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Effect of Permeability on Foam-model Parameters and the Limiting Capillary Pressure

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SUMMARY

Accurate modelling of foam rheology on the field scale requires detailed understanding of the correlation between the fundamental properties of foam and the scalable parameters of the porous medium. It has been experimentally observed that foam experiences an abrupt coalescence when the capillary pressure in the porous medium approaches a certain value referred to as the “limiting capillary pressure”, P_c^* . Current foam models that treat foam texture implicitly mimic this fundamental behaviour with a so-called dry-out function, which contains adjustable parameters like f_{mdry} and $epdry$ (in the STARS foam simulator). Parameter f_{mdry} (called S_w^* in other models) represents the water saturation corresponding to the limiting capillary pressure P_c^* and $epdry$ determines the abruptness of foam coalescence as a function of water saturation. In this paper, using experimental data, we examine the permeability-dependence of these parameters. We find that the value of f_{mdry} decreases with increasing permeability. We also find that, for the data examined in this paper, the transition from high-quality regime to low-quality regime is more abrupt in lower-permeability rocks. This implies that in high-permeability rocks foam might not collapse abruptly at a single water saturation; instead there is range of water saturation over which foam weakens. In addition, we address the question of whether P_c^* is dependent on formation permeability. We estimate P_c^* from data for foam mobility in vs. foam quality, and find, as did Khatib et al. (1988), who introduced the limiting capillary pressure concept, that P_c^* can vary with permeability. It increases as permeability decreases, but not enough to reverse the trend of increasing foam apparent viscosity as permeability increases.

Introduction

Gas-injection enhanced oil recovery (EOR) projects often suffer from poor volumetric sweep because the density and viscosity of the injected gas are smaller than those of the oil under reservoir conditions (Lake *et al.*, 2014; Bedrikovetsky, 1996). As a result a large portion of the oil is not contacted by the injected gas and the ultimate recovery of the reservoir remains low. Foam can potentially solve this shortcoming by reducing gas mobility and increasing the viscous pressure drop in the flow direction that can act against gravity segregation (Kovscek and Radke, 1994; Rossen, 1996; Hirasaki and Lawson, 1985; Hirasaki, 1989; Farajzadeh *et al.*, 2010). Foam also has the unique advantage that under some conditions it spontaneously reduces gas mobility more in high-permeability layers than in low-permeability layers.

To predict the efficiency of a foam-displacement process on the field scale foam, models should reflect the most relevant physics of foam in porous media (e.g. the effects of permeability, surfactant concentration, oil saturation and composition, wettability, etc.). Several models have been proposed for modeling of foam in porous media (Farajzadeh *et al.*, 2012; Ma *et al.*, 2014; Lotfollahi *et al.*, 2015), which can be divided into two main categories. Population-balance (PB) models (e.g. Falls *et al.*, 1988; Friedmann *et al.*, 1991; Kovscek *et al.*, 1995; Kam *et al.*, 2007; Chen *et al.*, 2010) attempt to describe the processes that create and destroy the lamellae that separate gas bubbles, as well as the effect of bubble size on gas mobility. The second group of models represents the effect of bubble size implicitly, through a mobility-reduction factor that depends on saturations, superficial velocities and other factors (e.g. Patzek *et al.*, 1989; Mohammadi *et al.*, 1993; Cheng *et al.*, 2000). We call this second group "implicit-texture" (IT) models. PB models can be restructured to assume local equilibrium (LE) between the dynamics of lamella creation and destruction; all current IT models assume LE. These models contain adjustable parameters that are estimated by fitting the model to experiments (Cheng *et al.*, 2000; Boeije and Rossen, 2013; Rossen and Boeije, 2013; Ma *et al.*, 2013, 2014; Lotfollahi *et al.*, 2015). Several different foam models can fit the steady-state mobility of strong foam in a given porous medium to a satisfactory degree (Lotfollahi *et al.*, 2015). To apply these models on the field scale, these parameters should be able to represent foam in the variety of geological strata in a reservoir, and in particular as a function of permeability.

Most current foam models are built on the concept of the limiting capillary pressure (Khatib *et al.*, 1988), above which foam becomes unstable. The P_c^* concept in porous media is analogous to the concept of the critical disjoining pressure in bulk foam, above which a single foam lamella breaks (Aronson *et al.*, 1992; Exerowa and Kruglyakov, 1998; Farajzadeh *et al.*, 2008). Similarly, in foam-displacement experiments in a porous medium, as gas saturation rises the local capillary pressure increases; at a certain point the capillary pressure is so great that foam films do not survive and the foam becomes coarser in texture (Khatib *et al.*, 1988).

In an oil-free porous medium under steady-state conditions, foam exhibits two flow regimes depending on gas fractional flow (i.e. foam quality) (Osterloh and Jante, 1992; Alvarez *et al.*, 2001). If foam collapse is abrupt the transition between regimes occurs at a single water saturation S_w^* (the water saturation corresponding to P_c^* , see Figure 1). In the so-called high-quality regime the pressure gradient along the core is independent of gas superficial velocity, while in the so-called low-quality regime the pressure gradient is independent of liquid superficial velocity (Figure 2, left). However, if foam collapse is not abrupt, there is a range of water saturations over which foam becomes coarser in texture. In this case, the contour plot of pressure gradient shows a rounded transition between regimes rather than a sharp corner (Figure 2, right). For a constant total superficial velocity, the high-quality regime is the range of foam quality where the pressure gradient decreases with increasing foam quality, whereas in the low-quality regime the pressure gradient increases with increasing foam quality. By applying Darcy's law and treating foam as a single phase, an "apparent" viscosity of foam can be defined from the foam-induced pressure gradient.

$$\mu_f^{app} \equiv \frac{k \nabla P}{u_t} \quad (1)$$

where ∇P is the magnitude of pressure gradient, k is permeability, and u_t is total superficial velocity. The apparent viscosity of foam, or equivalently, the pressure gradient, is at a maximum where the line of constant total superficial velocity crosses the transition on a plot like Figure 2.

Figure 1 Limiting capillary pressure concept: In a porous medium capillary pressure is a function of water saturation. Meanwhile, the gas fractional-flow function depends on bubble size. At the limiting capillary pressure P_c^* foam breaks. At this fixed water saturation $S_w^* \equiv S_w(P_c^*)$ foam becomes coarser, mean bubble size increases, and gas fractional flow increases (Adapted from Khatib *et al.*, 1988.) The LE behavior of foam would follow the red curve at small S_w , then the dotted line, and then the green curve at large S_w on the right plot.

The limiting capillary pressure is believed to vary with surfactant type and concentration, electrolyte concentration, gas flow rate and porous medium permeability (Khatib *et al.*, 1988). Nonetheless, the exact dependence on these variables is not known. Our main objective here is to explore the dependency of the limiting capillary pressure on permeability, surfactant type and surfactant concentration. We use an IT foam model and examine its capacity to fit steady-state foam mobility in porous media. The permeability dependence of the parameters in the IT model will also be examined. To that end we will use some published steady-state foam data. These experiments have been conducted with rocks with different permeability and using different surfactant solutions. The structure of the paper is as follows. First, in Section 2, we describe the IT foam model and briefly explain methods to extract its relevant parameters. Next, in Section 3 we fit model parameters to steady-state foam data and establish the permeability-dependence of the parameters. In Section 4, the derived parameters are used to evaluate the relationship between rock permeability and the limiting capillary pressure. The effects of surfactant type and concentration on P_c^* are also discussed.

Figure 2 Pressure-gradient contours as a function of superficial velocities of gas and liquid calculated from the foam model presented in Section 2 for $epdry = 1 \times 10^5$ (left) (abrupt transition between regimes) and $epdry = 1 \times 10^2$ (right) (less abrupt transition). The other parameters are kept constant ($fmmob = 4 \times 10^4$, $fmdry = 0.30$, $fmcap = 2.46 \times 10^{-6}$, $epcap = 0.30$, and other parameters in Table A.1). Foam quality increases as one moves from lower left to upper right. Also plotted (dotted line) is a hypothetical scan of foam quality at a fixed superficial velocity of 15 ft/day.

Implicit-texture foam model

We focus our study on one steady-state implicit-texture foam model, that in STARS (Computer Modeling Group, 2012; Farajzadeh *et al.*, 2012; Ma *et al.*, 2013, 2014). In that model gas mobility is scaled by a function, FM , when foam is present:

$$\lambda_g^f = \lambda_g^{nf} \cdot FM = \frac{\lambda_g^{nf}}{1 + fmmob \prod_{i=1}^n F_i} \quad (2)$$

where λ_g is gas mobility and superscripts f and nf represent the cases with and without foam. Parameter $fmmob$ is the maximum (or reference) mobility reduction factor that could be achieved by foam when all conditions are favorable. The “ F_i ” functions in Eq. (2) reflect the effects of different physical parameters such as surfactant concentration, water saturation, oil saturation (and composition), capillary number, etc., on foam behavior in porous media. In this paper we focus on the dry-out and shear-thinning functions. The dry-out function is defined in STARS as

$$F_w = 0.5 + \frac{1}{\pi} \arctan(epdry(S_w - fmdry)) \quad (3)$$

Parameter $fmdry$ is the water saturation at which foam experiences significant coalescence and consequently in the limit of large $epdry$ it is equivalent to S_w^* . Parameter $epdry$ controls the sharpness of transition from the high-quality regime to the low-quality regime: when $epdry$ is very large the transition is sharp and foam collapses within a very narrow range of water saturation. When $epdry$ approaches infinity foam coalescence occurs at a single water saturation (S_w^*). In the most recent version of STARS, the parameter $fmdry$ is renamed $sfdry$, and $epdry$ is renamed $sfbet$ (Coombe, 2012). In that model $sfdry$ can be represented as a function of surfactant concentration, oil saturation, salt concentration, and capillary number. If one disables these other functionalities $sfdry$ is a constant and plays the same roll as $fmdry$ does above.

The shear-thinning function is defined as

$$F_{shear} = \begin{cases} 1 & N_{Ca} \left(= \frac{k \nabla P}{\sigma} \right) < fmcap \\ \left(\frac{fmcap}{N_{Ca}} \right)^{epcap} & N_{Ca} \left(= \frac{k \nabla P}{\sigma} \right) > fmcap \end{cases} \quad (4)$$

where, ∇P is the magnitude of pressure gradient, k is permeability, and σ is the surface tension. Parameter $fmcap$ should be set to the smallest capillary number expected to be encountered by foam in the simulation (Cheng *et al.*, 2000; Boeije and Rossen; 2013), and $epcap$ represents the extent of shear-thinning behavior. Newtonian behavior corresponds to $epcap = 0$ and positive $epcap$ to shear-thinning behavior. To obtain $epcap$ from experimental data, we use of the definition of apparent foam viscosity:

$$\mu_f^{app} = \frac{1}{\frac{k_{rw}(S_w)}{\mu_w} + \frac{k_{rg}^{nf}(S_w)}{FM \cdot \mu_g}} \quad (5)$$

where u_t is total superficial velocity, k_{rw} water relative permeability and k_{rg}^{nf} is gas relative permeability without foam. Combining Darcy’s law and Eq. (5) we obtain

$$\nabla P = \frac{u_i}{k} \frac{1}{\frac{k_{rw}(S_w)}{\mu_w} + \frac{k_{rg}^{nf}(S_w)}{\mu_g \left(1 + fmmob.F_w \cdot \left(\frac{fmcap}{k \nabla P / \sigma} \right)^{epcap} \right)}} \quad (6)$$

Procedures to fit data for μ_f^{app} as a function of foam quality to the IT foam model has been provided by several authors. We have modified the procedure explained by Ma *et al.* (2013, 2014) by adding shear-thinning function because in their approach smaller values can be assigned to *epdry*, although as shown later choosing very large values of *epdry* (as described by Cheng *et al.*, 2000; Boeije and Rossen, 2013), does not alter the general conclusions of this paper. The procedure used here, described in Appendix B, seeks a least-squares fit to all the μ_f^{app} data subject to weighting and constraints supplied by the user.

Dependence of foam-model parameters on permeability

We apply the data reported by Moradi-Araghi *et al.* (1997) to investigate the effect of rock permeability on foam-model parameters. The experiments were conducted with CO₂ at 98°F and 2000 psi with reservoir-core material of different permeabilities. The cores were 1 inch in diameter and 4.84 inch in length. The rocks were of similar type and belonged to the same unit from West Cowden reservoir. The overall pressure drop in each foam experiment was divided by that for water injection at $S_w = 1$ and reported as reduction factor (*RF*). Therefore, to calculate the apparent viscosity of foam in cp as a function of gas fraction or foam quality (f_g), the reported reduction factors were multiplied by water viscosity at experimental conditions:

$$\mu_{app}^f(f_g) = RF(f_g) \cdot \mu_w = 0.65 \times RF(f_g) \quad [\text{cp}] \quad (7)$$

Thus the apparent viscosity μ_{app}^f is the viscosity one would infer for foam at the given fractional flow if foam were treated as a single-phase fluid (see Eqs. (1) and (5)).

This data set is unusually valuable in presenting the behavior of a single foam formulation over a range of foam qualities in media of widely differing permeability. However, there is some concern that the entrance effect (Ransohoff and Radke, 1992) might affect these data. In particular, if the entrance length (distance from the inlet over which foam comes to full strength) varies systematically as a function of permeability, that would appear in our data as an effect of permeability on IT foam parameters. We present these results as the best available to date, pending results for longer cores that exclude the entrance effect.

Figure 3 presents the gas apparent viscosity as a function of foam quality for rocks with different permeabilities. The symbols are the experimental data and the lines are calculated using Eqs. (2) and (5) with the parameters listed in Table 1 and Table A.1 in Appendix A. At all foam qualities, the apparent viscosity of the gas increases with increasing permeability, i.e., foam appears to be stronger in the rock with higher permeability. Moreover, the existence of two regimes is clear for all permeabilities; the apparent viscosity of foam increases and reaches a maximum as the foam quality decreases from 1 (high-quality regime) and thereafter decreases with further decreases in foam quality (low-quality regime).

To extract foam-model parameters from experimental data, we assume all core plugs have similar wetting behavior and consequently fix the value of Corey exponent in the water relative-permeability function to $n_w = 4$ (see Eq. C.4). Foam-model parameters are sensitive to relative-permeability parameters, and in particular n_w can greatly impact the parameters (Ma *et al.*, 2014). Below we test sensitivity of our results to n_w . The total superficial velocity of the experiments, which is needed for the calculation of capillary number, is not given in the paper. We assume a value of 10 ft/d (3.5 E-5 m/s) in our calculations. However, the total superficial velocity used in the calculations affects only the value of *fmmob* and the other parameters remain the same if a different value is used, albeit by fixing the value of *fmcap*. Table 1 presents the model parameters that have been used to match the

experimental data on Figure 3. This set includes a fit allowing all parameters to vary (except for f_{mcap} , which is simply a reference capillary number for F_5 in Eq. (2)) and constraining $epcap > 0$ (i.e., not allowing for shear-thickening in the low-quality regime. The model fits using different constraints are shown in Table A.2 in Appendix A.

In all the data fits, the value of f_{mdry} decreases with increasing permeability. This is an indication that foam reduces water saturations to lower values as permeability increases. In fact, the values of f_{mdry} are remarkably similar for all model fits, as shown in Table A.2. Moreover, the fitted values of $epcap$ indicate that foam exhibits shear-thinning behavior, which becomes more pronounced as permeability of the rock increases. The model fit using the approach of Boeije and Rossen (2013), i.e., using very large $epdry$ values supports this conclusion as well (see Table 2).

Foam exhibits shear-thickening behavior in the lowest-permeability rocks for the experimental data assessed in this paper. We know of no physical justification for this behavior.

Direct comparisons of values of f_{mmob} are ambiguous; if behavior is non-Newtonian in the low-quality regime, the value of f_{mmob} is affected by the arbitrary choice of the reference state represented by f_{mcap} (Boeije and Rossen, 2013). A better comparison is that for $[f_{mmob} F_5]$ at a fixed pressure gradient; this represents the mobility reduction in the low-quality regime at that pressure gradient. In Table 1 and Table A.2 we show values of $[f_{mmob} F_5]$ at a pressure gradient of 10 MPa/m, which is representative of a pressure gradient seen in all the corefloods in this paper, and at 0.1 MPa/m, which would be more representative of field application. The value of this product at any pressure gradient is given by

$$f_{mmob} F_5 = f_{mmob} \left(\frac{f_{mcap}}{k \nabla p / \sigma} \right)^{epcap} \quad (8)$$

The finding that the product $[f_{mmob} F_5]$ increases with increasing permeability is surprising. This product represents the mobility reduction for wet foams, in the low-quality regime. It does not reflect the effect of capillary pressure on foam, which makes foam stronger in high-permeabilities in the high-quality regime (Khatib *et al.*, 1988). Instead it is the mobility of bubbles thought to be about the size of pores (Bertin, 1998; Alvarez *et al.*, 2001). Pores would be expected to be smaller in lower-permeability media, which suggests that there are more lamellae resisting flow per unit length in these media. The total resistance is a function of many factors, including gas trapping and the resistance to individual lamellae as they flow; none of these immediately suggest lower resistance to flow in lower-permeability media. If confirmed by further study, this paradox deserves further investigation.

Figure 3 Apparent viscosity of CO_2 foam made with 2000 ppm Chaser CD-1050 as a function of permeability.

Table 1 Parameter estimation with permeability-dependent $epdry$ for foam data in **Figure 3**.

Permeability [mD]	Total Velocity (ft/day)	$n_w = 4$						
		fmmob	epdry	fmdry	fmcap	epcap	fmmob.F ₅ VP=10MPa/m	fmmob.F ₅ VP=0.1MPa/m
551.5	20	25171.2	80.6	0.265	2.46E-05	0.242	14871.43	29400.36
551.5	5	17997.5	80.6	0.265	2.46E-05	0.242	18349.3	17775.4
551.5	10	21284.2	80.6	0.265	2.46E-05	0.242	8479.31	25844.04
169.8	10	2058.6	195.9	0.308	2.46E-05	2.24E-14	2058.6	2058.6
56.8	10	899.4	> 530	0.349	2.46E-05	2.24E-14	899.4	899.4
32.8	10	502.3	> 10000	0.367	2.46E-05	4.33E-14	502.3	502.3

The assumption of a very large $epdry$ value for all experiments does not lead to good match in the high-quality regime, particularly for the high permeability rock. An example is shown in Figure 4, on which we plot the curves obtained with different $epdry$ values and compare it to the experimental data. The convex trend observed in the high-quality regime in the experiment can be best matched with selecting $epdry = 80.6$. This implies that, for the set of data examined in this paper, the transition from high-quality to low-quality is not abrupt and foam does not collapse at a constant water saturation. For small $epdry$ values the *transition* water saturation (S_w^{tr}) is different than the *limiting* water saturation (S_w^*), which suggests that water fractional-flow function (f_w) decreases smoothly from S_w^{tr} to shock water saturation. Moreover, we notice from Table 1 as the permeability of rock decreases, the $epdry$ value increases. For low permeability cores the match can be obtained with choosing $epdry > 10000$.

If we neglect the curved behavior of foam in the high-quality regime, it is possible to fit the data with a single or permeability-independent large $epdry$ value. The parameters are listed in Table 2. Note that with change of $epdry$, the other parameter should also be adjusted.

Figure 5 shows a second set of experimental data for a different surfactant formulation and compares them to the model. Again, $epdry$ decreases with increasing permeability in

Table 3. Once again, foam exhibits shear-thinning behavior in the rock with high permeability. This conclusion, that $epdry$ decreases with increasing permeability, needs to be confirmed by further study, as Kapetas *et al.* (2015) found no correlation between $epdry$ and permeability in their experiments. Moreover, the original study of the limiting capillary pressure (Khatib *et al.*, 1988), which reported an abrupt transition between regimes, was conducted in beadpacks of enormous permeability (up to 10,000 Darcy).

Our main interest in this paper the effect of the permeability on P_c^* , which is reflected in $fmdry$ and as mentioned its value is relatively independent of the fitting procedure. The next Section addresses this issue.

Figure 4 Results of the model with different $epdry$ values compared to the experimental data.

Table 2 Parameter estimation with assuming a very large $epdry$ of 100000 for foam data in **Figure 3**.

Permeability [mD]	Total Velocity (ft/day)	$n_w = 4$				
		fmmob	epdry	fmdry	fmcap	epcap
551.5	10	47434	1.00E+05	0.261	2.46E-05	0.449
169.8	10	1964.5	1.00E+05	0.308	2.46E-05	2.22E-14
56.8	10	886.2	1.00E+05	0.349	2.46E-05	3.4E-14
32.8	10	502.3	1.00E+05	0.367	2.46E-05	4.33E-14

Figure 5 Apparent viscosity of CO_2 foam made with 2000 ppm foamer NES-25 as a function of permeability.

Table 3 Parameter estimation for foam data in Figure 5.

Permeability [mD]	Total Velocity (ft/day)	$n_w = 4$				
		fmmob	Epdry	fmdry	fmcap	epcap
496	10	4386	89.8	0.334	2.46E-05	2.22E-14
176	10	1566	358.3	0.348	2.46E-05	2.22E-14
50.2	10	620	> 5000	0.380	2.46E-05	2.32E-14
22.7	10	364	> 10000	0.397	2.46E-05	3.94E-13

Effect of permeability on limiting capillary pressure

The general form of the gas-water capillary pressure function can be expressed as follows (Brooks and Corey, 1964; 1966; Skjaeveland *et al.*, 1998; Li, 2004):

$$P_c = P_{c@S_{wc}} (1 - b S_{wD})^{-1/\lambda} \quad (9)$$

where $P_{c@S_{wc}}$ is the capillary pressure at the connate water saturation, S_{wc} , when drainage capillary pressure is used. S_{wD} is the normalized water saturation defined as:

$$S_{wD} = \frac{S_w - S_{wc}}{1 - S_{wc} - S_{gc}} \quad (10)$$

Parameter b is a constant and can be calculated from:

$$b = 1 - \left(\frac{P_{ce}}{P_{c@S_{wc}}} \right)^{-\lambda} \quad (11)$$

where P_{ce} is the entry capillary pressure of the porous medium and λ is the sorting factor. For a known value of S_w^* , the value of limiting capillary pressure, P_c^* , can be estimated using Eq. (9). If the wetting states of the rocks are similar the scaling of capillary pressure with permeability can be estimated using the Leverett J-function:

$$J(S_w) = C_J \frac{P_c^k(S_w)}{\sigma} \sqrt{\frac{k}{\phi}} \quad (12)$$

Therefore, to understand the effect of permeability on the limiting capillary pressure, the J-Leverett function was calculated from capillary pressure data for a Berea sandstone core with permeability of about 500 mD (Jorden and Campbell, 1984). The red curve in Figure 6 is obtained by using $P_{ce}=2$ kPa and $P_{c@S_{wc}}=1000$ kPa for $k = 551$ mD, and the relative-permeability parameters in Table A.1. The capillary-pressure curves of the other permeabilities were calculated using Eq. (12). Note that in this model connate water saturation is assumed to be at $P_{c@S_{wc}} = 1000$ kPa; and as a result the connate water saturation is also permeability-dependent (as indicated in **Figure 6**).

Figure 6 Calculated capillary pressure curves for rocks with different permeability; P_{ce} ($k=551\text{md}$) = 2 kPa, $P_{c@Swc} = 1000$ kPa, $\lambda = 2/3$.

Once the capillary pressures are known, taking the value of f_{mdry} (extracted from experiments) as the limiting water saturation, the limiting capillary pressure can be estimated for each set of foam data for different permeability rocks. Figure 7 shows the relation between P_c^* and rock permeability. The value of limiting capillary pressure increases as permeability decreases. Moreover, P_c^* varies linearly with $1/\sqrt{k}$. It is interesting to note that this scaling is independent of the value of ep_{dry} and choice of n_w in our model fits, as shown in **Figure 7**. To extract the data for $n_w=2$ we assumed that capillary pressure and relative permeability parameters can be related as described in Appendix C. In all cases the value of P_c^* increases as n_w increases.

A trend of decreasing P_c^* with increasing permeability can also be observed between the direct measurements of P_c^* and rock permeability of Khatib *et al.* (1988) shown in **Figure 8**. Rossen and Zhou (1995) came to a similar conclusion based on inferences from limited data, as did Rossen and Lu (1997) based on a wide survey of data (but not taking account of the distinction between the two foam flow regimes). This finding implies that P_c^* for foam in porous media depends not only on disjoining pressure but also on the permeability of the porous medium. Although P_c^* increases somewhat with decreasing permeability, the trend is still that foam is stronger (lower gas relative mobility) in rocks with greater permeability, as shown in Figure 3 and Figure 5. Decreasing P_c^* with increasing permeability implies diversion from a lower-permeability into a higher-permeability layer if layers are in capillary equilibrium (Rossen and Zhou, 1995; Rossen and Lu, 1997). Rossen and Lu (1997) found, though, that capillary equilibrium is difficult to achieve with foam. Rossen *et al.* (1999) report that, at short times, foam diverts flow from high- to low-permeability layers; at much longer times capillary-pressure gradients may drive water out of the higher-permeability layer and weaken foam there, especially if the viscous pressure gradient is modest.

Figure 7 Effect of permeability on the limiting capillary pressure. The points are extracted using parameters in Table 1 and Table 2.

Figure 8 Limiting capillary pressure measured by Khatib et al. as a function of permeability.

As mentioned earlier, a premise of the of the “limiting capillary pressure” concept is that the stability of static foam film or lamella is limited by a critical capillary pressure that is equal to the critical disjoining pressure. Dynamic systems can expect to have a limiting capillary pressure that is less than this critical capillary pressure because the foam lamella stretches as a foam lamella moves from a pore throat to a pore body and thus increase in area (Jimenez and Radke, 1988). If the lamella cannot recruit sufficient water from the Plateau border, the lamella must thin to compensate for the increase in area, i.e., to maintain constant volume. If the lamella can recruit water from the Plateau border, it does not thin as much. As the lamella thins, the disjoining pressure increases. If the lamella thins to the critical disjoining pressure, it becomes unstable and breaks. The limiting capillary pressure is equal to the critical disjoining pressure at static conditions. As the lamella velocity increases, the limiting capillary pressure must decrease to compensate for the increase in disjoining pressure as the

lamella stretches in passing from a pore throat to a pore body. As the rock permeability increases, the limiting capillary pressure can be expected to decrease because area of the lamella increases relative to the volume of water in the Plateau border. The Plateau border is the perimeter of the lamella and thus proportional to the characteristic dimension while the lamella area is proportional to the square of the characteristic dimension. Therefore, as permeability increases, there is less water, relatively speaking, nearby in the Plateau border to stabilize the lamella as it stretches.

Effect of properties of surfactant solution on limiting capillary pressure

The disjoining pressure depends on surfactant type and concentration, among other parameters. Foam films with greater disjoining pressure can survive a greater imposed capillary pressure. We observe from Figure 9 that surfactant type has also an influence on the strength of foam in porous media. The foam data on Figure 9 were fitted to the STARS model with the procedure explained in Appendix B and using the fitted $fmdry$ values the magnitude of P_c^* was estimated for each surfactant. These parameters are presented in Table 4. In addition, using the data reported by Ma *et al.* (2013), who studied the effect of surfactant concentration on foam in sandpacks, we find a trend of increasing P_c^* with increasing surfactant concentration (Table 5). The smaller values of the P_c^* values in Table 5 compared to those reported in Figure 7 are due to very high permeability of the rock used by Ma *et al.* experiments. A similar trend was reported by Apaydin and Kovscek (2002).

Figure 9 CO₂ foam data with different surfactant types in a core with permeability of 176 md (Moradi-Araghi *et al.*, 1997). The concentration of surfactant in each test was set to 2000 ppm.

Table 4 Estimation of foam parameters for data in Figure 9.

Surfactant	$n_w = 4$					
	fmmob	fmdry	epdry	fmcap	Epcap	P_c^* [kPa]
NES-25	1955.2	0.298	123.3	2.46E-5	-0.053	28.8
CD-1045	209.1	0.324	123.3	2.46E-5	-0.595	23.9
CD-128	631.27	0.362	113	3.88E-5	-0.320	19.5

Table 5 Effect of surfactant concentration on limiting capillary pressure. Data from Ma *et al.* (2013).

Surfactant concentration [wt%]	P_c^* [kPa]
1	1
0.5	0.5
0.02	0.45

Conclusions

- An implicit-texture foam model, specifically the STARS model examined in this study, can represent the effects of rock permeability and surfactant type and concentration on steady-state foam flow in porous media.
- For the experimental data investigated in this paper, the value of $epdry$ increases as rock permeability decreases. These data suggest that, for certain formulations in high permeability rocks, foam does not collapse abruptly at a single fixed water saturation. Rather, there is a range of water saturations over which foam weakens. The width of this range narrows with decreasing permeability in the data examined.
- For the cases examined, the Limiting Capillary Pressure, P_c^* , increases as permeability decreases, but not enough to reverse the trend of decreasing foam apparent viscosity as permeability decreases. Foam is still stronger in higher-permeability rock.

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Appendix A: Parameters fractional flow function and foam model

Table A.1 Relative permeability parameters and viscosities used in this paper.

k_{rw}^0	0.22	n_w/n_g	4/1.8
k_{rg}^0	1.0	λ	2/3
S_{wc}	k -dep.*	μ_w	0.65 [cP]
S_{gc}	0.05	μ_g	0.05 [cP]

The relative-permeability functions employed in this study are all of the form:

$$k_{rw} = k_{rw}^0 \left(\frac{S_w - S_{wc}}{1 - S_{wc} - S_{gc}} \right)^{n_w} \quad (A.1)$$

$$k_{rg} = k_{rg}^0 \left(\frac{1 - S_w - S_{gr}}{1 - S_{wc} - S_{gc}} \right)^{n_g} \quad (A.2)$$

where k_{rw} , k_{rg} , S_{wc} , S_{gr} , n_w and n_g are constants which vary per investigated data-set. The values of these constants used in this study are given in Table A.1. Additional properties used in the investigated data-sets are also given.

* - see Figure 6 for values

Table A.2 Estimated parameter for data on Figure 3 with different constraints in the fitting procedure.

Permeability [mD]	$n_w = 4$							constraint
	fmmob	epdry	fmdry	fmcap	epcap	$\frac{[fmmob \cdot F5]}{VP=10\text{MPa/m}}$	$\frac{[fmmob \cdot F5]}{VP=0.1\text{MPa/m}}$	
551.5	32813.4	85	0.265	2.46E-05	0.337	9108.53	42998.02	constant $fmcap$ $epcap > 0$
169.8	1941.8	368	0.308	2.46E-05	2.22E-14	1941.8	1941.8	
56.8	856.8	>3800	0.349	2.46E-05	3.83E-14	856.8	856.8	
32.8	442.9	>10000	0.367	2.46E-05	2.22E-14	442.9	442.9	
551.5	32813.4	85	0.265	2.46E-05	0.337	9108.53	42998.02	constant $fmcap$ fmmob increasing with increasing k
169.8	902.6	219.2	0.307	2.46E-05	-0.221	1612.29	582.69	
56.8	169.5	> 295	0.347	2.46E-05	-0.568	404.18	29.55	
32.8	36.2	> 10000	0.367	2.46E-05	-1.03	99.416	0.86	
551.5	32813.4	85	0.265	2.46E-05	0.337	9108.53	42998.02	constant $fmcap$ fmmob increasing with increasing k $epcap > -0.5$
169.8	902.6	219.2	0.307	2.46E-05	-0.221	1612.28	582.70	
56.8	205.7	> 354	0.347	2.46E-05	-0.5	442.03	44.20	
32.8	124.6	> 10000	0.362	2.46E-05	-0.5	203.47	20.34	
551.5	21284.2	80.6	0.265	2.46E-05	0.242	8479.31	25844.04	fmmob increasing with increasing k $epcap > 0$
169.8	2058.6	195.9	0.308	2.46E-05	2.24E-14	2058.6	2058.6	
56.8	899.4	> 530	0.349	2.46E-05	2.24E-14	899.4	899.4	
32.8	502.3	> 10000	0.367	2.46E-05	4.33E-14	502.3	502.3	
551.5	10063	61.4	0.266	3.15E-05	0.148	5945.33	11753.74	no constraint
169.8	1866.7	593	0.308	6.67E-05	-0.0069	1887.78	1828.74	
56.8	744.9	> 750	0.347	3.29E-04	-0.535	421.72	35.89	
32.8	143.7	> 10000	0.367	9.16E-05	-1.0007	102.88	1.025	

Table A.3 Parameter estimation with permeability-dependent $epdry$ for foam data in Figure 3 with $n_w=2$.

Permeability [mD]	Total Velocity (ft/day)	$n_w = 2$				
		fmmob	Epdry	fmdry	fmcap	epcap
551.5	10	89874.6	216.4	0.133	2.46E-05	0.476
169.8	10	5153.1	296.5	0.152	2.46E-05	0.12
56.8	10	1547.9	>450	0.173	2.46E-05	2.22E-14
32.8	10	852.8	>10000	0.184	2.46E-05	9.3E-8

Appendix B: Details of fitting procedure

The experimental data that are obtained in a foam scan experiments are a series of values of pressure gradient ∇p_i^{exp} [Pa] for a given gas fractional flow $f_{g,i}^{\text{exp}}$. First, we calculate the water saturation for each data point as follows. The relations for the calculation of the apparent viscosity and liquid velocity read

$$\mu_{\text{foam},i}^{\text{exp}} = \frac{k}{u_{t,i}} |\nabla p_i^{\text{exp}}|, \quad (\text{B.1})$$

$$u_{w,i} = (1 - f_{g,i}^{\text{exp}}) u_{t,i} = \frac{k_{rw} k}{\mu_w} |\nabla p_i^{\text{exp}}|, \quad (\text{B.2})$$

where $\mu_{\text{foam},i}^{\text{exp}}$ [Pa.s] is the apparent viscosity of foam, $u_{t,i}$ [m/s] is the total Darcy velocity, k [m²] is the absolute permeability, $u_{w,i}$ [m/s] is the Darcy velocity of the liquid phase, μ_w [Pa.s] is the viscosity of the liquid phase, and k_{rw} [-] is the relative permeability of the liquid phase. Eqs. (B.1) and (B.2) can be combined to obtain

$$k_{rw} = \frac{(1 - f_{g,i}^{\text{exp}}) \mu_w}{\mu_{\text{foam},i}^{\text{exp}}}. \quad (\text{B.3})$$

From the Corey-type relation for the liquid relative permeability (Eq. A.1), the liquid saturation is calculated as

$$S_{w,i} = \left(\frac{(1 - f_{g,i}^{\text{exp}}) \mu_w}{k_{rw}^0 \mu_{\text{foam},i}^{\text{exp}}} \right)^{1/n_w} (1 - S_{wc} - S_{gr}) + S_{wc}, \quad (\text{B.4})$$

Next, using the foam model parameters, we calculate the estimated values of $\mu_{\text{foam},i}$ and $f_{g,i}$ from this value of $S_{w,i}$. These values of the liquid saturation, which are calculated for each data point, are used in the definition of the objective function. The apparent viscosity of foam and gas fractional flow are calculated as follows

$$\mu_{\text{foam},i}(\mathbf{x}, S_w) = \left(\frac{k_{rw}(S_w)}{\mu_w} + \frac{k_{rg}^f(\mathbf{x}, S_w, \mu_{\text{foam},i})}{\mu_g} \right)^{-1}, \quad (\text{B.5a})$$

$$f_{g,i}(\mathbf{x}, S_w) = \left(\frac{k_{rg}^f(\mathbf{x}, S_w)}{\mu_g} \frac{\mu_w}{k_{rw}(S_w)} \right)^{-1}, \quad (\text{B.5b})$$

where $\mathbf{x} = [fmmob, epdry, fmdry, fmcap, epcap]$ are the foam model parameters. Parameter $fmcap$ is set to the minimum expected capillary number of the system. For given values of liquid saturation and foam model parameters, the following equation can be solved to obtain the apparent viscosity of foam:

$$M(\mu_{foam}, \mathbf{x}, S_w) = \mu_{foam} - \left(\frac{k_{rw}(S_w)}{\mu_w} + \frac{k_{rg}^f(\mu_{foam}, \mathbf{x}, S_w)}{\mu_g} \right)^{-1} = 0, \quad (B.6)$$

The objective function for the optimization of foam model parameters can be defined as

$$\Phi(\mathbf{x}) = \sum_{i=1}^{n_{data}} w_i \left(\mu_{foam,i}(\mathbf{x}, S_{w,i}) - \mu_{foam,i}^{exp} \right)^2, \quad (B.7)$$

where w_i [-] are weight factors, and $\mu_{foam,i}$ [Pa.s] is the solution of the following nonlinear equation for given values of liquid saturation and foam model parameters:

$$M(\mu_{foam}(\mathbf{x}, S_{w,i})) = 0. \quad (B.8)$$

Expression (B.7) is minimized by using a constrained least-square algorithm in the optimization toolbox of Matlab software (function 'lsqnonlin'). It must be noted that in the objective function routine, the apparent viscosity of foam is calculated by solving the nonlinear Eq. (B.8) in a separate sub-routine. The constraints are in the form of lower and upper bounds, which sets limits on the foam parameters, e.g., $fmdry > S_{wc}$

Appendix C: Relation between capillary and relative permeability

We assume that the water relative permeability can be calculated using the following correlation between the relative permeability and capillary pressure functions (Burdine, 1953)

$$k_{rw} = k_{rw}^0 \frac{\int_0^{S_w} dS_w / (P_c)^2}{\int_0^1 dS_w / (P_c)^2} \quad (C.1)$$

Inserting P_c from Eq. (8) we obtain the following relationship for calculation of the relative-permeability function:

$$k_{rw} = k_{rw}^0 \left(\frac{1 - (1 - bS_{wD})^{1+2/\lambda}}{1 - (P_{ce} / P_{c@Swc})^{-(2+\lambda)}} \right) \quad (C.2)$$

The following relationship can be established between the Corey exponent for water relative permeability and λ in the capillary pressure function:

$$n_w = 1 + \frac{2}{\lambda} \quad (C.3)$$

Therefore, Eq. (C.2) can be re-written as

$$k_{rw} = k_{rw}^0 \left(\frac{1 - (1 - b S_{wD})^{n_w}}{1 - (1 - b)^{n_w}} \right) \quad (C.4)$$

The following provides the relationship between S_w^* and P_c^* . This equation is obtained by inserting $S_w = S_w^*$ in Eq. (9):

$$S_w^* = S_{wc} + \frac{1 - S_{wc} - S_{gc}}{b} \left(1 - \left(\frac{P_c^*}{P_{c@S_{wc}}} \right)^{1/(1-n_w)} \right) \quad (C.5)$$