Transients in turbulent convective heat transfer to a flow of supercritical helium
Transients in turbulent convective heat transfer to a flow of supercritical helium

PROEFSCHRIFT

ter verkrijging van de graad van doctor aan de Technische Universiteit Delft, op gezag van de Rector Magnificus, prof. drs. P.A. Schenck, in het openbaar te verdedigen ten overstaan van een commissie aan gewezen door het College van Dekanen op 30 Januari 1990 te 16.00 uur

door

Rob van der Linden

geboren te Rotterdam,
wiskundig ingenieur.
Acknowledgement

The investigations in the program of the Foundation for Fundamental Research on Matter (FOM) has been supported by the Netherlands Technology Foundation (STW).
Stellingen

Voor het realiseren van een goede thermische stabiliteit van een supergeleidende kabel, gecombineerd met een hoge voortplantingssnelheid van de verstoring, dient de helium druk (in tegenstelling tot wat Cornelissen [1984] poneerde) hoog te zijn (10-15 bar).

De instantane verandering van het snelheidsprofiel in een buis ten gevolge van een passerende drukgolf is constant over de doorsnede van de buis. Alleen in het viskeuze wandgebied ($y^+ < 5$) is deze verandering kleiner.

Een adequate beschrijving van transiente warmteoverdrachts verschijnselen met behulp van een surface renewal model is mogelijk mits een goede schatting van de “renewal” tijd $\tau$ beschikbaar is.

De nieuwe keramische supergeleidende materialen zullen, ondanks de recente snelle ontwikkelingen, voorlopig niet bruikbaar zijn in de magneet technologie.

Ter voorkoming van ernstige blessures zou door beoefenaars van sporten als wielrennen en skien een cursus valbreken gevolgd dienen te worden.

Een milieu beleid gebaseerd op het terugdringen van het auto gebruik is zinloos zolang er nog hele stukken uniek natuurgebied opgeofferd worden aan wegenbouw.

Stofzuigers behoeven niet zoveel lawaai te maken, ze werken geruisloos net zo goed.

Het feit dat ouderdom met gebreken komt wordt te vaak als vanzelfsprekend beschouwd.

Door de bouw van subtropische zwemparadijzen zal het aantal voor de wedstrijd zwemsport geschikte zwembaden verminderen. Hierdoor zal het niveau van deze sport dalen.
Aan mijn ouders
## Contents

List of principle symbols

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Introduction</td>
<td>1</td>
</tr>
<tr>
<td>1.1 Background</td>
<td>1</td>
</tr>
<tr>
<td>1.2 Aims and topics of this study</td>
<td>6</td>
</tr>
<tr>
<td>1.3 Outline of the investigation</td>
<td>7</td>
</tr>
<tr>
<td>2 Literature survey</td>
<td>9</td>
</tr>
<tr>
<td>2.1 Stability of super conducting magnets</td>
<td>9</td>
</tr>
<tr>
<td>2.1.1 Introduction</td>
<td>9</td>
</tr>
<tr>
<td>2.1.2 Cryostability</td>
<td>12</td>
</tr>
<tr>
<td>2.1.3 Metastability</td>
<td>14</td>
</tr>
<tr>
<td>2.1.3.1 Static criteria</td>
<td>14</td>
</tr>
<tr>
<td>2.1.3.2 Dynamic criteria</td>
<td>18</td>
</tr>
<tr>
<td>2.2 Propagation velocity</td>
<td>28</td>
</tr>
<tr>
<td>2.3 Heat transfer to supercritical helium</td>
<td>32</td>
</tr>
<tr>
<td>2.3.1 Stationary heat transfer experiments</td>
<td>32</td>
</tr>
<tr>
<td>2.3.2 Transient heat transfer experiments</td>
<td>35</td>
</tr>
<tr>
<td>2.3.2.1 The SULTAN conductor</td>
<td>35</td>
</tr>
<tr>
<td>2.3.2.2 The experiments of Bloem</td>
<td>37</td>
</tr>
<tr>
<td>2.3.2.3 The experiments of Nick</td>
<td>43</td>
</tr>
<tr>
<td>2.4 Turbulent flow models</td>
<td>45</td>
</tr>
<tr>
<td>2.4.1 Turbulence models based on the Reynolds decomposition</td>
<td>46</td>
</tr>
<tr>
<td>2.4.1.1 The Reynolds decomposition</td>
<td>46</td>
</tr>
<tr>
<td>2.4.1.2 The Boussinesq hypothesis</td>
<td>48</td>
</tr>
<tr>
<td>2.4.1.3 Eddy viscosity models</td>
<td>49</td>
</tr>
<tr>
<td>2.4.1.4 Discussion</td>
<td>53</td>
</tr>
<tr>
<td>2.4.2 Surface renewal models</td>
<td>54</td>
</tr>
<tr>
<td>2.4.2.1 General description of the turbulent burst mechanism</td>
<td>54</td>
</tr>
<tr>
<td>2.4.2.2 The age distribution function</td>
<td>57</td>
</tr>
<tr>
<td>2.4.2.3 Models based on stagnant eddies</td>
<td>58</td>
</tr>
<tr>
<td>2.4.2.4 Models including convection effects</td>
<td>60</td>
</tr>
<tr>
<td>2.4.2.5 Discussion</td>
<td>67</td>
</tr>
</tbody>
</table>
3 Numerical model
3.1 Introduction
3.2 The flow equations
3.3 Boundary conditions
3.4 Differencing schemes
3.4.1 Time differencing schemes
3.4.2 Spatial differencing schemes
3.5 The finite difference equations
3.5.1 The continuity equation
3.5.2 The transport equations
3.6 Boundary condition implementation
3.7 Helium properties
3.8 The solution method

4 Experimental method
4.1 Experimental setup
4.2 The hotwire anemometer
4.3 The resistance thermometer
4.4 The heat flux sensor
4.4.1 Theory
4.4.2 The construction of a heat flux sensor
4.5 The electronic equipment

5 Transients with surface renewal model
5.1 Introduction
5.2 Stationary case - Constant fluid properties
5.3 Instationary case - Constant fluid properties
5.3.1 A step in the wall temperature
5.3.2 An arbitrary wall temperature
5.4 Variable fluid properties
5.4.1 A step in the wall temperature
5.4.2 An arbitrary wall temperature
6 Numerical computations

6.1 Stationary computations
   6.1.1 Laminar constant property flow
   6.1.1.1 Hydrodynamics
   6.1.1.2 Thermal conditions
   6.1.2 Turbulent constant property flow
   6.1.2.1 Hydrodynamics
   6.1.2.2 Thermal conditions
   6.1.3 Turbulent variable property flow

6.2 Transient simulations
   6.2.1 Induced flow
   6.2.2 Transient heat transfer
   6.2.2.1 Numerical description of experimental set-up
   6.2.2.2 The reference situation
   6.2.2.3 Variation of physical properties
   6.2.2.4 Discussion

6.3 Stability study
   6.3.1 Critical energy computations
   6.3.1.1 The reference situation
   6.3.1.2 Numerical parameter variation
   6.3.1.3 Flow parameter variation
   6.3.1.4 Disturbance parameter variation
   6.3.2 Propagation velocity computations

7 Air experiments

7.1 Introduction
7.2 Stationary flow
   7.2.1 Pressure gradient
   7.2.2 Flow velocity measurements
   7.2.3 Heat transfer

7.3 Transient flow
   7.3.1 Pressure wave generation
   7.3.2 Results of transient measurements
   7.3.3 Numerical simulations
      7.3.3.1 Introduction
      7.3.3.2 Simulation of the transient air experiments
8 Discussion and conclusions

8.1 Transient flow 233
8.2 Transient heat transfer 234
8.3 Stability of the SULTAN conductor 235

Appendix A 237

References
Summary
Samenvatting
Curriculum vitae
Nawoord
### List of principle symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Surface</td>
<td>$[m^2]$</td>
</tr>
<tr>
<td>B</td>
<td>Magnetic field</td>
<td>$[T]$</td>
</tr>
<tr>
<td>c</td>
<td>Thermodynamic speed of sound</td>
<td>$[m/s]$</td>
</tr>
<tr>
<td>$c_1$</td>
<td>Heat loss coefficient for heat flux sensor</td>
<td>$[W/K]$</td>
</tr>
<tr>
<td>D</td>
<td>Tube diameter</td>
<td>$[m]$</td>
</tr>
<tr>
<td>$D_w$</td>
<td>Wetted perimeter</td>
<td>$[m]$</td>
</tr>
<tr>
<td>$e$</td>
<td>Stability margin</td>
<td>$[J/m^3]$</td>
</tr>
<tr>
<td>f</td>
<td>Friction factor $\left{ \frac{1}{4} \frac{\tau_w}{(\frac{1}{2} \rho &lt;u^2&gt;)} \right}$</td>
<td>$[-]$</td>
</tr>
<tr>
<td>$f_c$</td>
<td>Copper to superconductor ration</td>
<td>$[-]$</td>
</tr>
<tr>
<td>$f_M$</td>
<td>Metal to helium ratio</td>
<td>$[m/s^2]$</td>
</tr>
<tr>
<td>g</td>
<td>Acceleration of the gravity</td>
<td>$[-]$</td>
</tr>
<tr>
<td>G</td>
<td>Flow rate</td>
<td>$[kg/m^2s]$</td>
</tr>
<tr>
<td>h</td>
<td>Heat transfer coefficient</td>
<td>$[W/m^2K]$</td>
</tr>
<tr>
<td>H</td>
<td>Enthalpy</td>
<td>$[J/kg]$</td>
</tr>
<tr>
<td>I</td>
<td>Current</td>
<td>$[A]$</td>
</tr>
<tr>
<td>J</td>
<td>Current density</td>
<td>$[A/m^2]$</td>
</tr>
<tr>
<td>j</td>
<td>Dimensionless current $\left{ I/I_c \right}$</td>
<td>$[-]$</td>
</tr>
<tr>
<td>k</td>
<td>Turbulent kinetic energy $\left{ \frac{1}{2} \overline{u'_1u'_1} \right}$</td>
<td>$[m^2/s^2]$</td>
</tr>
<tr>
<td>$r$</td>
<td>Approach distance</td>
<td>$[m]$</td>
</tr>
<tr>
<td>l</td>
<td>Thickness of the renewal layer</td>
<td>$[m]$</td>
</tr>
<tr>
<td>$l_n$</td>
<td>Mixing length</td>
<td>$[m]$</td>
</tr>
<tr>
<td>$l_n$</td>
<td>Normal zone length</td>
<td>$[m]$</td>
</tr>
<tr>
<td>$L_Q$</td>
<td>Disturbance length</td>
<td>$[m]$</td>
</tr>
<tr>
<td>$m$</td>
<td>Mass</td>
<td>$[kg]$</td>
</tr>
<tr>
<td>$\dot{m}$</td>
<td>Mass flow</td>
<td>$[kg/s]$</td>
</tr>
<tr>
<td>P</td>
<td>Pressure</td>
<td>$[N/m^2]$</td>
</tr>
<tr>
<td>$q''_w$</td>
<td>Heat flux per unit surface at the wall</td>
<td>$[W/m^2]$</td>
</tr>
<tr>
<td>$\mathcal{R}$</td>
<td>Electrical resistance</td>
<td>$[\Omega]$</td>
</tr>
<tr>
<td>R</td>
<td>Tube radius</td>
<td>$[m]$</td>
</tr>
<tr>
<td>r</td>
<td>Radial coordinate</td>
<td>$[m]$</td>
</tr>
<tr>
<td>S</td>
<td>Renewal frequency</td>
<td>$[1/s]$</td>
</tr>
<tr>
<td>t</td>
<td>Time</td>
<td>$[s]$</td>
</tr>
<tr>
<td>$t_Q$</td>
<td>Disturbance duration</td>
<td>$[s]$</td>
</tr>
<tr>
<td>T</td>
<td>Temperature</td>
<td>$[K]$</td>
</tr>
<tr>
<td>$u^*$</td>
<td>Friction velocity ${ \sqrt{\tau_w/\rho} }$</td>
<td>$[m/s]$</td>
</tr>
</tbody>
</table>
u  Axial velocity component  [m/s]
v  Radial velocity component  [m/s]
V  Voltage  [V]
v  Volume  [m³]
φ  Propagation velocity  [m/s]
α  Dimensionless propagation velocity  [-]
y  Distance from the wall (R - r)  [m]
z  Axial coordinate  [m]

**Dimensionless numbers**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Name</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fo</td>
<td>Fourier number</td>
<td>( \frac{\alpha t}{\pi^2} )</td>
</tr>
<tr>
<td>Ma</td>
<td>Mach number</td>
<td>( \frac{u}{c} )</td>
</tr>
<tr>
<td>Nu</td>
<td>Nusselt number</td>
<td>( \frac{h D}{\lambda} )</td>
</tr>
<tr>
<td>Pe</td>
<td>Péclet number</td>
<td>( \frac{\langle u \rangle D}{\alpha} = Re Pr )</td>
</tr>
<tr>
<td>Pr</td>
<td>Prandtl number</td>
<td>( \frac{\nu}{\alpha} )</td>
</tr>
<tr>
<td>Re</td>
<td>Reynolds number</td>
<td>( \frac{\langle u \rangle D}{\nu} )</td>
</tr>
<tr>
<td>Gr</td>
<td>Grashof number</td>
<td>( \frac{\rho \Delta \rho g D^3}{\mu^2} )</td>
</tr>
<tr>
<td>Pr_t</td>
<td>Turbulent Prandtl number</td>
<td>( \frac{\varepsilon_m}{\varepsilon_h} )</td>
</tr>
</tbody>
</table>

**Fluid properties**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Name</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>ρ</td>
<td>Density</td>
<td>[kg/m³]</td>
</tr>
<tr>
<td>μ</td>
<td>Dynamic viscosity</td>
<td>[Ns/m²]</td>
</tr>
<tr>
<td>κ_v</td>
<td>Bulk viscosity</td>
<td>[Ns/m²]</td>
</tr>
<tr>
<td>ν</td>
<td>Kinematic viscosity (ρμ)</td>
<td>[m²/s]</td>
</tr>
<tr>
<td>λ</td>
<td>Thermal conductivity</td>
<td>[W/mK]</td>
</tr>
<tr>
<td>α</td>
<td>Thermal diffusivity (( \frac{\lambda}{\rho C_p} ))</td>
<td>[m²/s]</td>
</tr>
<tr>
<td>C_p</td>
<td>Specific heat</td>
<td>[J/kgK]</td>
</tr>
<tr>
<td>ε</td>
<td>Penetration parameter (( \lambda \rho C_p ))</td>
<td>[J²/m⁴K²s]</td>
</tr>
<tr>
<td>χ</td>
<td>Heat transfer parameter</td>
<td>[( \lambda^4 (\eta C_p)^4 )]</td>
</tr>
</tbody>
</table>
Greek symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>α</td>
<td>Steckly parameter</td>
<td>[-]</td>
</tr>
<tr>
<td>β</td>
<td>$\frac{dT}{dV}$ for resistance thermometer (par.5.5)</td>
<td>[K/V]</td>
</tr>
<tr>
<td>δ</td>
<td>Penetration depth</td>
<td>[m]</td>
</tr>
<tr>
<td>ε</td>
<td>Dissipation of kinetic energy of turbulence</td>
<td>[m²/s]</td>
</tr>
<tr>
<td>$\varepsilon_m$</td>
<td>Eddy viscosity (νₜ)</td>
<td>[m²/s]</td>
</tr>
<tr>
<td>$\varepsilon_h$</td>
<td>Turbulent thermal diffusivity (αₜ)</td>
<td>[m²/s]</td>
</tr>
<tr>
<td>Φ</td>
<td>Viscous dissipation</td>
<td>[W/m³]</td>
</tr>
<tr>
<td>φᵥ</td>
<td>Volume flow</td>
<td>[m³/s]</td>
</tr>
<tr>
<td>φ(θ)</td>
<td>Age distribution function</td>
<td>[1/s]</td>
</tr>
<tr>
<td>θ</td>
<td>Age of fluid element near the wall</td>
<td>[s]</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>$\frac{\lambda}{C_p} + \frac{\mu_t}{Pr_t}$</td>
<td>[Nm/s²]</td>
</tr>
<tr>
<td>$\rho_{el}$</td>
<td>Electrical resistance</td>
<td>[Am]</td>
</tr>
<tr>
<td>$\mu_t$</td>
<td>Turbulent viscosity (ρ•ε₁)</td>
<td>[Nm/s²]</td>
</tr>
<tr>
<td>$\mu_{eff}$</td>
<td>Effective viscosity (μ + μ₁)</td>
<td>[Nm/s²]</td>
</tr>
<tr>
<td>τ</td>
<td>Contact time (Renewal rate)</td>
<td>[s]</td>
</tr>
<tr>
<td>$\tau_w$</td>
<td>Wall friction (−μ $\frac{δu}{δy}$</td>
<td>$y=0$)</td>
</tr>
</tbody>
</table>

Special functions

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_{ij}$</td>
<td>Kronecker delta</td>
<td>δᵢⱼ = 0 i≠j \ δᵢⱼ = 1 i=j</td>
</tr>
<tr>
<td>$u(x)$</td>
<td>Unitstep function</td>
<td>u(x) = 0 x &lt; 0 \ u(x) = 1 x &gt; 0</td>
</tr>
<tr>
<td>$J_x$</td>
<td>Bessel function of order x</td>
<td></td>
</tr>
</tbody>
</table>

Superscripts

- Mean value
- Fluctuating component
- $t_k$ Value at time level $t_k$
- $t_0$ Value at previous time level
- $t_{∞}$ Value two time levels back
Dimensionless wall parameter:

\[ y^+ = y \frac{u^*}{v} \]
\[ u^+ = \frac{u}{u^*} \]
\[ s^+ = s \frac{v}{(u^*)^2} \]
\[ \tau^+ = \tau \frac{(u^*)^2}{v} \]
\[ T^+ = (T_0 - T) \frac{\rho C_p u^*}{q_w} \]

Subscripts:

- ax: Value on the axis of the tube
- b: Bulk value
- c: Critical value
- cs: Current sharing
- cu: Copper
- He: Helium
- max: Maximum value
- pc: Pseudo critical value
- r: Radial
- ref: Reference value
- s: Stationary value
- sc: Superconductor
- tot: Total (copper + superconductor)
- w: Wall value
- z: Axial
- o: Ambient value

Miscellaneous:

\(<.>\) Averaged value over tube diameter (bulk value)
\{.\} Ensemble average
1 Introduction

1.1 Background

The requirements for the magnetic field of large magnet systems applied in modern technology become higher and higher. An example is the mineral processing industry where magnetic separation is already a well established technique. To be able to separate a wider range of materials and particle sizes effectively the magnetic field has to be increased. Also new techniques like magnetic resonance imaging (MRI) and high energy physics require constant high magnet fields. Other (future) applications of high field electromagnets can be found in the area of energy generation. Magneto-hydrodynamic (MHD) generators convert heat energy into electrical energy by forcing ionized hot combustion products through a magnetic field. Nuclear fusion reactors can only work safely when stable field strengths in the order of 10 T are available to confine the extremely hot ionized plasma.

Conventional electromagnets are not capable to generate fields in this order of magnitude. Because of the electrical resistance of the magnet windings the large electrical currents, needed to generate a large magnetic field, result in an enormous ohmic heat dissipation, making large water cooling systems inevitable. These water cooling systems make the magnet very big, heavy and very expensive.

With the discovery of the superconducting property of mercury below 4 K by Kamerling Onnes in 1911, the construction of a new type of magnets came in principle into reach. Due to the zero resistivity of superconducting materials the main problem in constructing large magnetic fields, being the large ohmic heat dissipation, can be eliminated completely. However a new problem arises. Superconductivity only occurs if the temperature $T$, for a given current density $J$ and magnetic field $B$ lies below a material dependent critical value. The maximum value of the temperature at fixed values of $J$ and $B$ for which the material is still in its superconducting state is called the critical temperature $T_c(J,B)$. In a similar way the critical current density $J_c(T,B)$ and critical field $B_c(T,J)$ are defined.

The early superconducting materials like mercury, lead and
alloys of, for example, lead and bismuth have only very low critical fields. Since the first attempts to build a superconducting magnet better alloys with much higher critical values have been found. In figure 1.1 the critical surface in the T,J,B-space is shown for niobium titanium (NbTi). A material with even higher critical values is Niobium tin (Nb₃Sn). However, this intermetallic compound is very brittle and for this reason used only in applications demanding magnetic fields or current densities which cannot be reached with the more manageable alloy NbTi.

![Figure 1.1: The critical surface for NbTi](image)

With the fabrication of NbTi wires, and later Nb₃Sn wires, the development of high field superconducting magnet systems was made possible. However due to the low value of the critical temperature of the superconducting materials used in magnet technology (NbTi : Tc(0,0) ≈ 9.3 K) these magnets must be operated at liquid helium temperatures. Despite the refrigeration costs large superconducting magnets are still more economic than their conventional equivalents. This is partly because of a reduction in size and partly because large watercooling systems and high powered current supplies are not necessary. In addition superconducting magnets are also capable of producing higher magnetic fields. This is shown in figure 1.2 where the critical
surfaces for NbTi and Nb$_3$Sn at a constant temperature are compared with the operating range of conventional water cooled electromagnets [Wilson(1981a)]. As a consequence superconducting magnet systems are being used in many applications all over the world. [Karkera(1984), Luguang(1987), van Dam(1988), Dam and Pieterman, and many others].

![Figure 1.2:](image)

Critical current densities of NbTi and Nb$_3$Sn at a constant temperature of 4.2 K. The shaded area represents the usual operating range of conventional electromagnets.

As long as the magnet temperature is kept below the critical temperature the electrical current in the superconductor will cause no heat dissipation. In this situation only heat leaks from the surroundings have to be compensated to ensure a stable operation of the magnet. In practice however, superconducting magnets are very susceptible to thermal disturbances. At a given operating temperature $T$ of the magnet the current density is normally chosen as close as possible to the critical value $J_c$. As a consequence the temperature margin $T_c - T$ is small. Because of the very low heat capacity of the winding materials on this range of temperatures, a sudden, small (local) heat release in the windings suffices to raise the temperature locally above the critical value. In this state the conductor is at that location no longer superconducting but normal conducting. The ohmic heat dissipated in this state can raise the temperature of the conductor further resulting in an expansion of the normal conducting zone. The only way to stop this avalanching process, which is referred to as the quenching of a magnet, is to de-energize the magnet.

In order to make the superconducting magnets reliable a lot of work has been done in the past to locate and eliminate all possible sources of heat disturbances in the conductor. The principle source of
heating in the winding called flux jumping was eliminated by the introduction of multi-filamentary conductors. In a recent study of Iwasa [1985] mechanical disturbances caused by the enormous Lorentz forces acting on the windings were found to be the principle cause of premature quenches nowadays. Conductor motions in the order of 10 μm (microslips) are able to produce enough energy to raise the conductor temperature above the critical value. Since it is impossible to eradicate these microslips entirely it is necessary to construct the magnet in a way that these heat disturbances can be controlled.

Figure 1.3 : Two types of conductors used in SULTAN

\[ a \]: The 8T SULTAN I conductor
\[ b \]: The 12T SULTAN II conductor

In large magnet systems this has lead to the construction of so-called composite conductors. In these conductors the energy dissipation in a normal conducting zone is reduced by embedding the superconducting filaments in a low resistance material with a relatively high thermal capacity like copper. In addition copper also functions as a construction material. To remove the dissipated heat from the conductor the windings have to be cooled. Here to one or more cooling channels are included in the conductor. Figure 1.3 shows two different composite conductors used in the SULTAN project at ECN (The Netherlands). At the cryogenic temperatures considered only helium can be used as a coolant. In technical applications generally helium at supercritical pressures (P > 2.27 bar) is used to prevent vapor locking in the long narrow cooling channels which can occur when using liquid helium.
An internally helium cooled superconducting magnet will be metastable. This means that in case of a large thermal disturbance the normal conducting zone will grow leading to a quench of the conductor (unstable situation), but in case of a small disturbance the conductor will be able to return to its superconducting state (stable situation). In the past several stability studies have been performed in order to find the maximum disturbance a conductor can recover from. These studies, which will be reviewed in paragraph 2.1 of the next chapter, are generally based on several simplifying assumptions. Due to these assumptions the results of these studies are too conservative.

An other important parameter in magnet design is the speed at which the normal zone expands in case of an unstable situation. This speed is called the propagation speed of a disturbance. A quench in the magnet is detected by measuring the voltage drop over the conductor. In stable operation this voltage drop must be nearly zero. During a quench the voltage drop grows proportionally to the normal zone length. A large propagation speed of the normal zone will result in a fast detection of a quench. This is necessary because the energy stored in the magnet is quickly dissipated in the normal conducting zone of the conductor. Excessively large local temperature which can destroy the magnet will occur unless the magnet is de-energized in time.

To optimize the stability of a magnet at a maximum performance level and ensuring a large enough propagation velocity in case of a quench, detailed knowledge of the transient variable property effects in both the conductor and turbulent helium flow is required.

In a previous study Cornelissen [1984] has developed a simulation model in which the transient one dimensional heat conduction in the superconductor, including current sharing effects and variable metal properties, is described. He used this model to study the dynamic stability of a conductor using a constant heat transfer coefficient to describe the cooling by the helium flow. The results of this study were compared with static stability criteria from literature. Cornelissen also developed a model to include transient variable property effects in the turbulent helium flow by coupling the conductor model to a two dimensional turbulent helium flow model. To verify this flow model stationary heat transfer simulation results have been compared with
stationary experimental heat transfer correlations since no experimental data on transient heat transfer to supercritical helium were available at the time. Using a trial and error method to compute the critical disturbance energy Cornelissen found that the transient heat transfer had a large impact on the magnet stability. The energy of the maximum disturbance in the conductor being considered was raised by more than a factor three due to these transient effects. To compute the propagation velocity Cornelissen coupled the conductor model to a one dimensional flow model. The influence of a thermally induced flow and a non-constant temperature in the normal zone were found to result in a time dependent propagation velocity.

1.2 Aims and topics of this study.

This work is an extension of the study made by Cornelissen. The principle objective of the project is to develop a model capable of predicting the stability of a superconducting magnet after a thermal disturbance, including the propagation velocity of the normal zone in case of a quench.

In this study the emphasis will be on the description of the transient turbulent flow- and heat transfer phenomena in the cooling channel of a conductor. The topics which will be studied are:

- transient heat transfer
- induced flow
- turbulence in transients
- stability and propagation velocity

In a parallel project of Bloem [1986] at the ECN (Netherlands Energy Research Foundation) data on transient heat transfer to supercritical helium at cryogenic temperatures have recently become available. In close cooperation these experiments have been simulated to validate the transient heat transfer model.

Since no experimental data on transient cryogenic helium flow fields are available an experiment on transient tube flow with a simple fluid (air) has been set up. With some minor adjustments the turbulent flow model can be used to simulate these air experiments. The experimental and numerical results will be compared to validate the
transient flow simulations.

In order to be able to validate the transient heat transfer computations a transient heat flux sensor has been developed which has been used in the air flow experiment.

Heat transfer to the helium flow is known to be dominated by the thermal resistance of a small fluid layer near the wall [Cornelissen (1984) , Bloem (1986)]. A good description of the turbulent processes near the wall is thus indispensable to get a good prediction of the heat transfer process. Following a suggestion of Cornelissen a phenomenological model describing the turbulence in the wall region, called the surface renewal model, has been studied.

To study the stability of a superconductor the transient two dimensional flow model has been coupled to the one dimensional conductor model of Cornelissen. Using a trial and error procedure the critical disturbance energy of a conductor can be computed for different values of the mass flow and ambient pressure. Also the influence of the duration and extension of the disturbance on stability are studied. In practice these parameters will be unknown but can be influenced by the conductor design. Using the same model transient effects on the propagation velocity at different mass flows and ambient pressures are investigated.

1.3 Outline of the investigation

In chapter 2 literature on stability of superconductors and related studies on topics like turbulence, including the surface renewal models, are discussed. Transient heat transfer data of Bloem (1986) are also discussed in this chapter.

The model describing the flow and heat transfer in a cooling channel is presented in chapter 3. The governing equations, being the continuity equation, the Navier-Stokes equations and the energy equation, are formulated in a two dimensional axisymmetric coordinate system. This reduces the actual three dimensional problem by one dimension. In practice this means that all channels are approximated by a circular geometry. A numerical description of the problem is found by
using a finite volume differencing method. Both time and space
discretisation are carried out using a first and a higher order
difference scheme. The description of the boundary conditions and the
temperature and pressure dependence of the fluid properties complete
the numerical formulation.

The experimental method and set-up for determining the transient
flow- and heat transfer characteristics in an air flow is given in
chapter 4. Temperature and velocity measurements are performed using a
hotwire technique. In order to measure the transient heat transfer a
fast heat flux sensor based on a hot film technique is developed.

In chapter 5 a surface renewal model will be developed which
incorporates the effect of an arbitrarily varying wall temperature on
the transient heat transfer. This model is an extension of the
stationary surface renewal models reviewed in chapter 2. The validity
of our new surface renewal model will be verified by comparing the
results with the transient heat transfer data found by Bloem [1986].

The chapters 6 and 7 deal with the results from our study. The
helium flow results are presented in chapter 6 and the results of the
transient turbulent air flow experiments and simulations in chapter 7.

At first in chapter 6 the propagation of the heat induced pressure
wave in the helium flow is compared to the thermodynamic speed of sound
for the different discretisation schemes discussed in chapter 4. The
transient heat transfer to the helium is studied by simulating
cryogenic experiments of Bloem [1986] (chapter 2). Finally the model is
applied to find the critical energy and propagation velocity of the
SULTAN magnet for different flow situations.

In chapter 7 the results of the experiments on the transient air
flow, as described in chapter 4, will be presented together with the
results of numerical simulations. The influence of the induced pressure
wave on the velocity profile in the tube as well as on the heat
transfer to the air flow will be investigated.

Finally, in chapter 8 the results and conclusions of this
investigation are discussed.
2 Literature survey

2.1 Stability of superconducting magnets

2.1.1 Introduction

Superconducting magnets suffer, as was pointed out in chapter one, from thermal disturbances which can locally rise the conductor temperature above the critical temperature. This results in a normal conducting zone in the magnet in which heat is produced due to Ohmic dissipation. This leads to a further rise of the conductor temperature and an expansion of the normal zone.

To avoid this avalanching heating process, known as "quenching" of the magnet, three kinds of measures must be taken:

-1- Elimination of the sources of the disturbances
-2- Reduction of the heat generation in a normal zone
-3- Cooling of the magnet

Nowadays some sources of disturbances, such as flux jumps, are well understood. Techniques for their elimination by means of a fine subdivision of the superconducting cable into many thin filaments are well developed and widely used. Other sources are understood only qualitatively. The overall effect of these disturbances is generally assumed to be an injection of a certain amount of energy into the wire. The spectrum of the injected energy pulses, however, is unknown. The nature of the principle disturbances is thought to be mechanical. Conductor motions caused by the enormous Lorentz forces acting on the windings and releases of strain energy imposed on the conductor by the cool-down of the magnet from room temperature to cryogenic temperatures are found to be the major sources of heat releases in a magnet [Iwasa(1985)]. In practice it is hard, if not impossible, to banish these disturbances from a superconducting magnet completely.

To be able to control the temperature rise after a thermal disturbance in the conductor the superconducting filaments are usually embedded in a low resistive material with a high thermal conductivity like copper. Because of the high resistivity of the normal conducting
"superconductor" material the current in a normal zone is (partly) transferred to the low resistive copper backup. As a consequence the ohmic dissipation in a normal conducting zone is reduced. A further reduction of the dissipation can be achieved by increasing the amount of copper backup.

Beside the reduction of the heat generation in a normal zone, copper has two other positive effects on the performance of the magnet. First, the high thermal conductivity of the copper ensures a fast redistribution of the energy released in the conductor and thus inhibits high local temperature peaks in the conductor. And second, the copper serves as a construction material for the coil. A combined copper/superconductor cable is more rigid than a pure superconducting cable. This is necessary to withstand the enormous mechanical strains due to the Lorentz forces and results in a reduction of the occurring disturbances.

However, embedding the superconducting filaments into copper does not suffice to avoid a quench. To allow a conductor to recover from a disturbance and return to its superconducting state \((T < T_c)\) the locally released energy has to be removed from the conductor by a coolant. Because of the low temperatures required to preserve superconductivity only helium can be used as a coolant.

Based on the cooling technique two types of conductors, referred to as bath- and forced- cooled conductors, can be distinguished.

In the bath-cooled type the coil is immersed in a bath of boiling liquid helium at atmospheric pressure. Heat transfer is governed by the evaporation of the helium, the vapour is pumped away. The temperature of the helium is constant \((4.2 \text{ K})\). A disadvantage of this cooling method is the deterioration of the heat transfer at a certain temperature difference between the conductor and the helium \((\approx 1 \text{ K})\). At this temperature difference a transition from nucleate to a much less effective film boiling occurs. In practice this cooling method is only applicable for small magnet systems.

The forced-cooled type, in which the conductor is internally cooled with a forced helium flow, is preferred in the construction of large magnets. To circulate the helium flow a copper channel is used. The copper channel with the superconducting filaments soldered on to it
is referred to as "internally cooled superconductor" (ICS). The SULTAN conductors shown in figure 1.3 are of the ICS type. An other type of a forced-cooled superconductor in which the superconducting filaments are situated inside the cooling channel is referred to as a "cable in conduit ICS" or "internally cooled cabled superconductor" (ICCS). The latter type of conductor has a much larger wetted perimeter and thus provides a better heat transfer than the ICS type. The ICS type, however, is more rigid and thus suffers from less mechanical disturbances [Dresner(1984)].

![Phase P-T diagram of Helium-4.](image)

**Figure 2.1:**

Phase P-T diagram of Helium-4.

- **C.P.** Thermodynamical critical point
- **TCC** Transposed critical curve. (T-location of $C_p$ maximum).

Since the total length of wire in a coil is very large the cooling channel of an internally cooled conductor is also very long (1 km or more). To avoid boiling effects in the channel (vapour locking) a single helium phase at supercritical pressures ($P > 2.2$ bar) is usually preferred over two phase (liquid-gas) helium as a coolant. Also superfluid helium (He II - see figure 2.1) has been recognized as an attractive coolant for forced-flow type conductors [Dresner(1984), Frederking(1987)]. In this study we will concentrate on conductors internally cooled with a forced flow of supercritical helium.

The thermal behavior of a superconductor after a disturbance
depends on the conductor design (amount of copper backup etc) and on
the operation conditions (magnetic field, transport current, ambient
temperature, cooling capacity etc) of the magnet in the steady state.
If, after a disturbance of any size, the conductor returns to its
original steady state this state is called stable. If every disturbance
will lead to a quench the steady state is called unstable. Sometimes a
conductor inhibits both stable and unstable performance dependent on
the size of the disturbance. The steady state of the conductor is then
called metastable.

2.1.2 Cryostability

To design stable performing superconducting magnets at given
operation conditions, criteria for predicting the stability of a magnet
design are indispensable. A review has been given by Cornelissen
[1984], here the main points will be summarized.

The first criterion with which stable performing magnets could
successfully be designed is known as the Stekly criterion
[Stekly and Zar(1965)]. This criterion requires that at every cross
section in the conductor heat removal exceeds the maximum heat
dissipation. Defining the Stekly parameter $\alpha_s$ by the quotient of the
maximum dissipation and the local cooling capacity:

$$\alpha_s = \frac{\rho_{e1, cu} I_c^2 (B, T_0)}{h D_w A_{cu} [T_c (B, I)-T_0]}$$  \hspace{1cm} (2.1)

the criterion reads:

$$\alpha_s < 1$$  \hspace{1cm} (2.2)

At given values of the copper electrical resistivity $\rho_{e1, cu}$, critical
current $I_c$, heat transfer coefficient $h$ and temperature margin $T_c - T_0$
between the critical and ambient temperature, this criterion prescribes
the required amount of copper backup, described by the copper cross
section $A_{cu}$, and the required cooling perimeter $D_w$. When relation (2)
is satisfied the magnet is stable against all disturbances for all
operating currents $I$ less than the critical value $I_c$.

If, for some magnet design, the value of $\alpha_s > 1$ stable performance
is still possible. The operation current in this case has to be limited
to values at which the maximum dissipation is reduced below the cooling capacity, or:

$$\alpha j^2 < 1$$  \hspace{1cm} (2.3)

in which \( j = I/I_e \). The maximum allowable current after relation (3) is called the recovery current \( I_r \).

The Stekly criterion is known to be much too conservative. In order to assure a stable performance of a magnet the Joule heat in the conductor is overwhelmed by cooling capacity. This kind of stabilization, which is often referred to as cryostability, leads to magnet designs with large copper to superconductor ratios and suffer from low recovery currents.

Maddock, James and Norris [1969] were able to derive a somewhat less restrictive stability criterion by considering the one dimensional energy equation, in which the effect of heat conduction to cooler areas in the conductor is incorporated. This equation reads:

$$\frac{A_{\text{tot}}}{D_w} \frac{\partial}{\partial x} \lambda \frac{\partial T}{\partial x} = q_{\text{He}}(T) - q_D(T)$$  \hspace{1cm} (2.4)

\( q_{\text{He}} \) represents the heat removal by the coolant and \( q_D \) the heat generation in the conductor. Instead of a local equilibrium between heat generation and removal, a global equilibrium is assumed. For a constant value of \( \lambda \) this leads to

$$\int [q_{\text{He}}(T) - q_D(T)] \, dT = 0$$  \hspace{1cm} (2.5)

In this theory one end of the conductor has to remain "cold" \((q_{\text{He}} = q_D = 0)\). At the other end of the conductor a constant temperature \( T_1 \), at which \( q_{\text{He}}(T_1) = q_D(T_1) \), is assumed. For a constant value of the heat transfer coefficient \( h \) the resulting Maddock criterion becomes:

$$\alpha j^2 < 2 - j$$  \hspace{1cm} (2.6)

Like in the Stekly criterion (3), the maximum allowable current after relation (6) is called the recovery current \( I_r \).
In a Stekly stable magnet all points of a normal zone disappear simultaneously because of the local equilibrium requirement. In a Maddock stable magnet it suffices if the ends of the normal zone propagate inward until finally the total normal zone has disappeared (cold end recovery). In spite of some improvement, Maddock stable magnets suffer from allowing only low current densities, too.

2.1.3 Metastability

In applications that require higher current densities or lower copper to superconductor ratios, no unconditionally stable magnets as described above can be designed. For these applications metastable magnets have to be used in which the stability of the magnet depends on the magnitude of the occurring disturbances in the conductor. Stable performance of such a metastable magnet at high current densities is possible as long as the maximum occurring disturbance in the conductor does not exceed a critical value. In the stability analysis of metastable magnets the boundary between stable and unstable performance of a magnet is searched for. For a given magnet design at specified operation conditions this leads to criteria for the allowed disturbances. In practice the occurring disturbance spectrum will be unknown and stable performance of the magnet design can not be guaranteed from a stability study. However, from a stability study the performance of different designs of a magnet at a prescribed operation can be compared and an optimal design (low costs, large allowed disturbances) can be obtained.

The stability criteria for metastable magnets can be divided into two classes. First there are static criteria in which the critical disturbance is determined by the solution of the stationary energy equation in the conductor. Second there are dynamic criteria in which also transient effects on stability are considered. Both classes will be considered separately below.

2.1.3.1 Static criteria

In a static stability analysis the solution of the stationary
energy equation (4) describing the superconductor plays a dominant role. This solution forms the boundary between a stable and an unstable region of operation. From this solution criteria are deduced which describe the critical state of a normal zone rather than the disturbance which initiated it. With the introduction of the dimensionless parameters

\[ \mathcal{I} = \frac{T - T_0}{T_c - T_0} \quad \mathcal{R} = \sqrt{\frac{\hbar D_w}{\lambda A_{101}}} \]  

(2.7)

in which the conductivity \( \lambda \) is assumed to be constant, the energy equation (4) in case of a constant helium temperature, can be written as:

\[ \frac{\partial^2 \mathcal{I}}{\partial \mathcal{R}^2} = \mathcal{I} - \alpha j^2 \ddot{\mathcal{I}}(\mathcal{I}) \]  

(2.8)

\( \alpha \) is the Stekly parameter defined in equation (1) and \( j = I/I_c \). \( \ddot{\mathcal{I}} \) describes the dependency of the dissipation in the conductor on temperature. Simplified \( \ddot{\mathcal{I}} \) can be represented by a step function:

\[ \ddot{\mathcal{I}} = 0 \quad \mathcal{I} < 1-j \\
\ddot{\mathcal{I}} = 1 \quad \mathcal{I} > 1-j. \]  

(2.9)

A better description of the heat generation is given by a current sharing model. In this model the critical current density \( J_c(B,T) \), at a fixed magnetic field \( B \), is assumed to vary linear with \( (T_c - T) \) (see figure 2.2).

![](image)

**Figure 2.2 - Current sharing model**

For a given current density \( J < J_c(B,0) \) there exists a temperature \( T_{cs} \) at which \( J = J_c(B,T_{cs}) \). Below this current sharing temperature the
superconductor can carry the whole current without Ohmic dissipation. Above the critical temperature $T_c$, the superconductor has a high electrical resistance and all current is transferred to the copper backup. In the intermediate region $T_{cs} < T < T_c$, the superconductor can carry only part of the total current without resistance. The current is then shared between the superconducting material and the copper backup. Only the part of the current carried by the copper will cause heat dissipation in the conductor. In dimensionless form this results in the following relation for the function $*$:

$$\begin{align*}
  * &= 0 & \mathcal{I} < 1 - j \\
  * &= (\mathcal{I} + j - 1)/j & 1 - j < \mathcal{I} < 1 \\
  * &= 1 & 1 < \mathcal{I}
\end{align*}$$

(2.10)

Keilin et. al.[1967,1970] characterized the critical normal zone by its length $l_n$. As long as the normal zone is shorter than $l_n$ the conductor will be stable. If the normal zone exceeds $l_n$ the conductor will be unstable. The value of $l_n$ as a function of the steckly parameter $\alpha_s$ and the dimensionless current $j$ they found by integrating the dimensionless energy equation (8). Employing a step function for the dissipation function $*$ (equation (9)) they arrived at:

$$l_n = \log_e \left[ \frac{\alpha_s j^2}{(\alpha_s j^2 + 2j - 2)^{1/2}} \right]$$

(2.11)

When current sharing is included in the analysis they give

$$l_n = (\alpha_s j - 1)^{-1/2} \arctan \left[ (\alpha_s j - 1)^{-1/2} \right]$$

(2.12)

This results are only valid in case the helium temperature is assumed to be constant like in a bath cooled conductor. In case of of a forced cooled conductor the helium temperature will rise along the conductor. Keilin et. al.[1970] presented graphical results on $l_n$ for this case.
Wilson and Iwasa [1978] solved the stationary energy equation (4) numerically assuming the two ends of the conductor to be "cold" \( q_{ne} = q_D = 0 \). Somewhere in the normal zone the temperature gradient equals zero. At this point heating and cooling do not need to be in equilibrium like in the Maddock criterion and recovery at higher current densities then allowed by relation (6) is possible. However, stability is restricted. For currents exceeding the recovery current, \( I > I_r \), the stationary temperature profile computed from the energy equation (4) represents the boundary between stable recovery and unstable growth of the normal zone. This profile is called the "Minimum Propagation Zone" (MPZ). Iwasa [1979] characterized the MPZ by its maximum (dimensionless) temperature \( S_m \). For constant values of the heat transfer coefficient \( h \) and the conductivity \( \lambda \) and including current sharing effects he arrived at:

\[
S_m = \begin{cases} 
\alpha_j^2 - (\alpha_j^2)^4 + \alpha_j^3 - 2\alpha_j^2 \right)^{1/2} & \alpha_j^3 < 1 \\
(1-j) [\alpha_j + (\alpha_j)^{1/2} \right] / (\alpha_j - 1) & \alpha_j^3 > 1 
\end{cases}
\] (2.13)

Analytical solutions of the dimensionless energy equation (8), including current sharing effects, have been obtained by Cornelissen and Hoogendoorn [1984]. They presented the stability criteria, for both a constant and a temperature dependent thermal conductivity \( \lambda = kT \), in terms of the critical energy, which can be found by integration of the critical temperature profile. In case of a constant conductivity \( \lambda \) the maximum temperature at \( x = 0 \) equals the maximum temperature \( S_m \) found by Iwasa (equation (13)). From the analytical solution also the length of the normal zone can be found. In case \( \alpha_j^3 > 1 \) the normal zone length equals the value predicted by Keilin et. al. (equation (12)). However, if \( \alpha_j^3 < 1 \), in which case the maximum temperature exceeds the critical value \( (S_m > 1) \), the solution of Keilin is not valid. Equation (12) presented by Keilin et. al. is thus only part of the solution.

Jayagumar [1987] used the same technique as Cornelissen and Hoogendoorn to find the critical energy. However he abandoned the boundary condition \( S = 0 \) for \( x \to \infty \) as has been used by Cornelissen and Hoogendoorn to find a unique solution. Jayagumar showed that this
condition leads to an increasing critical energy as the value of $hD_w$ is decreased. In the limiting case of a dry conductor ($hD_w = 0$) this gives the physically non-realistic result of an infinite critical energy. Instead of applying this boundary condition Jayagumar minimized the energy content of the possible temperature profiles. Since the solution of Cornelissen and Hoogendoorn is one of the possible profiles the critical energy found by Jayagumar is less than or equal to the value found by Cornelissen and Hoogendoorn. In case of a dry conductor this technique results in a constant, finite, value for the critical energy.

2.1.3.2 Dynamic criteria

In the static criteria described above, the stability of a magnet is judged by the properties of the critical, static temperature profile. Nowadays the general accepted standard for judging the stability of different magnet designs is the energy contents of the critical disturbance [Dresner(1980), Wilson(1981a)]. Next to the energy contents, also the extension and duration of a disturbance are important parameters. The most damaging disturbances are thought to be the ones which introduce the energy on a short time scale. About the spatial extent of disturbances occurring in a magnet very little is known. For stability analyses a differentiation between the two extremes, the distributed and the point disturbance, can be made. In a distributed disturbance heat conduction effects are not significant and the actual extent is unimportant as long as it is longer than the length of the MPZ. The magnitude of the disturbance is characterized by the energy per unit volume generally called the stability margin [Hoenig, Iwasa and Montgomery (1975)]. The stability of a magnet against a point disturbance may be characterized solely by its total energy (critical energy). The extent or distribution seem unimportant as long as it is less than the MPZ [Wilson(1981a)].

In the static criteria the critical disturbance is generally assumed to have the same energy content as the critical temperature profile. The influence of transient effects on the temperature profile development in the conductor after the occurrence of a disturbance are not taken into account. In practice, however, transient effects can have a large impact on the stability of a magnet. Especially in forced
cooled conductors stability can be dominating [Ishibashi(1979a), Dresner(1984), Cornelissen(1984), et.al.].

In this paragraph stability criteria will be reviewed which take transient effects on the critical disturbance into account. Three different kinds of transients can be distinguished:

-1- transient heat conduction in the conductor
-2- transient heat transfer to the coolant flow in an ICS
-3- transient flow phenomena in the cooling medium inside the channel of an ICS

To account for the influence on stability of the transient heat conduction after a disturbance, the instationary energy equation in the conductor has to be solved. This equation reads in analogy with the stationary equation (4):

\[ \rho C_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left( \lambda \frac{\partial T}{\partial x} \right) - \left[ q_{He}(T, T_{He}) - q_D(T) \right] \frac{D_w}{A_{tot}} \]  

(2.14)

The dissipation \( q_D \) (in dimensionless parameters) is described by relation (9) or (10). The cooling term \( q_{He} \) in equation (14) is described by:

\[ q_{He} = h (T - T_{He}) \]  

(2.15)

By solving this equation numerically for different disturbance energies the boundary between a stable and unstable situation can be found iteratively for an instantaneous disturbance of given extent [Keilin and Romanovsky(1982), Cornelissen and Hoogendoorn (1985b)]. The critical energy \( Q_c \) found by this procedure and the energy contents \( Q_s \) of the static temperature profile are related by:

\[ Q_c = Q_s + \iint (q_{He} - q_D) \, dt \, dx \]  

(2.16)

The integral term in relation (16) represents the unbalance between the dissipation \( q_D \) and cooling \( q_{He} \) in the conductor during the temperature profile development. For the critical disturbance a stationary temperature profile develops in which \( q_{He} = q_D \) as \( t \to \infty \). Depending on the size and distribution of the disturbance this term can have both
positive and negative values.

Figure 2.3 : Influence of the extension on the size of the critical disturbance, computed by:

- Keilin and Romanovski [1982]
- Meuris [1984]
- Ishibashi el.al. [1979]
- Cornelissen [1984]

Dynamic simulations solving the heat conduction equation (14) for a constant heat transfer coefficient h and including current sharing effects in the solid have been performed by several workers. In figure 2.3 the influence of the extension L_Q of the disturbance as computed by
Keilin and Romanovsky [1982], Meuris [1984], Ishibashi et al. [1979] and Cornelissen [1984] respectively is shown. From this figure it can be seen that for extensions larger than 50 to 80 mm the critical disturbance increases with \( L_Q \). Ishibashi et al. find this behavior also for small disturbance extensions while Cornelissen predicts a constant value. Keilin and Romanovsky as well as Meuris find an increase of the critical disturbance energy when the extension of the disturbance is further reduced.

Meuris [1984] and Ishibashi [1979] also report computations on the critical disturbance for varying disturbance durations \( t_Q \). The results are shown in figure 2.4. Varying the duration from 0 to 40 ms Meuris found an increase of the critical disturbance of some 13% at \( t_Q = 10 \text{ ms} \) up to over 300% at \( t_Q = 40 \text{ ms} \). Ishibashi et al. however report a much larger increase of about 450% at \( t_Q = 10 \text{ ms} \).

**Figure 2.4:** Influence of the duration on the size of the critical disturbance, computed by:

- **a** - Meuris [1984]
- **b** - Ishibashi et al. [1979]

In the above described stability analyses the helium temperature \( T_{He} \) in (15) is assumed to be constant. In practice this assumption is only valid in case of a bath cooled magnet. In a forced cooled conductor, using supercritical helium as a coolant, the bulk flow temperature will rise due to the limited heat capacity of the helium inside the narrow cooling channels. Note that due to this fact an ICS will never be cryostable because a large enough disturbance will rise.
the coolant temperature always above the current sharing temperature making recovery impossible. Also a stationary state for heat transfer will in general not exist in this case [Meuris(1984), Dresner(1984)]. Stability criteria for this case can therefore only be found from transient studies in which the rising helium temperature is accounted for by solving the energy equation for the coolant simultaneously with the energy equation for the copper/superconductor matrix. Ries [1981] used a one dimensional version of the helium energy equation. The helium flow velocity he assumed to be constant and equal to $u_{He}$ leading to:

$$\left(\rho C_p\right)_{He} \left[ \frac{\partial T_{He}}{\partial t} - u_{He} \frac{\partial T_{He}}{\partial x} \right] = q_{He}(T, T_{He}) D_w / A_{tot} \tag{2.17}$$

For the cooling term $q_{He}$ relation (15) is used, the temperature $T$ of the conductor is evaluated from (14). The heat transfer coefficient $h$ is assumed to be constant and its value is taken from stationary heat transfer correlations. Taking into account the temperature dependency of the fluid and metal properties, the problem is nonlinear and numerical integration of the equations with the aid of computers is essential from this level on. This means that no general applicable stability formula can be expected from these studies. These transient studies rather aim to identify the exact stability region of a specified conductor and can be used to compare different conductor designs.

In an ICS cooled with supercritical helium not only the temperature rise of the coolant but also the transient development of the temperature profile in the flow effects the heat transfer and thus the stability of the conductor. For small times after the metal temperature is raised due to a thermal disturbance in the conductor, the heat transfer to the coolant can be modeled with the penetration theory. In this theory the transient heat transfer coefficient $h_t$ is given as:

$$h_t \propto \sqrt{t} \tag{2.18}$$

For small values of the elapsed time $t$ after the initial disturbance, the transient heat transfer coefficient $h_t$ is much larger than the
stationary heat transfer coefficient $h$, due to the large temperature gradient near the wall. So, a large amount of energy will be cooled away shortly after the disturbance occurred. This will have a large positive effect on the stability of the conductor. Hoenig, Montgomery and Waldman [1979] included transient heat transfer in their computer code MITAD by simply adding $h_t$ to the stationary value $h$. Also in other computer codes transient heat transfer has been treated this way, however, the abrupt decrease in heat transfer when the temperature of the fluid near the tube wall cross the pseudo critical line (burnout) can not be described this way [Dresner(1984)]. To account for this effect the temperature profile in the tube has to be computed using a two dimensional energy model. The instantaneous heat transfer coefficient can then be computed from the local temperature gradient near the wall and approximation (18) is no longer needed.

In all above described models the fluid velocity in the cooling channel of a conductor has been assumed to be constant. To assure a large enough heat transfer, the fluid flow has to be highly turbulent. This requires a large fluid velocity and results in a high pumping power requirement. However, in experiments on the stability of an ICCS, Iwasa, Hoenig and Montgomery [1977] noted that the stability margin was nearly independent of the helium flow rate. Even a significant stability margin at zero flow rate was found! At first this unexpected, but very pleasant, result was ascribed to a rapid heat transfer due to transient conduction as described above. However, soon it was recognized that the increased stability was caused by a strong flow initialized by the expansion of the heated helium. Since then a lot of work on heating induced flow has been reported. For example flow velocities of several meters per second ($Re \approx 10^5$), large enough to provide sufficient cooling for recovery, have been reported by Lue, Miller and Dresner [1980].

In the same paper Lue, Miller and Dresner also reported another surprising phenomenon the so called multivalued stability. In experiments on the stability of an ICCS at low coolant mass flows, they observed more than one transition from a stable to an unstable
performance of the magnet when they increased the applied disturbance. In figure 2.5 the stability margin ($e$) is sketched as a function of ambient pressure $P$, transport current $I$ and helium flow velocity $v$. The open and cross-hatched areas denote the stable and unstable operation regions of the magnet respectively. As can be seen from figure 2.5b, for low flow velocities the magnet is stable against the smallest disturbances, unstable against somewhat higher disturbances and again stable for still higher disturbances. Finally the magnet is unstable against the highest disturbances.

![Diagram showing stability margin $e$ as a function of ambient pressure $P$, transport current $I$, and helium velocity $v$.]

Figure 2.5: Sketch of the stability margin $e$ as a function of ambient pressure $P$, transport current $I$ or helium velocity $v$.

To find an explanation for this unusual phenomenon, Lue, Miller and Dresner artificially broke the coupling between heat transfer and induced flow. At first they calculated the stability margin $e$, imagining the heat transfer coefficient $h$ to be externally imposed. The result is schematically shown in figure 2.6a. The second part of the problem is to find the heat transfer coefficient $h$ as a function of an externally imposed heat flux rate $e$. In this part of the problem the influence of the induced flow and the transient heat transfer effects have to be modeled. Also the negative effect of pseudo
boiling (blanketing of the surface with low density helium at temperatures near the pseudo critical line) on heat transfer has to be included. The resulting heat transfer curve is the line CDAB as sketched in figure 2.6b. For small values of the imposed heat flux $e$ the helium temperature will not reach the pseudo critical value or only when most of the energy has already been transferred. In this case the effect of the transient heat transfer will be large and the overall heat transfer coefficient will be high (arc CD). As $e$ increases pseudo boiling, with the consequent deterioration of heat transfer, will occur sooner. Transient heat transfer will be less effective and stationary heat transfer, governed by convection to the induced flow (represented by the line OAB), will become more important. As a consequence the overall heat transfer will drop (arc DA). When $e$ is increased beyond point A stationary heat transfer is the principal mechanism and $h$ will increase with $e$ following arc AB.

![Diagram a: Variation of $e$ with imposed $h$](image1)

![Diagram b: Dependence of $h$ on imposed energy $e$](image2)

**Figure 2.6:** Relation between stability margin $e$ and heat transfer coefficient $h$.

By combining figures 2.6a and b in figure 2.7 four different regions of operation can be distinguished. If the magnitude of $e$ exceeds that of point R (for example point 1) the heat transfer coefficient needed for a stable performance (fig 2.4a) is larger than the induced heat transfer coefficient (fig 2.4b). The cooling is thus
insufficient to cause recovery and the conductor is unstable. If \( e \) lies between point Q and R (say point 2) the opposite argument applies and the conductor is shown to be stable. Similar reasoning shows a stable performance in region OP and an unstable performance in region PQ. So in this situation the conductor exhibits a multiple stable character. The influence of the initial mass flow, ambient pressure and transport current can also be qualitatively explained using this model. For more details one is referred to the original paper of Lue, Miller and Dresner.

To be able to predict a multiple stable performance of a magnet it is very important to know the actual shape of the cooling curve 2.4b. Especially the depth of the dip at point A must be well predicted to find the unstable region QP (fig 2.5). In most numerical studies the coolant flow is described by a one dimensional model in which the heat transfer is described by a Dittus Boelter like correlation. To include transient conduction into the coolant a transient heat transfer coefficient as in equation (18) is usually included. This kind of modeling is, however, inadequate to describe heat transfer [Dresner (1984)]. Especially the abrupt decrease of the heat transfer coefficient as the temperature of the coolant near the wall passes through the pseudo critical line (burnout) cannot be described by a one dimensional model. This is one of the reasons why none of the available codes has been able to predict multiple stability. Therefore a two (or three) dimensional model for the coolant flow, as initiated by Cornelissen [1984], is needed to provide a better insight in the

Figure 2.7 :

Combined plot of stability curve (2.6a) and the curve of induced heat transfer (2.6b).
induced heat transfer phenomenon.

Figure 2.8: Schematic representation of the stability margin, on a logarithmic scale, as a function of the transport current [Miller(1985)].

To design a magnet it is of great importance to know the location of the multiple stability regimes. As can be seen from the sketch in figure 2.8, taken from a paper of Miller [1985], the stability margin (in arbitrary units) increases by a full order of magnitude when the transport current is decreased below $I_{\text{lim}}$, at which the multiple stability regime starts. The value of limiting current $I_{\text{lim}}$ is hard to predict in practice. A scaling rule according to the theory of Lue, Miller and Dresner has been given, for initially stagnant helium, by Miller et al. [1980]:

$$J_{\text{lim}} = C \left[ f_c \left( 1 - f_m \right) / f_M \right]^{1/2} \left[ \frac{T_c - T_0}{\rho_{e1, c \theta}} \right]^{1/2} \left[ \frac{L}{l_q} \right]^{1/5} D^{-1} \quad (2.19)$$

where $f_c$ is the copper to superconductor ratio in the conductor and $f_M$
the metal to helium ratio. \( t_q \) and \( l_q \) are the duration and extension of the disturbance respectively. \( D \) is the hydraulic diameter of the helium filled part of the cable and \( C \) an unknown constant. The dependency on the disturbance parameters \( t_q \) and \( l_q \) in (19) has been found by using the dittus-boelter relation, in which \( \text{Nu} = \frac{\text{Nu}}{\text{Re}^{1/5}} \), for relating the heat transfer coefficient to the induced flow velocity. From experiments, however, the 1/5 power has been found to be unsatisfactory. To represent the experimental results the 1/5 power in (19) has to be replaced by a 1/15 power [Dresner(1984)]. The value of the prefactor \( C \) is not well established yet. In case of a helium pressure of 5 bar and temperature of 4.2 K Miller [1985] reports \( C = 0.6 \) (SI units) while earlier for the same case a value of \( C = 1.0 \pm 0.1 \) had been found. In addition Miller pointed out that the value of the prefactor \( C \) has to be dependent on the initial state of the helium. As a result the exact value of the limiting current is hard to predict. Together with the fact that multiple stability only occurs over a very narrow current range [Lue, Miller and Dresner(1980)] this makes it very hard to find this phenomenon numerically, even if heat transfer is accounted for correctly.

2.2 Propagation velocity

In the previous paragraph criteria to predict the stability of a superconducting magnet have been reviewed. In case of high performance magnets stability was found to depend on the magnitude of the applied disturbances. Although a magnet will be designed to be stable against all expected disturbances, in practice the spectrum of occurring disturbances is unknown and the possibility of unstable behavior can not be fully excluded. If a quench occurs in a conductor the normal zone will expand and the conductor temperature will rise. The velocity at which the normal zone expands is known as propagation velocity. To detect a quench the voltage over the conductor is measured. If a certain threshold value is exceeded the magnet is de-energized by using a dump circuit. Since the voltage over the conductor is proportional to the length of the normal zone a high propagation velocity leads to a quick detection of a quench and consequently reduces the chance of damaging the magnet. So, not only stability but also the propagation
velocity in case of a quench has to be taken into account when judging
the performance of a magnet design.

To compute the propagation velocity of a normal zone the time
dependent energy equation for the conductor as described in paragraph
2.1 (equation 2.14) has to be solved:

\[
\rho C_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left( \lambda \frac{\partial T}{\partial x} \right) - \left[ q_{He}(T,T_{He}) - q_D(T) \right] D_w / A_{tot}
\]  \hspace{1cm} (2.14)

By assuming
- Constant metal properties \(\rho C_p\) and \(\lambda\)
- Constant helium (bath) temperature \(T_{He}\)
- Constant propagation velocity \(w\)

and using a step function at \(\mathcal{F} = \mathcal{F}_c\) for the generation function \(G\) like
in equation (9) (\(\mathcal{F}_c = 1-j\)), equation (14) can be solved analytically.
The dimensionless temperature \(\mathcal{F}\) is defined by equation (7). For the
boundary conditions one end of the conductor is assumed to be "cold"
(\(\mathcal{F} = 0\)). At the other, normal conducting, end of the conductor heat
dissipation and removal by the coolant are assumed to be in
equilibrium. At this end, which can be also be a symmetry point the
temperature is constant and taken equal to \(\mathcal{F} = \mathcal{F}_i\) (see figure 2.9).

\[\begin{array}{c}
\mathcal{F} \\
cold \hspace{2cm} hot \\
\end{array}\]

\[\begin{array}{c}
\mathcal{F} \\
cold \hspace{2cm} hot \hspace{2cm} cold \\
\end{array}\]

a: whole conductor \hspace{1cm} b: half conductor (symmetry)

Figure 2.9: boundary conditions in the conductor

In case of a constant heat transfer coefficient \(h\) the
dimensionless propagation velocity \(w\) defined by

\[
w = w \left[ \frac{A_{tot}}{h \lambda D_w} \right]^{1/2} \rho C_p
\] \hspace{1cm} (2.20)

has been calculated by Broom and Roderick [1960]. They arrived at

\[
w = (\mathcal{F}_i - 2\mathcal{F}) \left[ \mathcal{F}(\mathcal{F}_i - \mathcal{F}) \right]^{1/2}
\] \hspace{1cm} (2.21)

Several other workers arrived at the same formula. For \(\mathcal{F}\), the constant
temperature at the "hot" end of the conductor, usually the value
\( \mathcal{I}_i = \alpha_i^2 \) has been used. For the value of \( \mathcal{I}_c \) three different values have been reported. Cherry and Gittleman [1960] used the critical temperature (\( \mathcal{I}_c = 1 \)) while Keilin et al. [1967] used the current sharing temperature (\( \mathcal{I}_c = 1-j \)). A compromise between these two values (\( \mathcal{I}_c = 1-j/2 \)) has been proposed by Dresner [1978]. This value is supposed to be the most realistic value.

Transient heat conduction into the helium flow has been taken into account by Lvovsky and Lutset [1982]. The helium temperature profile they computed from a one dimensional heat conduction model

\[
\frac{\partial T_{He}}{\partial t} = \alpha_{He} \frac{\partial^2 T_{He}}{\partial y^2}
\]

resulting in a transient heat transfer \( q_{He} = h(T-T_b) + q_t \), where

\[
q_t = \left[ \frac{(\lambda \rho C_p)_{He}}{\pi} \right]^{1/2} \int_{-\infty}^{t} \frac{\partial T}{\partial \tau} (t-\tau)^{-1/2} \, d\tau
\]

represents the transient heat conduction into the helium. Substituting \( q_{He} \) in equation (14) an integro-differential equation results which can be solved analytically. Beside the Stekly parameter \( \alpha_i \) a new dimensionless parameter \( \varepsilon \) appears in the solution procedure. This parameter, defined as:

\[
\varepsilon = \left[ \frac{(\lambda \rho C_p)_{He} D_w}{h \rho C_p A_{t o t}} \right]^{1/2}
\]

characterizes the transient heat conduction intensity with respect to the steady state heat transfer to the coolant.

Assuming \( \mathcal{I}_i = \alpha_i^2 \) and \( \mathcal{I}_c = 1 \) the transient propagation velocity \( \alpha_t \) can be written as:

\[
\alpha_t = \alpha \left[ 1 + \frac{3}{8} \zeta \left( 1 -\left( 1 + \frac{16}{2\zeta} \right)^{1/2} \right) \right]^{1/2}
\]

in which \( \alpha \) is the "stationary" propagation velocity found in equation (21). The parameters \( \xi \) and \( \zeta \) in (25) are given by
\[ \zeta = \frac{\varepsilon^2}{\alpha \cdot j^2 - 2} \quad \text{and} \quad \zeta = \frac{\varepsilon^2}{\alpha \cdot j^2 - 1} \]

From (25) it can be seen that \( \omega \) equals \( \omega \) only if \( \varepsilon = 0 \), in which case transient heat conduction is negligible to the steady state heat transfer. If transient conduction cannot be neglected \( (\varepsilon > 0) \) the transient propagation velocity \( \omega \) is less than the stationary value \( \omega \) going to very low values as \( \varepsilon \) becomes very large (high transient heat transfer).

More advanced models, taking into account effects of current sharing [Altov et.al.(1977)], variable metal properties [Dresner (1976), Ishibashi et.al.(1979b)] and heating of the helium [Greene and Saibel(1969), Bald(1970)] have been reported. In these cases no analytical solution for the propagation velocity is available but a numerical method had to be used. This means that no general applicable formula emerge from these studies. However, a representation of the graphical results of Altov et.al. for the current sharing case, has been given by Turck [1980]:

\[ \omega = \frac{\alpha \cdot j^2 + j - 2}{(\alpha \cdot j^2 - 1)^{1/2}} + (\alpha \cdot j^2 - 1)^{1/2} \left[ 2 - 2 \left( \frac{1 - j}{1 + j} \right)^{1/2} - \left( \frac{1 + j}{1 - j} \right) \right] \]  (2.26)

with \( Y = \frac{1}{(1 + 8\alpha \cdot j^{1/2} - 1)(2\alpha)^{-1}} \). This solution comes near the solution of Dresner [1978], who used a step function at \( \mathcal{G} = 1 - j/2 \) for the generation function.

In all above described models the propagation velocity \( \omega \), and the temperature at the normal conducting end, \( \mathcal{G} \), have been explicitly been assumed to be constant. In a forced cooled conductor, however, the helium temperature will rise in time and an equilibrium between heat generation and removal will not exist. This means that \( \mathcal{G} \) can not be constant. Also the propagation velocity can vary in time. Cornelissen [1985c] and Marinucci et.al. [1979] solved the energy equation (14), including current sharing and variable metal properties, with a zero temperature gradient at the ends of the conductor. In this
way the temperature in the normal zone and the propagation velocity were not restricted and could change with time. The cooling, in both models has been accounted for by solving a one dimensional flow model. The heat transfer coefficient at the computed velocity has been taken from a Dittus-Boelter like correlation. The difference between both models lies in the energy equation for the coolant. Marinucci et. al. did not include the term $P \frac{du}{dx}$, describing the work done by pressure on compression of the fluid, in the energy equation whereas Cornelissen did. According to Cornelissen this term is quite important. Due to the temperature rise in the coolant, caused by compression of the fluid, the temperature margin at the edge of the normal zone decreases in time. As a result Cornelissen found a propagation velocity which increased in time. No time dependency of the propagation velocity has been reported by Marinucci et. al.

2.3 Heat transfer to supercritical helium

In the previous paragraphs of this chapter it has been pointed out that the knowledge of the heat transfer phenomenon to the supercritical helium is of great importance for the prediction of both the stability of and propagation speed in a super conducting magnet. In literature several numerical and experimental investigations on this subject are known. In this paragraph the principal stationary experiments, as reviewed by Cornelissen [1984], together with new experimental data will be reviewed. Most attention will be focused on transient heat transfer data which have recently become available. Especially the experiments carried out by Bloem [1986] on transient heat transfer from the cooling channel of the SULTAN conductor will get a lot of attention since these experiments have been done in collaboration with this study in view of verification of the numerical model presented in this thesis (see Chapter 6). Since also the stability analysis in this thesis will be performed on the SULTAN conductor, the configuration of this conductor will be specified here.

2.3.1 Stationary heat transfer experiments

The best known and most widely used correlation describing the
heat transfer to supercritical helium has been provided by Giarratano and Jones [1975]. From extensive series of measurements on a 200 mm long heated pipe with an inside diameter \( D = 2.13 \) mm the results on the overall stationary heat transfer coefficient has been correlated as:

\[
Nu = 0.0259 \, Re^{0.8} Pr^{0.4} \left( \frac{T_w}{T_b} \right)^{-0.716} \quad (2.27)
\]

This correlation is, however, only valid for low heat fluxes. To distinguish between high and low heat fluxes they defined a correlating factor \( \varphi_i \)

\[
\varphi_i = \frac{4zq}{(H_{pc} - H_0)GD} \quad (2.28)
\]

which can be interpreted as the quotient of the inserted heat up to position \( z \) and the heat required to bring the fluid from the initial enthalpy \( H_0 \) to the pseudo critical enthalpy \( H_{pc} \). In the experiments the imposed heat flux \( q \) has been varied from 80 to 7130 W/m\(^2\). The flow rate \( G \) has been taken 70, 120 and 220 kg/m\(^2\)s. At a pressure of 2.5 bar and inlet temperatures of 4 and 5 K correlation (27) has been shown to hold for all heat fluxes as long as \( \varphi_i < 0.3 \). For heat fluxes yielding higher \( \varphi_i \) values a deterioration of the experimental heat transfer coefficient to as low as 12 per cent of the predicted value by relation (27) has been observed. Especially near the pseudo critical temperature \( T_{pc} \), when \( T_b < T_{pc} < T_w \), degradation was high.

In similar measurements on a 200 mm long heated section of a straight, 1.25 mm inner diameter tube, Ito et.al. [1986] used

\[
\varphi_i = \frac{q}{GD} \left[ \frac{kJ}{kg \, m} \right] \quad (2.29)
\]

as a correlating parameter to describe the deterioration of heat transfer. For low heat fluxes, \( \varphi_i \) up to 20, the measured heat transfer coefficients could be well correlated by the Dittus-Boelter correlation

\[
Nu_{DB} = 0.023 \, Re^{0.8} Pr^{0.4} \quad (2.30)
\]

for pressures varying from 3 to 8 bar. Deterioration of heat transfer
occurred at a high heat flux rate, $\varphi_i = 40$, especially when the caloric mean temperature of the fluid was near the pseudo critical value.

In these experiments also the influence of gravitation (buoyancy) on heat transfer has been studied by comparing up and down flow results. At a high flow rate, $G = 40$ [kg/m$^2$s], no difference between up and down flow has been observed for $\varphi_i$ up to 40, which means that buoyancy effects are negligible. At lower a flow rate, $G = 20$ [kg/m$^2$s], and a high heat flux rate, $\varphi_i = 40$, buoyancy effects are observed when

$$\frac{Gr}{Re^{2.7}} > 10^{-5} \quad (2.31)$$

This is in agreement with observations of Jackson and Hall [1975]. In other criteria from Brassington and Cairns [1977] and Hall [1971], the value of the constant in (31) has been found to be somewhat higher ($2.4 \times 10^{-5}$ and $1.2 \times 10^{-4}$ respectively).

In this paper Ito et.al. do not give a general applicable heat transfer correlation.

A correlation that fits data better than the Giarratano correlation, specifically in the region of large deterioration near the pseudo critical temperature, has been obtained by Yaskin et.al. [1977] by combining the results of various different experiments. They arrived at:

$$\frac{Nu}{Nu_{DB}} = \left[1 - 0.2 \left(\frac{Nu}{Nu_{DB}}\right)\beta \Delta T\right]^2 \quad (2.32)$$

in which $\Delta T$ is the temperature difference between the wall and the bulk of the fluid, $\beta$ is the compressibility of the fluid defined by:

$$\beta = -\left[\frac{1}{\rho} \left(\frac{\partial \rho}{\partial T}\right)_p\right] \quad (2.33)$$

and $Nu_{DB}$ is the Dittus-Boelter defined in equation (30). For the derivation of relation (32) they considered an analogy between the expansion of the fluid near the wall and the case of a fluid flow with gas injection through a porous wall for which such a correlation is available.

An aspect of the heat transfer at low temperatures which has not been considered yet is the Kapitza resistance. This resistance
represents the thermal resistance encountered when phonons, carrying the thermal energy, pass from the metal to the fluid. At low temperatures the heat transfer from a metal to a fluid is restricted by the availability of phonons in the metal. Cornelissen [1984] estimated the Kapitza resistance in a superconducting magnet and found that the influence on heat transfer from the conductor to supercritical helium at temperatures above 4 K, as under study in this thesis, could be neglected. In the rest of this thesis the Kapitza resistance will not be taken into account.

2.3.2 Transient heat transfer experiments

2.3.2.1 The SULTAN conductor

In this thesis the behavior of an existing conductor, developed at the Netherlands Energy Research Foundation (ECN) and employed in the inner coil of the SULTAN magnet, will be studied. The SULTAN (SUpra Leiter Test ANlage) magnet is a high field test facility for conductor tests. This magnet has been build in a cooperation between ECN, SIN (Schweizerischer Institut fur Nuklearforschung) and CNEN (Comitato Nazionale per l’Energia Nucleare,Italy) [Elen et.al(1981), Franken and Spoorenberg(1981), Horvath et.al.(1983)]. CNEN developed a 6 Tesla outer coil. The inner coil, developed at ECN, has been used to enhance the magnetic field up to 8 Tesla. SIN provided the instrumentation.

The ECN inner coil is a 1.15 m long solenoid with an inner diameter of 1.08 m and an outer diameter of 1.26 m. The super conductor is wound in 10 layers of 125 turns, each layer wound from a single length conductor of about 450 m. The transport current in the conductor is 1860 A (j=I/I_c=0.535).

At ECN transient heat transfer experiments on the SULTAN inner coil conductor have been carried out by Bloem [1986]. These experiments, which will be used to verify the transient heat transfer computations in chapter 6, are reviewed in the next paragraph. In this paragraph the conductor specification will be described.

35
Figure 2.10: The SULTAN inner coil conductor.

The SULTAN conductor is a 16 strand Rutherford cable soldered on a rectangular copper tube (see figure 2.10). Each strand is composed of 3677, 36.5 \( \mu \text{m} \) diameter, superconducting filaments (NbTi) embedded in copper. The relevant geometry parameters are gathered in table 2.1. In table 2.2 and 2.3 the critical superconducting properties and the metal properties respectively are given.

<table>
<thead>
<tr>
<th>Table 2.1: Conductor geometry</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hydraulic diameter</td>
</tr>
<tr>
<td>Wetted perimeter</td>
</tr>
<tr>
<td>Helium cross section</td>
</tr>
<tr>
<td>Metal cross section</td>
</tr>
<tr>
<td>Fraction superconductor</td>
</tr>
<tr>
<td>Fraction copper</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 2.2: Superconducting properties of NbTi at ( B = 8.2 \text{T} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Critical temperature</td>
</tr>
<tr>
<td>Critical current density</td>
</tr>
</tbody>
</table>

36
Table 2.3: Metal properties

<table>
<thead>
<tr>
<th></th>
<th>Superconductor</th>
<th>Copper</th>
<th>Composite</th>
</tr>
</thead>
<tbody>
<tr>
<td>ρ</td>
<td>kg/m³</td>
<td>6700</td>
<td>8900</td>
</tr>
<tr>
<td>ρC_p</td>
<td>J/m³K</td>
<td>1507T + 14.3T³</td>
<td>96T + 6.6T³</td>
</tr>
<tr>
<td>λ</td>
<td>W/mK</td>
<td>0.02T</td>
<td>45T</td>
</tr>
<tr>
<td>ρ_{el}</td>
<td>Ωm</td>
<td>0.3 \times 10^{-6} (normal cond.)</td>
<td>0.5 \times 10^{-9}</td>
</tr>
</tbody>
</table>

To complete the conductor specification the flow inlet parameters like mass flow, inlet temperature and ambient pressure must be given. For the mass flow a typical value is 1 g/s, the inlet temperature is usually about 4.2 to 4.4 K. The ambient pressure can vary between 3 and 15 bar.

2.3.2.2 The experiments of Bloem

![Diagram](image)

Figure 2.11: Test section in experiments of Bloem [1986]

Bloem [1986] has studied the transient heat transfer from a conductor to the forced flow of supercritical helium inside the cooling channel. His test section, as shown in figure 2.11, consists of a 174 mm long rectangular OHFC copper tube soldered between two 90° bends of stainless steel (soldered length in each bend is 9.7 mm). The copper tube is the same as has been used in the SULTAN conductor described in the previous paragraph. In the SULTAN design a superconducting
Rutherford cable has been soldered onto the copper tube. The energy dissipation in this cable, however, is hard to control. Bloem omitted the Rutherford cable in his experiments. Instead he inserted a fixed amount of energy, \( \phi_{in} \), into the copper channel by using electrical heaters, made of manganin foil, which he glued on the four outer sides of the channel. The temperature of the tube has been measured with two fast response thermometers T1 and T2. The position of these thermometers Bloem has determined on basis of the thermal entrance length. This length, which is defined as the length of pipe required to reach a Nusselt number that is within 5% of the fully developed flow value, must be larger than 8 hydraulic diameters. Bloem situated the first thermometer at 65 mm from the bend which is 13 hydraulic diameters. The temperature readings of the two thermometers were found to differ less than 0.2 K, even at large temperature rises. Since also axial conduction of energy from the test section has been shown to be negligible because of the very low thermal conductivity of the stainless steel bends \((\lambda \approx 0.1 - 1 \text{ W/mK})\), the temperature in the test section has been supposed to be uniform. The instantaneous heat transfer, \( \phi_{He} \), to the helium flow can then be computed from the transient temperature measurements using:

\[
\phi_{He} = \phi_{in} - mC_p(T) \frac{dT}{dt}
\] (2.34)

in which the last term represents the heating of the tube. The heat capacity of the test section, \( mC_p(T) \), has been measured as a function of temperature and could be expressed by:

\[
mC_p = (4.0T^3 + 33.44T) \times 10^{-5} \text{ J/K}
\] (2.35)

All the experimental heat transfer phenomena have been related to the difference between the copper temperature T and the fluid bulk temperature \( T_b \), which has been shown to be almost constant and equal to the inlet bulk temperature \( T_0 \). This temperature can be measured with the same thermometers as used for measuring the copper temperature because before the heat generation in the manganin heaters is started the helium bulk temperature and the wall temperature are both equal to \( T_0 \).
The instantaneous heat transfer coefficient $h$ has been defined as

$$h = \frac{\phi_{He}}{D_w(T - T_0)}$$

(2.36)

Where $D_w$ is the wetted perimeter inside the tube. $\phi_{He}$ is the heat flow from the wall into the helium and is defined by equation (34). In (36) the heat transfer coefficient is correlated to the inlet bulk temperature, which is common in heat transfer research.

In his experiments Bloem varied the pressure, mass flow and inlet temperature of the helium flow, as well as the amount of heat inserted into the test tube, as shown in table 2.4. The duration of the applied heat pulses usually has been taken to be 10 or 20 milliseconds.

<table>
<thead>
<tr>
<th>Table 2.4 : Parameter variations in experiments of Bloem</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure</td>
</tr>
<tr>
<td>Mass flow</td>
</tr>
<tr>
<td>Temperature</td>
</tr>
<tr>
<td>Heat pulse</td>
</tr>
</tbody>
</table>

In the experiments the heat transfer in the first milliseconds is governed by heat conduction in the boundary layer of the super critical helium flow. After some time convection heat transfer caused by turbulence of the flow will become the important heat transfer mechanism. Bloem made a rough estimate of the "take over time", $t_t$, when this change of mechanism takes place, by comparing the developing thermal boundary layer thickness, $\delta_T$, with the thickness of the effective momentum boundary layer, $\delta_{eff}$. The developing thermal boundary layer computed from the heat penetration theory for a constant wall heat flux started at $t = 0$ is given by Carlslaw and Jaeger[1959] :

$$\delta_T = \sqrt{4at/\pi}$$

(2.37)

The friction factor for supercritical helium can be expressed by the same relation as for classical fluids [Junghans(1980)] :

$$f = 0.076 \Re^{-1/4}$$

(2.38)
Using (38) the effective momentum boundary layer becomes:

\[ \delta_{\text{eff}} = \frac{2D}{0.076 \text{Re}^{0.75}} \]  

(2.39)

The resulting expression for the take over time follows from the equations (37) and (39) by requiring \( \delta_{\text{r}} = \delta_{\text{eff}} \):

\[ t_{\text{r}} = \frac{\pi}{a} \left( \frac{D}{0.076} \right)^{2} \text{Re}^{-1.5} \]  

(2.40)

A correlation for fitting the transient heat transfer results using a power law curve, \( h \propto t^{n} \), has been searched by Bloem. Using the take over time \( t_{\text{r}} \) he arrived at the following relation for the transient Nusselt number \( \text{Nu}_{\text{tr}} \) at time \( t \):

\[ \text{Nu}_{\text{tr}} = C_{1} \text{Re}^{0.75} \left( \frac{t_{\text{r}}}{t} \right)^{1/n} \]  

(2.41)

The exponent had to be a function of the mass flow in order to avoid large discrepancies between the computed and measured values for \( \text{Nu}_{\text{tr}} \). For \( \dot{m} \) ranging from 1 to 3 gr/s, \( n \) is expressed by

\[ n = 500 \dot{m} + 2.5 \]  

(2.42)

with \( \dot{m} \) in kg/s. For the constant \( C_{1} \) theoretically a value of \( C_{1} = 0.038 \) was found. From the experimental results a value of \( C_{1} = 0.06 \) emerged. Relation (41) then predicts the heat transfer results within 10%.

The analytical solution for the heat transfer coefficient in case of a constant heat flux starting at \( t = 0 \) to a fluid with constant properties [Carlslaw and Jaeger (1959)] is

\[ h = \frac{1}{2} \left( \frac{\pi \lambda \rho C_{p}}{t} \right)^{1/2} \]  

(2.43)

This solution, valid for short times \( t \), is completely determined by the combination of the fluid properties

\[ \varepsilon = \lambda \rho C_{p} \]  

(2.44)

and the time \( t \) itself. Bloem used the parameter \( \varepsilon \), which is some kind of volumetric heat absorption coefficient, to explain the influence of ambient pressure and inlet temperature of the flow on the transient

40
heat transfer coefficient qualitatively. In figure 2.12 the parameter $\varepsilon$ is plotted against temperature for supercritical helium at three different pressures. At an inlet temperature of 4.2 K $\varepsilon$ is about the same for all three pressures. At this temperature the pressure influence on heat transfer has been found to be small, just as expected. At an inlet temperature of 5 K an increase in the heat transfer coefficient at a pressure of 3 bar is predicted by the parameter $\varepsilon$ and is also found experimentally. Also the observed deterioration of heat transfer at an inlet temperature of 6 K in the 3 bar case can be explained from figure 2.12.

Although the parameter $\varepsilon$ has emerged from the penetration theory, which is valid only for constant fluid properties and for small times, the heat transfer results from Bloem could qualitatively be well predicted by it.

![Diagram](image)

**Figure 2.12:** Variation of the parameter $\varepsilon = \lambda \rho C_p$ as a function of temperature for different pressures.
Figure 2.13:

Pressure transients in experiments of Bloem [1986] at
2 m (solid line) and
4 m (dotted line) from the disturbance.
In order to investigate the influence of heat transfer on the helium flow in the cooling channel Bloem developed a second test section. This test section is very similar to the first. However, after the stainless steel bend at the outlet of the tube (figure 2.11), a 4 meter long, spiralized tube has been added. Halfway and at the end of the spiral pressure sensors were soldered. With this set-up the existence of a pressure wave has been detected when the tube was pulse heated (Heat release of 34.4 W in 2 ms over a length of 50 mm). By comparing the times at which the pressure wave reaches the pressure sensors, the speed of sound in the helium flow could be found. The experimentally found values of the speed of sound agreed very well with the numerical values computed by McCarty [1973]. The strength of the induced pressure wave was typically between 0.01 and 0.05 bar (see figure 2.13). The values, extracted from the figure and gathered in table 2.5, are, however, very inaccurate because of the fluctuations on the static pressure.

<table>
<thead>
<tr>
<th>Ambient pressure (bar)</th>
<th>3</th>
<th>6</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Induced pressure (kPa)</td>
<td>4</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 2.5 : Strength of the induced pressure wave

2.3.2.3 The experiments of Nick

Nick [1986] has performed experiments on the transient heat transfer to a forced flow of supercritical helium, which are similar to the experiments of Bloem [1986] discussed in the previous paragraph. Nick developed a experimental set-up in which helium of 4.2 K at supercritical pressures (3 to 10 bar) was forced through a circular copper tube with an inner diameter of 4 mm. The flow Reynolds number varied between $10^4$ and $10^5$. Around the tube two copper cylinders are placed which are thermally insulated. By electrical heating energy has been inserted into one of the cylinders. This results in an increase of the temperature of the cylinder and consequently a heat transfer to the
helium flow. By measuring the temperature difference between the cold cylinder \( T = T_{He} = 4.2 \text{ K} \) and the heated cylinder with a thermocouple, the energy used for raising the copper temperature can be found. In the same way as in the experiments of Bloem (equation 34) the heat transfer to the helium flow is found by:

\[
\phi_{He} = \phi_{in} - \phi_B - \frac{mC_p(T)}{d_t} \frac{dT}{dt}
\]  \hspace{1cm} (2.45)

In (45) an extra term, compared to equation (34), emerges. This term, \( \phi_B \), represents the energy loss by conduction into the background. In the experiments this term could not be neglected and had to be estimated. The heat transfer coefficient is defined by equation (36).

With the electrical heater energy pulses varying from 2 to 20 kW/m² could be generated. The length of the heater could be varied from 4 to 16 mm (1 to 4 diameters). The heated length is thus much smaller than in the experiments of Bloem where a length of 175 mm (35 hydraulic diameters) was heated.

From his results Nick concluded that the length of the heated section had little influence. Also the system pressure and the Reynolds number of the flow did not have a large impact on the measured heat transfer.

The transient temperature data of Nick are, however, very inaccurate. Large fluctuations in the temperature and heat transfer curves make it hard to draw good conclusions from the experimental results. The results of experiments performed on a larger time scale (1 s) are less disturbed. From these experiments, in which both up- and down-ward flow in a vertical tube have been investigated, it has been found that buoyancy effects on heat transfer were negligible up to 0.1 s. For longer times heat transfer was significantly better in downward flow than in upward flow.
2.4 Turbulent flow models

For all Newtonian fluids which satisfy the continuum approximation (Kn < 0.1), the flow is described by the continuity equation (46) in combination with the Navier-Stokes equations (47). In case of a variable property flow in which the fluid properties depend on temperature, these equations are coupled to the energy equation (48). Following Bird, Stewart and Lightfoot [1960], using the enthalpy representation of the energy equation, these equations read:

\[
\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_j}{\partial x_j} = 0 \tag{2.46}
\]

\[
\frac{\partial \rho u_i}{\partial t} + \frac{\partial \rho u_i u_j}{\partial x_j} = -\frac{\partial P}{\partial x_i} - \frac{\partial \tau_{ij}}{\partial x_j} + S_m \tag{2.47}
\]

\[
\frac{\partial \rho H}{\partial t} + \frac{\partial \rho u_i H}{\partial x_i} = \frac{\partial P}{\partial t} + u_j \frac{\partial P}{\partial x_j} - \frac{\partial q_j}{\partial x_j} + \Phi + S_e \tag{2.48}
\]

In these equations the Einstein convention applies:
\[\phi_i \phi_i = \phi_1 \phi_1 + \phi_2 \phi_2 + \phi_3 \phi_3\]

The elements \(\tau_{ij}\) of the stress tensor can be written as:
\[
\tau_{ij} = \mu \left[ \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right] - (\kappa - \frac{2}{3} \mu) \frac{\partial u_k}{\partial x_k} \delta_{ij} \tag{2.49}
\]

with \(\delta_{ij}\) the Kronecker delta function and \(\kappa\) the bulk viscosity.

The source term \(S_m\) in (47) contains external body forces, like for example gravity, working on the fluid. Internal heat sources influencing the heat balance (48) are included in the source term \(S_e\). \(\Phi\) represents the viscous dissipation term and \(q\) is the energy flux vector.

In principle the equations (46) to (48), combined with the appropriate boundary- and initial conditions, and the constitution equations of the fluid, contain all detailed information about a fluid flow to be studied. In practice however, computation of a flow field from these equations is not straightforward. In only a few cases, like a laminar constant property pipe flow, an analytical solution does
exist. Most often a numerical formulation of the equations has to be solved using a digital computer. To do so, the flow region has to be divided into a discrete mesh with the mesh size smaller than the smallest length scale of the fluid motion. Also the time step between two discrete time levels must smaller than the smallest time scale of the fluid motion. In case of a turbulent flow the smallest time and length scale are the scales of the smallest eddies. These scales are orders of magnitude smaller than the macroscopic scales of the flow. As a consequence an enormous number of grid points and time steps are required to compute all flow characteristics. Such a task can not be handled by presently available computers. For this reason models are needed to describe the turbulent motion in the flow.

The most popular turbulence models are base on the Reynolds decomposition method, which divides all flow properties into a mean value and a fluctuating part. The "classical" models based on this principle will be described in paragraph 2.4.1.

Heat transfer from the tube wall to the flow under study is mainly dominated by the resistance of a thin fluid layer at the wall. To predict the stability of a superconductor a good description of the heat transfer- and turbulence processes near the tube wall is required. In the "classical" models the influence of the wall has been incorporated by using wall functions or by applying modifications to the freestream turbulence models. In paragraph 2.4.2 special wall turbulence models based on the turbulent burst or surface renewal principle will be reviewed.

2.4.1 Turbulence models based on the Reynolds decomposition

2.4.1.1 The Reynolds decomposition

The standard procedure to model a turbulent flow, originally proposed by Reynolds, is to decompose all instantaneous quantities \( \phi \) into a mean value \( \bar{\phi} \) and a fluctuating component \( \phi' \):

\[
\phi = \bar{\phi} + \phi'
\]  

(2.50)

In steady flow for the mean value a time average can be used. For variable density flows an alternative averaging process is the
mass-weighted time averaging or Favre averaging procedure [Cebeci and Smith (1974)]. In case of a time dependent flow these averaging processes can only be used if the time interval T over which the averaging takes place, is large compared to the time scale of the turbulent quantities to be modeled but still small compared to a transient time scale of the mean flow. An averaging process which can be used in all flows is ensemble averaging. Here the mean value is obtained bas the arithmetic mean over many identical, time dependent, experiments (see Landahl and Mollo-Christensen (1986)).

If a suitable averaging process has been chosen this Reynolds decomposition can be applied to the equations (46) to (48) by substituting equation (50) for all quantities. Averaging of these equation lead to the Reynolds equations (51) to (53). Averages of fluctuating components disappear because they are zero by definition. Averages of a product of fluctuating components are non zero because the statistic fluctuations are correlated.

- Continuity equation

\[
\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_j}{\partial x_j} = 0
\]  

(2.51)

- Navier-Stokes equation

\[
\frac{\partial \rho u_i}{\partial t} + \frac{\partial \rho u_i u_j}{\partial x_j} = - \frac{\partial P}{\partial x_i} - \frac{\partial}{\partial x_j} \left( r_{ij} + \sigma_{ij} \right) + S_{mi}
\]  

(2.52)

- Enthalpy equation

\[
\frac{\partial \rho H}{\partial t} + \frac{\partial \rho u_j H}{\partial x_j} = \frac{\partial P}{\partial t} + u_j \frac{\partial P}{\partial x_j} - \frac{\partial}{\partial x_j} \left( q_{ij} + \psi_{ij} \right) + \Phi + S_e
\]  

(2.53)

where

\[
\sigma_{ij} = - \rho \frac{u_i u_j}{\bar{u}^2} \quad \text{(Reynolds stresses)}
\]  

(2.54)

\[
\psi_k = - \rho \frac{u_k H'}{\bar{u}^2} \quad \text{(Turbulent heat flux rates)}
\]  

(2.55)

In the Reynolds equations (51) to (53) the overbar, denoting the mean value, has been omitted for brevity.

Fluctuations in the density $\rho'$ have been neglected. Cebeci and Bradshaw [1984] argue that in highspeed boundary layers the correlation
\( \rho'v' \) is of the same order as \( \rho v \), but in most other cases density fluctuations due to pressure and temperature fluctuations, can be neglected in boundary layer flows. Hinze [1975] gives a condition for which density fluctuations due to pressure fluctuations can be neglected:

\[
\text{Ma}^2 \ll 1 \tag{2.56}
\]

in which Ma is the Mach number of the flow. In case of a coolant flow through a cooling channel of a superconducting cable this condition is easily met.

Because of the non linearity of the equations (47) and (48) correlations of fluctuating components appear in the averaged equations. These correlations, the Reynolds stresses \( \sigma_{ij} \) in equation (52) and the turbulent heat flux rates \( \psi_k \) in equating (53) are extra unknown quantities in the equations. This means that there are more unknowns than equations. So the extra terms have to be modeled. This is called the "closure problem" of turbulence.

2.4.1.2 The Boussinesq hypothesis

The Reynolds stress tensor can, as the name already suggests, be interpreted as a stress tensor. In this interpretation the diagonal elements of \( \sigma \) are normal stresses which are compressibility terms. The non diagonal elements can be interpreted as shear stresses an play a dominant role in the transport of momentum.

In 1877 Boussinesq suggested that a turbulent flow could be regarded as having an enhanced viscosity. He modeled the Reynolds stress tensor much in the same way as the viscous stress tensor defining an eddy viscosity \( \varepsilon_m \) :

\[
\sigma_{ij} = \rho \varepsilon_m \left[ \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right] - \frac{2}{3} \rho k \delta_{ij} \tag{2.58}
\]

where \( k \) is the turbulent kinetic energy defined by :

\[
k = \frac{1}{2} \overline{u_i' u_i'} \tag{2.59}
\]

Unlike the molecular viscosity \( \mu \), which is a fluid property, the turbulent viscosity defined as :
\[ \mu_t = \rho \varepsilon_m \]  

(2.60)

is completely determined by the flow field.

Substituting (58) and (59) in equation (52), neglecting all unimportant stress terms, the derivative of the stress tensors can be replaced by:

\[ - \frac{\partial}{\partial x_j} \left[ \tau_{ij} - \sigma_{ij} \right] \approx \frac{\partial}{\partial x_j} \left[ \mu_{eff} \frac{\partial u_i}{\partial x_j} \right] \]  

(2.61)

The effective viscosity \( \mu_{eff} \) is the sum of the molecular and turbulent viscosity

\[ \mu_{eff} = \mu + \mu_t \]  

(2.62)

The turbulent heat flux rate \( \psi_k \) can be modeled in analogy with the Boussinesq hypothesis:

\[ \psi_k = -\rho \bar{u}_k \bar{H} = \rho \varepsilon_h \frac{\partial H}{\partial x_k} = \rho \frac{\varepsilon_m}{Pr_t} \frac{\partial H}{\partial x_k} \]  

(2.63)

The turbulent Prandtl number, \( Pr_t = \frac{\varepsilon_m}{\varepsilon_h} \), is found to be of order unity.

Sideman and Pinczewski [1975] give an extensive review of models describing \( Pr_t \). In view of the inconclusive nature of the existing experimental data they state that no particular correlation or model can confidently be recommended. The simple Reynolds analogy usually suffices for engineering calculations and is chosen by many workers for lack of confidence in other models. The Reynolds analogy in fact means that both heat- and momentum are transported as a result of the same eddy motion, i.e. \( \varepsilon_m = \varepsilon_h \) or \( Pr_t = 1 \).

The modeling of the eddy viscosity \( \varepsilon_m \), which is the only unknown left, is described in the next paragraph.

2.4.1.3 Eddy viscosity models

To compute the eddy viscosity \( \varepsilon_m \), a lot of models have been proposed varying from a simple algebraic or zero equation model to a model in which 28 additional differential equations have to be solved (Kolavandin). In analogy with the molecular, which is proportional to
the average velocity of the molecules and to the mean free path, the eddy viscosity is defined as:

$$\varepsilon_m = \mathcal{L} \nu$$ \hspace{1cm} (2.64)

$L$ is a length scale and $\nu$ a velocity scale. Some of the most popular models defining the scales are discussed briefly below.

* Zero equation model

The most simple model is the mixing length model of Prandtl [1945]. In this zero equation model the eddy viscosity is calculated from an algebraic equation so no additional differential equations have to be solved.

In this model the length scale $L$ is the mixing length $l_m$ which can be interpreted as the length scale of a typical eddy. The velocity scale $\nu$ is then defined as the velocity difference over an eddy:

$$\nu = \frac{l_m}{\frac{\partial u}{\partial y}}$$

For a pipe flow the mixing length can be calculated from Nikuradse's formula [Schlichting(1968)]:

$$l_m = R \left[ 0.14 - 0.08 \left( 1 - \frac{y}{R} \right)^2 - 0.06 \left( 1 - \frac{y}{R} \right)^4 \right]$$ \hspace{1cm} (2.65)

where $y = R-r$ the distance from the tube wall.

In the presence of a wall the turbulence is damped out. Van Driest [1956] found from experiment that this effect can be modeled by correcting the mixing length:

$$l_m' = l_m \exp\left[ - \frac{y^+}{A^+} \right] \hspace{1cm} A^+ = 26$$ \hspace{1cm} (2.66)

where

$$y^+ = \frac{u^*}{v} y$$

$$u^* = \sqrt{\frac{\tau_w}{\rho}}$$

is the friction velocity.

* One equation model

One of the major disadvantages of the mixing length model is that it assumes turbulence to be in local equilibrium. The model does not
account for convection and diffusion of turbulence. To overcome this
shortcoming more advanced models have been proposed in which the
velocity- and/or length-scale parameter in equation (64) are computed
from additional differential equations. These equations for the
correlations of the fluctuating components are found by manipulations
of the original Navier-Stokes equations. In these equations new unknown
correlations occur which have to be modeled.

An example of a model with one additional differential equation is
the k-model. This model, originally introduced by Prandtl [1945] and
Kolmochorov [1948], is also known as the Prandtl-Kolmochorov energy
model. In this model the velocity scale \( \nu \) is taken to be proportional
to the square root of the kinematic energy of turbulence \( k = \frac{1}{2} u_i u_i \):

\[
\nu = c_\mu \sqrt{k} \quad \text{where } c_\mu \text{ a constant}
\]  

(2.67)
The same length scale is the same as in Prandtl’s model.

An equation for \( k \) is found by multiplying the original
Navier-Stokes equations with the instantaneous velocity \( u_i \) and then
taking the average. After some manipulations the following equation
results [Tennekes and Lumley(1972)]:

\[
\frac{\partial k}{\partial t} + \rho u_j \frac{\partial k}{\partial x_j} = -\rho k \frac{\partial u_i}{\partial x_j} + \frac{\partial}{\partial x_j} \left[ \frac{\partial k}{\partial x_j} \right] - \rho \varepsilon \\
- \frac{\partial}{\partial x_j} \left[ \frac{\rho}{2} u_j u_i u_i + \frac{p}{2} u_j u_j \right]
\]  

(2.68)
The terms on the lefthand side represent the accumulation and
convection of turbulent kinetic energy. The first two terms on the
righthand side are a production and a diffusion term. The term
\( \rho \varepsilon = \frac{\partial u_j}{\partial x_j} \frac{\partial u_i}{\partial x_j} \) in (68) is a dissipation term. The last term, containing
the triple correlation and the pressure fluctuation, describes the
transport of turbulent energy by turbulent eddies and pressure
fluctuations. As such it is also referred to as a diffusion or
redistribution term. The modeling of this term and the production term
is described e.g. by Wolfshstein,Naot and Lin [1975]. With
\[
\frac{\partial}{\partial x_j} \left[ \frac{1}{2} \rho u_j u_j + p u_j \right] = \frac{\partial}{\partial x_j} \left[ \mu \frac{\partial k}{\partial x_j} \right]
\]  
(2.69)  
\[
-\rho k \frac{\partial u_i}{\partial x_j} = \mu_t \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)^2 = P_k
\]  
(2.70)  

the following equation results:  
\[
\rho \frac{\partial k}{\partial t} + \rho u_j \frac{\partial k}{\partial x_j} = \frac{\partial}{\partial x_j} \left[ \left( \mu + \frac{\mu_t}{\sigma_k} \right) \frac{\partial k}{\partial x_j} \right] + P_k - \rho \varepsilon
\]  
(2.71)  

leaving one constant, \( \sigma_k \), still to be determined.  

* Two equation model  

In a two equation model both the length- and velocity-scale are calculated from a differential equation. In most models the velocity scale is the same as in the above described k-model. For the length scale in most cases a differential equation for a variable \( \phi = k^m \) is constructed. The most popular model of this type is the k-\( \varepsilon \) model, using \( m = -1 \) and \( n = \frac{3}{2} \). After modeling the unknown terms in the \( \varepsilon \) equation a set of five unknown constants results [Rodi(1980)]. To close the model this set of constants is determined from experiments on well defined turbulent flows and from computer optimizations. In case of wall bounded flows, however, the k-\( \varepsilon \) model is not valid in the wall region where turbulence is damped out.  

One way to model the influence of the low Reynolds number wall region is to make use of wall functions. In this way the variations of the variables near the wall do not have to be calculated but are described by analytical profiles. This method requires detailed knowledge on the flow in the boundary layer. it is customary to use wall functions which are supposed to agree with universal profiles, valid for steady, fully developed, constant property turbulent flows. An other way to find wall functions is to model the wall region flow with a wall turbulence model like the Surface Renewal model which will be described in paragraph 2.4.2.  

An other approach is to use special low Reynolds number versions of the k-\( \varepsilon \) model. To account for the effects of the low turbulent flow near the wall some rather ad hoc assumptions like including damping
factor corrections in both the k and ε equation have been employed in these models.

2.4.1.4 Discussion

Turbulence models of different complexity, using the Reynolds decomposition as a starting point, have been described. The models varied from the simple mixinglength model, in which an scalar eddy viscosity is calculated from a simple algebraic equation, to complex models which model all Reynolds stresses separately. It is evident that a simple model like the mixinglength model can not describe all complicated features of a general turbulent flow. In theory the more complex a model the more features can be captured. For example the k-ε model describes the transport and dissipation of turbulence where the mixinglength model assumes local equilibrium. Both models however can only be used in flows where turbulence is equally distributed over all directions. For flow fields where turbulence can be anisotropic, the use of the more complex Reynolds stress models is required.

In practice however, a complex model does not need to be preferable over a simple one. As an example one is referred to an article of Rubesin et. al. [1977] in which a Reynolds stress model (not discussed here), a two equation eddy viscosity model and the mixinglength model are compared. Rubesin et. al. found that in most test cases the mixinglength model could well compete with the two other models.

In case of a steady flow of supercritical helium through a circular tube Cornelissen and Hoogendoorn [1985a] compared the heat transfer coefficients computed with the mixinglength model, the k model and k-ε model of Lam and Bremhorst [1981], with experimental correlations of Giarratano and Jones [1975] and Yaskin et. al. . The worst correspondence between the numerical and experimental heat transfer coefficients was found using the k-ε model. The mixinglength model gave the best predictions in this situation. The more complex k-model resulted in predictions that were equally good but not better. In addition the mixing length model always has the advantage of simplicity and higher computation speed. The model has extensively been tested and excellent results can be obtained using this model in case
of boundary layer- or pipe flows, as under study in this thesis.

2.4.2 Surface renewal models

In this paragraph some relevant models, based on a phenomenological description of turbulence in the wall region, will be described. In these models, which are valid only in the wall region of a flow, the Reynolds averaging procedure is not employed and no eddy viscosity and turbulent Prandtl number have to be introduced.

2.4.2.1 General description of the turbulent burst mechanism

It is well known that the diffusivity of momentum, heat and mass in a turbulent field are much lower in the vicinity of a solid boundary than in the bulk of the flow far away from the wall. In classical wall turbulence modeling a laminar sublayer postulated by Prandtl is often used. The velocity profile in this layer is supposed to be stationary and linear with the distance from the wall. The thickness of this layer in terms of the dimensionless wall distance $y^+$ as defined in equation (66) is $y^+ = 5$.

The existence of such a laminar sublayer is however not fully borne out by experimental evidence. Hydrodynamic studies in the wall region of turbulent flows indicate the existence of a relatively well ordered, large scale, periodic motion in this region. Hanratty [1956] concluded, from a visual study of dust particles in water near a solid surface by Page and Towend [1932] and mass transfer data of Lin et. al. [1953], that a laminar sublayer as proposed by Prandtl must be much thinner than expected or does not exist at all. Instead of a stationary sublayer a mosaic of periodically replaced local elements seems to prevail. According to a flowvisualisation study of Corino and Brodkey [1969] this periodic motion can be divided in three phases. In the “fluid inrush phase” an eddy from the turbulent core penetrates the wall region. The acceleration of the low momentum fluid in this region results in an abrupt ejection of the accelerated fluid back into the turbulent core region (ejection phase). In the following viscous sublayer development phase the viscous interaction with the wall decelerates the inrushing fluid. A time dependent viscous sublayer and
temperature boundary layer will develop until the cycle is completed by the inrush of a new eddy from the turbulent core. Einstein and Li [1956] described essentially the same renewal mechanism. Only, they suggested the renewal cycle was initiated by a spontaneous eruption of the eddies from the surface due to inherent instability. In both mechanisms the inrush- and ejection phase is thought to be instantaneous because the time required for the breakdown of the viscous sublayer is much smaller than the boundary layer development time. Whether the instability leading to the renewal of the fluid near the wall is caused by inrushing eddies or by spontaneous eruption is less relevant because all models only relate to the eddies during their contact time with the surface.

In the elementary renewal model of Higbie [1935] and later Danckwerts [1951] the heat (or mass) transfer during the development phase is described with a penetration model. The fluid elements which reside in the vicinity of the wall are assumed to be stagnant and stay in place for a time period \( \tau \). In this time a time dependent temperature boundary layer develops, described by the equation:

\[
\frac{\partial T}{\partial \theta} = \alpha \frac{\partial^2 T}{\partial y^2}
\]  

(2.73)

\[
T = T_0 \quad y = 0 \quad 0 < \theta < \tau
\]

\[
T = T_b \quad 0 < y < 1 \quad \theta = 0
\]

\[
T = T_b \quad y = 1 \quad 0 < \theta < \tau
\]

(2.74)

For small contact times \( \tau \) the heat (mass) penetration depth \( \delta \) is much smaller than the thickness \( l \) of the stagnant fluid layer. In the boundary conditions (74) the condition \( y = 1 \) can then be replaced by \( y \to \infty \).

In this description the contact time \( \tau \) of an eddy with the wall is a statistical variable. An age distribution function \( \phi(\theta) \) can be defined with \( \phi(\theta)d\theta \) representing the relative part of the contact surface covered with fluid elements having ages between \( \theta \) and \( \theta + d\theta \). An average property \( \psi \) per unit area is obtained by integrating the
local, instantaneous property $\psi_i$ over the entire surface.

Considering a surface consisting of different areas $a_i$, each characterized by a surface renewal frequency $S_i = \tau_i^{-1}$, Danckwerts [1951] calculated the averaged heat transfer coefficient:

$$ h = \sqrt{\alpha} \frac{\sum a_i \sqrt{S_i}}{\sum a_i} $$

This general model has, however, no practical use and will not be used.

Averaging over all areas $a_i$ corresponds with averaging over all ages $\theta$ if the renewal frequency is assumed to be the same for all areas : $S_a = S$. This results in the averaging process:

$$ \psi = \int_0^\infty \psi_i(\theta) \phi(\theta) d\theta $$

(2.75)

Substituting $\psi = \psi_i = 1$ leads to:

$$ \int_0^\infty \phi(\theta) d\theta = 1 $$

(2.76)

Modifications and extension to the elementary surface renewal model of Higbie and Danckwerts differ by the assumptions made regarding the nature of

- the governing equations and boundary conditions
- the age distribution function and renewal frequency.

The general equations for heat transfer to an eddy during its residence in the wall region assuming constant fluid properties were formulated by Thomas [1978]:

- Continuity equation:

$$ \frac{\partial u_j}{\partial x_j} = 0 $$

(2.77)

- Navier-Stokes equations:

$$ \frac{\partial u_i}{\partial \theta} + u_j \frac{\partial u_i}{\partial x_j} = \nu \frac{\partial^2 u_i}{\partial x_j^2} - \frac{1}{\rho} \frac{\partial P}{\partial x_i} $$

(2.78)

- Energy equation:

$$ \frac{\partial T}{\partial \theta} + u_j \frac{\partial T}{\partial x_j} = \alpha \frac{\partial^2 T}{\partial x_j^2} $$

(2.79)
with the appropriate boundary conditions dependent on the used model.

The renewal of the fluid elements at the transport surface is accounted for by the averaging process using the age distribution function.

2.4.2.2 The age distribution function

The most simple distribution function is the uniform age distribution function (Higbie [1935])

\[
\phi(\theta) = \begin{cases} 
\frac{1}{\tau} & 0 < \theta < \tau \\
0 & \theta > \tau 
\end{cases}
\]  \hspace{1cm} (2.80)

In this model all fluid elements which arrive at the transport surface have identical residence times \( \tau \).

Danckwerts [1951] proposed a distribution function in which the probability of replacement of a fluid element is independent of its age. The resulting distribution is a random one:

\[
\phi(\theta) = S e^{-S\theta}
\]  \hspace{1cm} (2.81)

\( S = \frac{1}{\tau} \) is the mean renewal frequency.

Danckwerts age distribution can be identified with the residence time distribution in a well mixed vessel. Generalizing this well mixed vessel analogy a gamma distribution function representing the residence time distribution for a system of \( m \) serial vessels was proposed by Perlmutter [1961]. Higbies uniform distribution results if \( m \) approaches infinity.

In case of the penetration model of Higbie (73) the age distributions (80) and (81) result in the heat transfer coefficient

\[
h = c_s \sqrt{\alpha S}
\]  \hspace{1cm} (2.82)

with \( c_s = 1 \) using Danckwerts random age distribution and \( c_s = 2/\sqrt{\pi} = 1.13 \) using Higbies uniform age distribution.
The difference in the mean heat transfer coefficient using these two age distributions is 13%. The resulting value using a gamma distribution lies within these extremes. The choice of the distribution function has no large impact on the calculated mean heat transfer rates and in many cases the convenient Danckwerts random distribution is used.

2.4.2.3 Models based on stagnant eddies

In the above described shallow penetration model of Higbie [1935] and Danckwerts [1951] the fluid elements are supposed to be stagnant. Substituting \( u_i = 0 \) in the general equations (77) to (79) the following equation results:

\[
\frac{\partial T}{\partial \theta} = \alpha \frac{\partial^2 T}{\partial x_j^2}
\]

(2.83)

Together with the condition

\[ T = T_b \quad \text{at } y=1 \quad l \rightarrow \infty \quad 0 \leq \theta \leq \tau \]

(2.84)

this model represents the heat penetration into an infinite solid. Hydrodynamic effects are incorporated in the renewal mechanism.

Some renewal models for heat transfer at a solid wall, using the same assumption of stagnant fluid elements in the wall region but with different boundary- or initial conditions, will be discussed in this paragraph.

* The film penetration model.

\[ T = T_b \quad \text{at } y=1 \quad 0 \leq \theta \leq \tau \]

(2.85)

In the film penetration model proposed by Dobbins [1958] and Toor and Marchello [1958] interfacial transport is restricted to periodically replaced fluid elements of finite thickness \( l \). Mathematically this is described by condition (85) which is identical to the original condition (74c). For large contact times or low Prandtl numbers heat will penetrate through this layer into the fully turbulent
core of the flow. A steady state linear temperature profile over the wall layer is reached if

\[ F_{0\tau} = \alpha \tau / l^2 \geq 0.6 \quad (2.86) \]

For large Fourier numbers \((F_{0\tau} \gg 1)\) the influence of the unsteady development of the temperature profile can be neglected and the average heat transfer coefficient will be proportional to \(\alpha\) following the film model:

\[ h \propto \alpha \quad (2.87) \]

For low Fourier numbers, representing low residence times or high Prandtl number fluids, the penetration depth is small compared to \(l\) and condition \((85)\) can be replace by \((84)\) resulting in the penetration model with \(h \propto \sqrt{\alpha}\) \((82)\).

The film thickness and the surface renewal frequency, which are dependent on the Reynolds number, have to be evaluated from experimental data [Sideman and Pinczewski(1975)].

* The random distance rejuvenation model.

In the above described models fluid elements which influence the transfer rate reach to the interface replacing all fluid at the interface. This is, however, an oversimplified assumption. Due to the no slip condition at a solid interface an inrushing eddy will not reach the wall but will come within a short distance \(\xi\) depending on the energy of the eddy, leaving an unreplenished layer at the wall. Harriot [1962] suggested a penetration model in which at random times inrushing eddies sweep away the heat accumulated in the boundary layer from a random distance \(\xi\) to infinity. The temperature profile from 0 to \(\xi\) is left unchanged. Mathematically this initial condition reads:

\[ T = F(y) \left[ 1 - u(y-\xi) \right] + T_1 u(y-\xi) \quad \theta = 0 \quad (2.88) \]

\(F(y)\) is the temperature profile before the renewal, \(u\) is the unit step function and \(T_1\) the temperature of the inrushing fluid.
The surface rejuvenation model (83) with condition (88) was solved analytically by Thomas et. al. [1975] using Danckwerts random age distribution (81) and a random approach distance distribution:

\[ P_x(t) = \overline{T_1} \exp(-4\overline{T}) \]  

(2.89)

The resulting overall heat transfer coefficient, assuming \( T_i = T_b \) and applying similar averaging processes like (75) for all stochastic variables, reads:

\[ \frac{h \overline{T}}{\lambda} = \beta \left[ \frac{j_{2\beta - 1}(2\beta)}{j_{2\beta}(2\beta)} - 1 \right] \]  

(2.90)

where \( \beta = \frac{\overline{d}/4\alpha T}{\overline{T}} \) and \( j_{\beta} \) is the Bessel function of the first kind of order \( \beta \).

2.4.2.4. Models including convection effects

All models described in paragraph 2.4.2.3 are based on the assumption of stagnant fluid elements residing in the near wall region. In general, however, a micro-hydrodynamic structure will exist near a solid wall. The assumption of stagnant eddies is only valid in case the convective effects within an eddy can be neglected compared to the temporal acceleration term. Models incorporating the effects of convection are reviewed in this paragraph.

* The quasi steady flow model.

Ruckenstein [1958,1969] assumed the wall in a turbulent flow to be covered with fluid elements of equal length \( X_0 \). Within a single fluid element transient effects are neglected compared to convection. The velocity normal to the wall assumed to be zero (no suction). This leads to a flow type within a fluid element analogous to a steady boundary layer flow over a short flat surface. After a length \( X_0 \) the boundary layer is ejected into the turbulent core (Renewal mechanism). In this model the general equations (77) to (79) reduce to:

60
\[
\frac{\partial u}{\partial x} = -u \frac{\partial^2 u}{\partial y^2} = -u \frac{\partial^2 u}{\partial y^2}
\]

(2.91)

With the boundary- an initial conditions
\[
\begin{align*}
    u &= 0 & T &= T_0 & x &= 0 & y > 0 \\
    u &= u_1 & T &= T_b & 0 \leq x < X_0 & y = 0 \\
    u &= u_1 & T &= T_b & 0 \leq x < X_0 & y \rightarrow \infty
\end{align*}
\]

(2.92)

Solving (91) with conditions (92) the mean heat transfer coefficient was found as a function of \( X_0 \). Eliminating \( X_0 \) by using Blasius' equation for the mean shear stress, Ruckenstein arrived at

\[
Nu = \left( \frac{u^*}{u_1} \right) \left( \frac{f}{2} \right)^{1/2} Re \ Pr^{1/3}
\]

(2.93)

The factor \( u^+ = \frac{u_1}{u^*} \) is unknown and must be approximated. Taking \( u_1 = u_b \), and substituting \( \left( u^+ \right)^2 = \frac{\tau_w}{\rho} = \frac{f}{2} \rho u_b^2 \), (93) reduces to the simple Reynolds analogy:

\[
Nu = \frac{f}{2} Re \ Pr^{1/3}
\]

(2.94)

In Ruckenstein's derivation, however, \( u^+ \) is a constant. Evaluating this constant from experimental mass transfer data of Harriot and Hamilton [1965] and taking \( f = 0.184 \ Re^{-0.2} \), Ruckenstein infers that

\[
Nu \approx 0.0096 \ Re^{0.9} \ Pr^{0.33}
\]

(2.95)

* Transient periodic viscous sublayer models.

In a transient-periodic sublayer model the fluid elements from the turbulent core enter the wall region at a uniform velocity \( u_1 \). Instead of coming to rest like in the Danckwerts type models described in paragraph 2.4.2.3, a velocity gradient develops in time due to the viscous interaction with the wall. This time dependent viscous motion ends with a sudden breakdown of the boundary layer and the instant
renewal of the fluid element near the wall, initiating a new cycle. A schematic picture of this microscopic flow structure presented by Black [1968, 1969] is shown in figure 2.14. By the renewal process the stress build up in the viscous sublayer is periodically relieved and energy is transferred from the wall region to the turbulent core.

Figure 2.14: Schematic development and breakdown of cyclic sublayer motion [after Black (1969)].

The periodical (in time as well as in space) breakdown of boundary layers generates and maintains a system of characteristic horseshoe vortex structures. Strong experimental support for this model is provided by the flowvisualisation study of Corino and Brodkey [1969] as described in paragraph 2.4.2.1.

The mathematical description of the velocity boundary layer development for a fully developed turbulent flow ($u_1 = u$, $u_2 = u_3 = 0$) is given by Einstein and Li [1956] and Hanratty [1956] analogous to the penetration model (73) for heat (mass) transfer of Danckwerts [1951]:

$$\frac{\partial u}{\partial \theta} = \nu \frac{\partial^2 u}{\partial y^2}$$ (2.96)

with the boundary- and initial condition given in the most general form, described in analogy with Harriotts surface rejuvenation model.
\begin{align}
    u &= F(y) \left[ 1 - u(y-y) \right] + u_1 u(y-y) \quad y > 0 \quad \theta = 0 \\
    u &= 0 \quad y = 0 \quad \theta > 0 \\
    u &= u_M \quad y = y_M \quad \theta > 0 
\end{align}

(2.97)

\( u_1 \) is the uniform velocity at the moment of inrush, \( F(y) \) is the velocity profile just before the renewal of the boundary layer. \( U_M \) is the, fixed, velocity at the outside boundary of the wall layer. \( \psi \) is the unit step function and \( \ell \) the statistical approach distance. Taking \( \ell = 0 \) the renewal penetration model results. The version for shallow layers \( (y_M \to \infty, u_M = u_1) \) was solved by Hanratty [1956] using Higbies uniform age distribution function (80) :

\[
    u^+ = u^+_1 \int_0^1 \text{erf} \left[ \frac{\sqrt{\pi}}{4} \frac{y^+}{u^+_1} \left( \frac{\theta}{\tau} \right)^{-1/2} \right] \text{d} \frac{\theta}{\tau} 
\]

(2.98)

From this expression the dimensionless velocity profile can be found from numerical integration.

As described in the previous paragraph, the assumption of \( \ell = 0 \), which means that the entire sublayer down to the wall is periodically renewed, is an oversimplification. Eddies from the bulk will penetrate to within a small random distance \( \ell > 0 \) from the wall.

Meek and Baer [1970], included this effect in the renewal-penetration model by suggesting a constant approach distance \( \ell \), defined in dimensionless wall coordinates by :

\[
    \ell^+ = 1.5
\]

This value is consistent with observations of Popovich and Hummel [1967] who found from visualization studies :

\[
    \ell^+ = 1.6 \pm 0.4
\]

A numerical solution for heat transfer was obtained by calculating the velocity- and temperature boundary layer development through several boundary layer growth cycles. The "steady state" solution was reached when the solution at the end of a cycle was the same as at the end of the previous cycle. The renewal period \( \tau \) was taken constant (Higbies
age distribution). After each cycle the temperature $T$ outside the thin wall layer, $y^+ < f^+$, was made equal to the bulk temperature. The temperature within the wall layer was left unchanged.

Ooms et. al. [1978] applied this model to a pipe flow with a first order endothermic irreversible chemical reaction, using $f^+ = 1.6$. In the turbulent core a mixinglength model was assumed to be applicable. Starting from a universal velocity profile a "steady state" was reached after three or four cycles. Good agreement with the Sieder and Tate [1936] correlation was found.

The dimensionless mean renewal frequency $S^+ = S u (u^+)^{-2}$ and the dimensionless inrush velocity $u_1^+ = u_1 (u^+)^{-1}$, which appear in the model of Hanratty as well as in the model of Meek, are related. Computing the time averaged shear stress $\tau_w$ gives:

$$u_1^+ = c_s^{-1} (S^+)^{-1/2} \tag{2.99}$$

with $c_s = 2/\sqrt{\pi}$ if Higbies uniform age distribution is used. Using Danckwerts random age distribution $c_s = 1$ results.

To close the model either $u_1^+$ or $S^+$ must be specified. By assuming the velocity $u_1$ of the incoming fluid elements equal to the velocity at the "break point", $y^+ = 30$, between the turbulent core and the buffer layer of the universal velocity profile near a wall, Hanratty suggested:

$$u_1^+ = 13.5 \tag{2.100}$$

According to Meek [1968] $\tau^+ = (S^+)^{-1}$ is a function of the Reynolds number. By using Prandtl's relation for the friction factor Meek found a relation between the bulk velocity $u_b$ used in the Reynolds number and the interface velocity $u_1$. This relation, as shown in figure 2.15, has been used by Ooms.

Experimental data on $S^+$ range from $(14)^{-2}$ to $(18)^{-2}$. Meek[1972] claims that

$$S^+ = (15.6)^{2} \tag{2.101}$$

is probably the best approximation.
In the above described models of Meek and Ooms the thin wall layer was never refreshed. Pinczewski and Sideman [1974] suggested to divide the penetrating eddies into two groups: The majority reaching down to \( \frac{1}{\tau} \) at a frequency \( \frac{1}{\tau} \) and the minority reaching the wall at a frequency at \( \frac{1}{\tau} \). The relation between \( \tau \) and \( \tau_1 \) is given by:

\[
\tau_1 = \frac{\tau}{6}
\]  
(2.102)

For \( b \) the value of \( b = 0.07 \) is recommended.

A new solution technique for the averaged heat- and momentum-transport in a fully turbulent boundary layer flow has been introduced by Thomas et. al. [1975,1978]. With this technique an analytical solution can be found for both the surface renewal \((\lambda=0)\) and the surface rejuvenation model, in which the thickness \( \lambda \) of the unreplenished sublayer is described by the random distribution \((89)\). The essential difference with other solution methods is that the averaging process over all statistical parameters in the model has been carried out before solving the equations. For the statistical age distribution the convenient Danckwerts random age distribution has been used.

Instead of merely setting the parameter \( u_1^+ \) to an arbitrary constant like Hanratty [1956] or setting the inrush velocity \( u_1^+ \) to the bulk velocity \( u_b \) like Thomas [1970], leading to \( u_1^+ = \left( \frac{f}{4} \right)^{1/2} \), the value of \( u_1^+ \) in this new technique is computed by coupling the dimensionless
velocity profile in the wall region to the universal logarithmic velocity profile

\[ u^+ = \frac{1}{\kappa} \ln y^+ + C \quad (2.103) \]

valid in the bulk region of the flow \((y^+ > y_M^+)\). By requiring continuity in \(u^+\) and \(\frac{du^+}{dy^+}\) at \(y^+ = y_M^+\), \(u_I^+\) and \(C\) are related. For a turbulent pipe flow the constants \(\kappa = 0.4\) and \(C = 5.5\) are well established.

For the surface renewal case, with \(y_M \to \infty\) used in the interfacial condition in (97), Thomas [1978] found

\[ u_I^+ \approx 16 \quad (2.104) \]

Using the same technique, but solving equation (96) with the interfacial condition in (97) at a finite value of \(y_M\), Thomas [1982] arrived at

\[ u_I^+ = 14.93 \quad (2.105) \]

for a boundary layer flow in which \(\kappa = 0.41\) and \(C = 5.0\). For a pipe flow, \(\kappa = 0.40\) and \(C = 5.5\), this parameter becomes

\[ u_I^+ = 15.77 \quad (2.106) \]

The value of the dimensionless interface distance \(y_M^+\) computed from this model has been found to be in de or order of 50.

The rejuvenation model, in which the statistical approach distance \(\xi\) is described by the random distribution (89) was solved by Thomas [1980]. For the interfacial condition in (97) \(y_M \to \infty\) has been used. The dimensionless mean approach distance \(\xi^+\) has been taken

\[ \xi^+ = 5 \quad (2.107) \]

resulting in

\[ u_I^+ = 15.21 \quad (2.108) \]

The Nusselt number obtained from these models can be described by:

\[ Nu = \frac{T_0 - T}{T_0 - T_b} \left[ \frac{f}{2} \right]^{1/2} \text{Re Pr} (T^+)^{-1} \quad (2.109) \]

Definition : \(T^+ = (T_0 - T) \rho c_p u^* (q^*)^{-1}\)

66
where \( q^*_{0} = -\lambda \frac{dT}{dy} \bigg|_{y=0} \) is the heat flux at the wall.

\( T^+ \) follows from the analytical solution for the dimensionless heat equation. For example in the shallow penetration model \( (y_m \to \infty) \)

\[ T^+_1 = u^+_1 \cdot \frac{Pr}{Pr} \]

Assuming \( u^+_1 = u^-_b \) and \( T^+_1 = T^-_b \), gives the Reynolds analogy:

\[ Nu = \frac{f}{2} \cdot Re \cdot Pr^{1/2} \quad (2.110) \]

### 2.4.2.5 Discussion

In case of a hydrodynamic- and thermodynamic- fully developed tube- or boundary layer flow several models based on the turbulent burst mechanism have been discussed. These models differ mainly in the prediction of the heat transfer dependency on the Prandtl number of the fluid. For example, in the quasi steady flow model of Ruckenstein the Nusselt number is proportional to the Prandtl number to the power \( \frac{1}{3} \) (equation (94)). In the transient periodic sublayer model the Nusselt number is a more complex function of the Prandtl number (equation (109)), however in the shallow penetration approximation this reduces to equation (110) in which the Nusselt number is proportional to the square root of the Prandtl number. From experiments the Nusselt number dependency on the Prandtl number of a fluid has been established for a wide range of Prandtl numbers. For large Prandtl numbers the \( \frac{1}{3} \) power relation (94) of Ruckenstein is supported by experiments over a wide Prandtl range, whereas for moderate Prandtl numbers ( \( Pr \approx O(1) \) ) experiments are fitted better by the square root of the Prandtl number as predicted in relation (110). The more general relation (109) for the transient periodic sublayer model can be used to describe the heat transfer for a wider range of Prandtl numbers depending on the model used to determine \( T^+ \). The surface renewal model of Thomas in which the whole boundary layer is periodically refreshed is valid in the range \( 1 \leq Pr < 5 \). For \( Pr \leq 1 \) the temperature boundary layer is thicker than the momentum boundary layer and the turbulent region offers an additional resistance to the heat transfer. However for \( 0.5 \leq Pr \leq 1 \) this resistance can be neglected [Sideman and Pinczewski(1975)].
Prandtl numbers larger than 5 the influence of an unreplenished sublayer at the wall cannot be neglected and a surface rejuvenation model has to be used to determine the value of $T^+$ in (110).

The Reynolds number dependency of the Nusselt number follows from the flow model from which the renewal rate $\tau$ is determined. In both the quasi stationary- (94) as in the transient sublayer- (110) model the Reynolds analogy for a tube flow, in which $\text{Nu}^+ = f \cdot \text{Re}^{0.2}$, evolved by assuming $u_i^+ = u_b^+ = (\frac{2}{f})^{1/2}$. For the renewal rate this leads to

$$\tau^+ = \frac{2}{f} = 43.48 \cdot \text{Re}^{0.2} \quad (2.111)$$

An other experimental relation for the renewal rate has been found by Laufer and Badri Narayanan [1971] and Rao et. al. [1971] by studying a developing boundary layer over a flat plate:

$$\hat{\tau} = \tau \frac{u_\infty}{\delta} = 5 \quad (2.112)$$

To be able to use this result in case of a tube flow $\delta$ and $u_\infty$ have to be substituted by $\delta = R = D/2$, and $u_\infty = u_{\text{max}}$ [Schlichting (1968)]. In inner coordinates (112) reads [Meek (1972)]:

$$\tau^+ = f \cdot \text{Re} \quad (2.113)$$

This differs from the results of the renewal models described in the paragraphs above in which the dimensionless renewal rate was found to be constant, with probably the best approximation from equation (101):

$$\tau^+ = 243 \quad (2.114)$$

In figure 2.16 the dimensionless renewal rate $\tau^+$ from the equations (111), (113) and (114) are compared with experimental data as collected by Thomas [1982]. The experimental data on a fully developed tube flow, represented by the open points, can not distinguish between relation (111) and (114) but do not support relation (113). The solid points, which have been obtained from experiments on a boundary layer flow over a flat plate, seem to be fitter better by relation (113). However, Repik and Sosedko [1976] concluded from their experiments on a flow over a flat plate that also in this case rather $\tau^+$ (114) instead of $\hat{\tau}$ (112) is a constant. Although there is still some confusion about which representation of the mean renewal rate is best, the analytical
relation for a fully developed tube flow, $\tau^+ = \text{constant}$, is not in contradiction with presently available experimental data.

Figure 2.16: Stationary renewal times as a function of Re
- Open points = Fully developed tube flow
- Solid points = Boundary layer flow over a flat plate

All models discussed in this paragraph are only valid for constant property fluids. An extensions of these models to a variable property fluid has been proposed by Thomas et. al. [1974]. For a fluid with a known temperature dependence of the viscosity and thermal conductivity, and constant density and specific heat, the velocity and temperature profile in the wall region were determined using an integral technique. Using Goldman like variables Ivlev et. al. [1979] extended this theory to fluids in which all properties vary arbitrarily with temperature.
Meek and Baer [1970] and Ooms et. al. [1978] computed numerically the
temperature- and velocity profiles near the wall of a tube by starting
at a arbitrary initial profile and computing several renewal cycles.
The stationary situation was reached when the profile just before the
moment of renewal was identical to the profile at the end of the
previous cycle. Although in this procedure the profiles change in time
only a stationary solution results. In the time dependent procedure the
only difference between two cycles is the initial profile in the thin,
unreplenished, sublayer near the wall \((y^+ < 1.5)\). No time dependent
mean transfer rates are found.

In literature no extensions of the turbulent burst theory to
transient flow situations have been found.

In chapter 5 of this thesis the stationary surface renewal theory
will be extended to describe the heat transfer in case of a time
dependent mean wall temperature. The mean flow is assumed to be
stationary and fully developed. This means that the renewal frequency
is constant in time and can be determined using a stationary renewal
model as discussed in the paragraphs above. For supercritical helium
the Prandtl number lies in the range \(0.5 \leq \text{Pr} \leq 5\). In this range the
transient periodic sublayer model can be used in which the influence of
an unreplenished sublayer can be neglected.
3 Numerical model

3.1 Introduction

In paragraph 2.4 the equations describing the flow of and heat transfer to a Newtonian fluid have been stated. In this chapter these equations will be applied to the case of a supercritical helium flow inside the cooling channel of an internally cooled superconducting cable, as under study in this thesis. The main aim is to arrive at a good transient heat transfer model for the superconducting cable.

In paragraph 3.2 the resulting governing equations after some simplifications will be described. The influence of turbulence on the momentum- and heat transfer in the flow will be described using Prandtl's mixing length model (paragraph 2.4.1). The appropriate boundary- and initial conditions for the problem will be specified in paragraph 3.3.

The governing equations, the Navier-Stokes equation for the axial velocity \( u \) (and radial velocity \( v \)), the enthalpy equation and the continuity equation, are coupled and non linear. Further also the fluid properties depend strongly on pressure and temperature (enthalpy). Consequently only a numerical solution to the problem can be found. Therefore the physical domain has to be divided into a finite number of discrete grid points. On these points an approximation of the continuous solution has to be defined for a finite number of discrete time levels. The accuracy of the approximation depends on the differencing scheme and the number of mesh points used.

For the discretization of the equations two types of numerical methods are available:

- Finite element methods
- Finite difference methods

In the finite element methods a variational method is applied in which functionals (e.g. energy) are minimized. In the finite difference methods the difference equations can be derived in two different ways.
The first approach is to approximate the derivatives in the differential equations by means of Taylor series expansion. In the second approach the difference equations, are derived by considering the balances of the conserved properties (mass, momentum and energy) over a control volume, which is a cell formed by the grid. This last method, which leads to similar equations as the Taylor expansion approach, will be applied in this thesis. For a comparison between finite element and finite difference methods the reader is referred to Minkowycz et.al. [1988].

In the numerical method applied in this thesis the independent variables in the flow computations are the axial and radial velocity components \( u \) and \( v \), enthalpy \( H \) and pressure \( P \). The difference equations for the continuity and transport equations will be derived in paragraph 3.5 using the control volume approach. Different kinds of differencing schemes, described in paragraph 3.4, will be considered.

Since pressure is described only implicitly by the governing equations a special procedure for computing this variable must be followed. The most widely used pressure algorithm is the SIMPLE/SIMPLER algorithm [Patankar and Spalding(1972), Patankar (1980)]. We used the improved version for strong compressible flows I-SIMPLE(R) developed by Cornelissen [1986]. Although this procedure will not be discussed in detail here, a note on the required grid must be made. Patankar pointed out that a normal grid, in which all variables are evaluated on the same points, can result in physically non realistic, alternating, pressure fields. To avoid this difficulty a staggered grid has been proposed in which the velocity points are shifted compared to the normal points (see figure 3.1). In this thesis this staggered grid, on which the SIMPLE(R) algorithm is based, will be used. In axial direction the mesh size \( \Delta z \) will be constant. In radial direction a variable mesh size will be employed. A high concentration of grid points in the region of high gradients near the wall is realized using a grid proposed by Schnurr [1977]. In this grid the distance between two adjacent points is given by:

\[
\Delta r_j = \Delta r_{j-1} / C_s^{1/2} \quad 1 \leq j \leq Nr \quad C_s = 2
\]  

where \( j=Nr \) denotes the last grid point near the wall. Care is taken.
that at least one point is situated within the laminar sublayer, $y^+ = 5$.

Figure 3.1: Staggered grid

To complete the discretization the boundary conditions have to be implemented. This will be discussed in paragraph 3.6.

As stated before, the helium properties are strongly dependent on pressure and enthalpy (temperature). In paragraph 3.7 the literature values will be presented as plotted curves, and the implementation in the numerical model will be discussed.

After the discretization of the governing differential equations, a coupled, non-linear set of difference equations results. The solution technique used will be presented in paragraph 3.8.
3.2 The flow equations

Following Cornelissen [1984] the cooling channel is supposed to be circular. This assumption makes it possible to formulate the problem in two instead of three dimensions. The governing equations (2.46-2.48) in cylindrical coordinates are given by Cornelissen:

* Continuity equation

\[
\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial z} + \frac{1}{r} \frac{\partial \rho v}{\partial r} = 0
\]  

(3.2)

* Axial momentum equation

\[
\frac{\partial \rho u}{\partial t} + \frac{\partial \rho uu}{\partial z} + \frac{1}{r} \frac{\partial \rho vu}{\partial r} = \frac{\partial P}{\partial z} + \frac{1}{r} \frac{\partial}{\partial r} \left[ \mu_{\text{eff}} \frac{\partial u}{\partial r} \right] + \frac{\partial}{\partial z} \left[ \mu_{\text{eff}} \frac{\partial u}{\partial z} \right] + S_{m, x}
\]  

(3.3)

* Radial momentum equation

\[
\frac{\partial \rho v}{\partial t} + \frac{\partial \rho uv}{\partial z} + \frac{1}{r} \frac{\partial \rho vv}{\partial r} = \frac{\partial P}{\partial r} + \frac{\partial}{\partial r} \left[ \mu_{\text{eff}} \frac{\partial v}{\partial r} \right] + \frac{\partial}{\partial z} \left[ \mu_{\text{eff}} \frac{\partial v}{\partial z} \right] + S_{m, r}
\]  

(3.4)

* Enthalpy equation

\[
\frac{\partial \rho H}{\partial t} + \frac{\partial \rho u H}{\partial z} + \frac{1}{r} \frac{\partial \rho v H}{\partial r} = \frac{\partial P}{\partial t} + u \frac{\partial P}{\partial z} + v \frac{\partial P}{\partial r} + \frac{\partial}{\partial r} \left[ r \Gamma_{\text{eff}} \frac{\partial H}{\partial r} \right] + \frac{\partial}{\partial z} \left[ r \Gamma_{\text{eff}} \frac{\partial H}{\partial z} \right]
\]  

(3.5)

In equation (5) the viscous dissipation term \( \Phi \) is neglected and no internal heat sources are assumed \((S_{v} = 0)\). The source terms \( S_{m} \) in the momentum equations contain gravity forces. For a horizontal flow or in case buoyancy effects can be neglected \((Gr/Re^{2.7} < 10^{-5} - \text{see paragraph 2.3 equation 2.31})\), the source terms \( S_{m} \) are equal to zero.

\[\mu_{\text{eff}} = \mu + \mu_{t} \text{ and } \Gamma_{\text{eff}} = \frac{\lambda}{C_{p}} + \frac{\mu_{t}}{Pr_{t}}\]

are the effective viscosity and heat diffusion coefficient respectively. The turbulent contribution \( \mu_{t} \) is computed from the mixing length model (paragraph 2.4.1) using the van Driest correction factor (2.66) to incorporate the influence of the wall on the mixing length (2.65). The value of the turbulent Prandtl number \( Pr_{t} \) is set to unity.

A further simplification of the governing equations can be achieved recognizing that in a high Reynolds number tube flow the boundary layer assumptions are valid [Sideman and Pinczewski (1975)]. Using these assumptions \((u > v, \frac{1}{r} \frac{\partial \phi}{\partial r} > \frac{\partial \phi}{\partial z})\) the set of governing
equations (2-5) are shown to reduce to [Schlichting (1968)]:

- Axial momentum equation

\[
\frac{\partial \rho u}{\partial t} + \frac{\partial \rho u u}{\partial z} + \frac{1}{r} \frac{\partial \rho v u}{\partial r} = \frac{\partial P}{\partial z} + \frac{1}{r} \frac{\partial}{\partial r} \left[ \mu_{\text{eff}} \frac{\partial u}{\partial r} \right] \tag{3.6}
\]

- Enthalpy equation

\[
\frac{\partial \rho H}{\partial t} + \frac{\partial \rho u H}{\partial z} + \frac{1}{r} \frac{\partial \rho v H}{\partial r} = \frac{\partial P}{\partial t} + u \frac{\partial P}{\partial z} + \frac{1}{r} \frac{\partial}{\partial r} \left[ \Gamma_{\text{eff}} \frac{\partial H}{\partial r} \right] \tag{3.7}
\]

together with the continuity equation (2).

From the radial momentum equation it is deduced that the pressure variation in radial direction is much less than the axial pressure drop. The pressure field in the tube can thus be described by a one dimensional variable \( P(z) \). In the axial momentum equation the pressure gradient \( \frac{\partial P}{\partial z} \) in (3) becomes \( \frac{dP}{dz} \) in the boundary layer approximation. The radial velocity \( v \) is no longer described by the radial momentum equation (4) but can be computed directly from the continuity equation (2).

From simulations on transient supercritical helium flow through a circular tube we found that the difference in the resulting solutions from the sets of equations (2-5) and (2,6,7) was indeed small. Even in extreme situations, low Reynolds number (Re=10^4) and high heat flux, the difference in the pressure and velocity fields computed from the two models was found to be less than 1%. The influence on the heat transfer was even less. The use of the boundary layer equations in our case is thus allowed.

3.3 Boundary conditions

To solve the problem appropriate boundary conditions have to be specified. In case of a transient calculation also initial conditions are required.

Boundary conditions must be specified for the wall, the inlet and the outlet. The pipe center is an axis of symmetry. At the wall a no slip condition applies and either the heat flux or the wall temperature is specified. In our case the wall temperature is specified, either as
a constant or time dependent as computed from the equations for the copper/superconductor matrix (paragraph 2.1). The local heat flux can be computed from the resulting local temperature gradient near the wall in the fluid. We assume a continuous temperature profile over the wall, which means that the Kapitza resistance (paragraph 2.3) is neglected. At the outlet the pressure is fixed. At the inlet either the axial velocity profile is fixed or the pressure is specified. In the latter case the velocity profile at the inflow is assumed to be developed (zero velocity gradient in axial direction). The radial velocities are taken as zero at the inlet. For the enthalpy at the inlet a uniform profile is given.

In transient calculations the initial flow field is assumed to be a fully developed pipe flow. The wall friction, determined by the flow field, specifies the initial pressure drop implicitly. The initial wall temperature is assumed to be equal to the inlet temperature of the flow. This means that initially there is no stationary heat flux and the fluid temperature in the whole tube equals the inlet temperature.

3.4 Differencing schemes

In a numerical solution procedure values of the variables are stored only in the grid points and at discrete time levels. The time differentials in the governing equations are thus unknown and must be approximated by a relation between the values at the computed time levels. The accuracy of the approximation depends on the differencing scheme used. In paragraph 3.4.1 several time differencing schemes will be discussed.

In a control volume method the spatial differentials in the governing equations are interpreted as fluxes over a boundary which separates the control volumes around two adjacent points. These fluxes have to be evaluated at the interfaces between the grid points. Since the variables are not defined there they have to be found from interpolation. This interpolation can be done in various different ways, dependent on the differencing scheme used. In paragraph 3.4.2 some spatial differencing schemes will be described.
3.4.1 Time differencing schemes

To demonstrate discretization of a time derivative using a time differencing scheme we will study the equation

$$\frac{\partial \Phi}{\partial t} = \mathcal{F}$$  \hspace{1cm} (3.8)

The time derivative in (8) can be expressed in the values $\Phi^k$ of $\Phi$ at the discrete time levels $t_k$, $k=1,2,...$, with the aid of Taylor expansions. For a two time level scheme the expression

$$\frac{\partial \Phi}{\partial t} = \frac{\Phi^{k+1} - \Phi^k}{\Delta t}$$  \hspace{1cm} (3.9)

is exact for one time $t$ with $t_k < t < t_{k+1}$. If (9) is substituted in the equation (8) the value of $\mathcal{F}$ has to be evaluated at the same time $t$, which means that also $\mathcal{F}$ must be interpolated between the values at $t_k$ and $t_{k+1}$.

In the present study a maximum of three time levels will be taken into account. A general formulation for a three time level scheme can be expressed by:

$$\frac{\alpha_1 \Phi^{k+1} + \alpha_2 \Phi^k + \alpha_3 \Phi^{k-1}}{\Delta t} = \alpha_4 \Phi^{k+1} + (1-\alpha_4) \Phi^k$$  \hspace{1cm} (3.10)

where $\Phi^k$ is the value of $\mathcal{F}$ evaluated at time level $t_k$. The time step $\Delta t$ between the time levels is assumed to be constant.

The most simple scheme is the two time level scheme where $\mathcal{F}$ is evaluated at the old time level $t_k$:

$$\alpha_1 = 1 \ ; \ \alpha_2 = -1 \ ; \ \alpha_3 = 0 \quad \text{and} \quad \alpha_4 = 0$$

This scheme is called an explicit scheme since the value of $\Phi$ at the new time level $t_{k+1}$ only depends on known quantities at the previous time level $t_k$. $\Phi^{k+1}$ in every point can thus be computed directly without solving any matrix equations. This scheme has, however, a big disadvantage. The time step $\Delta t$ between the time levels is limited for stability reasons. Especially in flows in which locally a very dense grid is required (e.g. in tube flows near the solid boundary) the
allowed time step can become very small. This makes this otherwise convenient scheme very expensive.

Schemes in which \( \alpha_4 \neq 0 \) are called implicit schemes because \( \Psi^{k+1} \) depends on the unknown values of \( \Psi \) in other points of the computation domain. To find \( \Psi^{k+1} \) all points have to be solved simultaneously. This requires a matrix conversion and is thus more expensive than a single explicit time step. However, in general much larger time steps are allowed which reduces the number of time steps required and consequently the computation time.

A two level implicit scheme is the fully implicit scheme of Liebmann. The coefficients in (10) for this scheme, which will be denoted by FI, are:

\[
\alpha_1 = 1 \quad \alpha_2 = -1 \quad \alpha_3 = 0 \quad \text{and} \quad \alpha_4 = 1
\]

The function \( \mathcal{F} \) is thus evaluated completely at the new time level \( t_{k+1} \). For a linear equation this scheme is unconditionally stable. This means that there is no limit to the size of the time step. The accuracy of the solution becomes better if the time step is decreased. The FI scheme is, however, only first order accurate which means that the error is proportional to \( \Delta t \).

A second order accurate scheme (error proportional to \( \Delta t^2 \)) is found when \( \mathcal{F} \) is evaluated midway between the two time levels. The coefficients for this Crank-Nicolson scheme are:

\[
\alpha_1 = 1 \quad \alpha_2 = -1 \quad \alpha_3 = 0 \quad \text{and} \quad \alpha_4 = 0.5
\]

Although this scheme is also unconditionally stable for a linear equation, physically non realistic solutions and oscillations in time around the solution have been found when the time step was too large [Patankar and Baliga(1978)].

An other second order scheme is the three time level backward differencing scheme (shortly B3) which can be derived by cutting the Taylor expansion after the second derivative:

\[
\Psi^k = \Psi^{k+1} - \Delta t \frac{\partial \Psi^{k+1}}{\partial t} + \frac{(\Delta t)^2}{2!} \frac{\partial^2 \Psi}{\partial t^2} + O[(\Delta t)^3] \quad (3.11)
\]

using central differences for the second derivative in (11) and
neglecting the third order terms the following coefficients in (10) emerge:

\[ \alpha_1 = 1.5 \ ; \ \alpha_2 = -2 \ ; \ \alpha_3 = 0.5 \ \ \text{and} \ \alpha_4 = 1 \]

This scheme is unconditionally stable for a linear equation.

In this thesis both the first order FI-scheme and the second order B3-scheme will be employed.

3.4.2 Spatial differencing schemes

In this paragraph the flux \( J_w \) of a variable \( \phi \) over the interface \( A_w \) between two adjacent points \( W \) and \( P \) will be studied. The fluxes over the interfaces with other neighboring points can be derived in a similar way.

The flux \( J_w \) at the interface between the points \( W \) and \( P \) generally consists of two contributions, a convective and a diffusive one.

The diffusive part defined by:

\[ J_{\text{diff}} = -(\Gamma_w A_w \frac{\partial \phi}{\partial z})_w \]  \hspace{1cm} (3.12)

is discretized using a central differencing scheme

\[ \frac{\partial \phi}{\partial z}_w = \frac{\phi_P - \phi_W}{\Delta z_w} \]  \hspace{1cm} (3.13)

\( \Delta z_w \) is the distance between the points \( W \) and \( P \).

If \( \phi \) is a variable defined at the normal grid points of the staggered grid (figure 3.1) the west interface is a staggered grid point. This means that the diffusivity at the west interface is unknown and must be interpolated. Considering the space between \( W \) and \( P \) to consist of two slabs of different diffusivities (see figure 3.2), \( (\Gamma_w \phi)_w \) can be computed from:

79
\[ \frac{\Delta z_w}{(\Gamma_\phi)_w} = \frac{\Delta z_{w'}}{(\Gamma_\phi)_p} + \frac{\Delta z_{w''}}{(\Gamma_\phi)_w} \] (3.14)

which is the harmonic mean of the values in the points W and P in case of a non equidistant grid. However, if \( \phi \) represents the axial velocity, W and P are shifted velocity points. In this case the west interface is a normal grid point and the diffusivity at this point is known and the interpolation (14) is not needed.

\[ \begin{array}{c}
\text{W} \\
\quad (\Gamma_\phi)_w \\
\quad (\Gamma_\phi)_p \quad \text{P}
\end{array} \quad \begin{array}{c}
\quad \Delta z_w \\
\quad \Delta z_{w'} \\
\quad \Delta z_{w''}
\end{array} \]

Figure 3.2 : Diffusivity on the west interface

The resulting expression for the diffusive flux is:

\[ J_{\text{diff}} = -(\phi_p - \phi_W) A_w \left( \frac{(\Gamma_\phi)_w}{\Delta z_w} \right) = -d_w (\phi_p - \phi_W) \] (3.15)

in which relation (14) has to be used to find the diffusivity \((\Gamma_\phi)_w\) when the west interface is not situated at a normal grid point.

The convective part of the flux \( J_w \) is defined by

\[ J_{\text{conv}} = (\rho u A)_w \phi_w = C_w \phi_w \] (3.16)

To express the west surface value \( \phi_w \) by using the known values of the variable \( \phi \) in the neighboring points many possibilities are available. In this thesis two of them will be described. The first method is known as the HYBRID scheme, the second method developed by Leonard [1981] is called the EXQUISITE scheme.

In the HYBRID scheme a high velocity and a low velocity case, determined by the value of the Péclet number, are distinguished. The
Peclet number is defined as:

\[ \text{Pe}_w = \frac{C_w}{d_w} \] (3.17)

where \(C_w = (\rho u A)_w\) and \(d_w = (I_{\phi}/\Delta z)_w\) as defined in the equations (16) and (15) respectively.

In a high speed flow \(|\text{Pe}_w| > 2\) the diffusive flux (15) can be neglected. The convective flux (16) is assumed to be fully determined by the upstream value of the variable \(\phi\). This leads to the UPWIND scheme expressed by:

\[ J_{\text{conv}} = \left[ \phi_W \text{Max}[C_w,0] - \phi_P \text{Max}[-C_w,0] \right] \] (3.18)

where \(\text{Max}[a,b]\) is a function which returns the maximum value of the arguments.

In a low speed flow \(|\text{Pe}_w| < 2\) the diffusive flux can not be neglected. For the convective flux central differencing is used resulting in:

\[ J_{\text{conv}} = C_w \left[ \frac{\phi_W + \phi_P}{2} \right] \] (3.19)

Combining the equations (15), (18) and (19) the total flux through the west interface approximated with the HYBRID scheme can in closed form be expressed by:

\[ J_w = \phi_W d_w \text{Max}[\text{Pe}_w,1+0.5 \text{ Pe}_w,0] \]
\[ - \phi_P d_w \text{Max}[\text{Pe}_w,1-0.5 \text{ Pe}_w,0] \] (3.20)

In the HYBRID scheme only two points straddling the control volume face are used to express the flux at the interface. Because of the upwind character of this scheme the interpolation is only first order accurate in place. Higher order accuracy can be reached when three points, two points straddling the control volume face together with one additional adjacent upstream point, are involved in the interpolation (see figure 3.3).
Leonard [1980] used a quadratic interpolation between the three points WW, W and P to estimate the value of the variable $\phi$ at the west interface. He claims that this so called QUICK (Quadratic Upwind Interpolation for Convective Kinematics) scheme is third order accurate. However, in this QUICK scheme, a non realistic local minimum at the west interface can occur due to the quadratic interpolation. Leonard [1981] adapted the QUICK scheme to avoid this problem. Instead of always using quadratic interpolation he used an exponential interpolation when the solution was monotonic (like in figure 3.3 where $\phi_{WW} < \phi_{W} < \phi_{P}$). This new scheme was called the EXQUISITE scheme. In describing the interpolation of $\phi_{w}$ with the EXQUISITE scheme only the case $u > 0$ will be considered. For $u < 0$ a similar result applies.

In order to decide whether a quadratic or an exponential interpolation has to be applied a decision parameter $\gamma$ is introduced:

$$
\gamma = \frac{\phi - \phi_{WW}}{\phi_{P} - \phi_{WW}}
$$

(3.21)

in particular for the west point $W$:

$$
\gamma_{W} = \frac{\phi_{W} - \phi_{WW}}{\phi_{P} - \phi_{WW}}
$$

(3.22)

and the west interface $w$:

$$
\gamma_{w} = \frac{\phi_{w} - \phi_{WW}}{\phi_{P} - \phi_{WW}}
$$

(3.23)
Figure 3.4: Normalized interfacial flux $\phi_w$

The choice of the correct interpolation for $\phi_w$ (and thus for $\phi$) at the interface depends on the value of $\phi_W$ in the west point. If $0 \leq \phi_W < 1$ the value in the west point lies between the values in the points WW and P. In this case an exponential scheme is used. For values of $\phi_W$ outside this interval a quadratic interpolation is used (see figure 3.4). In the region $\phi_W \in (-1,0) \cup (1,1.5)$, however, the lower curve of the shaded region has to be used to ensure a smooth transition from the different interpolations. Summarized the EXQUISITE interpolation scheme reads:

\[ \phi_w = \frac{3}{8} + \frac{3}{4} \phi_W \quad \phi_W \in (-\infty,-1) \cup (1.5, +\infty) \]  (3.24a)

\[ \phi_w = \frac{\sqrt[3]{\phi_W(1-\phi_W)^3 - \phi_W^2}}{1 - 2 \phi_W} \quad \phi_W \in (0,1)/(0.5) \]  (3.24b)

\[ \phi_w = \frac{3}{4} \quad \phi_W = 0.5 \]  (3.24c)

\[ \phi_w = \frac{3}{8} \phi_W \quad \phi_W \in (-1,0) \]  (3.24d)

\[ \phi_w = \phi_W \quad \phi_W \in (1,1.5) \]  (3.24e)

83
Now the normalized value \( \Phi_w \) is known, the west interface value \( \phi_w \) can simply be found from equation (21). This leads to:

\[
\phi_w = \Phi_w (\phi_p - \phi_{WW}) + \phi_{WW} \tag{3.25}
\]

Note that in case \( \phi_p \) equals \( \phi_{WW} \) the parameter \( \Phi \) does not exist. In this case the above applied analysis can not be used. Instead the QUICK scheme will be applied which leads to:

\[
\phi_w = \frac{1}{2} (\phi_p + \phi_W) - \frac{1}{8} \text{CURV} \tag{3.26}
\]

where the local curvature CURV is defined as

\[
\text{CURV} = \phi - 2 \phi + \phi \tag{3.27}
\]

Combining the equations (14) and (16) the total flux through the west interface for the EXQUISITE scheme becomes:

\[
J_w = -d_w (\phi_p - \phi_W) + C_w \phi_w \tag{3.28}
\]

where \( \phi_w \) will be calculated using the equations (25) or (26) depending on the equality of \( \phi_p \) and \( \phi_{WW} \).

3.5 The finite difference equations

In this paragraph the finite difference form of the continuity and transport equations will be derived using the control volume approach. The continuity equation will be studied in paragraph 3.5.1. The u-momentum and the enthalpy equation, which are basically the same, will be studied in paragraph 3.5.2. For the radial velocity a similar approach could be used [Cornelissen (1984)]. However, as has been pointed out in paragraph 3.2, the radial velocity in a tube flow, as under study in this thesis, can be found directly from the continuity equation.
3.5.1 The continuity equation

A finite difference form of the continuity equation (3) can be derived by integration of this equation in the numerical domain over the control volume around point P:

\[
\iint_P \frac{\partial \rho}{\partial t} \, dv + \iint_P \frac{\partial \rho u}{\partial z} \, dv + \iint_P \frac{1}{r} \frac{\partial \rho v}{\partial r} \, dv = 0
\]  
(3.29)

Assuming \( \frac{\partial \rho}{\partial t} \) constant within the control volume and applying the Gauss theorem (29), taking \( \rho u \) and \( \rho v \) constant over respectively the east/west and north/south surfaces, leads to:

\[
\frac{\partial \rho}{\partial t} v + (\rho u)_{\text{w}} - (\rho u)_{\text{e}} + (\rho v)_{\text{n}} - (\rho v)_{\text{s}} = 0
\]  
(3.30)

The first term in (30) represents the accumulation of mass in the control volume. The other terms can be interpreted as mass flows through the surfaces of the control volume. For the discretization of the accumulation term in (30) the FI or B3 time differencing scheme presented in paragraph 3.4.1 will be used:

- FI scheme: \( \frac{\partial \rho}{\partial t} v = \frac{\rho^{k+1} - \rho^k}{\Delta t} v \)  
(3.31)

- B3 scheme: \( \frac{\partial \rho}{\partial t} v = \frac{3\rho^{k+1} - 4\rho^k + \rho^{k-1}}{2 \Delta t} v \)  
(3.32)

3.5.2 The transport equations

In this paragraph the difference equation for the axial velocity \( u \) and the enthalpy \( H \) will be derived using the control volume method. At first the transport equations for \( u \)-momentum (2) and enthalpy (4), are written in a general form:

\[
\frac{\partial \rho \phi}{\partial t} = - \frac{\partial J_{\phi}}{\partial z} - \frac{1}{r} \frac{\partial r J_{\phi}}{\partial r} + S_{\phi}
\]  
(3.33)

In this equation \( \phi \) is either \( u \) or \( H \) and \( S_{\phi} \) is the source term for the variable \( \phi \). \( J_z \) and \( J_r \), which represent the total flux in axial and radial direction respectively, are defined by:
\[ J_z = \rho u \phi - \Gamma_\phi \frac{\partial \phi}{\partial z} \]  
\[ J_r = \rho v \phi - \Gamma_\phi \frac{\partial \phi}{\partial r} \]

The first righthand term of (34) represents the convection flux. The diffusion flux is accounted for by the second term, in which \( \Gamma_\phi \) is the diffusion coefficient for the variable \( \phi \). In table 3.1 the representation of \( S_\phi \) and \( \Gamma_\phi \) for the different variables \( \phi \) are gathered.

<table>
<thead>
<tr>
<th>Equation</th>
<th>( \phi )</th>
<th>( \Gamma_\phi )</th>
<th>( S_\phi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Axial impulse</td>
<td>( u )</td>
<td>( \mu + \mu_t )</td>
<td>( -\frac{\partial P}{\partial z} )</td>
</tr>
<tr>
<td>Enthalpy</td>
<td>( H )</td>
<td>( \lambda/C_p + \mu_t )</td>
<td>( \frac{\partial P}{\partial t} + u \frac{\partial P}{\partial z} )</td>
</tr>
</tbody>
</table>

In order to derive the difference equation for the variable \( \phi \) we will now consider a point \( P \) in the interior of the computation domain. In the two dimensional \( z,r \) space the control volume around point \( P \) is limited by the surfaces \( A_n, A_e, A_s \) and \( A_w \) which separate \( P \) from its neighboring points \( N,E,S \) and \( W \) respectively (see figure 3.5).

![2D control volume around point P](image)

*Figure 3.5 : 2D control volume around point P*
Integrating equation (33) over the control volume around point P gives:

\[
\iiint_P \frac{\partial \rho \phi}{\partial t} \, dv = -\iint_P \left[ \frac{\partial J_z}{\partial z} - \frac{1}{r} \frac{\partial r J_r}{\partial r} \right] \, dv + \iint_P S_\phi \, dv \tag{3.35}
\]

where \( dv = 2\pi \, dz \, dr \). By assuming \( \frac{\partial \rho \phi}{\partial t} \) and \( S_\phi \) constant over the whole control volume, and applying the Gauss divergence theorem on the first term on the righthand side, equation (35) reduces to:

\[
\frac{\partial (\rho \phi)}{\partial t} \nu = - (J_e - J_w) - (J_n - J_s) + (S_\phi)_P \nu \tag{3.36}
\]

where \( \nu = \iint dv \) is the two dimensional volume of the considered control volume. If the values of \( J_e, J_w, J_n \) and \( J_s \) represent the integrated fluxes over the relating surfaces.

\[
J_w = 2\pi \int w r J_z \, dr \tag{3.37}
\]

\[
J_s = 2\pi r_s \int s J_r \, dz \tag{3.38}
\]

For \( J_e \) and \( J_n \) similar expressions apply.

When it is assumed that the flux \( J_z \) does not vary over the west interface \( w \), equation (37) becomes:

\[
J_w = \left[ \rho u \phi - \Gamma_\phi \frac{\partial \phi}{\partial z} \right] \nu A_w \tag{3.39}
\]

where \( A_w = 2\pi \int rdr \nu \) is the surface of the west interface. A similar result for \( J_s \) emerges if \( J_r \) is assumed constant over the south interface:

\[
J_s = \left[ \rho v \phi - \Gamma_\phi \frac{\partial \phi}{\partial r} \right] \nu A_s \tag{3.40}
\]

with \( A_s = 2\pi r_s \int dz \), the surface of the south interface.

Using the difference schemes, described in paragraph 3.4, the difference equation for the variable \( \phi \) can now be found from equation
First the difference equation resulting from the HYBRID scheme will be derived. Here to the expression (20) for the flux $J_w$ through the west interface of the control volume is substituted in (36). For the other fluxes similar expressions are applied. After subtraction of the continuity equation (30) multiplied by $\phi_P$ the following equation, for the variable $\phi$ at the new time level, results:

$$a_P\phi_P = \sum_{nb} a_{nb} \phi_{nb} + b \quad \text{nb} \in (W,E,N,S)$$

(3.41)

where the summation is over the four neighboring points W,E,N and S of the central point P. When the source term $(S_{\phi})_P\nu$ is linearised by:

$$(S_{\phi})_P\nu = (S_c + S_v \phi_P)\nu \quad \text{with } S_v > 0$$

(3.42)

as described by Patankar [1980], the coefficients $a$ and the source term $b$ in (42) are described by [Cornelissen(1984)]:

$$a_W = d_w \max[Pe_w,1+Pe_w/2,0] \quad a_E = d_e \max[-Pe_e,1-Pe_e/2,0]$$

$$a_S = d_s \max[Pe_s,1+Pe_s/2,0] \quad a_N = d_n \max[-Pe_n,1-Pe_n/2,0]$$

$$a_P = \sum_{nb} a_{nb} - S_v \nu + a_i \quad \text{nb} \in (W,E,N,S)$$

(3.43)

$$b = S_c \nu + b_i$$

with

$$Pe_w = \frac{C_w}{d_w} \quad C_w = (\rho u A)_w \quad d_w = \frac{(\Gamma \phi A)_w}{\Delta z_w}$$

as defined in paragraph 3.4.2. For the values at the e,n and s interfaces similar expressions hold.

The only terms to be specified are $a_i$ and $b_i$ appearing in the coefficient $a_P$ and the source term $b$ respectively. The value of these two terms, which represent the time derivative in the right-hand side of equation (36), depends on the time differencing scheme used. From the FI and B3 scheme the following expressions emerge:
• FI scheme: \[ a_t = \rho_P \frac{v}{\Delta t} \] \[ b_t = a_t \phi_P^0 \] (3.44a)

• B3 scheme: \[ a_t = (4\rho_P^0 - \rho_P^\infty)v \] \[ b_t = (4\rho_P \phi_P^0 - \rho_P^\infty \phi_P^\infty) \] (3.44b)

The subscript $^0$ denotes the value at the previous time level, and $^\infty$ the value two time levels back.

For the EXQUISITE scheme a similar equation like (41) can be derived. Herein the fluxes in the east-west direction will be considered first. The expression $J_w - J_e$ in equation (36) will be derived considering the case $u > 0$. $u < 0$ gives a similar result. Further, to be able to use relation (25) for $\phi_w$ and $\phi_e$, it is assumed that $\phi_P \neq \phi_{WW}$ and $\phi_E \neq \phi_W$. Otherwise relation (26) has to be used and somewhat different coefficients occur in the resulting equation.

Substituting expressions like (28) for $J_w$ and $J_e$ and using relation (25) we arrive at:

\[ J_w - J_e = -\hat{a}_P \phi_P + \sum_{n_b} a_{nb} \phi_{nb} \quad \text{nb} \in (WW,W,E,EE) \] (3.45)

where

\[ a_W = d_w - (1-\bar{\phi}_e)C_e \quad a_{WW} = (1-\bar{\phi}_w)C_w \]
\[ a_E = d_e - \bar{\phi}_e C_e \quad a_{EE} = 0 \]
\[ \hat{a}_P = \sum_{n_b} a_{nb} - C_e + C_w \] (3.46)

$C$ and $d$ are defined in equation (43) and $\bar{\phi}$ is computed from equation (24).

In the same way $J_w - J_e$ could be expressed using the values of $\phi$ in the point P and its neighboring points SS,S,N and NN. However, we will use the EXQUISITE scheme to improve the upwind character of the HYBRID
scheme in the description of the induced flow in the tube. This phenomenon primarily takes place in the horizontal, east-west, direction. In the radial, north-south, direction the velocities are generally small (Pe ≪ 2). The HYBRID scheme then reduces to a central differencing scheme which is already second order accurate.

So, in practice the EXQUISITE scheme is only applied in the east-west direction. For the north-south direction the HYBRID scheme is used. After subtracting the continuity equation (30) multiplied with \( \phi_P \) the EXQUISITE scheme reduces to:

\[
a_P \phi_P = \sum_{n_b} a_{n_b} \phi_{n_b} + b \quad \text{nb} \in \{W,E,N,S,WW,EE\}
\]

(3.47)

where the summation is now over six neighboring points of the central point P. The coefficients \( a_{WW}, a_{W}, a_{E} \) and \( a_{EE} \) (for \( u > 0 \) and \( \phi_P \neq \phi_{WW}, \phi_E \neq \phi_E \)) are defined in (46). The other coefficients, \( a_N, a_S \) and \( a_P \) and the source term \( b \) are the same as in the HYBRID scheme (43). Only in the summation in the coefficient \( a_P \), also the points WW and EE have to be included.

3.6 Boundary condition implementation

In the previous paragraphs the general difference equations in an interior point of the computation domain has been derived. In this paragraph a point near a boundary of the computation domain will be studied using the boundary conditions as described in paragraph 3.3.

Two different types of boundary conditions have to be considered:

-1- Dirichlet condition : \( \phi = \text{Constant} \)

-2- Neumann condition : \( \frac{\partial \phi}{\partial n} = \text{Constant} \)

where \( n \) denotes the direction of the boundary surface.

If a Dirichlet condition applies the value of \( \phi \) in the point \( B \) situated at the wall is known and the term \( a_B \phi_B \), in which \( B=W,E, N \) or \( S \), can be included in the source term. In case of a Neumann condition the coefficient \( a_B \), describing the flux over the boundary, is set equal to zero. For a non-zero flux, like the applied heat flux at the wall, the value of the flux is added to the source term \( b \).
Due to the upwind character of the HYBRID scheme no condition at the outlet is required in case of a high Péclet number flow. For a low Péclet number flow a zero gradient condition at the outlet is assumed.

In computing the flux at the outlet boundary with the EXQUISITE scheme the value of $\phi$ at one point outside the computation region is required. This value can be prescribed by acquiring a zero gradient or a zero curvature at the outlet. We assumed a zero gradient condition at the outlet.

For the boundary conditions the inlet of the tube two extra upstream points B and BB outside the computation region are needed. For a Dirichlet condition the value in the point B is specified. To find the value in point BB, which is used to compute the flux through the west interface $w$, the gradient in point P is assumed to be zero. The resulting Dirichlet condition is:

$$\phi_B^{k+1} \text{ specified } \quad \phi_{BB}^{k+1} = \phi_P^k$$  \hspace{1cm} (3.48)

For a Neumann condition the values in the points B and BB upstream from point P must be specified to compute the flux through the west interface of the control volume around point P. Assuming a zero gradient condition this results in:

$$\phi_B^{k+1} = \phi_P^k \quad \phi_{BB}^{k+1} = \phi_P^k$$  \hspace{1cm} (3.49)

In figure 3.6 the described inlet conditions are shown.

![Diagram of inlet boundary conditions](image)

a : Dirichlet condition \hspace{2cm} b : Neumann condition

Figure 3.6 : Inlet boundary conditions

91
3.7 Helium properties

The helium properties can be described using correlations developed from experiments. However, evaluation of the fluid properties during the simulations would cost far too much computation time because of the complexity of the correlations. For this reason Cornelissen [1984] precalculated the properties for distinct values of the variables pressure $P$ and enthalpy $H$ and stored the values in a look-up table. During the simulations the value of a property at a specified pressure and enthalpy can be found from this table by interpolation between the stored values. Cornelissen showed that a linear interpolation gave a 1% accuracy at the used intervals $\Delta P=0.5$ bar and $\Delta H=1.0$ kJ/kg between the points in the table.

The properties that are stored in the table are the density $\rho$, viscosity $\mu$, thermal conductivity $\lambda$ and specific heat $C_p$. For convenience also the value of $\lambda/C_p$, as used in the enthalpy equation is stored. Since the properties are stored as a function of pressure and enthalpy also the temperature $T$ needs to be expressed in these variables. Finally, the thermodynamic speed of sound $c$, which can be computed from the other properties by

$$
\frac{1}{c^2} = \left( \frac{\partial \rho}{\partial P} \right)_S = \left( \frac{\partial \rho}{\partial P} \right)_H + \frac{1}{\rho} \left( \frac{\partial \rho}{\partial H} \right)_P
$$

has been stored. $S$ is the entropy.

![Helium temperature as a function of the flow enthalpy at different pressures](image)

Figure 3.7:
Table 3.2 : Stored helium fluid properties

<table>
<thead>
<tr>
<th>Helium property</th>
<th>Reference</th>
<th>Fig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature</td>
<td>T</td>
<td>3.7</td>
</tr>
<tr>
<td>Density</td>
<td>ρ</td>
<td>3.8a</td>
</tr>
<tr>
<td>Viscosity</td>
<td>μ</td>
<td>3.8b</td>
</tr>
<tr>
<td>Thermal conductivity</td>
<td>λ</td>
<td>3.8c</td>
</tr>
<tr>
<td>Specific heat</td>
<td>C_p</td>
<td>3.8d</td>
</tr>
<tr>
<td>λ/C_p</td>
<td>λ/C_p</td>
<td>--</td>
</tr>
<tr>
<td>speed of sound</td>
<td>c</td>
<td>3.8e</td>
</tr>
</tbody>
</table>

The correlations for the fluid properties are extensively described by Cornelissen and will not be repeated here. In table 3.2 the fluid properties stored in the table are gathered together with a reference to the literature from which the correlation has been found. For C_p no literature reference has been given because the values of this property are computed from the enthalpy correlation using

\[ dH = C_p \, dT \]  

(3.51)

neglecting the small pressure influence.

In figure 3.7 the variation of the temperature with enthalpy is shown. The fluid properties as a function of temperature for different pressures are shown in figure 3.8.

3.8 The solution method

The difference equations (41) or (47) for the variable \( \phi \), as derived in paragraph 3.5.2, can be written as a matrix equation:

\[ A \, \Phi = B \]  

(3.52)

where \( \Phi \) and \( B \) are vectors representing the unknown variable \( \phi \) and source term \( b \) in the computation points respectively. The matrix \( A \) contains the coefficients \( a \), as defined in (43) and (46). Through the fluid properties and the convection term these coefficients are still dependent on \( P, H \) and \( u \). The equations are thus coupled and non linear. In order to be able to solve (52) the equation has to be linearised.

93
Figure 3.8:

Helium properties as a function of temperature at different pressures

- $P = 3$ bar
- $P = 4$ bar
- $P = 5$ bar
- $P = 6$ bar
- $P = 9$ bar
- $P = 12.5$ bar
In this thesis an iterative procedure is used. The equation is linearised by computing the coefficient matrix $A$ and the source term $B$ using the values of the variables, and the consequent fluid properties, from the latest iteration. Solving the now linear matrix equation gives a new value of $\phi$. If this value has changed more than a prescribed criterion the solution is not converged yet and a new iteration is started. This is repeated until convergence is reached. A sketch of the total iteration procedure is shown in figure 3.9.

![Diagram of iteration procedure](image)

**Figure 3.9 : Iteration procedure**

At the beginning of the iteration cycle the heat flux from the conductor to the helium flow is computed from the temperature gradient near the wall. To increase the accuracy Cornelissen [1984] assumed a hyperbolic instead of a linear temperature profile between the wall and the nearest grid point in the helium. He arrived at:

$$ q = \lambda \frac{T_w - T_1}{y_1} \left[ \frac{T_w}{T_1} \right] $$

(3.53)

In order to preserve energy the same value for the heat flux $q$ is used in the matrix and the helium enthalpy equation.

The equations for all variables can be solved simultaneously. This leads to the correct solution of the linearised set of equations.
However, this requires much computation effort. Instead we applied a procedure in which the equations are solved sequentially. This results in a less accurate solution after one iteration. However, this is no problem since the coefficients in the equations change from iteration to iteration. A further simplification of procedure is obtained by solving the equations on one radial north-south line at a time, assuming the variables on the neighboring east-west lines to be known. In this way a simple tri-diagonal matrix equation results which can simply be solved using the Tri Diagonal Matrix Algorithm [Patankar and Spalding (1970)]. The errors, introduced by the linearisation and simplifications of the matrix equation, can be made arbitrarily small by increasing the number of iterations.

Finally the procedure for computing the radial velocity from the continuity equation (30) must be described. Rewriting (30) gives:

\[
(\rho V A)_n = \left[ \frac{\partial \rho}{\partial t} \right]_P v + (\rho u A)_w - (\rho u A)_e + (\rho v A)_s
\]  \hspace{1cm} (3.54)

At the center of the tube the south surface of the control volume around the first normal grid point coincides with the tube center. At this point the radial velocity \( v_s \) equals zero for symmetry reasons. The right hand side of (54) is then completely determined and the radial velocity at the north interface can be computed. For the adjacent grid point in radial direction \( v_n \) of the previous grid point is the south interface velocity \( v_s \) of the new grid point. Again only \( v_n \) is unknown in equation (54) and can thus be solved. At the last grid point before the solid wall the same procedure still applies. However, the computed north interface velocity \( v_n \) at this point must be zero since no suction through the wall is considered. If a non zero value emerges for \( v_n \) at this point a mass error in the tube section exists. This can be used to test the convergence of the iteration procedure.

When all convergence tests are satisfied the values for the variables \( \phi \) are stored. After increasing the time \( t \) with the time incremental \( \Delta t \) a new iteration procedure to find the solution at the new time level is started. The initial guess for \( \phi \) is the value \( \phi^0 \), the final solution of the previous time level.
4 Experimental method

In a study where physical phenomena are simulated, verification of the model with experimental data is indispensable. In a model describing turbulent flows the governing physical processes must be simplified using turbulence models as described in paragraph 2.4. The reliability of these models can only be tested by comparing the obtained results with experimental data.

In case of a turbulent duct flow of cryogenic supercritical helium no experimental data on the turbulent flow field are available. Due to the low temperatures at which the processes in a superconductor take place, existing measuring techniques can hardly be used to obtain experimental information on the velocity and turbulence in the helium flow.

To be able to validate the flow simulations a much simpler experimental set-up, using air at room temperature instead of cryogenic helium, has been developed. In this set-up the heat induced pressure surge in the helium flow has been physically simulated by locally introducing an extra amount of air through the wall into the main stream. Clearly the transient flow situations in the laboratory set-up and in a cooling channel of a superconductor after a disturbance will differ. But with the mathematical model also the experimental situation can be simulated, satisfying the primary goal, being the validation of transient flow simulations as such.

The experimental set-up and measuring techniques will be discussed in this chapter.

4.1 Experimental set-up

In a rectangular aluminium duct (dimensions 6m*45mm*70mm) a stationary air flow is provided by means of suction with a ventilator at the outlet of the tube (see figure 4.1). At the inlet of the tube the air temperature is measured using a Platinum/Rhodium thermocouple. The mass flow can be found from a venturi meter at the inlet of the tube. For small pressure differences over the venturi meter the flow can be assumed to be incompressible. The mass flow as a function of the pressure difference over the venturi meter can be expressed by:
\[ \phi_m = (\rho <u> A) = \left(1 - \frac{A_i^2}{A_o^2}\right)^{-1/2} A_o \left[2\rho \Delta P\right]^{1/2} \]  

(4.1)

The surfaces \( A_i \) and \( A_o \) at the in- and out-let of the venturi meter respectively are \( A = A_o = 3.13 \times 10^{-3} \text{ m}^2 \) and \( A_i = 17.94 \times 10^{-3} \text{ m}^2 \). Solving the bulk velocity as a function of the pressure difference \( \Delta P \) over the venturimeter results in:

\[ <u> = 1.46 \sqrt{\Delta P/\rho} \]  

(4.2)

---

**Figure 4.1:** Experimental set-up

1. Stationary temperature sensor
2. Air inlet (Venturi meter)
3. Micro manometer
4. Pressure chamber
5. Manometer
6. Pressure airline
7. Quick opening magnetic valve
8. Traversing system
9. Air inlet (Venturi meter)
10. Heat flux sensor
11. Pressure sensor
12. Rubber
13. Ventilator (Suction)
14. Pressure regulator

A pressure surge on the stationary flow is generated by suddenly introducing an extra amount of air from a pressure vessel into the main stream. To realize this fast introduction of air, resulting in a sharp
pressure rise, the pressure vessel had to be separated from the tube by a valve which could be opened within a time period of about one millisecond. A valve which met this requirement was a Asco-magnetic valve type E030A17. (see figure 4.2). The diameter of the the valve opening is 12.5 mm. With this valve, operated at a maximum pressure difference of 0.35 bar over it, a pressure wave up to 600 Pa and velocity changes in the order of 1.5 m/s could be generated. The rise time of the pressure wave in the tube from zero to the maximum value, which originates from the opening of the valve and the pressure build up in the tube, was found to be about 3 milliseconds.

![diagram](image)

**Figure 4.2**: The magnetic valve (ASCO-E030A17)

The induced pressure wave will travel through the tube at the speed of sound (≈ 340 m/s). The measuring position is situated at 2 m from the pressure vessel which is placed 2 m from the tube inlet (see figure 4.1). In this way the reflections from the tube ends will reach the measuring position at the same time. In this time the original pressure wave has traveled a distance of 6 m. Measuring from the moment the pressure wave arrives at the measuring position to the moment the reflections from the tube ends return at this position this leads to a total measuring time of about \( \frac{6 \cdot 2}{340} \approx 12 \text{ ms} \). During this time the pressure in the pressure vessel must be constant. The minimum volume \( V_{\text{min}} \) of the vessel can be estimated from requiring that the pressure difference \( \Delta P_0 = P - P_0 \) (\( P_0 \) the ambient pressure) changes less than \( \varepsilon \). After a time \( t_\varepsilon \) a mass flow of

\[
\Delta m = \rho \phi_v t_\varepsilon
\]  

(4.3)
has left the vessel resulting in a pressure decrease $\Delta P$ in the vessel. Assuming the expansion of the (ideal) gas in the vessel is adiabatic, $\Delta P$ for a vessel with volume $V$ can be found from

$$\frac{m}{m - \Delta m} \left[ \frac{P}{P - \Delta P} \right]^{\frac{1}{\kappa}} \kappa = 1.4$$

(4.4)
in which $m = \rho V$ is the initial mass in the vessel. Rewriting (4) and using (3) gives:

$$v = \frac{\phi_v t_c \rho^{-1}}{1 - \left[ 1 - \frac{\Delta P}{P} \right]^{\frac{1}{\kappa}}}$$

(4.5)

Requiring that $\Delta P < \varepsilon \Delta P_0$ and recognizing that $\Delta P \ll P$ the minimum volume for the pressure vessel becomes:

$$V_{\text{min}} = \frac{P}{\Delta P_0} \frac{\kappa \phi_v t_c}{\varepsilon \rho}$$

(4.6)

At a pressure of 1.3 bar ($\Delta P_0 = 0.3$ bar) in the pressure vessel the mass flow $\phi_v$ has been found to be about $10^2$ m$^3$/s. The minimum volume to keep the pressure difference $\Delta P_0$ constant within 1 % during 12 milliseconds is:

$$V_{\text{min}} = \frac{1.3}{0.3} \frac{1.4 \ast 0.01 \ast 0.012}{0.01 \ast 1.16} \approx 60 \times 10^{-3} \text{ m}^3$$

The volume of the used pressure vessel has been taken equal to $V_{\text{min}}$. After every experiment the pressure in the vessel was restored with the aid of compressed air from a pressure air line.

At a distance of about 2 m ($\approx 36$ hydraulic diameters) from the pressure chamber the transient flow phenomena are measured. A Sensym pressure transducer type SX01D is used to measure the transient pressure wave. This sensor has a response time (10–90 %) of about 0.1 ms and can measure pressure differences up to 7 kPa. The velocity and transient temperature are measured with a hotwire anemometer as will be described in the paragraphs 4.2 and 4.3 respectively. To obtain transient heat transfer data a heat flux sensor based on a hotfilm technique has been developed. A description of this device will be given in paragraph 4.4. The electronic equipment and data handling will be discussed in paragraph 4.5.
4.2 The hotwire anemometer

Velocity data have been obtained with a hotwire probe operated using a constant temperature bridge and a Dantec mainunit (type 55M01).

The local velocity $u$ can be found from the bridge voltage $V$ by considering the heat balance in the probe. To keep the probe at a constant temperature the heat $Q$ which is convected from the heated wire to the flow must be balanced by the ohmic heat $P$ dissipated in the wire (Losses at the wire ends are neglected).
Noticing that from $Q \propto \text{Nu} (T_{\text{wire}} - T_0)$ and $P \propto V^2$ it follows that:

$$\frac{V^2}{T_{\text{wire}} - T_0} \propto \text{Nu}$$  \hspace{1cm} (4.7)

The Nusselt number $\text{Nu}$ depends on the local velocity around the wire. From experiments on convective heat transfer from small cylinders it has been found in literature [Collins and Williams(1959), Kramers (1946), Bourke(1969), King(1914)] that a general correlation for the Nusselt number can be written as:

$$\text{Nu} = C_1 + C_2 u^n \hspace{1cm} 0.45 < n < 0.50$$  \hspace{1cm} (4.8)

where $C_1$ and $C_2$ are constants.
Combining (7) and (8) the following relation results:

$$\frac{V^2}{\Delta T} = A + B u^n$$  \hspace{1cm} (4.9)

The constants $A, B$ and $n$ have to be found from a calibration of the hotwire probe. The influence of varying ambient temperature $T_0$ can be taken into account by the term $\Delta T = T_{\text{wire}} - T_0$.

To measure the flow phenomena two types of hotwire probes are used. In the bulk of the flow a straight general purpose probe (P11, Dantec - figure 4.3a) is used. Near the wall a right angle type of sensor (P04,Dantec - figure 4.3a) is used to be able to approach the solid boundary. Both tungsten wires have a diameter of 5 $\mu$m. The length of the wires on the P04 and P11 probes are 1 and 1.25 mm respectively.
From calibrations of the hotwire probes for air velocities between 0.5 and 10 m/s, taking 0.5 m/s steps, the following calibration constants in equation (9) are found:

<table>
<thead>
<tr>
<th>Probe</th>
<th>n</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>P11</td>
<td>0.455</td>
<td>0.0347</td>
<td>0.0183</td>
</tr>
<tr>
<td>P04</td>
<td>0.446</td>
<td>0.0270</td>
<td>0.0203</td>
</tr>
</tbody>
</table>

a  Straight general purpose type (P11)

b  Right angle type (P13)

Figure 4.3: Hot wire probes

For traversing the probe in the channel the probe support was coupled to a traversing system (figure 4.4). With this system it was possible to change the position of the probe with a 0.01 mm accuracy. The absolute position y from the wall however could only be determined with a 0.2 mm accuracy. Due to small misalignments of the wire axis, caused by a small rotation of the probe or small imperfections at the ends of the wire the distances $y_1$ and $y_2$ of both wire ends to the wall are not equal (figure 4.5). By traversing the probe to the wall until one end of the probe reaches the wall the difference $y_2 - y_1$ was estimated to be less than 0.2 mm. The effective distance $y$ at this point was taken $y = 0.1$ mm.
Figure 4.4:

Traversing system

1. Hotwire probe
2. Probe support
3. Traversing mechanism

Figure 4.5:

Traversing system
- Distance to the wall
4.3 The resistance thermometer

In paragraph 4.2 the temperature dependence of the resistance of a hotwire probe was used to keep the wire at a fixed temperature. The flow velocity could be evaluated from the heat dissipated in the wire.

In this paragraph transient temperature measurements are described using the same type of probes. Now, by applying a constant current to the probe, one measures the resistance of a wire, proportional to the wire temperature. If the applied current is low enough (depending on the probe used) the selfheating of the wire is negligible and the wire temperature equals the local flow temperature. The metal probes operated at a constant current mode are often referred to as resistance thermometers rather than hotwire probes.

The probe used in this investigation was a 1 $\mu$m diameter platinum wire (Dantec - straight probe 55P31). The length of the sensor is 0.4 mm. To operate the probe at a constant current a Dantec mainunit (55M01) was used in combination with a constant current (CCA) bridge.

For this probe a operation current $I < 0.3$ mA is recommended in the instruction manual.

4.4 The heat flux sensor

4.4.1 Theory

To measure the heat transfer to a transient airflow a heat flux sensor is needed with a response time in the order of milliseconds. From a study of Baines [1970], in which some commonly used types of unsteady heat flux sensors have been compared, it was concluded that for short measuring times thin- or thick film sensors were to be considered. The thin film sensor, being the more sensitive gauge, was selected for the present investigation.

A thin film sensor is essentially a metal film with a temperature dependent resistance. Two types of thin film sensors can be distinguished. With the first type, based on a coldfilm technique, the
film resistance is a measure for the surface temperature. The local heat flux can be obtained from this data through solving the unsteady heat conduction equation in the substrate on which the film is deposited. Van Heiningen et. al. [1985] have developed a sensor based on this technique with a resolution of 0.002 °C on measured instantaneous surface temperatures at sampling speeds up to 5000 Hz. However this type of sensors are only applicable if a temperature difference exists between the flow and the wall, on which the sensor is situated.

The second type of thin film sensor is based on a hotfilm technique. With this type of sensors the metal film is heated and kept at a constant temperature. To keep the metal film at a constant temperature the electrical power $P$ dissipated in the hotfilm must balance the convective heat transfer $Q$ to the flow together with the heat $Q_s$ conducted into the substrate on which the film is deposited:

$$ P = Q + Q_s \quad (4.10) $$

If $V$ is the voltage drop over the metal film and $R_s$ is its resistance at a constant sensor temperature $T_s$, then

$$ P = \frac{V^2}{R_s} \quad (4.11) $$

The convective heat transfer to the flow can be described by:

$$ Q = h (T_s - T) A \quad (4.12) $$

where $A$ is the surface of the hotfilm and $T$ the bulk temperature of the flow. The heat transfer coefficient $h$ is to be determined.

In first approximation the heatloss $Q_s$ through the substrate is proportional to the difference between the sensor temperature and the ambient temperature. If both are constant $Q_s$ can be considered to be constant. Combining this result with equations (10) to (12) the heat transfer coefficient can be found from the bridge voltage $V$ by:

$$ h = \frac{V^2 / R_s - Q_s}{(T_s - T) A} = \frac{V^2}{R_s \Delta T A} - h_0 \quad (4.13) $$

An estimate for $Q_s$, and thus for $h_0 = \frac{Q_s}{\Delta T A}$, can be found by isolating the hotfilm from the flow. All dissipated heat is then conducted into the substrate. $Q_s$ is related to the voltage drop $V_0$ by:

105
\[ Q_s = \frac{V_0^2}{\mathcal{H}_s} \]  

(4.14)

In literature the above described hotfilm technique has frequently been used for measuring skin friction (see for example Mathews and Poll [1985], Bellhouse and Schultz [1966]). By assuming a constant shear stress in the flow near the wall \( (\tau = \tau_w, \text{ Couette flow approximation}) \) and using Leveque's solution a relation between the wall shear stress and the heat flux to the flow can be found [Mathews and Poll]:

\[ h = \frac{\tau_w}{3} \]  

(4.15)

For a steady, fully developed turbulent flow the wall shear stress can be written as:

\[ \tau_w = \frac{1}{2} \rho \langle v \rangle^2 f \]  

(4.16)

with

\[ f = 0.184 \text{ Re}^{-0.2} \quad \text{Re} > 2 \text{ } 10^4 \]  

(4.17)

Substituting (16) and (17) into (15), using Re \( \therefore \) \( \langle v \rangle \) gives:

\[ \text{Nu } \therefore \text{ Re}^{0.6} \]  

(4.18)

This result can be compared with the experimental and numerical heat transfer data in case of a stationary flow.

4.4.2 The construction of a heat flux sensor

In this paragraph the construction of a heat flux sensor based on the hotfilm technique, discussed in the previous paragraph, will be described.

Following van Heiningen et. al. [1985] gold was selected as the thin film material. Platinum, as used by Bellhouse and Schultz [1966], was rejected because it is very hard to deposit and a nickel film has been found to be unstable in moist air [Van Heiningen et. al.]. To avoid imperfections in the gold film the substrate on which the film is deposited must be very smooth. The thermal conductivity \( \lambda \) of the substrate must be low in order to keep the heat loss to the background as low as possible.
As a substrate a 1 mm thick glass plate of $3\times30 \text{ mm}^2$ is used. Two silver wires with a diameter of 0.5 mm pierced through the glass plate serve as electrical contacts. A copper/constantan thermocouple is fixed in the middle of the backside of the glass plate. This construction is embedded in a 20 mm thick Rulon cylinder with a diameter of 50 mm (see figure 4.6). The gaps between glass and Rulon are filled with a aluminium ceramic RTC-70. After the ceramic has hardened the glass plate is made flush with the Rulon cylinder by scouring the surface.

A 250 Å film is deposited in vacuum on the glass surface using a mask that covers the Rulon surface. The first 50 Å of the film consists of copper/nickel to ensure a better contact between the glass surface and the 200 Å gold film. The reason Rulon has been chosen as a construction material lies in the fact that Rulon, unlike a lot of other synthetics, does not produce dust during the deposition in vacuum.

![Diagram](image)

a) sideview  

b) topview

1 - goldfilm $3\times30 \text{ mm}^2$ (50Å Copper/Nickel, 200Å gold)  
2 - glass $3\times30 \text{ mm}^2$ ($\lambda = 1.0 \text{ W/mK}$)  
3 - Rulon $D=50 \text{ mm}$ ($\lambda = 0.3 \text{ W/mK}$)  
4 - Silver wires $D=0.5 \text{ mm}$  
5 - Copper/Constantan thermocouple

Figure 4.6 : The heat flux sensor

The temperature dependence of the resistance of the produced hotfilm sensor is determined by measuring the resistance at different temperatures in a thermostat bath. After four calibration series the following relation resulted:

107
\[ R = \mathcal{R}_{\text{ref}} + \alpha (T - T_{\text{ref}}) \]  
with the coefficient \( \alpha = 0.0355 \, \Omega K^{-1} \). \( R_{\text{ref}} \) is about 29 \( \Omega \) at 20 °C.

In all experiments the sensor temperature is kept at 40 °C using a Dantec mainunit (type 55M01) in combination with a constant temperature bridge as has been used in the hotwire anemometry (paragraph 4.2).

To check the assumption of constant heat loss through conduction into the substrate the temperature in the substrate is monitored using a copper/constantan thermocouple. In stationary heat transfer experiments the heat flux to the flow must be corrected for changes in the heat conduction \( Q_s \) into the substrate. Assuming \( Q_s \) to be proportional to the temperature difference between the gold film and the thermocouple a heatloss constant can be defined as:

\[ c_i = \frac{Q_{s0}}{T_s - T_{th0}} \]  
(4.20)

\( Q_{s0} \) is the heat conducted into the substrate at a thermocouple temperature \( T_{th0} \), as estimated by equation (14). The heatloss at a thermocouple temperature \( T_{th} \) can be estimated using equation (20) by:

\[ Q_s = c_i (T_s - T_{th}) = Q_{s0} + c_i (T_{th0} - T_{th}) \]  
(4.21)

This leads to a correction \( \Delta h \) on the heat transfer coefficient computed from equation (13):

\[ h = \frac{V^2}{R_s \Delta T A} - h_0 - \Delta h \]

\[ \Delta h = \frac{c_i}{A} \frac{T_{th0} - T_{th}}{T_s - T} \]  
(4.22)

In figure 4.7 a schematic picture of the heat fluxes in the Rulon cylinder is shown. The heat \( Q_s \) conducted into the substrate can be split into two parts, namely \( Q_{sb} \) which is conducted through the Rulon to the background and \( Q_r \) which is conducted in radial direction and is transferred from the Rulon/Air interface into the flow.
Changes in the air flow influence the convective heat transfer $Q_r$ to the flow and consequently influences the temperature profile in the Rulon cylinder. As a result the thermocouple temperature and the total heat $Q_s$ conducted into the substrate will change. The adjustment of a new stationary temperature profile in the substrate after a change in the stationary velocity of the airflow takes about 10 minutes.

In case of an instationary flow the duration of one experiment takes about 12 milliseconds. In this time period the heat conducted from the gold film into the substrate $Q_s$ and the thermocouple temperature $T_{th}$ will not be influenced by the slow change in the temperature profile in the Rulon cylinder due to a change in $Q_r$. Consequently no corrections have to be made in this case. The time period between two instationary experiments however must be large enough for the temperature profile to be restored.

4.5 The electronic equipment

The central part of the electronic equipment (figure 4.8) is formed by a Signal Memory Recorder (Scientific Instruments - Basel). In this device transient analog electronic signals from the heat flux sensor, hotwire anemometer and pressure transducer, are converted to digital data which are store in memory. Four input channels with independent 12 bits A/D converters and a memory of 32k words are installed. The lowest sample rate for each converter is 4 $\mu$s (sample frequency 250 kHz).

To start the storing of data several triggering options including pre- and post-triggering were available. In the transient measurements pressure, being the most constant variable, was used as a triggering
signal. The recording of data was started when the pressure signal exceeded a predefined value. For stationary measurements manual triggering was used.

After the recording of data was completed the digitized voltages were read from the SMR into a HP9000 computer for storage and post processing. Together with this data extra parameters like the ambient temperature, needed for the interpretation of the voltages, were stored.

Hotwire data are converted to velocities using equation (9). The extra parameters needed for this conversion are the ambient temperature $T_0$ and the probe calibration constants $n, A$ and $B$.

![Diagram of equipment setup](image)

- **HF** - Heatflux sensor
- **HW** - Hotwire anemometer
- **RT** - Resistance thermometer
- **P** - Pressure transducer
- **S** - Voltage shifter

**Figure 4.8**: Electronic equipment

In a stationary flow the average velocity $\bar{u}$ and the root mean square (RMS) value of the velocity fluctuations $u'$ are computed from the velocity data $u_j$, $j = 1..N$ using

110
\[ \bar{u} = N^{-1} \sum_{j=1}^{N} u_j \]  

(4.23)

\[
RMS = (\bar{u}^\top \bar{u})^{1/2} = \left( \frac{1}{N - T} \sum_{j=1}^{N} (u_j - \bar{u})^2 \right)^{1/2}
\]

For the instationary measurements an ensemble averaging procedure is used:

\[
\{u_j\} = N_e^{-1} \sum_{i=1}^{N_e} \{u_i\}_j
\]

(4.24)

where \( \{u_i\}_j \) is the velocity at time \( t_j \) in experiment number \( i \). \( N_e \) is the total number of identical experiments.

In the same way an ensemble RMS on time \( t=t_j \), \( \{\text{RMS}\}_j \), can be defined by:

\[
\{\text{RMS}_j\} = \left\{ \frac{1}{N_e - T} \sum_{i=1}^{N_e} (\{u_i\}_j - \{u_j\}_j)^2 \right\}^{1/2}
\]

(4.25)

The voltage output of the resistance thermometer can be converted to temperatures using a linear relation:

\[
T = T_{ref} + \beta (V - V_{ref})
\]

(4.26)

To obtain absolute temperatures the output voltage \( V_{ref} \) at a known temperature \( T_{ref} \) must be determined. If only temperature fluctuations are of interest (26) can be written as:

\[
\Delta T = \beta \Delta V
\]

(4.27)

The proportionality constant \( \beta \) (\( \beta = \frac{dT}{dV} \)) which depends on the electronic circuit used, has to be found from calibration, as described in the Dantec instruction manual.

The voltage output of the heat flux sensor can be converted to heat transfer coefficients using equation (13). For stationary flow a correction for the changing substrate conduction can be computed from equation (22).

For instationary flow the voltage changes \( \Delta V \) are small compared to the mean voltage \( V \) over the sensor. Due to the limited voltage range of
the A/D converter in the SMR the voltage changes are stored with a low accuracy. This accuracy can be increased by subtracting the stationary voltage from the heat flux sensor signal using an electronic circuit as shown in figure 4.9.

![Diagram](image)

Figure 4.9: Voltage shifter for heat flux sensor

The recorded voltage $V_{\text{SMR}}$ is related to the real voltage drop $V$ over the heat flux sensor by

$$V_{\text{SMR}} = 0.515 \, V - 0.242 \, V_{\text{shift}} \quad (4.28)$$

By taking $V_{\text{shift}} = \frac{0.515}{0.242} \, V_0$ only voltage changes from $V_0$ are recorded and a high accuracy can be achieved.

In the conversion of the heat transfer data, using equation (13) or equation (22), the bridge voltage $V$ has to be calculated from equation (28). As extra parameters the shift voltage, the sensor resistance and the temperature difference between the sensor and the flow are required. For computing the correction $\Delta h$ from equation (22) also the change in the thermocouple temperature is needed.

The pressure transducer is calibrated using a closed pressure chamber which volume can be enlarged or reduced by a rotating piston. As a reference the pressure is measured using a manometer (type FC012). From the calibration the following relation between the amplifier output $V$ and the pressure $P$, relative to the ambient pressure, is found to be linear for $P \in (-4000, 4000)$ Pa:

$$P = 800 \, (V - V_{\text{offset}}) \quad (4.29)$$

In the conversion the voltage output $V_{\text{offset}}$ at zero flow is required as an extra parameter.
5 Transients with surface renewal model.

5.1 Introduction

In the mixing length model, as described in paragraph 2.4.1, the influence of the vicinity of a solid wall has been accounted for by correcting the mixing length (2.65) by the van Driest damping factor (2.66). Parallel to this approach an alternative method for describing the influence of a solid wall on turbulence has been employed in this thesis. In this approach the processes in the wall region are described using a turbulent burst model based on the instationary nature of the wall region flow as described in paragraph 2.4.2.

In this chapter the stationary surface renewal theory will be extended to account for a time dependent wall temperature. The mean flow is assumed to be stationary and fully developed. This means that the temperature field has no influence on the flow field, so the fluid density and viscosity must be independent of temperature. The (constant) mean renewal rate \( \bar{r} \) can then be found from a stationary renewal model as reviewed in paragraph 2.4.2. In the following analysis of the time dependent heat transfer, the renewal rate and the flow field are assumed to be known.

As a starting point the elementary surface renewal model of Danckwerts will be used. In this model the heat penetration in a single eddy near the wall is described by equation (2.73):

\[
\rho C_p \frac{\partial T}{\partial \theta} = \frac{\partial}{\partial y} \left[ \lambda \frac{\partial T}{\partial y} \right] \tag{5.1}
\]

The heat capacity \( \rho C_p \) and the conductivity \( \lambda \) can be dependent of the temperature in this formulation. The initial- and boundary conditions for the heat transfer to supercritical helium, \( 0.5 < \text{Pr} < 5 \), can be modeled by (see paragraph 2.4.2.5) equation (2.74):
\[ T = T_b \quad y > 0 \quad \theta = 0 \]
\[ T = T_b \quad y \rightarrow \infty \quad 0 < \theta < \tau \]  \hspace{1cm} (5.2)
\[ T = T_w(t) \quad y = 0 \quad 0 < \theta < \tau \]

\( t \) is the real time coordinate, \( \theta = t - t_0 \) is the age of the eddy. The time \( t_0 \) is the moment of inrush of the eddy and \( \tau \) is the statistical age of the eddy at the moment of renewal.

To compute the mean temperature profile at time \( t \) Danckwerts random age distribution function (2.81) is used in the averaging process (2.75):

\[ T(y,\theta,t,S) = \int_0^\infty T(y,\theta,t) \ S \exp(-\theta S) \ d\theta \]  \hspace{1cm} (5.3)

where \( S = (\tau)^{-1} \) is the mean renewal frequency.

In paragraph 5.2 equation (1) with conditions (2) will be solved in case of a stationary situation, \( T_w(t) = T_w \), and constant fluid properties. In paragraph 5.3 the extension to a time dependent wall temperature \( T_w(t) \) will be discussed. An analytical solution for the case of a sudden change in the wall temperature will be presented. For the case of an arbitrarily varying wall temperature an integral solution evolves which in general has to be solved numerically. In paragraph 5.4 also varying fluid properties (\( \rho C_p \) and \( \lambda \)) are included in the model. For this situation no analytical solution exists. The solution is given in an integral form which has to be solved numerically.

5.2 Stationary case - Constant fluid properties

For constant fluid properties equation (1) with conditions (2) has been solved by Thomas [1970] in case of a constant wall temperature:

\[ T(y,\theta) = (T_w - T_b) \ \exp\left[- \frac{1}{2} (\alpha \theta)^{1/2} \right] \]  \hspace{1cm} (5.4)

Using the averaging procedure (3), the mean temperature profile becomes:

114
\[ T(y, S) = (T_w - T_b) \exp \left[ -y \left( \frac{S}{Q} \right)^{1/2} \right] \]  

resulting in a mean heat flux:

\[ \overline{q}_w(S) = (T_w - T_b) \lambda \rho C_p S \]  

The stationary Nusselt number, defined by \( \text{Nu} = \frac{hD}{\lambda} \) with \( h = \frac{\overline{q}_w}{T_w} \), can be found from (6):

\[ \text{Nu} = (S^+)^{1/2} \left( \frac{D}{2} \right)^{1/2} \text{Re} \text{ Pr}^{1/2} = \text{Nu}_* \]  

where \( S^+ = S \nu (u^*)^{-2} = (\tau^+)^{1/2} \). \( u^* = (\tau_w/\rho)^{1/2} = u_b \left( \frac{D}{2} \right)^{1/2} \) is the friction velocity.

5.3 Instationary case - Constant fluid properties

5.3.1 A step in the wall temperature

In this paragraph the heat transfer to a constant property fluid will be studied after a sudden increase in the wall temperature (see figure 5.1). The bulk temperature \( T_b \) has been assumed to be equal to zero. Because of the linear nature of the problem this is allowed without loss of generality.

\[ \begin{array}{c|c|c}
T_w & T_2 & \text{Figure 5.1 : Wall temperature as a function of time.} \\
\hline
T_1 & 0 & \text{Step function} \\
\hline
\end{array} \]

For time \( t < 0 \) the stationary solution from paragraph 5.2 applies. For \( t > 0 \) two kinds of eddies reside in the wall region:

-1- Eddies with age \( \theta < t \):
These eddies have entered the wall region after \( t = 0 \). This means that during the whole residence time from \( \theta = 0 \) to \( \theta = t \) wall the wall temperature has been equal to \( T_2 \). The temperature profile in these eddies is described by the stationary solution (4) for a wall temperature \( T_w = T_2 \).
-2- Eddies with age \( \theta > t \):

When these eddies entered the wall region the wall temperature was equal to \( T_1 \). After a time \( \theta - t \) the wall temperature was increased from \( T_1 \) to \( T_2 \). The temperature profile in these eddies can be found by a superposition of the solutions for a constant wall temperature \( T_1 \) over a time period \( \theta \) and a wall temperature increase \((T_2 - T_1)\) over a time period \( t \). Using equation (4) this yields:

\[
T(y,\theta,t) = T_1 \text{ erfc}\left(\frac{1}{2}y (\alpha \theta)^{-1/2}\right) + (T_2 - T_1) \text{ erfc}\left(\frac{1}{2}y (\alpha t)^{-1/2}\right)
\]

Using the averaging procedure (3) the mean temperature becomes:

\[
T(y,\theta,S) = \int_0^1 T_2 \text{ erfc}\left(\frac{1}{2}y (\alpha \theta)^{-1/2}\right) \phi(\theta,S) \, d\theta + \int_{T_1}^\infty \left[ T_1 \text{ erfc}\left(\frac{1}{2}y (\alpha \theta)^{-1/2}\right) + (T_2 - T_1) \text{ erfc}\left(\frac{1}{2}y (\alpha t)^{-1/2}\right) \right] \phi(\theta,S) \, d\theta
\]

rewriting (9) leads to:

\[
T(y,\theta,S) = T_2 \exp\left(\frac{y}{2} \left(\frac{S}{\alpha}\right)^{1/2}\right) - (T_2 - T_1) \int_{T_1}^\infty \text{ erfc}\left(\frac{1}{2}y (\alpha \theta)^{-1/2}\right) S \exp(-\theta S) \, d\theta + (T_2 - T_1) \text{ erfc}\left(\frac{1}{2}y (\alpha t)^{-1/2}\right) \exp(-\theta t)
\]

The mean flux at \( t > 0 \) can found from (10) by calculating

\[
\bar{q}_w(t,S) = \dot{\lambda} \frac{dT}{dy}\bigg|_{y=0}
\]

resulting in:

\[
\bar{q}_w(t,S) = \sqrt{\lambda \rho C_p} S T_2 - K \int_{T_1}^\infty \frac{1}{2} (\alpha \theta)^{-1/2} S \exp(-\theta S) \, d\theta + K \frac{1}{2} (\alpha t)^{-1/2} \exp(-tS)
\]

116
where $K = 2\lambda (\pi)^{-1/2} (T_2 - T_1)$. Rewriting gives:

$$\bar{q}_w(\chi,S) = \frac{\lambda \rho C_p S}{\chi} \left[ T_2 + (T_2 - T_1) \left[ -\text{erfc}(\chi) + \frac{\exp(-\chi)}{\sqrt{\pi \chi}} \right] \right]$$

(5.11)

in which $\chi = tS = \frac{t}{\tau}$ is the dimensionless time parameter.

Finally, using equation (7), the Nusselt number $Nu = \frac{hD}{\lambda}$ with $h = \bar{q}_w/T_2$ can be written as:

$$Nu(\chi) = Nu_0 \left[ 1 + \frac{T_2 - T_1}{T_2} \left[ -\text{erfc}(\chi) + \frac{\exp(-\chi)}{\sqrt{\pi \chi}} \right] \right]$$

(5.12)

Figure 5.2: $Nu$ as a function of $\chi = t/\tau$ for different values of the parameter $C = T_1/T_2$

Studying the asymptotic behavior of the transient Nusselt number (12) it is found that for large values of the time parameter $\chi$ the
Nusselt number converges to the stationary value.

\[
\lim_{\chi \to \infty} \text{Nu}(\chi) = \text{Nu}_s
\]  

(5.13a)

and for small values of \(\chi\) :

\[
\lim_{\chi \downarrow 0} \text{Nu}(\chi) = \text{Nu}_s \lim_{\chi \downarrow 0} (\pi \chi)^{-1/2} \quad (T_1 = 0)
\]  

(5.13b)

For small values of \(\chi\) (and \(T_1 = 0\)) the Nusselt number equals the penetration solution in which \(\text{Nu} \sim \chi^{-1/2}\) [Carlslaw and Jaeger (1959)].

In figure 5.2 the Nusselt number as a function of \(\chi\) is shown for different wall temperature ratios \(C = T_1 / T_2\). For comparison reasons also the stationary solution (\(C=1\)) and penetration solution for \(T_1 = 0\) are included in the figure.

5.3.2 An arbitrary wall temperature

Using the same superposition principle as in paragraph 5.3.1 the heat transfer to a constant property fluid can be found in case of an arbitrarily varying wall temperature. For \(t < 0\) the wall temperature is assumed to be constant and equal to \(T_1\) (see figure 5.3).

\[
\begin{array}{c}
\text{T}_w \\
\hline
\text{T}_1 \\
0 \\
\hline
\text{t}
\end{array}
\]

Figure 5.3 : Wall temperature as a function of time.

- Arbitrary function

For this case the eddies at the wall at \(t > 0\) are again divided into two groups like in the case of the step in the wall temperature. The solution for the eddies with an age \(\theta > t\) is found by using the superposition principle of Duhamel :

\[
T(y, \theta, t) = T_w(t_0) \operatorname{erfc} \left[ \frac{1}{2} y (\alpha \theta)^{-1/2} \right]
\]

(5.14)

\[
+ \int_0^\theta \frac{dT_w(t_0 + \theta)}{d\theta} \operatorname{erfc} \left[ \frac{1}{2} y [\alpha (\theta - \theta')]^{-1/2} \right] d\theta'
\]

where \(t_0\) is the time at the moment of inrush of the eddy. In terms of the time \(t\) and the age \(\theta\) equation (14) reads :

118
\[ T(y,\theta,t) = T_w(t-\theta) \text{erfc} \left[ \frac{1}{2} y (\alpha \theta)^{-1/2} \right] \]
\[ + \int_0^\theta \frac{dT_w(t-t')}{d(t-t')} \text{erfc} \left[ \frac{1}{2} y (\alpha t')^{-1/2} \right] dt' \]

In case of a constant wall temperature, \( \frac{dT_w}{dt} = 0 \), the stationary solution (5) results.

By applying the averaging procedure (3) the mean temperature at all times \( t \) is found:

\[ T(y,t,S) = \int_0^\infty T_w(t-\theta) \text{erfc} \left[ \frac{1}{2} y (\alpha \theta)^{-1/2} \right] \phi(\theta,S) d\theta \]
\[ + \int_0^\infty \left[ \int_0^\theta \frac{dT_w(t-t')}{d(t-t')} \text{erfc} \left[ \frac{1}{2} y (\alpha t')^{-1/2} \right] dt' \right] \phi(\theta,S) d\theta \]

Changing the order of integration in the second term of (16) leads to:

\[ T(y,t,S) = \int_0^\infty H(t-\theta) \text{erfc} \left[ \frac{1}{2} y (\alpha \theta)^{-1/2} \right] S \exp(-\theta S) d\theta \]

with the function \( H \) defined by \( H(t) = T_w(t) + \frac{1}{S} \frac{dT_w(t)}{dt} \). For \( t < 0 \) where \( T_w = T_1 \) and \( \frac{dT_w}{dt} = 0 \), (17) reduces to the stationary solution (5).

By defining new variables \( \chi = tS \) and \( \xi = \theta S \), and the function \( H(\chi) = H_w(\chi) + \hat{T}_w(\chi) = T_w(t) + \frac{1}{S} \frac{dT_w(t)}{dt} \) the mean heat flux can be written as:

\[ \bar{q}_w(\chi,S) = \int_0^\infty \hat{H}(\chi-\xi) (\pi)^{-1/2} \xi^{-1/2} \exp(-\xi) d\xi \]

The transient heat transfer coefficient \( h \) is defined by the momentary heat flux divided by the momentary temperature difference between the wall and the bulk of the flow:

119
\[ h = \frac{\tilde{q}_w(\chi)}{T_w(\chi)} \quad T_w(\chi) \neq 0 \]

For the Nusselt number this leads to:

\[ \frac{\text{Nu}(\chi)}{\text{Nu}_s} = \frac{T_1}{T_w(\chi)} + \frac{1}{T_w(\chi)} \int_0^\infty \tilde{h}(\chi-\xi) (\pi \xi)^{-1/2} \exp(-\xi) \, d\xi \quad (5.19) \]

in which \( \tilde{h}(\chi) = \hat{h}(\chi) - T_1 \).

In general equation (19) has to be solved numerically in case of a given arbitrary wall temperature.

5.4 Variable fluid properties

In case of a variable property fluid, in which \( \rho C_p \) and \( \lambda \) depend on the fluid temperature, equation (1) can in general not be solved analytically. In this case an approximate solution using an integral method [Thomas et al. (1974), Ivlev et al. (1979)] or a numerical solution method has to be used. In this thesis a numerical solution method will be used.

In this paragraph we will assume that the solution \( \tau(y,\theta,t) \) of the instationary, variable property heat transfer problem (1) with conditions (2) is known. The mean heat flux and the heat transfer coefficient can then be determined from:

\[ q_w(\theta,t) = -\lambda \left. \frac{d\tau(y,\theta,t)}{dy} \right|_{y=0} \quad (5.20) \]

\[ \bar{q}_w(t,S) = \int_0^\infty q_w(\theta,t) \, S \exp(-\theta S) \, d\theta \quad (5.21) \]

and

\[ h(t,S) = \frac{\bar{q}_w(t,S)}{T_w(t)-T_b} \quad (5.22) \]

Notice that the problem is no longer linear because of the temperature dependency of the fluid properties. This means that the superposition principle of Duhamel is not applicable in this case.
5.4.1 A step in the wall temperature

In case of a sudden temperature step at the wall from \( T_1 \) to \( T_2 \) at \( t=0 \), assuming \( T_1 \) to be equal to \( T_b \), the solution (21) for the heat transfer can be simplified. First we recognize that all eddies with an age \( \theta < t \) have the same history, starting at a uniform temperature \( T_b \) and residing near a wall of constant temperature \( T_2 \). This means that the heat transfer to an eddy with age \( \theta < t \) is independent on the time \( t : q_w(\theta, t) = q_w(\theta) \). Secondly we recognize that the heat transfer to an eddy with an age \( \theta > t \) equals zero at times \( t < 0 \). This means that the heat transfer to such an eddy depends solely on the time \( t : q_w(\theta=t, t) = q_w(t) \). Substituting this idea into equation (21) the following equation results for the mean heat flux at time \( t \):

\[
\overline{q}_w(t, S) = q_w(t) \exp(-tS) + \int_0^t q_w(\theta) S \exp(-\theta S) \, d\theta \quad (5.23)
\]

In (23) only the heat flux \( q_w(\theta) \) to one eddy with an age in the range \( 0 < \theta < t \), has to be determined to be able to find the instationary mean heat flux at all times from 0 to \( t \). Numerically this is done by solving the temperature field from equation (1) at discrete time levels \( t_k, k=1,2,\ldots \) using helium properties. The heat flux to the eddy is found from equation (20). \( \overline{q} \) results from numerical integration of equation (23).

In figure 5.4 the Nusselt number \( \text{Nu} = \frac{hD}{\lambda} \) computed from the transient variable property surface renewal model is compared with the constant property solution (12) for \( T_2 \approx T_1 = T_b \) and an ambient pressure of 3 bar. In this case the fluid property variations are small and the two solutions are nearly identical as was expected. In the figure also the penetration solution (dotted line) is shown for comparison.

The stationary nusselt number \( \text{Nu}_s \) is defined by equation (7). The value of \( \tau^- \) is computed from equation (2.114):

\[
\tau^- = \tau \left( \frac{u^*}{v} \right)^2 = 243 \quad (2.114)
\]

which leads to:
Figure 5.4: Heat flux computed with the transient surface renewal model for $T_w \approx T_b < T_{pc}$

\[ \tau = 243 \frac{2}{f} D^2 \nu^{-1} Re^{-2} \quad (5.24) \]

The friction factor $f$ is computed from the expression for classical fluids

\[ f = 0.076 \, Re^{-0.25} \quad (5.25) \]

which, according to measurements of Junghans [1980], is also valid for supercritical helium.

To investigate the influence of the fluid properties the variable and constant property solutions are compared in case of a ambient pressure of 3 bar and varying wall and bulk temperatures. The pseudo critical temperature at this pressure is about 5.7 K. The results are shown in figure 5.5. A large deterioration of the heat transfer coefficient in the variable property case is found when $T_w > T_{pc}$ and $T_b \leq T_{pc}$. When also $T_b$ is raised above the pseudo critical value heat
transfer recovers again. This result is in qualitative agreement with
the experiments of Giarratano and Jones [1975] on heat transfer to
supercritical helium (paragraph 2.3.1).

\[ P = 1.8 \]
\[ T_b = 5.7 \]
\[ T_w = 4.2 \]
\[ \tau_m = 15.1 \]
\[ N_L = 225.4 \]

\[ P = 1.8 \]
\[ T_b = 8.2 \]
\[ T_w = 4.2 \]
\[ \tau_m = 15.1 \]
\[ N_L = 225.4 \]

\[ a : T_b < T_w < T_{pc} \]

\[ b : T_b < T_{pc} < T_w \]

\[ P = 1.8 \]
\[ T_b = 8.2 \]
\[ T_w = 5.7 \]
\[ \tau_m = 12.1 \]
\[ N_L = 627.0 \]

\[ P = 1.8 \]
\[ T_b = 8.2 \]
\[ T_w = 6.8 \]
\[ \tau_m = 6.0 \]
\[ N_L = 315.4 \]

\[ c : T_{pc} = T_b < T_w \]

\[ d : T_{pc} < T_b < T_w \]

Figure 5.5: Heat flux computed with the transient surface
renewal model for different values of $T_b$ and $T_w$.

123
5.4.2 An arbitrary wall temperature

In case the wall temperature is assumed to be constant up to \( t = 0 \) and varies arbitrarily for time \( t > 0 \) (see figure 5.3), equation (21) reduces to:

\[
\bar{q}_w(t,S) = q_w(t,t) \exp(-tS) + \int_0^1 q_w(\theta,t) \ S \exp(-\theta S) \ d\theta
\]  \hspace{1cm} (5.26)

In (26) the heat transfer to an eddy of age \( \theta \) at time \( t \) depends on the time \( t \). This means that at time \( t \) the heat flux to eddies of all ages \( 0 < \theta < t \) has to be computed at discrete time levels. This is a much larger task than solving equation (23) for a step in the wall temperature.

In order to test the model some experiments of Bloem [1986] (paragraph 2.3.2.2) are simulated. The wall temperature is computed from a zero dimensional temperature model

\[
\rho C_p \frac{\partial T_w}{\partial t} = Q_{in} - Q_{He}
\]  \hspace{1cm} (5.27)

\( Q_{in} \) is the constant heat flux applied to the copper tube in the experiments. The heat flux \( Q_{He} \) to the helium for an estimated value of the wall temperature \( T_w \) is computed by a numerical integration of equation (26). The fluxes \( q_w \) to the individual eddies in (26) are found from the computed temperature field as described in the previous paragraph. When \( Q_{He} \) for the guessed value of \( T_w \) is computed a new value for \( T_w \) results from equation (27). In general this value will differ from the previously guessed value. An iterative method has to be used to find the matching values of the wall temperature and the heat flux.

Table 5.1 : Computed renewal times (in ms)

<table>
<thead>
<tr>
<th>( \dot{m} ) g/s</th>
<th>Re</th>
<th>( \tau ) (24)</th>
<th>( \tau_{num} )</th>
<th>Re_{num}</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>1.7 ( 10^4 )</td>
<td>290</td>
<td>15</td>
<td>9.3 ( 10^4 )</td>
</tr>
<tr>
<td>1.0</td>
<td>5.5 ( 10^4 )</td>
<td>38</td>
<td>10</td>
<td>1.2 ( 10^5 )</td>
</tr>
<tr>
<td>3.0</td>
<td>1.7 ( 10^5 )</td>
<td>5</td>
<td>2.5</td>
<td>2.5 ( 10^5 )</td>
</tr>
</tbody>
</table>

In figure 5.6 the resulting temperature and heat transfer curves
for varying values of the renewal rate $\tau$ are shown. The pressure used is 6 bar and the bulk temperature is 4.2 K. The results are compared to the curves of Bloem for a mass flow of 0.3, 1.0 and 3.0 g/s respectively. For a renewal rate of 15, 10 and 2.5 milliseconds respectively the surface renewal curves are in good agreement with the curves of Bloem. However, the renewal rate for these mass flows computed from equation (24) are found to be much larger (see table 5.1).

Figure 5.6 : Transient surface renewal results for different renewal rates. - Varying wall temperature
Especially for low mass flows the agreement is bad. However, in these cases the influence of a thermally induced flow is large. In the surface renewal method this influence is not accounted for. When we model the induced flow by an increased stationary Reynolds number the numerical values of the renewal rate are correct at a Reynolds number equal to $Re_{num}$ as listed in the last column of table 5.1. In all cases the the induced Reynolds number, which is the difference between $Re_{num}$ and $Re$, is about equal to $8 \times 10^4$. A numerical simulation of the 0.3 g/s case with the full two dimensional model learned that the induced Reynolds number was however much lower, $\Delta Re = 10^4$. The description of the influence of the transient flow phenomena on the renewal rate by an increased stationary Reynolds number implicitly assumes that the velocity profile is fully developed. The induced flow, however, gives a nearly uniform velocity change in the whole cross section of the tube except very close to the wall. The influence of the transient effects on the renewal rate will thus be much larger than estimated above from the quasi stationary approach. The numerically found renewal rates $\tau_{num}$ in table 5.1 are thus probably realistic.

The derived transient, variable property, surface renewal model is found to give a qualitatively correct description of the heat transfer to supercritical helium. Both the phenomena after a sudden step in the wall temperature as the experiments of Bloem are described well. However, transient flow phenomena like a thermally induced flow, which has been found to have a large impact on the heat transfer to the helium flow inside the cooling channel of a superconductor, can not be predicted with this model. Although the model is promising it can in the present form not yet be used to predict the stability of a superconductor.
6 Numerical results

In this chapter the numerical results computed with the two dimensional simulation model described in chapter three will be presented. At first some simple flow situations will be computed in paragraph 6.1 in order to validate the computer code. In paragraph 6.2 transient effects will be studied. The propagation velocity of the heat induced pressure wave in the helium flow will be compared to the thermodynamic speed of sound. Transient heat transfer will be tested by simulating the experiments of Bloem [1986] as described in paragraph 2.3.2. In paragraph 6.3 the verified two dimensional flow model coupled to the one dimensional copper/superconductor model, as developed by Cornelissen [1984], will be used to study the stability of the SULTAN conductor (paragraph 2.3.2).

6.1 Stationary computations

In this paragraph stationary flow situations will be simulated starting with a simple laminar flow of a constant property fluid in paragraph 6.1.1. Two different flow types will be considered:

A hydrodynamic developed flow
A hydrodynamic developing flow

The heat transfer to these flow types will be studied under developed and developing thermodynamic conditions.

In paragraph 6.1.2 turbulent flows under the same conditions are considered. Variable property effects are tested in paragraph 6.1.3 by simulating the experiments of Giarratano and Jones [1975].

6.1.1 Laminar constant property flow

The simulations in this paragraph are performed on a tube with a diameter D of 6 mm. The fluid properties are taken from the table for supercritical helium at a pressure P = 12.5 bar and an enthalpy H = 15 kJ/kg (T = 4.226 K). The properties, which are assumed not to vary with pressure and enthalpy during the simulations, are gathered in table 6.1.

127
Table 6.1: Fluid properties in laminar simulations

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>$\rho$</td>
<td>154.9</td>
<td>kg/m$^3$</td>
</tr>
<tr>
<td>Viscosity</td>
<td>$\mu$</td>
<td>5.24 $10^{-6}$</td>
<td>Ns/m$^2$</td>
</tr>
<tr>
<td>Thermal conductivity</td>
<td>$\lambda$</td>
<td>24.19 $10^{-3}$</td>
<td>W/mK</td>
</tr>
<tr>
<td>Specific heat</td>
<td>$C_p$</td>
<td>3002</td>
<td>J/m$^3$K</td>
</tr>
<tr>
<td>Prandtl number</td>
<td>Pr</td>
<td>0.651</td>
<td>-</td>
</tr>
</tbody>
</table>

6.1.1.1 Hydrodynamics

In this section the velocity field and the pressure gradient in case of a developed and a developing flow will be studied. Both cases will be discussed separately below.

* Developed flow

In a developed flow the velocity profile is the same on every cross section of the tube ($\frac{\partial u}{\partial z} = 0$). Because the fluid properties are assumed to be constant the continuity equation dictates that the radial velocity must be equal to zero. In case of a laminar flow the axial velocity profile in a cross section can be found analytically. The well known result is a parabolic Poiseuille profile. The Fanning friction factor $f$, defined from:

$$\frac{dP}{dx} = 4f \frac{0.5 \rho \langle u \rangle^2}{D}$$

(6.1)

is, for this flow type equal to:

$$4f = 64 \text{ Re}^{-1}$$

(6.2)

From (1) and (2) the pressure drop $\Delta P$ over a length $L$ of the tube can be written as

$$\Delta P = \frac{32 \mu^2}{\rho D^3} \text{Re}L = C_1 \text{Re}L$$

(6.3)

Using the fluid properties from table 6.1 the constant $C_1$ is found to
be equal to $2.20 \times 10^{-5}$ N/m$^3$.

Numerical simulations are performed varying the Reynolds number of the flow in the range $Re \in (10,1000)$. The axial mesh size $\Delta z$ is set equal to $10^{-2}$ m and in radial direction the Schnurr grid defined in equation (3.1) is used taking the number of grid points equal to thirteen ($N_r=13$). In all computed situations the resulting velocity profile is found to fit the parabolic Poisseeulle profile very well. For the pressure drop the constant $C_1$ in (3), computed numerically for various Reynolds numbers, equals $2.22 \times 10^{-5}$ N/m$^3$. This result differs less than 1% from the theoretical value. Decreasing the number of radial grid points to 8 and increasing the axial mesh size by a factor 10 did not change the results.

![Graphs](image)

**a : Re = 67**

**b : Re = 670**

**Figure 6.1 :** Axial velocity profiles for a laminar developing flow. (Computed by Cornelissen [1984])

- Hydrodynamic entrance region

When the velocity at the inlet of the tube is assumed to be uniform the velocity profile in the tube will develop gradually to the developed Poisseeulle profile. The distance from the inlet of the tube to the location $z$, at which the profile has reached 99% of the developed value, is called the entrance length $L$. An analytical expression for the entrance length has been given by Langhaar [Rohsenow and Hartnett(1973)] :
During the development of the velocity profile Cornelissen [1984] found that, especially at low Reynolds numbers, a maximum in the velocity profile occurred which was not situated at the center line of the tube (see figure 6.1). Measurements by Sparrow et.al. [1967], however, indicate that the maximum of the velocity profile in the entrance region of a circular tube is always located on the center line (r=0). Also in semi-analytical computations of Langhaar [Rohsenow and Hartnett (1973)], which deviate less than 3% from the experimental results of Sparrow et.al., the maximum was always located at r=0.

![Diagram of normalized axial velocity profile](image)

**Figure 6.2:** Axial velocity profiles for a laminar developing flow computed at Re = 100 and Re = 1000.

For two different flow Reynolds numbers, Re=100 and Re=1000, we have calculated the developing velocity profile. When the velocity is scaled by the mean velocity \( <u> \) over the cross section defined by:
\[ <u> = \frac{\int u \, 2\pi r \, dr}{\int 2\pi r \, dr} \]  

(6.5)

identical velocity profiles for the same values of the parameter \( \frac{z}{Re \cdot D} \) result for both Reynolds numbers. In figure 6.2 the computed profiles are shown. No maxima occur outside the center line which is in agreement with the results of Langhaar. The deviating results of Cornelissen, which were computed with basically the same numerical code as used in our simulations, are found to be a consequence of the boundary conditions for the radial velocity, which had not been implemented correctly in the computer code, and do not have a physical meaning.

The pressure drop beyond the entrance length, \( z > L \), becomes equal to the developed value given by equation (3). In figure 6.3 the computed pressure drop along the tube is shown. The dotted line denotes the developed pressure drop. It can be seen that the value for the entrance length, given in equation (4) and represented by the dashed line, is a good estimate.

![Pressure drop graph](image)

Figure 6.3: Pressure drop in a laminar developing flow.
6.1.1.2 Thermal conditions

In this section the heat transfer to the above described laminar flow fields will be studied. In studying the heat transfer to a developed flow we can distinguish between thermally developed and thermally developing conditions. In case of a developing flow only thermally developing conditions apply. The different situations are described below.

* Thermally developed conditions

A thermally developed flow is defined as a flow in which the reduced temperature is independent of the axial coordinate \( z \)

\[
\frac{\partial}{\partial z} \left( \frac{T_w - T}{T_w - <T>} \right) = 0
\]  

(6.6)

in which \( T_w \) is the wall temperature. The temperature \( <T> \), the mean cup temperature in a cross section, is defined by:

\[
<T> = \frac{\int uT \ 2\pi r dr}{\int u \ 2\pi r dr}
\]  

(6.7)

in which the integration is performed over the cross section.

Table 6.2 : Nusselt numbers in developed laminar flow under thermally developed conditions

<table>
<thead>
<tr>
<th>Wall condition</th>
<th>Numerical</th>
<th>Analytical</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant temperature</td>
<td>3.62</td>
<td>3.656</td>
<td>-1</td>
</tr>
<tr>
<td>Constant flux</td>
<td>4.357</td>
<td>4.364</td>
<td>-0.1</td>
</tr>
</tbody>
</table>

Two different types of thermal conditions in developed flows, depending on the wall conditions, exist. In the first case the wall temperature is constant (\( \frac{\partial T}{\partial z} = 0 \)). In the second situation the heat flux from the wall to the flow is constant (\( \frac{\partial \phi}{\partial z} = 0 \)). For both cases the heat transfer coefficient \( h \), expressed in the Nusselt number
\[ \text{Nu} = \frac{hD}{\lambda}, \] is analytically found to be independent from the Reynolds number of the flow \cite{Rohsenow_and_Hartnett_1973}. Numerically the same developed flow situations as described in paragraph 6.1.1.1 are simulated for both wall conditions. The results are in good agreement with the analytical values, see table 6.2.

- Thermally developing conditions

Starting with a uniform temperature profile at the inlet of the tube the heat transfer to the flow will change with the distance from the inlet. Rohsenow and Hartnett \cite{Rohsenow_and_Hartnett_1973} have presented analytical solutions for the Nusselt number \( \text{Nu}_z \) at location \( z \) for both a developing and a developed flow. The results are given in terms of a dimensionless distance \( X \) defined by

\[ X = \frac{2z}{\text{Pr} \cdot \text{Re} \cdot D} \quad (6.8) \]

In both cases a constant heat flux from the wall to the laminar flow is assumed. Numerically these situations are simulated using 200 grid points situated at a distance \( \Delta z^+ = 10^3 \). In case of a Reynolds number equal to 100, and using table 6.1 this leads to a mesh size equal to \( \Delta z = 2.2 \times 10^4 \) m.

\begin{center}
\begin{tabular}{c}

\textbf{Figure 6.4}:

Nusselt number in a laminar developing flow.

\end{tabular}
\end{center}
The results are shown in figure 6.4. The dotted and the solid line are the numerical solutions for the developed and the developing flow case respectively. The analytical values from Rohsenow and Hartnett are included in the figure for comparison. The dashed line is the analytical fully developed Nusselt number for a constant heat flux, as given in table 6.2.

In the developing flow case the error is less than 2% on all locations. The overall agreement between our results and the literature data in the developed flow case is also good. Only at small values of \( X \) the error can get as large as 17% (\( X = 2 \times 10^{-3} \), second grid point from the inlet). Further in the tube, however, the error quickly reduces until at \( X = 10^{-1} \) (grid point 100) the difference between the numerical and analytical solution is only 0.7% . In paragraph 6.1.2.2 we will see that for a turbulent flow, in which we are basically interested, our results compare even better with literature data. For the heat transfer to a developed turbulent flow the difference with experiments is less than 2% on all locations. The thermal entrance region of a developed flow is thus described adequately by our model.

6.1.2 Turbulent constant property flow

A similar test series as described in the previous paragraph for a laminar flow situation is studied here for a turbulent flow. Unlike in a laminar flow the heat transfer to a turbulent flow depends on the Prandtl number of the fluid. In order to find the Prandtl number dependency of the heat transfer to a turbulent flow, three different situations are simulated. The fluid properties are again taken from the supercritical helium table. The Prandtl number variation for the fluid is hereby limited to values within \( 0.5 < \text{Pr} < 5 \). The values of fluid properties, as well as the Prandtl numbers are gathered in table 6.3. Also the pressure and enthalpy corresponding with the properties are included. In the simulations the properties are assumed to remain constant.
Table 6.3: Fluid properties in turbulent simulations

<table>
<thead>
<tr>
<th>Serial number</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prandtl nr.</td>
<td>0.70</td>
<td>1.00</td>
<td>3.00</td>
</tr>
<tr>
<td>Pressure</td>
<td>12.5</td>
<td>12.5</td>
<td>3.0</td>
</tr>
<tr>
<td>Enthalpy</td>
<td>23.5</td>
<td>49.0</td>
<td>29.0</td>
</tr>
<tr>
<td>Density</td>
<td>132.</td>
<td>73.7</td>
<td>50.8</td>
</tr>
<tr>
<td>Viscosity</td>
<td>4.05</td>
<td>3.29</td>
<td>0.68</td>
</tr>
<tr>
<td>Thermal cond.</td>
<td>27.4</td>
<td>24.7</td>
<td>14.6</td>
</tr>
<tr>
<td>Specific heat</td>
<td>4.73</td>
<td>7.50</td>
<td>2.14</td>
</tr>
</tbody>
</table>

The velocity field and the pressure drop in the tube are independent of the Prandtl number. For the investigation of the hydrodynamics only one set of properties (serial number 1) will be used.

6.1.2.1 Hydrodynamics

For a turbulent flow no analytical solutions for the velocity field and the pressure drop can be found. To validate the turbulent flow simulations the results have to be compared to experimental correlations found in literature. Only the developed flow situation will be considered here.

On the developed velocity profile in a turbulent flow a lot of experimental data is available. These data can be correlated by a linear profile very close to the wall and a logarithmic (law of the wall) profile further from the wall. In dimensionless wall variables $y^+$ and $u^+$, which are defined in equation (2.66) the velocity profile can be written as:

\[ u^+ = y^+ \quad \text{for} \quad y^+ < y_1^+ \]  
\[ u^+ = \frac{1}{\kappa} \ln(y^+) + C \quad \text{for} \quad y^+ > y_1^+ \]  

(6.10a)  
(6.10b)

Different values of $\kappa$, $C$ and $y_1^+$ in (10) are known from literature. Coles [1955] suggested
\[ \kappa = 0.41 \quad C = 5.1 \quad y_1^+ = 11.2 \]  
(6.11)

Von Karman [1939] splitted the profile in three parts. Equation (10a) he applied for \( y^+ < 5 \) \( (y_1^+ = 5) \). For higher values of \( y^+ \) he defined two logarithmic profiles

\[ \kappa = 0.40 \quad C = 5.5 \quad \text{for} \quad y^+ > 30 \]  
(6.12)

\[ \kappa = 0.20 \quad C = -3.05 \quad \text{for} \quad 5 < y^+ < 30 \]

In our numerical model we have used the mixing length model of Prandtl (see paragraph 2.4.1.3) in which the mixing length is corrected by the van Driest correction factor \( C_D \) defined in equation (2.66) as:

\[ C_D = \exp \left[ -\frac{y^+}{L_0} \right] \]  
(2.66)

The dimensionless velocity profile suggested by van Driest using the correction factor \( C_D \) can be found by integrating equation (13) [Bejan(1984)]:

\[ \frac{du^+}{dy^+} = \frac{2}{1 + \left[ 1 + 4(\kappa y^+)^2 \right] (1-C_D)^2} \]  
(6.13)

The difference between the resulting profiles computed from equation (13) and the many variations of equation (12) suggested by various workers have been shown to be small [Sideman and Pinczewsiki(1975)].

In case of a tube flow a correction of the logarithmic profile like suggested by Reichardt [1951] is needed in the bulk of the flow:

\[ u^+ = \frac{1}{\kappa} \ln(y^+) + C + \ln \left[ \frac{3(1-\xi)}{2(1-2\xi^2)} \right] \]  
(6.14)

In this empirical correlation \( \kappa = 0.4 \) and \( C = 5.5 \) has been used. The parameter \( \xi \) is the radial position divided by the radius of the tube : \( \xi = r/R \). The last term in (14) is small in the wall region and only gives an influential contribution in the bulk of the fluid near the center of the tube.

The pressure drop in the tube can be described using the friction
factor $f$ defined in equation (1). For a developed turbulent flow in a smooth tube the Von Karman-Nikuradse relation applies [Rohsenow and Hartnett(1973)]:

$$(4f)^{1/2} = -0.8 + 0.87 \ln \left( \frac{\text{Re}}{(4f)^{1/2}} \right) \quad (6.15)$$

Well known approximations of this relation are:

$$4f = 0.316 \text{ Re}^{-0.25} \quad 5 \times 10^3 < \text{Re} < 3 \times 10^4 \quad (6.16a)$$

$$4f = 0.184 \text{ Re}^{-0.20} \quad 3 \times 10^4 < \text{Re} < 10^6 \quad (6.16b)$$

The numerical velocity profiles and friction factors are compared with the above described correlations for five different Reynolds number flows. In the computer code the wall friction is computed from the velocity gradient near the wall using a linear interpolation. This is only valid if the last grid point near the wall is situated in the laminar sublayer $y^+ < 5$. The grid used in the computations is the Schnurr grid as defined in equation (3.1). The number of grid points $N_r$ and the parameter $C_2$ in (3.1) used for the various Reynolds numbers are gathered in table 6.4. The value of $y^+$, computed using a guessed value for the friction factor from equation (16), is taken equal to 2.5. For this value the best friction factor results were found.

Table 6.4 : Grid parameters at simulated Reynolds numbers

<table>
<thead>
<tr>
<th>Serial nr.</th>
<th>Re</th>
<th>Nr</th>
<th>$C_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$10^4$</td>
<td>10</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>$3 \times 10^4$</td>
<td>13</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>$10^5$</td>
<td>16</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>$3 \times 10^5$</td>
<td>19</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>$10^6$</td>
<td>15</td>
<td>3</td>
</tr>
</tbody>
</table>

In figure 6.5 the computed velocity profiles for the five Reynolds number flows are shown. For comparison the correlation of Van Driest (13) is included in the figure (solid line). For all Reynolds numbers this correlation is well predicted by the numerical results. For
Re = 10^5 the numerical result is also compared to the Bejan correlation (14) (dotted line). Although the Bejan correlation is always somewhat higher than our result (C = 5.0 would fit our results better) the increase of u^+ in the bulk of the flow, found in our simulation, is in good agreement with the description of Bejan.

![Graph](image)

Figure 6.5 : Dimensionless velocity u^+ as a function of the dimensionless distance y^+ to the wall in a fully developed flow.

The computed friction factors are compared with the experimental correlations (15) and (16). For the first two Reynolds numbers, Re ≤ 3 10^4, the computed friction factors agree within 2.5% with approximation (16a). For the other flows the agreement with approximation (16b) is within 3% . Comparing the numerical results with the Von Karman-Nikuradse correlation (15) the agreement is found to be even better. The difference is then less than 1.5% for all Reynolds numbers except Re = 10^4 where the difference is some 5% . In general it can be concluded that the friction factor can be computed well with the used grid, where the last point near the wall is within y^+ = 2.5.
6.1.2.2 Thermal conditions

- Thermally developed conditions

The heat transfer to a developed turbulent flow under thermally developed conditions is in general described by a relation between the Nusselt, Reynolds and Prandtl number like:

\[ \text{Nu} = C_N \text{Re}^n \text{Pr}^m \]  \hspace{1cm} (6.17)

The constants \( C_N, n \) and \( m \) can vary. Unlike in the laminar flow situations no distinction between a constant temperature or constant heat flux condition at the wall has to be made for turbulent flows. For Prandtl numbers over 0.5, as studied here, the difference between the corresponding Nusselt numbers is less than 5%. For Reynolds numbers larger than \( 3 \times 10^5 \) the Nusselt numbers in case of a constant wall temperature or a constant wall heat flux are identical.

Assuming that heat and momentum transfer are completely equivalent when the Prandtl number is equal to one, which means that the boundary layers thicknesses for both momentum and heat are identical, relation (17) can be written as

\[ \text{Nu} = \frac{f}{2} \text{Re} \]  \hspace{1cm} (6.18)

Relation (18) is known as the Reynolds analogy and is valid only for \( \text{Re} > 3 \times 10^4 \) (Bird(1960)). This relation has been extended to include the effect of Prandtl number variation. The resulting relation is known as the Chilton-Colburn relation and reads:

\[ \text{Nu} = \frac{f}{2} \text{Re} \text{Pr}^{1/3} \]  \hspace{1cm} (6.19)

Using approximation (16b) for the friction factor \( f \) the well known Sieder-Tate equation results:

\[ \text{Nu} = 0.023 \text{Re}^{0.8} \text{Pr}^{1/3} \]  \hspace{1cm} (6.20)
Many other variations of the general equation (17) are known. The power \( n \) for the Reynolds number is usually equal to 0.8 like in relation (20). The power \( m \) for the Prandtl number is less well established. Besides the value 1/3, equation (19) and (20), also the value 0.4 (the Dittus-Boelter relation) is often used. In Rohsenow and Hartnett [1973] numerical computations of the Nusselt number over a large Prandtl and Reynolds number range, performed by Kays and Leung, are tabulated which are believed to be accurate within a few percent. For Prandtl numbers near unity these values are better correlated by a Prandtl number power \( m = 0.6 \):

\[
\text{Nu} = 0.022 \, \text{Re}^{0.8} \, \text{Pr}^{0.6}
\]  

(6.21)

In figure 6.6 the computed Nusselt numbers as a function of the Reynolds number are shown for the three Prandtl numbers given in table 6.3. The simulated Reynolds are the same as in the previous paragraph (see table 6.4). Our results (solid points) are almost identical to the tabulated values of Kays and Leung and are thus also believed to be accurate within a few percent.

![Figure 6.6: Nusselt number as a function of the Reynolds number in a fully developed flow.](image_url)
Thermally developing conditions

For both a developed and a developing flow with a Reynolds number equal to $10^5$ and a Prandtl number equal to 0.7, Rohsenow and Hartnett [1973] have graphically presented the heat transfer in the thermal entry length of the tube. In figure 6.7 the Nusselt number divided by the fully developed value as computed above, is shown for the developed flow case. The developing flow case is shown in figure 6.8. In both figures the open points connected by the dotted line are the values extracted from the plots given by Rohsenow and Hartnett. Our numerical results (solid line) are in good agreement with these data.
In figure 6.7 the maximum difference is about 2 percent while in figure 6.8 the difference is not larger 4 percent.

The numerical results are obtained using two different methods for solving the radial velocity. In the first method the radial velocity is computed from the continuity equation (3.2) as described in paragraph 3.8. In the second method the transport equation (3.4) is solved using the same technique as described in paragraph 3.5.2 for the axial velocity and the enthalpy. Both methods yield identical results. For this case the boundary layer assumptions made in paragraph 3.2 are thus found to be valid and both methods can be used. However, solving the continuity equation instead of the transport equation for the radial velocity has the advantage that the total computation time is reduced by some 15 percent.

6.1.3 Turbulent variable property flow

In this paragraph stationary heat transfer to a turbulent helium flow in case of varying fluid properties is tested by simulating the experiments of Giarratano and Jones [1975] as described in paragraph 2.3. The same cases as have already been simulated by Cornelissen [1984] are repeated here. In this way the influence of some changes in the computer code can be tested by comparing the results.

In all experiments of Giarratano and Jones the ambient pressure was equal to 2.5 bar. In the simulations we performed the temperature of the helium at the inlet of the tube is 4.2 K (enthalpy 9950 J/kg) what is well below the pseudo critical value of 5.4 K.

To correlate the results the Goldmann parameter $\Phi$, which can be derived from manipulations of the Dittus-Boelter correlation (2.30), was found to give a good description of the influences of the varying properties on the stationary heat transfer [Cornelissen(1984)]. The Goldmann parameter is defined by:

$$\Phi = \frac{qD^{0.2}}{G^{0.8}} \quad (6.22)$$

where $G = \tau h/A$ [kg/m²s] is the mass flow of the helium and $q$ [W/m²] the
applied heat flux. D is the diameter of the tube which is equal to 2.13 mm like in the experiments of Giarratano and Jones. Two different values of the Goldmann parameter, $\Phi = 10$ and $\Phi = 30$, are considered. For the mass flow the values, $G = 10.5$ and 111.2 are used. The corresponding heat fluxes follow from equation (22).

![Diagram showing Nusselt number as a function of bulk enthalpy from numerical results compared to the Giarratano and Yaskin correlation](image)

Figure 6.9: Nusselt number as a function of the bulk enthalpy from numerical results compared to the Giarratano and Yaskin correlation

Like Cornelissen we found from simulations that the results at a constant Goldmann parameter are nearly independent of the mass flow but vary strongly when the Goldmann parameter is changed. In figure 6.9 the numerical results for the two different simulated case ($\Phi = 10$ and $\Phi = 30$) are compared to the experimental correlations of Giarratano (2.27) and Yaskin (2.32). The Yaskin correlation, which has been found by correlating the heat transfer results near the pseudo critical temperature from various workers, fits our data far better than the correlation of Giarratano. Since the Yaskin correlation is known to give a good description of experimental heat transfer data (paragraph 2.3) our transient model is thought to give a good description of the variable property effects on stationary heat transfer.
6.2 Transient simulations

Two types of transient effects will be studied in this paragraph. In paragraph 6.2.1 the pressure wave, induced by the expansion of the helium in the heated region of the flow, will be studied. The propagation speed of the wave will be compared to the theoretical value which is the thermodynamic speed of sound. Transient heat transfer will be studied by simulating the experiments of Bloem [1986] in paragraph 6.2.2.

6.2.1 Induced flow

Characteristics of a supercritical helium flow at cryogenic temperatures are almost impossible to find experimentally. This means that it is hard to verify numerical simulations of these flows. The only feature of a heat induced flow in a cooling channel of a superconductor which can be verified by experiments in supercritical helium is the pressure wave. When one wants to verify the computational model for the flow and temperature fields a simpler experimental model has to be developed. In chapter 7 data on a transient airflow in the experimental model described in chapter 4 will be presented together with simulations, using the computation model adapted to that transient situation.

In this paragraph we will concentrate on the induced pressure wave. It is theoretically known that the propagation speed of the wave has to be equal to the thermodynamic speed of sound c. The value of c at a specified fluid pressure and enthalpy can be computed from the fluid properties as described in paragraph 3.7 using equation (3.50). Besides the propagation of the top of the wave we will numerically also compare the propagation of two other points of the computed wave with the speed of sound. The location of these points illustrated in figure 6.10 where the pressure wave as a function of the axial distance z is shown at a certain time \( t = t_1 \). The top \( z_{\text{Top}} \) is the point were, at this time, the pressure has its maximum value. The foot \( z_{\text{Foot}} \) is the position at which the pressure just starts to deviate from zero. Numerically a deviation of 1% of the value of the top is used. The
intersection of the extrapolated slope at the inflection point with the x-axis is the place of the third point called the slope, \( z_{\text{SI}} \), of the wave. At a time \( t = t_2 = t_1 + \Delta t \) the wave has propagated and the positions \( z_{\text{Top}}, z_{\text{SI}} \) and \( z_{\text{Foot}} \) will be changed. The propagation velocity of the top is computed from these data using:

\[
c_{\text{Top}} = \frac{z_{\text{Top}}(t_2) - z_{\text{Top}}(t_1)}{t_2 - t_1}
\] (6.23)

For the slope and the foot equation (23) applies if \( z_{\text{Top}} \) is substituted by \( z_{\text{SI}} \) and \( z_{\text{Foot}} \) respectively. In a similar way also a figure of the pressure on a fixed position as a function of time can be used to find the propagation velocities.

![Diagram](image)

**Figure 6.10**: Illustration for the computation of the positions \( z_{\text{Top}}, z_{\text{SI}}, \) and \( z_{\text{Foot}} \) of the wave.

In the simulations of the pressure experiments of Bloem [1986] as described in paragraph 2.3.2 also the strength of the pressure wave characterized by the maximum pressure rise can be compared to the experimental values given in table 2.5.
Figure 6.11: Computed pressure wave at different times as a function of the distance $z$ from the disturbance.
In the results of Cornelissen [1984] the slope of the induced pressure wave had a propagation speed which largely exceeded the theoretical value due to numerical dispersion (see figure 6.11a). In order to minimize this effect we simulated the same situation with a smaller time step. The result is shown in figure 6.11b. As can be seen the slope of the wave in this case is already much steeper which means that the propagation velocity of the slope is reduced. In order to improve the result further also a simulation with a different time discretisation scheme, the B3 scheme as described in chapter 3, is performed. From figure 6.11c it can be seen that this scheme indeed results in a steeper slope, but after the top of the wave now non realistic wiggles occur in the solution.

In the continuation of this paragraph two different test series will be studied. In the first series the influence of the numerical parameters \( \Delta z \) and \( \Delta t \) on the propagation speed will be studied using both the FI and the B3 time differencing scheme. In the second series the pressure experiments of Bloem will be simulated. Both the propagation velocity as the strength of the wave will be studied using the FI and B3 time differencing schemes as well as the EXQUISITE and HYBRID spatial differencing schemes.

At first we will study the influence of the numerical parameters \( \Delta z \) and \( \Delta t \) on the propagation speed of the pressure wave for both the B3 and FI time schemes. As a test situation we will study a 6 meter long part of the SULTAN conductor as defined in tables 2.1 and 2.3 (paragraph 2.3.1). The electrical transport current is set equal to zero. A length of 0.2 meter in the middle of the tube will be heated during 1 ms. The total inserted heat in the tube is 0.02 J. The ambient fluid pressure is set equal to 12.5 bar. Further a fluid inlet temperature of 4.2 K and a mass flow of 2 g/s is selected. The thermodynamic speed of sound at this temperature and pressure is 304 m/s.

For a mesh size of 0.1 m and a time step of 1 ms the propagation speed of the top \( c_{\text{top}} = 300 \) m/s when the FI scheme is used. The B3 scheme gives a 20 percent too low value \( c_{\text{top}} = 240 \) m/s. The
propagation speed of the slope, however, is much better for the B3 scheme than for the FI scheme (395 and 600 m/s respectively). As can be seen in table 6.5 the slope computed with the FI scheme propagates at almost twice the thermodynamic speed of sound. Because of the inaccuracy of the simulations the speed of the foot of the wave could not be determined. Varying the mesh size from 0.1 to 0.05 and 0.025 m has no influence on the computed propagation speed of the pressure wave. Decreasing the time step from 1 to 0.5, 0.25 and 0.1 ms, however, considerably improves all results (Table 6.5). The speed of the top almost equals the theoretical value for all cases. The error in the speed of the slope and the foot of the wave are reduced to respectively 7 and 12 percent at a time step of 0.1 ms using the B3 scheme. For the FI scheme the differences are larger, respectively 40 and 62 percent at a time step of 0.25 ms. At the 0.1 ms time step no convergence of the computation could be reached in the FI case.

To describe the propagation velocity of the pressure wave adequately the B3 scheme is superior over the FI scheme. For both schemes a reduction of the time step considerably improves the results. However, the prediction of the propagation speed of the top of the wave, which is the most important feature of the wave, is good for all simulated situations.

Table 6.5 : propagation speeds for different time differencing schemes at decreasing time step together with the deviation from the theoretical value (304 m/s).

<table>
<thead>
<tr>
<th>Δt</th>
<th></th>
<th>top</th>
<th>Slope</th>
<th></th>
<th>Foot</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>F1</td>
<td>B3</td>
<td></td>
<td>F1</td>
</tr>
<tr>
<td>1.00</td>
<td></td>
<td>300</td>
<td>240</td>
<td></td>
<td>600</td>
</tr>
<tr>
<td></td>
<td></td>
<td>~0%</td>
<td>~20%</td>
<td></td>
<td>~100%</td>
</tr>
<tr>
<td>0.50</td>
<td></td>
<td>310</td>
<td>290</td>
<td></td>
<td>470</td>
</tr>
<tr>
<td></td>
<td></td>
<td>~2%</td>
<td>~5%</td>
<td></td>
<td>~55%</td>
</tr>
<tr>
<td>0.25</td>
<td></td>
<td>305</td>
<td>305</td>
<td></td>
<td>420</td>
</tr>
<tr>
<td></td>
<td></td>
<td>~0%</td>
<td>~0%</td>
<td></td>
<td>~40%</td>
</tr>
<tr>
<td>0.10</td>
<td></td>
<td>290</td>
<td>-</td>
<td></td>
<td>325</td>
</tr>
<tr>
<td></td>
<td></td>
<td>~5%</td>
<td>-</td>
<td></td>
<td>~7%</td>
</tr>
</tbody>
</table>

In a second test series the experiments of Bloem, on pressure
transients using test section 2 as described in paragraph 2.3.2.2, are simulated. In the numerical simulations a 6 meter long tube (D = 6 mm) is heated in the middle over 5 cm during 2 ms. The heat input is 34.4 W. The fluid at the inlet has a temperature of 4.2 K and a velocity of 1 m/s. For the numerical parameters Δz = 0.05 m and Δt = 0.25 ms is used. Besides the propagation speeds, as described above, in this case also the strength of the induced pressure wave can be compared to experimentally obtained values (see table 2.4).

![Simulation of the pressure experiments of Bloem[1986]](image)

**Figure 6.12:** Simulation of the pressure experiments of Bloem[1986]

- Comparison of the pressure waves computed with four different differencing schemes

At first the 6 bar experiment is simulated using four different combinations of the numerical time and spatial differencing schemes described in chapter 3. In figure 6.12 two typical pressure plot are shown. One as a function of the distance from the disturbance and one as a function of time. The only difference between the EXQUISITE and the HYBRID spatial differencing schemes is found after the pressure maximum. The pressure computed with the EXQUISITE scheme is somewhat higher there than the pressure computed with the HYBRID scheme. For the slope and the consequent determination of the propagation speed of the wave the choice of the spatial differencing scheme is not important. In

149
the rest of this thesis we will therefore only use the simpler HYBRID scheme. The choice of the time differencing scheme is much more important. As was found before the B3 scheme suffers from less numerical dissipation than the FI scheme. The slope of the computed wave is much steeper when the B3 instead of the FI scheme is used. After the top, however, the B3 scheme introduces some non realistic wiggles in the solution.

Table 6.6 : propagation speeds for different time differencing schemes at three different pressures together with the deviation from the theoretical value.

<table>
<thead>
<tr>
<th></th>
<th>3 bar</th>
<th>6 bar</th>
<th>10 bar</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FI</td>
<td>B3</td>
<td>FI</td>
</tr>
<tr>
<td>Theory</td>
<td>212</td>
<td>234</td>
<td>275</td>
</tr>
<tr>
<td>Top</td>
<td>210</td>
<td>205</td>
<td>230</td>
</tr>
<tr>
<td>~1%</td>
<td>~4%</td>
<td>~2%</td>
<td>~2%</td>
</tr>
<tr>
<td>Slope</td>
<td>265</td>
<td>230</td>
<td>295</td>
</tr>
<tr>
<td>~25%</td>
<td>~8%</td>
<td>~26%</td>
<td>~7%</td>
</tr>
<tr>
<td>Foot</td>
<td>285</td>
<td>240</td>
<td>330</td>
</tr>
<tr>
<td>~35%</td>
<td>~13%</td>
<td>~40%</td>
<td>~17%</td>
</tr>
</tbody>
</table>

For the 3 and 10 bar cases similar results are found as for the 6 bar case. The propagation speeds computed at the three different pressures are gathered in table 6.6. Again the B3 scheme is found to be superior over the FI scheme. Not much difference between the results for the three ambient pressures is found. The propagation of the top is predicted correctly for all cases. The slope and the foot propagate too fast. The difference is respectively about 25 and 40 percent for the FI and about 8 and 15 percent for the B3 scheme.

The numerical maximum values of the pressure wave at the center of the tube \( z = 0 \) and at \( z = 2 \text{ m} \) are compared to the experimental values in table 6.7. The values at the center are in good agreement with the experiments. Due to the numerical dispersion the values at 2 meters from the disturbance computed with the FI scheme are much too small. The results for the B3 scheme are much better.

150
Table 6.7: Comparison of the computed pressure maxima with the measured values of Bloem

<table>
<thead>
<tr>
<th>P (bar)</th>
<th>( \Delta P_{\text{measured}} ) at 2m (kPa)</th>
<th>Computed values</th>
<th>( \Delta P ) at 0m (kPa)</th>
<th>( \Delta P ) at 2m (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>F1</td>
<td>B3</td>
<td>F1</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>4.2</td>
<td>4.6</td>
<td>1.9</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>2.4</td>
<td>2.4</td>
<td>1.5</td>
</tr>
<tr>
<td>10</td>
<td>2</td>
<td>2.2</td>
<td>2.0</td>
<td>1.3</td>
</tr>
</tbody>
</table>

In chapter 3 it has been argued that the radial velocity can be computed directly from the continuity equation instead of from the Navier-Stokes equation (3.4). In paragraph 6.1.2 it has been shown that this assumption is valid for a stationary turbulent constant property flow. In order to check whether this assumption is also valid for an instationary situation the 6 bar simulation has been repeated computing the radial velocity from the Navier-Stokes equation. The results for both solution methods were nearly identical which means that also for a transient variable property flow the radial velocity can be solved directly from the continuity equation.

Figure 6.13: Simulation of the pressure experiments of Bloem[1986]
- Variation of the number of radial grid points
In the above described simulations of the 6 bar experiment of Bloem the number of radial grid points was 15. To find out whether this is enough or not a simulation with 18 radial grid points has also been performed. The resulting pressure wave after 8 ms as a function of the distance z from the disturbance is shown in figure 6.13 for both the FI and B3 scheme. The influence on the solution is rather small which means that 15 radial grid points is sufficient for this simulation. The only significant difference between the solutions is that the wiggles in the B3 scheme seem to be shifted. On the propagation velocity this has no influence.

In this thesis our main interest concerns with the heat transfer to the super critical helium flow. Therefore we will also compare the total heat flux from the tube wall to the flow in the 6 bar case as computed with the different differencing schemes. In figure 6.14 the total heat flux to the helium flow as a function of time is shown. The influence of the different schemes on heat transfer is rather small. The maximum deviation is 5 percent.

![Figure 6.14](image)

Figure 6.14: Simulation of the pressure experiments of Bloem[1986]
-Comparison of the total heat flux to the helium flow computed with four different differencing schemes.

It can be concluded that by decreasing the time step and using the B3 scheme the propagation of the pressure wave can be described
adequately with our transient flow model. Using the FI scheme results in a larger numerical dissipation than the B3 scheme. The B3 scheme, however, introduces some non realistic wiggles in the solution. The choice of the spatial differencing scheme has been found to have only minor influence on the computed pressure wave. When heat transfer is studied also the influence of the time differencing scheme used is small (5%)

6.2.2 Transient heat transfer

In order to be able to predict the stability of a super conducting magnet, cooled with a turbulent flow of supercritical helium, the transient heat transfer to the flow must be described adequately by the simulation model. Verification of the transient heat transfer simulations is a necessity. Only recently a large number of experimental data on the subject has been presented by Bloem [1986]. The numerical description of the experimental set-up is discussed in paragraph 6.2.2.1.

In close cooperation with Bloem a number of experiments have been selected to be simulated with our numerical model. In paragraph 6.2.2.2 a reference situation will be defined. This situation will be tested extensively, varying several numerical parameters. Variation of the physical parameters as there are the ambient pressure, mass flow, inlet temperature and applied heat flux, will be studied in paragraph 6.2.2.3.

6.2.2.1 Numerical description of experimental set-up

In the experiments of Bloem, as described in paragraph 2.3.2, the flow of supercritical helium is forced through a rectangular copper tube. Numerically a two dimensional axisymmetric flow model is used. This means that the rectangular cooling channel has to be transformed to a circular tube in the simulations. In the transformation it is impossible to keep both the wetted perimeter and the helium cross section unchanged at the same time. For the heat transfer the wetted perimeter $D_w$ is the most important parameter. In order to keep this parameter unchanged the diameter $D_{num}$ of the circular tube has to be
defined as

\[ D_{\text{num}} = \frac{1}{\pi} D_w = 6.1 \times 10^{-3} \text{ m} \quad (6.24) \]

in which \( D_w = 0.0191 \text{ m} \) as specified in table 2.1. The relation between the resulting numerical helium cross section \( A_{\text{num}} \) and the experimental value \( A_{\text{exp}} = 0.231 \times 10^{-4} \text{ m}^2 \) (table 2.1) can be expressed using the hydraulic diameter \( D_{\text{exp}} \) of the rectangular tube which is defined as

\[ D_{\text{exp}} = \frac{4 A_{\text{exp}}}{D_w} = 4.8 \times 10^{-3} \text{ m} \quad (6.25) \]

Using the relations (24) and (25) we arrive at

\[ \frac{A_{\text{num}}}{A_{\text{exp}}} = \frac{\frac{\pi}{4} D_{\text{exp}}^2}{\frac{\pi}{4} D_{\text{exp}} D_{\text{num}}} = \frac{D_{\text{num}}}{D_{\text{exp}}} = 1.21 \quad (6.26) \]

The numerical helium cross section in case the wetted perimeter is kept constant is thus 21 percent larger than the experimental value. Because of this discrepancy the mean fluid velocity \( \langle u \rangle \), the flow Reynolds number \( \text{Re} = \frac{\langle u \rangle D}{v} \) and the mass flow \( \dot{m} = \rho \langle u \rangle A \) can not all be kept invariant under the transformation from the experimental to the numerical situation. A choice between keeping the velocity or the Reynolds number and the mass flow the same in both situations has to be made. For large times when a stationary situation has been reached (\( \text{Nu} = \text{Nu(Re)} \)) or when the heating of the bulk of the helium flow becomes significant, the Reynolds number and the mass flow have to be kept unchanged. However, for short times heat transfer is governed by the transient processes in the vicinity of the wall. In this case the velocity profile near the wall, initially dictated by the mean velocity of the flow, is more important than the bulk flow parameters Reynolds number and mass flow. Since we are interested in the transient development of the heat transfer to the flow the mean velocity \( \langle u \rangle \) of the flow has been taken equal to the experimental value. The influence on the stationary heat transfer can be estimated using a stationary heat transfer correlation in which the Nusselt \( \text{Nu} = \frac{hD}{\lambda} \) is proportional to \( \text{Re}^{0.8} \) (see paragraph 6.1.2). For the heat transfer coefficient \( h \) this can be rewritten to

\[ h \propto D^{-0.2} \langle u \rangle^{0.8} \quad (6.27) \]
Since the numerical value for $<u>$ has been taken equal to the experimental value the relation between the experimental and numerical stationary heat transfer coefficient is given by

$$\frac{h_{\text{num}}}{h_{\text{exp}}} = \left( \frac{D_{\text{num}}}{D_{\text{exp}}} \right)^{0.2} = 0.95$$ (6.28)

This means that the influence of the transformation from a rectangular to a circular tube on the computed heat transfer coefficient is estimated to be about 5 percent for long times. For small times the influence is thought to be less.

In the experiments of Bloem neither the heat flux from the wall to the flow nor the wall temperature is fixed. Only the applied heat flux to the copper tube is specified. To simulate the experiments it is thus necessary also to compute the wall temperature. This is done by solving the heat conduction equation (2.14) for the copper tube using the one dimensional copper/super conductor model of Cornelissen [1984]. The heat generation function $G$ in this case only contains the constant heat input which is applied over a length of 0.174 m. The strength is different for the various experiments. The heat flux $Q$ to the flow is computed from the temperature gradient in the helium near the wall. The physical properties, $mC_p$ and $\lambda$, of the tube depend on the temperature of the tube. For the heat capacity $mC_p$ the experimentally determined relation (2.35) is used. The thermal conductivity of the test section has not been measured by Bloem. In the simulations the same relation

$$\lambda_{\text{Cu}} = 45 \text{ T}$$ (6.29)

as for the SULTAN conductor (table 2.1) is used.

6.2.2.2 The reference situation

From the data of Bloem one experiment is selected to act as a reference situation. In table 6.8 the physical conditions describing this situation are gathered. The mass flow of $\dot{m} = 1 \text{ g/s}$ refers to the experimental situation. At the inlet temperature $T_{in} = 4.2 \text{ K}$ and a pressure $P = 6 \text{ bar}$ the matching mean velocity is $<u> = 0.3 \text{ m/s}$. In the numerical situation this leads to a mass flow of $1.2 \text{ g/s}$ and a Reynolds
number of about $6 \times 10^4$.

Table 6.8 : The reference situation - Physical parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ambient pressure P</td>
<td>6</td>
</tr>
<tr>
<td>Mass flow rate m</td>
<td>1</td>
</tr>
<tr>
<td>Inlet temperature T</td>
<td>4.2</td>
</tr>
<tr>
<td>Applied heat flux q</td>
<td>4.9</td>
</tr>
<tr>
<td></td>
<td>bar</td>
</tr>
<tr>
<td></td>
<td>g/s</td>
</tr>
<tr>
<td></td>
<td>K</td>
</tr>
<tr>
<td></td>
<td>kW/m²</td>
</tr>
</tbody>
</table>

In a numerical simulation of a physical process also some numerical parameters have to be specified. In our simulations the time step $\Delta t$, the axial mesh size $\Delta z$ and the number of grid points $N_z$ and $N_r$ in axial and radial direction respectively are relevant. In order to find some suitable values for these parameters they are varied until the effect on the solution is small. In table 6.9 the resulting parameter values are gathered.

In paragraph 6.2.1 it has been found that using the B3 time differencing scheme has no advantage compared to the FI scheme when heat transfer under study. Since the B3 scheme introduces non realistic wiggles in the solution we will use the FI scheme here, and in all other heat transfer simulations.

Table 6.9 : The reference situation - Numerical parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time step $\Delta t$</td>
<td>1.0</td>
</tr>
<tr>
<td>Axial mesh size $\Delta z$</td>
<td>3.48</td>
</tr>
<tr>
<td>Axial grid points $N_z$</td>
<td>40</td>
</tr>
<tr>
<td>Radial grid points $N_r$</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>ms</td>
</tr>
<tr>
<td></td>
<td>cm</td>
</tr>
<tr>
<td></td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>-</td>
</tr>
</tbody>
</table>

For the radial grid the Schnurr grid (3.1), with the parameter $C_s$ equal to 2, is used. The grid point nearest to the wall is then situated at $y^+ = 2.5$ like in paragraph 6.1.2. In axial direction an equidistant grid is used. The total computed tube length is thus about 1.4 m and the number of points within the heated section of 17.4 cm is 5.
In figure 6.15 the influence of the number of radial grid points is shown. Reducing the number of grid points from 15 to 13, by which the dimensionless distance to the wall is doubled from $y^+ = 2.5$ to 5.0, has a large impact on the solution. However, when the number of grid points is increased to 20, ($y^+$ reduced to 0.4) the solution only changes in the first three milliseconds. Further increase of the number of grid points did not change the results. It can be concluded that using 20 grid points in radial direction suffices to describe the transient heat transfer to the helium flow. With only a small reduction of the accuracy also 15 points can be used. When 13 grid points are used the first grid point from the wall is situated at the edge of the laminar sublayer and the control volume around this point stretches even to $y^+ = 10$. The laminar sublayer is then not described adequately and the computed flow fields as well as the heat transfer to the flow will not be correct.

In figure 6.16 the time step and mesh size are varied. The mesh size is reduced by a factor 4, keeping the total length of the tube constant by increasing the number of grid points to 80. The other line is the solution in case the time step is reduced by a factor 2. As can be seen from the figure all three solutions are nearly identical. Also enlarging the tube length did not change the solution. The time step, mesh size and tube length are thus well chosen.
The computed heat transfer coefficient and wall temperature for the reference situation, defined by the physical and numerical parameters as specified in the tables 6.8 and 6.9 respectively, are compared to the experimental results of Bloem in figure 6.17. A good overall agreement is found. The maximum deviation between the numerical

Figure 6.16:
Reference situation
- Influence of the time step $\Delta t$ and the mesh size $\Delta z$.

Figure 6.17: Comparison of experimental and numerical temperatures and belonging heat transfer coefficients.
- Reference situation.
and experimental heat transfer coefficients is 10 percent of the experimental value. The origin of this difference will be discussed in paragraph 6.2.2.4.

6.2.2.3 Variation of physical parameters

In this paragraph the accuracy of the numerical simulations is tested for situations in which one of the physical conditions is changed compared to the reference situation. The tests can be divided into four series:

1: Mass flow variation
   Fig. 6.17 vs. Fig. 6.18, 6.19
2: Pressure variation
   Fig. 6.17 vs. Fig. 6.20, 6.21
3: Temperature variation
   Fig. 6.17 vs. Fig. 6.22, 6.23
4: Heat flux variation
   Fig. 6.17 vs. Fig. 6.24

- Mass flow variation

   When the mass flow is varied it must be noted that also the number of radial grid points has to be adapted. In order to satisfy the requirement that \( y^+ = 2.5 \) for the grid point near the wall the number of grid points for a mass flow of 3 g/s has to be increased from 15 to 18. For a mass flow of 0.3 g/s only 12 points would be sufficient. However, to describe the transient heat transfer effects adequately more points are required. Increasing the number of points again it was found that also for this case of low mass flow 15 radial grid points were needed.

   From the figures 6.17, 6.18 and 6.19 it can be seen that the results from our numerical model correspond better to the experimental values for a large mass flow than for a small mass flow, especially in the second part of the experiment (t > 10 ms). The error of about 30 percent in the heat transfer coefficient for large times at a mass flow of 0.3 g/s is reduced to less than 5 percent at a mass flow of 3 g/s. For small times (t < 10 ms) there is not much difference in the deviation between the numerical and experimental results. The maximum error is 10 percent for the heat transfer coefficient in all three cases.
Figure 6.18:
Comparison of experimental and numerical temperatures and belonging heat transfer coefficients.
- Low mass flow
  \((m = 0.3 \text{ g/s})\)

Figure 6.19:
Comparison of experimental and numerical temperatures and belonging heat transfer coefficients.
- High mass flow
  \((m = 3.0 \text{ g/s})\)

Figure 6.20:
Comparison of experimental and numerical temperatures and belonging heat transfer coefficients.
- High pressure
  \((P = 10 \text{ bar})\)
• Pressure variation

To test the influence of the fluid property variations two
different ambient pressures are simulated here. For the high pressure
case \((P = 10\ \text{bar})\) the property variations are small. This case can
only be compared with the 6 bar case for times less than 10
milliseconds, because at a 10 bar pressure no measurements were
performed with a 20 millisecond pulse (note the different time axis in
figure 6.20). In this time interval the results in the 10 bar case are
excellent. The errors is almost reduced to zero.

![Graph](image)

Figure 6.21 :
Comparison of experimental
and numerical temperatures
and belonging heat transfer
coefficients.
- Low pressure
  \((P = 3\ \text{bar})\)

For the low pressure case \((P = 3\ \text{bar})\) the accuracy of the
numerical results was found to be very poor. Due to the large
temperature dependency of the fluid properties at this pressure the
radial grid was found to be to coarse to describe the variations near
the wall. Increasing the number of grid points improved the results
considerably. Using 20 instead of 15 radial grid points the overall
deviation between the experimental and numerical heat transfer
coefficient is reduced to some 10 percent (see figure 6.21). The
influences of the fluid property variations on the transient heat
transfer can thus be described adequately by the numerical model
developed in this thesis when a sufficient number of radial grid points
is used.

161
Temperature variation

Besides the inlet temperature of 4.2 K Bloem also performed measurements at an inlet temperature of 5 and 6 K. The main difference between the results for the reference situation and the higher inlet temperature cases (figures 6.22 and 6.23) is the correspondence between the measured and calculated heat transfer coefficients for times larger than about 10 milliseconds. The measured values for both the 5 and 6 K cases are higher than in the reference situation. The computed values, however, are nearly the same in all three situations. This leads to an increase in the discrepancy in the results from 10 to about 25 percent. All other deviations are the same as in the reference situation.

Figure 6.22:
Comparison of experimental and numerical temperatures and belonging heat transfer coefficients.
- High inlet temperature
  (T = 5 K)

Figure 6.23:
Comparison of experimental and numerical temperatures and belonging heat transfer coefficients.
- High inlet temperature
  (T = 6 K)
• Heat flux variation

Increasing the heat flux from about 5 kW/m² up to almost 10 kW/m² (figure 6.24) leads to the worst correspondence between experimental and simulated results. The difference between the numerical and experimental heat transfer coefficient in the second part of the experiment is about 30 percent. In the reference situation this error was only 10 percent. In the first part (t < 10 ms) the results in both cases are about the same.

Figure 6.24:

Comparison of experimental and numerical temperatures and belonging heat transfer coefficients.
- High heat flux
  \( Q = 9610 \text{ W/m}^2 \)

6.2.2.4 Discussion

In general the heat transfer coefficients agree within some 15 percent except for the low mass flow and the high heat flux case in which a 30 percent deviation is found for large times. In all simulations the experimental heat transfer coefficient after 20 milliseconds is found to be larger than the numerically computed value. A deviation of about 10 percent can be explained by considering three different aspects of the geometry of the test section.

First: We found in paragraph 6.2.2.1 (equation 27) that describing the three dimensional, rectangular tube by a two dimensional, cylindrical tube results in a 5 percent reduction of the stationary heat transfer coefficient. This means that the asymptotic
value of the numerical and experimental curve will differ by some 5 percent. From the surface renewal theory in paragraph 5.4 we learned that the effective renewal frequency in the reference situation is about 10 milliseconds. This means that after 20 millisecond heat transfer is governed by stationary turbulent convection rather than by transient penetration and that a 5 percent deviation between the experimental and numerical results is expected.

Second: The experimental test tube is situated between two bends where the numerical tube is straight. This means that the at the entrance of the test section the numerical velocity profile is fully developed but the experimental profile is not. Bloem reduced the influence of the bend on the temperature measurement by locating the thermometer at 13 diameters from the tube inlet. In case of a hydrodynamic developed flow this is sufficient to assure a developed temperature profile (see paragraph 2.3.2.2). However the entrance length required to reach a developed velocity profile after a 90 degree bend is larger, 20-40 diameters [Rohsenow and Hartnett(1973)]. This means that the bend still influences the measurement through the velocity profile at the location of the thermometer. Also the increased heat transfer in the entrance region of the test section due to the turbulence generated by the bend will influence the measurement. Because of the good thermal conductivity of the copper heat will be conducted to the high heat transfer region at the entrance of the test section leading to a reduction of the temperature at the location of the thermometer. The heat transfer coefficient, computed from equation (2.34), will consequently be too high.

Third: At the location of the thermometer the tube is not heated. This results in a local minimum of the copper temperature under the thermometer. The temperature readout will consequently be lower than the mean temperature of the tube. The estimated heat flux to the helium flow, resulting from the mean temperature using equation (2.34), and the corresponding heat transfer coefficient will be too high. However, a three dimensional simulation of the stationary temperature profile in the copper tube learned that the influence of the thermometer on the heat transfer coefficient is less than 1 percent and can thus be neglected.
In the numerical simulations buoyancy effects are assumed to be neglectable. However, when this assumption is not correct the computed heat transfer coefficients will be too low compared to the experimental values. In order to verify whether buoyancy is neglected rightly or not the condition (2.31) stated in paragraph 2.3.1 must be checked for all simulations. Using the definitions \( \text{Gr} = \rho \Delta \rho g L^3/\mu^3 \) and \( \text{Re} = \rho <u>D/\mu \) for the Grashof and Reynolds number, where the length scale \( L \) equals the tube radius \( R = D/2 \) and \( g = 9.8 \text{ m/s}^2 \) is the gravitational acceleration, (2.31) can be rewritten as:

\[
\frac{\rho(T_b) - \rho(T_w)}{\rho(T_b)} < K \ 10^{-5} \ \frac{8 <u>^2 \text{Re}^{0.7}}{gD}
\]

(6.30)

The value of \( K \) equals 1.0 following Ito. According to Brassington and Cairns a value of 2.4 applies while Hall suggested \( K = 12 \) (see paragraph 2.3.1). The maximum value of \( \Delta \rho/\rho \) computed from the equation (30) for the three different values of the constant \( K \) are stated in table 6.10. In the last column the estimated value of \( \Delta \rho/\rho \) at the end of the experiment is given.

**Table 6.10 : Buoyancy criteria for the simulated experiments.**

<table>
<thead>
<tr>
<th>( \Delta \rho/\rho )</th>
<th>( &lt;u&gt; ) m/s</th>
<th>K=1.</th>
<th>eq (29)</th>
<th>K=2.4</th>
<th>K=12.</th>
<th>( \Delta \rho/\rho ) Exp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference</td>
<td>0.29</td>
<td>0.24</td>
<td>0.58</td>
<td>2.89</td>
<td>0.6</td>
<td>0.7</td>
</tr>
<tr>
<td>( \dot{m} = 0.3 \text{ g/s} )</td>
<td>0.09</td>
<td>0.01</td>
<td>0.02</td>
<td>0.11</td>
<td>0.5</td>
<td>0.7</td>
</tr>
<tr>
<td>( \dot{m} = 3.0 \text{ g/s} )</td>
<td>0.88</td>
<td>5.0</td>
<td>10.0</td>
<td>55.0</td>
<td>0.5</td>
<td>0.8</td>
</tr>
<tr>
<td>( P = 3 \text{ bar} )</td>
<td>0.31</td>
<td>0.30</td>
<td>0.73</td>
<td>3.65</td>
<td>0.5</td>
<td>0.4</td>
</tr>
<tr>
<td>( P = 10 \text{ bar} )</td>
<td>0.28</td>
<td>0.19</td>
<td>0.47</td>
<td>2.33</td>
<td>0.5</td>
<td>0.4</td>
</tr>
<tr>
<td>( T = 5 \text{ K} )</td>
<td>0.32</td>
<td>0.30</td>
<td>0.73</td>
<td>3.66</td>
<td>0.5</td>
<td>0.4</td>
</tr>
<tr>
<td>( T = 6 \text{ K} )</td>
<td>0.37</td>
<td>0.47</td>
<td>1.13</td>
<td>5.65</td>
<td>0.5</td>
<td>0.4</td>
</tr>
<tr>
<td>( Q = 10 \text{ kW/m}^2 )</td>
<td>0.29</td>
<td>0.24</td>
<td>0.58</td>
<td>2.89</td>
<td>0.5</td>
<td>0.4</td>
</tr>
</tbody>
</table>

The Ito condition (equation 30 - K=1) predicts buoyancy effects for all experiments except the high mass flow case (\( \dot{m} = 3 \text{ g/s} \)). However, equation (30) is only part of the condition. Ito only observed buoyancy in his experiments when the flow rate was low and the heat flux was
high. Following Ito $G = 20 \text{ kg/m}^2\text{s}$ is a low flow rate where a value of 40 is large. A value of 40 kJ/kg·m for the Ito parameter $\phi_i$ defined in equation (2.29), denotes a large heat flux case. In case of a value of 20 the heat flux is called low. From table 6.11 we learn that only the low mass flow case combines a low flow rate with a high value of the Ito parameter which means that buoyancy effects will be significant only in this situation. Returning to table 6.10 we find that the other conditions confirm this conclusion. The only case for which also buoyancy is predicted is the high heat flux case. The Brassington and Cairns condition (equation 30 - $K=2.4$) stating $\Delta \rho/\rho < 0.58$ is only just violated in the experiment ($\Delta \rho/\rho \approx 0.8$). Buoyancy effects are not expected to be large here.

Table 6.11: Values for the flow rate and the Ito parameter in the experiments of Bloem [1986]

<table>
<thead>
<tr>
<th>$&lt;u&gt;$</th>
<th>$G$</th>
<th>$\phi_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>m/s</td>
<td>kg/m$^2$s</td>
<td>kJ/kgm</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reference</td>
<td>0.29</td>
<td>42</td>
</tr>
<tr>
<td>$\dot{m}=0.3\ \text{g/s}$</td>
<td>0.09</td>
<td>13</td>
</tr>
<tr>
<td>$\dot{m}=3.0\ \text{g/s}$</td>
<td>0.88</td>
<td>125</td>
</tr>
<tr>
<td>$P=3\ \text{bar}$</td>
<td>0.31</td>
<td>42</td>
</tr>
<tr>
<td>$P=10\ \text{bar}$</td>
<td>0.28</td>
<td>42</td>
</tr>
<tr>
<td>$T=5\ \text{K}$</td>
<td>0.32</td>
<td>42</td>
</tr>
<tr>
<td>$T=6\ \text{K}$</td>
<td>0.37</td>
<td>42</td>
</tr>
<tr>
<td>$Q=10\ \text{kW/m}^2$</td>
<td>0.29</td>
<td>42</td>
</tr>
</tbody>
</table>

In figure 6.25 the heat transfer coefficients for $\dot{m} = 0.3\ \text{g/s}$ is compared to the reference situation. From this figure we learn that the experimental heat transfer coefficient for large times remains the same for the two flow rates where the numerical value decreases with the mass flow as is expected. This effect, which causes the relative large discrepancy of 30 percent between the experimental and numerical result, can be explained by the effect of buoyancy on the experimental heat transfer.
Figure 6.25: Comparison of the heat transfer coefficients for the reference situation and the low mass flow case.

Figure 6.26: Comparison of the heat transfer coefficients for the reference situation and the high heat flux case.
In the high heat flux case the Ito parameter equals 45 (table 6.11). At this value a deterioration of the stationary heat transfer is expected. In the reference situation the Ito parameter $\varphi_1 = 22$ which is a low value. This means that in this case no deterioration is expected. When we look at figure 6.26 we see that numerically the heat transfer coefficient at large times is indeed much lower for the high heat flux case than for the reference situation. Experimentally no such deterioration is found what explains the increased discrepancy between experiment and simulation for this case. The reason for this unexpected high experimental heat transfer is believed to be a buoyancy effect although no large buoyancy is expected for this case.

A last cause for the difference between experiments and simulations we will give here is the uncertainty of the thermal conductivity of the helium, which is about 5 percent according to Hands and Arp [1981]. Since the heat flux at the wall is computed from the temperature gradient near the wall using

$$\phi = -\lambda \frac{dT}{dy}$$

an error in the thermal conductivity $\lambda$ results in an equally large error in the computed heat transfer.

Regarding the above described discrepancies between the experiments and the simulations, numerical and experimental results are in good agreement. The principal source for the deviation between the numerical and experimental results has been found to be the differences in the experimental and simulated geometry of the test section (rectangular versus circular tube and the effects of the bend in the experimental tube). The larger errors for the low mass flow and the high heat flux cases (about 30% for large $t$) are found to be a result of heat transfer enhancement by buoyancy effects occurring in the experimental results. This effect has not been incorporated in our simulation model. For the situations in which buoyancy can be neglected the simulation model developed in this thesis is found to be an adequate tool for describing the transient heat transfer to a forced flow of supercritical helium.
6.3 Stability study

In this paragraph the stability of the SULTAN conductor, as specified in paragraph 2.3.2, will be studied. Therefore the two dimensional flow model is coupled to the one dimensional conductor model as developed by Cornelissen [1984]. Both the critical energy (paragraph 6.3.1) in case of a stable performance, as the propagation velocity of the normal zone (paragraph 6.3.2) in case of an unstable performance of the magnet will be studied.

6.3.1 Critical energy computations

At first, in paragraph 6.3.1.1 a reference situation will be defined. For this particular situation the critical disturbance \( Q_c \) is determined by trial and error. If for some disturbance \( Q \) the conductor returns after some time to its superconducting state, the conductor is stable and in the next calculation the disturbance is raised. If the calculated normal conducting zone (\( T_{cu} > T_c \) - see figure 2.2) then expands with time the conductor quenches. The conductor is then unstable and a lower disturbance must be assumed to find the critical value. The procedure is stopped when \( \Delta Q = Q_{unstable} - Q_{stable} \) is less than a prescribed value. The resulting critical energy is defined as

\[
Q_c = \frac{1}{2} (Q_{stable} + Q_{unstable})
\]

with an error of

\[
\varepsilon_Q = \frac{1}{2} \frac{\Delta Q}{Q_c} \cdot 100\%
\]

In paragraph 6.3.1.2 a variation of some numerical parameters starting from the reference situation will be performed to find the numerical inaccuracy in the solution. This inaccuracy should be less than the above defined error \( \varepsilon_Q \).

To find the influence of the system pressure and the Reynolds number on the critical energy, the trial and error procedure described above will be repeated in paragraph 6.3.1.3 for different values of these flow parameters.

Finally in paragraph 6.3.1.4 the influence of the duration and
extension of the disturbance are also studied in this way.

6.3.1.1 The reference situation

As a reference situation a 200 mm section of the Sultan inner coil as defined in paragraph 2.3.2 is chosen, operating at an ambient pressure of 12.5 bar and a temperature of 4.2 K. The flow Reynolds number is set at Re = 10$^5$ which equals a mass flow of 2.5 g/s and a mean velocity of 0.578 m/s.

The tube is divided in 40 cross sections resulting in an axial mesh size of 5 mm between two adjacent cross sections. The time step between two time levels is 1 ms. The applied disturbance is extended over two grid points situated in the middle of the tube and lasts one time step. The relevant parameters in this reference situation are gathered in table 6.12.

Table 6.12a : Reference situation - Physical parameters

<table>
<thead>
<tr>
<th>Pressure</th>
<th>P</th>
<th>12.5</th>
<th>bar</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature</td>
<td>T</td>
<td>4.2</td>
<td>K</td>
</tr>
<tr>
<td>Reynolds number</td>
<td>Re</td>
<td>10$^5$</td>
<td>-</td>
</tr>
<tr>
<td>- Mass flow</td>
<td>$\rho$</td>
<td>2.5</td>
<td>g/s</td>
</tr>
<tr>
<td>- Mean velocity</td>
<td>$&lt;u&gt;$</td>
<td>0.587</td>
<td>m/s</td>
</tr>
</tbody>
</table>

Table 6.12b : Reference situation - Physical parameters

<table>
<thead>
<tr>
<th>Time step</th>
<th>$\Delta t$</th>
<th>1</th>
<th>ms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mesh size</td>
<td>$\Delta z$</td>
<td>0.005</td>
<td>m</td>
</tr>
<tr>
<td>Tube length</td>
<td>L ($=40\Delta z$)</td>
<td>0.2</td>
<td>m</td>
</tr>
<tr>
<td>Duration of disturbance</td>
<td>$t_Q$ ($= \Delta t$)</td>
<td>1</td>
<td>ms</td>
</tr>
<tr>
<td>Extension of disturbance</td>
<td>$L_Q$ ($= 2\Delta z$)</td>
<td>0.01</td>
<td>m</td>
</tr>
</tbody>
</table>

In figure 6.27 the length of the normal conducting zone is shown as it develops in time after a disturbance Q has been applied between 0 and 1 ms. In case the applied disturbance is 17 mJ or below, this normal length becomes zero after some time. The total conductor is then
back in its superconducting state and the heat generation is stopped. The conductor is thus stable against these disturbances. Unstable situations, in which the normal zone expands with time, are found when the applied disturbance is taken 18 mJ or above. The critical energy in this reference situation, therefore, is

\[ Q_c = \frac{1}{2} (17 + 18) = 17.5 \text{ mJ} \]

with an inaccuracy of

\[ \varepsilon_Q = \frac{0.5}{17.5} \cdot 100\% \approx 3\% \]

Table 6.13: Comparison of the critical energy computed with models of different complexity.

| Static (const. metal prop.) | 5.3 |
| Static (var. metal prop.) | 6.4 |
| Dynamic (\( L_Q \downarrow 0 \)) | 4.1 |
| Transient (Cornelissen) | 15 |
| Transient (this study) | 17.5 |

Figure 6.27:

Reference situation
Critical disturbance computation.

- X - Q = 20 mJ
- □ - Q = 18 mJ
- Δ - Q = 17 mJ
- ○ - Q = 15 mJ
In this case not only the 17 and 18 mJ cases are studied, but also a calculation has been performed at the critical disturbance of 17.5 mJ derived from these two cases. The result, as plotted in figure 6.27 (dotted line), shows that the normal zone is nearly constant for a long time. This means that this situation is indeed quite near the critical situation in which the normal zone does not change because dissipation and cooling are in equilibrium.

From literature we can find the value of the critical energy computed without incorporating transient effects. In table 6.13 these values (taken from Cornelissen [1984]) are compared to the value computed with our full transient model. In can be concluded that transient flow and heat transfer increases stability considerably. Due to these effects the critical energy is raised by about a factor three. The value of 15 mJ found by Cornelissen for the fully transient situation has been determined rather roughly. Cornelissen therefore advised more extensive simulations in the 13-17 mJ range. From our simulations this resulted in a value of 17.5 mJ, slightly higher than Cornelissen had estimated.

6.3.1.2 Numerical parameter variation

The sensitivity of the solution of the reference situation to the variation of numerical parameters, as there are the tube length, the mesh size and the time step, is studied here. The applied disturbance is taken equal to 17.5 mJ, the critical value. In this case small differences in the calculated heat flux lead to large changes in the solution for long times because of the critically stable character of the solution. The influence of the numerical parameters is therefore more pronounced than it would be in a more stable situation.

- Tube length variation

First of all the influence of the limited tube length is examined. Keeping the mesh size fixed the tube length is doubled by taking twice as much axial grid points. This leads to identical results. In the case of a small mesh size, 2 instead of 5 mm, doubling of the tube length
also does not change the computed results. From this it is concluded that the tube length has been taken large enough for this heat transfer problem and increasing it will not change the calculated critical disturbance significantly. However, when the mesh size or the extension of the disturbance is large the heated region is only a few grid points from the tube ends and an influence of the tube ends can be expected. In order to avoid this influence the tube length is doubled when the influence of the mesh size or the extension of the disturbance is studied.

- **Mesh size variation**

![Graph](image)

**Figure 6.28:**

Reference situation
Mesh size variation

- $\Delta z = 2$ mm
- $\Delta z = 5$ mm (Ref)
- $\Delta z = 10$ mm

In figure 6.28 the reference situation, mesh size $\Delta z = 5$ mm, is compared with results for mesh sizes of 2 and 10 mm. Decreasing the mesh size from 5 to 2 mm gave only minor changes in the calculated results. The difference is well within the 3% limits found in the stability analysis for the reference situation and indicated by the results for the reference situation with disturbances of 17 and 18 mJ. Increasing the mesh size to 10 mm gives larger differences with the reference situation, even for short times. Thus, the 10 mm mesh size is judged to be too coarse. The 5 mm mesh size is found to be accurate
• Time step variation

An other important parameter is the time step $\Delta t$. The results computed with three different time steps are compared in figure 6.29, viz. $\Delta t = 0.5$ ms, $\Delta t = 1.0$ ms (reference situation) and $\Delta t = 2.0$ ms. In all three cases the disturbance is applied in the same time span, $t_Q = 2$ ms.

For times below 15 ms the curves B and C (figure 6.29) are almost identical. Curve A, for the largest time step, is initially somewhat lower than the other two. It is important that the transient effects are described adequately. Because the applied disturbance is near the critical value small errors introduced just after the disturbance will accumulate in time and can later lead to large deviations. A 2 ms time step is therefore found to be too large. For large times curves B and C deviate. With a time step of 0.5 ms a stable solution results while the 1.0 ms time step gives a unstable solution. However, as stated before these differences occur due to the critically stable character of the solution at 17.5 mJ. The effect on the critical disturbance is well within the 3% limits given by the 17 and 18 mJ disturbance cases calculated with a time step of 1 ms.

Figure 6.29:

Reference situation
Time step variation

- $\Delta t = 2.0$ ms
- $\Delta t = 1.0$ ms (Ref)
- $\Delta t = 0.5$ ms

174
6.3.1.3 Flow parameter variation

Using the same technique as described for the reference situation, the critical disturbance is calculated in case of different flow Reynolds numbers and system pressures here. To explain the results gathered in figure 6.30, the two heat transfer mechanisms involved in the cooling process (see paragraph 2.1.3), penetration and turbulent convection, will be described first.

![Critical Disturbance vs Pressure](image)

Figure 6.30: Critical disturbance dependence on pressure and Reynolds number.

For short times heat has to penetrate through the laminar boundary layer by conduction. The influence of turbulence on the heat transfer will start after some time once a considerable amount of heat has penetrated the transition region. The penetration heat transfer coefficient at constant fluid properties is given by equation (2.43):

\[ h(t) = C_1 (\lambda \rho C_p/t)^{1/2} = C_1 (\varepsilon/t)^{1/2} \]  

(6.33)
where the constant \( C_1 = (\pi/4)^{1/2} \) or \( C_1 = \pi^{-1/2} \) respectively in case of a fixed heat flux at the wall and a fixed wall temperature. The parameter \( \varepsilon \) as defined by Bloem[1986] is:

\[
\varepsilon = \lambda \rho C_p \tag{6.34}
\]

The stationary turbulent heat transfer coefficient can be described by the Dittus-Boelter correlation (see paragraph 6.1.2)

\[
Nu = C_2 \text{Re}^{0.8} \text{Pr}^{0.4} \quad C_2 = 0.023 \tag{6.35}
\]

or

\[
h = \frac{C_2^2}{D} \text{Re}^{0.8} \chi \tag{6.36}
\]

The parameter \( \chi \) is defined as

\[
\chi = \lambda^{0.6} (\eta C_p)^{0.4} \tag{6.37}
\]

and depends, like the parameter \( \varepsilon \) as defined in equation (34), solely on the fluid properties of the helium.

The transition from penetration to turbulent convective heat transfer takes place around a certain take-over time \( t_t \). This time is estimated by Bloem [1986] by comparing the transient thermal boundary layer with the effective momentum boundary layer (see paragraph 2.3.2). Cornelissen [1984] estimated \( t_t \) by comparing the transient and stationary heat transfer coefficients given in the equations (33) and (36). He arrived at

\[
t_t = \left( \frac{C_1 D}{C_2} \right)^2 A^{-1} \text{Re}^{1.6} \text{Pr}^{0.8} \tag{6.38}
\]

Using the parameter \( \varepsilon \) and \( \chi \) this can be rewritten as

\[
t_t = \left( \frac{C_1 D}{C_2} \right)^2 \text{Re}^{1.6} \varepsilon \chi^{-2} \tag{6.39}
\]

• Reynolds number variation

Decreasing the Reynolds number even to zero still a high value of \( Q_e \) is found, about 85% of the value at \( Re = 0.5 \times 10^5 \) (figure 6.30). To explain this it must be noted that already an amount of about 5 mJ is
needed to raise the conductor temperature to the current sharing
temperature $T_{cs}$. Disturbances below 5 mJ will thus cause no dissipation
and therefore always lead to a stable solution. The surplus of the
critical disturbance is due to conduction in the solid and the
influence of heat transfer to the coolant. This heat transfer is
enhanced, compared to the heat conduction in stagnant helium, by an
induced flow due to the thermal expansion of the helium.

For high Reynolds numbers the critical energy is increased
considerably. In the reference situation ($P = 12.5$ bar) the critical
disturbance of 17.5 mJ for $Re = 10^5$ is almost doubled in the case
$Re = 1.5 \times 10^5$ ($Q_c = 33$ mJ). This increase is due to the fact that the
heat accumulated in the conductor at short times is later cooled away
more efficiently because of the increased turbulence in the flow. It is
thus possible to increase the conductor stability considerably by
applying large enough flow Reynolds numbers. The problem, however, is
the large pumping power required to generate large mass flows in the
narrow cooling channels of the conductor. This limits the Reynolds
number in practice.

* Ambient pressure variation

If at a constant Reynolds number the ambient pressure is varied
between 3 and 12.5 bar, a minimum for the critical energy is found at
about 6 bar (see figure 6.30). To explain this behavior the heat
balance is considered. If the critical disturbance $Q_c$ is applied at
time $t = 0$, the total momentary heat flux $q_{He}(t)$ to the coolant is in
balance after some time $t_0$ with the momentary dissipation $q_d(t)$ in the
conductor. The energy stored in the conductor is the constant for
t > $t_0$. This amount of energy $Q_s$ depends on the stationary, turbulent
convective heat transfer. Since the Reynolds number is constant the
main parameter determining $Q_s$ is $\chi$ (see equation 36).

For time smaller than $t_0$ the heat flux will be larger than the
heat dissipated in the conductor. The conductor energy will decrease in
this period. The transient energy $Q_t$ cooled away for times up to $t_0$ is
equal to (see equation 2.16):

$$Q_t = \int_0^{t_0} \left[ q_{He}(t) - q_d(t) \right] dt$$

(6.40)
Figure 6.31: Heat transfer parameters $\varepsilon$ and $\chi$.

This energy depends strongly on the penetration heat transfer described by the parameter $\varepsilon$. The critical disturbance can be found by adding this energy decrease for $t < t_0$, $Q_t$, to the conductor energy $Q_s$ in the equilibrium state:

$$Q_s = Q_s + Q_t$$  \hspace{1cm} (6.41)

In table 6.14 the values of $Q_o$, $Q_s$ and $Q_t$ are gathered as they are computed for the reference case of $Re = 10^5$, $Q = Q_e$, at different ambient pressures. From table 6.14 it can be seen that both the conductor energy $Q_s$, depending on $\chi$, and the transient energy decrease $Q_t$, depending on $\varepsilon$, have a minimum at a pressure of about 6 bar.

In figure 6.31 the parameters $\varepsilon$ and $\chi$ depending on the fluid properties only, are plotted as a function of temperature for different values of the system pressure. Both $\varepsilon$ and $\chi$ are low over a wide temperature range at a pressure of 6 bar. This accounts for the low critical disturbance at this pressure. The large peak values of $\varepsilon$ and $\chi$
Table 6.14: Heat balance for the critical disturbance

<table>
<thead>
<tr>
<th>P (bar)</th>
<th>$Q_c$ (mJ)</th>
<th>$Q_s$ (mJ)</th>
<th>$Q_t$ (mJ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>20.5</td>
<td>9</td>
<td>11.5</td>
</tr>
<tr>
<td>4</td>
<td>19</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>17</td>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td>6</td>
<td>15</td>
<td>7.5</td>
<td>7.5</td>
</tr>
<tr>
<td>9</td>
<td>15.5</td>
<td>7.5</td>
<td>8</td>
</tr>
<tr>
<td>12.5</td>
<td>17.5</td>
<td>8</td>
<td>9.5</td>
</tr>
</tbody>
</table>

At a pressure of 3 bar have a large positive influence on the cooling. This explains the enhanced heat transfer at low pressures.

6.3.1.4 Disturbance parameter variation

In practice little is known about the disturbances which can occur in a superconductor. In our analysis the disturbance has been applied in 1 ms, extended over a length of 10 mm. In the reference situation this resulted in a critical disturbance of 17.5 mJ. However, varying the extension or duration of the disturbance will change this value. The influence of these parameters is discussed here.

- Disturbance extension variation

If the extension of the disturbance is varied the initial conductor temperature changes. This will influence both the total heat flux to the helium, $q_{He}(\Delta t)$, and the total dissipation in the conductor, $q_D(\Delta t)$, just after the disturbance has been applied. This will thus influence the stability of the conductor for this disturbance. To examine the influence of the disturbance extension $L_Q$ qualitatively, a parameter $R_Q$ is defined:

$$R_Q = \frac{q_D(\Delta t)}{q_{He}(\Delta t)} \quad \text{with} \quad \Delta t \downarrow 0 \quad (6.42)$$

If this parameter increases, the dissipation grows compared to the cooling just after the disturbance has been applied, and the conductor
will be more unstable. This means that the critical disturbance decreases for increasing values of \( R_Q \) and vice versa.

Consider the initial copper temperature to be raised uniformly to a temperature \( T \) over the total disturbance length \( L_Q \). Since both \( q_D \) and \( q_{He} \) are then proportional to the disturbed length \( L_Q \) the only variable in equation (42) is the initial temperature \( T \). The heat flux \( q_{He} \) to the coolant is proportional the temperature difference \( (T - T_{He}) \). The expression for the dissipation \( q_d \) in the conductor depends on the value of the conductor temperature \( T \). From the current sharing model (2.10), described in paragraph 2.1.3, it can be found that in the current sharing region, \( T_{cs} < T < T_c \), the dissipation is proportional to the temperature difference \( T - T_{cs} \). For higher temperatures the dissipation is constant, for lower temperatures the dissipation is zero. Substituting this information in equation (42) gives:

\[
R_Q = \begin{cases} 
C \frac{T_c - T_{cs}}{T - T_{He}} & T_c < T \\
C \frac{T - T_{cs}}{T - T_{He}} & T_{He} < T < T_c < T < T_{cs} \\
0 & T_{He} < T < T_{cs}
\end{cases}
\]  

(6.43)

Where \( C = \frac{q_{dM}}{h(T_c - T_{cs})} \) and \( q_{dM} \) is the maximum dissipation in the conductor. The \( R_Q \) parameter as a function of temperature is sketched in figure 6.32.

![Figure 6.32: The \( R_Q \) parameter as a function of temperature.](image)

180
Now suppose that for a disturbance extension \( L_Q = L_1 \) the temperature of the conductor is initially raised to \( T = T_1 \) when the critical disturbance \( Q = Q_{c1} \) is applied. When we vary \( L_Q \), keeping \( Q \) constant, also \( T \) and \( R_Q \) will change which means that the balance between the dissipation and the cooling is disturbed. If \( T_{cs} < T_1 < T_c \) (large disturbance extensions) both the dissipation and the heat flux to the coolant will change as \( L_Q \) is varied. A small increase of \( L_Q \) will lead to a small decrease of the conductor temperature and, as we can see from figure 6.32 this will result in a decrease of the \( R_Q \) parameter. The effective cooling in this situation is better than in the original situation and the conductor will be stable against the disturbance \( Q \). For this situation the critical disturbance will be higher than \( Q_{c1} \). In other words for large values of the disturbance extension \( Q_c \) increases with \( L_Q \). Using similar reasoning for the situation \( T = T_2 > T_c \), which can occur for small disturbance extensions, we find that in this range \( Q_c \) decreases for increasing \( L_Q \).

From the above described reasoning using the \( R_Q \) parameter we may expect that increasing the disturbance extension \( L_Q \) will decrease the value of the critical disturbance until the extension becomes too large and the temperature \( T \) drops below \( T_c \). For larger extensions \( Q_c \) will increase with \( L_Q \). The value of \( L_Q \) for which \( T = T_c \) is calculated to be about 100 mm.

In table 6.15 the critical disturbances calculated for different values of the disturbance extension are gathered. The critical disturbance decreases slowly for extensions up to about 50-75 mm and grows for larger extensions. This result is in good agreement with the simplified qualitative description using the \( R_Q \) parameter.

We can also compare our results, computed at a current density \( j = I/I_c \) equal to 0.535 and a Steckly parameter \( \alpha_s \approx 10 \), with the results of dynamic simulations at a constant heat transfer coefficient \( h \), as reviewed in paragraph 2.1.3.2. A good qualitative agreement is found with the results of Keilin and Romanowsky [1982], Meuris [1984] and Cornelissen [1984]. In their results also a minimal value for the critical disturbance is found at a disturbance extension of 50-80 mm. The value of the critical disturbance, however, is much lower than the
value found in our simulations as is expected because they did not include transient heat transfer effects in their computations. Ishibashi et.al. [1979], however, found a linear increase of the critical energy with $L_Q$. In their simulations the maximum temperature of the conductor is only 0.6 times the critical value. In this situation the $R_Q$ parameter also predicts $Q_c$ to increase with $L$ over the whole range $L > 0$ since for all simulated disturbance extensions the conductor temperature does not exceed the critical value ($T < T_c$ for all $L$). The result of Ishibashi et.al. is thus not in contradiction with our results.

Table 6.15 : Disturbance extension variation results

<table>
<thead>
<tr>
<th>Extension $L_Q$ (mm)</th>
<th>Critical disturbance $Q_c$ (mJ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>17.5</td>
</tr>
<tr>
<td>10</td>
<td>17.5</td>
</tr>
<tr>
<td>20</td>
<td>17</td>
</tr>
<tr>
<td>50</td>
<td>16.5</td>
</tr>
<tr>
<td>75</td>
<td>16.5</td>
</tr>
<tr>
<td>85</td>
<td>17</td>
</tr>
<tr>
<td>90</td>
<td>17.5</td>
</tr>
<tr>
<td>95</td>
<td>18</td>
</tr>
<tr>
<td>100</td>
<td>18.5</td>
</tr>
</tbody>
</table>

*Disturbance duration variation*

When the duration $t_Q$ of the disturbance is increased, keeping the total amount of energy input constant, no significant change in the critical disturbance for $t_Q$ in the interval $<0.5 \text{ ms}, 4 \text{ ms}>$. For larger disturbance durations an increase in the critical disturbance is observed. The critical disturbance distributed over 10 ms is found to be about 19 mJ, which is 1.5 mJ or about 13% higher than in the reference situation ($t_Q = 1 \text{ ms}$).

The reason for this enhanced stability when the disturbance is spread out over a larger period in time is that the coolant has more time to remove the applied disturbance before the dissipation in the
conductor starts. When the duration of the disturbance is long enough the coolant will remove so much of the heat before the end of the disturbance that the conductor never reaches the current sharing temperature. Since in this case no dissipation occurs the conductor is stable and the critical disturbance will be much higher than in the case of a short disturbance duration.

Comparing our results to the results of dynamic simulations with a constant heat transfer coefficient by Meuris [1984] (see figure 2.4a) we find in both cases an increase of about 13% in the critical energy when the duration of the disturbance is increased to 10 ms. An increase of over 450% as found by Ishibashi et.al. is in contradiction with both our results and the results of Meuris.

6.3.2 Propagation velocity computations

In the previous paragraph the stability of a superconductor, expressed in the value for the critical disturbance energy, has been studied. For disturbance energies below the critical value the normal conducting zone will shrink and finally disappear. For larger disturbances, however, the normal conducting zone will expand. Compared to the cool down time of the magnet, which is typically in the order of weeks, the temperature rise in the conductor during a quench is very fast. As a consequence the mechanical stresses, slowly build up during the cooling down of the magnet, are released quickly and the conductor can get seriously damaged. A quick detection of a quench is thus a necessity to protect the magnet. In practice a quench is detected by measuring the voltage drop over the conductor. When this voltage drop, which is proportional to the normal zone length, rises above a certain threshold value the conductor is de-energized. For a quick detection of a quench it is thus necessary that the voltage drop rises quickly, or in other words that the normal zone expands fast.

In this paragraph the influence of transients on the propagation velocity of the normal conducting zone in case of a quench will be studied. Again the SULTAN inner coil conductor is used in the simulations.

At first the propagation velocity at the reference situation,
defined in table 6.12a will be studied. The numerical parameters differ
from the values given in table 6.12b. To be able to study both the
early transient effects and the propagation velocity for larger times a
variable time step is used. A total time of 100 ms is simulated divided
in 5 steps of 1 ms, 10 steps of 2 ms and 15 steps of 5 ms. The
propagation velocity is computed from

$$\phi(t) = \frac{\Delta L}{\Delta t}$$  \hspace{1cm} (6.44)

$\Delta L$ is the change in the length of the normal zone in the time span $t-\Delta t$
to $t$. Since the conductor temperature is only known on discrete points
in space the position of the edges of the normal zone, where $T = T_{cs}$,
has to be found from the computed temperature profile by interpolation.
If the propagation of the edges of the normal zone is much less than
the mesh size $\Delta z$ the propagation velocity can only be computed very
inaccurately because of the large error in the value of $\Delta L$. This limits
the value of the mesh size. As a first estimate of the mesh size we use
$\Delta z = 5 \text{ mm}$. At a time step of 5 ms this means that at propagation
velocities larger than 1 m/s at the edges propagate at least one grid
point per time step.

The disturbance is applied in the first time step (1 ms) over 20
grid points (100 mm). In order to get a unstable situation the
disturbance energy must be larger than the critical value which is
18.5 mJ for this case (Table 6.15). A value of 100 mJ is selected.
Later also 25 and 500 mJ will be used to find the influence of the
initial energy of the disturbance.

Table 6.16 : Numerical parameters for the reference situation
in the propagation velocity simulations

<table>
<thead>
<tr>
<th>time step</th>
<th>1 ms ( 5 steps)</th>
</tr>
</thead>
<tbody>
<tr>
<td>mesh size</td>
<td>2 ms ( 10 steps)</td>
</tr>
<tr>
<td></td>
<td>5 ms ( 15 steps)</td>
</tr>
<tr>
<td></td>
<td>5 mm ( 100 points)</td>
</tr>
<tr>
<td></td>
<td>Var (2*10 points)</td>
</tr>
<tr>
<td>Applied disturbance</td>
<td>100 mJ</td>
</tr>
<tr>
<td>-- extension</td>
<td>100 mm (20 points)</td>
</tr>
<tr>
<td>-- duration</td>
<td>1 ms (1 step)</td>
</tr>
</tbody>
</table>

184
In order to avoid influences of the tube ends the length of the tube has to be much larger than the size of the normal zone. To accomplish this requirement a large number of grid points are needed which leads to a large computation effort. To minimize this effort without losing accuracy an variable mesh size is used at the tube ends. In the middle of the tube 100 grid points at a constant mesh size $\Delta z$ are used. At both ends of the tube 10 extra points are added. The distance between two of these extra points is every time doubled compared to the distance between the previous points. In this way a much larger tube is simulated than what is possible with a equidistant grid. In order to check the influence of the length of the tube the simulation has been repeated with 200 instead of 100 equidistant grid points. The resulting propagation velocities were identical, which means that the tube is large enough. The relevant parameters for the above described simulation are gathered in table 6.16.

![Graph](image)

--- $\Delta z = 5.0 \text{ mm}$

(Ref - Table 6.6)

--- $\Delta z = 2.5 \text{ mm}$

**Figure 6.33**: Propagation velocity as a function of time.

In figure 6.33 the resulting propagation velocity for the above described reference situation is shown. For short times $\cdots$ is large but
decreases fast. At this stage the temperature profile in the conductor develops. For larger times the normal zone expands at a velocity of about 2 m/s which slowly increases with time. For times up to 25 ms the propagation velocity fluctuates. The time step in the computation is in this stage 1 or 2 ms instead of 5 ms as for larger times. The mesh size is too coarse here. Therefore the simulation is repeated for a smaller mesh size of 2.5 mm (dashed line). Because of the increased computation effort only a smaller time period has been simulated. A smooth curve now results due to the better interpolation of the size of the normal zone. Although the curve for $\Delta z = 5 \text{ mm}$ shows fluctuations the accuracy compared to the $\Delta z = 2.5 \text{ mm}$ case is rather good and a mesh size of 5 mm will be used in all simulations to save computation time.

![Graph showing temperature profiles](image)

**Figure 6.34:** Reference situation: Metal temperature profiles.

For this situation also the temperature profile in the conductor is shown in figure 6.34. From this figure we learn that initially the maximum temperature decreases. After about 25 ms the maximum temperature starts to increase again. Due to the convection of hot
Figure 6.35: Reference situation: Heat flux profiles.

Fluid heat transfer downstream is less effective than upstream as can be seen from figure 6.35. This causes the downstream edge of the normal zone \((T > T_e)\) to propagate faster than the upstream edge and results in an asymmetric temperature profile for large times results. The development of the heat transfer coefficient is shown in figure 6.36. Initially the heat transfer coefficient is high in the heated zone denoted by the dotted lines but soon decreases sharply until a nearly constant value of about 1 kW/m²K results for large times. As the temperature profile becomes broader also the heat transfer coefficient outside the initially heated region decreases. For large times a nearly uniform value results over the whole region.

In figure 6.37 the influence of the applied disturbance on the transient development of the normal zone is shown. Three different disturbances, 25, 100 (Reference situation) and 500 mJ, are compared. For the low disturbance case, \(Q = 25\) mJ, the propagation velocity
Figure 6.36: Reference situation: Heat transfer coefficients.

decreases fast in the first milliseconds. Even negative values emerge, which means that the normal zone shrinks. This means that this disturbance comes near the critical value. However, after about 10 ms the normal zone expands again. For the larger disturbances the propagation velocity is always positive.

After some 50 ms the propagation velocity is the same for all three cases. The size of the normal zone, however, is different because of the transient development. The applied disturbance thus influences the size of the normal zone but has no influence on the propagation velocity after some time.

To find the influence of the ambient pressure on the expansion of the normal zone also simulations at pressures of 3 and 6 bar are performed. The results are compared with the computed propagation velocity in the 12.5 bar case in figure 6.38. The transient development for all three cases does not show much difference. The propagation velocity after 100 ms, however, depends strongly on the fluid pressure. The propagation velocity of 2 m/s for the 12.5 bar case decreases to 1.7 and 1 m/s when the pressure is decreased to respectively 6 and
3 bar. The highest critical disturbances have been found at low pressures (see paragraph 6.3.1). However, these high stability values are combined to a low propagation velocity. At high pressures, $P > 6$ bar, increasing the pressure leads to an increase in both the critical energy as the propagation velocity. This means that a high
system pressure is favorable from a stability point of view and also assures a quick detection of a quench.

Finally in figure 6.39 it is shown that a decrease of the Reynolds number from $10^5$ to $2 \times 10^4$ results in an increase of the propagation velocity from 2 to about 3.5 m/s. A low Reynolds number thus improves the propagation velocity in case of a quench. However, the stability of the conductor is better for higher Reynolds numbers. The optimal mass flow must thus be found by comparing the importance of these conflicting demands. Considering the relative high stability in case of a low mass flow and the high pumping power required to realize a high mass flow in the thin cooling channel of the conductor a low mass flow seems to be recommendable.

Figure 6.39:
Reynolds number variation.
7 Air experiments

7.1 Introduction

In this chapter the results of experiments performed on a transient in the turbulent air flow in a 6 m long test tube as described in chapter 4 will be presented. The primary goal of these experiments is the investigation of a transient flow, induced by a pressure wave, which can be simulated with the mathematical model presented in this thesis. To be able to compare the numerical and experimental results the computation model, as described in chapter 3, had to be slightly adapted in order to fit the experimental situation. The helium property table has been replaced by the formulas for the air properties as described in appendix A. Further also the injection of air by the sudden opening of the magnetic valve connecting the pressure vessel with the test tube has to be implemented as a boundary condition in the model.

In paragraph 7.2 measurements on a stationary air flow will be discussed. Transient flow situations will be studied in paragraph 7.3.

7.2 Stationary flow

In order to check the experimental equipment some experiments on a stationary airflow have been carried out. In paragraph 7.2.1 the pressure gradient in the tube is determined as a function of the Reynolds number of the flow. Velocity measurements, performed with the hot wire probes are described in paragraph 7.2.2. The heat flux sensor, developed in paragraph 4.4, is tested in paragraph 7.2.3 by measuring the stationary Nusselt number at different flow Reynolds numbers.

7.2.1 Pressure gradient

The pressure gradient over the test tube has been determined using the pressure transducer as described in chapter 4. With this transducer the difference \( \Delta P_0 = P_0 - P \) between the pressure in the tube and the ambient pressure at different distances \( x \) from the inlet of the tube
have been measured. The pressure gradient is found by comparing $\Delta P_0$ measured at location $x$ with the value measured at a reference location $x_{\text{ref}}$:

$$\frac{dP}{dx} = \left| \frac{\Delta P_0(x) - \Delta P_0(x_{\text{ref}})}{x - x_{\text{ref}}} \right|$$ (7.1)

For $x_{\text{ref}}$ we have selected the point $x_{\text{ref}} = 5$ m, which is near the outlet of the test tube ($x_{\text{outlet}} = 6$ m). By repeating the measurements for different flow velocities $<u>$ the dependency of the pressure gradient on $<u>$ is found. The value of the mean flow velocity is computed from the pressure difference $\Delta P$ over the venturi meter at the inlet of the tube using equation (4.2):

$$<u> = 1.45 \sqrt{\frac{\Delta P}{\rho}}$$ (4.2)

![Graph showing pressure gradient vs. velocity](image)

**Figure 7.1:** Stationary pressure gradient

In figure 7.1 the experimental results for the pressure gradient $\frac{dP}{dx}$ are shown for three different distances $x$ from the inlet of the
tube. Using a curve fitting program the data can be correlated by the expression:

\[
\frac{dP}{dx} = 0.445 \langle u \rangle^{1.756}
\] (7.2)

with an accuracy of 3%.

In chapter 6 the theoretical relation for the pressure gradient in a smooth tube has been given as:

\[
\frac{dP}{dx} = 4f \frac{0.5 \rho \langle u \rangle^2}{D}
\] (6.1)

where \(D = 54.8 \text{ mm}\) is the hydraulic diameter of the tube. For Reynolds numbers up to \(3 \times 10^4\), which in our case corresponds with \(\langle u \rangle\) up to about \(8 \text{ m/s}\), the friction factor \(f\) can be substituted by the Blasius equation (6.16a), \(4f = 0.316 \text{Re}^{-0.25}\). Using air properties at \(20^\circ\text{C}\) relation (6.1) becomes:

\[
\frac{dP}{dx} = 0.447 \langle u \rangle^{1.75}
\] (7.3)

Our experimental correlation (2) agrees very well with relation (3). The difference is less than 1 % for all flow velocities. Both curves are shown in figure 7.1 for comparison reasons.

In order to check the implementation of the air properties, described in appendix A, in the helium flow model as developed in this thesis, the stationary air flow has also been simulated numerically. The resulting pressure gradients are correlated by:

\[
\frac{dP}{dx} = 0.45 \langle u \rangle^{1.73}
\] (7.4)

which is in good agreement with the theoretical relation (3). However, the dependency on \(\langle u \rangle\) is slightly lower. At a velocity of \(10 \text{ m/s}\) this results in a 4 % deviation from the theoretical value. Compared to our experimental relation the difference is also about 4 %. From figure 7.1 we conclude that the relations (2),(3) and (4) all fit the bulk of the experimental data well.
7.2.2 Flow velocity measurements

In this paragraph the stationary air velocity data measured with the hot wire anemometer as described in paragraph 4.2 will be discussed. Both the mean velocity $\bar{u}$ as the Root Mean Square value of the fluctuating part $u'$ of the velocity ($\text{RMS} = (\bar{u}'^2)^{1/2}$) will be studied.

At first we have positioned the straight hot wire probe P11 at the center line of the tube. The voltage signal from the probe has been stored in the Signal memory recorder (SMR) at a rate of one sample per 100 $\mu$s using 8 k samples per experiment.

![Figure 7.2: Relation between the bulk and maximum velocity in a cross section of the tube.](image)

By comparing the mean hot wire signal with the venturi meter output we were able to find the relation between the maximum velocity $u_{\text{mx}}$ and the bulk velocity $<u>$ in the tube for different flow Reynolds numbers. In figure 7.2 we have combined our results (open points) with experimental data of Stanton and Pannel (solid points connected by the dashed line) for a circular tube as presented in Rohsenow and Choi [1961]. The solid line, computed using a curve fitting program, connects our data for $\text{Re} > 10^4$. In general a good agreement is found
Figure 7.3: Turbulent intensity in the center of the tube as a function of the flow Reynolds number.

for all Reynolds numbers. For Re > 10^4 our results are only slightly (∼ 1 %) higher than the values found by Stanton and Pannel.

From the same experiments we can also find the RMS value of the velocity fluctuations in the center of the tube at different flow Reynolds numbers. In figure 7.3 these data are presented in terms of the turbulent intensity Tu (%) with

\[ Tu = \frac{RMS}{u} \times 100\% \]  \hspace{1cm} (7.5)

In the figure also the results of similar experiments on a tube flow of supercritical Carbon dioxide performed by Bourke et.al. [1969] are presented. The uncertainty area for these experimental data is indicated by the dashed lines. For Reynolds numbers larger than 18000 our results lie above the experimental range of Bourke et. al. This difference, which gets as high as 30 % at Re = 3 \times 10^4, might be caused by the fact that we used a rectangular tube where Bourke et.al. used a circular one.
Figure 7.4: Velocity profiles at three different Reynolds numbers.

By traversing the hot wire probe it is possible to find the variation of the velocity and the RMS over the tube cross section. In order to be able to measure at positions close to the wall the right angle P04 probe has been used. The measurements have been done for three different flow Reynolds numbers, $Re = 0.9 \times 10^4$, $Re = 2.1 \times 10^4$ and $Re = 2.7 \times 10^4$.

In figure 7.4 the resulting velocity profiles are shown. The curves shown are computed from equation (6.10b), the logarithmic profile for a fully developed flow. The values for the parameters $\kappa$ and $C$ have been taken 0.4 and 5.5 respectively, giving:

$$u^+ = 2.5 \ln(y^+) + 5.5$$

(7.6)

Rewriting (6) gives:

$$\frac{u}{u_{ax}} = \frac{u^+}{u_{ax}^+} = \frac{2.5 \ln(y^+) + 5.5}{2.5 \ln(y_{ax}^+) + 5.5} = 1 + \frac{\ln(y^+/y_{ax}^+)}{\ln(y_{ax}^+) + 2.2}$$

196
Figure 7.5: Normalised velocity profiles compared with literature data.

or

\[
\frac{u}{u_{ax}} = 1 + A \ln \left( \frac{y}{y_{ax}} \right) \tag{7.7}
\]

The coefficient \(A = (\ln y_{ax}^+ + 2.2)^{-1}\) will depend on the Reynolds number of the flow. The variation of \(A\) computed using the Blasius equation (6.16a) and air properties at 20°C are gathered in Table 7.1.

<table>
<thead>
<tr>
<th>(Re)</th>
<th>(y_{ax}^+)</th>
<th>(A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(9 \times 10^4)</td>
<td>238</td>
<td>0.1303</td>
</tr>
<tr>
<td>(2.1 \times 10^4)</td>
<td>500</td>
<td>0.1188</td>
</tr>
<tr>
<td>(2.7 \times 10^4)</td>
<td>623</td>
<td>0.1158</td>
</tr>
</tbody>
</table>
As can be seen from the figure, relation (7) with the coefficient \( A \) from table 7.1 fits the experimental data well.

In figure 7.5 the velocity profiles divided by the center velocity \( u_{\infty} \) are compared with literature data of Hussain and Reynolds, taken from Akbari et.al.[1986], and Bourke et.al.[1971].

The data of Hussain and Reynolds are obtained from a fully developed air flow in a channel. The Reynolds number of the flow is \( 1.4 \times 10^4 \). Bourke et.al. performed their experiments in a water flow through a circular tube at a Reynolds number of \( 1.8 \times 10^4 \) using a laser doppler anemometer. In both cases the results agree very well with our experimental data.

![Figure 7.6: RMS profiles for different Reynolds numbers.](image)

By considering the fluctuations in the velocity signal the same series of measurements can be used to find the RMS profile in a cross section of the tube. The results are gathered in figure 7.6. For the high Reynolds number cases, \( Re = 2.1 \times 10^4 \) and \( 2.7 \times 10^4 \), the RMS increases
almost linear from the center of the tube to the wall. Only in the \( \text{Re} = 0.9 \times 10^4 \) case measurements have been carried out close enough to the wall to find the decrease of the RMS down to zero when the wall is approached. The maximum RMS value in this case has been found at \( y/y_{ax} \approx 0.05 \) (\( y^+ \approx 12 \)).

Figure 7.7 : Normalized RMS profile compared to literature data.

Akbari et.al.[1986] have presented turbulence measurements of Laufer in a two dimensional channel flow. The data are given in terms of \( k/u_{ax}^2 \), where \( k = \frac{1}{2}(u'u'_1) \) is the turbulent kinetic energy. Recognizing that the fluctuations on our, one dimensional, velocity signal are in fact composed of velocity fluctuations in two of the three directions (only fluctuations parallel to the hot wire are not measured) the value for the RMS = \( (u'u')^{1/2} \) found from our measurements can be compared to \( (2k)^{1/2} \).

In figure 7.7 we compare our data for \( \text{Re} = 2.7 \times 10^4 \) with the data of Laufer (\( \text{Re} = 3.1 \times 10^4 \)) in terms of RMS/\( u_{ax} \) = \( (2k/u_{ax}^2)^{1/2} \). The increase

199
of the RMS from the center to the wall of the tube in both cases agree well. The values of Laufer, however, are slightly higher than our results, especially near the wall.

In the center of the tube the value of RMS/\( u_{ax} \) is by definition the local turbulence intensity Tu as has been studied above. The value of 4.1% we found (figure 7.7) as well as the value of 4.4% given by Laufer, agree well with our data of Tu as presented in figure 7.3. The turbulence intensity values found by Bourke et al., presented in the same figure, seem to be too low.

7.2.3 Heat transfer

In this paragraph the heat transfer from the hot-film heat flux sensor, developed in this thesis, to the stationary air flow will be studied.

As has been discussed in paragraph 4.4 a large part of the dissipated heat in the sensor disappears by conduction through the substrate. In a stationary situation this heat loss depends on the air flow velocity in the tube and will thus influence the heat transfer results. A correction for the heat transfer coefficient, based on the temperature \( T_{th} \) measured with a thermocouple at the back of the glass plate, has been presented in equation (4.20). In the correction procedure a heat loss coefficient \( c_l \) (equation 4.18) is defined. In order to be able to compute this coefficient at first the heat loss through the substrate, \( Q_s \), will be determined. In equation (4.12) \( Q_s \) has been defined as:

\[
Q_s = \frac{V_0^2}{R_s}
\]  

(4.12)

where \( R_s \) is the electrical resistance of the sensor and \( V_0 \) the voltage required to keep the sensor at a constant temperature \( T_s \) in case the heat flux to the air is zero. This voltage is measured by insulating the sensor with a low thermal conductivity foam. At a sensor temperature of \( T_s = 40^\circ C \) \( (R_s = 29.14 \Omega) \) \( V_0 \) has been found to be 2.907 V. With equation (4.12) this gives a heat loss through the substrate of
\( Q_s = 0.290 \text{ W} \)

The thermocouple temperature at the back of the glass plate reads 30.35°C in this case. Using equation (4.18) this leads to a heat loss coefficient \( c_i \) equal to:

\[
    c_i = \left( \frac{Q_s}{T_s - T_{th}} \right)_o = \frac{0.290}{40 - 30.35} = 0.030 \text{ W/K} \tag{7.8}
\]

Substituting equation (8) in (4.20) the correction \( \Delta h \) on the measured stationary heat transfer coefficient as a function of the change in the thermocouple temperature, \( \Delta T_{th} = T_{th0} - T_{th} \), becomes:

\[
    \Delta h = \frac{c_i}{A} \frac{\Delta T_{th}}{T_s - T} = 16.9 \Delta T_{th} \tag{7.9}
\]

For the flow temperature the value \( T = 20.3°C \) has been used. This temperature was nearly constant during the whole experiment. The surface \( A \) of the sensor is \( 9 \times 10^{-5} \text{ m}^2 \).

Table 7.2 : Experimental heat transfer coefficients

<table>
<thead>
<tr>
<th>Re</th>
<th>( \Delta T_{th} )</th>
<th>( \Delta h )</th>
<th>( h_{\text{meas}} )</th>
<th>( h_{\text{corr}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.24 ( \times 10^3 )</td>
<td>0.54</td>
<td>9.1</td>
<td>53.0</td>
<td>43.9</td>
</tr>
<tr>
<td>9.13 ( \times 10^3 )</td>
<td>0.73</td>
<td>12.4</td>
<td>71.4</td>
<td>59.0</td>
</tr>
<tr>
<td>1.28 ( \times 10^4 )</td>
<td>0.90</td>
<td>15.2</td>
<td>87.5</td>
<td>72.3</td>
</tr>
<tr>
<td>1.64 ( \times 10^4 )</td>
<td>1.10</td>
<td>18.6</td>
<td>101.7</td>
<td>83.1</td>
</tr>
<tr>
<td>1.65 ( \times 10^4 )</td>
<td>1.14</td>
<td>19.3</td>
<td>102.6</td>
<td>83.3</td>
</tr>
<tr>
<td>2.01 ( \times 10^4 )</td>
<td>1.19</td>
<td>20.1</td>
<td>114.0</td>
<td>93.9</td>
</tr>
<tr>
<td>2.37 ( \times 10^4 )</td>
<td>1.29</td>
<td>21.8</td>
<td>125.8</td>
<td>104.0</td>
</tr>
<tr>
<td>2.65 ( \times 10^4 )</td>
<td>1.36</td>
<td>23.0</td>
<td>135.5</td>
<td>112.5</td>
</tr>
<tr>
<td>2.83 ( \times 10^4 )</td>
<td>1.38</td>
<td>23.4</td>
<td>140.7</td>
<td>117.3</td>
</tr>
</tbody>
</table>

After removing the insulating foam the heat transfer has been measured at different flow Reynolds numbers. In table 7.2 the measured
heat transfer coefficients as a function of the Reynolds number have been gathered. Also the change in the thermocouple temperature and the consequent correction $\Delta h$ (equation 9) on the measured data have been included in the table. The increase of $\Delta T_{th}$ indicates that not only the heat flux to the air flow but also the heat loss through the substrate increases when the flow Reynolds number is raised. By comparing the heat loss and the total dissipated heat in the sensor it is found that indeed a large part of the dissipated heat is conducted through the substrate. For $Re = 5.24 \times 10^3$ and $Re = 2.83 \times 10^4$ a heat loss of 84 and 67 % of the dissipated heat has been found respectively.

![Graph showing Nusselt number (Nu) vs. Reynolds number (Re)](image)

**Figure 7.8:** Stationary heat transfer relation for the heat flux sensor. Comparison between experimental and numerical results.

In figure 7.8 the resulting values of the Nusselt number, $Nu = \frac{hD}{\lambda}$, using the corrected value $h = h_{corr}$ as given in table 7.2 are shown as a function of the Reynolds number. The solid line, represented
by:

$$\text{Nu} = 0.506 \, \text{Re}^{0.605}$$  \hspace{1cm} (7.10)

has been found by using a curve fitting program.

In the figure also some numerical results, computed with the simulation program, are shown. In the simulations the wall temperature has been increased to 40°C along a distance of 3 mm. The stationary heat transfer coefficient along this heated length has been determined for different Reynolds numbers. Unlike in the simulations of the heat transfer experiments of Bloem [1986], as described in paragraph 6.2.2, in this situation the numerical tube diameter has been taken equal to the hydraulic diameter of the experimental tube. The wetted perimeter in the experimental and numerical situation are thus not the same. This is, however, not a problem since the heat flux from the wall is not important here as it was in the experiments of Bloem. As a consequence the problem concerning the conversion of the three dimensional experimental tube to the two dimensional numerical situation, as described in paragraph 6.2.2, does not play a role here.

Using a curve fitting program the numerical results can be described by the relation:

$$\text{Nu} = 0.456 \, \text{Re}^{0.608}$$  \hspace{1cm} (7.11)

Over the whole Reynolds number range shown in figure 7.8 the numerical results are only some 5% lower than the experimental values, which is a very satisfactory result for both the experiments as for the numerical simulations.

Comparing the experimental and numerical relations (10) and (11) with the theoretical relation (4.16) we find that the agreement is good. The dependency of the Nusselt number on the Reynolds number in both cases is only slightly higher than the 0.6 power in equation (4.16). We may conclude that the developed heat flux sensor is capable of measuring the stationary heat transfer to the air flow adequately when the described correction procedure is used. The simulation model in which the helium properties are replaced by air properties proves to be capable to predict the heat transfer to a stationary turbulent air flow.
7.3 Transient flow

In this paragraph the transient flow phenomena induced by the sudden injection of an extra amount of air from the pressure vessel into the mainstream in the tube will be studied.

In paragraph 7.3.1 at first the pressure wave generation will be studied. Especially the volumetric air flow rate from the vessel has to be determined. This parameter will be used as a boundary condition in the simulation model.

The induced pressure wave and its influence on the velocity and temperature in the tube as well as the heat transfer to the air flow will be studied in paragraph 7.3.2 experimentally.

Finally in paragraph 7.3.3 the experiments described in paragraph 7.3.2 will be simulated numerically using a slightly adapted helium flow model.

7.3.1 Pressure wave generation

As described in chapter 4 a pressure wave on the stationary flow situation in the tube is generated by suddenly introducing an extra amount of air through the wall into the main stream. This extra air is initially stored under pressure in a vessel which is separated from the tube by a fast opening magnetic valve (see figure 4.1). The induced pressure wave in the tube depends on the introduced flow $\phi_v$ from the vessel into the main stream. In order to be able to simulate the transient flow situation numerically, $\phi_v$ as a function of the pressure $P_v$ in the vessel has to be estimated. Since the duration of an experiment is typically in the order of 10 ms we are interested in the flow from the vessel in a short period after the opening of the valve. To measure this transient flow we have placed a 0.7 m long circular tube ($D=2cm$) axially on the outlet opening of the valve. At the end of this tube the outflow velocity of the air has been determined using the hot wire anemometer as described in paragraph 4.2. Direct measurement of the velocity at the opening of the valve has been found to be too inaccurate because of large velocity fluctuations. At the end of the circular tube ($L=35D$) these fluctuations are smoothed out and more accurate measurements are possible.
Figure 7.9: Outflow velocities from pressure vessel.

In figure 7.9 the velocity signals for different pressures in the vessel are shown. The rise time of the velocity ($v=0$ to $\text{max.}$), consisting of the opening time of the valve and the time to build up the pressure change in the tube, is found to be about 4 ms, independent of the pressure in the vessel. After the initial velocity rise a pronounced maximum and minimum value of the velocity occur. The time difference between these extremes is 4 ms, which is exactly the time period needed by the pressure wave front to travel twice the tube length ($2L = 1.4$ m) at the speed of sound of 340 m/s. This phenomenon is thus clearly caused by the reflection of the pressure wave on the tube ends.

A second maximum in the velocity signal occurs after a time period dependent on the pressure in the vessel. To be able to understand this we have to study the temperature of the air flow from the vessel. Due to the expansion of the air when the magnetic valve is opened the temperature of the air flow from the vessel drops. The time this cold air reaches the end of the tube (the position of the hot wire anemometer) depends on the flow rate and thus on the pressure in the vessel. The velocity signals shown in figure 7.9 are computed from the hot wire voltage output assuming a constant flow temperature. At the moment the cold air from the vessel reaches the hot wire anemometer a
correction for the change in flow temperature is needed. We have measured the change in the flow temperature using the resistance thermometer as described in paragraph 4.3. The resulting changes in the flow temperature for different pressures in the vessel are shown in figure 7.10. The recording of the temperature data started at the moment the cold air reaches the resistance thermometer. The time axis in figure 7.10 is thus shifted compared to figure 7.9 where t=0 represents the moment the pressure wave reaches the end of the tube (≈2ms after the opening of the valve). Using equation (4.9) and the temperature changes from figure 7.10 to correct the velocity measurements, the second maximum disappears. This extreme was indeed caused by the influence of the flow temperature on the velocity measurement.

![Temperature change graph](image)

**Figure 7.10**: Temperature changes due to the expansion of air from the pressure vessel.

After this correction the flow from the vessel can now be evaluated from the data presented in figure 7.9. The mean flow velocity \( \bar{V} \) in the tube is determined by averaging the velocity from the time the velocity becomes constant to the time the cold air starts to influence the velocity measurement. The resulting velocities and consequent
volumetric flow rates \( (\phi_v = \dot{V}) \) as a function of the pressure difference \( P_v - P_0 \) between the vessel and the tube are presented in table 7.3.

In order to get significantly large disturbances in the transient experiments the introduced flow, and thus the pressure in the vessel, must be taken as large as possible. In chapter 4 we found that the magnetic valve has been guaranteed for pressure differences up to 0.35 bar. In our experiments we used \( P_v - P_0 = 0.34 \) bar which leads to a flow of about 10 l/s.

Table 7.3 : Volumetric flow rate from the vessel at different pressures

<table>
<thead>
<tr>
<th>( P_v - P_0 ) (bar)</th>
<th>( \bar{V} ) (m/s)</th>
<th>( \phi_v ) (l/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.45</td>
<td>37.9</td>
<td>11.9</td>
</tr>
<tr>
<td>0.40</td>
<td>35.5</td>
<td>11.2</td>
</tr>
<tr>
<td>0.35</td>
<td>33.3</td>
<td>10.5</td>
</tr>
<tr>
<td>0.30</td>
<td>30.8</td>
<td>9.7</td>
</tr>
<tr>
<td>0.25</td>
<td>28.5</td>
<td>9.0</td>
</tr>
<tr>
<td>0.20</td>
<td>25.4</td>
<td>8.0</td>
</tr>
<tr>
<td>0.15</td>
<td>22.9</td>
<td>7.2</td>
</tr>
<tr>
<td>0.08</td>
<td>17.0</td>
<td>5.3</td>
</tr>
</tbody>
</table>

7.3.2 Results of transient measurements

In the transient experiments the pressure vessel is situated 2m from the entrance of the tube and 2m upstream from the measuring devices (see figure 4.1). The signals from the measuring devices are stored in the Signal Memory Recorder with a sample time of 5 \( \mu \)s (sample frequency 200 kHz). For one experiment 8k samples are recorded distributed from \( t = -20 \) to \( +20 \) ms by using a pretrigger option, \( t = 0 \) indicates the moment the pressure wave arrives at the measuring position.
In order to average out incidental fluctuations occurring in a single experiment every measurement has been repeated at least ten times. The resulting signal from a series of measurements is found by applying the ensemble averaging procedure (equation 4.24) described in chapter 4.

In the experiments four different initial flow conditions, one stagnant and three turbulent ones, have been studied. The mean velocities of the turbulent air flows are respectively:

- \( <u> = 2.5 \text{ m/s} \) (Re = 0.9 \( 10^4 \) ; \( \phi = 7.9 \text{ l/s} \))
- \( <u> = 5.1 \text{ m/s} \) (Re = 1.9 \( 10^4 \) ; \( \phi = 16.1 \text{ l/s} \))
- \( <u> = 7.5 \text{ m/s} \) (Re = 2.7 \( 10^4 \) ; \( \phi = 23.6 \text{ l/s} \))

In figure 7.11 the measured pressure waves, generated by applying an extra flow of 10 l/s (P_\text{v} - P_0 = 0.34 \text{ bar}) from the pressure vessel, are compared for these four situations. The rise time (P=0 to max.) is found to be 4 ms. The maximum value of the pressure wave is found to be slightly dependent on the Reynolds number of the flow. Increasing Re from zero to \( 2.7 \times 10^4 \) we measure a decrease of the pressure top from 680 to 620 N/m². At Re = 0 the induced flow is symmetric around the position of the air inlet (z = 2 m). This means that the maximum value of the pressure wave in this case is the same for up-stream (z < 2 m) and down-stream (z > 2 m) positions: \( \Delta P_\text{up} = \Delta P_\text{down} = \Delta P_0 = 680 \text{ N/m}^2 \). When a stationary flow in the tube is provided the symmetry between the up and down stream positions is broken. Besides the pressure increase in the tube the introduced air flow will now also lead to an increased resistance for the main flow and a consequent pressure drop \( \Delta P_\text{m} \) at the position of the inlet. For the pressure maxima up- and down-stream we find then:

\[
\Delta P_\text{up} = \Delta P_0 + \frac{1}{2} \Delta P_\text{m}
\]
\[
\Delta P_\text{down} = \Delta P_0 - \frac{1}{2} \Delta P_\text{m}
\]

Describing the pressure drop \( \Delta P_\text{m} \) by a resistance number \( K_w \) defined by:

\[
\Delta P_\text{m} = K_w \frac{1}{2} <u>^2
\]
Figure 7.11: Measured pressure changes for different Reynolds numbers.

as is common in engineering, we find $P_{\text{down}} = 620 \text{ N/m}^2$ at a Reynolds number of $2.7 \times 10^4$ ($<u> = 7.5 \text{ m/s}$) when:

$$K_w = 4.3$$ (7.14)

is used. The values of $P_{\text{down}}$ for the other two Reynolds number flows, computed using the relations (12), (13) and (14), then become respectively 673 and 653 N/m$^2$. This is in good agreement with the measured values presented in figure 7.11.

An other peculiarity in the pressure signals is the decrease of the pressure at about $t = 7 \text{ ms}$. Since the first reflection from the tube ends is expected to reach the measuring position at about $t = 12 \text{ ms}$ no pressure change was expected before this time. An explanation for this phenomenon will be discussed after the velocity changes have been studied.
Figure 7.12: Velocity changes induced by the pressure wave measured on the tube axis for different values of the Reynolds number.

In figure 7.12 the velocity change at the center of the tube induced by the pressure wave is shown. For all Reynolds numbers the velocity increase due to the pressure wave is found to be between 1.4 and 1.5 m/s (a variation of 6%) increasing to about 1.7 m/s at $t = 12$ ms. Increasing Re from zero to $2.7 \times 10^4$ has been found to result in a decrease in the pressure top of about 9%. Assuming that $u^2$ is proportional to $\Delta P$ a decrease of about 4.5% in the maximum velocity would be expected. However, this decrease has not been found in our measurements because the variation in the measured maximum velocities is larger than this small effect.

By traversing the hot wire probe in the cross section of the tube the influence of the pressure wave on the velocity profile has been studied. Only the case with a main flow Reynolds number of $0.9 \times 10^4$ has been measured. From figure 7.13a we learn that in the bulk of the flow
Figure 7.13: Velocity changes induced by the pressure wave measured at $Re = 0.9 \times 10^4$ on different distances from the wall.

The velocity transient has the same magnitude on all positions. For values of $y/y_+ \leq 0.022$ ($y^+ \approx 4$) the influence of the wall becomes noticeable (figure 7.13b). Not only the maximum velocity change decreases when the wall is approached, but also a slight decrease is
found for increasing time in the period \( t > 4 \text{ ms} \) after the initial velocity rise. In figure 7.14 the velocity changes are gathered as a function of the distance \( y \) from the wall. In order to reduce the influence of the velocity fluctuations the values of the velocity changes have been found by averaging \( \Delta u \) over a time span from 4 to 7 ms. From the resulting curve in figure 7.14 it can be concluded that the decrease of the velocity change in the vicinity of the wall \( y/y_{ax} < 0.03 \) \( (y^+ < 5) \) is logarithmic. In the bulk of the fluid, \( y/y_{ax} > 0.03 \), a constant value for the velocity change applies. The interesting result is that the influence of the wall on the velocity profile induced by the pressure wave is limited only to the viscous sublayer \( y^+ < 5 \).

![Graph](image)

**Figure 7.14**: Measured velocity changes at \( Re = 0.9 \times 10^4 \) as a function of the distance from the wall.

When the pressure wave reaches its (constant) maximum value the mean velocity in the tube cross section will also become constant. The velocity profile, however, will still change in time. The stationary,
logarithmic, velocity profile and the superposed (almost) constant velocity change will develop in time until a new stationary, logarithmic, velocity profile prevails. In the development of this new profile the velocity in the bulk of the flow will rise while near the wall the velocity decreases. Using equation (6) for the stationary velocity profile the increase of the velocity at the center of the tube in the new stationary situation can be estimated by computing the velocity at the Reynolds numbers before and after the pressure wave. For all cases a velocity increase of 1.7 m/s, compared to the change of 1.4 m/s at t = 4 ms, is found. The development of the new velocity is governed by a turbulent diffusion process which can be characterized by the turbulent diffusion coefficient \( \mu_t \). The turbulent diffusion time scale can be estimated by:

\[
t = \frac{1}{4} \frac{L^2}{v_t}
\]  

(7.15)

An estimate for \( \mu_t/\mu \) we have found from our numerical simulations. Using a curve fitting program we arrived at:

\[
\frac{\mu_t}{\mu} = 7.5 \times 10^{-3} \, \text{Re}^{0.89}
\]  

(7.16)

what results in values of \( \mu_t/\mu \) between 20 and 70 in the considered Reynolds number range. Substituting (16) into equation (15) and estimating the length scale L by \( R/2 \) results in:

\[
t = 2.1 \frac{D^2}{v} \, \text{Re}^{-0.89} = 420 \, \text{Re}^{-0.89}
\]  

(7.17)

For all simulations this time scale is between 40 and 100 ms which is much larger than the duration of our experiment. The fast increase of u in time found for all Reynolds numbers in figure 7.12 must thus have an other cause. This cause can be made clear by comparing the velocity and pressure signals after normalization as presented in figure 7.15. The pressure is divided by \( \Delta P_{\text{down}} \) as defined in equation (12). For the velocity a normalization factor 1.4, being the velocity just after the pressure reaches its maximum, is used. For all four Reynolds number cases we find that the normalized pressure and velocity signals are nearly identical up to \( t = 6.5 \) ms. For larger times the pressure is decreased as has already been noticed in figure 7.11. From figure 7.15 we now see that at the same time the pressure starts to decrease the
Figure 7.15:

Normalized pressure and velocity measurements.
velocity increases. At \( t = 12.5 \text{ ms} \) the changes in the pressure and velocity are both about 15\% in all cases. It is clear that these changes have to be coupled. An increasing velocity due to a decreasing pressure can only be found when a negative pressure wave traveling upstream passes the hot wire anemometer. This means that the pressure wave which passes the hot wire at \( t = 0 \text{ ms} \) has to reflect after a distance \( L \) at an open boundary returning after 6.5 ms as a negative wave at the measuring position. At a speed of sound equal to \( c = 340 \text{ m/s} \) the distance \( L \) can be computed from \( 2L = t \cdot c \) resulting in \( L \approx 1.1 \text{ m} \). Re-examining the experimental set up learned that the hole in the wall at 0.95 m from end of the tube, which had been used for the stationary pressure measurements (paragraph 7.2.1), had not been properly closed. This hole which is about 1.1 m downstream from the measuring position caused the reflection of the pressure wave and the consequent pressure decrease and velocity increase at \( t \approx 6.5 \text{ ms} \). The measured data for times larger than 6.5 ms after the pressure wave reaches the measuring position are thus polluted and can not be used for comparison with numerical data.

In figure 7.15 there is another phenomenon which has to be explained. In the stagnant flow case the velocity signal is retarded up to \( t = 3 \text{ ms} \) compared to the pressure signal. For the other cases (\( \text{Re} > 0 \)) no delay in the velocity signal is observed. In the case \( \text{Re} = 0 \) the heat dissipated from the hot wire is not convected away like
in the other cases (Re > 0). As a consequence the temperature of the surrounding stagnant air is raised in this situation. At the moment the velocity starts to increase at first this heated air is blown along the hot wire. The voltages measured with the hot wire anemometer are converted to velocities using equation (4.9):

\[ \frac{V^2}{\Delta T} = A + B u^n \]  

(4.9)

in which \( \Delta T \) is the temperature difference between the wire and the surrounding air. Since we assume the fluid temperature to be constant, \( \Delta T \) in the stagnant flow case is overestimated. Until the heated air has passed the hot wire the velocities found from (4.9) will consequently be too low. The lag in the velocity signal found in figure 7.15a is thus not a physical phenomenon but is caused by the error in the conversion of the hot wire signal.

An other temperature effect which may influence the velocity measurements is caused by the compression of the fluid when the pressure rises. Theoretically this effect can be estimated by assuming the transient flow to be adiabatic. Because of the short duration of the experiment this is a reasonable assumption. For an adiabatic process the temperature and pressure are related by the Poisson law:

\[ T^K P^{1-K} = \text{Constant} \]  

(7.18)

For an ideal gas \( \kappa = C_p/C_v = 1.4 \). Rewriting (18) leads to:

\[ \Delta T_{sd} = T_0 \left[ \frac{P_0 + \Delta P}{P_0} \right]^{\frac{\kappa - 1}{\kappa}} - 1 \propto \frac{\Delta P}{\rho C_v} \]  

(7.19)

In case of stationary situation with \( P_0 = 1 \) bar and \( T_0 = 293 \) K, equation (19) predicts a temperature increase of 0.54 K at a pressure difference \( \Delta P = 650 \) N/m².

To verify the adiabatic flow assumption a few experiments have been performed in which the pressure and temperature are measured simultaneously. In all cases the temperature change in the tube follows the pressure change closely. The values of the measured temperature change were all found to be within 3% of the adiabatic value computed from equation (19).
In the conversion of the voltage output from the hot wire anemometer to velocity data (equation 4.9) the difference between the wire and fluid temperature $\Delta T$ is assumed to be equal to the initial value which is about 200 K. As we have seen above the air temperature changes about 0.54 K due to the compression by the pressure wave. The value of $\Delta T$ is thus changed by 0.27%. Using equation (4.9) the influence of this error in $\Delta T$ is found to result in an error of 1% in the measured velocity. We have neglected this small error.

Simultaneously with the pressure and velocity in the tube also the transient heat transfer to the air flow has been measured using the heat flux sensor as developed in paragraph 4.4. The voltage output V of the sensor can be converted into transient heat transfer coefficients $h$ using equation 4.13:

$$h = \frac{V^2/\mathcal{R}_s - Q_s}{(T_s - T)} A$$

$$A = 9 \times 10^{-5} \text{ m}^2$$

(4.13)

The temperature $T_s$ and consequently also the resistance $\mathcal{R}_s$ of the resistance are kept constant in time. Also the heat loss $Q_s$ through the substrate is assumed to be constant during the short measuring time.

For the conversion the bulk temperature $T$ of the fluid has to be known as a function of time. However, it was not possible to measure the temperature together with the other quantities. An approximation $h_{x0}$ of $h$ has been found from equation (4.13) by assuming $T = T_0$. Afterwards the heat transfer coefficient has been corrected for the transient temperature increase $\Delta T_{sd}$ (equation 19) using:

$$h = h_{x0} \left( \frac{T_s - T_0}{T_s - T} \right) = h_{x0} \left( T_s - T_0 \frac{T_s - T_0}{T_s - T + \Delta T_{sd}} \right)$$

(7.20)

In figure 7.16 the transient change $\Delta h = h - h_s$ from the stationary value $h_s$ is shown for the three initially turbulent flow cases ($Re > 0$). The change in the heat transfer coefficient for $0 < t < 4 \text{ ms}$ is almost independent of the Reynolds number of the flow. For larger times the largest values of $\Delta h$ are found for the lowest Reynolds number. This is easily explained by looking at the dependency of the stationary heat transfer coefficient on the Reynolds number as has been found in paragraph 7.2.3. In equation (10) we found that
Figure 7.16: Changes in the heat transfer coefficient induced by the pressure wave for different Reynolds numbers.

\[ \text{Nu} = \frac{h D}{\lambda} \] is proportional to \( \text{Re}^{0.605} \). This means that for a constant increase \( \Delta \text{Re} \) of the Reynolds number the change in the heat transfer coefficient is larger for low than for high values of \( \text{Re} \). A better comparison between the measured heat transfer coefficients than from figure 7.16 can be made by normalizing the presented results.

Table 7.4: The stationary value \( h_s \) and the change \( \Delta h_s \) of the heat transfer coefficient after an increase of the Reynolds number \( \text{Re}_0 \) with \( \Delta \text{Re} = 0.55 \times 10^4 \).

<table>
<thead>
<tr>
<th>\text{Re}_0 \times 10^4</th>
<th>h_s</th>
<th>\Delta h_s</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9</td>
<td>58.5</td>
<td>19.4</td>
</tr>
<tr>
<td>1.9</td>
<td>90.1</td>
<td>15.3</td>
</tr>
<tr>
<td>2.7</td>
<td>113.9</td>
<td>13.3</td>
</tr>
</tbody>
</table>
suitable normalization factor for $\Delta h$ is the difference $\Delta h_s$ between the stationary value of the heat transfer coefficients $h_s$ for $t \to \infty$ ($Re = Re_0 + \Delta Re$) and the initial value $h_s$ at $t < 0$ where $Re = Re_0$ (see table 7.4). For all flow situations the maximum value of $\Delta h/\Delta h_s$ is 1. This value will be reached as $t \to \infty$.

![Normalized heat transfer coefficients](image)

Figure 7.17: Normalized heat transfer coefficients.

The heat transfer results after normalization of $\Delta h$ by $\Delta h_s$ are shown in figure 7.17. From this figure we find that at the moment the mean velocity $<u>$ in the tube gets constant ($t = 4$ ms) $\Delta h$ in the largest Reynolds number case ($Re = 2.7 \times 10^4$) has reached about 55% of its maximum value. In the other cases ($Re = 1.9 \times 10^4$ and $Re = 0.9 \times 10^4$) the relative change in $\Delta h$ is, about 45% and 40% respectively. The effect of the velocity changes in the bulk of the flow on the temperature gradient near the wall are largest for the highest Reynolds number. This effect is explained by the fact that the thickness $\delta_T$ of the temperature boundary layer near the wall reduces as the Reynolds number of the flow is increased. At high Reynolds numbers changes in
the bulk of the flow will thus penetrate faster through the boundary layer than at low Reynolds numbers leading to a faster change in the heat transfer at the wall. Also the fast response of $\Delta h$ for the higher Reynolds number cases at the moment that the mean velocity in the tube gets constant ($t \approx 4$ ms) can be explained by the same reasoning. For larger times ($t > 4$ ms) the heat transfer increases in all situations until a new stationary situation is reached where $h = h_\infty$.

In figure 7.17 all variables are made dimensionless except the time $t$. For the normalization of the time axis the governing time scale in the process should be used. For the first part of the experiment, $t < 4$ ms, we found that $\Delta h$ changed faster when the value of $Re$ was increased. The governing time scale $t_n$ is thus proportional to $Re^n$ $n > 0$. Due to the uncertainties in our heat transfer experiments the value of $n$ could not be determined from the measured data. Also in the second part of the experiment, $t > 4$ ms, the governing time scale could not be found. In order to find this time scale, which describes the asymptotic behavior for large times, experimental data over a time range in the order of this time scale should be available. Assuming that the heat transfer process for large times is described well by the turbulent diffusion time scale $t = (R/2)^2/\nu_t$ (equation 15), the measuring time should be in the order 40 to 100 ms. Our measurements are thus much to short to be able to give information over the asymptotic behavior as $t \to \infty$.

Finally we will investigate the change in the turbulence level of the flow at the axis of the tube. As has been described in chapter 4 two different methods for determining the RMS of the velocity signal can be distinguished. From a single experiment the RMS can be found by a time averaging procedure (equation 4.23):

$$\text{RMS} = \left[ \frac{1}{N-1} \sum_{j=1}^{N} (u_j - \overline{u})^2 \right]^{1/2} \quad (4.23)$$

in which $u_j$ is the instantaneous velocity at $t_j < t < t_j + \Delta t$ with $\Delta t$ the sample time. $N$ is the number of samples involved in the sampling procedure and $\overline{u}$ the time averaged velocity. This method, however, is only applicable when the mean velocity is constant in the time period over which the averaging is carried out. For an instationary flow, in
which the mean velocity changes in time, an ensemble averaging procedure is more appropriate. The ensemble RMS \( \{\text{RMS}_j\} \) in the sample time interval \( t_j < t < t_j + \Delta t \) is then found by comparing \( N_e \) velocity signals \( u_i \) of identical experiments using equation (4.25):

\[
\{\text{RMS}_j\} = \left[ \frac{1}{N_e-1} \sum_{i=1}^{N_e} [u_i - \{u_j\}]^2 \right]^{1/2}
\] (4.25)

As has been described above the velocity signals presented in figure 7.12 are ensemble averages over 10 experiments. The ensemble RMS computed from the same experiments using equation (4.25) are shown in figure 7.18.

![Figure 7.18](image)

Figure 7.18: Ensemble RMS values computed from the velocity signals at the tube axis.

For the stagnant flow case the RMS in the stationary situation \( (t < 0) \) is zero as expected. After the pressure wave reaches the measuring position at \( t = 0 \) the RMS starts to increase until at \( t = 4 \) ms the maximum value is reached at the same moment the pressure
wave reaches its maximum. Besides some small fluctuations the RMS is constant and equal to 0.10 m/s for t > 4 ms. At an induced velocity of 1.5 m/s this leads to a turbulent intensity of about 6.5%. Comparing this value with the value for a stationary flow at the same velocity, as has been presented in figure 7.3, we find that this value, although high, is within the experimental uncertainty area. This means that at the axis of the tube the stationary turbulence level seems to be reached very quickly, the RMS follows the <u> within a fraction of a millisecond. This is a surprising result since for the production of turbulence velocity gradients (shear) are required. The mean velocity induced by the pressure wave, however, is constant over the whole cross section except for a small region near the wall. The turbulence produced in the wall region is transported to the bulk of the flow by diffusion. The time scale of this process has been found to be about 40-100 ms. This means that the fast turbulence increase in the bulk can not be caused by production in the wall region. Since the mean induced velocity in the bulk has been found to be constant also local velocity gradients can not be the source of the turbulence production. As a consequence it is likely that the measured fluctuations on the velocity signal are induced directly by pressure fluctuations. The fluctuations on the pressure signals shown in figure 7.11 seem, however, not large enough to account for the observed RMS values. This is due to the fact that in figure 7.11 ensemble averages of the pressure signals are shown in which the high frequent fluctuations are smoothed out. Reexamining the original pressure data, fluctuations in the order of 10 to 15% are found. Assuming that P ∝ u² the pressure fluctuations result in velocity fluctuations of 5 to 7.5%. This is in the same order of the observed RMS values. We may conclude that the fluctuations on the pressure signal are responsible for the observed RMS values of our velocity signals.

7.3.3 Numerical simulations

7.3.3.1 Introduction

In order to be able to simulate the transient air experiments numerically the original helium flow model has to be slightly extended.
A new boundary condition describing the sudden air injection through the wall has to be introduced. In practice this condition is provided by describing a constant radial velocity at the wall on the position of the pressure vessel. In the numerical model this leads to an extra source term in the transport equations for the points near the wall. Also the mass balance (3.30) near the inflow has to be corrected for the non zero mass flow \((\rho v A)_{n}\) at the north interface of a point near the wall.

In the experimental situation the air is introduced through a small hole in the bottom of the tube. Because of the cylinder symmetry of the numerical model the air inlet in this case has to be distributed over the whole surface of the cross section. When the surface of the air inlet is kept the same as in the experimental set-up,

\[
\frac{1}{4}\pi D^2 = A_{\text{exp}} = A_{\text{num}} = \pi D \Delta z,
\]

the maximum allowed grid size \(\Delta z\) becomes about 0.5 mm. This results in an enormous amount of grid points to simulate the entire 6m long tube. Therefore we have increased the grid size and reduced the velocity of the introduced air flow by a factor 20 keeping \(\phi_v = \varphi A\) constant. In this way only a reasonable amount of grid points (\(N_z = 60, \Delta z = 0.1\) m) is required.

An other consequence of the cylinder symmetry in the simulation model is that the cross section \(A_{\text{num}} = \frac{1}{4}\pi D^2\) of the numerical tube differs from the experimental cross section \(A_{\text{exp}}\). The diameter \(D\) of the numerical tube has been taken equal to the hydraulic diameter \(D_h = 54.8\) mm. This results in:

\[
\frac{A_{\text{num}}}{A_{\text{exp}}} = \frac{0.25\pi(54.8)^2}{45 \cdot 70} = 0.75
\]

(7.21)

This means that when we keep the Reynolds number (and the mean velocity \(<u>\)) of the stationary flow in the numerical equal to the value in the experimental situation the mass flow \(m = \rho <u> A\) is numerically 25\% lower than experimentally. In the same way also the numerically induced transient flow is the same as in the experiments only when the applied mass flow from the vessel is 25\% lower than experimentally. In the simulations the value of \(\phi_v\) has thus to be set equal to 7.5 l/s instead of 10 l/s like in the experiments. At the mesh size used to simulate the experiments described below, \(\Delta z = 0.1\) m, the boundary condition for the radial velocity at the inlet position becomes \(\bar{v} = 0.4425\) m/s.
7.3.3.2 Simulation of the transient air experiments

The transient air experiments presented in paragraph 7.3.2 have been simulated with the above described adapted numerical model. In the simulations the FI scheme (see chapter 3) has been selected to describe the time differencing. In order to restrict the numerical dispersion occurring in the solutions for this scheme a small time step equal to $\Delta t = 0.25$ ms has been used. As has been found in paragraph 6.2.1 a smaller time step gives rise to problems concerning the converge of the solution. In all simulations the Schnurr grid (equation 3.1) has been employed with the parameter $C_s$ equal to 2 and the number of radial grid points $N_r$ equal to 20.

Figure 7.19: Computed pressure changes for different Reynolds numbers.
In the simulations we have studied the same Reynolds numbers for the main flow as have been used in the experiments:

- \( \text{Re} = 0.0 \ 10^4 \)
- \( \text{Re} = 0.9 \ 10^4 \)
- \( \text{Re} = 1.9 \ 10^4 \)
- \( \text{Re} = 2.7 \ 10^4 \)

For these situations the mean velocity is the same as in the experimental situation because the hydraulic diameters are the same. The mass flow, however, is proportional to the surface \( A \) of the cross section. Since \( A_{\text{exp}} > A_{\text{num}} \) (equation 21) the numerical mass flow is 25\% smaller than in the experimental situation.

The presented results are always computed at a distance of 2 m from the air inlet.

The computed pressure increase \( \Delta P = P - P_0 \) for the stagnant flow case is found to be 655 Pa (see figure 7.19). This is only about 4\% lower than the experimental value of 680 Pa. The decrease of \( \Delta P \) for increasing Reynolds numbers, as has been observed in the experiments, is also found numerically. At \( \text{Re} = 2.7 \ 10^4 \) the value of \( \Delta P \) is reduced to 610 Pa which is within 2\% of the experimental value of \( \Delta P = 620 \) Pa. The numerical and experimental pressure waves, gathered in figure 7.20, are found to be in close agreement. The numerical rise time (\( P=0 \) to \( \text{max.} \)), however, is found to smaller than in the experimental situation. Due to the numerical dispersion effects a larger numerical rise time would have been expected. However, the found effect is caused by the difference between the experimentally injected air flow at the location of the pressure vessel and the numerical description of this flow in the simulation program. Numerically the velocity of the injected air is assumed to reach its maximum value instantaneously at the moment the magnetic valve opens. In the experimental situation, however, the valve will not open instantaneously and consequently it will take some time before the flow from the vessel reaches its maximum value. As a consequence the pressure wave in the tube will experimentally build up slower than numerically. The rise time in both situations can thus not be compared. A comparison of the measured and computed effects of the pressure wave after the maximum value has been reached can be done.
Figure 7.20: Comparison of the measured and computed pressure transient.

At first we will investigate the influence of the pressure wave on the velocity in the tube. From the presented results in figure 7.21 we find that the induced velocity on the axis of the tube depends on the Reynolds number of the flow. However, the differences between the results for the different Reynolds number cases are rather small. Increasing Re from zero to $2.7 \times 10^4$ results in a velocity decrease of
Figure 7.21: Velocity changes induced by the pressure wave computed on the tube axis for different Reynolds numbers.

only 0.07 m/s which is about 4% of $\Delta u$. In the experiments the induced velocity seemed to be independent of Re. However, due to the uncertainty in the measurements a dependency on Re as small as in the computed results can not be detected from the experiments.

The size of the induced velocity $\Delta u$ on the axis of the tube, as presented in figure 7.21, has been computed to be about 1.6 m/s. Experimentally a somewhat lower value, between 1.4 and 1.5 m/s (see figure 7.22), has been found although the pressure wave is slightly higher. Considering the fluctuations and the possible errors in the experimental velocity data the overall agreement is pretty good.
Figure 7.22: Comparison of the measured and computed velocity transient.

The induced velocities on different distances $y$ from the wall are compared for $Re = 0.9 \times 10^4$ in figure 7.23. In the bulk of the flow the velocity development is almost independent of $y$ (figure 7.23a). From $y/y_{ex} \approx 0.03$, however, the influence of the wall becomes noticeable. After the maximum is reached at $t \approx 4$ ms the velocity reduces again due to the friction influence of the wall. In the wall region, $y/y_{ex} < 0.02$ as shown in figure 7.23b, the influence of the wall becomes so strong.
that also the maximum value of the velocity change $\Delta u$ becomes lower. These results agree very well with the experimental dependency on the distance $y$ as presented in figure 7.13. Experimentally the influence of the wall also becomes significant at $y/y_{\infty} \approx 0.03$. The decrease of the maximum velocity when the wall is approached is, however, a little smaller than we have found numerically but the difference falls within the experimental error band as can be seen in figure 7.24. In this figure the change in the numerical velocity profiles on three different times ($t = 4$, 7 and 10 ms) are compared with the experimental velocity increase after division by the value $\Delta u_{\infty}$ at the axis of the tube. The experimental data, as presented before in figure 7.17, are averaged values over the time period from 4 to 7 ms. The experimental error, caused by the uncertainty of 0.1 mm in the position $y$ of the hot wire probe relative to the wall, has also been marked in the figure. Comparing the numerical and experimental data we see that in both cases the decrease of the velocity near the wall starts at a distance of $y/y_{\infty} \approx 0.03$ ($y^+ \approx 5$). In the in the bulk of the flow and for values of
Figure 7.24: Computed profiles for the velocity change at $Re = 0.9 \times 10^4$ compared with experimental data.

$y/y_{ax}$ down to 0.01 the agreement is found to be within 5%. Closer to the wall the experimental velocity change exceeds the numerical value. When the error in the hot wire position is taken into account also for these data the agreement with the numerical values is good.

Due to the compression of the air by the pressure wave the temperature of the flow is increased. By assuming the flow to be adiabatic in paragraph 7.3.2 this temperature change $\Delta T$ has been found to be expressed by equation (19). At a flow temperature of $20^\circ C$ (293 K) a pressure change of 655 Pa as has been found for the stagnant flow case equation (19) results in a temperature change $\Delta T$ equal to $0.55^\circ C$. This value also emerged from our numerical simulations. For all other simulated flow situations the change in the flow temperature has also been found to be exactly equal to the adiabatic value given by equation (19).
Figure 7.25: Changes in the heat transfer coefficient induced by the pressure wave computed for different Reynolds numbers.

Finally we will compare the numerical heat transfer results with the experimental results as presented in figure 7.16. In the numerical heat transfer coefficients as shown in figure 7.25 we observe a decrease just after \( t = 0 \). The cause for this decrease has been found to be a small temperature increase near the wall due to compression of the fluid. This causes the temperature gradient near the wall to decrease and consequently also the heat transfer. Due to the fluctuations in the measured heat transfer signals this effects could not be observed experimentally.

The global trend of the numerical results is the same as in the experimental results. After a sharp rise of \( \Delta h \) which seems to be the about the same for all situations the changes in the heat transfer coefficient become dependent on the Reynolds number as has been found experimentally. Also the value of \( \Delta h \) is of the same order as in the measurements.
For the measurements it has been found that by a normalization of $\Delta h$ by $\Delta h_0$ a better comparison of the results for different Reynolds numbers is possible. In figure 7.26 both the numerical and experimental results after normalization are shown. The qualitative agreement between the experimental and numerical results is over the whole range quite good. The numerical changes in $\Delta h$, however, are somewhat smaller than has been found experimentally. Part of the difference is due to the decrease of $\Delta h$ found in the numerical simulation just after the pressure wave arrives at the measuring position. The rest of the difference falls within the experimental inaccuracy.

Figure 7.26: Comparison of the measured and computed heat transfer transients after normalization.

From the presented experimental and numerical transient heat transfer results we may conclude that the simulation model describes the heat transfer in a transient turbulent air flow well. Since also the computed pressure, velocity and temperature changes in the flow were in good agreement with the experimental results our two dimensional flow simulation model has been shown to be a good tool for predicting transient phenomena in a turbulent flow of a variable property fluid.
8 Discussion and conclusions

8.1 Transient flow

Theoretically the propagation velocity of a pressure wave travelling through a fluid is equal to the thermodynamic speed of sound. From our simulations on the heat induced pressure wave in a turbulent supercritical helium flow the numerical propagation velocity of the wave is found to be in good agreement with the theoretical value. Due to numerical dispersion the foot of the wave propagates somewhat too fast, leading to a small decrease of the maximum value of the pressure. This numerical dispersion effect has been found to be reduced when the time step is decreased and when a higher order time differencing scheme, like the B3 scheme, is used. The difference between the first order FI scheme and the second order B3 scheme has been illustrated by comparing our results with pressure maxima measured by Bloem [1986]. At the place of the thermal disturbance both the FI and B3 scheme give a good prediction of the maximum value of the pressure. At two meters distance the agreement between the results of the B3 scheme and the experimental results is still good. The prediction of the pressure maximum computed with the FI scheme, however, is much too low. A disadvantage of the B3 scheme is the fact that this scheme produces unrealistic oscillations called wiggles in the solution after the pressure maximum. In the helium flow case the induced wave is pulse shaped and the wiggles do not influence the pressure maximum or the propagation speed and the B3 scheme is to be preferred. In the air flow situation the pressure wave is more like a step function in time and the wiggles occur at the moment the maximum value is reached. This influences both the value and the time of occurrence of the maximum. In this case the FI scheme yields more realistic results as long as the time step is small enough.

The spatial differencing scheme has much less influence on the computed pressure wave. By replacing the first order upwind scheme by the third order Exquisite scheme in our model almost the same results are found. For our application the Exquisite scheme has no advantage over the much more economic Upwind scheme and has not been used.
The pressure wave travelling through the fluid influences the structure of the flow. In the bulk of the flow the velocity is increased by a value $\Delta u$ independent on the distance $y$ from the wall. Near the wall the velocity change is found to be reduced by the friction effects in a very small region: $y/y_{ax} < 0.03$ ($y^+ < 5$). This means that the friction of the wall influences the pressure induced velocity change only in the viscous sublayer of the flow. This effect, which we have found from our air flow experiments, is also predicted well by our numerical simulation model.

After the pressure has reached its maximum value ($t \approx 4$ ms) the Reynolds number in a cross section reaches a new constant value. Within the cross section the velocity profile then evolves slowly to the stationary turbulent velocity profile belonging to the new Reynolds number. The time scale of this turbulent diffusion process is about ten times larger than the transient time scale at the conditions of our tests.

Besides the velocity also the turbulence in the tube is affected by the pressure wave. However, due to the velocity fluctuations induced by pressure fluctuations the change in the turbulence level generated by the increasing velocity in the tube could not be measured.

8.2 Transient heat transfer

The transient heat transfer to a turbulent flow after a change in the heat flux from the wall can for a short period be described by a penetration model. After a long time period a new stationary situation will evolve in which the heat transfer can be described by a time independent Nusselt-Reynolds relation. The transient penetration and the stationary situation as well as the transition period can be described adequately with a new, transient, surface renewal model developed in this thesis. The values of the parameter $\tau$, the renewal time, found afterwards by comparing our results for different values of $\tau$ with experimental results of Bloem [1986], are much smaller than the values emerging from stationary surface renewal theories as given in literature.

The experiments of Bloem have also been used to validate the two dimensional flow model developed in chapter 3. Taking the geometry
differences between the experimental and numerical situation into account the results of our simulated describe the transient heat transfer experiments of Bloem adequately. Only in case of a low mass flow or a high imposed heat flux the agreement between the experimental and numerical results for large times is poor. This has been found to be caused by buoyancy effects occurring in the experiments which have not been incorporated in our simulation model.

In our air flow experiment the transient heat transfer after a change in the flow velocity has been investigated. At large Reynolds numbers the thermal boundary layer is thin and the changes in the flow will penetrate faster to the wall than for low values of the Reynolds number. The heat transfer after a change in the flow will thus adapt faster to the new flow situation as the Reynolds number is increased. This effect has been found both experimentally, using a self-designed heat flux sensor based on the hot-film technique, and numerically, using our two dimensional flow model. Also for this situation a good agreement between the heat transfer results computed with our two dimensional flow model and the experimental results is found.

The two dimensional flow model developed in this thesis has been validated by the experiments of Bloem and our own air flow experiments. In all cases good agreement between the measured and computed results has been found. Our simulation model has shown to be a good tool for predicting the hydrodynamic and heat transfer phenomena in a turbulent pipe flow under transient conditions.

8.3 Stability of the SULTAN conductor

As an application of the simulation model developed in this thesis we have studied the stability of the SULTAN magnet of ECN. For a reference situation \((P = 12.5 \text{ bar}, \ Re = 10^5, \ T = 4.2 \text{ K})\) we have computed the critical disturbance. The value of \(Q_c = 17.5 \text{ mJ}\) is slightly higher than the rough estimate of 15 mJ found by Cornelissen [1984]. The critical energy found from models which do not include the effects of flow transients on heat transfer are typically a factor 3 to 4 lower than our estimate.
By varying the Reynolds number of the flow a relatively high stability is found for low values of Re due to the influence of the heat induced flow in the tube. As a result the critical energy is almost constant for Re < 0.5 \times 10^5. For higher values of the Reynolds number the critical energy increases with Re. A high mass flow in the tube is thus favorable for the stability of the conductor. However, in order to realize a high mass flow a large pumping power is required. A second disadvantage is that the propagation speed in case of a quench reduces for increasing mass flows. Considering the still relatively high stability at low Reynolds numbers a low helium mass flow in the conductor is recommended.

The stability of the conductor has also been found to depend strongly on the ambient pressure of the helium in the tube. A minimum in \( Q_e \) has been found at \( P = 6 \) bar. The largest critical disturbances have been found for low pressures. However, at low pressures the propagation velocity of a quench is much lower than for higher pressures. An other disadvantage of a low ambient pressure is the fact that the total pressure drop over the conductor can be as high as 2 bar. In order to keep the pressure above the critical value (2.27 bar) the pressure at the inlet must be at least 2 bar higher. At an outlet pressure just above the critical value, say 3 bar, the inlet pressure must be 5 bar which is quite near the value of 6 bar for which the critical disturbance \( Q_e \) has its minimum. As a result the value of \( Q_e \) will depend strongly on the location of the disturbance in the conductor. At the outlet of the conductor the value of \( Q_e \) will be high due to the low helium pressure at this position. However, the critical disturbance will decrease considerably when the disturbance is released near the inlet of the conductor. As a consequence the stability of the whole conductor is worse than at a high pressure. Combined with the advantage of a high propagation speed it can be concluded from our investigation that a high helium pressure (\( P = 10-15 \) bar) results in the best performance of the conductor.
Appendix A - Air properties

Considering air to be an ideal fluid the density can be described by:

\[ \rho = K \frac{P}{T+273.15} \quad \left[ \frac{\text{kg}}{\text{m}^3} \right] \]  \hspace{1cm} (A.1)

At a pressure \( P = 10^5 \text{ Pa} (=1 \text{ bar}) \) and a temperature \( T = 20^\circ \text{C} \) the density equals 1.2 \( \text{kg/m}^3 \). The resulting value of the constant \( K \) is thus 3.5 \( 10^{-3} \text{ Ks}^2/\text{m}^2 \).

For the relation between enthalpy and temperature equation (4.45) applies:

\[ \mathrm{d}H = C_p \; \mathrm{d}T \]  \hspace{1cm} (4.45)

The specific \( C_p \) is independent of the temperature and pressure of the fluid:

\[ C_p = 1.0 \quad \left[ \frac{\text{kJ}}{\text{kg} \; \text{K}} \right] \]  \hspace{1cm} (A.2)

The thermal conductivity and the viscosity are also nearly independent of pressure. A linear dependency on the temperature (\( T \) in \( ^\circ \text{C} \)) is assumed [Janssen and Warmoeskerken(1982)]:

\[ \lambda = 0.024 + 74 \times 10^{-6} \; T \quad \left[ \frac{\text{W}}{\text{mK}} \right] \]  \hspace{1cm} (A.3)

\[ \mu = 17.2 \times 10^{-6} (1 + 0.00285 \; T) \quad \left[ \frac{\text{Ns}}{\text{m}^2} \right] \]  \hspace{1cm} (A.4)
References


Arp, V., New forms of the state equations for helium, Cryogenics, 14, 593, (1974)


Bourke, P.J., Brown, C.G. and Drain, L.E., Measurement of Reynolds shear stress in water by laser anemometry, Disa information, 12, 21, (1971)


Collins, D.C., and Williams, M.J., two dimensional convection from heated wires at low Reynolds numbers, J. Fluid Mech., 6, (1959)


Cornelissen, M.C.M. and Hoogendoorn, C.J., Propagation velocity for a force cooled superconductor, Cryogenics, 25, 185, (1985c)
Danckwerts, P.V., Significance of liquid film coefficients in gas absorption, Ind & End. Chem., 43, 1490, (1951)
Dresner, L., Propagation of normal zones in composite super conductors, Cryogenics, 16, 675, (1976)
Giarratano, P.J. and Jones, M.C., Deterioration of heat transfer to supercritical helium at 2.5 atmosphere, Int. J. Heat Mass transfer, 18, 649, (1975)
Hall, W.B., Heat transfer near the critical point, Adv. in heat transfer, 7, 1, (1971)
Hanratty, T.J., Turbulent exchange of mass and momentum with a boundary, A.I.Ch.E.J., 2,(3), 359, (1956)
Ishibashi, K., Wake, M., Kobayashi, M. and Katase, A., Propagation velocity of normal zones in a SC braid, Cryogenics, 19, 467, (1979b)
Ito, T., Takate, Y., Kasao, D., Yamaguchi, M. and Hara, T., Forced convection heat transfer to supercritical helium flowing in a vertical straight circular tube, AIChe symposium series, 82(251), 86, (1986)
Iwasa, Y., A critical current margin design criterion for high performance magnet stability, Cryogenics, 19, 705, (1979)


Jackson,J.D. and Hall,W.B., Influences of buoyancy on heat transfer to fluids flowing in vertical tubes under turbulent conditions, Turbulent forced convection in channels and bundles, 613, McGraw-Hill, (1975)


Junghans,D., Friction factor for flow of supercritical helium in a straight tube, Cryogenics, 20, 633, (1980)


Karman,T. von, The analogy between fluid friction and heat transfer, Trans ASME, 61, 705, (1939)


Keilin,V.E. and Romanovsky,V.R., The dimensionless analysis of the stability of composite superconductors with respect to thermal disturbances, Cryogenics, 6, 313, (1982)


Kramers,H., Physica, 12, (1946)

Laufer, J. and Badri Narayanan, M.A., Mean period of the turbulent production mechanism in a turbulent boundary layer, Phys. Fluids, 14(1), 182, (1971)
McCarty, R.D. Thermophysical properties of helium 4 from 4 to 3000 R with pressures to 15000 PSIA, NBS technical note 622, (1972)
Maddock, B.J., James, G.B. and Norris, W.T. Superconductive composites heat transfer and steady state stabilization, Cryogenics, 9, 261, (1969)
Marinucci, C., Hilal, A., Zellweger, J. and Vecsey, G., Quench studies of the swiss LCT conductor, Proc. 8th symp. on engineering problems of fusion research, San Francisco, (1979)
Mc Carty,R.D., Thermodynamics properties of helium 4 from 2 to 1500 K at pressures to $10^8$ Pa, J. Phys. Chem. Ref. Data, 2(4), 923, (1973)
Meek,R.L., Ph D Thesis, University of Utah, Salt Lake City, USA, (1968)
Meek,R.L., Mean period of fluctuations near the wall in turbulent flows, A.I.Ch.E.J., 18, 854, (1972)
Nick,W., Transienter Wärmeübergang an turbulent strömendes über kritisches helium, Thesis, University of Karlsruhe, Germany, (1986)


Prandtl, L., Über ein neues formelsystem für die ausgebildete turbulenz, Nachrichten von der akad. der wissenschaft in Goettingen, (1945)


Reichardt, H., Die Grundlagen des turbulenten Wärmeüberganges, Arch. Gesamte Wärme-tech, 2, 129, (1951)


Rodi, W., Turbulence models and their applications in hydraulics, Presented at the IAHR section on fundamentals of division II : Experimental and Mathematical Fluid Dynamics, (1980)


Rubesin, M.W. et al., A critique of some recent second order turbulence closure models for compressible boundary layers, AIAA 15th aerospace science meeting, 77-128, 1-19, (1977)


Toor, H.L. and Marchello, J.M., Film penetration model for mass and heat transfer, A.I.Ch.E.J., 4 (1), 97, (1958)
Turck, B., About the propagation velocity in superconducting composites, Cryogenics, 20, 146, (1980)
Wilson, M.N. and Iwasa, Y., Stability of superconductors against localized disturbances of limited magnitude, Cryogenics, 17, (1978)

Wilson, M.N., Some basical problems in superconducting magnet design, IEEE transactions on magnetics, 17(5), (1981b)


Yaskin, L.A. et.al, A correlation for heat transfer to supercritical helium in turbulent flow in small channels, Cryogenics, 17, 549, (1977)
SUMMARY

Transients in turbulent convective heat transfer to a flow of supercritical helium

In this thesis the features of a turbulent pipe flow with heat transfer from the wall under transient conditions are investigated both experimentally and numerically.

Numerically a two dimensional flow model is developed with which the velocity profile and the fluid temperature in the tube can be described in detail. The turbulence in the flow is accounted for by using the mixing length model.

Stationary heat transfer to a fully developed flow can also be described with a so called surface renewal model. In this study this model is extended to be able to describe the transient heat transfer phenomena in case of an arbitrarily varying wall temperature.

Both models are validated using transient heat transfer measurements in a supercritical helium flow of Bloem [1986]. The results of the two dimensional model are in good agreement with the experiments. A good description of the experimental data is also possible using the transient surface renewal model. However, in this model a parameter, the renewal time \( \tau \), has to be specified. For the prediction of this parameter no adequate model is available. The value of \( \tau \) could only be estimated afterwards by fitting the results to experimental data. It was found to be lower than values as given in literature.

With the two dimensional model also the hydrodynamics of the transient flow can be investigated. Under cryogenic circumstances, like in case of the supercritical helium flow through the cooling channel of a conductor, only the size and the propagation velocity of the pressure wave can be measured. Using these data the influence of the time- and spatial-differencing schemes used in the model are investigated.

The influence of the induced pressure wave on the other flow phenomena can only be studied experimentally under simpler conditions.
For this reason an experiment in a turbulent airflow in a tube has been done. A pressure wave in the flow is induced by the injection of an extra amount of air through the wall at a fixed location. The influence of the pressure wave on the velocity profile, the temperature in the flow and the heat transfer to the flow has been investigated. For all these flow phenomena a good agreement between the measured and computed results has been found.

For the velocity profile it has been shown that the instantaneous velocity change is constant over the cross section of the tube. Only for the viscous wall region \((y^+ < 5)\) the initial change decreases. With time turbulent diffusion gradually brings the logarithmic velocity profile back.

The two dimensional model is shown to be a good tool for predicting both the hydrodynamics in and the heat transfer to a turbulent flow under transient conditions.

As an application of the model the stability of the SULTAN conductor, developed at ECN, is computed. For different Reynolds numbers and helium pressures the critical values and the propagation velocities of the disturbance in the conductor are computed. A good stability combined with a high propagation speed of the disturbance is found at high helium pressures. The stability of the conductor can be increased by increasing the Reynolds number of the flow. However, because of the relatively high critical disturbance at low Reynolds numbers and the reduction of the propagation velocity of the quench as \(Re\) is increased a low flow Reynolds is to be preferred. A low Reynolds number has the additional advantage that the pressure drop over the tube is much lower than for high \(Re\) values.
SAMENVATTING

Transienten in turbulente convectieve warmte overdracht
daar een stroming van superkritisch helium

In dit proefschrift zijn de eigenschappen van een turbulente pijp
stroming met warmtetransport aan de wand onder transiente condities
zowel experimenteel als numeriek onderzocht.

Numeriek is een twee dimensionaal stromingsmodel ontwikkeld
waarmee het stromingsprofiel en de vloeistof temperatuur in de buis
gedetailleerd worden beschreven. De turbulentie in de stroming wordt in
rekening gebracht met behulp van het mengweglengte model.

De stationaire warmteoverdracht naar een volledig ingestelde
stroming kan ook beschreven worden met een zogenaamde "surface renewal"
model. In dit onderzoek is een uitbreiding van dit model ontwikkeld
waar ook de transiente warmteoverdrachtsverschijnselen bij een
willekeurig varierende wand temperatuur mee kunnen worden beschreven.

Beide modellen zijn gevalideerd met behulp van transient
warmteoverdrachtsmetingen aan een supercritische helium stroming van
Bloem [1986]. Het two dimensionele model geeft goede overeenkomst met
de metingen. Het transiente surface renewal model is ook in staat de
gemeten resultaten goed te beschrijven. In dit model moet echter een
parameter, de "renewal" tijd $\tau$, gespecificeerd worden. Voor de
schatting van deze parameter is echter geen adequaat model beschikbaar
zodat $\tau$ alleen achteraf uit fitting met experimentele resultaten
bepaald kon worden. De gevonden waarden waren lager dan de waarden
gegeven in de literatuur.

Met het twee dimensionele model kan ook de hydrodynamica van de
transiente stroming onderzocht worden. Onder cryogene omstandigheden
zoals in het geval van de superkritische helium stroming in een
koelkanaal van een geleider kan alleen experimentele informatie over de
hoogte en de voortplantingsnelheid van de drukgolf verkregen worden.
Met behulp van deze gegevens is de invloed van de gebruikte tijd- en
plaats- differentie schema’s in het model onderzocht.

De invloed van de ge induceerde drukgolf op de overige
stromingseigenschappen kan experimenteel alleen onder eenvoudiger omstandigheden bestudeerd worden. Hiertoe is een experiment met een turbulente luchtstroming in een pijp uitgevoerd. Een drukgolf in de stroming wordt geïnduceerd door het injectoren van een extra luchtstroom op een vaste positie in de pijp. De invloed van een drukgolf op het snelheidsprofiel, de luchttemperatuur en de warmteoverdracht naar de stroming is onderzocht. Voor al deze stromingseigenschappen is een goede overeenkomst tussen de gemeten en berekende resultaten gevonden.

Voor het snelheidsprofiel is gevonden dat de instantane verandering constant is over de doorsnede van de buis. Alleen in het viskeuze wand gebied \( y^+ < 5 \) neemt de initiele verandering af. Voor toenemende tijd keert het snelheidsprofiel geleidelijk terug naar het logaritmisch profiel onder invloed van turbulente diffusie.

Het twee dimensionele simulatie model is instaat gebleken om zowel de hydrodynamica in als de warmteoverdracht naar een turbulente stroming onder transiente condities te beschrijven.

Een toepassing van het model is gevonden in de bepaling van de stabiliteit van de SULTAN geleider, ontwikkelt op het ECN. Voor verschillende Reynolds getallen en helium drukken zijn de kritische waarde en de voortplantingssnelheid van de verstoring in de geleider berekend. Hieruit bleek dat voor een goede stabiliteit gecombineerd met een hoge voortplantingssnelheid van de verstoring een hoge helium druk aan te bevelen is. De stabiliteit van de geleider kan worden verhoogd door het Reynolds getal op te voeren. Dit wordt echter afgeraden omdat de kritische verstoring bij lage Reynolds getallen al relatief groot is terwijl bij het verhogen van Re de propagatiesnelheid van de verstoring snel verminderd. Verder heeft een laag Reynolds getal het bijkomende voordeel dat de drukval over het koelmateriaal van de geleider veel lager is dan bij hoge Re waarden.
Curriculum vitae

9 Maart 1961 Geboren te Rotterdam
1967-1969 Lagere school te Rotterdam
1969-1973 Lagere school te Heinenoord
1973-1979 Atheneum te Oud Beijerland
5 Juni 1979 Eindexamen Atheneum
1979-1984 Technische Hogeschool Delft
  - Afdeling der algemene wetenschappen
  - Onderafdeling der wiskunde
1983 Eerste graads onderwijsbevoegdheid wiskunde.
1983-1984 Afstudeerwerk in de vakgroep Toegepaste Analyse
  uitgevoerd op het Rotterdamsch Radiotherapeutisch
  Instituut te Rotterdam.
15 Nov. 1984 Doctoraal examen wiskundig ingenieur; (cum laude)
  Het afstudeeronderwerp was "Rekenmethodes voor
dosisbepaling bij bestraling van tumoren met hoog
energetische electronen bundels".
1985-1989 Promotieplaats Technische Universiteit Delft
  - Faculteit der technische natuurkunde
  - Vakgroep transportverschijnselen
  - Subgroep warmtetransport
  onder begeleiding van Prof.ir. C.J. Hoogendoorn.
1989-
  Research medewerker in de sectie Proces technologie
  van DSM Research (Geleen)
NAWOORD

Bij het verschijnen van dit proefschrift wil ik graag van de gelegenheid gebruik maken een ieder te bedanken die op engelei wijze aan het tot stand komen ervan bijgedragen heeft.

Mijn promotor Prof. ir. C.J. Hoogendoorn ben ik zeer erkentelijk voor zijn adviezen en opbouwende kritiek tijdens dit onderzoek.


Een speciaal woord van dank gaat uit naar Rimmert Hoekstra die altijd voor mij klaar stond, ook als er tot diep in de nacht door gewerkt moest worden.

Verder wil ik ook Jaap Beekman hier noemen voor onder andere zijn hulp zowel op computer gebied als bij het vervaardigen van de electronica voor de experimenten.

Naast de bovenstaande personen dank ik ook de rest van mijn vakgenoten binnen WT voor de prettige sfeer in de groep gedurende de afgelopen jaren.

Wim Bloem, die vanuit de vakgroep Lage Temperaturen werkzaam was bij het ECN in Petten, ben ik zeer erkentelijk voor de prettige samenwerking.

Tenslotte wil ik Erik Zeef bedanken voor zijn waardevolle adviezen op taalkundig gebied en zijn steun tijdens het schrijven van dit proefschrift.

Pa en Ma, zonder jullie steun had ik dit proefschrift nooit kunnen schrijven. Bedankt.