THE ANALYTICAL HIERARCHY PROCESS APPLIED FOR DESIGN ANALYSIS

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ABSTRACT

Being an intelligent activity, design is a complex process to accomplish. The complexity stems from the elusive character of this activity, which cannot be explained in precise terms, in general. In a design process, the determined relationships among the design elements provide important information to understand the role of each element with respect to others thereby improving the design. For this aim the method of analytical hierarchy process (AHP) is employed which provides hierarchical priorities of the design elements with respect to the parsed design goal. The priority information is extended to establish hierarchical relations among the elements as a novel approach to employ in architectural design process.

Keywords: design analysis, attribute relations, analytical hierarchy process (AHP)

INTRODUCTION

Design is inherently knowledge-based, intelligent behavior. This implies the cognition of the design task and the relevant environment with the complex interrelationships involved. Therefore, the achievement of a satisfactory design is dependent on personal qualities including the intelligence and aptitude of the designer, next to his/her design knowledge. In particular, in the architectural design this situation is more accentuated due to the richness of the design information to be integrated into the design. Seemingly, there can be a variety of ways for this integration so that each design can be substantially different. However, with the stipulations of the whole design information given, the solution space can be drastically narrowed down so that the desirable reconcilable design in search becomes hard to obtain. This challenging aspect of a design task suggests the need of some auxiliary means in a design process to overcome such high level problems which involves advanced mathematical methods such as evolutionary search, combinatorial optimization, eigenvalues and eigenvectors for the dimensionality reduction etc. Such methods can be borrowed from the domain of exact sciences and should be integrated into the architectural design for design enhancement. This is not alien to architectural design since architecture and mathematics are two major domains respectively in soft sciences and exact sciences respectively and they are organically related since their origin. To demonstrate the need of borrowing methods from exact sciences an example will be given from architectural design. For example, in an urban design exercise, the shadow of the buildings can be minimized in variety of ways. However, if one considers densely located high-rise buildings, the same minimization
efforts can become an elaborate task. Namely, the design minimizing the shadow effects requires a careful architectural deliberation. This situation can be made more complex if one endeavors to consider all potential unwanted effects due to shadowing and searches for an acceptable design solution. In this case, the relations of the shadowing aspect to other design attributes have to be known or realistically determined. These relations even may have conflicting effects with respect to the objective. With this basic example, it is tried to emphasize the importance of the valid attribute relations in a design endeavor. By means of estimating the attribute relations in design, design quality can be improved matching the design goals in optimal way. Note that, without attribute relation information, a design can still be obtained in one way or other that it seemingly observes some of the design goals. However, it is difficult to verify if these observations are valid enough, referring to “best” solutions among the vast seemingly “acceptable” design solutions.

ATTRIBUTE RELATIONS DETERMINATION

A number of attributes are involved in a design process. It is essential to estimate the effect of each attribute to the outcome to keep the design under control thereby avoiding the probable uncertainties on the quality of design. However, this simple statement is not an easy task to verify, due to the complexity involved. The complexity mentioned is twofold. Firstly, the number of attributes is generally high so that the number of attribute relations can be explosively high. Secondly, the relations themselves can be complex due to indirect relations, which appear to be seemingly direct. This situation occurs due to the attributes, which are never considered however they play role in the interrelations of the observed attributes. Therefore they are referred to as hidden attributes.

The above mentioned complexity issues and associated attribute relations determinations may be illustrated as shown in Figures 1 and 2 (Kocaturk 2004). These figures are related to an ongoing research searching the cognitive model of a designer. Figure 1 illustrates the complex relations among the variables, which are aggregated in three categories as formal aspects, structural aspects and production aspects in a generative design process.

In an architectural design the establishment of above illustrated relations to understand the effect of each design variable on the design outcome is an issue. Once these relations are established, they can be used for making a knowledge model, which can be used as an expert for a general use concerning the same categorical design, say free-form building design. In the literature the importance of this issue is pointed out. However, a firm clue addressing how to tackle this issue is still pending due to very soft nature of the problem. Terminologically such issues are termed as ill-defined problems, which require special treatment for solution. Such soft problems of soft sciences can effectively be treated by the special methods of the exact sciences. One method of interest for this research is the Analytical Hierarchy Process (AHP). Implementation of the method of AHP for the attribute relation determination is described below.
Figure 1: Schematic description of interdependent relations among the multidimensional design aspects and time-dependent progressive design designated as generative design.

Figure 2. Detailed description of interdependent relations among the multidimensional design aspects.
Analytical Hierarchy Process

In architectural design process to handle uncertainty, imprecision and subjectiveness can be carried out basically by means of probability theory and/or fuzzy set theory. The former focuses on the random nature of the decision-making process while the latter concerns the subjectivity and imprecision of human behavior. The other approach is the analytical hierarchy process (AHP) involving qualitative data and dealing with the uncertainty, imprecision and subjectivity. The AHP method is a technique developed by Saaty (1980) to compute the priority vector, ranking the relative importance of factors being compared. The only inputs to be supplied by the expert in these procedures are the pairwise comparisons of relative importance of factors, taken two at a time. This means, in an environment of complex relationships among the variables, one follows the principle of “divide and rule”. If we denote the expert input comparing the \(i\)th variable with respect to the \(j\)th variable by \(a_{ij} = w_i/w_j\), then the relative importance of the \(j\)th factor with respect to the \(i\)th factor is represented as \(1/a_{ij} = w_j/w_i\). Note that, in an environment with high number complex relations to make a judicious relational assertion is not easy. However, to make a simple comparison between any two attributes and to make a judgment is much easier for an expert. The \([n \times n]\) matrix obtained by arranging these pairwise comparison ratios is termed the reciprocal judgment matrix and designated as \(A\) where \(n\) is the number of factors subjected to pairwise comparison. The diagonal elements of \(A\) matrix are all unity. Since we take the reciprocals, we have to fill the upper diagonal elements which are altogether \(n(n-1)/2\).

\[
A = \begin{bmatrix}
    w_1 & w_2 & \cdots & w_n \\
    w_1 & \frac{w_2}{w_1} & \cdots & \frac{w_n}{w_1} \\
    \vdots & \ddots & \ddots & \vdots \\
    \frac{w_n}{w_1} & \cdots & \frac{w_n}{w_n} & 1
\end{bmatrix} = \begin{bmatrix}
    1 & a_{12} & \cdots & a_{1n} \\
    \frac{1}{a_{12}} & 1 & \cdots & \frac{1}{a_{1n}} \\
    \vdots & \ddots & \ddots & \vdots \\
    \frac{1}{a_{1n}} & \cdots & \frac{1}{a_{1n}} & 1
\end{bmatrix}
\]

(1)

The details of this technique are given by Saaty (1980, 2000). The comparison and highlighting its strength is reported in the literature (Saaty and Vargas 1984). The principal eigenvector \(W\) of \(A\) is computed by solving the eigenvalue problem \(AW = \lambda_{\text{max}} W\)

where \(\lambda_{\text{max}}\) is the principal or largest real eigenvalue of \(A\). The normalized eigenvector corresponding to \(\lambda_{\text{max}}\) is the priority vector \(P\). The beauty of the AHP operation can be appreciated by considering the tolerance of the method allowed during making the expert judgment. That is, some deviations in the expert judgments do not critically effect the final outcome. AHP is applied in many fields up to now, such as the economic analysis, urban or regional planning and forecasting etc, (Vargas 1990), knowledge model validation (Ciftcioglu 2003).

Illustrative Example

To exemplify the utilization of AHP for the relational attribute determination, let us consider figure 2 to determine the relations of structural aspects to formal aspects. In the first
step, let us choose the geometrical description variable of the formal aspects and in this context let us make expert judgment for the structural aspects. The structural aspects are

- Loads \( (w_1) \)
- Materials \( (w_2) \)
- Form and space geometry \( (w_3) \)
- Structural design approach \( (w_4) \)
- Structural behaviour \( (w_5) \)

Only for illustrative purposes, we might make expert judgments about ratio in connection to the relevance to geometrical description as follows:

\[
\begin{align*}
&w_2/w_1=0.8; \quad w_3/w_1=0.8; \quad w_4/w_1=0.5; \quad w_5/w_1=0.7; \\
&w_3/w_2=0.9; \quad w_4/w_2=0.3; \quad w_5/w_2=1.0; \\
&w_4/w_3=0.5; \quad w_5/w_3=1.0; \\
&w_5/w_4=1.8;
\end{align*}
\]

The rationale about these ratios is due to expert judgment. For instance for \( w_2/w_1=0.8 \) is selected comparing the relevance of loads to geometrical description to relevance of material to geometrical description. Load and material are closely related in the context of geometry and therefore they have weights at the same order. However, loads get slightly higher priority because in the present case, it precedes the selection of the material. Therefore the ratio is asserted to be 0.8. The same consideration applies for the ratio \( w_3/w_1 \) and therefore it is also 0.8. The structural design approach is an optional process. Therefore it has less priority compared to loads. Therefore \( w_4/w_1=0.5 \). Structural behaviour is an important consideration in the structure and its priority is comparable with loads being slightly inferior to loads. Therefore \( w_5/w_1=0.7 \). For the rest of the ratios the similar rationales prevail. Accordingly, the reciprocal judgments relevant to geometrical description becomes

Table 1. Reciprocal ratios of expert judgments for the attribute relations among Structural Aspects attributes

<table>
<thead>
<tr>
<th></th>
<th>Loads</th>
<th>Material</th>
<th>Form and space geometry</th>
<th>Structural design approach</th>
<th>Structural behaviour</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Loads</strong></td>
<td>1</td>
<td>1/.8</td>
<td>1/.8</td>
<td>1/.5</td>
<td>1/.7</td>
</tr>
<tr>
<td><strong>Material</strong></td>
<td>.8</td>
<td>1</td>
<td>1/.9</td>
<td>1/.3</td>
<td>1</td>
</tr>
<tr>
<td><strong>Form and space geometry</strong></td>
<td>.8</td>
<td>.9</td>
<td>1</td>
<td>1/.5</td>
<td>1</td>
</tr>
<tr>
<td><strong>Structural design approach</strong></td>
<td>.5</td>
<td>.3</td>
<td>.5</td>
<td>1</td>
<td>1/1.8</td>
</tr>
<tr>
<td><strong>Structural behaviour</strong></td>
<td>.7</td>
<td>1</td>
<td>1</td>
<td>1.8</td>
<td>1</td>
</tr>
</tbody>
</table>

The reciprocal judgment matrix, then, is given by
The largest eigenvalue $\lambda_{\text{max}} = 5$ and the corresponding priority vector is

$$p^T = [0.246 \quad 0.243 \quad 0.206 \quad 0.103 \quad 0.202]$$

The another essential property of AHP method is the embedded consistency check in the method. Namely, for fully consistent expert judgments $\lambda_{\text{max}}$ is the same as the number of variables being considered. In the above illustrative example the number of variables is $n=5$, and the largest eigenvalue is $\lambda_{\text{max}} = 5$, which indicates the ideal consistency of the judgments, though this is a mere illustrative example. Note that this is not a contrived example. The architectural design example is taken from the literature and the AHP method is applied to it. However, this outcome is still not to be surprised. This is simply due to the sound and consistent rationale exercised for forming the reciprocal judgment matrix. In a very natural way, sound expert knowledge yields the accurate reciprocal judgment matrix and thereby providing firm attribute relations in a design. The inconsistencies among the judgments are checked by $\lambda_{\text{max}}$ and in the case of excessive deviation, the expert is warned to review his/her judgments. At it was mentioned before, some certain degree of inconsistencies among the judgments are tolerated in the method and the priority hierarchy attached to the attribute relations does not alter. In the above example the relations between structural aspects and the geometrical description of the formal aspects is established. As to figure 2, the same procedural computation can be applied to establish the relations between structural aspects and form generation approach, structural aspects and architectonic expression, structural aspects and critical aspects of the form. When these computations have been done, then we obtain the following matrix the columns of which are the eigenvectors indicating the priorities in each computation above.

In Table 2, the third priority column is given by (4) and this is explicitly indicated in the table. Note that, the variable structural design approach in the pairwise expert judgment considered to be least significant in the context of geometrical description and it has indeed the deemed lowest priority relative to others, in this example. The result of the AHP corroborates this as $p_{43} = .10$. This intuitive information is substantiated as 10 per cent with a mathematical precision. For the bi-directional relations, the formal aspects and structural aspects should be interchanged and the same computations should be carried out. The result of this process is indicated in Table 3 by means of the elements $p_{a_{ij}}$.

Table 2. Reciprocal ratios of expert judgments for the attribute relations between Structural Aspects and Formal Aspects
### FORMAL ASPECTS

<table>
<thead>
<tr>
<th>Loads</th>
<th>Material</th>
<th>Form and space geometry</th>
<th>Structural design approach</th>
<th>Critical aspects of form</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_{sf11}$</td>
<td>$p_{sf21}$</td>
<td>$p_{sf31}$</td>
<td>$p_{sf41}$</td>
<td>$p_{sf51}$</td>
</tr>
<tr>
<td></td>
<td>$p_{sf12}$</td>
<td>$p_{sf22}$</td>
<td>$p_{sf32}$</td>
<td>$p_{sf42}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$p_{sf33}$</td>
<td>$p_{sf43}$</td>
<td>$p_{sf52}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$p_{sf41}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$p_{sf51}$</td>
</tr>
</tbody>
</table>

Table 3. Reciprocal ratios of expert judgments for the attribute relations between Formal Aspects and Structural Aspects

### STRUCTURAL ASPECTS

<table>
<thead>
<tr>
<th>Loads</th>
<th>Material</th>
<th>Form and space geometry</th>
<th>Structural design approach</th>
<th>Structural behavior</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_{fs11}$</td>
<td>$p_{fs21}$</td>
<td>$p_{fs31}$</td>
<td>$p_{fs41}$</td>
<td>$p_{fs51}$</td>
</tr>
<tr>
<td></td>
<td>$p_{fs12}$</td>
<td>$p_{fs22}$</td>
<td>$p_{fs32}$</td>
<td>$p_{fs42}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$p_{fs33}$</td>
<td>$p_{fs43}$</td>
<td>$p_{fs52}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$p_{fs41}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$p_{fs51}$</td>
</tr>
</tbody>
</table>

The priorities in Table 2 and Table 3 can be represented as priority matrices as shown below where the matrix dimensions are $[4 \times 5]$ and $[5 \times 4]$ respectively. The matrix elements as priorities are to be computed for each case and therefore they are different, in general.
These two priority matrices form the bi-directional attribute relations between formal aspects and structural aspects. To illustrate further attribute relations where aspects are not directly related but they are indirectly related. Consider Figure 1(a) where suppose there is no direct relation between production aspects and formal aspects. These two aspects are related to one another via structural aspects. In this case the relations being sought are determined as follows. First, the priority matrix of production-to-structural aspects is computed via AHP, as described in the illustrative example above. Second, the priority matrix of structure-to-formal aspects is computed in the same way. As to Figure 2, these matrices will have the dimensions of \( [3 \times 5] \) and \( [5 \times 4] \) respectively as indicated below where the matrix elements are individually computed for both matrices.

\[
P_{\text{ps}} = P_{\text{production-to-structural}} = \begin{bmatrix} p_{ps11} & p_{ps12} & p_{ps13} & p_{ps14} & p_{ps15} \\ p_{ps21} & p_{ps22} & p_{ps23} & p_{ps24} & p_{ps25} \\ p_{ps31} & p_{ps32} & p_{ps33} & p_{ps34} & p_{ps35} \end{bmatrix}
\]

\[
P_{\text{sf}} = P_{\text{structural-to-formal}} = \begin{bmatrix} p_{sf11} & p_{sf12} & p_{sf13} & p_{sf14} \\ p_{sf21} & p_{sf22} & p_{sf23} & p_{sf24} \\ p_{sf31} & p_{sf32} & p_{sf33} & p_{sf34} \\ p_{sf41} & p_{sf42} & p_{sf43} & p_{sf44} \\ p_{sf51} & p_{sf52} & p_{sf53} & p_{sf54} \end{bmatrix}
\]

The priority matrix bearing the relations from production aspects to formal aspects is obtained by means of the product of the priority matrices \( P_{ps} \) and \( P_{sf} \) above. This is indicated below as \( P_{psf} \) matrix with the dimensions of \( [3 \times 4] \), as it should be.

\[
P_{psf} = P_{ps} \times P_{sf} = P_{production-to-formal} = \begin{bmatrix} p_{psf11} & p_{psf12} & p_{psf13} & p_{psf14} \\ p_{psf21} & p_{psf22} & p_{psf23} & p_{psf24} \\ p_{psf31} & p_{psf32} & p_{psf33} & p_{psf34} \end{bmatrix}
\]

For bi-directional relations one has to consider the priority matrices \( P_{fs} \) as formal to structural and \( P_{sp} \) as structural to production, respectively as shown below where each row gives the relations of each formal aspects component to the production aspects. Note that, above result can be verified by computing the relation matrix \( P_{fp} \) directly in a similar way as described in Table 2 considering the formal aspect and production aspects pair. The attribute relations in the case of ramification of aspects as sub-aspects in the form a tree structure can be computed indirectly in contrast with the computations made directly as this is the case in this work. This is briefly described below. Consider the following new
structure as shown in figure 3 where formal aspects contain four attributes and the production aspects contain three attributes as shown in Figure 2.

\[
P_{FS} = P_{\text{FORMAL TO STRUCTURAL}} = \begin{bmatrix}
P_{f11} & P_{f12} & P_{f13} & P_{f14} & P_{f15} \\
P_{f21} & P_{f22} & P_{f23} & P_{f24} & P_{f25} \\
P_{f31} & P_{f32} & P_{f33} & P_{f34} & P_{f35} \\
P_{f41} & P_{f42} & P_{f43} & P_{f44} & P_{f45} \\
\end{bmatrix}
\]

\[
P_{SP} = P_{\text{STRUCTURAL TO PRODUCTION}} = \begin{bmatrix}
P_{s11} & P_{s12} & P_{s13} \\
P_{s21} & P_{s22} & P_{s23} \\
P_{s31} & P_{s32} & P_{s33} \\
P_{s41} & P_{s42} & P_{s43} \\
P_{s51} & P_{s52} & P_{s53} \\
\end{bmatrix}
\]

and finally

\[
P_{FS} \times P_{SP} = P_{FP} = P_{\text{FORMAL TO PRODUCTION}} = \begin{bmatrix}
P_{fp11} & P_{fp12} & P_{fp13} \\
P_{fp21} & P_{fp22} & P_{fp23} \\
P_{fp31} & P_{fp32} & P_{fp33} \\
P_{fp41} & P_{fp42} & P_{fp43} \\
\end{bmatrix}
\]

Note that the hypothetical structure in figure 3 is different than that given in figure 2. In figure 3, \( p_1 \) and \( p_2 \) are the priorities as to formal aspects and production aspects with respect to structural aspects.

Figure 3. Detailed description of interdependent relations among the multidimensional design aspects

The expert judgment ratios for formal aspects in the context of structural aspects are asserted as follows:
\[ \frac{w_2}{w_1} = 0.5; \frac{w_3}{w_1} = 0.6; \frac{w_4}{w_1} = 1; \]
\[ \frac{w_2}{w_2} = 1.2; \frac{w_3}{w_2} = 1.8; \]
\[ \frac{w_4}{w_3} = 0.9; \]
giving the expert judgement ratio matrix \( A_{\text{formal}} \) as
\[
\begin{bmatrix}
1 & 1/5 & 1/6 & 1 \\
5 & 1 & 1/2 & 1/1.8 \\
6 & 1.2 & 1 & 1/9 \\
1 & 1.8 & 9 & 1
\end{bmatrix}
\]
and the same ratios for comfort are asserted to be
\[ \frac{w_2}{w_1} = 1; \frac{w_3}{w_1} = 1.4; \]
\[ \frac{w_3}{w_2} = 1.75; \]
giving the expert judgment ratio \( A_{\text{production}} \) matrix as
\[
\begin{bmatrix}
1 & 1 & 1/1.4 \\
1 & 1 & 1/1.75 \\
1.4 & 1.75 & 1
\end{bmatrix}
\]
The corresponding priority vectors are computed as
\[
P^T_{\text{formal}} = [0.327, 0.167, 0.231, 0.275]
\]
\[
P^T_{\text{production}} = [0.291, 0.270, 0.439]
\]
The relational matrix \( R_{\text{formal-production}} \) as to from safety to comfort is computed from
\[
R_{\text{formal-production}} = P^T_{\text{formal-structure}} \times P^T_{\text{production-structure}} \times p_1 \times p_2 = [0.327, 0.167, 0.231, 0.275] \times \begin{bmatrix} 0.0952 & 0.0883 & 0.1436 \\ 0.0486 & 0.0451 & 0.0733 \\ 0.0672 & 0.0624 & 0.1014 \\ 0.0800 & 0.0743 & 0.1207 \end{bmatrix}
\]
assuming that \( p_1 \) and \( p_2 \) in figure 3 are both equal. In the same way, the relational matrix \( R_{\text{production-formal}} \) as to from production to formal is computed from
\[
R_{\text{production-formal}} = P^T_{\text{production-structure}} \times P^T_{\text{formal-structure}} \times p_1 \times p_2 = [0.291, 0.270, 0.439] \times \begin{bmatrix} 0.0952 & 0.0486 & 0.0672 & 0.0800 \\ 0.0883 & 0.0451 & 0.0624 & 0.0743 \\ 0.1436 & 0.0733 & 0.1014 & 0.1207 \end{bmatrix}
\]
By means of the direct and indirect method of relational attribute computations exemplified above, one can establish all the attribute relations in a design process of any complexity. In these computations, where expert knowledge plays the essential role for the pairwise
CONCLUSIONS

Categorically, the importance of the determination of attribute relations can be seen in two ways. These are from the ontology viewpoint and from the viewpoint of real-time design assistance by computer. These are briefly discussed below.

From the ontology viewpoint and Figure 3 can be considered as a taxonomic description of design information. Definitely such a description is helpful since it categorizes the information as abstraction thereby providing easy survey. However, such representations should be supported by semantic information for knowledge sharing. Also the semantic information may help for searching essential information being sought within such comprehensive information scheme. In this respect, semantic network associated with the ontology is a hot spot of research and it is in progress for development due to challenging effort required. Semantic information makes the ontology scheme explosively complex. The attribute relations qualify the items in the ontology indicating their semantic relations to the others. In this sense attribute relations next to their own important role, they may play the role of semantic labels. For instance, if a certain mode of design approach is altered during the production, the implication of this on the entire project is easily identified with a mathematical precision and the confidence about the design outcome is maintained.

Attribute relations provide a knowledge model of the design variables. By means of this model, the effect of the variable change on the design outcome can be monitored. Considering the limitations imposed, design can easily be verified against the limitations during the design process in real-time so that the design is guided real-time not to violate the design requirements. The relational map can be used as a knowledge base of an expert system, and such an expert system can serve as intelligent real-time coordinator.

This work aims at to point out that architectural design can benefit from the interaction with the exact sciences and to exemplify this statement. It addresses the attribute relation determination for a number of aspects in a given design task with the specified design attributes. In this context, definitely mathematics play the essential role to establish firm foundations for the design where architectural creativity is supported with reasoning leading the to a secure accomplishment as opposed to design, prone to uncertainties about the outcome. The method implemented is known to be analytical hierarchy process. The method has a number of diverse applications especially in soft science areas due to its ability to deal with ill-defined problems. However, for architectural design, its use via the attribute relations in the context of both semantic web and intelligent tools aiming to conceptual, collaborative and intelligent design is a novel and a pioneering endeavor.

REFERENCES


