Numerical Analysis for Fatigue Life Prediction on Railroad RCF Crack Initiation

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ABSTRACT

In the present paper, a numerical procedure for surface crack initiation analysis based on the critical plane approach is proposed. The complex stress/strain state of wheel and rail (W/R) contact is analysed by means of sub-modelling approach together with the transient contact nodal loads obtained from dynamic W/R rolling contact analysis. Meanwhile, a novel refining method is developed and utilized to discretise the full- and sub- solid models. In these models, realistic wheel and rail contact geometries and bilinear kinematic hardening material are both taken into account. The validity of the sub-modelling procedure has been successfully confirmed by simulation results comparison and cutting boundary stress verification. Moreover, the crack initiation analysis are implemented with integration of three advanced fatigue life models. Based on the results, it can be noticed that the surface initial crack orientation may differ from each other under different critical plane definitions. Also, it is worth mentioning that the numerical procedure is effective and efficient enough for doing crack initiation analysis.

1. INTRODUCTION

Wheel and rail interface is one of the most delicate element of the vehicle-track system, which requires to sustain heavy axle loads more than 10 tones over an contact area in the order of 1 cm² [1]. As a consequence, it is unavoidable for the damage to occur under repeated rolling contact loads. It is reported that wheel/rail service life is primarily determined by wear, plastic flow and rolling contact fatigue[2]. Initially, wear and plastic flow was considered as the major problem. However, with the improved material quality and better profile design, the initial problem tends to change from wear to rolling contact fatigue. Generally, the material deterioration process of rolling contact fatigue (RCF) related defects can be categorized into three phases[3, 4]: (I) micro-crack initiation; (II) macro-crack propagation; (III) complete fracture. Till now, although significant progress has been made for dealing with RCF damage problems, fatigue life prediction on railway rolling contact bodies is not completely understood and has been attracting more and more research resources from researchers and engineers[5-11].

Traditionally, simple engineering formulas[5, 6] incorporated with some Multi-body system (MBS) software were used to predict the fatigue life of the rails. Those engineering formulas are based on the shakedown theory which cannot be utilised to explore the damage mechanism inside both rails and wheels. Recently, with the development of ever increasing computer capacity, the finite element (FE) methods integrated with the complex multi-axial fatigue life models have already become popular in fatigue life assessment [7-11] due to its striking versatility. However, the mesh size of such FE models is not allowed to be refined to a desired level for crack initiation analysis due to the fact that the dimensions of wheels and rails are considerably bigger than the ones of the crack. A study in [7] introduces a sub-modelling technique into fatigue life prediction on wheels to overcome this problem. But the feasibility of sub-modelling technique on W/R rolling contact analysis is not verified. Moreover, quasi static loads instead of dynamic loads are applied on the sub-model, which is questionable for the accuracy and effectiveness of sub-modelling approach.
In current paper, a numerical procedure for surface micro-crack initiation analysis is proposed by means of sub-modelling technique together with dynamic loads. To start with, the main procedure of sub-modelling simulation is outlined in section 2. Also, the validity of the sub-modelling procedure is confirmed through comparison with the dynamic simulation results and consistency inspection on the cutting boundaries. Following that, the procedure and results for crack initiation analysis will be demonstrated in section 3. In the end, concluding remarks and outlooks will be drawn.

2. FINITE ELEMENT MODEL AND RESULTS VERIFICATION

An accurate determination of stress and strain fields in W/R contact should be the first step to the damage mechanism understanding and failure prediction. Thus, the sub-modelling analysis procedure, full- and sub-model as well as the verification of sub-modelling technique will be elaborated in detail in this section.

2.1 SUB-MODELLING ANALYSIS PROCEDURE

Sub-modelling technique known as the cut-boundary displacement method (also known as the specified boundary displacement method) is based on St. Venant’s principle, which states: the difference in effects due to two statically equivalent loadings becomes insignificant as the distance from the load application increases. It is commonly used to produce satisfactory results in a specific region of interest.

The main procedure for W/R contact sub-modelling analysis is shown in figure 2. To start with, wheel-rail transient frictional rolling contact analysis is required to do initially according to the method developed by the authors in [14]. The nodal interface loads (restored in NCFORC result file) obtained from dynamic analysis will be used as an input for the settlement of coarse full-model boundary conditions. Here, only the rail is considered in the coarse full-model analysis and the nodal interface loads in substitution for the functionality of the wheel are applied on the rail contact patches. After finishing the coarse full-model analysis, refined sub-model will be created and its boundary nodes will be stored into a file for displacement interpolation with the full-model result file. The calculated displacement on the cut boundary of the full-model will be used as boundary conditions for sub-modelling analysis. By doing the cut boundary verification and stress-strain post-processing analysis, the sub-modelling simulation would be finalized.

Figure 1. schematic graph of sub-modelling problem. a) coarse meshed full-model; b) refined sub-model in stress concentration region (shown superposed over full-model), red lines denote the sub-model cutting boundaries over full-model.

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![Flow diagram of sub-modelling analysis procedure.](image)


2.2 W/R DYNAMIC-, FULL- AND SUB-MODELS

According to the sub-modelling analysis procedure discussed in the previous section, three models, namely dynamic W/R contact model, rail full-model and rail sub-model, are required to achieve our research goal.

From figure 3 (a) and (b), it can be seen that both realistic wheel and rail geometry are considered in dynamic model. Two considered counter parts are a S1002 wheel profile and a 54E1 rail. The sprung mass is lumped and supported by a group of
springs and dampers of the first primary suspension. The axle load is chosen as 20t. The length of the track is limited to 1640mm which can cover the distance between two sleepers. The details of the dynamic model specifications can refer to [14].

While for the full model, it can be observed from figure 3 (c) and (d) that only rail model is considered. The rail dimensions and mesh size remain the same as the one is dynamic model for applying the nodal interface loads obtained from dynamic simulation results. Regarding sub-model, its dimension is approximately restrained to be 60×60×15 mm block, which is considerably smaller the full-model. In order to satisfy the requirement of crack initiation analysis, the mesh size in the sub-model can be refined to be 0.2mm, which is five times smaller than the mesh size in full model. Herein, all these three models are discretized by using the nested transition mapped hexahedral refining method, which is developed by the presented authors. The details of the refining method can be found in [14]. For the model the coordinate directions are: the Z-axis (longitudinal direction) is parallel to the direction in which wheel travels, the Y-axis is the vertical direction and the X-axis is the lateral direction.

Figure 3. Rail FE model. a) Dynamic model – side view; b) Dynamic model – cross sectional view; c) Full-model – side view; d) Full refined region – top view; e) Sub-model – side view; f) Sub-model - isometric view. (Notation: represents the location of sub-model with respect to full model.)

The completed sub-model contains 282,000 three-dimensional hexahedral brick-type elements as shown in figure 3(f). It takes about 4 hours for a single 3.10 GHz Pentium 16 processor computer to finish the sub-model analysis. The results of dynamic simulation will be skipped in this paper, since it has been discussed in detail in [14]. While for the results of full- and sub-model analysis, they will be presented and discussed in the following sections.

2.3 SUB-MODEL VERIFICATION

Following the solving procedure described in figure 2, sub-modelling analysis is conducted to investigate the contact behavior under operational loading.

2.3.1 STRESS RESULTS COMPARISON

From figure 4, the comparison of Von Mises stress and shear stress distribution obtained from dynamic model, full-model and sub-model analysis is observed. It can be noticed that the distributions of Von Mises and shear stress from full model and sub-model analysis is almost the same, and only a negligible deviation of the stress amplitude occurs. While for the results from dynamic analysis, there is a slight difference on both the stress distribution and its magnitude. The main reason caused the deviation from the dynamic simulation can be attributed to the fact that only the nodal interface applied on the rail full model and no nodal displacement are involved. However, it can be concluded that both the results from the three simulations are comparable, which means the sub-modelling technique is able to be used in wheel-rail contact analysis.

Figure 4. Stress distribution (Unit: MPa) from full-, sub- and dynamic-model. (Notation: “a” represents the Von Mises stress on rail surface from top view; “b” refers to Von Mises stress mapped on the longitudinal-vertical plane; “c” is the shear stress mapped on the longitudinal-vertical plane; “1”, “2” and “3” refer to results from full model, sub model and dynamic model respectively.)
2.3.2 CUTTING BOUNDARY VERIFICATION

The final step is to verify that the cut boundaries of the sub-model are far enough away from the concentration. It can be done by comparing results (stresses, strains etc.) along the cut boundaries with those along the corresponding locations of the coarse model. The cross sectional boundary of the sub-model and the corresponding cutting boundary is illustrated by red curves in figure 6 (a) and (b).

Fig. 5. a) Path on sub-model; b) path on full model.

From figure 6, it can be observed that the Von Mises stress distribution on the cross sectional boundaries are in good agreement for the full- & sub-model, which indicates that proper cross sectional boundaries have been chosen.

3 CRACK INITIATION ANALYSIS

It is reported that cracks initiate and grow on certain planes and the normal strains to those planes, assist in the fatigue crack growth process [12]. Base on this phenomenon, many different critical plane approaches are proposed to predict the most dangerous plane for the initial crack to grow. In this section, the main procedure and results for crack initiation analysis will be highlighted and discussed.

3.1 CRACK ANALYSIS METHOD

For an arbitrary stress/strain history of the point on the contacting surface(as shown in figure 8(a)), the critical plane of the fatigue crack initiation are not known in advance. It is noticed that the general 3D state of stress can be decomposed into one normal stress and one shear stress component respectively on an arbitrary material plane.

![Figure 8.](image)

Figure 8. a) an arbitrary 3D stress/strain state of a point on the nail surface; b) an arbitrary material plane of the stress/strain space;

Assuming the stress and strain history is given, the steps of crack initiation analysis may be outlined as follow:

1). The material planes that are candidate for the maximum damage critical plane is determined by computing the transformation matrices for a given set of planes and angles.

2). The stress tensor $\sigma(t)$ and strain tensor $\varepsilon(t)$ at each history point is described as follows:

$$\sigma(t) = \begin{bmatrix} \sigma_{xx}(t) & \tau_{xy}(t) & \tau_{xz}(t) \\ \tau_{yx}(t) & \sigma_{yy}(t) & \tau_{yz}(t) \\ \tau_{zx}(t) & \tau_{zy}(t) & \sigma_{zz}(t) \end{bmatrix},$$

$$\varepsilon(t) = \begin{bmatrix} \varepsilon_{xx}(t) & \gamma_{xy}(t) & \gamma_{xz}(t) \\ \gamma_{yx}(t) & \varepsilon_{yy}(t) & \gamma_{yz}(t) \\ \gamma_{zx}(t) & \gamma_{zy}(t) & \varepsilon_{zz}(t) \end{bmatrix}$$

In the above tensor, $\sigma_{xx}(t)$, $\sigma_{yy}(t)$, $\sigma_{zz}(t)$ are the normal stress components and $\varepsilon_{xx}(t)$, $\varepsilon_{yy}(t)$, $\varepsilon_{zz}(t)$ are the normal strain components. Whereas $\tau_{xy}(t)$, $\tau_{yz}(t)$ and $\tau_{xz}(t)$ are the shear stress components and $\gamma_{xy}(t)$, $\gamma_{yz}(t)$, $\gamma_{xz}(t)$ are the shear strain components. According to the schematic graph adopted in figure 8 (b), the normal unit vector $\mathbf{n}$ of on the material plane $\Delta$ can be defined through angles $\theta$ and $\phi$, where the latter is the angle between $\mathbf{n}$ and $Z$ axis, while $\phi$ is the angle between $X$ axis and the projection of unit vector $\mathbf{n}$ on plane $OXY$. The unit vector defining
the orientation of the material plane can be expressed as follows:

\[
\mathbf{n} = \begin{bmatrix}
    n_x \\
    n_y \\
    n_z
\end{bmatrix} = \begin{bmatrix}
    \sin(\theta) \cos(\phi) \\
    \cos(\theta) \\
    \sin(\theta) \sin(\phi)
\end{bmatrix}
\]  (2)

According to the definition reported above, the normal and shear stress/strain amplitude on the material plane can then be calculated respectively as:

\[
\sigma_n = \mathbf{n} \cdot \sigma \cdot \mathbf{n}, \quad \tau_n = \left\| \mathbf{n} \cdot \mathbf{\varepsilon} - \sigma_n \mathbf{n} \right\|
\]
\[
\varepsilon_n = \mathbf{n} \cdot \mathbf{\varepsilon} \cdot \mathbf{n}, \quad \gamma_n = \left\| \mathbf{n} \cdot \mathbf{\gamma} - \varepsilon_n \mathbf{n} \right\|
\]  (3)

3). The fatigue parameter are computed according to critical plane approaches, which will be discussed in section 3.2 by rotating the material planes. Considering symmetry characteristic of material planes, \( \theta \) is limited to \([0, \pi / 2]\), \( \phi \) is restricted to \([0, \pi]\).

4). Identify the material planes with the maximum fatigue parameter, which will be the potential crack initiation plane.

3.2 RESULTS & DISCUSSIONS

After performing the sub-model analysis, crack initiation analysis will be implemented using the method described in section 3.1. The obtained stress results and predicted potential crack initiation planes will be presented and discussed in this section.

3.3.1 STRESS RESULTS

According to the schematic of wheel-rail rolling contact shown in figure 7(a), this numerical study allows the determination of the stress distribution under moving contact loads, which can be clearly observed from figure 7(b).

Meanwhile, it is noticed that those stress/strain shifts can easily be transformed into stress-time and strain-time histories for the nodes lying into the contact patch.

From figure 8 (a) and (b), one material point (as denoted by a red solid circle) suffering from the maximum Von Mises stress response, will be chosen as the most potential crack initiation site according to the Von Mises yield criterion for ductile material.

In figure 9, the stress-time history can be observed. The coordinate does not indicate the real time and is the time step in sub-modelling analysis. It can be seen that all the normal stress components \( \sigma_{xx} \), \( \sigma_{yy} \), \( \sigma_{zz} \) are almost completely in compression over the whole time history. The magnitude of vertical normal stress \( \sigma_{yy} \) can reach to as large as 1200MPa, which has already significantly exceeded the yield stress limit (480MPa) of the rail material. For the shear stress components, the stress amplitude of \( \tau_{xy} \) and \( \tau_{xz} \) is slightly varying around zero, which is much smaller in comparison with \( \tau_{yz} \). This phenomenon can be explained by the applied traction force on the wheel.
Figure 10 shows the strain components variation with respect to time step. It can be noticed that the strain-time history is consistent with stress-time history. Moreover, it clearly shows that both the stress and strain history are not proportional, which indicates that the maximum normal stress/strain and maximum shear stress/strain do not occur simultaneously.

3.3.2 INITIAL CRACK ORIENTATION

It is assumed by Smith, Watson and Topper (SWT model) that the crack tends to initiate on the plane with maximum normal strain energy, which is shown as formula (4):

$$FP_{\text{max}} = \max_{\Delta} \left( \sigma_n \frac{\Delta \varepsilon}{2} \right)$$  (4)

Where $\sigma_n$ is the normal stress on a plane $\Delta$, $\Delta \varepsilon$ is the strain range on that plane.

Besides, it is postulated that the crack will propagate along the plane of maximum shear strain by Fatemi–Socie (FS model)[12]. The fatigue damage parameter is described as formula (5):

$$FP_{\text{max}} = \max_{\Delta} \left[ \frac{\Delta \gamma_{\text{max}} + J \Delta \tau}{\Delta} \right]$$  (5)

Where $\Delta \gamma_{\text{max}}$ is the shear strain amplitude on a plane $\Delta$, $\sigma_{n,\text{max}}$ is the maximum normal stress on that plane $\Delta$ and $k$ is the material parameter, $k=1.0$ in this case.

Recently, Jiang & Sehitoglu (JS model) [9] developed an model based on SWT relation. In the model, both the shear stress/strain and normal stress/strain components are used to describe the reason for crack initiation. Its expression is shown as:

$$FP_{\text{max}} = \max_{\Delta} \left( <\sigma_{\text{max}} > + J \Delta \gamma \Delta \tau \right)$$  (6)

in which $\Delta \tau$ is the shear stress range, $\Delta \gamma$ is the shear strain range. $< >$ denotes the MacCauley bracket (i.e. $< x > = 0.5(|x| + x)$). $J$ is a load and material-dependent parameter, equal to 0.25 for bainitic alloy or 0.3 manganese steel[9]. Here, to illustrate that is a material constant, we use 0.2 to take the place of $J$ [13].

Fig. 11. Fatigue parameter variation with plane orientation during one load-cycle. a) SWT model; b) FS model; c) JS model. (Color notation: from dark red to dark blue, the possibility of crack initiation is gradually decreasing).
Using the stress/strain time history obtained from sub-modelling analysis, the initial crack orientation is investigated by using three of the most popular crack initiation criteria according to the crack analysis method discussed in section 3.1.

Figure 11 shows the contour plot of maximum fatigue parameter under different material plane. Red color means the dangerous material planes for crack initiation, while blue area indicates the safe ones. From red to blue the possibility of crack initiation is gradually decreasing. It can be noticed that the results of the SWT model are completely different from the other two models. It can be interpreted by the assumptions of SWT, which assumes the crack will propagate along the plane with maximum normal strain energy. Considering the fact that the compressive normal stress/strain state of W/R interaction will obstruct the crack to grow, the feasibility of SWT criteria on crack initiation analysis is doubted. While for the other two models, the fatigue parameter distribution is similar but still slight difference exists. That is because that FS model is based on maximum shear strain theory, while JS model is depending on superposition of positive normal strain energy and shear strain energy.

Moreover, it can be found from figure 11 (b) that two potential region for crack initiation exists for FS model, when $\phi$ lies in $[0, 20], [160, 180]$ and $\theta$ falls in $[60, 90]$. Regarding JS model, one most dangerous region $\theta \in [0, 20], \phi \in [0, 100]$ occurs. If the contour plots were mapped onto two dimensional rail longitudinal cross section and rail top surface, the potential initial crack orientation can be clearly seen from figure 12.

From figure 12, it clearly shows that crack will initiate with a deep angle from the rail surface predicted by FS model. While for JS model, crack may initiate along the path, which will formulate a shallow angle from 0° to 20° with the rail top surface. While for the top view, the two predictions differ from each other even larger.

It is not easy to conclude that whether FS model or JS model fits the real W/R contact case, since it is depending more parameters such as material properties, loads etc. Thus, more field tests and experimental research are required to verify the predicting results.

4 CONCLUSION

This paper presents a numerical procedure of sub-modelling application on wheel/rail crack initiation analysis. It can be concluded that:

1) With the application of sub-modelling technique, a considerable efficiency and accuracy of the simulation results could be obtained.

2) Through the comparison with dynamic- and full-model results, it is confirmed that the sub-modelling procedure is trustworthy and can be used for wheel/rail contact analysis.

3) The crack initiation plane can be varied under different fatigue life models. Only FS model and JS model are recommended to do the crack initiation analysis on wheel/rail interaction.

In the future work, three-dimensional surface- and subsurface- crack might be developed on the basis of present work to study the wheel/rail rolling contact damage mechanism.

5. REFERENCES

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