TECHNISCHE HOGESCHOOL DELFT

LUCHTVAART- EN RUIMTEVAARTTECHNIEK

DELFT UNIVERSITY OF TECHNOLOGY
DEPARTMENT OF AEROSPACE ENGINEERING

Memorandum M-526

FIRST ORDER SECOND MOMENT ANALYSIS
OF THE BUCKLING OF SHELLS WITH
RANDOM IMPERFECTIONS

I. Elishakoff*
S. van Manen**
P.G. Vermeulen***
J. Arbocz****

November 13 84

- * Department of Aeronautical Engineering, Technion-IIT, Haifa, Israel.
- ** TNO-Institute for Building Materials and Structures, Rijswijk, The Netherlands.
- *** Fokker-Schiphol Oost, Amsterdam, The Netherlands.
- **** Department of Aerospace Engineering, Delft University of Technology, Delft, The Netherlands.

ABSTRACT

First order second moment analysis is given for nominally circular cylindrical shells possessing axisymmetric as well as general nonsymmetric random imperfections. Results of measurements of initial imperfection Fourier coefficients are used to construct their second order statistical properties. "The MIUTAM" computer code is applied for determination of buckling loads. Results of reliability calculations are compared with those delivered by the Monte Carlo method and are found to be in good agreement.

INTRODUCTION

A probabilistic approach to buckling analysis of structures has been first suggested in a study of imperfection sensitive structures by Bolotin [1], who postulated, in brief, that the random buckling load Λ_s of a structure can be expressed as a function of a finite number of random variables $\bar{X}_i$ representing the initial imperfection Fourier coefficients. Thus

$$\Lambda_s = \varphi(\bar{X}_1, \bar{X}_2, \dots, \bar{X}_n) \quad (1)$$

where n is the number of the imperfection modes retained. Given a particular function φ , the probability density of Λ_s can be derived from the joint probability density of the $\bar{X}_i$'s.

Bolotin applied this method to a cylindrical panel under a uniform compressive load along its curved edges, with the initial imperfections being represented by a single normally distributed amplitude parameter. A single-term Galerkin approximation yielded an equation of type (1). Makaroff [2] utilized a simplified expression of form (1) to derive the statistical properties of the buckling loads. However, to use Amazigo's [3] words, "it is a non-trivial problem to obtain a relation of type (1) and to perform the above analysis for $n > 2$, say. It is this difficulty that limits the effectiveness of this method." Indeed, there is no formula of type (1) in the literature for multiparametric imperfections, so that the direct, analytical evaluation of the buckling loads seems to be a formidable task.

On the other hand, procedures have been developed [4-8], dealing with the determination of the buckling loads numerically. We limit ourselves to mentioning the Multi-Mode Analysis (MIUTAM) by Arbocz and Babcock [5], the well-known general purpose code STAGS [6, 7] and the Dutch multi-purpose finite element package DIANA [8]. In addition, recently the results of extensive initial imperfection

surveys have been directly incorporated into the probabilistic analysis of shells with random imperfections [9,10], without resorting to the number of restrictive assumptions utilized in the literature on the probabilistic buckling of imperfect structures.

The question which arises in this context is as follows:

- "Is it possible to develop a simple but rational method of checking the reliability of the shells, making use of some statistical measures of the imperfections involved, in order to provide an estimate of the structural reliability without recourse to the cumbersome and time consuming Monte Carlo method?"

Of course, even having the relation of type (1), it would be an enormous task to use it for the reliability calculations due to the cumbersome integration in a multidimensional space. Alternatively, the first order second moment approach [11,13] is known for many years to those engaged in the probabilistic analysis of structures. This method, requiring only the knowledge of mean-values and elements of variance-covariance matrix of basic variables (imperfection Fourier coefficients), will be adopted in this paper.

ANALYSIS

The cornerstone of the method is the knowledge of the deterministic state equation

$$Z = \varphi(\bar{X}_1, \bar{X}_2, \dots, \bar{X}_n) \quad (2)$$

where the nature of the so-called performance function $\varphi(\dots)$ depends on the type of the structure and the limit state considered, then the equation

$$Z = 0 \quad (3)$$

determines the failure boundary, $Z < 0$ implies failure and $Z > 0$ indicates non-failure (successful performance). Then the use of the first order second moment method requires linearization of the function φ_x at the mean point, and the knowledge of the distribution of the random vector $\bar{X}$. Relatively simple calculations are to be performed if $\bar{X}$ is normally distributed. If $\bar{X}$ is not distributed normally, an appropriate normal distribution has to be substituted instead of the actual one. In our case, we are interested in knowing the reliability of the structure at any given load λ , that is

$$R(\lambda) = \Pr(\Lambda_s > \lambda) \quad (4)$$

Here a function Z can be defined as follows:

$$Z(\lambda) = \Lambda_s - \lambda = \varphi(\bar{X}_1, \bar{X}_2, \dots, \bar{X}_n) - \lambda \quad (5)$$

where λ is the applied deterministic load. The question arises, whether due to the absence of a straightforward deterministic relation connecting Λ_s and the $\bar{X}_i$'s the first order second moment analysis is unfeasible?

Indeed, it is impossible to perform such an analysis analytically. It can be done, however, numerically as it was performed for a different problem in Ref. 14. To combine the numerical codes with the mean-value first order second moment method, we need to know the lower order probabilistic characteristics of Z . In the first approximation, the mean value will be determined as follows:

$$\begin{aligned} E(Z) &= E(\Lambda_s) - \lambda \\ &= E[\varphi(\bar{X}_1, \bar{X}_2, \dots, \bar{X}_n)] - \lambda \approx \varphi[E(\bar{X}_1), E(\bar{X}_2), \dots, E(\bar{X}_n)] - \lambda \end{aligned} \quad (6)$$

This corresponds to the use of the Laplace approximation of the moments of the nonlinear functions. The value of

$$\varphi[E(\bar{X}_1), E(\bar{X}_2), \dots, E(\bar{X}_n)] \quad (7)$$

should be calculated numerically by either of the codes [5], [7] or [8]. It corresponds to the deterministic buckling load of the structure possessing mean imperfection amplitudes.

The variance of Z is given by

$$\begin{aligned} \text{Var}(Z) &= \text{Var}(\Lambda_s) \\ &\approx \sum_{j=1}^n \sum_{k=1}^n \left(\frac{\partial \varphi}{\partial \bar{X}_j} \right)_{\bar{X}_j=E(\bar{X}_j)} \left(\frac{\partial \varphi}{\partial \bar{X}_k} \right)_{\bar{X}_k=E(\bar{X}_k)} \cdot \text{COV}(\bar{X}_j, \bar{X}_k) \end{aligned} \quad (8)$$

Calculation of the derivatives $\partial \varphi / \partial \bar{X}_j$ are performed numerically by using the following numerical differentiation formula:

$$\frac{\partial \varphi}{\partial \bar{X}_j} \approx \frac{\varphi(\bar{X}_1, \bar{X}_2, \dots, \bar{X}_{j-1}, \bar{X}_j + \Delta \bar{X}_j, \bar{X}_{j+1}, \dots, \bar{X}_n) - \varphi(\bar{X}_1, \bar{X}_2, \dots, \bar{X}_n)}{\Delta \bar{X}_j} \quad (9)$$

at values of $\bar{X}_j = E(\bar{X}_j)$. To find these derivatives it is necessary to carry out n calculations of the buckling problem.

Having estimated $E(Z)$ and $\text{Var}(Z)$ the estimate for the probability of failure will be

$$P_f(\lambda) = \Pr(Z < 0) = \Phi(-\beta) \quad (10)$$

at the load level λ . In Eq. (10)

$$\beta = E(Z)/\sqrt{\text{Var}(Z)}$$

is the reliability index. Accordingly, reliability will be estimated as

$$R(\lambda) = \Pr(Z > 0) = 1 - \Phi(-\beta) = \Phi(\beta) \quad (11)$$

NUMERICAL ANALYSIS

For the actual calculations we used the data associated with the so-called B-shells [15]. The geometrical and material properties of the B-shells are provided in Table 1.

Since the measurements only included 41 points in axial direction and 49 points in circumferential direction, therefore it is not feasible to compute those Fourier coefficients the order of which exceeds the "cut-off" values of $k=20$ and $\ell=24$. For these coefficients the Donnell-Imbert imperfection model was utilized.

$$\bar{X}_{k,\ell} = \frac{\bar{X}}{k^r \ell^s} \quad (12)$$

where k is the number of axial half-waves and ℓ is the number of circumferential full waves. Values of measured imperfections are given in Table 2. Also shown is the complete imperfection model used with an indication which coefficients were actually measured and which ones were extrapolated in accordance with Eq. (12). The parameters $\bar{X}$, r and s are determined by least square fitting, the distribution of the measured Fourier coefficients.

The mean vector and the variance-covariance matrix of the Fourier coefficients, treated as random variables, are given in Table 3. In order to apply the first order second moment method, at first the mean buckling load has to be calculated. For this purpose the Multi-Mode Analysis is utilized (see Appendix A). The result of the calculation of the mean buckling load is given in Fig. 1. $E(\Lambda_s) = 0,746$ (i.e. 74,6% of the classical buckling load). The mean buckling load calculated via the Monte Carlo method is $E(\Lambda_s) = 0,739$, thus the difference between

mean buckling loads, delivered by these two methods is only 0,007 or 0,95%.

Next the derivatives were calculated. For the increment of the Fourier coefficients in Eq. (9), one percent of their original values are used, so that $\Delta \bar{X}_j = 0.01 \bar{X}_j$. The calculated derivatives are listed in Table 4. In this study the increments of end-shortening were chosen in such a way that the limit loads were found to an accuracy of 0.0001.

The results of the mathematical expectation and variance of Z are:

$$E(Z) = 0,746 - \lambda$$

$$\text{Var}(Z) = 0.0175$$

With formula (11) the reliability is calculated directly. In fig. 2 the reliability functions calculated with the Monte Carlo method and with first order second moment method are both given (for derivation of reliability by the Monte Carlo method see Appendix B). At first sight these curves are in good agreement. However, in the higher reliability region, which is important for design load derivation, the deviation is more noticeable.

CONCLUSIONS

From the results of the calculations it may be concluded that:

- a. the first order second moment method can be successfully used for determining the reliability function of axially compressed shells,
- b. the number of buckling load calculations necessary for the first order second moment method is significantly less then with the Monte Carlo method,
- c. the mean buckling loads due to both methods are in excellent agreement, but still higher than the experimental value. This is caused by the simplified deterministic buckling load analysis. Since the present method does not need as many calculations as the Monte Carlo method using a more advanced and expensive (in terms of the computer time) method can be utilized,
- d. the reliability functions delivered by both methods are in good agreement.

Work on using the level II "design point" iterative method is underway presently and the results will be reported in due course.

ACKNOWLEDGEMENTS

This research has been supported by STW (Stichting voor Technische Wetenschappen) and has been carried out during I.E.'s stays at TH-Delft in the summers of 1983 and 1984. This aid and warm hospitality are gratefully appreciated. Partial support to this study has been provided by the Technion's Foundation for Promotion of Research.

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Appendix A

Multi-mode Analysis for Buckling Load Calculation

Using the sign convention defined in Fig. 1. the Donnell type nonlinear equations for imperfect stiffened cylindrical shells can be written [16] (W is positive inward).

$$L_H(F) - L_Q(W) = -\frac{1}{R}W_{,xx} - \frac{1}{2}L_{NL}(W, W+2\bar{W}) \quad (A.1)$$

$$L_Q(F) + L_D(W) = \frac{1}{R}F_{,xx} + L_{NL}(F, W+\bar{W}) \quad (A.2)$$

where the linear operators are

$$\begin{aligned} L_D() &= D_{xx}(),_{xxxx} + D_{xy}(),_{xxyy} + D_{yy}(),_{yyyy} \\ L_H() &= H_{xx}(),_{xxxx} + H_{xy}(),_{xxyy} + H_{yy}(),_{yyyy} \\ L_Q() &= Q_{xx}(),_{xxxx} + Q_{xy}(),_{xxyy} + Q_{yy}(),_{yyyy} \end{aligned} \quad (A.3)$$

and the nonlinear operator is

$$L_{NL}(S, T) = S_{,xx}T_{,yy} - 2S_{,xy}T_{,xy} + S_{,yy}T_{,xx} \quad (A.4)$$

Commas in the subscripts denote repeated partial differentiation with respect to the independent variables following the comma. The stiffener properties have been "smeared" out to arrive at effective bending, stretching and torsional stiffnesses. The stiffener parameters $D_{xx}, H_{xx}, Q_{xx}, D_{xy}, \dots$ etc. are defined in [4]. Here $\bar{W}$ is the initial radial imperfection, W is the component of displacement normal to the shell midsurface and F is the Airy stressfunction.

Let us represent the $(m+1)$ th approximation to a solution of the above equations by

$$\begin{aligned} W_{m+1} &= W_m + \delta W_m \\ F_{m+1} &= F_m + \delta F_m \end{aligned} \quad (A.5)$$

where

$W_m, F_m = m^{th}$ approximation to the solution,

$\delta W_m, \delta F_m =$ correction to the m^{th} approximation.

Substituting into Eqs. (A.1) and (A.2) and neglecting products of the correction quantities yields a set of linear partial differential equations for determining the correction terms. If one represents the initial imperfections by

$$\bar{W} = \tau \sum_{i=1}^{N_1} \bar{W}_{i0} \cos i\bar{x} + \tau \sum_{k,l=1}^{N_2} \bar{W}_{kl} \sin k\bar{x} \cos l\bar{y} \quad (A.6)$$

where $\bar{x} = \pi x / L$ and $\bar{y} = y / R$, then the linearized governing equations admit separable solutions of the form

$$\begin{aligned} \begin{Bmatrix} W_m \\ \delta W_m \end{Bmatrix} &= \tau \begin{Bmatrix} W_v \\ 0 \end{Bmatrix} + \tau \sum_{i=1}^{N_1} \begin{Bmatrix} W_{i0} \\ \delta W_{i0} \end{Bmatrix} \cos i\bar{x} + \tau \sum_{k,l=1}^{N_2} \begin{Bmatrix} W_{kl} \\ \delta W_{kl} \end{Bmatrix} \sin k\bar{x} \cos l\bar{y} \\ \begin{Bmatrix} F_m \\ \delta F_m \end{Bmatrix} &= \frac{ERt^2}{c} \begin{Bmatrix} -\lambda \bar{y}^2 / 2 \\ 0 \end{Bmatrix} + \frac{ERt^2}{c} \sum_{i=1}^{N_1} \begin{Bmatrix} F_{i0} \\ \delta F_{i0} \end{Bmatrix} \cos i\bar{x} + \frac{ERt^2}{c} \sum_{k,l=1}^{N_2} \begin{Bmatrix} F_{kl} \\ \delta F_{kl} \end{Bmatrix} \sin k\bar{x} \cos l\bar{y} \end{aligned} \quad (A.7)$$

where $W_v = \frac{-v}{c} \frac{\bar{H}_{xx}}{1+\mu_1} \lambda$; $c = \sqrt{3(1-v^2)}$

The unknown coefficients are determined by Galerkin's procedure yielding a set of linear algebraic equations in terms of the unknown correction terms. In matrix notation

$$\begin{aligned} [A] \{ \delta F \} + [B] \{ \delta W \} &= -\{ E^{(1)} \} \\ [C] \{ \delta F \} + [D] \{ \delta W \} &= -\{ E^{(2)} \} \end{aligned}$$

To obtain the buckling load for a given imperfect cylindrical shell one begins by making an initial guess for $\{W\}$ and $\{F\}$ at a small initial load level λ . Iteration is then carried out until the correction vectors are smaller than some preselected value. The converged solutions then are used as the initial given at

the next higher axial load level $\lambda + \Delta\lambda$. The entire process is repeated for increasing values of the axial load parameter λ . The nonlinear analysis then will locate the limit point of the prebuckling states. By definition the value of the loading parameter λ corresponding to the limit point will be the theoretical buckling load.

It is shown in [4] that the solution satisfies the circumferential periodicity. It also contains details of the coefficient matrices A,B,C and D and the error vectors $E^{(1)}$ and $E^{(2)}$.

Appendix B

Monte Carlo Method for Reliability Derivation

This approach, which has been initiated for the imperfection-sensitivity of structures by Elishakoff [17,18] in a consistent manner, treats the Fourier coefficients of the Fourier series, in which the initial imperfections are decomposed as random quantities. From the measured initial imperfections of N sample shells, the sample mean and sample covariance matrix are calculated by ensemble averaging:

$$\vec{x}^{(m)} = \frac{1}{N} \sum_{n=1}^N \vec{x}^{(n)} \quad (B.1)$$

$$V^{(m)} = \frac{1}{N-1} \sum_{n=1}^N \left(\vec{x}^{(n)} - \vec{x}^{(m)} \right) \left(\vec{x}^{(n)} - \vec{x}^{(m)} \right)^T \quad (B.2)$$

Where superscript "m" denotes 'measured'.

An advantage of this method is that the statistical parameters of the initial imperfections are estimated from the real measurements on the shell profiles. The only assumption made is that the Fourier coefficients have a multivariate normal distribution. The estimated covariance matrix is then decomposed into a choleski decomposition by:

$$V^{(m)} = C C^T \quad (B.3)$$

With a random number generator a set of $N_s \gg N$ random vectors $\vec{R}$ with dimension p is generated, all from a normally distributed population with zero mean and unit variance. With these random vectors, N_s new realizations of imperfect shells can be simulated via:

$$\vec{x}^{(s)} = C \vec{R}^{(s)} + \vec{x}^{(m)} \quad (B.4)$$

Where superscript "s" denotes 'simulated'.

These new realizations are used as an input for a deterministic buckling load analysis. These calculations yield N_s buckling loads. From the histogram of buckling loads one then calculates the reliability function $R(\lambda)$ via frequency interpretation (i.e, fraction of an ensemble). Here $R(\lambda)$ is defined as the probability that the random buckling load Λ will exceed the specified load λ . For detailed discussion of the Monte Carlo method in context of the probabilistic

mechanics, one can consult the recent combined textbook-monograph [19].

LIST OF SYMBOLS

C	lower triangular matrix
D_{xx}, D_{xy}, D_{yy}	effective bending stiffnesses
E	modulus of Elasticity
$f(\dots)$	deterministic function
F	Airy stress function
k, ℓ	wavelengths
H_{xx}, H_{xy}, H_{yy}	effective stretching stiffnesses
L, L_{HA}	length
n	number of modes retained
N	number of measured shells
N_s	number of simulated shells
Q_{xx}, Q_{xy}, Q_{yy}	effective torsional stiffnesses
P_f	probability of failure
P_{exp}	experimental buckling load
r	empirical constant
R	shell radius
$\vec{R}$	standard normal vector
$R(\lambda)$	reliability
s	empirical constant
t	shell thickness
V	variance-covariance matrix
$\bar{W}$	radial imperfection from perfect circular cylinder
W	radial displacement - positive inward
$Var(\dots)$	variance
$\vec{x}$	imperfection vector
$\bar{X}$	empirical constant
$\bar{X}_1$	imperfection Fourier coefficients
Z	performance function
β	reliability index
ν	Poisson's ratio
λ	nondimensional applied load
Λ_s	nondimensional random buckling load
$\varphi(\dots)$	deterministic function
$\Phi(\dots)$	standard normal distribution function

TABLES

	R	t	L	L _{HA}	P _{exp}
B-1	101.60	0.2050	196.85	171.45	11326.0
B-2	101.60	0.1852	144.78	121.92	7178.5 *
B-3	101.60	0.1491	140.97	121.92	-
B-4	101.60	0.2634	140.97	121.92	16661.2

For all shells $E = 1.065 \times 10^5 \text{ N/mm}^2$ and $\nu = 0,3$

* shell had a visible stable prebuckle at a spot where the surface had been scratched by a sharp object

Note: all geometrical dimensions in (mm),

L_{HA} - length used for harmonic analysis

Table 1 - Geometrical and material data on the group of B-shells (6)

	B-1	B-2	B-3	B-4
A _{2,0}	-0.010809	-0.027238	-0.089906	-0.017560
A _{4,0}	0.022578	-0.007836	-0.025508	-0.009239
C _{1,2}	0.417400	0.392870	0.741280	0.222900
C _{1,6}	-0.077872	-0.143490	0.017483	0.077668
C _{1,8}	-0.263690	-0.009405	0.112470	0.101510
C _{1,10}	0.036568	0.043628	-0.245610	-0.008853
C _{2,3}	-0.101290	0.034018	-0.064766	-0.001887
C _{2,11}	0.009732	-0.008685	-0.028261	0.013545

Table 2 - Imperfection model used and values of the Fourier coefficients

-.0364	1	.1319								
-.0050	2	.0566	.0402							
.4436	3	-.7074	-.1916	4.686						
-.0316	4	-.0926	-.0810	-.0872	.9670					
-.0148	5	-.3646	-.3327	.6154	.9956	3.057				
-.0436	6	.4771	.1986	-2.478	-.6529	-1.372	1.868			
-.0335	7	.0384	-.0518	-.5978	-.0833	.5647	.2623	.3710		
-.0034	8	.0646	.0272	-.3739	.0205	-.1497	.2068	.0022	.0369	

mean vector

covariance matrix multiplied by 100

Table 3 - The sample mean vector and sample covariance matrix.

x_j	$\partial\varphi_s/\partial x_j$
$A_{2,0}$	.09668
$A_{4,0}$	.00340
$C_{1,2}$	-.01854
$C_{1,6}$	-.05687
$C_{1,8}$	-.24686
$C_{1,10}$	-.08183
$C_{2,3}$	.01314
$C_{2,11}$	-.07173

Table 4 - The derivatives of φ_s with respect to the Fourier coefficients

Figure 1 - Response curve with mean imperfections in the $\lambda - \delta_{NL}$ plane

Figure 2 - The reliability curves for the group of B-shells, calculated via the Monte Carlo method and the mean value, first order second moment method