sequence spread spectrum technique. In spite of the capability of DS-CDMA to combat multiple access interference and multipath, it suffers from the requirement of synchronization. Indeed no communication in DS-CDMA is possible until acquisition of synchronization has been achieved. The second issue is the near-far effect. Since users are geographically separated, a receiver, trying to detect the \( i \)th signal, might be much closer to \( k \)th transmitter rather than the \( i \)th transmitter. Therefore, if each user transmits with equal power the signal from the \( k \)th transmitter will arrive at the receiver in question with a larger power than the received power from the \( i \)th transmitter. This particular problem is often severe and some sort of power control schemes must be applied to mitigate its effect. Meanwhile it should be mentioned that only DS-CDMA, (not frequency hopping) suffers from the near-far effect.

### 5.2.3 Error Probability

Referring to Fig. 5.3 the sampled signal of (5.49) is compared with the threshold of zero, and the decision is made upon the sign of \( Z_i^{(m)} \). If it is positive a bit of "1" is presented and a bit of "0" otherwise. The average probability of error is evaluated by assuming \( b_{\epsilon 0} = 1 \) and computing \( \text{Prob}[Z_i^{(m)} < 0] \). In order to simplify the notation we neglect the subscript \( i \) in (5.49). Now the probability of error is

\[ P(E) = \text{Prob} \left[ \frac{A T}{2} a_m + \frac{A T}{2} I + n < 0 \right] \]  

(5.59)

which means

\[ P(E) = \int_0^\infty \text{Prob} \left[ \frac{A T}{2} a_m + \frac{A T}{2} I + n < 0 \mid a_m = a \right] f_A(a) \, da \]  

(5.60)

where \( f_A(a) \) is the pdf of amplitude \( a \). Using the variance of the Gaussian noise term expressed in (5.58), equation (5.60) is written as
$$P(E) = \int_0^\infty Q\left[\sqrt{\frac{A^2 T}{N_0}} (a + I)\right] f_A(a) \, da$$  \hspace{1cm} (5.61)$$

where $Q(x)$ is given by \( \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp\left(-\frac{y^2}{2}\right) \, dy \). Now (5.61) can be calculated as

$$P(E) = \int_0^\infty \int_0^\infty Q\left[\sqrt{\frac{A^2 T}{N_0}} (a + I | I = \xi)\right] f_A(a) f_G(\xi) \, da \, d\xi$$  \hspace{1cm} (5.62)$$

where $f_G(\xi)$ is the pdf of the interference and contains self multipath and mutual multiple access interferences. Referring to (5.51) and (5.52) the interference is the sum of $N_k - 1 + N_K(K-1)$ terms of disturbances. Considering independent components of disturbances, the pdf of $\xi$ is the convolution of pdf of $N_k K - 1$ random variables. One approximation for distribution of multiuser interference is to invoke the Gaussian assumption. In this regards using the distribution of path amplitudes and Gaussian (normal) distribution of $f_G(\xi)$ the probability of error of (5.62) is calculated. The accuracy of this approximation when the number of interfering components are large is good enough, however, some improved Gaussian approximations as a quick indicator of sufficiency of the standard approximation is reported in [29].

Another method of evaluating the probability of error of DS-CDMA is based on Gauss Quadrature Rule (GQR). According to this method the average probability of error is obtained numerically by evaluating the moments of the interference and weighting the quadrature rule [30], i.e., the probability of error of (5.62) is written as

$$P(E) = \int_0^\infty \sum_l W_l Q\left(\sqrt{\frac{A^2 T}{N_0}} (a + I_l)\right) f_A(a) \, da$$  \hspace{1cm} (5.63)$$

Where $W_l$ and $I_l$ are weights and nodes of the quadrature rule [31]. The accuracy of GQR increases with increasing of the number of terms. However, as the number of terms increases the complexity also increases. Consideration of twelve terms of approximation is reported to be sufficient for the evaluation of error probability [32].
5.3 PERFORMANCE EVALUATION OVER MEASURED CHANNELS

In this section we discuss the explained direct sequence spread spectrum multiple access technique over indoor communication channels, with measured responses described in chapter 2, and evaluate the performance of the system.

In applying DS-CDMA over indoor channels binary phase shift keying (BPSK) signaling with data rate of 390 kbps and transmission bandwidth of 200 MHz were assumed. Chip duration was selected 10 nsec. The discrete time impulse response of the channel was used (see section 2.4). Referring to multipath statistics of the data base in Table 2.2, the rms delay spread at each antenna separation is much smaller than the duration of data, this ensures that intersymbol interference (ISI) of data symbol is negligible, however, code sequences are subject to ISI. Using chip duration of 10 nsec, and 2.56 μsec data bit duration, the spreading factor is 256. Gold codes with the sequence length of 255 were used [33].

Referring to section 2.2, in each building and each antenna separation 1, 2, 5, 10 and 15 active users were considered. These users (portables) communicate with the base while traveling through the channel, paving 1.5 m. (Each location comprises 75 profiles which covers 75×2cm = 1.5 m). A star connected network for K portables with the speed of 25 cm/sec was assumed. In each case the average probability of bit error over the ensemble of measured profiles when the portables move through the channels was evaluated. A total number of 12000 profiles of impulse responses of channels was used. Because of different power levels, the results without power control were very poor, therefore, the average power control was considered by normalizing the output power of each profile in each antenna separation to unity [22]. In Figs. 5.4 and 5.5 the performance with the different number of active users over areas with antenna separation of 5 and 10 m of Building A, and in Fig. 5.6 for antenna separation of 10 m of Building B are shown. In Figs. 5.7 and 5.8 the performances averaged over all locations of all antenna separations of Building A and Building B are shown, respectively. Average performance over all locations of all antenna separations of the two buildings are shown in Fig. 5.9. These figures reveal the effect of increasing the number of active users on the degradation of performance of the system [34].
Figure 5.4  Average bit error probability versus signal to noise ratio for different number of active users. Averaged over locations with the antenna separation of 5 m of Building A.

Figure 5.5  Average bit error probability versus signal to noise ratio for different number of active users. Averaged over locations with the antenna separation of 10 m of Building A.
Figure 5.6  Average bit error probability versus signal to noise ratio for different number of active users. Averaged over locations with the antenna separation of 10 m of Building B.

Figure 5.7  Average bit error probability versus signal to noise ratio for different number of active users. Averaged over all locations and antenna separations of Building A.
Figure 5.8  Average bit error probability versus signal to noise ratio for different number of active users. Averaged over all locations and antenna separations of Building B.

Figure 5.9  Average bit error probability versus signal to noise ratio for different number of active users. Averaged over all locations and antenna separations of both buildings.
The effect of changing the sequence length of spreading codes is shown in Figs. 5.10 and 5.11. In these figures the performance for antenna separation of 5 m with 10 active users of Building A, and for antenna separation of 10 m of Building B with five active portables and with 127 and 255 chips/code are shown, respectively. The longer the code length the more the processing gain and the better the performance [35].

![Graph showing the effect of sequence length on BER](image)

**Figure 5.10**  Effect of changing the sequence length of the spreading codes (127 and 255). Building A, antenna separation 5m; 10 active portables.

Performance of DS-CDMA indoor radio system with BPSK modulation over lognormal fading channel is analyzed in [36], where similar results of the degradation of bit error performance, caused by increasing the number of users, are reported. This results are based on measured indoor radio channel data of [37] at 910 MHz, and using 472 impulse response profiles.
Figure 5.11 Effect of changing the sequence length of the spreading codes (127 and 255). Building B, antenna separation 10m; 5 active portables.

5.4 CONCLUSIONS

DS-SSMA is attractive to combat fading of the indoor channel, as well as providing other users' access. In this chapter these capabilities of DS-SSMA technique were explained. Performance of DS-CDMA in terms of average probability of bit error and with BPSK modulation was assessed when portables move through the channels, using the large data base of indoor propagation channels (chapter 2) with lognormal multipath amplitude fading. At each point of the portable the indoor channel was modeled by its impulse response as the linear time invariant filter. The matched filter receiver was considered. Gold codes were used for spreading the signals. Average power control was assumed by normalizing the output power of each channel to unity. The effect of multiple active users as well as changing the code length was investigated. Indoor channel varies drastically from one location to another. It necessitates the power control. Multiple active users cause degradation in performance. Increasing the processing gain (or code length of spreading sequence) improves the performance at the price of spectrum.
REFERENCES OF CHAPTER 5


CHAPTER 6

MULTICARRIER TRANSMISSION IN
INDOOR RADIO COMMUNICATION
SYSTEMS

Indoor radio communication has a main role in successful implementation of Personal Communication Services (PCS). With the high rate of growth of indoor communication systems it is necessary to use the radio spectrum more efficiently. Multi-Carrier (MC) method, also called Orthogonal Frequency Division Multiplexing (OFDM), is a modulation technique which provides spectral efficiency. In this transmission technique the spectra of subchannels overlap each other while satisfying orthogonality, giving rise to optimum spectral efficiency. Multipath fading associated with indoor radio propagation channels degrades the performance of digital communication systems. However, since in the MC system transmission is parallel, the pernicious
effect of fading is spread over many bits, which leads to lower sensitivity of this technique to the distortion of the indoor channel. Besides the longer symbol duration due to parallel transmission of the MC technique has the added advantage to work in impulsive noise environments. The other advantage is the efficient implementation of the modulator and demodulator by the Fast Fourier Transform (FFT) algorithm.

The idea of multi-carrier transmission appeared more than two decades ago [1]-[2]. However, with the advent of new technologies in Digital Signal Processing, there has been a great interest in using this technique in digital communication systems [3]. A cellular mobile system based on OFDM technique has been analyzed and simulated in [4]. The advantage of using MC transmission over multipath fading mobile channels is reported in [5]. The performance of OFDM/FM modulation in a digital mobile Rayleigh fading channel is evaluated in [6]. In [7] the bit error performance of a MC system with a differential detection scheme is analyzed and simulated, and in [8] the bit error probability of the OFDM technique in a Rician multipath fading environment is analyzed. The transmission characteristics of a MC signal in digital broadcasting systems are investigated in [9] and in [10] the bit error performance of a MC system in a mobile fading channel is presented.

In this chapter section 6.1 explains the principles of multicarrier modulation technique. The spectral efficiency as well as the efficient implementation of this technique by the use of the Fast Fourier method is discussed in this section. Transmission of the multicarrier signal through the channel is described in section 6.2. First the linear channel is considered and the influence of channel parameters on the received signal and ultimately on the detected bits is investigated. In the other part of this section the behavior of transmission of the MC signal through the indoor multipath fading channel is studied, and the intersymbol interference caused by the multipath nature of the channel as well as the interference due to different carriers are investigated. In particular, the effect of the channel on losing the orthogonality between the carriers is discussed and the performance of the system in terms of average error probability is analyzed. In section 6.3 the performance of the orthogonal multicarrier technique is evaluated over a large data base of the indoor radio propagation channel (which was described in chapter 2). The influence of changing the number of carriers as well as the guard interval on the average bit error rate is investigated and the degradation of the performance caused by carrier phase errors is explored. In section 6.4 concluding remarks are provided.
6.1 MULTICARRIER MODULATION TECHNIQUE

In serial transmission, sequences of data are transmitted as a train of serial pulses. However, in parallel transmission each bit of a sequence of $M$ bits modulates a carrier. In the multicarrier technique the transmission is parallel. The block diagram of this technique is shown in Fig. 6.1. In the modulator the input data with rate $R$ is divided into $M$ parallel information sequences with rate $R/M$. Each sequence modulates a subcarrier.

![Diagram](image)

**Figure 6.1** Block diagram of multicarrier system. a) Modulator  b) Demodulator
The frequency of the $m$th carrier is

$$f_m = f_0 + \frac{m}{T} \quad m = 0, 1, 2, \ldots, M-1$$  \hspace{1cm} (6.1)$$

where $f_0$ is the lowest frequency of the carriers and $T$ is the symbol duration. The multicarrier transmitted signal is written as

$$s(t) = \sum_{m=0}^{M-1} \sum_{i=-\infty}^{\infty} d_m(i) e^{j2\pi f_m (t-iT)} p(t-iT)$$  \hspace{1cm} (6.2)$$

where $d_m(i)$ is the symbol of the $m$th subchannel at time interval $iT$. For BPSK and QPSK modulation $d_m(i)$ is ±1 and ±1±j, respectively. $p(t)$ is the response of the transmitter filter for which a rectangular pulse with duration $T$ and amplitude 1 is assumed. The spectra of MC signal can be obtained by considering the power spectra of a sequence of identically independent distributed rectangular pulses having period of $T$. This results into $Tsinc^2(fT)$. Using the shift property, the Fourier transform of the MC signal (6.2) becomes

$$S(f) = \sum_{m=0}^{M-1} T \text{sinc}^2(f-f_m)T$$  \hspace{1cm} (6.3)$$

This spectrum is represented in Fig. 6.3. From this figure it is clear that adjacent spectra overlap. The bandwidth of the baseband multicarrier signal is given by

$$B_{MC} = f_0 + \frac{M}{T} \left( f_0 - \frac{1}{T} \right) = \frac{M+1}{T} = \frac{M}{T_1}$$  \hspace{1cm} (6.4)$$

where $T_1$ is the time duration of original signal. With this technique we can take a number of tones (carriers), and subsequently increasing the symbol duration without increasing the bandwidth. When number of carriers ($M$) becomes large (i.e., $M \to \infty$), the OFDM power spectrum tends to a perfect rectangular spectrum with a cutoff frequency of $1/T_1$.

Referring to (6.1) it is seen that the subcarrier frequencies are separated by multiples of $1/T$. Meanwhile, in a multicarrier system the carriers are orthogonal, i.e.,

$$\frac{1}{T} \int_{0}^{T} e^{j2\pi f_{m_1} t} e^{-j2\pi f_{m_2} t} \, dt = \begin{cases} 1 & \text{if } m_1 = m_2 \\ 0 & \text{otherwise} \end{cases}$$  \hspace{1cm} (6.5)$$
Figure 6.2 The spectrum of a multicarrier signal.  

(a) each carrier  

(b) resultant  

Therefore, considering ideal transmission, and in spite of overlapping of the spectra, detection of the signal in one subchannel gives no output for any other subchannel.

6.1.1 Spectral Efficiency

By considering the symbol rate of $R=1/T$, the bit rate of $Q$-ary system is $R \log_2 Q$. Since in the MC transmission each subchannel has more time duration, each subcarrier rate is $(R/M) \log_2 Q$. Referring to Fig. 6.2 and the bandwidth of the MC signal of (6.4), the spectral efficiency of a MC system defined as the bit rate per unit bandwidth [4] becomes

$$
\text{spectral efficiency} = \frac{R \log_2 Q}{B_{MC}} = \frac{R \log_2 Q}{M+1} = \frac{\log_2 Q}{1 + \frac{1}{M}}
$$

(6.6)

Accordingly, the $Q$-ary digital modulation scheme using OFDM can achieve a bandwidth efficiency of about $\log_2 Q$ bit/sec/Hz when a large number of carriers is used.
6.1.2 Discrete Fourier Transform

The multicarrier modulator and demodulator can be effectively constructed by using Fast Fourier Transform (FFT) methods. Referring to (6.2) the transmitted MC signal within a time interval of $iT$ is written as

$$s(t) = \sum_{m=0}^{M-1} d_m e^{j2\pi f_m t}$$  \hspace{1cm} (6.7)

By assuming $t=nT_1$ and using (6.1) we obtain

$$s_n = \sum_{m=0}^{M-1} d_m e^{j\frac{2\pi}{M} mn} \quad 0 \leq n \leq M-1$$  \hspace{1cm} (6.8)

where $s_n \Delta t(nT_1)$. Therefore, the sequence $s_n$ is the inverse discrete Fourier transform (IDFT) of input data $d_m$. In fact MC modulation is performed by making a block of $M$ consecutive input symbols and computing the IDFT of these blocks. In the same way, demodulation can be performed using DFT techniques. These properties, in association with the Fast Fourier transform (FFT), are used in Digital Signal Processing for a full digital implementation of a MC modulator and demodulator. The block diagram of the MC system using discrete Fourier transforms are shown in Fig. 6.3.

![Block diagram of multicarrier system using DFT.](image)

According to (6.8) the amplitude of the MC signal is the summation of $M$ phasors that varies in time. This makes OFDM signals very sensitive to nonlinearities of the power amplifier. As a matter of fact, the requirement for highly linear transmitters capable to handle high peak to average ratios is one of the drawbacks of the MC technique [11]. This problem is important when the MC technique is used for broadcasting on wide areas. However, in indoor wireless communication applications, due to the lower power levels this requirement may be less of a
problem. This shortcoming of MC system may be alleviated by amplifying the signal of each subcarrier and then summing the outputs.

6.2 MC TRANSMISSION THROUGH THE CHANNEL

6.2.1 Linear Channel

In this section we study the behavior of the MC signal when it is transmitted through the channel. We consider here a linear channel with transfer function $H(f)$. At the frequency $f_m$ this is written as

$$H(f) = H_m e^{j\Phi_m} \quad (6.9)$$

where $H_m = |H(f_m)|$ and $\Phi_m = \tan^{-1}(\text{Im} \ H(f_m)/\text{Re} \ H(f_m))$. If $1/T < B_{MC}$, we can assume that $H(f)$ does not change too much over subchannels. Therefore, in a first approximation the output of the channel $r(t)$ can be written as

$$r(t) = \sum_{m=0}^{M-1} d_m H_m e^{j(2\pi f_d t + \Phi_m)} \quad 0 \leq t \leq T \quad (6.10)$$

According to the block diagram of the MC system shown in Fig. 6.3, samples of the signal $r(t)$ are applied to the DFT. Therefore, the output of the DFT is

$$z_i = \sum_{k=0}^{M-1} r_k e^{-j\frac{2\pi k i}{M}} = \sum_{k=0}^{M-1} \left( \sum_{m=0}^{M-1} d_m H_m e^{j\Phi_m} e^{j\frac{2\pi m k}{M}} \right) e^{-j\frac{2\pi k i}{M}} \quad (6.11)$$

$$= \sum_{k=0}^{M-1} \sum_{m=0}^{M-1} d_m H_m e^{j\Phi_m} e^{j\frac{2\pi k (m-i)}{M}} = \begin{cases} d_i H_i M e^{j\Phi_i} & \text{if } m = i \\ 0 & \text{otherwise} \end{cases}$$

where $r_k=r(kT_1)$. Therefore, the estimate of the data at the receiver is obtained by [1]

$$\hat{d}_i = \frac{1}{M \bar{H}_i} z_i e^{-j\Phi_i} \quad (6.12)$$

From (6.12) the influence of the amplitude and the phase of the channel on the detected bit is clear.
Another approximation of the channel is a linear frequency dependence of the magnitude and phase of the channel around frequency \( f_m \). In this case the transfer function of the channel is approximated by [1]

\[
H(f) = |H_m| e^{j \phi_m + 2\pi \alpha_m (f - f_m)}
\]

where \( H_m = |H(f_m)| \), \( \phi_m = \angle H(f_m) \) and \( \alpha_m \) is the slope of the magnitude of the transfer function of the channel at the frequency \( f_m \),

\[
\alpha_m = \frac{d |H(f)|}{df} \bigg|_{f=f_m}
\]

and \( t_g = \) is the group delay of the channel at the frequency \( f_m \),

\[
t_g = \frac{1}{2\pi} \frac{d \angle H(f)}{df} \bigg|_{f=f_m}
\]

Considering the pulse \( p(t) \) with unit amplitude in the interval \([0, T]\) and by convolution of the multichannel signal of \( d_m e^{j2\pi f_m t} p(t) \) with the impulse response of the channel (i.e., the inverse Fourier transform of (6.13)), it is shown in [1] that the resultant signal is

\[
y(t) = \text{Re} \left[ d_m e^{j2\pi f_m t} p(t) \ast h(t) \right]
\]

\[
= d_m H_m \cos(2\pi f_m t + \phi_m) p(t - t_g) + \frac{\alpha_m}{2\pi} \sin(2\pi f_m t + \phi_m) \frac{d}{dt} p(t - t_g)
\]

It is seen that due to channel the \( m \)th carrier is modified by amplitude and phase of the channel as well as the group delay \( t_g \). The second term in (6.16) shows the distortion which is due to variations of \( H(f) \) and is a potential source of interchannel interference. In [1] it is shown that if \( T \) is large enough, depending on \( T_1 \) and the maximum range value of \( t_g \), there exists a time \( t_0 \) such that for the time interval \( t_0 \leq t \leq t_0 + T \)

\[
\frac{d}{dt} p(t - t_g) = 0
\]

and the effect of interchannel interference can be suppressed. This is similar to the guard interval in the fading channel which we discuss in the next section.
6.2.2 Fading Channel

In this section we study the behavior of the MC signal when it is transmitted through the indoor multipath fading channel. In chapter 1 the mathematical modeling of the indoor radio propagation channel was explained in detail. By transmission of the MC signal of (6.2) through the channel with the impulse response of \( h(t) \) of (1.2), the received signal \( r(t) \) (output of the channel), is

\[
r(t) = s(t) * h(t) + n(t)
\]

where \( n(t) \) is additive white Gaussian noise with a spectral density height of \( N_0/2 \). The first term of (6.18) is written as

\[
s(t) * h(t) = \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} \sum_{i=-\infty}^{\infty} a_n d_m(i) e^{j[2\pi f_n (t - \tau_n - i T) + \theta_n]} p(t - iT - \tau_n)
\]

where \( a_n, \tau_n \) and \( \theta_n \) are respectively, the amplitude, time of arrival, and phase of multipath components and \( N \) is the number of path components of the impulse response of the channel. In the receiver the recovery of data associated with the carrier \( f_k \) is performed by taking the decision variable \( z_k \) as

\[
z_k = \int_{0}^{T} r(t) p(t) e^{-j2\pi f_k t} dt
\]

which can be written as

\[
z_k = \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} \sum_{i=-\infty}^{\infty} a_n d_m(i) e^{-j[2\pi f_n (i T + \tau_n) + \theta_n]} \int_{0}^{T} e^{jt(2\pi (m-k)/T)} p(t - iT - \tau_n) p(t) dt + \int_{0}^{T} n(t) p(t) e^{-jt2\pi k/T} dt
\]

For BPSK signaling it becomes
\[ z_k = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} a_n (d_m (-1) \int_0^{\tau_n} \cos \left[ 2 \pi (f_m-f_k) t - \phi_{m,n} \right] dt + d_m (0) \int_0^{\tau_n} \cos \left[ 2 \pi (f_m-f_k) t - \phi_{m,n} \right] dt) + \int_0^{T} n(t) \cos (2 \pi f_k t) dt \]  

(6.22)

where \( \phi_{m,n} = 2 \pi f_m \tau_n - \theta_n \). Equation (6.22) can be written as

\[ z_k = \sum_{n=0}^{N-1} a_n (d_k (-1) \int_0^{\tau_n} \cos \phi_{m,n} dt + d_k (0) \int_0^{\tau_n} \cos \phi_{m,n} dt) \]

\[ + \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} a_n (d_m (-1) \int_0^{\tau_n} \cos \left[ 2 \pi (f_m-f_k) t - \phi_{m,n} \right] dt + d_m (0) \int_0^{\tau_n} \cos \left[ 2 \pi (f_m-f_k) t - \phi_{m,n} \right] dt) + \int_0^{T} n(t) \cos (2 \pi f_k t) dt \]  

(6.23)

Without loss of generality we can assume that receiver is matched to the first path of the multipath received signal, i.e., we assume \( \tau_0 = 0 \) and \( \theta_0 = 0 \). Therefore,

\[ z_k^{(0)} = a_0 d_k (0) T + \sum_{n=1}^{N-1} a_n (d_k (-1) \int_0^{\tau_n} \cos \phi_{k,n} dt + d_k (0) \int_0^{\tau_n} \cos \phi_{k,n} dt) \]

\[ + \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} a_n (d_m (-1) \int_0^{\tau_n} \cos \left[ 2 \pi (f_m-f_k) t - \phi_{m,n} \right] dt + d_m (0) \int_0^{\tau_n} \cos \left[ 2 \pi (f_m-f_k) t - \phi_{m,n} \right] dt) + \int_0^{T} n(t) \cos 2 \pi f_k t dt \]  

(6.24)

where \( d_m (-1) \) and \( d_m (0) \) indicate the previous and current symbols respectively, which are transmitted at the carrier \( f_m \). Equation (6.24) can be written as the following
\[ z_k^{(0)} = a_0 d_k(0) T + \sum_{n=1}^{N-1} a_n [d_k(-1) X_{k,k}^n + d_k(0) \dot{X}_{k,k}^n] \]

\[ + \sum_{n=0}^{N-1} \sum_{m=1}^{M-1} a_n \{ d_m(-1) [X_{m,k}^n + Y_{m,k}^n] + d_m(0) [\dot{X}_{m,k}^n + \dot{Y}_{m,k}^n] \} + w_k \]  

(6.25)

where

\[ w_k = \int_{0}^{T} n(t) \cos \frac{2\pi k}{T} t \, dt \]  

(6.26)

and

\[ X_{k,k}^n = \cos \phi_{k,n} \quad R_{k,k}(\tau_n) \quad \dot{X}_{k,k}^n = \cos \phi_{k,n} \quad \dot{R}_{k,k}(\tau_n) \]

\[ X_{m,k}^n = \cos \phi_{m,n} R_{m,k}(\tau_n) \quad \dot{X}_{m,k}^n = \cos \phi_{m,n} \quad \dot{R}_{m,k}(\tau_n) \]

\[ Y_{m,k}^n = \sin \phi_{m,n} R_{m,k}(\tau_n) \quad \dot{Y}_{m,k}^n = \sin \phi_{m,n} \quad \dot{R}_{m,k}(\tau_n) \]  

(6.27)

and the partial cross correlations are given by [8]

\[ R_{k,k}(\tau) = \int_{0}^{T} dt = \tau \]

\[ \dot{R}_{k,k}(\tau) = \int_{0}^{T} dt = T - \tau \]

\[ R_{m,k}(\tau) = \int_{0}^{T} \cos \frac{2\pi (m-k) t}{T} dt = T \quad \frac{\sin \frac{2\pi (m-k) \tau}{T}}{2\pi (m-k)} \quad m \neq k \]

\[ R'_{m,k}(\tau) = \int_{0}^{T} \sin \frac{2\pi (m-k) t}{T} dt = T \quad \frac{1 - \cos \frac{2\pi (m-k) \tau}{T}}{2\pi (m-k)} \quad m \neq k \]

\[ \dot{R}_{m,k}(\tau) = \int_{0}^{T} \cos \frac{2\pi (m-k) t}{T} dt = -T \quad \frac{\sin \frac{2\pi (m-k) \tau}{T}}{2\pi (m-k)} \quad m \neq k \]

\[ \dot{R}'_{m,k}(\tau) = \int_{0}^{T} \sin \frac{2\pi (m-k) t}{T} dt = -T \quad \frac{1 - \cos \frac{2\pi (m-k) \tau}{T}}{2\pi (m-k)} \quad m \neq k \]  

(6.28)

From (6.25) it is seen that the second term is due to intersymbol interference (ISI) caused by the multipath channel. The third term relates to the loss of orthogonality between subcarriers also due to multipath fading of the indoor channel, which is the intercarrier interference ICI. The last term
is due to noise. Accordingly, (6.25) can be rewritten as

$$z_k^{(0)} = \text{desired signal} + \text{ISI} + \text{ICI} + \text{noise}$$  \hspace{1cm} (6.29)

where

$$\text{ISI} = \sum_{n=1}^{N-1} a_n \left[ d_k(-1) X_{k,k}^n + d_k(0) \hat{X}_{k,k}^n \right]$$  \hspace{1cm} (6.30)

and

$$\text{ICI} = \sum_{n=0}^{N-1} \sum_{m=n+1}^{M-1} a_n \left[ d_m(-1) X_{m,k}^n + Y_{m,k}^n \right] + d_m(0) \left[ \hat{X}_{m,k}^n + \hat{Y}_{m,k}^n \right]$$  \hspace{1cm} (6.31)

One of the features of the multicarrier technique is to cope with the frequency selectivity of the channel. This is achieved by consideration of the guard interval. By exerting the guard period, the transmitted signal duration $T$ is divided into two parts, i.e., a guard interval $T_G$ and an effective symbol duration of $T'$, that is

$$T = T_G + T'$$  \hspace{1cm} (6.32)

In this case the impulse response of the receiving filter will have a rectangular shape but with duration of $T-T_G$ and the distance between carriers (i.e., $\Delta f = f_{m+1} - f_m$) will be $1/T'$ [3]. By considering the guard period greater than the maximum excess delay of the propagation channel the effect of ISI could be suppressed. However, by applying a guard period, due to the different durations for the transmitting and receiving filters, an optimal matched filter condition is not completely fulfilled, and a fraction of the transmitted power is sacrificed in order to avoid ISI. Besides, this reduces the bandwidth efficiency by a factor of $10\log(1+T_G/T')$ dB.

### 6.3 ERROR PROBABILITY

The error probability is calculated as the sampled signal of (6.24) is less than zero assuming a 1 has been transmitted or

$$P(E) = \text{Prob} \left( z_k^{(0)} < 0 \mid d_k(0) = 1 \right)$$  \hspace{1cm} (6.33)

Referring to the Eqs. (6.29) - (6.31) and by considering interference terms that comprise random path amplitudes, time of arrivals, and phases of the multipath channel and considering a large
number of components, the decision variable can be approximated as a Gaussian random variable. With this assumption, the error probability is calculated as

\[
P(E) = Q \left( \sqrt{\frac{(a_0 T)^2}{\text{Var}(z_k^0)}} \right)
\]

(6.34)

where \( Q(.) \) is the same as (4.43). Referring to (6.29), the variance of \( z_k^{(0)} \) contains the variance of ISI, ICI and noise terms. The variance of ISI is calculated as (see Appendix E)

\[
\sigma_{ISI}^2 = \sum_{n=1}^{N-1} \sigma_{a_n}^2 \left[ \frac{1}{\tau_n} - T \frac{T}{2} + \frac{T^2}{2} \right]
\]

(6.35)

where \( \sigma_{a_n}^2 \) is the expected value of \( a_n^2 \). For ICI, as shown in Appendix F, the variance is

\[
\sigma_{ICl}^2 = \frac{T^2}{2\pi^2} \sum_{n=0}^{N-1} \sigma_{a_n}^2 \sum_{i=-k,i \neq 0}^{M-k-1} \frac{1}{i^2}
\]

(6.36)

The variance of noise term, as shown in (5.58), is \( N_0 T/4 \). Considering independent interference terms, the total variance of disturbances is calculated as

\[
\text{Var}(z_0^0) = \frac{T^2}{3} \sum_{n=1}^{N-1} \sigma_{a_n}^2 + \frac{T^2}{2\pi^2} \sum_{n=0}^{N-1} \sigma_{a_n}^2 \sum_{i=1}^{M-1} \frac{1}{i^2} + \frac{N_0 T}{4}
\]

(6.37)

The probability error of (6.34) should be averaged over the probability density function of the path amplitude \( a_0 \).

6.4 PERFORMANCE EVALUATION OVER MEASURED CHANNELS

The orthogonal frequency division multiplexing technique was applied over a large database of indoor radio propagation channel (described in chapter 2). A discrete-time version of impulse responses (with 5 nsec. binwidth) was used. BPSK signal with a data rate of 1.6 Mbps was considered. A different number of carriers and different values of guard intervals were investigated. The discrete Fourier transform (DFT) was used for the MC modulator and demodulator. The DFT is a bank of filters which also limits the power of filtered noise in the
receiver and is also performed by the fast methods. Performance is evaluated in terms of average error probability versus signal-to-noise ratio. In each patch of data (i.e., each location of each antenna separation) the average impulse response was obtained and then the performance of the MC system was evaluated and averaged over all locations with the same antenna separation [12]-[13]. A total of 12000 profiles of impulse responses was used. In Figs. 6.4 - 6.6 the performance of the MC system averaged over all profiles of the antenna separations of 5 and 10 meters of Building A and the antenna separation of 5 meters of Building B are revealed, respectively. The averaged performance over all antenna separations of Building A is depicted in Fig. 6.7 and for both buildings in Fig. 6.8. From these figures it is seen that with increasing the number of carriers the performance gets better. However, for large number of carriers the performance enhancement is not large. Similar results are reported in [8] using the MC technique in a channel having two path components.

![Graph showing the relationship between average bit error probability and SNR for different numbers of carriers.](image)

**Figure 6.4** Average bit error probability versus signal-to-noise ratio for a different number of carriers (M). Averaged over all locations with an antenna separation of 5 m, (Building A).

In Figs. 6.9 - 6.10 the effect of exerting the guard interval is shown. These curves show the average bit error rate over all locations with an antenna separation of 5 meters of Building A and 10 meters of Building B, versus the guard interval for different signal to noise ratios, respectively. The BER averaged over all antenna separations of both buildings as a function of guard period is depicted in Fig. 6.11. From Figs. 6.9 - 6.11 it is clear that increasing the guard interval improves the performance of the system. This is because of the effect of multipath spread of the indoor channel. On the other hand with increasing the guard period the probability of bit error gets worse. This is due to losing some signal power by increasing the guard period. Therefore, there is an optimum guard interval to minimize the probability of bit error. This value for the averaged
Figure 6.5 Average bit error probability versus signal-to-noise ratio for a different number of carriers (M). Averaged over all locations with an antenna separation of 10 m, (Building A).

Figure 6.6 Average bit error probability versus signal-to-noise ratio for a different number of carriers (M). Averaged over all locations with an antenna separation of 5 m, (Building B).
Figure 6.7 Average bit error probability versus signal-to-noise ratio for a different number of carriers (M). Averaged over all antenna separations, (Building A).

Figure 6.8 Average bit error probability versus signal-to-noise ratio for a different number of carriers (M). Averaged over all antenna separations of both buildings.
Figure 6.9  Performance of the MC system versus the guard period for different values of signal-to-noise ratio. Number of carriers=4. Averaged over all locations with an antenna separation of 5m, (Building A).

Figure 6.10  Performance of the MC system versus the guard period for different values of signal-to-noise ratio. Number of carriers=4. Averaged over all locations with an antenna separation of 10m (Building B).
Figure 6.11 Performance of the MC system versus the guard period for different values of the signal-to-noise ratio. Number of carriers = 4. Averaged over all antenna separations of both buildings.

performance over both buildings is about 250 nsec., [12]-[13].

Degradation of the performance of the MC system due to frequency offset is reported in [14] - [17]. In [14] - [16] the influence of the static phase error on the performance of MC signal in the AWGN channel is discussed, and in [17] a maximum likelihood estimator for the frequency offset is designed and its performance is analyzed. The effect of carrier phase error in the MC transmission is also investigated in Fig. 6.12. This figure depicts the error probability averaged over all locations with an antenna separation of 5 meters of Building B versus signal-to-noise ratio for two constellations of BPSK and QPSK signals, and for fixed number of carriers and uniform phase error. For a fixed signal-to-noise ratio and phase error variance it is seen that QPSK has a higher bit error rate than BPSK. For a fixed number of carriers and for a QPSK scheme the average performance for two different variances of phase error is shown in Fig. 6.13. The influence of the variance of phase error on the symbol error rate is clear from this figure. For QPSK modulation the average performance over an antenna separation of 5 meters of Building A and for the fixed value of the phase error variance for multi-carrier and single carrier (M=1) cases is revealed in Fig. 6.14. It is seen that multi-carrier transmission is more sensitive to phase noise than the single carrier. This is mainly due to the increase of intercarrier interference and subsequently because of lowering the orthogonality between subcarriers in the case of carrier phase errors. Similar sensitivity results of the MC technique to the carrier frequency offset are
reported in [16]. One justification for the sensitivity of the MC system to phase noise is when the phase noise is modeled as a Wiener process [15]. In this case the variance of phase noise increases with time and therefore, comparing with a single carrier transmission, the MC system is more sensitive to the phase noise error.

**Figure 6.12** Performance of the MC system for BPSK and QPSK constellations. Phase error variance = 0.008, number of carriers = 8. Averaged over all locations with an antenna separation of 5m, (Building B).

**Figure 6.13** Performance of the MC system for QPSK signaling and two different phase error variances. Number of carriers = 8. Averaged over all locations with an antenna separation of 5m, (Building A).
Figure 6.14 Performance of the multi and single carrier transmission for a fixed error variance of 0.008. QPSK signaling. Averaged over all locations with an antenna separation of 3m, (Building A).

6.5 CONCLUSIONS

Multicarrier parallel orthogonal transmission was discussed in this chapter. Its spectral efficiency and capability to cope with the frequency selectivity of the channel as well as its efficient implementation with FFT algorithms were explained. The performance of the multicarrier system in terms of the average probability of bit error versus signal-to-noise ratio was evaluated over an empirical large data base of an indoor radio propagation channel at two dissimilar office buildings. In a fixed bandwidth the effect of increasing the number of tones was investigated. It was shown that the performance improves with increasing the number of carriers. However, the improvement of performance for the large number of carriers is not large. Application of a guard period in the MC transmission and its effect on the performance of the system was investigated. Trade off between bit error probability and guard period in achieving an optimal value of guard interval was observed. The influence of a carrier phase error on the MC system performance was studied. It was shown that in a fixed bit rate, the performance degradation of MC transmission due to carrier phase errors is higher than single transmission and increases with increasing the size of the signal constellation. In addition, the more sensitivity of the MC technique to phase errors may necessitate the differential demodulation schemes. The Mc signal also undergoes a nonlinear distortion in the transmitter, which might be severe in broadcasting applications.
REFERENCES OF CHAPTER 6


CHAPTER 7

MC/CDMA IN INDOOR RADIO COMMUNICATION SYSTEMS

In this chapter a communication system based on a combination of Multi-Carrier (MC) modulation and direct sequence Code Division Multiple Access (CDMA) technique is presented. Such a combination is called MC/CDMA [1] and has been investigated in some recent papers [2]-[10]. It is expected that by proper choice of the modulation parameters, such as the number of chips, number of carriers, the sequence structure, etc. such a system can combine the advantages of both techniques, i.e., the spectral efficiency, increased symbol duration of MC as well as the multiple access and multipath rejection capability of CDMA. There are several ways to combine both modulations. As will be explained in the following sections, we will restrict ourselves to the case where spreading is performed after multitone modulation. In section 7.1 this combined modulation scheme is described and the spectrum as well as the bandwidth of
MC/CDMA signal is explained, followed by the choice of modulation parameters in section 7.2. The reception of MC/CDMA signal is mentioned in section 7.3, where the received signal and the structure of the receiver are explained. The effect of interfering components on the received MC/CDMA signal is investigated in section 7.4, followed by the bit error probability in section 7.5. Performance results of this modulation technique over the measured indoor channels are presented in section 7.6 and conclusions are summarized in section 7.7.

7.1 MC/CDMA MODULATION SCHEME

The primary concern of CDMA is the multiple access capability. However, the performance of the system is limited by multiuser interference. Furthermore, a remarkable spreading factor is required for a good performance of the system. On the other hand as stated in the previous chapter, the OFDM technique has a high spectral efficiency and ability to cope with the frequency selectivity of the channel. In this section the combination of direct sequence CDMA and MC techniques is discussed. The block diagram of MC/CDMA system is shown in Fig. 7.1, which contains the following stages:

1) Mapping: The input bit stream is passed through a mapping device, which associates to each group of \( a \) consecutive bits an equivalent letter from a given alphabet of \( 2^a \) symbols. (This could be a binary PSK (\( a=1 \)), or QPSK (\( a=2 \)) modulator, for example). The symbols at the output are expressed by their inphase and quadrature values or equivalently, by complex base band symbols, whose sampling period is \( aT_i \), where \( T_i \) is the initial sampling time.

2) Serial to parallel convertor: This symbol stream is then split into \( M \) parallel streams which results in a sampling time of \( T = aMT_i \) in each stream.

3) MC modulation: As described in the previous chapter multicarrier modulation or orthogonal frequency division multiplexing (OFDM) is performed by rectangular pulse shaping with duration \( T \) multiplied by the corresponding carriers whose frequencies are given by

\[
f_m = f_0 + \frac{m}{T} \quad \text{for} \quad 0 \leq m \leq M - 1
\]

(7.1)

where \( f_0 \) is the lowest frequency.
4) CDMA spreading: Following to the OFDM modulation the signals are sent to the DS-CDMA modulator which spreads the MC signal by multiplying it with the specific pseudo-random code sequence of the user.

The multicarrier modulated signal of the kth user is expressed as

\[
s_k(t) = \sum_{m=0}^{M-1} \sum_{i=0}^{\infty} d_{m,k}(i) e^{j2\pi f_c(t-iT)} P_T(t-iT) \cdot c_k(t-iT)
\]  

(7.2)

where \( M \) is the number of tones, \( d_{m,k}(i) \) is the symbol of the \( m \)th subchannel at the time interval \( iT \) of the \( k \)th user, \( P_T(t) \) is a rectangular pulse of duration \( T \) and amplitude 1, and \( c_k(t) \) is the spreading code of \( k \)th user, already described in (5.23). The spectra of the MC/CDMA signal can be considered by referring to the spectrum of MC signal of (6.3), convolved with the spectrum of the spreading code \( c_k(t) \). If we take the width of the first lobe of the power spectrum of the signals as a good approximation of their spectra, the bandwidth of the MC/CDMA signal is approximated as

\[
B_{MC/CDMA} = \frac{M+1}{T} + \frac{1}{T_c}
\]  

(7.3)

where \( T_c \) is the chip length and \( T=LT_c \), (\( L \) is the length of the spreading code). Equation (7.3) is written as

\[
B_{MC/CDMA} = \frac{M+1}{T} + \frac{L}{T} = (1 + \frac{L+1}{M}) \frac{1}{T_1}
\]

\[ - \left( \frac{1 + \frac{L}{M}}{\alpha T_1} \right) \]  

(7.4)
The first term in (7.4), \(1/\alpha T_j\), is the bandwidth of signal carried out by one carrier and the second term, \(L/\alpha M T_j\), is the increase of bandwidth due to use of spread spectrum code. For the case of BPSK signaling and a large length of the spreading codes, the bandwidth of the MC/CDMA signal is \((1+L/M)/T_j\). Considering the equal code sequence duration with the MC signal duration, (7.4) can be written as

\[
B_{MC/CDMA} = (1 + \frac{M}{L}) \frac{1}{T_c} = R_{chip} + \frac{R_{bit}}{\alpha}
\]  

(7.5)

where \(R_{chip}\) and \(R_{bit}\) are the chip and bit rates, respectively. Referring to the spectrum of OFDM/CDMA signal we see that partial spectra overlap, but in ideal conditions for one user, the signal will be perfectly recovered, since the orthogonality is conserved, see also Eq. (6.5). However, it should be noted that for more than one user, global orthogonality on all of the users cannot be achieved in an asynchronous communication model, since it is hard to find orthogonal codes for all possible relative delay values. Therefore, similar to the case of CDMA, one can expect a degradation of the system performance by increasing the number of users. This issue will be more discussed in section 7.4.

### 7.2 THE CHOICE OF MODULATION PARAMETERS

There are a large number of possible combinations for the choice of the modulation parameter and we have to impose supplementary conditions on the resulting signal to obtain performances that can be compared. One kind of condition that is used in [1] for synchronous systems in multipath channels, is to keep the code length constant while increasing the number of tones at a constant bit rate. In indoor environments and low data rates, the transmission delay is less than the chip duration. The system is then said to be quasi-synchronous and codes with good correlation properties on this delay range, like Hadamard codes, can be used for synchronous systems, which leads to a good suppression of intersymbol and multiple access interferences on each carrier. However, in this chapter the scope of our work is to study the performance of OFDM/CDMA system for uplink transmission without any synchronization between users. We will then use codes having good correlation properties for all relative delay values, like Gold codes. Hence, performances will improve with greater code lengths and the advantage of MC will lie on the possibility of taking longer code sequences on each carrier. In order to make results comparable, a constant chip duration is considered. The second parameter
we fix the bit rate before modulation. These two imposed constraints, i.e., constant chip duration and bit rate, define the relationship between the other parameters (i.e., the number of carriers, number of chips and α). Then the total symbol duration becomes

\[ M(\alpha T_1) = LT_c \]
\[ T_c = \frac{\alpha M}{L} = \text{Const.} \]  
(7.6)

Therefore, the ratio \( M/L \) is kept constant, once the mapping parameter has been selected. Considering long code lengths \( L > M \), and the above constraint, the bandwidth of the MC/CDMA signal is approximately \( 1/T_c \), see (7.5). Hence, with constant chip duration, the total MC/CDMA signal bandwidth is kept constant.

### 7.3 RECEPTION OF THE MC/CDMA SIGNAL

In section 7.1 the block diagram of MC/CDMA modulation was shown. In the OFDM/CDMA system, which we consider in this chapter, each one of the \( K \) users of the system sends its own data independently and without any synchronization with other users. The MC/CDMA modulated signal of each user is passed through its channel. This is shown in Fig. 7.2. The baseband received signal \( r(t) \), is the sum of the all signals of users plus noise

\[ r(t) = \sum_{k=1}^{K} s_k(t) * h_k(t) + n(t) \]  
(7.7)

where \( h_k(t) \) is the impulse response of the channel corresponding to the \( k \)th user, i.e.,

\[ h_k(t) = \sum_{N_k=0}^{N_k-1} a_{k,n} \delta(t - \tau_{k,n}) e^{i\theta_{k,n}} \]  
(7.8)

where, \( a_{k,n} , \tau_{k,n} , \theta_{k,n} \) and \( N_k \) are the random amplitude, time of arrival, phase and number of multipath components of the \( k \)th impulse response of the indoor channel, (see also chapter 1). Substituting of (7.8) in (7.7) and using (7.2) yields

\[ r(t) = \sum_{k=1}^{K} \sum_{m=0}^{M-1} \sum_{n=0}^{N_k-1} \sum_{i=-\infty}^{\infty} a_{k,n} d_{k,m}(i) e^{i[2\pi f_m(t-iT-\tau_{k,n})+\phi_{k,n}]} c_k(t-iT-\tau_{k,n})p_T(t-iT-\tau_{k,n}) + n(t) \]  
(7.9)

Referring to the block diagram of MC/CDMA system of Fig. 7.2, which contains the channel and
receiver, and considering detection of the data of user \( k' \), the received signal is first despread by code \( c_k(t) \) and then is derived into \( M \) branches. Each branch is associated with a given carrier. Filtering and sampling is performed on each branch and eventually with the parallel/serial converter the serial data are extracted. The decision variable corresponding to the user \( k' \) and carrier \( m' \) is given by

\[
z_{k',m'} = \int_0^T r(t) e^{-j2\pi f_{m'}t} c_{k'}(t) \, dt
\]

Using (7.9) it can be written as

\[
z_{k',m'} = \sum_{k=1}^{K} \sum_{m=0}^{M-1} \sum_{n=0}^{N_k-1} \sum_{i=-\infty}^{\infty} a_{k,n} d_{k,m}(i) \int_0^T e^{j(\theta_{k,n} - 2\pi f_m t \tau_{k,n} - iT)} e^{j2\pi(f_m - f_{m'})t} c_k(t - \tau_{k,n} - iT) c_{k'}(t) \, dt + \int_0^T n(t) c_{k'}(t) e^{-j2\pi f_{m'}t} \, dt
\]

Denoting \( \theta_{k,n} - 2\pi f_m \tau_{k,n} = \Phi_{k,m,n} \), and using (7.1), we have

\[
z_{k',m'} = \sum_{k=1}^{K} \sum_{m=0}^{M-1} \sum_{n=0}^{N_k-1} \sum_{i=-\infty}^{\infty} a_{k,n} d_{k,m}(i) e^{j\Phi_{k,m,n}} \int_0^T e^{j2\pi(m-m')t} c_k(t - \tau_{k,n} - iT) c_{k'}(t) \, dt + \int_0^T n(t) c_{k'}(t) e^{-j2\pi f_{m'}t} \, dt
\]

Considering \( d_{k,m}(0) \) and \( d_{k,m}(-1) \) as the current and previous information being transmitted from \( k \)th user on the \( m \)th subchannel, respectively, (7.12) becomes

\[
z_{k',m'} = \sum_{k=1}^{K} \sum_{m=0}^{M-1} \sum_{n=0}^{N_k-1} a_{k,n} e^{j\Phi_{k,m,n}} \left[ d_{k,m}(-1) \int_0^T e^{j2\pi(m-m')t} c_k(t - \tau_{k,n}) c_{k'}(t) \, dt + d_{k,m}(0) \int_{\tau_{k,n}}^T e^{j2\pi(m-m')t} c_k(t - \tau_{k,n}) c_{k'}(t) \, dt \right] p_T(t - iT - \tau_{k,n}) \, dt + \int_0^T n(t) c_{k'}(t) e^{-j2\pi f_{m'}t} \, dt
\]

Denoting the terms in the bracket of (7.13) as
Figure 7.2  MC/CDMA technique in indoor communication system.
\[
R_{k,k',m,m'}(\tau) = \int_0^\tau e^{j2\pi(m-m')t/T} c_k(t-\tau)c_{k'}(t)\,dt \\
\hat{R}_{k,k',m,m'}(\tau) = \int_\tau^T e^{j2\pi(m-m')t/T} c_k(t-\tau)c_{k'}(t)\,dt
\]

(7.14)

the decision variable \( z_{k',m'} \) becomes

\[
z_{k',m'} = \sum_{k'=1}^K \sum_{m'=0}^{M-1} \sum_{n=0}^{N_c-1} a_{k,n} d_{k,m}(i) e^{j\Phi_{k,n}} [d_{k,m}(-1) R_{k,k',m,m'}(\tau_{k,n}) \\
+ d_{k,m}(0) \hat{R}_{k,k',m,m'}(\tau_{k,n})] + w_{k',m'}
\]

(7.15)

where

\[
w_{k',m'} = \int_0^T n(t)c_{k'}(t)e^{-j2\pi m't/T}\,dt
\]

(7.16)

Considering \( \tau_{k,n} \) as

\[
\tau_{k,n} = q T_c + \tau'
\]

(7.17)

where \( q=[\frac{\tau_{k,n}}{T_c}] \), and \( \lfloor x \rfloor \) is the integer part of \( x \), and \( 0 \leq \tau' \leq T_c \), one can decompose the integration intervals in (7.14) into smaller intervals where the code sequence products have constant values. We obtain then

\[
R_{k,k',m,m'}(\tau) = \sum_{l=0}^q c_{k,l} c_{k',l+L-(q+1)} S_{m,m'}^{l}(\tau') \\
+ \sum_{l=0}^{q-1} c_{k,l} c_{k',l+L-q} S_{m,m'}^{l}(\tau')
\]

(7.18)

and
The Interfering Terms

\[ \hat{R}_{k,k',m,m'}(\tau) = \sum_{l=q+1}^{L-1} c_{k,l} c_{k',l-q} S^{l}_{m,m'}(\tau') \]
\[ + \sum_{l=q}^{L-1} c_{k,l} c_{k',l-q} \hat{S}^{l}_{m,m'}(\tau') \]  \hspace{1cm} (7.19)

where the small scale partial cross correlations \( S^{l}_{m,m'} \) and \( \hat{S}^{l}_{m,m'} \) are

\[ S^{l}_{m,m'}(\tau') = \int_{T_{c} + \tau'}^{(l+1)T_{c}} e^{j\frac{2\pi (m-m')}{T} t} dt = \frac{Te^{j\frac{2\pi (m-m')}{T} \tau'}}{j2\pi (m-m')} \bigg[ e^{-j\frac{2\pi (m-m')}{T} \tau'} - 1 \bigg] \]  \hspace{1cm} (7.20)

\[ \hat{S}^{l}_{m,m'}(\tau') = \int_{lT_{c} + \tau'}^{(l+1)T_{c}} e^{j\frac{2\pi (m-m')}{T} t} dt = \frac{Te^{j\frac{2\pi (m-m')}{T} \tau'}}{j2\pi (m-m')} \bigg[ e^{-j\frac{2\pi (m-m')}{T} \tau'} - e^{-j\frac{2\pi (m-m')}{T} \tau'} \bigg] \]

We note that when \( m=m' \), these small scale partial correlations reduce to \( \tau' \) and \( T_{c} - \tau' \) respectively, which are the classical expressions of \( R_{k,k'}(\tau) \) and \( \hat{R}_{k,k}(\tau) \) developed by Pursley for the CDMA system [11].

7.4 THE INTERFERING TERMS

We assume the receiver is matched to the first path of the multipath received signal of (7.9) of \( k' \)th user on the subcarrier \( m' \). Therefore, referring to (7.15), the corresponding decision variable is written as

\[ z_{k',m}^{(0)} = \text{desired signal} + \text{ISI} + \text{ICI} + \text{MAI} + \text{noise} \]  \hspace{1cm} (7.21)

where

\[ \text{desired signal} = a_{k',0} \cdot d_{k',m'}(0) \cdot T \]  \hspace{1cm} (7.22)

The intersymbol interference (ISI) is

\[ \text{ISI} = \sum_{n=0}^{N_{c}-1} a_{k',n} e^{j\Phi_{k',m',n}} \left[ d_{k',m'}(-1) \cdot R_{k',k',m',m'}(\tau_{k',n}) + d_{k',m'}(0) \cdot \hat{R}_{k',k',m',m'}(\tau_{k',n}) \right] \]  \hspace{1cm} (7.23)

the interchannel interference (ICI) is
ICL = \sum_{m=0}^{M-1} \sum_{n=0}^{N_{k'}} a_{k',n} e^{j\phi_{k',m,n}} [d_{k',m}(-1) R_{k,k',m,m'}(\tau_{k',n}) + d_{k',m}(0) \hat{R}_{k,k',m,m'}(\tau_{k',n})]

(7.24)

and the multiple access interference (MAI) is expressed as

MAI = \sum_{k=1}^{K-1} \sum_{k' \neq k} \sum_{m=0}^{M-1} \sum_{n=0}^{N_{k}} a_{k,n} e^{j\phi_{k,m,n}} [d_{k,m}(-1) R_{k,k',m,m'}(\tau_{k,n}) + d_{k,m}(0) \hat{R}_{k,k',m,m'}(\tau_{k,n})]

(7.25)

In the following each possible interference contribution is outlined.

i) ISI: This kind of interference is due to nonideal characteristics of the multipath fading channel which introduces an influence of the former symbols on the decision taken for the actual symbol on the same carrier. The extension in time domain is grossly proportional to the ratio \(\tau_{rms}/T\), where \(\tau_{rms}\) is the rms delay spread of the channel (see chapter 2, Eq. (2.3)). Hence, there will be a transfer from interfering terms toward useful terms when the symbol duration (the number of carriers) increases. Another reason for decreasing the ISI with increasing the number of carriers is also that we can use longer codes having better multipath rejection properties.

ii) ICI: As explained in the previous chapter the multipath fading nature of the channel introduces the influence of the symbols from other carriers on the decision made for the symbols of the carrier \(m'\). The use of rectangular pulse shaping in the modulation scheme leads to an ICI contribution proportional to the number of carriers. It should be noted that increasing the number of carriers will also decreases the interfering ISI terms from other carriers.

iii) MAI: This interference is due to presence of other users in the communication system and thus will increase with the number of users. Increasing the number of tones will allow better code performances and thus reduces this kind of disturbance. The considerations made about ISI and ICI remain also valid since MAI term involves both ISI and ICI.

7.5 ERROR PROBABILITY

Considering the decision variable of (7.21) and using the standard Gaussian approximation of the interfering terms, the error probability is found as
\[ P(E) = Q \left( \sqrt{\frac{(a_{k',0} T)^2}{\sigma^2}} \right) \]  

(7.26)

where \( Q(x) \) is given by \( \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} \exp \left( -\frac{y^2}{2} \right) \, dy \), and \( \sigma^2 \) is the variance of interfering terms (including ISI, ICI, MAI and noise). Assuming independent interference terms it can be shown that [3]

\[ \sigma^2 = \frac{T^2}{12L^3} \sum_{k=2}^{K} r_{k,1} + \frac{T^2}{4\pi L^3} \sum_{k=2}^{K} \sum_{m=0}^{M-1} \frac{\mu_{k,1}(0) - \mu_{k,1}(1)}{(m-m')^2} + \frac{N_0 T}{4} \]  

(7.27)

where

\[ r_{k,i} = \sum_{l=0}^{L-1} \{ D_{k,i}(l-L) + D_{k,i}(l-L)D_{k,i}(l-L+1) \]  
\[ + D_{k,i}(l-L+1) + D_{k,i}(l+1) \} \]  

(4.28)

and

\[ \mu_{k,i}(0) = \sum_{l=0}^{L-1} D_{k,i}(l-L+1) + D_{k,i}(l+1) \]  

\[ \mu_{k,i}(1) = \sum_{l=0}^{L-1} D_{k,i}(l-L)D_{k,i}(L-l+1) + D_{k,i}(l)D_{k,i}(l+1) \]  

(7.29)

\( D_{k,i}(l) \) is the aperiodic cross correlation functions for the sequences \( c_{k,i} \) and \( c_{i,j} \) defined by [10]

\[ D_{k,i}(l) = \begin{cases} 
\sum_{j=0}^{L-1-l} c_{k,j} c_{i,j+l} & 0 \leq l \leq L-1 \\
\sum_{j=0}^{L-1-l} c_{k,j-l} c_{i,j} & -(L-1) \leq l \leq 0 \\
0 & |l| \geq L
\end{cases} \]  

(7.30)

The error probability of (7.26) should be averaged over the probability density function of the path amplitude \( a_{k',0} \).
7.6 PERFORMANCE RESULTS OVER THE MEASURED CHANNELS

MC/CDMA technique was employed in the indoor radio communication system. BPSK signaling with the rate of 6.6 Mbps was considered. Spreading of signals was carried out by using Gold codes [12]. As in the DS-CDMA system, the performance increases with greater code length and the advantage of OFDM lies on the possibility of taking larger code sequences on each carrier. As mentioned in the section 7.2, in order to make the results comparable, the parameters of chip duration and bit rate were kept constant. The chip duration was selected 10 nsec., (2 times the binwidth of the measurements). These two fixed parameters (chip period and bit rate), make the study of behavior of the other parameters (i.e., the number of carriers \( M \) and number of chips \( L \)) more tractable. In fact these conditions provide (7.6) and therefore the ratio \( M/L \) is kept constant once the mapping parameter \( \alpha \) has been chosen. The values of the parameters in (7.6) are indicated in Table 7.1. It is seen that the number of carriers that was used are powers of 2. This is interesting from DSP point of view, since it allows utilization of the FFT algorithms to perform the OFDM modulation.

Just like the DS-CDMA system in the combination of MC and DS-CDMA techniques, the near-far problem exists. Therefore, in our research the average power control was considered by normalizing the output power of each channel to unity.

Table 7.1 The number of carriers and corresponding number of chips for the MC/CDMA system with constant bandwidth.

<table>
<thead>
<tr>
<th>( M )=Number of carriers</th>
<th>( L )=Number of Chips ((\alpha=1)), BPSK Modulation</th>
<th>( L )=Number of Chips ((\alpha=2)), QPSK Modulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>15</td>
<td>31</td>
</tr>
<tr>
<td>2</td>
<td>31</td>
<td>63</td>
</tr>
<tr>
<td>4</td>
<td>63</td>
<td>127</td>
</tr>
<tr>
<td>8</td>
<td>127</td>
<td>255</td>
</tr>
<tr>
<td>16</td>
<td>255</td>
<td>511</td>
</tr>
<tr>
<td>32</td>
<td>511</td>
<td>1023</td>
</tr>
</tbody>
</table>
The MC/CDMA received signal entails different kind of interferences. In order to study the performance of MC/CDMA technique in indoor communication systems, the behavior of the interfering terms was investigated using the large data base of indoor propagation channel (explained in chapter 2). In Fig. 7.3 the variance of intersymbol interference (ISI) term is sketched against the number of carriers for different antenna separations of Building A (with different average rms delay spread). The same is repeated for the different antenna separations of Building B in Fig. 7.4.

**Figure 7.3** Variance of ISI versus the number of carriers. Building A, solid line: ant. sep. of 5m, (av. $\tau_{\text{rms}}=15.4$ ns), dotted line: ant. sep. of 30m, (av. $\tau_{\text{rms}}=32.4$ ns).

**Figure 7.4** Variance of ISI versus the number of carriers. Building B, solid line: ant. sep. of 10m, (av. $\tau_{\text{rms}}=18.4$ ns), dotted line: ant. sep. of 20m, (av. $\tau_{\text{rms}}=23.1$ ns).
From these figures it is obvious that by increasing the number of tones the variance of ISI is decreasing. However, increasing the \textit{rms} delay spread increases the power of ISI term. Decreasing the power of ISI with increasing the number of carriers is completely justified, since the greater the number of carriers the greater the symbol duration and the lower the effect of the other bits on the current symbol. As mentioned earlier another disturbance which affects the received MC/CDMA signal is ICI. The variance of ICI term has been determined using the large data base of the indoor channel and is illustrated in Figs. 7.5 and 7.6 for different antenna separations of Building A and B, respectively. Again it is seen that the power of interference from the other carriers on the tone under consideration decreases with increasing the number of the carriers. Meanwhile, the larger number of carriers also decreases the ISI terms of other carriers.

\begin{figure}[h]
    \centering
    \includegraphics[width=0.5\textwidth]{vari.png}
    \caption{Variance of ICI versus the number of carriers. Building A, solid line: ant. sep. of 5m, dotted line: ant. sep. of 30m.}
\end{figure}

As explicated before, in the MC/CDMA system due to the access of other users to the channel, a disturbance from other users is introduced to the desired signal. The variation of the power of multiple access interference versus the number of tones for different number of users is shown in Fig. 7.7. The power of MAI increases with the number of users. Increasing the number of carriers will allow better code performances and thus reduces this kind of interference. Therefore, one can obtain better performance with a fixed number of users in the system for a fixed performance. The consideration made about ISI and ICI remain valid since the MAI term contains both ISI and ICI.
The total interference (ISI, ICI and MAI) added on the power basis has been illustrated in Fig. 7.8. Comparing the power of interfering terms of Figs. 7.3, 7.5 and 7.7, it is clear that the variance of MAI term dominates the interference power (even in the case of 2 users). Therefore, although the influence of each interference term decreases with increasing the number of carriers, the power of multiple access disturbance (which increases with the number of users) limits the performance of the system. It should be mentioned that the results reported in Figs. 7.3-7.8 have been averaged over several different code sequences used for spreading of the signals.
Figure 7.8  Power of interference versus the number of carriers. Building A, ant. sep. of 5m, 2 users.

Figure 7.9  Bit error probability versus number of carriers for different signal-to-noise ratios. Building A, ant. sep. of 5m, 2 active users.

The performance of MC/CDMA in terms of bit error probability has been investigated. In Figs. 7.9 and 7.10 the performance of the system versus the number of tones for different signal to noise ratios are revealed. It is seen that for the large number of carriers the BER does not improve too much, this is due to saturation of the power of interfering terms with the number of tones, that was described in Figs. 7.3-7.7. Meanwhile, Figs. 7.9 - 7.10 display that the performance of MC/CDMA system depends strongly on the number of users of the system. Indeed, since the contribution of the power of multiple access interference is dominant with
respect to the other disturbances, it limits the performance of the system. To this end, utilization of the multiuser cancellation methods is suggested, since it remarkably improves the performance of the MC/CDMA system.

![Graph showing bit error probability versus number of carriers for different signal-to-noise ratios.
](image)

**Figure 7.10** Bit error probability versus number of carriers for different signal-to-noise ratios. Building A, ant. sep. of 5m, 4 active users.

### 7.7 CONCLUSIONS

In this chapter a communication system based on the combination of multicarrier and DS-CDMA modulations was discussed. Regardless from the advantages of CDMA system (i.e., fighting multipath fading and providing multiple access), it suffers from the limited capacity as well as spectral inefficiency. On the other hand, MC technique uses the spectrum efficiently, combats frequency selectivity of the channel and is implemented easily by fast Fourier methods. Investigation of the combination of these two techniques by using the data base of indoor channel indicates that although MC/CDMA modulation provides the advantages of the two methods, its performance is mainly limited by multiple access interference. Utilization of the methods of cancellation of multiuser interference is a promising approach for this modulation scheme, however, it increases the complexity of the system. Meanwhile, the sensitivity of MC technique to the frequency offset and phase noise, which consequently leads to the degradation of the performance of MC/CDMA system is another issue that should be considered in the ratio of complexity/performance compromise. In the light of this a differential demodulation scheme, with rather simple implementation is proposed to obtain.
REFERENCES OF CHAPTER 7


CHAPTER 8

CONCLUSIONS AND RECOMMENDATIONS

During this dissertation two main parts of the wireless indoor communication systems were studied. In the first part the propagation of the radio signal in the indoor environments was investigated. Mathematical modeling of the propagation channel with the impulse response method was carried out and characterization of the signal amplitude and phases of the indoor channel was performed, based on the large data base of 12 000 impulse response profiles of the channel. In the second part, the spread spectrum multiple access technique with the spectral efficient modulation scheme in indoor wireless communication systems were investigated. This research was reinforced by utilization of the empirical data base of the indoor communication channel. Based on this categorization, the results of the research are divided in two parts which are recapitulated in the following.

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8.1 CONCLUSIONS ON THE PROPAGATION CHANNEL

Indoor propagation channels were modeled as linear filters. The impulse response functions of these filters characterize the channel. Parameters of the impulse response of the indoor radio propagation channel are the amplitudes, time of arrivals, phases and number of multipath components. Utilization of the large data base of the indoor radio propagation channel indicates that the number of multipath components in each impulse response is a normally distributed random variable with a mean value that increases with increasing the antenna separations. A modified Poisson distribution shows a good fit to the arrival time of the multipath components. Amplitudes are lognormally distributed over local and global areas, with a log-mean which decreases almost linearly with increasing excess delay. Deterministic and random phase increment models show a good fit to the phase of the multipath components. For the small displacement of the receiving antenna the amplitudes of multipath components are correlated. The rms delay spread is normally distributed with mean values that increase with increasing antenna separations.

8.2 RESULTS ON THE SPREAD SPECTRUM MODULATION TECHNIQUES

Direct sequence code division multiple access technique was applied over indoor radio channels. Application of this method is suggested in indoor since it can combat the fading of the channel as well as provide the multiple access capability. However, due to the near/far problem, a stringent power control algorithm is essential for the successful implementation of DS-CDMA in indoor environments, which consequently increases the complexity of the system. Enlargement of the code length of the spreading sequence improves the performance of the system at the price of the spectrum. The indoor channel varies drastically from one location to another. It also necessitates the power control. Performance of the DS-CDMA technique in indoor radio systems is limited by self multipath and multi-user interferences. The contribution of multi-user interference dominates the disturbances, and mainly restricts the performance of the system.

Orthogonal frequency division multiplexing scheme is a promising technique in the indoor communication systems. It provides the spectral efficiency and capability to cope with the frequency selectivity of the channel. Meanwhile, it is implemented efficiently by using FFT algorithm. Results of the application of the multi-carrier technique using the data base of the
indoor channel show that the performance of the system improves with increasing the number of carriers. However, the improvement of the performance for large number of carriers is not large. Introducing the guard period in the MC transmitted signal suppresses the effect of ISI of the channel. The trade off between the bit error probability and the guard period leads to the optimal value of the guard interval. The carrier phase error has a detrimental effect on the performance of the MC transmission in indoor environments. Results show that the MC technique is more sensitive to phase noise than single carrier transmission. Degradation of the performance of the MC transmission due to carrier phase noise increases with the signal constellation. Meanwhile, MC transmission has the limitation of nonlinear distortion in the transmitter.

Results of the combination of the multicarrier and direct sequence CDMA techniques show that regardless of the advantages of the two systems (i.e., fighting multipath - providing multiple access associated with the CDMA, and spectral efficiency - coping with the frequency selectivity of the channel corresponding to the MC), the performance of the MC/CDMA modulation system is mainly limited by multi-user interference. Meanwhile, just like the MC system, the sensitivity of the MC/CDMA technique to the frequency offset and phase noise degrades the performance of the system.

8.3 RECOMMENDATIONS FOR THE FUTURE WORKS

8.3.1 Temporal Variation of the Indoor Channel

As mentioned in chapter 1, the indoor radio propagation channel can be investigated by studying the spatial or temporal variations of the channel. In the spatial variation study of the channel, fluctuations due to motion of the portable antenna in a static environment is explored. It was explained earlier that in this dissertation, the measurements, modeling, and performance evaluation of indoor communication systems have been carried out based on consideration of the spatial variations of the channel. In the general indoor environment, due to motion of people, equipments and portable, the channel also changes with time. Therefore, in addition to the spatial study of the channel, temporal variation of the indoor environment, when both transmitting and receiving antennas are stationary and the medium is changing, should be investigated. The temporal variations of the channel may cause drastic changes in the signal level (deep fades up to 20 dB [1]-[2]), which subsequently influences the performance of the system. This study is
important for the applications such as high rate wireless computer communications when both terminals are fixed but motion of people results in multipath distortions in the received signal. The next step is to combine these two variations together in order to provide a model to describe both kinds of variations of the channel that can also be used to simulate the channel under diversified sets of conditions. In this connection the data base of 11520 seconds of narrow-band CW envelope fading signals, reported in [3]-[4] could be a valuable asset in complement to the spatial variation data base of 12000 impulse response profiles (described in chapter 2), which were used in this dissertation.

8.3.2 General Purpose Propagation Simulator

Based on the large data base of the indoor radio propagation channel, the analysis and modeling of the indoor channel have been carried out in this thesis. A novel aspect of this research is the unique empirically driven phase model described in chapter 4. The extensive measurements and modeling effort of the channel, which have taken more than 6000 man-hours of work, has paved the way for developing a unique simulation package for the indoor radio propagation channel [5]. The reliability of this propagation simulator is bolstered by the size of the data base (12 000 impulse response profiles), which is the largest in its kind, added with the elaborate statistical methods and modeling endeavour discussed in this dissertation. The proposed software package is a general purpose channel simulator. Basic features and capabilities of this simulation software are described below:

1. Large scale path losses. The user specifies his choice of path loss model. If unable to do so, the package generates and outputs a path loss value derived from one of the available models described in [1]. From this path loss value and the input parameters specified by the user (transmitted power, antenna gains, antenna separation, etc.), the average received power at a location is calculated.

2. Complete impulse response of the channel at spatially separated points. This is the core and most important feature of the simulation package. It draws from extensive modeling of the arrival time sequences, the amplitude sequences, and the novel developed phase sequences. The user specifies antenna separation, sampling distance, and the number of impulse response profiles to be generated. The user may also specify the

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1 Main part of this subsection has been written using [5].
topography (LOS or NLOS), or leave it to the package to generate consistent sets of impulse response profiles that are typical of the given environment. Knowing the antenna separation the delay axis is adjusted properly, and given the information in item 1 above, the amplitudes are shifted to their correct value. All necessary correlations and interdependencies between parameters are taken into account internally without involving the user. The resulted sequences of impulse responses corresponding to a given "local" area can be used directly in the simulations involving performance analysis.

3. CW narrowband fading waveforms. Given the antenna separation and topography of the environment a consistent segment of a CW narrowband fading waveform of arbitrary length is generated. Such a waveform should accurately reproduce statistical properties of the measured data.

4. RMS delay spread of the channel. The user of the propagation simulation package supplies input parameters such as antenna separation, sampling distance, and length of the track. The simulation package generates and outputs a consistent set of delay spread values typical of the environment using a previously developed model of rms delay spread. With this direct method, hundreds of thousands of delay spread values can be generated efficiently on a workstation.

5. Generating noise samples. Specifying the noise level of a given environment, the package generates additive Gaussian noise samples and combines them with the narrowband or wideband channel responses. With additional modeling, other types of noise such as impulsive noise typical for some environments, or noise initiated from fluorescent lamps can also be generated.

6. Temporal variation of the channel. This option can also be incorporated in the propagation simulator by using the results of the recent research on the topic [3]-[4]. However, prior to its inclusion to the software, further analysis and modeling of the temporal variation of the indoor channel based on data base of [3] should be carried out, (see also the previous subsection).

The user friendly software package will be a valuable tool in the research and development of indoor wireless communication systems. Using the simulated channel model, the entire communication link (transmitter-channel-receiver) can be simulated on a computer under
realistic propagation conditions. A list of typical applications of this propagation simulator follows:

- Analysis of various pulse shaping and modulation techniques for candidate indoor communication systems
- Investigation of intersymbol interference (ISI) and bit error rate calculations under diversified sets of conditions
- Performance evaluations of different channel coding schemes
- Performance and capacity analysis of the different variants of the spread spectrum multiple access techniques
- Investigation of the adjacent and cochannel interference
- Studies of various diversity schemes for the indoor wireless communications

This indoor propagation software will be more efficient than the other available indoor radio simulation package [6], since it is based on much more measurements (over small and large distances), and invokes more sophisticated models for generating the amplitudes, time of arrivals and phases of the multipath components, (see chapters 2-4). Meanwhile, it should be emphasized that this indoor propagation simulator can be well combined with the existing mobile radio propagation simulator (SURP)\(^2\) in order to provide a general purpose wireless propagation package. Such an overall simulator package (covering indoor and outdoor) will be a precious tool for the research and development in the radio communications, and because of its wide spectrum of usage, can be utilized in the different kind of applications (e.g., in both "microscopic" and "macroscopic" levels). Due to lack of such a means in the wireless community and because of the rapid growth of radio communications, development of this general purpose simulation package of wireless propagation channel is strongly recommended.

\(^2\) Simulation of Urban Radio Propagation (SURP) is a mobile propagation simulator based on extensive urban measurements, that has been widely used in the design and analysis of outdoor communication systems. (See references [7] and [8]).
8.3.3 Spread Spectrum Multiple Access Techniques

As mentioned in the outset of chapter 5, making use of the spread spectrum techniques provides multiple access capability. The spreading of the signals can be performed by direct sequence code division multiple access (DS-CDMA) method, which was carried out in this dissertation, or by other common types (such as frequency hopping or hybrid methods). Referring to chapter 5, one of the major shortcomings of the DS-CDMA technique is the near/far problem. The solution to this problem is the power control algorithm. Invoking of the power control scheme ensures that signals received from users' transmitters near base station do not overwhelm the signal received from distant users. Equally important, power control ensures that each user transmits at only the power level required to achieve the desired transmission performance, which has the advantage of preserving longer battery life in the portable and of reducing interference to other users [9]. However, implementation of a fast and accurate power control algorithm in a fully connected random access network is a tough issue. Meanwhile recent research shows that only 1 dB error in the implementation of the power control, may drastically degrade the performance of the system [10]-[11]. In chapter 3 it was revealed that the signal amplitude fading of the indoor propagation channel has a lognormal distribution. One approach for the utilization of the power control scheme in the analysis and design of direct sequence spread spectrum indoor communication systems relies on this lognormal distribution. Accordingly, by using the inverse lognormal function on the amplitudes of the transmitted signal, the received power from all the transmitters will be equalized and the power control algorithm will be simply implemented. In this regard the parameters of the distribution (i.e., the mean and standard deviation of local and global amplitudes, which were described in chapter 3), will be extremely useful.

Another variant of the CDMA is based on the frequency hopping scheme. In the frequency hopping (FH)-CDMA, the carrier is caused to shift frequency in a pseudo-random manner. Application of the FH/CDMA technique in the mobile and particularly in indoor communication systems is attractive, since FH/CDMA is robust to near/far effect. However, it is more vulnerable to multipath interference than DS-CDMA [12]-[13]. In this aegis, studying of the frequency hopped methods (slow and fast hopping) in indoor wireless system and assessment of the system performance in terms of error probability and capacity, using the mentioned data base of indoor propagation channel is recommended. Extending this research to the investigation of hybrid DS/FH-CDMA in indoor wireless communication systems is another challenging issue. By this combination technique while the main problem of power control is removed, the protection
against multipath and multiple access interferences is provided by the DS and FH schemes, respectively. Again, the data base will have a salient role in this research.

As explained in chapter 5, in the study and investigation of the spread spectrum technique in indoor communication systems, we considered a matched filter receiver. It should be noted that this receiver is suboptimal in multipath channels. The optimum receiver for processing of the wideband received signal in multipath environment is a RAKE receiver [14]. The RAKE receiver has an inherent diversity capability of the multipath signal [15]. In the light of this, besides from the investigation of the external diversity techniques (e.g., multiple antennas, frequencies, etc.), studying the RAKE reception in digital indoor radio is suggested. This research will be enriched by the utilization of the realistic channel data.

8.3.4 MC/CDMA Technique

Combination of two modulation techniques (i.e., MC and DS-CDMA), was discussed in chapter 7. Referring to Fig. 7.1 this novel modulation technique is implemented by performing OFDM modulation on the signal followed by spreading the spectrum using pseudo-random sequences. Another method suggested for MC/CDMA modulation technique in [16] is based on spreading of the data by several code sequences and then OFDM modulation of each spread data. This method is carried out by copying the data of each user into several branches and then multiplication of each parallel stream by one chip of a spreading code followed by MC modulation of each branch and then summing the outputs. Application of this method in the frequency nonselective Rayleigh indoor channel is reported in [17]. In [18] the performance of these two approaches of the MC/CDMA modulation techniques, in a fixed bandwidth, is compared, indicating the latter over performs the former, in a nonselective fading channel. Further analysis of this method of combination of the MC and CDMA techniques is suggested, prolonged with the selection of different reception methods. Comparison of the performance of this method of MC/CDMA in the frequency selective indoor channel using the data base of the indoor propagation channel, is the subject of a future study.

Another challenging issue in this relation is the combination of the MC modulation technique with the frequency hopping CDMA. This new combination technique is quite promising. Since by using the MC technique the advantages described in chapter 6 (i.e., spectral efficiency, combating the frequency selectivity of the channel and suppressing ISI) is achieved. On the other hand, because the spreading of the MC signal is performed by frequency hopping
method, this combined technique will be robust to near/far effect (no power control required). Meanwhile, as we saw in the chapter 7, the main limiting factor on the performance of the MC/DS-CDMA system is the multiple access interference. Utilization of the frequency hopping CDMA with the MC technique has the added advantage of less sensitivity to multiple access interference which will lead to a performance of this technique better than MC/DS-CDMA. Investigation of the performance of this novel modulation method in the indoor communication systems is strongly recommended. Meanwhile, just like what was explained for the MC/DS-CDMA, two approaches can be considered in the case of MC/FH-CDMA, these are 1) MC followed by FH/CDMA, and 2) copying data of each user, and give the data a FH spread followed by OFDM modulation.

In chapter 5 it was explained in detail that in the CDMA system, the output of the matched filter contains residual interference from other users. In military applications of the spread spectrum techniques, the bandwidth is not limited, and therefore, the processing gain can be large enough and the number of users small enough to result in a low level of interference. However, in personal communications this kind of interference is important and due to scarce of bandwidth and the large number of users, it limits the performance of the CDMA as well as MC/CDMA systems. To handle this problem, multi-user detection and cancelation techniques have been proposed by many researchers [19]-[20]. Some of these detectors are optimal (with the disadvantage of computational complexity) and some are suboptimal. Application of adaptive multi-user CDMA detectors and cancelers are reported in [21]. In [22] the idea of the bootstrapped decorrelating algorithm is proposed for the adaptive cancelation of the multi-user interference, which has the advantage that it does not require knowledge of the cross correlation matrix of the different signals at the output of the matched filter. Study and investigation of the multi-user detection-cancelation techniques in the CDMA and MC/CDMA systems for improving the capacity and performance, while having a moderate complexity, is another challenging issue to contemplate.

Regarding the code sequences for spreading the signals, we used Gold codes (see chapter 5). These codes have good correlation properties in the CDMA systems [23]-[24]. However, these codes may not be optimal for the MC/CDMA systems. Referring to section 7.5 it is seen that the expressions of the partial cross correlations in a MC/CDMA system is much more complex than in the CDMA system. In this regards more constraints are expected to be fulfilled in a MC/CDMA system, since the correlation properties should be good for all possible combinations of the carriers. Therefore, studies in order to find sets of code sequences particularly designed for
the MC/CDMA systems are suggested to be carried out. Since, spreading of the signal is an important part of the MC/CDMA technique, investigation for achieving the proper codes having low cross correlations is quite essential in the successful utilization of this novel modulation scheme. On the other hand, in whole of our research on the spread spectrum multiple access techniques we considered rectangular base-band waveforms in the transmitter. However, researches show that making use of the sine pulse, instead of the rectangular, decreases the multiple access interference by about 12%, [25]. Selection of the optimum base-band waveforms subject to minimization of the out of band energy and second moment bandwidth of a multiple access system, leading to the prolate spheroidal wave functions, for AWGN channels, is reported in [26]. Similar results are presented for the DS-CDMA systems in [27]. Selection of the optimal waveforms in the MC/CDMA system in order to minimize the effect of the multiple access interference, is another topic for future research.
REFERENCES OF CHAPTER 8


References of Chapter 8


A.1 Goodness of Fit Figures for Local Amplitude Distributions

![Empirical & Theoretical Distribution Functions](chart)

* Rayleigh, K-S Criterion=0.167353
- Lognormal, K-S Criterion=0.0354954
- Nakagami, K-S Criterion=0.169709
x Rice, K-S Criterion=0.16756
o Weibull, K-S Criterion=0.210139
- Suzuki, K-S Criterion=0.269533

Confidence Level 90 percent
K-S Critical Value=0.0917332

**Figure A.1.1** Empirical and theoretical local amplitude distributions. Building B, antenna separation 5 meters, location 1, excess delay 100 nsec.
Figure A.1.2 Empirical and theoretical local amplitude distributions. Building B, antenna separation 10 meters, location 10, excess delay 150 nsec.

Figure A.1.3 Empirical and theoretical local amplitude distributions. Building A, antenna separation 20 meters, location 10, excess delay 25 nsec.
**Figure A.1.4** Empirical and theoretical local amplitude distributions. Building A, antenna separation 30 meters, location 15, excess delay 125 nsec.

**Figure A.1.5** Empirical and theoretical local amplitude distributions. Building B, antenna separation 5 meters, location 5, excess delay 150 nsec.
Figure A.1.6 Empirical and theoretical local amplitude distributions. Building B, antenna separation 10 meters, location 3, excess delay 200 nsec.

Figure A.1.7 Empirical and theoretical local amplitude distributions. Building B, antenna separation 20 meters, location 4, excess delay 75 nsec.
### A.2 Goodness of Fit Tables for Local Amplitude Distributions

**Table A.2.1** Local amplitude distributions. Building A, antenna separation 10m, locations 6-10. "I" means the distribution passed the K-S test and "0" means it did not.

<table>
<thead>
<tr>
<th>Excess Delay (Nsec.)</th>
<th>25</th>
<th>50</th>
<th>75</th>
<th>100</th>
<th>125</th>
<th>150</th>
<th>175</th>
<th>200</th>
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**Table A.2.2** Local amplitude distributions. Building A, antenna separation 20m, locations 11-15. "I" means the distribution passed the K-S test and "0" means it did not.

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<th>Excess Delay (Nsec.)</th>
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Table A.2.6  Local amplitude distributions. Building B, antenna separation 30m, locations 11-15. "1" means the distribution passed K-S test and "0" means it did not.

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A.3 Goodness of Fit Figures for Global Amplitude Distributions

Figure A.3.1  Empirical and theoretical global amplitude distributions. Building A, antenna separation 30 meters, excess delay 125 nsec.

Figure A.3.2  Empirical and theoretical global amplitude distributions. Building B, antenna separation 10 meters, excess delay 225 nsec.
### A.4 Tables of Wilcoxon Test for Global Amplitude Distributions

#### Table A.4.1 The value of $\omega^2$ (Wilcoxon Test) for global amplitude distributions. Building A, antenna separation of 5 meters.

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#### Table A.4.2 The value of $\omega^2$ (Wilcoxon Test) for global amplitude distribution. Building A, antenna separation 20 meters.

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Table A.4.3  The value of $\omega^2$ (Wilcoxon Test) for global amplitude distributions. Building A, antenna separation of 30 meters.

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Table A.4.4  The value of $\omega^2$ (Wilcoxon Test) for global amplitude distributions. Building B, antenna separation of 5 meters.

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Table A.4.6  The value of $\omega^2$ (Wilcoxon Test) for global amplitude distributions. Building B, antenna separation of 20 meters.

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A.5 Figures of Mean and Standard Deviation of Amplitudes

![Graph 1](image1)

**Figure A.5.1** Mean of amplitudes versus excess delay. Averaged over all locations with antenna separation 20 meters of Building A.

![Graph 2](image2)

**Figure A.5.2** Mean of amplitudes versus excess delay. Averaged over all locations with antenna separation 10 meters of Building B.
Figure A.5.3 Standard deviation of amplitudes versus excess delay. Averaged over all locations with antenna separation 20 meters of Building A.

Figure A.5.4 Standard deviation of amplitudes versus excess delay. Averaged over all locations with antenna separation 10 meters of Building B.
A.6 Tables of Mean and Standard Deviation of Amplitudes

Table A.6.1 Mean of amplitudes at different excess delays. Different locations of Building B, antenna separation of 30 meters. All values in dB.

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* 0 means no component present for calculating of mean and variance at that delay.
Table A.6.2 Standard deviation of amplitudes at different excess delays. Different locations of Building B, antenna separation of 30 meters. All values in dB.

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* 0 means no component presents for calculating of mean and variance at that delay.
APPENDIX B

Derivation of (4.25).

Using (4.21) and (4.23) and introducing $A_{k,n}$ and $B_{k,n}$ as

\[ A_{k,n} = \sum_{m=1}^{M} \alpha_{k,n,m} \cos \beta_{k,n,m} \]  

\[ B_{k,n} = \sum_{m=1}^{M} \alpha_{k,n,m} \sin \beta_{k,n,m} \]

(B.1)  

(B.2)

gives

\[ \phi_{k,n} = \tan^{-1} \left[ \frac{A_{k,n} \sin \theta_{0,n} + B_{k,n} \cos \theta_{0,n}}{A_{k,n} \cos \theta_{0,n} - B_{k,n} \sin \theta_{0,n}} \right] \]  

(B.3)

by denoting

\[ \zeta_{k,n} = \tan^{-1} \left( \frac{B_{k,n}}{A_{k,n}} \right) \]  

(B.4)

(B.3) becomes

\[ \phi_{k,n} = \theta_{0,n} + \zeta_{k,n} \]  

(B.5)
APPENDIX C

Derivation of (4.41)

Since the phase increments $\varphi^{(III)}_{k,n}$ are independent, variance of $\zeta_{k,n}$ is

$$\sigma_{\zeta}^2 = E[\zeta_{k,n}^2] = \sum_{i=0}^{k} \left[ \sigma_{\varphi^{(III)}_{i,n}} \right]^2 = \sum_{i=0}^{k} \sigma_{\max}^2 \left( 1 - e^{-b_s \frac{2}{X}} \right)$$

(C.1)

Considering position of profile $s_i = iD$, ($D$ is sampling distance), we have

$$\sigma_{\zeta}^2 = \sum_{i=0}^{k} \sigma_{\max}^2 \left( 1 - e^{-bD_s \frac{2}{X}} \right)^2$$

(C.2)

Let us denote $e^{-bD_s \frac{2}{X}} \Delta x$, therefore (C.2) becomes

$$\sigma_{\zeta}^2 = \sum_{i=0}^{k} \sigma_{\max}^2 \left( 1 - x_i \right)^2 = \sum_{i=0}^{k} \sigma_{\max}^2 \left( 1 - 2x_i + x_i^2 \right)$$

(C.3)

second and third terms of (C.3) can be written as

$$\sum_{i=0}^{k} x_i^2 = \frac{1 - x^{2(k+1)}}{1 - x^2}$$

(C.4)

$$\sum_{i=0}^{k} x_i = \frac{1 - x^{k+1}}{1 - x}$$

(C.5)

Substituting (C.4) and (C.5) in (C.3) gives (4.41).
APPENDIX D

Derivation of (4.42).

Since \( \phi_{\alpha,n} \) and \( \zeta_{k,n} \) are independent, the probability density function of the sum is the convolution of pdf of each variable. Instead of convolving \( \text{Unif}[-\pi, \pi] \), with zero mean Gaussian distribution, it is easier to convolve derivative of \( \text{Unif}[-\pi, \pi] \), which is two Dirac functions with amplitude \( 1/2\pi \) and \(-1/2\pi \) located at \(-\pi \) and \( \pi \) respectively, with Gaussian distribution and then integrate. Suppose \( f_1(x) \) be \( \text{Unif}[-\pi, \pi] \) corresponding to \( \phi_{\alpha,n} \) and \( f_2(x) \) zero mean Gaussian distribution with variance \( \sigma^2 \), associated to \( \zeta_{k,n} \). Denoting pdf of \( \phi_{k,n} \) as \( f(x) \), we have

\[
f(x) = \frac{1}{2\pi} \int_{-\infty}^{x} \left[ f_2(y + \pi) - f_2(y - \pi) \right] dy
\]

which is

\[
f(x) = \frac{1}{2\pi \sigma \sqrt{2\pi}} \int_{-\infty}^{x} (e^{-\frac{(y+\pi)^2}{2\sigma^2}} - e^{-\frac{(y-\pi)^2}{2\sigma^2}}) \, dy
\]

by denoting \( (y+\pi)/\sigma \Delta u \) and \( (y-\pi)/\sigma \Delta v \) we have

\[
f(x) = \frac{1}{2\pi} \left[ \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{u^2}{2}} du - 1 \right] \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{v^2}{2}} dv
\]

which is

\[
f(x) = \frac{1}{2\pi} \left[ 1 - Q\left(\frac{x+\pi}{\sigma}\right) - 1 + Q\left(\frac{x-\pi}{\sigma}\right) \right] = \frac{1}{2\pi} \left[ Q\left(\frac{x-\pi}{\sigma}\right) - Q\left(\frac{x+\pi}{\sigma}\right) \right]
\]

By removing modulo \( 2\pi \), the CDF of \( \phi_{k,n} \) will be the same as (4.42).
APPENDIX E

Derivation of (6.35).
Referring to (6.30) the mean of the ISI term is

$$
E\{ISI\} = E \sum_{n=1}^{N-1} a_n [d_k(-1)X_{k,k}^n + d_k(0)\tilde{X}_{k,k}^n] = \sum_{n=1}^{N-1} E\{a_n [d_k(-1)X_{k,k}^n + d_k(0)\tilde{X}_{k,k}^n]\}
$$

(E.1)

Due to independence of $a_n$ with $d_k$ and $X_{k,k}$ and $\tilde{X}_{k,k}$ it becomes

$$
E\{ISI\} = \sum_{n=1}^{N-1} a_n [d_k(-1)\overline{X}_{k,k}^n + d_k(0)\overline{\tilde{X}}_{k,k}^n]
$$

(E.2)

Since $E[d_k(-1)]=E[d_k(0)]=0$, the mean of ISI will be zero. For the calculation of variance we have

$$
\sigma_{ISI}^2 = E \sum_{n_1=1}^{N-1} \sum_{n_2=0}^{N-1} a_{n_1}a_{n_2} [d_k(-1)X_{k,k}^{n_1} + d_k(0)\tilde{X}_{k,k}^{n_1}] \cdot [d_k(-1)X_{k,k}^{n_2} + d_k(0)\tilde{X}_{k,k}^{n_2}]
$$

(E.3)

Again, because $a_n$ and $d_k$ are independent of $X_{k,k}$ and $\tilde{X}_{k,k}$, and due to the independence of data bits at different intervals, (E.3) can be written as

$$
E\{ISI\} = \sum_{n_1=1}^{N-1} \sum_{n_2=0}^{N-1} a_{n_1}a_{n_2} [d_k(-1)^2X_{k,k}^{n_1}\tilde{X}_{k,k}^{n_2} + d_k(0)^2\overline{X}_{k,k}^{n_1}\overline{\tilde{X}}_{k,k}^{n_2}]
$$

(E.4)

which is

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\[ \sigma_{ISI}^2 = \sum_{n_1=1}^{N-1} \sum_{n_2=0}^{N-1} \frac{a_{n_1} a_{n_2}}{a_{n_1} a_{n_2}} \left[ X_{k,k}^n X_{k,k}^n + X_{k,k}^n X_{k,k}^n \right] \]  

(E.5)

Considering independent path components, that is  
\[ E[a_{n_1} a_{n_2}] = \sigma_{n_1}^2 \delta(n_1 - n_2) \]  
Eq. (E.5)

becomes

\[ \sigma_{ISI}^2 = \sum_{n=1}^{N-1} \sigma_{a_n}^2 \left[ (X_{k,k}^n)^2 + (X_{k,k}^n)^2 \right] \]  

(E.6)

Using (6.27) the first term in the bracket of (E.6) is calculated as

\[ (X_{k,k}^n)^2 = E[\cos^2 \phi_{k,n} R_{k,k}^2(\tau_n)] = E\left( \frac{\tau_n^2}{2} \right) + E\left( \frac{\tau_n^2}{2} \cos(4\pi f_k \tau_n - 2\theta_n) \right) \]  

(E.7)

where  
\[ \phi_{k,n} = 2\pi f_k \tau_n - \theta_n \]  

By denoting

\[ g(\tau_n, \theta_n) \triangleq \tau_n^2 \cos(4\pi f_k \tau_n - 2\theta_n) \]  

(E.8)

(E.7) is written as

\[ (X_{k,k}^n)^2 = \frac{\tau_n^2}{2} + \frac{1}{2} E\left[ g(\tau_n, \theta_n) \right] \]  

(E.9)

The second term of (E.9) can be calculated as

\[ E[g(\tau_n, \theta_n)] = E_{\theta_n}[E_{\tau_n}[g(\tau_n, \theta_n)|\theta_n]] \]  

(E.10)

Omitting the subscript \( n \) for the ease of notation.
\[ E_{\tau} \left[ g(\tau, \theta) | \theta \right] = \int_{\tau} \tau^2 \cos(4\pi f_k \tau - 2\theta) f_T(\tau) \, d\tau \]

\[ = \cos 2\theta \int_{\tau} (\tau^2 \cos 4\pi f_k \tau) f_T(\tau) \, d\tau + \sin 2\theta \int_{\tau} (\tau^2 \sin 4\pi f_k \tau) f_T(\tau) \, d\tau \]

\[ = \alpha_1 \cos 2\theta + \alpha_2 \sin 2\theta \]

(E.11)

where \( f_T(\tau) \) is the probability density function of \( \tau \). By assuming a uniform \((0,2\pi)\) distribution for \( \theta \), Eq. (E.11) results

\[ E[g(\tau, \theta)] = E_{\theta}[\alpha_1 \cos 2\theta + \alpha_2 \sin 2\theta] = \alpha_1 E[\cos 2\theta] + \alpha_2 E[\sin 2\theta] = 0 \]  

(E.12)

Therefore, (E.7) yields

\[ \overline{(X_{k,k}^n)^2} = \tau_n \]  

(E.13)

In a same manner one can show that

\[ \overline{(X_{k,k}^n)^2} = \frac{T^2}{2} - T \tau_n + \frac{\tau_n}{2} \]  

(E.14)

Substituting (E.13) and (E.14) in (E.6) results into

\[ \sigma_{f_{SL}}^2 = \sum_{n=1}^{N-1} \sigma_n^2 \left[ \tau_n^2 - T \tau_n + \frac{T^2}{2} \right] \]  

(E.15)

which is (6.35).


APPENDIX F

Derivation of (6.36).
Referring to (6.31), ICI can be written as

\[ ICI = \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} a_n^m \{ d_m(-1) U_{m,k,n} + d_m(0) V_{m,k,n} \} \tag{F.1} \]

where

\[ U_{m,k,n} = X_{m,k}^n + Y_{m,k}^n \]
\[ V_{m,k,n} = X_{m,k}^n - Y_{m,k}^n \tag{F.2} \]

Similar to what was carried out for the calculation of the mean of ISI term, one can show that \( E[ICI] = 0 \). The variance of ICI can be calculated as

\[ \sigma^2_{ICI} = E \sum_{n_1=0}^{N-1} \sum_{m_1=0}^{M-1} \sum_{n_2=0}^{N-1} \sum_{m_2=0}^{M-1} a_{n_1} a_{n_2} \left[ d_{m_1}(-1) U_{m_1,k,n_1} + d_{m_1}(0) V_{m_1,k,n_1} \right] \left[ d_{m_2}(-1) U_{m_2,k,n_2} + d_{m_2}(0) V_{m_2,k,n_2} \right] \tag{F.3} \]

Using similar assumptions as Appendix E, (F.3) can be written as

\[ \sigma^2_{ICI} = \sum_{n_1=0}^{N-1} \sum_{m_1=0}^{M-1} \sum_{n_2=0}^{N-1} \sum_{m_2=0}^{M-1} \frac{\bar{a}_{n_1} \bar{a}_{n_2}}{M} \left[ d_{m_1}(-1) \frac{d_{m_2}(-1)}{U_{m_1,k,n_1} U_{m_2,k,n_2}} \right] \frac{d_{m_1}(0) d_{m_2}(0)}{V_{m_1,k,n_1} V_{m_2,k,n_2}} \tag{F.4} \]

or

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\[ \sigma_{ICI}^2 = E \sum_{n_1=0}^{N-1} \sum_{n_2=0}^{N-1} \sum_{m=0, m \neq k}^{M-1} a_{n_1} a_{n_2} \left[ U_{m,k,n_1} U_{m,k,n_2} + V_{m,k,n_1} V_{m,k,n_2} \right] \]  

(F.5)

Using the independent path components it becomes

\[ \sigma_{ICI}^2 = \sum_{n=0}^{N-1} \sum_{m=0, m \neq k}^{M-1} a_n^2 \left[ U_{m,k,n}^2 + V_{m,k,n}^2 \right] \]  

(F.6)

and

\begin{align*}
U_{m,k,n} &= (X_{m,k})^n + (Y_{m,k})^n + 2 X_{m,k} Y_{m,k} \\
V_{m,k,n} &= (\dot{X}_{m,k})^n + (\dot{Y}_{m,k})^n + 2 \dot{X}_{m,k} \dot{Y}_{m,k}
\end{align*}

(F.7)

By the assumption of independent uniform distributions for \( \tau \) and \( \theta \) in the interval of \([0, T]\) and \([0, 2\pi]\) respectively, it can be shown that

\begin{align*}
(X_{m,k})^n &= (\dot{X}_{m,k})^2 = \frac{T^2}{16 \pi^2 (m - k)^2} \\
(Y_{m,k})^n &= (\dot{Y}_{m,k})^2 = \frac{3 T^2}{16 \pi^2 (m - k)^2} \\
X_{m,k} Y_{m,k} &= \dot{X}_{m,k} \dot{Y}_{m,k} = 0
\end{align*}

(F.8)

Therefore, the variance of ICI term is

\[ \sigma_{ICI}^2 = \sum_{n=0}^{N-1} \sum_{m=0, m \neq k}^{M-1} \frac{1}{2} a_n^2 \frac{T^2}{\pi^2 (m - k)^2} \]  

(F.9)

or

\[ \sigma_{ICI}^2 = \frac{T^2}{2 \pi^2} \sum_{n=0}^{N-1} \sum_{i=k}^{M-k-1} \frac{1}{l^2} \]  

(F.10)

which is the same as (6.36).
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<th>Definition</th>
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<td>Additive White Gaussian Noise</td>
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<td>Binary Phase Shift Keying</td>
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<td>Code Division Multiple Access</td>
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APPENDIX H

LIST OF MAJOR SYMBOLS

The main mathematical symbols are listed in this Appendix. Symbols are independent in the different chapters. Therefore, all the major symbols of each chapter are listed and defined, separately. Symbols are sorted alphabetically. Greek characters are sorted based on their pronunciation, rather than the Greek alphabet.

Chapter 1

A \quad \text{Amplitude of the narrowband CW signal}

a_i \quad \text{Amplitude of the } i\text{th multipath component}

\delta \quad \text{Dirac delta function}

\Delta f \quad \text{Sampling frequency}

f \quad \text{Frequency (Hz)}

G(z) \quad \text{Transfer function of the linear filter for modeling of the indoor channel in the frequency domain}

h(t, \tau) \quad \text{Impulse response of linear time variant filter of indoor radio propagation channel}

h(t) \quad \text{Impulse response of time invariant filter of indoor radio propagation channel}

H(f) \quad \text{Frequency response of the indoor channel}

H_k \quad \text{The } k\text{th sample of the transfer function of the indoor channel}

j \quad \sqrt{-1}

K \quad \text{Number of the samples of the frequency response of the indoor channel}

N \quad \text{Number of the multipath components}

n(t) \quad \text{Additive white Gaussian noise}

v_k \quad \text{The } k\text{th sample of the noise}

\omega_0 \quad \text{Carrier frequency (rad/sec.)}

\omega \quad \text{Frequency (rad/sec.)}

p \quad \text{Number of poles of the filter for modeling of the indoor channel in the frequency domain}

p_i \quad \text{The } i\text{th pole of the filter for modeling the indoor channel in the frequency domain}

\phi \quad \text{Phase of the narrowband CW signal}

R \quad \text{Autocorrelation function of the frequency response of the indoor channel}

s(t) \quad \text{Transmitted signal}

\sigma^2 \quad \text{Variance of the input noise to the filter for modeling of the indoor channel in the frequency domain}
$T_k$ The $k$th sample of the transfer function of the indoor channel added by noise
$	au_i$ Arrival time of the $i$th path component
$\theta_i$ Phase of the $i$th path component
$y(t)$ Output of the indoor radio propagation channel

Chapter 2

$a_k$ Amplitude of the $k$th multipath component
$I$ Number of paths occurring in a given time interval
$\lambda(t)$ Mean arrival rate of the multipath components at a time $t$
$\mu$ Integral of mean arrival rate over a given time interval $T$
$N$ Length of the window function
$t_a$ Arrival time of the first path
$\tau_{rms}$ RMS delay spread
$\bar{\tau}$ Mean excess delay
$\tau_k$ Arrival time of the $k$th path component
$w(n)$ The window function

Chapter 3

$a$ ln10/10
$\alpha$ Level of confidence
$\chi^2$ Criterion of the Chi-square test
$D_N$ Maximum distance between the theoretical and empirical distributions in K-S test
$\Delta$ Distance between the theoretical and empirical distribution functions
$E[.]$ Expected value
$e_{N,a}$ Critical value of the K-S test
$\eta^*$ Criterion of the Mean Square Error test
$f(r)$ Probability density function of the amplitude fading signal $r$
$F(x)$ Cumulative distribution function of $x$
$F_N(x)$ Empirical cumulative distribution function of $x$
$\Gamma(x)$ Gamma function of $x$
$H(x,...)$ Confluent hypergeometric function
$I_0(x)$ Modified Bessel function of the first kind and zeroth order
$j$ $\sqrt{-1}$
$m$ Parameter of the Nakagami distribution, $m = \Omega^2/\text{Var}(r^2)$
$\mu$ Mean of the amplitude fading signal
$N$ Sample size
$\Omega$ Expected value of $r^2$ in the Nakagami distribution
$\omega^2$ Criterion of the Wilcoxon test
$\psi'(m)$ Digamma function = d ln$\Gamma(m)/dm$
$\psi''(m)$ Trigamma function = $d^2$ ln$\Gamma(m)/dm^2$
$r$ Amplitude of the fading signal
$\bar{r}$ Expected value of $r$
$S$ Sample mean
$S'$ Sample mean in log scale
$\sigma$ Standard deviation of the amplitude fading signal
$u(.)$ Unit step function
$V$ Sample variance
$V'$ Sample variance in log scale
The $i$th sample of the amplitude data

The $i$th sorted value of the amplitude data samples

Chapter 4

$a_i$ Amplitude of the $i$th multipath component

$a_{k,n,m}$ Real attenuation coefficient of the reflected path from the $m$th scatterer on the $n$th ellipse in the $k$th profile

$\rho_{k,n,m}$ Normalized distance of the $k$th position of portable from the $m$th scatterer on the $n$th ellipse, of the corresponding distance in the initial position of the portable

$c$ Speed of light

$D$ Sampling distance

$\Delta \rho_{k,n,m}$ Normalized distance of the $k$th position of portable from the $m$th scatterer on the $n$th ellipse, of the corresponding distance in the $(k-1)$st position of the portable

$f_m$ Maximum Doppler shift (portable speed / wavelength)

$G_1$ Number of computations for generating initial phases of Model I

$G_2$ Number of computations for generating each uniform distributed initial phases of Model II

$h_k(t)$ Impulse response of the indoor propagation channel at the $k$th position of the portable

$j \sqrt{(-1)}$ Final position number of the portable

$K$ Average number of correlated path components in a fixed excess delay

$\lambda$ Wavelength

$M$ Number of scatterers around each confocal ellipse

$N_k$ Number of multipath components at the $k$th profile

$NC_I$ Number of computations for all excess delays and profiles of Model I

$NC_{II}$ Number of computations for all excess delays and profiles of Model II

$N$ Average number of path components

$p_n$ Half of the major axis of the $n$th ellipse

$\Phi_k$ Resultant phase of the signal in the $k$th profile (position of portable)

$\Phi_{k,n}$ Phase of the path component at the $n$th excess delay and $k$th profile

$\Phi_{(I),k,n}$ Phase increment at $n$th excess delay and $k$th profile of the Model I

$\Phi_{(II),k,n}$ Phase increment at $n$th excess delay and $k$th profile of the Model II

$\Psi_{k,n,m}$ Angle between the line connecting the $m$th scatterer on the $n$th ellipse to the $k$th position of the portable

$Q(x)$ Standard $Q$ function $= (2\pi)^{-1/2} \int_{x}^{\infty} e^{-\alpha^2/2} d\alpha$

$q_n$ Half of the minor axis of the $n$th ellipse

$r$ Half of the focal distance of the confocal ellipses

$R_k$ Resultant envelope of the signal in the $k$th profile (position of portable)

$R_{k,n}$ Path amplitude of the signal at $n\tau_0$ excess delay and $k$th profile

$S_{n,m}$ The $m$th scatterer on the $n$th ellipse

$s_k$ Spatial separation between $(k-1)$st and $k$th profiles

$\sigma$ Standard deviation of phase increments of the Model II

$\sigma_{\text{max}}$ Maximum standard deviation of the phase increments of the Model II

$t$ Ratio of the number of computations of Model II to the number of computations of Model I

$\tau_0$ Binwidth of the measurements

$\tau_i$ Arrival time of the $i$th path component
Appendix H

\[ \theta_i \quad \text{Phase of the } i \text{th path component} \]
\[ \text{Unif}(a, b) \quad \text{Uniform distribution function between } a \text{ and } b \]
\[ U_1 \quad \text{Number of computations for updating each path phase component in Model I} \]
\[ U_2 \quad \text{Number of computations for updating each path phase component in Model II} \]
\[ \zeta_{k,n} \quad \text{Phase increment at the } k \text{th profile and } n \text{th excess delay with reference to the initial path phase} \]

Chapter 5

\[ A \quad \text{Amplitude of the transmitted signal} \]
\[ a_{k,n} \quad \text{Amplitude of multipath component at the } n \text{th excess delay of } k \text{th profile} \]
\[ b_{n}(t) \quad \text{Binary data signal of the } k \text{th portable} \]
\[ b_{k,i-1} \quad \text{Previous data bit of the } k \text{th user} \]
\[ b_{k,0} \quad \text{Current data bit of the } k \text{th user} \]
\[ B_{SS} \quad \text{Bandwidth of the spread spectrum signal} \]
\[ c_{k}(t) \quad \text{Spreading code of the } k \text{th user of the DS-CDMA system} \]
\[ f_a(a) \quad \text{Pdf of the amplitude fading signal } a \]
\[ f_{\Sigma}(\xi) \quad \text{Pdf of the total interference} \]
\[ I_{\text{SMP}} \quad \text{Self multipath interference} \]
\[ I_{\text{MAMP}} \quad \text{Mutual multiple access multipath interference} \]
\[ I_{i} \quad \text{Self multipath plus mutual multiple access multipath interference} \]
\[ j \quad \sqrt{-1} \]
\[ K \quad \text{Number of users} \]
\[ M \quad \text{Processing gain} \]
\[ n(t) \quad \text{Additive white Gaussian noise} \]
\[ N_0/2 \quad \text{Two sided spectral density height of the noise} \]
\[ N_k \quad \text{Number of multipath components at } k \text{th profile} \]
\[ \omega_0 \quad \text{Carrier frequency} \]
\[ P(E) \quad \text{Bit error probability} \]
\[ p(t) \quad \text{Unit amplitude rectangular pulse in the interval } [0, T] \]
\[ P_T_c(t) \quad \text{Unit amplitude rectangular pulse with duration } T_c \]
\[ \varphi_k \quad \text{Phase of } k \text{th user (portable)} \]
\[ \Pi(t/T) \quad \text{Rectangular pulse with amplitude 1 and duration } T \]

\[ Q(x) \quad \text{Standard Q function} \]
\[ = (2\pi)^{-1/2} \int_{x}^{\infty} e^{-\xi^2/2} d\xi \]
\[ r(t) \quad \text{Received signal at the output of the channel} \]
\[ R_v(\tau) \quad \text{Autocorrelation function of the random process } v(t) \]
\[ R_{k,i}(\tau) \quad \text{Partial cross correlation of the } k \text{th and } i \text{th spreading codes at time } \tau \]
\[ \hat{R}_{k,i}(\tau) \quad \text{Partial cross correlation of the } k \text{th and } i \text{th spreading codes at time } \tau \]
\[ \text{sinc}(x) \quad (\sin \pi x) / (\pi x) \]
\[ s_k(t) \quad \text{Transmitted signal of the } k \text{th user} \]
\[ T \quad \text{Duration of data} \]
\[ T_c \quad \text{Chip duration} \]
\[ \tau_{k,n} \quad \text{Arrival time of multipath component at } n \text{th excess delay of } k \text{th profile} \]
\[ \theta_{k,n} \quad \text{Phase of multipath component at } n \text{th excess delay of } k \text{th profile} \]
\[ Z_i \quad \text{Output of the matched filter matched to the } i \text{th signal} \]
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_n$</td>
<td>Amplitude of the $n$th multipath component</td>
</tr>
<tr>
<td>$\sigma_m$</td>
<td>Magnitude slope of the transfer function of the channel at the frequency $f_m$</td>
</tr>
<tr>
<td>$B_{MC}$</td>
<td>Bandwidth of the MC signal</td>
</tr>
<tr>
<td>$\hat{d}_i$</td>
<td>Estimate of the data bit on the $i$th carrier at the receiver</td>
</tr>
<tr>
<td>$d_m(-1)$</td>
<td>Previous transmitted symbol on the carrier frequency $f_m$</td>
</tr>
<tr>
<td>$d_m(0)$</td>
<td>Current transmitted symbol on the carrier frequency $f_m$</td>
</tr>
<tr>
<td>$d_m(i)$</td>
<td>Symbol of the $m$th subchannel at time interval $iT$</td>
</tr>
<tr>
<td>$\Delta f$</td>
<td>Distance between two consecutive carriers</td>
</tr>
<tr>
<td>$f_0$</td>
<td>Lowest frequency of the carriers</td>
</tr>
<tr>
<td>$f_m$</td>
<td>The $m$th frequency of the carriers</td>
</tr>
<tr>
<td>$h(t)$</td>
<td>Impulse response of the channel</td>
</tr>
<tr>
<td>$H(f)$</td>
<td>Transfer function of the linear channel</td>
</tr>
<tr>
<td>$j$</td>
<td>$\sqrt{-1}$</td>
</tr>
<tr>
<td>$M$</td>
<td>Number of carriers of the MC system</td>
</tr>
<tr>
<td>$N$</td>
<td>Number of multipath components</td>
</tr>
<tr>
<td>$n(t)$</td>
<td>Additive white Gaussian noise</td>
</tr>
<tr>
<td>$N_{ps}/2$</td>
<td>Two sided spectral density height of the noise</td>
</tr>
<tr>
<td>$p(t)$</td>
<td>Response of the transmitter filter (rectangular with amplitude 1 and duration $T$)</td>
</tr>
<tr>
<td>$P(E)$</td>
<td>Bit error probability</td>
</tr>
</tbody>
</table>

$$Q(x) = (2\pi)^{-1/2} \int_{x}^{\infty} e^{-\alpha^2/2} d\alpha$$

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$</td>
<td>Input data rate</td>
</tr>
<tr>
<td>$r(t)$</td>
<td>Received signal at the output of the channel</td>
</tr>
<tr>
<td>$r_k$</td>
<td>Sample of the received signal $r(t)$ at $kT_1$</td>
</tr>
<tr>
<td>$R_{m,k}(\tau)$</td>
<td>Partial cross correlation between carriers $m$ and $k$ at time $\tau$</td>
</tr>
<tr>
<td>$\hat{R}_{m,k}(\tau)$</td>
<td>Partial cross correlation between carriers $m$ and $k$ at time $\tau$</td>
</tr>
<tr>
<td>$s(t)$</td>
<td>Multicarrier transmitted signal</td>
</tr>
<tr>
<td>$s_n$</td>
<td>Sample of the transmitted signal $s(t)$ at $nT_1$</td>
</tr>
<tr>
<td>$S(f)$</td>
<td>Spectrum of the MC signal</td>
</tr>
<tr>
<td>$\sin(x)$</td>
<td>$\sin(\pi x)/(\pi x)$</td>
</tr>
<tr>
<td>$\sigma_{ISI}^2$</td>
<td>Variance of the intersymbol interference</td>
</tr>
<tr>
<td>$\sigma_{ICl}^2$</td>
<td>Variance of the interchannel interference</td>
</tr>
<tr>
<td>$\sigma_n^2$</td>
<td>Expected value of $a_n^2$</td>
</tr>
<tr>
<td>$T$</td>
<td>Symbol duration</td>
</tr>
<tr>
<td>$T_1$</td>
<td>Time duration of the original data</td>
</tr>
<tr>
<td>$T_G$</td>
<td>Guard period</td>
</tr>
<tr>
<td>$T^*$</td>
<td>Effective symbol duration</td>
</tr>
<tr>
<td>$t_{g_m}$</td>
<td>Group delay of the channel at the frequency $f_m$</td>
</tr>
<tr>
<td>$\tau_n$</td>
<td>Arrival time of the $n$th path component</td>
</tr>
<tr>
<td>$\theta_n$</td>
<td>Phase of the $n$th path component</td>
</tr>
<tr>
<td>$z_k$</td>
<td>Decision variable for detecting data on the $k$th carrier</td>
</tr>
</tbody>
</table>
Chapter 7

\( a_{kn} \) Amplitude of multipath component at the \( n \)th excess delay of the \( k \)th profile

\( B_{MC/CDMA} \) Bandwidth of the MC/CDMA signal

\( c_g(t) \) Spreading code of the \( k \)th user

\( d_{mk}(i) \) Symbol of the \( m \)th subchannel at the time interval \( iT \) of the \( k \)th portable

\( d_{mk}(-1) \) Previous symbol of the \( m \)th subchannel of the \( k \)th portable

\( d_{mk}(0) \) Current symbol of the \( m \)th subchannel of the \( k \)th portable

\( f_0 \) Lowest frequency of the carriers

\( f_m \) The \( m \)th frequency of the carriers

\( h_k(t) \) Impulse response of the channel of the \( k \)th portable

\( j \sqrt{-1} \)

\( K \) Number of users

\( L \) Length of the spreading code

\( M \) Number of carriers

\( n(t) \) Additive white Gaussian noise

\( N_{o/2} \) Two sided spectral density height of the noise

\( N_k \) Number of multipath components at the \( k \)th profile

\( P(E) \) Bit error probability

\( P_T(t) \) Rectangular pulse with duration \( T \) and amplitude 1

\( Q(x) \) Standard \( Q \) function \( = (2\pi)^{-1/2} \int_{\infty}^{\infty} e^{-\alpha^2/2} d\alpha \)

\( r(t) \) Received signal at the output of the channel

\( R_{\text{chip}} \) Chip rate

\( R_{\text{bit}} \) Bit rate

\( R_{k,k',m,m'}(\tau) \) Partial cross correlation of the code sequences \( k \) and \( k' \) on the carrier \( m \) and \( m' \) at the time \( \tau \)

\( \hat{R}_{k,k',m,m'}(\tau) \) Partial autocorrelation of the code sequences \( k \) and \( k' \) on the carrier \( m \) and \( m' \) at the time \( \tau \)

\( s_k(t) \) Multicarrier spread transmitted signal of the \( k \)th user

\( \text{sinc}(x) = (\sin(\pi x))/(\pi x) \)

\( S_{m,m'}^{\perp} \) Small scale partial cross correlation of the code sequences \( k \) and \( k' \) on the carrier \( m \) and \( m' \)

\( \hat{S}_{m,m'}^{\perp} \) Small scale partial cross correlation of the code sequences \( k \) and \( k' \) on the carrier \( m \) and \( m' \)

\( \sigma^2 \) Variance of the interference

\( T \) Symbol duration \( = LT_c \)

\( T_c \) Chip duration

\( T_1 \) Initial sampling time

\( \tau_{kn} \) Arrival time of multipath component at the \( n \)th excess delay of the \( k \)th profile

\( \theta_{kn} \) Phase of multipath component at the \( n \)th excess delay of the \( k \)th profile

\( z_{k',m'} \) Decision variable corresponding to the user \( k' \) and carrier \( m' \)
Samenvatting (Summary in Dutch)

In de afgelopen jaren heeft draadloze communicatie een aanzienlijk groei doorgemaakt. Wereldwijd groeien onderzoek naar en ontwikkeling van radio communicatie exponentieel, resulterend in een groot aanbod van draadloze produkten op de markt. Digitale communicatie, eerst gebruikt in militaire toepassingen wegens de bestendigheid tegen storing en afluisteren, is nu hard op weg naar commerciële toepassingen. De superioriteit van de prestatie van digitale communicatie systemen boven analoge, samen met de technologische vooruitgang op het gebied van solid-state micro-elektronica, hebben deze trend ondersteund. Het gebruik van digitale technieken in draadloze communicatie, die een goede kwaliteit bieden en lage kosten hebben maar ook diensten aanbieden aan een groot aantal gebruikers, biedt goede vooruitzichten op de realisatie van ongelimiteerde "Personal Communication Services (PCS)". Draadloze "indoor-communicatie" is de transmissie van spraak en data naar mensen die zich bewegen binnen gebouwen, woonwijken, supermarkten, winkelcentra, enzovoort. Het omvat zowel de verbinding binnen een gebouw tussen draagbare computers, zicht systemen, kassa's, etc. als de communicatie tussen vaste basis stations en bewegende robots in een produktie- of fabrieksomgeving. Indoor-communicatie speelt een sleutelrol in de implementatie van het PCS.

Draadloze indoor-communicatie staat bloot aan "meerweg fading", dat de prestatie van het systeem ernstig verslechtert. Het modelleren van het "indoor propagatie kanaal" maakt het mogelijk om de zender en ontvanger zo te ontwerpen dat het effect van fading wordt gereduceerd. Daarom is een gedetailleerde karakterisering van het radiopropagatiekanaal een belangrijke vereiste voor het succesvol ontwerpen van "indoor draadloze communicatie systemen". In het eerste deel van deze dissertatie, dat de hoofdstukken 1-4 omvat, wordt de statistische modellering en de karakterisering van het indoor radiopropagatiekanaal uitgevoerd. De resultaten zijn gebaseerd op wiskundige analyse, simulatie en metingen. De grote empirische "database" die gebruikt is in deze dissertatie bevat 12000 impuls responses van het kanaal, verzameld in twee verschillende kantoor omgevingen. Vanwege de grote snelheid waarmee de draadloze communicatie groeit, wordt het nodig om het spectrum efficiënter te gebruiken. Spread spectrum is een techniek die niet alleen efficiënt gebruik van het spectrum biedt (doordat extra gebruikers dezelfde band mogen gebruiken als de andere gebruikers die hetzelfde spectrum al benutten), maar ook fading van het indoor kanaal bestrijdt. In het tweede gedeelte van dit proefschrift, hoofdstukken 5-7, is de prestatie van spread spectrum modulatie technieken geëvalueerd in een indoor-communicatie systeem, waarbij gebruik gemaakt wordt van de eerder genoemde database.

Hoofdstuk 1 beschrijft de wiskundige modellering van het indoor radio propagatie kanaal en de "impuls respons methode". Hoofdstuk 2 beschrijft de details van de meetcampaegne, die geleid heeft tot de grote database van de metingen van het indoor draadloze kanaal, het meetplan, de meetprocedure en de meettechniek. Hoofdstuk 3 beschrijft de uitvoering van de statistische modellering van de "signaal amplitude fading". Amplitude-fading in de indoor multipad omgeving kan beschreven worden

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door verschillende verdelingen, afhankelijk van het gebied bestreken door de metingen, de aan- of afwezigheid van dominerende sterke componenten en enkele andere omstandigheden. De belangrijkste kandidaat-verdelingen zijn beschreven en uitgebreide tests zijn gebruikt om te controleren of deze verdelingen passen bij de amplitude-verdeling van de grote empirische database. Het is bekend dat de prestatie van digitale communicatie systemen in multipad omgevingen erg gevoelig is voor de statistische eigenschappen van de fase van het ontvangen signaal. Daarom rapporteert hoofdstuk 4 de resultaten van de diepgaande modellering van de veranderingen van de ontvangen signaal-fase voor digitale transmissie binnen gebouwen. Er worden twee fase-modellen voorgesteld en in detail bestudeerd. De prestatie van deze modellen zijn geëvalueerd door middel van uitgebreide computer simulaties en door gebruik te maken van de database.

Hoofdstuk 5 bestudeert "direct sequence spread spectrum" in de multipad fading indoor kanaalomgeving en evaluateert de prestatie ervan met behulp van de gemeten data. De "multicarrier modulatie techniek", die het spectrum efficiënter gebruikt, is onderzocht in hoofdstuk 6. Bij deze parallelle transmissie techniek overlappen de spectra van de sub-kanalen elkaar terwijl aan de orthogonaliteit voldaan is, resulterend in de spectrale efficiëntie. De voordelen van deze techniek en speciaal de bestrijding van de frequentie-selectiviteit van het kanaal zijn verklaard. De prestatie van de multicarrier transmissie is geëvalueerd door gebruik te maken van realistische indoor kanalen. Hoofdstuk 7 combineert de twee technieken (spread spectrum en multicarrier modulatie) om een hoge spectrale efficiëntie en capaciteit te bereiken. Ook hier is de prestatie bekeken voor de gemeten kanalen. Hoofdstuk 8 geeft afsluitende opmerkingen en aanbevelingen voor toekomstig werk.
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CURRICULUM VITAE

Homayoun Nikookar was born in 1962, in Iran. He received his B.Sc. degree and M.Sc. degree (with distinction) both in Electrical Engineering from Sharif University of Technology, Tehran, Iran, in February 1986 and December 1987, respectively. He joined Telecommunications and Traffic Control Systems Group of the Delft University of Technology in September 1993 and International Research Center for Telecommunications-Transmissions and Radar of the Delft University of Technology in July 1994, as a research fellow, working towards his Ph.D. degree.

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