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VELOCITY-INVERSION IN SPACE-TIME DOMAIN

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ABSTRACT

An approximate method to calculate the impulsive wave motion generated by a point source in a horizontally layered configuration is presented. The configuration consists of a stack of layers between two half-spaces where the source and the receiver are located in the upper half-space. We assume that the generalized ray-constituent has travelled from source to receiver through a single medium with some appropriately chosen root-mean-square wave speed. Subsequently, an approximate expression for the reflected field in space and time as a function of the medium parameters is arrived at. Using this closed-form expression a very simple inversion scheme is arrived at.

INTRODUCTION

One of the techniques to calculate the impulsive wave motion in layered media is the Cagniard-De Hoop method (e.g. Drijkoningen and Fokkema, 1987). Through the method, the total wave motion is expressed as a time convolution of the strength of the source and the space-time Green's function of the configuration. The space-time Green's function is expressed as the superposition of generalized rays. Each generalized-ray constituent travels from the source and reaches, after successive reflections and transmissions at the interfaces, the point of observation. The space-time expression for these constituents consists of a single integral over a real, finite range, the integrand of which is an algebraic function of a certain complex ray parameter. The so-called Cagniard-De Hoop path maps this complex ray parameter to the real time parameter. Except, for the simplest case where the generalized-ray constituent travels in a single medium, the Cagniard-De Hoop paths must be determined with the aid of numerical techniques.

In the present paper, we avoid the numerical determination of the Cagniard-De Hoop paths. We assume that the generalized ray-constituent has travelled from source to receiver through a single medium with some appropriately chosen root-mean-square (RMS) wave speed. A vertical RMS slowness is introduced, and all vertical slownesses are expanded in terms of this vertical RMS slowness. An approximate closed form expression for the generalized ray-constituent in space and time as a function of the medium parameters is then arrived at. For a representative configuration, we present numerical results of the space-time Green's function using both the Cagniard-De Hoop method and the method based on our RMS approximation.

Further, we show that our closed form RMS approximation for the generalized-ray constituent leads to a very simple inversion scheme, where the wave speeds of the layers can be reconstructed within a single trace. We finally present some results of the inversion scheme applied to synthetic data of the Cagniard-De Hoop technique. Our scheme is superior to inversion schemes based on the low-contrast Born approximation (Fokkema and Van den Berg, 1988).

DESCRIPTION OF CONFIGURATION

We consider a horizontally stratified medium. The wave speed c is a function of the vertical x_3 -coordinate and independent of the horizontal coordinates x_1 , x_2 , and the time coordinate t . The wave speed $c = c(x_3)$ is taken piecewise constant (Fig.1). The impulsive wave motion $u = u(x_1, x_2, x_3, t)$ is generated by a concentrated source at $\{0, 0, x_3^{(S)}\}$. This source starts to act

Fig. 1. The configuration.

at $t = 0$. A receiver is located at $\{x_1^{(R)}, x_2^{(R)}, x_3^{(R)}\}$. We consider the case that both source and receiver are located in the upper homogeneous half-space with $c = c_0$. The wave motion satisfies the inhomogeneous wave equation

$$\partial_1^2 u + \partial_2^2 u + \partial_3^2 u - c^{-2} \partial_t^2 u = -Q(t) \delta(x_1, x_2, x_3 - x_3^{(S)}), \quad (1)$$

where the source-strength $Q(t)$ is understood to be zero when $t < 0$. The Cagniard-De Hoop analysis now takes advantage of the invariance of the configuration with respect to time and with respect to the horizontal coordinates.

CAGNIARD-DE HOOP SOLUTION FOR PRIMARIES

In the Cagniard-De Hoop analysis, we use the one-sided Laplace transform with respect to time

$$\hat{u} = \int_0^{\infty} u \exp(-st) dt, \quad s > 0, \quad (2)$$

and subsequently a Fourier transform with respect to the horizontal coordinates

$$\tilde{u} = \int_{-\infty}^{\infty} dx_2 \int_{-\infty}^{\infty} \hat{u} \exp[is(\alpha_1 x_1 + \alpha_2 x_2)] dx_1 \quad (3)$$

with the inverse Fourier transform

$$\hat{u} = \left(\frac{s}{2\pi}\right)^2 \int_{-\infty}^{\infty} d\alpha_2 \int_{-\infty}^{\infty} \tilde{u} \exp[-is(\alpha_1 x_1 + \alpha_2 x_2)] d\alpha_1. \quad (4)$$

The solution of the Cagniard-De Hoop method with primary reflections only is obtained as

$$\left(\frac{s}{2\pi}\right)^2 \tilde{u}(\alpha_1, \alpha_2, x_3^{(R)}, s) = s \hat{Q}(s) \tilde{G}(\alpha_1, \alpha_2, x_3^{(R)}, s), \quad (5)$$

in which $x_3^{(R)}$ is the observer level and $\hat{Q}(s)$ is the Laplace transform of $Q(t)$. The spectral representation of Green's function $\tilde{G} = \tilde{G}(\alpha_1, \alpha_2, x_3^{(R)}, s)$ is given by a superposition of the direct contribution and the contributions of reflections from the interfaces $x_3 = x_3^{(n)}$, $n=0,1,2,\dots,N$, as

$$\tilde{G} = \tilde{G}_{\text{dir}} + \sum_{n=0}^N \tilde{G}_n, \quad (6)$$

with

$$\tilde{G}_{\text{dir}} = \frac{1}{8\pi^2} \frac{\exp[-s \gamma_0 |x_3^{(R)} - x_3^{(S)}|]}{\gamma_0}, \quad (7)$$

$$\tilde{G}_n = \frac{1}{8\pi^2} \Gamma_n \Pi_n \frac{\exp[-s \sum_{m=0}^n \gamma_m h_m]}{\gamma_0}, \quad (8)$$

in which the reflection factor Γ_n is given by

$$\Gamma_n = (\gamma_n - \gamma_{n+1}) / (\gamma_n + \gamma_{n+1}), \quad (9)$$

the vertical slowness is given by

$$\gamma_n = (c_n^{-2} + \alpha_1^2 + \alpha_2^2)^{1/2}, \quad (10)$$

and the product of transmission factors of the traversed interfaces is given by

$$\Pi_n = \Pi_{m=0}^{n-1} (1 - r_m^2) = (1 - r_0^2)(1 - r_1^2) \dots (1 - r_{n-1}^2). \quad (11)$$

In Eq. (8), h_m denotes the vertical wave path that the wave has traversed in the layer with wave speed c_m and is given by

$$\begin{aligned} h_0 &= 2x_3^{(0)} - x_3^{(R)} - x_3^{(S)}, \\ h_m &= 2(x_3^{(m)} - x_3^{(m-1)}), \quad m = 1, 2, 3, \dots \end{aligned} \quad (12)$$

The total vertical geometric path is given by

$$z_n = \sum_{m=0}^n h_m. \quad (13)$$

In the Cagniard-De Hoop technique, the result in the space-time domain is obtained by inspection. Except, for the simplest case where the generalized ray constituent of Eq. (8) travels in a single medium, a Cagniard-De Hoop path must be determined with the aid of a numerical technique. In the next section, we discuss an approximation in which we assume that the generalized ray-constituent has travelled from source to receiver through a single medium with some appropriately chosen root-mean-square (RMS) wave speed. This avoids the numerical computations of Cagniard-De Hoop paths.

RMS APPROXIMATION OF THE VERTICAL WAVE PROPAGATION

The vertical wave propagation term $\sum_m \gamma_m h_m$ can, for real α_1 and α_2 , be approximated within an upper bound and a lower bound. For real α_1 and α_2 , we are able to write (Van den Berg and Fokkema, 1988)

$$(c_n'^{-2} + \alpha_1^2 + \alpha_2^2)^{1/2} z_n \leq \sum_{m=0}^n \gamma_m h_m \leq (c_n''^{-2} + \alpha_1^2 + \alpha_2^2)^{1/2} z_n, \quad (14)$$

where c_n' is the root-mean square wave speed defined as

$$c_n' = (\sum_{m=0}^n c_m h_m)^{1/2} / (\sum_{m=0}^n c_m^{-1} h_m)^{1/2} \quad (15)$$

and c_n'' is the root-mean square wave speed defined as

$$c_n'' = (\sum_{m=0}^n h_m)^{1/2} / (\sum_{m=0}^n c_m^{-2} h_m)^{1/2}. \quad (16)$$

An approximation for the vertical wave propagation is given by

$$\sum_{m=0}^n \gamma_m h_m \approx \gamma_n^{\text{rms}} z_n, \quad (17)$$

with

$$\gamma_n^{\text{rms}} = ((c_n^{\text{rms}})^{-2} + \alpha_1^2 + \alpha_2^2)^{1/2}, \quad (18)$$

where

$$c_n^{\text{rms}} = (c_n' c_n'')^{1/2}, \quad (19)$$

is the mean value of c_n' and c_n'' . Using the result of Eq. (17) in our expression of the generalized ray-constituent of Eq. (8), we obtain

$$\tilde{G}_n = \frac{1}{8\pi^2} \Gamma_n \Pi_n \frac{\exp[-s \gamma_n^{\text{rms}} z_n]}{\gamma_0}, \quad n = 1, 2, 3, \dots \quad (20)$$

In order to express Γ_n , Π_n and γ_0 in terms of γ_n^{rms} , we write

$$\gamma_m = \gamma_n^{\text{rms}} \left(1 + \frac{c_m^{-2} - (c_n^{\text{rms}})^{-2}}{2(\gamma_n^{\text{rms}})^2} + O[(\gamma_n^{\text{rms}})^{-4}] \right), \quad (21)$$

by which Γ_n results into

$$\Gamma_n = \frac{1}{4} (c_n^{-2} - c_{n+1}^{-2}) (\gamma_n^{\text{rms}})^{-2} + O[(\gamma_n^{\text{rms}})^{-4}]. \quad (22)$$

and

$$\Pi_n = 1 + O[(\gamma_n^{\text{rms}})^{-4}]. \quad (23)$$

We then end up with the following spectral representation for the generalized ray-constituent

$$\tilde{G}_n = \frac{1}{8\pi^2} \left(\frac{B_n}{(\gamma_n^{\text{rms}})^2} + O\left[\frac{1}{(\gamma_n^{\text{rms}})^4}\right] \right) \frac{\exp[-s \gamma_n^{\text{rms}} z_n]}{\gamma_n^{\text{rms}}}, \quad (24)$$

with

$$B_n = \frac{1}{4}(c_n^{-2} - c_{n+1}^{-2}). \quad (25)$$

Using the Cagniard-De Hoop analysis, the approximated space-time Green's function is obtained by inspection as

$$G = G_{\text{dir}} + \sum_{n=0}^N G_n, \quad (26)$$

in which

$$G_{\text{dir}} = (4\pi R_{\text{dir}})^{-1} H(t - R_{\text{dir}}/c_0), \quad (27)$$

$$G_n = (4\pi)^{-1} \{B_n \eta_n(t) + O[\zeta_n(t)]\} H(t - R_n/c_n^{\text{rms}}), \quad (28)$$

with the distance between receiver and source,

$$R_{\text{dir}} = [r^2 + (x_3^{(R)} - x_3^{(S)})^2]^{1/2} > 0, \quad (29)$$

and the horizontal offset between receiver and source

$$r = [(x_1^{(R)})^2 + (x_2^{(R)})^2]^{1/2} > 0, \quad (30)$$

and the travel distance receiver and source via the reflection level $x_3^{(n)}$

$$R_n = [r^2 + z_n^2]^{1/2} > 0, \quad (31)$$

Further, in Eqs. (27)-(28), $H(t)$ is the unit step function defined by

$$H(t) = \begin{cases} 1, & t > 0, \\ 0, & t < 0, \end{cases} \quad (32)$$

and the functions $\eta_n(t)$ and $\zeta_n(t)$ are given by

$$\eta_n(t) = z_n t [t^2 - r^2/(c_n^{\text{rms}})^2]^{-3/2}, \quad (33)$$

$$\zeta_n(t) = \frac{1}{2} z_n t \{ (3r^2 - 2z_n^2) t^2 - 3r^2 R^2 / (c_n^{\text{rms}})^2 \} [t^2 - r^2/(c_n^{\text{rms}})^2]^{-7/2}. \quad (34)$$

Note that the travel time

is our approximation of the exact travel time. Although we do not need the c_n -function in our approximation, we do take it into account in our inversion scheme. From Eqs. (28), (33) and (34), we observe that our expansion of all vertical slownesses in terms of the vertical RMS slowness is equivalent with an expansion of the space-time domain representation of the Green's function such that each higher order term exhibits a higher degree of time variation. After determination of the space-time Green's function $G = G(r, x_3^{(R)}, t)$, the time-domain representation for the wave motion follows from Eq. (5) as

$$u(r, x_3^{(R)}, t) = \partial_t \{Q(t) * G(r, x_3^{(R)}, t)\}, \quad (36)$$

in which $*$ denotes convolution.

Now, we shall present some numerical results for the nine-layer configuration of Table 1. For this configuration, Drijkoningen and Fokkema (1987) have computed the space-time Green's function by using the exact Cagniard-De Hoop technique with primary reflections only. In Fig. 2, our approximate results are compared with their results. The dashed line represents the exact results of the Cagniard-De Hoop technique with primaries only. We observe that the results of our simple approximation are in good agreement with the much more complicated exact Cagniard-De Hoop technique for primaries only.

Table 1. The location of the interfaces and the wave speed of the layers for the nine-layer configuration.

n	$x_3^{(n)}$ (m)	c_n (m/s)
0	70	1500
1	100	1600
2	135	1700
3	175	1800
4	210	1700
5	260	1600
6	340	1900
7	415	2000
8	-	2200

Fig. 2. The approximation of the space-time Green's function $G - G_{\text{dir}}$ for the nine-layer configuration; $x_3^{(S)} = x_3^{(R)} = 7.5$ m and $r = 50$ m.

INVERSION

Let us now consider the inversion scheme. In order to obtain the Green's function $G(t) = G(r, x_3^{(R)}, t)$ the inversion procedure consists of a deconvolution of the data with the known source signature. We first determine the jumps in the curve of $G(t)$. These time instants are the arrival times T_n , $n=0,1,2,\dots,N$. The last time instant considered in the data is defined as T_{N+1} . In each time interval $T_n < t < T_{n+1}$, $n=0,1,2,\dots,N$, we then perform a simple least-square-error fit of the right-hand side of Eq. (26)-(28) to the data. Then, the values of B_n are determined from the minimization procedure of the quantities

$$\int_{T_n}^{T_{n+1}} |g_n(t) - B_n n_n(t)|^2 dt,$$

in which

$$g_n(t) = 4\pi \{G(t) - G_{dir}(t)\} - \sum_{m=0}^{n-1} B_m \eta_m(t). \quad (37)$$

The values of B_n are obtained as

$$B_n = b_1 / a_{11}, \quad (38)$$

in which, for each n ,

$$a_{11} = \int_{T_n}^{T_{n+1}} \eta_n^2(t) dt, \quad (39)$$

$$b_1 = \int_{T_n}^{T_{n+1}} \eta_n(t) g_n(t) dt. \quad (40)$$

These integrals are calculated numerically, using a simple rectangular integration rule over the time samples.

Subsequently, the values of the wave speed are then obtained as

$$c_{n+1}^{-2} = c_n^{-2} - 4B_n \quad (41)$$

The RMS-velocity is determined recursively as

$$c_n^{rms} = (c'_n c''_n)^{1/2}, \quad (42)$$

with

$$c'_n = \left(\sum_{m=0}^n c_m (T_m - T_{m-1}) \right)^{1/2} / \left(\sum_{m=0}^n c_m^{-1} (T_m - T_{m-1}) \right)^{1/2} \quad (43)$$

and

$$c''_n = \left(\sum_{m=0}^n (T_m - T_{m-1}) \right)^{1/2} / \left(\sum_{m=0}^n c_m^{-2} (T_m - T_{m-1}) \right)^{1/2}, \quad (44)$$

in which

$$T_{-1} = r/c_0. \quad (45)$$

Note that these definitions of c'_n and c''_n differ from Eqs. (15) and (16), but it has been verified numerically that the final results for c_n^{rms} do not differ significantly. The present definitions are more advantageous in the inverse scheme, since we only can determine the arrival times from the data.

As soon we have determined c_n^{rms} , the precise locations of z_n (and hence h_n) are obtained as a result:

$$z_n = (R_n^2 - r^2)^{1/2}. \quad (46)$$

with

$$R_n = T_n c_n^{\text{rms}}, \quad (47)$$

In this way a very simple inversion scheme is arrived at. The only a priori information we have used is the knowledge that the horizontally stratified medium consists of a stack of homogeneous layers. This assumption complies with the presence of jumps in the data corresponding to the arrival times T_n of the reflected waves from the interfaces.

The inversion procedure can be improved by taking into account the dependence of G on $\zeta_n(t)$. Although, in Eqs. (26)-(28), the explicit dependence of G on $\zeta_n(t)$ has not been prescribed, we can improve the inversion by determination of B_n from the minimization procedure of the quantities

$$\int_{T_n}^{T_{n+1}} |g_n(t) - B_n \eta_n(t) - C_n \zeta_n(t)|^2 dt,$$

in which

$$g_n(t) = 4\pi \{G(t) - G_{\text{dir}}(t)\} - \sum_{m=0}^{n-1} \{B_m \eta_m(t) + C_m \zeta_m(t)\}. \quad (48)$$

where C_n is an efficiency parameter to arrive at optimum values for B_n . We are however not interested in the final results for C_n . The values of B_n and C_n are determined from the algebraic system of two linear equations

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} B_n \\ C_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}, \quad (49)$$

in which, for each n , a_{11} and b_1 follows from Eqs. (39)-(40), while

$$a_{12} = a_{21} = \int_{T_n}^{T_{n+1}} \eta_n(t) \zeta_n(t) dt, \quad (50)$$

$$a_{22} = \int_{T_n}^{T_{n+1}} \zeta_n^2(t) dt, \quad (51)$$

$$b_2 = \int_{T_n}^{T_{n+1}} \zeta_n(t) g_n(t) dt. \quad (52)$$

After determination of B_n , the values of z_n , R_n and c_n^{rms} follow from Eqs. (41)-(47).

In Fig. 3, the reconstructed velocity profile is shown. As starting point, we have taken the synthetic data of the exact Cagniard-De Hoop technique of Fig. 2. In Fig. 3, the exact profile is presented as the dashed line. The result of the simple inversion scheme of Eqs. (37)-(40) is presented as the dotted line (Note that this scheme is almost identical to the one of the RMS Born approximation of Fokkema and Van den Berg, 1988), while the result of the improved inversion scheme of Eqs. (48)-(52) is presented as the solid line. We observe that the improved inversion scheme reconstructs the wave speed very well.

Fig. 3. Reconstructed wave speed from the Cagniard-De Hoop data of Fig. 2.

CONCLUSIONS

We have derived a simple closed-form expression for the space-time domain scattered field in a layered structure. The results of the forward modeling are in good agreement with the synthetic data of the Cagniard-De Hoop technique for primaries only. The inversion scheme is really simple and performs very well on synthetic data. The inversion operator based on our approximation is a local operator. The inversion is performed while marching on in time. The inversion process is of the order of a few seconds on a IBM Personal Computer AT.

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