Stochastic Open Pit Design with a Network Flow Algorithm: Application at Escondida Norte, Chile

Master of Science Thesis

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TUDelft

June 2011
Stochastic Open Pit Design with a Network Flow Algorithm: Application at Escondida Norte, Chile

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Thesis M. Sc., June 2010

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Thanks to:
Roussos Dimitrakopoulos for guidance and supervision during the thesis, and the COSMO laboratory for the chance to work with them.
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Abstract
In the optimization of open pit mine design, the Lerchs-Grossmann algorithm is the industry standard, although network flow algorithms are also well suited, efficient, and known. The stochastic version of the conventional (deterministic) network flow algorithm is based on the use of multiple simulated realizations of the ore deposit, thus accounting for geological uncertainty. In comparison, the conventional pit optimization methods use only one estimated or average-type model of the deposit and assume it represents the exact deposit in the ground. The use of multiple scenarios results in the ability to generate risk profiles in terms of both grade and material types for pit designs and production schedules. This thesis focuses on the application of the stochastic maximum flow algorithm for multiple ore processing destinations at the Escondida Norte copper mine, Chile. The case study shows the optimal pushback layout minimising geological risk during the life-of-mine. The limitation of this method is that it uses only a part of the local joint uncertainty of the block grades and material types. However, it can be extended to account for simulated commodity price forecasts as well as discounting.
Uittreksel
In de optimalisatie van mijnbouwkundige operaties wordt het Lerchs-Grossmann algoritme standaard in de industrie gebruikt, alhoewel “network flow” algoritmes goed toepasbaar, efficiënt and bekend zijn. De stochastische versie van de conventionele (deterministische) “network flow” algoritme is gebaseerd op het gebruik van meerdere realisaties van het ertslichaam. Dit zorgt ervoor dat er rekening wordt gehouden met de onzekerheid van de geologie. In vergelijking, conventionele open pit optimalisatie methoden gebruiken slechts een geraamd of gemiddeld model van het ertslichaam en neemt aan dat dit model een exacte representatie is van het ertslichaam onder de grond. Het gebruik van meerdere mogelijkheden van het ertsmodel resulteert in de mogelijkheid om risico profielen te genereren van het ertsgehalte en materiaal type voor de dagbouw ontwerpen en productie schema.

Deze thesis focust op een toepassing van het stochastische maximum flow algoritme voor een erts met meerdere verwerkingsstromen voor de Escondida Norte kopermijn in Chili. De casus toont het optimale pushback ontwerp bij een minimaal geologisch risico gedurende de “life-of-mine”. De limit van deze methode is dat zij alleen maar gebruikt maakt van een gedeelte van de locale onzekerheid voor het ertsgehalte en de materiaal typen. De methode zou kunnen worden uitgebreid met discontering, metaalprijssimulaties en -voorspellingen.
1. Introduction

Open pit mine design may be seen as an optimization process, where the net present value (NPV) of metal production is maximized. This is based on assigning a mining block to a certain period of extraction. The process assumes various constraints, such as the maximum mining and processing capacities, reserves, slope constraints, and so on.

In this thesis, different optimization methods are described, including the conventional (deterministic) Lerchs-Grossman algorithm and network flow algorithms as well as stochastic extension with multi-destinations.

1.1 Open Pit Optimization

The optimization of a mine design may be formulated as a complex, large scale mixed integer program (MIP) (Ramazan and Dimitrakopoulos, 2004, 2006; Ramazan, 2007; Hustrulid and Kuchta, 2006; Zuckerberg et al., 2010). However large MIP’s are hard to solve and limited by the numbers of mining blocks (binary variables) used. Given these practical limitations, the established conventional practice considers a pit design in three steps. The first step of this process is to generate the optimal ultimate pit, by implementing the Lerchs-Grossman (L-G) algorithm (Lerchs and Grossman, 1965) as a graph closure problem. During this step, the ultimate pit is generated from the orebody model. In the second step, nested pits are created within this ultimate pit by changing the capacities of the arcs between the nodes of the graph; this process is termed pit parameterization. In the third step, the nested pits are combined to obtain a pushback design, and later on, a production schedule is added. Pushbacks are generated by combining nested pits so as to maximize the net present value of the pit design (pit limit and pushbacks).

The conventional open pit optimization methods have several limits. First, they use a single average type fixed orebody model; this model does not account for any uncertainty (Ravenscroft, 1992; Dowd, 1997; Dimitrakopoulos et al., 2002, 2007a, 2007b). An additional problem is the large differences in pushback sizes, known as the gap (Meagher et al., 2011). Conventional methods are based on the assumption that a single estimated orebody model or any other input is the actual one; this can lead to misleading optimization results (Dimitrakopoulos et al., 2002).
New methods, including the method employed herein, use multiple realisations of a deposit to quantify risk on the metal content and material type, thus establishing economic values of mining blocks. Optimisers that work with multiple simulations of an orebody will give a distribution of models instead of a single model estimate, as with conventional methods.

### 1.2 A Review of Methods for Open Pit Design

The common conventional open pit optimization methods are reviewed next. Their shortcomings with respect to risk are discussed in the technical literature (Ravenscroft, 1992; Dowd, 1994, 1997; Dimitrakopoulos et al., 2002, 2007a, 2007b).

All of these methods are based on the economic block values (EBV), which is the main parameter to consider if a block in a mine is ore or waste. The calculation of this block value contains several parameters, for example grade, tonnage, metal (selling) price and treatment cost, processing cost, recovery, mining cost, etc. The equations of the economic block values are shown below:

\[
EBV = GR \times T \times Rec \times (Pr - TC) - T \times PC \times PCAF - T \times MC \times MCAF \tag{1}
\]

\[
EBV = -T \times MC \times MCAF \tag{2}
\]

where \( EBV \) is the economic block value of the block, \( GR \) is the grade, \( T \) is the block tonnage, \( Rec \) is the recovery of the ore processing, \( Pr \) is the metal price of the commodity, \( TC \) is the treatment charge and selling costs, \( PC \) is the processing cost, \( PCAF \) is the process cost adjustment factor and is based on the additional processing cost for a certain block (for example a different material type), \( MC \) is the mining cost, and \( MCAF \) is the mining cost adjustment factor, which increases with distance (depth) from the processing plants. \( PCAF \) and \( MCAF \) are not always available; when available these factors are simulated (\( PCAF \)) or assigned bench by bench during the orebody modelling and pit design. The \( EBV \) and the other parameters can differ for each block.

Equation 1 is valid for ore, \( EVB \geq 0 \). And equation 2 is valid for waste, \( EBV < 0 \), where only the mining cost will be considered. When a mining operation has different process destinations for ore blocks, multiple economic block values are calculated.
1.2.1 Lerchs-Grossman Algorithm and Variants

The Lerchs-Grossman algorithm (Lerchs and Grossman, 1965) was the first optimization method used to design large open pits in a reasonable amount of time (Zhao and Kim, 1992). It is used in mining optimization software as the industry standard, for example Gemcom’s Whittle software (Whittle 1998a, 1998b, 1999), to find the optimal pit and pushbacks. The method works as follows: first, a directed graph (Bondy and Murty, 1976) is constructed with the nodes of the orebody, the blocks in the orebody model. These connected blocks have certain constraints, for example precedence and slope constraints. The method builds, by connecting the blocks, a tree with a dummy node and strong and week arcs between the nodes, as shown in Figure 1.1. When the constraints are satisfied, the pit has the maximum closure graph at a certain scaled capacity. The Lerchs-Grossman algorithm is well documented in the technical literature (Lerchs and Grossman, 1965; Zhao and Kim 1992; Seymour, 1995; Hustrulid and Kuchta 2006). Figure 1.1 shows the working of the Lerchs-Grossman algorithm (L-G) (Meagher et al., 2010). In step one, the blocks/nodes are connected to the dummy node, X0, with arcs from X0. Step two shows the initial normalized tree, the positive strong (ps) arcs are plus arcs supporting a strong branch, positive weak (pw) are plus arcs supporting a weak branch. Step three depicts merging branches X4 and X6; the arc between X0 and X6 is removed. Minus weak (mw) denotes a minus arc supporting a strong arc. Step four shows the tree when all the weak branches above X6 are merged. Step five shows the final graph closure with the strong branches connected to the dummy node.
A modified version of the Lerchs-Grossmann Algorithm developed by Seymour (1995) uses the same approach. The method incorporates what is known as parameterization. Open pit parameterization produces the maximum valued pit as a function of another parameter, in this algorithm, pit volume. It does not deal with only one single tree as in the L-G algorithm; it contains a set of several branches (sub trees), where the branch strength is determined by dividing its value by its mass. A “threshold” value is used to determine if a branch is weak or strong, and by altering the threshold value, a series of nested pits can be created. All the strong branches combined form the normalized tree of the L-G algorithm when the threshold is set to a minimum. The method is slow for large-size open pits, compared with L-G, and, in addition, does not address the gap problem. Also, during the optimization process, heuristics are needed to produce a feasible schedule.
1.2.2 Mixed Integer Programming (MIP) and Blasor

Blasor is a long-term mine planning and optimization software developed by the BHP Billiton’s R&D group (Stone et al., 2004; Menabde et al., 2007), and it is based on mixed integer programming (Zuckerberg et al., 2010). Blasor is designed to handle multi-pit operations where ore is blended. Its goal is to find the optimal mining sequence, which maximises discounted cash flow. Blasor takes the market tonnage, grade and the ore quality into account, the mining sequences, the life-of-mine, where the discounted cash flow is maximized, and thus the ultimate pit limit. Blasor requires input constraints for an operation, for instance slope constraints, mining rates and the capacity of the down stream supply chain, market tonnage, ore quality (blended), and grade constraints (cut off grade). The material is, during the calculation process, assigned to bins according to the properties of blocks (called clumps). The material in these clumps is considered homogeneous and will be used further in calculations. The steps of the MIP in Blasor are briefly explained below.

1. Aggregation of blocks, using binning. Ore blocks are allocated into clumps.
2. Mixed integer program is run, so a schedule can be acquired. Then, this schedule is used to design the pushbacks of the mine.
3. Mining pushbacks design from schedule using a fuzzy smoothing algorithm.
4. Valuation of the optimal panel sequence. After the pushback design, panels are designed; these are between the pushbacks and the benches in the mining operation. The panels are based on tonnages, discounted cash flow, market, and process constraints.

Blasor can find the optimal solution in the complex optimization of open pit mines, for example a multiple pit operation. However, as with any other method, some compromises and simplifications have to be made during the design of the mining pushbacks, so as to generate solutions in a reasonable amount of time.

1.3 Deterministic Network Flow Algorithm

The network flow algorithm, NFA (Ahuja and Orlin, 1989), is an algorithm for designing an open pit using the maximum flow (minimum cut) of arcs between the nodes, under the context of graph closure (Picard, 1976). The nodes are the blocks in the mining blocks in the orebody model. This includes the orebody but also the topographic surface. Arcs are constrained between
the nodes; these arcs all bear a capacity. The goal of the maximum flow algorithm is to cut the arcs which carry the least capacity (Goldberg, 1988; Gallo et al., 1989; Faaland et al., 1990).

In the network flow algorithm, all mining blocks are seen as nodes; they are data points in three-dimensional space. These nodes are connected to each other by capacity arcs. In order to run the algorithm, a source and a sink node are established. The source and sink nodes are data points created outside the data set. The arcs in the directed graph $G=(V,A)$, where a node in the graph represents a block in the orebody model, have a capacity based on the economic block value; the source node is connected by an arc to the positive EBV, ore, and the sink node to a negative EBV, waste (Hochbaum and Chen, 2000; Hochbaum, 2001, 2003, 2004 and 2008). Arcs are the connections between the nodes, for example between blocks that are on top of each other or have a connection to the source and sink. In addition to the arcs to the source and sink based on EBV, there are arcs between the blocks; these are the slope constraints and have an unlimited capacity. This is because overlying blocks have to be mined before the extraction of the target block. Since there is no direct path from the source node to the sink node, a cut has to be chosen to design the open pit. This cut will be a minimum cut so that the set of capacity arcs which will be cut bear the minimum possible capacity; in other words, the sum of the capacity of arcs cut is minimal. This means there is a minimal amount of ore outside the pit and there is a minimal amount of waste in the open pit (the minimum cut graph).
Figure 1.2: An example of the use of a network flow algorithm (NFA) on a single scenario conventional (estimate) orebody model
2 A Stochastic Network Flow Algorithm

A stochastic version of the network flow algorithm (SNFA) uncertainty (Albor and Dimitrakopoulos, 2009) in the EBV is used to account for the geological risk in open pit design (Meagher et al., 2009; Meagher, 2011). Simulated equally probable models of an orebody are used by SNFA to manage uncertainty (Dimitrakopoulos et al. 2002; Leite and Dimitrakopoulos, 2007; Leite, 2008; Dimitrakopoulos and Ramazan, 2008). Instead of working with one precise and possible wrong model, the calculations are done on a set of scenarios (Meagher et al., 2009, 2010; Chatterjee et al., 2009, 2010). This maximizes the net present value (NPV) for an open pit, given the uncertainty from limited data and orebody models.

The method works as follows: When the simulations are run, the economic block value of each block in each simulation is calculated. Then, the blocks with a negative EBV will be connected to a sink which is the same for all the simulations (one single sink node). The same is done with the blocks with a positive economic block value; these are connected to a single source node and the precedence constraints have to be honoured. In addition, blocks that have the same location in the grid (x, y, z) will have infinite number of capacity arcs between them. This new constraint exists because the same blocks in the grid have to be either inside or outside the pit for each of the simulations since only one pit is being designed.

The next step of the algorithm is to merge the blocks at the same position to one block (node), which has maximum one arc from the source node and one arc to the sink node. Where these arcs have the capacity of the sum of the capacity of the arc at the same location in the single simulations (Meagher et al., 2010). This will result in a single graph (matrix), which makes the process less computationally demanding, because the number of nodes is massively reduced and only one minimum cut calculation is needed. When more simulations are added to the process, the capacity of the arcs will noticeably increase.

For example there are three different simulations, all of these are equal probable to occur, figure 2.1a. Thus when the network flow algorithm is used three different ultimate pit designs will be generated, figure 2.1b. These ultimate pits could all be the actual ultimate pit. By using a
stochastic data input, thus multiple scenarios, an ultimate pit can be generated taking this geological risk into account, as shown in Figure 2.2a to Figure 2.2b.

The limitation of this method is that it only looks at differences, and so distributions, of single blocks while the local uncertainty is uncertainty of a neighbourhood, and thus contains multiple blocks. By considering the uncertainty for each single block separately only a part of the local uncertainty is addressed.

Figure 2.1a: Example of network flow algorithm with multiple orebody simulations (NFA)

Figure 2.1b: Example of network flow algorithm with multiple orebody simulations (NFA)
**Figure 2.2a:** Example of network flow algorithm with multiple orebody simulations (SNFA)

**Figure 2.2b:** Example of network flow algorithm with multiple orebody simulations (SNFA)
2.1 An Extended Stochastic Network Flow Algorithm with Multi-Process Destinations

The past work on the stochastic network flow algorithm uses two rock destinations, ore and waste. However, in most mining operations, there are multiple destinations for the excavated material. The ore could, for instance, be sent to a mill, where it is crushed and won by flotation or a heap-leaching pad. The proposed algorithm will use the best economic block value of each process and assign it to the most valuable destination, which will result in the highest NPV over the mine life. The goal of this additional part of the algorithm is to place the mining block within the most likely group of process destinations, over all the simulations. The latter is done by forming arcs between the simulations with an indefinite capacity between the blocks at the same location in each simulation. This step of the process considers slope constraints and the mining and processing capacities. The method has the following steps:

1) Calculate EBV’s.
2) Select the best process for the block based on EBV (best pick).
3) Establish arcs between all the nodes (slope) and simulation (location and process).
4) Merge the values of all blocks.
5) Find the minimum cut based on the probability of the arcs.
6) Design the ultimate pit limits.
7) Design the pushbacks.

2.2 Mathematical Representation of Network Flow Algorithm for Multiple Process Destinations

Maximum flow can be described mathematically, showing that the limit of the optimal ultimate pit calculation is equivalent to the maximum graph closure mentioned. The ultimate pit limit and pushback design can be described as a graph, which is defined by the directed graph \( G=(V, A) \), where each block with a value \( c_i \) becomes a node in \( V \) and the directed arc in \( A \) is formed node \( i \) to node \( j \) if block \( j \) overlies block \( i \) (Meagher et al., 2009). This sets an extraction precedence of block \( j \) over block \( i \). Therefore the solution to this problem lies in finding the set of \( V' \subseteq V \), including maximum value nodes of the nodes along with all successors such that \( \Sigma_{v' \subseteq V} c_{ijk} \) is maximized, leading to the maximum closure on the graph (Hochbaum and Chen, 2000; Chandran and Hochbaum, 2009). Johnson (1968) established the relationship between the ultimate pit limit and the maximum flow problem. This relationship was presented

Maximize:

\[ Z = \sum_{i \in V} c_i x_i \]  

Subject to:

\[ x_i - x'_i \leq 0, i' \in \xi_i, i \in V \]  

\[ x_i \in \{0, 1\}, i \in V \]

Where \( x_i \) is a binary variable equal to one when the node \( i \) is inside the closure and zero otherwise. \( \xi_i \) is the set of successors of the block or node \( i \). And \( c_i = h_i + \lambda q_i \) reflects the modified block value in the parameterization, where \( h_i \) represents the economic dependence of \( \lambda \) and \( q_i \) represents the economic parameters linearity depending on \( \lambda \).

The maximum closure problem includes the open pit production capacity constraints through Lagrangian relaxation of the production capacity constraints. This leads to the classical maximum closure problem. In this approach, a source (s) and sink (t) node are augmented to the directed graph, which leads to the related graph \( \bar{G} = (\bar{V}, \bar{A}) \) such that \( \bar{V} = V \cup \{s, t\} \), where \( \bar{V} \) is the series of nodes connected with the source and sink. Therefore, the related graph consists of a set of positive value \( (c_i \geq 0) \) nodes \( V^+ = \{i \in V | c_i \geq 0\} \) and negative value \( (c_i < 0) \) nodes \( V^- = \{i \in V | c_i < 0\} \), representing ore and waste blocks in the deposit with, respectively, a set of arcs \( A \cup \{(s, v) | v \in V^+\} \cup \{(v, t) | v \in V^-\} \). The capacity of all the arcs in \( A \) is set to infinite \( (\infty) \) and the capacity of all the arcs connecting the sources is \( |c_v| \), such that \( c(s, v) = c_v \) for \( v \in V^+ \) and \( c(v, t) = -c_v \) for \( v \in V^- \). Thus, the source set of the minimum cut separating source and sink is also the maximum closure in the related graph.

In the proposed method, multiple ore destinations could be chosen. Therefore, Equations (3)-(5) have to be written, to consider the mining capacity constraints and the multiple destinations for the ore, as follows:
Maximize: \[ C_i = \sum_{k \in \mathcal{K}} c_{ik} \]  \hspace{1cm} (6)

\[ Z = \sum_{i \in \mathcal{V}} C_i x_i \]  \hspace{1cm} (7)

Subject to: \[ x_i - x'_i \leq 0, i' \in \mathcal{I}, i \in \mathcal{V} \]  \hspace{1cm} (8)

\[ \sum_{i=1}^{I} T_i x_i \leq b \]  \hspace{1cm} (9)

\[ x_i \in \{0,1\}, i \in \mathcal{V} \]  \hspace{1cm} (10)

Here, \( c_{ik} \) is the block value of each block \( i \) in process \( k \), including the waste dump. \( C_i \) is the maximum value of each block \( i \), taken from one of the processes, and \( b \) is the production capacity in tonnes of material of the pit in each period. \( Z_i \) is the best economic process for the block and thus the value which is used for the block. And \( x_i \) is one when the process is selected and zero otherwise.

The Lagrangian relaxation (Wang and Sevim, 1993; Asad and Dimitrakopoulos, 2010) of the pit production capacity constraints leads to the classical maximum closure. If \( \lambda \geq 0 \) are the multipliers associated with Equation (10), then a modification of the relaxation is needed, presented as:

Maximize: \[ C_i = \sum_{k \in \mathcal{K}} c_{ik} \]  \hspace{1cm} (11)

\[ Z = \sum_{i \in \mathcal{V}} [C_i - \lambda T_i] x_i \]  \hspace{1cm} (12)

Subject to: \[ x_i - x'_i \leq 0, i' \in \mathcal{I}, i \in \mathcal{V} \]  \hspace{1cm} (13)

\[ \sum_{i=1}^{I} T_i x_i \leq b \]  \hspace{1cm} (14)

\[ x_i \in \{0,1\}, i \in \mathcal{V} \]  \hspace{1cm} (15)

The solution to the relaxation is possible through an application of the parametric maximum flow algorithm over a number of iteration of \( \lambda \) values selected systematically, such that the pit limit
and pushbacks are developed (Dagdelen and Francois-Bangorcon, 1982; Francois-Bangorcon, 1984; Coleou 1989).

The equations show that the (stochastic) network flow algorithm uses only the information, and thus the probabilities, stored in the arcs between the nodes; therefore, not all the information available in the input data, the simulations, is used simultaneously. This means that the SNFA does not look at all the feasible combinations of the block values in the parts of the orebody. That is considered during the optimization and thus uses limited information for determining the probability of the blocks to have a given value.

2.4 Sequential Pushback Design

When the ultimate pit is found, a push re-label algorithm is used to find the different pushbacks within the ultimate pit (Meagher et al. 2011; Chatterjee et al., 2009). To generate these nested pits inside the ultimate pit, a parameterization of the minimum cut algorithm is implemented, through a Lagrangian relaxation. The method used in the L-G method can be used in the minimum cut algorithm, since both of the algorithms have comparable features: arcs bearing a load. When the method is used, the $\lambda$ must be monotonically increasing or decreasing in order to find pits with similar sizes, so as to avoid the gap-problem. When the EVB are directly multiplied by $\lambda$, an increasing $\lambda$ value will generate pits from small to large until the ultimate pit is reached, when $\lambda$ is one. The $\lambda$ is a value between zero and one, and the mining and processing cost and value of the nodes or blocks are multiplied by this factor. This creates several nested pits from a very low $\lambda$, to a $\lambda$ of one. When the parameterization, the $\lambda$, is used for mining and processing costs, the pits will become smaller with an increasing $\lambda$, starting with the calculation of the ultimate pit until there is no pit left. Multiple parameterizations are also used for the process, which will result, for example, in a combination of a lower metal value and a higher processing cost. The multiplication is used on the ultimate pit. Thus, the pit that has the maximum flow and the best ore process is selected for every block or node.

Changing the $\lambda$ value of the Economic Block Values to $\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_n$ where $\lambda_1 < \lambda_2 < \lambda_3 < \ldots < \lambda_n$ results in the nested pits $P_1, P_2, P_3, \ldots, P_n$ with the sizes of the pits $P_1 < P_2 < P_3 < \ldots < P_n$. In this thesis, a steady constant increasing $\lambda$ value was chosen, and the capacity of the arcs between the source and the nodes is multiplied by this value, creating a scaling in the values of the ore
blocks. The arcs from the nodes to the sink node (waste) are kept as-is, because the objective of the algorithm is to scale the economic value of the ore blocks.

The advantages of the push re-label Algorithm are well described in the case of an open pit design from numerous orebody simulations (Meagher et al., 2011). The results show that the blocks that have less variability in the economic block values across the different simulations will have a larger chance of being selected for the first pushback than blocks which vary in grade between the different simulations. In other words, low-risk blocks, blocks with a high probability of a certain value, have a higher chance of being in an earlier pushback, so they will be extracted earlier than high-risk blocks. The case is the same for high-value blocks versus blocks with a lower value, since there is more profit in the high-value blocks, because high-value blocks lose more money over time in the discount rates when compared with low-value blocks. Thus, the selection for blocks in the first pushbacks are at equilibrium between the probability of having a certain (positive) economic value and the high of the value of the ore block, where the value comes from the grade, mining and processing costs, and the recovery of the metal.

From this it becomes clear that the $\lambda$ value has a large influence on the pit size, and the steps between the $\lambda$ values define the different sizes of the pits. When a $\lambda$ larger than one is selected, the pit can be larger than the found ultimate pit. So the selection of the $\lambda$ has to be done carefully. Otherwise, large gaps (variation in pit sizes) will occur and the pushback design/mining sequence will be unfeasible. In addition, the number of pushbacks is important, because it defines the number of periods/units for mining. In these periods, the mining and processing parts of the operation are expected to work at their optimal (maximum) capacity.
3 Application at a Copper Mine: Escondida Norte, Chile

The Escondida Norte mine is one of the largest copper mines in the world and is located in Northern Chile in the Atacama Desert, approximately 170 km from Antofagasta, at an altitude of 3100 meters above sea level. The mine covers a circular area with a diameter of 2850 meters and has a maximum depth of 850 meters from the surface. The mine has two pits; this case study only looks at the northern pit of the mine, Escondida Norte. All the data, including the simulations for the grades, recoveries and ore types were provided by BHP-Billiton. The case study uses this provided simulations.

![Figure 3.1: Location of Escondida Norte copper mine, Chile](Bing Maps, Microsoft Cooperation, 2011)

In this case study, the approach of the previous section is used to design the optimum ultimate pit and pushback design. The results of the stochastic network flow algorithm, obtained from the algorithm on the simulations, will subsequently be compared with pit design for a single average type orebody model, representing the conventional pit optimization approach. Secondly, the results from the stochastic network flow algorithm will be compared to the L-G Algorithm, the industry standard method.

3.1 Description of the Deposit and Mine

The deposit is a homogeneous porphyry copper mineralization with the majority of the copper grades between 0.2 and 1 %. Due to weathering the deposit contains an amount of enriched ore,
so with a higher copper grade. Thus it is expected to have very similar simulations. Due to this the results between the stochastic method and conventional method will have a minor difference than case studies done before with SNFA. So this case study will show an application of the stochastic method on a multi destination deposit and at a homogeneous deposit. And so show that the method is applicable on these homogeneous deposits.

The mine contains four types of ore, which can be sent to the milling plans, the acid leaching or bio-leaching pads, or the waste dump. The deposit contains two different material types, copper sulphides and copper oxides. These materials have different production processes (milling-flotation or bio-leaching vs. acid leaching) and thus different recoveries for each process. These are taken into account for the calculation of the EBV of the deposit. Due to the limitation of the capacity of the algorithm, because a matlab network flow plug-in is used, only the pushbacks 3 to 7 of the Escondida Norte pit are considered, because pushback 1 and 2 have already been mined out. And when pushbacks 8, 9 and 10 are added to the dataset, the algorithm cannot be run.

The mining capacity for Escondida Norte is 180Mt a year (including stripping), the milling capacity 43 Mt annually, the acid leaching process capacity 11 Mt per year, and there is an unlimited capacity for the bioleach process. The copper produced at the northern pit is 0.6 Mt annually. The Cu grades (%) and their distribution in the pit are shown, for one simulation, in Figure 3.2. The figure shows as expected a more or less homogeneous deposit, the grades do not vary much. This is expected since Escondida Norte consists out of mainly copper porphyries, which are known to be homogeneous. The simulations displayed also show the parts with the enriched ore, where the copper grades are higher. Since the deposit is homogeneous the simulations are not deviating a lot from each other, thus there is a lot of consistency between the simulations. This will result very similar ore and waste tonnages it each simulation for the pit and pushback design. The mining blocks (SMU) are 25m by 25m by 15m (9375 m³). The figure shows a top view of the part of the orebody used in the rest of the case study, so the original pushbacks three to seven. The axis system shows the rotation of the model where the y-axe is north, x-axe is east and the z-axe is the altitude.
The block tonnages and their distributions are shown in Figure 3.3. The average block tonnage is ~23,000 Tonnes. On the slopes, some blocks have lower tonnages. This is because the block is already partly mined, and so has a lower tonnage. Since the data only has single size blocks, the tonnages of the blocks is the only tool that enables us to see the difference between complete and partial blocks.

For the stochastic approach, 15 equal-probable simulations of the orebody are used to design the optimum ultimate pit and the pushbacks, by using McGill University’s COSMO stochastic mine planning laboratory’s in-house software for the stochastic network flow algorithm. These simulations where delivered by BHP-Billiton form the drillhole data. This drillhole data is not used in this case study since the author had no access to it, but the original data, before extraction, consisted around 30 drillholes. The drill hole data is used to simulate the copper grades in the orebody model. All these simulations are scenarios, which has an equal probability to occur.
Figure 3.2: Top view of the grades (% Cu) for pushback 3 to 7 from the original Escondida Norte data (simulation 1-4)
Figure 3.3: Top view on the tonnages of the blocks (25x25x15m$^3$) in the Escondida Norte deposit, location and distribution (original pushback 3 to 7)

<table>
<thead>
<tr>
<th>Minimum Coordinate (m)</th>
<th>x</th>
<th>y</th>
<th>z</th>
</tr>
</thead>
<tbody>
<tr>
<td>17100</td>
<td>112450</td>
<td>2607.5</td>
<td></td>
</tr>
<tr>
<td>Maximum Coordinate (m)</td>
<td>19950</td>
<td>115275</td>
<td>3447.5</td>
</tr>
<tr>
<td>Block Size (m)</td>
<td>25</td>
<td>25</td>
<td>15</td>
</tr>
<tr>
<td>Maximum Number of Blocks</td>
<td>115</td>
<td>114</td>
<td>57</td>
</tr>
<tr>
<td>Total Number of Blocks</td>
<td>219903</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Minimum Coordinate (m)</th>
<th>x</th>
<th>y</th>
<th>z</th>
</tr>
</thead>
<tbody>
<tr>
<td>17150</td>
<td>113175</td>
<td>2637.5</td>
<td></td>
</tr>
<tr>
<td>Maximum Coordinate (m)</td>
<td>19825</td>
<td>115150</td>
<td>3432.5</td>
</tr>
<tr>
<td>Block Size (m)</td>
<td>25</td>
<td>25</td>
<td>15</td>
</tr>
<tr>
<td>Maximum Number of Blocks</td>
<td>103</td>
<td>80</td>
<td>54</td>
</tr>
<tr>
<td>Total Number of Blocks</td>
<td>61862</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3.1: Ranges of the original data and the data of original pushback 3 to 7 for Escondida Norte (distance from the origin)
Most of the copper grades of the deposit range between 0.00% (waste) and 4.5% (ore) as shown in Figure 3.4. While the tonnages are mainly between 21,000 tonnes and 25,000 tonnes per block, there are some blocks on the slopes with a far lower tonnage, which are actually only partial blocks to be simulated as complete ones (25x25x15m).
The input values for the calculation of the economic block values are listed in Table 3.2. The mining cost adjustment factor is given for each block in the simulations and ranges from 1.000 to 1.217, these where also provided by BHP-Billiton. The processing cost adjustment factor is not available; it is set to one.

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Copper price</td>
<td>2.50</td>
<td>$/lb</td>
</tr>
<tr>
<td>Treatment Charge</td>
<td>0.27</td>
<td>$/lb</td>
</tr>
<tr>
<td>Electro Winning Solvent Extraction</td>
<td>0.30</td>
<td>$/lb</td>
</tr>
<tr>
<td>Mining Costs</td>
<td>1.75</td>
<td>$/tonne</td>
</tr>
<tr>
<td>Milling Costs</td>
<td>6.50</td>
<td>$/tonne</td>
</tr>
<tr>
<td>Bio leaching Costs</td>
<td>1.75</td>
<td>$/tonne</td>
</tr>
<tr>
<td>Acid Leaching Costs</td>
<td>4.50</td>
<td>$/tonne</td>
</tr>
</tbody>
</table>

*Table 3.2: Prices and costs for Escondida Norte*

When the equations (1) and (2) are used the models displayed are found. An example of a calculation is the following:

\[
EBVLC = GR*T*Rec*(Pr-TC)-T*PC*PCAF-T*MC*MCAF = 0.02*21,000t*0.80*(2.50$/lb-0.27$/lb)*2204.623lb/t-21,000t*6.50$/t*1-21,000t*1.75$/t*1.205=1,651,879.92$-136,500$-44,283.75$ = 1,471,096.17$

\[
EBVWaste= -T*MC*MCAF = 21,000t*1.75$/t*1.205 = -44,283.75$

Figure 3.5a: Top view on the economic block value of each block for the four processes in the original pushbacks 3 to 7 in simulation 1-4
Figure 3.5b: Top view on the economic block value of each block for the four processes in the original pushbacks 3 to 7 in simulation 1-4
Figure 3.5 shows the economic block values at each location; there is a visible correlation between the ore grade in Figure 3.2 at each location and the economic block value. The highest grades and economic block values are in the middle of the existing pit. When the economic block values are plotted versus the grades of the copper of the deposit, as seen in Figure 3.6, a more or less linear relation becomes visible. This major correlation is a bit deviated since the mining costs deviate, based on the mining costs adjustment factor (mcaf). The blocks in the cloud below this correlation line are probably caused by the differences in tonnages, as displayed in Figure 3.3. This is because lower block tonnages contain smaller (absolute) amounts of copper, resulting in a lower value of the block. The high grade ore and the bottom of the graphs (waste) is caused by the fact that there are two kinds of copper ore, sulphide-based with a high recovery in the milling/flotation circuits, LC and LS, which have different recoveries and a lower recovery in the bio-leaching processes (RM); and oxide-based ore, which can be extracted with an acid/oxide leaching process (OX).
Figure 3.6: The economic block value (EBV) versus the copper grade of each block (25x25x15m³) for the values from one simulation (SIM01) (original pushback 3 to 7)
3.2 Pit Limit and Pushbacks Design

Now that the EBVs have been found, the next step is to design the ultimate pit and then scale the nested pits in order to find the different pushbacks. The mine and plants have several production limits, which are displayed in Table 3.3.

<table>
<thead>
<tr>
<th>Process</th>
<th>Limit (365 d/yr production)</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Milling, Los Colorados (LC)</td>
<td>21.9</td>
<td>Mt/yr</td>
</tr>
<tr>
<td>Milling, Laguna Seca (LS)</td>
<td>21.9</td>
<td>Mt/yr</td>
</tr>
<tr>
<td>Bio Leaching (RM)</td>
<td>Unlimited</td>
<td></td>
</tr>
<tr>
<td>Acid (Oxide) Leaching (OX)</td>
<td>11.0</td>
<td>Mt/yr</td>
</tr>
<tr>
<td>Mining (Incl. stripping)</td>
<td>182.5</td>
<td>Mt/yr</td>
</tr>
</tbody>
</table>

*Table 3.3: Production limits for Escondida Norte*

From the original data, only the first four pushbacks that are still in the pit will be considered. This would be enough for 15 to 18 years of mining and processing when both limits are considered. In this case study, the mining limit is reduced to be around the expected limit, which is dictated by the processing capacity. An estimated 55% of the material in the deposit will be sent to the milling process, based on rock type and designated process stream. These processes have a capacity of 43.8 Mt/yr. Since material from the mine cannot be stockpiled an extraction rate of 80 Mt/yr is used for the calculation, since stripping of the deposit is not needed. When taking into account the expected (average) stripping ratio, the total processing capacity (excluding bio-leaching) is 54.8 Mt. With the found stripping ratio, the mine production of ore and waste has to be around 80 Mt on a yearly basis, which is similar on the yearly mine production acquired based on only the milling processes. So the Algorithm was run on pushbacks 3 to 7 of the Escondida Norte pit, since pushback 1 and 2 have already been mined out. And pushbacks 8, 9 and 10 are not considered. This pit contains $1.42 \times 10^9$ tonnes of material, which is good for 17.8 years of production at the expected and current mining rate. This part of the total orebody model contains about 62,000 mining blocks.

3.2.1 Design of Sequential Pushbacks

To produce the sequential pushbacks, infinite capacity arcs have to be set in order to determine and define the sequence of extraction. The input parameters are the critical slope angles in a number of directions and the number of benches to be mined, in this case study 54 benches; this data is given in Table 3.4. With this data, a file will be made describing the relations between the blocks. The data was put in to a program arcs.exe and the outcomes are displayed in Figure 3.7
and it shows a visualisation of the precedence for a block at the lowest bench and the three-dimensional precedence of this block taken into account the slope constrains. Where the average slope error is caused by the fact that only complete blocks can be extracted from the deposit, so block definite and not geotechnical average slope error.

<table>
<thead>
<tr>
<th>Direction</th>
<th>Slope Angle Data Escondida Norte measured (degrees) (Provided by BHP-Billiton)</th>
<th>Slope Angle Measured in Existing Pit Figures Escondida Norte (degrees) (extracted from the data set)</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>35</td>
<td>35</td>
</tr>
<tr>
<td>S</td>
<td>-</td>
<td>35</td>
</tr>
<tr>
<td>E</td>
<td>-</td>
<td>33</td>
</tr>
<tr>
<td>W</td>
<td>41</td>
<td>40</td>
</tr>
<tr>
<td>NW</td>
<td>-</td>
<td>30</td>
</tr>
<tr>
<td>SE</td>
<td>33</td>
<td>35</td>
</tr>
<tr>
<td>NE</td>
<td>-</td>
<td>25</td>
</tr>
<tr>
<td>SW</td>
<td>35</td>
<td>45</td>
</tr>
</tbody>
</table>

Table 3.4: Azimuth and dip of the slopes of Escondida Norte

<table>
<thead>
<tr>
<th>Slope Angle Data Escondida Norte measured (provided by BHP-Billiton)</th>
<th>Slope Angle Measured in Existing Pit Figures Escondida Norte (extracted from the datasets)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Blocks</td>
<td>10867</td>
</tr>
<tr>
<td>Number of arcs</td>
<td>501</td>
</tr>
<tr>
<td>Average slope error</td>
<td>0.4; 0.4</td>
</tr>
</tbody>
</table>

Figure 3.7: Top view of “cone” for each block at the lowest level of the data set and arc information for the azimuth and dip
In order to make pushbacks, a series of nested pits will be generated until the ultimate pit (3.2.2) is generated where lambda, as explained before, will be increasing from zero, no pit, to one, the ultimate pit. Due to the capacity constraints of the network flow algorithm, an overall slope angle of 45 degrees is used, resulting in a cone with a 45-degree slope on all sides. The decision was made on the fact that a 45-degree slope is steep but could exist in an actual operation.

The results are displayed in the Tables and Figures below. The total life of mining this part of Escondida Norte is about 15.4 years, and in that period the four pushbacks will be extracted, instead of the five where the original data was divided in; this is around 1,232 Mega tonnes of ore and waste material, taking into account that it is based on the mining capacity of 80 Mt per year, not on the process capacity for each process. Unfortunately, the pushback design has some unfeasible characteristics, especially the location of the blocks in the pushbacks. Some of these blocks are disconnected from the rest of the pushback. In order to extract them mining equipment should be relocated for the extraction of a single block, what is very costly. Figure 3.11 shows that the pushbacks have sizes which are similar this is caused by the fact that the network flow algorithm is using scaling during the pushback design, this means that the \( \lambda \) is selected in such a way that the pushback are more or less equal sized. All the figures will use the new designed pushbacks one to four, which would be pushbacks three to six of the original design, but since the original pushbacks one and two where not considered, they will not be used in the figures.

*Figure 3.8: Top view of the new pit with the pushbacks (SNFA)*
Figure 3.9: The pushbacks in the cross-sections of the pit with the number of blocks from the pit origin (SNFA)

Table 3.5: Pit and pushback sizes for Escondida Norte, generated by the SNFA from the original data (pushback 3 to 7)
Figure 3.10: Pit sizes for Escondida Norte, generated by SNFA from the original pushbacks 3-7

Figure 3.11: Pushback sizes for Escondida Norte, generated by SNFA from the original pushbacks 3 to 7
Figure 3.12a: Tonnage curves for Escondida Norte based on the pushback design generated with SNFA, from the original pushbacks 3 to 7

Figure 3.12b: Tonnage curves for Escondida Norte based on the pushback design generated with SNFA, from the original pushbacks 3 to 7
Figure 3.12c: Tonnage curves for Escondida Norte based on the pushback design generated with SNFA, from the original pushbacks 3 to 7

Figure 3.12 displays the tonnage curves for the ore, all processes, and waste, where the expected curve is the average tonnage of the 15 simulations. The simulations differ a bit, and especially in the fourth pushback, while the overall tonnage is the same for any single simulation. The variation in the fourth pushback could be caused by the facts that the material is deeper underground, and so less information is available for the simulations and that there is more material that has to be mined in this pushback, which can be ore or waste since it is the grade is around the cut-off grade of the material. But unfortunately it is not sure since the drillhole data was not available.

The other output from the network flow algorithm determines the stripping ratio, which is expected to increase during the mining of the pushbacks. This is shown in Figure 3.13. The expected and average stripping ratios of the 15 different simulations are slowly increasing. This is caused by the extra material (waste) which has to be mined in the pushbacks. The higher part at the second pushback is caused by the amount of overburden (waste), which has to be mined at the top of the slope, as displayed in the figures before.
Figure 3.13: The stripping ratios for the new phase of the Escondida Norte mine (SNFA)

The total cumulative expected undiscounted cash flow is $14.4 \times 10^9$ where the “discounted” cash flow is $10.3 \times 10^9$ at an 8.5% discount rate. These are shown in Figure 3.14a and 3.14b. Figure 3.14c shows the difference between the simulations and the average, which is set to one, of these simulations, which is around 5%, so the simulations are close to each other and there is not that much variation between them. The values of the case flows are within the expected range. This means the method works well and uses the highest economic block values of each block in the deposit. Since the main factor in this case study is to optimize the pit/pushback designs the targets (processing limits) are not always honoured.
Figure 3.14: The undiscounted and “discounted” cash flows of the new design (SNFA)
### 3.2.2 Generation of the Optimal Ultimate Pit

From the data of the blocks, the more precise the economic block value, the more efficiently the ultimate pit will be generated. Unfortunately, the network flow algorithm generator was not able to handle the data set with the given arcs. So a standard 45-degree slope angle was used to generate the ultimate pit. This pit contains about 54,000 of the 62,000 blocks and is shown in Figure 3.15. The total tonnage mined during the operation before this ultimate pit is reached is 1,232 M tonnes. And it will take 15.4 years to mine this amount of rock.

![Figure 3.15: The new ultimate pit for the original pushbacks 3 to 7 of the Escondida Norte operation (Cross-sections at E-W 19250m and N-S 114375 m of the origin) (SNFA)](image)

### 3.3 Comparison to the Conventional Methods

The next step is to compare the founded values with the conventional processes. The following two methods will be used: (1) network flow algorithm on a single deposit, where the mean values are taken, also called the NFA deposit; and (2) the use of a Lerchs-Grossman algorithm implemented in Whittle software.

#### 3.3.1 A Single Estimate Deposit, Deterministic Network Flow Algorithm

The results of the stochastic method with 15 simulations will be compared with the results of the network flow algorithm on a single deposit. In this case, the estimate is used. In the estimate, the values of the 50 original simulations are averaged in a single model. Of this model, the economic...
block values of each of the four processes are calculated. And the algorithm is run on one orebody model with the four processes. The results of these are shown in the Table and Figures below.

The pit layout in the of the NFA deposit looks similar to the one extracted from the simulations. As the Figures show, the highest value ore is mined first. It is located on the northern side of the pit (dark blue). The total tonnage is 1,232 Mt, which is the same as in the ultimate pit from the risk-based method. Also, the mine life of 15.4 years is the same. The pushback design shows the same unfeasible characteristics, as discussed before.

**Figure 3.16:** Cross-sections of the new pushback designs of Escondida Norte based on the original pushback 3 to 7 (Estimate EBV LC and NFA)
Figure 3.17: Cross-sections of the new pushback designs of Escondida Norte (SNFA and NFA) based on the five original pushbacks (3 to 7)

<table>
<thead>
<tr>
<th>Pit</th>
<th>Blocks in Pit</th>
<th>Blocks in Pushback</th>
<th>Tonnage per Pushback</th>
<th>Life of Pushback [yr]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8396</td>
<td>1</td>
<td>8396</td>
<td>189 M</td>
</tr>
<tr>
<td>2</td>
<td>24294</td>
<td>2</td>
<td>15898</td>
<td>361 M</td>
</tr>
<tr>
<td>3</td>
<td>39544</td>
<td>3</td>
<td>15250</td>
<td>356 M</td>
</tr>
<tr>
<td>Ultimate Pit</td>
<td>53564</td>
<td>4</td>
<td>14020</td>
<td>326 M</td>
</tr>
<tr>
<td>Total</td>
<td>53564</td>
<td>53564</td>
<td>1,232 M</td>
<td>15.39</td>
</tr>
</tbody>
</table>

Table 3.6: New Pit and pushback sizes for the NFA of Escondida Norte based on the original pushbacks 3 to 7
Figure 3.18: New pit sizes for Escondida Norte (SNFA and NFA)

Figure 3.19: New pushback sizes for Escondida Norte (SNFA and NFA)
Figure 3.20: Tonnage curves per newly generated pushback for Escondida Norte (NFA)

Figure 3.20 shows that the extraction is more or less linear, so there is not a large variation in pushback size. So the scaling of the pushback sizes in done well.
Figure 3.21: Histograms of EBV for LC (mill) process for simulation 1 (SNFA) and the estimate (NFA)

The graph of the pit sizes and the pushbacks show similar geometric structures for the NFA deposit and the one extracted from 15 simulations, with SNFA. But the NFA shows less extreme fluctuations and has a smaller gap problem. This is caused by the fact that the grades of all the simulations are averaged and so smoother, so the blocks have fewer fluctuations in economic block value. This makes it “easier” for the algorithm to assign them to a certain pushback, which will result in pushbacks of very similar sizes.

The stripping ratios of the estimated orebody, NFA, are lower than the ones for the simulations, SNFA. This could be caused by the fact that this orebody model is a combination of the simulations, and so will generally create a smoothed, more uniform distribution of the grades. In this case study, it could result in a positive EBV for most of the blocks inside the deposit, resulting in a low stripping ratio. And due to this process, the stripping ratios for the NFA orebody are not the same as the stripping ratios for the SNFA, where the simulations gave a
skewed distribution, where the majority of the material is lower grade, and there is a more normal distribution of the estimation with most of the economic block value of the material in the middle of the distribution, as shown in Figure 3.21. The correct distribution can be found when the distribution of the block grades (or value) is compared to the grades of the composite drillhole data, which was not available for this case study.

**Figure 3.22:** The stripping ratios for SNFA (expected) and NFA for the new pushback design

**Figure 3.23:** The cumulative undiscounted and “discounted” cash flows (SNFA and NFA) for the new pushback design
Figure 3.23 above shows the cumulative Cash Flow for the estimated orebody (NFA) and the simulations (SNFA) discussed in the previous paragraph. The function shows that the cash flow of the estimate of the orebody, in the basic network flow algorithm, is almost the same as the one from that expected from the stochastic network flow algorithm ($14.3 \text{ B and } $10.3 \text{ B})--around 0.5% difference in the undiscounted and “discounted” case (discount rate: 8.5%), so absolute respectively $77 \text{ M and } 440 \text{ M}. So it shows that using the NFA or SNFA makes no difference for the cash flow, but there are some differences for the stripping ratio.

For the risk analysis, 15 simulations, the same as the ones used before, are run through the NFA schedule. The results are shown in the Figures below. They show that the expected ore quantity is lower and the expected amount of waste is higher. However, the cash flows are very similar when compared with the expected value of the SNFA.

![Figure 3.24: Risk analyses on the new pushback design, ore tonnage (NFA)](image)
Figure 3.25: Risk analyses on the new pushback design, waste tonnage (NFA)

Figure 3.26: Risk analyses on the new pushback design, undiscounted cash flow (NFA)

Figure 3.27: Risk analyses on the new pushback design, cumulative cash flow (NFA)
3.3.2 Industry standard, Lerchs-Grossman Algorithm

As the industry standard, Whittle Software is the established way in the mining industry to design open pits and find the pushback sizes, tonnage, stripping ratios and cash flows. The program works with a Lerchs-Grossman Algorithm and uses a single model ore deposit, in this case the NFA orebody model. A note has to be made about the fact that Whittle will only use one recovery for each process over all the blocks, while the data contains a recovery for each process in each block. The input variables for Whittle are showed in Table 3.7. The main parameters are the same as shown before, costs and prices, but the recovery factor is an overall (average) recovery factor for each ore type in each process, where in the NFA the recovery is given for each separate block. The recovery also bears the difference between the treatment charge (selling cost) and the Electro Winning Solvent Extraction, for the leaching processes.

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slope</td>
<td>45</td>
<td>degrees</td>
</tr>
<tr>
<td>Mining cost</td>
<td>1.75</td>
<td>$/tonne</td>
</tr>
<tr>
<td>Mine recovery</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Mine dilution</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>CAF</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Rehab</td>
<td>0</td>
<td>$/tonne</td>
</tr>
<tr>
<td>MCAF</td>
<td>model</td>
<td></td>
</tr>
<tr>
<td>Metal Price</td>
<td>2.50</td>
<td>$/lb</td>
</tr>
<tr>
<td>Selling cost</td>
<td>0.27</td>
<td>$/lb</td>
</tr>
<tr>
<td>Discount rate</td>
<td>8.5</td>
<td>%</td>
</tr>
<tr>
<td>Mine limit</td>
<td>80</td>
<td>Mt/yr</td>
</tr>
<tr>
<td>Process limit</td>
<td>0</td>
<td>Mt/yr</td>
</tr>
<tr>
<td>Ore selection</td>
<td>Cash flow</td>
<td></td>
</tr>
<tr>
<td>Total pits</td>
<td>36</td>
<td></td>
</tr>
<tr>
<td>Pushback definition</td>
<td>2, 6, 11, 36</td>
<td></td>
</tr>
</tbody>
</table>

*Table 3.7: Input data for Whittle Software®*
<table>
<thead>
<tr>
<th>Process type</th>
<th>Rock type 2</th>
<th>Rock type 3</th>
<th>Rock type 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mill LC</td>
<td>6.50 $/tonne</td>
<td>6.50 $/tonne</td>
<td></td>
</tr>
<tr>
<td>Recovery</td>
<td>0.8172</td>
<td>0.7835</td>
<td></td>
</tr>
<tr>
<td>Mill LS</td>
<td>6.50 $/tonne</td>
<td>6.50 $/tonne</td>
<td></td>
</tr>
<tr>
<td>Recovery</td>
<td>0.8241</td>
<td>0.7867</td>
<td></td>
</tr>
<tr>
<td>Bio leaching</td>
<td>1.75 $/tonne</td>
<td>1.75 $/tonne</td>
<td>1.75 $/tonne</td>
</tr>
<tr>
<td>Recovery</td>
<td>0.3851</td>
<td>0.4870</td>
<td>0.4424</td>
</tr>
<tr>
<td>Acid leaching</td>
<td></td>
<td></td>
<td>4.50 $/tonne</td>
</tr>
<tr>
<td>Recovery</td>
<td></td>
<td></td>
<td>0.6928</td>
</tr>
</tbody>
</table>

*Table 3.7 cont.: Input data for Whittle Software®*

Figure 3.28 shows that the pit with the Lerchs-Grossman algorithm differs a bit from the pits designed with the algorithm, for example in the E-W-direction, where a little hill is displayed in the middle of the pit. It also shows that several blocks are not connected to the blocks of the same pushback, this characteristic is discussed in the sections before.
Figure 3.28: Cross-Sections of Escondida Norte’s new pit and pushback design for the SNFA (left) and L-G (right) based on the original pushbacks 3 to 7

Stochastic network flow algorithm

Lerchs-Grossmann (Whittle)
Figure 3.29: Cumulative pit sizes from the SNFA and L-G results for the new pit and pushback design based on the original pushbacks 3 to 7

Figure 3.30: New pushback sizes of the SNFA and the L-G results based on the original 5 PB
Figures 3.30 and 3.31 above show the cumulative pit size and pushback sizes. It becomes clear from this graph that the second last pushback is larger (i.e., it contains more tonnage) than the one done with the proposed algorithm. This means that the Lerchs-Grossman algorithm has a larger gap problem than the stochastic network flow algorithm, making the latter a better option. The table below shows that the total tonnage mined is a bit less in the Lerchs-Grossman algorithm, than both the SNFA and the NFA. This also results in a shorter life time of the mine, since the production capacity is only limited by the mining capacity, 80 Mt/yr.
Table 3.8: New pit and pushback sizes for the L-G of Escondida Norte

<table>
<thead>
<tr>
<th>Pit</th>
<th>Blocks in Pit</th>
<th>Pushback</th>
<th>Blocks in Pushback</th>
<th>Tonnage per Pushback</th>
<th>Life of Pushback [yr]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6047</td>
<td>1</td>
<td>6047</td>
<td>139 M</td>
<td>1.74</td>
</tr>
<tr>
<td>2</td>
<td>19845</td>
<td>2</td>
<td>13798</td>
<td>317 M</td>
<td>3.97</td>
</tr>
<tr>
<td>3</td>
<td>38946</td>
<td>3</td>
<td>19101</td>
<td>439 M</td>
<td>5.49</td>
</tr>
<tr>
<td>Ultimate Pit</td>
<td>53301</td>
<td>4</td>
<td>14355</td>
<td>330 M</td>
<td>4.13</td>
</tr>
<tr>
<td>Total</td>
<td>53301</td>
<td></td>
<td>53301</td>
<td>1,226 M</td>
<td>15.32</td>
</tr>
</tbody>
</table>

Figure 3.32: The stripping ratios of the pushbacks from SNFA and L-G for the new pushback design, based on the original pushbacks 3 to 7

Figure 3.32 shows a stripping ratio between the expected stripping ratio in the stochastic network flow algorithm and the conventional L-G algorithm on the estimate of the deposit, resulting in a selection process which will mine more waste in order to make a higher cash flow, as displayed in Figure 3.33.
The cash flow is lower than in the calculations of the basic network flow algorithm and SNFA. The improvement of the (S)NFA is 4.4% when compared to L-G data for the undiscounted expected cash flow ($13.8 \times 10^9$) and 3.0% for the "discounted: expected cash flow ($10.0 \times 10^9$) in absolute values it is respectively around $633 \times 10^6$ and $309 \times 10^6$.

When the risk analysis is performed, no spectacular differences occur, as shown in Figures 3.34-3.37. But it shows that the expected ore quantity is a bit lower and the expected amount of waste is a bit higher. The variation of the possible scenarios, the risk analysis of the pushback design with the simulations, is overall similar and thus in the same range. But it shows a larger variation.
in the third pushback and a lower variation in the fourth pushback. This is unwanted since a high variation means a higher uncertainty and so a higher risk. Normally this risk is delayed to the last phase of extraction; since the costs of the risks is lower in a later stage of the operation.

![Ore Tonnage](image1)

*Figure 3.34: Risk analyses on the new pushback design, ore tonnage (L-G)*

![Waste Tonnage](image2)

*Figure 3.35: Risk analyses on the new pushback design, waste tonnage (L-G)*
Figure 3.36: Risk analyses on the new pushback design, cash flow per pushback (L-G)

Figure 3.37: Risk analyses on the new pushback design, cumulative cash flow (L-G)
4 Conclusions

The proposed algorithm, SNFA, which works with multiple simulations, has slightly better results than the conventional methods, especially L-G. The small different, less than in other examples of the SNFA, is expected since copper porphyry deposits, like this one, are relative homogeneous, this results in smaller variations of the ore grades and so the EBV of blocks in the deposit. Since the deposit is relative homogeneous, the difference of the ore grades and EBVs between simulations and the estimation of the orebody model will be small. This will result in similar pushback designs and result of tonnages and cash flows. It means that the extended SNFA for the multiple processes is not out performing the deterministic methods; on the other hand it also does not under for these methods. In other words the SNFA can be used for a multi process destination deposit without corrupting the results.

The real benefit of the proposed method is the fact that it gives a range of potential values, which also could be achieved by a risk analysis of the design with the simulations. This risk-based feature is used to deal with extreme values, both positive and negative. Where the deterministic methods only use a single model, which often does not show such extreme values and so neglects them. The total tonnage in all the methods is similar, and so are their pushback sizes, as seen in the expected values in Figures 4.1-4.3. Since this application is done on a homogeneous deposit it does, unfortunately, not show its full potential, but still shows (the lack of) the geological risk of the deposit and improved the knowledge on the orebody in comparison with the deterministic methods.

Although the total pit sizes are similar, the difference in the pushback sizes for the L-G is large, much larger than the proposed method, showing a large gap problem. This is a big issue for equipment planners since a large gap problem complicates their work since it requires flexibility of the equipment fleet, which is not available due to the costs.
**Figure 4.1:** Pit sizes for the three methods discussed herein

**Figure 4.2:** Pushback sizes for the three methods discussed herein
The stripping ratios for the proposed method are much larger than for the conventional methods. This is caused by the fact that the grades and values for the blocks are averaged and so smoothed in the conventional methods, so fewer blocks are considered as waste.

*Figure 4.3: Ore and waste tonnage curves for the three methods discussed herein*
The undiscounted and "discounted" cash flows show better results for the risk-based method when multiple simulations of the ore deposit are used. The proposed risk-based method is, in this case study, not better than the use of the basic network flow algorithm (0.4%), but is still 4.3% better than the industry standard method, Lerchs-Grossmann.
Figure 4.5: The cumulative undiscounted and “discounted” cash flows for the three methods discussed herein

A risk analysis with the 15 simulations does show some differences, so for this deposit the method of selecting pushbacks has no major effect on the cash flow, or NPV. But it shows that the actual amount of ore is lower than expected and the actual amount of waste is higher than expected from the NFA and L-G calculations. The simulations show the risk and give the range of the ore and waste tonnage, which makes the use of multiple realizations of the orebody an effective tool for showing the risks and alternations that can occur.
**Figure 4.6:** Risk analyses ore tonnage for the three methods discussed herein

**Figure 4.7:** Risk analyses waste tonnage for the three methods discussed herein

**Figure 4.8:** Risk analyses cumulative cash flow for the three methods discussed herein
Figure 4.9: Cross-Sections to the pit for the three methods discussed herein, where E-W and N-S are the distance from the origin.
5 Future Work

This thesis shows that the use of the stochastic network flow algorithm and the use of simulations improves the knowledge of the orebody, since the uncertainty and the variation of the orebody is defined. Although in a relative homogeneous deposit, like Escondida Norte this uncertainty is relatively low.

However, the method only accounts for geological risk and not for other risks like market or costs. Thus, improvements in the optimisation could be found in those areas, like the use of simulations for commodity prices (Schwartz, 1997) and production costs, energy, for example. Another area for improvement is the reduction of the computational needs, so that the algorithm will run faster and the orebody model can be larger, with more data points. The network will work the same but each block has more possibilities for its economic block value. Another area of interest is the feasibility of the higher pushbacks numbers, since the L-G and (S)NFA give very spread out results; blocks within one pushback are located every where in the pit and not all of them connected to each other. These results cannot be directly applied in the operation, since the equipment cannot be relocated for extraction for only a few blocks in one area.
References
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