Numerical Investigations of Motions and Drift Forces on Different Bodies Using the DELFRAC Program

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CONTENTS

Introduction.

Description of bodies.

Calculation conditions.

Description of dimensions of output data.

Calculations of the hemisphere and conclusions.

Calculations of the box and conclusions.

Calculations of the tanker and conclusions.

Conclusions.

Acknowledgements.

References.
INTRODUCTION.

The program DELFRAC that is based on the three-dimensional potential theory is investigated in order to analyse results of computations. Results from a comparative study of numerical predictions of motions due to waves and of the mean drift forces are presented and discussed. Comparisons with experimental and theoretical results of other authors are discussed. Some conclusions about the use and development of the program DELFRAC are made.

DESCRIPTIONS OF BODIES.

In order to analyse the results obtained using the program DELFRAC three bodies have been selected:

- hemisphere;
- box;
- tanker.

Two of these have rather classical shape, the last one being a real ship shape, which is a practical case.

All these bodies are shown on the graphs (fig. 1 - 3), including the panels.

The first body is a floating or fixed hemisphere of radius \( R = 10 \text{ m} \). The mass coefficients of body in air in surge, sway and heave are 1.0 m for every direction and the radius of gyration for roll, pitch and yaw are 5.0 m. The centre of gravity is located at the waterline of the hemisphere. The wetted surface of the body is discretized by 55 panels per quadrant or 220 for the whole body. For calculation of drift forces 6 or 12 waterline elements were used.

The next body is a parallelepipedic barge of 90 m length, 90 m breadth with a draft of 40 m. The mass coefficients of body in air in surge, sway and heave are 1.0 for every direction and the radius of gyration for roll is 33.04 m; the radius of gyration for pitch is 32.09 m; the radius of gyration for yaw is 32.92 m. The calculation point is at gravity centre located at \( z = -10.62 \text{ m} \).

The wetted surface of box is discretized by two numbers of panels:

- 27 panels for quadrant;
- 225 panels for quadrant.
For calculation of the drift forces using the near-field method 6 or 18 waterline elements respectively were applied. The table below shows the number of panels per wave length at different periods.

<table>
<thead>
<tr>
<th>Frequency, rad/sec</th>
<th>0.1</th>
<th>0.4</th>
<th>0.8</th>
<th>1.0</th>
<th>1.2</th>
<th>1.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period, sec</td>
<td>62.8</td>
<td>15.7</td>
<td>7.85</td>
<td>6.28</td>
<td>5.23</td>
<td>3.93</td>
</tr>
<tr>
<td>Wave length, m</td>
<td>6160.7</td>
<td>385.04</td>
<td>96.26</td>
<td>61.61</td>
<td>42.78</td>
<td>24.07</td>
</tr>
</tbody>
</table>

According to the conclusions of G. Delhommeau (see ref. [11]) the mesh refinement must be greater than 6 panels per wave length in all directions. It can be seen that the second mesh is more suitable to obtain correct calculation results at almost every period. The first mesh is not satisfying the previous condition at every wave period. And we could see that the drift forces in the first case are not in good agreement even at rather low wave frequencies.

For the practical case a tanker was investigated. The main dimensions of the tanker are shown in tab. 2.

### MAIN PARTICULARS OF THE 200,000 DWT TANKER

<table>
<thead>
<tr>
<th>Designation</th>
<th>Unit</th>
<th>Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length between perpendiculars</td>
<td>m</td>
<td>309.98</td>
</tr>
<tr>
<td>Breadth</td>
<td>m</td>
<td>47.17</td>
</tr>
<tr>
<td>Depth</td>
<td>m</td>
<td>29.60</td>
</tr>
<tr>
<td>Draft fore</td>
<td>m</td>
<td>18.90</td>
</tr>
<tr>
<td>Draft aft</td>
<td>m</td>
<td>18.90</td>
</tr>
<tr>
<td>Displacement</td>
<td>tons</td>
<td>240.697</td>
</tr>
<tr>
<td>Centre of buoyancy forward of Section 10</td>
<td>m</td>
<td>6.61</td>
</tr>
<tr>
<td>Centre of gravity above keel</td>
<td>m</td>
<td>13.32</td>
</tr>
<tr>
<td>Metacentric height</td>
<td>m</td>
<td>5.78</td>
</tr>
<tr>
<td>Longitudinal radius of gyration</td>
<td>m</td>
<td>77.47</td>
</tr>
<tr>
<td>Transverse radius of gyration</td>
<td>m</td>
<td>17.00</td>
</tr>
<tr>
<td>Natural heave period</td>
<td>sec.</td>
<td>11.80</td>
</tr>
<tr>
<td>Natural pitch period</td>
<td>sec.</td>
<td>10.80</td>
</tr>
</tbody>
</table>
The calculations were made for the following cases given in tab. 3

Tab. 2 (continued)

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Natural roll period</td>
<td>sec</td>
</tr>
<tr>
<td>Midship-section coefficient</td>
<td></td>
</tr>
<tr>
<td>Waterplane coefficient</td>
<td></td>
</tr>
<tr>
<td>Block coefficient</td>
<td></td>
</tr>
</tbody>
</table>

The centre of gravity is located 5.58 m below the mean waterline.

The wetted surface of the tanker is meshed by 194 facet elements per half body, meaning that the total number of panels is 388. For this case a total of 92 waterline elements was applied.

**CALCULATION CONDITIONS.**

In our numerical investigation we obtain the results on the regular waves. In this case, the amplitude of wave is 1 m. The circular frequencies of oscillation of the incident waves $\omega$ varied from 0.05 and 2.5 rad/sec corresponding to wave periods of 125.6 to 2.51 seconds. This range is larger that is used in practice because we tried to obtain also the asymptotic results.

The calculations are made for several wave directions which depended on the planes of symmetry: for the cases of a hemisphere and a box, the wave directions are 90 and 135 degrees; for the case of a tanker 90, 135, 160 and 180 degrees of wave propagation were applied.

In every computational case we have had some specific conditions:

- **hemisphere**

  All of the results are obtained for infinite water depth and for one draft. In order to compare the drift forces calculations were carried out for the floating and fixed conditions.

- **box**

  In the case of a floating box, the influence of number of panels on the convergence of the results as well as the influence of the position of a centre of gravity were determined.

The calculations were made for the following cases given in tab. 3:
THE CASES OF THE CALCULATION OF THE FLOATING BOX

Tab. 3

In almost cases the water depth is infinite. Calculations were made with the same box discretized by 27 panels per quadrant for the 60 meters of water depth (with the centre of gravity $z_g = 10.62$ m). Results were also obtained for the fixed box with 225 facet elements and the same dimensions.

- tanker

In the tab. 4 the cases of the calculations of a tanker are shown.

THE CASES OF THE CALCULATION.

Tab. 4

The program of investigation of all of bodies consisted of the comparisons of the following data:

- hydrodynamic coefficients (added mass and damping);
- wave exciting forces;
- motions of bodies in regular waves;
- drift forces and moment obtained based on two methods.
DESCRIPTIONS OF DIMENSIONS OF OUTPUT DATA.

Output files of DELFRAC contain the dimensional values. In order to compare the results they were made non-dimensional in accordance with the data of other authors in the following way:

- **hemisphere**

  Non-dimensional added mass \(- A_i / (\rho r^{3})\),
  where \( A_{ii} \) - the dimensional added mass,
  \( i = 1, 2, 3 \),
  \( \rho \) is the mass density of water,
  \( r \) is the radius of hemisphere.

  Non-dimensional damping coefficients \(- B_i / (\rho \omega r^{3})\),
  where \( B_{ii} \) - the dimensional damping coefficient for the same \( i \) - number;
  \( \omega \) is the circular frequency of oscillation of the incident waves.

  Non-dimensional amplitudes of motions \( \eta_i / \zeta_s \),
  where \( \eta_{i} \) - dimensional amplitudes of motions,
  \( i = 1, \ldots, 6 \),
  \( \zeta_s \) is the incident wave amplitude.

  Non-dimensional horizontal wave exciting force \(- F_i / (\rho g r^{2} \zeta_s )\),
  where \( F_{i} \) is a dimensional wave exciting force,
  \( g \) is the acceleration of gravity.

  Non-dimensional vertical wave exciting force \(- F_3 / (\rho g \pi r^{2} \zeta_s )\),
  where \( F_{3} \) is the same dimensional one.

  Non-dimensional drift forces \(- F_i / (\rho g r \zeta_s^{2})\),
  where \( F_{i} \) is \( i \)-th dimensional drift force.

  All of results are presented as a function of non-dimensional wave frequency \( \omega^{2} r / g \),
  where \( \omega \) is the circular frequency of oscillation of the incident waves.

- **box**

  Non-dimensional added mass \(- A_{ii} / (\rho V)\),
  where \( A_{ii} \) - the dimensional added mass,
  \( i = 1, 2, 3 \),
  \( \rho \) is the mass density of water,
Volume of box \( V \) is the volume.

Non-dimensional added moments of inertia - \( A_{ii} / (\rho \ V \ L^2) \), where \( A_{ii} \) is the dimensional added moments, 
\( i = 4, 5, 6 \),
\( L \) is the length of the box.

Non-dimensional damping coefficients - \( B_{ii} / (\rho \ V \sqrt{g/L}) \), where \( B_{ii} \) is the dimensional damping coefficient 
\( i = 1, 2, 3 \).

Non-dimensional damping coefficients - \( B_{ii} / (\rho \ V \ L^2 \sqrt{g/L}) \), where \( B_{ii} \) is the dimensional damping coefficient 
\( i = 4, 5, 6 \).

Non-dimensional amplitudes of motions \( \eta_i / \zeta_a \), where \( \eta_i \) is dimensional amplitudes of motions, 
\( i = 1, \ldots, 3 \),
\( \zeta_a \) is the incident wave amplitude.

Non-dimensional amplitudes of motions \( \eta_i \ L / \zeta_a \), where \( \eta_i \) is dimensional amplitudes of motions, 
\( i = 4, \ldots, 6 \),
\( \zeta_a \) is the incident wave amplitude, 
\( L \) is the length of the box.

Non-dimensional wave exciting forces - \( F_i / (\rho g \ V \zeta_a / L) \), where \( F_i \) is a dimensional wave exciting force, 
\( g \) is the acceleration of gravity.

Non-dimensional wave exciting moments - \( F_i / (\rho g \ V \zeta_a) \).

Non-dimensional mean drift forces - \( F_i / (\rho g L \zeta_a^2) \).

All of the results are presented as a function of period of the incident waves.

- tanker

Non-dimensional amplitudes of motions \( \eta_i / \zeta_a \), where \( \eta_i \) is dimensional amplitudes of motions, 
\( i = 1, \ldots, 6 \),
\( \zeta_a \) is the incident wave amplitude.
Non-dimensional drift forces and moments - $F_i / (\zeta_i^2)$,
where $F_i$ is i-th drift force or moment in tons/m$^2$ or tons*m/m$^2$,
$i = 1, \ldots, 6$.

All of results are presented as a function of a circular frequency of oscillation of the incident waves ($\omega$) or as a function of a non-dimensional wave frequency $\omega \sqrt{L/g}$, where $L$ is the length of the tanker, $g$ is the acceleration of gravity.

CALCULATIONS OF THE HEMISPHERE AND CONCLUSIONS

We are obtained the comparisons the results of calculations using the program DELFRAC and theoretical and experimental results of the following authors:

RESULTS OF CALCULATION OF THE FLOATING HEMISPHERE

<table>
<thead>
<tr>
<th>Name of authors</th>
<th>Zhongsheng</th>
<th>Hulme</th>
<th>Aanesland</th>
<th>Kudou</th>
</tr>
</thead>
<tbody>
<tr>
<td>Added mass</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>Damping</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>Wave forces</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Motions</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>Drift forces</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

In the case of the fixed hemisphere, we used the following results (see tab. 6):

RESULTS OF CALCULATION OF THE FIXED HEMISPHERE

<table>
<thead>
<tr>
<th>Name of authors</th>
<th>Aanesland</th>
<th>Kudou</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wave forces</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>Drift forces</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>
In figures 4 through 7 the dimensional hydrodynamic coefficients are presented as functions of the circular wave frequency. In figures 8 - 9 we are shown the results of amplitudes of motions in beam waves and in bow quartering waves.

The results of the calculation of drift forces using the near- and far-field methods are shown in graphs 10 and 11. As can be seen both methods give the same results for the whole range of wave frequencies. From these results it may be concluded that as the wave frequency increases the heave motion increases and at the same time the mean horizontal drift force increases (see, for instance, fig. 8 and 10). The maximum heave motions occurs at a slightly lower wave frequency than the maximum horizontal drift force. In this range of frequencies the effects due to diffraction and body motions on the wave drift forces are increasing. At higher frequencies the body motions decrease continuously to become zero at frequencies tending to infinity. In the range of these frequencies the effects of body motions on the drift force decrease rapidly and in the limit only effects due to diffraction remain. At high frequencies there is some limit value of the mean horizontal drift force depended on the body which will be shown later.

The next set of figures show the non-dimensional forces and amplitudes of motions for the floating hemisphere.

Non-dimensional surge and heave added mass and damping coefficients as obtained from the program DELFRAC and calculations of Hulme and Zhongsheng are compared in Figures 12, 13, 14 and 15. In each figure the data are given to a base of the non-dimensional wave frequency. The agreement between the results of different authors are quite good. The same convergence of results of the motion amplitudes is shown in figures 16 and 17. In these figures the experimental results of Kudou are in good agreement with the numerical results of DELFRAC. Analysis of these results affirm that in case of this classical shape of a body the influence of viscosity is negligible.

In this investigation we also compared the results of experiments and calculations of horizontal and vertical wave exciting forces which are obtained by Aanesland, Hulme, Zhongsheng and by the program DELFRAC. The results are given in Figures 18 and 19. In fig. 18 the existence of the irregular frequency can be seen which correspond to the non-dimensional wave frequency $\omega^2 r/g = 2.5$ for the vertical direction. The same result was obtained by Zhongsheng (see ref. [7]). For the horizontal direction the first irregular frequency is observed at the non-dimensional wave frequency $\omega^2 r/g = 4.0$, i.e. the same result as obtained by Zhongsheng.

The most interesting conclusions are made from comparisons of the horizontal and vertical drift forces. The comparison of non-dimensional horizontal drift forces is given in the figure 20. From this graph it can be concluded that the drift forces can be computed with reasonable accuracy based on the near- or far field methods. In general the far-field method gives the more stable result. All results are in good agreement with the results of other authors. Scatter of Kudou's experimental points of the drift force can be observed but the theoretical results of Kudou are the same as obtained by the program DELFRAC.

In figure 21 the good convergence of non-dimensional drift force to the high frequency asymptotic value can be seen which for this case amounts to 0.667. This asymptotic value can be used from $\omega^2 r/g > 3.0$. As can be seen in the same figure for
the rather wide range of frequencies we have several irregular frequencies (see fig. 21).

Special attention was paid to determining the influence of the number of waterline elements on the computation results. It is noticed that this influence is significant. In the case when the number of waterline element is double the number of facets close to the waterline it is necessary to use the parameter Lenfac=0 (about parameter Lenfac see ref. [10]). In the case of Lenfac=4, the number of waterline element must be the same as the number of facet element close to the waterline. The summary information of the different cases of computation of drift forces is given in the tab. 7.

Input data for calculation of drift forces using DELFRAC

<table>
<thead>
<tr>
<th>Number of top facets close to waterline</th>
<th>n</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of waterline elements</td>
<td>2n</td>
<td>n</td>
</tr>
<tr>
<td>Parameter Lenfac</td>
<td>0</td>
<td>4</td>
</tr>
</tbody>
</table>

From the results presented in this section it can be concluded that the mean wave drift forces on a hemisphere can be computed by both discussed methods with reasonable accuracy using the program DELFRAC.

In order to determine the influence of motions on drift forces the same results were calculated for the case of the fixed hemisphere. The horizontal and vertical drift force transfer functions are given in figure 22. As in the case of the floating hemisphere the drift forces obtained by using the near- and far-field methods have the same values.

The comparison of results of Kudou, Aanesland and by DELFRAC is shown in figure 23, where all values are presented in non-dimensional form. It can be seen that all computation results are completely similar and as in the case of the floating hemisphere some scatter of experimental data is observed.

In figure 24 the influence of motions on the horizontal drift forces are shown for the floating and fixed hemisphere. The significant difference between these forces can be seen over a rather wide range of wave frequencies, especially at the natural heave frequency of the hemisphere. This difference can be explained due to the influence of the heave motion. Up to the wave frequency $\omega = 1$ rad/sec the drift force on the fixed hemisphere exceeds the value for the floating hemisphere and after this frequency the influence of motions increases, the force on floating hemisphere becomes larger. The same irregular frequencies occur for both calculated forces.

A similar influence of the heave motion is shown in figure 25, in which the vertical drift forces for the floating and fixed hemispheres are presented. Significant differences are observed up to the wave frequency 1.1 rad/sec, after which the results become the same in both cases.
The curves presented in figures 26 - 27 show that the drift forces on the hemisphere in bow quartering waves are completely similar to those of the figures 24 - 25 and may indicate the same influence of motions on forces as in the previous case.

To come to a conclusion, the satisfying correlation found between results of computations and experiments confirms the general applicability of the theory used in the program DELFRAC for predicting the motions and the drift forces on floating and fixed hemisphere.

CALCULATIONS OF A BOX AND CONCLUSIONS

In order to compare the wave forces and motions of a box the results of O.M. Faltinsen and F.C. Michelsen (see ref. [2]) who have also presented experimental data were used.

In the table 8 the natural frequencies of box for different values of the vertical position of the centre of gravity are shown as applied in numerical computations.

<table>
<thead>
<tr>
<th>Centre of gravity (from WL), m</th>
<th>+10.62</th>
<th>-29.38</th>
<th>-10.62</th>
</tr>
</thead>
<tbody>
<tr>
<td>Natural frequency, rad/sec</td>
<td>0.404</td>
<td>0.227</td>
<td></td>
</tr>
<tr>
<td>Natural period, sec</td>
<td>15.54</td>
<td>27.67</td>
<td></td>
</tr>
</tbody>
</table>

'-' means that in this case the metacenteric height is negative.

In figures 28 - 35 the dimensional hydrodynamic coefficients of a floating box are given. We can discuss only surge (sway), heave and pitch (roll) hydrodynamic coefficients because of the equal dimensions of a box in longitudinal and transverse directions. It is obvious that surge, heave and yaw coefficients do not depend on the centre of gravity. But in the case of pitch we have a significant influence of the vertical position of the centre of gravity, as is shown in figures 36 - 37.

The results of comparative analysis of box the discretized by 27 or 225 panels per quadrant are given in figures 38 - 40. For sway and heave added mass no differences are seen between both curves. But in the case of the roll damping moment a significant influence of the number of panels is observed and shown in figure 40.

In figures 41 - 42 the non-dimensional amplitudes of motions of a box for beam waves and different centres of gravity are shown. It can be seen that the heave amplitude does not depend on a centre of gravity. For sway and roll the influence of the latter is significant due to the different natural frequencies. That is why the effect of the roll motion of the box with the centre of gravity -10.62 meters is less compared to the case with the centre of gravity -29.38 meters. In the latter case at resonance of roll motions, the influence of roll motion on the other motions is sensitive (see fig. 44).
Figure 43 shows the influence of the number of facet elements on the amplitudes of motions for the case of $z_g = -10.62$ m. The greatest influence is shown on the roll motions, less - on heave, and no influence on sway motions. It may be noticed that this influence occurs near the natural frequency of the roll and heave motions respectively. This fact can be explained by the influence of the number of panels on the damping and exciting wave forces. The same result is shown in figure 44 for the case of $z_g = -29.38$ m. For this case the influence of panels is less. Also it is be noted that an increase of the amplitudes of motions is related to an increase or decrease of the number of panels (see fig. 43, 44).

The results of calculations of the wave exciting forces and moments are presented in figures 45 - 46. From these figures the existence of irregular frequencies even at rather low wave frequencies can be observed ($\omega > 0.8$ rad/sec). This fact can be explained by the considerable dimensions of the box. The influence of the vertical position of the centre of gravity on the roll wave exciting moment is illustrated in figure 47. Figure 48 shows the influence of the number of panels on the same moment. As can be seen, the roll wave exciting moment is more sensitive to the number of panels than the hydrodynamic coefficients, excluding roll damping.

Figures 49 - 52 illustrate the drift force transfer functions calculated by the near-field and the far-field methods for the different cases of centre of gravity. As in the case of wave exciting forces here it is observed that the results vary considerably from wave frequency $\omega > 0.7$ rad/sec because of several irregular frequencies. The figures seem to indicate that the drift forces have a strong dependence on the heave motions. There is a significant increase of the drift forces around the heave natural frequency of the box ($\omega = 0.404$ rad/sec and of course not depending on centre of gravity). On comparing the curves obtained by the near-field and the far-field methods it was found that up to the frequency $\omega = 0.7$ rad/sec the results are quite similar but after this frequency the results are not realistic. For such a large box the predicted motions at high frequencies are less probable.

Analysing the figure 53 in which the horizontal drift force transfer functions are presented for $z_g = -29.38$ m and the different numbers of panels it can be noticed the considerable influence of the latter on the forces.

The effect of the number of panels on the drift forces is best illustrated by figures 54 through 57, in which the drift forces and moment are given for two numbers of panels - 27 and 225 per quadrant of the box. In the last graph 57 the summary information about the horizontal drift forces in beam sea is given for the case of centre of gravity -10.62 m. As is shown in all cases (near- and far-field methods) there is a visible difference between the forces obtained with 27 and 225 panels. The results show that both methods are sensitive to the mesh used, but in general, the far-field method gives more stable results even for less facet elements.

The influence of the number of waterline elements was also checked. There are no significant differences between the results of calculations of the drift forces with different numbers of waterline elements as in the case of the hemisphere (and further it can be seen in the case of a tanker).

Generally, it is clear that the prediction of wave forces and first order motions of such a large box is rather adequate as is confirmed in figures (58) - (74).

The comparison was made for all of hydrodynamic coefficients, wave and drift forces and motions. First the accuracy of computation of hydrodynamic forces is discussed.
Figure 58 illustrates the results of calculations of surge added mass using DELFRAC. Here added masses are given as a function of wave period. The results of calculations by Faltinsen based on 3D theory and strip theory are shown in the same figures as well as the experimental points. On comparing the curves the good agreement was found between the theoretical results of Faltinsen and those obtained using DELFRAC. All theoretical curves are rather close to the experimental data excluding the case of calculation using the 2D method. This result demonstrates the necessity of using the 3D theory for the case of rather large body.

The same good agreement as found in the previous case can be seen in figure 59, where the heave added masses are presented. Unfortunately, we cannot say the same about the pitch added moment (see graphs 60 - 62). This fact may be explained partially by the lack of information about the mass moment of inertia and the centre of gravity.

The results for the yaw added moment given in figure 63 are also in good agreement with Faltinsen’s results and experimental data.

The results of comparison of damping coefficients are given in figures 64 - 68. All surge damping curves are quite similar and in good correlations with the experimental data, excluding the curve obtained by strip theory. However, in the case of heave and especially pitch damping the convergence between theoretical and experimental results is not so strong (see fig. 65 - 68). This may be due to viscous effects, but it should be pointed out also that damping is small and difficult to determine experimentally with any great degree of accuracy.

In figure 69 only the theoretical results of calculation of horizontal wave exciting force as a function of wave period for beam waves are shown. The 3D results of Faltinsen and from the program DELFRAC are exactly the same. It may be concluded that this kind of force can be calculated perfectly for every shape of body. In all cases (see also fig. 70 and 71, where the horizontal wave exciting force for bow quartering waves and the vertical wave exciting force for head waves are shown) there is an absolute convergence of theoretical and experimental results obtained by different authors.

The comparison of heave amplitudes of motions in beam waves is given in figure 72. As in the all previous cases experimental results confirm the accuracy of the theory, excluding the results of strip method calculations.

The satisfactory agreement between calculated drift forces and experimental values can be seen in fig. 73. In the same figure the asymptotic value of the non-dimensional horizontal drift force is shown. Based on this figure, it can be said that for rather short periods (from $\tau < 6$ sec.) the asymptotic drift force may be used. With respect to the two theoretical methods used, it may be concluded that both give the same adequate result in a realistic range of periods. Figure 74 once again illustrates the previous conclusions concerning the calculations of drift forces.

In order to analyse the influence of the motions of the box on drift forces calculations for the fixed box have been carried out. The curves obtained by the two discussed methods are presented in figure 75 for beam waves and in figure 76 for bow quartering waves. As it can be seen the behaviour of drift forces in this case is quite different than in the case of the free-floating box which can be explained by the strong influence of heave motions in the second case. The results obtained by DELFRAC by the far-field method are close to Faltinsen’s results derived by the same method.

Some difference is also it is observed between near-field and far-field methods from the wave period $\tau < 11$ sec. for beam waves and from $\tau < 14$ sec. for bow
quartering waves. In practical cases it is preferable to compare the results of both methods.

All of the previous results are valid for infinite water depth. The last investigation is made in order to compare hydrodynamic forces of a box for cases of infinite and limited water depth. A water depth of 60 meters was selected, i.e. the ratio water depth / draft is 1.5. The results of comparisons of hydrodynamic coefficients are given in figures 77 - 80. The strongest dependence of water depth is observed on the heave added mass of the box which increases with decreasing water depth. No other significant influence was noticed. Indeed, some hydrodynamic coefficients are completely the same in both cases (for example, surge, heave and yaw added masses and damping).

In summary, it can be concluded that for dynamic analysis of behaviour of large three-dimensional structures only a 3D method based on a source technique is adequate. Using a two-dimensional strip theory can give wrong results.

The choice of number of the panels to describe a body can be made after calculations with increasing numbers of panels. The criterion for necessary number of panels may be selected after analysis of the convergence of calculation results. In the case of a large box 225 panels per one quadrant is more appropriate than 27, especially for the calculation of drift forces which are more stable in the case of the large number of panels.

Two methods of calculation of drift forces have been found successful. In most cases the near-field method based on pressure integration is less stable than the far-field method. But in the case of a box both methods give the similar results.

It is obvious that the values of rotational hydrodynamic coefficients as well as other forces and motions depend on the position of centre of gravity. As it is shown in this study such influence can be strong and may change the forces and the amplitudes of motions significantly.

Considerable influence of water depth on heave added mass of a box has been demonstrated. No great differences between the other hydrodynamic coefficients for infinite and limited water depth are observed.

CALCULATIONS OF A TANKER AND CONCLUSIONS

In the case of a tanker, we compared the results of computations with the experimental data from MARIN report (see ref. [5], [6]) which contained the following data:
EXPERIMENTAL RESULTS FROM MARIN REPORT FOR DIFFERENT WAVE DIRECTIONS

For a correct comparison it is necessary to keep in mind that the amplitudes of the longitudinal and transverse drift forces from DELFRAC are in kN / m², and from MARIN report - in tons / m².

Hydrodynamic coefficients of a tanker obtained using DELFRAC are shown in figures 81 - 86 (added masses) and figures 87 - 92 (damping coefficients) for different water depths. Having analysed these graphs two cases of influence of water depth were found. First of these is connected with an increase of hydrodynamic coefficients in a rather narrow range of wave frequencies when the water depth decreases. As seen from figures 81 - 92, surge, sway, yaw added masses as well as sway and yaw damping increase rapidly when \( \omega < 0.6 \) rad / sec. For surge, heave roll and pitch damping this increase is observed at the high frequency range, when \( \omega < 1.1 \) rad / sec., i.e. at the most interesting range for practical cases. Some coefficients increase significantly: heave and pitch damping - 100 % compared to infinite water depth; roll damping - more than 300 % for the same case. The increase of other coefficients is less (see, for instance, fig. 87 - surge damping coefficient, fig. 88 - sway damping coefficient, fig. 92 - yaw damping coefficient). For this case the limit value of coefficients at high frequencies does not depend on water depth and equals to the value for infinite water depth.

Another case of water depth influence is shown in figures 83, 84 and 85, where heave, roll and pitch damping are presented respectively. As can be seen the influence of water depth exists at every frequency. Significant influence is observed at rather low frequencies (0.2 - 0.3 rad/sec) but the limit value of these hydrodynamic coefficients at high frequencies not equal to the value for infinite water depth. This effect of water depth is more remarkable because it exists over the whole range of wave frequencies.

Analysing the results of calculated hydrodynamic forces of the tanker we note that the lowest irregular frequencies appear at practical frequencies, e.g. \( \omega = 1.1 \) rad / sec, and even 0.9 rad / sec, which correspond periods 5.71 sec and 7.0 sec respectively.

<table>
<thead>
<tr>
<th>Water depth, m</th>
<th>82.50</th>
<th>37.80</th>
<th>30.20</th>
<th>22.68</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heave motion</td>
<td>180, 160</td>
<td>180, 135</td>
<td>180, 160, 135</td>
<td>180, 160, 135</td>
</tr>
<tr>
<td>Pitch motion</td>
<td>180, 160</td>
<td>180, 135</td>
<td>180, 160, 135</td>
<td>180, 160, 135</td>
</tr>
<tr>
<td>Surge motion</td>
<td>180, 160</td>
<td>180, 135</td>
<td>180, 160, 135</td>
<td>180, 160, 135</td>
</tr>
<tr>
<td>Roll motion</td>
<td>160</td>
<td>135, 90</td>
<td>160, 135, 90</td>
<td>160, 135, 90</td>
</tr>
<tr>
<td>Sway motion</td>
<td>160</td>
<td>135, 90</td>
<td>160, 135, 90</td>
<td>160, 135, 90</td>
</tr>
<tr>
<td>Yaw motion</td>
<td>160</td>
<td>135</td>
<td>160, 135</td>
<td>160, 135</td>
</tr>
<tr>
<td>Longitudinal drift force</td>
<td>-</td>
<td>180, 135</td>
<td>180, 135</td>
<td>180, 135</td>
</tr>
<tr>
<td>Transverse drift force</td>
<td>-</td>
<td>135, 90</td>
<td>135, 90</td>
<td>135, 90</td>
</tr>
<tr>
<td>Yaw drift moment</td>
<td>-</td>
<td>135</td>
<td>135</td>
<td>135</td>
</tr>
</tbody>
</table>
In order to illustrate the influence of water depth on the exciting forces and moments the graphs 93 and 94 are shown. This influence can be seen only at the range of rather low frequencies for sway excited force and for a wide frequency range (up to 1.1 - 1.2 rad/sec) for the roll excited moment.

The effect of the influence of water depth on the motions of the tanker is illustrated in fig. 95 through 100 for several wave directions. In general, this effect is connected with the increase of the amplitudes of motions at rather low frequencies, as it is shown, for instance, in fig. 95 and 98. But, when the added masses increase strongly and the natural periods change in the same way, the motions also change over a wide range of wave frequencies (see, for instance, fig. 96 in which the amplitude of heave motion in beam sea, or fig. 100 in which the pitch motion in head sea are shown). In the last case, for the limit water depth the natural frequency usually moves to the low frequency and the motions become larger because of the damping.

In our opinion, the peak value of roll motion shown in figure 97 should be reduced to more realistic value. This means that viscous damping should be added in this calculation. This is important not only for this case, but also for most practical shapes of body.

The mean drift forces were calculated and are represented in figures 101 - 110 as a drift force transfer function for different water depths to a base of wave frequencies. In general, satisfactory agreement between two curves of drift forces calculated by the near-field and the far-field methods is obtained. The best agreement is found for the transverse drift force for different water depths and wave propagation 90 degree (see, for instance, fig. 101, 103, 104, 106). For arbitrary wave propagation the curves of drift forces calculated by the two methods are slightly different. This fact may probably be explained by more complicated interaction between the waves and the moving body.

In the other series of figures 111 - 123 the influence of water depth on the mean drift forces and yaw moment is demonstrated. From these figures it can be seen that for beam waves the influence of water depth is related to the increase of the horizontal drift forces at the natural frequency of heave motion when the water depth decrease (see fig. 111 and 112). At frequencies far from the heave natural frequency, no significant influence is observed.

Results were also obtained for the vertical drift forces. In the case of limit water depth the vertical drift force even changes sign compared with the same force for deep water (see fig. 113 and 118). As can be seen, for instance, in fig. 113, for the rather shallow water the peak of the vertical drift force occurs at a wave frequency of 0.4 rad/sec. This fact can be explained that for the case of limit water depth this frequency is the natural heave frequency. That is why the influence of the vertical motions on the drift force is greatest. In this case the second component of the mean second order drift forces on the tanker, connected with integration of pressures over the oscillating surface, is dominant and contributions III and IV have rather small values. However, for the case of infinite water depth the vertical natural frequency moves to higher frequencies and at frequency \( \omega = 0.4 \) rad/sec there is no resonance of heave motion, consequently, the second contribution of drift force is not so remarkable compared with other contributions. The total drift force also has a small value. These two cases are demonstrated below in table 8.
CONTRIBUTIONS OF TOTAL VERTICAL DRIFT FORCE ON A TANKER, WAVE DIR. 90 DEG., FREQ. 0.4 RAD/SEC.

<table>
<thead>
<tr>
<th>Designation</th>
<th>Water depth 22.68 m</th>
<th>Infinite water depth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical natural freq. rad/sec</td>
<td>0.380</td>
<td>0.527</td>
</tr>
<tr>
<td>Contribution I</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Contribution II</td>
<td>-2.53 * 10^4</td>
<td>-3.041 * 10^3</td>
</tr>
<tr>
<td>Contribution III</td>
<td>5.7 * 10^3</td>
<td>6.49 * 10^3</td>
</tr>
<tr>
<td>Contribution IV</td>
<td>2.48 * 10^3</td>
<td>-2.85 * 10^3</td>
</tr>
<tr>
<td>Total force, KN/m²</td>
<td>-1.716 * 10^4</td>
<td>5.996 * 10^3</td>
</tr>
</tbody>
</table>

The most unstable results are obtained for yaw drift moment which are difficult to interpret (see, for instance, fig. 114).

For other wave directions the effect of influence of water depth is less, but has the same effect as for beam sea.

As in the case of a hemisphere special attention was paid to determine the influence of the number of waterline elements on the computation results. As follows from figure 124 the influence is significant. In the case when the number of waterline element is double the number of facets close to the waterline it is necessary to use the parameter Lenfac=0 (about parameter Lenfac see ref. [10]). In the case of Lenfac=4, the number of waterline element must be the same as the number of facet elements close to the waterline.

To analyse the accuracy of the theoretical method used in DELFRAC a number plenty of comparisons were made with the MARIN experimental results. In figures 125 - 151 the correlation between the calculated curves and experimental points of amplitudes of motions for different water depths and several wave directions can be seen. In general, this agreement is quite good, but in some cases of roll motions differences are found (see, for instance, the amplitudes of roll motions for 22.68 m and 82.5 m water depths and several wave directions). In our opinion, this difference is related to viscous effects which are more significant in this case but these are not included in the present theory. The theoretical amplitudes of the all other motions have a perfect convergence with experimental results.

The drift forces and moments are plotted in figures 152 -166 as a force transfer function to a base to non-dimensional wave frequency described earlier. As from the previous graphs we can see satisfactory agreement between theoretical and experimental results for all cases of water depth. This very important conclusion confirms the general applicability of two methods: pressure integration method (or near-field method) and Maruo-Newman formulation (or far-field method) for the prediction of mean second order forces and moments.

Concerning the effects due to the water depth it may be underlined that this effect generally depends on the heave motions and increases with decreasing of water depth. The increase of motions and particularly drift forces at the frequencies close to the natural heave frequency are also shown.
CONCLUSIONS.

From the results of calculations using DELFRAC, the comparisons and the analysis of the results the following main conclusions can be drawn with respect to the hydrodynamic and wave drifting forces as well as the use of the program DELFRAC:

- the program DELFRAC is able to predict the first order quantities with an accuracy of the order of 10%, which is generally sufficient. An improvement of this program can be made by taking into account viscous damping at resonance in order to obtain an improvement in the estimation of the response.

- drift forces and moments can be calculated by two methods: direct pressure integration method described by prof. J. Pinkster in [1] and known as near-field method and method of conservation of momentum or Maruo-Newman far-field method. For the three forms of the bodies the mean forces and moments are compared. The correlation found between results of computations and experiments confirms the general applicability of both theories for predicting the second order forces on a wide range of hull forms.

- a strong influence of the position of the centre of gravity and of the water depth on the motions and drift forces of arbitrary free-floating body were found and are presented in this work.

- investigation of the influence of the motions on the drift forces is made. A considerable dependence of this kind of forces on motions is shown which is mainly due to the heave motions. It can be concluded that the main component of the mean horizontal drift force is related to the relative wave elevation for the rather large body.

The following conclusions concern the use of DELFRAC:

- the results of computation are very sensitive to the number of facet elements used to approximate the body. The choice of the number of facets used for computations is a compromise between the quality of the results obtained and the time (and costs) of computations. In general, it is necessary to repeat several computations with increasing number of panels. The convergence between the results obtained can give an optimal number of panels. Also to obtain a reasonable accuracy in obtaining of the second order quantities (e.g. drift forces), we must have a rather refined mesh. In this case 6 - 8 panels per wave length are the minimum as it can be demonstrated in this work.

- one of the finding of this study is related to the influence of the number of waterline elements on the drift forces calculated by using the near-field method. In order to obtain the correct results it is necessary to use the special factor Lenfac = 4, when the number of waterline elements is the same as the number of top facet elements.
the computer time is very dependent on the total number of panels used. In the case of a hemisphere discretized by 55 panels per quadrant the time on a PC AT 386 DX 40 is about one minute and 10 seconds per frequency; the same calculation of a box with 27 panels per quadrant takes a half a minute, with 225 panels - 20 minutes and with 319 panels - one hour and 40 minutes. In the case of a tanker with 194 facet elements for a half of a body the time is about 9 minutes.

finally, the ability of DELFRAC to solve a radiation/diffraction problem and for predicting the second order forces on a wide range of hull forms was demonstrated.

The further improvement of this program is to take into account viscous damping at resonance to get an estimation of the response. This can be done by experimental means in wave tanks or by computation codes solving Navier-Stokes. The other improvement can be connected with the addition of calculation of vertical drift force by using the far-field method to have more results for comparisons. Next important case concerns the influence of current on drift forces which is described, for instance, by V. Aanesland or O. Faltinsen. Finally research on the problem with forward speed must be carried out.

ACKNOWLEDGEMENTS.

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The author also thanks all colleagues from the Shiphydrodynamics Laboratory of Delft University of Technology for their useful and kind helps and co-operations.

REFERENCES.


Fig. 1

The hemisphere with total 220 panels.
Two cases of a box with total 108 and 900 panels each.
Fig. 3

The tanker with total 388 panels.
Added mass of a hemisphere

Fig. 4
Damping of a hemisphere

Fig. 5
Added Mass of Hemisphere

Fig. 6
Damping moments of hemisphere

Fig. 7
Motions of hemisphere, wave dir. 90 deg.

Fig. 8
Motions of hemisphere, wave dir. 135 deg.

Fig. 9
Horizontal drift force on a floating hemisphere, wave dir. 90 deg.

Fig. 10
Horizontal drift force on a floating hemisphere, wave dir. 135 deg.

Fig. 11
Surge Added mass of hemisphere

\[ A_{11}(\rho^2) \]

\[ \omega^2 r/g \]

Fig. 12
Surge damping coefficient of hemisphere

Fig. 13
Heave Added Mass of Hemisphere

\[ A_{33}(\omega^2 \rho g) \]

Fig. 14
Heave damping coefficient of hemisphere

\[ \frac{B_{33}}{\rho \omega^2 r^2} \]

\[ \omega^2 r/g \]

Fig. 15
Non-dimensional Heave Amplitude of a Hemisphere

Fig. 16
Non-dimensional Sway Amplitude of a Hemisphere

Fig. 17
Heave exciting force on a hemisphere, wave dir. 90 deg.

Fig. 18
First-order linear horizontal wave excitation force on a hemisphere

Fig. 19
Horizontal drift force on a floating hemisphere

Fig. 20
Non-Dimensional Horizontal Drift Force on a Floating Hemisphere

Fig. 21
Drift forces on a fixed hemisphere

Fig. 22
Horizontal drift force on a fixed hemisphere

Fig. 23
Horizontal drift forces on a hemisphere, wave dir. 90 deg. (DELFRAC)

Fig. 24
Vertical drift forces on a hemisphere, wave dir. 90 deg. (DELFRA

Fig. 25
Horizontal drift forces on a hemisphere, wave dir. 135 deg. (DELFRAC)

Fig. 26
Vertical drift forces on a hemisphere, wave dir. 135 deg. (DELFRAI)

Fig. 27
Added mass of box

Fig. 28
Added moments of inertia of a floating box

Fig. 29
Fig. 30

Added moments of inertia of box

Wave frequency, r/s

Added moment of inertia in tons·m²

- Roll & Pitch
- Yaw
Added moments of inertia of a floating box, 225 panels, \(z_g = -10.62\) m

![Graph depicting added moments of inertia vs. wave frequency.](image)

Wave frequency, r/s

Fig. 31
Damping of a box

Fig. 32
Damping moments of floating box

Wave frequency, r/s

Damping moment in kN.m/s

Fig. 33
Damping moments of box

Fig. 34
Damping moments of a floating box, 225 panels, $Z_g = -10.62$ m

Fig. 35
Roll added moment of inertia of a floating box

Fig. 36
Roll damping moment of a floating box

Fig. 37
Added mass of a floating box

Fig. 38
Damping Roll & Pitch Coefficients of a floating box

Fig. 39
Roll damping moment of a box

![Graph showing roll damping moment of a box with wave frequency on the x-axis and damping moment in KNm/s on the y-axis. The graph includes data points for 225 panels and 27 panels with wave frequency ranging from 0 to 2.5 rad/sec and damping moment ranging from 0 to 4x10^5. The damping moment peaks at various wave frequencies.]

Zg = -10.62 m

Fig. 40
Motions of box, wave dir. 90 deg.

Fig. 41
Motions of a floating box, wave dir. 90 deg.

Fig. 42
Motions of a floating box, wave dir. 90 deg., $Z_g=10.62\text{m}$

![Graph showing wave frequency vs. motion parameters]

Fig. 43
Motions of a box, wave dir. 90 deg.

Fig. 44
Linear wave exciting forces on a floating box, wave dir. 90 deg.

Fig. 45
Linear roll wave exciting moment on a floating box, wave dir. 90 deg.

Fig. 46
Wave exciting moment on a floating box, wave dir. 90 deg.

Fig. 47
Linear roll wave exciting moment on a floating box, wave dir. 90 deg.

Fig. 48
Horizontal drift forces on a floating box, wave dir. 135 deg.

Fig. 49
Horizontal drift force on a floating box, 225 panels

Fig. 50
Horizontal drift force on a floating box, wave dir. 90 deg.

Zg = -10.62 m; 225 panels

Fig. 51
Horizontal drift force on a floating box, wave dir. 135 deg.

$Z_g = -10.62\ m$; 225 panels

Fig. 52
Horizontal drift force on a floating box

![Graph showing horizontal drift force](image)

**Fig. 53**
Transverse drift force on a box, wave dir. 90 deg.

Wave frequency, rad/sec

Zg = -10.62 m

Near-field method

Fig. 54
Vertical drift force on a box, wave dir. 90 deg.

![Graph showing force transfer function in KN/m² against wave frequency in rad/sec.]

- Zg = 10.62 m
- Near-field method

Fig. 55
Transverse drift force on a floating box, wave dir. 90 deg.

Zg = -10.62 m
Far-field method.

Fig. 56
Transverse drift force on a floating box, wave dir. 90 deg.

![Graph showing force transfer function in KN/m² vs. wave frequency in rad/sec.]

Zg = -10.62 m

Fig. 57
Surge added mass coefficients of floating box

Fig. 58
Heave added mass coefficients of floating box

Fig. 59
Pitch added mass of floating box

![Graph showing pitch added mass of floating box]

- **DELFRAC**
- Calcul. Faltinsen, 48 elem.
- Calcul. Faltinsen, 108 elem.
- Calcul. Faltinsen, 8 offset point
- Experiment Faltinsen

Fig. 60
Pitch added mass of floating box

![Graph showing the relationship between wave period and pitch added mass](image)

Fig. 61
Pitch added mass of a floating box

![Graph showingPitch added mass of a floating box](image)

$\frac{A_{sg} (\rho V L^2)}{\text{Wave period, s}}$

Fig. 62
Fig. 63

Yaw added mass of floating box
Surge damping of floating box

Fig. 64
Heave damping of floating box

![Diagram showing the relationship between wave period and heave damping for different calculations and experiments.](Fig. 65)
Pitch damping coefficients of floating box

Fig. 66
Pitch damping coefficients of floating box

![Graph showing pitch damping coefficients versus wave period.](image)

**Fig. 67**
Pitch damping moment of a floating box

Fig. 68
Non-dimensional wave exciting force on a floating box, wave dir. 90 deg.

Fig. 69
Non-dimensional wave exciting force on a floating box, wave dir. 135 deg.

\[ \frac{F}{(h^2, r, \theta, \phi)} \]

**Fig. 70**
Non-dimensional wave exciting force on a floating box, wave dir. 90 deg.

Fig. 71
Heave amplitude of a floating box, wave dir. 90 deg.

Fig. 72
Non-dimensional horizontal drift force on a floating box, wave dir. 90 deg.

Zg = -10.62 m; 225 panels

Fig. 73
Non-dimensional horizontal drift force on a floating box, wave dir. 135 deg.

$Z_g = -10.62 \text{m}$; 225 panels

$F_z/\left(\rho g L^2\right)$ vs Wave period, s

Fig. 74
Non-dimensional horizontal drift force on a fixed box, wave dir. 90 deg.

\( F_z / (\rho g L^2) \)

Wave period, s

Fig. 75
Non-dimensional horizontal drift force on a fixed box, wave dir. 135 deg.

$Z_g = -10.62 \text{ m}$; 225 panels

\[ F_z / (p g L^2) \]

Fig. 76
Heave Added Mass of box

Fig. 77
Added mass moments of inertia of box

Fig. 78
Damping of box

Fig. 79
Fig. 80
Surge added mass of a tanker

![Graph showing surge added mass of a tanker with different water depths.](image)

Fig. 81
Sway added mass of a tanker

![Sway added mass graph](image)

Fig. 82
Heave added mass of a tanker

Fig. 83
Roll added moment of inertia of a tanker

Fig. 84
Pitch added moment of inertia of a tanker

Fig. 85
Fig. 86

Yaw added moment of inertia of a tanker

Added moment in tons * m²

Wave frequency, rad/sec

Infinite water depth
Water depth 82.50 m
Water depth 30.20 m
Water depth 22.68 m
Surge damping coefficient of a tanker

Fig. 87
Sway damping coefficient of a tanker

Fig. 88
Heave damping coefficient of a tanker

Fig. 89
Roll damping moment of a tanker

Fig. 90
Pitch damping moment of a tanker

Wave frequency, rad/sec

Damping moment in KN·m/sr

- Infinite water depth
- Water depth 82.50 m
- Water depth 30.20 m
- Water depth 22.68 m

Fig. 91
Yaw damping moment of a tanker

Fig. 92
Exciting wave force of a tanker, wave dir. 90 deg.

Fig. 93
Exciting wave moment on a tanker, wave dir. 90 deg.

Fig. 94
Sway motions of a tanker, wave dir. 90 deg.

\[ \frac{\eta_2}{\eta_a} \]

- \( \triangledown \) \( \triangledown \) Infinite water depth
- \( \bullet \) \( \bullet \) Water depth 82.50 m
- \( \bullet \) \( \bullet \) Water depth 30.20 m
- \( \Delta \) \( \Delta \) Water depth 22.68 m

Wave frequency, rad/sec

Fig. 95
Heave motions of a tanker, wave dir. 90 deg.

Fig. 96
Roll motions of a tanker, wave dir. 90 deg.

Fig. 97
Surge motions of a tanker, wave dir. 180 deg.

Fig. 98
Heave motion of a tanker, wave dir. 180 deg.

Fig. 99
Horizontal drift force on a tanker, wave dir. 90 deg.
Horizontal drift force on a tanker, wave dir. 180 deg.

Wave frequency, rad/sec

Fig. 102
Horizontal drift force on a tanker, wave dir. 90 deg.

![Graph showing horizontal drift force as a function of wave frequency.](image)

Water depth 82.50 m

Wave frequency, r/s

Force transfer function in KN/m²

- Near-field meth.
- Far-field meth.

Fig. 103
Horizontal drift force on a tanker, wave dir. 135 deg.

Water depth 82.50 m

Wave frequency, r/s

Force transfer function in KN/m$^2$

Fig. 104
Horizontal drift force on a tanker, wave dir. 180 deg.

Water depth 82.50 m

Fig. 105
Horizontal drift force on a tanker, wave dir. 90 deg.

Water depth 30.2 m

Force transfer function in KN/m^2

Wave frequency, r/s

Fig. 106
Longitudinal drift force on a tanker, wave dir. 135 deg.

Fig. 107
Longitudinal drift force on a tanker, wave dir. 160 deg.

![Graph](image)

- **Near-field meth.**
- **Far-field meth.**

Water depth 30.2 m

**Fig. 108**
Horizontal drift force on a tanker, wave dir. 180 deg.

Fig. 109
Horizontal drift force on a tanker, wave dir. 180 deg.

Water depth 22.68 m

Fig. 110
Horizontal drift force on a tanker, wave dir. 90 deg.

Near-field method

Fig. 111
Transverse drift force on a tanker, wave dir. 90 deg.

Wave frequency, rad/sec

Force transfer function in KN/m²

- Infinite water depth
- Water depth 62.50 m
- Water depth 30.20 m
- Water depth 22.68 m

Far-field method

Fig. 112
Vertical drift force on a tanker, wave dir. 90 deg.

Fig. 113

Wave frequency, rad/sec

Near-field method

Force transfer function in KN/m²
Fig. 114
Yaw drift moment on a tanker, wave dir. 90 deg.

Far-field method

Fig. 115
Transverse drift force on a tanker, wave dir. 135 deg.

Wave frequency, rad/sec

Near-field method

Fig. 116
Transverse drift force on a tanker, wave dir. 135deg.

Fig. 117
Vertical drift force on a tanker, wave dir. 135 deg.

Near-field method

---

Wave frequency, rad/sec

**Fig. 118**
Yaw drift moment on a tanker, wave dir. 135 deg.

Fig. 119
Yaw drift moment on a tanker, wave dir. 135 deg.

![Graph showing yaw drift moment with different water depths and wave frequencies.](image)

Fig. 120
Longitudinal drift force on a tanker, wave dir. 180 deg.

Fig. 121
Longitudinal drift force on a tanker, wave dir. 180 deg.

Fig. 122
Vertical drift force on a tanker, wave dir. 180 deg.

Fig. 123
Horizontal drift force on a tanker, wave dir. 90 deg.

Factor Lenfac = 4

Water depth 22.68 m

Fig. 124
Sway motion of a tanker, wave dir. 90 deg.

Water depth 22.68 m

Fig. 125
Heave motion of a tanker, wave dir. 90 deg.

Fig. 126
Roll motion of a tanker, wave dir. 90 deg.

Water depth 22.68 m

Fig. 127
Surge motion of a tanker, wave dir. 135 deg.

![Graph showing surge motion](image)

Water depth 22.68 m

Fig. 128
Sway motion of a tanker, wave dir. 135 deg.

Water depth 22.68 m

Fig. 129
Heave motion of a tanker, wave dir. 135 deg.

Water depth 22.68 m

Fig. 130
Roll motion of a tanker, wave dir. 135 deg.

Fig. 131
Pitch motion of a tanker, wave dir. 135 deg.

Fig. 132
Fig. 133

Water depth 22.68 m
Surge motion of a tanker, wave dir. 160 deg.

Water depth 22.68 m
Sway motion of a tanker, wave dir. 160 deg.

Water depth 22.68 m

Fig. 135
Heave motion of a tanker, wave dir. 160 deg.

Water depth 22.68 m

Fig. 136
Roll motion of a tanker, wave dir. 160 deg.

Water depth 22.68 m

Fig. 137
Pitch motion of a tanker, wave dir. 160 deg.

Water depth 22.68 m

Fig. 138
Yaw motion of a tanker, wave dir. 160 deg.

![Graph showing wave frequency vs. $\eta_0/\eta_n$](image_url)

Water depth 22.68 m

Fig. 139
Surge motion of a tanker, wave dir. 180 deg.

Fig. 140

Water depth 22.68 m
Heave motion of a tanker, wave dir. 180 deg.

Fig. 141
Pitch motion of a tanker, wave dir. 180 deg.

Water depth 22.68 m

Fig. 142
Surge motion of a tanker, wave dir. 160 deg.

Water depth 82.50 m

Fig. 143
Sway motion of a tanker, wave dir. 160 deg.

Water depth 82.50 m

Fig. 144
Heave motion of a tanker, wave dir. 160 deg.

Fig. 145
Roll motion of a tanker, wave dir. 160 deg.

Fig. 146
Pitch motion of a tanker, wave dir. 160 deg.

Fig. 147
Yaw motion of a tanker, wave dir. 160 deg.

Water depth 82.50 m

Fig. 148
Surge motion of a tanker, wave dir. 180 deg.

Fig. 149
Heave motion of a tanker, wave dir. 180 deg.

Water depth 82.50 m

Fig. 150
Pitch motion of a tanker, wave dir. 180 deg.

Water depth 82.50 m

Fig. 151
Transverse drift force on a tanker, wave dir. 90 deg.

Figure 152

Water depth 22.68 m

Wave frequency, r/s

Fig. 152
Longitudinal drift force on a tanker, wave dir. 135 deg.

Wave frequency, r/s

Water depth 22.68 m

Fig. 153
Transverse drift force on a tanker, wave dir. 135 deg.

Water depth 22.68 m

Fig. 154
Yaw drift moment on a tanker, wave dir. 135 deg.

![Graph showing yaw drift moment vs. wave frequency](image)

\[ M_{V}/L^2 \]

Wave frequency, \( r/s \)

Water depth 22.68 m

Fig. 155
Longitudinal drift force on a tanker, wave dir. 180 deg.

Fig. 156
Transverse drift force on a tanker, wave dir. 90 deg.

Fig. 157
Longitudinal drift force on a tanker, wave dir. 135 deg.

Water depth 30.2 m

Fig. 158
Transverse drift force on a tanker, wave dir. 135 deg.

Water depth 30.2 m

Fig. 159
Fig. 160

Yaw drift moment on a tanker, wave dir. 135 deg.

- DELFRAC (near-field meth.)
- DELFRAC (far-field meth.)
- Calcul. MARIN, 352 facet elements
- Experiment MARIN, regular waves

Water depth 30.2 m
Longitudinal drift force on a tanker, wave dir. 180 deg.

Fig. 161
Transverse drift force on a tanker, wave dir. 90 deg.

Water depth 82.50 m

Fig. 162
Longitudinal drift force on a tanker, wave dir. 135 deg.

Water depth 82.50 m

\[ \frac{F_x}{\zeta^2} \]

\[ \omega \sqrt{\text{L/g}} \]

Fig. 163
Transverse drift force on a tanker, wave dir. 135 deg.

Fig. 164

Water depth 82.50 m

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Fig. 164
Yaw drift moment on a tanker, wave dir. 135 deg.

Water depth 82.50 m

\[ \frac{M_y}{c_A^2} \]

\[ \omega^* \sqrt{L/g} \]

Fig. 165
Longitudinal drift force on a tanker, wave dir. 180 deg.

Water depth 82.50 m

Fig. 166