LASER SCANNING MODELLING of a CESSNA CITATION for Computational Fluid Dynamics (CFD) Studies

MSc. Geomatics Thesis

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SUMMARY

In Aerospace Engineering the field of Computational Flow Dynamics (CFD) studies the aerodynamic behaviour of aircrafts. What is currently being used to perform CFD simulations are Computer Aided Design (CAD) models of the airplanes, which are usually low-detail industrial design models. Research of new methods for improving the results of the simulating process is of great importance. One method that can be tested in this direction is the creation of more detailed models of the actual airplanes for CFD use.

Such a model can be constructed via reverse engineering techniques. Out of the many available methods, laser scanning is the most suitable for a project like this. This is due to the fact that laser scanning has the advantage of acquiring a mass amount of object points in a short period of time and with high accuracy.

TU Delft has the necessary resources for carrying out such a project. A Cessna Citation II belonging to the Aerospace Engineering Faculty of TU Delft, used for teaching and scientific purposes, was measured. A CAD design model of this airplane was also available. In addition, the Optical and Laser Remote Sensing Chair of TU Delft provided a Z+F Imager 5003 laser scanner. This is a phase scanner that can easily capture 120,000 points per second in X-, Y- and Z-coordinates.

The measurements took place during the course of one day in a hangar at Schiphol East, where the Cessna is situated. The chosen measurement set-up used 12 scanning positions that ‘surrounded’ the airplane from different heights and provided complete coverage of the object. All the scans had overlap with one another and targets were placed that could be seen from more than one scanning positions. As a result, 180 million points depicting the Cessna along with the complete hangar were acquired from the 12 scans.

The separate point clouds from the different scans were afterwards combined into one final and cohesive result. This was possible via a procedure called registration, which is based on the overlap between the scans. Registration was performed with the Iterative Closest Point (ICP) algorithm, which fit together the overlapping parts. The final outcome was a complete reconstruction of the measurement site. The Cessna was then digitally separated from the rest of the site and the result was a point cloud of 13.6 million points belonging to the airplane alone. Although sufficient for the visualisation needs of the project, the ‘Laser Cessna’ has parts with small point coverage and an explicit value of its accuracy is not available.

The final processing step involved the modelling of a part of the Cessna point cloud. 3D modelling techniques for the right wing were tested but were not successful. In the end, 2D modelling techniques were used that entailed the division of the wing in profiles. Through the points of each profile, a set of 3 B-splines was fit that resembled the original NACA airfoils of the wing. The separate ‘laser airfoils’ were then connected with a series of meshes that created the final 3D visualisation.

The project illustrated that laser scanning can lead to the creation of models more detailed than the original CAD ones. However, initial CFD testing showed that the simulation results were not equally improved. The problem lies within the chosen modelling technique and the nature of the algorithms used in CFD. Further investigation on modelling techniques more suitable for CFD studies, is expected to generate improved aerodynamic results. At the same time, additional research can show if the advanced laser models can prove to be helpful in aerodynamic studies.
PREFACE

CFD studies are a relatively new and constantly developing field in aerospace engineering. They provide simulations of the aerodynamic behaviour of an object without the use of wind tunnels. The benefits from CFD studies are obvious; in the future it is expected that it will be possible to create complete databases for airplanes before they are even built. Such advancements can render flight testing obsolete.

On the other hand there is a new method for 3D shape measurements called laser scanning. Its main advantage is that it can use laser beams to gather millions of points from an object. This process is carried out in a small amount of time and with high accuracy. Laser scanning is already becoming extremely popular in reverse engineering projects.

Thus, the idea behind this thesis was born. The main objective of this project is to study how laser scanning can be used for improving the results of CFD studies. This entails the creation of a geometrical model of an existing airplane from laser measurements. Towards this goal, the Cessna Citation of the Aerospace Faculty of TU Delft was scanned.

The work was conducted from February till October 2006, as part of the graduation project for the M.Sc. in Geomatics program of TU Delft. The thesis was an interdisciplinary effort between two parties; the section for Optical and Laser Remote Sensing within the Department of Earth Observation & Space Systems (DEOS) and the Control & Simulation Department. Both these departments belong to the faculty of Aerospace Engineering of TU Delft.

I really think that I could write a whole new thesis just by thanking all the people I want to. Alas, this thesis is already 200 pages long, which means that I will have to settle with just the few names I am presenting here. My deepest apologies to all of those not mentioned. You know who you are…

First and foremost my deepest thanks go to the Greek scholarship foundation I.K.Y. Their continuous financial support was integral for my successful studies in Delft. My supervisor Alexander Bucksch was the driving force behind this project and his help was invaluable. Sorry for being so stubborn all the time and for not arranging a meeting with the 3 Greek amazons… Ben Gorte had the original idea for the thesis and was always very supportive behind every stage. I appreciate your constructive remarks and opinions. It is also a great honour to have Professor Teunissen approving the content of my work. Furthermore the support from Frank van den Heuvel, my original supervisor, during my first steps in Delft means a lot to me.

However this was an interdisciplinary project and I want to acknowledge the contribution of some people from the Control & Simulation department. First of all Joao Oliveira was my main connection with all things Aerospace. Thanks for giving me simple and understandable guidelines in a field so different from mine. The same goes for Wim Vos. Good luck with your PHD. I would also like to thank Steven Hulshof for providing me the original CAD model of the Cessna. Professor Mulder was very enthusiastic from the beginning with this project and I deeply appreciate it. Henk Lindenburg went to great lengths to arrange everything we asked for our visits in the hangar. Menno Klaassen and Ferdinand Postema stayed up till 9 p.m. so that I could complete my scans. I have only gratitude for you…

The help from my group was priceless as well. Jane van Ree and Steven Sablerolle not only assisted with the measurements, but also gave valuable input
during the analysis phase of the project. And I just cannot say enough good things about Martin… apart from giving me lessons in programming you made the whole thesis a fun thing to do. Thank you my friend…

There are also two people in Greece that I have always on my mind when I look back at what I have achieved in Delft. Ms. Petsa and Mr. Karras… you are and always will be my mentors. Thank you for always believing that I could do something more…

I have met so many people during my stay in Delft that I treasure. Nevertheless I will single out 3 persons. Yu Chen… thanks for allowing me to find out about your culture. Niki… apart from being the best cook I have ever met, you offered me a shelter during my nomad days. You are a true friend and I feel fortunate for knowing you… And of course… Christos… without you I would not even be here. You are the reason I had this wonderful experience in Delft and your presence was like having family away from home… I knew things would only get better just by having you around… stay strong no matter what… you are meant for great things…

Last but certainly not least… mom… dad… Taki… I am coming home…

Adamantios Kagkaras
Delft, October 2006
“We’re all faced throughout our lives with agonizing decisions, moral choices. Some are on a grand scale, most of these choices are on lesser points. But we define ourselves by the choices we have made. We are, in fact, the sum total of our choices. Events unfold so unpredictably, so unfairly, human happiness does not seem to be included in the design of creation. It is only we, with our capacity to love that give meaning to the indifferent universe. And yet, most human beings seem to have the ability to keep trying and even try to find joy from simple things, like their family, their work, and from the hope that future generations might understand more…”

*Woody Allen from* “Crimes and Misdemeanours”
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1. Introduction

In Aerospace Engineering, Computational Fluid Dynamics (CFD) studies is a rapidly advancing field. Fluid dynamics is the science that studies the behaviour of fluids (liquids and gases) when they are forced. CFD covers the use of advanced computational methods for the analysis and solution of fluid dynamics problems. CFD studies require the availability of a high precision CAD model of the object under study. In TU Delft, the Control and Simulation group of the Aerospace department is conducting CFD studies for the university’s Cessna Citation.

However, a very high quality CAD model of the actual geometry of the plane is not at hand. CFD studies are currently being carried out with a design CAD model provided by Cessna. Although this model is adequate for basic CFD studies it has small detail and is only consisting of 50,000 points. This model is not representative of the actual plane, since there are always differences between the design phase and the final product. Furthermore the actual Cessna Citation has already been flying for more than 10 years and its body has been strained considerably. Thus, compared to the already available model, a more detailed representation of the actual geometry of the plane could provide valuable information in this field.

The required model is expected to be constructed via reverse engineering techniques. All the basic 3D shape measurement methods are considered. Only laser scanning is fitting for the needs of such a project. The main constraints in this task are the big size of the object and its very limited availability time-wise. It is clear that the method used should give the maximum amount of data with very high accuracy and in a very fast manner. That is exactly what laser scanning can provide, since it has the advantage of acquiring a mass amount of object points in a very short period of time. However another very popular technique, Close Range Photogrammetry (CRP), will be briefly studied. CRP deals with the measurement of objects in the real world via photographs and can give very high accuracies, like the ones required.

This is where the project involves the close cooperation between the Control and Simulation group and the Optical and Laser Remote Sensing (OLRS) group of TU Delft. The OLRS group has a laser scanner by Zoller+Frohlich available and will be responsible for the first part of the project. In this stage the necessary measurements of the Cessna will be taken. The product of this procedure will be the creation of a point cloud representative of the plane. Specific parts of this point cloud will then be modeled. The point cloud and available models resulting from this process will then be used by the Control and Simulation group for their CFD studies.

The OLRS group will try to ensure a satisfactory result that can be used by the Control and Simulation group for their studies. The main objective for the first part of the project is to create a product that is more detailed than the already available CAD design model. For the second part the Control and Simulation group will evaluate the produced model. Thus, for this interdisciplinary project, the main research question is formulated as follows: *Can laser scanning lead to 3D models, which improve the results of CFD studies?*

This thesis focuses on the measurement part. The project is viewed as a good opportunity by the OLRS group to test laser scanning in practice. The reason behind this is that laser scanning is a very new technique and many of its aspects are currently
unknown. This makes for a fascinating field of study. Hence, the main research question for the measurement part is: Can laser scanning be used for the creation of a model for the Cessna Citation of the Aerospace faculty that is more detailed than the CAD design model currently used for CFD studies?

To answer this question the project can be divided in three phases. Each phase has separate questions to answer:

1) How can laser scanning be used to make a highly accurate 3D model of a Cessna Citation (design phase)?
   - What are the advantages of laser scanning techniques?
   - Can other alternative techniques be considered?
   - What is the most cost-effective and fastest solution for our specifications?
   - What precision can be achieved with the proposed solution?

2) What are the results of the laser scanning procedure (execution phase)?
   - What was the set-up used for the scanning?
   - What problems were met during the use of laser scanning?
   - What are the general considerations for the technique in this specific application?

3) What does the analysis of the product provide (processing phase)?
   - How can a unified point cloud be obtained?
   - What is the quality of the point cloud?
   - What modeling algorithms can be used?
   - What is the best way for modeling the specific object?
   - What is the quality of the obtained model?
   - What are the differences of the Laser model with the original CAD model?

Regarding the structure of this report, in chapter 2, the measurement project will be explained. It contains an introduction to CFD studies as the motivation behind this project and the reasoning behind choosing laser scanning as the most appropriate solution. Chapter 3 is dedicated to the measurement of the plane. It contains a presentation of the laser scanner and the Cessna, followed by a feature on the measurement set-up and the actual measurements. In chapter 4 the first part of the processing of the laser scans is explained. This is where the registration phase for the point clouds is given. This is where the registration phase for the point clouds is explained. The processing phase continues in chapter 5, where the modelling part of the procedure is presented. Finally, in chapter 6, a summary and all the conclusions derived by this research will be presented. Also included are recommendations for further study on this topic.
2. The Measurement Project

As it was already mentioned in the introductory chapter, this thesis is describing a reverse engineering project of an airplane. This chapter will deal with the basic presentation of all the aspects of this project. In chapter 2.1 the motivation behind the project will be featured, along with a small introduction into CFD studies. Next, in chapter 2.2, an overview of five main 3D shape measurement techniques will be given. Finally, the reasons behind choosing laser scanning for this project will be presented in chapter 2.3.

2.1 Motivation for the Project

In this section the motivation behind this project will be presented. In chapter 2.1.1 an introduction to CFD studies will be given. In chapter 2.1.2 the available CAD design model of the Cessna will be presented.

2.1.1 CFD Studies

The most basic part of Aerospace Engineering is the analysis of fluid dynamics problems. Computational Fluid Dynamics (CFD) studies the use of advanced computational methods for their solution. It is only recently that CFD studies have shown huge progress, due to the big advancements in computer technology. The use of computers is necessary for these studies because all the methods used for analysis of fluid dynamics problems are very labour intensive. Although CFD methods are outside the scope of this thesis, a small overview will be made in the next part. There are two types of methods used for CFD, the mesh-based and the non mesh-based methods:

Mesh-based: They are the most commonly used methods for CFD. In this case the object of interest is formed by a volume mesh or grid. Then, suitable algorithms are applied to solve the equations of fluid motion that emulate the air flow. Most popular equations are [nasa]:

- Euler Equations: The Euler Equations describe how the velocity, pressure and density of a moving fluid are related. The equations are named in honor of Leonard Euler, who studied fluid dynamics problems in the mid-1700's. The equations are a set of coupled differential equations and they are solved with calculus methods. The Euler equations neglect the effects of the viscosity of the fluid. Thus a solution of the Euler equations is only an approximation to a real fluids problem. However, problems like the lift of a thin airfoil at low angle of attack, can be modelled sufficiently with the solution Euler equations.

- Navier-Stokes Equations: The Navier-Stokes Equations describe how the velocity, pressure, temperature, and density of a moving fluid are related. They are named by G.G. Stokes and M. Navier, who derived these equations independently in the early 1800's. The equations are extensions of the Euler Equations and include the effects of viscosity on the flow. That is why a solution of Navier-Stokes Equations provides a better model for reality in fluids problems than Euler equations. That is why problems
like the growth of the boundary layer on a flat plate, are solved with these equations.

**Non mesh-based:** Less commonly used are non mesh-based methods:

- **Smoothed particle hydrodynamics:** It is a method that simulates fluid flow by dividing the fluid in smaller discrete elements.

- **Spectral methods:** In this technique the original partial differential equations describing the fluid flow are transformed onto globally smooth functions (e.g. Legendre or Chebyshev polynomials). Fast Fourier transformation is commonly used.

- **Lattice Boltzmann methods:** It is a technique used for the computational modelling of a wide variety of complex fluid flow problems. It is a discrete computational method based upon the Boltzmann equation. It considers a typical volume element of fluid to be composed of a collection of particles. These particles are represented by a particle velocity distribution function for each fluid component at each grid point. The time is counted in discrete time steps and the fluid particles can collide with each other as they move, possibly under applied forces. The time-average motion of the particles is consistent with the Navier-Stokes equation [nist].

For further information about the aforementioned CFD techniques, the reader is encouraged to consult specialised handbooks [Anderson 84] [Katz 00].

Especially in Aerospace Engineering CFD studies are of great importance in the design phase of an airplane. This is due to the fact that CFD can be used for a preliminary study on the aerodynamic properties of an aircraft before it is actually built. That is why what is currently being used to do these simulations are Computer Aided Design (CAD) models. These models are usually the original aircraft design models from the industry. During CFD testing with a CAD model, a calculation and a flow model error are produced. The calculation error corresponds to the method and the flow model error to the errors in the CFD model used. In the end the final CFD results are given (see figure 2.1 below). The conclusions based on these results can be used for the improvement of the initial designs.

![Fig. 2.1 CFD Testing Flowchart](image)

However, in reality there are many differences between a CAD model and a built airplane. The design model will always be different than the actual product coming out of an assembly line. Most attachments of an aircraft are bolted together, turning the aircraft skin into a blunt surface. Furthermore, an airplane’s body is strained by the many hours of flying. Finally, due to the computational cost of running CFD models, it is not uncommon for design models to have really rough approximations for specific parts of the aircraft skin. That is why CFD studies are not considered fully adequate for aerodynamic studies and actual flight testing is still performed. CFD models will provide
a qualitative a-priori database of knowledge about the aircraft aerodynamic model. Nevertheless, accurate models can only be achieved from flight testing results, because they reflect reality [Staveren 03]. During flight testing the actual airplanes produce an experimental error and give a final flight test result, as shown in the following figure:

![Flight Testing Flowchart](image)

There is no use in establishing the best out of the two techniques as each one has its own supporters due to their corresponding advantages and disadvantages. Even though flight testing can be considered more accurate, CFD can be used for testing which is impossible or too dangerous to perform in reality. Additionally, the fact that CFD studies can be used before even construction takes place is a huge bonus. Aerospace Engineering supports that in the end both tests should be performed and results should be compared with each other to obtain the best conclusions.

The differences in this aforementioned comparison can be summarised in two basic error sources; the **shape error** and the **aeroelastic deflection error**:

**Shape Error:** The CAD design model used for CFD studies is correspondent to the state of the Cessna while being constructed. This is called the jig-shape of the plane, due to the jigs used during manufacturing. Any survey performed on the Cessna now will be done with the plane on jacks. This is called the jack-shape of the Cessna and is altered from the jig-shape. Apart from that the actual plane has antennas and other modifications on it that are not depicted in the design model. This changes the shape significantly from the original jig-shape. Thus, the total difference in the shapes of the two models is defined here as a shape error.

**Aeroelastic deflection error:** The aeroelastic deflection is the deformation of the wings and fuselage of a plane during flying [Juliana 01]. Although this is true for real airplanes, it is common practise in CFD for a specific airplane shape to be studied. This is also the case with the airplane design model supplied by Cessna. The CFD studies performed on the design model assume the Cessna in a solid state. Thus the previously mentioned jig shape is the one studied so far in simulations. That is why the aeroelastic deflection and the error that comes with it can only be measured in test flights.

The whole connection described above between CFD testing and test flights is summed up in the flow chart in figure 2.3:
The prevailing idea until now is that CFD studies will always be a step behind because the models used are too simple compared to the reality of a constructed airplane. But it is logical to assume that this can change if the model used for CFD studies is more accurate and representative of the real aircraft. If there is a way to put the actual airplane geometry in the place of the CAD design model it would be possible to have the same comparison between flight testing and CFD testing as before. The difference this time is that this comparison can be performed without having to do the actual flight testing. This comparison does not give the aeroelastic deflection error, since it can only be detected in flight testing, but it can provide the shape error. Hence in theory, carrying out CFD studies of the actual geometry and comparing them with the original studies on a design model, is enough to measure the shape error. This is explained in the following flow chart in figure 2.4:
This idea was the natural outcome of the needs of the Control & Simulation department of the Aerospace Engineering Faculty of TU Delft. This faculty is responsible for managing the Cessna Citation II that belongs to the TU Delft and is used for scientific and teaching purposes. Nevertheless, for performing CFD studies the only resource available is a CAD model from the design phase of the airplane.

2.1.2 The Cessna CAD Design model

The available CAD design model was provided by Cessna. Although it is considered adequate for basic applications, engineers and researchers of the Control & Simulation department have found it lacking in some areas:

- Symmetry
- Low Detail
- Wing design
- Points of interest

Symmetry: The provided model is completely symmetrical. Actually only half of the plane is given in the original model. In order to get the complete model a mirror image of the available points was created. The symmetry of the Cessna is something that certainly does not correspond to reality.

Low Detail: The whole model consists of only approximately 50,000 points. These points create a profile-like grid structure. The problem is that in some parts of the Cessna this grid is dense while in some others really sparse. Thus it is understandable that the model is low in detail.

Wing Design: In aerospace engineering airfoils are used for designing the wings. Airfoil is the shape of the cross-section of a wing (see figure 2.5 below). A series of subsequent airfoils gives in the end the final wing shape.

![Fig. 2.5 Airfoil [source: laesiworks]](source)

There are different types of airfoils used for different types of aircrafts. The most commonly used standard for aircraft wing design is the NACA airfoils. The shape of the NACA airfoils is described using a series of digits. The 4 and 5-series digits are the most popular. These digits are used for inputting the three parameters utilized for the generation of the airfoils. These parameters are: the maximum thickness, the design lift
coefficient and the maximum camber [aerospaceweb]. A complete introduction to the NACA airfoils is given in Appendix 2.

In the Cessna Citation II a 5-digit series NACA profile is used. This is the 23012 airfoil. This airfoil has a maximum thickness of 12%, a design lift coefficient of 0.3, and a maximum camber located 15% back from the leading edge. The 23012 NACA profile is shown in figure 2.6 below:

![Fig. 2.6 The 23012 NACA airfoil](pagendarm)

The main advantage with this airfoil is that it has a smooth shape. This makes computer calculations for CFD studies not as labour-intensive as other solutions might have been. However this is also a problem because such an airfoil is generated by a mathematical function. It does not take into account many parameters, e.g. the construction procedure of the plane or the years of flying that have strained the wings. It is clear that the real shape of the wings will have differences from the original NACA profiles.

**Points of interest:** Finally, some curvature points in the fuselage and the wings are not very detailed in the original model. These points are the ones that show the connection of the wings and the turbines with the fuselage of the Cessna. In these cases engineers from the Control & Simulation department had to add some points in order to get a complete result.

All the above factors make the need of getting a more accurate model of the Cessna even more imperative. This available CAD design model is featured in the figure 2.7 below:

![Fig. 2.7 The Cessna CAD design model](pagendarm)

In this point the Optical & Laser Remote Sensing (OLRS) Chair of TU Delft comes in. The Cessna when not flying is situated in a hangar at Schiphol East in
Amsterdam. The goal of the project from the OLRS part is to find the best way to make a more accurate model of the airplane than the already existing one. It is basically a reverse engineering project and that is why all the available 3D shape measurement techniques have to be considered.

2.2 Overview of 3D Shape Measurement Techniques

3D shape measurement is a field with vast applications in many different areas. These vary from industrial metrology and cinematography, to cultural heritage applications and reverse engineering, just to name a few. Especially now, 3D shape measurement is rapidly developing due to continuous advancements in technology, which offers different techniques to utilize. All these different techniques have their own advantages and disadvantages. This fact does not make one method better than the other. It just means that there are more suitable techniques for different applications and this is what the current chapter will explore.

There are five basic 3D shape measurement techniques:

- **Surveying**,  
- **CMM measuring**,  
- **Close Range Photogrammetry**,  
- **Structured Light**,  
- **Laser scanning**.

These methods will be given an overview in the following corresponding chapters.

### 2.2.1 Surveying (Total Station)

The most typical way of making accurate measurements in the real world is by means of surveying. Instruments used in such projects have been around for many years and are constantly improving with more features. What is currently commonly used is an instrument called the total station. The total station combines the angle measuring capabilities of a normal theodolite with the distance measuring capability of an electronic distance meter (EDM). This allows the user to determine a specific spatial position with pinpoint accuracy. Commonly used in construction sites and for creating topographic maps, the total station is one of the most accurate instruments available for 3D shape measurements. A typical total station is shown in figure 2.8 below:
The way a total station is used is by being mounted on a tripod and leveled with extreme accuracy. A prism is then placed on the point of interest. Through the telescope the operator finds and focuses on the prism. The EDM sends out a red or infrared laser beam that strikes the prism and is bounced back. The distance from the total station to the object is calculated from the time-of-flight of the beam. At the same time the vertical and horizontal angles between the position of the total station and the point are determined. These three values are enough to calculate the spatial coordinates of the specific point. The latest versions of the total station actually have no need for a prism; the beam can be bounced back directly from the object for a range of couple hundreds of meters without any external help. This makes measuring with total station a non-tactile method.

The total station’s greatest advantage is the accuracy that can be achieved, which can easily reach millimeter levels. But apart from that there is one basic disadvantage with this method; it is point oriented which means that only a small amount of data can be acquired at a given time frame. There are no images as in photogrammetry that can be used for massive data collection or for interpretation. That makes this technique usable in practice for very specific applications, like determining control points on an object for photogrammetric projects.

Some typical accuracy characteristics of a total station are featured in the table 2.1 below. The total station series in question is the Leica TPS1200:

| Table 2.1 Leica TPS1200 series Accuracy Characteristics (source: www.leica.com) |
|---------------------------------|-----------------|-----------------|-----------------|-----------------|
| **Angle measurement**          | **Type 1201**   | **Type 1202**   | **Type 1203**   | **Type 1205**   |
| Accuracy standard deviation, ISO 17123-3 | N, V           | 3'' (0.12'')    | 2'' (0.06'')    | 1'' (0.04'')    | 0.5'' (0.02'')  |
| Method                          | Absolute, both axes, spherical           | Absolute, both axes, spherical           | Absolute, both axes, spherical           | Absolute, both axes, spherical           |
| **Compass error**               | Working range: | 4'' (0.07'')    | 4'' (0.07'')    | 4'' (0.07'')    | 4'' (0.07'')    |
| Setting accuracy                | 0.5'' (0.2'')  | 0.5'' (0.2'')   | 1.0'' (0.4')    | 1.5'' (0.6'')   |
| Method                          | Centralized dual axis compensator       | Centralized dual axis compensator       | Centralized dual axis compensator       | Centralized dual axis compensator       |

| **Distance measurement (IR)**   | **Type 1200**  | **Type 1202**  | **Type 1203**  | **Type 1205**  |
| Range, average atmospheric conditions | Round prism (CRPA) | 3000 m | 3000 m | 3000 m | 3000 m |
|                                | 360° reflector (CRB): | 1000 m | 1000 m | 1000 m | 1000 m |
|                                | Multiline (CRPOL) | 1200 m | 1200 m | 1200 m | 1200 m |
| Reflective tape (600 m x 1000 m) | 250 m | 250 m | 250 m | 250 m |
| Shortest measurable distance    | 1.5 m          | 1.5 m        | 1.5 m        | 1.5 m        |
| Method                          | Phase measurement (central, visible, infrared) | Phase measurement (central, visible, infrared) | Phase measurement (central, visible, infrared) | Phase measurement (central, visible, infrared) |

<table>
<thead>
<tr>
<th><strong>PinPoint R100/R500 reflectorless distance measurement (RL)</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Range, average atmospheric conditions</td>
</tr>
<tr>
<td>Shortest measurable distance</td>
</tr>
<tr>
<td>Long range to round prism (CRPA)</td>
</tr>
<tr>
<td>Accuracy, standard deviation, ISO 17123-4</td>
</tr>
<tr>
<td>Accuracy, standard deviation, ISO 17123-4</td>
</tr>
<tr>
<td>Accuracy, standard deviation, ISO 17123-4</td>
</tr>
<tr>
<td>Laser dot size</td>
</tr>
<tr>
<td>Method</td>
</tr>
<tr>
<td>Method</td>
</tr>
</tbody>
</table>
2.2.2 Coordinate Measuring Machine (CMM)

The CMMs are mechanical systems designed to use a measuring probe for determination of the coordinates of points on the surface of an object. The way these machines work is by using the probe to touch the point of interest. The position of the probe can then be tracked with a very high accuracy by the CMM. There is a wide variety of CMMs, depending on the type of trackers and probes used. On the one hand there are big stationary CMM tables with articulated arms carrying the probes, and on the other there are CMMs with hand-held probes and portable trackers. One of the latter is shown in the figure 2.9 below:

![Fig. 2.9 Leica Laser Tracker with T-Probe (source: leica)](image)

The main advantage of CMMs is the very high accuracy that can be achieved which can reach μm levels. This is a direct result of using the probe for measurements, since contact digitizers give better results than non-contact ones. Another advantage is that the probe enables inspection of hidden points and areas that cannot be easily reached with other solutions. However, the tactile nature of this method is also its ultimate problem. It is easy to imagine how difficult measuring big objects can be. The main drawbacks are the slow measuring speed and the point-oriented nature of the method. All these factors make CMMs a rather specialised solution used mainly in industrial production lines. In these cases, extremely high accuracies are of utmost importance, i.e. in assembling different airplane parts together.

Some typical characteristics of a CMM are featured in the table 2.2 below. The CMM in question is the Leica Laser Tracker series.
2.2.3 Close Range Photogrammetry

The science of Photogrammetry deals with the techniques used to obtain representations of real world objects from photographs. Photogrammetry is a field known for some time now and constantly developing. Its main application is found in aerial projects, for the production of maps. But the methods used for aerial purposes have been optimized, especially in recent years, for close distances. These methods have formed the new field of close-range photogrammetry.

The concept behind close range photogrammetry is simple; a photograph is considered to be expressed geometrically by the model of central projection. According to this model, points of the 3D world are projected onto a plane via the optical rays that come through the projection point. This point can be considered in this case as the central point of the camera lens [Petsa, 98]. This principle can be explained schematically by the following figure:
However one image is not enough to make a 3D model of an object. This can be explained geometrically according to the central projection model; it can be said that the intersection of the rays from at least two different images of the same object provide the representation of the actual object in 3D space. What that means is that, if two or more images of an object from different angles are available, it is possible, in principle, to determine mathematically the 3D shape of the pictured item [Dermanis 90]. This concept can be explained for the case of four images in figure 2.11 below:

Of course there is a whole procedure that needs to be followed from using some photographs to arriving to actual measurement results. Marking of specific points in all the images has to be performed and correspondences between these points have to be established [Hanke 03], just to name a few. The steps of a photogrammetric procedure are a lot and varying depending on the application at hand and in the end this is what the field of photogrammetry covers. However, since in this project photogrammetry was only used in a limited scope, these steps are not going to be presented here. What will follow is a general description of advantages and disadvantages of this method.

Some of the main advantages are that photogrammetry is a non tactile process, from which many points can be acquired. The level of detail that can be achieved is very high because the method is flexible and can be easily adapted to each project’s needs.
Furthermore, the nature of image acquisition means that the actual data collecting procedure can take very little time. This is supported also by the fact that there is no real need now for heavy, specialised photogrammetric cameras to get very high accuracy results. Advancements in digital cameras mean that low-cost off-the-shelf equipment is quite adequate for the majority of applications. Finally, images can also be used for other purposes apart from measuring, e.g. forming an archive. This is usually the case in cultural heritage applications.

On the other hand photogrammetry is not really applicable with objects that do not have sufficient texture. The points acquired are only detail points that can be easily distinguished, like edges and cracks. In addition, even though the data collection can be fairly quick, the processing of the images is usually very labour-intensive. Also preparations for the measurements can prove to be complicated in some cases. For example in many projects control points have to be determined. This is done usually by means of surveying. With all these factors under consideration, it is evident that photogrammetry also has some considerable disadvantages.

It is worth to mention though that close range photogrammetry is still regarded as the most standard method of performing elaborate 3D shape measurements. Because of its proved functionality, it was decided to perform a small scale photogrammetric survey of a limited part of the airplane. The goal of this assignment was to show the differences between photogrammetry and laser scanning not only in theory, like they are presented here, but also in practise. This task was conducted as part of the course Digital Photogrammetry. Its complete description and results are featured in Appendix 1, along with a more in depth look at the principles of close range photogrammetry.

### 2.2.4 Structured light

One major problem with classic close range photogrammetric techniques is the low level of automation that can be achieved with the currently available tools. All the procedures concerning close range photogrammetry (which are described in detail in Appendix 1), entail measurements that must be done in every image separately. Procedures like, for example, establishing correspondences between matching points, need to be defined by the user. The structured light technique is basically a different approach on a typical close range photogrammetric case and can provide a solution to this problem.

The structured light technique is based on the projection of a specific pattern upon the objects of interest. This pattern is projected via a beamer and recorded either via photographs or video. What is special in this case is that the used patterns are specifically designed to be distinguishable by a local coding strategy [Maas 92]. These patterns can range from an array of points to a series of fringes. Some examples of structured light set-ups along with a sample pattern are displayed in figure 2.12:
What structured light offers is a way to get to a more automated process in regular photogrammetric procedures. For example in point marking, the selection of all the necessary points can be a very tedious and time consuming stage for the user. In addition, this whole process can involve a fair amount of errors, since the selection of points is subject to the perception of the user and can, in the end, be a highly subjective matter.

However the patterns used in structured light are specially designed so that codewords are assigned to a set of pixels [Salvi 03]. Every coded pixel has its own codeword. Thus there is a direct mapping from the codewords to the corresponding coordinates of the pixel in the pattern. The codewords are numbers mapped in the pattern by using grey levels, colour or geometrical representations. Detection of a specific point in the image is easy now. It is only necessary to encode a single axis, since a 3D point can be obtained as follows; by intersecting two lines (i.e. when both pattern axis are coded) or intersecting one line (the one which contains a pixel of the camera image) with a plane (i.e. when a single pattern axis is coded).

The main disadvantage of structured light is not being able to choose specific detail points of the object, like edges and cracks. This of course has another advantage, which is that objects with insufficient texture can also be measured, e.g. simple planar surfaces. Actually the problem of not detecting explicit points can be overcome in some extent by selecting denser patterns that generally fit each project’s needs. Consequently, many more points can be acquired with this method than with traditional photogrammetry, where the number of points is limited to the detail points.

In conclusion, structured light is a technique that basically ‘upgrades’ close range photogrammetry. That is why it is widely referred as active triangulation, whereas
photogrammetry is referred to as passive triangulation. In a way it could be said that this method is a bridge between photogrammetry and laser scanning, since many of the new characteristics (e.g. the acquisition of a great amount of points in a pattern) are inherent in laser scanning. The main disadvantage though of this technique is that explicit points cannot be obtained. Moreover the use of a beamer makes the whole set-up rather elaborate. This makes planning and executing a measurement procedure a pretty complicated task.

### 2.2.5 Laser scanning

Laser Scanning is the latest technique used for 3D shape measurements. Although laser technology has been around for a long time, its commercial use for measuring started only in recent years. As it was the case with photogrammetry, the first applications of laser scanning where mainly airborne with laser scanners mounted on helicopters and the results of the scanning process used for topographic maps. However, the same technology was also used and improved to produce scanners suitable for close range applications.

Terrestrial laser scanners work as follows; the scanner is mounted on a tripod, for achieving a fixed position. Then the laser light is used as an outgoing signal from the scanner. This outgoing signal is reflected on an object and received back by the scanner. From this reading the distance and the angle (horizontal and vertical) between the scanner and a point on the object can be determined. All the needed calculations are performed at a high rate, in a systematic pattern and in a specific region. Thus a scan consisting of many points on an object is created. These measurements are adequate so that for each scanned point its 3D coordinates can be determined. The coordinate system in this case is local and established by the position of the scanner at the time of the measurement process.

More specifically the laser beam generated inside the scanner is deflected via a system of rotating mirrors to a certain direction. Depending on the type of scanner this system may use two rotating mirrors or one rotating mirror and a rotating measuring device. The figure 2.13 below features these set-ups:

![Laser scanner set-ups](image)

**Fig. 2.13** (Left) The interior of a laser scanner with two moving mirrors. (Right) The interior of a laser scanner with one rotating mirror and a rotating measuring device; 1: the laser signal is send from here, 2: the deflection of the laser signal caused by the mirror, 3: the rotating mirror, 4: the rotating measuring device (source: [rieglusa])
The outgoing laser signal may then be reflected on an object but only a part of this reflected signal is received back from the scanner, as shown in figure 2.14. That is why also the intensity of the received signal is measured. In this way, i.e. if the measured intensity is within some threshold values, it can be assured that the received signal is the actual echo of the outgoing signal.

![Fig. 2.14 Measuring principle of a terrestrial laser scanner (source: [Claassen 03])](image)

There are three main types of scanners used for close range purposes and this categorisation is based on the method used for the distance measuring. These different types are:

- **Triangulation scanners**
- **Time-of-flight scanners**
- **Phase scanners**

Although they all use laser beams as a basis for measurements, their principles in measuring distances differ considerably. What will follow is a description of these three separate systems.

**Triangulation scanners**: The concept behind these scanners is basically a specific case of the structured light technique presented in chapter 2.2.4. In image 2.12 one of the featured set-ups used a laser beam and a line camera. This set-up is also incorporated into the triangulation scanners. Even though this is a structured light technique, it is also considered a laser scanning method, due to the use of laser for measurements. A detailed description of the used measuring principle follows:

Triangulation scanners use triangulation for measuring the distance between scanner and object. This triangulation is created by a laser, a mechanical base, a rotating mirror and a CCD-camera inside the scanner. In this case the triangle is formed for each point by the length of the base between the mirror and the CCD, along with the two angles formed by the mirror, the CCD and the object. The distance can then be determined with a simple use of the sine law. The set-up described above is featured in figure 2.15 in the next page:
The main measure of accuracy with a triangulation scanner is the intersection quality, which depends, as with any other triangulation, on the base length vs. object distance ratio. That understandably has an impact on the measurement range. Thus triangulation scanners offer very high accuracy but only in small ranges. Actually the accuracy of the determined distance between instrument and object (\(\sigma_R\)) decreases with the square of this distance (\(R\)) [Boehler & Marbs 02]:

\[
\sigma_R \approx R^2 \quad (2.1)
\]

In practice that makes triangulation scanners ideal for a maximum range of 10 meters and small objects in general.

**Time-of-flight:** Also called pulse scanners, they use another way for measuring the distance. The scanner sends the pulse which is then reflected by the object. Part of the outgoing signal is received back by the scanner. The time lapse between sent and received pulse, or time-of-flight, is measured and used for calculating the distance. Since the velocity of a laser pulse in the air is known (\(c = 299,792,458 \text{ m/s}\)), the time-of-flight (\(t_f\)) can be used to determine the distance between the scanner and the object (\(R\)):

\[
R = c \cdot \frac{t_f}{2} \quad (2.2)
\]

This principle of measuring is presented in figure 2.16:
The range accuracy for a specific angle in these scanners is considered to be independent of distance but this is not entirely true. As it was shown in figure 2.14, there is a loss of intensity in the return signal. That means that it is important for the outgoing signal to have a high enough intensity so that it can provide a strong enough return signal for a given range. In practise this entails using higher pulse energies for longer ranges. Another way to increase accuracy is by using multiple shots for one point and in the end taking their average.

However, the main problem with the time-of-flight scanners is that the reflectivity of the objects affects the accuracy of the measurements. Hence the reflectivity of the objects in question has to always be taken into account before any measurement is planned. The main elements that affect reflectivity are the construction material of the object and the colour covering it. It is still a matter of investigation and research how do specific materials and colours react, but generally glass surfaces and black colours pose serious problems in laser measurements.

**Phase scanners:** These scanners also use the outgoing and received laser signal for measurements, but this time the phase differences between them are measured and not time. The signal is emitted by the scanner with a certain phase angle. After it is reflected by the object it returns back to the scanner with a different phase angle. That happens because the phase angle varies in time and by measuring this variation also the distance can be calculated.

This phase difference between the two signals is determined with the help of a modulation of the laser signal with a harmonic wave, as featured in figure 2.17 in the next page:
The main advantage of the phase scanners is that they measure distances continuously, since the used phase angles also differ continuously in time. This makes the phase scanner much faster than a time-of-flight scanner. There is no need anymore for measuring the travel time between scanner and object. In addition, the accuracy is increased because shorter wave lengths are used. The accuracy is also in this case not dependent on the distance but rather on the object’s reflectivity in the scanner’s wavelength. Nevertheless, the main disadvantage of a phase scanner is that the shorter wave lengths used also affect the maximum range of the measurements. At the moment the maximum distance that can be measured with phase scanners is around 50 meters.

Furthermore there is a main drawback to every laser scanner. There are certain health risks that can be posed from the infrared laser used. The problem is that direct contact of the eyes with the laser beam can cause, in some extreme cases, blindness. Generally these risks are considered to be very low, especially when precautionary measures have been taken.

2.3 Choosing the right method

In the previous five sections the main methods for 3D shape measuring were introduced. All these techniques have their unique advantages and disadvantages, which make them ideal for different tasks. So in order to make the right decision about which method to use, the parameters concerning this project have to be taken into account.

This project entails making a very accurate survey of a Cessna Citation. The main considerations for this, like any other measuring process, is how to get the best result possible for the object in question in a given time schedule. In this case the main problem is the time limitations. The Cessna Citation in this specific time frame is being used for
teaching purposes from the Department of Aerospace Engineering of TU Delft. Additionally, the plane is also co-owned by the NLR, which is conducting its own test flights in the same time period. That means that the Cessna will only be available for short time periods of one or two days the most.

Furthermore there is the actual plane as an object to consider. First of all the Cessna is a small airplane, compared to a big commercial airliner. Apart from that, the shape of the plane is obviously very aerodynamic, with hardly any edges or specific detail points. Finally there is also the matter of the highest accuracy possible that is needed. This can be translated to acquiring the most data possible from the surface of the Cessna, which can subsequently give a better result in the processing stage.

All the aforementioned concerns lead to the following reasoning when it comes to choosing the correct method. A total station could provide the necessary accuracy but its point-oriented nature makes this method too slow to even consider it for a complete survey of the plane. On the other hand a CMM can give even higher accuracies but is also hindered by its point-based philosophy and its small capacity for automation.

Then there is close range photogrammetry which normally should be the first choice for a project like this. It can offer as much accuracy as desired just by designing a different network with the right amount of photos. If everything is designed well beforehand, it can also be completed very quickly, since taking pictures is a very fast procedure. But there are also other problems with this specific case. Photogrammetry is ideal when having a building or a monument to survey, with many detail points. Alas, the Cessna is basically one big collection of curved surfaces and has no considerable texture. This would make the processing of the data very difficult, unless a large amount of targets were to be placed on the surface of the plane. However this procedure would take a lot of time for planning and executing, which is not available.

Maybe one of the solutions would be to use the structured light method. The projection of a pattern on the plane would eliminate any reason to use targets. Nevertheless, this would make the creation of the needed network a very elaborate procedure, much more than setting up a photogrammetric one. Thus the time needed for measurements would increase.

In the end there seems to be only one method available that can deliver within the constraints of this project and this is laser scanning. Laser scanning can offer a huge amount of data in the shortest time possible. One day should be enough to perform a complete survey of the plane. It also needs far less preparation work than the other methods. Although it cannot give the same high accuracy as photogrammetry or any other of the aforementioned methods, it can still deliver a much better result than the model already used for CFD studies.

A summary of all the factors presented above is featured in table 2.3 in the following page:
Table 2.3 Comparison of 3D measurement techniques

<table>
<thead>
<tr>
<th></th>
<th>Total Station</th>
<th>CMM</th>
<th>Structured Light</th>
<th>Photogrammetry</th>
<th>Laser Scanning</th>
</tr>
</thead>
<tbody>
<tr>
<td>Suited for Points</td>
<td>YES</td>
<td>YES</td>
<td>NO</td>
<td>YES</td>
<td>NO</td>
</tr>
<tr>
<td>Suited for Edges</td>
<td>YES</td>
<td>YES</td>
<td>NO</td>
<td>YES</td>
<td>NO</td>
</tr>
<tr>
<td>Suited for Surfaces</td>
<td>NO</td>
<td>NO</td>
<td>YES</td>
<td>NO</td>
<td>YES</td>
</tr>
<tr>
<td>Range</td>
<td>5 Km</td>
<td>0.05-2 m</td>
<td>0.5-50 m</td>
<td>0.5 - 50 meters</td>
<td>1 - 500 meters</td>
</tr>
<tr>
<td>Precision</td>
<td>mm</td>
<td>micrometer</td>
<td>Pattern dependent (can reach mm)</td>
<td>Scale dependent (can reach sub-mm)</td>
<td>3 mm</td>
</tr>
<tr>
<td>Speed</td>
<td>Very slow</td>
<td>Very slow</td>
<td>Medium</td>
<td>High</td>
<td>Medium</td>
</tr>
<tr>
<td>Costs</td>
<td>High</td>
<td>Very High</td>
<td>Low</td>
<td>Low</td>
<td>Very High</td>
</tr>
<tr>
<td>Specific Problems</td>
<td>Required object knowledge</td>
<td></td>
<td></td>
<td>Surface texture required for finding homologous points</td>
<td>No return signal from absorbing or specularly reflecting surfaces</td>
</tr>
<tr>
<td>Specific Advantages</td>
<td>No need for prism in latest models</td>
<td>Capability to capture very small irregularities</td>
<td>Images can be used for texture mapping</td>
<td>High amount of data acquisition (120,000 points/sec)</td>
<td></td>
</tr>
</tbody>
</table>

From the three choices available the **triangulation scanner** provides the highest accuracy. However its very small range is a negative aspect when it comes to the limited time issue, since more scans would be required for a full coverage. That leaves the **time-of-flight** and **phase scanners** for consideration but since the object is relatively small, it will be well within a range of 50 meters. Thus it would be considered overkill to use a time-of-flight scanner which has a much bigger range and, in the end, would make the whole measurement procedure slower. The phase scanner can gather more points than the other scanners in the same time frame and its range is more than enough for the needs of this project. All these qualities are what finally make a phase scanner the most appropriate tool for this task.
3. Measuring the Airplane

In chapter 2 the motivation behind carrying out the current project was presented, along with the reasoning behind choosing laser scanning as the optimal solution for the measuring procedure. The two integral parts for this project to take place are the airplane and the laser scanner used for the measurements. Thus, in this chapter, these two elements along with the measurements will be presented. In chapter 3.1 the Z+F Imager 5003 laser scanner is featured, followed by the Cessna Citation and its specifications in chapter 3.2. In addition the measurement set-up is featured in chapter 3.3. Moreover the hypotheses and the expectations from using this set-up are discussed in chapter 3.4. Finally a small description of the actual measurement procedure is given in chapter 3.5.

3.1 The Measuring Device: The Z+F Imager 5003 laser scanner

Zoller+Froehlich is a medium sized company situated in Wangen, Germany. It was founded in 1963 and by now has expanded enough to have subsidiaries in Great Britain and North America. Z+F has been developing the Imager series of laser scanners. In this project the 5003 model is used. In 2003 a cooperation agreement was made between Z+F and Leica Geosystems. Now the Leica HDS 4500, which is identical to the Imager 5003, is sold under the Leica brand [zf-laser].

The Imager 5003 is a phase scanner whose continuous measuring of distances, described in chapter 2.2.5, enables it to be faster than regular pulse scanners. This characteristic makes it possible to capture 120,000 points per second. What is acquired for these points in a measurement process are x-, y- and z-coordinates and received intensity readings. The point coordinates are polar, which means that they are calculated by measuring two angles and a distance. The general specifications of the Z+F Imager 5003 are featured in table 3.1 below:

<table>
<thead>
<tr>
<th>Table 3.1 Overview of Z+F Imager 5003 Specifications [zf-laser]</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>IMAGER 5003</strong></td>
</tr>
<tr>
<td><strong>FEATURES</strong></td>
</tr>
<tr>
<td>Laser measurement system</td>
</tr>
<tr>
<td>Range</td>
</tr>
<tr>
<td>Resolution Range</td>
</tr>
<tr>
<td>Measurement rate</td>
</tr>
<tr>
<td>position accuracy</td>
</tr>
<tr>
<td>Angle accuracy</td>
</tr>
<tr>
<td>Linearity error (range accuracy)</td>
</tr>
<tr>
<td>Optical transceiver</td>
</tr>
<tr>
<td>Laser output power (CW)</td>
</tr>
<tr>
<td>Beam divergence</td>
</tr>
<tr>
<td>Beam diameter (mm circular)</td>
</tr>
<tr>
<td>Laser wavelength</td>
</tr>
<tr>
<td>Deflection unit</td>
</tr>
<tr>
<td>Vertical field of view</td>
</tr>
<tr>
<td>Horizontal field of view</td>
</tr>
<tr>
<td>Vertical resolution</td>
</tr>
<tr>
<td>Horizontal resolution</td>
</tr>
<tr>
<td>Vertical maximum scanning</td>
</tr>
<tr>
<td>speed</td>
</tr>
<tr>
<td>-------------------------------------------</td>
</tr>
<tr>
<td>Max. # points vertical (per 360 deg)</td>
</tr>
<tr>
<td>Max. # points horizontal (per 360 deg)</td>
</tr>
<tr>
<td>Middle resolution</td>
</tr>
<tr>
<td>Scanning time at middle resolution</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Dimensions and weight</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Scanner (w x d x h)</td>
<td>300 x 180 x 500 mm</td>
</tr>
<tr>
<td>weight scanner</td>
<td>16 kg</td>
</tr>
<tr>
<td>accu size (w x d x h)</td>
<td>240 x 260 x 300 mm</td>
</tr>
<tr>
<td>weight accu</td>
<td>16 kg</td>
</tr>
<tr>
<td>number of cables</td>
<td>2, rotation part of the scanner</td>
</tr>
<tr>
<td>Separate components (yes/no)</td>
<td>No</td>
</tr>
</tbody>
</table>

What can be seen from the specifications is that there are two settings available in the Imager 5003, one for measurements in the range of 25.2 meters and one for measurements in the range of 53.3 meters. That is feasible because there are actually two systems implemented in the scanner, the LARA25200 and the LARA53500 (LARA is the name for the Z+F developed Laser Measurement System). Their difference is the length of the longest wave length of the modulated wave used, which in the end has an effect in the maximum measurable distance. So, the LARA25200 has a 25.2 meter and the LARA 53500 a 53.5 meter range. On a side note, it can be observed that these ranges are far smaller than those available when using a pulse scanner.

Finally, the operational setup used for laser scanning is not always convenient. That is because it is a heavy system; the scanning unit itself has a weight of 16 kg and its size is 30 x 18 x 35 cm, resembling a big total station. This unit is mounted on a tripod with wheels for easier moving. The power needed for the system to work comes from a 24V battery, which can also be mounted on the tripod. Its size is 24 x 26 x 30 cm and has a weight of 16 kg. Alternately, the system can be powered directly by a nearby power outlet. Also a laptop is needed to control the scanner with the appropriate software provided by Z+F. These separate units are connected with the scanner via cables. The following figure features the previously described setup:
3.2 The Measured Object: The Cessna Citation II

The faculty of Airospace Engineering of TU Delft has an airplane used for its scientific purposes and teaching needs. This is a Cessna Citation II, a light twin-engine jet aircraft. Jets like that have also the nickname business jets because they are commonly used for transporting small groups of business people. In the image below the TU Delft Cessna can be seen:

Fig. 3.2 The TU Delft Cessna (source: [steenhouver])

What is a really important characteristic of this aircraft is that it can support a wide variety of modifications. This is the reason that makes this plane very useful for scientific purposes. It is commonly used with remote sensing configurations, including largeformat aerial photography as well as data collection for digital cameras, hyperspectral, multispectral, and LIDAR systems. Below the main characteristics of the Cessna are featured along with some blueprints [aoc]:

<table>
<thead>
<tr>
<th>General characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Crew: 2</td>
</tr>
<tr>
<td>• Capacity: 10</td>
</tr>
<tr>
<td>• Length: 14.39 m (47 ft 3 in)</td>
</tr>
<tr>
<td>• Wingspan: 15.91 m (52 ft 3 in)</td>
</tr>
<tr>
<td>• Height: 4.57 m (15 ft 0 in)</td>
</tr>
<tr>
<td>• Wing area: 31.8 m² (343 ft²)</td>
</tr>
<tr>
<td>• Empty weight: 3655 kg (8060 lb)</td>
</tr>
<tr>
<td>• Max takeoff weight: 6850 kg (15,100 lb)</td>
</tr>
<tr>
<td>• Powerplant: × 2 Pratt &amp; Whitney Canada JT15D4Bs turbofans, 11.1 kN (2500 lbf) each</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Performance</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Cruise speed: 746 km/h (403 kt)</td>
</tr>
<tr>
<td>• Range: 3700 km (2000 nm)</td>
</tr>
<tr>
<td>• Service ceiling: 45,000 ft (13,700 m)</td>
</tr>
<tr>
<td>• Rate of climb: 3040 ft/min (15.4 m/s)</td>
</tr>
</tbody>
</table>
This Cessna has been already flying for more than 10 years. The constant modifications being carried out on the airplane for the different experiments have strained its body. It was mentioned in chapter 2.2.5 that different materials and colours can greatly affect the results of a laser scanning procedure, thus one of the main points of interest for this project are the materials used and the colours covering the body of the plane. The majority of the Cessna, body and wings, are made out of aluminium. Only specific parts of the plane are made of different materials. One of these materials is fiber-glass in the nose of the plane, where the radar is installed. Radar signal cannot penetrate aluminium, so, for it to work properly, the aluminium was replaced with fiber-glass. Another small part where fiber-glass is being used is in the connection of the fuselage with the tail. Other materials used are titanium for a big part of the turbines and also rubber for the front part of the wings. Finally, all the windows are made out of special clear plastic. In the following image all these specific parts are featured:
Regarding the colours, photographs show that the majority of the Cessna is covered in white. Nevertheless, a further up-close inspection of the airplane has to be made. TU Delft’s Cessna Citation II, when not flying, is situated in a hangar at the east part of Schiphol airport in Amsterdam. This is where the airplane will be measured. The hangar with the Cessna is shown in figure 3.5 below:

![Fig. 3.5 TU Delft’s Cessna Citation II inside hangar at Schiphol East](image)

The tail of the plane along with the low part of the fuselage is blue. There are also blue and red stripes on the fuselage. These details can be seen in figure 3.6 below:

![Fig. 3.6 Tail part of Cessna Citation](image)
In addition, as it was stated previously, the wings have a part of their surface covered with rubber. These rubber parts of the wings have black color. Furthermore, another part of the wings has a silver aluminum finish, while the turbines feature also a similar surface. Both these details are featured in the following images:

![Fig. 3.7 (Left): Black, rubber surface in front part of wing (Right): Silver aluminum surface in front part of wing and turbine](image)

### 3.3 The Measurement Set-up

The main goal when designing the measurement set-up is to choose the least amount of scanning positions that can give a complete coverage of the Cessna. For this to be achieved the conditions in the measurement site must be considered. The plane will be measured inside the hangar. Therefore the whole arrangement of the hangar needs to be taken into account. An overview of the hangar is featured in the following image:

![Fig. 3.8 Hangar overview with Cessna position](image)

The Cessna is situated there along with another airplane, so that makes the site a bit confined. However this cannot in any way hinder the choosing of scanning positions. That means that the plane can be easily covered with several positions.
around it. Two are the things that have to be decided during the measurement set-up design:

- **Target positions**
- **Laser scanner positions**

**Target positions:** One concern that has to be addressed during the design phase of the measurement set-up is the use of targets. As it will be explained extensively in chapter 4, the eventual processing of the separate laser scans will result into a unified point cloud. This is possible by combining the different overlapping point clouds. The procedure is called *registration* and can be greatly helped by situating targets in strategic positions. The best positions for the targets are places where they can be visible by more than one scans. This way the aforementioned combination of point clouds can be carried out far more easily and in some cases even automatically, with the use of proper software.

In the current project the hangar, where the plane is situated, can prove to be very helpful for the registration process. The reason behind this is because the hangar is essentially a confined space. In situations like these, targets can easily be placed and distributed in the whole measurement area. Apart from that, the surrounding can also be used to find well defined points of interest, even in places where it would be impossible to place targets. Thus, the vast choice of points and surfaces that can be used makes the hangar a very good location for measuring the Cessna.

**Laser scanner positions:** The laser scanner should be placed in positions that provide a complete coverage of the plane. This is especially crucial for the correct survey of the two pairs of wings, the main and the tail ones. That means that the Cessna must be divided in three areas:

- **Main part**
- **Low part**
- **Upper part**

**Main part:** The main part of the plane can easily be covered with scanning positions surrounding it. Four scans are enough with the scanner standing on a tripod at a regular height.

**Low part:** The low positions should have the scanner as close to the ground as possible. That means that the scanner will be put on the ground without a tripod. The scanning positions should cover the low part of the fuselage and the wings.

**Upper part:** These should be at a level above the plane. Thus data will be gathered from the top of the fuselage along with the upper parts of the main and tail wings.

It is decided that the whole hangar will be scanned. Such scans will provide for a better registration. Thus, the laser scans will be covering an area of 360°. Points of interest from the whole hangar, along with strategically placed artificial targets will be used. With these factors in mind, the setup featured in the following image is decided:
The regular height level position will be at a normal height of about 1.60 m to 1.70 m. These stations are 1 to 4. By surrounding the airplane in four corners, they can achieve maximum coverage of the bulk of the Cessna fuselage.

The low positions are numbered from 5 to 10. Positions 5 and 6 are covering the front-low part of the fuselage. Stations 8 and 9 are covering not only the rear-low part of the fuselage, but also the lower part of the tail wings. Finally, positions 7 and 10 are specifically placed to get data from the lower parts of the main wings. The height should be as close to the ground as possible and is expected to be around 30 cm. In addition, the plane will be raised in some manner, so that better coverage of the low part can be achieved. This will be possible with the use of jacks, which can elevate the plane by 40 cm.

The high positions are stations 11 and 12. These scans will give an overview of the whole site and can cover the upper part of the tail and main wings. They also cover the top part of the fuselage. The height of these positions should be around 3 m, which is enough to have an overview of the complete airplane.
Apart from the scanning positions, the plan has some tentative placements for the registration targets. The proposed positions for the targets ensure that they are visible by a maximum amount of scans. The scanning positions are also thought out in a manner that not only provides a complete coverage, but also gives the overlap necessary for the registration.

3.4 Hypotheses – Expectations

All the elements that could be thought out beforehand regarding the set-up are planned. In this section the expected results and problems are going to be discussed. Furthermore, measures to solve these problems are proposed. The main concerns with this measurement project are the following:

- Laser scanner Imager 5003 problems with specific colours
- Laser scanner Imager 5003 behaviour with specific materials
- Laser scanner Imager 5003 behaviour in special conditions

Laser scanner Imager 5003 problems with colours: In chapter 2.2.5 the effect of different colours of surfaces in the laser scanning process was mentioned. This field has not been studied extensively yet and there is much research currently being carried out [Ree 06]. That is why in many cases only results from former experience can be considered. In the case of the Cessna there are 5 colours covering its surface:

1. **White**: This colour presents no problems in laser scanning
2. **Red**: This colour presents no problems in laser scanning
3. **Blue**: This colour presents some difficulties when it comes to laser scanning. The density of points returned from a blue surface is considerably smaller than surfaces with other colours. This behaviour is more evident when the scanning angle is 90°. These problems with the colour blue are probably due to the infrared wavelength of the emitted laser pulse.
4. **Black**: This colour presents a big problem in laser scanning. Black colours absorb the energy emitted from the laser. This results in little returning signal and noisy points.
5. **Silver**: This colour also presents a problem in laser scanning. Silver is a shiny surface. Shiny surfaces have specular or ‘mirror-like’, characteristics. This means that when an incoming signal hits the object at a specific angle, it will be reflected from it with the same angle. In this case no laser points are detected.

The most effective way to solve a problem like this is to mask the parts that pose a danger to produce such results. This can be achieved by using material of a ‘neutral’ colour to cover the surfaces in question. Such material could be a white duct-tape. What is also crucial is to find a tape of a very small thickness, so that it can be applied uniformly on the wings. This way their actual shape will not be affected and, consequently, neither will the final scanning results.

Laser scanner Imager 5003 behaviour with specific materials: Laser scanning results can show different behaviour depending on the material scanned, as mentioned previously. In the presentation of the airplane in chapter 3.2 it was featured that aluminum is the main material used. Some parts are made of fiber-glass, rubber
and titanium. Experience shows that these materials do not present any alarming behaviour. The only parts that could prove problematic are the windows, which are made from a special type of clear plastic. However these parts can also be covered in the same way as the wing surfaces, to avoid such complications.

**Laser scanner Imager 5003 behaviour in special conditions:** Scanning the low parts of the plane may prove to be problematic. The main concern is that, while scanning, there needs to be a safety distance between the scanner and the object. This distance is one meter and if it is not kept the scan is not carried out. To successfully scan the low part of the plane would mean to put the scanner below the plane in such a distance.

This is understandably a serious undertaking. The only possible way to achieve it is to raise the plane on jacks. The maximum height that the plane can be elevated with the use of jacks is 40 cm. Thus, it may be difficult to take scans from the low positions.

### 3.5 The Measurement Procedure

In this section a short description of the actual measurement procedure will be given. Only one day was needed for the measurements to be completed. They took place in mid-March and a crew of four people from the OLRS group worked on them. Additionally two members of the maintenance staff of the plane helped. The measurement procedure consisted of the following steps:

1. **Setting up the equipment (see figure 3.10)**
2. **Placing the targets according to the set-up plan (see figure 3.11)**
3. **Raising the plane on jacks (see figure 3.12)**
4. **Masking parts of the plane that would cause problems in scanning**
5. **Scanning**
6. **Additional detail scans**

![Fig. 3.10 Step 1: Setting-up the laser scanner](image-url)
The first 3 parts were preparatory work for the actual measurements and did not take more than 30 minutes to complete. Problems began when the rest of the process started. The people working in the hangar had to be informed about the measurement procedure and its hazards. As it was mentioned in chapter 2.2.5, there are certain health risks that can be posed from laser scanning. Namely the risk is that
direct contact of the eyes with the laser beam can cause, in some extreme cases, blindness.

The Cessna is co-owned by TU Delft and the NLR (Nationaal Lucht - en Ruimtevaartlaboratorium). On the day of the measurements a crew from NLR was working on the plane. When informed in detail about the scanning dangers, the NLR team was reluctant to allow the OLRS group to work. All the assurances given to them were not satisfying. That had repercussions on the work carried out that day, since measurements could only be carried out when no other crew was working on the plane. Furthermore, due to this lack of co-operation, step 4 of the plan was not allowed to be carried out. Thus the black and silver surfaces of the Cessna were not covered. In the end only 3.5 hours were used for scanning out of the whole day.

5. Scanning: Scanning was carried out according to the set-up presented in chapter 3.3. Due to the limited time available, the scanning procedure needed to be completed quickly. That is why it was decided to do the scans in medium resolution. Only the overview scans 11 and 12 would be in high resolution. The first four scanning positions (scans 1-4 at a regular height) presented no difficulties (see figure 3.13 below).

![Fig. 3.13 Scanning position 1](image)

Problems began with the low scans 5 to 10. As mentioned in chapter 3.4, the minimum scanning distance was expected to create some trouble with the placing of the scanner below the Cessna. The scanner could not be placed on the ground without giving a minimum distance error. That prompted the crew to raise the scanner above the ground just enough so that the low scans could be completed. Eventually the setup shown in figure 3.14 was used:
The elevated scanning positions 11 and 12 were left after that. In this phase a metal staircase was successfully used for putting the laser scanner on. This staircase is used for the entering of passengers into big airliners. The final setup is featured in the figure 3.15 below:

6. Additional detail scans: It is normal in every project to perform additional scans when there are areas that are not covered properly. This was also the case here. One of the things possible during the measurement process was to check a small preview of the scan results. That check did create a suspicion about a specific part of the Cessna where the results might not be satisfactory. This part was the connection of the wings with the fuselage.
 Normally scanning positions 1 and 2 should give points from that area but it seems that the incidence angle was not favourable enough to give back a result. That prompted the team to look for some additional positions where a number of complementary scans could be made. It seemed that elevated scans were needed for that area. Unfortunately scans 11 and 12 could not cover this part due to the placement of the turbines. That meant that the new elevated scans should be carried out closer to the wings and in a position not obstructed by any other airplane parts.

The best possible choice was to perform two additional scans 13 and 14 placed in front of the wings. The concept behind the additional scans can be seen in figure 3.16 below, along with the areas that seem to have very small point coverage:

![Fig. 3.16 Concept of additional scans 13 and 14 for better coverage of the problematic area](image)

Unfortunately time was already limited. There was no way to work any longer with the laser scanner and complete the proposed scans. Consequently, these additional measurements could not be carried out.
4. Registration

In chapter 2 the measurement project and the reasoning behind it were featured. In chapter 3 the laser scanner and the Cessna were presented, followed by a detailed description of the scanning procedure. Now all the acquired data need to be processed. This chapter will illustrate the first step in the processing phase, which is called registration. In this stage the point clouds from the separate laser scans are transformed into one unified result. This is achieved by means of combining the different overlapping point clouds. The algorithm used for performing this task is the Iterative Closest Point algorithm (ICP). Thus, in chapter 4.1 the ICP algorithm will be presented, while in chapter 4.2 an overview of the acquired laser scans will be given. In chapter 4.3 the process of registering the laser scans will be described. Chapter 4.4 features some attempts for optimizing the registration, while in chapter 4.5 the registered Cessna is presented. Finally chapter 4.6 describes the procedure followed for putting the laser scanned Cessna point cloud with the CAD design model into the same coordinate system.

4.1 Iterative Closest Point (ICP) algorithm

The ICP algorithm, which was presented by Chen and Medioni [Chen 91] and Besl and McKay [Besl 92], is considered the standard method for aligning 3D models based solely on the geometry of their points. Other existing methods are similar and based on aligning objects from the models and not point clouds. The concept behind ICP is the following.

Two point clouds are taken and an initial guess for their relative rigid-body transformation is made, by means of some initial corresponding points. The transformation is then iteratively refined by repeatedly generating pairs of corresponding points on the two clouds and minimizing an error metric.

ICP is ideal for registering point clouds coming from laser scans, where usually there are overlapping scans of the same object from different directions. The analytical description of the ICP algorithm is featured below [San-Jose 04]:

Let $S$ be the data shape (source shape) and $M$ the model shape (target shape) to be registered with. Both shapes are considered to be in point set form, if not already available in that form. Thus, let $N_S, N_M$ be the number of points in the shapes. $S$ and $M$ are respectively defined by the $N_S$-tuple $S = \{s_1, s_2, ..., s_{N_S}\}$ and the $N_M$-tuple $M = \{m_1, m_2, ..., m_{N_M}\}$. There is not a pre-established correspondence between points $s_i$ and $m_i$.

ICP finds the rigid body transformation [Besl 92], rotation $R$ and translation $t$ that aligns the source with the target by minimizing the distance metric:

$$J(R,t) = \sum_{i=1}^{N} \left\| y_i - Rs_i - t \right\|^2 \quad (4.1)$$
is a point on the surface of the target model $M$ that corresponds to the point $s_i$ and $C$ is a correspondence operator. The problem that needs to be solved is the correspondence between source points and target points. For that an iterative search is needed, where in each iteration, a correspondence is established by means of the correspondence operator $C$ mentioned above. Then, the transformation that minimizes a mean square metric is computed. The computed transformation is then used for the transformation of the data points. This process is repeated until a minimum error threshold is achieved or a maximum number of iterations is reached.

The basic point matching algorithm [Sharp 99] is the following:

Again it is assumed that $S$ is a set of $N_S$ points $\{s_1,\ldots,s_{N_S}\}$ and $M$ is the model. Let $\|s - m\|$ be the Euclidean distance between point $s \in S$ and $m \in M$. Let $CP(s,M)$ be the closest point in $M$ to the scene point $s$.

1. Let $T^{[0]}$ be an initial estimate of the rigid transformation.
2. Repeat for $k = 1\ldots k_{\text{max}}$ or until convergence:
   1. Compute the set of correspondences
      \[ C = \bigcup_{i=1}^{N_S} \{(s_i, CP(T^{[k-1]}(s_i), M))\} \]
   2. Compute the new Euclidean transformation $T^{[k]}$ that minimizes the mean square error between point pairs in $C$.

As the solution is based on least squares, a variance matrix provides a measure of quality for the process. By now there are many variations of this basic algorithm. All these extensions and additions were performed over time to improve some of the drawbacks of the ICP. These modifications have been made to affect the following six stages of the algorithm [princeton]:

1. **Selection** of some set of points in one or both point clouds.
2. **Matching** these points to samples in the other point cloud.
3. **Weighting** the corresponding pairs appropriately.
4. **Rejecting** certain pairs based on looking at each pair individually or considering the entire set of pairs.
5. **Assigning an error metric** based on the point pairs.
6. **Minimizing** the error metric.

There are problems which are commonly encountered with ICP, like the fact that unequal uncertainty among points is not considered in the process [San-Jose 04]. A number of modifications used to overcome some of the most important drawbacks is presented below:

- correspondence between a point and a tangent plane to overcome the lack of an exact correspondence between the two sets [Chen 91],
- making the algorithm more robust to the influence of outliers and features lacking correspondences [Zhang 94],
- using a weighted least-square error metric to overcome the unequal uncertainty among points [Dorai 97], and
- matching between features using a metric trading off distance and feature similarity (based local shape invariances) [Sharp 99].

Testing the advantages and disadvantages of the different variants is a vast subject that cannot be presented thoroughly in the scope of this thesis. However in [Rusinkiewicz 01] there is an extensive comparison of all the effective variants of the ICP algorithm. One of the most basic concerns with ICP is the speed and the quality of convergence of the algorithm. In [Rusinkiewicz 01] it is proven that the speed of the ICP method is mainly affected by the type of error metric used and the type of algorithm used in the matching stage. Thus, for different error metrics there are differences in the behaviour of the ICP algorithm.

This is evident for typical error metrics like the point-to-point and point-to-plane presented above [Chen 91]. It is proven that the traditional point-to-point error metric has substantially slower convergence than a point-to-plane metric. In addition a projection-based algorithm also proves faster than the more standard closest-point algorithm in the matching process.

4.2 Overview of acquired laser scans

The result of every laser scanning procedure is the acquisition of a massive amount of points, appropriately called a point cloud. In the current project, the twelve scanning positions provided twelve separate point clouds. Apart from previewing the result of each scan before further processing, also the amount of obtained points is available.

As it was mentioned in the presentation of the laser scanner in chapter 3.1, there are many possibilities when it comes to perform the actual scans. The basic choice affecting the procedure is what resolution the scans will have. In addition, the scanning range is also playing a role in the time needed for a scan to take place. The choices with the used set-up regarding the resolution were between preview, medium, high and super high, and regarding the range were close (25.2 m) and far (53.5 m). Understandably, each added level of detail means more points for the final scan. At the same time though the drawback for a higher amount of points is a longer time for scanning. It is important to notice that the choice made here does not affect the quality of the final result, i.e. there is no compromise being made on the quality of the obtained points. When talking about the same project, points from a medium setting and points from a high setting have the same accuracy. The only change is the size of the final point cloud.

In this case, although the initial plan was to have all the scans in high resolution, the problematic conditions on the site forced the crew to make the most time effective choice. It was decided that the majority of the scans would be carried out in medium resolution and close range, due to the proximity of the scanner to the plane and its small size (14 m) compared to the range (25.2 m). Only the elevated scans 11 and 12 were taken in high resolution and far range. This decision was based on the grounds that these two were overview scans tying all the other ones, so the maximum amount of points provided would prove helpful. In the end the gain in time was significant: 1:42 minutes for medium scans and around 5:30 minutes for high resolution scans. Nevertheless, the difference in the final result is equally considerable: 10 million points for medium scans and 40 million points for high
resolution scans, giving a total of 180 million points. The table below features the above information for each scan in detail:

<table>
<thead>
<tr>
<th>Scanning Position</th>
<th>Number of Points</th>
<th>Resolution</th>
<th>Range (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9,980,882</td>
<td>Medium</td>
<td>25.2</td>
</tr>
<tr>
<td>2</td>
<td>10,006,485</td>
<td>Medium</td>
<td>25.2</td>
</tr>
<tr>
<td>3</td>
<td>10,123,892</td>
<td>Medium</td>
<td>25.2</td>
</tr>
<tr>
<td>4</td>
<td>10,089,532</td>
<td>Medium</td>
<td>25.2</td>
</tr>
<tr>
<td>5</td>
<td>10,000,596</td>
<td>Medium</td>
<td>25.2</td>
</tr>
<tr>
<td>6</td>
<td>9,898,791</td>
<td>Medium</td>
<td>25.2</td>
</tr>
<tr>
<td>7</td>
<td>9,897,419</td>
<td>Medium</td>
<td>25.2</td>
</tr>
<tr>
<td>8</td>
<td>9,979,066</td>
<td>Medium</td>
<td>25.2</td>
</tr>
<tr>
<td>9</td>
<td>9,986,069</td>
<td>Medium</td>
<td>25.2</td>
</tr>
<tr>
<td>10</td>
<td>9,925,519</td>
<td>Medium</td>
<td>25.2</td>
</tr>
<tr>
<td>11</td>
<td>40,396,914</td>
<td>High</td>
<td>53.5</td>
</tr>
<tr>
<td>12</td>
<td>40,151,851</td>
<td>High</td>
<td>53.5</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>180,437,016</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### 4.3 Registering the Scans

In chapter 4.2 an overview of the acquired laser scans was given. These twelve scans are going to be processed with the software package Cyclone ver. 5.4. This is a very popular program for cloud alignment produced by Leica. It is provided along with every Z+F laser scanner as part of their agreement. Although registration could be performed with other packages as well (e.g. Geomagic), Cyclone was chosen. This is because Cyclone provides better tools for registration along with a number of quality control indicators resulting from the process [Ioannidis 05]. This enables the user to get a better control of the process.

The way registration in Cyclone works is by fitting together the overlapping parts of the scans with the help of the ICP algorithm, which was presented in the previous chapter. The concept behind ICP is to take the two overlapping point clouds and make an initial guess for their relative rigid-body transformation. This is achieved with the determination of some initial corresponding points by the user. The transformation is then iteratively refined by repeatedly generating pairs of corresponding points on the two clouds and minimizing an error metric. Thus, constraints are being created between the separate pairs of overlapping point clouds. In the end these constraints are used for a final, global computation, which provides a singular, registered point cloud.

A short description of the whole procedure followed is presented below:

- **Creating a new project:** The new registration project Cessna Citation is created in Cyclone by inputting the scans presented in chapter 4.2.
- **Determining the scanworlds:** In Cyclone terminology a Scanworld is the point cloud that comes from one scanning position. In this project 12 scanworlds are defined, since there were 12 scanning positions used. Also 12 controlspaces, i.e. the corresponding 3D spaces of each point cloud, are defined. Both scanworlds and controlspaces are distinguished from 1 to 12 according to the sequence used in the scanning procedure (see fig. 3.9).
• **Adding constraints:** Cyclone provides several methods for adding constraints. For an automatic addition of constraints, Cyclone can use registration labels. In most cases, constraints that result from Registration labels come from specialised targets (called HDS Targets) that have been placed in overlapping areas of scans for the specific purpose of using them for Registration. Some HDS Targets may have also been imported from a reference coordinate system (e.g., survey data).

In this project such target were not used, so the creation of constraints was a manual procedure. One alternative for manual adding constraints is with the use of objects. This is only possible if these objects are added to the controlspace for each scanworld and establishes that these two objects are equivalent. This means that the objects have to be modelled beforehand in each scanworld separately.

This procedure was considered too labour-intensive for this project. That is why the second alternative for manual addition of constraints was used; the determination of cloud constraints.

• **Cloud constraints:** Cloud constraints do not differentiate between clouds and meshes in Cyclone. This means that cloud constraints can be used to align a cloud with a mesh mode also. In this case though there are only 12 point clouds. The special Cloud Constraints Wizard Tool was used for the creation of the constraints. Its layout is featured in figure 4.1 below:

![Registration with Cyclone: Defining constraints between scans 4 and 10](image-url)
This tool makes the whole process easier and works in the stages described below:

1. The scanworld pairs that are overlapping are specified. In this project the 12 scans gave 32 overlapping pairs.
2. The Constraint viewers in the Registration window show the two controlspaces that contain the overlapping clouds.
3. Three or more matching points on the two clouds need to be picked. In this project an average of 4 or 5 points were picked for each overlapping pair. For picking the best possible points for the initial alignment, the following simple rules were followed:
   - Points were chosen that are easily identified. Good choices include corners, and intensity markings. Such markings are the targets used and described in chapter 3.3 (see figure 4.2 below). Bad choices are points in the middle of a flat wall or in the middle of a smoothly curving surface without any identifiable features.
   - Points were chosen that match the pick points in the other scanworld as closely as possible. That means that the chosen points should be easily identifiable in both scanworlds. The closer the corresponding points are, the better the quality of the match will be in the end.
   - The points were spatially distributed over a large volume. Ideally three points should form a very large triangle. That is why the entire overlapping area is inspected so that the picked points cover as much of that area as possible. This ensures that the initial alignment aligns all the overlapping areas as closely as possible. Furthermore, when more than three points are picked, it is best for the alignment that they are asymmetrical.

After the constraint for two scanworlds was defined, a visual check of the result was performed. The connected scanworlds unified point cloud is available as an interim result by Cyclone (see fig. 4.3 in the following page).
Fig. 4.3 (Top) Point cloud from scan 12, (Middle) point cloud from scan 11, (Bottom) constrained point cloud from scans 11 and 12

- **Registering**: The next step is to perform the registration. The registration command computes the optimal alignment transformations for each scanworld in the project so that all constrained objects are aligned as closely as possible. Statistics for the successful registration are available in the end via a diagnostics window. The quality of the procedure is shown in this diagnostics.

- **Creating a registered scanworld and modelspace**: The final step is to create a new scanworld from the registration. This scanworld
includes the component scanworlds of the registration in a common coordinate system. The resulting modelspace gives the visual outcome of the procedure.

The above procedure was followed for this project. The final product can be seen in the images below:

Fig. 4.4 The registered hangar
The registration diagnostics for the 32 constraints used are featured in the following table:

<table>
<thead>
<tr>
<th>Constraint #</th>
<th>Scanworlds</th>
<th>Error (m)</th>
<th>Error Vector (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2-3</td>
<td>0.002</td>
<td>0.017 (aligned)</td>
</tr>
<tr>
<td>2</td>
<td>2-5</td>
<td>0.001</td>
<td>0.017 (aligned)</td>
</tr>
<tr>
<td>3</td>
<td>2-1</td>
<td>0.002</td>
<td>0.015 (aligned)</td>
</tr>
<tr>
<td>4</td>
<td>2-7</td>
<td>0.002</td>
<td>0.018 (aligned)</td>
</tr>
<tr>
<td>5</td>
<td>2-8</td>
<td>0.001</td>
<td>0.014 (aligned)</td>
</tr>
<tr>
<td>6</td>
<td>2-11</td>
<td>0.002</td>
<td>0.016 (aligned)</td>
</tr>
<tr>
<td>7</td>
<td>3-4</td>
<td>0.001</td>
<td>0.016 (aligned)</td>
</tr>
<tr>
<td>8</td>
<td>3-5</td>
<td>0.001</td>
<td>0.015 (aligned)</td>
</tr>
<tr>
<td>9</td>
<td>3-7</td>
<td>0.002</td>
<td>0.018 (aligned)</td>
</tr>
<tr>
<td>10</td>
<td>3-11</td>
<td>0.003</td>
<td>0.019 (aligned)</td>
</tr>
<tr>
<td>11</td>
<td>4-5</td>
<td>0.001</td>
<td>0.015 (aligned)</td>
</tr>
<tr>
<td>12</td>
<td>4-1</td>
<td>0.001</td>
<td>0.017 (aligned)</td>
</tr>
<tr>
<td>13</td>
<td>4-6</td>
<td>0.001</td>
<td>0.014 (aligned)</td>
</tr>
<tr>
<td>14</td>
<td>4-10</td>
<td>0.001</td>
<td>0.016 (aligned)</td>
</tr>
<tr>
<td>15</td>
<td>4-12</td>
<td>0.001</td>
<td>0.019 (aligned)</td>
</tr>
<tr>
<td>16</td>
<td>5-6</td>
<td>0.002</td>
<td>0.014 (aligned)</td>
</tr>
<tr>
<td>17</td>
<td>5-7</td>
<td>0.002</td>
<td>0.017 (aligned)</td>
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<tr>
<td>18</td>
<td>5-11</td>
<td>0.002</td>
<td>0.018 (aligned)</td>
</tr>
<tr>
<td>19</td>
<td>1-6</td>
<td>0.002</td>
<td>0.018 (aligned)</td>
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<tr>
<td>20</td>
<td>1-9</td>
<td>0.000</td>
<td>0.014 (aligned)</td>
</tr>
<tr>
<td>21</td>
<td>1-10</td>
<td>0.001</td>
<td>0.015 (aligned)</td>
</tr>
</tbody>
</table>
The error column features the error for each constraint. It gives the quality of the alignment between two scanworlds, i.e. how well the ICP algorithm was implemented. The mean absolute error for all the enabled constraints is 0.001 m. On the other hand the error vector gives the error in the baselines between the scanworlds after the registration has been completed. The mean error vector is 0.0167 m. The reason for the difference between these errors is not entirely clear. Cyclone is an expensive software package and a ‘black box’ to its users. According to the manual [Cyclone 06] there is no problem when the error is close to the mean value, which is the case here. Furthermore the recommendations for the error vector value are not very precise. It can be around 1 cm with the use of the specialized HDS targets but it can be higher for complex objects.

In the end the manual remains vague about the accuracy. Although the mean error vector value seems a bit high, the mean error has a very low value. It is assumed that the ICP algorithm implementation in Cyclone uses a point-to-plane metric which gives better results than a standard point-to-point metric [Rusinkiewicz 01]. This explains the difference between the two errors, since the error vector is not affected by this. Eventually it is suggested by the manual that the most important check is the visual. As it can be seen from the figures above, the result is very accurate.

### 4.4 Attempts for an optimized registration

In the registration presented in the previous chapter there are five constraints that were not used. The scan pairs 9-10, 9-12, 10-12, 8-9 and 6-10 were not taken into account in the global computation (not aligned). To improve the result it was attempted to find a solution that would use all the constraints. Two approaches were taken for this.

First of all the picked point in the specific constraints were improved. This time points that were more distinguishable were chosen. Additionally more points were picked. Secondly the Cyclone manual was consulted. For an optimized cloud alignment there is a parameter called Cloud Reg Subsampling Percentage. This parameter specifies a speed-versus-accuracy trade-off in the performance of the Optimize Cloud Alignment command. The default is 3%, which is effective and efficient for typical cases. In general, a value of 100% may give results up to two times as accurate as a value of 3%. The downside is that it take approximately ten times as long [Cyclone 06].

In both these cases all the constraints were finally used in the registration. The results are featured in the following images:
It is clear that the results are far worse. The added new constraints were not good for the final registration. The diagnostics showed that the error was in some cases around 2 to 3 meters, hence the outcome presented in the images. It seems that these constraints were not good from the beginning. In the end that poses no problem, since all 12 scans are used for the original registration from other constrained pairs. So, no information is lost in the original registration. Due to this and the good accuracy results, it is considered fine for the purposes of this project.

4.5 The registered Cessna Citation II

The whole hangar was used for the registration of the 12 point clouds but only the actual airplane is of interest for this project. Cyclone was used for removing all the
points that were not part of the Cessna. Furthermore some outlier points were cleared from the surface of the plane. The result is shown in the following images:

Fig. 4.8 Three views of the registered Cessna Citation II
Out of the approximately 180 million points of the 12 scans, the registered Cessna uses 13,678,050. The result can be said that is truly impressive. The point cloud is really close to the original plane. The coloured points are showing the intensity of the reflected laser signal from the scans. At first glance the set-up seems to have been successful; the Cessna is captured from every angle and the resulting outcome looks complete. A further inspection shows that there are some problematic areas.

The fears concerning the low scanning positions (see chapter 3.4) are confirmed. The scanner was too close to the plane to get a nice coverage. Unfortunately there are not points from the whole low part of the plane captured. This problem is obvious in the low section of the fuselage (see figure 4.9 below).

![Fig. 4.9 Lower part of the fuselage not covered with points](image)

It is also clearly evident in the wings, where another problem also arises. The black rubber area in the front part of the wings has many gaps and is very noisy due to the absorbing characteristics of the surface (see figure 4.10).

![Fig. 4.10 Wing view from below: Small coverage of the low part of the wing (the red points are corresponding to this area) and noisy points and gaps in the front part (yellow circle)](image)
In addition there are also problems with the connection of the wings with the fuselage, as it was suspected (see figure 4.11 below). The additional two scans proposed (see chapter 3.5) would have sidestepped this issue. Finally there also seems to be a low coverage in the nose of the plane (see figure 4.11 below). The reason for this seems to be the same as it was for the wing connections with the fuselage. Normally scanning positions 3 and 4 (see figure 3.9) should give points from the nose area but it seems that the incidence angle of the laser was not favourable enough to give back a result.

Apart from these minor problems the rest of the plane is captured completely and the result is considered satisfactory. The outcome is even more impressive when the small amount of time available for the scanning (3.5 hours) is taken into account.

### 4.6 Putting the laser scanned Cessna and CAD model into the same coordinate system

When Cyclone performs a registration, the user can choose any of the scanning positions as the coordinate system origin. Depending on this choice, all the points of the final, unified point cloud will be corresponding to this specific scanning position. The problem is that none of these coordinate systems is the same as the one of the CAD model available. If there is to be any kind of comparison of the CAD model with laser scanned Cessna obtained from the registration, it would be very helpful if they were into the same coordinate system.

The coordinate system picked to be the origin is the one belonging to the CAD model. This is due to the following two reasons. First of all the CAD design model provided by Cessna is symmetrical, as featured in figure 4.11:
In this case the y-axis is dividing the plane in half along its fuselage. All the other coordinate systems coming from the 12 scanning positions are randomly scattered around the Cessna. However the CAD model origin is in a central and specific position that can prove helpful for further processing. The second reason is that the product of this thesis is going to be used from engineers in the Control & Simulation department for CFD studies. It is understandable that it would be much more convenient for them to continue these studies at the improved model if it had the same coordinate system as the previous one.

The approach taken to put the laser scanned plane into the CAD model’s coordinate system was to perform a new registration. In the original registration a new scanworld with the CAD model was imported. This time additional constraints were created between the 12 scans and the CAD model. Due to lack of points only 9 scans out of the 12 were constrained. The procedure followed was the same as the one described previously in chapter 4.3. Corresponding pick points on the plane were distinguished for the constraints, as featured in the image below:
Unfortunately, although constraints were created, a registration could not be carried out in the end. The additional constraints were not aligned and the CAD model could not be incorporated to the previous registration. The explanation behind this result is simple. The CAD model consists of only 50,000 points, while the scans have a minimum of 10 million points. The ICP algorithm used for the registration cannot function properly due to the very big point difference between the scanworlds. The CAD model has too few points for the point cloud fitting that ICP performs. Furthermore, the picked points were only in the area of the plane and not the whole hangar. Thus the overlapping area was too small to get a good result.

After that it was decided to take a different approach. This time a big registration would not be performed. Rather a single constraint would be created between two scanworlds. The first would contain the CAD design model and the second would have the laser scanned Cessna separate from its environment, as it was featured in chapter 4.5. Cyclone was again used for picking corresponding points on the plane. The difference in this case was that the registration was not executed in the end. Only the interim result of the constraint was taken. In this procedure 7 points were used on the Cessna so that the result would be as accurate as possible. A complete fit was not possible anyway because the two objects are not the same. However the final result (see figure 4.13 below) was impressive and suited the needs of this project perfectly.

Fig. 4.13 CAD design model (green) and laser scanned Cessna (coloured) in the same coordinate system
5. Modelling

The measurement project and the reasoning behind it were featured in chapter 2. In chapter 3 the laser scanner and the Cessna were presented, followed by a detailed description of the scanning procedure. In chapter 4 the first step in the processing phase, called registration, was illustrated. In this chapter the next step in the data processing will be presented; modelling. The concept of ‘modelling’ varies depending on the type of project where it is used. For this thesis the meaning of ‘modelling’ is geometric modelling, as in creating a representation of reality for visualisation purposes. Thus, from now on, the word ‘modelling’ will be used in this context.

Thus, in chapter 5.1 the mathematical background used in modelling will be presented. In chapter 5.2 the process behind modelling the right wing of the CAD point cloud will be featured. A 3D approach for modelling the right wing of the laser point cloud is described in chapter 5.3, while in chapter 5.4 a 2D approach is presented for the same area. Finally in chapter 5.4 a comparison is made between the CAD and the laser models.

5.1 Mathematical background used for modelling

In this section the mathematical background needed for the modelling part of this thesis will be presented. In chapter 5.1.1 the principles of triangulation will be featured. Chapter 5.1.2 contains the definition of B-Splines, while chapter 5.1.3 is dedicated to the least squares fitting of points with surfaces and curves.

5.1.1 Triangulation

Triangulation is a standard modelling technique and core part of almost all reconstruction programs. A triangulation converts a given set of points into a consistent polygonal model (mesh). During this operation the input data are partitioned into simplices. Then faces representing the analysed surface are generated, which meet only at shared edges. This is done by dividing the measured domain into many small ‘elements’. Triangulation can be performed in 2D, 2.5D or in 3D, depending on the type of input data.

2D Triangulation: In this case the input data are 2D points. The result of the triangulation procedure is a generation of triangles that intersect only at shared edges and vertices. Most common construction algorithm is the Delaunay triangulation. Its definition follows below. Also defined are the basic concepts of the convex hull and voronoi diagrams.

**Convex hull:** The convex hull of a set of points $S$ in $n$ dimensions is the intersection of all convex sets containing $S$. The convex hull $C$ is given by the expression [mathworld]:

$$C = \left\{ \sum_{j=1}^{N} \lambda_j p_j : \lambda_j \geq 0 \text{ for all } j \text{ and } \sum_{j=1}^{N} \lambda_j = 1 \right\}, \quad (5.1)$$
where: \( N \) is the number of points in the set; \( p_1, \ldots, p_N \) are the set points; \( \lambda \) are non-negative coefficients for each point that sum up to 1.

A visualisation of a convex hull for a set of points \( S \) is shown in figure 5.1 below:

**Voronoi diagrams:** Voronoi diagrams are also called the Thiessen diagrams. Let \( S \) be a set of \( n \) sites in Euclidean space of dimension \( d \). For each site \( p \) of \( S \), the Voronoi cell \( V(p) \) of \( p \) is the set of points that are closer to \( p \) than to other sites of \( S \). I.e. a point \( q \) lies in the cell corresponding to a site \( p_i \in P \) if and only if:

\[
\text{Euclidean Distance (}q,p_i\text{)} < \text{Euclidean Distance (}q,p_j\text{)} \quad \text{for each} \quad p_i \in P, \ j \neq i
\]

The Voronoi diagram \( V(S) \) is the space partition induced by Voronoi cells [inria]. A voronoi diagram for a set of points \( S \) is featured in figure 5.2 below.

**Delaunay triangulation:** The Delaunay triangulation of \( S \) is the geometric dual of the Voronoi diagram of \( S \): two sites of \( S \) are linked by an edge in the Delaunay triangulation if and only if their cells are incident in the Voronoi diagram of \( S \) [inria]. Two basic characteristics are that:

i) the boundaries of the triangles is the convex hull of the set of points,

ii) the circle circumscribed about a Delaunay triangle has its center at the vertex of a Voronoi polygon.

A delaunay triangulation for the same set of points \( S \) is shown in figure 5.2 below:

**Fig. 5.2:** Voronoi diagram (left) and Delaunay triangulation (right) of the same set of points. In Voronoi, each region consists of the part of the plane nearest to that node. Connecting the nodes of the Voronoi cells that have common boundaries forms the Delaunay triangles [Remondino 03].
2.5D Triangulation (TIN): This type of triangulation is common in geodesy. Also called a mesh, TIN stands for Triangulated Irregular Network. The name comes from the regularly or almost randomly distributed generated surface. The 2.5D triangulation applies here because input data is a set of points $P$ in a plane along with a real and unique elevation function $f(x, y)$ at each point $(x, y) \in P$.

A 2.5D triangulation creates a linear function $F$ interpolating $P$ and defined on the region bounded by the convex hull of $P$. For each point $p$ in $P$, $F(p)$ is the weighted average of the elevation of the vertices of the triangle that contains $p$ [Remondino 03]. The Delaunay triangulation is commonly used for meshing. An example is shown in figure 5.3 below:

![Fig. 5.3 TIN example](csiss)

3D Triangulation: Also called tetrahedralization. Here the input data are 3D points and they are partitioned into a collection of tetrahedra that meet only at shared faces. Most commonly used is the Delaunay tetrahedron which is defined as follows [iue]:

Let again $S$ be a finite set of points in a sub-domain $\Omega^3$ of the 3-dimensional space $R^3$. Four non-coplanar points $p_i, p_j, p_k$ and $p_l$ form a Delaunay tetrahedron $T$ if and only if there exists a location $x \in \Omega^3$, which is equally close to $p_i, p_j, p_k$ and $p_l$ and closer to $p_i, p_j, p_k, p_l$ than to any other $p_m \in S$. The location $x$ is the center of a 3-dimensional sphere which passes through the points $p_i, p_j, p_k, p_l$ and which contains no other points $p_m$ of $S$. In the 3-dimensional space $R^3$ this sphere is unique. This is the circumsphere of the Delaunay tetrahedron $T$: 

![Fig. 5.3 TIN example](csiss)
An example of tetrahedralization is shown in the figure 5.4 below:

5.1.2 B-Splines

B-splines are a general type of the Bezier curves. B-splines have control points which are associated with the basis function and it is defined by the following equation [cl.cam]:

\[ P(t) = \sum_{i=1}^{n+1} N_{i,k}(t)P_i \quad \text{for} \quad t_{\text{min}} \leq t \leq t_{\text{max}} \quad (5.4) \]

where:

- \( n+1 \) is the number of control points.
- \( P_1, P_2, ..., P_{n+1} \) are the control points.
- \( k \) is the order of the curve (degree \( k-1 \)). It must satisfy: \( 2 \leq k \leq n+1 \).
- \( N_{i,k} \) are the basis functions of order \( k \) (degree \( k-1 \)).

The order \( k \) must be at least 2 (linear), and can be no more than \( n+1 \) (the number of control points). The order of the curve (linear, quadratic, cubic,...) is therefore not dependent on the number of control points. This is the difference with the Bezier curves, where \( k \) must always equal to \( n+1 \).

Equation (5.4) defines a piecewise continuous function. A knot vector \( (t_1, t_2, ..., t_{k+(n+1)}) \), must be specified. These are necessary for determining the values of \( t \) at which the curve pieces join. It is essential that:

\[ t_i \leq t_{i+1}, \forall i \quad (5.5) \]
The \( N_{i,k} \) depend only on the value of \( k \) and the values in the knot vector. \( N \) is defined recursively as:

\[
N_{i,1}(t) = \begin{cases} 1, & t_i \leq t \leq t_{i+1} \\ 0, & \text{otherwise} \end{cases} \tag{5.6}
\]

\[
N_{i,k}(t) = \frac{t-t_i}{t_{i+k}-t_i} N_{i,k-1}(t) + \frac{t_{i+k}-t}{t_{i+k}-t_{i+1}} N_{i+1,k-1}(t)
\]

The following properties are true for these equations:

- Each \( N_{i,k}(t) \) depends only on the \( k+1 \) knot values from \( t_i \) to \( t_{i+k} \).
- \( N_{i,k}(t)=0 \) for \( t < t_i \) or \( t \geq t_{i+k} \) so \( P_i \) influences the curve only for \( t_i \leq t \leq t_{i+k} \).
- \( P(t) \) is a polynomial of order \( k \) (degree \( k-1 \)) on each interval \( t_i \leq t < t_{i+1} \). Across the knots \( P(t) \) is continuous. \( P(t) \) is \( C^{k-2} \) continuous in all its derivatives between the knots.
- \( P(t) \) is validly defined for \( t_{\min} \leq t < t_{\max} \), where \( t_{\min} = t_k \) and \( t_{\max} = t_{n+2} \).

An example of a Quadratic B-Spline is shown in the following figure:

![Figure 5.5 A Quadratic B-Spline (n=3, k=3): Control points are in blue and knots are in black (top right). In the right the basis polynomials are featured. The actual B-spline curve is composed of \((n-k+2)\) segments painted in different colours. Corresponding \( t \) intervals (in the right window) are painted in the same colours [ibiblio].](image)

**Knot vectors:**

The shapes of the \( N_{i,k} \) basis functions are determined entirely by the relative spacing between the knots \((t_0, t_1, \ldots, t_{n+k})\). Scaling or translating the knot vector has no effect on the shapes of the basis functions and the B-spline. Generally there are three kinds of knot vectors [ibiblio]:

- Uniform: These are knot vectors for which:

\[
t_{i+1} - t_i = \text{constant}, \forall i \tag{5.7}
\]

The Uniform Knot Vector is derived by (5.6) for \( t_i = i \):
\[ N_{i,1}(t) = \begin{cases} 1, & i \leq t < i + 1 \\ 0, & \text{otherwise} \end{cases} \tag{5.8} \]

\[ N_{i,k}(t) = \frac{t-i}{k-1} N_{i,k-1}(t) + \frac{i+k-t}{k-1} N_{i+1,k-1}(t) \]

- **Open uniform:** They are uniform knot vectors which have \( k \)-equal knot values at each end:

\[ t_i = t_1, \quad i \leq k \]

\[ t_{i+1} - t_i = \text{constant}, \quad k \leq i < n + 2 \tag{5.9} \]

\[ t_i = t_{k+(n+1)}, \quad i \geq n + 2 \]

- **Non-uniform:** This is the general case, where the only constraint is that \( t_i \leq t_{i+1} \).

### 5.1.3 Least Squares fitting of points with surfaces and curves

In general, when given a set of points \( \mathbf{P} \) in 3 dimensions, the goal is to find a surface \( \mathbf{X}(u,v) \) such that the sum of Euclidean distances from all points in \( \mathbf{P} \) to their corresponding locations on \( \mathbf{X}(u,v) \), are minimized. Thus for \( m \) points in the point cloud there is:

\[ \sum_{i=1}^{m} \| \mathbf{P}_i - \mathbf{X}(u_i, v_i) \|^2 = \min \tag{5.10} \]

This guarantees the best-fit surface with minimum error for the given input parameters. Hence for \( n \) parameters the least squares method involves creating a consistent system of equations, given in matrix form as [Teunissen 00]:

\[ y_{mx1} = A_{mx1} x_{mx1} + e_{mx1} \tag{5.11} \]

where \( y \) is the vector containing the observations, \( x \) is the vector containing the parameters, \( A \) is the design matrix connecting the parameters with the observations and \( e \) is the error vector. The minimization featured in (5.10) is possible by minimizing the error vector \( e = y - Ax \):

\[ \min_x (y - Ax)^T (y - Ax) \tag{5.12} \]

From calculus it is known that \( \hat{x} \) is a solution of (5.12) if \( \hat{x} \) satisfies:

\[ \frac{\partial E}{\partial x} (\hat{x}) = 0 \quad \text{and} \quad \frac{\partial^2 E}{\partial x^2} (\hat{x}) \text{ positive-definite} \tag{5.13} \]
where $E(x)$ is given as:

$$E(x) = (y - Ax)^T (y - Ax) = y^T y - 2y^T Ax + x^T A^T Ax \quad (5.14)$$

The first-order and second-order partial derivatives of $E(x)$ give:

$$\frac{\partial E}{\partial x}(x) = -2A^T y + 2A^T Ax \quad \text{and} \quad \frac{\partial^2 E}{\partial x^2}(x) = 2A^T A \quad (5.15)$$

Equating the first equation of (5.15) with zero gives:

$$A^T A \hat{x} = A^T y \quad (5.16)$$

Since the system is consistent (5.16) has a unique solution. This solution is found from the inversion of the normal matrix $A^T A$:

$$\hat{x} = (A^T A)^{-1} A^T y \quad (5.17)$$

The matrix $\frac{\partial^2 E}{\partial x^2}$ of (5.15) is also positive-definite. Consequently this solution $\hat{x}$ is the minimizer of (5.14). The vector $\hat{x}$ is known as the least-squares estimate of $x$, since it produces the smallest possible value of the sum-of-squares function $E(x)$.

Weights to specific points can be applied by incorporating a weight matrix $W$. In this case the solution to the minimization problem is given by the formula:

$$\hat{x} = (A^T WA)^{-1} A^T Wy \quad (5.18)$$

In addition, for the complete description of the quality of the least squares estimation, the two moments are computed. The first is the mean $E\{y\}$ and the second is the variance matrix $D\{y\} = Q_y$, which denotes the measurement variability.

The above correspond to a system of linear equations relating the observations with the parameters. It is more common though for the observations to be non-linearly related to the unknown parameters. In this case the equations need to be linearised first before use. Furthermore initial values are needed for the parameters. The above procedure is then computing corrections for the initial parameter values via an iterative procedure. The computed corrections are added to the previous values, thus improving them. With each iteration these corrections become smaller, until they become negligible.

Following are two examples of least squares fitting through points; fitting of a plane and fitting of a B-Spline.

**Least Squares fitting of plane:** Given a set of $n$ points in 3 dimensions, it is asked for a plane to be fitted through them. The equation for a plane is:

$$z = ax + by + c \quad (5.19)$$

In order to determine the plane parameters $a$, $b$ and $c$, a least squares adjustment is performed. The height measurements are used for observations. The resulting system of equations is the following:
The solution is given by the equation: \( \hat{x} = (A^T A)^{-1} A^T y \)

When the computed parameters are used for the plane, the following plot is the result:

**Fig. 5.5 Fitted plane through points**

---

**Least Squares fitting of B-Spline:** Given a set of data points \( \{(s_k, S_k)\} \), it is asked for a B-Spline curve to be fitted through them. \( S_k \) are the sample data and \( s_k \) are the sample times, so that \( s_0 < s_1 < ... < s_m \). A B-spline curve that fits the data is parameterized by \( t \in [0,1] \), so the sample times are mapped to the domain \( t_k = (s_k - s_0)/(s_m - s_0) \).

A B-Spline of degree \( d \) and basis functions \( N_{i,d}(t) \) is defined by the formerly presented equation (5.4):

\[
P(t) = \sum_{0}^{n} N_{i,d}(t)P_i \quad \text{for} \quad t_{\min} \leq t \leq t_{\max}
\]

For the fitting process the control points \( P_i \) are unknowns that have to be determined. These control points are collected in a vector:

\[
\hat{P} = \begin{bmatrix} P_0 \\ P_1 \\ \vdots \\ P_n \end{bmatrix}
\]
The same happens with the samples $S_k$:

$$\hat{S} = \begin{bmatrix} S_0 \\ S_1 \\ \vdots \\ S_m \end{bmatrix} \quad (5.22)$$

For a specified set of control points, the least-squares error function between the B-spline curve and the sample points is the following scalar-valued function [Eberly 05]:

$$E(\hat{P}) = \frac{1}{2} \sum_{k=0}^{m} \left[ \sum_{j=0}^{n} N_{j,d}(t_k)P_j - S_k \right]^2 \quad (5.23)$$

The term $\sum_{j=0}^{n} N_{j,d}(t_k)P_j$ signifies the point on the B-spline curve at a specific sample time $t_k$. This error function measures the total sum of squared distances between sample points $S_k$ and their corresponding curve points. The goal of the least squares fit is to minimize this error by using the best control points possible.

From calculus it is known that the function $E$ will have a global minimum when all its first-order partial derivatives are equal to zero. These partial derivatives in terms of the control points $P_i$ are:

$$\frac{\partial E}{\partial P_j} = \sum_{k=0}^{m} \left( \sum_{j=0}^{n} N_{j,d}(t_k) - S_k \right) N_{i,d}(t_k) \Rightarrow$$

$$\frac{\partial E}{\partial P_j} = \sum_{k=0}^{m} \sum_{j=0}^{n} N_{i,d}(t_k) N_{j,d}(t_k)P_j - \sum_{k=0}^{m} N_{i,d}(t_k)S_k \Rightarrow (5.24)$$

$$\frac{\partial E}{\partial P_j} = \sum_{k=0}^{m} \sum_{j=0}^{n} \alpha_{ik}\alpha_{jk}P_j - \sum_{k=0}^{m} \alpha_{ik}S_k$$

with $\alpha_{rc} = N_{c,d}(t_r)$ for $0 \leq i \leq n$. By taking the partial derivatives equal to zero, the result is the following system of equations:

$$0 = \sum_{k=0}^{m} \sum_{j=0}^{n} \alpha_{ik}\alpha_{jk}P_j - \sum_{k=0}^{m} \alpha_{ik}S_k = A^TA\hat{P} - A^T\hat{S} \quad (5.25)$$

with $A = [\alpha_{rc}]$ a matrix with $n+1$ rows and $m+1$ columns and $A^T$ the transpose matrix of $A$. This system of equations has the known form of a least-squares problem presented above; for solving the system $Ax = b$ in a least squares sense, the minimization leads to the system $A^TAx = A^Tb$.

As shown previously in equation $(5.17)$ the normal solution for $(5.25)$ is:

$$\hat{P} = (A^TA)^{-1}A^T\hat{S} \quad (5.26)$$
The problem though with the normal solution is that the matrix inversion used can be ill-conditioned. That happens when the eigenvalues of the matrix are zero. The result in this case is that the normal solution cannot give a result. That is why in these situations a different approach for solving the system must be taken.

The $A^T A$ is a symmetric matrix. Furthermore the $A$ matrix is banded. This is a special type of sparse matrix with a contiguous set of upper bands and lower bands with non-zero elements. The diagonal has also non zero elements. All the remaining elements are zero. The best method for solving such ill-conditioned systems is with Cholesky decomposition [cornell]. By definition, given a symmetric positive-definite matrix $A$, the Cholesky decomposition is an upper triangular matrix $O$ such that:

$$A = V^T V$$

In this case the Cholesky decomposition will be:

$$A^T A = V V^T \quad (5.27)$$

Then the linear system will be:

$$V V^T \hat{P} = A^T \hat{S} \quad (5.28)$$

Here $V$ is lower triangular and $V^T$ upper triangular. For getting a numerically stable solution now, first the $V$ matrix is inverted and then the $V^T$ separately. With this method the ill-conditioning problems encountered with the normal least squares solution are resolved.

5.2 Modelling the right wing of the CAD point cloud

In chapter 2.1.2 the available CAD design model was presented. This model consists of points arranged in a grid-like manner. This structure is helpful for CFD studies. Especially in the wing section, the points are organized in profiles that correspond to the NACA airfoil 23012 (see chapter 2.1.2 and appendix 2 for more information).

In this chapter a small part of the existing CAD point grid will be modelled. The reason behind this is to get a better visualization of the actual aircraft. The available point grid is sparse in some places and dense in others. The result is a model that does not give a very accurate impression of the Cessna. Thus this model needs to be improved.

Modelling the whole airplane is a huge undertaking, hence a specific area needed to be chosen. In aerospace engineering the wings are the most important parts of the plane. Actually in CFD studies the behaviour of a whole aircraft can be monitored almost completely just by testing the airflow performance of the wings. That is why the part chosen for modelling was the right wing. Since the original cloud is symmetrical it made no sense to perform the same task for both wings. Images of the chosen area are shown in the next page:
The best way of modeling this point grid is by using a mesh to connect the separate profiles. The result will then be an accurate representation of the wing, with no gaps between the profiles. The program Cyclone, used for the registration process, was chosen for the mesh creation. The reasons for that were its available tools for creating TINs and its very good visualization results. The procedure followed consisted of the following steps:

I. Dividing the profiles in polylines
II. Creating mesh between neighbouring polylines
III. Correcting the mesh
I. Dividing the profiles in polylines: There is no automated way to generate a correct mesh through the whole wing. Algorithms tested from programs like Matlab created a random triangulation that did not correspond to the actual wing. What needs to be done is to divide this task into smaller parts. That is why the generation of the mesh must first start between neighbouring profiles.

The existing model is already consisting of points representing NACA airfoils 23012. Initially the points must be specified in each airfoil for which the mesh will be created. In Cyclone this can be achieved by creating polylines connecting points of each profile. The goal is to take parallel polylines with the same amount of points in neighbouring profiles. The mesh can then be created between these parallel polylines. The objective of this method is to divide the wing in ‘panels’, as shown in the example featured in figure 5.8:

![Fig. 5.8 Two views of a mesh created between two neighbouring wing profiles: The 3 polylines created in the left profile are connected with the parallel 3 polylines created for the right profile. Thus, this part of the wing is divided in 3 panels.](image)
II. Creating mesh between neighbouring polylines: After specifying the polylines in all the profiles the next step is to connect them with a TIN. In this way the separate meshes form the needed panels. This procedure is shown in detail in the following image:

![Image of connecting polylines with meshes](image)

**Figure 5.9 Creating model of the wing: Connecting neighbouring NACA profiles with meshes and creating panels**

A main concern when building this model is how accurate it is going to be. This is depending on the number of panels used. The best solution would be to create as many panels as possible with the available points. However this solution is very complicated and labour-intensive.

A more suitable solution is to create more panels in areas of the profiles where there is more curvature. The flat areas of the wing on the other hand can be depicted very well with a smaller amount of panels. In this way the description of both the curved parts and the flat parts of the wing remains accurate. The way the profiles are divided with this concept, is shown in the image below:

![Image of NACA profile division](image)

**Fig. 5.10 Division on NACA profile in 4 parts according to curvature for modeling**

The maximum curvature is featured in the front part of the airfoil. This area is divided in an upper and a lower section for easier triangulation. The rest of the airfoil is also divided in upper and lower parts, thus creating 4 sections. In each section the generated meshes created 4 panels. The results are shown in the following images:
III. Correcting the mesh: It is common when creating a mesh for the result to need some refinements. Usually there are imperfections in the triangles or errors in the surface [Remondino 03]. That means that corrections must be done for improving the final result. These correcting operations are usually manual and can vary from small triangle editing to bigger corrections of surface errors.

This was also the case here. During the generation of the meshes there were cases where holes in the mesh were evident. In other instances unwanted triangles were created between points that were not supposed to be connected. An example of this error is shown in the figure 5.15 below:

These problems were especially encountered in the front part of the wing where the curvature was higher. That is the main reason there were so many panels used for modeling this area. All the remaining unwanted triangles were deleted manually in most cases. Triangle insertion was also used for filling up holes in the mesh. These triangles were inserted by constructing polylines that surrounded the specific areas.
The final outcome of the process presented above was a modelled right wing consisting of 647 lines and 629 meshes. It is shown in the images below:

Fig. 5.16 Three views of final right wing model
5.3 3D approach for modelling the right wing of the laser point cloud

In chapter 4.5 the obtained laser Cessna point cloud was presented. The final point cloud consists of approximately 13.6 million points arranged in a random manner. The result though is not complete, since there are parts of the airplane where the point coverage is lacking. Especially in the wing section, there are problems with the front part and some areas in the lower part (see chapter 4.5 for more information).

In this chapter a small part of the registered Cessna point cloud will be modelled. This is done because the product from the registration is just a collection of random points from the Cessna. However, these points can be fitted with some type of mathematical surfaces or lines. When this happens two things are achieved. Firstly a more accurate visualization of the aircraft is obtained. Secondly, fitting a suitable surface reduces the errors from the registration procedure. During registration an error is produced that signifies the quality of the separate point cloud merging. Using an appropriately fitted surface averages these errors [Cyclone 06]. Thus the result will be a model that offers a recreation of the Cessna that is more accurate than the simple point cloud.

Modelling the whole airplane is again not considered, because it is a huge task. Hence a small area needs to be selected from the whole plane. In chapter 5.2 the right wing was used due to the importance of the wings in aerospace engineering and specifically CFD studies. That is why the same part is chosen in this case as well. Furthermore, with this choice, a comparison of the two models will be possible afterwards. Images of the chosen area are shown below:

![Fig. 5.17 Two front views of the right wing laser points](image-url)
The approach followed in this chapter is to treat this problem in 3D. That means that surfaces will be fitted through the points of the wing. The idea is that a suitable surface can be fitted through more points, thus making the procedure easier. The steps followed are presented below.

I. Divide the wing in parts suitable for fitting.
II. Fit surfaces through parts.
III. Connect surfaces and evaluate result.

I. Divide the wing in parts suitable for fitting: While modelling the CAD point grid it was discovered that there was not one way to create a TIN for the whole wing. The same is the case here. The laser points of the right wing are too many and the object too complicated for one surface to be adequate for fitting. That means that it should be divided into smaller parts. This division can be performed in many ways, depending on what surfaces are used for fitting.

The initial idea is to use simple planar surfaces for fitting. The reason for that is that this is the easiest surface fitting possible. Furthermore, planar surfaces can cover for a big part of the wing, especially its rear sections (see figure 5.18 below). In the parts where the curvature is bigger, like the front part, the wing can be divided into smaller parts suitable for planar surfaces. Another choice could be to fit cylinders or other curved surfaces in these areas.

![Fig. 5.18 The rear parts (in blocks) of the NACA airfoil can be fitted with planes: the bigger the division, the higher the accuracy of the final product](image)

The division of the wing can be done in many ways. An example is shown in the following image:

![Fig. 5.19 Initial division of wing in 2 parts: front and tail. Further division will be performed afterwards for more detail and accuracy](image)
II. Fit surfaces through parts: The mathematical background behind fitting planar surfaces through points was presented in detail in chapter 5.1.3. From the previous step the wing is also divided in smaller areas. Now, with the use of Matlab ver. R2006a, the following algorithm is implemented (all the programs written in Matlab are presented in appendix 4):

1. **Load data (3D points of the wing part)**
   The 3D points of a specific part are input as data (see figure 5.20 below).

![Fig. 5.20 Part of the lower area in the tail of the wing](image)

2. **Create and solve standard Least Squares A-form**
   The system of equations (5.20) is created, as featured in chapter 5.1.3.

3. **Find Residuals**
   A way to evaluate the quality of the acquired solution is to calculate the residuals. A histogram of the residuals can be created, as shown in figure 5.21 below:

![Fig. 5.21 Histogram of residuals with gaussian line fitted](image)
4. **Find plane bounded by convex hull of points**

The previous steps have just provided the equation of the best fitted plane. But this plane is suited for the specific area of the data points. Thus it needs to be bounded, creating a ‘panel’ of sorts. This is achieved by firstly finding the points that make for the convex hull of the data (see chapter 5.1.1). Then the coordinates of these points on the best fitting plan are calculated, thus determining the boundaries of the panel (see figure 5.22 below).

![Fig. 5.22 Panel created by bounding the plane with a convex hull](image)

**III. Connect surfaces and evaluate result:** The procedure described above must be performed for every separate part of the wing. In the end the result is a collection of panels that needs to be connected. Thus the final model of the wing is created. The panels were exported in Cyclone for better control. The result after performing the procedure for the tail part of the wing is shown in the images below:

![Fig. 5.23 Top view of fitted panels in the tail part of the wing](image)
What can be seen is that the result is not really satisfactory. The panels appear to be disconnected and the result does not give a good visualization of the wing. ‘Snapping’ the panels together in Cyclone proved to be a difficult procedure that did not always work. That is because the differences between the separate panels were in many cases too big to provide a good connection. This method was tried for different sizes of flat areas of the wing but the results remained unsatisfactory. Due to this outcome in the tail part, the method was not investigated any further for the rest of the wing. Another approach to solve this problem was decided.

5.4 2D approach for modelling the right wing of the laser point cloud

In chapter 5.3 a 3D approach was used for modelling the laser Cessna point cloud. The result was not satisfactory, thus a new approach had to be decided. The modelling performed on the CAD point grid in chapter 5.2 was based on connecting the NACA profiles with meshes. Thus the modelling needed the separate airfoils in order to be completed. This approach can also be taken for the laser point cloud.

The idea at this time is to divide the laser point cloud not in areas for surface fitting, but in profiles. These profiles can then be fitted in 2D with suitable lines. Then the airfoils can be connected, just like the CAD point grid in chapter 5.2. The steps of the followed procedure were implemented in Matlab and are shown below:

I. Divide Laser point cloud in profiles
II. Fit lines that match NACA airfoils through profile points
III. Place 2D profiles in 3D space
IV. Connect profiles with mesh

I. Divide Laser point cloud in profiles: The point cloud of the right wing has unfortunately some areas without adequate point coverage. This is especially evident in the low part of the wing, as shown in figure 5.25:
That meant that to get enough points for creating airfoils the whole wing could not be taken into account. That is why only half of it is considered in this task. The used part of the wing is shown in the following image:

The laser point cloud was ‘aligned’ with the CAD point grid (see chapter 4.6). The advantage from this procedure is that the two point clouds have the same coordinate system. The chosen system was the one corresponding to the CAD point grid. This system is very practical, because its X-axis is a symmetry axis that divides the Cessna in two along the fuselage. Furthermore the Y-axis of the coordinate system is parallel to a virtual axis that goes through the wings. This last property can be used for the division of the wing.

First of all the wing point cloud can be translated into the origin of the coordinate system. This translation is achieved by finding the center of the point cloud. The mean in the X-, Y- and Z-direction is used. Then the wing point cloud is translated by subtracting the mean values from all the points. What has been achieved with this is to make the Y-axis of the coordinate system the symmetry axis of the wing.
Now the division in profiles is an easy task. They can be created by taking ‘beans’ along the Y-axis. The width of the beans dictates the distance between the profiles. In this case a 10 cm width was chosen, because the corresponding profiles of the CAD model had a bigger gap between them. Thus, a 10 cm width will give more profiles and a more detailed outcome.

II. Fit lines that match NACA airfoils through profile points: The previous step gave 35 profiles. Each profile is considered separately in this step in 2D, along its X- and Z-axis. Through the points of each one, a line must be fitted that corresponds to a NACA airfoil (see appendix 2 for more information on NACA airfoils). The problem that needs to be solved is what line can describe a NACA airfoil accurately.

This task becomes easier when the points of a profile are divided in smaller parts. What needs to be represented accurately is the curvature of the airfoil. That is why a division in 3 parts was chosen, as shown in figure 5.27 below:

These smaller parts can easily be fitted with curves. Then the three curves can be connected seamlessly to provide a unified airfoil. B-spline curves were presented in chapter 5.1.2. They are a very good choice for this task for two reasons. Firstly they display very good behaviour in gaps of data. This is very helpful for the wing since its low part has areas with small amount of points. Secondly the fitting of a B-spline can be controlled by a series of parameters. The main parameters examined in this task are weights and the degree of the B-spline. By applying weights a B-spline can pass from specific points. Additionally, when choosing a high degree for the B-spline, the fitting solution will correspond to as many data points as possible. Thus, by choosing an appropriate degree for the B-spline, the resulting curve can be as smooth or as uneven as it is desired.

A method was created and implemented in Matlab for fitting B-splines through the 3 parts and getting an airfoil for each profile. The steps of this method are presented below:

1. Divide the profile initially in two parts: upper and lower

Before dividing the profile in 3 parts, an initial in upper and lower parts is necessary. The mean line in the Z-axis is calculated and used as a measure for the separation. An example is shown in figure 5.28 in the next page:
2. **Fit initial B-Splines through upper and lower parts.**
   A first fitting of B-splines is carried out. This initial fitting is going to help in finding the points of the front part. An 8th degree B-spline is used.

3. **Find the point on the upper B-spline where the first derivative is zero (zeropoint).**
   By determining this point on the upper B-spline, the point of curvature change of the curve is also calculated. This will be a good reference for defining the front part of the airfoil.

4. **Consider the zeropoint as a measure for distinguishing the front part points.**
   The zeropoint is used for determining the front part. Due to the problematic coverage of this area of the wing (see chapter 4.5) there is a peculiar situation. In some profiles there are enough points left from the zeropoint and in others not that many. That means that due to practical reasons this point cannot always be considered as the start of the front part. However it can be taken as a reference (e.g. front part is taken as the 6/5 of the distance between most left point and zeropoint). Thus in each profile a different division is made, according to the availability of data. An example is shown in the image below:
5. **Rotate points of front part by 90° to fit a B-spline.**
With the 3 parts now defined, the fitting can begin. The problem with the front part is that a regular least squares fitting through the existing points will not give the desired shape. That is why the first step is to rotate the points by 90°. In most of the cases the fitting is done with a 5th degree B-spline, due to the lack of available points. Weights are given to the division points so that the curve will pass from them. The result for one profile is shown below:

![Fig. 5.29 B-spline through rotated front part](image)

6. **Fit B-Splines through the new upper and lower parts.**
The other two parts must also be fitted with B-splines. The Upper part has very good point coverage in all the cases, thus the fitting is done with an 8th degree B-spline. The Lower part has worst point coverage. In some profiles there are considerable gaps in this area. Furthermore the points of this part have more noise. That is why it was decided to fit a 6th degree B-spline, which gives smooth results. Weights are given to the division points and the most right point. This way the curve passes from them.

7. **Rotate front part B-Spline and connect with other two B-Splines.**
All the curves are available. It is essential to rotate the curve from the front part to its original position. The weights helped so that the all the B-splines pass through the same points. The final outcome for profile 28 is shown in figure 5.30 below:

![Fig. 5.30 Fitted airfoil for points of profile 28](image)
The decision for using the specific B-splines that were used was based on two criteria. On one hand the result had to be as representative of the point cloud as possible. On the other hand a smooth result would be very helpful for the CFD studies. The difficulty of the computations is increasing for complex airfoil shapes (see chapter 2.1.2). Thus the choice of the degree of the curves was decided according to these factors after many trials. The result is the best possible outcome that accomplishes both these goals. Especially for the lower part where the point cloud has more noise, the 6th degree used featured the best balance between these criteria. A comparison between different degrees of B-splines can be seen in figure 5.31:

![Fig. 5.31 (Top) 6th degree B-spline, (Bottom) 8th degree B-spline: The 6th degree B-spline gives a more smooth result than the 8th degree one, while at the same time passing through all the points](image)

In appendix 3, the resulting airfoils from the B-spline fitting for each of the 35 profiles are featured.

**III. Place 2D profiles in 3D space:** In the previous steps the X- and Z-values were calculated for the fitted airfoils. In order to place them in 3D space the Y-value must also be known. The mean Y-value of each bean was used for each profile. The points were exported to Cyclone and the result is featured in the following images:
The result is considered very satisfactory, since it reproduces the wing very faithfully. However there are some problems with it. The lack of points in the front part of the wing had repercussions on the quality of the fitted curves. This is evident when placing the profiles in their corresponding positions, as shown in figure 5.33 below:

During the presentation of the scanning procedure, the problem that the laser presented with black surfaces was featured (see chapter 3.4). Unfortunately the measurement conditions did not allow for the crew to deal with this problem. Thus the result of the fitting process for the front part is not accurate. Subsequently the outcome is a wing with inconsistencies in its front section.

**IV. Connect profiles with mesh:** In chapter 5.2 the CAD point grid of the wing was modelled with the use of TINs. The same procedure will be followed here for the fitted airfoils of the laser points. Again the reason behind this is to get a better visualization of the fitted wing.

Cyclone and its tools were once more used for the modelling. The procedure was the same as the one followed in chapter 5.2. Hence, the profiles were divided in
polylines. Afterwards, the neighbouring polylines corresponding to the airfoils were connected by creating meshes.

Two were the differences this time. Firstly, the profiles were consisting of more points, thus making for more detailed polylines. As a consequence the meshes created based on these polylines needed very few corrections. Secondly it was decided not to model the front surfaces of the wing. This was due to the accuracy problems of the fitted curves on this part of the wing. The final result is featured in the following images:

![Figure 5.34](image.jpg) Three views of the modelled wing laser points
5.5 2D Comparison of laser and CAD model

After the fitting process, the model from the laser point cloud is available. The final step is to compare this model with the original CAD design point grid. The two models are shown in figure 5.35 below:

![Figure 5.35 Red points show the laser model and green the original CAD model](image)

A first visual comparison shows two obvious points. Firstly, the laser model has more airfoils than the CAD model. Actually the number of airfoils of the laser data can be defined by the user according to the project's needs. This is done by choosing an appropriate bean size for the division of the points. In this case 10 cm was chosen in order to get a denser airfoil grid. Secondly, each laser model airfoil has more points than its corresponding CAD wing profile.

A more concrete way of comparing the two models is to consider their corresponding airfoils. That means that a 2D approach for the comparison will be used, like the modelling phase. The concept behind this comparison is based on the following two steps:

I. Registering the separate airfoils to estimate the differences for the whole wing section.
II. Calculating differences between corresponding points in the two models to get a sense of specific differences on the wing surface.

I. Registering the separate airfoils to estimate the differences for the whole wing section: In chapter 4.1 the registration procedure was presented. Registration was used for fitting the separate scans together and for getting the final point cloud. This can also be used for comparing two different point clouds of the same object.

In chapter 4.6 a registration was conducted between the CAD and the laser model. The result was unsuccessful due to the big difference in the number of points between the two models (CAD = 50,000, Laser = 13.6 million). What could be done to overcome this problem is to select a number of points from the bigger laser database. This would create a sample considerably smaller than the original point...
cloud. Luckily, this was already achieved for the wing in the 2D modelling approach presented in chapter 5.3. The wing is now divided in profiles like the original CAD point grid. Such sample of the laser wing point cloud is representative of the whole wing. Thus the registration is now feasible. The ICP algorithm used in the registration process is a least squares approach, hence the quality of the registration is provided by a covariance matrix and a mean error [Rabbani 06]. The mean error is considered for comparing the two models.

The algorithm used for this part is the following:

1. **Find the corresponding airfoils**: The separate models shown in figure 5.35 have differences. The most basic is that the laser model is more detailed than the CAD model. Actually it has almost double the amount of airfoils. The problem is that a comparison can only be done between corresponding wing profiles. Since the CAD model has less airfoils it is used as a basis for the comparison. The profiles from the laser model closer to the profiles of the CAD model in the Y-axis are selected as the corresponding ones. This distance varies but is in all cases smaller than 3 cm. In the end 16 comparison pairs are created.

2. **Use ICP algorithm for each pair**: The differences between the airfoil pairs are now visible. In chapter 2.1.2 the shape error between the jig-shape and the jack shape of the plane was introduced. This error is visible in the airfoil pair shown in figure 5.36:

The scanned wing is clearly elevated compared to the same wing on the CAD model. This is due to the jacks being placed under the wings during laser scanning. However the ICP algorithm can combine the two profiles, thus providing an initial comparison of the models. The result of using ICP for 10 iterations is shown in figure 5.37:
What is clear from this image is the inconsistency problem with the front part of the wing that was presented in chapter 5.3. While the rest of the laser profile is more close to the CAD one, the front part features bigger dissimilarities. Because that was the case for almost all airfoil pairs, the ICP algorithm was also used for laser profiles without their front problematic part. The evaluation of this registration will give a better measure of the differences between the two models. The result is shown in the following figure:

3. **Evaluate mean error**: The procedure described in step 2 was performed for each of the 16 pairs. The results for all the airfoils are presented in appendix 5. Each registration produced a mean registration error, which indicates the quality of the fitting of the two wing profiles. The following table presents these results:
<table>
<thead>
<tr>
<th>Comparison #</th>
<th>Airfoil Pairs (CAD – Laser)</th>
<th>Mean Registration Error for Complete Airfoils (m)</th>
<th>Mean Registration Error for Airfoils with no front part (m)</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>(1-3)</td>
<td>0.01628</td>
<td>0.01441</td>
</tr>
<tr>
<td>2</td>
<td>(2-7)</td>
<td>0.01552</td>
<td>0.01375</td>
</tr>
<tr>
<td>3</td>
<td>(3-11)</td>
<td>0.01900</td>
<td>0.01662</td>
</tr>
<tr>
<td>4</td>
<td>(4-15)</td>
<td>0.01930</td>
<td>0.01581</td>
</tr>
<tr>
<td>5</td>
<td>(5-18)</td>
<td>0.01974</td>
<td>0.01570</td>
</tr>
<tr>
<td>6</td>
<td>(6-21)</td>
<td>0.01908</td>
<td>0.01505</td>
</tr>
<tr>
<td>7</td>
<td>(7-24)</td>
<td>0.01796</td>
<td>0.01488</td>
</tr>
<tr>
<td>8</td>
<td>(8-26)</td>
<td>0.01759</td>
<td>0.01453</td>
</tr>
<tr>
<td>9</td>
<td>(9-28)</td>
<td>0.01697</td>
<td>0.01403</td>
</tr>
<tr>
<td>10</td>
<td>(10-29)</td>
<td>0.01854</td>
<td>0.01421</td>
</tr>
<tr>
<td>11</td>
<td>(11-30)</td>
<td>0.01951</td>
<td>0.01645</td>
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<td>(12-31)</td>
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<td>0.01472</td>
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<td>(14-33)</td>
<td>0.01765</td>
<td>0.01467</td>
</tr>
<tr>
<td>15</td>
<td>(15-34)</td>
<td>0.01649</td>
<td>0.01305</td>
</tr>
<tr>
<td>16</td>
<td>(16-35)</td>
<td>0.01581</td>
<td>0.01350</td>
</tr>
<tr>
<td><strong>MEAN:</strong></td>
<td></td>
<td><strong>0.01786</strong></td>
<td><strong>0.01483</strong></td>
</tr>
</tbody>
</table>

What is again clear from the registration error results is the problem with the front part of the airfoils. This area is responsible for increasing the final error significantly. Table 5.1 shows that the mean error is 3 mm less when this area is not considered in the calculations. However, even in the best case, there is a standard 1.5 cm registration error between two airfoils. This is attributed to the fact that the laser airfoils are thicker than the CAD ones. This is not only evident in the example shown in figure 5.37, but also in the rest comparison pairs (see Appendix 5 for complete comparison catalogue).

That leads to the conclusion that the airfoils of the constructed wing have a bigger maximum thickness parameter than the original design. Thus the first comparison step shows that the actual wing is thicker than the original design model wing by approximately 1.5 cm.

**II. Calculating differences between corresponding points in the two models to get a sense of specific differences on the wing surface:** The second step of the comparison process is more detailed. In this stage the points from the two airfoils will be compared. That means that the corresponding points for one airfoil pair must be determined first. After comparing those, the procedure is carried out for all the wing profile pairs. Complete results of this procedure are featured in appendix 5.

The method used is the following:

1. **Take registered airfoil pairs:** The airfoils used in this stage will be the registered ones that were the result of the first part of the comparison.

2. **Find corresponding points of laser airfoil to CAD airfoil:** The Laser model is more detailed than the CAD one. That means that to do a comparison, a sample of the Laser airfoil points must be taken first. This sample will be the closest points from the Laser airfoil to the CAD one. Thus, corresponding pairs of points from the two profiles are created.
This is possible by calculating the distances between one point of the CAD airfoil and all the points of the Laser one. The minimum distance is then determined. This distance indicates the corresponding point of the Laser profile to the specific CAD one. This process is carried out for all points of the CAD airfoil. The result is shown in the following figure:

![Registered Airfoils and Closest Points](image)

**Fig. 5.39 Top: Registered Airfoils (RED: Laser, BLUE: CAD)**

**Bottom: Closest points of laser to CAD airfoil**

3. **Compute distances of closest laser points to CAD airfoil:** The differences between the airfoils are calculated now. These are considered to be the distances of one sampled laser point to the line segment defined by the two closest CAD airfoil points to it (see figure 5.40).
This distance is calculated for all the points of the laser airfoil.

4. **Evaluate differences for each airfoil pair:** In the previous step the differences for all the points were calculated. The histogram can now be created for these values. The histogram of the example pair featured in the previous steps is shown below:

5. **Plot Cad airfoils with colours indicating differences:** The final step is to visualize the calculated differences. For this purpose a colourbar plot is created of the CAD wing profile. The colours of each point indicate the size of the difference. The colourbar plot of the currently used example is:
This procedure is performed for all the comparison pairs (see appendix 5). After taking all the separate 2D profiles, the result is a CAD wing with colours indicating its deviation from the Laser wing. This outcome is featured in figure 5.43:

![Fig. 5.43 CAD wing with colours indicating deviations from laser wing (in meters)](image)

The following comments can be made by examining this plot:

1. From the 1st stage of the comparison it was established that in all cases the Laser airfoils are thicker from the CAD ones. Thus all the indicated differences in the plot have an outward direction, as featured in figure 5.44. This is consistent for all areas apart from the problematic front part. In these sections the points from both models are intertwined.

![Fig. 5.44 Differences have an outward direction (RED outside BLUE)](image)

2. No conclusions can be made by examining the separate difference plots for each airfoil pair (see appendix 5). No specific distributions can be distinguished. When taking all the point differences into account the mean is 8 mm and the histogram is the following:
As expected the bigger differences are presented in the front part of the wing. These can reach 2.5-2.8 cm. That is due to the problematic nature of this area (see chapter 4.5). These differences are featured with black circles in figure 5.46. Nevertheless there is a region in the front part of the wing where the differences are small (see red circle in figure 5.46). This is because in that part there is no rubber surface.
4. The rest of the wing features smaller differences. In general their majority does not surpass 1 cm, as it was featured in the histogram 5.45. The subsequent trends are distinguished in the main part of the wing (see figure 5.47 below):

Fig. 5.47 (Top) Top view of airfoils with marked area. (Bottom) Profile from that area. The difference remains constant in this part, signifying a smooth wing section

i. Image 5.47 features the first trend. The marked area shows the end of the wing. In general the colour in this area is constant, indicating a constant difference of approximately 1 cm. This is corresponding also with profiles of that area. This constant change signifies the different thickness between CAD and Laser model.

ii. Image 5.48 shows two other parts of the top section of the wing marked. In these areas the colour of the points is lighter than the area that connects them. According to the colourbar, the points in this connecting area have
smaller differences than the other two. This can be translated to a dent in this section, which is also visible in the corresponding profiles of this area.

Fig. 5.48 (Top) Top view of airfoils with 2 marked areas. (Bottom) Profile from that area. The difference is smaller in the section connecting the marked areas. This signifies a dent in the wing.

This dent happens to be in a specific part of the wing; the gap created by the aileron situated in this section. This is very interesting because it shows that the modelling technique used managed to depict this area. The laser scanner obtained points from the gap. Then the used B-splines passed from this part. However, at the same time this gap is smooth due to the properties of B-splines. Hence the dent is a representation of this gap. This detail of the wing is featured in figure 5.49:
iii. The same situation as in ii is occurring in the bottom part of the wing. Image 5.50 shows one part of the low section of the wing marked. In this area the colour of the points is darker than in its adjacent areas. Again this means that this connecting area has points with smaller differences than the other two. This can be translated to a dent in this section, which is also noticeable in the corresponding profiles of this area.
Fig. 5.50 (Top) View of airfoils with marked area in the low part. (Bottom) Profile from that area with adjacent sections marked. The difference is smaller in the marked section than its adjacent areas in the profile. This signifies a dent in the wing.

The reason for the existence of this dent is the same as in ii. This time the low part of the gap due to the aileron is depicted. As a result of using B-splines, the gap is smoothed. Hence, with the chosen modelling technique, the gap is represented by this dent.
6. Conclusions and Recommendations

This thesis project consists of 6 chapters. After the introductory chapter 1, the measurement project and the reasoning behind it were featured in chapter 2. In chapter 3 the laser scanner and the Cessna were presented, followed by a detailed description of the scanning procedure. In chapter 4 the first step in the processing phase, called registration, was illustrated. In chapter 5 the next step in the processing phase was presented; modelling. Finally, in this chapter the conclusions made out of all the previous work are featured. Thus, in chapter 6.1 the actual conclusions are presented. Then, following in chapter 6.2, are the recommendations for further research on this subject.

6.1 Conclusions

The original idea behind this project came from fact that there was only a low detail model available of the TU Delft’s Cessna Citation. This model was the CAD design model offered by Cessna. However the airplane is available and a reverse engineering technique could be used for obtaining a more detailed model of the actual Cessna geometry. Laser Scanning is a new reverse engineering technique that is currently under extensive study from the scientific community. It was considered the most appropriate solution for this type of project. Thus the main research question of this thesis was the following: Can laser scanning be used for the creation of a model for the Cessna Citation of the Aerospace faculty that is more detailed than the CAD design model currently used for CFD studies?

To answer this question an assumption is made. The need for this assumption is the result of the following: The registration outcome presented in chapter 4.5 was considered successful according to Cyclone’s specifications. However Cyclone did not give an explicit accuracy value for the Laser Cessna model. The mean registration error was 1 mm but this only shows the quality of fitting the point clouds together. The mean error vector of 1.67 cm shows the error in the baselines connecting the scanning positions. However it is not certain how this affects the final model accuracy.

On the other hand the scanner’s range accuracy is 3 mm according to Z+F’s specifications. Furthermore the modelling procedure carried out in chapter 5.4 improved the accuracy of the point cloud. This is due to the fact that the fitting surfaces average out the point errors. According to these factors a general comment can be made that the final accuracy of the model should be at the 1 cm level.

Regardless of that there are some undeniable facts:

- The whole scanning procedure was carried out under the same conditions from all stations. No problems were recorded during scanning.
- The material scanned was the same for the majority of the object (aluminium). The coating used to cover the Cessna was also the same for the whole plane. The results showed that this case poses no problems with laser scanners. Hence no inconsistencies are possible when scanning the same object under the same conditions. This leads to the conclusion that bumps and dents recorded by the laser scanner in the aluminium parts are corresponding to reality.
- Inconsistencies can only exist for problematic materials. In this project such inconsistencies were evident in the rubber parts of the wings.
Most noticeable is the fact that a close inspection of the actual wing shows that it does not have the entirely smooth surface suggested by the NACA profiles. There are two main features that the design model does not have. Firstly, the strain after many hours of flying has created some bumps and dents to the wings. Secondly, the actual wing features gaps in areas where different parts are connected (e.g. flaps and ailerons). These two characteristics are evident in the constructed wing.

All the above facts lead to the following assumption: Even though the exact accuracy of the obtained Laser model is not known explicitly, it is assumed that the result is representative of the actual Cessna.

Under this assumption the answer to the main research question is yes. The laser scanned model is more detailed than the original CAD design model. This is due to a number of reasons:

- The scans provided a huge amount of data for the whole plane. Processing of the data provided more airfoils than the original CAD model. In addition each airfoil consisted of more points than the CAD wing profiles.
- The scans managed to capture the existing anomalies on the surface of the wings. The wings in the original CAD model were smooth and based on the mathematical description of the NACA profiles.
- The final Laser Cessna model is not symmetrical, corresponding to the actual plane. On the other hand the CAD model was symmetrical.
- Problems with the scans were presented (e.g. the rubber parts of the wings). However these problems were expected and there were available solutions to tackle them. Unfortunately conditions on the site were not helpful for these solutions to be applied. Thus, under better circumstances laser scanning would provide even better results.

The outcome of this thesis is a new Cessna point cloud and a new right wing model. These products can be used by the Control & Simulation group for CFD studies. Due to the problems with the front part of the wing, it is illogical to have a 3D testing of the laser wing. Nevertheless, a 2D CFD test can be performed on a specific airfoil (see Appendix 6). The testing results will be used for answering the main research question of this joint project: Can laser scanning lead to 3D models, which improve the results of CFD studies?

Finally, some secondary conclusions can be made for the three separate phases of the project, as they were defined in chapter 1:

**Design Phase:**
- Laser Scanning has the advantage of acquiring a huge amount of data in a very small time frame.
- Alternative techniques that could be considered for this project are close range photogrammetry or structured light. Furthermore, a combination of methods (e.g. laser scanning and close range photogrammetry) could also be tested (see appendix 1 for more information).
- This project needed a very detailed survey of the Cessna in the fastest manner possible. Under these conditions, the fastest and most cost-effective solution was by far laser scanning.
- According to the factory specifications of the Z+F scanner, a range accuracy of 3 mm and an angle accuracy of 0.01 degrees can be achieved.
Execution Phase:
- The set-up used for measuring consisted of 12 scanning positions in 3 levels; low, regular and high. In general the positions should surround the Cessna from every angle in order to provide complete point coverage of the plane (see chapter 3.3 for a detailed description).
- Problems were met when cooperation was needed for the laser measurements. The conclusion is that laser scanning is still a very new technique and people are strongly biased against it.
- The main considerations with this specific project were two: scanner behaviour with black and silver surfaces and scanner behaviour in low positions (see chapter 3.4 for detailed description). The conclusion is that black and silver surfaces are problematic during laser scanning. Furthermore the close distance of the scanner to the object is another parameter that can pose trouble in laser measurements.

Processing Phase:
- Registration can be used for getting a unified point cloud out of separate scans. Registration uses the ICP algorithm which utilizes least squares to minimize the distances between corresponding areas.
- The quality of the point clouds is depending on the accuracy of the laser scanner. Thus, 3 mm range accuracy is expected for the separate point clouds.
- Modelling can be performed in 3D or 2D. In 3D, surfaces are fitted through the data points. In 2D only two dimensions of the points are considered and lines are fitted through them. A 3D result can then be obtained by connecting the separate lines with meshes (see chapter 5.4 for detailed description).
- In this project a 2D modelling technique was used for the wing of the plane. A 3D approach was attempted, but the object was too complicated for this method to give a satisfactory result. Thus the 2D approach was more suitable in this case.
- No explicit measures can be given for the quality of the final model. The mean registration error of 1 mm only shows the quality of the point cloud fitting. The mean error vector of 1.67 cm shows the error in the baselines connecting the scanning positions. However it is not clear how this affects the quality of the final object. After modelling, the final accuracy should be around 1 cm.
- The main difference with the CAD model is that the laser model has thicker airfoils by an average value of 1.5 cm. Furthermore, two dents are detected in the laser model. The first one is on the top part and the second on the low part of the wing (see chapter 5.5 for more details).

6.2 Recommendations

The results presented in chapter 6.1 are significant. However this thesis is just an approach to a very interesting and vast subject; how can geomatics in general help CFD studies. Furthermore there were some compromises that had to be made in some stages of this project. These two reasons are the incentive for making the following recommendations. These are suggestions and they are proposed with the hope that they can be a starting point for further research in this field. In addition, they offer
possible solutions to problems that were encountered during the project but were not able to be applied.

The recommendations are divided in two main parts; the ones regarding the measurement phase and the ones regarding the data analysis phase. The categorisation is made because these are the main stages of the project. Changes proposed in these stages can affect the results of the research considerably. Finally, some questions are posed. These questions are considered to be interesting research topics in this field.

Measurement Phase:

- **Repetition of Scanning Procedure:** Although the obtained laser scans gave a satisfactory result, the Laser Cessna had some problems. Even though the outcome was impressive for 3.5 hours of scanning, the final Cessna model was not complete. The low point coverage in specific areas was presented extensively in chapter 4.5. Furthermore, the lack of points in the front of the wings had repercussions in the B-spline fitting (see chapter 5.3). Due to these problems it would be very helpful if the scanning procedure was repeated under more favourable conditions. This time the measurements should be conducted according to plan. This means that:
  1. Problematic areas need to be covered with appropriate material, as it was originally planed (see chapter 3.4). Better reflective materials can be used for covering the black and silver parts of the Cessna.
  2. Time for detail scans should be available. The obtained scans should be inspected on site and additional scans in problematic cases should be conducted.
  3. A way to make complete scans of the low part of the plane needs to be devised. Two methods could make this possible. One option is to use bigger jacks to raise the plane at least one meter high. Another option is to disable the safety distance setting of the scanner (see chapter 3.4) and to place it on the ground as close to the Cessna as possible. Ideally both these measures should be enough to get a complete coverage of the low part.
  4. High detail scans could be performed for the total project and not only for specific scans. This way a bigger amount of data will be available.

- **Using other Laser Scanners:** The Z+F Imager 5003 laser scanner used was suitable for the needs of this project. However different measurement conditions mean that different scanners could also be used. First of all other phase scanners could be considered, like the LS 4200 from Faro. This way different scanners can be tested in practise and under the same circumstances. Another idea is to use other types of laser scanners (see chapter 2.2.5). Triangulation scanners would give higher accuracy than phase ones, but more time is needed for the measurements. Even time-of-flight scanners could be tested for this project. The results from all these different sources could be compared with the phase scanners and their differences could be highlighted.

- **Using a combination of measurement methods:** Although laser scanning was suitable for the conditions of this project, other measurement methods could also be considered. In appendix 1 a photogrammetric survey of a part of the plane is presented. There, a strong case is made about the
benefits of combining the accuracy of photogrammetry with the huge amount of data of laser scanning. Such a combination could also be used here, especially for breaklines, which photogrammetry can survey more accurately. Additionally, a structured light system could be used for this project. The results could then be compared with the ones from laser scanning.

- **Measuring Control Points:** Although the registration result from the laser scans is considered successful, there is no exact measure of the accuracy of the final model. The mean registration error of 1 mm only shows how well the fitting was performed. The mean error vector of 1.67 cm shows the error in the baselines connecting all the scanning positions. However it is not clear how this affects the quality of the registered Cessna. In general errors in laser scanning is a new field under continuous study. That is why in new laser measurements it would be helpful to determine control points. These control points could be placed on the plane or the surroundings. A total station could be used for measuring them accurately. Alternatively also the laser scanning positions could be measured with a total station. The comparison of the registration results with the total station coordinates could give a measure of the scanning accuracy.

**Data Analysis Phase:**

- **Use different 2D approaches when modelling the airfoils:** In chapter 5.4 a 2D approach was taken for modelling the separate wing profiles. The points from each profile were divided in 3 parts and B-splines were fitted through them. In the end the 3 B-splines were connected to get the final result. What can change in this approach are three aspects:
  1. *The division of each point cloud:* The division in 3 parts was decided because it was sufficient for giving the curvature of a NACA airfoil. If bigger detail is needed in the recreation of the wing, the point cloud could be divided in more pieces. Especially in the presented case, a separate section for the aileron would be helpful. This way the fitted curves from each part would give a more accurate representation of the wing.
  2. *The curves that are fitted through each part:* The decision of fitting B-splines of 8th and 6th degree for the profiles was based on two factors; the point availability and the desired smoothness of the final airfoil. If higher accuracy is needed, more complex curves can be used for fitting. Higher degree polynomials can ‘pass’ through more points of the cloud. Thus the final result would be more representative of the laser points. However the airfoils might be, at the same time, not smooth enough for good CFD results.
  3. *Create more airfoils out of the laser point cloud:* The available point cloud of the wing was divided into beans of 10 cm width. This width can be changed depending on the desired detail of the final model and the available data of the point cloud. Smaller width, e.g. 5 cm, will give the double number of airfoils in the final wing model.

- **Use different 3D approaches when modelling the airfoils:** In chapter 5.3 a 3D approach was taken for modelling the wing. It was divided in smaller parts where planar surfaces were fitted. The result was the creation of
‘panels’ which would then be connected to give the final wing. However the result was not satisfactory. A different 3D approach would be to use more complex shapes to fit though the point cloud. Suitable spheres, cylinders or ellipsoids could be fitted with least squares method through bigger parts of the wing. At the same time they would be more appropriate for describing the inherent curvature changes of the object.

- **Use NACA airfoils for fitting:** When fitting each profile’s points in the 2D approach, it was necessary to divide the cloud in parts. However in appendix 2 the mathematical description of the NACA airfoil was given. This can be used instead of separate B-splines for fitting through the points. A least squares method can be used. This way the division is not needed and the result could be a different NACA airfoil than the one used.

- **Model the rest of the plane:** Due to time constraints, only the wings of the plane were modelled. That decision was based on the fact that the wings are the most important part studied in CFD. However the point cloud of the whole Cessna is available. It would be very interesting for the fuselage to also be modelled. This way a complete model of the plane would be obtained.

**Questions:**

- Can airplanes be constructed in a more cost-effective manner with the use of better models?
- How can the errors in the more complex parts of the plane be sufficiently quantified (e.g. antennas)?
References

Publications


**Internet Sites**


Appendix 1

Photogrammetric Survey
(also for the course Digital Photogrammetry ge4502)

This part of the thesis is dedicated to the application of close range photogrammetry for a small part of the surveyed airplane. This decision was made for one main reason. Even though laser scanning was considered more appropriate for the objectives of this project, close range photogrammetry is still regarded as the most standard method for performing elaborate 3D shape measurements. This is also the reason for performing a close range photogrammetric survey of a part already scanned by a laser. In this way the differences between the two methods can be clearly noticed in practise. What needs to be mentioned is that the scope of this study is rather limited. This is due to the fact that it is being conducted as a part of the course Digital Photogrammetry for the MSc of Geomatics. Therefore, the goal is not to make an assessment of accuracy differences between the two techniques, but rather to actually confirm their generally perceived and documented advantages and disadvantages.

Hence, in section A the principles of close range photogrammetry will be featured. This will be followed by a presentation of the measurement project and the network used for the measurement procedure in section B. In section C the results from the processing of the measurements will be given. Finally, section D has the conclusions from this study and a discussion on the findings.

A. Principles of Close Range Photogrammetry

Photogrammetry is the science that deals with the techniques used for obtaining representations of real world objects from images. Photogrammetry is a field known for some time now and constantly developing. Its main application is found in aerial projects, for the production of maps. However the methods used for aerial purposes have been optimized, especially in the latest years, for close distance projects. These methods have formed the new field of close-range photogrammetry.

The concepts used are the same in both platforms. Thus, a photograph is considered to be expressed geometrically by the model of central projection. According to this model, points of the 3D world are projected onto a plane via the optical rays that come through the projection point. This point can be considered as the central point of the camera lens in this case [Peta 98]. This principle applies also to human eyesight (where the eyes play the role of the projection point), and it can be explained schematically by figure A.1:

![Fig. A.1 Central projection of point (P-p) and line segment (D-d) in space](image)
But one image is not enough to make a 3D model of an object. This is also the case with our eyes. By having two eyes, the sense of depth perception is acquired. This can be explained geometrically and according to the proposed model as follows; the intersection of the rays from the two different images of the same object (the two different points of view each eye offers), gives the representation of the 3D object. This is the basis for stereoscopic vision which is featured in figure A.2:

![Fig. A.2 Stereoscopic Vision](Petsa_98)

What this means is that, if two or more images of an object from different angles are available, it is feasible, in principle, to determine mathematically the 3D shape of the pictured item. This concept can be explained for the case of four images by the figure A.3 below:

![Fig. A.3 Intersection of rays from 4 images gives 3D points](Wang_90)

Of course there is a whole procedure that needs to be followed to get to the final results and this is what the field of photogrammetry is covering. The steps of a photogrammetric procedure are a lot and varying, depending on the application. What will follow is the description of the stages encountered for the method used in this specific case, namely bundle adjustment. For further information about photogrammetric techniques, the reader is encouraged to consult specialised handbooks.

The first stage of a photogrammetric project is named the \textit{inner orientation} [Wang 90]. As mentioned before, the camera follows the central projection model. This model is described by the position of the projection point (or main point) in relation with the projection plane and can be described by three parameters: the two
coordinates of the main point in the plane \((x_0, y_0)\) and the focal length of the lens \((c)\).

There are also other parameters needed because the central projection model is not strictly applied in a camera. In reality, there are errors in the manufacturing of the camera and its lenses that cause a deviation from this model. The main added parameter considered in this study will be the lens distortion. This is an error that causes the edges of an image to ‘bend’, due to construction characteristics of the lens.

Finally, what is achieved with the inner orientation process is the recreation of the camera geometry, as shown in figure A.4:

![Fig. A.4 Parameters of Inner Orientation](image)

The process of determining the inner orientation of a camera is called calibration. During this process, pictures of a field of points with known coordinates, called control field, are used. By indicating the points in each image and their exact coordinates, the needed parameters are extracted. The calibration is then valid for every project carried out with the same camera, as long as this camera is considered geometrically stable.

After the inner orientation is defined for the used camera, the stage of the exterior orientation follows [Bender 71]. Here, all the images used for the project must be oriented, each one in respect to the others. Thus, the intersections of the rays of each image can be reproduced and the model of the pictured object can be created. The exterior orientation of each image is described by six parameters: three coordinates in space of the projection point \((X, Y, Z)\) and three rotations of the image \((O, P, K)\) in the direction of each one of the three measurement axis. The exterior orientation can describe completely the position of an image in 3D space. Consequently, it is possible to recreate the model of the specific object, from the ray intersections of the oriented images. An example of this procedure is provided in figure A.5:

![Fig. A.5 Exterior orientation](image)
The way the exterior orientation can be determined, is by detecting the matching points of the object of interest in all the used images. These points are called corresponding points. By determining a substantial number of these, the parameters needed for the orientation are obtained [Dermanis 90]. One aspect that must be particularly noticed during this procedure is the value of the intersection angles. These angles must be as perpendicular as possible, because the smaller they are, the more likely it is for the results to be erroneous. This can be understood if one considers that with small intersection angles, a small error in their value translates to a large displacement of the actual intersection. Of course, it is impossible to always achieve such angles, so generally intersection angles bigger than 30° are considered adequate.

With all the parameters known it is easy now to select the necessary points for creating the model of the object of interest. If scale is of importance it is necessary to have some additional information about the object. This could vary depending on the accuracy needed and the type of project. Thus, it could be measurements of some distances on the real object or even precisely defined control points with the use of a total station. The former is adequate if only the scale is of importance. However the latter is essential if the position of the object has to be determined in a pre-existing 3D coordinate system.

It is a common practice to perform the exterior orientation procedure for all the images while doing the calibration process described above. In this case, i.e. when a complete solution of the problem at hand with a least square approach is attempted, the method is called bundle adjustment. Although all the parameters can be determined, a priori knowledge of some of the parameters is desired. These are usually the interior orientation parameters. This way a better solution for the system is provided.

B. The measurement project

The Cessna was presented in detail in chapter 3.2. The thesis also showed in detail how a laser scanning setup was used for making a complete survey of the plane. To make a comparison of the two techniques, a specific part of the airplane had to be chosen. It was decided to pick a part that featured a breakline in the surface of the airplane. This would also test how photogrammetry handles this type of cases. In the end, the part chosen was the connection of the right wing with the fuselage of the Cessna, as shown in figure B.1:

![Fig. B.1 View of the photogrammetrically surveyed area](image-url)
The choice was based on the fact that, generally, the airplane does not feature many breaklines. The connection of the wings with the fuselage is by far the most prominent one. Furthermore that was also one of the problematic parts of the laser scanning survey. Apart from the breakline, also points from the adjacent area of the fuselage and the wing are measured.

After deciding on the area, the next step is to decide on what measuring device to use. In this case the measuring device is a camera and the OLRS chair has a Canon EOS 350D used for such applications. This is a Digital Single-Lens Reflex (DSLR) camera and has a plethora of specifications. However its main characteristics which are of interest for designing a photogrammetric network are its resolution and its focal length. This camera has an 8.0 megapixel resolution and a focal length which varies between 18 mm and 55 mm. The camera is featured in figure B.2 below:

![Fig. B.2 The Canon EOS 350D](image)

What is proven very useful is that this camera has been calibrated before. This means that the procedure is not going to be carried out again during the bundle adjustment. This will help immensely during the data processing that is going to follow. Furthermore the 8.0 megapixel image capability also helps a lot, since higher resolution can provide better accuracy at longer distances from the object. However one thing that needs to be decided in advance is the used focal length. The reason is that the adjustment works more efficiently when only one camera configuration is considered.

The design of the photogrammetric network is the most basic step in the whole process. The chosen set-up is going to be a series of four-station convergent networks. The reasoning behind the chosen set-up is featured in the book Close Range Photogrammetry and Machine Vision by K.B. Atkinson. In the chapter for network design [Fraser 96], it is stated that even though a stereoscopic configuration is basic for a photogrammetric network, the geometry produces a very inhomogeneous precision, especially affecting the accuracy in the depth direction. The situation is greatly improved when convergent stations are added in such a way that all points are seen from each image. That leads to a specific set-up which is considered optimal; a four-station convergent geometry. In this case homogeneous and near-isotropic precision is obtained. Furthermore, it is much easier to detect measurement errors in such configurations, making these networks much more reliable. In figure B.3 there is a comparison between the two types of set-ups:
The way this specific set-up is going to be used in this project, is by taking as many four-station configurations needed to cover the entire area in question. The adjacent configurations should overlap one with the other in order to get a complete result in the bundle adjustment process. An example of this set-up is shown in the following figure:

![Fig. B.4 Set-up of adjacent four-station configurations](image)

With the general set-up of the camera positions known, the calculations for their exact placement in the network need to take place. As mentioned before in chapter 2.2.3, one of the main advantages of close range photogrammetry is that it is a flexible measuring technique. That means that it can offer whichever accuracy a project needs with the use of the appropriate network. For this project an added bonus is the state-of-the-art camera used, which with its 8.0 megapixel resolution can offer extremely high accuracies at longer distances from the object. Thus, to make the necessary calculations, the final configuration of the camera needs to be fixed. It is decided to use the 55 mm focal length and the highest resolution for the images. This gives 3,456 pixels by 2,304 pixels images for the camera CMOS sensor of 22.2mm by 14.8mm.

For the precision of triangulations in a convergent, multi-station network, the general formula below is used as a preliminary indicator:

$$\overline{\sigma}_c = \frac{q}{\sqrt{k}} S \sigma = \frac{q}{\sqrt{k}} d \sigma_a \quad (1.1)$$

with $\overline{\sigma}_c$ the r.m.s. value of XYZ object point co-ordinate standard errors; $S$ the scale number, given as a mean object distance, $d$, divided by the camera principal distance, $c$; $\sigma$ is the image co-ordinate standard error; $\sigma_a$ is the corresponding angular measurement standard error; $q$ is a design factor expressing the strength of the basic camera station configuration; and $k$ the average number of exposures at or near each station [Fraser, 96].
With formula (1.1) and an initial configuration of the camera already decided upon, all the required data exist for designing the network. To show the capability for high accuracy a desired precision $\sigma_c$ of 0.1mm is chosen! The rest of the parameters are as follows: the manual image measurement precision $\sigma$ is taken for 0.5 pixels; principal distance $c$ is 55 mm; only one exposure is considered from each station, so $k=1$; it is assumed that optimal geometry is achieved, so $q = 0.6$.

Firstly the $\sigma$ must be expressed in metric units. Hence:

For a pixel size of \[
\frac{22.2\text{mm}}{3456 \text{pix}} = \frac{14.8\text{mm}}{2304 \text{pix}} = 0.0064\text{mm},
\]
so \[\sigma = 0.5 \text{pix} \cdot 0.0064\text{mm} = 0.0032\text{mm}\]

Now the following form of formula (1.1) is used to find the maximum distance of the camera from the object:

\[
d_{\text{max}} = \frac{\sigma_c c \sqrt{k}}{q \sigma} \quad (1.2)
\]

By substituting the known parameters the result is:

\[
d_{\text{max}} = \frac{0.1\text{mm} \cdot 55\text{mm} \cdot \sqrt{1}}{0.6 \cdot 0.0032\text{mm}} \Rightarrow d_{\text{max}} = 2.865\text{m}
\]

Now the $d_{\text{max}}$ can be used for the calculation of the scale. To begin with, the camera geometry at the time of an image acquisition has to be considered, as it is shown in figure B.5:

![Fig. B.5 Camera geometry during image acquisition in x and y direction](image_url)
From the known triangle rule the result is:

\[
\frac{22.2\text{mm}}{55\text{mm}} = \frac{x}{2865\text{mm}} \Rightarrow x = 1.156\text{m} \quad \text{and} \quad \frac{14.8\text{mm}}{55\text{mm}} = \frac{y}{2865\text{mm}} \Rightarrow y = 0.771\text{m}
\]

And thus the scale is:

\[
S = \frac{22.2\text{mm}}{1156\text{mm}} = \frac{14.8\text{mm}}{771\text{mm}} \Rightarrow S = \frac{1}{52}
\]

Finally a measure of the obtained accuracy can be calculated, by determining the precision \(\sigma_c\) in pixel size for the final configuration. So for \(\sigma_c = 0.1\text{mm}\) in reality, the corresponding figure in the image will be:

\[
\frac{0.1\text{mm}}{52} = 0.001923\text{mm}, \text{ which in pixel size is } \frac{0.001923\text{mm}}{0.0064\text{mm}} = 0.3\text{pixels}
\]

This gives an initial impression of the extreme accuracy that can be achieved, since 0.1mm in reality will translate to 0.3 pixels in the image!

Now all the necessary planning is made for the actual measurement phase of the project. The camera will be stationed at a distance of 2.86 meters and the network design will have geometry of consecutive convergent four-stations with overlap, as shown previously in figure B.4. The final distance was decided after considering a variety of options. However this was deemed the most appropriate, since for the same camera and \(\sigma_c = 0.5\text{mm} \Rightarrow d_{\text{max}} = 14.32\text{m}\), while for \(\sigma_c = 0.3\text{mm} \Rightarrow d_{\text{max}} = 8.59\text{m}\). These distances are too long, especially considering the confined space of the hangar where the Cessna is situated.

The photogrammetric measurements took place at a separate day than the laser scan in mid-May. The procedure was far easier this time and far less elaborate. This was due to the fact that a camera is far easier to handle than a laser scanner. Thus, only the author was needed for the measurements. This time the process was also not hindered in any way by workers on the site.

The main preparation that needed to be done before taking the pictures was to create a texture that would allow for easier targeting of points at the data processing stage. As shown in figure B.1, the surface of interest is mainly white, with some red and blue stripes, along with a silver surface at the front of the wing. Furthermore, the shape of the plane is too curved to give many detail points that can be easily distinguished in different images. That is why there were two ways used to create the much needed texture. First of all 6 targets were placed on the whole surface of the area in question; four on the fuselage and two on the wing. These will be used apart from providing detail points to also make possible some measurements of distances on the Cessna. The distances will be used later for scaling in the data processing phase. Finally, also some pieces of tape were glued in various parts of the surface to provide simple detail points. Special care was taken to use it on places of interest like on the breakline that needs to be surveyed in more detail. The final look of the area before the measurements took place is featured in figure B.6:
In the end, four consecutive four-station configurations were needed to cover the complete area of interest, as shown in image B.4. Of course more than the essential 16 images were obtained, since one of the advantages of using a camera is that many pictures can be taken instantaneously. Afterwards, the ones better suited for the specific project are chosen. In total 30 pictures were taken.

Finally the distances between the targets were measured. A standard measuring tape was used, but all the distances were measured two times and an average was taken. In general, a total station could be used for such a task, but one was not available at that time. In addition this level of accuracy was not considered necessary for the scope of this study. These measurements were 13 overall and will be used to give a measure of accuracy in the data processing stage.

C. The data processing

The software used for performing all the photogrammetric calculations was Photomodeler 5.0. The procedure followed by this program is the bundle adjustment described in detail in section A.

The inner orientation of the camera is solved if the camera has been already calibrated. Luckily the camera used was calibrated for a 55mm focal length. All the interior orientation parameters are featured in the table C.1:
Table C.1 Interior orientation parameters of Canon EOS 350D with 55mm focal length

<table>
<thead>
<tr>
<th></th>
<th>Canon EOS 350D 55 mm setting</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Focal Length</strong></td>
<td>52.3955 mm</td>
</tr>
<tr>
<td><strong>Format Size</strong></td>
<td>W: 22.1736</td>
</tr>
<tr>
<td></td>
<td>H: 14.7828</td>
</tr>
<tr>
<td><strong>Principal Point</strong></td>
<td>X: 11.5626</td>
</tr>
<tr>
<td></td>
<td>Y: 7.7223</td>
</tr>
<tr>
<td><strong>Lens Distortion</strong></td>
<td>K1: -2.192e-005</td>
</tr>
<tr>
<td></td>
<td>K2: -2.193e-008</td>
</tr>
<tr>
<td></td>
<td>K3: 0.000e+000</td>
</tr>
<tr>
<td><strong>Image Size</strong></td>
<td>3456 pixels</td>
</tr>
<tr>
<td></td>
<td>by 2304 pixels</td>
</tr>
<tr>
<td><strong>CMOS Size</strong></td>
<td>22.2 mm</td>
</tr>
<tr>
<td></td>
<td>by 14.8 mm</td>
</tr>
</tbody>
</table>

With the interior orientation parameters known, the internal geometry of the camera is also known. What is needed now is the exterior orientation of each picture for the determination of all the camera stations. From the 30 pictures taken the 16 were chosen, in such a way so that they fit the four-station convergent geometry described in chapter B. In the end, the intersecting rays for each point from all the separate images will create the final 3D model. This is how Photomodeler also works. All the images are opened and then it is the task of the user to mark all corresponding points from the overlapping pictures. In the end all these points, along with the parameters from the inner orientation, are taken into account in the least square solution of the problem. A sample of the manual work or marking corresponding points with Photomodeler is shown in figure C.1 below:

![Fig. C.1 Marking points with Photomodeler](image)

A sample of the points chosen can also be seen in figure C.1 above. The breakline created by the connection of the wing with the fuselage is marked in the parts where tape was put. The rest pieces of tape provide points in the completely white surface of the Cessna. The actual targets are well visible and offer clear points for marking. Finally there are some detail points from the airplane as well, e.g. weldings like the one zoomed in the image.

Eventually 92 points were marked. From the initial bundle adjustment, the residuals of these points in pixel size are available. By consulting these results some...
necessary corrections can be made. These corrections mostly entail performing a more
detailed marking of points with big residuals. Generally it is optimal when this value
remains below 5 pixels. In the end 86 points were used in the final adjustment. Six
had to be discarded because they were giving bad references.

The results were considered satisfactory. Out of the 86 points, the residuals
range from a minimum of 0.46 to 3.83 pixels, which is impressive when one considers
that 0.3 pixels correspond to 0.1 mm in reality. The RMS of the residuals is even
more impressive, as it is ranging from 0.37 to 2.26 pixels. The mean of the RMS
residual values is 1.24 pixels, which in reality is translated into 0.41 mm!

The bundle adjustment solution gave also the camera stations. In figure C.2
below, one can see the four-station convergent geometry of the network used. This
geometry is not prevalent in all the length of the network due to some images not
being clear. That prompted the author to use some off the additional images taken.
However, the result is still very close to the original plan:

What was left after the last step was to give scale to the final result. That can
be done by choosing two points and assigning to them the actual distance measured.
Then the whole model is scaled and additional measurements are made in order to
give a measure of the obtained accuracy. The distance used for the scaling is the one
between targets 1 and 4 (see Fig. B.6). It was chosen because it covers almost the
whole area of interest. After that the distances measured in reality are also measured
in the obtained model. The result of this comparison can be seen in table C.2 below:

<table>
<thead>
<tr>
<th>Distances</th>
<th>Model (m)</th>
<th>Reality (m)</th>
<th>Difference (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5-4</td>
<td>0.4900</td>
<td>0.4892</td>
<td>0.8</td>
</tr>
<tr>
<td>5-3</td>
<td>0.4300</td>
<td>0.4306</td>
<td>0.6</td>
</tr>
<tr>
<td>5-1</td>
<td>0.9484</td>
<td>0.9492</td>
<td>0.8</td>
</tr>
<tr>
<td>5-2</td>
<td>1.3760</td>
<td>1.3762</td>
<td>0.2</td>
</tr>
<tr>
<td>6-2</td>
<td>1.0748</td>
<td>1.0760</td>
<td>1.2</td>
</tr>
<tr>
<td>6-1</td>
<td>0.7450</td>
<td>0.7458</td>
<td>0.8</td>
</tr>
<tr>
<td>6-3</td>
<td>0.6110</td>
<td>0.6104</td>
<td>0.6</td>
</tr>
<tr>
<td>6-4</td>
<td>0.9510</td>
<td>0.9498</td>
<td>1.2</td>
</tr>
<tr>
<td>4-3</td>
<td>0.5610</td>
<td>0.5605</td>
<td>0.5</td>
</tr>
<tr>
<td>4-1</td>
<td>Scale Distance</td>
<td>1.1915</td>
<td>-</td>
</tr>
<tr>
<td>3-1</td>
<td>0.6320</td>
<td>0.6308</td>
<td>1.4</td>
</tr>
<tr>
<td>3-2</td>
<td>1.1140</td>
<td>1.1132</td>
<td>0.8</td>
</tr>
<tr>
<td>1-2</td>
<td>0.5030</td>
<td>0.5040</td>
<td>1.0</td>
</tr>
</tbody>
</table>
These results show that the setup used indeed managed to provide a very high level of accuracy. The distances between the targets presented above are distributed over the whole area and the mean of all the differences is 0.825 mm. In the end what was achieved was to create a photogrammetric network that offers higher accuracy than the one obtained by the laser scanning solution.

The final result can be seen in the following images. The images show the breakline created by the connection of the wing with the fuselage, along with the targets used, and part of the coloured stripes:

Fig. C.3 Comparison: (Top) Part of surveyed area; (Bottom) Photogrammetric outcome
D. Conclusions

In the course of this study a photogrammetric survey was conducted for a small part of the TU Delft Cessna Citation. In chapter A the principles of close range photogrammetry were presented, followed by the description of the measurement project in chapter B. Finally the results of the photogrammetric data processing were presented in detail in chapter C. Since this part of the plane was already surveyed with the use of a laser scanner, this project is suited for pointing out differences between these two techniques. In the following figure, the survey results of the two methods are shown:
The image above gives an initial impression of the most basic difference; it is evident that laser scanning provides a bigger amount of data than photogrammetry. In addition the surface texture of an object is of no importance, as opposed to photogrammetry. This also means that there is no need for extensive marking of the object. Lastly, laser scanning is more suited for measuring surfaces than photogrammetry (even though it has problems with absorbing colours and specularly reflecting materials).

On the other hand it was shown in chapter C that the network used gave results of sub-mm accuracy. This is impossible with laser scanning. Furthermore the actual acquisition of the data in photogrammetry (i.e. taking pictures) is much faster than laser scanning. Moreover these pictures can be used for texture mapping. Also important is the fact laser scanners are still bulky and expensive equipment. At the same time the cost and size of high resolution digital cameras is constantly decreasing. Finally, photogrammetry is much better at detecting specific detail points and breaklines than laser scanning.

An overview of the differences is given in table D.1 in the following page:
Table D.1 Laser scanning vs. photogrammetry comparison

<table>
<thead>
<tr>
<th></th>
<th>Laser Scanning</th>
<th>Photogrammetry</th>
</tr>
</thead>
<tbody>
<tr>
<td>Suited for Points</td>
<td>NO</td>
<td>YES</td>
</tr>
<tr>
<td>Suited for Edges</td>
<td>NO</td>
<td>YES</td>
</tr>
<tr>
<td>Suited for Surfaces</td>
<td>YES</td>
<td>NO</td>
</tr>
<tr>
<td>Range</td>
<td>1 - 500 meters</td>
<td>0.5 - 50 meters</td>
</tr>
<tr>
<td>Precision</td>
<td>3 mm</td>
<td>Scale Dependent (can reach sub-mm)</td>
</tr>
<tr>
<td>Speed</td>
<td>Medium</td>
<td>High</td>
</tr>
<tr>
<td>Costs</td>
<td>High</td>
<td>Low</td>
</tr>
<tr>
<td>Specific Problems</td>
<td>No return signal from absorbing or specularly reflecting surfaces</td>
<td>Surface texture required for finding homologous points</td>
</tr>
<tr>
<td>Specific Advantages</td>
<td>High amount of data acquisition (120,000 points/sec)</td>
<td>Images can be used for texture mapping, Good for breaklines &amp; detail points</td>
</tr>
</tbody>
</table>

In the end both techniques feature their own strengths and weaknesses which make them suitable for different types of applications. As an example cultural heritage applications can be considered. In case where architecture needs to be represented, detection of sharp edges and lines is very important. Thus photogrammetry seems to be more appropriate. In contrast, if modeling sculptures and smooth surfaces is of interest, then laser scanning is more fitting [Agnello et al 05].

That is why the trend today is to have the ‘best of both worlds’ by integrating the two methods together. Already a lot of research is being carried out in this direction with very promising results. In [Balletti et al 2005] a hybrid sensor was used with the following combination; a high-performance long-range laser scanner with a wide field-of-view was combined with a calibrated high-resolution digital camera firmly mounted onto the scanning head. This integration provided a very convenient, efficient and powerful system for automatically generating accurately textured high-resolution 3D models in architectural applications. One could say that, even though lately laser scanning seems to be more popular in this field, the future lies within this type of hybrid sensors which can provide advantages from both methods.

References:


Appendix 2

The NACA Airfoils

Airfoil design in the past was an arbitrary procedure. The designers were solely based on past experience with known shapes when making designs. Experiments carried out with modifications to those shapes gave the final result.

This changed in the 1930s with the publishing of a report by the National Advisory Committee for Aeronautics (NACA) entitled *The Characteristics of 78 Related Airfoil Sections from Tests in the Variable Density Wind Tunnel*. After systematic testing, this report came to the conclusion that the most efficient airfoil shapes shared similarities between them. The primary variables that affect those shapes are four:

- the maximum slope of the airfoil mean camber line (m),
- the position of this maximum camber (p),
- the maximum thickness distribution (t) above and below the mean camber line.
- and the maximum length of the airfoil defined as the maximum chord (c).

The NACA airfoils are categorized in series depending on the digits used for the description of the variables. The simple ones are the 4- and 5-digit series and the more sophisticated ones are the 6-, 7- and 8-digit series. The use of the four above variables is enough to generate all the simple 4- and 5-digit NACA airfoils, as shown below:

![Fig. 1 Generation of NACA airfoil with the use of four parameters](image)

What follows is a presentation of the calculation of the 5-digit series airfoils. The 23012 used in the Cessna Citation II is such an airfoil.

5-Digit Series

As an example the 23012 NACA profile used in the Cessna Citation II will be considered. The first digit, when multiplied by 3/2, yields the design lift coefficient ($c_l$) in tenths. The next two digits, when divided by 2, give the position of the maximum camber (p) in tenths of chord. The final two digits again indicate the maximum thickness (t) in percentage of chord. Thus the NACA 23012 has a...
maximum thickness of 12%, a design lift coefficient of 0.3 and a maximum camber located 15% before the leading edge.

The coordinates of a 5-digit airfoil are calculated by following the procedure presented below:

1. Decide on the ‘step’ of the value \( x \) from 0 to the maximum chord \( c \).
2. Compute the mean camber line coordinates for each \( x \) location using the following equations:

\[
y_c = \frac{k_1}{6} [x^3 - 3mx^2 + m^2(3 - m)x] \quad \text{from} \quad x = 0 \quad \text{to} \quad x = p
\]
\[
y_c = \frac{k_1m}{6}(1 - x) \quad \text{from} \quad x = p \quad \text{to} \quad x = c
\]

Since \( p \) is known, the values of the parameters \( m \) and \( k_1 \), requested above, can be determined by using the table below:

<table>
<thead>
<tr>
<th>Mean-line designation</th>
<th>Position of max camber (( p ))</th>
<th>( m )</th>
<th>( k_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>210</td>
<td>0.05</td>
<td>0.0580</td>
<td>361.400</td>
</tr>
<tr>
<td>220</td>
<td>0.10</td>
<td>0.1260</td>
<td>51.640</td>
</tr>
<tr>
<td>230</td>
<td>0.15</td>
<td>0.2025</td>
<td>15.957</td>
</tr>
<tr>
<td>240</td>
<td>0.20</td>
<td>0.2900</td>
<td>6.643</td>
</tr>
<tr>
<td>250</td>
<td>0.25</td>
<td>0.3910</td>
<td>3.230</td>
</tr>
</tbody>
</table>

3. Calculate the thickness distribution above (+) and below (-) the mean line by plugging the value of the maximum thickness (\( t \)) into the following equation for each of the \( x \) coordinates:

\[
\pm y_i = \frac{t}{0.2} (0.2969\sqrt{x} - 0.1260x - 0.3516x^2 + 0.2843x^3 - 0.1015x^4)
\]

4. Calculate the final coordinates of the upper and lower surface of the airfoil by using the following formulas:

Upper Part: \( x_U = x - y_i \sin \theta \)

\[
y_U = y_c + y_i \cos \theta
\]

Lower Part: \( x_L = x + y_i \sin \theta \)

\[
y_L = y_c - y_i \cos \theta
\]

with \( \theta = \arctan \left( \frac{dy_c}{dx} \right) \)

Source: Airospaceweb Organisation: 
Appendix 3

B-Spline fitting through wing profiles

In chapter 5.4 the process of fitting B-splines through the laser points was presented. A laser scanned part of the right wing of the Cessna was divided in 35 profiles. The points from each profile were then fitted with a series of 3 B-splines. This series of B-splines was chosen because it emulates very closely the shape of the NACA airfoils used for wing design.

In this appendix the result of the fitting process for each profile will be presented. What follows are images containing the original laser points along with their fitted B-splines. The horizontal and vertical axes correspond to the X- and Z-axis respectively. Units are in meters. The laser points are in black colour. The B-splines fitted through the profiles have the following colours:

- Front Part: Magenta
- Upper Part: Yellow
- Lower Part: Green

Furthermore, the caption of each image contains the degree of the B-splines used for the fitting. In the cases of the first 10 profiles there were not enough points in the low part of the wing to have a successful fitting:

Profile 1: Not enough points for fitting
Profile 3 (Upper Part: 8th deg, Front Part: 3rd deg)
(Horizontal Axis = X, Vertical Axis = Z, Units = meters)

Profile 4 (Upper Part: 8th deg, Front Part: 3rd deg)
(Horizontal Axis = X, Vertical Axis = Z, Units = meters)
Profile 5 (Upper Part: 8th deg, Front Part: 3rd deg)
(Horizontal Axis = X, Vertical Axis = Z, Units = meters)

Profile 6 (Upper Part: 8th deg, Front Part: 3rd deg)
(Horizontal Axis = X, Vertical Axis = Z, Units = meters)
Profile 7 (Upper Part: 8th deg, Front Part: 3rd deg)
(Horizontal Axis = X, Vertical Axis = Z, Units = meters)

Profile 8 (Upper Part: 8th deg, Front Part: 4th deg)
(Horizontal Axis = X, Vertical Axis = Z, Units = meters)
Profile 9 (Upper Part: 8\textsuperscript{th} deg, Front Part: 4\textsuperscript{th} deg)
(Horizontal Axis = X, Vertical Axis = Z, Units = meters)

Profile 10 (Upper Part: 8\textsuperscript{th} deg, Lower Part: 6\textsuperscript{th} deg, Front Part: 4\textsuperscript{th} deg)
(Horizontal Axis = X, Vertical Axis = Z, Units = meters)
Profile 11 (Upper Part: 8th deg, Lower Part: 6th deg, Front Part: 4th deg)
(Horizontal Axis = X, Vertical Axis = Z, Units = meters)

Profile 12 (Upper Part: 8th deg, Lower Part: 6th deg, Front Part: 5th deg)
(Horizontal Axis = X, Vertical Axis = Z, Units = meters)
Profile 13 (Upper Part: 8th deg, Lower Part: 6th deg, Front Part: 5th deg)
(Horizontal Axis = X, Vertical Axis = Z, Units = meters)

Profile 14 (Upper Part: 8th deg, Lower Part: 6th deg, Front Part: 5th deg)
(Horizontal Axis = X, Vertical Axis = Z, Units = meters)
Profile 15 (Upper Part: 8\textsuperscript{th} deg, Lower Part: 6\textsuperscript{th} deg, Front Part: 5\textsuperscript{th} deg)  
(Horizontal Axis = X, Vertical Axis = Z, Units = meters)

Profile 16 (Upper Part: 8\textsuperscript{th} deg, Lower Part: 6\textsuperscript{th} deg, Front Part: 4\textsuperscript{th} deg)  
(Horizontal Axis = X, Vertical Axis = Z, Units = meters)
Profile 17 (Upper Part: 8th deg, Lower Part: 6th deg, Front Part: 5th deg)
(Horizontal Axis = X, Vertical Axis = Z, Units = meters)

Profile 18 (Upper Part: 8th deg, Lower Part: 6th deg, Front Part: 5th deg)
(Horizontal Axis = X, Vertical Axis = Z, Units = meters)
Profile 19 (Upper Part: 8th deg, Lower Part: 6th deg, Front Part: 5th deg)
(Horizontal Axis = X, Vertical Axis = Z, Units = meters)

Profile 20 (Upper Part: 8th deg, Lower Part: 7th deg, Front Part: 5th deg)
(Horizontal Axis = X, Vertical Axis = Z, Units = meters)
Profile 21 (Upper Part: 8th deg, Lower Part: 7th deg, Front Part: 5th deg)
(Horizontal Axis = X, Vertical Axis = Z, Units = meters)

Profile 22 (Upper Part: 8th deg, Lower Part: 6th deg, Front Part: 5th deg)
(Horizontal Axis = X, Vertical Axis = Z, Units = meters)
Profile 23 (Upper Part: 8th deg, Lower Part: 6th deg, Front Part: 5th deg)
(Horizontal Axis = X, Vertical Axis = Z, Units = meters)

Profile 24 (Upper Part: 8th deg, Lower Part: 6th deg, Front Part: 5th deg)
(Horizontal Axis = X, Vertical Axis = Z, Units = meters)
Profile 25 (Upper Part: 8th deg, Lower Part: 6th deg, Front Part: 5th deg)
(Horizontal Axis = X, Vertical Axis = Z, Units = meters)

Profile 26 (Upper Part: 8th deg, Lower Part: 6th deg, Front Part: 5th deg)
(Horizontal Axis = X, Vertical Axis = Z, Units = meters)
Profile 27 (Upper Part: 8th deg, Lower Part: 6th deg, Front Part: 5th deg)
(Horizontal Axis = X, Vertical Axis = Z, Units = meters)

Profile 28 (Upper Part: 8th deg, Lower Part: 6th deg, Front Part: 5th deg)
(Horizontal Axis = X, Vertical Axis = Z, Units = meters)
Profile 29 (Upper Part: 8th deg, Lower Part: 6th deg, Front Part: 5th deg)
(Horizontal Axis = X, Vertical Axis = Z, Units = meters)

Profile 30 (Upper Part: 8th deg, Lower Part: 7th deg, Front Part: 3rd deg)
(Horizontal Axis = X, Vertical Axis = Z, Units = meters)
Profile 31 (Upper Part: 8th deg, Lower Part: 7th deg, Front Part: 3rd deg)
(Horizontal Axis = X, Vertical Axis = Z, Units = meters)

Profile 32 (Upper Part: 8th deg, Lower Part: 7th deg, Front Part: 5th deg)
(Horizontal Axis = X, Vertical Axis = Z, Units = meters)
Profile 33 (Upper Part: 8\textsuperscript{th} deg, Lower Part: 7\textsuperscript{th} deg, Front Part: 5\textsuperscript{th} deg)  
(Horizontal Axis = X, Vertical Axis = Z, Units = meters)

Profile 34 (Upper Part: 8\textsuperscript{th} deg, Lower Part: 6\textsuperscript{th} deg, Front Part: 5\textsuperscript{th} deg)  
(Horizontal Axis = X, Vertical Axis = Z, Units = meters)
Profile 35 (Upper Part: 8th deg, Lower Part: 7th deg, Front Part: 5th deg)
(Horizontal Axis = X, Vertical Axis = Z, Units = meters)
Appendix 4

Matlab scripts

function bestfitplane
% Find the best fitting plane through points'

% load data
data = load ('tailpart4low.xyz');
x = data(:,1); % x coordinates
y = data(:,2); % y coordinates
z = data(:,3); % z coordinates
n = length (data)

% Plot of 3D points
figure(1);
scatter3(x,y,z);

% Create and solve standard Least Squares A-form
for i=1:n
    A(i,1)=x(i,1);
    A(i,2)=y(i,1);
    A(i,3)=1;
    Y(i,1)=z(i,1);
end;
XX=(inv(A'*A))*(A'*Y);
a=XX(1,1)
b=XX(2,1)
c=XX(3,1)

% Find Residuals
R=Y-A*XX;

% Plot Residual and fit histogram
figure(2);
histfit(R,100);

% Plot plane bounded by convex hull
figure(3);
k = convhull(x,y);
plot(x(k),y(k),'r-',x,y,'b+');

% Export Boundary Points of fitted plane to file
fd = fopen ('boundaries.txt','wt');
for i=1:m,
    fprintf(fd,'%f  %f  %f
', x(k(i)), y(k(i)), z(i));
end
fclose(fd);

function [profiles xi yi zi profy]=makeProfiles(data, resolution)
% Generate profiles out of XYZ data
xi=data(:,1); % x coordinates
yi=data(:,2); % y coordinates
zi=data(:,3); % z coordinates
n = length (data)
xi=x-(max(xi)+min(xi))/2;
yi=y-(max(yi)+min(yi))/2;

139
\[ zi = zi - (\text{max}(zi) + \text{min}(zi))/2; \]

\[ \text{minX} = \text{min}(xi); \]
\[ \text{minY} = \text{min}(yi); \]
\[ \text{minZ} = \text{min}(zi); \]

\[ \text{maxX} = \text{max}(xi); \]
\[ \text{maxY} = \text{max}(yi); \]
\[ \text{maxZ} = \text{max}(zi); \]

\[ \text{mysize} = (\text{maxX} - \text{minX})/\text{resolution} \]
\[ \text{matrixSize} = \text{ceil}(\text{mysize}); \]

\[ \text{profiles} = \text{zeros(length}(xi), 1); \]
\[ \text{profy} = \text{zeros(length}(xi), 1); \]

\[ \text{for } i = 1:\text{length}(yi) \]
\[ \quad \text{tempArrPos} = (yi(i) - \text{minY})/\text{resolution}; \]
\[ \quad \text{arrPos} = \text{ceil}(\text{tempArrPos}) + 1; \]
\[ \quad \text{profiles}(i) = \text{arrPos}; \]
\[ \quad \text{profy}(i) = \text{minY} + (\text{arrPos} - 0.5) \times \text{resolution}; \]
\[ \text{end} \]

\textbf{function} \[ [x \text{ mean upper lower}] = \text{getMeanFromPoints(xi, yi, zi, profiles, profileid, doplot)} \]
\textbf{\%} Returns mean of last profile

\[ \text{\% Define order of polynomial} \]
\[ N = ?; \]
\[ X = [xi \ yi \ zi]; \]

\textbf{for } i = 1:\text{length}(profileid) \]
\[ c = X(\text{find}(\text{profiles}==\text{profileid}(i)),:); \]

\[ \text{\% Split in upper and lower} \]
\[ \text{mostleft} = c(\text{find}(c(:,1)==\text{min}(c(:,1))),:); \]
\[ \text{mostright} = c(\text{find}(c(:,1)==\text{max}(c(:,1))),:); \]

\[ \text{\% line=ax+b} \]
\[ b = \text{mostleft}(3); \]
\[ a = ((\text{mostright}(3) - \text{mostleft}(3))/(\text{mostright}(1) - \text{mostleft}(1))); \]

\[ \text{upper} = c(c(:,3) > (b+a*(c(:,1)-\text{mostleft}(1))),:); \]
\[ \text{lower} = c(c(:,3) <= (b+a*(c(:,1)-\text{mostleft}(1))),:); \]

\[ \text{\% Fit polynomial} \]
\[ \text{Pupper} = \text{polyfit}(\text{upper(:,1)}, \text{upper(:,3)}, N); \]
\[ \text{Plower} = \text{polyfit}(\text{lower(:,1)}, \text{lower(:,3)}, N); \]

\[ x = \text{mostleft}(1):0.01:\text{mostright}(1); \]
\[ \text{Yupper} = \text{polyval}(\text{Pupper}, x); \]
\[ \text{Ylower} = \text{polyval}(\text{Plower}, x); \]

\[ \text{mean} = (\text{Yupper} + \text{Ylower})/2; \]

\textbf{if} \[ \text{doplot} == 1 \]
\[ \text{figure; } \]
\[ \text{hold on} \]
\[ \text{plot(upper(:,1), upper(:,3), 'b.'); } \]
\[ \text{plot(lower(:,1), lower(:,3), 'r.'); } \]
\[ \text{plot(x, Yupper, 'r-'); } \]
\[ \text{plot(x, Ylower, 'b-'); } \]
\[ \text{plot(x, mean, 'g-'); } \]
\[ \text{plot(x, mean, 'k-'); } \]
\[ \text{axis equal} \]
\textbf{end}
function drawProfile(xi, yi, zi, profiles, profileid)
    % Draw a profile with profile id
    X=[xi yi zi];
    for i=1:length(profileid)
        % Take only one profile
        figure;
        c=X(find(profiles==profileid(i)),:);
        plot(c(:,1), c(:,3), 'b.'
        title(profileid(i))
        axis equal;
    end

function dycdx=meanDerivative(x,yc)
    % Calculate the first derivative of the mean
    % yc=mean
    for i = 1:length(x)
        if i==1
            dycdx(i)=0;
        else
            dycdx(i)=(yc(i)-yc(i-1))/(x(i)-x(i-1));
        end
    end

function fitsplines
    % Fit 3 splines through profile points to create airfoil
    close all
    [profiles xi yi zi profyy]=makeProfiles(X,0.1);
    % % For all profiles:
    % doprofiles=unique(profiles)
    % For one profile
    doprofiles=[28];
    for prof=1:length(doprofiles)
        profile=doprofiles(prof)
        drawProfile(xi, yi, zi, profiles, profile)
        [x mean upper lower]=getMeanFromPoints(xi, yi, zi, profiles,profile,0);
        profy=unique(profyy(find(profiles==profile)))
        if (length(lower)>10 & length(upper)>10)
            figure;
            hold on
            plot(upper(:,1), upper(:,3), 'b.'
            plot(lower(:,1), lower(:,3), 'r.'
            axis equal
            hold off
            mostright1=find(xi==max(lower(:,1)));
            if length(mostright1)>1
                mostright1=mostright1(1);
            end
            % Create Weight Matrices for Upper/Lower Part
            wl1=ones(1,length(upper));
            wl1(1)=10000;
w1(length(upper))=10000;
w2=ones(1,length(lower));
w2(1)=10000;
w2(length(lower))=10000;

% % Fit the spline
upsp=spap2(1, 7, upper(:,1), upper(:,3),w1);
losp=spap2(1, 7, lower(:,1), lower(:,3),w2);

% % Plot Splines
figure;
fnplt(upsp,'k')
fnplt(losp,'b')

% Calculate the first derivative of upper part
xxx=min(upper(:,1)):0.01:max(upper(:,1));
yyy=fnval(upsp,xxx);
yyyy=fnval(losp,xxx);
for i = 1:length(xxx)
    if i==1
dycdx(i)=NaN;
    else
dycdx(i)=(yyy(i)-yyy(i-1))/(xxx(i)-xxx(i-1));
    end
end
dycdx;

% Find point of division of points
Zeropoint=find(dycdx<=0.0000001);
Zeromin=round(min(Zeropoint)*(2.5/5)) % Here you can change the point dividing the front part
upper=[xxx(Zeromin),0,yyy(Zeromin);upper;xi(mostright1),0,zi(mostright1)];
lower=[xxx(Zeromin),0,yyy(Zeromin);lower;xi(mostright1),0,zi(mostright1)];

% Create Weight Matrices for Upper/Lower Part
w1=ones(1,length(upper));
w1(1)=10000;
w1(length(upper))=10000;
w2=ones(1,length(lower));
w2(1)=10000;
w2(length(lower))=10000;

% % Fit the spline
upsp=spap2(1, 8, upper(:,1), upper(:,3),w1); % Here you can change degree of spline fitted in upper part
losp=spap2(1, 6, lower(:,1), lower(:,3),w2); % Here you can change degree of spline fitted in lower part

% Find points of front part
X2=[xi yi zi];
c=X2(find(profiles==profile),:);
I=(find(c(:,1)<=xxx(Zeromin)));
front=c(I,:);
front1=[xxx(Zeromin),0,yyy(Zeromin);front;xxx(Zeromin),0,yyyy(Zeromin)];
plot(front(:,1), front(:,3), 'k.')

% Create Weight Matrix
w3=ones(1,length(front1));
w3(1)=10000;
w3(length(front1))=10000;

% Rotate the front by 90 degrees
R=[0 1;-1 0];
frontR=front1(:,[1, 3])*R;
```matlab
figure;
plot(frontR(:,1), frontR(:,2),'b.');
axis equal
hold on

% Fit spline to Front Part
frsp=spap2(1, 5, frontR(:,1), frontR(:,2),w3); % Here you can change
degree of spline fitted in front part
fnplt(frsp)
xx=min(frontR(:,1)):.01:max(frontR(:,1));
yy=fnval(frsp,xx);
frvalR=[xx',yy'];
frval=frvalR*inv(R);

% Remove points right of Minzero point
frval=frval(frval(:,1)<=xxx(Zeromin),:);

% Add the minzero point to make a nicely connected line
frval=[xxx(Zeromin),yyy(Zeromin);frval;xxx(Zeromin),yyyy(Zeromin)];

% Get values for all splines
xx=xxx(Zeromin):0.01:xi(mostright1);
upval=fnval(xx, upsp);
loval=fnval(xx, losp);
figure;
hold on
plot(upper(:,1),upper(:,3),'k.') %plot original points
plot(lower(:,1),lower(:,3),'k.') %plot original points
plot(frval(:,1), frval(:,2), 'm-')
plot(xx, upval, 'y-')
plot(xx, loval, 'g-')
axis equal
plot(xxx(Zeromin),yyy(Zeromin),'o')
plot(xxx(Zeromin),yyyy(Zeromin),'o')
title(profile)

% Export Spline Points of Profile to File
fd = fopen (sprintf('profile%d.txt', profile),'wt');
for i=1:length(xx),
    fprintf(fd,'%f  %f  %f
', xx(i),profy, upval(i));
end
for i=1:length(xx),
    fprintf(fd,'%f  %f  %f
', xx(i),profy, loval(i));
end
for i=1:length(frval),
    fprintf(fd,'%f  %f  %f
', frval(i,1),profy,frval(i,2));
end
fclose(fd);
else
disp ('Too few points for this profile!')
end

end

function d=points2line(p,x1,x2)
% d=points2line(p,x1,x2)
% Returns perpendicular distance from point to a line segment.
% p is a vector with the x- and y-coordinate of the point
% x1 is a vector with the coordinates of the begin point
% x2 is a vector with the coordinates of the end point
```
xdiff=x2(1)-x1(1);
ydiff=x2(2)-x1(2);
diffdist=sqrt(xdiff^2+ydiff^2);
d=abs(xdiff*(x1(2)-p(:,2))-ydiff*(x1(1)-p(:,1)))/diffdist;
end

close all

% Load CAD and Laser point profiles
cad=load('Cadprof16.txt');
scan=load('2Scanprof35.txt');

% Plot the airfoils
figure(1);plot(cad(:,1), cad(:,3),'b.');hold on; plot(scan(:,1), scan(:,3),'r.');axis equal
figure(2);scatter3(cad(:,1), cad(:,2), cad(:,3),'b.');hold on; scatter3(scan(:,1), scan(:,2), scan(:,3),'r.');axis equal

% Do the ICP (Using program registerICP from Tahir Rabbani’s toolbox)
Tt=registerICP(cad, scan, 10, 100);
scantrans=xformxyz(scan, Tt);

% Plot fitted airfoils
figure(3);
plot(cad(:,1), cad(:,3),'b.');hold on; plot(scantrans(:,1), scantrans(:,3),'r.');axis equal

% Compute differences in airfoil points and find minimum
for j=1:length(cad);
    d = sqrt((cad(j,1)-scantrans(:,1)).^2 + (cad(j,3)-scantrans(:,3)).^2);
    I(j)=find(d==min(d));
    err(j)=min(d);
end

% Plot closest points of Laser airfoil to Cad airfoil
figure(4);
plot(cad(:,1), cad(:,3),'b.');hold on; plot(scantrans(I,1), scantrans(I,3),'r.');axis equal

% Compute distances of closest Laser points to Cad airfoil (errors)
for j=1:length(cad);
    if j==length(cad)
        k=1;
    else
        k=j+1;
    end
    errnew(j)=points2line([scantrans(I(j),1)
    scantrans(I(j),3)], [cad(j,1) cad(j,3)], [cad(k,1) cad(k,3)]);
end

% Plot histogram with errors
figure(5);
hist(errnew,10);

% Export errors to File
fd = fopen ('zzz.txt','wt');
for i=1:length(cad),
    fprintf(fd,'%f %f %f %f
',
    cad(i,1), cad(i,2), cad(i,3), errnew(i));
end
fclose(fd);

% Plot Cad airfoils with colours indicating errors
figure(6);
scatter3(cad(:,1), cad(:,2), cad(:,3), 10, errnew, 'filled');
axis equal;grid off
colorbar('southoutside');
Appendix 5

Comparison of CAD model & Laser model airfoils

In chapter 5.5 the process of comparing the CAD model and Laser model airfoils was presented. Registration between corresponding airfoil pairs was used as a first step. Then the points of the CAD model were compared with the nearest points from the Laser model. Due to the problematic nature of the front part of the laser airfoils, registrations were also carried out without them. However, comparisons took the complete profiles into account.

In this appendix the result of this process for each wing profile pair will be presented. The following images are featured for all the pairs:
1. Airfoils in original position
2. Airfoils after Registration
3. CAD Airfoil with Laser airfoil without front part after Registration
4. Registered airfoils with nearest points from Laser to CAD airfoil
5. Histogram of differences between airfoil points
6. Colourbar chart of CAD airfoil points indicating the differences

In the first 2 pairs the laser airfoil had no lower part. A complete comparison between these airfoils is not possible. Thus only the registration was performed in these cases and not the point difference calculation.

A table with a description of the comparison pairs follows. After that, the formerly described images for each pair are featured. The content of each image is indicated by its caption.

<table>
<thead>
<tr>
<th>Airfoil Comparison Pairs</th>
<th>Comparison Pair</th>
<th>Airfoil Pairs (CAD – Laser)</th>
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</thead>
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<tr>
<td>1</td>
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<tr>
<td>16</td>
<td>(16-35)</td>
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</tr>
</tbody>
</table>
PAIR 1:

Pair 1: Airfoils in original position (Horizontal Axis = X, Vertical Axis = Z, Units = meters, Red = Laser Model, Blue = CAD Model)

Pair 1: Airfoils after Registration (Horizontal Axis = X, Vertical Axis = Z, Units = meters, Red = Laser Model, Blue = CAD Model)
Pair 1: CAD Airfoil with Laser airfoil without front part after Registration
(Horizontal Axis = X, Vertical Axis = Z, Units = meters,
Red = Laser Model, Blue = CAD Model)

PAIR 2:

Pair 2: Airfoils in original position (Horizontal Axis = X, Vertical Axis = Z, Units = meters,
Red = Laser Model, Blue = CAD Model)
Pair 2: CAD Airfoil with Laser airfoil without front part after Registration
(Horizontal Axis = X, Vertical Axis = Z, Units = meters, Red = Laser Model, Blue = CAD Model)
PAIR 3:

Pair 3: Airfoils in original position (Horizontal Axis = X, Vertical Axis = Z, Units = meters, Red = Laser Model, Blue = CAD Model)

Pair 3: Airfoils after Registration (Horizontal Axis = X, Vertical Axis = Z, Units = meters, Red = Laser Model, Blue = CAD Model)
Pair 3: CAD Airfoil with Laser airfoil without front part after Registration
(Horizontal Axis = X, Vertical Axis = Z, Units = meters, Red = Laser Model, Blue = CAD Model)

Pair 3: Registered airfoils with nearest points from Laser to CAD airfoil
(Horizontal Axis = X, Vertical Axis = Z, Units = meters, Red = Laser Model, Blue = CAD Model)
Pair 3: Histogram of differences between airfoil points
(Horizontal Axis = Differences in meters, Vertical Axis = Number of points)

Pair 3: Colourbar chart of CAD airfoil points indicating the differences
(Horizontal Axis = X, Vertical Axis = Z, Units = meters)
PAIR 4:

Pair 4: Airfoils in original position (Horizontal Axis = X, Vertical Axis = Z, Units = meters, Red = Laser Model, Blue = CAD Model)

Pair 4: Airfoils after Registration (Horizontal Axis = X, Vertical Axis = Z, Units = meters, Red = Laser Model, Blue = CAD Model)
Pair 4: CAD Airfoil with Laser airfoil without front part after Registration
(Horizontal Axis = X, Vertical Axis = Z, Units = meters, Red = Laser Model, Blue = CAD Model)

Pair 4: Registered airfoils with nearest points from Laser to CAD airfoil
(Horizontal Axis = X, Vertical Axis = Z, Units = meters, Red = Laser Model, Blue = CAD Model)
Pair 4: Histogram of differences between airfoil points
(Horizontal Axis = Differences in meters, Vertical Axis = Number of points)

Pair 4: Colourbar chart of CAD airfoil points indicating the differences
(Horizontal Axis = X, Vertical Axis = Z, Units = meters)
PAIR 5:

Pair 5: Airfoils in original position (Horizontal Axis = X, Vertical Axis = Z, Units = meters, Red = Laser Model, Blue = CAD Model)

Pair 5: Airfoils after Registration (Horizontal Axis = X, Vertical Axis = Z, Units = meters, Red = Laser Model, Blue = CAD Model)
Pair 5: CAD Airfoil with Laser airfoil without front part after Registration
(Horizontal Axis = X, Vertical Axis = Z, Units = meters, Red = Laser Model, Blue = CAD Model)

Pair 5: Registered airfoils with nearest points from Laser to CAD airfoil
(Horizontal Axis = X, Vertical Axis = Z, Units = meters, Red = Laser Model, Blue = CAD Model)
Pair 5: Histogram of differences between airfoil points  
(Horizontal Axis = Differences in meters, Vertical Axis = Number of points)

Pair 5: Colourbar chart of CAD airfoil points indicating the differences  
(Horizontal Axis = X, Vertical Axis = Z, Units = meters)
PAIR 6:

Pair 6: Airfoils in original position (Horizontal Axis = X, Vertical Axis = Z, Units = meters, 
Red = Laser Model, Blue = CAD Model)

Pair 6: Airfoils after Registration (Horizontal Axis = X, Vertical Axis = Z, Units = meters, 
Red = Laser Model, Blue = CAD Model)
Pair 6: CAD Airfoil with Laser airfoil without front part after Registration
(Horizontal Axis = X, Vertical Axis = Z, Units = meters, Red = Laser Model, Blue = CAD Model)

Pair 6: Registered airfoils with nearest points from Laser to CAD airfoil
(Horizontal Axis = X, Vertical Axis = Z, Units = meters, Red = Laser Model, Blue = CAD Model)
Pair 6: Histogram of differences between airfoil points
(Horizontal Axis = Differences in meters, Vertical Axis = Number of points)

Pair 6: Colourbar chart of CAD airfoil points indicating the differences
(Horizontal Axis = X, Vertical Axis = Z, Units = meters)
PAIR 7:

Pair 7: Airfoils in original position (Horizontal Axis = X, Vertical Axis = Z, Units = meters, Red = Laser Model, Blue = CAD Model)

Pair 7: Airfoils after Registration (Horizontal Axis = X, Vertical Axis = Z, Units = meters, Red = Laser Model, Blue = CAD Model)
Pair 7: CAD Airfoil with Laser airfoil without front part after Registration
(Horizontal Axis = X, Vertical Axis = Z, Units = meters, Red = Laser Model, Blue = CAD Model)

Pair 7: Registered airfoils with nearest points from Laser to CAD airfoil
(Horizontal Axis = X, Vertical Axis = Z, Units = meters, Red = Laser Model, Blue = CAD Model)
Pair 7: Histogram of differences between airfoil points
(Horizontal Axis = Differences in meters, Vertical Axis = Number of points)

Pair 7: Colourbar chart of CAD airfoil points indicating the differences
(Horizontal Axis = X, Vertical Axis = Z, Units = meters)
PAIR 8:

Pair 8: Airfoils in original position (Horizontal Axis = X, Vertical Axis = Z, Units = meters, Red = Laser Model, Blue = CAD Model)

Pair 8: Airfoils after Registration (Horizontal Axis = X, Vertical Axis = Z, Units = meters, Red = Laser Model, Blue = CAD Model)
Pair 8: CAD Airfoil with Laser airfoil without front part after Registration
(Horizontal Axis = X, Vertical Axis = Z, Units = meters, Red = Laser Model, Blue = CAD Model)

Pair 8: Registered airfoils with nearest points from Laser to CAD airfoil
(Horizontal Axis = X, Vertical Axis = Z, Units = meters, Red = Laser Model, Blue = CAD Model)
Pair 8: Histogram of differences between airfoil points
(Horizontal Axis = Differences in meters, Vertical Axis = Number of points)

Pair 8: Colourbar chart of CAD airfoil points indicating the differences
(Horizontal Axis = X, Vertical Axis = Z, Units = meters)
PAIR 9:

Pair 9: Airfoils in original position (Horizontal Axis = X, Vertical Axis = Z, Units = meters, Red = Laser Model, Blue = CAD Model)

Pair 9: Airfoils after Registration (Horizontal Axis = X, Vertical Axis = Z, Units = meters, Red = Laser Model, Blue = CAD Model)
Pair 9: CAD Airfoil with Laser airfoil without front part after Registration
(Horizontal Axis = X, Vertical Axis = Z, Units = meters, Red = Laser Model, Blue = CAD Model)

Pair 9: Registered airfoils with nearest points from Laser to CAD airfoil
(Horizontal Axis = X, Vertical Axis = Z, Units = meters, Red = Laser Model, Blue = CAD Model)
Pair 9: Histogram of differences between airfoil points
(Horizontal Axis = Differences in meters, Vertical Axis = Number of points)

Pair 9: Colourbar chart of CAD airfoil points indicating the differences
(Horizontal Axis = X, Vertical Axis = Z, Units = meters)
PAIR 10:

Pair 10: Airfoils in original position (Horizontal Axis = X, Vertical Axis = Z, Units = meters, Red = Laser Model, Blue = CAD Model)

Pair 10: Airfoils after Registration (Horizontal Axis = X, Vertical Axis = Z, Units = meters, Red = Laser Model, Blue = CAD Model)
Pair 10: CAD Airfoil with Laser airfoil without front part after Registration
(Horizontal Axis = X, Vertical Axis = Z, Units = meters, Red = Laser Model, Blue = CAD Model)

Pair 10: Registered airfoils with nearest points from Laser to CAD airfoil
(Horizontal Axis = X, Vertical Axis = Z, Units = meters, Red = Laser Model, Blue = CAD Model)
Pair 10: Histogram of differences between airfoil points
(Horizontal Axis = Differences in meters, Vertical Axis = Number of points)

Pair 10: Colourbar chart of CAD airfoil points indicating the differences
(Horizontal Axis = X, Vertical Axis = Z, Units = meters)
PAIR 11:

Pair 11: Airfoils in original position (Horizontal Axis = $X$, Vertical Axis = $Z$, Units = meters,
Red = Laser Model, Blue = CAD Model)

Pair 11: Airfoils after Registration (Horizontal Axis = $X$, Vertical Axis = $Z$, Units = meters,
Red = Laser Model, Blue = CAD Model)
Pair 11: CAD Airfoil with Laser airfoil without front part after Registration
(Horizontal Axis = X, Vertical Axis = Z, Units = meters, Red = Laser Model, Blue = CAD Model)

Pair 11: Registered airfoils with nearest points from Laser to CAD airfoil
(Horizontal Axis = X, Vertical Axis = Z, Units = meters, Red = Laser Model, Blue = CAD Model)
Pair 11: Histogram of differences between airfoil points
(Horizontal Axis = Differences in meters, Vertical Axis = Number of points)

Pair 11: Colourbar chart of CAD airfoil points indicating the differences
(Horizontal Axis = X, Vertical Axis = Z, Units = meters)
PAIR 12:

Pair 12: Airfoils in original position (Horizontal Axis = X, Vertical Axis = Z, Units = meters, Red = Laser Model, Blue = CAD Model)

Pair 12: Airfoils after Registration (Horizontal Axis = X, Vertical Axis = Z, Units = meters, Red = Laser Model, Blue = CAD Model)
Pair 12: CAD Airfoil with Laser airfoil without front part after Registration
(Horizontal Axis = X, Vertical Axis = Z, Units = meters, Red = Laser Model, Blue = CAD Model)

Pair 12: Registered airfoils with nearest points from Laser to CAD airfoil
(Horizontal Axis = X, Vertical Axis = Z, Units = meters, Red = Laser Model, Blue = CAD Model)
Pair 12: Histogram of differences between airfoil points
(Horizontal Axis = Differences in meters, Vertical Axis = Number of points)

Pair 12: Colourbar chart of CAD airfoil points indicating the differences
(Horizontal Axis = X, Vertical Axis = Z, Units = meters)
Pair 13: CAD Airfoil with Laser airfoil without front part after Registration
(Horizontal Axis = X, Vertical Axis = Z, Units = meters, Red = Laser Model, Blue = CAD Model)

Pair 13: Registered airfoils with nearest points from Laser to CAD airfoil
(Horizontal Axis = X, Vertical Axis = Z, Units = meters, Red = Laser Model, Blue = CAD Model)
Pair 13: Histogram of differences between airfoil points
(Horizontal Axis = Differences in meters, Vertical Axis = Number of points)

Pair 13: Colourbar chart of CAD airfoil points indicating the differences
(Horizontal Axis = X, Vertical Axis = Z, Units = meters)
PAIR 14:

Pair 14: Airfoils in original position (Horizontal Axis = X, Vertical Axis = Z, Units = meters,
Red = Laser Model, Blue = CAD Model)

Pair 14: Airfoils after Registration (Horizontal Axis = X, Vertical Axis = Z, Units = meters,
Red = Laser Model, Blue = CAD Model)
Pair 14: CAD Airfoil with Laser airfoil without front part after Registration
(Horizontal Axis = X, Vertical Axis = Z, Units = meters, Red = Laser Model, Blue = CAD Model)

Pair 14: Registered airfoils with nearest points from Laser to CAD airfoil
(Horizontal Axis = X, Vertical Axis = Z, Units = meters, Red = Laser Model, Blue = CAD Model)
Pair 14: Histogram of differences between airfoil points
(Horizontal Axis = Differences in meters, Vertical Axis = Number of points)

Pair 14: Colourbar chart of CAD airfoil points indicating the differences
(Horizontal Axis = X, Vertical Axis = Z, Units = meters)
PAIR 15:

Pair 15: Airfoils in original position (Horizontal Axis = X, Vertical Axis = Z, Units = meters, Red = Laser Model, Blue = CAD Model)

Pair 15: Airfoils after Registration (Horizontal Axis = X, Vertical Axis = Z, Units = meters, Red = Laser Model, Blue = CAD Model)
Pair 15: CAD Airfoil with Laser airfoil without front part after Registration
(Horizontal Axis = X, Vertical Axis = Z, Units = meters, Red = Laser Model, Blue = CAD Model)

Pair 15: Registered airfoils with nearest points from Laser to CAD airfoil
(Horizontal Axis = X, Vertical Axis = Z, Units = meters, Red = Laser Model, Blue = CAD Model)
Pair 15: Histogram of differences between airfoil points
(Horizontal Axis = Differences in meters, Vertical Axis = Number of points)

Pair 15: Colourbar chart of CAD airfoil points indicating the differences
(Horizontal Axis = X, Vertical Axis = Z, Units = meters)
PAIR 16:

Pair 16: Airfoils in original position (Horizontal Axis = X, Vertical Axis = Z, Units = meters,
Red = Laser Model, Blue = CAD Model)

Pair 16: Airfoils after Registration (Horizontal Axis = X, Vertical Axis = Z, Units = meters,
Red = Laser Model, Blue = CAD Model)
Pair 16: CAD Airfoil with Laser airfoil without front part after Registration
(Horizontal Axis = X, Vertical Axis = Z, Units = meters, Red = Laser Model, Blue = CAD Model)

Pair 16: Registered airfoils with nearest points from Laser to CAD airfoil
(Horizontal Axis = X, Vertical Axis = Z, Units = meters, Red = Laser Model, Blue = CAD Model)
Pair 16: Histogram of differences between airfoil points
(Horizontal Axis = Differences in meters, Vertical Axis = Number of points)

Pair 16: Colourbar chart of CAD airfoil points indicating the differences
(Horizontal Axis = X, Vertical Axis = Z, Units = meters)
Appendix 6

CFD testing

In the introductory chapter the main research questions of this thesis were presented. This thesis covers the first part of the project. In the first part the main objective was to create a model of the Cessna that is more detailed than the already available CAD design model. After succeeding in that, the Control and Simulation group should evaluate the CFD behaviour of the produced model. Thus the main research question for this interdisciplinary project is the following: *Can laser scanning lead to 3D models, which improve the results of CFD studies?*

Some initial CFD testing was conducted by the Control and Simulation group with the model; a 2D test was carried out with one airfoil. These early results will be presented in this appendix. Although they have a limited extent, they can give a preliminary idea about the answer to the main research question.

The conclusions presented in chapter 6 were based on the following assumption: *Even though the exact accuracy of the obtained Laser model is not known explicitly, it is assumed that the result is representative of the actual Cessna.* This assumption is also going to be made here. There are two main issues raised from the CFD testing: the connection of the separate airfoil parts and the existence of the dents in its tail part. These issues will be presented in the corresponding two sections. They will be followed by a section with conclusions.

A. Connection of airfoil parts

Due to the problems with the front part of the wing (see chapter 5.4) a 3D CFD test of the wing did not seem logical. That is why a 2D CFD test was conducted for one of the airfoils. Airfoil 11 from the center of the modeled wing was chosen and the result is shown in the following plot:

![Fig. 1 Plot of air pressure along the scan & CAD airfoil surface](image-url)
Figure 1 shows a $C_p$ plot. These plots show the pressure along the surface of a wing profile. The profiles featured in the plot are the laser airfoil 11 (in red) and its corresponding airfoil from the CAD model (in blue). $C_p$ is a pressure coefficient defined as:

$$C_p = 1 - \left( \frac{Q}{Q_{inf}} \right)^2$$  \hspace{1cm} (1)

where: $Q$ is the norm of the flow velocity at some point; $Q_{inf}$ is the norm of the flow velocity far away from the wing profile or the body that is perturbing the flow.

According to the Bernulli equation, pressure is related to the square of the flow velocity. This means that the pressure coefficient can be computed from velocity. When there is a stagnation point, i.e. when $Q = 0$, the flow velocity decreases to zero and the pressure builds up to its maximum value, $C_p = 1$. On a wing profile, this happens generally close to the leading edge and in the inner surface (for positive angles of attack). Close to the leading edge but now on the upper surface, there is usually the highest velocity. The flow is going around the profile taking the upper route but remains attached to the wing surface. Thus, for a particle of fluid going around that track at high speed, there is a significant centripetal force applied to it in the form of pressure. In the regions where the velocity is highest and the pressure is minimum, a vacuum is created. When the flow pressure is smaller than the atmospheric pressure ($atm \text{ pressure} \Rightarrow Q = Q_{inf} \Rightarrow C_p = 0$), the flow velocity is higher than $Q_{inf}$ and the $C_p$ is negative. The area between the $C_p$ plots for the upper surface and lower surface of the profile gives the aerodynamic force. The 1st order moment of integration gives also the aerodynamic moments relative to some point in space. A typical $C_p$ plot for a NACA profile is shown in figure 2:

![Fig. 2 $C_p$ plot for a typical NACA profile](image)

Returning in figure 1, the blue line shows the plot for the NACA profile used in the CAD model. The red line shows the plot for the laser airfoil. It is evident that the cad plot looks more like the typical plot. There are some differences in the suction part and the trailing edge part, where a small ripple is presented when compared to a
typical plot. However, this ripple can be repaired by augmenting the number of points and the smoothness of the line that connects the points.

The laser profile on the other hand has more differences when compared to the typical plot. These differences are related with the modelling method used for obtaining the airfoils. This method of creating airfoils from the laser point cloud was presented in detail in chapter 5.4. The laser points of each airfoil were divided in three parts. B-splines were fitted through each part and then connected to give the final result.

When looking at the $C_p$ scan plot (red line), one can see that there are some peaks in the plot. This happens in the transition zone from one B-spline to another and is attributed to the fact that the profile surface is not continuous. In an actual flow, the kinds of surfaces that generate peaks in the pressure create a lot of turbulence. Hence, the flow changes dramatically around the wing profile and a lot of the aerodynamic properties of the airfoil are lost. This means that the laser wing profile behaves more like a blunt body and not so much like a wing.

The following comments can be made about this case:

- Continuity between the three B-splines was not taken into account during the modelling process. However B-splines are $C^{k-2}$ continuous across their knots ($k$ = order of the spline). That translates into very smooth curves, especially when one considers that 8th, 6th and 5th degree B-splines were used for the fitting. While connecting the separate B-splines, the concern was for them to pass through the same connection points. This was considered accurate for the visualisation purposes of the first part of the thesis.

However the CFD studies are greatly affected by discontinuities on the wing. This is due to the nature of the CFD techniques. Nevertheless the used modelling method would be sufficient if the there were enough points covering the wing. A correct fitting through adequate points would ensure an accurate and smooth representation of the wing. Unfortunately there was small point coverage for the majority of the front part of the wing. This meant that the fitting process in this area was based on educated guess. Consequently, the connection of the three B-splines could not be smooth enough in these cases. The difference is evident when comparing these airfoils with airfoils coming from areas with complete point coverage:
Fig. 3 Airfoil fittings from profiles 13 and 32: In profile 32 the points cover the whole area and the fitting result is smooth and resembles a NACA profile. In profile 13 the point coverage is smaller and the result less smooth than profile 32.

- One way to overcome the continuity problem is to ensure the continuity between the three B-splines. So, in areas with low point coverage, the modelling technique should be enhanced. First or higher order continuity should be incorporated between the three curves. This means that the connection points should be replaced with connection zones. The zones contain points from the both connecting curves. The smoothness of the final result depends on the order of continuity chosen; higher order of continuity gives a smoother transition between the B-splines.

- Finally the chosen number of points of each B-spline can enhance the end product. The three curves of the airfoil were evaluated for a 1 cm step each. This interval can be shortened. This operation could prove especially useful for the front part, where the B-spline is fitted through a smaller interval than the other two parts. In the end, a more detailed airfoil will be obtained.

B. Dents on the wing

In chapter 5.4, the fitted wing profiles were presented. The main characteristic of each profile were two dents. Those were attributed to the existence of the aileron in that specific area. However the aileron gap signified by the dents, especially the lower one, is exaggerated. It is believed that such a big dent is not a characteristic of the actual wing.

Regarding this problem, the following comments can be made:

- Problems with point coverage from the lower scans were evident (see chapter 3.5). These problems were caused by the close proximity of the scanner to the Cessna. This led to situations like the ones shown in figure 4, where ‘shadows’ and other data gaps are obvious:
Nevertheless the assumption made in the beginning stated that, despite the problems, the laser points are representative of the actual Cessna. This assumption is supported by specific examples like the one shown in figure 5:

The image shows the high detail that is achieved with laser scanning. The profile of this area contains points from the metallic joint that connects the aileron with the rest of the wing.

- The laser scanner was in the same line of sight with the aileron gap during the measurement procedure. This led to a good coverage of points inside the gap, despite its small size. These points were taken into account during the modelling procedure, thus creating the big dent (see chapter 5.4).
- Another reason could cause the dent to become bigger. The position of the aileron during the scan would affect the final fitting result. Although the aileron was in a horizontal position, it could have been slightly tilted. Thus, if the aileron was tilted downwards, the result after the modelling procedure would be a bigger dent in the final airfoil.
In the end the main reason for the exaggerated dent is the chosen modelling technique. The reasoning behind choosing it was that it could provide a smooth representation of the aileron gap. This was indeed achieved, even though the final dent may not be very close to the actual wing.

C. Conclusions

In this appendix a limited in scope CFD study was presented. The purpose was to give an initial answer to the main research question of this interdisciplinary project: *Can laser scanning lead to 3D models, which improve the results of CFD studies?*

It is obvious that a definitive answer to this question cannot be given. This is due to two reasons. Firstly, the CFD testing carried out in this appendix had a small extent. Secondly, only one modelling technique was studied in this thesis. This method gave satisfactory results for the visualization part of the project. However the CFD studies presented in section A showed that the created model does not have good aerodynamic properties. This is largely caused by the lack of points in the front part of the wing and the discontinuities between the separate parts of the airfoil.

Hence it can be said that the modelling technique presented in this thesis does not improve the results of CFD studies. Nevertheless this is only one modelling method out of many that can be investigated. It was recommended in chapter 6 that other methods should also be tested. One of the most important findings of this thesis is that laser scanning can offer very high detail data from the airplane (e.g. in section B the metallic joint that connects the aileron with the rest of the wing was featured). This level of detail is greater than what is currently used in CFD studies. It would be a very interesting field of study to see how such details (e.g. small parts like antennas) can be incorporated in a complete CFD testing procedure.

Laser scanning can prove to be very helpful in aerodynamic studies. Its main contribution is the huge amount of data that can be used according to each project’s needs. Further research on modelling techniques more suitable for CFD studies is expected to generate improved aerodynamic results.