STELLINGEN
behorende bij het proefschrift:

"A Three Dimensional Method for the Calculation of the Unsteady Ship Wave Pattern using a Neumann - Kelvin Approach"

van

Carlo van der Stoep

1. Alhoewel niet is voldaan aan de eis dat de te gebruiken splines van een oneven graad moeten zijn, is bij de benadering van de Prandtl integro-differentiaal operator toch een fout van $O(h^2)$ te verwachten.

2. Om de verkregen matrix vergelijking snel succesvol te kunnen 'pre-
conditioneren' is een kolom georiënteerde nummering van de pa-
nelen noodzakelijk.
 Dit proefschrift, hoofdstuk 4.

3. Het volgnummer, dat altijd meegegeven wordt bij een nieuwe re-
lease van een computer applicatie, is niet alleen een hulp bij het op de juiste wijze sorteren van de programmatuur maar ook een voorzichtige schatting van het aantal dagen dat de argeloze ge-
bruiker, na de installatie van het nieuwe pakket, kwijt is om zijn programmatuur weer werkend te krijgen.

4. Het gebruik van Padé approximatie bij de berekening van de zo-
genaaamde 'wave resistance Green functie' is zinvol.
 Dit proefschrift, hoofdstuk 3.

5. De opmerking van Nakos over de moeilijkheden die ondervonden worden bij de ontwikkeling van een Neumann-Kelvin schema voor de berekening van de instationaire scheepsgolven is slechts ten dele waar.
 Dit proefschrift, hoofdstuk 1.
 Nakos, D.E. (1990). Ship wave patterns and motions by a three di-
ensional Rankine panel method. Ph.D. Thesis. MIT.

6. Uit het feit dat de formatie-plaats van een AIO op 70% gesteld is, mag men niet concluderen dat de gemiddelde AIO kleiner van stuk is dan de gemiddelde wetenschappelijke medewerker.

8. De moeilijkheden die ontstaan bij het opstellen van het zogenaamde 'muurtje' bij voetbal maken duidelijk dat de huidige definitie van de meter in de praktijk lastig hanteerbaar is.

(De meter is gedefinieerd als: de lengte die gelijk is aan 1650763,73 golfdalingen in het luchtedige van de straling, overeenkomend met de overgang tussen de niveaus 2p₁₀ en 5d₅ van de kryptonisotoop ⁸⁶Kr.)

9. Het genereren van een random positie op een cirkelschijf wordt meestal, ten onrechte, geïllustreerd met: "het gooiien van pijlen op een dartbord". De pijlen, gegoooid door een darter, zullen op dit dartbord echter slechts een gebied \((R,\phi)\) raken dat gedefinieerd wordt door:

\[ \phi \in \left( \frac{\pi}{2} - \frac{\pi}{20}, \frac{\pi}{2} + \frac{\pi}{20} \right) \quad \text{en} \quad R \in (9.5, 10.5) \]

Hetgeen natuurlijk nauwelijks random te noemen is.
10. Net als in alle andere takken van sport, kent de bridge-sport de onderverdeling in goede en minder goede spelers. Het paradoxale van deze benaming bij de bridge-sport is dat juist de minder goede spelers de betere spelers voor onoverkomelijke problemen kunnen stellen.

De technisch juiste (met grootste kans op het behalen van 2 slagen) speelwijze om de volgende kaartcombinatie te behandelen is: in slag 1 een kleintje naar de 9 spelen (indien links geen honneur verschijnt).

Noord:
AB9

West: ???

Oost: ???

Zuid:
xxx

Stel nu dat links de H verschijnt. West, een goede tegenstander, zal van HTx de H inleggen (falsecard). De minder goede westspeler zou juist van HVx de H kunnen inleggen. De juiste speelwijze zal dus van de speelsterkte van de tegenstanders afhangen. Ook dit laatste, het op de juiste wijze inschatten van de speelsterkte van de tegenstanders, vormt een niet onaanzienlijk deel van de tactiek van de wedstrijdbridger.

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Proefschrift

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Nomenclature

Coordinates:

\[ x = (x, y, z) \]  Coordinate system moving with velocity \( U \)
\[ x \]  Direction of the forward velocity
\[ y \]  Lateral direction
\[ z \]  Vertically upward
\[ n \]  Unit vector normal to \( \Sigma \) in outward direction
\[ \alpha \]  Motion vector

Physical properties:

\[ B \]  Beam at midship
\[ \Sigma \]  Total wetted surface of the body
\[ L \]  Length at water line
\[ H \]  Draft at midship
\[ V_S \]  Ship volume
\[ g \]  Gravitational acceleration

Non dimensional parameters:

\[ \epsilon \]  Principal small parameter
\[ C_B \]  Ship block coefficient\((V_S/BLH)\)
\[ Fn \]  Froude number\((U/\sqrt{gL})\)
\[ Re \]  Reynolds number\((UL/\nu)\)
\[ C_R \]  Residual resistance coefficient\((R/\frac{1}{2}\rho U^2\Sigma)\)
NOMENCLATURE

$C_D$  Total resistance coefficient($D/\frac{1}{2}\rho U^2\Sigma$)
$C_F$  Frictional resistance coefficient($F/\frac{1}{2}\rho U^2\Sigma$)
$C_W$  Wave resistance coefficient($W/\frac{1}{2}\rho U^2\Sigma$)

Flow data:

$U$  Ship speed
$p$  Pressure
$\rho$  Density
$\nu$  Viscosity
$\zeta(x,y)$  Elevation of the free surface
$\zeta_0(x,y)$  Steady wave elevation
$\zeta_1(x,y)$  Unsteady wave elevation
$z_a$  Oscillator amplitude
$S_A$  relative motion composed of pitch, heave and undisturbed wave amplitude
$\Delta S_A$  swell-up
F.S.  Free surface
$\bar{\phi}$  Steady wave potential
$\tilde{\phi}$  Unsteady wave potential
$\Phi$  Total potential
$\omega$  Frequency of motion
$\omega_e$  Frequency of encounter
$\gamma$  Dipole strength
$\sigma$  Source strength

Miscellaneous:

$G(x,\xi)$  Green function
$T_n(x)$  Chebychev polynomial of order $n$
$J_n(x), K_n(x), Y_n(x)$  Bessel functions of order $n$
Chapter 1

General Introduction

Fluid motions are basically governed by three kinds of forces: Inertial, Viscous and Gravitational. In order to estimate the actual full-scale flow from the scale model, the actual force have to be corrected by a suitable multiplicative factor. The problem is that all three forces act simultaneously and they each have their own multiplicative factor. For instance, using the length scale \( L \), the three forces are of order of magnitude \( L^2 \), \( L \) and \( L^3 \) respectively. To predict the forces of the full-scale model from tests with the small-scale model, it is helpful to form the ratios of the three forces. Because any two of these ratios are sufficient to define the third, only the well-known Froude number, the ratio of the inertial to the gravitational force and the Reynolds number, the ratio of the inertial to the viscous force will be used (Newman [28]).

At the end of the last century, Froude stated his hypothesis that the ship resistance coefficient can be expressed as the sum of two components: the residual drag coefficient, depending on the Froude number and the frictional drag coefficient, depending on the Reynolds number:

\[
C_D(Re, Fn) = C_F(Re) + C_R(Fn)
\]  

(1.1)

For almost every hull form, provided the ship velocity is not too low, the dominant part of the residual drag is associated with wave resistance. The viscous forces in the fluid are confined to a thin boundary layer close to the ship hull and can be treated separately.
The neglect of viscosity in the Navier-Stokes equations results in the Euler equations. Under the additional assumptions of constant density \(\rho\) and irrotational flow this leads to the class of Potential flows. Under the assumption of a potential flow the stationary problem of calculating the ship wave resistance is described by the Laplace equation, subject to conditions on the ship hull and the free surface. Many theorems and computer programs have been developed to solve this problem. For instance computer algorithms based on finite difference methods and algorithms based on boundary element methods. The latter can be classified in two categories, based on the choice of the singularity:

**Kelvin wake sources.** These are originally due to Brard(1972). One of the major advantages of this approach is the elimination of the free surface from the domain over which the resulting integral equation is defined. Also the so-called radiation condition is automatically satisfied.

**Rankine sources.** Dawson(1977) was one of the first to use the Rankine source as the elementary singularity. One of the advantages of this method is the flexible use of different free surface conditions. One of the disadvantages is the introduction of a distortion of the wave system due to the free surface discretization. Also errors are introduced by imposing the radiation condition numerically. See also Sclavounos & Nakos(1988), Piers(1983) and Raven(1988).

The estimation of a ship's speed and power was usually based on still water performance. However, in order to be able to predict ship performance in seaway, it is also desirable to be able to solve the instationary problem of a sailing ship. For example a ship sailing in waves or an oscillating vessel. In preliminary design studies the use of a fast computer algorithm could help to assist the traditional model testing. With the considerable increase in computer speed and resources, the first numerical solutions of 3-D seakeeping problems appeared. Recently Nakos(1990) and Bertram(1990) reported work with the Rankine source approach.

All of the other existing algorithms are based on the Neumann-Kelvin approach. According to Nakos(1990):

"Further developments of the unsteady scheme, stumble, however, on the additional and numerical difficulties in the evaluation of the unsteady Kelvin source potential, which serves as the Green function."

One of the drawbacks of the Rankine panel method was not being able to model the propagation of the complicated unsteady ship wave pattern by a
discretized free surface. Recently Nakos and Sclavounos (1990) proposed a free surface discretization scheme that is free of numerical damping and introduces small numerical dispersion, while providing flexibility in the choice of the linearization of the free surface condition.

In this thesis a different approach will be used. For the solution of the unsteady problem, the solution method of the steady state problem will be used. Particularly the additional numerical difficulties are reduced so they can be handled more easily. The unsteady wave pattern will be calculated, especially the part near the vessel itself. This is important for the relative motion between the vessel and the free surface, shipping of green water, bow slamming and propeller clearance. Shipping of green water, for instance, is assumed to occur whenever the relative motion amplitude is greater than the freeboard (see Blok [6]).

For the prediction of the wave resistance use can be made of far-field approximations for the evaluation of the (added) resistance. Also a method based on direct integration of pressure can be used. This direct pressure method is the approach which will be followed here and is similar to Pinkster [30].

One of the main problems in this thesis will be the calculation of the dynamic swell-up (and as a result the added resistance) of a ship in seaway. Dynamic swell-up is the effect of water being pushed up around a bow higher than can be accounted for by considering heaving, pitching and incident wave alone (see also figure (1.1)). The swell-up is given by the quotient of the amplitude of relative motion ($\zeta_1$) and the oscillator amplitude ($z_a$).

![Diagram](image)

Figure 1.1: The dynamic swell-up.

Some work in this area has been done by Blok [6] and Tasaki [38]. Tasaki
was the first to introduce the term swell-up. On the basis of experiments he obtained an empirical formula for the calculation of the dynamic swell-up, provided that the wave frequency, ship speed and ship block coefficient $C_B$ are between some given limits.

**Overview:**

This thesis deals with the hydrodynamical problem of a ship sailing at forward speed. Of practical interest will be features such as shipping of green water, (added) resistance, sinkage, trim and water height. The mathematical formulation based on smallness of the unsteady movements will be given in the first chapter. A new free surface condition will be given, that connects the steady flow to the unsteady flow. A linearization round the 'known' steady solution will lead to a free surface condition. The body boundary condition will be expanded round a mean position with respect to the small displacement parameter $\epsilon$, to be defined appropriately in Chapter 2.

For the solution of the two coupled integral equations as a result from a Neumann-Kelvin approach an expansion in terms of the small oscillation frequency $\omega$ has been chosen. The body boundary condition is also expanded in this small parameter $\omega$. A modified Neumann-Kelvin formulation has been found, using only the steady state Green function for the calculation and evaluation of the ship wave resistance problem.

In Chapter 3 the computational aspects are dealt with. The integral equation will be solved using a boundary element method. The wetted body will be divided into triangular panels to solve the equations. One of the most time consuming aspects of the problem is the calculation of the Green function. This is one of the major problems when using a Neumann-Kelvin approach (as has been mentioned by Nakos in his Ph.D. Thesis).

In Chapter 4 the stability and convergence of the resulting matrix equation will be analysed. Some of the aspects of these equations, like singular values and convergence of the solution algorithm will be dealt with.

In Chapter 5 some computational results will be presented. The forward speed problem has been solved for a specific hull form: the Wigley hull. At the Delft Hydromechanics Laboratory measurements on a Wigley hull have been
performed. The results from these measurements will be compared with the theoretical results presented here. In the last chapter conclusions and recommendations for future developments will be given.

The research has been carried out in cooperation with MARIN and the Delft Hydromechanics Laboratory.
Chapter 2
Mathematical Formulation

2.1 Introduction

We consider a ship moving horizontally in still water of infinite depth and a constant velocity $U$. Viewed from an inertial frame $(x, y, z)$ attached to the ship, there is an incident flow of velocity $U$ in the direction of the negative $x$-axis. It is assuming that the fluid is inviscid, incompressible, irrotational and free from surface tension effects, wave breaking and cavitation. By virtue of these assumptions, the velocity vector $V$ of the fluid can be represented as the gradient of a potential function $\Phi$:

$$V = \nabla \Phi \quad (2.1)$$

The problem will be formulated in a right handed Cartesian coordinate system $Oxyz$ moving with the ship. The $x$-axis is pointing astern and the $z$-axis vertically upward.

![Figure 2.1: The moving coordinate system.](image)

The free surface elevation is defined by a function $z = \zeta(x, y, t)$. For practical applications, the vessel is performing in seaway. This means that there are incoming waves present or some sort of forced oscillation exists.
This means that there is not only a steady (time independent) but also an unsteady (time dependent) part of the velocity potential present.

The total velocity potential $\Phi_{total}$ can be split in a steady (the sum of the uniform flow and the stationary wave part) and an unsteady part:

$$\Phi_{total}(x, t) = \underbrace{-Ux + \bar{\phi}(x)}_{\text{steady}} + \underbrace{\tilde{\phi}(x, t)}_{\text{unsteady}}$$  \hspace{1cm} (2.2)

$\Phi_{total}$ has to satisfy the following conditions:

- It has to satisfy the Laplace equation.

$$\Delta \Phi_{total} = 0 \quad \text{in the fluid region} \quad , z \leq \zeta(x, y)$$  \hspace{1cm} (2.3)

- The physical nature of the free surface requires two boundary conditions: firstly the kinematic boundary condition, which states that the normal velocity of the fluid equals the normal velocity of this free surface and secondly the dynamic condition which requires that at the free surface the pressure should be atmospheric and independent of the position on the free surface (see figure 2.2).

![Figure 2.2: The free surface conditions.](image)

The free surface $S$ is given by the equation:

$$\Upsilon(x, y, z; t) = 0$$  \hspace{1cm} (2.4)

In our case the free surface is given by the equation $z = \zeta(x, y; t)$, so $\Upsilon = z - \zeta(x, y; t)$.

This free surface should have the property that any particle which is once on the surface will remain on it. The total (particle) derivative is given by:
2.1. **INTRODUCTION**

\[
\frac{d\zeta}{dt} = u \frac{\partial \zeta}{\partial x} + v \frac{\partial \zeta}{\partial y} + w \frac{\partial \zeta}{\partial z} + \frac{\partial \Phi}{\partial t} = 0 \quad \text{on } S \tag{2.5}
\]

Thus equation (2.5) results in the so-called kinematic boundary condition:

\[
-\Phi_x \frac{\partial \zeta}{\partial x} - \Phi_y \frac{\partial \zeta}{\partial y} + \Phi_z - \frac{\partial \zeta}{\partial t} = 0 \quad \text{on } S \tag{2.6}
\]

The dynamic boundary condition can be derived from Bernoulli’s law:

\[
p = p_{\text{atm}} - \rho g z - \rho \frac{\partial \Phi}{\partial t} - \frac{1}{2} \rho \nabla \Phi \cdot \nabla \Phi + C(t) \tag{2.7}
\]

\((p_{\text{atm}} \text{ may be taken equal to zero and } C(t) = \frac{1}{2} \rho U^2 \text{ without loss of generality})\)

So at the free surface \(\zeta(x,y,t)\) the exact\(^1\) dynamic and kinematic boundary conditions read:

\[
\begin{aligned}
\begin{cases}
\frac{\partial \Phi}{\partial t} + \frac{1}{2} \nabla \Phi \cdot \nabla \Phi + g z = \frac{1}{2} U^2 & \quad \text{at } z = \zeta \\
\frac{\partial \zeta}{\partial t} - \frac{\partial \Phi}{\partial x} \cdot \frac{\partial \zeta}{\partial x} - \frac{\partial \Phi}{\partial y} \cdot \frac{\partial \zeta}{\partial y} = 0 & \quad \text{at } z = \zeta
\end{cases}
\end{aligned} \tag{2.8}
\]

- Finally the boundary condition on the ship hull:

\[
\frac{\partial \Phi}{\partial n} = V_n, \quad \text{at } \Sigma \tag{2.9}
\]

when \(V_n\) is the normal velocity of the body.

**Remark**

The solution also has to satisfy a so-called radiation condition. For this, the reader is referred to section (2.2.3).

\[\square\]

---

\(^1\)One has to assume that the fluid is ideal and that the surface tension is negligible, which indeed is the case. The surface tension for air-water is about 0.07 N/m (see Newman [28]).
2.2 Linearization of the boundary conditions

As can be observed from equation (2.8) the main difficulty in this problem is to find a solution of the Laplace equation satisfying boundary conditions at a free surface which is still unknown. One way of dealing with this problem could be in using a direct discretization scheme. For the complete time dependent problem this would involve some sort of iterative solver: at every time step a new solution must be computed and before this is possible, the free surface has to be calculated iteratively as well. Obviously this would lead to an algorithm which is very time consuming.

In 1898, Michell was the first to obtain an approximating analytical solution for this problem. His 'thin' ship theory has become worldwide famous. Michell and others after him obtained his perturbation scheme using the slenderness\(^2\) of the ship. Another small parameter could be the velocity of the ship. If the Froude number, based on the length of the ship, is a small parameter, an expansion with respect to this parameter can be obtained. Brandsma ([7],[8]) has developed a computational method to obtain the wave resistance of full ship forms. Further work can be found in Baba [2], Sakamoto [33], Hermans [17] and many others.

To derive an approximating solution of the problem we continue by linearizing the free surface condition (2.8) and the body boundary condition (2.9).

2.2.1 The free surface condition

The free surface condition (2.8) consists of two parts:
The dynamic boundary condition:

\[
\frac{\partial \Phi}{\partial t} + \frac{1}{2} \nabla \cdot \nabla \Phi + gz = \frac{1}{2} U^2 \quad \text{at } z = \zeta(x,y,t) \quad (2.10)
\]

and the kinematic boundary condition:

\[
\frac{\partial \Phi}{\partial z} - \frac{\partial \zeta}{\partial t} - \frac{\partial \Phi}{\partial x} \cdot \frac{\partial \zeta}{\partial x} - \frac{\partial \Phi}{\partial y} \cdot \frac{\partial \zeta}{\partial y} = 0 \quad \text{at } z = \zeta(x,y,t) \quad (2.11)
\]

\(^2\)Note the difference between thin and slender; Thin means that the beam is small compared to all other characteristic lengths of the problem, by slender we mean that the transverse dimensions of the body are small compared to its length.
2.2. **LINEARIZATION OF THE BOUNDARY CONDITIONS**

The complicated non-linear nature of the two free surface conditions (2.10) and (2.11) prohibits the development of an exact solution. Therefore a method of approximation is required.

First we give a simple example of the linearization procedure. For instance in the case of small unsteady potentials and the forward speed equal to zero one way of dealing with this problem could be neglecting the second and higher order terms in $\Phi$ and $\zeta$.

Then the kinematic boundary condition (2.11) leads to:

$$\frac{\partial \Phi}{\partial z} = \frac{\partial \zeta}{\partial t}$$

(2.12)

This approximations means that the vertical velocity of fluid equals the vertical velocity of the free surface.

Substituting the $\zeta$ for $z$ in equation (2.10) leads for the linearized free surface height to:

$$\zeta = -\frac{1}{g} \frac{\partial \Phi}{\partial t}$$

(2.13)

combining equations (2.12) and (2.13) leads to the well known free surface condition:

$$\frac{\partial^2 \Phi}{\partial t^2} + g \frac{\partial \Phi}{\partial z} = 0 \quad \text{at } z = 0$$

(2.14)

Of course this free surface condition should be imposed on the actual free surface $z = \zeta$, but in this linearized theory it makes no difference.

□

We now proceed with the more general case. In order to be able to solve the steady and its perturbed wave problem, we introduce a more complicated linearization by making further assumptions. We assume that the unsteady wave part is due to a small displacement of the ship. So when the body is displaced from its equilibrium position, the deflection of any point of the hull is assumed to be small (see also section 2.2.2).

In many cases a small parameter $\epsilon$ is involved. For instance $\epsilon$ can be the slenderness of the ship or the magnitude of the instationary disturbance. In this thesis we consider the case that the instationary potential is small compared to the stationary potential. Therefore we replace $\tilde{\Phi}(x, t)$ by $\epsilon \tilde{\Phi}(x, t)$ in equation (2.2).
We will now obtain an asymptotic solution for the free surface condition. Let $|\epsilon| \ll 1$ and expand the free surface elevation around a 'known' solution $\zeta = \zeta_0(x, y)$:

$$\zeta(x, y, t) = \zeta_0(x, y) + \epsilon \zeta_1(x, y, t) + \ldots$$  \hspace{1cm} (2.15)

Denote the total velocity potential by $\Phi_{\text{total}}$, and split this total potential in a steady and an unsteady part as follows:

$$\Phi_{\text{total}}(x, t) = -Ux + \Phi(x) + \epsilon \Phi(x, t)$$  \hspace{1cm} (2.16)

All the terms in the free surface condition (2.10) and (2.11) have to be expanded (see Appendix A for details). This leads to the following free surface condition:

For the zeroth order non-linear problem (the steady state solution):

$$\phi_x + \frac{1}{2g} \nabla(-Ux + \hat{\phi}) \cdot \nabla[\nabla(-Ux + \hat{\phi}) \cdot \nabla(-Ux + \hat{\phi})] = 0$$  \hspace{1cm} at $z = \zeta_0$  \hspace{1cm} (2.17)

with a first order linear approximation of the wave height $\zeta_0$:

$$\zeta_0(x, y) = \frac{U}{g} \Phi_x(x, y, 0)$$  \hspace{1cm} (2.18)

and for the first order linear problem (the unsteady state solution):

$$g(\phi_x - \phi_{xx}\zeta_0 - \phi_y\zeta_0y) - \phi_{xx}(\phi_t + \nabla(-Ux + \hat{\phi}) \cdot \nabla\hat{\phi}) +$$
$$2\nabla(-Ux + \hat{\phi}) \cdot \nabla\phi_t + \nabla(-Ux + \hat{\phi}) \cdot \nabla[\nabla(-Ux + \hat{\phi}) \cdot \nabla\hat{\phi}] +$$

$$\phi_{tt} = 0$$  \hspace{1cm} at $z = \zeta_0$  \hspace{1cm} (2.19)

with the first order wave elevation given by (see equation A.14):

$$\zeta_1 = -\frac{1}{g} \left\{ \frac{\partial \hat{\phi}}{\partial t} + \nabla(-Ux + \hat{\phi}) \cdot \nabla\hat{\phi} \right\}$$  \hspace{1cm} (2.20)
2.2. LINEARIZATION OF THE BOUNDARY CONDITIONS

Regrouping of equation (2.19), by leaving the terms which are independent of the steady state solution \( \tilde{\phi} \) to the left, gives us:

\[
g \tilde{\phi}_z - 2U \tilde{\phi}_{xt} + \tilde{\phi}_{tt} + U^2 \tilde{\phi}_{xx} = \mathcal{L}(U; \tilde{\phi})\{\tilde{\phi}\} \quad \text{at} \quad z = \zeta_0
\]  

(2.21)

Where \( \mathcal{L}(U; \tilde{\phi}) \) denotes a linear differential operator acting on \( \tilde{\phi} \) as defined in equation (2.19). This operator can be found in appendix B.

The linearization procedure (of the F.S.C.) can be performed with respect to either the undisturbed uniform flow or the double-body flow (see also Raven [32]). The former approach has been used in this thesis, the latter has been used by Sakamoto and Baba [33]. After some rather strong demands (the unsteady wave part must be of order \( O(U^n) \) with \( 3 < n < 7 \)) Sakamoto and Baba obtained two independent linearized equations for the unsteady steady free surface conditions:

\[
\frac{\partial \phi_0}{\partial z} + \frac{1}{g} \left\{ u_{r0} \frac{\partial}{\partial x} + v_{r0} \frac{\partial}{\partial y} \right\}^2 \phi_0 = D(x, y) \quad \text{at} \quad z = \zeta_r
\]  

(2.22)

\[
\frac{1}{g} \left\{ \frac{\partial}{\partial t} + u_{r0} \frac{\partial}{\partial x} + v_{r0} \frac{\partial}{\partial y} \right\}^2 \phi_1 + \frac{\partial \phi_1}{\partial z} = 0 \quad \text{at} \quad z = \zeta_r
\]  

(2.23)

where (see Sakamoto and Baba [33]):

\( \phi_r \): velocity potential representing double body flow in calm water. This is regarded as the base flow on which the following perturbation potentials are superimposed.

\( \phi_0 \): velocity potential representing steady wave making.

\( \phi_1 \): velocity potential representing unsteady wave making.

\( u_{r0}, v_{r0} \): velocity components of double body flow at \( z=0 \).

\[
D(x, y) = \frac{\partial u_{r0}\zeta_r}{\partial x} + \frac{\partial v_{r0}\zeta_r}{\partial y}
\]

\( \zeta_r \): wave height of double body flow.

\[\square\]

In this thesis we are especially interested in the formulation of the influence of the steady part \( \tilde{\phi} \) on the unsteady part \( \tilde{\phi} \), so we will use equations (2.17) and (2.19).
2.2.2 The body boundary condition

The body boundary condition for the stationary potential reads as: (see also equation (2.9))

$$
\frac{\partial \Phi}{\partial n} = V_n \quad \text{at the body } \Sigma \quad (2.24)
$$

or equivalently for the steadily sailing ship:

$$
n \cdot \nabla (-U x + \tilde{\phi}) = 0 \quad \text{at } \Sigma \quad (2.25)
$$

Next we consider the calculation of the instationary part (see also for instance Timman [39] et al.). The displacement of the ship hull is assumed to be small compared to the length of the ship. We write:

$$
x - x' = \epsilon \tilde{\alpha}(x, t) \quad (2.26)
$$

where \( \epsilon \) is a small parameter, \( x \) denotes the coordinate system moving with velocity \( U \) in the direction of the positive \( x \)-axis, and \( x' \) denotes the coordinate system fixed to the ship. The body boundary condition states that: at the instantaneous position of the hull, the normal velocity of the fluid is equal to the normal velocity of the hull.

Expansion of \( \Phi \) around the mean position with respect to a small parameter \( \epsilon \) yields:

$$
\Phi_{\text{momentary}} = \Phi_{\text{total}} + \epsilon \tilde{\alpha} \cdot \nabla \Phi_{\text{total}} \quad (2.27)
$$

So for the body boundary condition this leads to:

$$
n \cdot \nabla (\Phi_{\text{total}} + \epsilon \tilde{\alpha} \cdot \Phi_{\text{total}}) = \epsilon \ n \cdot \frac{\partial \tilde{\alpha}}{\partial t} \quad (2.28)
$$

Since on \( \Sigma \) we have that \( n \cdot \nabla (-U x + \tilde{\phi}) = 0 \) equation (2.28) reduces to:

$$
\frac{\partial \tilde{\phi}}{\partial n} = n \cdot (\frac{\partial \tilde{\alpha}}{\partial t} - \nabla (\tilde{\alpha} \cdot \nabla (-U x + \tilde{\phi}))) \quad (2.29)
$$

Now using the assumption of \( \alpha(x, t) \) and \( \tilde{\phi}(x, t) \) to be oscillatory:

$$
\dot{\alpha}(x, t) = \Re\{\alpha(x) \cdot e^{-i\omega t}\}
\tilde{\phi}(x, t) = \Re\{\tilde{\phi}(x) \cdot e^{-i\omega t}\} \quad (2.30)
$$

This finally leads to the following body boundary condition:
2.2. LINEARIZATION OF THE BOUNDARY CONDITIONS

\[
\frac{\partial \tilde{\phi}}{\partial n} = -n \cdot (i\omega \alpha + \nabla (\alpha \cdot \nabla (-U x + \tilde{\phi})))
\]  
(2.31)

and from the free surface condition (2.21):

\[-\omega^2 \tilde{\phi} + 2i\omega U \tilde{\phi}_x + U^2 \tilde{\phi}_{xx} + g \tilde{\phi}_z = L(U; \tilde{\phi})\{\tilde{\phi}\} \quad \text{at } z = \zeta_0
\]  
(2.32)

where \( L(U; \tilde{\phi}) \) denotes the linear differential operator acting on \( \tilde{\phi} \).

2.2.3 The radiation condition

In this thesis we are dealing with the situation of a body which is oscillating with a frequency \( \omega \) and moving steadily with velocity \( U \). Besides the boundary conditions given in section (2.2) an additional condition has to be imposed: a condition at infinity to ensure a unique solution.

For instance a body moving steadily has no waves (fluid motion) far ahead and far below of the body. An oscillating body has no waves far below of the body, but outgoing waves at infinity in all directions (radiation condition).

In our case, the body is both oscillating and moving steadily, so the situation is more complicated. For a small velocity \( U \) the situation will have the tendency to behave like in the case of an oscillating body. (i.e. a radiation condition). However for larger velocities \( U \), the situation will have the tendency to behave like a steadily moving body and the condition for the steady state case has to be imposed.

This situation will be taken care of by formulation of an initial value problem. The solution obtained as we let \( t \to \infty \) is the desired solution (see Wehausen [43] and others).

Remark:

A simplification of the steady free surface condition (2.17) can be obtained if \( \tilde{\phi} \) is small compared to \( U x \). This is true for thin or slender ships. In this case the steady state problem is called a Neumann-Kelvin problem. The linearized free surface condition \( (U^2 \tilde{\phi}_{xx} + g \tilde{\phi}_z = 0) \) is that used by Lord Kelvin. The exact body boundary condition on the ship hull is of Neumann's type.

In the next section we will solve the instationary problem by using Green's theorem.
2.3 The leading equations

2.3.1 The singularity distributions

A Green function $G(x, \xi; U)$ is introduced as a solution of Laplace's equation, representing a source of oscillating strength in uniform motion. In this section we will construct a solution of the Neumann-Kelvin problem in terms of an integral distribution of Kelvin wave sources. We will apply Green’s theorem to the domain as can be seen in figure (2.3).

![Figure 2.3: The mathematical domain.](image)

Let $\Sigma$ denote the wetted surface of the hull. $C_f$ is the closed contour (waterline) intersected by $\Sigma$ in the free surface. $C_f$ is the closed contour intersected by the free surface in a certain vertical prism. Let $\Sigma_{Fi}$ be the part of the free surface located inside the hull and $\Sigma_{Fe}$ the free surface outside $\Sigma$, with $\Sigma_{Fe}$ the part of $\Sigma_{Fe}$ inside the prism. Let $D_i$ denote the interior of the closed surface $\Sigma_i + \Sigma_{Fi}$ and $D_e$ the interior of the closed surface $\Sigma_e + \Sigma_{Fe} + \Gamma +$ the area interior to the prism in the plane $z = -\infty$. $C_\infty$ is the intersection of
the free surface and an infinite prism.

We choose the function $G$ such that it fulfills the following adjoint homogeneous free surface condition:

$$ -\omega^2 G - 2i\omega U G_{\xi} + U^2 G_{\xi} + gG_{\zeta} = 0 \quad \text{at } \zeta = 0 $$

(2.33)

with the function $G$ written as:

$$ G = \left( \frac{1}{r} - \frac{1}{r_1} \right) + \Psi(x, \xi; U) $$

(2.34)

where $r$ denotes the distance between the source and the field point and $r_1$ denotes the distance between the field point and the image of the source above the free surface. In the following we will apply Green's theorem to a problem in $D_i$ inside $\Sigma$ and to the problem in $D_e$ outside $\Sigma$. Combining the formulation inside and outside the ship we obtain a description of the potential function defined outside $\Sigma$ by means of a source and vortex distribution (see equation (2.47)).

In the same way as Brad [9] we consider the following integrals:

$$ I_E = \iint_{\Sigma_F + \Sigma_e + \Gamma} (\phi_e \frac{\partial G}{\partial n_e} - G \frac{\partial \phi_e}{\partial n_e}) dS $$

(2.35)

and

$$ I_I = \iint_{\Sigma_F + \Sigma_i} (\phi_i \frac{\partial G}{\partial n_i} - G \frac{\partial \phi_i}{\partial n_i}) dS $$

(2.36)

then the following is valid:

$$ I_E = \begin{cases} 
4\pi \phi_e & x \in D_e \\
0 & x \in D_i 
\end{cases} $$

(2.37)

$$ I_I = \begin{cases} 
4\pi \phi_i & x \in D_i \\
0 & x \in D_e 
\end{cases} $$

(2.38)

At $\Sigma_F$ and $\Sigma_E$ equations (2.32) and (2.33) are valid:

$$ \frac{\partial \phi}{\partial n} = \frac{1}{g} \left\{ \mathcal{L}(\phi) + \omega^2 \phi - 2i\omega U \frac{\partial \phi}{\partial \xi} - U^2 \frac{\partial^2 \phi}{\partial \xi^2} \right\} $$

(2.39)

$$ \frac{\partial G}{\partial n} = \frac{1}{g} \left\{ \omega^2 G + 2i\omega U \frac{\partial G}{\partial \xi} - U^2 \frac{\partial^2 G}{\partial \xi^2} \right\} $$

(2.40)
Combining equations (2.39) and (2.40) leads for equation (2.35) to:

\[
\left( \phi_e \frac{\partial G}{\partial n_e} - G \frac{\partial \phi_e}{\partial n_e} \right) = \frac{\omega^2}{g} \phi_e G + \frac{2i\omega}{g} U \phi_e \frac{\partial G}{\partial \xi} - \frac{U^2}{g} \phi_e \frac{\partial^2 G}{\partial \xi^2} + \\
- \frac{1}{g} \mathcal{L}(\phi)G - \frac{\omega^2}{g} \phi_e G + \frac{2i\omega}{g} U \frac{\partial \phi_e}{\partial \xi} G + \frac{U^2}{g} \frac{\partial^2 \phi_e}{\partial \xi^2} G \tag{2.41}
\]

Regrouping the expressions at the right hand side of equation (2.41) leads to the following formulation:

\[
+ \frac{2i\omega}{g} U \frac{\partial (\phi_e G)}{\partial \xi} + \frac{U^2}{g} \frac{\partial (\frac{\partial \phi_e G}{\partial \xi} - \phi_e \frac{\partial G}{\partial \xi})}{\partial \xi} - \frac{1}{g} \mathcal{L}(\phi_e)G \tag{2.42}
\]

and almost the same for equation (2.36). The \( \Sigma_{F_e} \) part of equation (2.35) leads to:

\[
\int \int_{\Sigma_{F_e}} (\phi_e \frac{\partial G}{\partial n_e} - G \frac{\partial \phi_e}{\partial n_e})dS = \int_{C^\infty - C_f} \frac{2i\omega}{g} U \phi_e G d\eta + \\
- \int_{C^\infty - C_f} \frac{U^2}{g} (\frac{\partial \phi_e}{\partial \xi} G - \phi_e \frac{\partial G}{\partial \xi})d\eta - \frac{1}{g} \int \int_{\Sigma_{F_e}} \mathcal{L}(\phi_e)G dS \tag{2.43}
\]

and for the \( \Sigma_{F_i} \) part:

\[
\int \int_{\Sigma_{F_i}} (\phi_i \frac{\partial G}{\partial n_i} - G \frac{\partial \phi_i}{\partial n_i})dS = \int_{C_f} \frac{U^2}{g} (\frac{\partial \phi_i}{\partial \xi} G - \phi_i \frac{\partial G}{\partial \xi})d\eta + \\
+ \int_{C_f} \frac{2i\omega}{g} U \phi_i G d\eta \tag{2.44}
\]
2.3. **THE LEADING EQUATIONS**

Adding equations (2.43) and (2.44) results in:

\[
\frac{\partial}{\partial n_i} = - \frac{\partial}{\partial n_e} = \frac{\partial}{\partial n}
\]

\[
I_E + I_I = \iint_{\Sigma} \left( \frac{\partial \phi_e}{\partial n} - \frac{\partial \phi_i}{\partial n} \right) G dS - \frac{2i\omega}{g} U \int_{C_j} (\phi_e - \phi_i) G d\eta + \\
- \frac{1}{g} \int_{\Sigma_{F_e}} L(\phi_e) G dS - \frac{1}{g} \int_{\Sigma_{F_i}} L(\phi_i) G dS + \int_{\Sigma} (\phi_i - \phi_e) \frac{\partial G}{\partial n} dS + \\
+ \frac{U^2}{g} \int_{C_j} \left( \frac{\partial \phi_i}{\partial \xi} - \frac{\partial \phi_e}{\partial \xi} \right) G d\eta + \frac{U^2}{g} \int_{C_j} (\phi_e - \phi_i) \frac{\partial G}{\partial x} d\eta
\]

(2.45)

The source and dipole strength are defined as:

\[
\gamma = \phi_e - \phi_i \\
\sigma = \frac{\partial \phi_i}{\partial n} - \frac{\partial \phi_e}{\partial n}
\]

(2.46)

equation (2.45) will now transform into:

\[
4\pi \phi(x) = - \iiint_{\Sigma} \gamma(\xi) \frac{\partial G(x, \xi)}{\partial n_\xi} dS_\xi - \iiint_{\Sigma} \sigma(\xi) G(x, \xi) dS_\xi + \\
- \frac{2i\omega}{g} U \int_{C_j} \gamma(\xi) G(x, \xi) d\eta + \frac{U^2}{g} \int_{C_j} \gamma(\xi) \frac{\partial G(x, \xi)}{\partial x} d\eta + \\
+ \frac{U^2}{g} \int_{C_j} (-\alpha_t \frac{\partial \gamma}{\partial t} - \alpha_r \frac{\partial \gamma}{\partial r} + \alpha_n \sigma(\xi)) G(x, \xi) d\eta + \\
- \frac{1}{g} \int_{F.S.} L(\phi) G(x, \xi) dS_\xi
\]

(2.47)
with:

\[
\begin{align*}
\tau &= t \times n \\
\alpha_t &= \cos(Ox, t) \\
\alpha_\tau &= \cos(Ox, \tau) \\
\alpha_n &= \cos(Ox, n)
\end{align*}
\]  

(2.48)

where \(t\) is the tangent to the waterline.

So the potential is generated by the following five singularity distributions (see also Brard [9]):

1. A distribution of simple sources over \(\Sigma\) with strength \(\sigma\) per unit area.
2. A distribution of double sources over \(\Sigma\) with strength \(\gamma\) per unit area.
3. A distribution of double sources over the waterline with strength \(\frac{U^2}{g} \gamma\) per unit arc length.
4. A distribution of simple sources over the waterline with strength \(\frac{2i\omega}{g} U \gamma + \alpha_n \sigma - \alpha_t \frac{\partial \gamma}{\partial t} - \alpha_\tau \frac{\partial \gamma}{\partial \tau}\) per unit arc length.
5. A distribution of simple sources over the entire free surface with strength \(\frac{1}{g} \mathcal{L}(\phi)\).

There is still a free choice in \(\gamma\) and \(\sigma\). We can make \(\phi_0 \equiv \phi_i (\gamma(\xi) = 0)\) at the boundaries. This means that the tangential velocities are now continuous at the boundaries, but the normal velocities are discontinuous (see also the remark on the next page).

With a choice of \(\gamma(\xi) \equiv 0\), the following expression will be obtained:

\[
4\pi \tilde{\phi}(x) = - \iint_{\Sigma} \sigma(\xi) G(x, \xi) dS_\xi + \frac{U^2}{g} \int_{C_0} \alpha_n \sigma(\xi) G(x, \xi) d\eta +
\]

\[
+ \frac{1}{g} \int_{F.S.} \mathcal{L}(\tilde{\phi}) G(x, \xi) dS_\xi
\]

(2.49)
2.3. THE LEADING EQUATIONS

The potential $\tilde{\phi}(x)$ is expressed in simple sources only!

Using the body boundary condition (2.31) we may obtain a description of the potential function $\tilde{\phi}$ by means of a source distribution of the following form:

$$4\pi \frac{\partial \tilde{\phi}}{\partial n} = 2\pi \sigma(x) - \int \int_{\Sigma} \sigma(\xi) \frac{\partial G(x, \xi)}{\partial n_x} dS_\xi +$$

$$+ \frac{U^2}{g} \int_{C_j} \alpha_n \sigma(\xi) \frac{\partial G(x, \xi)}{\partial n_x} d\eta - \frac{1}{g} \int \int_{F.S.} \mathcal{L}(\tilde{\phi}) \frac{\partial G(x, \xi)}{\partial n_x} dS_\xi$$

(2.50)

with $\frac{\partial \tilde{\phi}}{\partial n}$ (see equation (2.31)) given by:

$$\frac{\partial \tilde{\phi}}{\partial n} = -n \cdot (i\omega \alpha + \nabla(\alpha \cdot \nabla(-U \cdot x + \tilde{\phi})))$$

(2.51)

**Remark**

We consider the following potential $\Phi$:

$$\Phi = \int \int_{\Sigma} \sigma \frac{dS}{r}$$

(2.52)

The following is valid (Kellogg [21]): *if the density $\sigma$ of the distribution on $\Sigma$ is continuous at $x$, the normal derivative of the potential $\Phi$ approaches limits as $X$ approaches $x$ along the normal to $\Sigma$ at $x$ from either side.*

These limits are:

$$\left( \frac{\partial \Phi}{\partial n} \right)_+ = -2\pi \sigma(x) + \int \int_{\Sigma} \sigma(\xi) \frac{\partial 1/r}{\partial n} dS_\xi$$

(2.53)

$$\left( \frac{\partial \Phi}{\partial n} \right)_- = +2\pi \sigma(x) + \int \int_{\Sigma} \sigma(\xi) \frac{\partial 1/r}{\partial n} dS_\xi$$

(2.54)

□

So when using the $\frac{1}{r}$ sources the resulting equations looks like:

$$+2\pi \sigma(x) - \int \int_{\Sigma} \sigma \frac{\partial G(x, \xi)}{\partial n_x} dS_\xi + ... = V_n$$

(2.55)
and when the \( \frac{1}{\tau} \) sources are used (as in Brard [9]) the resulting equation transforms into:

\[
-2\pi \sigma(x) - \int_{\Sigma} \sigma \frac{\partial G(x, \xi)}{\partial n_x} dS_\xi + \cdots = V_n
\]  

(2.56)

2.3.2 Expansion of source strength and potential function.

We will now consider small values of \( \omega \). From model test experiments one generally concludes that a large part of the dynamic swell-up of a ship originates at the lower frequencies. So the vessel is in a nearly steady state motion, but still swell-up occurs.

On the basis of experiments Tasaki [38] obtained an empirical formula to evaluate the height of the dynamical swell-up (see also Blok [6]):

\[
\frac{\Delta S_A}{S_A} = C \cdot \left[ \frac{\omega^2 L}{g} \right]^{1/2}
\]  

(2.57)

with:

\[
C = \frac{1}{3} (C_B - 0.45)
\]  

(2.58)

this formula should only be valid for the following:

\[
0.16 < F_n < 0.29
\]

\[
1.6 < \frac{\omega^2 L}{g} < 2.6
\]  

(2.59)

\[
0.6 \leq C_B \leq 0.8
\]

Now the following approach is justified. We can solve the unsteady state problem by expansion of all the relevant physical parameters in this small parameter. We will use an expansion of \( \sigma \), \( \phi \) and \( G \) in the small parameter \( \omega \):

\[
\sigma(x) = \sigma_0(x) + \omega \sigma_1(x) + \cdots
\]

\[
\phi(x) = \phi_0(x) + \omega \phi_1(x) + \cdots
\]

\[
G(x, \xi) = G_0(x, \xi) + \omega G_1(x, \xi) + \cdots
\]  

(2.60)
2.3. THE LEADING EQUATIONS

Substitution of these expansions in equations (2.49) and (2.50) leads for the first order problem to:

\[-4\pi n \cdot \nabla(\alpha \cdot \nabla(-U x + \bar{\phi})) = 2\pi \sigma_0(x) - \int_\Sigma \sigma_0(\xi) \frac{\partial G_0(x, \xi)}{\partial n_x} dS_\xi +\]

\[+ \frac{U^2}{g} \int_{C_f} \alpha_n \sigma_0(\xi) \frac{\partial G_0(x, \xi)}{\partial n_x} d\eta - \frac{1}{g} \int_{F.S.} L_0(\phi) \frac{\partial G_0(x, \xi)}{\partial n_x} dS_\xi\]  \hspace{1cm} (2.61)

where $\phi_0$ is given by:

\[-4\pi \hat{\phi}_0(x) = - \int_\Sigma \sigma_0(\xi) G_0(x, \xi) dS_\xi + \frac{U^2}{g} \int_{C_f} \alpha_n \sigma_0(\xi) G_0(x, \xi) d\eta +\]

\[+ \frac{1}{g} \int_{F.S.} L_0(\hat{\phi}) G_0(x, \xi) dS_\xi\]  \hspace{1cm} (2.62)

and for order $\omega$:

\[2\pi \sigma_1(x) - \int_\Sigma \sigma_1(\xi) \frac{\partial G_0(x, \xi)}{\partial n_x} dS_\xi + \frac{U^2}{g} \int_{C_f} \sigma_1(\xi) \frac{\partial G_0(x, \xi)}{\partial n_x} d\eta =\]

\[-4\pi n \cdot i\alpha \int_\Sigma \sigma_0(\xi) \frac{\partial \psi_1(x, \xi)}{\partial n_x} dS_\xi - \frac{U^2}{g} \int_{C_f} \alpha_n \sigma_0(\xi) \frac{\partial \psi_1(x, \xi)}{\partial n_x} +\]

\[+ \frac{1}{g} \int_{F.S.} (L_1 + L_0 \frac{\partial G_0(x, \xi)}{\partial n_x} + L_0 \frac{\partial \psi_1(x, \xi)}{\partial n_x}) dS_\xi\]  \hspace{1cm} (2.63)

where $\hat{\phi}_1$ is given by:

\[-4\pi \hat{\phi}_1(x) = - \int_\Sigma (\sigma_0(\xi) \psi_1(x, \xi) + \sigma_1(\xi) G_0(x, \xi)) dS_\xi +\]

\[+ \frac{U^2}{g} \int_{C_f} \alpha_n (\sigma_0(\xi) \psi_1(x, \xi) + \sigma_1(\xi) G_0(x, \xi)) d\eta +\]

\[- \frac{1}{g} \int_{F.S.} (L_0 + L_1) G_0(x, \xi) + L_0(\hat{\phi}_0) \psi_1(x, \xi) dS_\xi\]  \hspace{1cm} (2.64)
2.3.3 Expansion of body boundary condition.

The right hand side of equation (2.31) contains a vector $\alpha^k$:

$$\alpha^k(x, t) = \begin{cases} 
\alpha_k(t) \ i_k & k = 1, 2, 3 \\
\alpha_k(t) \ (i_k \times x) & k = 4, 5, 6 
\end{cases} \quad (2.65)$$

Here $\alpha_k(t)$ is the deflection in translational motion for $k = 1, 2, 3$ and for $k = 4, 5, 6$, $\alpha_k(t)$ represents rotation angles about the $x_{k-3}$-axis. See figure (2.4).

![Body motions diagram]

**Figure 2.4: Body motions.**

Combining equation (2.65) with equation (2.31) leads to the body boundary condition.

So for $k=1$ (surge, translation parallel to the longitudinal axis) this looks like:

$$\frac{\partial \phi}{\partial n} = -n_1(i\omega + \bar{\phi}_{xx}) - n_2\bar{\phi}_{xy} - n_3\bar{\phi}_{xz} \quad (2.66)$$

For $k=2$ (sway, translation in the lateral direction):

$$\frac{\partial \phi}{\partial n} = -n_1\bar{\phi}_{xy} - n_2(i\omega + \bar{\phi}_{yy}) - n_3\bar{\phi}_{yz} \quad (2.67)$$
2.3. **THE LEADING EQUATIONS**

\[ k=3 \text{ (heave, translation in the vertical direction)}: \]
\[
\frac{\partial \phi}{\partial n} = -n_1 \tilde{\phi}_{xz} - n_2 \tilde{\phi}_{yz} - n_3 (i\omega + \tilde{\phi}_{xz}) \tag{2.68}
\]

\[ k=4 \text{ (roll, rotational motion about } x_1\text{-axis)}: \]
\[
\frac{\partial \phi}{\partial n} = n_1 (z \tilde{\phi}_{xy} - y \tilde{\phi}_{xz}) + n_2 (zi\omega + z \tilde{\phi}_{yy} - \tilde{\phi}_x - y \tilde{\phi}_{yz}) + \\
+ n_3 (-yi\omega + \tilde{\phi}_y + z \tilde{\phi}_{yz} - y \tilde{\phi}_{xz}) \tag{2.69}
\]

\[ k=5 \text{ (pitch, rotational motion about } x_2\text{-axis)}: \]
\[
\frac{\partial \phi}{\partial n} = n_1 (-zi\omega - z \tilde{\phi}_{xx} + \tilde{\phi}_x + x \tilde{\phi}_{xy}) + n_2 (-z \tilde{\phi}_{xy} + y \tilde{\phi}_{yz}) + \\
+ n_3 (x i\omega + U - \tilde{\phi}_x - z \tilde{\phi}_{xz} + x \tilde{\phi}_{xz}) \tag{2.70}
\]

\[ k=6 \text{ (yaw, rotational motion about } x_3\text{-axis)}: \]
\[
\frac{\partial \phi}{\partial n} = n_1 (yi\omega + y \tilde{\phi}_{xx} - \tilde{\phi}_y - x \tilde{\phi}_{xy}) + n_2 (-x i\omega - U + \tilde{\phi}_x + \\
+ y \tilde{\phi}_{xy} - x \tilde{\phi}_{yy}) + n_3 (y \tilde{\phi}_{xx} - x \tilde{\phi}_{yz}) \tag{2.71}
\]

When \( \tilde{\phi} \ll 1 \) then the expansion (2.60) will start with leading order \( \omega \) \( (\sigma = \omega \sigma_1 + \omega^2 \sigma_2 + \cdots) \) and then equation (2.61) will transform into (for surge motion):
\[
2\pi \sigma_1 - \int_{\Sigma} (\sigma_1 \frac{\partial G_0}{\partial n} + \sigma_0 \frac{\partial G_1}{\partial n}) + \cdots = -n_1 \tag{2.72}
\]

When \( \tilde{\phi} = O(1) \) then the expansion will start with leading order zero \( (\sigma = \sigma_0 + \omega \sigma_1 + \cdots) \) and equation (2.61) will look like:
\[
2\pi \sigma_0 - \int_{\Sigma} \sigma_0 \frac{\partial G_0}{\partial n} + \cdots = -n_1 \tilde{\phi}_{xx} - n_2 \tilde{\phi}_{xy} - n_3 \tilde{\phi}_{xz} \tag{2.73}
\]

It should also be noted that for the calculation of the steady wave potential \( \tilde{\phi} \) the same matrix equation (2.61) as for the unsteady potential \( \tilde{\phi} \) can be used! But of course the right hand side for \( \phi_n \) now looks like:
\[
\frac{\partial \phi}{\partial n} = n_1 U \tag{2.74}
\]
2.4 (Added) Resistance.

If the potential $\Phi$ is known, the pressure in a point in the fluid can be calculated using Bernoulli's equation:

$$ p = p_{atm} - \rho g z - \rho \frac{\partial \Phi}{\partial t} - \frac{1}{2} \rho \nabla \Phi \cdot \nabla \Phi + C(t) \quad (2.75) $$

In order to determine the resistance of a vessel, two different lines of approach can be followed (see Pinkster [30]):

1. Using momentum considerations. The change of momentum of the fluid is equated to the mean force. Placing the momentum control volumes at infinity use can be made of the knowledge of the far field behaviour of the potential flow (see for instance Maruo [24] and Huijsmans [18]).

2. Direct integration of pressure. This approach will be followed in this thesis. The underlying approach is similar to Pinkster [30].

The fluid forces can be determined by direct integration of the pressure:

$$ F = - \iint_S p \, n \, dS \quad (2.76) $$

and for the moment:

$$ M = - \iint_S p (x \times n) \, dS \quad (2.77) $$

in which:

$p = \text{fluid pressure}$

$n = \text{outward normal}$

$S = \text{wetted surface}$

$x = \text{coordinate of surface element}$

The oscillating movement of the ship is defined by the sum of the translational and the angular motions (see also equation (2.65)):
2.4. (ADDED) RESISTANCE.

\[ x = x_g(t) + \alpha(x, t) \]  
(2.78)

Where \( x_g(t) \) denotes the motion vector of the centre of gravity of the body.

Now the first order (steady state) force is given by:

\[ F = -\int \int_S -\rho g z + \rho U \dot{\Phi}_x + \frac{1}{2}\rho (\dot{\Phi}_x^2 + \dot{\Phi}_y^2 + \dot{\Phi}_z^2) n dS \]  
(2.79)

If the vessel is in an oscillatory motion, the pressure in each point can be found using a Taylor expansion of this point round the mean position:

\[ p|_{x=x(1)} = p|_{x=x(0)} + (x(1) - x(0)) \cdot \nabla p + ... \]  
(2.80)

Using this, Bernoulli's equation (2.75) leads to:

\[ p|_{x=x(1)} = -\rho g z(0) - \rho g z(1) \]
\[ -\rho \frac{\partial \Phi(0)}{\partial t} - \rho (x(1) - x(0)) \cdot \nabla \frac{\partial \Phi(0)}{\partial t} \]
\[ -\frac{1}{2} \rho (\nabla \Phi(0) \cdot \nabla \Phi(0)) - \frac{1}{2} \rho (x(1) - x(0)) \cdot \nabla (\nabla \Phi(0) \cdot \nabla \Phi(0)) \]  
(2.81)

Using Pinkster [30] equation (III-68) and further, the following equation for the second order force can be found:

\[ F = -\int \int_{S_{inst}} p n dS \]  
(2.82)

The instantaneous wetted surface \( S_{inst} \) is split in two parts: a constant part \( S_0 \) up to the static waterline on the hull and an oscillating part \( s \) between the static waterline on the hull and the wave profile. The force due to the part \( s \) is given by:

\[ F_s = -\int \int_s p n dS \]
\[ = -\int_{WL z=z_0+z_a} \int_{z=z_0}^{z=z_0+z_1} p n dz dl \]  
(2.83)
At the waterline $WL$ the following is valid:

$$\rho g(\zeta_0 + \zeta_1) = -\rho \frac{\partial \Phi}{\partial t} - \frac{1}{2} \rho \nabla \Phi \cdot \nabla \Phi$$  \hspace{1cm} (2.84)

so equation (2.83) transforms into:

$$F_s = - \int_{WL} \int_{z=\zeta_0+z_a} ^{z=\zeta_0+\zeta_1} (-\rho gz + \rho g(\zeta_0 + \zeta_1)) n \, dz \, dl$$  \hspace{1cm} (2.85)

The time average of this force is given by:

$$\overline{F_s} = - \int_{WL} (\frac{1}{2} \rho g \zeta_1^2 - \rho g \zeta_1 z_a) n \, dl$$  \hspace{1cm} (2.86)

Now the second order (unsteady state) force is given by the sum of the following six components (see Pinkster [30]):

1. Wave elevation:

$$-\frac{1}{2} \rho g \int_{WL} \zeta_1^2 n \, dl + \rho g \int_{WL} \zeta_1 z_a n \, dl$$  \hspace{1cm} (2.87)

2. The first order velocity:

$$- \int \int_{S_0} -\frac{1}{2} \rho |\nabla \Phi|^2 n dS$$  \hspace{1cm} (2.88)

3. Product of first order motion ($\vec{X}^{(1)}$) and gradient of first order potential:

$$- \int \int_{S_0} -\rho(\vec{X}^{(1)} \cdot \nabla \Phi_t) n dS$$  \hspace{1cm} (2.89)
4. Product of first order motion and gradient of squared velocity:

\[-\iint_{S_0} -\frac{1}{2} \rho \langle \tilde{X}^{(1)} \cdot \nabla(\nabla \Phi \cdot \nabla \Phi) \rangle n dS \] (2.90)

5. Product of first order angular motions and inertia forces:

\[\tilde{\alpha} \times (M \cdot \tilde{\omega}_g) \] (2.91)

6. Unsteady potential:

\[-\iint_{S_0} -\rho \Phi_t n dS \] (2.92)

In our case only the time average is of importance, so for the first contribution this is:

\[\frac{1}{T} \left( -\frac{1}{2} \rho g \int_{W_L} \zeta_1^2 n d l \right) \] (2.93)

The first order wave elevation is given by (see equation A.14):

\[\zeta_1 = -\frac{1}{g} \left\{ \frac{\partial \tilde{\phi}}{\partial t} + \nabla (-U_x + \tilde{\phi}) \cdot \nabla \tilde{\phi} \right\} \] (2.94)

with the use of the summation convention

\[f_i g_i = \sum_{i=1}^{3} f_i g_i \] (2.95)

where the subscript \(i\) denotes differentiation with respect to \(x_i\).

This can be written as:

\[\zeta_1 = -\frac{1}{g} \left\{ \frac{\partial \tilde{\phi}}{\partial t} - U \tilde{\phi}_x + \tilde{\phi}_t \tilde{\phi}_x \right\} \] (2.96)

Now consider heave motion (mode = 3). The motion and potential are given by:

\[\alpha(x,t) = \Re\{\alpha(x) \cdot e^{-i\omega t}\} \]
\[\tilde{\phi}(x,t) = \Re\{\tilde{\phi}(x) \cdot e^{-i\omega t}\} \] (2.97)
\[ \tilde{\phi}(x, t) = \Re\{\hat{\phi} \cos(\omega t) - i\hat{\phi} \sin(\omega t)\} \]  

For heave mode the quantity \( \hat{\phi} \) is pure imaginary, so this leads to (We replace \( \hat{\phi} \) by \( i \hat{\phi} \)):

\[ \tilde{\phi} = \hat{\phi} \sin(\omega t) \]  

\[ \alpha(x) = (0, 0, 1)^T z_a \cos \omega t \]

so the first order wave elevation (equation 2.96) is now given by:

\[ \zeta_1 = -\frac{1}{g} \{\omega \hat{\phi} \cos \omega t - U \hat{\phi}_x \sin \omega t + (\hat{\phi}_i \hat{\phi}_i) \sin \omega t\} \]

Most of the unsteady forces (2.87) - (2.92) can be calculated easily. For instance equation (2.90) produces:

\[
\begin{pmatrix}
0 \\
0 \\
z_a \cos \omega t
\end{pmatrix}
\cdot
2
\begin{pmatrix}
(-U + \hat{\phi}_x + \hat{\phi}_x \sin \omega t)(\hat{\phi}_{xx} + \hat{\phi}_{xx} \sin \omega t) \\
(\hat{\phi}_y + \hat{\phi}_y \sin \omega t)(\hat{\phi}_{yy} + \hat{\phi}_{yy} \sin \omega t) \\
(\hat{\phi}_z + \hat{\phi}_z \sin \omega t)(\hat{\phi}_{zz} + \hat{\phi}_{zz} \sin \omega t)
\end{pmatrix}
\]

In this equation only products like \( \sin \omega t \cos \omega t \) occur.

In order to calculate the time average, the following three relations should be remembered:

\[ \frac{1}{2\pi} \int_0^{2\pi} \sin t dt = \frac{1}{2\pi} \int_0^{2\pi} \cos t dt = 0 \]  

\[ \frac{1}{2\pi} \int_0^{2\pi} \cos^2 t dt = \frac{1}{2\pi} \int_0^{2\pi} \sin^2 t dt = \frac{1}{2} \]

\[ \frac{1}{2\pi} \int_0^{2\pi} \sin t \cos t dt = 0 \]
2.4. **(ADDED) RESISTANCE.**

So contributions in the mean force can only originate from the \((\sin^2 t)\)-terms and the \((\cos^2 t)\)-terms.

For pitch motion \((\text{mode} = 5)\) \(\ddot{\phi}\) is given by:

\[
\ddot{\phi} = \hat{\phi} \cos(\omega t)
\]  

\[
\alpha(x) = (z, 0, -x)^T \theta_a \cos \omega t
\]

So equation (2.90) leads to:

\[
\theta_a \cos \omega t \begin{pmatrix} z \\ 0 \\ -x \end{pmatrix} \cdot 2 \begin{pmatrix} (-U + \ddot{\phi}_x + \hat{\phi}_x \cos \omega t)(\ddot{\phi}_{xx} + \hat{\phi}_{xx} \cos \omega t) \\ (\ddot{\phi}_z + \hat{\phi}_z \cos \omega t)(\ddot{\phi}_{zz} + \hat{\phi}_{zz} \cos \omega t) \end{pmatrix}
\]

Now the time average is given by:

\[
\theta_a \{ z((-U + \ddot{\phi}_x)\ddot{\phi}_{xx} + \hat{\phi}_x \ddot{\phi}_{xx}) - x(\ddot{\phi}_x \ddot{\phi}_z) \} 
\]  

In the case of equation (2.87) - (2.92) this leads to:

**HEAVE:**

1. \[ \overline{\zeta_1^2} = \frac{1}{g^2} \left\{ \frac{1}{2} \omega^2 \ddot{\phi}^2 + \frac{1}{2} U^2 \ddot{\phi}_x^2 + \frac{1}{2} (\ddot{\phi}_t \ddot{\phi}_t)^2 - U \ddot{\phi}_x (\ddot{\phi}_t \ddot{\phi}_t) \right\} \]

   \[ \overline{\zeta_1 z_a} = -\frac{1}{2g} \omega z_a \dot{\phi} \]

2. \[ \overline{|\nabla \Phi|^2} = \frac{1}{2} (\ddot{\phi}_x^2 + \ddot{\phi}_y^2 + \ddot{\phi}_z^2) \]

3. \[ \overline{\langle \ddot{X}^{(1)} \cdot \nabla \Phi_t \rangle} = \frac{1}{2} z_a \omega \ddot{\phi}_z \]

4. \[ \overline{\langle \ddot{X}^{(1)} \cdot \nabla (\nabla \Phi \cdot \nabla \Phi) \rangle} = 0 \]

5. \[ \overline{\alpha \times (M \cdot \dot{x}_g)} = 0 \]

6. \[ \overline{\Phi_t} = 0 \]
CHAPTER 2. MATHEMATICAL FORMULATION

PITCH:

1. \[ \ddot{\zeta}_1^2 = \frac{1}{g^2} \left\{ \frac{1}{2} \omega^2 \dot{\phi}_x^2 + \frac{1}{2} U^2 \dot{\phi}_x^2 + \frac{1}{2} (\dot{\phi}_y \dot{\phi}_i)^2 - U \dot{\phi}_y (\dot{\phi}_y \dot{\phi}_i) \right\} \]

\[ \ddot{\zeta}_1 \ddot{z}_a = \frac{1}{2g} \omega z \theta_a \dot{\phi} \]

2. \[ |\nabla \Phi|^2 = \frac{1}{2} (\dot{\phi}_x^2 + \dot{\phi}_y^2 + \dot{\phi}_z^2) \]

3. \[ (\ddot{X}^{(1)} \cdot \nabla \Phi) = 0 \]

4. \[ (\ddot{X}^{(1)} \cdot \nabla (\nabla \Phi \cdot \nabla \Phi)) = \theta_a \{ z ((-U + \Phi_x) \ddot{\phi}_{xx} + \Phi_x \ddot{\phi}_{xx}) - x (\Phi_x \ddot{\phi}_x)_x \} \]

5. \[ \ddot{\alpha} \times (M \cdot \ddot{x}_g) = 0 \]

6. \[ \ddot{\Phi}_i = 0 \]

The dependence on the different parameters \( \omega \) and \( z_a \) can be found in the following table:

<table>
<thead>
<tr>
<th></th>
<th>heave</th>
<th></th>
<th>pitch</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>frequency</td>
<td>amplitude</td>
<td>frequency</td>
</tr>
<tr>
<td>Force-I</td>
<td>( \sim (\omega^2 + ...) )</td>
<td>( \sim z_a^2 )</td>
<td>( \sim (\omega^2 + ...) )</td>
</tr>
<tr>
<td>Force-Ilh</td>
<td>( \sim \omega^2 )</td>
<td>( \sim z_a^2 )</td>
<td>-</td>
</tr>
<tr>
<td>Force-Ip</td>
<td>-</td>
<td>-</td>
<td>( \sim \omega )</td>
</tr>
<tr>
<td>Force-II</td>
<td>( \sim \omega^2 )</td>
<td>( \sim z_a^2 )</td>
<td>( \sim \omega )</td>
</tr>
<tr>
<td>Force-III</td>
<td>( \sim \omega^2 )</td>
<td>( \sim z_a^2 )</td>
<td>-</td>
</tr>
<tr>
<td>Force-IV</td>
<td>-</td>
<td>-</td>
<td>( \sim (U + \omega) )</td>
</tr>
<tr>
<td>Force-V</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Force-VI</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 2.1: The dependence of added resistance on amplitude and frequency.
Chapter 3
Computational Aspects

3.1 Solution of the integral equation.

The solutions obtained by the singularity distributions are the result of solving two coupled integral equations. Equations (2.61) and (2.62) can be solved using an iterative scheme. In this scheme use has been made of the numerical evaluation of the wave resistance Green function as done by Newman ([26]+[27]).

![Diagram showing panel influence](image)

Figure 3.1: Panel influence.

The numerical solution of equation (2.61) and (2.62) is carried out using a finite element method. The wetted body $\Sigma$ is divided into $N$ triangular panels (see figure (3.1)).
Integration is done using a piecewise constant variation of the source strength \( \sigma(\xi) \).

\[
\iint_{\Sigma} = \sum_{j=1}^{N} \iint_{e_j} \sigma(\xi) \frac{\partial G(x, \xi)}{\partial n_x} dS_\xi
\]  

(3.1)

In this way a set of \( N \) linear equations for the \( N \) source strengths \( \sigma_j \) is obtained. So the main structure of the integral equation (2.50) looks like:

\[
U_{n_1}^{(1)} = 2\pi \sigma(x^{(1)}) - \sigma(\xi^{(1)}) \frac{\partial G(x^{(1)}, \xi^{(1)})}{\partial n_x^{(1)}} S(1) + \cdots - \sigma(\xi^{(N)}) \frac{\partial G(x^{(1)}, \xi^{(N)})}{\partial n_x^{(1)}} S(N)
\]

\[
U_{n_1}^{(2)} = 2\pi \sigma(x^{(2)}) - \sigma(\xi^{(1)}) \frac{\partial G(x^{(2)}, \xi^{(1)})}{\partial n_x^{(2)}} S(1) + \cdots - \sigma(\xi^{(N)}) \frac{\partial G(x^{(2)}, \xi^{(N)})}{\partial n_x^{(2)}} S(N)
\]

\vdots

\[
U_{n_1}^{(N)} = 2\pi \sigma(x^{(N)}) - \sigma(\xi^{(1)}) \frac{\partial G(x^{(N)}, \xi^{(1)})}{\partial n_x^{(N)}} S(1) + \cdots - \sigma(\xi^{(N)}) \frac{\partial G(x^{(N)}, \xi^{(N)})}{\partial n_x^{(N)}} S(N)
\]

with the normal vector \( n \) at element number \( (i) \) denoted by:

\[
n^{(i)} = \begin{pmatrix} n_1^{(i)} \\ n_2^{(i)} \\ n_3^{(i)} \end{pmatrix}
\]  

(3.2)

\( S(i) \) denotes the area of element number \( (i) \) and \( G(x, \xi) \) the Green function. The matrix equation now reads as:

\[
A \sigma = \tilde{V}_n^{(k)}
\]  

(3.3)

where \( \tilde{V}_n^{(k)} \) depends on the \( (k) \)-th oscillation mode.
3.1. SOLUTION OF THE INTEGRAL EQUATION.

So for each element we need to evaluate the following integrals:

1. For the evaluation of $\phi(x)$ when $\sigma(x)$ is known:

$$
\int_{e_k} \frac{1}{r} dS_\xi, \quad \int_{e_k} G(x, \xi) dS_\xi, \quad \int_{l_k} G(x, \xi) d\eta
$$

(3.4)

2. For the calculation of the integral equation:

$$
\int_{e_k} \frac{\partial}{\partial n_x} \frac{1}{r} dS_\xi, \quad \int_{e_k} \frac{\partial G(x, \xi)}{\partial n_x} dS_\xi, \quad \int_{l_k} \frac{\partial G(x, \xi)}{\partial n_x} d\eta
$$

(3.5)

with $e_k$ an area element and $l_k$ a waterline element. Numerical errors occur at different places:

- Ship hull panelling.

- The numerical integration of the Green function.

- Numerical integration of the waterline integral; errors occur here as a result of the singular behaviour of the Green function at the free surface.

The Green function $G$ contains a $1/r$-term. The integration of these $1/r$-terms is carried out by a subroutine developed at MARIN [12] using analytical expressions of those integrals in order to avoid large errors from numerical integration for points close to the panel.
3.2 Generating the grid

3.2.1 Hullform definition

In this thesis, most of the computations have been performed on two hull forms: a Wigley hull and a modified Wigley hull. In this section the generation of the grid for these two hull forms are discussed. Of course, other ship hull forms can be implemented without much difficulties.

![Figure 3.2: Wigley hull contour plot.](image)

A right-handed coordinate system is defined by:

![Diagram with axes](image)

with:

\[ O: \text{ amidships in the waterplane} \]
\[ \xi: \text{ longitudinal, positive forwards} \]
\[ \eta: \text{ lateral, positive to port side} \]
\[ \zeta: \text{ vertical, positive upwards} \]  \hspace{1cm} (3.6)

Then the non-dimensional hull form of the model (a two-sided parabolic hull form) is defined by (see figure (3.2)):

\[ \eta = (1 - \zeta^2) \cdot (1 - \xi^2) \]  \hspace{1cm} (3.7)
3.3. **THE GREEN FUNCTION**

The principal dimensions of this specific Wigley hull are given by: \( \frac{L}{B} = 10 \) and \( \frac{L}{H} = 16 \).

In modern analysis also a modified Wigley hull definition is used:

\[
\eta = (1 - \zeta^2) \cdot (1 - \xi^2) \cdot (1 + a_2 \cdot \xi^2 + a_4 \cdot \xi^4) + \\
\lambda \cdot \xi^2 \cdot (1 - \zeta^8) \cdot (1 - \xi^2)^4
\]

(3.8)

with:

\[
a_2 = 0.2 , \text{ for all models} \\
a_4 = 0.0 , \text{ for all models} \\
\lambda = 1.0 , \text{ for models with } C_M = 0.9090 \\
= 0.0 , \text{ for models with } C_M = 0.6667
\]

(3.9)

The mid-body of this modified Wigley hull is fatter then the mid-body of the standard Wigley hull. The shape of the modified Wigley hull looks more like the shape of commercial vessels then the standard Wigley hull.

### 3.2.2 Panel division

In our calculation the wetted body \( \Sigma \) in divided into triangular panels. This has been done in such a way that a symmetrical grid with respect to the \( y = 0 \) plane has been obtained. The panel arrangement can be observed in figure (3.3).

### 3.3 The Green function

#### 3.3.1 Behaviour of the Green function.

In Wehausen and Laitone [43] the Green function of an oscillating source is given:

\[
G(x, \xi; U) = \frac{1}{r} - \frac{1}{r_1} + \Psi(x, \xi; U)
\]

(3.10)

with the function \( \Psi \) given by:

\[
\Psi(x, \xi; U) = \frac{2g}{\pi \frac{1}{2}} \int_0^{\frac{1}{2}\pi} \int_{L_1} F(\theta, k) d\theta dk \\
+ \frac{2g}{\pi} \int_{\frac{1}{2}\pi}^{\pi} \int_{L_2} F(\theta, k) d\theta dk
\]

(3.11)
where $F(\theta, k)$ is given by:

$$
F(\theta, k) = \frac{k e^{k(z + \zeta + i(x - \xi) \cos \theta)} \cos(k(y - \eta) \sin \theta)}{gk^2 - (\omega + kU \cos \theta)^2}
$$

(3.12)

the paths $L_1$ and $L_2$ are given in figure (3.4).

With $k_1 - k_4$ the poles of $F(\theta, k)$:

$$
\sqrt{gk_1} = \frac{1 - \sqrt{1 - 4\tau \cos \theta}}{2\tau \cos \theta} \omega \quad \theta \in (0, \frac{\pi}{2})
$$

(3.13)

$$
\sqrt{gk_2} = \frac{1 + \sqrt{1 - 4\tau \cos \theta}}{2\tau \cos \theta} \omega \quad \theta \in (0, \frac{\pi}{2})
$$

(3.14)

$$
\sqrt{gk_3} = \frac{1 - \sqrt{1 - 4\tau \cos \theta}}{2\tau \cos \theta} \omega \quad \theta \in (\frac{\pi}{2}, \pi)
$$

(3.15)

$$
\sqrt{gk_4} = -\frac{1 + \sqrt{1 - 4\tau \cos \theta}}{2\tau \cos \theta} \omega \quad \theta \in (\frac{\pi}{2}, \pi)
$$

(3.16)

With $\tau = \frac{U \omega}{g}$, (so $\tau \ll 1$ in this thesis) this leads to:

$$
\sqrt{gk_1} = \sqrt{gk_3} \approx \omega + O(\omega^2)
$$

(3.17)
3.3. THE GREEN FUNCTION

\[ \sqrt{gk_2} = -\sqrt{gk_4} \simeq \frac{g}{U \cos \theta} + \omega + O(\omega^2) \] (3.18)

When for instance \( U \to 0 \) then \( k_2 \) and \( k_4 \) will go to \( \infty \) and the paths \( L_1 \) and \( L_2 \) will coincide with their poles located at \( k_1 = k_3 = \omega^2/g \), see also figure (3.5).

\[ k_1 = k_3 \quad L_1 = L_2 \]

Figure 3.5: The location of the poles when \( U \to 0 \).

When \( \omega \to 0 \) the poles \( k_1 \) and \( k_2 \) will move to the origin and the paths \( L_1 \) and \( L_2 \) can be seen in figure (3.6). However this is not correct. When \( \omega \to 0 \) a factor \( k \) can be removed from the function \( F(\theta, k) \) and looks like this:

\[ F_{\omega=0}(\theta, k) = \frac{ek(z + \zeta + i(x - \xi) \cos \theta) \cos(k(y - \eta) \sin \theta)}{g - kU^2 \cos^2 \theta} \] (3.19)

with only one pole, located at \( k_2 = k_4 = \frac{g}{U^2 \cos^2 \theta} \).
3.3.2 Calculation of the Green function.

Newman ([26] + [27]) has published two papers in the Journal of ship research concerning the evaluation of the wave resistance Green function: one for the calculation of the double integral and one for the calculation of the single integral on the centreplane. The Green function is written in the following form (see Wehausen and Laitone [43]):

\[
G = \frac{1}{R_0} - \frac{1}{R} - 4 \pi \int_0^{\infty} \int_0^{kz\cos(kx\cos\theta)\cos(ky\sin\theta)} e^{-kz\cos(kx\cos\theta)\cos(ky\sin\theta)} \frac{dkd\theta}{k\cos^2\theta - 1}
\]

\[
- 4 \int_0^{\frac{1}{2}\pi} e^{-z \sec^2\theta \sin(x \sec\theta \cos(y \sec^2\theta \sin\theta)} \sec^2\theta d\theta
\]

(3.20)

The quantities are defined as can be seen in figure (3.7).

with \( R = \sqrt{x^2 + y^2 + z^2} \).

As has been done by Newman, this integral can be split in a Double and a Single integral as follows:

\[
\text{Double} = \frac{2}{\pi} i \int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi} \int_0^{+\infty} \cos \psi \frac{e^{-kz + i k|x|\sec \psi + ky \tan \psi}}{k - \cos^2 \psi + i \epsilon} dk d\psi \quad (3.21)
\]
3.3. **THE GREEN FUNCTION**

Figure 3.7: Location of field-, source- and image point.

\[
\text{Single} = 4iH(-x) \int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi} \sec^2 \theta e^{-z \sec^2 \theta} + ix \sec \theta + i|y| \sec^2 \theta \sin \theta \, d\theta
\]

(3.22)

3.3.3 **Double integral**

The double integral as given by equation (3.21) will be approximated by Chebychev polynomials as done by Newman. In order to approximate a function \( f \) of one variable \( x \) in the normalized range \([-1, +1] \), the Chebychev polynomial of order \( n \) is defined by:

\[
T_n(x) = \cos(n \arccos x) \quad -1 \leq x \leq +1
\]

(3.23)

so the function \( f(x) \) is approximated by:

\[
f(x) = \sum_{m=0}^{N} c_m T_m(x)
\]

(3.24)

The coefficients \( c_m \) can be found according to:
\[ c_m = \frac{\epsilon_m}{N} \sum_{n=0}^{N} f(x_n) T_m(x_n) \]  
(3.25)

with \( \epsilon_0 = 1, \epsilon_m = 2 \), the double prime indicates that the first and the last terms in this summation are multiplied by \( \frac{1}{2} \). The coordinates \( x_n \) are given by:

\[ x_n = \cos\left(\frac{n\pi}{N}\right) \]  
(3.26)

(see Fox and Parker [14]).

For the 3D-case the situation is completely equivalent. In equation (3.21) logarithmic singularities are present when \( R = 0 \). These singularities must be subtracted and approximated first in order to improve the convergence of the approximation. The final approximation is given by:

\[ D \approx S + \sum_{i=0}^{16} \sum_{j=0}^{16} \sum_{k=0}^{8} C_{ijk} T_i[f(r)] T_j(-1 + \frac{4}{\pi} \theta) T_{2k}(\frac{2}{\pi} \alpha) \]  
(3.27)

The function \( f(r) \) is defined so that the interval \((0, \infty)\) transforms into the interval \((-1, +1)\), see figure (3.8).

\[ \alpha \text{ and } \theta \text{ are defined as:} \]

\[ x = R \sin \theta \]
\[ z + iy = R \cos \theta e^{i\alpha} = \rho e^{i\alpha} \]  
(3.28)
3.3. *THE GREEN FUNCTION*

$T_i, T_j$ and $T_{2k}$ are Chebychev polynomials. $S$ is the logarithmic part of the double integral. The Chebychev coefficients $C_{ijk}$ are calculated and tabulated by Newman. Also the differentiated Green function has to be evaluated, which contains terms like $\partial G(x, \xi) / \partial n_x$ (see equation (2.61)). Each part of the expansion (3.27) has to be differentiated and evaluated analytically. The following terms have to be evaluated:

$$\frac{\partial D}{\partial x} = A \cdot \frac{\partial D}{\partial \alpha} \tag{3.29}$$

with the transformation matrix $A$ given by:

$$A = \begin{pmatrix}
\frac{\partial \theta}{\partial x} & \frac{\partial \alpha}{\partial x} & \frac{\partial R}{\partial x} \\
\frac{\partial \theta}{\partial y} & \frac{\partial \alpha}{\partial y} & \frac{\partial R}{\partial y} \\
\frac{\partial \theta}{\partial z} & \frac{\partial \alpha}{\partial z} & \frac{\partial R}{\partial z}
\end{pmatrix} \tag{3.30}$$

And also for the singular part $S$. For example the next figure (3.9) is obtained. The singular character is well shown here.

### 3.3.4 Single integral

After changing the variable of integration to $s = \sec \theta$ and $t = \tan \theta$, the following expression equivalent to (3.22) is obtained.

$$f(x, y, z) = -4H(-x) \int_{-\infty}^{\infty} \sin \{(x + yt)\sqrt{1 + t^2}\} e^{-z(1 + t^2)} dt \tag{3.31}$$

Numerical integration of this equation is rather difficult because of the rapidly oscillatory integrand as can be observed from figure (3.10).

In order to avoid time-consuming numerical integration Bessho [5] has derived two complementary Neumann series (a summation of different kind of Bessel functions) to evaluate these integrals.

The single integral as given by equation (3.22) is evaluated by Newman at the centreplane, i.e. a special case where the source and field point are in the same longitudinal plane. This is especially important when analyzing thin ships. The case $y \neq 0$ will be dealt with later. For instance with the use of Padé approximations. The centreplane integral looks like this:
Figure 3.9: Function plot of the Double integral.

\[ S(x, y, z) | y = 0 = -8H(-x) \int_0^{\frac{1}{2} \pi} \sec^2 \theta e^{-z \sec^2 \theta} \sin(x \sec \theta) d\theta \]  

(3.32)

In each of the different x-z regions (see figure (3.11)), the integral will be approximated differently.

**Region A:** (small z) an expansion involving differentiated Bessel functions of the second kind:

\[ S \approx \frac{1}{\sqrt{z}} F(\xi) - \sum_{n=0}^{\infty} \frac{1}{n!} z^n \frac{d^{2n}}{dx^{2n}} \left[ \frac{\pi}{2} Y_1(x) + \frac{1}{x} \right] \]  

(3.33)

where \( Y_1(x) \) denotes the Bessel function of the second kind, \( \xi \) equals \( \frac{1}{2} \frac{x}{\sqrt{z}} \) and \( F(\xi) \) Dawson’s integral:

\[ F(x) = e^{-x^2} \int_0^x e^{t^2} dt \quad 0 \leq x < \infty \]  

(3.34)
3.3. THE GREEN FUNCTION

Region B: an expansion in Neumann series, products of Bessel functions of the first kind and modified Bessel functions of the second kind:

\[ S = \frac{1}{2}e^{-\frac{1}{2}z} \sum_{n=0}^{\infty} (-1)^n J_{2n+1}(x) \left[ K_n\left(\frac{1}{2}z\right) + K_{n+1}\left(\frac{1}{2}z\right) \right] \]  \hspace{1cm} (3.35)

where \( J_n(x) \) denotes the Bessel function of the first kind and \( K_n(z) \) the modified Bessel function of the second kind.

Region C: large distances form the origin, steepest-descent expansion. The final expansion looks like this:

\[ S \simeq -\frac{1}{2}ie^{h(0)} + i\delta \sum_{n=0}^{\infty} (2n + 1) B_n\left(\frac{2}{\rho}\right)^{n+\frac{1}{2}} (\beta + i\alpha)^n \Gamma(n + \frac{1}{2}) \]  \hspace{1cm} (3.36)
where only the coefficient $B_n$ has to be evaluated.
For all the expansions in the different domains (A)-(C), the differentiated Green function has to be evaluated. An example for the Green function can be seen in figure (3.12). The wave character is well observed here.

3.4 Padé approximations.

3.4.1 Introduction

The basis for the Padé approximation technique is the formal Taylor series expansion. From this basis a Padé approximation can be found. It is also possible for a Taylor expansion to be divergent and the Padé expansion to be convergent and vice versa.
As an example we will use the Taylor expansion of the exponential function and the Euler function.
3.4. PADÉ APPROXIMATIONS.

\[ e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{1}{2} x^2 + \frac{1}{6} x^3 + \cdots \quad r_c = \infty \quad (3.37) \]

\[ E(x) = \int_0^\infty \frac{e^{-t}}{1 + xt} dt = 1 - x + 2x^2 - 6x^3 + \cdots \quad r_c = 0 \quad (3.38) \]

The idea of Padé expansion is to approximate the function by a rational function of the following form:

**Definition** (see Baker [3],[4])

We denote the \([L,M]\) Padé approximant of \(f(x)\) by:

\[ \left[ \frac{L}{M} \right] = \frac{P_L(x)}{Q_M(x)} \quad (3.39) \]
where \( P_L(x) \) is a polynomial of degree at most \( L \) and \( Q_M(x) \) is a polynomial of degree at most \( M \). The formal power series of \( f(x) \) reads as:

\[
f(x) = \sum_{i=0}^{\infty} a_i x^i
\]  

(3.40)

When we require:

\[
f(x) - \lfloor L/M \rfloor = O(x^{L+M+1})
\]

(3.41)

Then the coefficients of \( P_L \) and \( Q_M \) can be found according to the following scheme (Baker):

\[
\begin{align*}
a_0 & = p_0 \\
a_1 + a_0 q_1 & = p_1 \\
a_2 + a_1 q_1 + a_0 q_2 & = p_2 \\
& \vdots \\
a_l + a_{l-1} q_1 + \ldots + a_0 q_l & = p_l \\
a_{l+1} + a_l q_1 + \ldots + a_{l-m+1} q_m & = 0 \\
& \vdots \\
a_{l+m} + a_{l+m-1} q_1 + \ldots + a_l q_m & = 0
\end{align*}
\]  

(3.42)

\[
a_n \equiv 0 \quad n < 0 \\
q_1 = 1 \\
q_j \equiv 0 \quad j > M
\]

for instance the \([1/1]\) approximant for \( e^x \) reads as:

\[
e^x \approx \frac{2 + x}{2 - x}
\]  

(3.43)

inevitable a pole occurs at \( x = 2 \). So the Padé approximants seems worse here. If equation (3.43) is evaluated, the meaning of equation (3.41) is better understood:
3.4. **Pade approximations.**

\[
\frac{2 + x}{2 - x} = 1 + x + \frac{1}{2}x^2 + \frac{1}{4}x^3 + \cdots \tag{3.44}
\]

The difference with the Taylor expansion begins at the \(O(x^3)\)-term.

The Euler function evaluated at \(x = 1\) is wildly oscillating (when using Taylor expansion). The answer however for \(E(1)\) is known: \(E(1) \simeq 0.5963\). The Taylor expansion never reaches this value. The Pade approximants however do. The \([2/2]\) approximant reads as:

\[
[2/2](x) = \frac{1 + 5x + 2x^2}{1 + 6x + 6x^2} \tag{3.45}
\]

So \(E(1) \simeq 0.6154\), only five Taylor terms have been used to get this accuracy. The Taylor expansion leads to the result:

\[Taylor_5(1) = 20.\] \tag{3.46}

The \([6/6]\) approximant is even better: \(E(1) \simeq 0.5968\). Pade approximations can also be used when calculating continued fractions, using the \([M/M]\) and the \([M/M + 1]\) Pade approximants (see Baker [3]). For instance the Taylor approximation of the following function reads as:

\[
\frac{1}{2}(\sqrt{x + 1} + 1) \simeq 1 + \frac{1}{4}x - \frac{1}{16}x^2 + \cdots \quad r_c = 1 \tag{3.47}
\]

and the continued fraction approximation is given by:

\[
\frac{1}{2}(\sqrt{x + 1} + 1) \simeq 1 + \frac{\frac{1}{4x}}{1 + \frac{1}{4x} \cdots} \quad r_c = \infty \tag{3.48}
\]

The advantage in mainly due to the fact that the series (3.47) has a range of convergence \(r_c = 1\), whereas the continued fractions (3.48) have a \(r_c\) of the whole complex plane. This is true as long as \(x\) is not on the real axis for \(x \in (-\infty, -1]\) see figure (3.13).

So Pade approximant could lead to valuable results when Taylor expansion fails (see also Hermans and van Gemert [16]). In the next paragraph use will be made of this when evaluating the single integral at \(y \neq 0\).
3.4.2 Padé approximations in the single integral

The single integral to be approximated is given by equation (3.22) repeated here:

\[
\text{Single} = 4iH(-x) \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sec^2 \theta e^{-z \sec^2 \theta} + ixe^\theta + iy \sec^2 \theta \sin \theta d\theta
\]

(3.49)

Only the real part will be used:

\[
-8H(-x) \int_{0}^{\frac{\pi}{2}} \sec^2 \theta e^{-z \sec^2 \theta} \sin(x \sec \theta) \cos(y \sec^2 \theta \sin \theta) d\theta
\]

(3.50)

which will be denoted by:

\[-8H(-x) f(x, y, z)\]

(3.51)
3.4. **PÄDÉ APPROXIMATIONS.**

changing of variable to \( s = \sec \theta \) leads to:

\[
f(x, y, z) = \int_{1}^{\infty} \frac{se^{-z s^2}}{\sqrt{s^2 - 1}} \sin(sx) \cos(y s \sqrt{s^2 - 1}) ds \tag{3.52}
\]

Taylor expansion of \( f(x, y, z) \) leads to:

\[
f(x, y, z) = f(x, 0, z)_{y=0} + y \frac{\partial f(x, y, z)}{\partial y} \bigg|_{y=0} + \frac{1}{2} y^2 \frac{\partial^2 f(x, y, z)}{\partial y^2} \bigg|_{y=0} + \cdots \tag{3.53}
\]

applying this to (3.52) leads to:

\[
f(x, y, z) = \int_{1}^{\infty} \frac{se^{-z s^2}}{\sqrt{s^2 - 1}} \sin(sx) ds
\]

\[
-\frac{1}{2} y^2 \int_{1}^{\infty} s^3 \sqrt{s^2 - 1} e^{-z s^2} \sin(sx) ds + \mathcal{O}(y^4) \tag{3.54}
\]

Newman [27] uses only the first term of the Taylor approximations (the thin ship term) in his article. So expanding further we might be able to get more accurate results.

Equation (3.54) is the formal Taylor series expansion. Numerical investigation reveals the fact that this series is not converging very well. Perhaps Padé approximation could lead to valuable results. Numerical experiments give raise to the following table (see table (3.1)):

Examine for instance the first row in this table. The Taylor(Newman) expansion of the function leads to a value of the calculated integral of 0.5179. Using Padé approximation leads to surprisingly good results in some cases.
<table>
<thead>
<tr>
<th>$(x, y, z)$</th>
<th>$f(x, y, z)$</th>
<th>Taylor</th>
<th>$\text{Pade}[2/2]$</th>
<th>$\text{Pade}[0/4]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(.1,.1,.1)$</td>
<td>.30124</td>
<td>.5179</td>
<td>.3019</td>
<td>.3031</td>
</tr>
<tr>
<td>$(.1,.5,.1)$</td>
<td>.0053</td>
<td>$2.7 \times 10^2$</td>
<td>.0073</td>
<td>-.0018</td>
</tr>
<tr>
<td>$(.1,.9,.1)$</td>
<td>.0029</td>
<td>$1.1 \times 10^5$</td>
<td>.006</td>
<td>-.00012</td>
</tr>
<tr>
<td>$(.1,.9,.9)$</td>
<td>.00252</td>
<td>.0044</td>
<td>.00254</td>
<td>.00255</td>
</tr>
<tr>
<td>$(.5,.1,.9)$</td>
<td>.2132</td>
<td>.2132</td>
<td>.2132</td>
<td>.2132</td>
</tr>
<tr>
<td>$(.5,.9,.5)$</td>
<td>.1608</td>
<td>2.11</td>
<td>.168</td>
<td>.177</td>
</tr>
</tbody>
</table>

Table 3.1: *Comparison between different approximation techniques.*
Chapter 4

Solution of the Matrix Equation

4.1 Introduction

In order to obtain accurate results for the forward speed problem a sufficient number of panels for the discretization of the ship hull has to be used. In most of our calculations we used 48 panels in the $x$-direction and 8 panels in the $y$-direction. So the total number of element will be 768. This is also the matrix size. If we increase the number of panels, a direct solver (like LU-decomposition) for the matrix equation cannot be used anymore. So some sort of iterative method has to be used. In this chapter some suitable iterative methods will be discussed.

4.2 The matrix equation

In order to solve the ship wave problem the following two equations are important (see also equation (2.49) and equation (2.50)):

$$4\pi \ddot{\phi}(x) = - \int_{\Sigma} \sigma(\xi)G(x, \xi) dS_\xi + \ldots \quad (4.1)$$

$$2\pi \sigma(x) - \int_{\Sigma} \sigma(\xi) \frac{\partial G(x, \xi)}{\partial n_x} dS_\xi + \ldots = Vn \quad (4.2)$$

Once the potential $\phi(x)$ is known, the water height can be found according to equation (2.18):
\[ \zeta_0(x) = \frac{U}{g} \phi_x(x, y, 0) \]  

(4.3)

After discretization of these equations the following matrix equations are obtained:

The system influence matrix for the determination of the source strength \( \sigma \) (equation (4.2)) is given by:

\[ K \cdot \sigma = b \]  

(4.4)

with \( \sigma_i \) the source strength at panel number \( i \).

The potential \( \phi \) can be written as (equation (4.1)):

\[ \phi = P \cdot \sigma \]  

(4.5)

with \( \phi_i \) the potential at panel number \( i \).

The wave height \( \zeta \) (equation (4.3)) becomes:

\[ \zeta = X \cdot \sigma \]  

(4.6)

\( \zeta_i \) is the waterheight at the waterline panel. The matrix structure for the matrix \( K \) looks like (with \( K_{ij} \) the influence of a source at panel \( j \) upon panel \( i \).):

\[
K = \begin{pmatrix}
(A) & (B) \\
(B) & (A)
\end{pmatrix}
\]  

(4.7)

So at (A) we have:

\[ K(i, j) = K(i + \frac{N}{2}, j + \frac{N}{2}) \]  

(4.8)

and at (B) we have:

\[ K(i, j) = K(i - \frac{N}{2}, j + \frac{N}{2}) \]  

(4.9)

For the symmetrical problem the following block-structure can be observed: (see also figure (4.1)).

\[
\begin{pmatrix}
a & b & e & f \\
c & d & g & h \\
e & f & a & b \\
g & h & c & d
\end{pmatrix}
\begin{pmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4
\end{pmatrix}
= 
\begin{pmatrix}
b_1 \\
b_2 \\
b_3 \\
b_4
\end{pmatrix}
\]  

(4.10)
4.3. CONVERGENCE OF THE SOLUTION ALGORITHM.

with a right hand side vector of \((b_1, b_2, b_1, b_2)^T\), the solution will be the vector \((x_1, x_2, x_1, x_2)^T\), for the matrix now reduces to:

\[
\begin{pmatrix}
  a+e & b+f \\
  c+g & d+h
\end{pmatrix}
\begin{pmatrix}
  x_1 \\
  x_2
\end{pmatrix}
=
\begin{pmatrix}
  b_1 \\
  b_2
\end{pmatrix}
\] (4.11)

4.3 Convergence of the solution algorithm.

In order to investigate the convergence behaviour of the matrix the singular values have been plotted (see figure 4.2). The singular values of a matrix \(K\) are the square roots of the eigenvalues of the matrix \(K^TK\) \((\sigma^2(K) = \lambda(K^TK))\).

From this figure a few things can be noticed:

- The condition number \((\sigma_{max}/\sigma_{min})\) of this matrix is very small (so is very good), it is about 3.2.
Figure 4.2: Singular values of Matrix $K$.

- There is an enormous range in singular values which are almost the same. For instance the $150^{th}$ singular value is 12.68 and the $170^{th}$ is 12.66. We think that this behaviour is typical for integral operators of this type. A sudden change in singular values near the begin and the end of the spectrum and slowly varying values in the middle. For a differential operator just the opposite can be observed: slowly changing values at the begin and at the end of the spectrum and a relatively rapid change in the middle.

Most of the problems we have been dealing with in this thesis have been solved using only a relatively small number of panels (about 800). If we increase the number of panels, a direct solver (like LU-decomposition) for the matrix equation $K \cdot \mathbf{g} = \mathbf{b}$ cannot be used anymore. So some sort of iterative method has to be used. Not only does the error increase rapidly using LU-decomposition, but also the amount of work. LU-decomposition needs $O(n^2)$ and iterative methods like CG, ART or SIRT only $O(n^{3/2})$.

Because of the special structure of the singular values a method like CG could be most successful. The rate of convergence of CG is given by:

$$|x^i - \hat{x}| \leq 2C^i|x^0 - \hat{x}|$$ (4.12)
with $C$ given by:

$$C = \frac{\sqrt{c_1} - 1}{\sqrt{c_1} + 1}$$  \hspace{1cm} (4.13)

and $c_1$:

$$c_1 = \frac{\lambda_{\text{max}}}{\lambda_{\text{min}}}$$  \hspace{1cm} (4.14)

But each time a singular value is approximated by CG, this value can be 'removed' from the spectrum and the value of $c_1$ can be adjusted. If there exists a large cluster of singular values which are almost the same, the convergence will increase rapidly. Some startup delay may occur, but this can be fixed by preconditioning the matrix $K$ (see also van der Vorst and van der Sluis [36],[41]).

Due to the block structure of the matrix $K$ (see figure (4.1)) the LU decomposition of $K$ will almost have the same structure as the matrix $K$ itself (see figure (4.3)). Of course this specific block structure of the matrix $K$ only occurs when the panels are generated as indicated in figure (3.3).

So the matrix equation $K \cdot \sigma = b$ can be preconditioned using only the lower part of the matrix $K$. What you are doing is to approximate the matrix $K$ using the following:

$$K = LU \approx \tilde{L} \tilde{U}$$  \hspace{1cm} (4.15)

with $\tilde{L}$ the actual lower triangular part of the matrix $K$ and with $\tilde{U}$ the identity matrix $I$. The matrix $\tilde{L}$ is easy to invert because of its lower triangular form. So instead of solving the original equation:

$$K \cdot \sigma = b$$  \hspace{1cm} (4.16)

we now solve the preconditioned equation:

$$\tilde{L}^{-1} K \cdot \sigma = \tilde{L}^{-1} b$$  \hspace{1cm} (4.17)

This preconditioned matrix $\tilde{L}^{-1} K$ should be much easier to handle using a CG algorithm. A first approximation for the solution $\sigma$ is $\tilde{L}^{-1} b$. In the case of a symmetrical problem, the same preconditioning could be applied. The matrix
\[ \frac{1}{2} \tilde{K} \text{ (see equation (4.11)) is almost a lower triangular matrix, so preconditioning should work even better.} \]

If the matrix size exceeds the central storage capacity of the computer, only parts of the matrix can be kept in its memory, the rest will have to be stored on the hard disk unit or some other secondary storage. (As an example the central memory of the HP 9000/835, which has been used for most of our calculations, is 16Mb. The maximum matrix size is about 1400x1400 using double precision calculations.)

The matrix \( K \) is stored in computer memory in a number of pages (a certain amount of memory). Only parts of the matrix \( K \) are stored at once. If the computer algorithm needs parts of the matrix \( K \) which are not in this direct memory, the page that has been the longest time out of use will be swapped out and the new page will be overlayed on its space (this is called a page fault). Storing a matrix is usually done in a row-wise fashion. An algorithm
like standard LU-decomposition with partial pivoting uses the computer memory in almost random order. Winter [44] describes a method of partitioning the matrix $K$ in submatrices in order to keep the process of swapping data from the secondary to the central memory to an absolute minimum.

An iterative method should be used which doesn't have the disadvantage of continuously swapping data from secondary to central memory. Two so-called 'row action methods' are briefly discussed here. The first is ART (Algebraic Reconstruction Techniques) first invented by Kaczmarz (1937) and the second is a closely related method, called SIRT (Simultaneous Iterative Reconstruction Techniques). A description of these methods can be found in van Dijke [13].

An initial approximation $x^{(0)}$ is projected onto the first hyperplane corresponding to the first equation of the matrix $\mathbf{A}$ ($x^{(1)}$ is a least squares solution of the first row-matrix equation). This first approximation $x^{(1)}$ will be mapped onto the second row of the matrix $\mathbf{A}$ and so on. This concept of ART is made visible in figure (4.4).

![Figure 4.4: A geometrical interpretation of ART.](image)

SIRT almost uses the same algorithm as ART, but now with an averaging process. SIRT first computes the corrections for all the rows simultaneously. Averaging these corrections for all the $n$-rows leads to the first update of $x^{(0)}$. 


4.4 Conclusions

In this chapter the stability and convergence of the solution of the matrix equation is described. If we increase the number of panels to discretise the ship hull, a direct solver (like LU-decomposition) cannot be used anymore. So some sort of iterative method like ART, SIRT or CG has to be used. Also in the case of a matrix equation with complex-valued coefficients a method like CGS or CGSTAB (see Sonneveld and van der Vorst [42] and also Kuyper [23]) could be useful.

In our case the Laplace equation has been solved using Green functions. These functions already 'contain' information about the solution of the matrix equation. This could be an explanation for the smooth behaviour of the singular values. The CG algorithm can be used without much adjustments. Using only 1/r-sources, a less smooth behaviour could be expected. In the extreme case of solving the Laplace equation directly (using finite differences) the CG algorithm will converge very slowly.
Chapter 5
Computational Results

5.1 Introduction

In this chapter results of computations of wave profiles and wave resistance will be compared with results obtained from measurements and with other theorems. (Tsutsumi [40], Dawson [11] and Kitazawa [22]).

The computer tests have been performed on two hull types: The Wigley hull ($\alpha_2 = 0.0$) and a modified Wigley hull ($\alpha_2 = 0.2$) (see equation (3.8)).

Results of computations with the former hull type are used to compare with the results obtained by the authors mentioned above (see section 5.2.2), results of computations obtained with the latter hull type are used to compare with the results from measurements which have been performed at the Delft Hydromechanics Laboratory (see section 5.2.4). The tests at this laboratory include both steady and unsteady measurements.

5.2 Steady motion

5.2.1 Wave profiles of Wigley hull

Some examples for a parabolic Wigley hull (Shearer [35]) will be calculated. The calculations have been performed with the following parameters for the Wigley hull:

$$\alpha_2 = 0. \quad \frac{L}{B} = 10. \quad \lambda = 0.$$

$$\alpha_4 = 0. \quad \frac{L}{H} = 16. \quad (5.1)$$
Figure 5.1: Hull side wave profiles of WIGLEY.
5.2. **STEADY MOTION**

As has been noted before, the calculation of the steady and the unsteady wave potential can be done using the same matrix kernel. For the steady wave potential $\phi$, matrix equation (2.61) together with the right hand side (2.74) have been used. The solution of this matrix equation is the source strength $\sigma(\xi)$. The integral equations (2.61) and (2.62) are two coupled equations. We would need an iterative scheme to solve these two equations. In this thesis only the first step of this iteration scheme has been performed. In the case of thin ships ($\bar{\phi}$ is small compared to $Ux$) this is a valid approximation. Now using equation (2.62) the potential $\bar{\phi}$ can be calculated. A first order approximation for the wave height $\zeta(x, y)$ can be found using the following:

$$\zeta(x, y) = \frac{U}{g} \bar{\phi}_x(x, y, 0) \quad (5.2)$$

The potential $\bar{\phi}$ and the source strength $\sigma$ are known (have been calculated). The wave height or equivalently $\bar{\phi}_x$ can be calculated using approximately the same formulas as Kellogg [21]:

$$\frac{\partial \phi}{\partial x} = \frac{1}{2} \sigma(x)n_1 + \int \int_{\Sigma} \sigma(\xi) \frac{\partial G}{\partial x} dS_{\xi}$$

$$\frac{\partial \phi}{\partial y} = \frac{1}{2} \sigma(x)n_2 + \int \int_{\Sigma} \sigma(\xi) \frac{\partial G}{\partial y} dS_{\xi}$$

$$\frac{\partial \phi}{\partial z} = \frac{1}{2} \sigma(x)n_3 + \int \int_{\Sigma} \sigma(\xi) \frac{\partial G}{\partial z} dS_{\xi} \quad (5.3)$$

The differentiated Green functions $\partial G/\partial x_i$ are known because they have also been used in order to calculate matrix equation (2.61). So when setting up the computer program a data set with the $\partial G/\partial x_i$ must be preserved. Now the wave heights can be calculated easily.

5.2.2 **Comparison with measurements I**

For a series of Froude numbers (0.20 - 0.45) the steady wave potential $\bar{\phi}$ of a Wigley hull have been calculated as can be seen in figure (5.1). The results have been compared with the measured and calculated values of Kitazawa and Kajitani [22]. In our calculations a grid size of 32 x 8 has been used, so discrepancies could be also due to this relatively large grid size. Especially near the bow this could lead to substantial errors. See for instance figure (5.3). For
a specific value of the Froude number (Fn = 0.348), the calculation can be compared with values calculated by Tsutsumi [40] and Dawson [11] as can be seen in figure (5.2).

![Graph of Wigley hull - Wave Profile for Fn=0.348](image)

Figure 5.2: Wigley hull - Wave Profile for Fn=0.348

5.2.3 Convergence aspects

In order to test the convergence of the algorithm, several grid sizes are considered. Results are shown in figure (5.3). These calculations have been performed on grid sizes of 24 x 8, 32 x 8 and 60 x 16 respectively.

The values near the bow agree more with the measured values. Starting from the bow the wave height first increases a little and then decreases rapidly as can be seen from the measurements and can also be observed in figure (5.3). It can also be observed that only near the ship bow the wave height changes substantially when decreasing the grid size. So local grid refinement near the ship bow could lead to more accurate results. But we have to be careful not to deform the triangular panels too much. If the ratio of the width to the height of the panel is too small, singular behaviour occurs at those panels (see also figure (5.4)). In this figure, the closer the panels are to the bow, the closer they are to each other.
5.2. STEADY MOTION

Figure 5.3: Wave profile with increasing grid sizes ($Fn = 0.20$).

Figure 5.4: Wave profile with locally refined grid(I).

Figure 5.5: Grid independence of the algorithm ($Fn = 0.266$).
In all our calculations the wetted body $\Sigma$ is divided into *triangular* panels (see figure (3.1)). In order to test the grid-independence of the algorithm, the symmetry with respect to the $y \equiv 0$ plane has been removed. Two grids have been tested: a symmetrical and an antisymmetrical grid. Results of this can be observed in figure (5.5). The algorithm seems to be relatively grid-independent.
5.2.4 Comparison with measurements II

At the Delft Hydromechanics Laboratory measurements on four Wigley hulls have been performed. The water depth of the basin was 2.50 m, the length was 150 m and the width 4.60 m. The model set-up and oscillator are shown in this photo. The first two hull forms have been tested by Gerritsma [15] in 1988. Two models with a L/B ratio of 10. and 5. respectively; both with a midship area coefficient $C_M = 0.9090$. From september 1990 until june 1991 two other Wigley hull forms have been tested. The same L/B ratios have been used, but with another midship area coefficient ($C_M = 0.6667$).

For each model four types of experiments have been carried out:

A. Forced heave and pitch oscillations in still water.
B. Wave force and moment measurements in regular waves.
C. Resistance, sinkage and trim in still water.
D. Heave and pitch motion and added resistance in regular waves.

Wherever applicable the following physical parameters have been used:

Froude numbers: $Fn = 0.2, 0.3, 0.4$ (part C. also $Fn = 0.15, 0.25, 0.35, 0.45$)
Frequencies of oscillations: $\omega = (1.00, 2.00), 3.00 \cdots 12.0 \text{ rad/s}$
Amplitudes of oscillation:

heave: \( z_a = 0.025 \) and \( 0.050 \) m.
pitch: \( \theta_a = 0.026 \) and \( 0.052 \) rad.

Wave amplitudes: \( \zeta_a = 0.010, 0.020 \) and \( 0.030 \) m.

Wave lengths: \( \lambda/L = 0.50 \cdots 2.00 \) m.

The experiments carried out in part A have also been recorded on video tape. An example of this can be seen in figure (5.6). The calculated computer program results have been marked with a white +.

Figure 5.6: Wave height measurements \((F_n = 0.20)\). The station numbers as can be observed in the photograph, range from 20 to 15. These station numbers are mapped on the graph on the mathematical domain \(-1.0\) to \(-0.5\) respectively.
5.2.5 Resistance

If the potential function $\Phi$ is known, the pressure in a point in the fluid can be calculated using Bernoulli's equation (2.75).

An example of this pressure distribution on one side of the hull can be observed in figure (5.7).

![Contour plot of the pressure on the hull.](image)

The wave resistance can be obtained by integration of these pressures per panel over the ship hull.

The wave resistance of the Wigley hull as a function of the total number of panels can be seen in figure (5.8).

At model test experiments usually the total resistance of the vessel is measured. The wave resistance can be extracted from these measurements in different ways:
• A complete analysis of the wake (wake survey).

\[ D = \frac{1}{2} \pi \rho U^2 \int_{-\pi/2}^{\pi/2} |A(\theta)|^2 \cos^3 \theta d\theta \]  
(5.4)

with \( A(\theta) \) the wave amplitude.

• Some sort of empirical method.
5.2. **STEADY MOTION**

In our investigations use have been made of the latter method. As has been mentioned before in Chapter 1, this empirical method is based on Froude's hypothesis. The drag on a ship hull can be expressed as the sum of a 'flat-plate' frictional drag, depending on the Reynolds number, plus a residual drag, depending on the Froude number:

\[ C_D(Re, Fn) = C_F(Re) + C_R(Fn) \]  \hspace{1cm} (5.5)

On almost every ship hull form, the residual drag is dominated by wave resistance. The frictional drag \( C_F(Re) \) is now estimated by experiments. Results of these experiments are so-called empirical curves: Shoenherr, ITTC & ATTC (International and American Towing Tank Conferences). The most widely used (ITTC) is given by:

\[ C_F(Re) = \frac{0.075}{(10 \log Re - 2)^2} \]  \hspace{1cm} (5.6)

So the difference between this line and the measured total resistance is an estimate for the wave resistance. (The reader is referred to figure (5.9) for an illustration. The measurements marked with an asterisk(*) have been performed at the Delft University of Technology.)

Use have been made of the following:

\[ Re = \frac{UL}{\nu} \hspace{1cm} Fn = \frac{U}{\sqrt{gL}} \]

\[ C_D = \frac{D}{\frac{1}{2} \rho U^2 \Sigma} \hspace{1cm} \nu = 1.07 \times 10^{-6} \text{ m}^2/\text{s} \hspace{1cm} (T = 17^\circ C) \]  \hspace{1cm} (5.7)

Now the wave resistance can be calculated. The results can be seen in table (5.1) and in figure (5.10). As a reference also the total resistance of the Wigley hull has been measured when this model is free to trim and sinkage (see figure 5.11).
Figure 5.9: The total drag coefficient of a Wigley hull as a function of the Reynolds number. The ITTC line (International Towing Tank Conference) is the flat-plate frictional drag curve.

<table>
<thead>
<tr>
<th>Fn</th>
<th>Re</th>
<th>ITTC</th>
<th>$C_{D, meas.}$</th>
<th>$C_{W, meas.}$</th>
<th>$C_{W, calc.}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>$3.04 \times 10^6$</td>
<td>$3.73 \times 10^{-3}$</td>
<td>$4.24 \times 10^{-3}$</td>
<td>$0.51 \times 10^{-3}$</td>
<td>$0.44 \times 10^{-3}$</td>
</tr>
<tr>
<td>0.3</td>
<td>$4.56 \times 10^6$</td>
<td>$3.45 \times 10^{-3}$</td>
<td>$5.32 \times 10^{-3}$</td>
<td>$1.87 \times 10^{-3}$</td>
<td>$1.48 \times 10^{-3}$</td>
</tr>
<tr>
<td>0.4</td>
<td>$6.08 \times 10^6$</td>
<td>$3.28 \times 10^{-3}$</td>
<td>$5.64 \times 10^{-3}$</td>
<td>$2.36 \times 10^{-3}$</td>
<td>$1.50 \times 10^{-3}$</td>
</tr>
</tbody>
</table>

Table 5.1: Calculated and measured wave resistance.

These measured values can be compared with our theorem and the theorem of for instance Tsutsumi. In this figure two different hull form are used! The normal Wigley hull (marked with *) which have been calculated with our computer program and by Tsutsumi, and the modified Wigley hull (marked with o) which have been calculated with our computer program and have been measured at the Delft University of Technology.
5.2. STEADY MOTION

Figure 5.10: Measured and calculated wave resistance.

Figure 5.11: Measured total resistance, the difference between a model which is kept fixed and an model which is free to trim and sinkage.
CHAPTER 5. COMPUTATIONAL RESULTS

5.3 Unsteady motion

5.3.1 Wave profile of Wigley hull

For the unsteady motion $\ddot{\phi}$ the water heights for the six different types of motion have been calculated using the same matrix equation (2.61) as for the unsteady motion $\ddot{\phi}$ but now with different right hand sides (for instance equation (2.66)). see figure (5.12).

For this figure the following right hand sides for the unsteady wave motion $\ddot{\phi}$ have been used:

\[
\text{surge} : -\omega n_1 \quad \text{roll} : \omega (zn_2 - yn_3) \\
\text{sway} : -\omega n_2 \quad \text{pitch} : Un_3 \\
\text{heave} : -\omega n_3 \quad \text{yaw} : -U n_2 \\
\tag{5.8}
\]

Of course as mentioned in the paragraph dealing with the incorporation of the body boundary condition, these right hand sides are not complete. For instance when $\ddot{\phi} = O(1)$, the right hand side for surge looks like:

\[
-n_1 \ddot{\phi}_{xx} - n_2 \ddot{\phi}_{xy} - n_3 \ddot{\phi}_{xz} \\
\tag{5.9}
\]

But already some interesting features of the unsteady wave character or the dynamic swell-up factor can be observed here. In these figures there is a sway-yaw and a pitch-heave correspondence. This can also be seen from the right hand side of equation (5.8). But a different order for the magnitude: sway $O(\omega)$ and yaw $O(U)$ is evident.

5.3.2 Comparison with measurements.

At the Delft Hydromechanics Laboratory measurements on a Wigley hull have been performed. Froude number from 0.10 - 0.40 and oscillations from 0.00 - 8.00 rad/s have been tested (see figures (5.17 - 5.20)). A comparison with the calculated values will be given here.

5.3.3 Dynamic swell-up

In order to calculate the swell-up, the wave height has to be calculated. At a specific station number the wave height in pitch motion as a function of time can be observed in figure (5.24). The wave height is given by equation (A.14):

\[
\zeta_1(x, y, t) = \frac{1}{g} \left\{ \omega \dot{\phi} \sin \omega t + U \dot{\phi}_x \cos \omega t - (\ddot{\phi}_1 \dot{\phi}_1) \cos \omega t \right\} \\
\tag{5.10}
\]
Figure 5.12: Wave heights for the unsteady motion due to unit oscillation ($Fn = 0.30$).
The swell-up coefficient is defined as:

\[ SUC = \frac{s_a}{z_a} \quad (5.11) \]

with \( s_a \) the amplitude of the relative motion and \( z_a \) the oscillator amplitude. As a reference the calculated quasi-static values for different oscillator amplitudes \( z_a \) have also been given (see figures 5.13 and 5.15). The oscillator frequency is going to zero.

### 5.3.4 Added resistance

Also the added resistance has been measured (see figures (5.17) - (5.20)). The added resistance is the difference between the measured mean total resistance and the stationary total resistance. The total resistance is given by the sum of the frictional drag and the residual drag. The main part of the added resistance however will consist of residual drag. A few features can be observed from figures (5.17) - (5.20) with the use of table (2.1):

- The added resistance for heave mode is proportional to the squared oscillator amplitude \( z_a \). (table (2.1), second column):

\[ R_{add} \sim c_1 z_a^2 + ... \quad (5.12) \]

- The measured added resistance for pitch mode doesn't have such obvious dependence on the amplitude as should be expected from table (2.1), final column.

- For heave mode the frequency dependence can be observed from (table (2.1), first column):

\[ R_{add} \sim c_1 \omega^2 + c_2 \omega^3 + ... \quad (5.13) \]

- In pitch mode, even for the frequency 'equal' zero there is still added resistance (table (2.1), third column):

\[ R_{add} \sim c_1 U + c_2 \omega + c_3 \omega^2 + ... \quad (5.14) \]

\( c_1, c_2 \) and \( c_3 \) are constants independent of \( \omega \) and \( z_a \).
In order to solve the matrix equations (2.61) and (2.62), use has been made of a piecewise constant variation of the unknown source strength $\sigma(\xi)$. It is relatively easy and accurate to calculate the single derivatives of the velocity potential $\phi(x)$ (like $\phi_x$, $\phi_y$, and $\phi_z$), (see also equation (5.3)).

In the right hand side (equations (2.66) - (2.71)) of the matrix equation also second derivatives of the function $\phi(x)$ (like $\phi_{yz}$) are necessary. In the implementation of the computer program this has not been done (see equation (2.72)). So accurate results for the added resistance can be expected for pitch mode because the right hand side (equation (2.70)) is of the order $O(U)$. This is also observed in our calculations for the added resistance. (see figures (5.23), (5.21) and (5.22)).

Using a higher order approximation for the source strength $\sigma(\xi)$ a better approximation for the added resistance is expected! Discrepancies between the theoretical and the measured results can also be due to tank wall interference. The hydrodynamical forces, moments and motions are considerably affected by tank wall interferences, particularly in the low frequency region. Since the forces we are considering here are quadratic phenomena, a small change in motion amplitude or phase due to interference effects may cause a large change in these values.
Figure 5.13: $Fn = 0.20$, heave.

Figure 5.14: Wave resistance, $Fn = 0.20$.

Figure 5.15: $Fn = 0.30$, heave.

Figure 5.16: Wave resistance, $Fn = 0.30$. 
Figure 5.17: Measured total resistance, $Fn = 0.20$, heave.

Figure 5.18: Measured total resistance, $Fn = 0.30$, heave.

Figure 5.19: Measured total resistance, $Fn = 0.20$, pitch.

Figure 5.20: Measured total resistance, $Fn = 0.30$, pitch.
CHAPTER 5. COMPUTATIONAL RESULTS

Figure 5.21: Measured and calculated added resistance.

Figure 5.22: Measured and calculated added resistance.

Figure 5.23: Calculated added Resistance, heave mode.

Figure 5.24: Calculated wave height, pitch mode.
Chapter 6

Concluding Remarks

In this thesis we presented an asymptotic method for the calculation of the added resistance of an oscillating ship sailing in otherwise calm water. As a result of the investigations presented in this study the following concluding remarks can be made:

- The method we have used here is a modified Neumann-Kelvin formulation. Using this we were able to calculate the added resistance, dynamic swell-up of the unsteady problem using the steady state characteristics. We have shown that our assumption, the influence of the stationary on the instationary part is of importance, was true. (Chapter 2).

- For the calculation of the oscillating Green function use have been made of a first order approximation. For the calculation of this wave resistance Green function an algorithm has been developed which is similar to the algorithms as described by Newman([26],[27]). The so-called single integral in the Green function is evaluated by Newman at the centreplane. The use of Padé approximations revealed that this is a useful extension. (Chapter 3).

- If we increase the number of panels, some sort of iterative method has to be used to solve the matrix equation. Because of the smooth behaviour of the singular values of the matrix the CG algorithm can be used efficiently. Due to the special block structure of the matrix, precondition is easy. (Chapter 4).

- From the calculation of the added resistance a few things can be noticed:
  
  - The added resistance for heave mode is proportional to the squared oscillator amplitude $z_a$. 
CONCLUDING REMARKS

- The measured added resistance for pitch mode doesn't have such obvious dependence on the amplitude as could be expected from theory.
- For heave mode the added resistance is proportional to the squared oscillator frequency $\omega$.
- In pitch mode, even for the frequency 'equal' zero there is still added resistance.

For pitch mode a piecewise constant source strength $\sigma(\xi)$ is accurate. But in heave mode a better approximation for the added resistance using a higher order approximation for the source strength is expected. (Chapter 5).
Appendix A

Expansion of the Free Surface Condition

All the terms in the free surface conditions (2.10) and (2.11) have to be expanded:

\[
\frac{\partial \Phi}{\partial t} \bigg|_{z = \zeta} = \left. \frac{\partial \Phi}{\partial t} \right|_{z = \zeta_0} + \epsilon \zeta_1 \frac{\partial}{\partial z} \left( \frac{\partial \Phi}{\partial t} \right) \bigg|_{z = \zeta_0} + \ldots
\]

\[
= \epsilon \frac{\partial \tilde{\Phi}}{\partial t} + O(\epsilon^2) \quad \quad (A.1)
\]

\[
\nabla \Phi \cdot \nabla \Phi = \left( -U + \frac{\partial \tilde{\phi}}{\partial x} + \epsilon \frac{\partial \tilde{\phi}}{\partial x} \right)^2 + \left( \frac{\partial \tilde{\phi}}{\partial y} + \epsilon \frac{\partial \tilde{\phi}}{\partial y} \right)^2 + \left( \frac{\partial \tilde{\phi}}{\partial z} + \epsilon \frac{\partial \tilde{\phi}}{\partial z} \right)^2
\]

\[
\nabla \Phi \cdot \nabla \Phi \bigg|_{z = \zeta} = \left. \nabla \Phi \cdot \nabla \Phi \right|_{z = \zeta_0} + \epsilon \zeta_1 \frac{\partial}{\partial z} (\nabla \Phi \cdot \nabla \Phi) \bigg|_{z = \zeta_0} + \ldots
\]

\[
= U^2 + (\frac{\partial \tilde{\phi}}{\partial x})^2 - 2U \frac{\partial \tilde{\phi}}{\partial x} - 2\epsilon U \frac{\partial \tilde{\phi}}{\partial x} + 2\epsilon \frac{\partial \tilde{\phi}}{\partial x} \frac{\partial \tilde{\phi}}{\partial x} + \]

\[
+ \left( \frac{\partial \tilde{\phi}}{\partial z} \right)^2 + 2\epsilon \frac{\partial \tilde{\phi}}{\partial z} \frac{\partial \tilde{\phi}}{\partial z} + \left( \frac{\partial \tilde{\phi}}{\partial y} \right)^2 + 2\epsilon \frac{\partial \tilde{\phi}}{\partial y} \frac{\partial \tilde{\phi}}{\partial y} + \]

\[
+ 2\epsilon \zeta_1 \left\{ -U \frac{\partial^2 \tilde{\phi}}{\partial x \partial z} + \frac{\partial \tilde{\phi}}{\partial x} \frac{\partial^2 \tilde{\phi}}{\partial x \partial z} + \frac{\partial \tilde{\phi}}{\partial z} \frac{\partial^2 \tilde{\phi}}{\partial z^2} + \frac{\partial \tilde{\phi}}{\partial y} \frac{\partial \tilde{\phi}}{\partial y \partial z} \right\}
\]

\[
+ O(\epsilon^2) \quad \quad (A.3)
\]
\[ z = \zeta_0 + \epsilon \zeta_1 + \mathcal{O}(\epsilon^2) \quad (A.4) \]

\[ \frac{\partial \Phi}{\partial z} \bigg|_{z=\zeta} = \frac{\partial \Phi}{\partial z} \bigg|_{z=\zeta_0} + \epsilon \zeta_1 \frac{\partial}{\partial z} \left( \frac{\partial \Phi}{\partial z} \right) \bigg|_{z=\zeta_0} + \ldots \]
\[ = \frac{\partial \Phi}{\partial z} + \epsilon \frac{\partial \Phi}{\partial z} + \epsilon \zeta_1 \left( \frac{\partial^2 \Phi}{\partial z^2} + \frac{\partial^2 \tilde{\Phi}}{\partial z^2} \right) \]
\[ = \frac{\partial \Phi}{\partial z} + \epsilon \frac{\partial \Phi}{\partial z} + \epsilon \zeta_1 \frac{\partial^2 \Phi}{\partial z^2} + \mathcal{O}(\epsilon^2) \quad (A.5) \]

\[ \frac{\partial \zeta}{\partial t} \bigg|_{z=\zeta} = \frac{\partial \zeta}{\partial t} \bigg|_{z=\zeta_0} + \epsilon \zeta_1 \frac{\partial}{\partial t} \left( \frac{\partial \zeta}{\partial t} \right) \bigg|_{z=\zeta_0} + \ldots \]
\[ = \frac{\partial \zeta_0}{\partial t} + \epsilon \frac{\partial \zeta_1}{\partial t} + \mathcal{O}(\epsilon^2) \quad (A.6) \]

\[ \frac{\partial \Phi}{\partial x} \bigg|_{z=\zeta} = \frac{\partial \Phi}{\partial x} \bigg|_{z=\zeta_0} + \epsilon \zeta_1 \frac{\partial}{\partial x} \left( \frac{\partial \Phi}{\partial x} \right) \bigg|_{z=\zeta_0} + \ldots \]
\[ = -U + \frac{\partial \tilde{\Phi}}{\partial x} + \epsilon \frac{\partial \tilde{\Phi}}{\partial x} + \epsilon \zeta_1 \left( \frac{\partial^2 \Phi}{\partial x \partial z} + \frac{\partial^2 \tilde{\Phi}}{\partial x \partial z} \right) + \ldots \]
\[ = -U + \frac{\partial \Phi}{\partial x} + \epsilon \frac{\partial \Phi}{\partial x} + \epsilon \zeta_1 \frac{\partial^2 \Phi}{\partial x \partial z} + \mathcal{O}(\epsilon^2) \quad (A.7) \]

\[ \frac{\partial \zeta}{\partial x} \bigg|_{z=\zeta} = \frac{\partial \zeta}{\partial x} \bigg|_{z=\zeta_0} + \epsilon \zeta_1 \frac{\partial}{\partial x} \left( \frac{\partial \zeta}{\partial x} \right) \bigg|_{z=\zeta_0} + \ldots \]
\[ = \frac{\partial \zeta_0}{\partial x} + \epsilon \frac{\partial \zeta_1}{\partial x} + \epsilon \zeta_1 \frac{\partial^2 \zeta_0}{\partial x \partial z} + \mathcal{O}(\epsilon^2) \quad (A.8) \]

\[ \frac{\partial \Phi}{\partial y} \bigg|_{z=\zeta} = \frac{\partial \Phi}{\partial y} \bigg|_{z=\zeta_0} + \epsilon \zeta_1 \frac{\partial}{\partial y} \left( \frac{\partial \Phi}{\partial y} \right) \bigg|_{z=\zeta_0} + \ldots \]
\[ = \frac{\partial \Phi}{\partial y} + \epsilon \frac{\partial \Phi}{\partial y} + \epsilon \zeta_1 \left( \frac{\partial^2 \Phi}{\partial y \partial z} + \frac{\partial^2 \tilde{\Phi}}{\partial y \partial z} \right) \]
\[ = \frac{\partial \Phi}{\partial y} + \epsilon \frac{\partial \Phi}{\partial y} + \epsilon \zeta_1 \frac{\partial^2 \Phi}{\partial y \partial z} + \mathcal{O}(\epsilon^2) \quad (A.9) \]
\[
\frac{\partial \zeta}{\partial y} \bigg|_{z=\zeta} = \frac{\partial \zeta}{\partial y} \bigg|_{z=\zeta_0} + \epsilon \zeta_1 \frac{\partial}{\partial z} \left( \frac{\partial \zeta}{\partial y} \right) \bigg|_{z=\zeta_0} + \ldots \\
= \frac{\partial \zeta_0}{\partial y} + \epsilon \frac{\partial \zeta_1}{\partial y} + \epsilon \zeta_1 \frac{\partial^2 \zeta_0}{\partial y \partial z} + O(\epsilon^2) \quad (A.10)
\]

In order to obtain insight in the order of magnitudes in the above equations we will use a dimensional analysis:

\[
x' = x/L \quad y' = y/L \quad z' = z/L \\
\phi' = \bar{\phi}/UL \quad \bar{\phi} = \tilde{\phi}/UL \quad t' = \omega t \quad \zeta' = \zeta/L 
\quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad (A.11)
\]

Equation (2.10) transforms into the following equation:

\[
-\omega U \frac{\partial \tilde{\phi}}{\partial t'} + U^2 \frac{\partial \tilde{\phi}}{\partial x'} + U^2 \frac{\partial \phi'}{\partial x'} \frac{\partial \phi'}{\partial x'} + U^2 \frac{\partial \phi'}{\partial y'} \frac{\partial \phi'}{\partial y'} + U^2 \frac{\partial \phi'}{\partial z'} \frac{\partial \phi'}{\partial z'} +
\]

\[
L \zeta_1' \left\{ \frac{U^2}{L} \frac{\partial^2 \phi'}{\partial x' \partial x'} + \frac{U^2}{L} \frac{\partial \phi'}{\partial x'} \frac{\partial^2 \phi'}{\partial x' \partial z'} + \frac{U^2}{L} \frac{\partial \phi'}{\partial x'} \frac{\partial^2 \phi'}{\partial z' \partial z'} + \frac{U^2}{L} \frac{\partial \phi'}{\partial y'} \frac{\partial^2 \phi'}{\partial z' \partial y'} \right\}
\]

\[+ gL \zeta_1' \quad \text{at } z = \zeta_0 \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad (A.12)
\]

In order to be able to handle this formula, and especially the one when extracting \( \zeta_1 \), some moderate demands for \( U \) have to be made (\([U^2] \ll [gL])\). Then equation (A.12) leads to (leaving the \( x' \) for \( x \)):

\[
\frac{\partial \tilde{\phi}}{\partial t} - U \frac{\partial \tilde{\phi}}{\partial x} + \frac{\partial \tilde{\phi}}{\partial x} \frac{\partial \tilde{\phi}}{\partial x} + \frac{\partial \tilde{\phi}}{\partial y} \frac{\partial \tilde{\phi}}{\partial y} + \frac{\partial \tilde{\phi}}{\partial z} \frac{\partial \tilde{\phi}}{\partial z} + g \zeta_1 = 0 
\quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad (A.13)
\]

The other equation remains unchanged. Equation (A.13) leads to the following expression for \( \zeta_1 \):

\[
\zeta_1 = -\frac{1}{g} \left\{ \frac{\partial \tilde{\phi}}{\partial t} + \nabla (-U x + \phi) \cdot \nabla \tilde{\phi} \right\} 
\quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad (A.14)
\]

Now remember the following equations for the differential of a function \( f \):

\[
\frac{\partial}{\partial x} \left[ f(x, y, z) \right] \bigg|_{x=\xi} = \left( \frac{\partial f}{\partial x} + \frac{\partial f}{\partial z} \frac{\partial \zeta}{\partial x} \right) \bigg|_{x=\xi} 
\quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad (A.15)
\]
\[
\frac{\partial}{\partial y}[f(x, y, z)]_{z=\xi} = \left(\frac{\partial f}{\partial y} + \frac{\partial f}{\partial z} \frac{\partial \xi}{\partial y}\right)_{z=\xi}
\]

(A.16)

For the function \(\zeta_0\) this leads to:

\[
\frac{\partial \zeta_0}{\partial x} = -\frac{1}{2g} \left\{ \frac{\partial}{\partial x} (\nabla(-Ux + \phi) \cdot \nabla(-Ux + \phi)) + \frac{\partial}{\partial z} (\nabla(-Ux + \phi) \cdot \nabla(-Ux + \phi)) \frac{\partial \zeta_0}{\partial x} \right\}
\]

(A.17)

\[
\frac{\partial \zeta_0}{\partial y} = -\frac{1}{2g} \left\{ \frac{\partial}{\partial y} (\nabla(-Ux + \phi) \cdot \nabla(-Ux + \phi)) + \frac{\partial}{\partial z} (\nabla(-Ux + \phi) \cdot \nabla(-Ux + \phi)) \frac{\partial \zeta_0}{\partial y} \right\}
\]

(A.18)

and for the function \(\zeta_1\):

\[
\frac{\partial \zeta_1}{\partial t} = -\frac{1}{g} \left\{ \frac{\partial^2 \phi}{\partial t^2} + \nabla(-Ux + \phi) \cdot \nabla \frac{\partial \phi}{\partial t} \right\}
\]

(A.19)

\[
\frac{\partial \zeta_1}{\partial x} = -\frac{1}{g} \left\{ \frac{\partial^2 \phi}{\partial x \partial t} + \frac{\partial^2 \phi}{\partial z \partial t} \frac{\partial \zeta}{\partial x} + \frac{\partial}{\partial x} (\nabla(-Ux + \phi) \cdot \nabla \phi) + \frac{\partial}{\partial z} (\nabla(-Ux + \phi) \cdot \nabla \phi) \frac{\partial \zeta}{\partial x} \right\}
\]

(A.20)

\[
\frac{\partial \zeta_1}{\partial y} = -\frac{1}{g} \left\{ \frac{\partial^2 \phi}{\partial y \partial t} + \frac{\partial^2 \phi}{\partial z \partial t} \frac{\partial \zeta}{\partial y} + \frac{\partial}{\partial y} (\nabla(-Ux + \phi) \cdot \nabla \phi) + \frac{\partial}{\partial z} (\nabla(-Ux + \phi) \cdot \nabla \phi) \frac{\partial \zeta}{\partial y} \right\}
\]

(A.21)
Combining these two equations finally leads to the following expressions for the free surface condition:

Everything with order $\epsilon^0$

$$
\frac{\partial \tilde{\phi}}{\partial z} + \frac{1}{2g}(-U + \frac{\partial \tilde{\phi}}{\partial x}) \cdot \frac{\partial}{\partial x} (\nabla(-Ux + \tilde{\phi}) \cdot \nabla(-Ux + \tilde{\phi})) + \frac{1}{2g} \frac{\partial}{\partial z} (\nabla(-Ux + \tilde{\phi}) \cdot \nabla(-Ux + \tilde{\phi})) \cdot \frac{\partial \zeta_0}{\partial x} \cdot (-U + \frac{\partial \tilde{\phi}}{\partial x}) + \frac{1}{2g} \frac{\partial}{\partial y} (\nabla(-Ux + \tilde{\phi}) \cdot \nabla(-Ux + \tilde{\phi})) \cdot \frac{\partial \tilde{\phi}}{\partial y} + \frac{1}{2g} \frac{\partial}{\partial y} (\nabla(-Ux + \tilde{\phi}) \cdot \nabla(-Ux + \tilde{\phi})) \cdot \frac{\partial \zeta_0}{\partial y} \cdot \frac{\partial \tilde{\phi}}{\partial y} = 0 \tag{A.22}
$$

And everything with order $\epsilon$

$$
- \frac{1}{g} \left\{ \frac{\partial \tilde{\phi}}{\partial t} + \nabla(-Ux + \tilde{\phi}) \cdot \nabla \tilde{\phi} \right\} \frac{\partial^2 \tilde{\phi}}{\partial z^2} + \frac{1}{g} \left\{ \frac{\partial^2 \tilde{\phi}}{\partial t^2} + \nabla(-Ux + \tilde{\phi}) \cdot \frac{\partial \tilde{\phi}}{\partial t} \right\} \right.
+ \frac{1}{g} (-U + \frac{\partial \tilde{\phi}}{\partial x}) \left\{ \frac{\partial^2 \tilde{\phi}}{\partial x \partial t} + \frac{\partial^2 \tilde{\phi}}{\partial x \partial t} \frac{\partial \zeta_0}{\partial x} + \frac{\partial \zeta_0}{\partial x} \nabla(-Ux + \tilde{\phi}) \cdot \nabla \tilde{\phi} \right.
+ \frac{\partial \zeta_0}{\partial y} \nabla(-Ux + \tilde{\phi}) \cdot \nabla \tilde{\phi} + \frac{\partial}{\partial z} \nabla(-Ux + \tilde{\phi}) \cdot \nabla \tilde{\phi} \left\{ \frac{\partial^2 \tilde{\phi}}{\partial y \partial t} + \frac{\partial^2 \tilde{\phi}}{\partial z \partial t} \frac{\partial \zeta_0}{\partial y} \right\}
+ \frac{\partial \tilde{\phi}}{\partial z} - \frac{\partial \zeta_0}{\partial x} \frac{\partial \tilde{\phi}}{\partial x} - \frac{\partial \zeta_0}{\partial y} \frac{\partial \tilde{\phi}}{\partial y} = 0 \quad \text{at} \quad z = \zeta_0 \tag{A.23}
$$

Regrouping of the equations leads for the first to:

$$
\tilde{\phi}_z + \frac{1}{2g} \nabla(-Ux + \tilde{\phi}) \cdot \nabla[\nabla(-Ux + \tilde{\phi}) \cdot \nabla(-Ux + \tilde{\phi})] = 0 \quad \text{at} \quad z = \zeta_0 \tag{A.24}
$$

and for the second:

$$
g(\tilde{\phi}_z - \tilde{\phi}_x \zeta_0 - \tilde{\phi}_y \zeta_0) - \tilde{\phi}_{zz}(\tilde{\phi}_t + \nabla(-Ux + \tilde{\phi}) \cdot \nabla \tilde{\phi}) + 2 \nabla(-Ux + \tilde{\phi}) \cdot \nabla \tilde{\phi}_t + \nabla(-Ux + \tilde{\phi}) \cdot \nabla[\nabla(-Ux + \tilde{\phi}) \cdot \nabla \tilde{\phi}] + \tilde{\phi}_{tt} = 0 \quad \text{at} \quad z = \zeta_0 \tag{A.25}
$$
Appendix B

The Differential Operator

In chapter 2 a linear differential operator $\mathcal{L}$ was mentioned. This operator is given by:

$$
\begin{align*}
&g\ddot{\phi}_z + \ddot{\phi}_x \ddot{\phi}_x \ddot{\phi}_{xx} + \ddot{\phi}_x \ddot{\phi}_y \ddot{\phi}_{xy} + \ddot{\phi}_x \ddot{\phi}_z \ddot{\phi}_{xz} - \ddot{\phi}_x U \ddot{\phi}_{xx} + \ddot{\phi}_y \ddot{\phi}_x \ddot{\phi}_{xy} \\
&+\ddot{\phi}_y \ddot{\phi}_y \ddot{\phi}_{yy} + \ddot{\phi}_y \ddot{\phi}_z \ddot{\phi}_{yz} - \ddot{\phi}_y U \ddot{\phi}_{xy} - \ddot{\phi}_{zz}(\ddot{\phi}_t - U \ddot{\phi}_x + \ddot{\phi}_x \ddot{\phi}_x + \ddot{\phi}_y \ddot{\phi}_y \\
&+\ddot{\phi}_z \ddot{\phi}_z) + \dddot{\phi}_t - 2U \dddot{\phi}_{xt} + 2\ddot{\phi}_x \dddot{\phi}_{xt} + 2\ddot{\phi}_y \dddot{\phi}_{yt} + 2\ddot{\phi}_z \dddot{\phi}_{zt} + U^2 \dddot{\phi}_{xx} \\
&-U \dddot{\phi}_{xx} \ddot{\phi}_x - U \dddot{\phi}_x \ddot{\phi}_{xx} - U \dddot{\phi}_{xy} \ddot{\phi}_y - U \dddot{\phi}_y \ddot{\phi}_{xy} - U \dddot{\phi}_{xz} \ddot{\phi}_z - U \dddot{\phi}_z \ddot{\phi}_{xz} \\
&-U \dddot{\phi}_x \ddot{\phi}_{xx} + \dddot{\phi}_x \ddot{\phi}_{xx} + \dddot{\phi}_x \ddot{\phi}_{xx} + \dddot{\phi}_x \ddot{\phi}_{xy} \dddot{\phi}_{xy} + \dddot{\phi}_x \ddot{\phi}_y \dddot{\phi}_{xy} + \dddot{\phi}_x \ddot{\phi}_z \dddot{\phi}_{xz} \\
&+\dddot{\phi}_x \ddot{\phi}_z \dddot{\phi}_{xz} - U \dddot{\phi}_y \ddot{\phi}_{xy} + \dddot{\phi}_y \ddot{\phi}_y \dddot{\phi}_{xy} + \dddot{\phi}_x \ddot{\phi}_x \dddot{\phi}_{xy} + \dddot{\phi}_y \ddot{\phi}_y \dddot{\phi}_{xy} + \dddot{\phi}_y \ddot{\phi}_y \dddot{\phi}_{yy} \\
&+\dddot{\phi}_y \ddot{\phi}_y \dddot{\phi}_{yz} + \dddot{\phi}_y \ddot{\phi}_z \dddot{\phi}_{yz} - U \dddot{\phi}_z \ddot{\phi}_{xz} + \dddot{\phi}_z \ddot{\phi}_z \dddot{\phi}_{xz} + \dddot{\phi}_z \ddot{\phi}_x \dddot{\phi}_{xz} + \dddot{\phi}_z \ddot{\phi}_y \dddot{\phi}_{yz} + \dddot{\phi}_z \ddot{\phi}_y \dddot{\phi}_{yz} \\
&+\dddot{\phi}_z \ddot{\phi}_y \dddot{\phi}_{yy} + \dddot{\phi}_z \ddot{\phi}_z \dddot{\phi}_{zz} + \dddot{\phi}_z \ddot{\phi}_z \dddot{\phi}_{zz} \tag{B.1}
\end{align*}
$$

and with $\ddot{\phi}_x = 0$ and $\dddot{\phi}_{zz} \neq 0$ this could lead to:

$$
\begin{align*}
&\dddot{\phi}_{tt} + g\ddot{\phi}_z - 2U \dddot{\phi}_{xt} + 2\nabla \dddot{\phi} \cdot \nabla \dddot{\phi}_t + (U^2 - 2U \dddot{\phi}_x + \dddot{\phi}_{xx}) \dddot{\phi}_{xx} \\
&+ 2(-U + \dddot{\phi}_x) \dddot{\phi}_y \dddot{\phi}_{xy} + \dddot{\phi}_y \dddot{\phi}_{yy} + [2(-U + \dddot{\phi}_z) \dddot{\phi}_{xz} + 2\dddot{\phi}_y \dddot{\phi}_{xy} \\
&-(-U + \dddot{\phi}_x) \dddot{\phi}_{xx}] \dddot{\phi}_x + [2(-U + \dddot{\phi}_x) \dddot{\phi}_{xx} + 2\dddot{\phi}_y \dddot{\phi}_{yy} - \dddot{\phi}_y \dddot{\phi}_{zz}] \dddot{\phi}_y \\
&-\dddot{\phi}_{zz} \dddot{\phi}_t = 0 \tag{B.2}
\end{align*}
$$
Appendix C

Computer Program Outline

A computer program, called SLOSPO (SLowly OSCillating POTential), has been developed at the Delft University of Technology.

Given a forward speed $U$ and a ship hull, the program calculates the potential flow around the ship. It uses of a singularity distribution on the ship hull and the ship waterline (see chapter 2). The program calculates the waterheight along the ship hull, the wave resistance and is able to calculate the added resistance and swell-up as well.

The computer program outline is given in figure C.1. The calculation of the Green function is carried out in two specially developed subroutines called greendbl and greensgl. The subroutine greendbl calculates the double integral part in the Green function according to tables as has been done by Newman [26], (see the description in section 3.3.3). The subroutine greensgl calculates the single integral in the Green function, see also figure C.2 and section 3.3.4.
Figure C.1: Computer program outline.
Figure C.2: Outline of Green function calculation.
Appendix D
Measuresments: Wigley Hull

Every number in this tables denotes the pixel coordinate resulting from the video-tape measurements. At runnumber 116t; at station number 17; the pixel value of the waterline is 376. So the waterline is 376-366 (the value at run 000) is 10! Every pixel is 0.139 cm, so the waterline depth is -1.39 cm.

| runno. | model parameters
<table>
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Table D.1: The model parameters.
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<tr>
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</table>

Table D.2: The results from video tape. In this table the extensions t(top), b(bottom) or m(middle) denote the position of the vessel at this specific stage of one oscillation.
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</tr>
</tbody>
</table>

Table 4.3: More results from video tape. (values in cm.)
References


REFERENCES


REFERENCES


Acknowledgements

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Also I like to thank J.A. Pinkster and J.M.J. Journée of the Delft Hydromechanics Laboratory for being able to carry out model experiments. The actual experiments have been carried out by A.J. van Strien.

Finally I like to thank M.C.A. van Dijke of the University of Utrecht for his valuable comments on solving the matrix equation iteratively.
Summary

"A three dimensional method for the calculation of the unsteady ship wave pattern using a Neumann-Kelvin approach."

The main problem in this thesis is the calculation of the wave resistance of an oscillating ship sailing in otherwise calm water. As a result of the oscillating movement, the ship will encounter an added resistance. In order to calculate this added resistance it is necessary to calculate the resistance of the ship when it is sailing in calm water: the stationary wave resistance.

In order to solve this problem a few assumptions have been made: the fluid has negligible viscosity, the density of the fluid is constant and the fluid is irrotational. These assumptions lead to the class of Potential flows. Under the assumption of a potential flow the stationary problem of calculating the ship wave resistance is described by the Laplace equation and boundary conditions on the ship hull and the free surface (this is the boundary between the fluid and the air above). Once the potential flow has been determined, the pressure, resistance, wave height, trim and sinkage at the ship can be calculated. Some methods to solve these problems have been discussed in this thesis.
The method which has been used in this thesis is originally due to Brard (1972). This method uses Kelvin sources, which are solutions of the Laplace equation and subject to the linearized free surface condition. After linearization of the free surface condition, the superposition principle can be applied. This means that every linear combination of Kelvin sources also satisfies this Laplace equation and the free surface condition. The Kelvin sources are distributed continuously over the ship hull. Using the individual strength (the linear combination) of each Kelvin source, the second boundary condition, on the ship hull, can be met.

We assume that the total velocity potential can be written as the sum of a stationary (time independent) and an instationary (time dependent) part. Especially the influence of the stationary on the instationary part will be of importance. The instationary problem is the description of the behaviour of a ship oscillating round a stationary position. As an alternative for the strip-theory (Ogilvie and Tuck (1969)) an expansion of the total potential in terms of a small parameter \( \omega \) (the frequency of oscillation) has been used. This approach is justified by measurements (Blok (1983)). These measurements show that for instance the dynamic swell-up has a very weak frequency dependency. Dynamic swell-up is the effect of water being pushed up around the bow higher than can be accounted for by considering heaving, pitching and incident wave alone.

This approach, a linearization round a known stationary solution, leads to a modified Neumann-Kelvin formulation. It has been shown that the velocity potential can be represented as an area source singularity distribution over the ship hull and a source line distribution on the ship waterline. Using the body boundary condition we obtain an integral description of the potential function by means of a source distribution. Expansion of the source strength and the potential function into the small parameter \( \omega \) leads to a formulation where for the calculation of the steady wave potential and the unsteady wave potential the same Neumann-Kelvin Green function can be used.
The original Neumann-Kelvin problem is to find a continuously distributed source distribution which satisfies the body boundary condition. Once this source distribution is known, the potential function can be calculated. From this potential function the pressure, resistance and the water heights can be found. The source strength is dependent on the ship geometry and ship speed. The determination of an analytical solution for even the simplest geometrical forms is almost impossible. Therefore a numerical approximation for the solution of the problem has been found. The ship hull is divided into a number of panels (discretized). On each panel a specific source density distribution is assumed. Usually the source density distribution on an individual panel will be a constant. The influence of this panel on every other panel on the ship hull is calculated (the Green function). In order to obtain a good numerical approximation of the analytical solution the number of panels has to be sufficiently large. However the more panels are used to discretize the ship hull, the more (computer) time it will take to solve the problem.

Because, after discretization of the problem, the Green function has to be evaluated many times, we need a fast algorithm for the calculation of this Green function. The algorithms for the calculation of the Green function, used in this thesis, are based on algorithms developed by Newman (1987). Parts of these algorithms are only suitable for the calculation of the Green function when applied to thin ships. In this thesis these algorithms have been extended with the use of Padé approximations. Results of this can be found in Chapter 3.

In chapter 4 the stability and convergence of the solution of the matrix equation is described. If we increase the number of panels to discretize the ship hull, a direct solver (like LU-decomposition) cannot be used any longer. So some sort of iterative method like ART, SIRT or CG has to be used. It is also explained in this chapter that because of the specific block structure of the panel influence matrix, the matrix equation can be preconditioned. The preconditioning should lead to an even more successful use of the CG algorithm.
The method which has been described in this thesis has been implemented in a computer program. Results of this computer program are compared with measurements performed at the Delft Hydromechanics Laboratory. At the stationary case the measured values are slightly underestimated. The wave resistance and phase however are predicted pretty well. Because measurements at low oscillation frequencies are very difficult to perform, the measurements have been conducted at somewhat higher frequencies (at $\omega = 2.0$ rad/s). The qualitative results agree very well. Although an underestimation occurs with respect to the measured values.
Samenvatting

"Een drie dimensionale methode voor de berekening van de instationaire scheepsgolven met behulp van een Neumann-Kelvin aanpak."

Het probleem dat centraal staat in dit proefschrift is de berekening van de golfweerstand van een oscillerend schip dat vaart in verder kalm water. Het schip ondervindt ten gevolge van de oscillerende beweging een zogenaamde extra weerstand. Teneinde deze extra weerstand te kunnen bepalen is het ook nodig de weerstand die het schip heeft ten gevolge van de stationaire beweging, het varen alleen, te berekenen.

We doen nu een aantal aannames: de viscositeit van de vloeistof is te verwaarlozen, de dichtheid is constant en de vloeistof is rotatievrij. Er kan nu een zogenaamde potentiaal formulering gevormd worden. De potentiaal stroming kan beschreven worden door de Laplace vergelijking. Bij deze Laplace vergelijking worden randvoorwaarden gedefinieerd: randvoorwaarden op de scheepshuid en randvoorwaarden op het vrije vloeistof oppervlak (de grens tussen het water en de lucht erboven). Als de potentiaal stroming eenmaal bekend (berekend) is kunnen andere grootheden zoals onder meer druk, weerstand, golfhoogte, trim en inzinking van het schip kunnen berekend worden. Verschillende methoden om dit probleem op te lossen zijn in dit proefschrift besproken.
SAMENVATTING

De methode waar in dit proefschrift voor gekozen is, is oorspronkelijk afkomstig van Brad (1972). Deze methode maakt gebruik van zogenaamde Kelvin bronnen. Kelvin bronnen zijn oplossing van de Laplace vergelijking. Ook voldoen ze aan de gelineariseerde vrije vloeistof oppervlakte conditie. Aangezien nu ook (na linearisatie van de vrije vloeistof oppervlakte conditie) het superpositie beginsel geldig is, is het zo dat elke lineaire combinatie van Kelvin bronnen voldoet ook aan de Laplace vergelijking en de vrije vloeistof oppervlakte conditie. De Kelvin bronnen vormen een continue verdeling over de scheepshuid. Door de sterkte (de lineaire combinatie) van de Kelvin bronnen te variëren kan ook aan de tweede randvoorwaarde, namelijk die op de scheepshuid, worden voldaan.

We nemen aan dat de totale potentiaal geschreven kan worden als de som van een tijdsonafhankelijk (stationair) en een tijdsafhankelijk (instationair) deel. Vooral de invloed van het tijdsonafhankelijke deel op het tijdsafhankelijke deel zal hier van belang zijn. Het instationaire probleem bestaat uit het beschrijven van het gedrag van een schip dat oscilleert rond een stationaire positie. Als alternatief van de zogeheten strip-theorie (Ogilvie en Tuck (1969)), wordt hier gekozen voor een expansie van de totale potentiaal naar een kleine parameter $\omega$: de oscillatie frequentie. Metingen (Blok (1983)) rechtvaardigen een dergelijke aanpak omdat bijvoorbeeld een grootte als dynamic swell-up redelijk frequentie ongevoelig is. Dynamic swell-up is het effect dat water hoger langs de boeg wordt opgestuwd dan verklaard kan worden door alleen het stampen van het schip en de inkomende golven te beschouwen. Ook bij lage oscillatie frequenties vindt dit verschijnsel plaats.

Uit deze gekozen aanpak, een linearisatie rond een bekende stationaire oplossing, is een gemodificeerde Neumann-Kelvin formulering voortgekomen. De Neumann-Kelvin formulering beschrijft de potentiaal stroming met behulp van een integraal voorstelling. De potentiaal wordt geschreven als de som (integraal) van Kelvin bronnen. Invullen van de randvoorwaarde op de scheepshuid lever een integraal voorstelling op voor de bronsterkte. Voor zowel de stationaire als de instationaire berekening kan nu de stationaire Neumann-Kelvin Green functie gebruikt worden. De instationaire effecten, het oscilleren, zijn nu verwerkt in het rechterlid van de verkregen integraalvergelijking.
Het oorspronkelijk Neumann-Kelvin probleem is het vinden van een continue bronverdeling over de scheepssromp. Indien de bronsterkte bekend is, kan ook de potentiaalfunctie berekend worden. Uit deze potentiaalfunctie kunnen nu andere grootheden zoals bijvoorbeeld druk, weerstand en waterhoogten bepaald worden. De sterkte van de bronverdeling is afhankelijk van de vorm van het schip en van de snelheid waarmee het schip vaart. Omdat zelfs voor de meest simpele geometrische vormen het vinden van een analytische oplossing al een onmogelijke taak is, is een numerieke benadering voor de oplossing van het probleem gevonden. Hiertoe wordt de scheepshuid onderverdeeld in een aantal panelen (gediscretiseerd). Over deze panelen wordt een bepaalde bronverdeling verondersteld. Deze bronverdeling per paneel kan 'iedere' verdeling aannemen, maar meestal wordt gekozen voor een constante verdeling per paneel of een stuksgewijs lineaire verdeling per paneel. Van ieder paneel wordt nu de invloed bepaald op alle andere panelen van het schip (de zogenaamde Green functie). Om een goede benadering te verkrijgen van de analytische oplossing zal het aantal panelen waaruit het schip is opgebouwd voldoende groot gekozen moeten worden. Naarmate het aantal panelen echter toeneemt zal de hoeveelheid rekentijd, de tijd die de computer nodig heeft om tot een oplossing te komen, ook sterk toenemen.

Aangezien, na discretisatie van het probleem, de Green functie veelvuldig berekend zal moeten worden, is een effectief (snel) algoritme nodig. De algoritmen die in dit proefschrift zijn ontwikkeld zijn gebaseerd op routines ontwikkeld door Newman (1987). Aangezien delen van dit algoritme alleen geschikt zijn voor berekening van de Green functie bij slanke schepen, is ook gekeken naar een uitbreiding van het algoritme met behulp van zogenaamde Padé approximatie. Resultaten van deze aanpak zijn terug te vinden in hoofdstuk 3.

In hoofdstuk 4 wordt de stabiliteit en convergentie van de oplossing van de matrix vergelijking beschreven. Indien het aantal panelen dat gebruikt wordt voor de onderverdeling van het schip zeer groot wordt, is een directe oplosmethode voor het stelsel, zoals LU-decompositie, niet meer betrouwbaar. Er is aangegeven wat voor soort iteratieve methoden (ART, SIRT, CG) voor het oplossen van de matrix vergelijking gebruikt kunnen worden. Ook is aangegeven hoe het gegeven stelsel 'gepreconditioneerd' kan worden om succesvol bepaalde algoritmen, zoals CG, te kunnen gebruiken.
De methode zoals beschreven in dit proefschrift is geïmplementeerd in een computer programma. Resultaten hiervan worden vergeleken met metingen die verricht zijn bij onder andere de afdeling Maritieme Techniek Delft. De stationaire resultaten geven voor de berekende golfhoogten een lichte onderschatting ten opzichte van de gemeten waarden. De golfweerstand en de fase zijn echter wel correct. Aangezien metingen bij lage oscillatie frequenties praktisch moeilijk realiseerbaar zijn, zijn de metingen uitgevoerd bij iets hogere frequenties (vanaf $\omega = 2.0$ rad/s). Kwalitatief zijn goede resultaten verkregen, er is hier echter ook weer sprake van een onderschatting ten opzichte van de gemeten waarden.
Curriculum Vitae

